Proceedings of the 17th International Symposium on OPERATIONAL RESEARCH in Slovenia

SOR '23

Bled, Slovenia

September 20-22, 2023

Edited by:

S. Drobne • L. Zadnik Stirn • M. Kljajić Borštnar • J. Povh • J. Žerovnik

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Slovenian Society INFORMATIKA (SDI)
Section for Operational Research (SOR)

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Preface

This volume, Proceedings of the 17th International Symposium on Operational Research, called SOR'23, contains papers presented at SOR'23 (https://sor.fov.um.si/), organised by the Slovenian Society INFORMATIKA (SSI), the Section for Operational Research (SOR), the University of Maribor, Faculty of Organisational Sciences, Kranj, Slovenia (UM FOV), and the University of Ljubljana, Faculty of Mechanical Engineering, Ljubljana, Slovenia (UL FS), held in Bled, Slovenia, from September 20 to September 22, 2023. The volume contains blind peer-reviewed papers or abstracts of papers presented at the symposium.

The opening address at SOR'23 was delivered by Prof. Dr. Lidija Zadnik Stirn, President of SOR, representatives from SSI, UM FOV and UL FS, Assist. Prof. Dr. Tea Šestanović, President of Croatian Operational Research Society (CRORS), Prof. Dr. Marc Sevaux, former President of EURO and Dr. Sarah Fores, manager of EURO, and presidents/representatives of some others Operational Research Societies from abroad.

SOR'23 is the scientific event in the field of Operational Research, another in the traditional series of biennial international OR conferences organised in Slovenia by SDI-SOR. It is the continuation of sixteen previous symposia. The main objective of SOR'23 is to promote knowledge, interest and education in the field of OR in Slovenia, Europe and worldwide in order to build the intellectual and social capital essential for maintaining the identity of OR, especially at a time when interdisciplinary cooperation is proclaimed as particularly important for solving problems in today's challenging times. By joining IFORS and EURO, SDI-SOR has also agreed to collaborate with different disciplines, i.e., to balance the depth of theoretical knowledge in OR and the understanding of theory, methods, and problems in other fields within and outside OR. We believe that SOR'23 creates the advantage of these goals, contributes to the quality and reputation of OR by presenting and sharing new developments, opinions and experiences in the theory and practise of OR.

SOR'23 was highlighted by five distinguished keynote speakers. The first part of Proceedings SOR'23 contains invited abstracts, presented by five outstanding scientists: Prof. Suresh P. Sethi, The University of Texas at Dallas, Naveen Jindal School of Management, Center of Intelligent Supply Networks, Texas, United States of America, Prof. Marc Sevaux, Université Bretagne Sud, Centre de recherche, Lab-STICC, Lorient, France, Prof. Mirjana Pejić Bach, University of Zagreb, Faculty of Economics and Business, Department of informatics, Zagreb, Croatia, Assist. Prof. Andrej Kastrin, University of Ljubljana, Faculty of Medicine, Institute of Biostatistics and Medical Informatics, Ljubljana, Slovenia, Dr. Victor Magron, Institute of Mathematics from Toulouse, Laboratoire d'analyse et d'architecture des systèmes (LAAS-CNRS), Équipe POP, Toulouse, France.

The Proceedings includes 96 papers or abstracts by 198 authors. Most of the authors of the contributed papers came from Slovenia (63), then Croatia (35), Hungary (20), Serbia (16), Slovak Republic (13), Czech Republic (11), Poland (9), Spain (7), Turkey (6), United States of America (4), Germany (3), France (3), Italy (2), Austria (1), Israel (1), Pakistan (1), Republic of North Macedonia (1), The Netherlands (1), and United Kingdom (1). The papers published in the Proceedings are divided into Plenary Lectures (5 abstracts), eight special sessions: Applications of OR in Agricultural Economics (8 papers), Applications of OR in Industry and Mechanical Engineering (4 papers), Artificial Intelligence in Business: Obstacles and Perspectives (4 papers), Discrete Optimization Methods and Models for Real-world Problem Domain (15 papers), Game Theory (5 papers), Industry & Society 5.0: Optimization and Learning in Human and Industrial Environments (9 papers), Social

Innovations in Ageing Studies Supported by OR Models (7 papers), Unravelling the Business Models of Sharing Economy by Applying Methods of OR and Statistics (3 papers), and six sessions: Econometric Models and Statistics (4 papers), Human Resources (5 papers), Finance and Investments (8 papers), Location and Transport, Graphs and their Applications (7 papers), Mathematical Programming and Optimization (5 papers), and Multi-Criteria Decision-Making (7 papers).

Proceedings of the previous sixteen International Symposia on Operational Research organised by the Slovenian Section on Operational Research, listed at https://www.drustvo-informatika.si/sekcije-drustva?stran=publikacije-sor, are indexed in the following secondary and tertiary publications: Current Mathematical Publications, Mathematical Review, Zentralblatt Mathematik/ Mathematics Abstracts, MATH on STN International and CompactMath, INSPEC. It is expected that Proceedings SOR'23 will be covered by the same bibliographic databases.

The success of the SOR'23 scientific events and the present conference proceedings is due to joint efforts. On behalf of the organisers, we would like to express our sincere gratitude to all those who assisted in the preparation of the event. Without the dedication and advice of the active members of the Slovenian Operations Research Section, we would not have been able to attract so many top-class speakers from all over the world. Many thanks to them. In addition, we would like to express our deepest gratitude to the prominent keynote speakers, the members of the Programme and Organising Committees, the reviewers who contributed to the quality of SOR'23 with their useful suggestions, the Section Chairs and all the numerous people - far too many to list individually here - who helped in the organization of the 17th International Symposium on Operational Research SOR'23 and the compilation of this proceedings. Finally, we thank the authors for their efforts in preparing and presenting the papers that made the 17th Symposium on Operational Research SOR'23 a success.

We would like to give special thanks to MDPI AG, Mathematics, EURO, Project EuroCC2, EXCELLERAT P2 - The European Centre of Excellence for Engineering Applications, Rudolfovo - The Scientific and Technological Centre in Novo Mesto, and Project L7-3188 for their valuable support.

Ljubljana and Kranj, September 20, 2023

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INSIGHTS INTO THE GROWTH AND IMPACT OF OPERATIONS RESEARCH IN SLOVENIA OVER THE PAST 30 (60) YEARS: BUILDING INTELLECTUAL AND SOCIAL CAPITAL IN THE ENVIRONMENT OF OR FROM THE PERSPECTIVE OF THE ACTIVITIES OF SSI-SOR

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Abstract: The article deals with general data about the Slovenian Section of Operations Research (SOR), which is under the umbrella of the Slovenian Society INFORMATIKA (SSI), i.e., SSI-SOR. The history of operations research (OR) in Slovenia, starting in 1964, the foundation of SSI-SOR in 1992/1993 and the series of OR symposia in Slovenia with the first one in autumn 1993 are presented. Thus, this year SSI-SOR celebrates the 30th anniversary and the 30th anniversary of the first international symposium (conference) on OR, organized by SSI-SOR in Slovenia. Below are some facts explaining the activities of OR in Slovenia, especially at the international level, and the mission of SSI-SOR. The most important and internationally recognized publications of SSI-SOR are listed, including proceedings, monographs, and special journal issues which are examined. We conclude the article with suggestions from SSI-SOR for the future.

Keywords: history of OR in Slovenia, 30 years of SSI-SOR, mission of SSI-SOR, proceedings of SOR symposia, monographs, special issues of journals of SSI-SOR

1 INTRODUCTION

1.1 Operations research in science and first beginnings of OR in Slovenia

Operations/Operational Research (OR), also Management Science, Business Analytics ... belongs to the field of Information and Management Sciences and represents a typical interdisciplinary field of work. Research on OR involves experts from engineering, business, organization, information, environmental, social, and other sciences. The approaches in OR represent an indispensable management tool that specifies a set of methods for identifying, formalizing, algorithmizing, and implementing problems and their solutions in all areas of operations. OR mainly deals with quantitative methods and generates solutions from which responsible decision makers can choose (Winston 2010).

The birth of OR dates back to 1943, when the problem of optimizing the transport of the American fleet was solved as part of the "operation" against Japan. This is where the name of the discipline comes from (OR). The fact of the matter is, of course, that many of the methods we use today in the field of OR realm go back much further. For more information on the origins of OR and OR in general, see Hillier and Lieberman (1995), for example.

The rapid development of OR methods as well as their increasing application in almost all fields has fostered the integration of OR experts to join forces at national and global levels. For example, the International Federation of Operational Research Societies (IFORS) was officially founded in 1959. In 1960, IFORS organized the first international conference in Aix-en-Provence. At this point, IFORS already had 10 members (https://www.ifors.org/history). Today, IFORS has 54 members, i.e., national societies of OR (https://www.ifors.org/national-societies/). In 1975 the European association OR, the Association of European Operational Research Societies (EURO), was founded under the umbrella of IFORS (https://www.euro-online.org/web/pages/197/short-history-of-euro). In the same year, the first EURO conference took place in Brussels.

OR came to Slovenia in the early 1960s, although a few experts were familiar with the evidence before then. At that time, the Slovenian Business Association and the Slovenian Chamber of Commerce organized a symposium in Ljubljana, Slovenia, entitled "Mechanographics (Data processing) and OR" with 30 lectures/papers. The symposium consisted of two parts, the first dealing with the problems of data acquisition and processing, the second with OR. In the first part, representatives of various companies and institutions reported on the increase in the amount of information, which requires a more rational way of data acquisition and processing. The speakers reported on experiences and problems in data acquisition and processing. Some also presented practical experiences and theoretical concepts. This latter duality underlined the solid link between theory and practice. In the second part, OR was presented as a new science for decision support in many fields. Several practical examples were given from industrial production, site selection, transport, agriculture, investment and health care, where linear programming and a two-stage production process supported by a linear programme dominated as methods. The papers are printed in the proceedings of the conference (Zbornik Mehanografija in operacijsko raziskovanje 1964). The program and the organizing committee were led by Prof. Rupnik and Prof. Vadnal. This symposium marked the beginning of systematic research, teaching and consultancy activities in the field of OR in Slovenia. The main messages of the symposium were two: i) the link between data acquisition/processing and OR is necessary, as OR needs solid numerical data about the problem to be solved, and ii) successful OR is only possible in teamwork.

In 1967, the Association of Economists of Yugoslavia and the Association of Economists of Slovenia held a conference in Bled, Slovenia, entitled "Consultation on the use of OR methods in organizations/institutions in Yugoslavia" with 33 papers. A collection of papers was published in (Zbornik 1967). The problems presented at this conference were from manufacturing, banking, transport, agriculture, food industry and tourism. Among the methods, linear programming was predominant, while queuing, network, branch-and-bound and dynamic programming methods were covered, as well as some extensions of the simplex method.

The year 1974 marked the beginning of the traditional Yugoslav symposia of OR, known as SYM-OP-IS. These symposia were organized at the Yugoslav level by the Faculty of Organizational Sciences, the Institute of Industrial Economics and the Mihajlo Pupin Institute (all from Belgrade, Yugoslavia). The symposia were held every year in Herceg Novi. Between 150-200 participants from all over Yugoslavia attended, some also from abroad. Contributions to the SYM-OP-IS symposia were published in proceedings, which were issued regularly as part of each symposium. For example, (SYM-OP-IS 1986) contains the full papers of 154 talks in the following areas: Mathematics, Programming, Combinatorial Optimization, Networks, Graphs, Multicriteria Analysis, Stochastic Processes, Simulations, Forecasting, Information Systems, Expert Systems, Manufacturing, Transportation, Traffic, and Business Applications. Many OR fellows from Slovenia, which was then part of Yugoslavia, actively participated in SYM-OP-IS every year until 1990.

In 1974, a Master's programme in Operations Research began at the Faculty of Economics at the University of Ljubljana, which a few years later was expanded into a PhD (doctoral) programme in Information and Management Sciences. The students of this programme were graduates of economics, mathematics, physics, mechanical engineering, electrical engineering, civil engineering, law, sociology and other subjects, who after graduation worked or are still working in the field of operations research at universities, institutes and research departments of companies.

In the same year, Prof. Viljem Rupnik published the book Outline to Operations Research in Slovenian (Rupnik 1974). It is the first comprehensive book in Slovenian in the field of operations research. In order to reach a wider readership, the book does not follow the strict pattern of a textbook, which usually presupposes some knowledge of mathematics and statistics. Rather, it is an overview of problems and touches on various tools/methods as a secondary objective. It is thus a systematic explanation of a number of serious theoretical and practical issues affecting operations research.

1.2 Foundation of SSI-SOR and its integration into the international environment

In 1991, the Croatian Association OR (CRORS) was founded and colleagues from Croatia organized the 1st International Conference on OR (KOI'91). Many experts in the field of OR in Slovenia participated in this conference. Following the example of our colleagues from Croatia, the Slovenian experts in OR founded their own association, the Section for Operations Research (SOR) under the umbrella of the Slovenian Society for INFORMATIKA (SSI), i.e., SSI-SOR, at the end of 1992, which today has 107 members.

SSI-SOR is a forum for researchers and practitioners from all areas of OR and related fields, across disciplines and programmes in resource management, networks, tools such as linear and nonlinear programming, discrete and combinatorial optimization, stochastic decision making, multicriteria optimization, strategic games, inventory theory, graph theory, dynamic optimization, systems management, control theory and others, information and education. At their first meeting, the founders of SSI-SOR clearly outlined the group's goal, which is to support and promote research, development, application, and education in operations research, which encompasses mathematics, economics, computer science, statistics, environmental economics and systems theory, as well as several other disciplines. Therefore, the interdisciplinary and applied science character of OR is one of the main concerns of SSI-SOR. The main activities supporting the mission of SSI-SOR, i.e., increasing the visibility and impact of OR and working more closely with educational institutions, industry, government, and international institutions, can be understood as follows (Zadnik Stirn and Drobne 2022):

i) Publication of basic and applied research in the form of reports, articles, and monographs. This activity has been carried out with great success by the members of SSI-SOR during the years (1992-2023) at national and especially at international level. The members of SSI-SOR have written a number of articles for proceedings of international conferences OR in Slovenia, edited these proceedings, have written and edited five monographs, have been members of editorial boards of international journals with impact factor, reviewers, have written numerous articles published in national and international journals in the field of OR, etc.

ii) Joining international associations dealing with OR, such as IFORS, EURO

iii) Active participation in the organization of national and international meetings. Members of SSI-SOR have implemented this activity intensively over the years by organizing international symposia in Slovenia, participation in EURO and IFORS conferences, conferences in Croatia, Austria, Germany and other OR societies, participation in informatics conferences, statistics conferences organized by the Croatian Statistical Society, IFIP TC7 conferences, Multicriteria Decision Making conferences organized by the Faculty of Economics in Katowice, Poland, and many others. Members of SSI-SOR have served on committees, commissions, and sections of OR, co-organized conferences, and presented papers at conferences; many members of SSI-SOR have also been invited speakers at these conferences.

iv) Integration of the contents of OR into study processes. Topics or modules from OR have been implemented in several curricula, but there is still much opportunity for more intensive implementation of OR at all levels of study.
v) Familiarize businesses and the public with the possibility of using OR.

Since SSI-SOR was founded, its members have been active in a variety of areas (Zadnik Stirn, 2010). Among the most important activities is the organization of international symposia on operations research in Slovenia, known as SOR'93, SOR'94 … SOR'21, and the anniversary symposium SOR'23, which was held in Bled, Slovenia, in September 2023, dedicated to the 30th anniversary of SSI-SOR. Originally, these symposia were organized annually; later, in 1997, after an agreement was reached with the Croatian OR Society (CRORS), they are organized every two years in one country each, Slovenia and Croatia. The members of SSI-SOR are also co-organizers of the annual informatics meetings in Slovenia and the biannual symposia on OR in Croatia. They also actively participate in the conferences INFORMS, IFORS, EURO and other OR.

SSI-SOR is the 47th member of IFORS and was admitted to IFORS in 2007, while it became the 30th member of EURO in 2008. It was welcomed during the IFORS conference in Sandton at EURO (Itmann 2008).

2 REVIEW OF SSI-SOR PUBLICATIONS

2.1. International conferences at OR and proceedings

In 1993, SSI-SOR, with the support of the Ministry of Science and Technology of the Republic of Slovenia and the Faculty of Economics of the University of Ljubljana, organized the 1st Symposium on OR, SOR'93, at the national level. The report of SSI-SOR on SOR'93 is available at https://www.drustvo-informatika.si/sekcije-drustva/SOR/porocilaSOR.

In the period 1993-2023, SSI-SOR organized a total of 17 international symposia in the field OR. Since 1994, these symposia have been held with international participation, which has increased year by year. A brief overview of the keynote speakers and sessions, as well as the countries from which the keynote speakers came, can be found in Table 1. The symposia were supported by the members of Slovenian universities, ministries of the Republic of Slovenia, foreign institutions, associations and societies (Croatian, German, Austrian, Hungarian, Polish OR societies and others), EURO, IFORS and many other institutions and individuals.

The SOR symposia are the first scientific event in the field of OR in Slovenia, and provide an international forum for scientific exchange at the frontiers of OR, mathematics, statistics, economics, engineering, education, environment, computer science and other fields.

Since 1995, SDI-SOR has organized a symposium every two years in agreement with HDOI, which means that SDI-SOR and HDOI alternately organize OR international symposia every year, one year in Slovenia, the other year in Croatia. Thus, SDI-SOR has organized (until now): SOR'93, SOR'94, SOR'95, SOR'97, SOR'99, SOR'01, SOR'03, SOR'05, SOR'07, SOR'09, SOR'11, SOR' 13, SOR'15, SOR'17, SOR'19, SOR'21 and SOR'23. The papers presented at these symposia were peer reviewed and published in proceedings of each symposium (Proceedings of the 17th … (2023), (2021), (2019) … (1993)). The Proceedings are also available in electronic form at https://www.drustvo-informatika.si/sekcije-drustva?stran=publikacije-sor, which can also be accessed from the Symposia website https://sor.fov.um.si/publications/. Detailed reports on the symposia can be found at https://www.drustvo-informatika.si/sekcije-drustva.

A total of 1110 peer-reviewed papers presented at SOR symposia were published in 17 proceedings indexed in Current Mathematical Publications, Mathematical Review, Zentralblatt fuer Mathematik/Mathematics Abstracts, MATH on STN International and CompactMath, INSPEC, and others. The maximum number of papers presented at SOR'21 was 118, while the minimum number presented at SOR'95 was only 21. The average number of papers presented and published in the proceedings was 65.29 papers per symposium. A total of 105 keynote lectures were given by authors from 26 countries. The presentations at the symposia were divided into sessions. There were 157 sessions at all symposia, with a maximum of 19 (11 special sessions and 8 contributed sessions) at SOR'21, which is an average of 9.06 sessions per symposium. There we can also see which sessions were the most frequent, recurring, and of special interest last time, and we can get an idea of the content of the papers presented. The papers presented and published there were written by 1964 authors. The highest number of authors was recorded at SOR'21 (240), the lowest at SOR'95 (28), and the average number of authors was 110.51 per symposium. Authors came from all over the world, most of them of course from Slovenia and neighbouring countries (Croatia, Hungary, Austria, Italy, Czech Republic, Slovak Republic, Germany …). The highest number of countries was exhibited at SOR'17 (25), the lowest at SOR'93 (only 1, as it was a national symposium), the average number of countries per symposium is 12.56.

Table 1: Review of the scientific papers in the Proceedings of SOR in 1993-2023 according to the themes/sessions.

Proceedings	Keynote speakers	Special sessions	Sessions
SOR'23 (96 papers)	1. Sevaux EURO/(FR) 2. Sethi (USA) 3. Magron (FR) 4. Pejić Bach (CRO) 5. Kastrin (SI)	1. Applications of OR in Agricultural Economics (8 papers) 2. Applications of OR in Industry and Mechanical Engineering (4 papers) 3. Artificial Intelligence in Business: Obstacles and Perspectives (4 papers) 4. Discrete Optimization Methods and Models for Real-world Problem Domain (15 papers) 5. Industry & Society 5.0: Optimization and Learning in Human and Industrial Environments (9 papers) 6. Game Theory (5 papers) 7. Social Innovations in Ageing Studies Supported by OR Models (7 papers) 8. Unravelling the Business Models of Sharing Economy by Applying Methods of OR and Statistics (3 papers)	1. Econometric Models and Statistics (4 papers) 2. Human Resources (5 papers) 3. Finance and Investments (8 papers) 4. Location and Transport, Graphs and their Applications (7 papers) 5. Mathematical Programming and Optimization (5 papers) 6. Multi-Criteria Decision-Making (7 papers)
SOR'21 (118 papers)	1. Ban (A) 2. Kojić (CRO) 3. Patrinos (B) 4. Sethi (USA) 5. Gros (SI)	1. Application of OR in Smart Cities (6 papers) 2. Computational Mathematical Optimization (13 papers) 3. Data Science - Methodologies and Case Studies (10 papers) 4. Graph Theory and Algorithms (2 papers) 5. High-Performance Computing and Big Data (3 papers) 6. Industry & Society 5.0: Optimization in Industrial and Human Environment (6 papers) 7. International Projects in OR (2 papers) 8. Lessons Learned from the COVID-19 Pandemic (8 papers) 9. Logistics and Sustainability (9 papers) 10. OR in Ageing Studies and Social Innovations (5 papers), 11. OR in Agricultural Economics and Farm Management (5 papers)	1. Econometric Models and Statistics (6 papers) 2. Environment and Social Issues (5 papers) 3. Finance and Investments (6 papers) 4. Location and Transport, Graphs and their Applications (5 papers) 5. Mathematical Programming and Optimization (6 papers) 6. Multi-Criteria Decision-Making (10 papers) 7. Theory of Games (3 papers) 8. Problems Approaching OR (3 papers)

Proceedings	Keynote speakers	Special sessions	Sessions
SOR'19 (106 papers)	1. Bratko (SI) 2. Čižmešija (CRO) 3. Illes (HU) 4. Jozefowska EURO/(PL) 5. Praprotnik (SI)	1. Application of OR in Agriculture and Agribusiness Management (5 papers) 2. Formal and Behavioral Issues in MCDM (7 papers) 3. Graph Theory, Algorithms (12 papers) 4. High-Performance Computing and Big Data (4 papers) 5. Optimization in Human Environments (7 papers) 6. System Modelling & Soft OR (5 papers) 7. Towards Industry 4.0 (5 papers)	1. Econometric Models and Statistics (10 papers) 2. Environment and Social Issues (6 papers) 3. Finance and Investments (11 papers) 4. Location and Transport, Graphs (4 papers) 5. Mathematical Programming and Optimization (9 papers) 6. Multi-Criteria Decision-Making (6 papers) 7. Human Resources (4 papers) 8. Production and Management (6 papers)
SOR'17 (93 papers)	1. Bogaerts PRACE/(BE) 2. Leopold-Wildburger (AU) 3. Perc (SI) 4. van Wassenhove (FR) 5. Zekić-Sušac (CRO)	1. Advances in Modelling and Statistical Research of the Western Balkan Countries in the Times of Economic Crisis (8 papers) 2. High-Performance Computing and Big Data and General OR Topics (8 papers) 3. Logistics (5 papers) 4. MCDM – Software and Applications (6 papers) 5. Metaheuristic Optimization (5 papers) 6. Industrial Engineering and Services (5 papers) 7. MRP and Related Systems Approach to Systems Optimization and Control with Applications (5 papers)	1. Econometric Models and Statistics (10 papers) 2. Environment and Human Resources (6 papers) 3. Finance and Investments (5 papers) 4. Location and Transport, Graphs, and their applications (6 papers) 5. Machine Learning (4) 6. Mathematical Programming and Optimization (8 papers) 7. Multiple Criteria Decision Making (4 papers) 8. OR Perspectives: Where we have been, where we can go (3 papers)
SOR'15 (93 papers)	1. Ben Tal (IL) 2. Cabello (SI) 3. Cozzini (IT) 4. Gvozdenović (RS) 5. Weber, Savku, Pinheiro, Azevedo (TR) 6. Wei, Tang (SE)	1. Qualitative Multiple Criteria Decision Making (6 papers) 2. Inventory Research (7 papers) 3. Metaheuristic Optimization (7 papers) 4. Big Data (4 papers)	1. Mathematical Programming and Optimization (7 papers) 2. Graphs and their Applications (5 papers) 3. Multiple Criteria Decision Making (5 papers) 4. Econometric Models and Statistics (10 papers) 5. Production (7 papers) 6. Finance and Investments (7 papers) 7. Location and Transport (7 papers) 8. Environment and Human Resources (9 papers) 9. OR Perspectives (6 papers)
SOR'13 (61 papers)	1. Jukić (CRO) 2. Klavžar (SI) 3. Petitjean (FR) 4. Sotitrov (NLl)		1. Mathematical Programming and Optimization (14 papers) 2. Graphs and their Applications (10 papers) 3. Econometric Models and Statistics (5 papers) 4. Finance and Investments (6 papers) 5. Location and Transport (6 papers) 6. Multiple Criteria Decision Making (8 papers) 7. Production and Inventory (3 papers) 8. Creative core FIS - Simulations (5 papers)
SOR'11 (53 papers)	1. Anderson (DK) 2. Gerhardt, Hamacher, Ruzika (DE) 3. Gurtjahr (AU) 4. Koster (DE) 5. Lukač (HR) 6. Pferschy (AU)		1. Graphs and their Applications (3 papers) 2. Production and Inventory (12 papers) 3. OR Applications in Telecommunication and Navigation Systems (3 papers) 4. Finance and Investments (6 papers) 5. Multiple Criteria Decision Making (6 papers) 6. Pascal2 session (3 papers) 7. Mathematical Programming and Optimization (3 papers) 8. Econometric Models and Statistics (6 papers) 9. Location and Transport (5 papers)

Proceedings	Keynote speakers	Special sessions	Sessions
SOR'09 (61 papers)	1. Babić (CRO) 2. Csendes (HU) 3. Grubbstroem (SE) 4. Sniedovich (AU) 5. Trzaskalik (PL) 6. Yuan (SE)		1. Discrete Mathematics and Optimization (9 papers) 2. Multicriteria Decision Making (6 papers) 3. Scheduling and Control (5 papers) 4. Finance and Investments (5 papers) 5. Production and Inventory (5 papers) 6. Location and Transport (6 papers) 7. Environment and Human Resources (6 papers) 8. OR Perspectives (2 papers) 9. Statistics (10 papers)
SOR'07 (68 papers)	1. Boljunčić, Neralić (CRO) 2. Bomze (AT) 3. Gavalec, Plavka (CZ), (SK) 4. Boeckenhauer, Hromkovič (CH) 5. Povh (SI) 6. Stougie (NL) 7. Zadnik Stirn (SI)		1. Networks (5 papers) 2. Stochastic and Combinatorial Optimization (5 papers) 3. Algorithms (3 papers) 4. Multicriteria Decision Making (4 papers) 5. Scheduling and Control (4 papers) 6. Location Theory and Transport (4 papers) 7. Environment and Human Resource Management (5 papers) 8. Duration Models (5 papers) 9. Finance and Investment (7 papers) 10. Production and Inventory (7 papers) 11. Education and Statistics (5 papers) 12. OR Communications (7 papers)
SOR'05 (63 papers)	1. Boehm (AT) 2. Manger (CRO) 3. Rupnik (SI) 4. Rupnik, Sundać (SI), (CRO) 5. Castelli, Pesenti, Ukovich (IT)		1. Scheduling and Control (4 papers) 2. Stochastic and Combinatorial Optimization (4 papers) 3. Algorithms (7 papers) 4. Environment and Human Resources (7 papers) 5. Location Theory and Transport (10 papers) 6. Finance and Investment (8 papers) 7. Multicriteria Decision Making (5 papers) 8. Networks (4 papers) 9. Production and Inventory (5 papers) 10. Education and Statistics (3 papers)
SOR'03 (49 papers)	1. Cechlarova (SK) 2. Koechel (DE) 3. Luptačik (AT) 4. Šimundić (CRO) 5. Zlobec (I&S), Compton, Vuong (CA)		1. Algorithms (6 papers) 2. Location Theory and Transport (6 papers) 3. Finance (4 papers) 4. Environment and Human Resources (4 papers) 5. Production and Inventory (8 papers) 6. Scheduling and Control (7 papers) 7. Multicriteria Decision Making (3 papers) 8. Education and Statistics (4 papers) 9. Open OR Problems (2 papers)
SOR'01 (59 papers)	1. Rendl (AT) 2. Šorić (CRO) 3. Schaerf, Di Gaspero (IT) 4. Shawe-Taylor (GB) 5. Ferrari, Manzini, Regattieri, Persona (IT)		1. Algorithms (11 papers) 2. Optimization ((5 papers) 3. Scheduling and Control (4 papers) 4. Networks (4 papers) 5. Production (7 papers) 6. Finance (6 papers) 7. Environment and Human Resources (6 papers) 8. Dynamic Systems (3 papers) 9. Education and Statistics (4 papers) 10. Current Projects in Slovenia and Croatia (4 papers)
SOR'99 (43 papers)	1. Burkard, Fortuna (AT) 2. Mitra, Koutsoukis (GB) 3. Neralić (CRO) 4. Schields (US) 5. Zimmermann (CZ)		1. Optimization and Control (4 papers) 2. Hot Lines Panel Section (4 papers) 3. OR Applications (9 papers) 4. Modelling (9 papers) 5. Production and Inventory (6 papers) 6. Network Analysis (6 papers)

Proceedings	Keynote speakers	Special sessions	Sessions
SOR'97 (58 papers)	1. Becker, Keuhrer, Leopold-Wildburger (DE), (AT) 2. Grubbstroem (SE) 3. Zimmermann (CZ)		1. Wood Processing and Agriculture (10 papers) 2. Optimization (9 papers) 3. Economic (Systems) Modelling and Control (7 papers) 4. Production (6 papers) 5. Business (System) Modelling and Control (6 papers) 6. Traffic and Transportation (4 papers) 7. OR in Transitional Economies (4 papers) 8. OR Experiences and Practical Solutions (4 papers) 9. Inventory (3 papers) 10. Last Minute Section (2 papers)
SOR'95 (21 papers)	Hill (GB)		Uniform session (20 papers)
SOR'94 (31 papers)	1. Csebfalvi (A&G) (HU) 2. Komlosi (HU) 3. Marinović (CRO) 4. Neralić (CRO) 5. Sethi, Taskar, Zhang (CA) 6. Varga (HU)		1. Production (9 papers) 2. Transport (2 papers) 3. Mathematical Programming (6 papers) 4. Various OR Applications (5 papers) 5. Computer Programs (4 papers)
SOR'93 (37 papers)	1. Barle, Grad (SI) 2. Rupnik (SI)		1. Production (10 papers) 2. Transport (5 papers) 3. Mathematical Programming (4 papers) 4. Multicriteria Decision Making and Environment (6 papers) 5. Various OR Applications (7 papers) 6. Computer Programs (3 papers)

2.2 Monographs from SSI-SOR

Most members of SSI-SOR are affiliated with universities or institutes. Therefore, the members are mostly researchers in the field of operations research and related fields. Some of them, together with their colleagues from abroad, mostly from Croatia, have compiled their work in the form of monographs. Since 1998, SSI-SOR has published 5 monographs: Rupnik (1998), Rupnik et al. (2000), Zadnik Stirn et al. (2005), Rupnik and Sundać (2005) and Rupnik (2013).

The monograph (Rupnik 1998) is divided into six chapters that follow the Introduction and end with the cited literature. The first chapter provides the foundations of the theory of economic integrability, while the second and third chapters diagnose horizontal and vertical economic integrability. The fourth and fifth chapters present the prediction of horizontal and vertical economic integrability. In the last (sixth) chapter, the author establishes the usefulness of the integration methodology in the past and in the future.

The monograph Solutions to Production Problems (Rupnik et al. 2000) consists of a total of 27 chapters divided into five parts. In it, 33 authors from Slovenia and Croatia discuss the production process using simulations in production, combinatorial optimization of production, technical production planning, economic production planning, and techno-economic production planning.

In addition, the monograph Selected Decision Support Models for Production and Public Policy Problems (Zadnik Stirn et al. 2005) contains 8 chapters written by 12 authors from Slovenia and Croatia. It presents both the theory and the application of models based on new OR methods for solving problems in production and management (mainly in public administration). The chapters are as follows: economic modelling and analysis/measurement of quality, multicriteria tools for evaluation of socio-economic and environmental programmes, bounds of common functions on polyhedra - a new approach to global optimization, planning process with stochastic demand, optimization of material cutting process, optimization of scheduling in production process, analysis of the impact of road network development on daily migration (case of Slovenia) and analysis of efficiency of state administration. The monograph was reviewed by L. Neralić and S. Indihar. The introduction to the monograph was written by L. Zadnik Stirn and L. Ferbar.

In their monograph, authors Rupnik and Sundać (2005) state that the neoliberal and monetarist doctrine opened the way for the unfettered development of globalization. This has had a number of positive and negative effects. Society has readily accepted the positive effects, but wonders whether it should also accept the negative effects that lead to unemployment, environmental degradation, economic insecurity, stratification of society and even disease and death. The authors model the system of worldwide globalisation using a model based on metamathematics and simulations. Using the model, the authors find that uncorrected neoliberal globalization, supported by the doctrine of monetarism, can lead to terrible differentiation of humanity. Based on the model, the authors also make suggestions to preserve the positive effects of globalization and to eliminate or at least partially rehabilitate the negative ones.

The monograph (Rupnik 2013) is aimed at economists, social scientists, and many others who want to reflect on current socio-economic phenomena. The author recognizes that capital plays an important role in development in reaching consensus on key development issues and advocates for sustainable development. He proposes an "ethical" economy that includes the so-called information society as an important dimension of society. In addition to empirical approaches, his findings and recommendations are also based on multiparametric mathematical models. The preface is followed by 17 chapters dealing with neoliberal globalization (conflicts, consequences and possible corrections), the existential triangle of society consisting of capital, nature and labour, and an analysis of the interaction between them. In his conclusions, he proposes an ethical economy as a "remedy". The preface to the monograph was written by M. Krisper.

As we can see from the references, some monographs or their parts are in Slovene. We would like to add that for the Slovenian technical language, i.e., the Slovenian terminology in the field of OR, the publications in Slovenian language are indispensable and of great importance.

2.3 Special journal issues from SSI -SOR

In 1994, SSI-SOR joined the Austrian, Czech, Croatian, Hungarian, and Slovak societies for operations research. Under the leadership of the Austrian colleagues, the Central European Journal for Operations Research and Economic was launched and published until the end of 1998. From 1999, the journal was published by Physica and later by Springer Verlag under the current name Central European Journal of Operations Research (CJOR). The journal was indexed in the Web of Science in 2007. It is currently on the borderline between the first and second half of journals by impact factor in the Web of Science category Operations Research and Management Science (Jablonsky et al. 2022).

Since 1997, the members of SSI-SOR have been co-editors of CJOR. In 2011, the first special issue with guest editors was published by SSI-SOR. In 2013, 2015, 2017, 2019, 2021, and 2023, the members of SSI-SOR published further CJOR special issues on international symposia organized by SSI-SOR. The bibliometric analysis of the papers published in the first six SSI-SOR special issues of CJOR is presented in Kastrin et al. (2021). In this paper, the authors analyse and virtualize the annual dynamics of the number of published papers, highlight the most prolific authors and countries, present the papers with the highest impact in terms of number of citations, and examine the keyword match network divided into eight clusters. The results of the basic analysis are based on the full set of 67 articles from the six special issues of SDI-SOR, while the extended analysis includes only the 63 articles from the WoS collection and Dimensions.ai. In terms of publication type, all articles are considered original scientific contributions.

The latest issue of CJOR (CJOR Volume 32, September 2023, Issue 3) is again an SSI-SOR special issue and contains 16 articles presented in more detail in Povh et al. 2023.

The number of articles in all seven CJOR special issues edited by the editors of SSI-SOR, and the names of the authors can be found in Table 2. Table 2 shows that the number of articles published in the CJOR special issues of SSI-SOR is increasing. This is due to the increased visibility of the work of SDI-SOR in Central Europe and the increasing importance of CJOR (higher impact factor), which means that there is more interest in publishing in this journal.

Table 2: Number of articles and names of authors in CJOR special issues edited by SSI-SOR in 2011-2023.

Year	Number of articles	Author(s)
2011	8	Bogataj et al. 2011; Grošelj et al. 2011; Hentsch and Köchel 2011; Ivanov et al. 2011; Jurun and Pivac 2011; Kovačič and Bogataj 2011; Matis and Kohani 2011; Žerovnik and Žerovnik 2011
2013	7	Dumičić et al. 2013; Govorčin et al. 2013; Hvalica 2013; Kovačič and Bogataj 2013; Kramberger et al. 2013; Toth and Kresz 2013; Mladenović et al. 2013
2015	12	Agrež and Damij 2015; Cechlárová et al. 2015; Dumičić et al. 2015; Gaspars-Wieloch 2015; Janáček 2015; Kovačić et al. 2015; Dalpasso and Lancia 2015; Moeini et al. 2015; Shao and Vesel 2015; Hunjet et al. 2015; Nguyen and Chassein 2015; Rudec and Manger 2015
2017	11	Bala et al. 2017; Režnar et al. 2017; Gaspars-Wieloch 2017; Nikolakopoulos and Ganas 2017; Bohanec et al. 2017; Mihelčić and Bohanec 2017; Jánošíková et al. 2017; Janáček and Kvet 2017; Chocholatá and Furková 2017; Shao et al. 2017; Hladik 2017
2019	14	Garajová et al. 2019; Tavakoli and Klavžar 2019; Trzaskalik et al. 2019; Kadoić et al. 2019; Ligardo-Herrera et al. 2019; Sternad Zabukovšek et al. 2019; Jánošíková et al. 2019; Brelih et al. 2019; Pavlovčič Prešeren et al. 2019; Jordan et al. 2019; Furková 2019; Jakšić et al. 2019; Čampelj et al. 2019; Bokal and Steibacher 2019
2021	15	Hladík 2021; Tomanová and Holý 2021; Jakšić et al. 2021; Cechlárová et al. 2021; Trzaskalik 2021; Jukić and Sabo 2021; Matejaš et al. 2021; Janáček and Kvet 2021; Vrankić et al. 2021; Drnovšek et al. 2021; Furková and Chocholatá 2021; Zakrajšek et al. 2021; Campuzano-Bolarin et al. 2021; Povh and Žerovnik 2021; Smole et al. 2021
2023	16	Nagy and Varga 2023: Hladik 2023; Gaspars-Wieloch 2023; Perič et al. 2023; Garajova and Rada 2023; Trazskalik 2023; Gabrovšek et al. 2023; Čegovnik et al. 2023; Osz and Hegyhati 2023; Lukać 2023a; Lukać 2023b; Milavec Kapun et al 2023; Varga and Madari 2023; Pejić Bach et al. 2023; Krpan 2023; Kašparova 2023

In 2012, SSI-SOR was invited by the editors of Business Systems Research (BSR) to publish a special issue (SI). BSR is an academic journal that focuses on research findings in economics and business systems. In addition, BSR also considers research that combines economics with other scientific fields such as information systems, mathematics and social sciences. BSR examines a variety of business decisions, processes and activities in the context of the actual business environment as well as within a systems approach. The journal is indexed in Scopus, Web of Science (ESCI-WoS) and Portal of Croatian Scientific and Professional Journals. It is currently indexed as Q3 in Scopus for Economics, Econometrics and Finance. SSI-SOR has published six special issues since 2012: 2012, 2014, 2016, 2018, 2020 and 2022. The special issues of BSR, i.e., SI of BSR, have focused on recent advances in the field of Operations Research and Management Science (OR/MS), with particular emphasis on linking OR/MS with other areas of quantitative and qualitative methods in a multidisciplinary framework (Drobne et al 2022).

The SI of BSR Issue 2022, Vol. 13/3 contains ten papers presenting advances and new techniques (methods) in operations research (OR) and their application in various fields, including risk management, mathematical programming, game theory, gravity, spatial analysis, logistics, circular economy, continuous improvement, sustainability, e-commerce, forecasting, Gaussian processes, linear regression, multi-layer perception and machine learning. The authors are from Portugal (5), Croatia (3), and Slovenia (2).

Eleven papers were selected for the SI of BSR Issue 2020, Vol. 11/2, presenting improvements and new techniques (methodology) of OR and their application in various fields of economics, spatial sciences, smart mobility, higher education, human resources, environment, agriculture and social networks. The authors of these papers were from Croatia (3), Slovenia (3), Hungary (1), Portugal (1), and the Czech Republic (1), while two papers were written by authors from different countries: one team was from the Netherlands, Slovenia and Spain, the other from Norway, Hungary and Slovenia.

Nine papers received a positive review for SI from BSR 2018, Vol. 9/2. They present improvements and new methods in OR and their application in various fields of economics, spatial science, and site assessment. The authors are from Slovenia (3), Croatia (3), Norway (1), Turkey (1), one paper was written by authors from Croatia and the Republic of North Macedonia, and one by authors from Croatia and Singapore.

Seven papers dealing with OR techniques such as the Intramax method, energy analysis, multivariate analysis and DEA and their application in different areas of economics were selected for SI of BSR 2016, Volume 7/2. The authors are from Croatia (4), Slovenia (2) and the Czech Republic (1).

Six papers dealing with high-dimensional classification problems using machine learning, stock markets using illiquidity measures, optimization of information systems using LP, using statistical methods for small business performance, the Intramax method for studying functional regions, and queueing models for optimizing call centre performance were selected for SI of BSR 2014, Vol. 5/3. Four authors are from Croatia and two from Slovenia. Three authors from Slovenia and three from Croatia contributed to the first SI of BSR published by SSI-SOR. They studied inflation dynamics, the use of the gravity model for the case of labour commuters, the theory of multicriteria group models, management analysis based on an innovative statistical approach, neural networks, and novel heuristics for solving the garbage packing problem.

SSI-SOR is co-editor of the journal Uporabna informatika (in Slovenian; translation: Applied Informatics; https://uporabna-informatika.si/ui, a Slovenian journal of computer science/OR. Its mission is to inform the professional public and users about the latest achievements in informatics/OR in Slovenia and worldwide. A special merit of the journal is the information about Slovenian research projects and European documents that form the basis for trends in informatics/OR and inevitably influence our environment.

3 A FEW OTHER SSI-SOR ACTIVITIES

- SSI-SOR members actively participate in all EURO conferences where they organize sections, are members of programme committees, invited speakers, co-editors of proceedings, members of EURO commissions, and representatives of the EURO Council.
- SSI-SOR members actively participate in all IFORS conferences where they organize sections, are members of programme committees, representatives of IFORS Council and corresponding members of IFORS News.
- SSI-SOR members cooperate with the Czech Society of OR; for example, in 1998 at the 11th joint Czech-German-Slovak Conference: Mathematical Methods in Economics and Industry in Liberec, where they also gave invited presentations.
- In 1998, SSI-SOR co-organized the Austrian-Croatian-Slovenian workshop in OR in Seggauberg, Austria.
- SSI-SOR members actively participate with invited talks, contributions, as reviewers, programme committee members and section leaders in all international conferences organized by HDOI, Croatia, i.e., International Conferences on Operational Research KOIxx, where xx refers to KOI 1992 … KOI 2022.
- SSI-SOR members actively participate in IFIP TC7 conferences with contributions, as members of programme committees and are members of the TC7 Board Council.
- Last but not least, members of SSI-SOR actively participate in the Days of Slovenian Informatics (DSI) with contributions, as reviewers, programme committee members and section chairs - unfortunately to a lesser extent in recent years.

At this point we must mention that the members of SSI-SOR are also involved to some extend in editing and reviewing the interactive Internet dictionary Islovar (http://islovar.org/islovar), which is also recognized as a reference dictionary for the public.

In 2010, the activities of SSI-SOR were published in the Wiley Encyclopaedia of Operations Research and Management Science, edited by J. J. Cochran (Zadnik Stirn 2010).

4 AWARDS AND RECOGNITIONS FOR SSI-SOR MEMBERS

Members of SSI-SOR have also received awards and recognitions for their contributions to the field of informatics and operational research. These also include:

- SSI's recognition for the development of international cooperation and exchange of achievements in the field of OR in 2007.
- The honorary membership of OEGOR (Austrian Society for OR) in 2018.
- A recognition for lifetime achievement in computer science in Slovenia by the Slovenian society INFORMATIKA and the promotion of SSI-SOR in the international environment in 2019 and 2022.
- Donald Michie-Alan Turing Award for lifetime achievement in computer science in 2020.

5 CONCLUSIONS

Congratulations on the 30th anniversary of SSI-SOR! This is a remarkable milestone that reflects the years of dedication, hard work, and common goals of all members. May SSI-SOR continue to prosper and make a positive impact for many more years to come. Let's celebrate the achievements of SSI-SOR and look forward to a bright future filled with even more success and accomplishments. Cheers to 30 years of excellence from SSI-SOR!

Even though the work done by the members of SSI-SOR in the 30 years of the existence of the OR section is extensive, there are still a number of tasks that need to be done according to the goals set. So there are still many challenges, such as:

- Continuation of the publications and the organization of the international OR Symposia (SOR) by the SSI-SOR members.
- Continuation of SSI-SOR collaboration with the College of Ljubljana, College of Maribor and College of Primorska (different departments, such as UL EF, UL FS, UL FDV … UM FOV and UM EPF …) and searching for new connections within the faculties, institutes and research organizations.
- Continued active participation in the international associations IFORS, EURO, IFIP and others.
- Continued networking with foreign OR associations (Croatian, Austrian, German, Czech, Hungarian, Slovak, Polish, American and others) and search for new connections, especially in joint research projects and publication and dissemination of results (conferences and other forms of meetings).
- Presentation of SSI-SOR in Wikipedia on the page EURO (http://en.wikipedia.org/wiki/Association_of_European_Operational_Research_Societies).
- Appointment of a group of volunteers to work on the terminology of OR; the basis could be the publication Terminološki rečnik iz opérationih istraživanja, SYM-OP-IS'83, Faculty of Organizational Sciences, Belgrade, 1983, 514 pages, 9 languages (the Slovenian part was contributed by M. Omladič, J. Dekleva and A. Vadnal), and many foreign terminological dictionaries.
- Integration of the content of OR into the study processes. In this respect, SSI-SOR has been partially successful. Topics or modules from OR have been implemented in several curricula, but there is still much opportunity for more intensive implementation of OR at all levels of study.
- Familiarize businesses and the public with the possibility of using OR. SSI-SOR has not yet been active enough in this area.
- Increase membership also by involving colleagues in joint projects and students through theses, student projects, and their participation in meetings of national and international stakeholders through conferences and other forms.
- Closer involvement within the SDI in various areas: in organizing, leading sections, presenting papers, roundtables, workshops in the SSI, working on terminology dictionaries, joint scientific projects, and public outreach.
- Active activity in the field of public relations, especially in the scientific field through domestic and foreign journals (continuation of cooperation with CJOR, BSRJ ...).

Some of the topics discussed in this article have already been presented in recent publications/reports from SSI-SOR (Povh et al. 2023; Zadnik Stirn and Drobne 2022; Kastrin et al. 2021).

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Professor Dr. Janez Grad - 90th Anniversary

Summary *Professor Dr. Janez Grad has graduated in mathematics at the Ljubljana university, has started as a research assistant and soon after held leading positions in research and university institutions in Slovenia and abroad. He received a PhD in mathematics and has been a professor at Ljubljana university and in United States. His work includes several scientific papers on decision support, data base management systems, informatics, computer science, mathematics and operations research. He is a productive author, has published numerous papers, has authored quite a few books and proceedings and has been granted awards and recognitions for his work. A special thanks goes to him for having been one of the initiators to establish the charter for operations research at Slovenian Society INFORMATIKA. Besides his notable work in mathematics and informatics he is also a respectable naturalist: his work on bumblebees is widely appreciated.*

Keywords *Prof. Janez Grad, anniversary, achievements, contributions, impact, reflections, congratulations.*

Prof. Dr. Janez Grad at the presentation of his book on rare bumblebee species in Slovenia (Grad et al., 2016; Source: Archive of Celje Mohorjeva Družba)

Professor Dr. Janez Grad was born in Petelinje near Ljubljana in 1933. After graduating from high school, he studied mathematics at the University of Ljubljana and graduated in 1958. In 1958 he started as a research assistant at the Jožef Stefan Institute in Ljubljana. He was then appointed head of the Computer Centre of the Republic in 1969. From 1972 he was head of the Computer Centre of the University of Ljubljana. His rich professional career includes work for the Federal Statistical Office in Belgrade, for the Institute for Radiation and Nuclear Physics in Bonn and at the University of Calgary. He received an MSc in mathematical physics from the University of Birmingham in 1968 and a PhD in mathematics from the University of Zagreb in 1973. From 1972 to 1978 he was Assistant Professor and since 1979 Full Professor at the University of Ljubljana. From 1983 to 1985 he was Vice-Dean of the Faculty of Economics at the University of Ljubljana. In 1986 he was a visiting professor at the School of Business at Indiana University, USA. From 1991 to 2000 he was Head of the Postgraduate Programme in Informatics and in Information Management Sciences at the Faculty of Economics in Ljubljana and Full Professor of Informatics at the Faculty of Administration in Ljubljana. He is now retired.

For more than forty years, Professor Grad has worked in various institutions and companies as a manager and consultant, but above all he was an excellent teacher and an active and creative researcher. His research area is very important for the development of the interdisciplinary trinity: mathematics/informatics, operations research and computer science. His research efforts have been devoted to decision support system technologies (Barle et al., 1999; Damij et al., 2008), especially in the field of optimisation and simulation problems (Damij and Grad, 1995), on the one hand, and database management systems (Biloslavo and Grad, 2003) and database development, on the other. His research efforts focused on the use of graph theory in the process of database normalisation, object-oriented databases and decision tables. A significant part of *his research focused on the theory and practise of operations research (Grad et al., 1986; Intihar Štemberger and Grad, 2001; Grad, 2001; Arsham, 2007). He developed many linear programming programmes and algorithms based on simplexes, interior point algorithms, sparce matrices and sensitivity analysis (Arsham et al., 2003, 2005; Damij and Grad, 2006). Professor Grad's bibliography includes more than 400 items. He is the author or co-author of 60 scientific papers, has edited and co-edited more than 52 professional publications, has published more than 80 papers in international conference proceedings, is the author of 14 books and more than 25 textbooks, and has written more than 40 research reports. In his teaching career, he was mentor to 15 PhD students, to 36 MSc candidates, and to several undergraduate students. Professor Grad has edited several proceedings in the field of information science, serves on the editorial boards of the journals Informatica and Applied Informatics, and has been a reviewer for several scientific articles submitted to national and international conferences. Details can be found at: https://bib.cobiss.net/bibliographies/si/webBiblio/bib201_20230828_185018_a1464163.html.*

His important work did not go unnoticed. During his long and dedicated career, Professor Grad was honoured with several recognitions. He received an award for a lecture at the FCIP International Symposium and a gold plaque from the University of Ljubljana. In 1995, he was awarded a recognition by the Slovenian society INFORMATIKA for lifetime achievement in informatics, and in 2014 he received the Donald Michie and Alan Turing Award for lifetime achievement in mathematics and informatics.

