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Are we in a √-shaped or a W-shaped recession?

We apply a new factor model for the estimation of recession probabilities in the euro area and Slovakia. The model is based on standard monthly indicators of real activity. In the case of Slovakia, we also use timely available data on the value of client transactions in the payment system SIPS, which is run by the NBS. The results for the euro area and Slovakia suggest that the probability of a technical recession with a drop of GDP in two consecutive quarters is negligible. In other words, the quarterly growth profile of GDP during the corona crisis will look more like a square root symbol (√) and a W shape is unlikely based on the latest data.

Warning: The results of this note do not represent the official forecast of the NBS.

The new model for estimating recession probabilities[1] is **based on monthly indicators**, which allows us to evaluate **information that becomes available gradually.** Timely information is crucial for estimating current quarter GDP before the official figures are published. The multivariate setup is consistent with classical definitions of the business cycle, which state that turning points between phases of the cycle should be observed in several macroeconomic variables simultaneously.[2] The carefully selected indicators focus on real activity, which allows the model to identify **recessions of different origins** without conditioning them on specific shocks, e. g. shocks originating from the financial sector.

The time-varying depth of recession in this model allows **recessions of flexible shapes** following the set of indicators included. As a further advantage, the model is compatible with **alternative definitions of recessions**, which are comparable across countries, such as i) a period of low growth below potential, ii) a drop of GDP in one quarter, or iii) a technical recession with a decrease in GDP in two consecutive quarters. In the present discussion note, we will use the latter definition for simplicity.

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[1] The topic of estimating recession probabilities relates to our Discussion note No. 46 from 2017, where we applied the basic univariate Markov switching model of GDP without additional indicators.
[2] See Burns and Mitchell (1946) and the related stream of literature.

A Markov Switching Dynamic Factor Model

We estimate a Markov switching dynamic factor model (MS-DFM). The model was developed in a working group of the European System of Central Banks and is described in Leiva-Leon et al. (2020)[3] in more detail. The parameters and unobserved components are estimated using Bayesian methods (Monte Carlo Markov Chain simulations), which allows depiction of the parametric uncertainty around estimated variables and forecasts using a fan chart.

The **MS-DFM** is composed of a system of five main equations, including the monthly growth rates of unobserved GDP and four real activity indicators. The movement of the variables is explained by an unobserved common factor and indicator-specific error terms. The common factor has a constant mean in the regime of expansion. Whereas in the recession regime the mean of the factor is the sum of a constant and a flexible component (depth of recession), where the latter follows a random walk process. The quarterly growth of GDP is assumed to be observable only in the third month of each quarter, when data for all three months of the particular quarter are available. Quarterly GDP growth can be approximated by the weighted average of five consecutive monthly growth rates that are unobserved and are linked to the common factor.[4] For a more detailed description of the model's equations see the Technical appendix.

Data

Similarly to Leiva-Leon et al. (2020), we included the following monthly indicators of the **euro area**: industrial production, the value of exports of goods traded within the euro area, the value of exports of goods traded outside the euro area, and new orders in manufacturing. The selected variables concentrate on the activity of global value chains in industry, which are the largest players in European trade and influence the euro area business cycle to a great extent.[5] At the time of preparing this note, the indicators mentioned were available up to November 2020.

In the model for **Slovakia,** we included the monthly value of exports of goods, industrial production index and the economic sentiment indicator (ESI),[6] which are standard indicators

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[3] The authors implemented the model element of time-varying depth of recessions introduced by Eo and Kim (2016) in the Markov switching dynamic factor model of Camacho et al. (2018). We are grateful for Leiva-Leon et al. (2020) for sharing their program codes. For a more detailed treatment of the structure of the model see the Technical appendix of this note.

[4] It is a standard approximation introduced by Mariano a Murasawa (2003). Intuitively, in order to describe the developments of a monthly variable between two quarters, we need data for six months or five monthly differences. For more details on the derivation of this approximation see the Technical appendix of this note.

[5] The availability of appropriate monthly indicators for the euro area as a whole is somewhat limited, as new data are published with a longer lag and time series tend to be shorter compared to data for particular member countries. We could have considered ESI or retail sales, for example, as alternatives to new orders in manufacturing.