The account of Professor Grad's work and life would not be complete without mentioning his lifelong interest and deep knowledge of bumblebees (Grad and Toplak, 2018; Grad and Gradišek, 2018; Grad and Oštir, 2021; Gradišek et al., 2023). In 2016, a book on rare bumblebee species in Slovenia was published and Professor Grad is one of the three authors of the book, which also shows him as a person with great interest in nature (Grad et al., 2016).

Let me conclude this paper with two very personal remarks. First, I am deeply grateful to the editors, especially Professor Lidija Zadnik Stirn (Zadnik Stirn, 2003), for allowing me to use their work to compile this report. Without their efforts, it would have been next to impossible for me to produce it. Secondly, I am really glad that Professor Grad has lived to see how his colleagues and friends value him and how his work is appreciated and respected by his peers.

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Ljubljana, September 5, 2023 *Niko Schlamberger*

Professor Dddr. Viljem Rupnik - 90th Anniversary

Summary *This article pays tribute to the remarkable life and work of Prof. Dddr. Viljem Rupnik on the occasion of his 90th birthday. Throughout his illustrious life, Prof. Rupnik has had a lasting impact on society, inspired countless people, and made significant contributions to his field. This paper recognizes these remarkable achievements, highlights his life's work, and examines Prof. Rupnik's influence on the respective fields of operational research, mathematics, economics, and beyond.*

Keywords *Prof. Dddr. Viljem Rupnik, anniversary, achievements, contributions, impact, reflections, congratulations.*

Prof. DDDR. Viljem Rupnik, with SOR'17 Proceedings, (Source: Delo 21.03.2018, Ljubljana, Slovenia)

There are several reasons why professionals in the field of Operational Research, Mathematics, Economics and Engineering should at least briefly stop by the nine crosses of Prof. Dddr. Viljem Rupnik, which he reached on May 21, 2023, even though we know that this event is not an ordinary milestone for him, which means nothing more than one of his current and future working days. Since Prof. Rupnik has not withdrawn from work in retirement, this year's life anniversary by no means tears him away from the tasks he has started and planned. After all, his anniversary is first and foremost a work anniversary. Those who know Prof. Rupnik know that it cannot be any other way, we just want it to stay that way for a long time.

Prof. Rupnik is known as a teacher, mentor, researcher, consultant, publicist, director, all that and more in the field of economic, mathematical, and technical sciences and beyond.

Prof. Rupnik has earned three degrees and three doctorates. First, he graduated in music, then in 1956 in applied and theoretical mathematics at the Faculty of Natural Sciences and Mathematics of the University of Ljubljana, and finally in 1961 in Economics at the Faculty of Economics of the University of Ljubljana. He received his PhD in Economics from the Faculty of Economics in 1963, in Mathematics from the Faculty of Science and Mathematics of the University of Ljubljana in 1978, and in Chemical Engineering from the Faculty of Technology of the University of Zagreb, Croatia, in 1986. He was a Ford Scholar and was trained by R. L. A. Ackoff at the Center for Management at the University of Pennsylvania in Philadelphia and by R. Bellmann at the University of Southern California, USA, in 1964-65 (Enciklopedija Slovenije, 1996).)

In 1956 Prof. Rupnik was offered employment at the Academy of Music and at the University of Ljubljana, Faculty of Economics. He faced a big decision what to choose for his life path, what direction to take, art or science. He chose science and accepted the position at the university; he has been a full professor there since 1974. Initially, he was engaged in mathematical modeling of microeconomic structures, and after his return from the United States, he continued his work on the management and non-terminal control of linear systems. At the Research Center of the Faculty of Economics, he developed the mathematical theory of economic integration, *production theory as an application of general mathematical systems theory, and proved the existence theorems of multicriteria controllability of dynamical systems. Through his hard work, he contributed to the field with influential publications. He published (some with his colleagues/collaborators) about 70 technical and scientific articles, 10 books, and several textbooks (https://bib.cobiss.net/bibliographies/si/webBiblio/bib201_20230826_102840_ a1781603.html). I mention here only a few. very significant for the field of mathematics, economics, operations research, and society at large.*

As early as 1964, he became known for the optimization model for optimal complex planning of agricultural production by the method of bilinear programming (Vadnal and Rupnik, 1964). He and Prof. Vadnal successfully applied mathematics to the solution of concrete economic problems. They proved that mathematics could solve economic problems. When times are tight, when there are many constraints, mathematics could be a useful tool. The situation in the economy, especially in the food industry, was difficult at that time, there was a lack of this and that. The first problem they encountered was the low live weight gain of cattle and the high cost of feed. Using the optimization method, they proposed a cost-optimal feed mixture from the raw materials of the time, which quickly increased self-sufficiency in meat. Their solutions/proposals were adopted by all Slovenian livestock farms at that time. As Prof. Rupnik (Delo, 2018): "At that time, the Chamber of Commerce of Slovenia established a special office for business and market research, as a service provider for the optimization of production in companies. Today, however, science in general is not directly related to business as it used to be. Problems are dealt with in the false consciousness that we live in abundance and that we can sin."

In the following years his publications were devoted to mathematical models of production processes, information-based management systems for business decisions and spatial planning, and mathematically oriented microeconomic theory of production (Lalić and Rupnik, 1971, Rupnik 1973, 1975, 1983). In 1978 Prof. Rupnik published a new approach to continuous dynamic linear programming (Rupnik, 1978) and in 1996 methodological research on the theory of economic integrability factors and their practical modeling (Rupnik, 1996).

In 1974, Prof. Rupnik published the book Outline to Operations Research, in Slovenian (Rupnik 1974). It is the first more comprehensive book in Slovenian in the field of Operations Research. Aiming at a wider readership, the book does not follow the strict pattern of a book that usually assumes some knowledge of mathematics and statistics. Rather, it is an overview of the problems and touches on various tools/methods as a secondary objective. Thus, it is a systematic explanation of several serious theoretical and practical issues affecting operations research.

It is important to stress that Prof. Rupnik is not only a mathematician and economist, but much more, as he deals with broader issues of humanity (Rupnik, 2005, 2013. 2017). In his recent monograph (Rupnik, 2013) on globalization, Professor Rupnik addresses not only economic development but also social development. He looks for a balance between capital, labor, and nature. He analyzes all these factors in detail. He looks for interactions between them and treats them meaningfully as an ethical triangle, because he assumes that the collapse of the system can be prevented only by an ethical and reasonable approach to the "use" of all three factors; in fact, he proposes a development within the framework of the so-called ethical economy. He derives his findings from multiparametric mathematical models and empirical questions. This approach of Prof. Rupnik to the problems of globalization is the basis for further development and a creative attitude, especially in relation to work and the preservation of nature for our descendants, both individuals and larger groups.

Finally, his theoretical and practical reflections on mathematical models, economics, production, decision making, and other topics have not gone unnoticed. They appeared in publications of the Chamber of Commerce of Slovenia, in the journal Finance, in the publications of numerous seminars, national and international conferences, and also as research *reports and expert opinions (https://bib.cobiss.net/bibliographies/si/webBiblio/bib201_20230826_102840_a1 781603.html).*

The areas of his research in mathematical, economic, and technical sciences are also described in more detail in (Nemec, 2023).

His educational work is also very important. About 30,000 prospective economists attended his school at the Faculty of Economics in Ljubljana. Prof. Rupnik's transfer of knowledge to his students is unique and enormous. He mentored 45 master's students and 42 doctoral students. He and his colleagues also wrote textbooks and manuals for students in economics and operational research, many of which are so arbitrary that they have served and continue to serve students and researchers in many other fields (social sciences, engineering, and natural sciences) to acquire knowledge in mathematics, operational research, and economics. Among the first comprehensive textbooks on applied mathematics were those published in 1977 and 1978 (Arih et al., 1977, 1978), followed by several others, such as (Bogataj et al., 1988).

For more than forty years, Prof. Rupnik has worked for various institutions and companies as a manager and consultant. As Consultant for business and industry (1959-) he has carried out more than 200 industrial projects for production optimization and beyond. In 1970-1976 he was director of the Research Center of the Faculty of Economics in Ljubljana; 1981-1984 he was consultant at ICPE (International Center for Management of Public Enterprises in Developing Countries based in Ljubljana) and 1987-1989 consultant at Milan Vidmar Institute for Energy Economics. Since 1992 he has been a director of INTERACTA, a business information technology company.

For the development of Operations Research in Slovenia it is very important to mention that Prof. Rupnik was the initiator and co-founder of the master's program in Operations Research and the PhD program in Information Business Studies, both at the Faculty of Economics of the University of Ljubljana.

In 1992/1993, Prof. Rupnik co-founded the Slovenian Section on Operational (SOR) Research and was its president for many years. He was co-organizer of several symposia on operations research in Slovenia and co-editor of the proceedings of these symposia.

In addition, Prof. Rupnik was, of course, an editor of journals and monographs and a reviewer of numerous scientific papers published in journals with a high scientific index.

Prof. Rupnik's many years of high-profile scientific and professional works have received widespread attention and public recognition. All the awards speak for the scope and, above all, the importance of the jubilarian's work. He was one of the first to receive the Kraigher Prize for his achievements in the field of economics/political economy, for introducing operational methods into economics. Prof. Rupnik received the Slovenian Chamber of Commerce Award, then called the Boris Kraigher Award, in the first group of beneficiaries in 1969, at the young age of 35, together with his colleague Prof. Vadnal, "for the efficient and successful theoretical and applied development of operational research, which contributed to progress in the organization and management of production and operation of economic organizations." In 2000, Prof. Rupnik was awarded a recognition for lifetime achievements in informatics in Slovenia by the Slovenian Society INFORMATIKA. After his retirement, he was appointed professor emeritus at the University of Ljubljana (2003).

At the beginning of his tenth decade, may he be comforted by the certainty that the furrows he has plowed in the fields of operational research, mathematics, and economics will still bear useful fruit for generations to come. Therefore, we researchers in the field of operations research, mathematics and economics must be wholeheartedly grateful to Prof. Rupnik and wish him many more healthy, fruitful, and happy years.

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Ljubljana, September 5, 2023 *Lidija Zadnik Stirn*

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The 17th International Symposium on
Operational Research in Slovenia

SOR '23

Bled, SLOVENIA
September 20 - 22, 2023

Plenary Lectures

KNOWLEDGE DISCOVERY BY LITERATURE MINING: FROM SERENDIPITY TO COMPUTATIONAL CREATIVITY

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Abstract: Since the inception of written thought, human knowledge has steadily increased, as have the number and size of published works. The output of the scientific community has doubled every nine years over the past decades. The National Library of Medicine, for example, adds more than 3,000 papers daily to MEDLINE, the world's leading bibliographic database in the life sciences. Working in such information overload, researchers can miss valuable segments of knowledge. Machine extraction of relevant knowledge is an important research activity today, with the challenging task of linking diverse scientific information into coherently interpretable knowledge.

Knowledge discovery from scientific publications (also called literature-based discovery [LBD]) is a methodology for automatic generation/validation of research hypotheses. The main goal of LBD is to uncover hidden, previously unknown relationships from existing knowledge. The general framework for LBD is based on three literature concepts: A, B, and C. For example, suppose that a researcher has found a link between disease A and a gene B. Let us further assume that another researcher has studied the effect of compound C on gene B. The use of LBD may suggest an AC relationship, indicating that substance C may be treating disease A. Such a latent relationship may provide a hypothesis for a potential, yet undiscovered relationship. The LBD methodology was popularized by Swanson, who discovered that dietary fish oil could be used to treat Raynaud's disease, which is characterized by reduced blood flow to the extremities causing pain and cold sensations.

In this talk, we give a general overview of the LBD field, briefly introduce different methodological approaches, and discuss recent approaches such as knowledge graph completion and representation learning.

Keywords: text mining, natural language processing, literature-based discovery

SPARSE POLYNOMIAL OPTIMIZATION: THEORY AND PRACTICE

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Abstract: Polynomial optimization methods often encompass many major scalability issues on the practical side. Fortunately, for many real-world problems, we can look at them in the eyes and exploit the inherent data structure arising from the input cost and constraints. The first part of my talk will focus on the notion of "correlative sparsity", occurring when there are few correlations between the variables of the input problem. The second part will present a complementary framework, where we show how to exploit a distinct notion of sparsity, called "term sparsity", occurring when there are a small number of terms involved in the input problem by comparison with the fully dense case. At last but not least, I will present a very recently developed type of sparsity that we call "ideal-sparsity", which exploits the presence of equality constraints. Several illustrations will be provided on important applications arising from various fields, including computer arithmetic, robustness of deep networks, quantum entanglement, optimal power-flow, and matrix factorization ranks.

Keywords: polynomial optimization, semidefinite programming, moment-sum of squares hierarchy, correlative sparsity, term sparsity, ideal sparsity, robustness of deep networks, optimal power-flow, quantum entanglement, matrix factorization rank

OPERATIONS RESEARCH MEETS ARTIFICIAL INTELLIGENCE: INTERSECTION OR UNION

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Abstract: Operations research (OR) and artificial intelligence (AI) are fast-growing methods in methodology development and practical deployment. However, the debate is on the relations between these two fields, whether they are related or subordinated. The talk aims to present the results of the systematic literature review of OR and AI methods and to identify the applications that merge operations research and artificial intelligence methods. The talk will review the main topics extracted with the text mining approach. The findings shed extensive light on how OR and AI have evolved together. Finally, using the survey findings as a springboard, several avenues are suggested down which future research may go, including such themes as new approaches to research and techniques.

Keywords: interdisciplinarity, text mining, topic mining, AI, OR

JULIA, A PROGRAMMING LANGUAGE FOR OPERATIONS RESEARCH

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Abstract: Julia is a modern python-like programming language created in 2009. The presentation will show how this new language is easy to learn and use. The large library of packages shows that the language can cover all aspects of Operations Research from simple heuristics programming, MILP modeling, data manipulation to artificial Intelligence. Julia is the perfect language for research as well as for teaching to many students. Experience shows that non-computer science students get used to it in a few weeks and show new programming skills rapidly.

Keywords: operations research, programming language, julia

PROMOTING ELECTRIC VEHICLES: REDUCING CHARGING INCONVENIENCE AND PRICE VIA STATION AND CONSUMER SUBSIDIES

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Abstract: Operational decisions for a government are i) to incentivize EV ownership by a direct consumer subsidy, a station subsidy that reduces charging inconvenience, or by both subsidies; and ii) to minimize subsidy expenditure or to maximize EV adoption level. Environmental and energy independence concerns lead to government subsidies for electric vehicles (EVs). We model the interactions between the government and the charging supplier as a Stackelberg game and study the optimal structure of subsidies by incorporating charging inconvenience. We prove that this inconvenience is decreasing convex in the number of stations. In the expenditure minimization case, the optimal policy depends on the government adoption target and the charging station construction cost. If the adoption target is below a threshold that depends on the construction cost, the government provides pure consumer subsidy or no subsidy; otherwise, a combination of consumer and station subsidies is optimal. As the station construction cost increases, the charger builds fewer stations, regardless of the subsidy type. We establish that expenditure minimization and adoption maximization yield the same subsidy policy if the charging inconvenience is linear. In a real-life case, we find numerically that a station subsidy alone is optimal if the construction cost is not low but the adoption target is low. Besides, a long driving range significantly reduces the need for subsidies if the construction cost is high. In contrast, a long charging time necessitates high expenditure mainly allocated to a station subsidy.

Keywords: government subsidies, convex charging inconvenience, electric vehicles, sustainable operations

The 17th International Symposium on
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SOR '23

Bled, SLOVENIA
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Special Session 1:

Applications of OR in Agricultural Economics

DOES ECONOMIC SITUATION CAUSE LAND ABANDONMENT? ESTIMATING ECONOMIC VIABILITY OF FARMING IN A SUB-MEDITERRANEAN REGION IN SLOVENIA

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Abstract: Land abandonment is increasingly frequent in European marginal areas with high nature value due to poorer socioeconomic status and growing conditions, with better policy solutions needed. We use Farm model to determine farm economic viability in a case of Slovenian Sub-Mediterranean region facing land abandonment. We identified seven typical farms and analysed economic indicators founded on production plan level using mathematical programming. Out of seven farms four cannot live from farming due to small size, making their socio-economic sustainability questionable. Policy measures targeting land access and local asset commercialisation are needed to secure the future of these farms.

Keywords: Farm model, Linear programming, Land abandonment, Karst

1 INTRODUCTION

Land abandonment is becoming an increasingly pressing issue in Europe and occurs particularly frequently in High Nature Value Farmland in the Mediterranean Europe, where traditional extensive agriculture has slowly been discontinued in recent decades, which has negative consequences both for biodiversity and society [1]. While generalisation is difficult due to high local variation, land abandonment is most frequently caused by socio-economic drivers, while ecological drivers and improper land management can also contribute [2].

Despite several policy measures within the Common Agricultura Policy (CAP) that can provide additional revenue to farms in marginal areas, CAP has had little impact on decreasing abandonment rates [3]. To prepare more targeted and efficient policies against land abandonment, better understanding of the local drivers, including the economic situation of local farms, is needed. One of the methods for policy impact assessment and analysing farm economic situation that is gaining prominence are bio-economic farm models [4]. Farm models are typically based on mathematical programming. Detailed input data at the farm level are used to optimise farm production activities and obtain maximal profit [5]. Such an approach allows for impact assessment at the farm level and can thus include various differences between farms, such as specialisation, size, intensity and working conditions. These are usually typical farms that are representative for a certain group of farms [6].

In this study we use the Farm model, developed by Žgajnar et al. [7], to analyse the economic situation of typical farms on Karst, Sub-Mediterranean region in Slovenia with high rates of land abandonment. The purpose of our study is to better understand economic situation as well as reasons for agricultural abandonment on Karst and provide policy recommendations for encouraging agriculture in the selected area.

2 METHODS

2.1 Description of the area

Karst is a 618 km^2 (61,800 ha) limestone karst plateau with Sub-Mediterranean climate in the south-west Slovenia. The area had previously been almost completely deforested and covered with dry grassland, associated with high biodiversity. Since the 19th century, afforestation with black pine and overgrowth of abandoned agricultural land began, so that 73% of the area is now covered by forests [8]. In 2019, there were 1,045 farms with over half less than 5 ha in size, together cultivating 8,677 ha of agricultural land, mostly permanent grasslands.

2.2 Farm model

We used a Farm model, developed by Žgajnar et al., [7], which is built in Excel spreadsheets with Visual Basics (VBA). The model is composed of three components, namely the Model calculations (MC) from Agricultural Institute of Slovenia, Typical agricultural holdings model (TAH), and the Farm model (FM).

MC are originally simulation models that estimate costs and economic situation of different agricultural products and can be adapted to varying production conditions and intensity. In the FM, MC are thus adapted to reflect the conditions on the specific farm and enable the calculation of production coefficients for each activity included in the farm production plan. TAH are models that represent hypothetical farms (at the level of production plan) across all agricultural sectors and subsectors in Slovenia, and are presented in [7] in more detail. In this study, the TAH most similar to Karst farm production conditions were selected and used as a starting point of the modelling process, with their attributes adjusted to local conditions in the next step. Finally, the FM uses data from TAH and MC to calculate the production plan for each farm with linear programming (LP), while maximising expected gross margin, subject to technical and production conditions reflecting local production conditions. The expected gross margin is the difference between variable costs and the sum of market revenue and subsidy payments, excluding voluntary measures.

To gather detailed data about local farm characteristics that were further used to adjust the TAH to local conditions, we conducted two focus groups with local agricultural advisors. In such a manner we identified additional seven typical farms and defined their utilised agricultural land and its structure, production technology and production conditions, available workforce, CAP measures and infrastructure. We analysed them with FM, where majority of the technical and objective function coefficients were calculated based on MC.

3 RESULTS AND DISCUSSION

We identified seven typical farms, of which there is one small (VIT_0001) and one medium viticultural farm (VIT_0005); one small (SUC_0010) and one large suckler cow farm (SUC_0040); one small (SHB_0045) and one large sheep breeding farm (SHB_0240); and a small farm that solely produces hay with mowing (PGL_0005) (Table 1).

All the livestock farms are based on permanent grassland and put their animals on pastures and produce some hay. Due to dry climate and stony karst landscape the production is generally very extensive. Pastures are grazed with low livestock densities (0.2 livestock units per ha on average) and grasslands are mown only once a year and are mostly unfertilised. The pasture and hay are both of low nutritional value and as only large sheep farm buys concentrated feed, the average growth of both sheep and cattle (850 g/day) is low. Hay meadows and pastures also have rather low average harvest of about 1.7 t/ha, or when fertilised, 2.2 t/ha. On the other hand, viticulture farms have relatively better production results with average harvest of 7,000 kg/ha or 4,000 kg/ha for a superior quality wine. While the large cattle, sheep and viticultural farm all enable one person to be employed on the farm with the help of family members, other farms do not provide enough income nor work for a full employment.

Table 1: Characteristics of typical farms

	SUC_0040	***SUC_0010***	***SHP_0240***	***SHP_0045***	***PGL_0005***	***VIT_0001***	***VIT_0005***
Livestock (no. animals)							
Suckler cows	40	10					
Heifers	8	2					
Sheep			240	45			
Plant production – (ha)							
Pasture	69.0	12.6	64.8	4.8			
Hay meadow (1 mow)	46.0	5.12	55.2	3.2	5.0	1.6	
Meadow + autumn pasture		14.7					
Viticulture (ha)							
Vineyard						1.0	2.0
Vineyard – superior quality						0.5	2.5
Utilised agri. area (ha)	115.0	32.0	120.0	8.0	5.0	3.1	4.5
Own permanent grassland	115.0	32.0	3.0	8.0	5.0	1.6	
Leased grassland			117.0				
Own vineyard						1.5	4.5
Own workforce (FTE)	1.5	0.6	1.2	0.3	0.1	0.4	1.2

In table 2 we present main economic results for analysed farms. In all cases, except viticultural farms (in these, according to data from the field, the prices of wines are above average for individual quality of wine), budget payments significantly contribute to a relatively favourable indicator (EGM/FTE). However, especially on small farms, the EGM is too low and does not allow for normal development. Due to extensive production with pasturing that requires less work, the EGM is particularly good per working hour. However, it should be emphasized here that in this case, this is an effective workforce, which is often smaller than the available family labour on the farm. The latter is especially true for smaller farms, which would significantly worsen the indicator in a case that all family labour would be considered. Despite the relatively good EGM, the economic and social sustainability is questionable for farms that do not provide full-time employment, as it is challenging for these farms to cover larger fixed costs, such as new mechanisation or infrastructure. Such farms also require considerable work aside regular jobs, which makes their maintenance less attractive for future generations.

Many farms already meet the criteria for organic production or agri-environmental schemes (AES), but the smaller farms do not always join as the extra revenue does not pay off transaction costs, which are equal regardless the farm size [9]. The results indicate that land abandonment is not primarily driven by low farming incomes, but may be more related to the size of the farms that are too small for providing an employment. Given that previous studies have pointed to poor land accessibility in the area, motivated smaller farms may lack opportunities to expand and make a living from agriculture, instead choosing to abandon farming. Additional policy measures to prevent further land abandonment should therefore focus less on providing additional income support and more on creating a wider environment that supports agricultural and rural development, including land reform, adding value to local products, natural and cultural heritage as well as building infrastructure [10].

Table 2: Economic indicators of typical farms (EGM = expected gross margin, FTE = full time equivalent)

	SUC_0040	***SUC_0010***	***SHP_0240***	***SHP_0045***	***PGL_0005***	***VIT_0001***	***VIT_0005***
Revenues (€)	**80,983**	**23,501**	**66,140**	**10,246**	**2,623**	**20,391**	**65,752**
Market revenues (€)	44,589	11,147	34,170	7,874	1,234	19,899	63,954
Subsidies (€)	36,394	12,353	31,970	2,272	1,389	492	1,798
Variable costs (€)	36,607	9,520	32,607	3,223	859	7,355	23,443
EGM (€)	**44,376**	**13,981**	**33,532**	**7,023**	**1,764**	**13,036**	**42,309**
EGM in €/hour	15.94	13.32	15.00	11.23	16.96	17.85	19.94
Indicators on FTE							
Revenue €/FTE	52,374	40,316	53,253	29,485	45,373	50,252	55,782
EGM €/FTE	28,699	23,984	26,999	20,209	30,521	32,127	35,894

4 CONCLUSIONS

The modelling approach based on the integration of three independent models (FM, MC, and TAH), proved to be effective in the case of the analysis. The optimization potential of LP allowed us to balance the material flows at the level of each production plan. The production plan is thus technologically appropriate and enables the simulation of economic and technical indicators in the given extensive conditions for each farm type. The economic indicators point to the lack of socio-economic sustainability of small farms, calling for policies that enable farm enlargement and better commercialisation of local products.

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MEASURING THE EFFICIENCY OF SPAIN'S WINERIES THROUGH DATA ENVELOPMENT ANALYSIS

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Abstract: The research evaluates the efficiency of wineries in Spain using the DEA (Data Envelopment Analysis) model. The study utilizes the CCR (Charnes-Cooper-Rhodes) model within DEA, which assumes constant returns to scale. Input and output data from 848 wineries were collected and normalized for analysis. The results indicate that only one winery operated at maximum efficiency, achieving the best possible results compared to others. The findings highlight the importance of efficient processes, resource allocation, and management practices in achieving optimal performance. Further research could explore efficiency comparisons among wineries with different activities, such as diversification into tourism.

Keywords: efficiency measurement, DEA model, Spain's wineries, agrarian economics

1 INTRODUCTION

Performance efficiency is essential for organizations because it allows them to maximize resource usage [7], cut expenses [11], boost production [4], acquire a competitive edge [3], boost customer happiness [2], encourage sustainability [6], and realize total organizational effectiveness. In today's dynamic and competitive business world, it is essential to success and long-term sustainability. Many methods exist (qualitative and quantitative) for efficiency evaluation across various fields. Among the typical forms of qualitative efficiency evaluation methods are classified metrics of KPIs (Key Performance Indicators), different Benchmarking analyses, the framework of a Balanced Scorecard, etc.

The Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) methods represent typical quantitative methods for measuring the effectiveness of organizations. SFA is a parametric performance evaluation method that considers technical inefficiency and random error. It determines the inefficiency of decision-making units concerning an estimated production frontier. The methodology was developed by economists Arie Kapteyn and Jan van den Berg in the late 1970s. A non-parametric technique for evaluating the relative effectiveness of decision-making units is called DEA. It considers the capacity of various departments to transform inputs into outputs to compare their efficiency. DEA was developed by Abraham Charnes, William W. Cooper, and Edwardo Rhodes in the late 1970s [1]. In a time when agricultural and other economic sectors are mainly focused on reducing negative environmental impacts and increasing positive effects on people and capital, there is an even greater need for studying efficiency [5], [8], [9].

Sistema de Análisis de Balances Ibéricos, commonly known as SABI, a financial database and analysis system that provides comprehensive financial information and analysis of companies in Spain and Portugal, was the source of data for our research on measuring the efficiency of wineries in Spain [10]. Through the database, we have accessed 2.880 wineries in Spain that have their wine cellars in all the country's wine regions on that part of the Iberian Peninsula.

After the introductory part, where we present the importance of efficiency research, there follows a chapter on methodology, where we present the development of the used DEA model. The results section follows. The article concludes with a summary of the main findings, advantages, limitations, possibilities of application and direction for further research.

2 DATA COLLECTION AND METHODOLOGY

After collecting and editing the data, the DEA model was created: first, we defined the objective function and the constraints that encompass the efficiency evaluation process. This was followed by preparing data for analysis by normalizing inputs and outputs. This step ensures that the variables are comparable and that the DEA model can accurately estimate efficiency.

In the case of evaluating the efficiency of Spanish wineries, we focused on data from 2019. We could not access the latest data after the coronavirus period; for this, we used a database from the period before the outbreak of the epidemic, as the situation during the epidemic was quite different from the more normal one. We defined input and output data for the model. For the input data, we considered the number of employees in the winery in 2019, while the output data were 1) business revenue in 2019 and 2) profit per employee in 2019.

Among all 2.880 wineries, we eliminated all those for which we could not obtain all the data representing our inputs and outputs. Furthermore, we also cancelled those with a negative profit in 2019 (negative profit is understood as management inefficiency). This left us with 848 companies (wineries) in the set for further analysis, based on which we checked the mentioned efficiency.

The DEA model was developed using the MaxDEA 7 tool, Beijing Realworld Software Company Ltd. Within the DEA model methodology, the CCR model was used. The CCR model assumes constant returns to scale (CRS), assuming the DMUs operate at an optimal scale and any input/output quantities changes do not affect their efficiency. The model seeks to maximize the efficiency scores of the DMUs while ensuring that the weighted sum of their outputs does not exceed the sum of their inputs [1]. Mathematically, the CCR model can be formulated as a linear programming problem.

The formula for evaluating efficiency in the CCR model is expressed:

$$Max \frac{\sum_{r=1}^{n}(u_{rb})(y_{rb})}{\sum_{k=1}^{m}(v_{kb})(x_{kb})} \tag{1}$$

whereby the following conditions must be observed:

$$\frac{\sum_{r=1}^{n}(u_{rb})(y_{rj})}{\sum_{k=1}^{m}(v_{kb})(x_{kj})} \leq 1 \text{ for each unit } j$$

$$u_{rb}, v_{kb} \geq \varepsilon \text{ for each unit } r, k \tag{2}$$

y_{rj} = output vector *r* built with unit *j*
x_{kj} = input vector *k* built with unit *j*
u_r = output weight *r* on basic unit *b*
v_i = input weight I on basic unit b
j = number of DMU
r = number of outputs
k = number of inputs
ε = small positive number

3 RESULTS

3.1 CCR-I model

The efficiency score generated by the CCR model for each DMU represents the ratio of its weighted sum of outputs to the weighted sum of inputs (Charnes et al., 1978). An efficiency score of 1 indicates that the DMU is operating efficiently, utilizing its inputs effectively to generate results. Scores below 1 indicate relative inefficiency, with room for improvement. According to Charnes et al., (1978) when input and output-oriented technological efficiencies are equivalent, the technology demonstrates consistent returns to scale (CRS). The technology is inefficient if variable returns to scale (VRS) characterizes the technology, and this equality does not hold for each group of inputs and outputs.

Table 1: Results of input and output-oriented CCR model.

alternative (Winery)	***CCR – I***
DMU 1	0,137348
DMU 2	0,770799
DMU 3	1
DMU 4	0,337550
DMU 5	0,146867
DMU 6	0,108621
DMU 7 (average value)	0,009035
DMU 8 (lowest value)	0,000002

The results of the CCR-I model indicate that a winery is efficient when it is impossible to increase outputs without reducing residual outcomes or increasing any input. From the set of all companies analyzed, the DEA analysis of the CCR model showed total operational efficiency in only one winery. Table 1 shows a collection of the six most efficient (out of 848) wineries we got during the analysis (DMU 1 – DMU 6). The average score of 0,770799 (DMU 2) shows that the analyzed winery could operate at 77 % of the current output level with unchanged input quantities. Winery could increase its output by 23 % with unchanged inputs.

4 CONCLUSION

The DEA (Data Envelopment Analysis) performance measurement model is designed to assess the relative performance of decision units (DMUs) in a data set. The results of our analysis show that a particular DMU operates with maximum efficiency within a given input-output framework. In other words, it achieves the best possible results relative to the other units in the analysis. It is possible for a winery to implement effective processes, resource allocation strategies, and management practices. However, the effects of their technologies, resources, market position, and other expertise they manage also probably play an essential role. When interpreting the results, we should also consider that we used one input and two output data, which is a weakness for this type of analysis. For more accurate calculations, it would be necessary to check other DEA analysis models and supplement them with the AHP method. In the future, we could compare the effectiveness of DEA between wineries that only grow and sell grapes or wines compared to those that diversify their activity with other complementary activities (for example, tourism).

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PRICE VOLATILITY AND ITS IMPACT ON FARM OPERATIONS, AN EXAMPLE OF ANALYSIS WITH A FARM MODEL

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Abstract: Inflation, conflicts around the world have huge impacts on prices of agricultural inputs and outputs. The paper addresses preliminary analysis of economic indicators on typical agricultural holding with arable production in Slovenia for the period 2018-2022. For this purpose farm model was applied. Results show on high volatility in economic indicators through the years, especially in the years 2021 and 2022. Value of coefficient of variation for expected gross margin (0.61) confirms unstable economic conditions. This shows the high risk of arable farming, despite the significant rise of prices of agricultural products. Thus, it will be very important for farms that want to increase their production to include these price risks in their managment.

Keywords: farm model, linear programming, economic indicators, agricultural inputs, typical farms, price volatility

1 INTRODUCTION

In the wake of energy crises and climate change, supply chains for agricultural inputs are becoming more volatile. The war in Ukraine has led to a global increase in gas prices and a decline in fertilizer supplies. This aggression has exacerbated energy and agricultural resource crises in global supply chains and prices for natural gas, oil, and coal have risen rapidly [4]. Energy, especially oil and petroleum derivatives, is considered as an important factor of production in economy. They are widely used to supply various sectors such as transport and agriculture [8]. Prices for commodities such as grains and vegetable oils have reached record highs since 2021, even surpassing the levels of the global food price crises of more than a decade ago. Now, the war in Ukraine has driven prices even higher [3]. Economically viable farm households contribute to the resilience of agricultural and food systems and are a prerequisite for the provision of food, agricultural landscape services and environmental goods [6]. The current practice of sustainability assessment at the farm level considers quantitative data and financial ratios [9].

Agricultural models are an appropriate tool to assess the economic indicators at the farm level and to monitor development trends in individual segments of the sector as well as the impact of different policies at farm level [7]. Farm-level modelling enables simultaneous consideration of production, price and policy information [5].

2 FARM MODEL

For our research, Farm model [10] based on model calculations (MC) prepared by Agricultural institute of Slovenia was used [1]. The model is based on mathematical programming and has independent modules for individual phases of analysis. Together, it enables a comprehensive analysis of the farm production plan. The majority of the operations are automated and performed by Visual Basic (VBA) macros, which allows relatively simple microeconomic analyses and the adaptation of MC to the production conditions of the analysed farms. MC are independent simulation models for agricultural activities that allow estimation of input consumption and, consequently, production costs for each agricultural product based on the specified (selected) initial technological parameters.

In the version of the farm model that was used for this study, the model is based on linear programming (LP) (1) to (3). This is a single-objective approach in which the decision maker is assumed to make decisions primarily based on a single objective in the face of changing circumstances when determining a production plan, considering all production constraints at the farm level. The matrix of production possibilities thus represents an example of the determination of an (optimal) production plan, where we focus on maximizing the objective function (1). In our case, this was the expected gross margin, which we assumed to be one of the most important objectives influencing decision making, along with the characteristics (type) of the farm.

The characteristic of LP is that the solution can change rapidly during optimization. In the case of price changes, it can be even more drastic. Therefore, sepecial attention has been put to crop rotation and limmited possible changes of production plan.

$$\mathrm{maxEGM} = \sum_{j=1}^{n} c_j x_j \tag{1}$$

$$\sum_{j=1;f=1}^{n;r} a_{ij} x_j \leq b_i \qquad \text{for all } i = 1 \text{ to } m \tag{2}$$

$$x_{j \geq 0} \qquad \text{for all } j \tag{3}$$

The idea is to optimize production plan, calculating values of production activities - variables (x_j) using a LP, maximizing the expected gross margin (maxEGM). In our example, the coefficients of the objective function (c_j) represent the expected gross margin (EGM) for each production activity in the model. It is calculated as six months average of variable costs and revenues for each production activity, with the same probability for each period within 2018 and 2022.

2.1 Typical arable farm cultivating 70 ha of arable land

The main objective of this study is to illustrate how changes in input and output prices (price volatility) affect farm economic outcomes and how such an exercise could be modelled with farm model. For the purposes of this analysis, we choose ten consecutive six months periods in last five years. Prices of agricultural inputs are monitored and updated regularly on monthly bases, and are recalculated for this timeframe [1]. Research with the farm model was conducted on a typical agricultural holding with arable farming (TAH-A), which is specialized in crop production [10]. Main production activities on analysed farm are production of cash crops as barley, corn, wheat, soybeans, rapeseed and buckwheat (Table 1).

In Table 1 we can see the abstract of production plan for analysed farm, developed in another study [10]. It cultivates 70 ha of arable land and is well equipped with machinery. The workload on such a farm accounts for 0.5 full-time equivalent (FTE). It is representative of the semi-professional medium-sized farms in Slovenia, of which there are many in Slovenia and in which we are interested in. Namely, it is certainly representative for a group of farms that will in the future try to increase the volume of production (ha) in order to maintain its existence.In this respect, these economic fluctuations are even more important, as it is a significant price risk. After all, most of these farms will increase their production on account of the leased areas and risk assessment from this point of view is important for successful business.

Table 1: Production plan for analysed typical arable farm

Production activities	*Unit*	Typical arable farm *Scope (unit)*	*Yield (kg/ha)*	*$GM_{2018-2020}$ (EUR)*
Barley	(ha)	14.0	6000	322.6
Wheat	(ha)	17.5	6500	444.0
Buckwheat	(ha)	2.8	1000	101.5
Corn	(ha)	28	11000	129.3
Soybeans	(ha)	3.5	3500	716.6
Rapeseed	(ha)	7.0	3400	397.1
Set-aside	(ha)	3.5		
Arable land on farm	(ha)	70.0		
Labour on farm	(FTE)	0,5		

Further we present the economic results for the analysed periods and deviations from the average three-year situation (Baseline). With this, we want to show how important the real-time recalculation of economic indicators is and how the events of this five-year period significantly affect the efficiency of production.

Table 2: Economic indicators for typical 70 ha arable farm

Economic indicators (EUR)	***Typical 70 ha arable farm in Slovenia***					
	EUR					
	Baseline	***% of baseline***				
	2018-2020	***118-618***	***718-1218***	***119-619***	***719-1219***	***120-620***
Total revenue	104905	103.7	103.6	95.8	95.8	96.9
Total variable costs	84064	96.1	97.3	97.4	96.8	99.1
EGM	20842	134.5	129.0	89.4	91.7	87.9
EGM/h	20.8	136.6	131.2	90.5	93.0	88.0
SD of EGM	12715					
		720-1220	***121-621***	***721-1221***	***122-622***	***722-1222***
Total revenue		96.1	132.1	132.1	160.0	160.0
Total variable costs		94.2	96.8	112.8	155.4	166.8
EGM		103.4	274.3	210.1	178.6	132.5
EGM/h		105.4	278.9	213.6	179.8	133.6

Farm generally achieves good economic results for the agricultural sector. With an hourly rate of 20.8 EUR/h, it achieves average results regarding arable production. In Table 2 we show economic indicators for two six month averages for each year (January-June, July-December). Each period is subject of optimization while the production plan can only be slightly changed within the existing constraints (crop rotation) of the farm. The results show high volatility in total revenue, variable costs and gross margin, especially in years 2021 and 2022. It should be noted that the crop yield in the model is unchanged in these years. In practice, we know that crop production depends largely on weather conditions, which have been very variable in recent years as well.

Result show that total revenues, total variable costs and expected gross margin for a typical farm in our research are relatively volatile in a 5 year period. As a Baseline, we considered three years average 2018-2020 [10], and the results per each six month period are expressed as a percentage change of the baseline. It can be seen that farm's revenues decreased by up to 4 % in 2019 and 2020. However, in 2021, revenues increased up to 32 % and 2022 was an extreme year with revenues up to 60 %. Assuming, of course, that the drought did not cause major crop losses. In parallel, total variable costs were below baseline until the second half of 2021, when they increased by up to 13% from July to December. In the first six months of 2022, total costs were over 55% and in the last six months even up to 67% higher, compared to the Baseline. All this had a significant impact on the EGM. EGM was highest in first six months of 2021, input price increases have not yet kept up with revenues. The trend of the hourly rate is similar, but the deviations are mainly due to slight changes in production plans between years. Coefficient of variation (CV) in our scenario is 0.61 which is another indicator that economic conditions were not stable. Results are consistent with data in Report on the state of agriculture, food, forestry and fisheries in 2021 which shows that agricultural commodity prices at the aggregate level are volatile and have increased significantly in 2021 [2].

3 CONCLUSIONS

The paper presents a case study of analysing economic indicators of a typical arable farm with a farm model. It is important to observe prices in detail in shorter periods of time, because average values of longer periods can hide many things and can be catastrophical for farmers. We were interested in an example of an analysis of the effects of changes in input and output prices on economic indicators at the farm level. The results show that recent years have been very volatile as far as input and output prices are concerned. Second half of 2022 shows total revenue can increase up to 60 % while total costs have increased up to 67 %. CV of EGM (0.61) confirms unstable conditions and shows high risk of arable farming despite the significant increase in the price of agricultural products. For farms that want to increase their production, it will therefore be very important to include these price risks in their management.

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BUILDING RESILIENT AGRICULTURAL SYSTEMS: A MULTI-STAKEHOLDER APPROACH FOR SUSTAINABLE TRANSFORMATION

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Abstract: The global population growth requires increased food production, but it faces challenges due to climate variability and environmental crises. Innovative approaches are proposed for transforming agricultural systems using regenerative practices and natural capital valuation. The "multi-stakeholder" approach enables informed decision-making and sustainable measure implementation. It incorporates diverse perspectives, expertise, and local knowledge, fostering effective research, policy development, and stakeholder engagement in agriculture and environmental planning. Emphasizing multi-stakeholder collaboration is essential for achieving sustainable agriculture and addressing global challenges.

Keywords: food production, regenerative practices, natural capital valuation, multi-stakeholder collaboration

1 LITERATURE REVIEW

The literature review discusses the impacts of climate change on agro-ecosystems, strategies for achieving sustainability in agriculture, challenges faced by farmers in implementing sustainable agricultural practices, and innovative approaches like regenerative practices and natural capital. Climate change poses a significant challenge to agriculture due to limited resources and extreme weather events (Jia, 2021). Conventional approaches contribute to climate change and negatively affect crop yields, livestock farming, and fisheries (FAO, 2021). Strengthening climate resilience is crucial to address these challenges (Holleman et al., 2020). Sustainable agriculture aims to preserve biodiversity, soil fertility, and mitigate climate change impacts (Slabe et al., 2018). Transforming agricultural systems is essential for efficiency and resource utilization (Borec et al., 2021). Shifting towards sustainable business models is vital for addressing environmental challenges (Barth et al., 2021). Specific on-farm measures contribute to achieving environmental and safety objectives (FAO, 2021). Farmers face economic challenges in implementing sustainable practices due to limited access to data and high costs (Jia, 2021). Implementing organic farming on a large scale may pose challenges (Balázs et al., 2023). Addressing these challenges requires a flexible and dynamic approach known as the agroecological transition (Jia, 2021). Regenerative farming offers a pathway to address climate change and empower farmers (Brown et al., 2022). Organizations embracing regenerative business models prioritize planetary health and societal well-being (Konietzko et al., 2023). Implementing regenerative practices may pose challenges, but financial incentives and considering ecosystem services can facilitate successful implementation (Schreefel et al., 2022; Farrell et al., 2022). Natural capital accounting is crucial for preserving ecosystem services and achieving sustainable goals (EU Commission, 2023).

2 MATERIALS AND METHODS

1.1 Multi-Stakeholder Collaboration

"OPTAIN - OPtimal strategies to retAIN and re-use water and nutrients in small agricultural catchments across different soil-climatic regions in Europe" is an EU-funded research project promoting the adoption of Natural/Small Water Retention Measures (NSWRMs) to address conflicts between agricultural water usage and other water demands. The project involves stakeholders from 14 case studies, scientific modelling, and optimization approaches to enhance understanding and maximize the benefits of NSWRMs. Objectives include identifying effective techniques for water and nutrient retention, optimizing NSWRMs for environmental and economic sustainability, and providing recommendations for sustainable water management in agriculture. Collaboration with local stakeholders aims to enhance adaptation to extreme events and reduce conflicts by implementing various water retention and efficiency measures in agriculture. Workshops are vital in the project, serving as a platform to engage stakeholders and gather necessary information. They aim to determine the best combinations of measures for water and nutrient retention, considering social and environmental goals. Through collaboration among scientific, non-scientific, and local partners, workshops facilitate understanding and co-creation of solutions for challenges related to soil vulnerability and water and nutrient cycles. Multi-Actor Reference Groups (MARGs) are established within each case study to ensure alignment with socio-economic context, legal aspects, and socially acceptable solutions. Workshop objectives include co-creation of knowledge, identification of effective measures, and determination of assessment indicators. Stakeholders from various backgrounds critically assess the evaluated measures, contributing to a more realistic assessment. The workshops promote mutual learning, reflection, dialogue, and shared understanding, aiming to identify solutions and facilitate structured collaboration within the case study. Establishing a collaborative workgroup and implementing a problem-solving process with diverse stakeholders involves several key steps. These include defining research objectives, identifying necessary information and data, and conducting a well-structured introductory workshop. An experienced moderator is crucial for effective group facilitation. Engagement methods such as focus group discussions, surveys/questionnaires, and semi-structured interviews can provide valuable insights. Stakeholder involvement is important throughout all project phases, including problem identification, assessment, scenario development, and solution implementation. The organization of the workgroup and participant selection are vital for successful collaboration. A visual diagram illustrating stakeholder engagement and workgroup establishment can represent the process (Figure 1) (van den Brink et al., 2021).

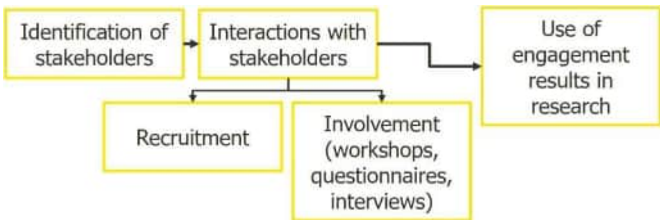

Figure 1 : The process of establishing a multi-stakeholder group and engaging stakeholders for research purposes (OPTAIN, 2021).

Stakeholder selection for workshops followed a flexible and pragmatic approach, considering diverse case study contexts. The establishment of MARGs varied due to differences in engagement norms and existing practices across case studies. Trust, power balance, equity, shared goals, and available resources influence stakeholder engagement and should be considered. Building relationships and trust takes time and continuity. Understanding stakeholder perspectives is crucial for meaningful engagement. MARGs should be adaptable to the local context, and workshop organizers must be aware of these factors to address local concerns effectively. Methods like focus group discussions, surveys/questionnaires, and semi-structured interviews can facilitate stakeholder engagement (OPTAIN, 2023, van den Brink et al., 2021).

3 RESULTS

The results of the OPTAIN project highlight the significance and benefits of a multi-stakeholder approach in agricultural research and decision-making. Key findings include:

- Involving diverse stakeholders is crucial for informed decision-making and the implementation of sustainable measures. Neglecting local stakeholders' opinions can lead to suboptimal outcomes. A holistic view and consideration of environmental impacts are essential when involving all relevant actors.
- Collaboration between researchers and stakeholders in the agricultural sector is proving effective in the OPTAIN project. The participation of interested parties contributes diverse perspectives and insights to research and decision-making processes. This collaborative approach is highly recommended for similar initiatives at different levels.
- The combination, strategic location, and proper implementation design of measures are critical for realizing the benefits of sustainable practices. Public financing and involvement are important for economically viable implementation. Public actors should assess measure suitability and align them with existing agricultural measures and strategic plans.
- The establishment of multidisciplinary reference groups, such as the MARG in OPTAIN, plays a pivotal role. The group allows practical stakeholders to critically evaluate measures, providing real-world data. It includes decision-makers, farmers, and organizations involved in agricultural and environmental management.
- Workshops facilitate knowledge exchange, foster closer connections among sectors, and enhance mutual understanding between researchers and stakeholders. They raise awareness of the impacts of sustainable measures and set the stage for collaborative action.
- Tailored approaches considering local environments and farmers' needs are crucial for optimal results. The multidisciplinary approach helps identify potential conflicts and explore compromises and solutions acceptable to all parties involved.

Continued utilization of the multi-stakeholder approach is strongly recommended in similar projects, policy development, and strategy formulation in agriculture and the environment. This approach promotes sustainable agriculture by incorporating diverse perspectives, expertise, and local knowledge. It ensures the comprehensive consideration of stakeholders' needs and interests, leading to inclusive and equitable decision-making processes. By embracing this approach, we can work towards a more sustainable future for agriculture, the environment, and the well-being of all stakeholders.

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OPTIMISED ROUTING OF THE BLUEBERRY CULTIVATING UNMANNED GROUND VEHICLE

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Abstract: Precision treatment of blueberries has recently gained high importance due to the value of the crop. Such treatment is intended to be performed by unmanned ground vehicles (UGVs). However, the autonomy of UGVs is constrained by technological limitations, requiring their tasks to be executed in optimal routes. This paper presents an ALNS-based approach for the optimisation of UGV routes and its evaluation on problem instances based on an actual blueberry field. We also identify possible alternative solution methodologies that will be evaluated in the future research.

Keywords: Precision agriculture, UGV, blueberries, optimal routing, ALNS, TSP.

1 INTRODUCTION

Blueberries are a high-value crop which requires a careful and dedicated treatment. To address this need, the BioSense Institute is developing an unmanned ground vehicle (UGV) specialized in precision monitoring, soil sampling, and spraying in blueberry plantations. The UGV autonomy is of major importance as it determines the efficiency of UGV deployment. Hence, determining the near-/optimal UGV route for tasks execution, considering the observed blueberry field, its rows and characteristics, the requested task locations within the field, and the starting point of UGV's execution, is essential.

We particularly analyse the problems of precision spraying and soil sampling performed by the UGV. In both cases, the UGV is required to visit certain points in the blueberry field and perform spraying or soil sampling. To reach those points, the UGV can only move between blueberry rows, without crossing them, as shown in

Figure 1. Changing rows is only permitted at the ends of each row. Furthermore, there is a drainage trench, located adjacent to each field row, preventing the UGV from approaching the row from that side. The trench is represented with a zigzag line in
Figure 1. Finally, the UGV's camera for precise positioning is looking on the left-hand side, with respect to the UGV movement direction. The camera must face the targeted blueberry row to ensure accurate UGV positioning. Therefore, we aim to approach each spraying/sampling point (SSP) so that the camera is looking in its direction, thereby avoiding UGV rotation and additional power consumption.

Figure 2 illustrates a desired, optimised UGV path. The green points and purple diamonds denote the allowed and visited projections of SSPs to inter-row corridors. At the former, the UGV approaches the SPP with the camera facing it, while the latter denote locations where the UGV must rotate to achieve precise positioning. The dark green lines denote the travelled path. The black X marks represent the SSPs projections that are not allowed due to the presence of the drainage trench. The red points and dashed lines denote the inter-row corridors that are not utilised.

Since the UGV should visit every SSP in the near-/optimal manner, with respect to the abovementioned physical characteristics of blueberry fields and utilised UGVs, this problem represents a customised version of the travelling salesman problem (TSP). In this paper, we provide the details of the extended problem and our solution approach.

The remaining of the paper is organised as follows. In Section 2, we analyse the related work. Sections 3 shows the details of problem modelling and solution algorithm. Section 4 presents the results and evaluation of our solution approach. Finally, Section 5 gives the conclusions and our intentions for the further research.

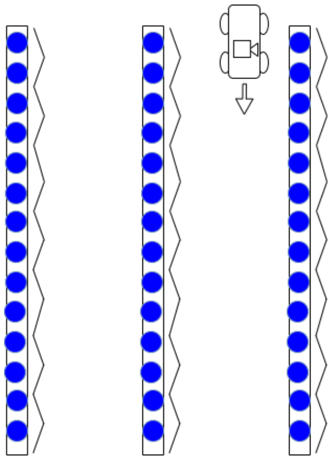

Figure 1: The appropriate UGV movement within the field rows.

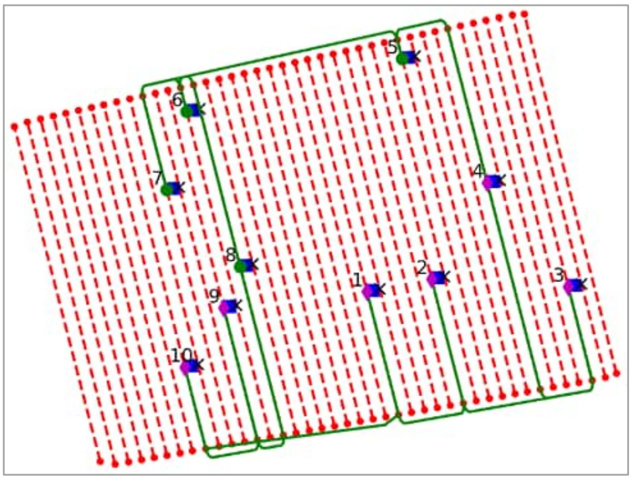

Figure 2: An optimised UGV route.

2 RELATED WORK

One of the most common approaches for TSP solution is the Lin-Kernighan's improvement heuristic, which belongs to the class of local search algorithms [3]. Here, the search starts from an arbitrarily selected tour, and, at each state, the algorithm tries to remove *k* tour arcs and insert *k* new ones to improve the solution cost. Such move is known as the *k-opt* move [3].

In general, metaheuristic algorithms such as simulated annealing (SA, [1]), tabu search [4], and genetic algorithms (GA, [2]) are also commonly employed to solve the TSP. Recently, adaptive large neighbourhood search (ALNS), a metaheuristic algorithm, has been used to solve the TSP with good results [5, 6]. This metaheuristic builds upon the SA framework and utilises a set of heuristic moves, called destroy and repair operators, to generate candidate solutions. The purpose of a destroy operator is to create a significantly different solution and hopefully push the search process into an unexplored part of the search space. Then, the repair operator is responsible for restoring the feasibility of the new solution, if needed. In each iteration, a pair of destroy-repair operators is randomly selected driven by their success rate up to that point, i.e., more successful operators have higher chances of being selected. If the operators create a better solution, their success rate is increased, otherwise, it is penalised.

Based on the benchmark tests, Laporte, Ropke, and Vidal suggest several other solution methods, such as GA, memetic algorithms, parallelised local search algorithms, and hybrid approaches combining GA and neighbourhood search [5]. These methods are located on the Pareto front representing the compromise between the average optimality gap and solution time. Consequently, all of them are considered good candidates for the solution of our problem.

3 SOLUTION METHODOLOGY

To account for the constraints additional to the original TSP, we perform specific data preparation. Each sampling point is projected onto a corresponding point in the inter-row corridor, on the opposite side of the drainage trench. The graph utilised in the TSP solving consists solely of these projection points. With that, we avoid approaching the rows from the side of the trench.

Rather than explicitly implementing the constraint that the precise positioning camera must look towards the sampled row, we solve the relaxed problem. Each rotation incurs a substantial penalising cost, which drives the search algorithm to avoid such situations. The benefit of such a solution approach is allowing the algorithm to investigate states, worse than the current solution, and escape local minima. Finally, the objective function accounts for an additional rotation cost at each location where the camera is faced opposite to the sampling point.

Thus, the mathematical model of our UGV routing problem is defined by extending the objective function of the original TSP model [1] with the cost of UGV rotations. The cost of travelled distance and UGV rotations are both expressed in the amount of utilised energy, in the same units, and combined in a single objective function with a plain sum. The model constraints are the same as in the original TSP model [1]. Due to limitations in paper length, we omit the mathematical formulation of the model, which is available upon request.

One of the simplest solution approaches for the TSP is the greedy algorithm [1]. In our case, the greedy algorithm starts at the SSP closest to the UGV's starting position. At each SSP, it selects the nearest unvisited SSP as the next destination. This process is repeated until all SSPs have been visited. Although it is quite naive, the greedy algorithm is capable of quickly finding the solutions that outperform than the man-made ones. In our solution approach, we utilise the greedy algorithm to generate an initial solution, which will be further improved.

Finally, we apply the heuristic based on the ALNS framework, starting from the initial solution. Following the findings of [3], we combine the pairs of destroy and repair operators into unified operators based on *k-opt* moves. In the current state of our research, we select *2-opt*, *3-opt*, and *4-opt* operators, while the *larger-k-opt* operators induce either a longer execution time without yielding better solutions, or even the algorithm being trapped in solution spaces distant from the best-found solution. We have also tested shuffle operators, which randomly permute subtours. However, these did not lead to any result improvements.

4 EXPERIMENT RESULTS

The ALNS parameter tuning included selecting the initial and final probabilities for accepting a worse solution, the strategy for temperature drop, the number of temperature drops, and the number of intra-temperature iterations. Figure 3 illustrates the achieved algorithm behaviour, demonstrating desired exploration and diversification at the beginning, followed by intensified local search in the most promising search region. The green line denotes the current solution, while the blue line represents the overall best-found solution.

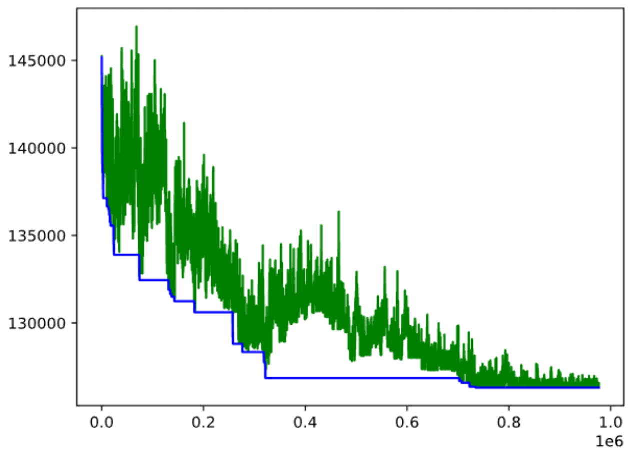

Figure 3: The desired algorithm execution. The x-axis denotes the iteration number, while the y-axis represents the objective function value.

For the algorithm evaluation, we have solved the three problem instances based on the model of a real blueberry field located in Babe, Serbia. To thoroughly assess algorithm's performance, the tested instances contain 10, 50, and 100 randomly generated SSPs. They are denoted UGV10, UGV50, and UGV100, respectively. Each problem instance is solved 50 times with randomly selected starting UGV locations around the field. Table 1 shows the statistics regarding the cost improvement and execution time for each instance. The tests were conducted on Intel(R) Core(TM) i5-9400 CPU @ 2.90GHz with 16 GB RAM.

Table 1. Developed algorithm solution statistics, i.e., average, minimal, and maximal solution improvement and time are shown for each problem instance.

Instance	***Average impr. [%]***	***Max. impr. [%]***	***Min. impr. [%]***	***Avg. sol. time [s]***	***Max. sol. time [s]***	***Min. sol. time [s]***
UGV10	16.47	22.58	12.52	81.43	90.91	79.67
UGV50	12.47	18.63	4.79	395.39	405.53	387.67
UGV100	7.76	11.22	4.26	759.68	776.95	749.06

5 CONCLUSIONS AND FUTURE RESEARCH STEPS

The evaluation of the current algorithm has demonstrated its suitability for the intended purpose of UGV routing optimisation. With the smaller number of SSPs, the cost improvement is significant while the running time is satisfactory. Moreover, the experiment results indicate that the running time increases linearly with the problem size, which is the same time complexity as obtained by the *k-opt* implementation by Helsgaun [3]. Furthermore, the current algorithm implementation uses only one CPU thread, which leaves space for further performance improvement.

In our future research, we intend to estimate the optimality gaps obtained by the current solution algorithm. Additionally, we aim to develop algorithms based on GA and hybrid approaches, as suggested in [5], and compare their performance with the presented algorithm in the observed case of UGV routing optimisation.

Acknowledgement

This work has been carried out in the scope of the H2020 FlexiGroBots project, which has been funded by the European Commission in the scope of its H2020 programme (contract number 101017111, https://flexigrobots-h2020.eu/).

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TYPICAL FAMILY DAIRY FARMS IN THE REPUBLIC OF CROATIA

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Abstract: The milk processing sector in the Republic of Croatia faces many challenges. To this end, we are developing a modelling approach that will enable analysis at different levels of the sector. In this paper, we present results of defining typical dairy farms using cluster analysis. Cluster analysis was performed on real data obtained from the Croatian Agency for Agriculture and Food. Hierarchical clustering and non-hierarchical clustering were performed using IBM SPSS Statistics. Two options are presented. The goal is to find the most acceptable typical dairy farms that will be used to create farm models based on the mathematical programming approach.

Keywords: cluster analysis, typical farms, dairy sector, mathematical programming, farm model

1 INTRODUCTION

Over the years, there has been a decline in the number of farms, livestock, and the amount of milk production. Small farms with a few animals are either closed or transferred to arable production [5]. As the situation in the milk production sector in the Republic of Croatia is far from good, and the measures implemented obviously do not lead to improvement, a more detailed analysis of the sector is needed. Therefore, the development of appropriate models is a logical follow-up to make better decisions. For decision-makers to have a better insight into what is happening on certain types of agricultural holdings and for their decisions to be based on facts, different models are needed [3].

The strategic plans of the EU member states place increasing emphasis on the use of models that enable simulation at the farm level or at the level of the selected aggregate [4]. They suggest that each member state should choose its own agricultural policy priorities and, in accordance with common EU-principles, determine the type, allocated funds and scope of individual measures. Since the effects of policies differ by farm type, the farm model becomes the main tool to generate scenarios or 'what-if' analyses and simulate how a particular scenario, for example a change in agricultural resource prices, agricultural or environmental policy, may affect a set of performance indicators [2]. In the farm models, mathematical programming is most often applied, as well as models based on econometric and simulation approaches [6]. As it is impossible to carry out the analysis at the level of every agricultural farm, it is carried out at the level of typical representatives. Agricultural farms are classified into smaller groups with common characteristics, which are called typical agricultural farms [1]. These are generally real or hypothetical farms that best represent the situation in a certain segment of a particular sector (representative households) and allow generalization at the aggregate level.

Before the actual creation and application of the farm model in the dairy sector in the Republic of Croatia, it is necessary to create typical farms focusing on dairy production, i.e., farms should be grouped according to main common characteristics. Therefore, the main purpose of this work is to obtain typical farms by applying cluster analysis based on available data following the example of similar studies. It does not require any assumptions about the number and structure of the groups into which the data will be distributed, but the grouping is done on the basis of similarity between the data of farms. The aim is to find the most acceptable solution that will help us to define typical farms and develop a farm model for each cluster.

2 METHODS

Cluster analysis was performed on real data obtained from the Croatian Agency for Agriculture and Food. The database consisted of 4.198 dairy farms that supply milk in the Republic of Croatia. After arranging the obtained database (connecting data from different farm databases, removing duplicate and inactive farms, etc.) there were 3.398 farms left for the analysis. Since in this analysis we focus on family farms, 67 of the farms have been excluded from the sample, since they have the status of a legal entity and form a special category. A separate cluster analysis will be done on them. Therefore, the final number of farms analysed is 3.331. These are family farms that delivered milk and as such were included in the resulting register (Table 1). IBM SPSS, Statistics V22.0 software package was used for statistics data processing and analysis. The variables and their descriptive statistics are presented in Table 1.

Table 1: Descriptive statistics for 3.331 dairy farms

Variable Name	**Variable**	**Mean**	**SD**	**Min**	**Max**
NOC	Number of cows	14,78	20,47	1,00	456,00
ADOM	Annual delivery of milk (kg)	73 257,69	146 015,79	21,00	2 799 071,00
NOPC	Number of plant cultures	6,29	2,27	1,00	18,00
AUC	Area under culture (ha)	23,68	31,68	0,15	469,05

The cluster analysis was first carried out in relation to all the mentioned variables. First, hierarchical (agglomerative) clustering was performed using Ward's method. Then non-hierarchical clustering, i.e., k-means algorithm, was performed [7]. Given that all variables are numerical the squared Euclidean distance was chosen as the distance measure. All algorithms were applied to standardized data. Among numerous solutions, two solutions were chosen as the final.

Cluster analysis solutions are not unique and depend on the application of different elements of the analytical procedure (e.g. hierarchical or non-hierarchical method, different algorithms of the same method). The solution also depends on the variables that were used as a basis for measuring similarity, so one should be careful about the impact of each decision when choosing variables.

3 RESULTS

The resulting clusters of farms will be the starting point in further analysis to define typical farms (production parameters, technologies etc.), which will be modeled with the farm model. Two options of clusters are presented in this paper. The first one describes the clusters in which all the mentioned variables are included (Table 1). In the second, only 2 variables were included in the analysis (NOC and ADOM) that are more significant for this analysis.

3.1 Case 1

The next step is to determine the optimal number of clusters using the dendrogram (see Fig. 1). The dendrogram suggests 10 clusters. As the number of clusters increases, the heterogeneity within clusters decreases. After the hierarchical one, a non-hierarchical k-means algorithm was applied to the data limited to a maximum of 10 clusters. Table 2 shows the structure of all clusters after the implementation of k-means.

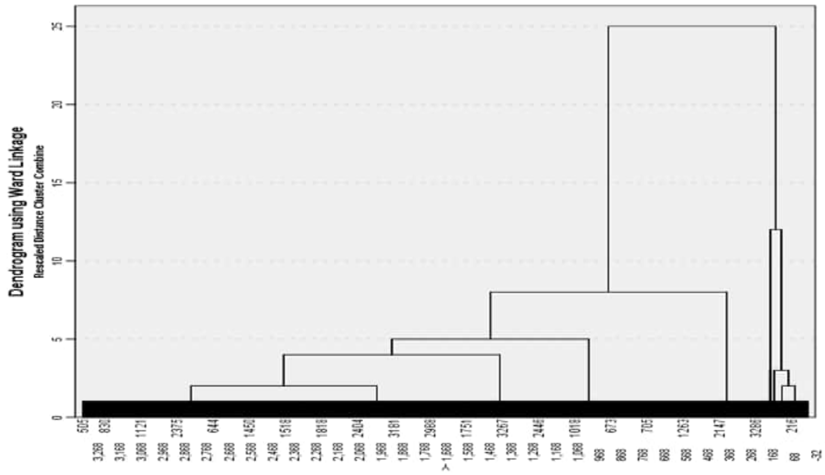

Figure 1: Dendrogram – Case 1

In this case 45% farms belong to cluster 1. The average number of cows in this cluster (7,70) is smaller than the average number of cows in the Republic of Croatia (14,78), the average annual milk delivery (30.107,85 kg) is lower than the national average of 73.275,69 kg, the average area of land per farm (10.67 ha) is also smaller than the national average. This implies that cluster 1 consists of very small farms. It can be seen that cluster 2 is also relatively similar to cluster 1, but farms in cluster 2 have much more land and plant cultures than farms in cluster 1. Namely, there are many farms with a few cows and a lot of lands, and these are not farms whose primary activity is milk production. For the stated reason, the variables AUC and NOPC are excluded from the analysis. The two largest farms are in special clusters (cluster 9 and 10).

Table 2: Cluster structure – Case 1

Clusters	***Variables***								
	Number of farms	***Average NOC***	***Min NOC***	***Max NOC***	***Average ADOM***	***Average NOPC***	***Average AUC***	***Yield per cow***	***Land area per cow***
1	1.496	7,70	1	40	30 107,85	4,56	10,67	3 973,20	2,09
2	873	8,41	1	25	33 634,14	7,84	15,13	4 020,36	2,70
3	310	17,85	1	53	77 299,30	10,24	44,31	4 409,71	3,57
4	464	24,99	6	56	122 814,55	5,96	34,39	5 096,74	1,69
5	25	46,52	8	111	216 429,72	8,80	220,43	4 831,20	8,04
6	124	53,60	23	124	342 978,66	7,10	70,73	6 569,65	1,45
7	27	98,26	64	157	738 311,22	6,78	110,40	7 793,64	1,19
8	10	201,50	153	238	1 543 495,06	6,80	192,35	7 671,41	0,95
9	1	317,00	317	317	2 799 071,00	8,00	469,05	8 829,88	1,48
10	1	456,00	456	456	2 560 156,00	6,00	370,27	5 614,38	0,81
	3 331	**14,78**	**1**	**456**	**73 257,69**	**6,29**	**23,68**	**4 329.80**	**2,34**

3.2 Case 2

Therefore, the variables that remained in the analysis are NOC and ADOM. A dendrogram was made using the same procedure as in Case 1, and the k-means algorithm was implemented. Table 3 shows the structure of all clusters after the implementation of k-means. Although the dendrogram suggested 7 clusters, an analysis was done with 10 clusters for the sake of comparison with Case 1. The obtained analysis (see Tab. 3) shows that the variable NOC for obtained clusters has a smaller range of disjoint intervals. The same holds for the ADOM variable since it is a correlation variable.

Table 3: Cluster structure – Case 2

Clusters	Variables								
	Number of farms	***Average NOC***	***Min NOC***	***Max NOC***	***Average ADOM***	***Average NOPC***	***Average AUC***	***Yield per cow***	***Land area per cow***
1	1.889	5,70	1	15	20 374,81	5,95	12,98	3 801,96	2,96
2	994	16,77	7	32	73 871,13	6,68	27,01	4 640,77	1,64
3	304	33,27	16	59	168 547,71	6,82	43,52	5 328,34	1,31
4	101	58,07	30	111	373 897,32	7,10	74,78	6 719,71	1,30
5	30	96,50	56	157	699 186,73	6,87	130,98	7 645,72	1,42
6	6	182,17	153	204	1 275.357,46	7,17	136,78	7 197,46	0,73
7	3	204,67	200	214	1 636 569,67	7,00	247,45	7 999,35	1,22
8	2	230,50	223	238	2 103 315,93	4,50	204,51	9 123,60	0,88
9	1	317,00	317	317	2 799 071,00	8,00	469,05	8 829,88	1,48
10	1	456,00	456	456	2 560 156,00	6,00	370,27	5 614,38	0,81
	3 331	**14,78**	**1**	**456**	**73 257,69**	**6,29**	**23,68**	**4 329.80**	**2,34**

4 CONCLUSIONS

By applying cluster analysis to the data of dairy farms in Croatia, several cases of relatively homogeneous clusters were obtained. These will be typical farms that will be described/defined in more detail at workshops with consultants and experts in the field and will be further adjusted and upgraded with the Slovenian farm model [6]. This model will help to evaluate various economic, technological, and environmental indicators with predetermined constraints. Typical farms have different levels of economic efficiency. E.g certain policy measure that would increase the profitability of one typical farm, does not mean that it would increase the profitability of another farm. So, the same measure is not equally effective for every typical farm. Such results could be useful to policy makers to get information on which dairy farm needs which type of measure and how much they can adapt to the given situation. Such more effective policy measures should encourage more economical farming.

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MULTI-CRITERIA MODEL FOR ASSESSMENT OF SPA SERVICE QUALITY

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Abstract: Supply quality of tourism services has been subject of numerous studies. In this paper we propose a methodology for assessment of supply quality of spas. It is based on combination of Likert scale questionnaires and multi criteria method DEXi. DEXi utilizes a multi-criteria decision analysis (MCDA) approach, which means it considers multiple criteria or factors when evaluating alternatives, qualitative criteria values and utility functions in the form of "if…then" decision rules. In this case we use a Likert scale questionnaire data as input data in the multi criteria model.

Keywords: supply quality, spa, DEXi, Likert scale

1 INTRODUCTION AND METHODOLOGY

The use of multiple criteria analysis (MCDA) to assess the quality of the SPA supply can address the issue of how the use of several criteria affects the evaluation of alternatives. In assessing predetermined criteria, it is feasible to employ both qualitative and quantitative values [11] Rozman et. Al 2017. The most appropriate method is qualitative evaluation of the criteria within the multi-criteria problem due to the particularities of service activities, including tourism. Determining the current state of the SPA supply quality is therefore a logical application of the DEX approach proposed by Bohanec et al. [2].

Model manipulation is possible using the decision-based approach using statistical tools, simulation, and optimization [1]. The following steps are taken when using these models to assessment problems: presenting the issues using hierarchy and decision rules; assessing the criteria and options when making decisions; examining the data generated from the model by sensing the analysis; or discussing the outcomes. The decision support concept is widely used in the tourism industry. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Decision EXpert (DEX), Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), and other more attribute-based approaches are utilized in this context. In tourism management, the application of several criterion methodologies is widely acknowledged.

Positive consumer feelings are significantly influenced by service quality, a sort of cognitive appraisal [3]. SERVQUAL, created by [2], has frequently been used to gauge how well customers feel their services are being received in various service contexts. However, SERVQUAL required to be tailored to the particular service situation, according to [4]. To develop a suitable instrument for measuring spa service quality, sufficient empirical data on the dimensions and features of spa services is still required.

Delivering treatments that elicit good feelings is crucial for spas because doing so increases client satisfaction [7] and repeat business [10]. The majority of spa research [5-6], [8], [12], etc. concentrate on consumers' motivation, expectations of service quality, satisfaction, preferences, and intentions to return.

2 DATA COLLECTION

The data fort attribute is transformed directly from Likert scale questionnaire. The basic purpose of the primary research was to find out how foreign tourists or visitors are satisfied with the tourist offer in northeastern Slovenia. The research was conducted using the survey method, whereby the questionnaire consisted of 13 questions, of which four were demographic questions.

In the first part of the questionnaire, it was determined on what basis the tourists decided to visit and what their reasons were, where they stayed and how they evaluated the accommodation. Their satisfaction with the services in the complex itself, how many times they had already been to SPA in northeastern Slovenia and how they traveled to the destination were also recorded. We were also interested in whether you would recommend visiting SPA in northeastern Slovenia to they friends or acquaintances, and what they liked most about the destination and what they missed. The second part of the questionnaire contained demographic data, namely about the gender, nationality, age and level of education of the respondent. The data obtained in this way were subsequently used to prepare the DEX model. The rounded average ordinal value from Likert scale was used as direct input into the DEXi model.

3 MULTI CRITERIA MODEL

We developed a multi criteria model based on DEXi method. DEXi (Decision EXpert for Industrial systems) is a software tool designed to provide decision support in complex problem-solving situations. It is a popular decision support system that helps users make informed decisions by organizing and analyzing information in a structured manner. DEXi utilizes a multi-criteria decision analysis (MCDA) approach, which means it considers multiple criteria or factors when evaluating alternatives. It allows decision-makers to define and weight various criteria based on their importance and then assess different alternatives against these criteria. The detailed description of DEXi in the context of measuring suppy quality in tourism is provided by Rozman et al. (2009). We used following hierarchy and scales for the spa service quality assessment (figure 1).

DEXi The multicriteria model for spa suppy quality.dxi 13. 06. 2023 Page 1

Scales

Attribute	Scale
The multicriteria model for spa suppy quality	**very poor**; poor; average; good; ***very good***
Services	**very poor**; poor; average; good; ***very good***
Offer of sport services	**very poor**; poor; average; good; ***very good***
Offer of wellness services	**very poor**; poor; average; good; ***very good***
Offer of various experiences	**very poor**; poor; average; good; ***very good***
Catering quality	**very poor**; poor; average; good; ***very good***
Sence of security	**very poor**; poor; average; good; ***very good***
General criteria	**very poor**; poor; average; good; ***very good***
»Value for money« ratio	**very poor**; poor; average; good; ***very good***
Access to tourist information	**very poor**; poor; average; good; ***very good***
Hospitality	**very poor**; poor; average; good; ***very good***
Environment and accomodation	**very poor**; poor; average; good; ***very good***
Tidiness of the surroundings	**very poor**; poor; average; good; ***very good***
Tidiness of hotel rooms	**very poor**; poor; average; good; ***very good***

Figure 1: Hierarchy and scales of the spa service quality assessment

Utility functions were determined with the weights due to larger number of possible decision rules combination. Initially we assumed equal importance of attributes (figure 2).

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Average weights

Attribute	Local	Global	Loc.norm.	Glob.norm.
The multicriteria model for spa suppy quality				
Services	25	25	25	25
Offer of sport services	25	6	25	6
Offer of wellness services	25	6	25	6
Offer of various experiences	25	6	25	6
Catering quality	25	6	25	6
Sence of security	25	25	25	25
General criteria	25	25	25	25
»Value for money« ratio	33	8	33	8
Access to tourist information	33	8	33	8
Hospitality	33	8	33	8
Environment and accomodation	25	25	25	25
Tidiness of the surroundings	50	13	50	13
Tidiness of hotel rooms	50	12	50	12

Figure 2: Utility functions

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Evaluation results

Attribute	SPA IN NE SLOVENIA
The multicriteria model for spa suppy quality	good
Services	good
Offer of sport services	good
Offer of wellness services	***very good***
Offer of various experiences	good
Catering quality	good
Sence of security	***very good***
General criteria	good
»Value for money« ratio	good
Access to tourist information	good
Hospitality	***very good***
Environment and accomodation	good
Tidiness of the surroundings	good
Tidiness of hotel rooms	good

Figure 3: Evaluation results

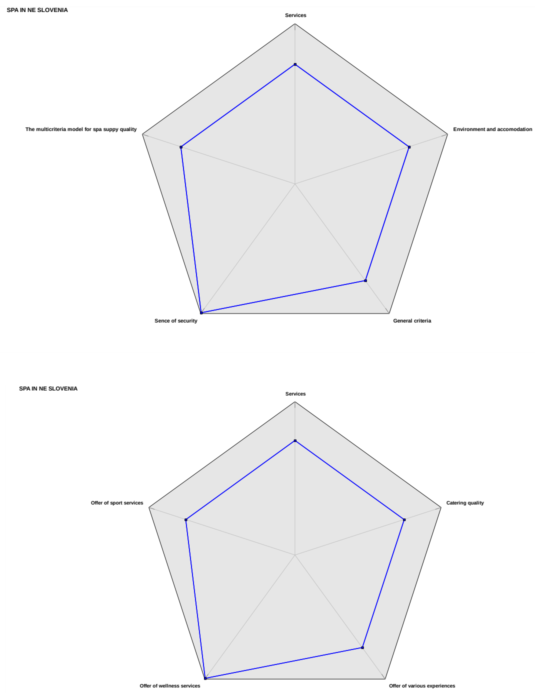

Figure 4: Assessment of selected SPA

Figure 3 and 4 shows a sample (case study) assessment of SPA in northeastern Slovenia

4 CONCLUSION

As it can be gleaned from figures 3 and 4 the DEXi provides qualitative assessment of spa service quality. The radar charts can be used for identification of strong and weak points through the entire hierarchy and in this way helps the analyst/decision maker to propose improvements based on assessment DEXi model provides a structured framework for analysis, particularly in complex situations where there are multiple criteria to consider as in the case of service quality. The DATA Likert scale questionnaire can be directly transformed for the use DEXi model. Although this model is in its initial phase of development we can recognize its potential in the analysis of service quality in spa management.

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THE USE OF OPERATION RESEARCH METHODS TO SUPPORT AGRICULTURAL POLICY

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Abstract: In this paper, we present an example of a modelling approach used to support agricultural policy decisions in the preparation of the strategic plan for the next programming period 2023-2027. The model is based on mathematical programming and exploits the optimization potential of LP. It is a specific approach that allows analysis at the level of the agricultural production plan and further aggregation of results at the sectoral level. In this paper, we present an example of a policy scenario analysis for the beef sector. It is shown that the approach used is useful and allows an ex-ante impact assessment. Given the scenario chosen, the results indicate an expected deterioration of the situation.

Keywords: linear programming, farm model, impact assessment, CAP reform, beef

1 INTRODUCTION

Agriculture faces many challenges, and the Common Agricultural Policy (CAP) seeks to support them and provide direction for development and transition to a fairer, healthier, and more environmentally friendly food system. The new programming period 2023-2027 therefore addresses many challenges. In addition to the income issue and a more equitable distribution of direct payments among farms, climate preferences, etc., the environment plays a particularly important role. As various studies show, reducing the environmental footprint of EU agriculture can only be achieved at the price of lower production and higher food prices [3], which has a significant impact on farms. While the results make an important contribution to the policy debate, the farm-level implications have generally been overlooked and additional farm-level analysis is needed, especially in the development of strategic plans that should consider the national and local characteristics of Member States.

The use of various methods to support policy makers has a long history. Reviewing the literature, we find various examples of their use, especially for ex-ante analysis, ranging from macro-sectoral analysis (models based mainly on the partial and general equilibrium approach), which were particularly common in the first period of analysis by agricultural economists, to the microsimulation models that emerged in the last period [1]. These are a type of microsimulation models commonly referred to as farm models. Such models provide a better understanding of decision making and management at the farm level and, on the other hand, give policy makers a better insight into what is happening on individual farms so that they can make better evidence-based decisions and thus achieve greater targeting.

In most cases, these are models based on the optimization potential of mathematical programming [4]. This includes the model IFM-CAP used by the European Commission in the EU. IFM-CAP is based on the Positive Mathematical Programming (PMP) approach and allows the evaluation of different policy impacts on existing aggregates and groups of farms [2]. Its main purpose is to evaluate and analyse the different impacts of the CAP on the economic and environmental performance of farms. Its main advantage is its EU-wide coverage. A farm-level impact analysis of the CAP for the post-2020 period has also been conducted with IFM-CAP [3].

In Slovenia we also have an microsimulation model based on mathematical programming that was used for this analysis. It is a farm model that enables analysis at the farm production plan level [5]. In addition to the optimization of the production plan, it allows various analyses as well as the evaluation of the impact of the selected CAP measures. For this study, we used models of typical farms (TAH) specialized in beef production, which are presented in more detail in Žgajnar et al [5].

2 MATERIAL AND METHODS

2.1 Farm model and aggregate analysis

The farm model used in this study is a tool based on a mathematical programming approach that allows analyses of the impact of policies at the farm production plan level and aggregate analysis at the sector level. This allows the use of different techniques in solving the production plan, which is the basic level of problem solving at the farm level. It is a tool that follows the modern trends in agro-economic analysis in this field and allows analysis at the TAH level [5].

In the present version, the deterministic LP is used. The developed matrix of production possibilities is an example of production planning, where we focus on finding the optimal combination of production activities, considering production constraints, to maximize gross margin (GM).

In calculating the economic indicators, we have included in the analysis the average prices of the three-year period 2018-2020, reducing the impact of inter-annual fluctuations that can otherwise have a significant impact on (market) revenues and even more on gross margin (GM) as two indicators we use to measure the impact of changes in our analysis.

In the remainder of this section (Table 1), we briefly present the models of TAHs based on beef production. We summarize the main characteristics of the studied farms, which are representative of the beef sector in Slovenia. They are presented in more detail in [6].

2.2 Scenario analysis

In the analysis, we simulated the expected effects of changes in CAP measures at the agricultural production plan level. In the analysis, we considered first pillar measures and less favoured areas (LFA) payments, while inclusion in voluntary farm environmental measures (organic schemes) was simplified and modelled based on data available at the time.

We conducted an impact assessment for coupled direct payments for beef. The purpose was to help policymakers determine what kind of impact direct payments have and whether it is justified to support the beef sector as a sector facing certain difficulties. Therefore, in the case of scenario 1 (S1), we considered all expected payments to which the farm would be entitled, while in scenario 2 (S2) we excluded coupled income support (CIS). In doing so, we analysed the impact of CIS on the economic indicators of cattle TAHs.

In a further step, we extrapolated these results to the sector level. The basic assumption of the scenario analysis was that the production plan can be changed only partially, and to a part and extent that does not change the production activities that define the type of farm (e.g., the number of cattle). This means that due to certain (favourable/unfavourable) conditions brought by a single scenario, the production plan can only be partially changed. Either it is a slightly changed distribution of production resources or an increased implementation of certain market activities.

3 RESULTS AND DISCUSSION

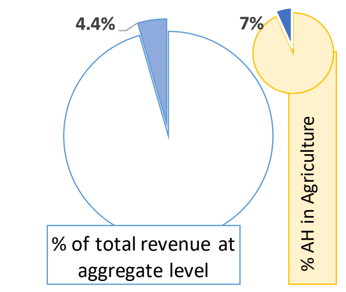
The main characteristics of TAHs specialized in beef fattening and the economic results of the scenario analysis are presented below. Beef as the dominant production activity is found on about 7% of farms in Slovenia. In the farm model, the whole sector is represented by 12 farms and contributes 4.4% to the total income.

As shown in Table 1, 90% of beef farms are smaller than the average Slovenian farm in terms of available area. Small herds predominate. Therefore, poor economic results were expected for these farms. As shown in Table 1, only farms with more than 25 beef achieve GM better than 10 €/hour. Very small farms (which represent 84% of Slovenian beef farms) with less than 6 cattle usually achieve less than 4 €/hour. According to the obtained results, the last farm (TAH12), which also produces hops, stands out in all economic indicators. This type of farm is typical only for one region in Slovenia. The other farms can be found all over Slovenia.

Table 1: Typical agricultural holdings specialised in beef farming in Slovenia and selected economic indicators

TAHs	Farms	Beef	FTE	Field	Permanent grass	GM	GM/h	GM/h	GM/h
	(No)	(No)	(1 800 h)	(ha)	(ha)	(EUR)	(EUR)	(EUR)	(EUR)
	Baseline (BL)							S1	S2
TAH1	600	1	0.13	0.00	1.00	262	1.1	1.1	1.1
TAH2	600	2	0.15	0.00	1.54	883	3.2	3.3	3.0
TAH3	600	3	0.17	0.00	2.02	1,344	4.5	4.5	4.1
TAH4	400	6	0.20	1.27	1.84	1,115	3.1	2.8	2.1
TAH5	400	8	0.22	2.38	0.92	1,721	4.4	4.2	3.3
TAH6	450	12	0.24	3.49	0.92	2,546	5.9	5.1	3.9
TAH7	250	17	0.32	5.29	0.92	4,933	8.5	7.9	6.6
TAH8	250	25	0.41	6.91	1.38	8,136	11.1	9.7	8.3
TAH9	30	60	0.54	6.13	9.90	5,973	6.1	5.6	3.0
TAH10	30	75	0.82	19.54	3.68	17,810	12.1	10.2	8.5
TAH11	18	150	1.33	42.00	5.52	33,504	14.0	11.2	9.2
TAH12	2	150	1.85	42.00	5.52	75,401	22.7	20.8	19.4
Total	**3 630**	**32 145**	**796**	**7 689**	**5 341**				

The importance of subsidies is very pronounced in beef farms. Budget payments in total account for more than 80% of GM and in many small farms even exceed the obtained GM. Therefore, the sector can be expected to be very sensitive to changes in CAP measures. As the results show, the reform will lead to a deterioration (Fig. 1), especially for larger farms (GM/hour) (Table 1). The positive effect of production-based support is evident in all TAHs (S2). The expected maintenance of CIS (S1) for this sector will only partially mitigate the negative effects. This is because without it, there is more deterioration, as can be clearly seen from Figure 1 and Table 1. The more negative impacts occur with more intensive breeding, which tends to occur on farms with a larger proportion of fields to total area, which also tend to have more favourable outcomes.



	Gross margin	Gross margin per FTE	Gross margin per working hour	Scenario	Gross margin per working hour	Gross margin	Revenues	Budgetary payments	LFA
	(EUR)	(EUR)	(EUR/h)		(EUR/h)	Share based on baseline			
	Baseline (BL)			After the CAP reform					
Minimum	262	1,978	1.1	S1	1.1	0.758	0.948	0.619	0.712
				S2	1.1	0.482	0.930	0.462	0.712
Average	2,344	10,692	5.9	S1	5.3	0.897	0.981	0.863	0.942
				S2	4.5	0.747	0.956	0.680	0.942
Maximum	75,401	40,802	22.7	S1	20.8	1.024	1.006	1.040	1.267
				S2	19.4	0.986	0.998	0.988	1.267

Figure 1: Summary of selected indicators for the beef sector in BL, S1 and S2

4 CONCLUSIONS

Based on the analysis, we can conclude that the modelling approach used has proven effective in providing various business insights (indicators) in the scenario analysis, both at the farm level and at the individual sector level. The use of mathematical programming techniques allows us to balance the material balances at the farm level in a relatively simple way, and in this way the production plan is technologically consistent and balanced. However, it has been shown that when simulating different CAP measures, especially in marginal cases, the sensitivity of the model can be problematic. Indeed, this is a problem of LP, where in some cases small (as well as larger) changes in conditions (CAP measures) can lead to completely different solutions. However, from the point of view of the modelling approach, a limitation has arisen, namely that in this way we can analyse only a certain part of the measures that do not significantly interfere with the production and breeding technology.

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Roman and Italian rainbow domination number of graphs

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Abstract: Combination of two types of domination, namely the rainbow domination and the Roman and Italian domination, is discussed in view of its suitability for nontrivial modeling of practical problems.

Keywords: rainbow domination, generalized Petersen graphs, Roman domination, Italian domination

1. Introduction

In combinatorial optimization, the domination problems are used to model the problems where we are keen to determine the minimum number of places in which to keep a resource such that every place either has a resource or is adjacent to the place in which the resource exists. It is quite common that in practical applications some additional constraints or desires are taken into account [14, 15].

Inspired by several facility location problems, Brešar, Henning and Rall in 2005 [4, 5] initiated the study of the t-rainbow domination problem. In short, the t-rainbow domination aims to locate resources of t types to nodes of the graph such that each node has all types of resources available in the neighborhood. For more details and references on rainbow domination we refer to the twin contribution in this proceedings [22].

Roman domination was inspired by the strategies for defending the Roman Empire presented by Stewart [20] and ReVelle and Rosing [18]. Since then, almost 100 papers have been published on this topic including studies of double Roman domination and several variations. Two generalizations, namely $[k]$-roman (called double Roman for $k = 2$) and Roman k-domination (also called Italian domination for $k = 2$), are of interest in cases where we model planning of organization of emergency services where local rescue teams often need support from neighborhood.

Here we first briefly recall these variations of the domination, and then discuss possible natural combinations that may provide good models for practical application.

Roman domination, double Roman and Italian domination

In the 4th century, old Roman Emperor Constantine was faced with a problem of how to defend his empire with limited number of armies. The decision was taken to allocate two types of military units to the empire provinces. Some units were able to move quickly from one province to another to respond to any attack. The second type were the local militia. These armies were permanently positioned in their home province. Emperor Constantine ordered that no legion should ever leave the province to defend the second, if in this case the first province remains undefended. Consequently, there were two armies at some provinces, and at some other provinces only local militia units were stationed. Some provinces had no permanent presence of an army, and were guarded by the armies from neighbouring provinces. While the classical problem is still of interest in military operations research [2], it also can be used to model and solve the problems where a time-critical service needs to be provided with some reserve. For example, a first aid emergency station should never send all its crew to answer a single emergency call. Similar reasoning applies when considering any emergency service implying high importance of the studies of these type of domination problems for importance for the optimization and efficient organization of society.

A natural model is to define a graph where the provinces are vertices and two provinces are adjancent if an army can move from a province to attacked province in a reasonably short time.

In a graph $G = (V, E)$, a set $D \subseteq V$ is a dominating set if every vertex in $V \setminus D$ is adjacent to a vertex in D. The *domination number* $\gamma(G)$ equals the minimum cardinality of a dominating set in G. A dominating set of cardinality $\gamma(G)$ is called a γ–set.

Roman domination was formally defined in [12, 18] as follows: For a graph $G = (V, E)$, let $f : V \longrightarrow \{0, 1, 2\}$ and let (V_0, V_1, V_2) be the ordered partition of V induced by f, where $V_i = \{v \in V(G) \mid f(v) = i\}$. Let $|V_i| = n_i$ for $i = 0, 1, 2$. Note that there exists a 1–1 correspondence between the functions $f : V \longrightarrow \{0, 1, 2\}$ and ordered partitions (V_0, V_1, V_2) of V. Thus, we will write $f = (V_0, V_1, V_2)$. A function $f = (V_0, V_1, V_2)$ is a *Roman dominating function* (RDF) if every vertex in V_0 is adjacent to a vertex in V_2. The weight of f is defined as:

$$w(f) = \sum_{v \in V} f(v) = n_1 + 2n_2.$$

The *Roman domination number*, $\gamma_R(G)$, equals the minimum weight of an RDF of G.

Recall that with Roman domination, one legion is required to defend any attacked vertex. What Beller et al. propose in [3] is a stronger version of Roman domination that doubles the protection by ensuring that any attack can be defended by at least two legions. In Roman domination at most two Roman legions are deployed at any one location. Clearly, the ability to deploy three legions at a given location provides a level of defense that is both stronger and more flexible, at less than the anticipated additional cost. It is called double Roman domination.

Roman domination has been extensively studied in the past, see surveys [10, 8, 7, 6, 11]. For double Roman domination, we refer to survey [19].

$[k]$-Roman domination has been defined by Ahangar et al. in [1]. Let $k > 1$ be an integer and f be a function that assigns labels (i.e. number of legions) from the set $\{0, 1, \ldots, k + 1\}$ to the vertices of G. The neighborhood $N(v)$ of a vertex $v \in V(G)$ is the set of all w that are adjacent to v, $wv \in E(G)$. The active neighborhood $AN(v)$ of a vertex $v \in V(G)$ with respect to f is the set of all vertices $w \in N(v)$ such that $f(w) > 1$. A $[k]$-Roman dominating function on a graph G, abbreviated $[k]$-RDF, is a function $f : V(G) \to \{0, 1, \ldots, k + 1\}$ such that for any vertex $v \in V(G)$ with $f(v) < k$, $f(v) + \sum_{u \in AN(v)} f(u) \geq |AN(v)| + k$. The $[k]$-Roman domination number is the minimal weight of a $[k]$-RDF.

Examples can be constructed showing that $[k]$–Roman domination functions may exist that provide the full cover at lower cost than multiplying the domination function. For example, a graph is called *double Roman* if $\gamma_{dR}(G) = 3\gamma(G)$. In general $\gamma_{dR}(G) \leq 3\gamma(G)$, and it is a question of interest to characterize graphs that are double Roman. ($\gamma_{dR}(G)$ denotes the double Roman, i.e. [2]-Roman domination number of G.)

Note that $[k]$-Roman domination should not be confused with some other generalizations of Roman domination. Alternative relevant answers to multiple attacks, or, multiple emergency calls, have been defined. In the Italian domination problem, each vertex labeled must have the labels of the vertices in its closed neighborhood sum to at least two [9, 16]. Later, the Roman k-domination [17] is studied by Kammerlin et al. (Roman 3-domination is also called double italian domination [21], so perhaps Roman k-domination may be called $(k - 1)$-Italian domination. Furthermore, the same invariant is sometimes called Roman $\{k\}$-domination.) For later use, let us explicitly write up the definition of the Roman $\{k\}$-domination. Roman k-dominating function f assigns labels from the set $\{0, 1, 2, \ldots, k\}$ such that for any vertex v, $f(v) + \sum_{u \in N(v)} f(u) \geq k$. Roman k-domination number of a graph is the minimal weight of such a function, $\gamma_{Rk}(G) = \sum_{u \in V(G)} f(u)$.

2. Roman rainbow and Italian rainbow domination

While on one hand, the two concepts have some similarities, it is clear that they are essentially different. Differences between the 2-rainbow domination and Roman domination are studied in [13]. Furthermore, recall that there are slight but important differences between double roman and Italian domination. Here we are going to define and briefly discuss some combinations. For motivation, the next two examples of applications are obvious.

Example 1. Emergency services and rescue services are either addressing certain types of emergencies, or deal with ad hoc emergencies as part of their normal responsibilities. The availability of emergency services depends very heavily on location. Usually, different services are needed in case of an emergency, often police, fire brigade and emergency medical service are alerted. The success thus depends on response times of all the services involved. In some cases, more than one rescue team of each kind are needed.

Example 2. A construction company carries out several simultaneous projects in many locations. There are several services that may be required frequently and cannot be precisely planned in advance, such as equipment repair. Therefore, these services should be available in time to avoid longer disruptions to the work process. Again, depending on severity of the event, various services and larger teams may be needed.

$[k]$-roman t-rainbow domination

We use functions that assign integer vectors to vertices, c.f. $f : V(G) \to \{0, 1, \ldots, k+1\}^t$, i.e. each vertex v is assigned a vector with t entries $f_i(v) \in \{0, 1, \ldots, k+1\}$, $i \in \{1, 2, \ldots, t\}$.

Definition. A *[k]-roman t-rainbow dominating function* on a graph $G = (V, E)$ is a function $f : V(G) \to \{0, 1, \ldots, k+1\}^t$. Furthermore, each component function f_i is a $[k]$-Roman dominating function of G.

More formally, a $[k]$-roman t-rainbow dominating function has the following property. Let vertex $v \in V(G)$ be a vertex with $f_i(v) < k$. The i-th active neighborhood $AN_i(v)$ with respect to f is the set of all vertices $w \in N(v)$ such that $f_i(w) > 1$. Then $f_i(v) + f_i(AN_i(v)) \geq |AN_i(v)| + k$. In other words, each resource (service) is fully available in the neighborhood of v.

Note however that, because each component function f_i is a $[k]$-roman dominating function, we can in fact study and optimize each concept separately. It is not clear how one can in this case naturally incorporate the feature of rainbow domination, namely that a vertex that is assigned nonempty set does not neccesarily have all colors in the neighborhood.

t-rainbow Roman k-domination (t-rainbow $(k-1)$-Italian domination)

Definition. A *t-rainbow Roman k-dominating function* on a graph $G = (V, E)$ is a function $f : V(G) \to \{0, 1, \ldots, k\}^t$ with the following property: if $f(v) = (0, 0, \ldots, 0)$ then $f_i(v) + \sum_{u \in N(v)} f_i(u) \geq k$, for each $i \in \{1, 2, \ldots, t\}$.

Remark. Replacing the property with "if $f_i(v) = 0$ then $f_i(v) + \sum_{u \in N(v)} f_i(u) \geq k$, for each $i \in \{1, 2, \ldots, t\}$", the definition *would imply* that each f_i is a Roman k-dominating function.

There are of course many other possibilities to combine the concept of rainbow and roman domination. Among the two written here, we can conclude that the second, based on Italian generalization of the Roman domination brings some more room for improvement of solutions when considering all the services concurently. In particular, these means that in a practical situation, we would have two types of vertices, the normal vertices that need complete service in case of an event, and the service nodes, i.e. vertices that provide one (or more) services to the neighborhood.

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HEATING WITH SOLID FUEL IN SLOVAK DWELLINGS: A GWR APPROACH

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Abstract: This paper deals with the spatial regression analysis for the 79 Slovak districts regarding the "low-carbon and greener Europe" priority. The ratio of dwellings heated with the solid fuel (in %) is considered as a dependent variable. As for independent variables, the average nominal monthly salary of an employee (in Euro), population density per km^2 and houses with the gas connection (in %) were used. Estimates of parameters from the global linear model indicate, that while increasing the average salary or increasing gasification, respectively, leads to a decrease in the ratio of households heated with solid fuel, the effect of increasing population density is the opposite. The spatial heterogeneity was considered using the geographically weighted regression (GWR) approach which enabled to explore the spatial differentiation of the individual model parameters across the Slovak districts.

Keywords: heating, solid fuel, Slovakia, spatial, geographically weighted regression (GWR)

1 INTRODUCTION

The strategic document Europe 2020 [4] included a priority focused on the environment as one of the basic priorities. This priority was focused on sustainable growth, support for more efficient use of resources with the aim of achieving greener and more competitive economies. The aspect of ecology (the so-called "green" aspect) has also been considered in the current framework of the EU cohesion policy for the period 2021-2027. This policy is focused on five investment priorities - a smarter Europe, a low-carbon and greener Europe, a more connected Europe, a more social Europe and a Europe closer to citizens [3]. Considering the focus of our paper, we will concentrate on the priority of low-carbon and greener Europe.

A significant contribution to improving the quality of air, water or the environment in general is the increase in energy efficiency and the support of renewable energy sources [3]. Therefore, ambitious goals aimed at reducing CO_2 emissions and transition from fossil fuels to renewable energy sources have been set both at the national and international level. Decarbonisation and the associated reduction of greenhouse gas emissions is also a challenge for heating. The air in Slovakia is most polluted by fine dust particles. The biggest source of these pollutants in Slovakia is not the industry, but the heating of dwellings with solid fuel. The 2021 Housing Census in Slovakia even pointed out the fact that up to 66.22% of Slovak dwellings are heated with gas, followed by heating with solid fuel (21.32% of dwellings). In order to improve the air quality and to support the process towards low-emission energy sources used for heating, the Slovak government has introduced the subsidy mechanisms to support these measures [2].

Regarding the environmental priority, the spatial aspect also plays an important role. In this paper, we assume an uneven distribution of dwellings heated with solid fuel across 79 Slovak districts (regions), that is the spatial heterogeneity problem. The main aim of the paper is consideration of this spatial effect following a Geographically Weighted Regression (GWR) approach. Thus, this paper is intended to investigate the spatial differentiation of the individual model parameters.

The rest of the paper is organized as follows: section 2 deals with methodological background (GWR approach) of the study. Section 3 presents data and empirical results and section 4 closes with concluding remarks.