[6] The index is composed of confidence indicators in industry, construction, retail trade, services and of consumers.

of real activity.[7] The fourth monthly variable we used is the value of client transactions in the retail payment system SIPS, which is run by the National Bank of Slovakia. This variable has been neglected in similar analyses so far, but has proved useful. Our selection of indicators reflects the openness of the Slovak economy to the rest of the world (exports and industrial production). The selection also takes domestic demand into account via i) client payments between individuals and firms and ii) a set of soft indicators from several sectors of the economy (ESI). Client payments and ESI have a further advantage from the early release of new data right at the end of each month. We used the latter two series up to December 2020 in the present analysis, while industrial production and exports were available up to November of the same year.

Results

According to the model, **the probability of recession** with a drop of GDP in two consecutive quarters at the recent turn of the year **is negligible (Figure 1)**. **The short-term forecast is positive** for the euro area and it shows a mild slowdown in the case of Slovakia (**Figure 2**). In other words, a W-shaped recession is unlikely at the moment and the quarterly growth profile of GDP is **more similar to a square root symbol ($\sqrt{}$).** Sluggish dynamics of the Slovak economy (GDP levels are shown in Figure 5 in the appendix) can also be observed in monthly indicators at the end of last year (the right-hand panel of Figure 4 in the appendix). The model evaluated these developments as an elevated probability of the monthly recession regime (about 40% in December, the right-hand panel of Figure 3 in the appendix). This implies a potential risk of a mild drop in GDP in the final quarter of 2020.

Figure 1 Probability of a decrease in GDP in two consecutive quarters
Euro area   Slovakia
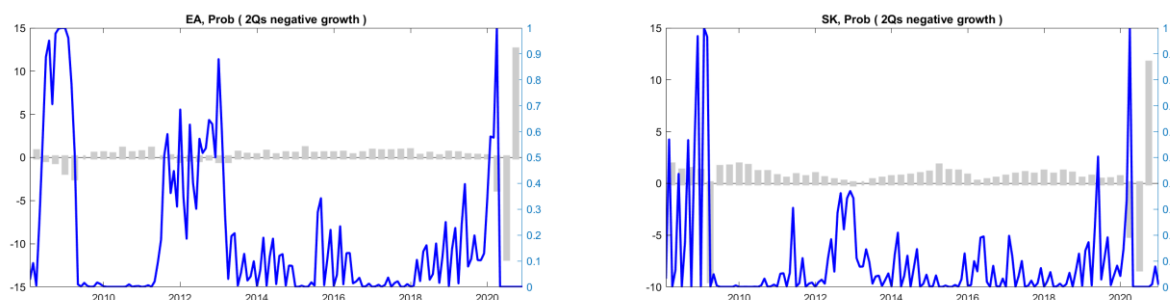
Note: Left axis – for grey bars, seas. adj. growth rates of GDP in % q/q, right axis – for blue lines, probability of a decrease in GDP in two consecutive quarters.
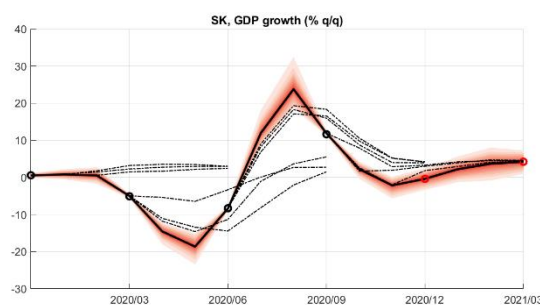
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[7] See for example Leiva-Leon et al. (2020), where an index of production, exports and a soft indicator are the most common variables included in their models for twelve countries.

Figure 2 Monthly updates of the GDP forecast for 1.Q 2020 – 1.Q 2021.
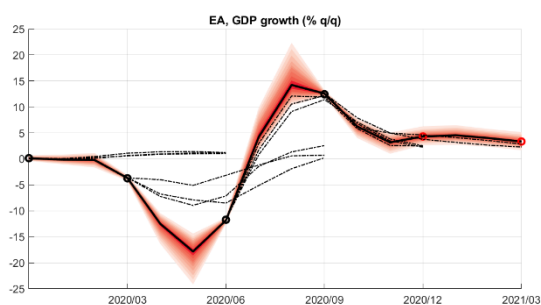
Euro area   Slovakia

Notes: *black circles* – observed GDP growth q/q in third months of each quarter, *red circles* – latest model forecast of GDP for 4.Q 2020 and 1.Q 2021 retrieved in mid-January 2021 (not the official forecast of the NBS), *black line* – monthly estimate of GDP growth, *red fan chart* – Bayesian parametric uncertainty around the monthly estimate, *black dashed lines* – previous monthly versions of the forecast.