2 METHODOLOGY – GWR APPROACH

The classical approach to model the relationship between the dependent variable and independent variables which might explain it, is to use the linear regression based on the ordinary least squares (OLS) estimation method. Even in regional data, the issue of the location of a region has not been automatically considered. Unlike the classical global linear regression, the GWR approach of Brunsdon, Fotheringham and Charlton (see [5]) enables to calculate the local linear regression estimates for each region and thus to capture the spatial non-stationarity (spatial heterogeneity) across analysed regions. The local GWR model is specified as follows [1], [5], [6]:

$$y_i = \boldsymbol{x_i}\boldsymbol{\beta_i} + \boldsymbol{\varepsilon_i} \tag{1}$$

where index $i = 1, \ldots, n,$ denotes the i-th region, y_i is the value of dependent variable at region i, $\boldsymbol{x_i}$ is a row vector of independent variables and $\boldsymbol{\beta_i}$ is a column vector of regression parameters at region i. The local regression parameters are functions of region i and can be estimated by the weighted least squares with $\boldsymbol{W_i}$ being the n x n diagonal weighting matrix at region i whose off-diagonal elements are zero and diagonal elements are calculated based on a spatial kernel function giving higher weight to the nearby regions in comparison to regions farther away [1], [5], [6]. Fotheringham et al. [5] distinguish GWR with fixed and adaptive spatial kernels, respectively. Nakaya [8] presents various possibilities how to determine the optimal value of the bandwidth of the spatial kernel function specifying the range of the search window.[1]

3 DATA AND EMPIRICAL RESULTS

This paper is focused on spatial regression analysis for the 79 Slovak districts regarding the environmental aspect. The ratio of dwellings heated with the solid fuel (in %) is considered as a dependent variable. As for independent variables, we used – the average nominal monthly salary of an employee (in Euro), population density per km^2 and houses with the gas connection (in %). The data were downloaded from the DATAcube database of the Statistical Office of the Slovak Republic (the average nominal monthly salary of an employee and population density per km^2) [10] and from the 2021 Housing Census in the Slovak Republic (ratio of dwellings heated with the solid fuel and number of houses with gas connection) [9][2]. Analyses, using the aforementioned data for 2021, were carried out in the free downloadable softwares GeoDa and GWR4.

Figure 1 illustrates the ratio of dwellings heated with the solid fuel (in %) across individual Slovak districts in order to visualise the unequally distribution over space. The districts in the western part of Slovakia and partly in the eastern part of Slovakia have the lowest representation, although the district with the highest representation of heating with solid fuel is the district of Košice III located in eastern part. The districts of central Slovakia are among the districts with a high share of solid fuel for heating (also due to lower gasification of the territory).

The basic spatial analysis based on the percentile map (Figure 1) is followed by the OLS estimation of the global linear regression model specified as follows:

$$solid\_fuel_i = \beta_0 + \beta_1 ln\,(wage)_i + \beta_2 ln\,(dens)_i + \beta_3 gas_i + \varepsilon_i \tag{2}$$

where $\beta_0, \beta_1, \beta_2, \beta_3$ are unknown model parameters, ε_i represents an error term. Estimates of individual global parameters (i.e. without regional differentiation) are in Table 1 (column: Linear model). All the estimated parameters were statistically significant at the 1 percent level

1 Statistical tests for spatial non-stationarity based on the GWR model are presented, e.g. in [7].

2 In the further text of the paper, the above mentioned variables are denoted as follows: „solid_fuel“, „gas“, „wage“ and „dens“. Variables „wage“ and „dens“ were used in form of natural logarithms.

of significance indicating the positive impact of population density and negative impact of the gas connection and of the average nominal monthly salary of an employee, respectively, on the analysed dependent variable.

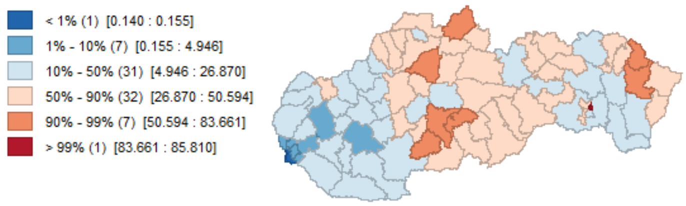

Figure 1: Percentile map for the ratio of dwellings heated with the solid fuel (in %)

To investigate the spatial non-stationarity across analysed regions, i.e. spatially varied effects of independent variables[3], the local GWR analysis was provided (the adaptive bi-square kernel based on 50 nearest neighbours was used):

$$solid\_fuel_i = \beta_{i0} + \beta_{i1} ln\,(wage)_i + \beta_{i2} ln\,(dens)_i + \beta_3 gas_i + \varepsilon_i \qquad (3)$$

Table 1: Estimation results of OLS regression and of GWR

Model	***Linear model OLS***	***GWR*** Minimum	Low. Quart.	Mean	Upp. Quart.	Maximum
β_0	513.412	220.269	256.094	338.216	443.730	456.734
β_1 ***(ln(wage))***	-67.239	-62.002	-60.158	-41.366	-26.402	-20.940
β_2 ***(ln(dens))***	7.673	-0.792	0.163	5.688	12.157	12.404
β_3 ***(gas)***	-0.663			-0.658		
AICc	639.499			628.161		
Adjusted R^2	0.492			0.573		

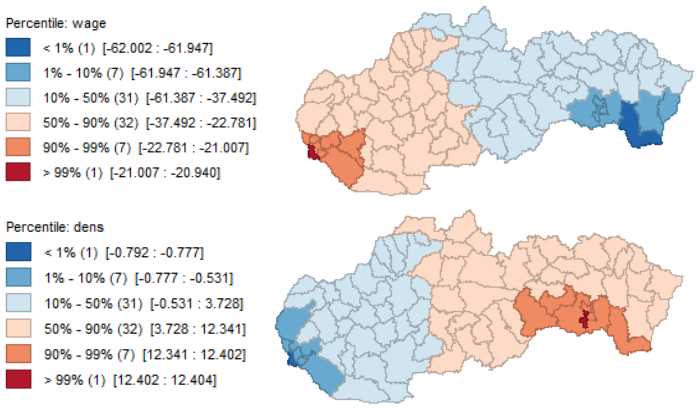

Figure 2: Mapping of local parameters of *ln(wage)* and of *ln(dens)* – GWR fit results

Selected GWR estimation results are presented in Table 1 (columns: GWR). The goodness-of-fit based on values of the adjusted R^2 and corrected Akaike Information Criterion (AICc) for

[3] Testing geographical variability of local parameters proved the global character of the *"gas"* variable.

the global linear regression model and local GWR model, respectively, indicate the evidence of improvement in the model performance.

The estimated set of local parameters from the GWR fit is mapped in Figure 2 which clearly documents the different model performance in individual regions. The impact of the first independent variable, the average nominal monthly salary of an employee on the ratio of dwellings heated with the solid fuel, was negative both in the global model fit as well as in the local GWR fit across all analysed regions. Although the population density shows a globally significant positive effect, there are several districts in the western and central part of Slovakia, where the effect was negative. Since the last independent variable, houses with the gas connection, was proved to be spatially invariable, i.e. global, its effect was the same across all analysed regions. The negative value of parameter β_3 from the GWR fit was in line with the global model fit results.

4 CONCLUSION

To deal with environmental issues, the spatial regression analysis of ratio of dwellings heated with the solid fuel was provided for the 79 Slovak districts (regions). As for the independent variables, the average nominal monthly salary of an employee, population density per km^2 and number of houses with gas connection were used. The global regression results for Slovakia as a whole confirmed the negative impact of the “wage” and “gas” variables and positive impact of the “dens” variable. The local GWR fit estimation results enabled to capture the spatial instability across the Slovak districts.

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CONTROL THEORY AND NUMERICAL ANALYSIS OF MAGNETIC FIELD INVOLVING MECHANICAL SYSTEMS

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Abstract:
The central topic of this work is the research of ways to improve the control and performance of magnetic field involving mechanical systems. Quality of control of such systems depends on the accuracy of field calculations. In case of non-simplest volumetric bodies it is often impossible to integrate the appearing expressions. This is why in engineering numerical approaches like the boundary element method are often implemented. One of the research challenges is the development of new operations research strategies and numerical algorithms that would uniformly approximate the field under given conditions.

Keywords: control theory, magnetic levitation, PID control, multi-objective optimization algorithm, nonlinearity identification

1 INTRODUCTION

There exist various applications of magnetic fields in engineering. Coil's design is crucial in magnetic resonance imaging [1]. Fast computational optimization of transcranial magnetic stimulation coil placement for individualized electric field targeting improves performance of such medical imaging systems [2]. These systems are used to non-invasively modulate activity of targeted brain networks via a magnetically induced electric field. Modelling of such complex systems requires new optimization approaches. For example, when modelling active magnetic bearings, a linearized model is usually used for determining the controller settings. However, the parameters of the model may vary according to the operating point. In order to determine robustness and stability of the closed-loop controlled active magnetic bearings system, the discussed parameters have to be calculated over the entire operating range [3].

The magnetic levitation set-up involves a well-known phenomenon often used for control system studies. Recently it has become important in a very wide range of industrial applications where magnetic suspension techniques can be applied. The best known ones are high-speed ground transportation [4] and high-speed bearings with reduced noise and friction [5].

Boundary element method is a set of universal numerical methods for solving boundary problems for partial differential equations [6]. With the use of layer potentials the original boundary value problem for the differential equation is reduced to an integral equation on the boundary of the domain. The main advantage of boundary element method is the possibility of discretization of only the boundaries of the studied area. Thus, the dimension of the problem, and hence the order of the system of algebraic equations, decreases by one. In external boundary value problems boundary element method automatically satisfies the conditions at infinity, in contrast to final difference method or final element method, which reduces the computation time. The decrease in dimension by one in some problems in engineering has a decisive influence on the choice of this solution method [7].

2 METHODOLOGY

A certain way to look at the magnetic field caused by a permanent magnet is the use of imaginary magnetic charges. This method is based on the analogy of the electrostatic and the magnetostatic fields. It is convenient to use in the absence of conduction currents $\sum \mathbf{j} = 0$. Since $\operatorname{div} \mathbf{B} = 0$

$$\mathbf{B} = \mu_0 \left(\mathbf{H} + \mathbf{M} \right), \tag{1}$$

where $\mathbf{B}$ is the magnetic flux density vector, $\mathbf{H}$ is the magnetic field strength vector, $\mathbf{M}$ is the magnetization vector and μ_0 is the vacuum permeability. Let's formally introduce the notation $\rho_m = -\operatorname{div} \mathbf{M}$ as the density of imaginary magnetic charges. Then

$$\operatorname{div} \mathbf{B} = \mu_0 \left(\operatorname{div} \mathbf{H} - \rho_m \right) = 0. \tag{2}$$

This and the fact that $\operatorname{rot} \mathbf{H} = \mathbf{j}$ and $\mathbf{j} = 0$ consecutively give us the following equations

$$\begin{cases} \operatorname{div} \mathbf{H} = -\operatorname{div} \mathbf{M} = \rho_m \\ \operatorname{rot} \mathbf{H} = 0. \end{cases}, \tag{3}$$

These two equations can be compared to the electrostatics equations

$$\begin{cases} \operatorname{div} \mathbf{E} = \dfrac{\rho}{\epsilon_0}, \\ \operatorname{rot} \mathbf{E} = 0, \end{cases}, \tag{4}$$

where ρ is the density of electrical charges and ϵ_0 is the dielectric permeability of vacuum. As can be seen there is a complete analogy between equations (3) and (4).

Therefore the original magnetostatics problem can be replaced as an equivalent problem of electrostatics. By finding the solution of (4) and performing the formal constitution $\mathbf{E} \to \mathbf{H}$ and $\rho/\epsilon_0 \to \rho_m$ one can find the solution to the original problem (3).

To conclude this, after solving the analogy electrostatic problem one can write the formal substitution rule as follows:

$$\begin{cases} \mathbf{E} \to \mathbf{H}, \\ \rho/\epsilon_0 \to \rho_m, \end{cases} \tag{5}$$

to explicitly get the solution to the original magnetostatics problem.

A magnetic scalar potential u is analogous to an electric potential. It is used to calculate the magnetic field in case of no free currents. This can be used to determine the magnetic field of permanent magnets when their magnetization is known. To describe the magnetic field, a magnetic potential u is introduced, so the magnetic field induction is found as $\mathbf{B} = -\operatorname{grad} u$. This approach is appropriate, since in the region where the magnetic field is studied, the currents are usually small and their magnetic field can be neglected.

3 RESULTS

In a magnetic levitation (Maglev) train the main interest is the distribution of the magnetic field in the accelerating channel and in its immediate vicinity, excluding the elements of the magnetic system. The close-range solutions to such problems are not broadly studied in literature and the authors of this work aim to develop this field more.

The magnetic potential u is set in any region with zero current density and provides an expression of the magnetic field at a given point in space. The main benefit of this approach is that it allows us to reduce this problem to a boundary problem for a Laplace equation. This, in term, provides an opportunity to apply the powerful techniques developed for this equation, specifically the boundary element method.

Let us introduce in space the Cartesian coordinate system $x = (x_1, x_2, x_3) \in \mathbb{R}^3$. Let Γ be a simple smooth closed surface of class C^2 bounding a simply connected inner region D. Consider the exterior Neumann boundary value problem for the Laplace equation with a continuous boundary condition given on Γ

$$\begin{cases} \Delta u = 0, & u \in C^1(\overline{\mathbb{R}^3 \setminus D}) \cap C^2(\mathbb{R}^3 \setminus \overline{D}), \\ \left.\dfrac{\partial u(x)}{\partial \mathbf{n}}\right|_{\Gamma} = f(x), & x \in \Gamma, \ f(x) \in C^1(\Gamma), \\ u = O\left(\dfrac{1}{|x|}\right), & |x| \to +\infty, \end{cases} \tag{6}$$

where $\partial/\partial\mathbf{n}$ means the normal derivative [8] on the surface Γ from the outside at a point x and it is assumed that the function $u(x)$ has a normal derivative on the surface Γ. Let us find the solution of the Neumann problem in the form of the simple layer potential $\mathcal{V}_0[\mu](x)$

$$u = \mathcal{V}_0[\mu](x) = \frac{1}{4\pi} \int_{y \in \Gamma} \mu(y) \frac{1}{|x-y|} dS_y, \tag{7}$$

where $\mu = \mu(y) \in C^0(\Gamma)$ — is the density of the potential. The simple layer potential $\mathcal{V}_0[\mu](x)$ — is a harmonic function in the region $\mathbb{R}^3 \setminus \overline{D}$.

Normal derivative from outside to surface Γ is given by the expression [8]

$$\frac{1}{2}\mu(x) + \left.\frac{\partial \mathcal{V}_0[\mu](x)}{\partial \mathbf{n}_x}\right|_{\Gamma}, \ x \in \Gamma, \tag{8}$$

where $\mathcal{V}_0[\mu](x)/\partial\mathbf{n}_x|_\Gamma$ — is the direct value of the normal derivative of the simple layer potential for the Laplace equation on the surface Γ, while $\mathbf{n}_x$ — is the internal unit normal. Equating this expression to the function defined on Γ, we obtain the following equation

$$\frac{1}{2}\mu(x) + \left.\frac{\partial \mathcal{V}_0[\mu](x)}{\partial \mathbf{n}_x}\right|_{\Gamma} = f(x), \ x \in \Gamma. \tag{9}$$

The equation (9) is a 2D linear Fredholm integral equation of the second kind, which is known to be uniquely solvable [8, §28.3]. The reduction of number of dimensions by 1 tremendously saves computational time. This expression allows us to introduce the theory of numerical solutions of such equations, which is thoroughly developed, to a magnetic field problem. The magnetic potential density μ acquired by numerically solving equation (9) is later used in the expression (7) to get a value of the magnetic potential u at any point in 3D space. This approach does not require any order reduction techniques and the size of a boundary element does not need to get infinitely smaller when one approaches the immediate vicinity of a magnetized object.

4 CONCLUSION

Precision in calculation of magnetic field is important for control theory. Previously in our research it was shown that a more complicated magnetic force equation can be a big factor in achieving a better control for an open-loop unstable system [9]. In literature the BEM method for magnetic fields of complicated surfaces is usually addressed in the 2-dimensional case whereas in this work the authors consider specifically the 3-dimensional case. Furthermore, research will focus on the development and use of improved computational methods for increased accuracy of calculations of magnetic fields.

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THE ROLE OF RESOURCE COMPLEMENTARITY AND OPPORTUNISM IN STRATEGIC ALLIANCE PERFORMANCE

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Abstract: This paper aims to investigate the performance of strategic alliances in relation to resource complementarity and attitudes toward relationship with alliance partners. According to resource-based theory, resource complementarity is extremely important for strategic alliance success. However, since alliances often fail due to partners' opportunistic behavior, fostering collaborative relationships, omitting opportunistic behavior and perceiving partner as important is crucial for alliance success. To test proposed hypotheses a survey based on 31 large Croatian firms was conducted. The analysis was made using PLS-SEM. The results have shown that resource complementarity fully mediates the link between attitude toward alliance partner and alliance performance.

Keywords: knowledge**,** opportunism, performance, PLS-SEM, resource-based view, strategic alliances, strategy

1 INTRODUCTION

Strategic alliances refer to a whole series of different forms of relationships between firms. Although some firms have sufficient resources and capabilities to pursue their strategies independently, it is increasingly common for firms to collaborate with suppliers, distributors, manufacturers of complementary products, and sometimes with selected competitors [17].
The choice of partner is crucial for the success of a strategic alliance; it's perhaps the most important assessment managers should make when forming alliances. Many alliances break up before they reach their potential due to friction and conflict among partners. A good choice of a partner that possesses certain resources, can in the future determine the competitive advantage of a strategic alliance and lead to improved performance. Resource-based view (RBV) states that firms enter strategic alliances to gain access to valuable resources of other firms [3]. The resource-based view is focused on relationships between firms' internal resources and capabilities and competitive advantage [16]. Its assumptions are that companies within an industry are heterogeneous in terms of resources they control. Since resources may not be perfectly mobile, heterogeneity can be long lasting [1].

In the context of forming strategic alliance, the synergy effects of combining resources will be more unique and valuable when based on complementarity, than similarity [9].

In the next chapter, a relevant theory is presented and hypotheses are developed. Operationalization of the variables, research methodology and results are elaborated in the third chapter. Concluding remarks are given at the end of an article.

2 THEORY AND HYPOTHESES

Because of the path dependence, companies often can't generate new resources and capabilities through internal development. In that case, external mechanisms, such as alliances and acquisitions, are the best way to acquire new resources and capabilities [18]. A strategic alliance is the teamwork of several firms that together possess more resources, knowledge, and capabilities than as an individual firm. There are two motives for creating strategic alliances: obtaining new resources, and retaining and development of existing resources by combining them with other firm's resources [3].

The stages of alliance creation are identification, evaluation, negotiation, and implementation. Alliances are most often formed due to a lack of own resources, and potential opportunities and benefits from joining an alliance are analyzed before entering. After that, a decision is made about profitability, and if it is satisfactory, a strategic partner is found, and implementation begins. Once a possible alliance has been determined, negotiations start, and if they are successfully completed, alliance is implemented, which is the longest phase. Companies must be ready to enter an alliance because unpreparedness can easily lead to the failure of the alliance. Another important thing when joining an alliance can be the experience that companies have gained through previous alliances [8].

The main purpose of forming strategic alliances is to create added value. When properly organized, the obtained resources can create greater value for each alliance partner. Each partner must be able to create more value for himself through the alliance than he could do on his own. Alliances are based on reciprocity: partners acquire, exchange, or integrate specific business resources and competencies for mutual benefit. Motives that direct the company into alliances can be related to product, technology, marketing, protectionism, production, resources and competitiveness [10]. Successful firms understand that strategic alliances can be a powerful way to adapt in turbulent and uncertain times [12].

It is necessary for firms to take advantage of new competitive advantages that can arise from forming alliances and manifest through the synergy of all involved partners. But without trust, alliance cannot be successful. Trust can play three interrelated roles in interorganizational relationships: first, it can act as a barrier to opportunistic partner behavior; second, it can replace the management hierarchy; third, it can create a competitive advantage. Strategic alliances blur the boundaries between firms and make the interdependent [14]. The strategic interdependence of alliance partners makes opportunistic behavior extremely dangerous and reduces the likelihood of conflicts, which has a positive impact on the performance of the strategic alliance [4]. One of the key decisions in creating a strategic alliance is partner selection. It is very important that partners balance their contributions to the alliance in order to avoid conflict situations in which one participant dominates the other and thus destroys relations within the alliance itself, but also the possible success of the alliance on the market. The basic assumption of cooperation is the existence of a common market self-awareness and mutual trust between partner firms. It is important to highlight the importance of attitudes that companies have towards the alliance itself and how they see the future of their company within the alliance [8]. Therefore, we propose:

H1. Attitude toward relationship with alliance partner is positively related to alliance performance.

Strategic alliances enable firm to increase its resource base when its current resources and capabilities are not sufficient to achieve the desired outcomes. Resources refer to all assets, capabilities, organizational processes, firm's characteristics, information and knowledge that a firm uses in shaping and implementing strategies [1]. In this paper abovementioned definition is adopted, so when referencing to resources we also have in mind capabilities, knowledge and skills.

Alliances provide access to partner's resources, and therefore firms often look for partners who have the resource they lack [9]. Resource complementarity is related to partners' strategic interdependence [4] and strategic alliances are primarily motivated by opportunities to exploit complementarities between resources and capabilities possessed by different firms [5]. Firms that perceive their alliance partner as important for them, foster collaborative relationships and avoid opportunistic behavior will be more likely to share complementary resource with alliance partner. We propose:

H2. Attitude toward relationship with alliance partner is positively related to resource complementarity.

Resource and capability complementarity is important for the effective functioning of strategic alliance. Complementary of resources allow the firm to combine the acquired resources with its resource base. The synergy effects of combining resources and capabilities will more likely be unique and valuable if they are based on complementarity rather than similarity. The best alliances are the ones where partner are aligned in context of resource and goals. Complementary resources unable firm to combine them with their own resource base and in that was create new resource bundle that is unique and inimitable. Resource complementarity has singer effects and can positively impact long term performance [9].

H3. Partners' resource complementarity is positively related to alliance performance.

While resource complementarity facilitates mutual learning and synergy creation in strategic alliances, various factors can undermine knowledge-sharing and partner relationships, thereby inhibiting the value-realization potential of complementary resources [11]. The complementarity of partner's resources keeps them in a long term cooperation. It is linked to their perception of alliance importance and their willingness to resolve conflicts and avoid opportunism [13]. Resource complementarity positively moderates the trust–performance link [15], hence:

H4. The link between partners' resource complementarity and alliance performance is mediated by attitudes toward alliance partner.

3 METHODOLOGY AND RESULTS

Data was collected using a questionnaire. The sample is composed of large Croatian firms that were identified from the data by Croatian Chamber of Economy. Out of 436 firms, 48 replied to a survey, creating a response rate of 11%. However, only 31 participating firms had prior alliance experience, so only those answers were usable.

The resource complementarity (COMP) is assessed through resource, capability and knowledge complementarity, attitude toward relationship with alliance partner (RELA) through partner importance, non-opportunistic behavior and fostering cooperative relationships. The alliance performance (PERF) was operationalized by contribution to risk minimizing, skills building, gaining access to new industries, competition blocking and fulfilling the government's demands. A 5-piont Likert scale was used.

To analyze measurement model and test theory, a path analysis using partial least squares (PLS-SEM) was conducted. PLS-SEM was chosen due to small sample size and flexibility regarding multivariate normality. SmartPLS 4software was used for data analysis.

To test collinearity, Variance Inflation Factors (VIF) was analyzed. The results proved that there is no multicollinearity between variables problem since all values are between 0.2 and 5 as proposed by Hair et al. (2014), with VIF(RES)= 2.586 being the highest.

To test measurement model and confirm reflective constructs, individual reliability, composite reliability, convergent validity, and discriminant validity were tested. The results of reliability and convergent validity analysis are presented in Table 1. All factor loadings are statistically significant and above 0.6. Cronbach's α, as well as Rho_A and Rho_C, are above 0.7 as suggested by Hair et al. (2010), confirming reliability of constructs. Average variance extracted (AVE) values are above 0.5 proving that acceptable convergent validity is established.

Table 1: Reliability and convergent validity

	Cronbach's alpha	*CR (Rho_A)*	*CR (Rho_C)*	*AVE*
RELA	0.705	0.826	0.806	0.584
COMP	0.867	0.880	0.918	0.789
PERF	0.753	0.770	0.833	0.501

Discriminant validity was tested with Fornell-Larker criterion and heterotrait-monotrait (HTMT) ratio of correlations. Table 2 presents the results of discriminant validity analysis using Fornell-Larker criterion.

Table 2: Fornell-Larker criterion

	RELA	*COMP*	*PERF*
RELA	0.764		
COMP	0.477	0.888	
PERF	0.482	0.674	0.708

From the results presented in Table 2 it can be seen that the square root of AVE is higher than correlation between different constructs which leads to conclusion that discriminant validity is ensured. The same conclusion can be derived from observing HTMT ratio, which is below 0.8 (Table 3).

Table 3: HTMT ratio

	RELA	*COMP*	*PERF*
RELA			
COMP	0.496		
PERF	0.648	0.780	

Structural relationships were estimated using bootstrapping method (500 subsamples). To test mediation, propositions by Baron and Kenny (1986) and Hair et al. (2010) were followed. After it was established that individual relationships between analyzed variables are statistically significant, i.e. that there is statistically significant direct relationship between RELA and PERF, COMP and RELA, and COMP and PERF, the initial model is estimated containing only the direct impact from RELA on PERF. The tested relationship was significant (β=0.590, $p<0.001$), confirming hypothesis H1 stating that attitude toward relationship with alliance partner is positively related to alliance performance.

Next, model containing resource COMP as mediating variable and two additional paths was estimated. The impact of RELA on COMP was significant (β=0.432, $p<0.01$), confirming H2 which states that attitude toward relationship with alliance partner is positively related to resource complementarity. The impact of COMP on PERF was also positive and statistically significant (β=0.560, $p<0.001$) confirming H3 which says that partners'' resource complementarity is positively related to alliance performance.

After including RELA in the model, path coefficient between COMP and PERF decreased (β=0.259, $\Delta\beta$=-0.301) and was not statistically significant any more (p=0.155), confirming full mediation of COMP and hypothesis H4 according to which the link between partners' resource complementarity and alliance performance is mediated by attitudes toward alliance partner. Figure 1 presents the structural model with full mediation.

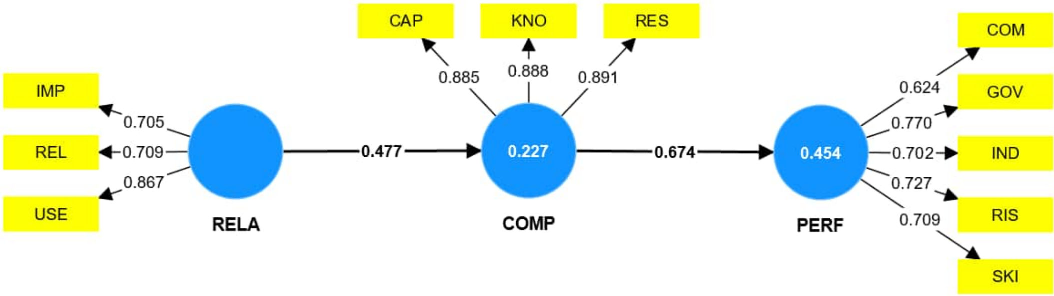

Figure 1: Structural model with full mediation

After including mediator in the model, the explained variance of endogenous construct PERF increased (ΔR^2=0.196, ΔR^2_{adj}=0.203), confirming the proposed mediation. f^2(PERF)=0.832, and f^2(COMP)=0.294, all indicating exogenous construct's large effect on endogenous construct.

4 CONCLUSION

Strategic alliances are becoming important mode of firm's development, since they can be used for expanding business, entering new industries, developing new products, technologies, or skills, without major risks or investments. Although there is substantial research on strategic alliances and the perquisites for their success, the contribution of this paper reflects in proposing and testing the model that incorporates impact of resource complementarity and attitudes toward alliance partner on alliance performance. In that context, if firms foster relation with alliance partner in a way that they perceive him as important, try to behave in a collaborative way and avoid opportunistic behavior the performance of alliance will be higher. Also, if alliance partners are behaving in this kind of way, they will be more willing to share their complementary resource with alliance partner, which should in turn also lead to higher alliance performance. All that was confirmed by PLS-SEM analysis bases on the sample of 31 Croatian company. The empirical results of this research can contribute to encouraging and understanding the role of partners' resource characteristics and attitude toward partner relationship for alliance success. The results can serve all firms engaging in strategic alliance, to better choose alliance partner and foster partner relationship.

Although this research contributes to the analysis of strategic alliances, several limitations must be acknowledged. This study was based on large firms operating Croatia, and the results could be different in other national contexts and should be interpreted cautiously. Also, the results could depend on the industry and vary in industries characterized by less dynamic environments. Future research could focus on more industries or countries and reveal contextual differences. The second methodological limitation of this study is related to the small sample size, which could also be solved in future research activities.

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FACTORS AFFECTING COVID-19 VACCINE UPTAKE OF YOUNG ADULTS: MACHINE LEARNING APPROACHES

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Abstract: This study aims to estimate a data-driven model for the predictors of COVID-19 vaccination uptake in the Federation of Bosnia and Herzegovina. The research was conducted on a snowball sample of respondents collected during December 2021 and divided into training and testing sets. Unlike previous studies, the classification problem was formulated, and empirical estimates on the training data sample were obtained using three different empirical methods: decision tree, logistic regression and support vector machines. Empirical findings pointed out potential side effects, a general attitude about the vaccine, the perception of COVID-19 as a non-threatening disease, the age of respondents, and visits to the doctor in the last six months as factor determinants of COVID-19 vaccination status. The prediction accuracy for each approach was evaluated on the testing sample. The best forecasting performance was found for the support vector machine, amounting to 92.03%. Forecasting accuracy for logistic regression amounts to 91.30%, and for decision tree, it amounts to 89.86%. The lowest forecasting accuracy was found for penalized logistic regression, amounting to 86.96%. Findings from this research suggest a need for more communication and education about the vaccine and the consequences of COVID-19 disease.

Keywords: COVID-19 vaccines; decision making; decision trees; machine learning; mass vaccination; logistic regression, support vector machine

1 INTRODUCTION

COVID-19 emerged as one of the strongest threats to modern societies, having a strong impact on worldwide health and, conversely, on doing business and education [1] [3] [7]. In late 2020 and early 2021, a vaccine against COVID-19 was introduced as an effective and most important measure against the COVID-19 pandemic. However, the vaccine was not fully accepted, and there is a recognized problem of vaccine hesitancy [12] [9] [14]. Afterwards, a substantial body of literature has emerged considering vaccine hesitancy in various population groups, but mainly within developed countries [14] [8]. At the same time, vaccination hesitancy is the most prominent among post-communist developing countries and threatens to undermine the success of coping with the disease. Athias and Macina pointed out the importance of considering the historical specificity of ethnic groups while designing health policy and communication [2]. Therefore, the issue of vaccination hesitancy is under-examined within post-socialist societies despite the topic's urgency. Furthermore, the empirical findings in the literature often rely on one empirical approach, and it is a well-known fact in the empirical literature that results always depend on data selection and model specification as well as the method used to estimate parameters. Framing measures against vaccination hesitancy requires a deeper understanding of the topic and calls for further research. Empirical evidence from different parts of the globe is still needed, and the ones from countries with the lowest vaccination rates, like post-socialist societies, are of special importance. Furthermore, existing empirical findings mostly rely on very few empirical approaches. Different empirical approaches must be challenged since results depend on data samples and model selection. As further illustrated in the methods section, this paper evaluates logistic regression, decision trees, and support vector machines. Furthermore, results from penalized logistic regression were compared with results from logistic regression. While targeting these two goals, this research aims to take a step ahead while following the theoretical framework from Eilers et al. [5]. Consequently, the aim of this research is twofold. Firstly, to provide results from a post-communist developing country (the Federation of Bosnia and Herzegovina) and secondly, to evaluate the performance of competing empirical approaches to forecasting vaccination status based on a predicting factor.

2 METHODOLOGY

Decision trees were first employed as a rather simple and interpretable supervised learning approach. The response variable is binary categorical with $K = 2$ classes representing vaccination status and p explanatory variables $X_1, \cdots, X_p$. The split or classification was selected with the greatest decrease in impurity. As a measure of node impurity in classification trees, the Gini index was used $(\sum_{k=1}^{C}(p_k \cdot (1 - p_k))$ where C represents the number of classes of predictor variables; afterwards, logistic regression with lasso regularisation was considered. Taking y as a dependent binary categorical variable representing vaccination status (1-vaccinated and 0-unvaccinated), $x_1, \cdots, x_n$ as predictors of y and $\beta_1, \cdots, \beta_n$ coefficients representing a potential link between predictors and dependent variable logistic regression can be assumed as in equation (1).

$$p(y|\boldsymbol{\beta}) = \frac{1}{1+e^{-(\beta_0+\beta_1 \cdot x_1+\cdots+\beta_n \cdot x_n)}} \tag{1}$$

Coefficients $\beta_1, \cdots, \beta_n$ were determined by minimising the negative log-likelihood function or logistic loss function in equation (2).

$$LL = \sum_{i=1}^{N}[-y_i \cdot \log(p(y_i|\boldsymbol{\beta})) - (1 - y_i) \cdot \log(1 - p(y_i|\boldsymbol{\beta}))] \tag{2}$$

Where y_i represents the observed value (1-vaccinated and 0-unvaccinated and $p(y_i)$ is a function of $(\beta_0, \cdots, \beta_n)$ representing the probability of being vaccinated based on the predictors $[x_{1i}, \cdots, x_{ni}]$ corresponding to respondent i.

However, in case of too many potential predictors problem of variable selection needs to be solved first, and for that purpose we use the least absolute shrinkage and selection operator (LASSO), which includes a penalty for regression coefficients and trades off between an increase in bias and a decrease in variance [11].

$$L_{lasso} = LL + \lambda \sum_{i=1}^{m}|\beta_i| \tag{3}$$

In Lasso regression, the sum of the absolute coefficient is penalised (L1 penalty), and for high values of the regularisation penalty (λ), some coefficients in LASSO regression may obtain a zero value. Therefore, LASSO regression reduces the problem of multi-colinearity and prevents over-fitting. To find the best λ, cross-validation was performed, and λ providing the lowest bias and variance was selected.

As another possible classification procedure support vector machine was considered [13]. More precisely, a support vector machine for two-class data sets that are separable by a linear classifier. The support vector machine defines the criterion for a decision surface that is maximally far away from any data point. Assuming a hyperplane defined by $\boldsymbol{w}$ and b, where $\boldsymbol{w}$ a decision normal vector perpendicular to the hyperplane and b represents intercept. Since there are many hyperplanes, by fixing b the one is selected. Therefore, any point on the hyperplane satisfies the equation:

$$\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} \;=\; -\,b \tag{4}$$

Where $\boldsymbol{w}$ represents the weight vector, $\boldsymbol{x}$ represents the input vector, and b represents bias.

Assuming the data set defined as $S = \{(\boldsymbol{x}_i, y_i)\}$, so each pair of data set consist of a point $\boldsymbol{x}_i$ and corresponding y_i representing a class. In the case of a support vector machine, the two classes are always named +1 and -1 rather than 1 and 0. In this research, $\boldsymbol{x}_{\boldsymbol{i}}$ represent a set of answers from one respondent and y_i its corresponding vaccination status (vaccinated or unvaccinated). The linear classifier is defined as in the equation:

$$f(\vec{x}) = sign(\boldsymbol{w}^T\boldsymbol{x} + b) \tag{5}$$

Therefore, the value +1 indicates one class, and -1 indicates another. Therefore, a linear line was fitted to separate the two classes and maximise the distances from either class. Hence, class 1 is on line $\boldsymbol{w}_{\boldsymbol{i}}\boldsymbol{x}_{\boldsymbol{i}} + b = 1$ and above the line while the classes -1is on line and below a line $\boldsymbol{w}_{\boldsymbol{i}}\boldsymbol{x}_{\boldsymbol{i}} + b = -1$ as formulated in the formula:

$$\begin{cases} class \;\;=\;\; 1, & \boldsymbol{w}_{\boldsymbol{i}}\boldsymbol{x}_{\boldsymbol{i}} + b \geq 1 \\ class \;\;=\;\; -1, & \boldsymbol{w}_{\boldsymbol{i}}\boldsymbol{x}_{\boldsymbol{i}} + b \leq -1 \end{cases} \tag{6}$$

The distance between the two classes is $\frac{2}{|\boldsymbol{w}|}$ and the preferred distance is the maximum one. Equivalently, $min\|\boldsymbol{w}\|^2$ that represents the problem of quadratic optimisation with linear constraints for all $\{(\boldsymbol{x}_{\boldsymbol{i}}, y_i)\}, y_i(\boldsymbol{w}^T\boldsymbol{x}_{\boldsymbol{i}} + b) \geq 1$. There is a unique solution or minimum in case all data points of one class are separable from those of the other. In real life, all points are rarely detachable, and the goal is to find a separating hyperplane that separates many, but not all, data points. Therefore, some misclassifications are tolerable. In such case, the optimisation problem can be expressed in the equation:

$$\min_{\boldsymbol{w},b} \frac{1}{2}\|\boldsymbol{w}\|^2 + C\frac{1}{n}\sum_{I=1}^{n}\varepsilon_i \tag{7}$$

With subject to $\begin{cases} y_i(\boldsymbol{w}^T\boldsymbol{x} + b) \geq (1 - \varepsilon_i) \\ \varepsilon_i \geq 0 \end{cases}$ $\quad \forall i = 1, \cdots, n.$ For some constant C.

With Lagrange multipliers (α_i, μ_i) constrained optimisation problem as the primal LaGrange function can be formulate1d as in equation (8)

$$\min_{\boldsymbol{w},b,\alpha,\mu}\left[\frac{1}{2}\|\boldsymbol{w}\|^2 + C\frac{1}{n}\sum_{I=1}^{n}\varepsilon_i - \sum_{i=1}^{n}\alpha_i[y_i(\boldsymbol{w}^T\boldsymbol{x} + b) - (1 - \varepsilon_i)] - \sum_{i=1}^{n}\mu_i\varepsilon_i\right] \tag{8}$$

Consequently, the function in equation (8) can be rewritten as in equation (9).

$$L(\boldsymbol{w}, b, \alpha, \mu) \tag{9}$$

Based on conditions $\frac{\partial L(\boldsymbol{w},b,\alpha,\mu)}{\partial \boldsymbol{w}}, \frac{\partial L(\boldsymbol{w},b,\alpha,\mu)}{\partial b}, \frac{\partial L(\boldsymbol{w},b,\alpha,\mu)}{\partial \alpha}$ and $\frac{\partial L(\boldsymbol{w},b,\alpha,\mu)}{\partial \mu}$, obtained solutions are $\boldsymbol{w} = \alpha_i y_i \boldsymbol{x}_i$ and $0 = C - \alpha_i - \mu_i$. A corresponding dual optimisation problem can be formulated as in the equation:

$$\max_{\alpha}\sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{J=1}^{n}\alpha_i\alpha_j y_i y_j \boldsymbol{x}_i^T\boldsymbol{x}_j \tag{10}$$

With constraints $\begin{cases} 0 = \sum_{I=1}^{n}\alpha_i\, y_i \\ 0 \leq \alpha_i \leq C\,, \forall i = 1, \cdots n. \end{cases}$

So,$\forall \boldsymbol{x}_i \exists \alpha_i$, $\alpha_i > 0$ for support vectors only, while for all other points $\alpha_i = 0$. Constant C controls the trade-off between maximising the margin and minimising the loss. In this paper, the regularisation parameter C and the best one was found using cross-validation.

Open-source R software was used with several ready-to-use libraries ("e1071" and "rpart") that support chosen algorithms. The questionnaire for this research was developed following Eilers et al. [5] and extended for some COVID-19-relevant issues; it is available on request from the authors. The research sample was a snowball sample of 692 adult respondents obtained via social networks and e-mail between December 3 and 15, 2021. The research sample can be considered highly representative of young adults in the Federation of Bosnia and Herzegovina. A description of the training and testing research sample and questionnaire is provided in the appendix, from Table A1 up to Table A4. The training sample consisted of 554 respondents who responded first, and the testing sample consisted of 138 respondents who provided their answers afterwards.

3 RESULTS

The decision tree was estimated and illustrated in Figure 1 following the proposed methodology. As illustrated in Figure 1, the overall probability of vaccination uptake in the training sample was 0.15. The perception of vaccination safety, measured by agreeing with the statement "the vaccine is not acceptable to me due to possible side effects" (x14), emerged as a question of importance. This finding is consistent with most previous studies [10] [12]. On a scale from 1 to 5, 79% of respondents indicated 3 or higher to express the extent to which they agree with this statement. Suppose the respondent indicated a 3 or higher number to agree with the statement that the probability of vaccination uptake amounts to 0.05. The respondent indicated 1 or 2, indicating a low degree of agreement with the statement that the probability of vaccination uptake rises to 0.49. For 21% of respondents, an important statement emerged: "I have a positive attitude about the vaccine" (x7). Respondents who supported this statement with 1, 2, or 3 belong to a group with a probability of being vaccinated of 0.2. In contrast, those who supported the statement with a 4 or 5 are more likely to be vaccinated. The probability of being vaccinated in that group amounts to 0.29. Another important statement for vaccination status appeared as the statement, "I think that I am healthy and that for me COVID-19 is a form of mild flu or cold" (x11), consistent with Fridman et al. [6], among others. The overall conclusion from the illustrated decision tree classifier pointed out that perceived side effects and perceptions of one's own health status were important drivers of COVID-19 vaccine uptake. Respondents who perceived themselves as healthier were more likely to uptake vaccines.

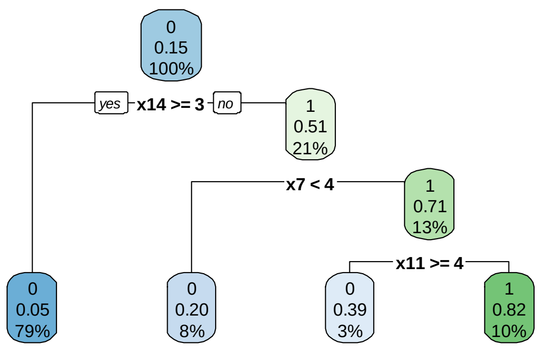

Figure 1: Decision tree of the vaccination status

Afterwards, estimates from logistic regression were obtained. As further described in the methodology section of this paper, variable selection for penalised logistic regression model was performed using LASSO. Out of the results from LASSO, 12 variables were shrunk to zero. Afterwards, one logistic regression model was estimated with the rest of the variables (15 variables) as independent variables and vaccination status dependent. In the estimated model, effects from some independent variables were not distinguishable from zero, based on z-test statistics. After removing non-significant regressors from the estimated penalised logistic regression model was summarised in Table 1. Estimates or β coefficients in logistic regression are reported as odds ratios. Respondents supporting the statement "the vaccine is not acceptable to me due to possible side effects " (x14) are less likely to uptake the vaccine. Estimates from logistic regression pointed out the importance of variable x24, "I have visited a doctor in the last 6 months". Respondents who visited a doctor in the last 6 months are more likely to uptake COVID–19 vaccine. Consistently with Sherman et al. [9], the age of respondents (x28) appeared as an important variable in the logistic regression model as well, suggesting that older respondents are more likely to uptake the COVID-19 vaccine.

Table 1: Penalised logistic regression model for vaccination status

Independent variable	Estimate (Std. error)	Std. error	z value	p-value
x14	-1.38	0.131	-10.52	< 0.000
x24	1.01	0.303	3.35	0.001
x28	0.06	0.011	5.34	< 0.000
AIC: 289.25				

Eventually, the optimal hyperplane based on the support vector machine defined by w and b was summarised, providing estimates for distinguishing between vaccinated and unvaccinated respondents (results are available on request from authors); however, which are less informative than estimates from decision trees and logistic regression. Therefore, logistic regression will be the preferred approach if we want to find and understand a link between vaccination status and various attitudes of respondent decision trees.

Table 2: Comparison of prediction accuracy; Note: 0-not vaccinated; 1-vaccinated

Predicted									
Approach:		Decision trees		Logistic regression		Logistic regression (penalised)		Support vector machines	
		0	1	0	1	0	1	0	1
Realised	0	112	1	112	1	110	3	112	1
	1	13	12	11	14	15	10	10	15
Prediction accuracy		89.86%		91.30%		86.96%		92.03%	

The data from the testing set are in rows, while predicted data from different forecasting approaches are in columns (Table 2). The best out-of-sample performance for support vector machines was found, followed by a decision tree and logistic regression. Nonetheless, each approach obtained forecasting accuracy higher than 86% and can be considered successful. Conclusively, different approaches should be considered to reveal a complete picture when dealing with factors affecting vaccination and resolving the estimation problem.

4 CONCLUSION

Several conclusions might be derived from the research presented in this paper. Firstly, empirical findings from this research pointed out potential side effects of the COVID-19 vaccine as a main factor related to COVID-19 vaccination hesitancy. Secondly, general attitudes about a vaccine and the perception of COVID-19 as a non-threatening disease are factors relevant to vaccination decisions. Thirdly, respondents who visited a doctor in the last six months were more likely to be vaccinated, while age increases the likelihood of being vaccinated. Perceived side effects of the COVID-19 vaccine appeared as a factor robust to methodology selection since its relevance does not depend on methodology selection. Support vector machines appeared to be the most accurate among the considered approaches. The penalized logistic regression approach had the lowest accuracy ratio, while the decision tree was between the two. However, the accuracy ratio for any approach evaluated in this research was above 85% and can be considered satisfactory. Based on empirical findings from this research, the support vector machine was a preferred approach for forecasting, while the results from penalized logistic regression and decision trees were more interpretable. Research findings from this paper revealed similar drivers of vaccination hesitancy in a post-socialist country to those in developed countries. Vaccination hesitancy seems to be a multidimensional issue, and a policy mix might be required to mitigate its adverse effects. Overall, findings from this paper undoubtedly pointed out a need for communication and education as part of the policy mix. Education and communication with the wider community might not be directed only towards the safety of the COVID-19 vaccine but also towards the severity of the COVID-19 disease.

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TOPIC MODELLING OF OPEN GOVERNMENT DATA IMPACT AREAS USING GPT 3.5 MODEL

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Abstract: The paper addresses the problem of assessing impact of Open Government Data (OGD) use. For this purpose, we analysed the reports on 697 OGD use cases from the European Data portal. First, we manually determined the impact areas of the use cases, which later served as a validation group for the extraction algorithms. Second, we used a Generative Pre-trained Transformer model (GPT-3.5) for topic modelling. The model showed promising results in extracting topics from the OGD use cases, outperforming previous techniques with an accuracy of 73%.

Keywords: open government data, topic modelling, OGD impact topics, data mining, GPT 3.5

1 INTRODUCTION

Open data is data whose content can be freely used, modified, and shared by anyone for any purpose [16] and in the context of the public sector is also called Public Sector Information (PSI) or Open Government Data (OGD) [27]. OGD represents one of the sources of open data published by the public sector in the form of databases or datasets, intending to promote transparency of operations, accountability, and the creation of added value [15]. Public sector organizations produce and process ever-increasing amounts of data, and the number of research and initiatives on the topic of open data is also increasing [2, 3, 21, 23, 26]. By making their databases accessible, public institutions also become more interesting for citizens' political participation [19], the establishment of companies and innovative services focused on citizens is encouraged [20], and the long-term aim is to ensure universal transparency of government information [9].

As Ubaldi [23] points out, one of the elements for creating the added value of OGD is data with a high impact on the public. There are currently many open data policies at different levels of government, while very little systematic and structured research has been conducted on their actual impact [10, 11, 18, 19, 20, 27]. Many other authors [1, 5, 21, 25] also recognize the problem of measuring and recognizing the impact of open data, the reasons for this, however, are mostly in the availability and low quality of data, costs, and legal obstacles, and users. We define the impact as an economic-social effect, which is defined by the British Council for Economics and Social Sciences [24] and defined as a demonstrable contribution to society and the economy and a benefit for individuals and organizations. Impact, whether scientific or economic-social, according to Penfield et al. [17], is monitored for reasons of accountability and funding, among others. By responsibility, we mean the importance of presenting the value of research to the government, stakeholders, and the general public, while by funding we mean mainly the contribution or assessment of the contribution that research has to society or the economy and helps in the allocation of support and financial resources where the intended effect is the biggest. The importance of OGD impact assessment is also highlighted by the index of the Organization for Economic Co-operation and Development (OECD) - OURdata 2019 [14], which emphasizes that the long-term impact of open data requires appropriate management frameworks, competent civil servants, high political commitment and recognition of a key role data and that without the introduction of a mechanism to monitor and assess the impact of open data, initiatives to open and process OGD may fade away. An additional argument for the importance of measuring the impact of OGD is also their potential and the size of the market, which amounts to 184 billion euros, the number of potential jobs in this sector - 1 million, and the potential for growth in individual industries, which in those with a high impact amounts to 15.7 percent [8].

As mentioned by Ferencek & Kljajić Borštnar [6], an opportunity to identify impact areas of OGD was recognized in individual open data use cases published by the Publications Office of the European Union. The goal of this paper is to extract the not yet existent open government impact areas from open data use cases using generative AI tools. Extracted topics will be validated against manually determined impact areas, which were set by a domain expert. Validation of topics will be performed by calculating semantic similarity of the topics.

2 METHODOLOGY

The methodological approach employed in this research is grounded in Design Science Research (DSR), with the overall aim to develop a ML model for automatic classification of OGD use cases into OGD topic taxonomy [7]. The first step in the classification model development is constructing an OGD topic taxonomy, where we followed the CRoss-Industry Standard Process for Data Mining (CRISP-DM) [4]. For this purpose we analysed the OGD use cases sourced from the European Data portal. We conducted a detailed manual analysis of all 697 use cases in the corpus and manually determined their impact areas, which served as a validation group for our extraction algorithms. Analysed use cases were unstructured PDF files the length of which varied from 431 to 1353 words.

Next, we pre-processed the data by lowercasing the corpus and lemmatization and removed stop words and noise. Stop words used were from the NLTK library whereas the noise removal function was written by us. The function took incoming documents and removed special characters and punctuation, numerical values, URLs, email addresses, and extra whitespaces by using regular expression. Finally, we performed text normalization with which we tokenized the sentences, calculated word frequencies using NTLK, identified near-identical words, and removed redundant words from the text.

For topic modelling, we used a Generative Pre-trained Transformer (GPT) which is a type of language model developed by OpenAI. GPT models are based on the Transformer architecture, which is a deep learning model designed for natural language processing tasks [22]. The GPT models, such as GPT-3.5, are trained on large amounts of text data to learn patterns and relationships in language. They can generate coherent and contextually relevant responses given a prompt or input text [12].

At the time of research, we used the latest GPT models which are GPT-3.5 [13], more specifically we used *text-davinci-003* and an extended, *gpt-3.5-turbo* model.

To use text-davinci-003 model in code, *openai.Completion.create()* function has to be used and parameters need to be defined. The basic parameters we have set up are engine, which is text-davinci-003, *max_tokens* which is the maximum number of tokens that can be created by the model (150 in our case), *temperature* which is the parameter that defines how creative (higher value) or deterministic (lower value) the answer is, and finally, the *question* parameter as a prompt from which the model generates the answer. Temperature value suggested to be used, based on Open AI recommendations [13] is 0.2 for Data Analysis Scripting and 0.7 for Creative writing.

To use gpt-3.5-turbo model, *openai.ChatCompletion.create()* function has to be used with the same parameters.

We modified the maximum number of tokens and the temperature to test, if there is any difference in the results. The question used in both models was "*What domain is this text about in one word:*".

Models were evaluated by calculating accuracy as a semantic similarity (Word Embedding similarity), by using spaCy small pipeline package.

3 RESULTS

3.1 text-davinci-003

The maximum number of tokens didn't provide any difference in model output, while temperature modification did. We tested *temperature*=0.7 and *temperature*=0.2 and in the entire corpus, there were 50 out of 697 documents, for which the topic was extracted differently (Appendix 1). The only shortcoming of the text-davinci-003 model was recognizing the country of origin or the name of the project as a topic due to the word's frequent occurrence in the document. In comparison with the manual extraction of the document topics by domain expert, the model outperformed any technique we tried in our previous research [6] with 73% accuracy.

3.2 gpt-3.5-turbo

Due to two major limitations of the free tier model use – the number of requests per minute (3) and ongoing timeouts due to the model's capacity limitation, we kept the value of *max_tokens* the same as with text-davinci-003 (150) and we used default *temperature* of 0.7. While text-davinci-003 had one shortcoming, gpt-3.5-turbo seemed to address it very well but had lower, 71% accuracy as visible in Table 1. Finally, impact areas presented in Appendix 2 were recognized by the models used and will serve us in developing open government impact areas taxonomy.

Table 1: Sample of different topics extracted based on both models used with Word Embedding similarity accuracy score.

	text-davinci-003 Temp. = 0.2	*Score*	*text-davinci-003 Temp. = 0.7*	*Score*	*gpt-3.5-turbo Temp. = 0.7*	*Score*
Ex. 1	Austria	0,5339	Covid-19	0,4073	Health	0,8075
Ex. 2	Security	0,7920	Law/Crime	0,4572	Public domain	0,4846
Ex. 3	Technology	1	Telecommunication	0,8435	Technology	1
Ex. 4	Emergency	0,3566	Safety	0,3780	Public Safety	0,494
Ex. 5	Regions	0,5817	Cities	0,5815	Tourism	0,8128
	Average score *(of 697 use cases)*	0,7302	**Average score *(of 697 use cases)***	0,7277	**Average score *(of 697 use cases)***	0,7140

4 CONCLUSION

The purpose of this paper was to automatically extract impact topics of OGD use cases. Two GPT-3.5 models were used: text-davinci-003 and gpt-3.5.-turbo. While the first model already outperformed models and techniques we used in past research with 73% accuracy, it had a limitation which was its inability to ignore the document country of origin and/or project name and extract it as the topic due to frequent appearance in the document. Gpt-3.5-turbo successfully addressed the shortcomings of its processor but extracted topics with 71% accuracy. While the domain expert extracted 26 impact areas, the generative AI models recognized much more underlying topics which will significantly improve the usefulness of the taxonomy. There are few limitations of this study. First, the availability of free use of Gpt, second, we only focused on extracting the main (high level) topics of the text, and availability of only one domain expert for validation of the extracted topics. Future steps are using other most frequently used techniques in the field of topic extraction, such as Non-negative Matrix Factorization (NMF), Hierarchical Dirichlet process (HDP), Probabilistic Latent Semantic Analysis (pLSA), and BERTopic/Top2Vec, OGD impact taxonomy development and validation with additional domain experts.

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Appendix 1: Sample of different topics extracted based on temperature parameter change.

	Temperature = 0.2	*Temperature = 0.7*
Example 1	Austria	Covid-19
Example 2	Security	Law/Crime
Example 3	Technology	Telecommunication
Example 4	Emergency	Safety
Example 5	Regions	Cities

Appendix 2: Extracted impact areas by the models used. Impact areas were also filtered out of country and/or project names.

Model	*Topics*	*No. of topics*
Manually extracted topics by domain expert	Agriculture, Business, Crime, Culture, History, Economy, Education, Energy, Engineering, Entertainment, Environment, Finance, Foods, Geography, Government, Health, Jobs, Justice, Living, Politics, Sanitation, Science, Social, Technology, Transportation, Waste	Originally 26
text-davinci-003 temp. 0.2	Accessibility, Agriculture, Android, Applications, Art, Auctions, Biking, Bioeconomy, Budgeting, Camping, Cities, Comics, Commodities, Companies, Corruption, Courts, COVID-19, Crime, Crowdfunding, Culture, Cycling, Data, Delivery, Democracy, Demography, Diversity, Earthquakes, Ecommerce, Economics, Economy, Education, Elections, Emergency, Energy, Engineering, Environmental, Equality, Events, Finance, Fire, Fishing, Flooding, Food, Forestry, Foundation, Gambling, Genetics, Geodata, Geodesic, Geographic, Geospatial, GIS, Governance, Government, Grocery, Health, Healthcare, Heritage, History, Homelessness, Housing, ICT, Imagery, Industry, Innovation, Insurance, Investigative, IoT, IT, Jobs, Journalism, Justice, Law, Legal, Legislation, Manufacturing, Mapping, Marketing, Meteorology, Migration, Mobility, Monitoring, Municipality, Museum, Music, Names, Nutrition, Open Data, Parking, Patents, Pathogen, Planning, Plans, Platform, Police, Politics, Population, Poverty, Privacy, Procurement, Property, Prospecting, Public, Public health, Quality, Real Estate, Recruitment, Recycling, Regional, Religion, Renewable, Renting, Research,	Originally 179 Filtered out 145

	Restrooms, Satellite, Science, Security, Seismology, Services, Smart Cities, Snow plows, Society, Software, Solar, Space, Sports, Statistics, Sustainability, Technology, Toilets, Tourism, Transparency, Transportation, Travel, Trees, UNESCO, Urbanism, Utilities, Veganism, Vehicles, Voting, Waste, Water, Weather, Website	
text-davinci-003 temp. 0.7	Accessibility, Agriculture, Android, Applications, Art, Auctions, Bioeconomy, Budgeting, Camping, Cities, City, Comics, Commodities, Companies, Corruption, Court, COVID-19, Crime, Crowdfunding, Culture, Cycling, Data, Databases, Delivery, Democracy, Demographics, Diversity, Earthquakes, Ecommerce, Economics Economy, Education, Elections, Energy, Engineering, Environmental, Equality, Events, Extractive, Finance, Fire, Fishing, Flooding, Floods, FOI, Food, Forestry, Foundation, Future, Gambling, Genetics, Geodata, Geodesic, Geography, Geospatial, GIS, Governance, Government, Grocery, Health, Healthcare, Heritage, History, Homelessness, Housing, ICT, Imagery, Industry, Innovation, Insurance, Investigative, IoT, IT, Jobs, Journalism, Justice, Law, Legal, Legislation, Manufacturing, Mapping, Marketing, Meteorology, Migration, Mobility, Municipality, Museum, Music, Names, Open Data, Parking, Parliament, Patents, Pathogen, Planning, Plans, Platform, Police, Politics, Population, Poverty, Privacy, Procurement, Prospecting, Public, Quality, Real estate, Recruitment, Recycling, Regional, Regions, Renewable, Renting, Restrooms, Safety, Satellite, Science, Seismology, Services, Smart, Smart Cities, Snow plows, Society, Software, Solar, Space, Sports, Statistics, Sustainability, Technology, Telecommunication, Tenders, Toilets, Tourism, Transparency, Transportation, Travel, Trees, UNESCO, Urbanism, Utilities, Vegan, Vehicles, Voting, Waste, Water, Weather, Websites	Originally 174 Filtered out 147
gpt-3.5-turbo temp. 0.7	Agriculture, Agriculture/Fisheries, Archaeology, Art and Culture, Auctions, Bioeconomy, Bioinformatics, Biotechnology, Business, Business/Economy, Business/Entrepreneurship, Business/Finance, Commodities, Consulting, Culture, Data, Data Analytics, Data Governance, Data Journalism, Data Science, Data visualization, Database, Demographics, Economics, Economy, Education, Emergency services, Energy, Engineering, Entertainment, Events, Extractive Industries, Finance, Finance/Economy, Food, Food and beverage, Food delivery, Food industry, Food Safety, Food waste, Forest fires, Forestry, Gambling, Genetics, Geodata, Geographic data, Geographic Information (GI), Geographic Information System (GIS), Geographical data, Geography, Geography/Geospatial data, Geospatial, Geospatial data, Geospatial technology, Geospatial/environmental data, Governance, Government, Government data, Government procurement, Government spending, Government transparency, Government/Open Data, Government/Public, Government/public sector, Governmental budget data, Health, Health and Wellness, Healthcare, Heritage, History, Homelessness, Immigration, Infrastructure, Insurance, IoT (Internet of Things), IT, Jobs/Employment, Journalism, Justice, Justice/Legal System, Law, Law enforcement, Legal, Legal information, Mapping, Meteorology, Music, Navigation, Open data, Open government data, Outdoor recreation, Parking, Patent, Politics, Procurement, Public Administration, Public domain, Public health, Public Restrooms, Public Safety, Public Services, Real estate, Real Estate/Housing, Recreation, Research, Retail, Sanitation, Satellite data, Science and Technology Security, Seismology, Smart cities, Smart City, Smart city technology, Society, Sports, Sports data, Statistics, Sustainability, Technology, Technology/ICT, Tourism, Transparency, Transportation, Travel, Travel/Camping, Urban data, Urban planning, Utilities, Weather	Originally 134 Filtered out 131

A MCDM APPROACH TO MACHINE LEARNING MODEL SELECTION: BITCOIN RETURN FORECASTING

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Abstract: Estimation of different machine learning (ML) models, i.e. feedforward neural networks (NNs), convolutional NNs and long short-term memory, where each has its own set of varying parameters can lead to enormous number of models that need to be evaluated. Using different performance measures, i.e. MSE and accuracy on train and test samples, makes the problem of selection of the optimal ML model a multicriteria decision making (MCDM) problem. Using combination of the conjunctive method for preliminary selection of the models and PROMETHEE method for the final ranking, the optimal ML models are selected. This approach is tested for Bitcoin return forecasting in period from 2016 to 2022.

Keywords: Bitcoin, feedforward neural networks, convolutional neural networks, long short-term memory, PROMETHEE, conjunctive method

1 INTRODUCTION

Machine learning models have been widely used in different applications, as well as in cryptocurrency forecasting. Their superior performances with respect to their parametric counterparts has been proved [12]. Among ML models most commonly used are the feedforward neural networks (FNNs), convolutional neural networks (CNNs) and long short-term memory (LSTM) models due to their proved dominance. Some research confirm CNN to predict better compared to LSTM and/or other ML architectures [6, 19]. Others confirm LSTM to have better predictability compared to other ML models [10, 9]. Therefore, the research provide contradictory results regarding the optimal ML method and this paper aims to reach a unique answer.

After the estimation of different FNN, CNN and LSTM models, where each has its own set of varying parameters can lead to enormous number of models that need to be evaluated. Usually they are compared using the mean squared error (MSE) on the train and test set. Additional important criterion is the accuracy of the prediction on the train and test set. Since the decision making process includes numerous ML models and more than one selection criterion, the task of selecting the optimal ML model can be viewed as MCDM problem.

Firstly, to narrow the number of models to approximately 25 best, a conjunctive method is used. To be chosen, a ML model must exceed a minumum cut-off value on all criteria. However, ranking of the ML models simultaneously according to either train or test, MSE or accuracy does not lead to the selection of the best ML model because the optimal ML model in-the-sample, can have poor out-of-sample performances on either criteria and *vice versa*. To achieve the best compromise solution, i.e., to select an optimal ML model, the preference ranking organization method for enrichment of evaluations (PROMETHEE) is used. PROMETHEE is combined with NNs in [1], [2] and [15] for different applications. Namely, in order to compare the system performance between the alternatives, [1] evaluated each alternative using PROMETHEE and find the proposed methodology to be an effective method. Similarly, [2] used firstly self-organizing map to cluster and prequalify the suppliers based on customer demand attribute and sustainability elements. Further, MCDM methods are used to rank the cluster of suppliers and showed the efficiency of proposed approach. [15] ranked the obtained NN models to reach a best compromise solution, i.e. optimal NN for inflation forecasting. Advantage of PROMETHEE lies in the pairwise comparison of the alternatives with respect to a number of criteria, allowing the decision maker to express the preference in the form of threshold parameters [17].

This paper therefore gives a methodological contribution to the development of ML models and their appropriate selection with MCDM approach. It provides a unique answer to which ML model to use which gives the optimal Bitcoin return predictions. Moreover, most of the papers test the proposed models in bullish or stable market conditions. This paper, however finds an optimal ML model in the downturn period of the cryptocurrency market.

The remainder of the paper is organized as follows. Section 2 describes the data and methodology. Section 3 presents empirical findings with a discussion of the results. Finally, conclusions and directions for future research are provided in Section 4.

2 DATA AND METHODOLOGY

2.1 Data

The data covers the period from 1.1.2016 to 31.3.2022. From the Bitcoin closing prices the log returns are calculated. Descriptive statistics for Bitcoin returns for the whole period is given in Table 1 along with normality, stationarity, independence and nonlinearity tests. The null hypothesis of the Jarque-Berra (JB) test, which assumes normal distribution, can be rejected at 1 % significance level. The null hypothesis that unit root is present in Bitcoin returns, can be rejected according to the augmented Dickey-Fuller (ADF) test at 1 % significance level. The null hypothesis of identically and independently distributed random variables can be rejected according to the Brock, Dechert, and Scheinkman (BDS) test. As the null hypotheses of Teräsvirta NN test can be rejected at 1 % significance level, it can be concluded that there is an inherent nonlinearity in Bitcoin returns.

min	max	mean	sd	skew	kurt	ADF	JB	BDS	TNN F
-0,464	0,225	0,0018	0,0397	-0,72	11,17	-12,19***	12406***	***	5,25***

Table 1: Descriptive statistics for Bitcoin returns

Using Bai-Perron structural break test one significant break is detected. Therefore, training set starts with 6.1.2016 while it ends with 12.3.2021, i.e. it consists of 1893 days. Training set is followed by a test set whose size corresponds to five percent of the size of the train set, i.e. 95 days. The test set is characterized by a bearish market. Therefore, this paper aims to find an optimal ML model in the downturn period of the cryptocurrency market. The Bitcoin closing prices and returns are given in Figure 1. Dependent variable in the ML models is the Bitcoin log return for the following trading day. The independent variables include internal factors: average block size, average block time, average hash rate, average transaction fee, moving average of close price as well as lag return and number of Tweets as an attractiveness measure. External factors include: S&P500, Chicago Board Options Exchange volatility index and Gold prices. Daily data for internal factors and attractiveness measure are obtained from Coinmetrics (charts.coinmetrics.io/network-data), while data for external factors is obtained from FRED (fred.stlouisfed.org).

2.2 Methodology

FNNs, CNNs and LSTM models are estimated and compared in their training and testing mean squared error (MSE) and accuracy (A) measures. FNNs consist of input, hidden, and output layers. Inputs and outputs are the independent and dependent variables, while the

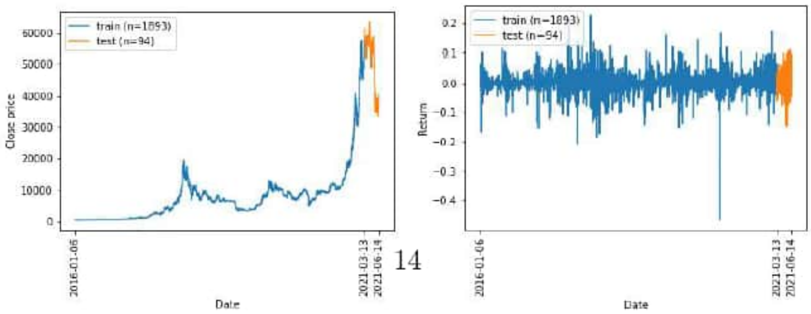

Figure 1: *Bitcoin closing prices and returns*

optimal number of hidden neurons has to be found. The backpropagation learning algorithm is used to estimate the weights. Explanation for FNN is given in [16]. LSTM consists of LSTM units that merge to form the LSTM layer. LSTM unit is composed of the cells having input, output, and forget gate which control the flow of information. It is given extensively in [4, 14]. CNNs have the advantage over other architectures which lie in a lower number of parameters to estimate. In CNN architectures there are convolutional, max-pooling, dropout, and fully connected FNN layer. The convolutional layer consists of the convolution (filtering) operation. Basic convolution operation is shown in [14].

The number of hidden units (h) is calculated based on working rules from [13], [11] and [8] given in [15] which depend on the number of observations in the train set, and number of independent and dependent variables in the model. Following the working rules the number of hidden neurons used for training is 3, 4, 7, 10, 12 and 25. The following configurations are identical for all ML types: optimizer is stochastic gradient descent (SGD); learning rates (λ) are 0.01, 0.001, 0.0001; loss function is mean square error (MSE); batch sizes (β) are 2, 32, set size; and number of epochs is 500.

Other FNN parameters used in analysis are: one hidden layer and tanh activation functions. LSTMs parameters are: one LSTM layer, one dense layer, and neurons given for FNN multiplied by 10, since LSTM learns better with much higher number of hidden neurons. CNN configuration is: one dimensional convolutional layer, MaxPooling1D layer for the max-pooling layer, tanh activation function, 32 filters, pool size is 2, kernel size is 540, and the same set of hidden neurons as in FNN. Kernel size is calculated with formula $\lfloor \frac{setsize}{100} \cdot 30 \rfloor$ [6].

Since 162 models are estimated, the problem of selecting the optimal ML model emerges. To solve this problem, firstly the conjunctive method is used to dichotomise ML methods into acceptable/unacceptable categories. Each model is acceptable as long as it satisfies the minimum cut-off values on each of 4 performance measures: training and testing MSE and accuracy. Approximately 25 ML models need to remain which will then be used in PROMETHEE for the final ranking. In conjunctive method [3] consider a set of n equally weighted independent criteria, m alternatives, i.e. ML models, and r a percent of the models which have to be rejected, then P_c, a probability that a randomly chosen model is above the conjunctive cut-off level, is:

$$P_c = (1 - r)^{\frac{1}{n}} \tag{1}$$

The data used for Conjunctive method is given in Table 2. From the remaining ML models, the models with the accuracy lower or equal to 0.5 are also excluded from further analysis due to their low prediction ability. Finally, 10 ML models entered the PROMETHEE.

PROMETHEE [5] compares and ranks the remaining ML models simultaneously on the 4 criteria, i.e. training and testing MSE and accuracy. Consider the following problem:

$$Max[f_1(a), f_2(a), ..., f_n(a)|a \in A_i] \tag{2}$$

Conjunctive method	Calculated values
n	4
m	162
r	0,85
P_c	0,62
Remaining ML models	35

Table 2: Data for Conjunctive method

where A_i is a finite set of $m = 10$ possible ML models, and f_j are $n = 4$ criteria. The result of the comparison between two ML models a and b , with respect to a particular criterion, has to be expressed in terms of preferences. A preference function P_j gives the intensity of preference of the ML model a over b. Out of six types of possible preference functions, the Gaussian is selected with the s parameter corresponding to standard deviation of each criterion. Additionally, for each criterion the relative importance (weight - w_j) is determined so that testing criteria weight the same as the train criteria [1]. The preference index π is defined as the weighted average of the preference functions P_j and it represents the intensity of preference of ML model a over b, when considering simultaneously all criteria, i.e.

$$\pi(a, b) = \frac{\sum_{j=1}^{n} w_j P_j(a, b)}{\sum_{j=1}^{n} w_j} \tag{3}$$

For each ML model a, positive $\Phi^+(a)$ and negative $\Phi^-(a)$ flows are defined, i.e.

$$\Phi^+(a) = \frac{1}{m-1} \sum_{x \in A} \pi(a, x), \Phi^-(a) = \frac{1}{m-1} \sum_{x \in A} \pi(x, a) \tag{4}$$

Positive flow expresses how an ML model a is outranking all the others. The higher $\Phi^+(a)$, the better the ML model. Negative flow provides a measure of the outranked character of ML model a. The lower the $\Phi^-(a)$, the better the ML model. That is PROMETHEE I partial relation. A total preorder (complete ranking without incomparabilities) or PROMETHEE II complete relation is the net outranking flow or the balance between the "power" and the "weakness" for each ML model. The higher the net flow the better the ML model, i.e.

$$\Phi(a) = \Phi^+(a) - \Phi^-(a) \tag{5}$$

3 Empirical results

FNN, CNN and LSTM are estimated in Python with the parameter settings defined in Section 2.2., yielding 162 models. After the initial filtering using the Conjunctive method given in Table 2, and after excluding the models with the accuracy (A) on the test set lower or equal to 0.5; finally 10 ML models entered the PROMETHEE method respectively (Table 3)[2].

The prediction period is characterized by a significant downturn in Bitcoin prices in the beginning of 2021, where the Bitcoin prices almost halved from 60000 $ in March and April 2021 to around 35000 $ in June. Bitcoin price again reached an all-time high in 2021, as values exceeded over 65000 $ in November 2021. Despite this turbulent period 10 ML methods were able to have excellent in-the-sample performances while retaining good predictive power. However, the best ML model in the bearish market is CNN. CNNs are ranked the highest by the net flow of PROMETHEE method and they appear 6 times in the top 10. Additionally, CNN reaches the highest accuracy on the test set of 58,8 %. Although LSTM have the lowest MSEs

[1] The results do not change significantly with different weights of criteria.

[2] Other results are available from the authors upon reasonable request

Model	h	λ	β	MSE_{train}	A_{train}	MSE_{test}	A_{test}	Φ	Φ^+	Φ^-
CNN32	10	0,001	32	0,001503	0,589	0,002354	0,541	0,30	0,34	0,04
CNN35	10	0,0001	32	0,001635	0,535	0,002252	0,588	0,26	0,37	0,11
CNN41	12	0,001	32	0,001474	0,582	0,002428	0,529	0,24	0,29	0,05
CNN50	25	0,001	32	0,001505	0,576	0,002376	0,506	0,05	0,23	0,18
LSTM40	120	0,001	2	0,001608	0,552	0,002148	0,532	0,01	0,12	0,11
LSTM49	250	0,001	2	0,001608	0,541	0,002169	0,532	-0,03	0,11	0,13
CNN53	25	0,0001	32	0,001637	0,542	0,002427	0,529	-0,07	0,09	0,15
FNN20	7	0,01	32	0,001613	0,554	0,002206	0,511	-0,13	0,08	0,21
CNN8	3	0,0001	32	0,001689	0,532	0,002296	0,518	-0,21	0,05	0,26
FNN48	25	0,01	1893	0,001617	0,536	0,002227	0,511	-0,43	0,10	0,53

Table 3: The best 10 ML models along with the parameter setting, performance measures and PROMETHEE flows

on the test set, their accuracy is somewhat lower, reaching 53,2 %. Therefore, they are ranked on positions 5 and 6 in the top 10. FNNs are ranked among the lowest, i.e. on the 8^{th} and 10^{th} place. To conclude, the optimal model for Bitcoin return forecasting, which provides stable prediction accuracies and forecasting performances is CNN. Its advantages come from their structure which incorporates FNN model after the convolutional layer, enabling lower number of hidden neurons. The proposed batch size is 32 as it is selected 7 out of 10 times; learning rate of 0.001 is selected in 5 our of 10 models; and finally, most ML models have relatively high number of hidden neurons which can be explained by an extremely volatile prediction period.

4 Conclusion

In this paper the estimated FNN, CNN and LSTM models are firstly filtered using conjunctive method to reduce the number of ML models and then ranked using PROMETHEE based on four criteria (train and test MSE and accuracy) to reach a final conclusion regarding the optimal ML model in the downturn period. The results yield CNN as the optimal model exhibiting stable prediction accuracies. The limitation of this study stems from the application of the proposed methodology for Bitcoin return forecasting.

5 Acknowledgements

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Maximizing Ad Campaign Effectiveness through TV Viewership Analysis: A Machine Learning Investigation

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Abstract:

This study utilizes scant data from media activity reports to assess the relationships between television (TV) viewing and other TV characteristics. This research aims to reveal those dependencies and identify suitable ad spots using two machine learning techniques, Random Forest and Generalized Additive Model (GAM), with smoothing splines. Predicting the most watched TV slots is the driving force behind this study since they are more likely to have the intended impact on an advertising campaign. The TV channel and the time of day have the biggest effects on viewership, according to both models. Although the GAM with smoothing splines can estimate non-linear behaviour, the Random Forest model is shown to be more accurate and reliable. The study advises employing GAM with higher levels of interaction to simulate the capabilities of the Random Forest model in future studies. Based on the proposed procedure, marketers can devise a model for prognosticating TV time slots with the highest viewership.

Keywords: Generalized Additive Models, Smoothing Splines, Random Forest, Interpretable Machine Learning, TV Ratings.

1 INTRODUCTION

The relationship between share-of-market (SOM) and share-of-voice (SOV), which was explored by [2] based on data on 1096 global brands, served as the inspiration for this study. He also created the foundation for the advertising intensiveness curve. His research established the relationship between SOV and SOM and [1] demonstrated how it applies to FMCG products. [4] provides yet another example of an empirical demonstration of this theory. Increasing viewing is one strategy to boost SOV. Knowing how and where to post an advertisement to get the most viewers is helpful in achieving this. Therefore, the primary goal of this essay is to comprehend the relationships between viewership (expressed with Gross Rating Points, or GRP) and TV ad qualities. The growth in viewing would consequently increase SOV and, thus, even SOM. Additionally, viewership increases the likelihood that the advertisement will result in the desired outcome (such as consideration, a purchase, or website visits). Marketing managers can profit from this information by being aware of which ad characteristics are associated with greater viewership.

2 ANALYZED DATA SAMPLE

The dependent variable in this study is the Gross rating point (GRP). It represents a percentage of a base population (specifically all potential adult television viewers in a market – usually all 15+ years) reached by a campaign or advertisement through a communication medium. The anonymized dataset covers TV ad details of a fast-moving consumer goods (FMCG) company in 2019. The dataset consists of $n = 3222$ observations about TV ads. It contains the viewership (in GRPs), the date and the time of each broadcasting, and four categorical variables describing the qualities of each spot: Following program type, TV channel, Break inside (describes whether the ad block is inside a program or between two different programs), Buying daypart (represents the time of day with the highest viewership).

3 METHODOLOGY

Smoothing splines are a flexible non-parametric regression technique used for estimating underlying relationships in data. They involve fitting a smooth curve by minimizing the sum of squared residuals subject to a penalty on the curvature (calculated by summing (integrating) the variability in the function g), effectively controlling overfitting. The smoothness of the curve is determined by a tuning parameter (λ), allowing it to adapt to the data's inherent noise and capture underlying trends more effectively.

$$\sum_{i=1}^{n}(y_i - g(x_i))^2 + \lambda \int g''(t)^2 dt \to \text{MIN}, \tag{1}$$

4 EMPIRICAL PART

As a benchmark model to comprehend the general dependencies in the dataset, a base GAM with a single smoothing spline for time (Model 1) was developed. Therefore, the estimation will be grounded in the following equation:

$$GRP_i = \beta_0 + s(time_i) + \sum_{j=2}^{n} \beta_j x_{ij} + u_i, \tag{2}$$

The sum in the equation takes all variables (x_ij) (all of them are dummy variables) except for the time variable (hence $j = 2 \dots n$). All β parameters and the parameters of smoothing splines are optimized to minimize the sum of the squares of residuals (SSR). The sort of program that follows, the TV channel, and the hour are some of the most important variables impacting viewership (GRP). Some TV channels cater to distinct viewers who have different viewing habits. By connecting time and TV channels, this presents a chance for development. Because each TV channel has a different target audience, the viewership based on time may vary. The enriched GAM for time interacting with the TV channel (Model 2) will adhere to the following equation:

$$GRP_i = \beta_0 + \sum_{k=1}^{l} s_k(time_i \times TV\ Channel_{ik}) + \sum_{j=2}^{n} \beta_j x_{ij} + u_i, \tag{3}$$

where there are l smoothing splines for the interaction with time, l is also the number of distinct TV channels in the dataset. The remainder of the equation has the same format as the first. The most important factor is still the next program kind. The most crucial TV Channels 2 and 5 interaction times are during commercial breaks. As can be seen in Table 1, the performance of such a model is much improved (34.2 % drop in root mean square error (RMSE) on the training sample).

The following part compares GAM with smoothing splines and interactions with random forests. Hyperparameters are tuned, impacting random forest performance. Cross-validation involves data split, model construction, and selection of optimal parameters based on the lowest validation MSE, with the final model tested for resilience.

The number of selected variables is set to 4, the number of trees is set to 1000, the minimum size of terminal nodes is set to 10, and the drawn sample size is set to 100 % based on the cross-validation. The mean absolute error (MAE), root mean square error (RMSE), and pseudo coefficient of determination (R^2 derived as $corr(y_i; \hat{y}_i)^2$) for the top-performing random forest are displayed in Table 1.

The partial dependence plot is one way to visualize the relationship between viewership (GRP) and the explanatory variables. They may be applied to any machine learning model and are not just particular to random forests. The basic idea is to illustrate each variable's

Table 1: The comparison of different metrics for GAM with a single smoothing spline (Model 1), GAM with smoothing splines and interaction (Model 2), and Random forest on the training and testing samples. %Δ stands for a relative difference between the metrics for the training and testing sample (the training sample is the reference value).

	Model 1			Model 2			Random forest		
Metrics	Training	Testing	%Δ	Training	Testing	%Δ	Training	Testing	%Δ
Pseudo R^2	0.6079	0.5955	-2.0 %	0.8656	0.8272	-4.4 %	0.9200	0.8891	-3.4 %
RMSE	0.9394	1.0085	+7.4 %	0.5905	0.6181	+4.7 %	0.4368	0.4600	+5.3 %
MAE	0.6596	0.6627	+0.5 %	0.3561	0.3760	+5.6 %	0.2343	0.2536	+8.2 %

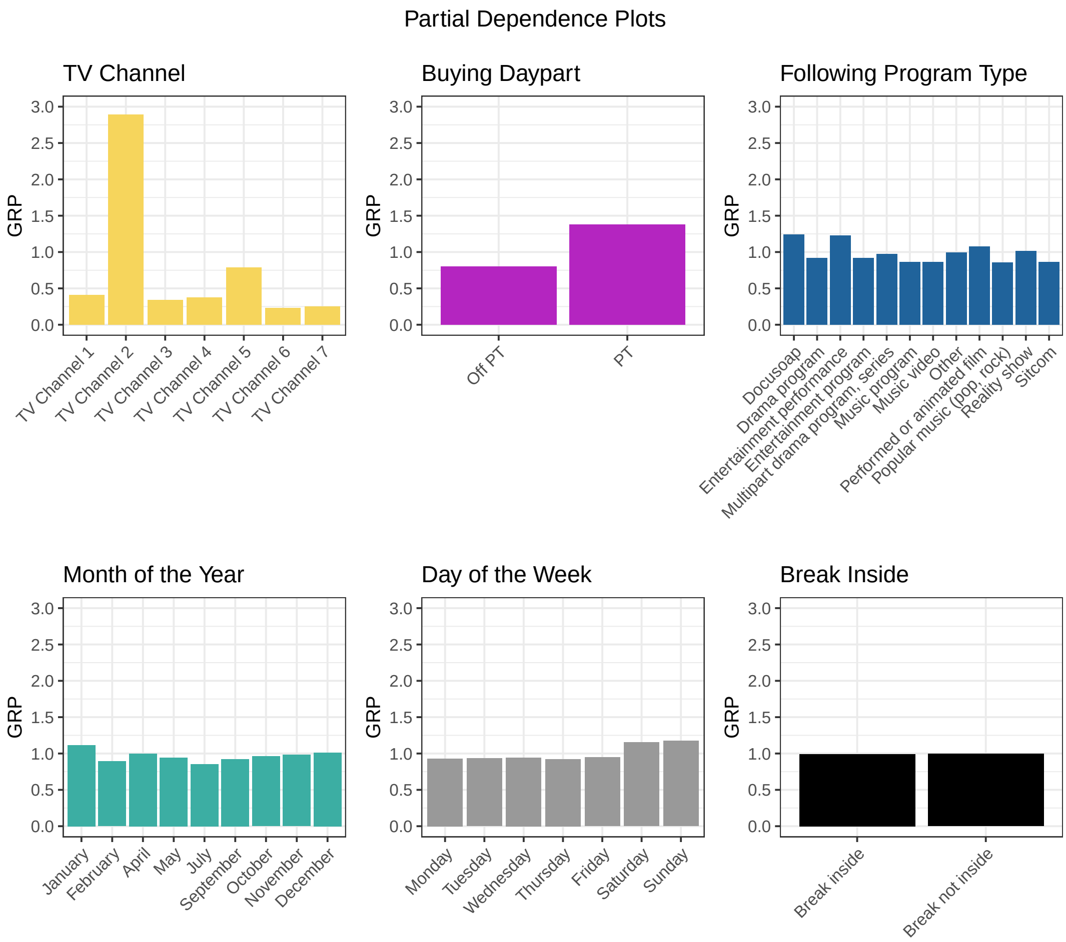

Figure 1: Partial dependence plots for the six explanatory variables.

average marginal impact while keeping other factors constant. In other words, if a given value of the explanatory variable is plugged into all of the observations, the displayed function at that value represents the average prediction [3]. Figure 1 displays the partial dependence charts. Estimating the impact of purchasing daypart using Model 2 and the random forest model is different. Unlike Model 2, where it is unimportant, it is the third most important variable in the random forest model. However, the day of the week and the TV channel stayed the same.

The accuracy evaluation is made based on the testing sample in Table 1. In every statistic, the random forest model surpasses all other models. It also performs better than the competition in training and testing samples. The relative difference between the training and testing samples is determined for each metric in Table 1 to assess robustness (the training metric's reference value). The basic principle behind this is that the difference between the training and testing samples will decrease the more resilient the model. As shown in Table 1, the least relative difference between the training and testing samples in RMSE is shown by Model 2 (GAM with smoothing splines and interactions), making it the most reliable model.

5 CONCLUSION

This research aims to pinpoint TV slots with the highest viewership, which offer a higher potential for attaining the desired impact in an advertising campaign. It was fulfilled by creating and comparing two models.

The random forest model proved the best option when the two methods were evaluated because of its higher accuracy and robustness. GAM could estimate the non-linear behaviour of time quite similarly to the random forest when smoothing splines and interaction were used. This suggests future research in GAM with n-level depth of interactions.

The TV channel and the time of day affected viewership most, according to the results of both the GAM with smoothing splines and interaction and the random forest model. Both models supported the widely held belief that TV station 2 has more viewers overall since it is the most watched local TV station and that viewership increases on weekends. The random forest model supplied more information. The following program kind that performs the best is an entertainment performance followed by a docusoap. Furthermore, it makes no difference whether the advertisement appears during a break because it has little impact.

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Discrete Optimization Methods and Models for Real-World Problem Domain

MANAGING AND OPTIMIZING CONTAINER FLOW IN PORT LOGISTICS

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Abstract: Efficient container management is pivotal to port logistics. This paper details a method for optimizing truck scheduling within a port, using a mathematical model that facilitates concurrent tasks under specific constraints and capacity limitations. The model's efficiency is showcased via simulated instances.

Keywords: scheduling, port logistics, container flow optimization, simulation, mixed-integer linear programming

1 INTRODUCTION AND MOTIVATION

Effective management of port logistics is a cornerstone of global trade, with the transit of shipping containers posing unique challenges. The challenges associated with port operations have been studied extensively, with researchers addressing various aspects. This includes issues like ship routing and stowage planning [8, 10], berth allocation and quay crane assignment [3, 9], vehicle dispatching and gate operations planning [1, 5], and container storage and yard crane scheduling [7, 4]. Each of these subproblems influences overall port efficiency, underscoring the importance of integrated and optimized approaches.

Efficient container handling and movement within ports require complex solutions. However, the optimization of container flow through coordinated scheduling of trucks and ships remains an area needing further research [6].

This paper proposes a mixed-integer linear programming (MILP) model to optimize the container flow within a port environment. The efficiency of this model is demonstrated on simulated instances, underlining its potential to improve port operations.

2 PROBLEM DESCRIPTION

The objective of our study is to improve the efficiency of port logistics through the optimization of container flow. Each truck carries one container, and the containers are loaded onto ships at the dock using a crane. The docks have a given capacity for containers. Constraints in this process include FIFO rules on all roads, one-way directionality, and gates and docks can proccess only 1 truck at a time. In this streamlined process, trucks first gather and queue at the external parking lot. They then proceed through the assigned gate, followed by an offloading phase at the dock. The process culminates with the loading onto the specified ship.

The port operates on a semi-continuous basis where the movement of vehicles begins as early as possible and continues under specific constraints. The capacity of roads, gates, and docks influences the flow of trucks, impacting the overall efficiency of port operations.

The mixed-integer linear programming (MILP) model we propose accommodates these complex features, coordinating simultaneous tasks to optimize container flow. The model allows the system to adapt to varying demands, improving the overall efficiency of the port.

3 FORMAL PROBLEM DEFINITION AND THE PROPOSED MODEL

3.1 Problem data

The problem data is formally represented as follows:

V is the set of all trucks, S is the set of all ships. Port structure is defined by the set of gates G and docks D. Due to space limitations, only the key parts of the mathematical model are presented here and the full model description is available in the appendix [2].

For each truck $v \in V$, gate $g \in G$, dock $d \in D$, ship $s \in S$ we set the following parameters (only the parameters that will be used later in the paper are shown here - the rest is available in the appendix).

- pp_v - planned arrival time of truck v to the parking lot.
- vc_v - unit cost of the use of truck v.
- sc_s - unit cost of waiting of ship s in the port.

3.2 Variables

We introduce 3 types of variables. *Assignment* variables are binary and denote the assignment of specific tasks within the port's operational structure, such as the allocation of trucks to specific gates and docks. *Time* variables are continuous and represent different time aspects concerning the movement and processing of trucks within the port, including arrival and departure times at various points in the port logistics chain. We also have binary *auxiliary* variables that help with complex constraint handling.

For the objective function we only require the following time variables and the rest are available in the appendix.

- $n'_{v,d}$ - departure time of truck v from dock d.
- $s_{s,d}$ - arrival time of ship s at dock d.
- $s'_{s,d}$ - departure time of ship s from dock d.

3.3 Constraints

The model includes several constraint categories, the full description of which is available in the appendix:

- **Assignment Constraints:** These ensure correct distribution of trucks, dictating which entities (gates, docks and corresponding roads) are paired with which vehicles (trucks, ships).
- **Time Constraints:** These control the timeline of operations, dictating when certain tasks should start or finish.

- **Assignment-Time Constraints:** These integrate task assignment and timeline, ensuring that scheduled assignments follow the corresponding temporal limitations.
- **Capacity Constraints:** These limit the volume of tasks assigned, ensuring that no entity is overloaded beyond its operational capacity.

3.4 Objective

Minimization of costs resulting from the cumulative operation time of all vehicles:

$$\begin{aligned} \text{Minimize} \sum_{v \in V} vc_v \left(\sum_{d \in D} n'_{v,d} - pp_v \right) + \\ \sum_{s \in S} \sum_{d \in D} sc_s (s'_{s,d} - s_{s,d}) \end{aligned} \tag{1}$$

4 NUMERICAL RESULTS

Obtaining real-world datasets featuring diverse sizes and structures can be challenging. To test the model, we utilized real data for ship arrivals, coupled with simulated data for truck arrivals specific to each ship. This simulation was based on the outcomes from Yang et al. [11]. Truck departures were scheduled for 5 hours post their respective arrival times.

The computational experiments for all instances were conducted using the Gurobi 10.0.1 solver on a PC equipped with an Intel Core i5-10210U, 1.6 GHz CPU, and 16 GB RAM. A time constraint of 1200 seconds was implemented for each test run.

Port structure was fixed for all the tests, with 3 gates, 4 docks and all the gates were connected to all the docks. For all the other parameters, such as the capacities of the roads and a parking lot, length of the roads and processing time of the facilities, we had a semi-random data.

Instances were given with the number of ships ranging from 1 to 5 and number of trucks ranging from 1 to 10. Instances with less than 5 trucks were solved to optimally in less than 2 seconds. Instances $(4,7), (4,9), (4,10)$ and $(5,10)$ could reach a suboptimal solution with gaps ranging from 54% to 67%. All the other instances were solved optimally in less than 15 seconds.

5 CONCLUSIONS

This paper introduced a problem concerning the effective management of container flow in port logistics, where deliveries from trucks are coordinated and loaded onto specific ships. A mixed-integer linear programming model was proposed to optimize the various processes within a port, from the truck's arrival at the parking lot to the container's loading onto the ship. The efficiency of this model was verified through the application on simulated instances that mirrored real-world circumstances. Results indicated that the model effectively schedules numerous transactions, offering operational advantages. Further work could explore the application of this model over longer planning periods and potentially integrate metaheuristic algorithms to handle instances with a larger volume of deliveries. It would also be beneficial to validate the model using real-world datasets.

6 Acknowledgements

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AUTOMATIC PLANNING OF VEHICLE AND DRIVER SCHEDULES FOR PUBLIC TRANSPORTATION: A CASE STUDY

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Abstract: The combined vehicle and driver scheduling problem consists of scheduling a fleet of vehicles and their drivers to cover a set of tasks at minimum costs. The tasks are given by predetermined time intervals and vehicles are supplied by different depots. There are several known mathematical models that can be used to solve this problem resulting valid schedules of both the vehicles and the drivers. In practical problems there are a lot of vehicle and driver requirements that can not be handled easily and usually have a significant computational demand. We present the results of our research and development project through a case study based on real life experiments at the Budapest Transport Corporation.

Keywords: optimization, vehicle and driver scheduling problem, public transport

1 INTRODUCTION

Operational costs represent a large proportion of the costs of public transport service companies. These costs – as main components – include fleet purchase costs, vehicle fuel and maintenance costs, and driver fees. In the past decades using decision support systems the considered models applied complete solutions of both optimization tasks for the public transportation companies. Vehicle and driver scheduling in public transport can be very complex. In theory, we are generally looking for a global optimum that minimizes both vehicle-related tasks and driver scheduling costs. These two types of costs affect each other, so it is usually the best way to handle the tasks together.

To solve this problem a so called combined vehicle and driver scheduling mathematical optimization models are used. Several versions of such methods are known in the literature. Recently, in the most widely used models the vehicle scheduling problem is formulated as a multicommodity network flow problem ([2], [7], [8]). In this model, the optimal schedule can be calculated as a solution of an integer programming problem. Other models for solving this task also exist. The problem can also be formulated as a set partition problem (see, for example, [9], [5]). For the driver part the best known technology is the so-called Generate and Select (GaS) method. In the first phase a large number of regular shifts are generated. In the selection phase, a subset of them that minimizes the cost and covers the trips is chosen. Usually, both phases require significant computation time. The amount of calculation highly depends on the number of trips and the complexity of working rules. The selection phase can be modeled as a set covering or set partitioning problem.

In 2005, Huisman et al. [6] extended the former combined models and algorithms of the single-depot case [3, 4] to the multi-depot version. This was the first general mathematical formulation of the combined multi-depot problem. In the paper [1] Békési and Nagy presented how the methods used in the above papers were adapted to develop a decision support system for the Budapest Transport Corporation. The aim of this project was to automatically calculate optimal or approximately optimal vehicle and driver schedules for a given list of trips based on the master data and the company specific requirements and parameters in compliance with labor regulations.

This paper overviews how the complete integration was implemented and what kind of specific developments were necessary to take into account all the practical requirements of the company.

2 OVERVIEW OF THE AUTOMATIC CALCULATION PROCESS

Based on [1] we overview the most important properties of the system, the requirements, the input data and parameters. To be able to automatically solve a given problem, all the information necessary for a calculation are stored in packages. Usually a package contains trips from a single line or a group of lines. Each input package contains the following information:

- data of the lines,
- end stations, depots and their parameters,
- trips with time and location information,
- number of the vehicles with their type and availability information,
- parameters of the labor regulations and break rules,
- parking capacities of the stations, parking areas and depots given in 5- minute time intervals for each vehicles type,
- driver change options, as well as break and detour permits.

The solution of the problem must satisfy the following requirements:

- exactly one vehicle and driver schedule must cover each trip,
- no more vehicles can be used than the number of them given in the package,
- vehicle and driver schedules must be regular,
- fuel consumption and parking rules must be satisfied,
- the required technological and compensatory times must be kept between two trips,
- driver change rules must be observed.

There are three phases in our process.

Phase 1. In the first phase of the calculation, the appropriate input data and parameters are read and a directed graph is generated. In this graph the trips are the nodes which are completed with departure and arrival depot vertices. In our model a multiple depots problem is handled, therefore more depot nodes being generated. Two nodes in the graph are connected by a directed arc if the trips representing them are compatible, i.e. they can be processed subsequently. The arc length always represents the net working time of the driver, which corresponds to the objective function. Short and long arcs are defined in [6]. Short arcs represent those shorter events when the driver remains with the vehicle, while the long arcs represent those events when the driver stops the vehicle in the parking place and the bus remains unattended.

While building the graph we take into account several "side-conditions", as technological and compensatory times, location information, vehicle types, labor rules and several parameters given in the package. These parameters influence the structure of the graph and so the solution itself. The most important parameters are the followings:

- the length of the permitted minimum and maximum working time in minutes,
- minimum and maximum average net working time in minutes,
- maximum and minimum length of breaks in minutes,
- maximum end station waiting time in minutes,
- minimum and maximum driver change time in minutes,
- latest departure time from the depots,
- earliest arrival time to the depots,
- earliest start time of divided rest periods.

The user can check the results of different scenarios by modifying these parameters, so the system helps the decision process to find the best alternative.

Phase 2. Having completed the graph, all regular driver schedules are generated. Only the schedules that correspond to all labor rules are accepted here. The generated driver schedules are stored. If too many schedules are generated, the process is stopped and a node contraction heuristic method is applied to decrease the complexity of the problem. There are also some parameters which limit the number of schedules.

Phase 3. In the third phase a mathematical model is constructed and solved. The details of the model can be found in [1]. In case of successful solution the results are read from the solver and they are sent back to the company's information system using an output package. This contains the vehicle and driver schedules with the required data. The information system applies the output data in the daily schedules of vehicles and drivers too.

3 NEW DEVELOPMENTS AND IMPROVEMENTS

3.1 Parallel schedule generation

For the generation of the regular shifts we use depth-first search algorithm started from the special departure depot vertices of the graph. The shift generation can be very time consuming so we implemented it as a parallel algorithm. At this stage the shifts are represented as paths in the graph starting at the departure depot vertex and ending at the arrival depot vertex. For the parallel generation as an initial step we generate and store all the possible prefixes of the paths from the departure depot vertex to a given small depth (e.g. 4) and store them in a list. The process can be summarized as follows.

- In the main part of the generation each thread selects an unprocessed path-prefix from the list, and generates all the possible regular shifts with this prefix.

- After finishing the process, the thread stores the generated shifts and selects a new unprocessed prefix and generates its shifts.
- If there are no unprocessed shifts, the thread stops.
- If all threads are stopped the generation is finished.

3.2 Smart node contraction

The basic way of node-contraction is the greedy strategy that is described in [1]. Using this strategy we remove many edges from the graph. As a consequence many break possibilities and many driver change possibilities are also lost. Sometimes it causes infeasibility. Recognizing this problem we developed a smart node-contraction algorithm. Depending on its parameter settings this algorithm keeps more break and driver change possibilities than the greedy strategy.

3.3 Parking constraints for more vehicle types

In our problem the parking capacities are defined also for variant vehicle categories. The categories differ by size. Smaller categories can also occupy the parking lots of the bigger ones. Bigger categories cannot use the parking lots of smaller categories. For handling this problem we had to introduce new variables and constraints in the mathematical model. The variables store the vehicle category that will be used for executing a given task. The constraints guarantee that the number of vehicles at the same location and in the same time interval will not exceeds the capacities.

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EDGES WHICH ARE CRITICAL FOR EMERGENCY SERVICE SYSTEMS

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Abstract: Detection of edges that are critical for a designed emergency service system is important when we study the vulnerability of such systems. In our work, we study several possible approaches regarding how to assign a critical value to each edge. This value represents the effect of the elongation of its traverse time on the whole emergency system. In experiments, we compare probabilistic and non-probabilistic approaches. We also suggest a method for setting the parameters in a probabilistic approach.

Keywords: Critical edges, transport network, p-median, gamma distribution.

1 INTRODUCTION AND DEFINITIONS

When local authorities plan the reconstruction of road segments or bridges, it is useful to know the impact of increased driving time on public service systems (emergency service system). For this reason, two quantities – the transportation performance and the changes of transportation performance - were defined and studied [6], [9], [3] and [5]. These quantities allow to assign a value to every edge. We call this number the critical value of the edge. With the help of these numbers, we can determine the edges with the highest impact on the functionality of the designed emergency system – the most critical edges. If we know the edges, which are the most critical for the emergency system (located ambulance stations), then we can properly react to the situation, when such edges are affected. For example, we can relocate ambulance stations or improve some road segments which belong to the detours. In our contribution, we study and compare two methods for the evaluation of edges by these values.