Apart from the latest forecast in Figure 2, we also depicted simulations of its earlier versions based on past data in monthly sequences (dashed lines). The simulations show that the gradual release of new data led to correct adjustments in the forecasts towards future GDP outcomes. Our exercise confirmed the **usefulness of timely data that signal turning points** in the performance of the economy, together with **models adapted to the analysis of such data**, for GDP forecasting purposes especially **in non-standard times**, such as the ongoing corona crisis.

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Appendix – additional figures

Figure 3 The probability of the monthly recession regime
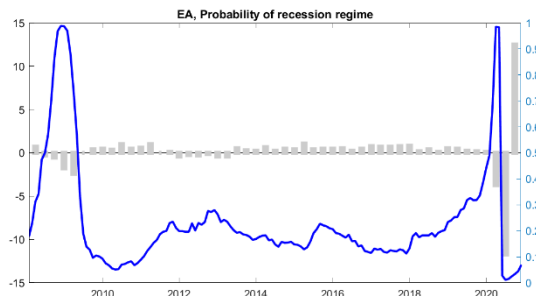
Euro area   Slovakia
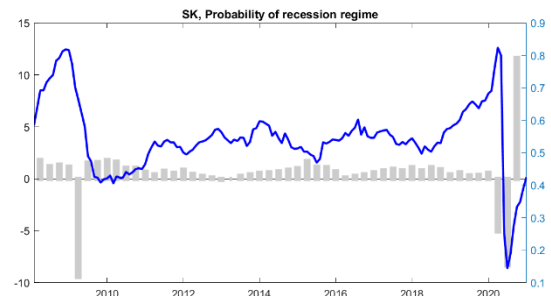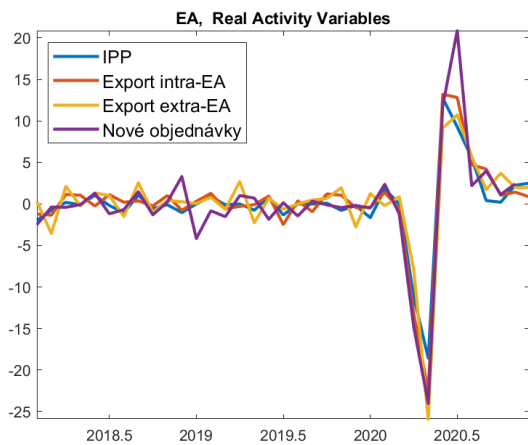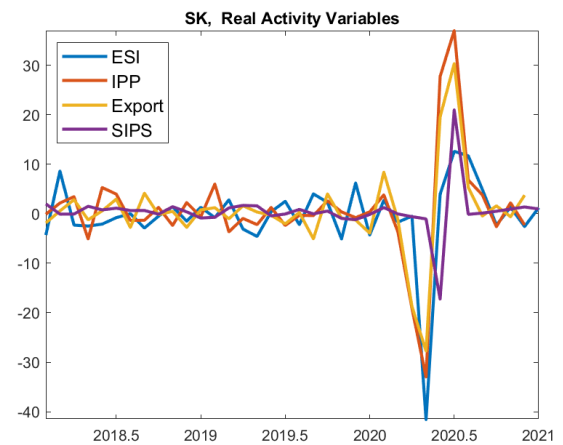

Notes: Left axis – for grey bars of seas. adj. GDP growth q/q in %, right axis – for blue lines of the probability of the monthly recession regime.