Transportation network $\boldsymbol{G}$ is ordered quadruple $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{w}, \boldsymbol{t})$, where $\boldsymbol{V}$ is the set of vertices, $\boldsymbol{E}$ is the set of edges, $\boldsymbol{w}(\boldsymbol{u})$ is the weight of the vertex $\boldsymbol{u} \in \boldsymbol{V}$, and $\boldsymbol{t}(\boldsymbol{e})$ is the driving time through the edge $\boldsymbol{e} \in \boldsymbol{E}$ - measured in minutes. We also consider the subset $\boldsymbol{S} \subset \boldsymbol{V}$ of the located emergency stations. The transportation performance [5] is the number

$$\boldsymbol{W} = \sum_{\boldsymbol{u} \in \boldsymbol{V}} \boldsymbol{w}(\boldsymbol{u}) \cdot \boldsymbol{d_G}(\boldsymbol{u}, \boldsymbol{S}),$$

where $\boldsymbol{d_G}(\boldsymbol{u}, \boldsymbol{S})$ is the driving time between vertex $\boldsymbol{u}$ and the nearest vertex from the set $\boldsymbol{S}$. Let the edge $\boldsymbol{e} \in \boldsymbol{E}$ be given. Let $\boldsymbol{x}$ be the extension of the driving time on $\boldsymbol{e}$. The total transportation performance with this extension is denoted by $\boldsymbol{W_e}(\boldsymbol{x})$. The change of total transportation performance is the function of one variable:

$$\boldsymbol{V_e}(\boldsymbol{x}) = \boldsymbol{W_e}(\boldsymbol{x}) - \boldsymbol{W}.$$

It was proved in [3] and [4] that $\boldsymbol{V_e}(\boldsymbol{x})$ is piece-wise linear, concave, and non-decreasing function with domain $\langle \boldsymbol{0}, \infty)$.

Example 1. We can consider the network $\boldsymbol{G}$ in Figure 1a. Weights of vertices are $\boldsymbol{w(u_1) = 3}$, $\boldsymbol{w(u_2) = 2}$ and $\boldsymbol{w(u_3) = 1}$. Set $\boldsymbol{S}$ contains only vertex $\boldsymbol{v}$ and the affected edge is $\boldsymbol{e = \{u_2, v\}}$. Graph of the function $\boldsymbol{V_e(x)}$ is in Figure 1b. The formula for this function is

$$\boldsymbol{V_e(x) = \begin{cases} 5x, if\ x \in \langle 0, 1); \\ 2x + 3, if\ x \in \langle 1, 2); \\ 7, if\ x \in \langle 2, \infty). \end{cases}}$$

Algorithms for computation of this formula can be found in [6] and [5].

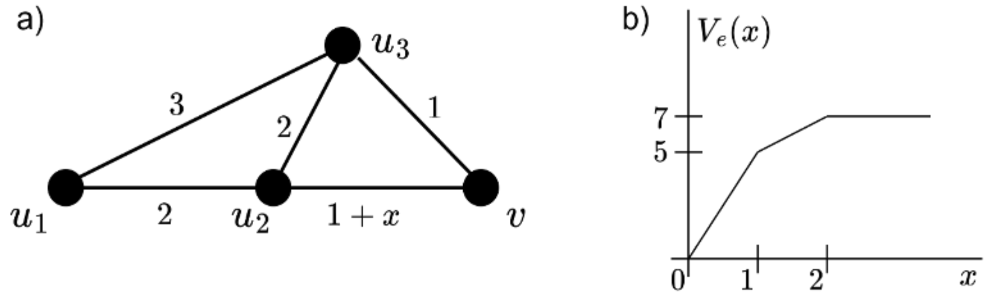

*Figure 1:*a) Network G from Example. b) Graph of the change of total transportation performance.

2 THE EVALUATION OF EDGES

When we have the formula for the function $V_e(x)$, we can assign a value for every edge e. We consider two approaches.

1) In the first version, we assign to each edge e the area under the function $V_e(x)$:

$$C_{S,1} = \int_0^T V_e(x)dx,$$

where T is an appropriate upper bound. In our tests, we use value $T = 60$ minutes.

2) If we consider the fact that the different values of delay occur with different probabilities, then we can assign to every edge the value:

$$\boldsymbol{C_{S,2} = \int_0^\infty V_e(x) f(x) dx,}$$

where $f(x)$ is a convenient probabilistic density function. In our tests, we use Gamma distribution [1], since its special case – Erlang distribution – is used for modelling of the waiting time in Poisson processes (and these processes are used, for example, in the modelling of occurrence of unexpected incidents in traffic). The probabilistic density function of Gamma distribution is

$$\boldsymbol{f(x) = \lambda^\alpha \cdot x^{\alpha-1} \cdot e^{-\lambda x} / \Gamma(\alpha),}$$

where λ, α are parameters of this distribution and $\Gamma(\alpha)$ is gamma function. In our tests, we use parameters $\lambda = 2$ and $\alpha = 4$.

3 COMPUTATIONAL RESULTS

As a testing benchmark, we use the transport network of Zilina self-governing region in Slovakia. This network has 432 vertices, where 315 of them represent municipalities with non-zero weights. The rest of them represents crossroads. Ambulance stations are located in exactly

29 municipalities. We computed the values $C_{S,1}$ and $C_{S,2}$ for all 494 edges of the network. The results of the tests can be seen in Table 1.

Table 1: Lists of five edges with largest values $C_{S,1}$ and $C_{S,2}$ in Zilina self-governing region.

$\boldsymbol{C_{S,1}}$			$\boldsymbol{C_{S,2}}$		
edge		*value*	*edge*		*value*
Trstena	Liesek	129 600	Zilina	crossr. 353	219.75
Bytca	crossr. 349	95 400	crossr. 411	Turcianske Teplice	213.63
crossr. 349	Hvozdnica	95 400	Martin	Kostany nad Turcom	200.09
crossr. 411	Turcianske Teplice	88 859	Trstena	crossr. 346	199.79
crossr. 327	Turzovka	81 000	Tvrdosin	crossr. 346	183.79

We can see that the results for values $\boldsymbol{C_{S,1}}$ and $\boldsymbol{C_{S,2}}$ are different. We suppose that it is more convenient to use criterion $\boldsymbol{C_{S,2}}$ for the detection of the most critical edges since this criterion also includes probabilities of driving time elongation. The setting of values for parameters $\boldsymbol{\lambda}$ and $\boldsymbol{\alpha}$ is an important question.

4 VALUES OF PARAMETERS

The optimal situation occurs if we have sufficiency of data for ambulance driving times on each road segment of the network. Then it is possible to use some statistical methods and create an appropriate probabilistic model (with density function $\boldsymbol{f(x)}$) for every road segment. However, this is not a usual situation. For most of the road segments, there is a lack of data. In such situations, we can consider models of dependency of driving time of fire engines and ambulances on distance from papers [8] and [2].

The authors of these works express the dependency of the median, mean and coefficients of variance of the driving time on the distance. For example, the dependency of the median can be expressed as follows:

$$\boldsymbol{m(d)} = \begin{cases} \boldsymbol{c\sqrt{d}, d \leq d_0} \\ \boldsymbol{ad + b, d > d_0} \end{cases},$$

where $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{d_0}$ are appropriate parameters derived from velocity, acceleration, and distance.

Relations between parameters $\boldsymbol{\lambda}$ and $\boldsymbol{\alpha}$ of gamma distribution and its mean, median and variance are well-known [7]:

$$\boldsymbol{mean = \frac{\alpha}{\lambda}},$$

$$\boldsymbol{median \approx \frac{\alpha}{\lambda} \cdot \left(1 - \frac{1}{3\alpha + 0.2}\right)},$$

$$\boldsymbol{variance = \frac{\alpha}{\lambda^2}}.$$

If we have estimations of the mean, median or variance, then we can derive the values of parameters $\boldsymbol{\lambda}$ and $\boldsymbol{\alpha}$ for any road segment of length $\boldsymbol{d}$. We prepare the computation of these parameters for all self-governing regions in Slovakia.

5 CONCLUSIONS

In our contribution, we study the detection of the most critical edges for designed emergency service systems. We suggest a method for the evaluation of each edge of the transport network, which is based on probabilistic models of ambulance driving time. In the near future, we plan to compute parameters of the probability (gamma) distribution for each road segment in our region (Zilina self-governing region) and finish the detection of edges that are most critical for the emergency system.

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DETECTION OF CRITICAL VERTICES FOR THE DESIGNED SERVICE SYSTEM

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Abstract: In our contribution, we introduce and study the topic of the detection of vertices, for which the elongation of their driving time has a large impact on the whole service system. In our approach, we replace the elongation of driving time through a given vertex by elongations on the edges that are incident with this vertex. This modification allows us to use ideas developed in our previous works for the detection of critical edges.

Keywords: Transportation network, critical vertex, service system, transportation performance.

1 INTRODUCTION

Increased travel time of the elements of transport network can have a marked impact on the functionality of the public service systems. Our previous research dealt with the detection of edges that are critical for the designed public service systems, which was based on the notions of total transportation performance and change of total transportation performance in transport networks. This contribution is an introduction to the problem of the detection of vertices which are the most critical for public service systems. We concern ourselves with the detection of individual vertices because the detection of critical subsets is a problem with exponential complexity.

An ordered quadruple $G = (V, E, w, t)$ is a transport network, if V is a set of vertices (municipalities and crossroads), E is a set of edges (road segments), $w(u)$ is the weight of the vertex $u \in V$ (number of inhabitants), and $t(e)$ is the driving time through the edge $e \in E$ (for our purposes measured in minutes). Subset $U \subset V$ represents the set of service system customers (vertices with non-zero weight). Subset $S \subset V$ is the set of located service centers. We say that an edge e is incident with the vertex v, if v is the end-vertex of e ($v \in e$). The total transportation performance [6] is

$$W = \sum_{u \in U} w(u) \cdot d_G(u, S),$$

where $d_G(u, S)$ is the distance between u and the nearest vertex from S. Let us suppose that due to unusual circumstances, the driving time through a vertex $v \in V$ be elongated by a value $2x$. If v is an "end-vertex" – it means located service center ($v \in S$) or served customer ($v \in U$), then we will use the elongation time x. The total transportation performance with this elongation on the vertex v will be denoted by $W_v(x)$. The change of total transportation performance will be denoted by

$$V_v(x) = W_v(x) - W.$$

The properties of this function for edges and sets of edges were studied in [3], [4] and [7]. In [4], it was proved that the change of total transportation performance $V_Y(x_1, \cdots, x_k)$ is a continuous, piece-wise linear and concave function of k variables, where $Y = \{e_1, \cdots, e_k\}$ is the set of affected edges and $x_1,\cdots,x_k$ represents the extensions of driving times on the edges from Y. In [5], the algorithm for the computation of the formula for functions $V_Y(x_1)$ and $V_Y(x_1, x_2)$ is given. For $k > 2$ edges, the computation of this formula is more complicated and only few exceptions, with fast algorithm, can be found. One of them is the situation, when the affected edges e_1,⋯, e_k are exactly those, which are incident with the vertex v. It is possible to show that simplification $x_1 = x_2 = \cdots = x_k = x$ leads to the equality

$$V_Y(x, \cdots, x) = V_Y(x) = V_v(x).$$

Consequently, $V_v(x)$ is a continuous, piece-wise linear and concave function of one variable.

The situation can be seen in the following example.

Example 1. Let the network G be given (Figure 1a). Customers are represented by vertices u_1 and u_2 with weights $w(u_1) = 2$ and $w(u_2) = 3$. The service center is located in the vertex w and the affected vertex is v. The graph of the function $V_v(x)$ can be seen in Figure 1b.

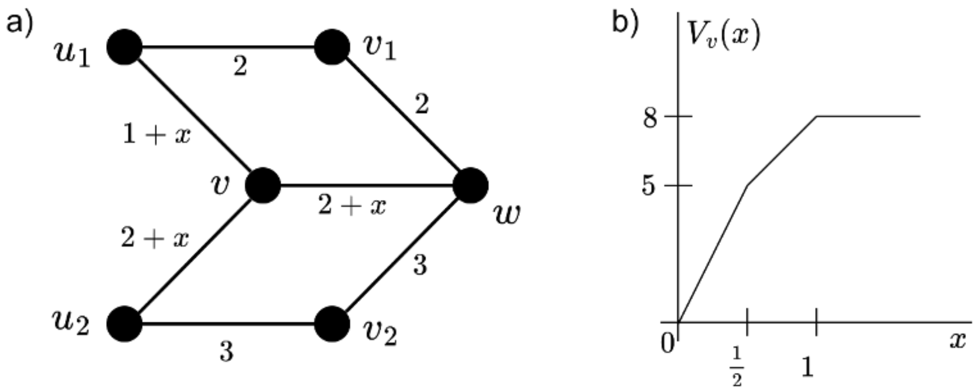

*Figure 1:*a) Network G from Example 1. b) Graph of the change of total transportation performance.

The formula for $V_v(x)$ is

$$V_v(x) = \begin{cases} 10x, if\ x \in \langle 0, 1/2), \\ 2 + 6x, if\ x \in \langle 1/2, 1), \\ 8, if\ x \in \langle 1, \infty). \end{cases} \quad (1)$$

2 ALGORITHM FOR THE COMPUTATION OF FUNCTION $V_v(x)$

For the computation of the formula for $V_v(x)$, we can use the following modification of the algorithm from paper [5] (network $G - e$ is replaced by network $G - v$ and step 3 is modified):

1. For every customer $u \in U$, compute values $d_G(u, S)$ and $d_{G-v}(u, S)$.
2. Create a list of vertices $u_1, \cdots, u_k \in U$, for which $0 < d_{G-v}(u_i, S) - d_G(u_i, S) < \infty$.
3. For $i = 1, \cdots, k$, order the values $(d_{G-v}(u_i, S) - d_G(u_i, S))/2$ into the non-decreasing sequence.
4. Omit repeated values to obtain increasing sequence x_1,⋯,x_m.
5. Add values $x_0 = 0$ and $x_{m+1} = x_m + 1$ to this sequence.
6. For $i = 0,1, \cdots, m + 1$, compute values $y_i = V_v(x_i)$.
7. For each interval $\langle x_i, x_{i+1})$, compute parameters a_i and b_i of function $y = a_i x + b_i$.

Values $y_i = V_v(x_i)$ from step 6 can be computed as follows:
A driving time through every edge, which is incident with vertex $\boldsymbol{v}$, is extended by value $\boldsymbol{x_i}$. We denote this network by $\boldsymbol{G_i}$. Then

$$\boldsymbol{W_v(x_i)} = \sum_{\boldsymbol{u \in U}} \boldsymbol{w(u) \cdot d_{G_i}(u, S)},$$

$$\boldsymbol{V_v(x_i) = W_v(x_i) - W}.$$

It was mentioned earlier that for $\boldsymbol{i = 0, \cdots, m}$, the function $\boldsymbol{V_v(x)}$ is linear on each interval $\langle \boldsymbol{x_i, x_{i+1}} \rangle$. Then

$$\forall \boldsymbol{x} \in \langle \boldsymbol{x_i, x_{i+1}} \rangle \;\; \boldsymbol{V_v(x) = y = a_i x + b_i}.$$

It is a known fact from analytical geometry that the coefficients $\boldsymbol{a_i}$ and $\boldsymbol{b_i}$ of the function

$$\boldsymbol{y = a_i x + b_i}$$

can be computed in the following way:

$$\boldsymbol{a_i} = \frac{\boldsymbol{y_{i+1} - y_i}}{\boldsymbol{x_{i+1} - x_i}},$$

$$\boldsymbol{b_i} = \frac{\boldsymbol{y_i \cdot x_{i+1} - x_i \cdot y_{i+1}}}{\boldsymbol{x_{i+1} - x_i}}.$$

Coefficients $\boldsymbol{a_m}$ and $\boldsymbol{b_m}$ for interval $\langle \boldsymbol{x_m}, \infty \rangle$ are the same as for interval $\langle \boldsymbol{x_m, x_{m+1}} \rangle$.

This is an algorithmic approach to the computation of a formula for the function $\boldsymbol{V_v(x)}$. When we want to finish our computations and determine the most critical vertices, we need to assign to each vertex a certain number, which we call the critical value of the vertex, and it is denoted by $\boldsymbol{C_S(v)}$. The computation of this value involves a probabilistic density function which simulates the probability of elongation of travel time through the vertex (for example a crossroad or a municipality). The determination of the appropriate density function will be part of our future research.

Example 2. We apply this algorithm to the network and values from Example 1.
1) $d_G(u_1, S) = 3$, $d_{G-v}(u_1, S) = 4$, $d_G(u_2, S) = 4$ and $d_{G-v}(u_2) = 6$.
2) $x_0 = 0$, $x_1 = 1/2$, $x_2 = 1$ and $x_3 = x_2 + 1 = 2$.
3) $y_0 = V_v(0) = 0$, $y_1 = V_v(1/2) = 5$, $y_2 = V_v(1) = 8$ and $y_3 = V_v(2) = 8$.
4) $a_0 = (5 - 0)/(1/2 - 0) = 10$ and $b_0 = (0 \cdot 1/2 - 0 \cdot 5)/(1/2 - 0) = 0$;
$a_1 = (8 - 5)/(1 - 1/2) = 6$ and $b_1 = (5 \cdot 1 - 1/2 \cdot 8)/(1 - 1/2) = 2$;
$a_2 = (8 - 8)/(2 - 1) = 0$ and $b_2 = (8 \cdot 2 - 1 \cdot 8)/(2 - 1) = 8$.
We can see that we obtain the same coefficients as in (1).

3 CONCLUSIONS

In our contribution, we introduce a method for the detection of vertices, which are critical for the functionality of the designed service system. For computation, we modify the notion of changes of transportation performance, which was used in our previous work for the purpose of the detection of critical edges. We provide an algorithm for the computation of the formula for this function. If we want to evaluate every vertex by a concrete number, it is necessary to find an appropriate mathematical (probabilistic) model for extensions of travel time through the vertices (such as municipalities and crossroads) of the network.

In [5], we assign to each edge the following value:

$$C_S(e) = \int_0^\infty V_e(x) \cdot f(x) dx,$$

where $f(x)$ is the probabilistic density function of the gamma distribution. We plan to use a similar approach for vertices. It means we can assign to each vertex v value:

$$C_S(v) = \int_0^\infty V_v(x) \cdot f(x) dx.$$

An open question is: Which probabilistic density function $f(x)$ can we use in this case? Our future research will focus on finding an answer to this question. We suppose that ideas developed in [1], [2] and [8] could provide us useful tools, which are necessary to complete the evaluation of vertices.

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OPTIMIZATION OF TREE BUCKING WITH QUALITY AND VOLUME REQUIREMENTS

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Abstract: Maximizing the value of harvested trees is an important objective of the forest industry. Bucking, the process responsible for converting a harvested tree into logs, plays a significant role in determining the ultimate value of a tree. This paper considers the problem of maximizing bucking value on the forest stand level, finding optimal bucking options for multiple tree stems simultaneously. The bucking of these trees has to satisfy a market demand for the various log types while meeting quality requirements of the corresponding tree segments used for the logs. This paper introduces a mathematical model to address the problem described above.

Keywords: tree bucking, optimization, quality constraints

1 INTRODUCTION

Tree bucking is an important process in the chain of transforming trees to lumber products. It is responsible for converting a batch of available tree stems into logs. The exact method and constraints of this procedure highly depend on the approach taken to address the problem. Most importantly, different approaches are applied in the case of optimizing the bucking of individual tree stems (buck-to-value) and bucking multiple stems of a forest stand while also considering demands (buck-to-order). Mathematical approaches for these problems have existed since the 1960s; Smith and Harrel proposed a linear programming formulation [5] for assigning heuristically derived bucking patterns to stems, while Pnevmaticos and Mann introduced a dynamic programming algorithm [3] for the optimization of single stems. More efficient approaches for generating cutting patterns were developed by Näsberg [2] and Sessions [4]. Approaches optimizing a forest stand usually integrate tree bucking with either production planning decisions [6] or optimizing price matrices [1] to maximize the extracted value from the stems. While there has been extensive work done for various special cases of this problem class, the case where quality details of each segment are know in advance has not been studied to our knowledge. This paper presents a mathematical model for bucking a set of non-uniform tree stems to service a given number of orders for logs of various classes. Each log class has a diameter and a quality requirement that the stem segments allocated to it have to satisfy. Preliminary computational results are given to show the efficiency of this proposed model.

2 PROBLEM DEFINITION

Consider the problem of cutting available tree stems into logs based on customer order requirements. Every available stem is unique with regards to length and dimensions, and a preliminary mapping of the position and size of defects in each steam is also available. Stems are broken down into segments of unit length. More formally, this means that for each segment, we know their diameter and quality information (number of defects) The incoming orders for logs have requirements for dimensions (length, diameter) and minimum quality (described by the maximum amount of total defects in the segments chosen for the log). The goal of the problem is to serve all customer orders by cutting logs from the available stems while leftover waste and the number of used stems is also minimized.

A simplified representation of the stem is based on the above description can be imagined as the following:

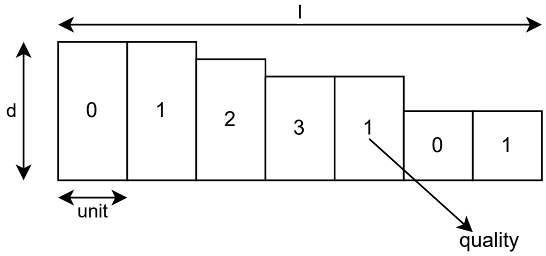

Figure 1: Representation of a tree stem and the quality of its segments

3 MATHEMATICAL MODEL

Based on the description in Section 2, the following notations have to be introduced :

- x_{ijk}: binary decision variable denoting if log of type i starts at segment j of tree stem k
- y_{ijk}: binary derived variable denoting if segment j of tree stem k is used for log type i
- z_k: binary derived variable denoting if tree stem k is used or not
- $l_T(k)$: length of tree stem k
- $w_T(j,k)$: diameter of segment j on tree stem k
- $f(j,k)$: quality (number of faults) in segment j of tree stem k
- $l_L(i)$: length log type i
- $w_L(i)$: required width/diameter length of log i
- $q(i)$: required quality (max number of faults) of log type i
- $o(i)$: total number of orders of log type i
- T: set of tree stems, L: set of log types, S_k: set of segments on tree stem k
- U: constant for the unit length of one segment

Using the above notation, an integer programming model for the problem can be formulated in the following way:

$$\min \sum_{k\in T}\sum_{j\in S_k} Uw_T(j,k)(z_k - \sum_{i\in L} y_{ijk})$$

s.t.

$$y_{ijk} = \sum_{m=j}^{j-l_L(i)+1} x_{imk}, \quad \forall i \in L, \forall k \in T, j \in T_k \tag{1}$$

$$x_{ijk}(j + l_L(i) - 1) \le l_T(k), \quad \forall i \in L, k \in T, j \in T_k \tag{2}$$

$$x_{ijk}(\sum_{m=j}^{j+l_L(i)-1} f(m,k)) \le q(i), \quad \forall i \in L, k \in T, j \in T_k \tag{3}$$

$$\sum_{k\in T}\sum_{j\in S_k} x_{ijk} = o(i), \quad \forall i \in L \tag{4}$$

$$\sum_{i\in L} y_{ijk} \le 1, \quad \forall k \in T, j \in T_k \tag{5}$$

$$w_L(i)y_{ijk} \le w_T(j,k), \quad \forall i \in L, k \in T, j \in T_k \tag{6}$$

$$\sum_{i\in L}\sum_{j\in S_k} y_{ijk} >= z_k/|S_k|, \quad \forall k \in T \tag{7}$$

$$\sum_{i\in L}\sum_{j\in S_k} y_{ijk} <= |S_k|z_k, \quad \forall k \in T \tag{8}$$

$$x_{ijk}, y_{ijk}, z_{jk} \text{ binary}, \quad \forall i \in L, k \in T, j \in T_k \tag{9}$$

The objective function aims to minimize the total volume of leftover tree segments. Segments belonging to unused trees are not considered. Constraint (1) registers for each segment on every stem whether it belongs to a log of type i or not (regardless of which exact log it is). Constraint (2) ensures that a log assigned to a stem will actually fit on the stem lengthwise. Constraint (3) controls the segments of the desired quality for each log. Constraint (4) guarantees the completion of the required number of orders for each log. Constraint (5) ensures that a segment that belongs to a log cannot be chosen as part of another log. Constraint (6) ensures the segments of the desired width for each log. Constraints (7) and (8) register for each stem if it has at least one used segment. This is needed for the objective function. Constraint (9) guarantees that our variables are binary.

4 PRELIMINARY RESULTS

The model has been tested on randomized input data so far. For each input instance, a set of n unique tree stems were generated, each with a maximum length of m segments (both n and m was chosen as multiples of 10 in the [10,100] interval). Diameter and quality data were also randomly assigned to each segment. Two sets of o incoming log orders were generated for each (n,m) pair; ones with 5 different lengths (3-7 segments) and with 10 different lengths (3-12 segments). Quality requirements and total numbers for these log orders were also randomly chosen. As this input generation setup does not guarantee feasibility with regards to the required quality of the log orders in some cases, inputs were generated until there were 10 feasible ones for each (n,m,o) tuple. The total number of logs to cut depended on the log order types. As the 10 log types were chosen from a longer length range, the total for these inputs was around $55 - 56\%$ of the logs in corresponding (n,m) instances with only 5 log types. While the inputs with 5 order types were solved under a minute on average for all (n,m) pairs, this was only possible for the instances of $n \leq 30$ and $m \leq 50$ in the cases with 10 log order types. Other instances took over 5-10 minutes on average to solve, with some higher n values even requiring more than 30-40 minutes in some cases.

5 CONCLUSIONS AND FUTURE WORK

This paper introduced a mathematical model for log bucking where quality information is available for each tree segment. The study considered a simplified representation of tree stems and quality, and was also tested on randomly generated instance sets without using any real-world distributions or data. These simplifications were introduced to show the efficiency of the approach. However, to make it applicable in real-world scenarios, further extensions and adaptations are necessary.

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DISTRIBUTED SYSTEM BASED SENSOR NETWORKS AND THE CONNECTED P-MEDIAN PROBLEM

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Abstract:

In wireless networks, efficient gateway deployment while serving demands from sensors and ensuring a seamless connection between them for load sharing is vital. To this end, we define a new problem in this paper and develop a baseline mathematical model to tackle it. Since the model comprises computationally inefficient cycle elimination constraints, an alternative model and an algorithmic procedure exploiting the baseline model are presented and a computational analysis is performed to test their performance.

Keywords: sensor networks, facility location, mathematical programming

1 INTRODUCTION

Distributed system based sensor networks are organized on two levels [8]. The first one is the level of sensors measuring the environment; they are connected to send data each other. The second level of the network is the level of gateways. They are forming a distributed system meaning that they should be able to store the data in a distributed way. Data from the sensors are travelling to the gateways, each sensor is arranged to a gateway node through a path determined by the routing. For the gateway placement the sensor network is given and our goal is to identify the gateway nodes in an optimal way. We suppose that the number p of gateways is given. For the communication cost, we can consider positive weights on the links of the network; however, we assume that a different cost an the links are defined as the weight between potential gateways. The process of data collection by the gateways correspond to the classical p-median problem [5]. However, because of the distributed system feature, the subnetwork formed by the median nodes needs to be connected by which we are facing the connected p-median problem.

A well-known related problem is the so-called *Connected Facility Location Problem* where the backbone means a Steiner Tree [4] connecting the median nodes, but no connectivity is required as their induced subgraph. In our approach, we handle the following problem on a sensor network: Given that each sensor has a communication demand, each gateway has a fixed installation cost, satisfying a demand from a gateway incurring transportation cost and building a connection line between deployed gateways causing a connection cost, identify the optimal p gateway locations forming a spanning tree and the optimal sensor assignments to each of these locations at a minimum total cost of opening, connection and transportation cost. We derive a novel mathematical model (baseline) for this problem. Since the model contains intractable cycle constraints, an alternative model and an algorithmic procedure exploiting the model is presented in this study to alleviate this difficulty. They are tested on randomly generated and benchmark instances.

2 PROBLEM DESCRIPTION

In the rest of the paper, we will use the standard concepts and notations of facility location problems [7]. In a weighted network, assume that each vertex $i \in V$ correspond to a customer location having some demand D_i. Each customer can be served by a facility in the network at a some cost which is the product of D_i and the shortest distance between the customer and demand node, d_{ij}. We call the cost transportation cost. In the classical p-median problem, we would like to find the location of p facilities such that all customer demands are satisfied at a minimum total transportation cost [5]. On top of that, these p facilities might need to share the workload by building a connected induced subgraph [6]. Because the installation of an edge $(i, j) \in E$ between facility $i \in F$ and $j \in F, j \in \delta_i$ where E is an edge set, F is a set of facilities and δ_i is a set of adjacent vertices of vertex i incurs an installation cost c_{ij}, we name the cost of the spanning tree over p nodes as connection cost. Finally, a facility could also have an opening cost, f_i. As a result, we seek to find a spanning tree over opened p facilities serving customers at a minimum total opening, transportation and connection cost. We name the problem as *fixed charge weighted connected p-median problem*

3 BASELINE MODEL

At first, we devise a new binary integer programming (BIP) model to solve our problem. Since it has some complications, a mixed integer programming (MIP) version of it will be introduced at section 4.1.

$$Minimize \sum_{i \in F} f_i y_i + \sum_{i \in V} D_i \sum_{j \in F} d_{ij} x_{ij} + \sum_{(i,j) \in E: i<j, i,j \in F} c_{ij} Z_{ij} \tag{1}$$

subject to

$$\sum_{j \in F} x_{ij} = 1 \qquad i \in V \tag{2}$$

$$x_{ij} \le y_j \qquad i \in V, j \in F \tag{3}$$

$$\sum_{j \in F} y_j = p \tag{4}$$

$$\sum_{(i,j) \in E: i<j, i,j \in F} Z_{ij} = p - 1 \tag{5}$$

$$\sum_{(i,j) \in E: i<j, i,j \in W} Z_{ij} \le |W| - 1 \qquad W \subset F, 2 \le |W| < p \tag{6}$$

$$Z_{ij} \le y_i \qquad (i,j) \in E : i < j, i \in F, j \in F \tag{7}$$

$$Z_{ij} \le y_j \qquad (i,j) \in E : i < j, i \in F, j \in F \tag{8}$$

$$y_i \le \sum_{j \in \delta_i, j \in F} Z_{ij} + Z_{ji} \qquad i \in F \tag{9}$$

$$Z_{ij} \in \{0, 1\} \qquad (i,j) \in E : i < j, i \in F, j \in F \tag{10}$$

$$x_{ij} \in \{0, 1\} \qquad i \in V, j \in F \tag{11}$$

$$y_j \in \{0, 1\} \qquad j \in F \tag{12}$$

4 SOLUTION METHODOLOGIES

4.1 Miller-Tucker-Zemlin Model (MTZ)

It is apparent that constraint (6) has exponentially many constraints for large amounts of p and $|V|$. To target this issue, we write an alternative constraint $u_i - u_j + pZ_{ij} \leq p - 1$ adopted from travelling salesperson problem [4] for subtour elimination where u_i is a dummy nonnegative real variable to ensure cycle breaking. In Section 3, the model is defined over undirected graph of Z_{ij} variables. Since our new constraints require a directed graph, we redefine the Z_{ij} variable as 1 if facility $i \in F$ is connected to facility $j \in F$ and 0 elsewhere. Hence, an edge directed from j to i can also have $Z_{ij} = 1$. In addition, constraint $\sum_{j \in F} Z_{ij} \leq y_i$ is also added to complement the alternative constraint in the directed graph. In such a case, given that a facility is opened at a vertex i, at most one outcoming arc from that facility to each facility candidate point is allowed which is a substitute for constraint (7) in the baseline model. Moreover, its usage also renders constraint (9) as invalid so the latter one is removed from the model where remaining equations are inherited from the baseline model wherein $i < j$ condition is lifted.

4.2 Relax-Cut-Repair Method (RCR)

According to preliminary experiments, constraint (6) in the baseline model causes intractability in terms of the model building time, even for small size instances. Hence, we devise the following method to attack the problem: When we *relax* the constraint, the problem is hoped to yield a cycle free solution in a much less amount of time. Cycle free solution has a high probability because of the combination of constraints (5) and (9). Should a cycle is encountered, a *no good cut* is inserted into the model to prevent the same cycle to be seen. At the end of the algorithm, we check whether the solution is connected. If not, we restore the feasibility by applying *Repair* Algorithm: We implement a Network Flow based formulation so as to find the Minimum Steiner Tree to enable the connection in the final solution. If this tree consists of p facilities, we stop. Otherwise, we use MTZ model to $p + r$ facilities obtained from Steiner Tree with size $p + r$ to get the final feasible solution.

5 NUMERICAL RESULTS

In order to test our methods, we utilize 2 graphs from OR-LIB [3] and 2 graphs from TSP-LIB [2]. Additional two graphs are generated with the help of Erdos-Renyi random graph generation procedure where probability of selecting an edge between two vertices is 0.30. We validated via code that all graphs are connected. For each graph, we tried $p = 5$ and $p = 10$ leading to 12 instances in total. The parameters f_i, D_i and d_{ij} are determined using uniform distribution. Afterwards, all-pairs shortest path algorithm is performed to find d_{ij} values. Note that one TSP-LIB graph is also set according to Erdos-Renyi procedure. At last, we set $c_{ij} = 100d_{ij}$ where $i, j \in F$. We solve all instances using C++ calling Cplex 12.7 solver on a PC with x64 Intel(R) Core(TM) i5-3210M 2.50 GHz CPU and 8 GB RAM. We tried different time limits as follows: 10 mins, 30 mins and 1 hr.

We have one small instance with 26 nodes and 100 edges. Other instances have varying node sizes with at least 100 nodes. The largest instance has 225 nodes with 7604 edges. Selecting a better performing method for each instance, the results suggest that 50% of the instances are solvable within the first 10 mins. Among the remaining ones, 1 instance is solved to optimality up to 30 mins. After 30 mins, 1 instance is successfully solved to optimality and 33% of the instances yield 3.33% gap on average under the better performing method where most of instances are comprised of more than 6000 edges.

When we compare these methods, MTZ mostly beats RCR when an instance is solvable under 10 mins. For larger instances, MTZ performs better when $p = 5$. Once p is raised to 10, none of these methods outperform each other significantly.

6 CONCLUSIONS

Some benchmark instances alongside randomly generated data are exploited to show the effectiveness of mathematical modeling based approaches. The results demonstrate that the methods are solvable for small cases and yield small solution gap after one hour. As a performance, MTZ defeats RCR slightly. Yet, they should be tested on larger datasets along with $p = 20$ facilities. Another possible extension is to apply LP rounding and approximation methods to these methodologies to find a good quality initial solution and provide them with it as a starting solution to speed up solution time and enhance the solution quality.

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OPTIMAL SCHEDULE OF A SPORT SHOOTING COMPETITION

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Abstract: The paper explains the problem of amateur sport shooting competition scheduling with a manual and an automated solution. The automated solution provides an optimal schedule to minimize the infrastructure and human resource occupation. The proposed programming model also minimizes the attendance of participating sport clubs, because considers that the amateur competitors want to spend minimum time necessary (with required overhead) for the competition and the team of a sport club usually travel together.

Keywords: scheduling, optimization, resource utilization

1 INTRODUCTION

Sports have worldwide interest. A professional sports events, for example Olympic Games and Professional Athletic Leagues, are different from amateur events in terms of available resources as well as the main target of the event organization. Professional events target as large audience as possible to maximize profit income, while amateur events target the realization of the event's sport result.

When organizing an amateur sport shooting competition, it is important to optimally utilize the available infrastructure and thereby rationalizing the operation and rental costs. The optimal utilization is also crucial in the case of the requested human resources, for example, range officers. With an optimal schedule the necessary organizational time as well as the requested human resources can be minimized. It is of importance not only from monetary point of view but also from the sports' professional quality point of view, namely the quality of the event can be raised with better schedules since for example with less tired range officers less mistakes can happen.

Scheduling of the sport shooting competitions include the place, date and time interval of the disciplines. As a consequence, the infrastructure as well as the required human resources are also determined. Besides, competitors are also included. Generation of the schedule is influenced by the number of disciplines, the required interval for the execution, the number of available firing points, and the number of registered competitors.

Scheduling in sports is usually supported with science based automated processes in case of professional leagues. [1] provided a coverage on various sports scheduling articles until 2010. More recently, [2] introduced an international sports timetabling competition. There the problem was to decide on a suitable time slot for each of the games to be played within a round-robin tournament while respecting some hard constraints and minimizing the penalties from violated soft constraints. [3] investigated various integer programming formulations for scheduling a round robin tournament. [4] proposed a dynamic approach for scheduling e-sports tournaments based on a modification of the Swiss system design. [5] provided a review of fundamental problems in sports scheduling and their formulations, followed by a survey of applications of optimization methods to scheduling problems in professional leagues. Despite the vivid scientific efforts supporting professional leagues and events, for amateur sports shooting tournaments schedules are still created manually without any automated support.

2 PROBLEM DEFINITION

The amateur sport shooting competition has the following problem definition hereinafter.
There are individual competitors. The individual competitors are part of teams delegated by sport clubs. One competitor can be a member of exactly one of the teams.

There are various disciplines. For the disciplines, the maximum duration is given, i.e., the competitor must complete the discipline within the time limit. One discipline can only be completed by the competitor at most once. Parallel disciplines should start at the same time but can finish at different time if lengths are not similar. Because competitors of shorter discipline must wait for competitors of longer discipline, this scheduling should be avoided, or lengths should be as close as possible.

Registration: one competitor can complete one or more disciplines.

There are multiple firing points available. One competitor completes one discipline at a given firing point.

Any disciplines could be performed by any competitors in any firing points parallelly.

Series: Synchronously started and parallelly executed disciplines in selected (possibly all) firing points. Must be handled together in one batch until the competitor executing longest discipline is not finished. In other words, the firing point cannot be used again until any of the firing points is occupied within the batch. In special cases, firing points may be grouped and thus the full firing range may be divided in smaller batches.

Let us consider the following aim of optimisation. Let us minimise the overall duration, while the competitors belonging to the same team should complete their registered disciplines as close to each other, as possible. In other words, the start and finish time of a sport club must be as close as possible.

The optimal schedule minimises the overall duration of the competition in a way that every competitor can execute the discipline where registered. When considering the objective function, the difference between the last and first series of teams and of disciplines should also be considered. Minimising the overall duration of the competition guarantees to minimize the corresponding cost of infrastructure and human resources. Further, minimising the difference between the last and first series guarantee the shortest participation of teams and disciplines.

3 ILLUSTRATION

Let us consider the following example as illustration.

- There are competitors: A, B, C, D, E, F and G.
    - Individual competitors A and E form Team 1.
    - Individual competitors B, C and D form Team 2.
    - Individual competitors F and G form Team 3.
- There are three disciplines
    - Discipline 1 with length of 4 time unit
    - Discipline 2 with length of 2 time unit
    - Discipline 3 with length of 1 time unit

- Registrations
    - Competitor A: Discipline 1, Discipline 2.
    - Competitor B: Discipline 1.
    - Competitor C: Discipline 2.
    - Competitor D: Discipline 2.
    - Competitor E: Discipline 1, Discipline 2.
    - Competitor F: Discipline 2.
    - Competitor G: Discipline 3.
- There are 5 firing points available: 1, 2, 3, 4, 5

<table>
<tr><th>Firing point</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td rowspan="4">Series 1</td><td rowspan="4">A-1</td><td rowspan="4">B-1</td><td>G-3</td><td rowspan="2">D-2</td><td rowspan="2">E-2</td></tr>
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<tr><td></td><td></td><td></td></tr>
<tr><td rowspan="4">Series 2</td><td rowspan="4">E-1</td><td rowspan="2">A-2</td><td rowspan="2">F-2</td><td rowspan="2">C-2</td><td></td></tr>
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Figure 1: A possible schedule of the illustrative example.

Solution given in Figure 1. is a possible, valid schedule, nevertheless it corresponds to a suboptimal solution. The length of the event is the maximum, both series have length of the longest discipline, further all teams must participate throughout the whole event, in other words, no one can arrive later than the beginning nor leave earlier than the end of the competition.

<table>
<tr><th>Firing point</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td rowspan="3">Series 1</td><td rowspan="3">A-1</td><td rowspan="3">B-1</td><td rowspan="3">E-1</td><td rowspan="2">C-2</td><td rowspan="2">D-2</td></tr>
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<tr><td></td><td></td></tr>
<tr><td rowspan="2">Series 2</td><td rowspan="2">A-2</td><td rowspan="2">F-2</td><td rowspan="2">E-2</td><td>G-3</td><td></td></tr>
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Figure 2: A possible schedule of the illustrative example with shorter team participation.

Figure 2 shows a solution where the participation of the teams is minimal, at the expense of the lengths of disciplines. The multi target optimization upon the total event length, the discipline length and the team participation length would not provide one best, optimal solution in general. The final output selection depends on whether the discipline length or the team participation length is more important for the organizers. This decision must be a weight parameter within the solution generator algorithm.

4 PROPOSED SOLUTION

We introduce the following variables:

- an integer variable is introduced to represent the number of competitors as part of a team registered for a specific discipline within a series;
- a continuous variable is introduced to represent time, note that 0 is the starting point of the competition;
- a binary variable is introduced to represent that there at least one competitor within the series registered for the discipline;
- a binary variable is introduced to represent that there at least one competitor as part of the team within the series;
- a variable for the first and for the last series of the team.

The model has the following number of variables:

$$NoS \cdot NoT \cdot NoR + NoS + NoS \cdot NoR + NoS \cdot NoT + NoT + NoT \quad (1)$$

the number of restrictions:

$$5 \cdot NoS \cdot NoT + 3 \cdot NoS \cdot NoR + NoT \cdot NoR + NoS \quad (2)$$

where

NoP	Firing points	
NoS	Number of series	$i \in \{1,2,\dots,NoS\}$
NoT	Number of teams	$j \in \{1,2,\dots,NoT\}$
NoR	Number of disciplines	$k \in \{1,2,\dots,NoR\}$

The objective function minimises the overall duration, while the start and finish time of the sport teams are also minimised. The corresponding weight parameters can be adequately adjusted.

The mixed integer linear programming model was developed and implemented in python language accessing FICO Xpress optimizer 41.01.06 API, community license. The computer details are as follows: processor Intel(R) Core(TM) i7-9750H CPU @ 2.60GHz Memory 16 GB. To test the model, real case data were used of a national sport shooting competition organised at Pécs. The model was solved optimally in less than 100 seconds.

Acknowledgement
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THE LAPLACIAN ENERGY OF SOME SPECIAL TREE FAMILIES

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Abstract: A conjecture by Radenković and Gutman [9] has been left. This conjecture is on the Laplacian energy of trees and partly solved in some papers. In this study, we show that the conjecture holds for some classes of trees with 5 to 15.

Keywords: Laplacian energy, total -electron energy, tree

1 INTRODUCTION

All the graphs considered in this study are finite, undirected and simple.

The Laplacian energy (LE) of a graph is introduced molecular-graph-based analog of the total π-electron energy (E). It is shown that the Laplacian energy and energy have a similar structure-dependency only when molecules of different sizes are compared, when a good linear correlation between them exists. Within classes of isomers, Laplacian energy and energy are either not correlated at all or (as in the case of acyclic systems) are inversely proportional. The total π-electron energy E, as calculated within the Hückel molecular orbital (HMO) model, is one of the most thoroughly studied quantum–chemical characteristics of large polycyclic conjugated molecules. Details on the theory and applications of E can be found in the literature and in the references cited therein. It was shown a long time ago that the various-electron descriptors of the HMO model, including E, can be calculated from the eigenvalues of the corresponding molecular graph. This definition served as a motivation for the definition of the so-called graph energy. Namely, whereas within the HMO model E is meaningful only in the case of a restricted class of molecular graphs.In view of this, the energy of a graph (also denoted by E) is defined as the sum of the absolute values of all eigenvalues of this graph, and this definition extends to all graphs. This seemingly insignificant change in the interpretation of the graph energy resulted in a great expansion of research in this area and has advanced the theory of total π-electron energy greatly.

In [9], Radenković and Gutman studied the correlation between the energy and the Laplacian energy of trees. They also computed the energy and the Laplacian energy for all trees up to 14 vertices. Moreover, they found that the energy and Laplacian energy of a tree are inversely proportional, and gave a conjecture on the Laplacian energy of trees. The conjecture relates the Laplacian energy of path, tree, and star graphs.

Trevisan et al. [11], showed that the conjecture is true for trees of diameter 3. In the same paper, they also showed that the conjecture works for all trees with at most 18 vertices.

In [5], the upper bound of this conjecture was proved. But the lower bound of this conjecture is still an open problem.

In [10], Rahman et al. extended this study up to diameter 4. Also in [3] Chang and Deng showed that the conjecture is true for some classes of trees with diameter 4 and 5.

1.1 Preliminaries

For a graph G with vertices $v_1, v_2, ..., v_n$ and adjacency matrix A, let $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$ be the eigenvalues of the adjacency matrix A. The Laplacian spectrum of G is the set of eigenvalues

$\mu_1 \geq \mu_2 \geq ... \geq \mu_n$ of the Laplacian matrix of G, given by $L = D - A$, where D is the diagonal matrix of vertex degrees.

In [6], the concept of energy of a graph G is defined by the following equation

$$E(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{1}$$

Given any graph G with n vertices and average degree $\overline{d}$, the Laplacian energy of G is defined by Gutman and Zhou [7], is given by

$$LE(G) = \sum_{i=1}^{n} |\mu_i - \overline{d}|. \tag{2}$$

A tree T_n is a undirected connected graph without cycle. A path graph P_n of order n is a tree with two vertices of vertex degree 1, and the other $n-2$ vertices of vertex degree 2. A star graph S_n of order n is a tree on n vertices with one vertex having vertex degree $n-1$ and the other $n-1$ vertices having vertex degree 1. A pendant vertex is a vertex of degree 1.

2 METHODOLOGY

Let T_n be a tree on n vertices. Fricke et al.[4], and Jacobs et al. [8] proposed an algorithm to calculate characteristic polynomials of T_n. The algorithm works by associating, with each vertex v, a rational function $a(v) = \frac{r}{s}$. Here r and s are members of the polynomial ring $Q[\lambda]$. These polynomials are computed bottom-up starting with leaves which are assigned $\lambda - 1$ (the tree can be rooted in an arbitrary way). Once all the children of v have been processed, v is assigned the function

$$a(v) = \lambda - d_v - \sum_{c \in C} \frac{1}{a(c)} \tag{3}$$

where C is the set of its children and d_v is the degree of v. After all vertices have been processed, we compute the Laplacian characteristic polynomial by taking the product of all functions $a(v)$:

$$\chi(\lambda) = \prod_{v \in V} a(v). \tag{4}$$

3 RESULTS

The aforementioned conjecture is as follow:

Conjecture 3.1 *Let T_n be a tree with n vertices.Then, $LE(P_n) \leq LE(T_n) \leq LE(S_n)$.*

In [5], Fritscher et al. proved the following theorem

Theorem 3.2 *Let T_n be a tree with n vertices such that $T_n \neq S_n$. Then, $LE(T_n) < LE(S_n)$.*

Let $T_n^{(i)}(p,q)$ be trees with $i(\; 5 \leq i \leq 15)$ diameters obtained by adding p and q number of pendant vertices to the pendant vertices of the P_{i-1} paths. Examples for these trees are given in Figure 1.

In this study, using above algorithm, we show that the left side of conjecture is satisfied for $T_n^{(i)}(p,q)$ trees classes between 5 and 15 in diameter, that is,

Theorem 3.3 *Let P_n be a path with n vertices. Then, $LE(P_n) < LE(T_n^{(i)}(p,q))$, for $\; 5 \leq i \leq$* 15.

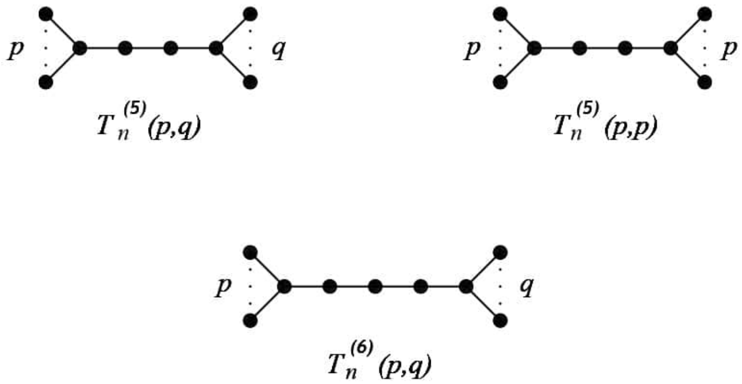

Figure 1: Trees with diameter 5 and 6

Sketch of the Proof: We use the algorithm described in subsection 1.2 to construct the Laplacian characteristic polynomials for $(T_n^{(i)}(p,q))$, for $5 \leq i \leq 15$. After having the characteristic polynomials for the aforementioned trees the eigenvalues of these trees can be find by straightforward calculation. Then by calculating the Laplacian energies of $(T_n^{(i)}(p,q))$, for $5 \leq i \leq 15$ we show that $LE(P_n) < LE(T_n^{(i)}(p,q))$, for $5 \leq i \leq 15$. Finally by combining the result of Theorem 1.2 and $LE(P_n) < LE(T_n^{(i)}(p,q))$, for $5 \leq i \leq 15$ we prove that $LE(P_n) \leq LE(T_n^{(i)}) \leq LE(S_n)$, for $5 \leq i \leq 15$.

4 DISCUSSION

In this study, we proved conjecture given by Radenković and Gutman in [9] for the class of trees (see for example Figure 1) with diameter d such that $5 \leq d \leq 15$. This conjecture relates the Laplacian energy of path, tree and star graphs.

5 ACKNOWLEDGEMENTS

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A general framework for modeling opinion formation

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Abstract: The paper introduces a general agent-based framework for modeling opinion formation in a group of people. It is based on the assumption that agent's opinion and confidence about his/her opinion may shift after an interaction with another agent. Such a shift depends on several variables. The work focuses on two of them – distance between opinions and distance between confidences of two interacting agents. Opinion and confidence changes are described with diagrams called opinion shift map and confidence shift map in the model. Properties of the opinion formation model are analyzed via numerous computer simulations. Advantages, limitations and future extension of the framework are discussed.

Keywords: opinion formation, opinion shift, agent-based modeling, computer simulation

1 INTRODUCTION

Rapid growth of information technologies arouses more and more curiosity about how people form their opinion. There is variety of issues to form opinion about – news, innovations, rules, products, statements etc. People usually form their opinion with the help of other individuals. Encounter with other person may result in opinion strengthening or shift. There are several papers devoted to the topic of opinion formation. For comprehensive review of the research area see, e.g., [1] or [3].

Inspired by the work [2], we describe a general agent-based framework that may serve as a tool to understand the process of opinion formation in a group of people. We use computer simulations to examine influence of its parameters on resulting opinion configurations.

2 OPINION FORMATION MODEL

Let us work with a numbered group of people. Since we build an agent-based model, we call them *agents*. We are interested in opinion shift of an agent i after an interaction with another agent j, which we call *advisor*. In [4] and [5] they show empirically that two important factors influence individual opinion formation process: distance between opinions of the two individuals $\Delta O_{ij} = |O_i - O_j| \in [0, 1]$, and the distance between confidences of these individuals about their opinion $\Delta C_{ij} = (C_i - C_j) \in [-1, 1]$. The negative value of ΔC_{ij} means, that the advisor has higher confidence. The model described in [2] uses both of these parameters. When an agent i meets an advisor j in a time step t one of the three outcomes will happen: the agent i either *keeps* her initial opinion ($O_i^{t+1} = O_i^t$), makes a *compromise* ($O_i^{t+1} = O_i^t + \delta \cdot |O_i^t - O_j^t|$), with some compromise rate $\delta \in (0, 1)$, or *adopts* advisor's opinion ($O_i^{t+1} = O_j^t$).

In the paper [2] they designed experiments to capture such a process. During experimental interactions of several participants, they were able to plot average opinion outcomes for various combinations of O_{ij}, C_{ij}. The resulting diagram is of a complex shape. Hence, a simplification they provided is to divide distances between opinions into three categories: Near, Intermediate and Far, which is the same approach as in [5]. This way, they reduce the diagram into three bands. We realized that, each band is actually an interval $[-1, 1]$, therefore for its complete description it is enough to mark sections of the intervals with resulting action **C**: compromise, **A**: adopt or **K**: keep. We call such a diagram *opinion shift map (OSM)*. An example of an OSM is shown in Figure 1 a).

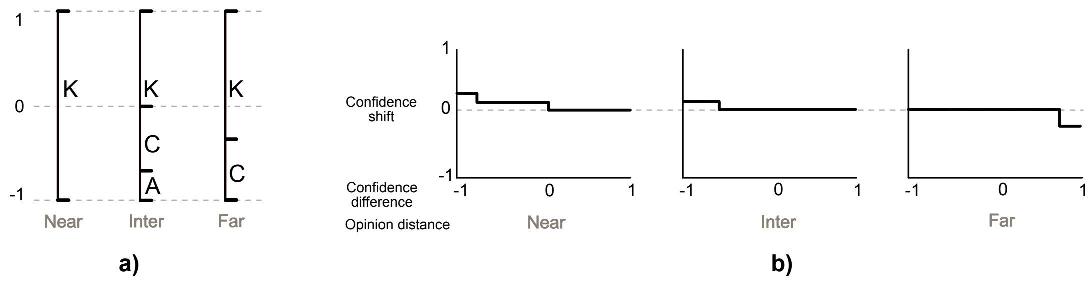


Figure 1: a) Example of an opinion shift map, b) Example of a confidence shift map. Both correspond to empirical findings in [2].

In [2] they explored opinion shift regarding general knowledge questions. However, for different types of information, the opinion shift map could look different. We realized that what was a way of simply visualizing the experimental results can actually be used to serve as a general framework for capturing opinion shift process in variety of situations.

Usually, our confidence in our opinion can shift after an encounter with an advisor. The change also depends on O_{ij}, C_{ij}. For example, a confident advisor with a different opinion can shake our confidence in our opinion if our confidence is weak. On the other hand a confident advisor with a similar opinion could increase our own confidence.

After generalization of [2], in each of the three cases of opinion distances (Near, Intermediate and Far) confidence shift can be approximated by a piece-wise linear function. We call such a set of functions *confidence shift map (CSM)*. As an example we show the CSM based on approximation of data collected from the experimental study in [2]. An example of a CSM is shown in Figure 1 b). Again, for situations other than general knowledge testing, CSM might look different.

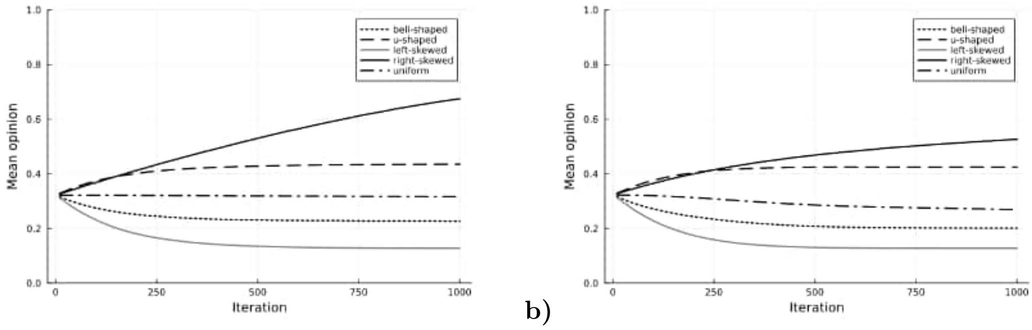
3 RESULTS

To test the features of the model we implemented it in the Julia language. For every simulation we use the population of size $N = 52$, which is the same size as in [2]. Opinions and confidences are represented as numbers coming from the interval $[0, 1]$. For the purpose of testing we use the opinion and confidence shift map based on [2]. Similarly, the compromise rate is set to $\delta = 0.4$. For now, the constants Near and Far are set to $0.2, 0.5$ respectively.

In each interaction step an agent is chosen randomly and an advisor from the group is randomly assigned to that agent. Based on $\Delta O_{ij}, \Delta C_{ij}$ agent's opinion is updated according to the OSM and CSM, which is the same for every agent. One simulation consists of 1 000 interaction steps since it is close to $\binom{N}{2}$. Because it is a stochastic process, we average our results from 1 000 simulation runs for each parameter combination.

To set initial opinions and confidences of agents, we tested several types of distributions on $[0, 1]$ – symmetric bell-shaped, symmetric U-shaped, uniform, left-skewed and right-skewed. For opinions the meaning of such distributions is straightforward. For confidences a distribution curve describes how high is the confidence of advisors holding certain opinion from $[0, 1]$.

First, we started with confidences that do not change in time. When distributions are symmetric the final mean opinion is also centered, i.e., around 0.5. When opinions and confidences are skewed left, than the final opinion tends to go left (towards 0). The most interesting case is when opinion minority with high confidence and opinion majority with low confidence are located at the opposite sides of the opinion spectrum. In the left of Figure 2 we can see the average trajectories of mean opinion in population for the initial opinions skewed left and confidences initialized according to various types of distributions. In the only case, when confi-



a) b)

Figure 2: Opinion formation process in time. The initial distribution of opinions is skewed left. Each mean opinion trajectory is averaged over 1 000 simulations for each initial distribution of confidences. The part a) shows the process when confidences of agents remain fixed to their initial values, b) shows the process for shifting confidences according to the introduced CSM.

dences are skewed right, the final mean opinion shifts to the other side of the opinion spectrum (is greater than 0.5).

Allowing shifting confidences does not always have a significant impact. For example, when an opinion distribution is symmetrical (U-shaped, bell-shaped) shifting confidences do not have visible influence. However, when initial opinions are skewed left, we can see obvious difference between fixed and shifting confidences for most of the confidence distributions. In such situations it is more difficult to shift opinions to the other side of the spectrum as seen in the right part of Figure 2. Presumably, it is because members of opinion majority support each other and their confidence also increases.

When is it possible for the minority to shift the opinion of the majority? We run simulations for various opinion confidences of the majority and minority. For the case of fixed confidences, the confidence of the majority must be low (around 0.2) and confidence of the minority must be high (around 0.8). When confidences can shift it makes the situation of opinion propagation even harder, since the majority supports themselves in their opinion. That is why difference between confidences of the two groups must be even larger.

The compromise rate δ specifies how much agent's opinion shifts towards advisor's opinion. The default value is $\delta = 0.4$. Let us return to the situation when opinions are distributed towards the left and confidences towards the right. We experimented with values of δ from 0.1 to 0.9. For fixed confidences the final mean opinion is basically the same for all values. However, the lower the compromise rate the larger the amount of steps needed for convergence of the final mean opinion. In the case of shifting confidences the parameter plays more important role. The smaller the number of compromise rate the lower is the final mean opinion. The difference the between final mean opinions in case of $\delta = 0.1$ and $\delta = 0.9$ is more than 0.2, which is one fifth of the length of the whole opinion spectrum. For lower values $\delta = 0.1, 0.2$ the final mean opinion does not exceed the half of the original opinion spectrum.

We executed even more tests regarding definition of constants Near and Far, number of interactions or size of population, which cannot be visually presented here due to the lack of space. At least, we can summarize them together with the above mentioned results. Initial opinion and confidence distribution matters. The most interesting case is when two groups are based on the opposite sides of the opinion spectrum. It is possible for minority to attract the opinion of majority, however, the majority must be much more confident in their opinion. Shifting of confidences can work both as an inhibitor or catalyst for opinion propagation. The number of interactions does not matter too much. After a certain number of interactions the final mean opinion reaches its equilibrium and fluctuates around it.

4 DISCUSSION

The paper introduced a generalization of the work [2]. However, forming opinion about geographical facts might not have the same sense of urgency as forming opinion about political or ethical matters. Hence, the resulting general framework is prepared for various types of opinion formation processes by introducing the concept of opinion and confidence shift map.

In more critical situations people might tend to form their opinion according to opinion leaders or significant others. Therefore, each advisor has potentially different importance to every agent. Several models mentioned in [3] implement importance in form of weights. We also intend to include such weights in our model, which will add a new dimension to the OSM and CSM.

The introduced framework is well suited for smaller groups of people where pairs of individuals have similar probability of interaction, e.g., a choir, study group, hobby club etc. For larger groups or for groups with more intricate communication system, it would be favorable to incorporate deeper knowledge about the communication networks among individuals. Such channels may differ according to the subject of opinion formation.

We showed several properties of a particular opinion formation model that resulted from definition of the opinion and confidence shift map according to [2]. The interesting behavior of the model depends on several parameters that were not explored in the previous paper. Namely, distribution of initial opinions and confidences, compromise rate, number of interactions, shifting confidences and changing the meaning of near and far opinions. Due to a limited space of the paper we picked only several interesting scenarios.

In future research it would be favorable to consider various types of opinion and confidence shift maps and their heterogeneous mixture in a population. The goal is to base opinion and confidence shift maps on experimental results connected to more critical areas of opinion formation than general knowledge. Exploring various patterns of multiple-agent interaction is also very relevant. Extensive computer experimentation with such models could shed more light on real opinion formation processes, especially processes that are difficult to be studied directly.

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SCATTER SEARCH FOR BI-CRITERIA PUBLIC SERVICE SYSTEM DESIGN

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Abstract: A public service system design problem with a pair of mutually exclusive criteria has many non-dominated solutions, which differ in accenting one of the criteria subject to keeping the other under some limit. The entire Pareto front of the system designs represents a complete set of designs, which is impossible to get improved. Due to complexity of obtaining the exact Pareto front, the metaheuristics are welcome tools to create a precise Pareto front approximation in an acceptable computational time. This contribution presents an implementation of an evolutionary heuristic of the scatter search type adapted for bi-criteria system designs to produce a precise approximation of the Pareto front.

Keywords: bi-criteria location problems, Pareto front approximation, directed search, scatter search

1 INTRODUCTION

Different heuristic algorithms, metaheuristics and other sophisticated evolutionary approaches to the optimization problems have been recently developed not only as a complement to the common exact methods, but their goal is to overcome their weaknesses [3, 4]. A specific weak point of almost all exact methods consists in the verification process, which consists in confirming that the best-found solution is the optimal one. Even if some modelling techniques, i.e. radial approach [2, 12], significantly accelerate the associated solving process, mentioned verification phase of the exact methods takes usually too much time. On the other hand, heuristic approaches enable us to obtain a good solution in considerably shorter time [5, 7].

Similar obstacle occurs also in bi-criteria location problems, which can be met in public service system designing. If there are two objectives to be optimized, a special set of solutions called Pareto front has to be searched for. As completing the exact Pareto front is very time demanding [8, 9], effective heuristics are applied to its determination [10, 11, 13]. In this paper, we focus on the design and implementation of an evolutionary heuristic approach to the Pareto front approximation based on the scatter search strategy. One member of the approximation corresponds to a solution of the public service system design problem. The solution is determined by a choice of p service center locations from the set of m possible locations.

2 SCATTER SEARCH SCHEME

The original scatter search algorithm consists of two phases. The first one creates a pair B and D of the reference sets. The set B contains a collection of good problem solutions from the point of a given objective function. The set D is intended for solutions, which should spread over the set of all feasible solutions as much as possible. The first phase employs various methods of intensification to improve the starting solutions from B and also tries to improve diversity of the set D to cover the outliers of the set of all feasible solutions.

The second phase generates new solutions by combining solutions from B and D to search the space between the chosen solutions. The newly assembled solutions are submitted to series of improving heuristics with the goal to improve the content of the set B. The best found solution is then returned as the result of scatter search algorithm. If this algorithm is applied on the problem, solution of which are represented by elements of the Euclidean space, then the new solution is constructed as convex combination of a few solutions selected from B and D, where weights of the solutions are proportional to objective function values of the solutions [6]. In the case of combinatorial problems, this approach must be replaced by some other way of combining the selected solutions, e.g. path-relinking method etc. Regarding the efficiency of the phases, the general finding is that the second phase takes much more time to improve the objective function value of the best-found solution by the same amount.

3 SCATTER SEARCH FOR BI-CRITERIA LOCATION PROBLEMS

Implementation of the scatter search algorithm for finding a good approximation of the Pareto front of public service system designs with two criteria must take into account several specifics in which the problem differs from the usual ones. First, the public service system design is a combinatorial problem, solution of which is given by a sub-set $\boldsymbol{y}$ of p service center locations from a set of m possible locations. In this contribution, we consider two objective functions $f_1(\boldsymbol{y})$ and $f_2(\boldsymbol{y})$ for a solution $\boldsymbol{y}$. The value $f_1(\boldsymbol{y})$ is the sum of traversing times between a demand location and the nearest available center location. The value $f_2(\boldsymbol{y})$ corresponds to the amount of demands located further than given radius T from the closest center location. Second, we do not search for the best public service system design, but for a set of non-dominated solutions, which approximates the Pareto front. The mutual dominance of solutions $\boldsymbol{y}$ and $\boldsymbol{x}$ is defined so that the solution $\boldsymbol{y}$ dominates the vector $\boldsymbol{x}$ if $f_1(\boldsymbol{y}) \leq f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{y}) \leq f_2(\boldsymbol{x})$ hold. The approximated Pareto front is the set of non-dominated solutions such that for any solution $\boldsymbol{x}$ there exists a solution $\boldsymbol{y}$ from the Pareto front which dominates $\boldsymbol{x}$ [1, 8, 9, 11].

That is why the set B of the suggested scatter search algorithm is represented by the currently best-found Pareto front approximation. The set B consists of non-dominated solutions $\boldsymbol{y}^1$, …, $\boldsymbol{y}^{|B|}$ of the public service system design problems, which are ordered according to the increasing values of f_2. Quality of the approximation given by current B can be evaluated by (1).

$$Q(B) = \sum_{k=1}^{|B|-1} \left(f_1\left(\mathbf{y}^k\right) - f_1\left(\mathbf{y}^{|B|}\right)\right)\left(f_2\left(\mathbf{y}^{k+1}\right) - f_2\left(\mathbf{y}^k\right)\right) \tag{1}$$

The input of the scatter search phase consists of a good approximation B obtained in the first phase by process of intensification and a set D of mutually non-dominated solutions obtained by a diversification process. To create a new solution employing solutions from B and D, a new approach has been suggested.

The set B is traversed sequentially so that the pair $\boldsymbol{y}^k, \boldsymbol{y}^{k+1}$ for $k = 1, \ldots, |B|-1$ is chosen from B for processing. The elements of D are ordered according to Manhattan distance from $\boldsymbol{y}^k$ in the two dimensional space of f_1 and f_2 and subsequently the first, second, up to the r-th closest solution $\boldsymbol{x}$ is appointed to be the starting solution of the following incrementing swapping heuristic minimizing the weighted criterion $a_1f_1(\boldsymbol{x}) + a_2f_2(\boldsymbol{x})$. The heuristic runs the procedure $Exch(\boldsymbol{x}, i, j)$, that returns solution $\boldsymbol{x}'$, in which the location j replaces the location i.

Heuristic($\boldsymbol{x}, \boldsymbol{a}, B$)

0. Set $F = a_1f_1(\boldsymbol{x}) + a_2f_2(\boldsymbol{x})$ and initialize $F^* = F$.
1. For each pair (i, j) in which $i \in \boldsymbol{x}$ and $j \notin \boldsymbol{x}$, perform the following operations.
   Update B by $Exch$ $(\boldsymbol{x}, i, j)$.
   Evaluate $E = a_1f_1(Exch\ (\boldsymbol{x}, i, j)) + a_2f_2(Exch\ (\boldsymbol{x}, i, j))$.
   If $F > E$, then set $F = E$, $i^* = i$, $j^* = j$.
2. If $F^* = F$, then set $\boldsymbol{y} = Exch\ (\boldsymbol{x}, i^*, j^*)$ and continue with step 1, otherwise terminate.

The values a_1 and a_2 are determined according to (2).

$$\begin{aligned} a_2 &= (f_1(\mathbf{y}^k) - f_1(\mathbf{y}^{k+1})) / (f_2(\mathbf{y}^{k+1}) * f_1(\mathbf{y}^k) - f_1(\mathbf{y}^{k+1}) * f_2(\mathbf{y}^k)) \\ a_1 &= (1 - a_2 * f_2(\mathbf{y}^k)) / f_1(\mathbf{y}^k) \end{aligned} \tag{2}$$

4 NUMERICAL EXPERIMENTS

All computational experiments reported in this study were performed using the 64-bit version of Java language embedded into the NetBeans environment. They were run on a common PC equipped with the 11th Gen Intel® Core™ i7 1165G7 2.8 GHz CPU and 40 GB RAM.

As the input dataset for this study, we made use of commonly used benchmarks described in [8, 9, 10, 11, 13], the origin of which comes from the road network of Slovakia, through which the urgent medical care is provided by emergency agencies. The list of regions, for which the experiments were performed, contains Nitra (NR), Trenčín (TN), Trnava (TT) and Žilina (ZA). The basic characteristics of used instances are summarized in Table 1. The column denoted by $|I|$ reports the cardinality of the set of candidates I, from which exactly p center locations are to be chosen. The last column of the table denoted by $Q(PF)$ contains the area evaluation of the complete Pareto front PF computed according to (1).

Table 1: Benchmarks characteristics.

Region	$\|I\|$	p	$Q(PF)$
NR	350	27	736846
TN	276	21	829155
TT	249	18	814351
ZA	315	29	407293

The following Table 2 summarizes the results of suggested heuristic approach to Pareto front approximation. Since the algorithm was run ten times for each problem instance and for each parameter setting, the table contains the average values of all studied characteristics. Row denoted by *CT* contains the average runtime of the methods in seconds. Symbol *NoD* is used to denote the cardinality of the set *D*. The number of solutions approximating the Pareto front is evaluated as *NoS* before and after the scatter search method. The last studied characteristic consists in gap, which expresses the relative difference between the values of *Q* computed according to (1) for the Pareto front and its approximation.

Table 2: Results of scatter search-based heuristics.

	Region NR				**Region TN**			
	$r = 1$	$r = 2$	$r = 4$	$r = 8$	$r = 1$	$r = 2$	$r = 4$	$r = 8$
CT [s]	31.7	63.9	121.5	249.5	13.0	25.7	51.9	108.7
NoD	42.1	41.6	43.4	41.3	32.7	34.8	31.6	31.9
start *NoS*	102.2	102.2	102.0	101.7	84.0	84.0	84.0	84.0
end *NoS*	104.4	106.5	104.7	104.0	90.4	90.2	91.1	91.7
start *gap* [%]	6.06	6.05	6.06	6.02	0.85	0.85	0.85	0.85
end *gap* [%]	5.68	5.75	5.58	5.20	0.81	0.82	0.81	0.78
	Region TT				**Region ZA**			
	$r = 1$	$r = 2$	$r = 4$	$r = 8$	$r = 1$	$r = 2$	$r = 4$	$r = 8$
CT [s]	5.3	10.8	21.6	43.6	18.7	37.6	75.9	153.0
NoD	28.5	29.4	29.1	30.3	33.4	36.4	35.7	38.1
start *NoS*	62.0	62.0	62.0	62.0	91.0	91.0	91.1	91.0
end *NoS*	62.4	64.0	64.0	64.0	91.9	92.1	92.1	92.0
start *gap* [%]	0.65	0.65	0.65	0.65	0.22	0.22	0.22	0.22
end *gap* [%]	0.07	0.00	0.00	0.00	0.20	0.19	0.19	0.21

5 CONCLUSIONS

This paper was focused on heuristics developed to approximate a Pareto front, which is a common output for decision-makers, if the associated location problem takes into account more than one objective function. The achieved results show that the suggested scatter search can produce a very precise approximation of the original Pareto front of service system designs in acceptably short computational time. Such a great a accuracy makes it suitable for practical applications.

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INTENSIFICATION AND DIVERSIFICATION FOR PARETO FRONT APPROXIMATION

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Abstract: Determination of the Pareto front of two criteria p-location problem solutions demands for enormous amount of computational time. That is the reason, why metaheuristics are used to obtain a good Pareto front approximation. Evolutionary metaheuristics efficiency depends on two characteristics of the input population. The first of them consists in quality of the starting solutions and the second one is diversity of the population. This contribution focuses on the way to obtain two reference sets, where the first one consists of very good solutions and represents the starting approximation of the Pareto front. The second set consists of mutually non-dominated solutions, which are not members of the approximation and form the diversed part of the starting population.

Keywords: bi-criteria location problems, Pareto front approximation, intensification, diversification

1 INTRODUCTION

Pareto front of public service system designs is the only exact result, which can be offered to a final decision maker responsible for negotiation between supporters of system solution and advocates of the fair approach to the worst situated minorities. Regardless of the accented criterion, the public service system design problem can be modelled and solved by means of integer linear programming [1, 2, 4, 5, 6] as one of common p-location problem. History of the approaches started with p-median problem and p-centre problem models, which use allocation variables and constraints. The development continued with radial approach [3, 11]. An effort to model a limited capacity of used facilities led to special models, which comply the property of real systems, where a randomly emerging demand is assigned to the nearest available centre instead of to the geographically closest one [9, 10, 11, 12]. Accepting all the generalizations, Pareto front can be found, but with enormous computing time requirements [7, 8]. This complexity inspired the idea to develop an evolutionary metaheuristics to obtain a good approximation of the Pareto front of two criteria public service system designs.

As the classical genetic algorithm [5] uses only selection process to search the set of feasible solutions, we focused on a more sophisticated evolutionary metaheuristic as the scatter search [5], which employs intensification heuristic to obtain set *BEST* of good starting solutions and introduces so called diversed reference set *DIV* to enlarge the searching process over a bigger part of the set of all feasible solutions.

Within this contribution, we suggest a special structure of the reference sets of the starting solutions together with an intensification method and the method, which enables to produce a diversed set with the given cardinality.

2 INTENSIFICATION

The process of intensification serves for creating of a set *BEST* of different problem solutions with very good values of an objective function in general. This contribution deals with the public service system design problem, where the design is given by a sub-set $\boldsymbol{y}$ of p service centres selected from a set of m possible service centre locations. The value of the system objective function $f_1(\boldsymbol{y})$ is sum of distances between a demand location and the nearest available service centre. The value of the fair objective function $f_2(\boldsymbol{y})$ equals to the number of demands located outside a radius T from the closest service centre. As the objectives are contradictory, there is no complete ordering on the set of all feasible solutions. The pair of objective functions enables to define a partial ordering called dominance. It is stated that solution $\boldsymbol{y}$ dominates solution $\boldsymbol{x}$ if $f_1(\boldsymbol{y}) \leq f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{y}) \leq f_2(\boldsymbol{x})$. The Pareto front of the problem solutions is the set of non-dominated solutions such that for any feasible solution $\boldsymbol{x}$ there exists an element $\boldsymbol{y}$ of the Pareto front which dominates $\boldsymbol{x}$.

The set *BEST* in our approach to Pareto front approximation will consist of mutually non-dominated solutions and it will contain two members of the Pareto front. The first one will be member with minimal value of f_2 and the second one will get the maximal value of f_2. The elements of *BEST* will be ordered according to f_2 in the increasing way. This ordering enables easy updating the set by arbitrary new solution. The current set *BEST* represents an approximation of the Pareto front. The process of intensification will gradually improve the current approximation. Quality of the approxmation given by ordered *BEST* members $\boldsymbol{y}^1$, …, $\boldsymbol{y}^{|BEST|}$ will be measured by (1).

$$Q(BEST) = \sum_{k=1}^{|BEST|-1} \left(f_1\left(\mathbf{y}^k\right) - f_1\left(\mathbf{y}^{|BEST|}\right)\right)\left(f_2\left(\mathbf{y}^{k+1}\right) - f_2\left(\mathbf{y}^k\right)\right) \tag{1}$$

To perform intensification process, we suggested so-called gradual refinement heuristic. This heuristic is based on step-by-step improvement of a current *BEST* by searching neighborhood of its member $\boldsymbol{y}^k$ and including admissible neighborhood member in the current approximation. The used neighborhood search heuristic is a swap algorithm performing the first admissible strategy, where each tested neighbor contribution is evaluated by the decrease of *Q*(*BEST*) caused by updating the current *BEST* by the tested neighbor.

The gradual refinement starts with the two-element initial *BEST* consisting of the two members of the Pareto front. The stepwise refinement processes solutions $\boldsymbol{y}^1$, …, $\boldsymbol{y}^{|BEST|}$ improving the set *BEST*. As updated *BEST* may contain some new solutions, the process is repeated so that the *BEST* that is the result of the performance of one process is used as the input *BEST* for the next application of the process. This cycle is stopped when no decrement of *Q* is obtained.

3 DIVERSIFICATION

The set *DIV* should be formed by such solutions, which are not included in *BEST* and which form maximally diversed set of solutions to cover a substantial part of the set of all feasible solutions. In addition, the objective functions of the solution should not be too bad, because these solutions are are intended for the improving process as a starting solutions.
Diversity of the solution set can be defined by many ways, where each of the approaches is based on the definition of the difference between two solutions. The difference can be defined as a distance in *m*-dimensional space, e.g. Hamming distance, or some other measure can be used. In this contribution, the diversity of the set *DIV* is computed by (2), where $d(\mathbf{y}, \boldsymbol{x})$ denotes the difference of solutions $\boldsymbol{y}$ and $\boldsymbol{x}$.

$$D(DIV) = \sum_{k=1}^{|DIV|-1} \sum_{j=k+1}^{|DIV|} d(\mathbf{y}^k, \mathbf{y}^j) / ((|DIV|-1)*|DIV|/2) \tag{2}$$

Contrary to [5], where random generation of the diversed set was recommended, we suggested our own approach, where we exploit the special operation *Crossover* developed specially for this type of *p*-location problems so that the resulting offspring were represented by sub-sets of exactly *p* service centre locations [7, 8]. *Crossover*($\boldsymbol{y}$, $\boldsymbol{x}$) returns the pair $\boldsymbol{u}$, $\boldsymbol{v}$ of subsets, for which the following rules hold. Intersection of $\boldsymbol{y}$ and $\boldsymbol{x}$ is contained in both subsets $\boldsymbol{u}$ and $\boldsymbol{v}$.

The elements from $\boldsymbol{x} \cup \boldsymbol{y}$ - $\boldsymbol{x} \cap \boldsymbol{y}$ are randomly distributed between $\boldsymbol{u}$ and $\boldsymbol{v}$ so that the numbers of locations in both sub-sets are the same.

The set *DIV* is derived from the set *BEST* so that pairs $\boldsymbol{y}^k, \boldsymbol{y}^{k+1}$ for $k = 1, \ldots, |BEST|-1$ are submitted to *Crossover* and the offspring are saved to a list *CAND*, after the *CAND* is filled up, filtering process of the offspring is performed. The filtering process excludes the elements, which are contained in *BEST* or can update the *BEST*. The remaining elements are used for addition to the current *DIV*, but the *DIV* can contain only non-dominated solutions. If the cardinality of *DIV* is less than a given limit, the process of *Crossover* application can be repeated for pairs $\boldsymbol{y}^k, \boldsymbol{y}^{k+t}$ from the current *BEST* for $t = 2, 3, \ldots$ etc.

4 NUMERICAL EXPERIMENTS

The numerical experiments reported in this case study were carried out using the 64-bit version of Java language embedded into the NetBeans environment. They were run on a PC equipped with the 11th Gen Intel® Core™ i7 1165G7 2.8 GHz CPU and 40 GB RAM.

As far as the set of used problem instances is concerned, we took the data from existing Emergency Medical Service (EMS) system in Slovakia implemented in the self-governing regions of Nitra (NR), Trenčín (TN), Trnava (TT) and Žilina (ZA). These datasets were used also in our previous research reported in [9, 10, 11, 12], in which concrete forms of used criteria are discussed, too.

The diversity of the *DIV* was computed according to (2) where $d(\boldsymbol{y}^k, \boldsymbol{y}^j)$ is normalized Manhattan distance between $\boldsymbol{y}^k$ and $\boldsymbol{y}^j$ in the two dimensional space of f_1 and f_2.The normalization is performed by differences between the highest and lowest values f_1 and f_2 of the elements of Pareto front. The following Table 1 summarizes the results of the first phase of the suggested method. The left part of the table contains the problem sizes defined by the cardinality of candidate set *I* and the number *p* of centres to be located. The middle column denoted by *PF_Area* brings the value of (1) for the entire Pareto front. The right part of Table 1 summarizes the first phase. Computational time in seconds is reported in the column denoted by *CT* [s], the number of solutions is denoted by *NoS* and the last column denoted by gap contains the difference in percentage between *Q*(*BEST*) and the area of the entire Pareto front.

Table 1: Results of the first phase of suggested method.

Reg.	|*I*|	*p*	*PF_Area*	*CT* [s]	*NoS*	*gap* [%]
NR	350	27	736846	11.27	102	6.07
TN	276	21	829155	5.93	84	0.85
TT	249	18	814351	2.05	62	0.65
ZA	315	29	407293	11.29	91	0.22

Table 2 contains the results of obtaining the *DIV* set approach performed with different number *t* of *Crossover* applications. Since each algorithm was performed ten times, we report the average values of studied parameters. *NoI* denotes the number of included solutions, *CT* denotes the computational time in seconds. *NoS* is used for the number of solutions in the set.

Table 2: Results of numerical experiments for *DIV* set obtaining.

Reg.	*t* = 1			*t* = 2			*t* = 3			*t* = 4		
	NoI	*CT* [s]	*NoS*	*NoI*	*CT* [s]	*NoS*	*NoI*	*CT* [s]	*NoS*	*NoI*	*CT* [s]	*NoS*
NR	94.60	0.02	102.20	200.30	0.04	101.90	291.20	0.07	102.00	367.70	0.13	102.00
TN	98.80	0.01	84.00	189.40	0.01	84.00	270.70	0.05	84.00	347.10	0.04	84.00
TT	64.60	0.01	62.00	125.90	0.01	62.10	185.70	0.03	62.00	246.10	0.02	62.00
ZA	101.20	0.02	91.00	177.10	0.03	91.00	236.70	0.05	91.00	295.10	0.07	91.00

5 CONCLUSIONS

This research paper was focused on the way to obtain two reference sets of *p*-location problem solutions. The first set consists of very good solutions and represents the starting approximation of the Pareto front. The second one consists of mutually non-dominated solutions, which are not members of the approximation and form the diversed part of the starting population. The obtained results have proved fast performance of the algorithm. The number of crossover applications has no impact on the diversity, but it significantly affects the computational time.

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Finding the best 2-OPT move on nearly random Euclidean TSP tours in average linear time

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Abstract

We describe an exact algorithm for finding the best 2-OPT move which, experimentally, was observed to take linear time on random and nearly-random tours for Traveling Salesman Problem Euclidean instances. To analyze its complexity, we introduce a family of heuristic procedures and discuss their performance in graphs whose edge costs are Euclidean distances between random points in the plane. We prove that, for any probability p, there is a heuristic in the family which can find the best move with probability at least p in average-time $O(n)$, and we also show that on all instances on which a heuristic finds the best move, the exact algorithm finds it in a smaller time. Notice that linear-time is indeed an optimal complexity for this problem, since to find the best move there are at least n edges to look at.

Introduction. The Traveling Salesman Problem [10, 6, 2], also known as the TSP, is in all likelihood the most well-known combinatorial optimization problem. Its objective is to identify a shortest Hamiltonian cycle in a complete graph of n nodes, weighted on the arcs. Without loss of generality, let the set of nodes be $\{1, \ldots, n\}$. We consider the symmetric version of the problem, and denote by $c(i, j) = c(j, i)$ the distance between two generic nodes i and j. Each solution of the problem is called a *tour*, and it is identified by a permutation $(\pi_1, \ldots, \pi_n)$ of the nodes. The length of a tour T, denoted by $c(T)$, is the sum of the lengths of the edges of the tour.

A very effective way to tackle hard combinatorial optimization problems such as the TSP [1, 7] is *local search*, a general paradigm for the minimization of an objective function f over a set $\mathcal{S}$ of feasible solutions. Let $N : \mathcal{S} \mapsto 2^{\mathcal{S}}$ be a map which associates to every solution $x \in \mathcal{S}$ a set $N(x)$ called its *neighborhood*. Starting at any solution x^0, local search samples the solutions in $N(x^0)$ looking for a solution x^1 better than x^0. If it finds one such solution, it iterates the same step, this time looking for x^2 in $N(x^1)$. It then continues this way until the current solution x^i satisfies $f(x^i) = \min\{f(x)|x \in N(x^i)\}$, i.e., it has reached a *local optimum*. Replacing x^i with x^{i+1} is called performing a *move* of the search, so that $N(x^i)$ is the set of all solutions reachable with a move from x^i. The total number L of moves applied to get from x^0 to the local optimum x^L is called the *convergence length*.

In the 2-OPT neighborhood for the TSP, a move $\mu(i, j)$ on a tour $T = (\pi_1, \ldots, \pi_n)$, is identified by two non-consecutive edges of the tour, namely $\{\pi_i, \pi_{i+1}\}$ and $\{\pi_j, \pi_{j+1}\}$, called the *pivots* of the move. The move consists in removing $\{\pi_i, \pi_{i+1}\}$ and $\{\pi_j, \pi_{j+1}\}$ and replacing them with $\{\pi_i, \pi_j\}$ and $\{\pi_{i+1}, \pi_{j+1}\}$, so as to obtain the new tour $T' = (\pi_1, \ldots, \pi_i, \pi_j, \pi_{j-1}, \ldots, \pi_{i+1}, \pi_{j+1}, \ldots, \pi_n)$.

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We define

$$\Delta(\mu(i,j)) := c(T) - c(T') = c(\pi_i, \pi_{i+1}) + c(\pi_j, \pi_{j+1}) - \big(c(\pi_i, \pi_j) + c(\pi_{i+1}, \pi_{j+1})\big).$$

The introduction of the 2-OPT neighborhood for the TSP dates back to the late fifties [4]. Local search based on this neighborhood is probably the most popular approach for the TSP. The obvious, most used, algorithm for finding the best 2-OPT move, i.e., a double-`for` cycle iterating over all $1 \leq i < j \leq n$ and taking $\Theta(n^2)$ time. Most research in the literature has not been devoted to trying to speed-up this quadratic algorithm, but rather to studying the the convergence length and the quality of the local optima that can be obtained. With respect to the Euclidean TSP, Chandra et al. [3] have shown that the convergence length is polynomial on average, while Englert et al. have described instances on which it becomes exponential [5]. As for the local optima quality, [3] shows that they are, with high probability, within a constant factor from the global optimum. Clearly, the complete enumeration algorithm for finding the best 2-OPT move is not only worst-case $\Theta(n^2)$, but its average-case is $\Theta(n^2)$ as well. Building on our previous research [9, 8], we propose a new algorithm for finding the best 2-OPT move which takes about $O(n)$ for Euclidean graphs whose vertices are points drawn u.a.r. in the unit square. Notice that this is the best possible complexity for finding an optimal move.

The strategy for moves enumeration. Without loss of generality, let us assume that the tour is $T = (1, \ldots, n)$. For $\alpha > 0$ let us call *α-large* any edge $\{i, i+1\}$ along the tour, such that $c(i, i+1) > \alpha$. In this paper we are going to follow a strategy that allows us not to enumerate all moves, but only those which are "good candidates" to be the best overall. The idea is quite simple, and it relies on a sequence of iterative improvements in which, at each iteration, there is a certain move (the current "champion") which is the best we have seen so far and which we want to beat.

Assume the current champion is $\hat{\mu} := \mu(\hat{\imath}, \hat{\jmath})$. Then, for any move $\mu(i,j)$ better than $\hat{\mu}$ it must be

$$\Delta(\hat{\mu}) < c(i,i+1) + c(j,j+1) - \big(c(i,j) + c(i+1,j+1)\big) \leq c(i,i+1) + c(j,j+1)$$

and hence

$$\left(c(i,i+1) > \frac{\Delta(\hat{\mu})}{2}\right) \vee \left(c(j,j+1) > \frac{\Delta(\hat{\mu})}{2}\right)$$

i.e., at least one of the move pivots must be $(\Delta(\hat{\mu})/2)$-large. Based on this observation, we will set-up an enumeration scheme which builds the moves starting from pivots that are $(\Delta(\hat{\mu})/2)$-large and then completing any such edge into a move by adding the second pivot of the move. Our basic steps are the *selection* and the *expansion* of the tour edges. The selection of an edge is simply the choice of an edge $\{i, i+1\}$. The expansion of $\{i, i+1\}$ is the evaluation of all moves $\mu(i,j)$, for $j \neq i$.

The (greedy) algorithm $\mathcal{A}_g$ make then use of a max-heap, in which we put each node $i \in \{1, \ldots, n\}$ together with the value $c(i, i+1)$, used to order the heap. At the beginning $\hat{\mu}$ is undefined, so we set $\Delta(\hat{\mu}) := -\infty$. Testing if there are still any large edges takes time $O(1)$ per test since we just need to read the value at the heap's root. The selection takes time $O(\log n)$ to maintain the heap property. The main loop terminates as soon as there are no longer any large edges. At the generic step, we pop the top of the heap, and, if it is large enough, we run the expansion, of cost $\Theta(n)$. Each time we find a move better than the current champion, we update its value. This way the termination condition becomes easier to satisfy.

Probabilistic analysis. We consider instances drawn at random according to the following distribution: draw u.a.r. n points $P_1, \ldots, P_n$ in the unit square and then sett the cost of each edge $\{i, j\}$ to the Euclidean distance between P_i and P_j. (Note that the edge lengths are not independent random variables since triangle inequality must hold.)

We denote by $t^n_{\mathcal{A}}(I)$ the number of elementary steps taken by the algorithm $\mathcal{A}$ on an instance I of size n and we define the associated random variable $T^n_{\mathcal{A}}$ as the time taken by $\mathcal{A}$ on a random instance of size n. The average-case complexity of the algorithm $\mathcal{A}$ is then defined as $\bar{T}_{\mathcal{A}}(n) := \mathbb{E}\,[\,T^n_{\mathcal{A}}\,]$.

To study our algorithm, we consider its (heuristic) variant which works as follows: Given an input I of size n, the heuristic first computes a threshold δ_n (i.e., depending only on n, and constant for a fixed n) and then it expands all and only the edges $\{i, j\}$ of the tour such that $c(i, j) > \delta_n$. Notice that there is an algorithm of this type for each possible function δ_n, and hence we can talk of a family of algorithms. Let us call a generic algorithm of this family ALG(δ_n).

Each algorithm ALG(δ_n) is a heuristic since there is no guaranteed that it will find the best move, but rather it will find it with a certain probability, depending on δ_n. In particular, ALG(δ_n) may fail because either (i) no edge is expanded (all arcs have cost $\leq \delta_n$) and hence no move is found; (ii) some edges are expanded, but the optimal move did not remove any edges of length $> \delta_n$ and so it won't be found.

The probability of failure can be controlled by a proper setting of δ_n. Intuitively, by lowering (increasing) δ_n we decrease (respectively, increase) the probability of errors. At the same time, we increase (respectively, decrease) the average time complexity of the algorithm, since more (respectively, less) edges get expanded. We have studied a way to balance these two conflicting objectives, namely, having a δ_n large enough so as to guarantee an average sub-quadratic algorithm, but small enough so as the probability of errors can be upper-bounded by any given constant.

We will proceed without proofs, for space limitations. Let $\Delta^*(I)$ denote the value of an optimal 2-OPT move on an instance I, and let us call I a *good instance* if there exist some move μ such that $\Delta(\mu) > 2\delta_n$. Then,

Lemma 1. *For every good instance I, ALG(δ_n) finds an optimal solution.*

Furthermore, under the conditions of the lemma, $\mathcal{A}_g$ runs faster than ALG(δ_n).

Lemma 2. *For every good instance I, it is $t^n_{\mathcal{A}_g}(I) \leq t^n_{ALG(\delta_n)}(I)$.*

Lemma 3. *Let D be the random variable representing the cost of the edge between two random nodes of the graph. If δ_n is chosen so that $\mathbf{Pr}[\,D > \delta_n\,] = \Theta(n^{-1})$, then $\bar{T}_{ALG(\delta_n)}(n) = \Theta(n)$.*

Lemma 4. *Let $1.055 < d \leq \sqrt{2}$ and let D be the distance between two random points drawn uniformly in the unit square. Then*

$$\mathbf{Pr}\,[\,D > d\,] \leq \frac{7}{16}\left(1 - \sqrt{d^2 - 1}\right)^4.$$

Corollary 1. *Let $\alpha > 0$ be a constant and define $\delta_n := \sqrt{2} - \alpha\, n^{-1/4}$. Then, the probability that an edge of a random tour is expanded is $\Theta(n^{-1})$, and the average-case complexity of ALG(δ_n) satisfies $\bar{T}_{ALG(\delta_n)}(n) = \Theta(n)$.*

We then describe a specific type of good moves, namely the *uncrossing* of crossing diagonal edges in a random tour. Such an uncrossing is always a move of high 2-OPT value, since two very long crossing edges (edges of length close to $\sqrt{2}$, say $\{A, B\}$ and $\{A', B'\}$, from the top-left corner to the bottom right corner of the unit square) get replaced by two very short edges between corner points (i.e., $\{A, A'\}$ and $\{B, B'\}$) We show that, asymptotically in n, the probability of having no good uncrossings tends to 0 for increasing α. This implies that for every $p \in [0, 1)$ we can find an α to set δ_n so that, asymptotically, the probability for an instance to be good is grater than p.

n	CE	$\mathcal{A}_g$	ALG(δ_n)	$\bar{f}(n)$	$\frac{CE}{\mathcal{A}_g}$
2,000	1,999,000	15,786	63,626	63,100	126.63
4,000	7,998,000	32,811	119,614	119,846	243.76
6,000	17,997,000	46,710	173,905	176,063	385.29
8,000	31,996,000	61,073	232,322	231,907	523.90
10,000	49,995,000	78,926	285,921	287,487	633.44
12,000	71,994,000	93,552	340,711	342,869	769.56
14,000	97,993,000	110,450	397,165	398,093	887.21
16,000	127,992,000	124,632	457,667	453,189	1026.95
18,000	161,991,000	141,852	513,313	508,177	1141.97
20,000	199,990,000	156,056	569,531	563,072	1281.52
22,000	241,989,000	169,574	618,985	617,886	1427.03
24,000	287,988,000	181,513	675,163	672,629	1586.59

Table 1: Finding the best move on a random tour. Results for Euclidean instances.

We conclude that ALG(δ_n) is a heuristic whose average-case running time is linear that succeeds on at least a fraction p of instances. This implies that, for at least a fraction p of all instances, $\mathcal{A}_g$ is dominated by an algorithm of linear average-case running time, where p can be made as close to 1 as we want.

Computational experiments. We compared experimentally the three algorithms ($\mathcal{A}_g$, ALG(δ_n) and complete enumeration) by looking at how many moves they evaluate on average, over 1000 runs. In particular, we generate 100 random instances and for each of them we generate 10 random tours on which we determine the best 2-OPT move. Given that the bookkeeping costs are dominated by the number of evaluations, looking at the number of moves which are evaluated gives a pretty precise idea of the ratios between the running times as well.

We have performed a similar set of experiments on random Euclidean instances. In Table 1 we can see that the greedy algorithm is from *two to three orders of magnitude* faster than complete enumeration when looking for the best move on a random tour on graphs with up to 24,000 nodes. The values are averages over 1,000 experiments for each size n, exactly as before. The fixed threshold algorithm has been run with $\delta_n = \sqrt{2} - 2.5/\sqrt[4]{n}$. We remark that the algorithm ALG(δ_n) found the optimal move, over *all* the 12,000 trials. The linear behavior of $\mathcal{A}_g$ can be appreciated by looking at the interpolating function $7.7\,n$.

Some test-bed instances on the repository TSPLIB [11] are of geometric nature, and we have tested our algorithm on those as well. In particular, there are some Euclidean instances (but they are not random, they correspond to some networks of world cities), and other are metric, not Euclidean, instances. We have selected the largest such instances (with the exception of `pla85900`, that, with $\geq 85,900$ nodes, was too big for our computer setting). The results are reported in Table 2 in the appendix. It can be seen that our method achieves a speed-up of two to three orders of magnitude in finding the best move on a random tour.

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Appendix

name	n	$\mathcal{A}_g$	CE	$\frac{CE}{\mathcal{A}_g}$
`euc2d/rl5915`	5,915	59,258	17,490,655	295.16
`euc2d/rl5934`	5,934	51,261	17,603,211	343.40
`ceil2d/pla7397`	7,397	48,665	27,354,106	562.08
`euc2d/rl11849`	11,849	98,457	70,193,476	712.93
`euc2d/usa13509`	13,509	104,147	91,239,786	876.06
`euc2d/brd14051`	14,051	170,286	98,708,275	579.66
`euc2d/d15112`	15,112	195,385	114,178,716	584.37
`euc2d/d18512`	18,512	174,374	171,337,816	982.58
`ceil2d/pla33810`	33,810	371,561	571,541,145	1538.21

Table 2: Finding the best move on a random tour on TSPLIB instances.

THE MOLDABLE TASKS IN CONTAINER PORT TERMINAL

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Abstract: We study the Berth and Quay Crane Alocation Problem (BAQCAP) as the moldable task scheduling problem. In the model the processing speed of a task is considered to be a non-linear strictly increasing and arbitrary function of the number of processors allocated to it. We present approximation algorithms which solved problem with bounds starting from processors allocation gave by lower bound which is a solution of the continuous version, i.e. where the tasks may require fractional quantities of the resources. We conducted computational experiments to evaluate the performance of the algorithm which showed that the algorithm gives solutuion not worst than 1.64 of lower bound for the problem.

Keywords: moldable task, approximation algorithm, berth allocation.

1 INTRODUCTION

Even though maritime transportation has increased modestly by 3.2 percent in 2021 (UNCTAD 2022), the pressure on the container ports for becoming more efficient is ever present. To this end, the ports should focus on the utilization of their limited resources, such as berths and quay cranes. Since the number of quay cranes allocated to a ship will affect its turnaround time, quay crane allocation is one of the major operational problems in container ports. Obviously this problem interacts with the other limited resource, that is the berth, due to physical constraints. This interaction necessitates the integration of berth allocation problem (BAP) with quay crane allocation problem (QCAP). As indicated in the literature, this integration can happen in different forms. In context of problems in port container terminal, other models were used rather than moldable tasks scheduling, with the exception of our earlier paper [2] where similar problem was considered and literature overview of the earlier papers was elaborated. Therefore we are presenting few recent papers only in which berth and quay crane allocation were considered together.

In [8] considered the BACAP under data uncertainties together with the Quay Crane setup time of shifting along the quay. The berth is continuous but quay and time are partitioned by a fixed length. The authors provided two robust optimization models to handle data uncertainties in QC productivity and proposed Genetic Algorithm and an insertion heuristic.

An integrated BAQCAP as scheduling problem with a heterogeneous set of cranes was considered in [1]. A mathematical model based on the relative position formulation (RPF) for the BA is presented which gives discrete of the time and space variables. For a such descrete values to derive good feasible solutions, a rolling horizon heuristic (RHH) together with a branch and cut approach are presented.

As we mentioned, in this paper we take the fact that berthing time of a ship depends on the number of quay cranes allocated to it. This dependence is by non-linear processing speed function described, which differs form the above models in the literature described, where either this function is linear or it is not considered at all. Of cource our approach still consider berth allocation together with quay crane allocation and is modeled as a moldable task scheduling problem. In our model the quay cranes operating along the berths are processors and the vessels are tasks to be processed by the processors without preemption. From the point of view of the port operator and the ship owners the main goal is to increase utilization of the berths and minimize the *turn-around time* of the ships. In the moldable task scheduling, this will be achieved by minimizing the maximum completion time of all the tasks, i.e., the schedule length.

Our approach except for shorten turn-around time of the ships can simultaneously increase the utilization of the quay cranes and decrease the waiting time of the containers.

2 MOLDABLE TASKS MODEL

Moldable task scheduling is one of the parallel task scheduling models documented in the literature [4]. The moldable task scheduling model was first proposed by Turek et al. [9] and later studied by Ludwig [6] and Mounié et al. [7]. The moldable task scheduling problem can be stated formally as follows: We consider deploying a set of m identical *processors* (quay cranes in BAQCAP) to execute a set $\mathcal{T}$ of n independent, non-preemptive (i.e., once a task starts its execution, it has to be processed on the processors it is assigned to without any interruption for the period of its processing time) *moldable tasks* (ships). Each moldable task (MT) has a processing time (an amount of work) that is dependent on the number of processors allocated for its execution. As the number of processors allocated to a task is a decision variable at the time when the scheduling decision is made by the definition of moldable tasks, we denote this processing time by $t_i(r) = p_i/f_i(r)$, where $f_i(r)$ is the non-decreasing processing speed function which relates processing speed of task T_i to the number of processors allocated, p_i is the amount of work associated with task T_i ($p_i = t_i(1)$, that is processing time of task T_i on one processor) and r is the number of processors allocated to task T_i, $1 \leq r \leq m$. The dependence of a moldable task's processing speed on the number of processors allocated to it is given as a discrete function; i.e., it takes values at integer points only. The criterion to be minimized is the schedule length (turn-around time of the ships in BAQCAP), which is denoted by $C_{max} = \max_i\{C_i\}$, where C_i denotes the completion time of task T_i.