Figure 4 Indicators of real activity
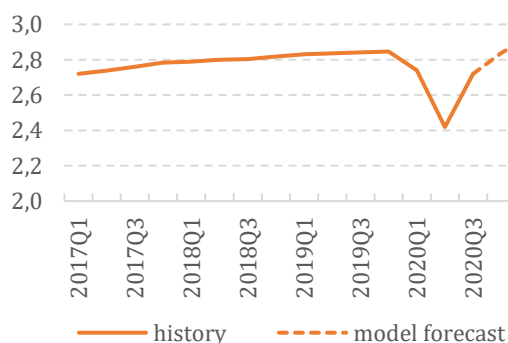
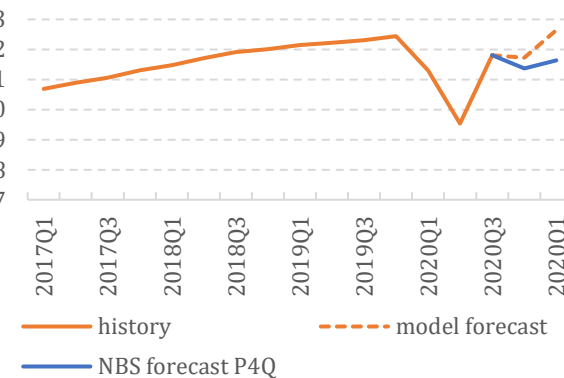Euro area   Slovakia

Notes: Seas. adj. monthly growth rates in %. IPP – industrial production index, ESI – economic sentiment indicator (monthly changes in the index), SIPS – the value of client payments in the payment system SIPS.

Figure 5 Levels of real GDP

Euro area (tril. eur)   Slovakia (bil. eur)

Notes: GDP chain-linked volumes, seas. adj.

Technical appendix

The MS-DFM is defined on monthly frequency as:

$$\begin{pmatrix} y_t \\ z_{1,t} \\ z_{2,t} \\ z_{3,t} \\ z_{4,t} \end{pmatrix} = \begin{pmatrix} \gamma_t \\ \gamma_{1,t} \\ \gamma_{2,t} \\ \gamma_{3,t} \\ \gamma_{4,t} \end{pmatrix} f_t + \begin{pmatrix} u_t \\ u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \end{pmatrix} \qquad (1)$$

where y_t is the unobserved monthly growth rate of GDP,

$z_{1,t}, \dots, z_{4,t}$ are monthly growth rates of real activity indicators,

and f_t is the common factor of GDP and the indicators.

The error terms in (1) follow an AR(2) process:

$$u_{i,t} = \psi_{i,1} u_{i,t-1} + \psi_{i,2} u_{i,t-2} + \varepsilon_{i,t} \qquad (2)$$

The common factor has a constant mean of μ_0 in regime $S_t = 1$ (expansion), while the mean in regime $S_t = 0$ (recession) is composed of a constant μ_1 and a flexible component x_t. The latter component sets the depth of recession.

$$f_t = \mu_0(1 - S_t) + \mu_1 S_t + S_t x_t + e_t \qquad (3)$$

$$e_t \sim N(0, \sigma_e^2)$$

$$\epsilon_{i,t} \sim N(0, \sigma_{\varepsilon,i}^2)$$

The flexible component follows a random walk process:

$$x_t = S_t x_{t-1} + (1 - S_t) u_t \qquad (4)$$

$$u_t \sim N(0, \sigma_u^2)$$

The expansion and recession regimes move according to a Markov process:

$$Prob(S_{t=i} \, / \, S_{t-1=j} \,, \Omega_{t-1}) = p_{i,j} \qquad (5)$$

The quarterly growth rate of GDP is defined using the approximation of Mariano and Murasawa (2003) as the weighted average of five consecutive monthly growth rates:

$$Y_t = \tfrac{1}{3} y_t + \tfrac{2}{3} y_{t-1} + y_{t-2} + \tfrac{2}{3} y_{t-3} + \tfrac{1}{3} y_{t-4} \qquad (6)$$

The approximation in (6) is derived using the formula for the geometric mean. We assume that the value of an indicator for a quarter is the geometric mean of three months. The quarterly growth rate is expressed as the difference in logarithms in consecutive geometric means. By

adding and subtracting the necessary terms we get equation (6), including the weights of each monthly difference of logarithms y_{t-k} (k=0,..,4).

Plugging expression (6) to the first row of the system in (1), we can rewrite the row to the quarterly frequency:

$$Y_t = \frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4} + \frac{1}{3}u_t + \frac{2}{3}u_{t-1} + u_{t-2} + + \frac{2}{3}u_{t-3} + \frac{1}{3}u_{t-4} \qquad (7)$$

where the quarterly growth rate of GDP, Y_t, is observed only in the third month of each quarter.