Since the problem of scheduling independent MT without preemption is NP-hard [5], research on this problem has focused on designing suboptimal algorithms. Turek et al. [9] showed that any λ-approximation algorithm for the two dimensional bin-packing problem can be polynomially transformed into a λ-approximation algorithm for the MT scheduling problem. Based on this result, Ludwig [6] developed a 2-approximation algorithm and Mounié et al. [7] developed an approximation algorithm with a worst case performance guarantee of $\sqrt{3}$ for this problem.

Since approximation algorithms are not viable for real-life problems, we took another approach to solve this MT scheduling problem. In order to explain this approach we note that we may consider our problem such that the processors represent a continuously divisible renewable resource bounded from above. With this observation, we utilize a continuous model developed for resource constrained scheduling problems [10] to find an approximate solution to discrete MT scheduling problem. We then propose a procedure to convert this approximate (and maybe infeasible) solution into a feasible one for the original problem. When we consider the processors as continuously divisible renewable resource, we can relax the integrality requirement of processors (resources) in the allocation process. Blazewicz et al. [3] showed that an optimal solution to such a problem in which the processing speed functions are interpolated by piecewise linear functions between integer points (the P-CNTN problem) can be found in polynomial time, assuming that the processing speed functions are all concave. It is worth noting that the solution of the P-CNTN problem provides a good starting point for the discrete model and the schedule length obtained is an absolute lower bound for the optimal discrete schedule.

3 SUBOPTIMAL ALGORITHM

In this section we present the solution procedure for solving BAQCAP which was modeled as a MT scheduling problem in Section 2. The discrete processing speed function is replaced by a continuous interpolation using piecewise linear functions between integer points [3]. Then we propose an algorithm to transform the schedule obtained from the continuous version into a feasible schedule for the discrete moldable task model. In the proposed algorithm, a rounding scheme for a non-integer allocation is first used. Next, the tasks are thoroughly packed using several steps of rounding off in order to achieve better utilization of the processors. These steps allow for a good average behavior, as is demonstrated in the computational experiments, for a wide range of task and processor parameters.

3.1 Longest Task First algorithm (LTF)

1 Solve the continuous version of the problem with fractional processors allocation to each task.

2 Round the allocations of the processors to a nearest integer number.

3 Sort tasks in non-decreasing order by the number of processors.

4 Put the longest task in the layer so that it starts at the moment $t = 0$ (of the layer's inner time) and is executed by the sequence of processors starting from the first one. Set the layer's length d to the length of this task.

5 Add the remaining tasks, starting from the longest one, to next processors at the moment $t = 0$. If any task doesn't fit because there aren't enough processors empty from the moment $t = 0$, skip this task.

6 Add the remaining tasks, starting from the longest one, just after those already scheduled. For each task, first try to place it on the sequence of processors starting from the first one. If it fails, move the sequence by one processor and try again and again. If a task doesn't fit anywhere because of its number of processors or its length, skip it.

7 If there are $r_x > 0$ empty processors, set the number of processors to the longest task. Move the rest of task to next processors by the number of those added to the longest one.

8 If there are any tasks left, go to step 1.

After sorting tasks for every layer we first try to add every task and then try to place them on any possible sequence of processors. That leads to the complexity of $O(n^2m)$.

4 COMPUTATIONAL EXPERIMENTS AND CONCLUSION

As mentioned, to construct a schedule for the non-preemptive moldable task scheduling problem, we first use the solution of the continuous problem with arbitrary and piecewise linear processing speed functions. Such approximations of these functions allow us to obtain a better lower bound on the optimal moldable solution. Furthermore, the allocation of the processors after rounding very rarely changes during the execution of Algorithm LTF.

We observe that Algorithm LTF behaves well for a wide range of task and processor parameters. In Figure 1 you can see that for all instances tested in our experiments, a ratio (makespan for LTF algorithm to a lower bound) doesn't exceed 1.64. It is worth to amphasize that the best practical algoritm has a ratio $\sqrt{3}$. The experiments show that when the number of tasks is significantly less to the number of processors, the optimal continuous solution approximate well the discrete moldable one. Thus, the continuous solution may be a good starting point for the construction of an optimal moldable schedule. It is interesting to note that this behavior is not affected much by changes in the shapes of the speed functions. This implies that our approach can be used to deal with real life problems at container terminals as the number of ships is usually significantly different from the number of the quay cranes at the port.

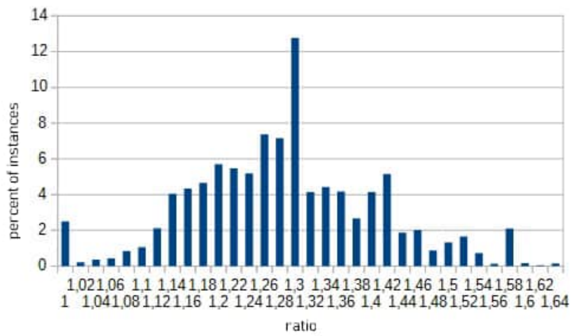

Figure 1: Solution ratio of Algorithm LTF

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Simulation Model For Cyclic Single Track Railway Problem

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Abstract:

Considered problem involves a single-track railway line that connects multiple stations. Trains can travel in both directions along the track, but since there's only one track, they must wait for each other to pass if they're going in opposite directions. Additionally, each station has a limited capacity, which means that trains cannot stop at a station if it's already full.

The objective of the scheduling problem is to determine the cyclic schedule with trains maximization of trains for the given set parameters the track, and ensuring that trains don't exceed station capacity for an assumed cycle time.

Simulation-based optimization methods use to simulate train movements and station capacity. Optimization algorithms are then used to find the effective schedule based on the simulation results. In this paper, the case of models of STRSP are considered and analyzing them from both the capacity point of view and the maximal number of trains that can be scheduled for the given single-track railway lines and cycles time. Finally, one can construct a simulation model that can automatically generates timetables for trains, including cyclic scheduling cases.

Keywords: cyclic train scheduling, single track, simulation.

1 INTRODUCTION

The problem arises when a set of trains need to travel along a single track with limited stations capacity. it is complicated by the fact that trains traveling in opposite directions cannot pass each other, and the number of trains that can be accommodated at each station is limited by the station's capacity. For the general case the problem is NP-hard.

A key factor in the problem is the maximum throughput rate, which is the maximum number of trains that can pass through a station per unit time. This rate is limited by the station's capacity and the speed of the trains, among other factors. The scheduling efficiency of the system depends on how well the maximum throughput rate for the given cycle time is utilized.

To solve the clasical single track scheduling problem, various optimization techniques can be used, such as mathematical programming, simulation, or heuristics. The optimal solution depends on the specific constraints and objectives of the problem, such as minimizing travel time, maximizing train throughput, or ensuring safety and reliability.

Efficient scheduling of trains on a single track with limited stations capacity is crucial for the safe and reliable operation of railway systems. The problem has applications in various fields, including transportation, logistics, and supply chain management. As a result, there is a literature on the single track scheduling problem, including various algorithms and models for solving it [1].

Then the simulation technique that uses computer models to simulate the operation of the railway system and evaluate different scheduling scenarios. Simulation can provide parameters and insights into system performance and help identify potential bottlenecks and improvements for dynamic and varying environment.

In practice, there are many factors that need to be taken into account when scheduling trains for transporting goods, such as the capacity of the stations, the speed and weight of the trains, the availability of tracks, the weather conditions, and the safety and reliability of the system [2].

The motivation for research on the single track scheduling problem often comes from real-world applications, such as the transportation of coal from mines to a harbor in the case of the Australian railway as a practical example. Therefore, finding optimal solutions to the single track scheduling problem is an important research topic with practical implications.

The paper considers some models and formulates capacity station dependencies. The results can be utilized for practical "what-if" analysis, where the lines are described by the number of stations, total travel time, and travel time between stations. The changes on the line and train parameters may cause the bottlenecks which should be avoided and the estimation of the maximum trains traveling in the cycle scheduling for the given time horizon.

2 PROBLEM FORMULATION AND MODELS

We are attempting to determine the maximum number of trains that can travel from the starting station to the ending station and back within a given time frame. Let us define:

T_s as the time at which the first train begins its journey. T_f as the time at which the last train completes its journey. We will be examining journeys that take place within the time interval $[T_s, T_f]$, subject to the following assumptions:

- The time taken to travel between neighboring stations is constant and is denoted by $t_{i,j}$.
- It is possible for at least one train to travel between any two stations within a given time.
- The start station (s) and the end station (e) have infinite capacity, denoted by cp_s and cp_e respectively.

2.1 Train maximization problem

One can define the following:

B - the length of the railway from the starting station to the ending station. There are a total of $B+1$ stations, including the starting and ending stations. Hence, the ending station is denoted by $e = B+1$.

T - the length of the time interval being considered, where $T = T_f - T_s$.

there are interested to finding $max, R_{(B,T)}$, which is the maximum number of trains that can travel the distance B within the given time T.

There may be generated several models for the same physical railway system. The travel times can be differ for example by maintenance cases and potential changes in the station capacity (potential investments).

2.2 Minimization of the total traveling time model with finite capacities

This model aims to minimize the total traveling time subject to finite capacities at each station.

Under above assumptions, one can formulate the following objective function for the particular model as a calculated sub-problem:

$$\min_{n_{i,j}} \sum_{i=1}^{e-1} \sum_{j=i+1}^{e} n_{i,j}(t_{i,j} + t_{j,i}) \tag{1}$$

This function minimizes the total traveling time of all trains on the railway. The variables are the number of trains n traveling between each pair of stations (i, j).

with capacity constraints the number of trains arriving at any station i cannot exceed its capacity cp_i, where $i = 1, 2, \ldots, e$. For the possible stations' capacity changes the total completion times can be represented as:

$$\sum_{j=1}^{i-1} n_{j,i} - \sum_{j=i+1}^{e} n_{i,j} \leq cp_i, \qquad i = 1, 2, \ldots, e \tag{2}$$

These constraints ensure that the number of trains traveling between each pair of stations does not exceed their capacity, and the total time taken by all trains does not exceed the time interval T.

Station capacity dependencies can be formulated as follows:

cp_i: the capacity of intermediate station i (i.e., the maximum number of trains that can be on the station at the same time), which is assumed to be finite. Our objective is to find the minimal cp_i for each station $i \in 2, 3, \ldots, e-1$ that allows the maximum number of trains to pass through within the given time.

The greedy strategy that gives always priority to the train that goes first, causing delays 0, 1, 2... respectively to the following trains provides such upper bound. One is able to present patterns that results with a sum of times equal to this bound or lower.

Let's denote:
$t_{s_i}-$time train i departures from starting station s, $t_{s_i} \in \{0, 1...\}$;
$t_{f_i}-$time train i reaches back the station s,
so $c_i = t_{f_i} - t_{s_i}$.
We can observe differences in t_{s_i} between these patterns.
$t_{s_i} = i$ for $i < B$ and $t_{s_i} = 2(i-1) + 1$ for $i \geq B$;
whereas:
$t_{s_i} = i$,
but in both cases $t_{f_i} = 2B + 2(i-1)$ and $c_i \leq 2B + i - 1$.

Note that any additional wait of the first train enlarges $\sum_{j=1}^{R} c_j$ (as it enlarges t_{f_i} for some i, without changing t_{s_i}).

It can be observe in the pattern presented below, that results with:

$$\min_j \sum_{j=1}^{R} c_j^{(3)} \leq \min_j \sum_{j=1}^{R} c_j^{(2)} \leq \min_j \sum_{j=1}^{R} c_j \tag{3}$$

3 Algorithmic simulation model

In order to implement a simulation algorithmic solution for the models described above, it is necessary to address the bottleneck problem. There are several methods to avoid bottlenecks, the one is chosen as to reserve at least one unit of capacity on stations where the capacity is greater than one. Stations with a capacity of one are reduced from the line, and the travel times in neighboring systems with capacities greater than one are increased accordingly. The simulation then calculates a feasible solution and generates a cyclic timetable. The algorithm takes into account potential collisions and conflicts, as described in previous literature [3]. For lines with defined integer travel times, the algorithm generates an effective cyclic train timetable, as illustrated in the example in Figure 1. To perform a "what-if" analysis, one can manipulate the station capacity parameter and the traveling time, to determine the optimal increase in capacity for a given instance of the railway line. The software simulation package was prepared in [4] and several experiments were performed.

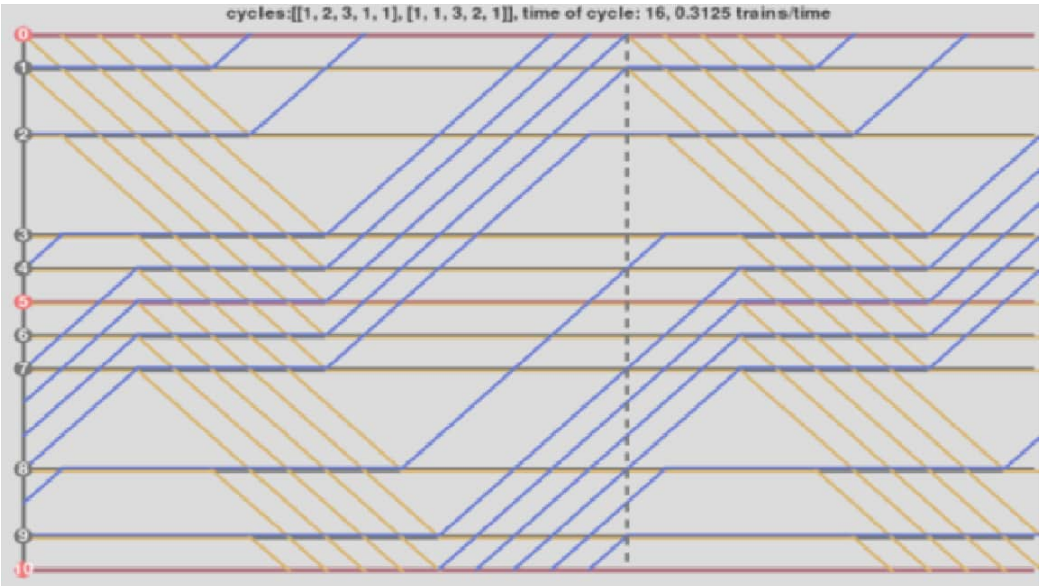

Figure 1: Example of time-table solution produced by the simulation.

4 Conclusions

Overall, the single track train scheduling problem with limited station capacity is a complex optimization problem that requires careful consideration of many factors, including train speeds (caused for example by the maintenance), station capacity, and safety constraints. By using appropriate optimization techniques, it is possible to find an optimal solution that minimizes travel time while cycle time and efficient train operations. The simulation algorithmic model generating the effective cyclic time-tables had been presented.

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Metaheuristic Methods for TV Advertisement Scheduling Problem

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Abstract: The paper concerns the optimization problem which arises in the process of planning TV advertising campaigns. The goal is to assign TV ads to commercial breaks in order to achieve the expected total viewership under a given set of constraints. The formal model of the problem is presented and expressed in terms of the scheduling theory. The solution methods based on list and metaheuristic approaches were proposed and validated in computational experiments, performed using test data generated based on real historical data.

Keywords: advertisement scheduling, mathematical model, metaheuristic algorithms

1 INTRODUCTION

Despite the increasing importance of marketing based on Internet technologies, TV advertising is still a powerful and useful tool used for promoting products and attracting customers. TV advertising is related to various research fields due its complex nature. It involves predicting viewership and viewers' behavior, then designing relevant pricing policies for commercial slots and determining their impact on the audience, particularly on target groups. Predicted viewership and prices are the main factors taken into account by companies or media houses while buying commercial time: their main goal is to achieve expected impact on viewers at the lowest cost or within the assumed budget. Finally, the TV program must be properly constructed by scheduling advertisements within commercial breaks to meet all domain specific constraints and contracted requirements (e.g., [3, 6, 8]).

In the presented research, we focus on the last mentioned aspect - planning commercial breaks - considered mainly from the point of view of TV channel owners, i.e., broadcasters. Our studies have been inspired and supported by real historical data available for scientific purposes at our institute. Within the presented research we proposed a formal definition of the problem and a few heuristic and metaheuristic approaches. They were validated and compared in the extended computational experiments, performed for test instances created based on historical data.

2 PROBLEM DEFINITION

The problem considered in the reported research concerns scheduling advertisements in commercial breaks on a few TV channels belonging to the same owner (non-competing channels) within a given planning horizon. Commercial breaks are divided into slots of equal length, which differ in the predicted viewership. It is higher at the beginning and at the end of breaks. Such prime slots are the most profitable for customers, and the most costly at the same time. Within breaks, a set of commercials must be scheduled. Particular advertisements must be broadcast several times in order to achieve the expected total viewership specified by customers. Moreover, for each commercial the minimum number of airings is given, as well as the expected percentage of broadcasts within prime slots. The number of possible repetitions of the same advertisements within a single break is limited. Additionally, the requested number of other commercials, which should be broadcast between two airings of the same advertisements, might be specified to avoid viewers' irritation. For marketing reasons, consecutive scheduling advertisements of similar products or services offered by competing brands or companies should be avoided. Finally, multiple broadcasting a given commercial spot should be planned within a certain time window, determined by the advertising campaign duration for a promoted product. Depending on the product type, for some commercials the subset of preferred TV channels on which they should be aired might be specified in order to reach specific target groups. Similarly, for some advertisements the broadcasting time is limited to certain time windows due to law restrictions.

To construct a solution for the considered TV channels there is necessary to determine the number of airings of each advertisement. Particular airings must be assigned to breaks and scheduled within these breaks, taking into account all constraints and requirements mentioned before. The quality of schedules might be evaluated from various points of view, since the advertisement scheduling problem is, by its nature, a multi-criteria optimization problem (e.g., [1]). Within our research, we focused on minimizing the cumulative objective function determined by: the total extension of the commercial breaks duration, the total duration of commercials broadcast after their due dates, and the total cost related to brand/companies incompatibilities for advertisements scheduled consecutively within one break, taking into account relative priorities of customers ordering advertising campaigns. While constructing a schedule some requirements concerning broadcasting advertisements might be considered as hard constraints, which must be fulfilled, others can be treated as soft constraints, which might be broken, but they influence the quality of this schedule.

The careful analysis of the problem of sequencing advertisements within commercial breaks showed similarity to the scheduling problems [5, 7], which allowed us to propose the mathematical model expressed in terms of the scheduling theory [4]. We formulated a problem of scheduling a set of jobs (advertisements) on a set of identical parallel machines (TV channels) with eligibility constraints and availability periods, modeling the real world constraints presented above.

3 METAHEURISTIC ALGORITHMS

The scheduling problem sketched above is obviously NP-hard [4, 10]. Due to the complex nature of the considered advertisement scheduling problem, involving various parameters, constraints and numbers of decision variables, heuristic and metaheuristic methods are natural choices for solving it (e.g., [2, 9, 11]). Within our research, we decided to compare the efficiency of a population-based method, namely genetic algorithm, with a trajectory based metaheuristic, namely simulated annealing [12].

The initial solutions for these approaches were provided by the specialized greedy list algorithms equipped with a simple local search engine. For the considered problem, constricting by list approaches a feasible solution meeting all problem constraints might be difficult or even impossible for some instances. For this reason, the list scheduling algorithms construct initial schedules focusing on the most important constraints only. Then they try to satisfy the remaining constraints formulated in the model, by applying a simple local search.

The solutions constructed by list algorithms provide starting points for further exploring the solution space by metaheuristics. Both proposed methods, simulated annealing (SA) and genetic algorithm (GA), are based on the classical framework of these approaches. SA starts with a single initial solution. A current solution is modified according to a move randomly chosen from a set of implemented moves, which insert, delete or swap some jobs in a schedule. The probability of accepting worse solutions by SA depends on the objective function deterioration and the temperature, which is controlled by one of two implemented cooling schemes: the logarithmic and arithmetic-geometric ones. On the contrary, GA starts with the whole population of solutions, which initially contains list schedules generated at the high randomness level. Then GA constructs new solutions using a crossover operator with multi-point crossing and modifies existing solutions using a mutation operator. GA keeps the constant size population, which modifications are controlled with recombination and mutation rates. Both methods stop after reaching a given termination condition such as exceeding an assumed number of iterations/generations or solution transformations, and exceeding a predefined time limit. Moreover, both methods can be stopped when a feasible solution is constructed.

4 COMPUTATIONAL EXPERIMENTS

The efficiency of all proposed methods was validated in extensive computational experiments. Test data were generated based on real historical data. They included the detailed descriptions of TV program and commercials broadcast on more than hundred TV channels, as well as the rich characteristic of viewership measured in particular time slots. This huge amount of rough data was analyzed, cleaned and preprocessed to make their further usage possible. The historical schedules were considered as reference solutions for instances generated based on original historical data, and they were used for validating the efficiency of the proposed methods. Moreover, based on the basic (original) instances, additional sets of instances were generated by relaxing some constraints. In this way we constructed 6 test sets of 120 instances differing in their difficulty. Particular instances contained from 1 to 6 non-competing channels, from 570 to 8077 commercial breaks, and from 4665 to 81606 adds per TV channel. The smallest instance involved 1 400 000 of decision variables.

As we have mentioned, due to the complex nature of the considered problem, the main goal for the proposed algorithms became constructing solutions fulfilling all constraints: hard and soft ones. The simple greedy list algorithms were able to find such solutions for no instance belonging the most difficult (original) data set. For easiest (most relaxed) data set such schedules were determined for 72% of instances. Metaheuristics were obviously much more powerful from this point of view. SA and GA fulfilled all constraints for all instances belonging to 4 among 6 data sets. GA appeared to be more efficient than SA, particularly for the most difficult instances. For this test set, SA found solutions fulfilling all constraints for ca. 25% of instances only, while GA was almost two times better. The superiority of the population based metaheuristic is also visible when comparing the quality of schedules (determined based on the criterion value and the percentage of violated constraints). Both methods, SA and GA, started from list schedules, but GA improved these solutions much more than SA. For easiest instances, GA improved the quality of ca. 50%, while SA of ca. 35%, respectively. However, for two most difficult test sets, the power to improve the solution quality was similar for both methods (of ca. 75%).

The running times of both metaheuristics were significant due to the huge number of parameters and constraints defined in the considered problem. However, both methods never reached the two-hour time limit per instance and stopped after the given (tuned) number of iterations/generations without improvement. GA required at most 50 minutes, while SA required less than 20 minutes. Greedy list algorithms, used as stand-alone procedures, were obviously much faster and consumed from 1 second to 8 minutes, depending on the variant used. It is worth to be mentioned that time efficiency is not crucial for the considered problem, since the process of scheduling advertisements is performed in off-line mode.

5 CONCLUSIONS

Within the presented research, we studied the advertisement scheduling problem from the point of view of a company (TV channel owners or media houses) constructing a schedule of commercials, taking into account requirements and preferences of customers ordering advertising campaigns. Based on the careful analysis of historical data we proposed the mathematical model and list algorithms, as well as simulated annealing and genetic methods solving it. This analysis made possible to distinguish the most important parameters and their characteristics, as well as to formulate the constraints crucial for solving the problem. The very demanding cleaning and pre-processing of historical data allowed us to generate test instances of various difficulty. Computational experiments showed the efficiency of both metaheuristic methods, which were able to construct solutions satisfying all constraints for most test sets of instances. The population-based genetic algorithm appeared to be slightly more effective than simulated annealing modifying a single solution in the search process. The proposed greedy list algorithms showed to be efficient tools for providing initial schedules in a short time. The obtained results will direct the further work on improving the optimization methods for the considered advertisement scheduling problem.

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Game Theory

DETERRING BRIBERY WITH SCOTCH HOLD'EM POKER

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Abstract: Corruption requires a coalition to form and reach an agreement. Is there a cheap way to stop any agreement from being reached? We find an optimal mechanism that resembles Poker. The players' hands are synthetic asymmetric information, and they create a lemons problem in the market for bribes. Our Poker mechanism is robust: it thwarts bribes regardless of the negotiation procedure, including alternating offers bargaining, Dutch auctions, arbitration, and so on. We embed our mechanism in regulatory approval and regulatory compliance settings. In the latter case, there is a trade-off between rewarding the agent for honesty and punishing the agent for non-compliance. This trade-off is resolved by rigging the Poker hand distribution against the agent.

Keywords: adverse selection, bribery, contagion, corruption, information design, information frictions, mechanism design, poker.

NON-COOPERATIVE BARGAINING ON DEBT RESTRUCTURING

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Abstract: An insolvent firm has liabilities towards a group of creditors. We analyze the problem of how to distribute the asset value of the firm among the creditors and the firm itself, using a dynamic non-cooperative bargaining model. We specify a bargaining protocol where a randomly selected active player can propose a coalition and a feasible asset allocation, where feasibility depends on the choice of the coalition. If the proposal is unanimously accepted by all members of the coalition at hand, then the related creditors leave the game. The firm remains an active player in the game as long as there are creditors remaining. We analyze the subgame perfect equilibria in stationary strategies. We show that when the discount factor goes to one, the equilibrium allocation is converging to the constrained equal awards rule, where creditors with smaller claims get a higher proportion of their claims rewarded than creditors with higher claims.

Keywords: insolvency, debt forgiveness, bankruptcy games, bargaining.

BOUNDING SOLUTION CONCEPTS OF INCOMPLETE COOPERATIVE GAMES

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Abstract: The computation of a solution concept of a cooperative game usually employs values of all coalitions. However, in some applications, values of some of the coalitions might be unknown due to high costs associated with their determination or simply because it is not possible to determine them exactly. We introduce a method to retrieve as much information as possible about the solution concept when only a portion of the values is given to us. The ideas of our method are demonstrated on different structures of incomplete information. Bounds are derived for different solution concepts including the Shapley value, the nucleolus, or the core. We show explicit formulas for bounds on some of the solution concepts and show there are different bounds based on additional information given about the game.

Keywords: incomplete cooperative games, solution concepts, Shapley value, tau-value, core

1 INCOMPLETE COOPERATIVE GAMES

A *cooperative game* (N,v) is given by a set of players $N = \{1,\dots,n\}$ and a characteristic function $v\colon 2^N \to \mathbb{R}$ with $v(\emptyset) = 0$. Value $v(S)$ of *coalition* $S \subseteq N$ can be interpreted as a possible worth of the coalition when its players decide to cooperate. The main goal is then to split $v(N)$ between the players, under the assumption that N forms. Different payoff distributions (*solution concepts*), represented as subsets of vectors $x \in \mathbb{R}^n$, follow different goals. When we want to impose *stability* (no subcoalition S is willing to leave N), one might employ *the core*

$$\mathcal{C}(v) = \{x \in \mathbb{R}^n \mid \sum_{i\in N} x_i = v(N) \text{ and } \sum_{i\in S} x_i \geq v(S) \text{ for } S \subseteq N\}.$$

Since the core might contain more than one $x \in \mathbb{R}^n$, *nucleolus* $\eta(v) \in \mathcal{C}(v)$ is further defined (see [5] for a proper definition). When *fairness* is required from the solution concept, *the Shapley value* $\phi(v) \in \mathbb{R}^n$, defined for $i \in N$ as

$$\phi(v) = \sum_{S\subseteq N\setminus i} \frac{|S|!(|N|-|S|-1)!}{|N|!}\left[v(S\cup\{i\}) - v(S)\right],$$

or the τ*-value* $\tau(v) \in \mathbb{R}^n$, defined as $\tau(v) = \alpha a^v + (1-\alpha)b^v$ satisfying $\sum_{i\in N}\tau_i(v) = v(N)$ is considered (see [3] for definition of a^v,b^v).

In real world situations, some of the values $v(S)$ might be unknown or too costly to acquire. To model this, denote $\mathcal{K} \subseteq 2^N$ the *set of coalitions with known value*, thus $(N,\mathcal{K},v)$ with $v\colon \mathcal{K} \to \mathbb{R}$ represents *incomplete cooperative game.* Further, it always holds $\emptyset \in \mathcal{K}$ and $v(\emptyset) = 0$.

In this paper, we suppose there is an underlying game $(N,\hat{v})$ accompanied by $(N,\mathcal{K},v)$ representing our partial knowledge about $(N,\hat{v})$, i.e. $\hat{v}(S) = v(S)$ for $S \in \mathcal{K}$. Further, from the essence of $(N,\hat{v})$, we know it comes from $C \subseteq \Gamma^n$ a subset of n-person cooperative games.

Definition 1.1 (C-extension) *Let $C \subseteq \Gamma^n$ be a class of n-person games. A cooperative game $(N,w) \in C$ is a C-*extension *of an incomplete game $(N,\mathcal{K},v)$, if $w(S) = v(S)$ for every $S \in \mathcal{K}$.*

C-extensions are simply all the candidates for $(N,\hat{v})$ based on the knowledge of $(N,\mathcal{K},v)$ and C. When we are not sure about the class C, we might even consider whether there is a C-extension (C*-extendability*). There are different incomplete games based on the structure of $\mathcal{K}$. First, *minimal* incomplete games constitute only of values of *singletons* $\{i\}$ for $i \in N$ and N. Games with $\mathcal{K} = \{S \subseteq N \mid i \in S\}$ for a fixed player i are called *player-centered.*

2 BOUNDS ON SOLUTION CONCEPTS

Imagine that we want to determine a solution concept $\mathcal{S}(\hat{v})$ of a cooperative game $(N, \hat{v})$. However all we know is $(N, \mathcal{K}, v)$ and that $(N, \hat{v})$ lies in $C \subseteq \Gamma^n$. The set of C-extensions of $(N, \mathcal{K}, v)$ represents all possible candidates for $(N, \hat{v})$, and $(N, \hat{v})$ is among these games. This means that by computing $\mathcal{S}(w)$ for every C-extension (N, w), we also compute $\mathcal{S}(\hat{v})$. The only problem is that we cannot distinguish which one of the solutions is $\mathcal{S}(\hat{v})$. Nevertheless, by considering the union of $\mathcal{S}(w)$ for all C-extensions (N, w),

$$\cup\mathcal{S}(C, \mathcal{K})(v) = \bigcup_{w \in C(v)} \mathcal{S}(w),$$

we are sure we do not leave $\mathcal{S}(\hat{v})$ out. Similarly, if we consider the intersection of all solutions $\mathcal{S}(w)$, denoted by $\cap\mathcal{S}(C, \mathcal{K})(v)$, we are getting a subset of $\mathcal{S}(\hat{v})$, thus

$$\cap\mathcal{S}(C, \mathcal{K})(v) \subseteq \mathcal{S}(v^*) \subseteq \cup\mathcal{S}(C, \mathcal{K})(v). \tag{1}$$

We call these the *strong solution* ($\cap$) and the *weak solution* ($\cup$). It is important to highlight that the payoff vectors from the strong solution are actually contained by $\mathcal{S}(\hat{v})$. The difference between the bounds and $\mathcal{S}(\hat{v})$ depends heavily on both C and $\mathcal{K}$. If for example $\mathcal{K} = 2^N$, all three sets coincide. If on the contrary $\mathcal{K} = \{\emptyset\}$, for most of the standard solution concepts, the relations become $\emptyset \subseteq \mathcal{S}(v^*) \subseteq \mathbb{R}^n$. Also, the more restrictive C is, the less C-extensions are considered, thus the closer the bounds are to one another. The ultimate goal is to find a compromise between information provided by $(C, \mathcal{K})$ and the strength of the bounds.

2.1 Main results

We studied the bounds for both minimal incomplete games and player-centered incomplete games. For minimal incomplete games, the amount of information represented by $(N, \mathcal{K}, v)$ seems rather insufficient as most of the bounds for most of the solution concepts are of form

$$\emptyset \subseteq \mathcal{S}(\hat{v}) \subseteq \mathcal{I}(v)$$

where $\mathcal{I}(v) = \{x \in \mathbb{R}^n \mid \sum_{i \in N} x_i = v(N) \text{ and } x_i \geq v(\{i\}) \text{ for } i \in N\}$. Since $\mathcal{S}(\hat{v}) \subseteq \mathcal{I}(v)$ holds for every game $(N, \hat{v})$ and most of the solution concepts $\mathcal{S}$, $\mathcal{I}(v)$ can be considered a trivial upper bound. Interestingly, the upper bound does not improve for the above mentioned solution concepts until we restrict to class of *positive* extension (for the definition of positive games, see [4]). More interestingly, the bounds remain trivial for the core even if we further restrict to *2-additive positive* extensions (see also [4]). As this set of C-extensions can be considered to be really restrictive, bounding of the core seems to be a challenging task under minimal information.

For player-centered incomplete games, there are two special extensions (N, v_0) and (N, v_1), which can be expressed as

$$v_0(S) := \begin{cases} v(S) & \text{if } S \in \mathcal{K}, \\ v(S \cup i) & \text{if } S \notin \mathcal{K}, \end{cases} \quad \text{and} \quad v_1(S) := \begin{cases} v(S) & \text{if } S \in \mathcal{K}, \\ 0 & \text{if } S \notin \mathcal{K}. \end{cases}$$

These games do not only play a central role in expressing the sets of different C-extensions but are also present in determining of various bounds of the solution concepts. For example, for the core, we get bounds in form

$$\mathcal{C}(v_0) \subseteq \mathcal{C}(\hat{v}) \subseteq \mathcal{C}(v_1).$$

Not only the bounds can be expressed as the core of special games but we know they cannot be improved upon because both (N, v_0) and (N, v_1) are possible candidates for $(N, \hat{v})$.

For the Shapley value ϕ (and similarly for the τ-value), the following bounds can be derived for player i (around which the game is centered),

$$\phi_i(v_1) \leq \phi_i(\hat{v}) \leq \phi_i(v_0)$$

and for $k \neq i$, we get bounds

$$\phi_k(v_0) \leq \phi_k(\hat{v}) \leq \phi_k(v_1).$$

Thus, these games can be considered as *best/worst-case* scenarios between the candidates. Not only player-centered games yield non-trivial bounds using our method, but the bounds tell us a lot about both the structure of the set of C-extensions as well as about the structure of the weak and the strong solutions.

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Three Variations on Money Pump, Common Prior, and Trade

Extended abstract

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There are two starting points of our analysis: 1) The Borel-Kolmogorov paradox (Kolmogorov, 1956), and 2) ”no sure win” principle (de Finetti, 1972).

The Borel-Kolmogorov paradox says that a prior cannot determine a posterior on a zero probability event. A Bayesian decision maker who has a prior, cannot update on zero probability events, hence the posterior cannot be deduced from the prior in every case. Therefore, we assume that posterior is the primitive, we give priority to the posterior over the prior. In other words, the framework of our analysis is information structure (type space), where the posteriors of the players are given.

The interpretation of probability we follow in this paper is the one which says a probability is a degree of belief. The ”no sure win” principle says that a gambler cannot win at every state of the world. de Finetti (1972) showed that the ”no sure win” principle implies that the probability is additive. Therefore, in this paper we assume that the players’ beliefs are additive set functions.

As in the case of the ”no sure win” principle, the answer for our question is not mathematical, it is rather decision theoretic, economic, and financial. In the single player setting the answer is clear, and related to the ”no sure win” principle. We consider the notion of money pump, which reflects the very same intuition as the ”no sure win” principle, and conclude that a probability distribution excludes the possibility of money pump if and only if it is disintegrable. Therefore, in the single player setting we conclude that a prior is a disintegrable probability distribution.

In the multiplayer setting, where we quest for the proper definition of common prior, the answer is not unequivocal. In this setting we apply all three lenses, decision theory, economics and finance, in our analysis. We analyse three variants of common prior (decision theory lens), trade (economic lens) and money pump (finance lens). Then our answer is that the proper definition of prior depends on the propose of the modeller, what variant fits her model the best.

Keywords: Information structure, Common prior, Trade, Money pump, Disintegrability, Conglomerability

ON THE CORE OF MANY-TO-ONE ASSIGNMENT GAMES

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Abstract: We study many-to-one assignment markets, or matching markets with wages. Although it is well-known that the core of this model is non-empty, the structure of the core has not been fully investigated. To the known dissimilarities with the one-to-one assignment game, we add that the bargaining set does not coincide with the core and the kernel may not be included in the core. Besides, not all extreme core allocations can be obtained by means of a lexicographic maximization or a lexicographic minimization procedure, as it is the case in the one-to-one assignment game.

On the positive side, the maximum and minimum core allocations are characterized in two ways: axiomatically and by means of the longest length paths and the shortest length paths in a given directed graph. Regarding the remaining extreme core allocations of the many-to-one assignment game, we propose a lexicographic procedure that, for each order on the set of workers, sequentially maximizes or minimizes each player's core payoff. This procedure provides all extreme core allocations.

Keywords: many-to-one matching markets, extreme core allocations, side-optimal allocations

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COMPARING OPTIMUM KMEDIAN AND MSS CLUSTERING WITH GROUND TRUTH CLUSTERING

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Abstract

Data clustering, also known as cluster analysis, aims to identify groups of data points with high internal (within-group) similarity and low inter-group similarity. This paper focuses on solving mixed integer linear programming formulations for the KMEDIAN and Minimum Sum of Squares Clustering Problem (MSS). We used the GUROBI solver to solve both models optimally for four data sets for which ground truth clustering was known. We compared the optimum results with each other and with ground truth clustering using the Adjusted Rand Index (ARI). Our results confirm that the optimum KMEDIAN and MSS clusterings are close to ground truth when the ground truth clustering has the geometry of well-separated ellipsoids, and that KMEDIAN is much easier to solve.

Keywords: exact clustering, integer programming

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1 INTRODUCTION

The task of finding groups of data points that share some internal (hidden) similarity is known as data clustering or clustering analysis, which is an important subfield of data mining and the common starting point of several advanced data analysis tasks. If we consider network data, then the task is also called community detection [6, 9].

Data clustering is essentially a task of partitioning a set of data points into subsets such that the data points within each subset are as similar as possible, while the data points in different subsets should be as different as possible. This high-level definition already shows that data clustering is a discrete optimization problem where the partitions into subsets are modelled by integer, usually binary, variables, while the group similarity and between-group dissimilarity should be expressed in the objective function, leading to a large number of different mathematical programming formulations of the clustering problem, see Subsection 2. Thus, when we talk about clustering algorithms, we need to specify the underlying (di)similarity measure.

In addition, we also need to specify the number of groups into which we want to group the data. Sometimes we can answer this question with the help of additional expertise. In these cases, we only need to calculate the assignment of data points into this number of groups. This is a rather rare situation, and usually the number of clusters needs to be determined during the cluster analysis. Some methods, such as the hierarchical clustering methods, determine the number of clusters during the clustering process. The other methods, such as k-means or k-median clustering, require this number as input, and there are many different criteria that can be used to determine the appropriate number of clusters, such as the 'elbow' method, gap statistics, silhouette method, the sum of squares method, etc., see e.g. [8, 10].

Clustering can be considered both an art and a science [14], it is often a part of exploratory research, and in general there is not just one good/optimal optimum clustering. Selecting a similarity measure, number of groups, and good assignment to clusters is often iterative based on a growing understanding of clustering.

In this paper, we focus on two specific formulations of mixed integer linear programming (MILP) formulations of clustering problems where the decision variables model assignments to clusters while the objective function represents the (i) sum of squares of distances to the group median (the KMEDIAN problem) and (ii) sum of squares of distances within the groups (the Minimum Sum of Squares Clustering Problem MSS). Both problems are formally introduced in Section 2 and are well known to be NP -hard problems, since there exist polynomial reductions from the 3- SAT to the KMEDIAN problem and from the densest cut problem to MSS [11, 2].

Once we have selected the number of clusters, the main question is to find the best grouping. Several clustering algorithms search for such solutions without explicitly considering the objective function, e.g., the well-known KMEANS algorithm assigns each data point to the closest centroid from the previous iteration and stops when the maximum number of iterations is reached or when the assignment stabilises. Here, the objective function is implicitly the sum of the squares of the distances to the centroid, but this function is not used in the usual way, i.e., to evaluate each solution and move to a new solution when the objective value improves.

Knowing the global optima of the KMEDIAN and MSS problems is very important to evaluate the performance of the heuristic algorithms for both problems. Also, it is interesting to see how the global optima for both clustering problems match the ground truth clustering, i.e., the clustering provided by the data provider.

This paper is organised as follows: In Section in Section 2 we write the KMEDIAN and the MSS problems as MILP problems, in Section 3 we present some medium-size size data sets for which the ground truth clustering is known, and present global solutions of the KMEDIAN and the MSS problems, together with an evaluation of how well they align with the ground truth clustering. In Section 4 we provide discussion and conclusions.

2 EXACT $0-1$ MODELS

Suppose we have N data points and a distance (or dissimilarity) matrix D. The elements of the matrix are denoted by d_{ij}. In our case, the distance matrix is the matrix of squared Euclidean distances. The KMEDIAN (or PMEDIAN) formulation first appeared in [12], and was later studied by many other researchers, see e.g. [4, 7].

The KMEDIAN problem on N data points can be formulated as the following MILP:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} d_{ij}^2 x_{ij} \to \min \tag{1}$$

s.t.

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad \forall\, i \in [N] \tag{2}$$

$$x_{ij} \le y_j \qquad \forall\, i,j \in [N] \tag{3}$$

$$\sum_{j=1}^{N} y_j = K \tag{4}$$

$$x_{ij} \ge 0 \qquad \forall\, i,j \in [N]$$

$$y_j \in \{0,1\} \qquad \forall\, j \in [N]$$

In this formulation, y_j is one if data point j is a cluster centroid, and x_{ij} is one if data point i is in the cluster whose centroid is data point j. Note that the constraint (4) ensures that we have exactly K clusters. According to (2), each data point is exactly in one cluster. By (3) the data point i can be assigned to the data point j ($x_{ij} = 1$) only if the data point j is a centroid ($y_j = 1$).

The Minimum Sum of Squares (MSS) Clustering problem also appears in [12]. The problem is usually solved with the so-called KMEANS algorithm, which is a heuristic algorithm that runs very fastly and has nice interpretation, so it is one of the most famous clustering algorithms. Exact algorithms are also known and are variants of exact algorithms for hard integer optimization problems, like branch and bound or branch and cut, see, for instance, [5].

Ágoston and E.-Nagy gave in [1] a mixed integer LP formulation, which is based on [3]. In this formulation, the cluster centers are not decision variables in a direct way, in the formulation only the distances between observation points appear, which gives the possibility to use other distances as well. The MSS formulation, proposed by Ágoston and E.-Nagy, is as follows:

$$\sum_{i,j} d_{ij}^2 \, z_{ij} \to \min \tag{5}$$

s.t.

$$\begin{aligned}
\sum_{j=1}^{N} z_{ij} &= 1 & \forall\, i \in [N] && (6)\\
z_{ij} &\le z_{ii} & \forall\, i \neq j \in [N] && (7)\\
z_{ij} &= z_{ji} & \forall\, i, j \in [N] && (8)\\
\sum_{i=1}^{N} z_{ii} &= K & && (9)\\
z_{ij} + z_{i\ell} - z_{j\ell} &\le z_{ii} & \forall\, i, j, \ell \in [N] && (10)\\
z_{ij} &\le \zeta_{ij} & \forall\, i, j \in [N] && (11)\\
z_{ii} - z_{ij} &\le 1 - \zeta_{ij} & \forall\, i, j \in [N] && (12)\\
(N - K + 1) z_{ij} &\ge \zeta_{ij} & \forall\, i, j \in [N] && (13)\\
(N - K + 1)(z_{ii} - z_{ij}) &\ge 1 - \zeta_{ij} & \forall\, i, j \in [N] && (14)\\
z_{ij} &\ge 0 & \forall\, i, j \in [N] && \\
\zeta_{ij} &\in \{0, 1\} & \forall\, i \neq j \in [N] && (15)
\end{aligned}$$

In this model, ζ_{ij} is one if the data points i and j are in the same cluster. Furthermore, it can be proven that for a feasible solution, z_{ij} is either 0 (data points i and j are assigned to different clusters) or is reciprocal to the cardinality of the corresponding cluster. Here we again fixed the number of clusters to K, see constraint (9). The constraint (11) is a kind of transitivity property, namely, if data points i and j are in the same cluster and the same is true for data points i and ℓ, then j and ℓ should also be in the same cluster. We also need to connect variables z_{ij} and ζ_{ij} (constraints (12)-(15)). In a feasible solution, $\zeta_{ij} = 1$ if and only if z_{ij} is positive.

3 RESULTS

3.1 Data

For our analysis, we used two synthetic data sets, created by us, and three benchmark data sets from the literature.

First, we created two data sets as follows: We set three vertices of an equilateral triangle as cluster centroids and generated 30 data points around each vertex (sampling from a multidimensional normal distribution). In the first case, the standard deviation was low, so the

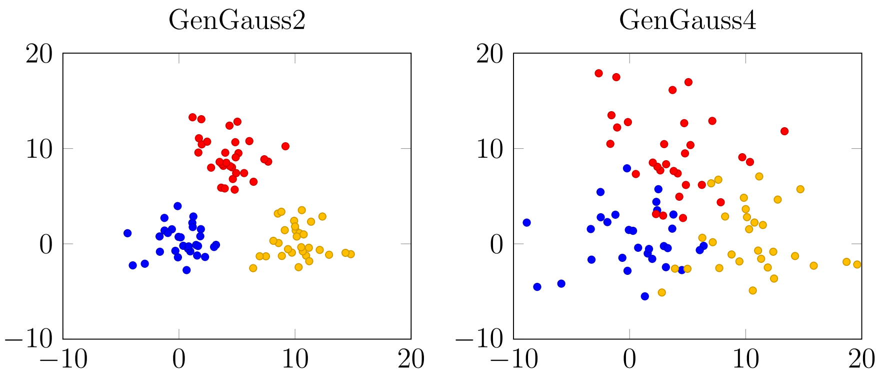

Figure 1: Scatter plots for synthetic data 'GenGauss2' and 'GenGauss4'

three groups did not overlap (GenGauss2), while in the second case, the standard deviation was slightly higher and the groups overlapped a little (GenGauss4). The two generated data sets can be seen in Figure 1 and are available to the readers on request.

In addition to synthetic data, we used three benchmark data sets: the well-known 'Iris' and 'Wine' data sets[1]. In the case of the 'Wine' data set, standardization was highly needed (variables were measured at different scales). In the case of 'Iris', the variables were expressed on a similar scale, so we computed the optimal solution for both the standardized and non-standardized versions.

For each of these data sets, we previously knew the true clustering, which is referred to as 'ground truth'. The number of clusters for both data sets is 3.

3.2 Numerical results

We implemented the described KMEDIAN and MSS formulation in Python (Spyder) environment. For formulating MILP problems, we used PuLP package and used GUROBI as a MILP solver. We allowed 10,000 seconds of running time.

We calculated the optimal solution for each of these data sets for cluster numbers between 2 and 9. Later we calculated the so-called Adjusted Rand Index (ARI) to measure (1) alignment between the optimum clusterings, obtained by solving the MSS and KMEDIAN problems, and the ground-truth clustering, and (2) alignment between both optimum clusterings. These values can be found in Tables 1–2.

4 DISCUSSION AND CONCLUSIONS

In this paper we have studied MILP formulations for two NP -hard clustering problems: KMEDIAN and MSS problem. Although they are very hard to solve, we managed to solve them using the MILP solver GUROBI for four data sets (two synthetic and two real), varying the number of clusters from 2 to 9.

The computational results unsurprisingly confirm the fact that the optimal clusterings for MSS and KMEDIAN coincide with the ground truth clusterings when the ground truth clusters have the shape of ellipsoids that are well separated. Otherwise, the clusterings differ, see Table 1. This was also observed in [13].

We have also shown that KMEDIAN gives very similar results to MSS for the data sets studied. Thus, if we want to compute an optimal clustering for a data set that is beyond the

[1]Available at UCI Machine Learning Repository

Data set	Number of clusters	Ground truth vs. KMEDIAN	Ground truth vs. MSS	KMEDIAN vs. MSS
GenGauss2	2	0.566	0,566	1
GenGauss2	3	1	1	1
GenGauss2	4	0.888	0,888	1
GenGauss2	5	0.744	0.750	0.960
GenGauss2	6	0.597	0.609	0.926
GenGauss2	7	0.542	0.550	0.956
GenGauss2	8	0.477	0.494	0.717
GenGauss2	9	0.429	0.440	0.886
GenGauss2	10	0.414	0.384	0.759
GenGauss4	2	0.310	0.384	0.276
GenGauss4	3	0.605	0.605	0.936
GenGauss4	4	0.507	0.495	0.519
GenGauss4	5	0.371	0.341	0.867
GenGauss4	6	0.310	0.324	0.730
GenGauss4	7	0.287	0.295	0.691
GenGauss4	8	0.266	0.272	0.556
GenGauss4	9	0.245	0.234	0.560
GenGauss4	10	0.216	0.209	0.632

Table 1: ARI values for synthetic data sets ('GenGauss2' and 'GenGauss4')

reach of state-of-the-art MILP solvers, we recommend using the KMEDIAN model, since it is a much simpler problem and therefore much easier to solve.

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Data set	Number of clusters	Ground truth vs. KMEDIAN	Ground truth vs. MSS	KMEDIAN vs. MSS
Iris	2	0.558	0.534	0.947
Iris	3	0.746	0.730	0.942
Iris	4	0.622	0.650	0.937
Iris	5	0.600	0.608	0.930
Iris	6	0.439	0.446	0.914
Iris	7	0.483	0.473	0.895
Iris	8	0.453	0.435	0.868
Iris	9	0.399	0.395	0.706
Iris	10	0.364	0.374	0.617
z_Iris	2	0.568	0.568	1
z_Iris	3	0.620	0.620	0,960
z_Iris	4	0.587	0.471	0,546
z_Iris	5	0.442	0.418	0,790
z_Iris	6	0.438	0.350	0,753
z_Iris	7	0.438	0.296	0,571
z_Iris	8	0.386	0.367	0,872
z_Iris	9	0.304	0.359	0,725
z_Iris	10	0.334	0.329	0,908
z_Wine	2	0.358	0.374	0.548
z_Wine	3	0.726	0.897	0.761
z_Wine	4	0.632	0.754	0.635
z_Wine	5	0.550	NA	NA
z_Wine	6	0.439	NA	NA
z_Wine	7	0.402	NA	NA
z_Wine	8	0.397	NA	NA
z_Wine	9	0.356	NA	NA
z_Wine	10	0.339	NA	NA

Table 2: ARI values for benchmark data sets (unstandardized and standardized 'Iris' ('Iris' and 'z_Iris' respectively) and standardized 'Wine')

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CLIMATE RISK INDICATORS FOR SMALL COMMUNITIES – THE EFFECT OF HEAT STRESS ON MORTALITY

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Abstract: Data driven decision making is of utmost importance for optimization of resource usage in building climate resilience. Due to demographic changes, health sector is among the highest priority domains where climate resilience has to be established. To aid this process in selected municipalities of the Drava river region, we investigate the health risks that statistically significantly increase due to heat waves. We propose indicators that could in municipalities exhibiting the risks be used to evaluate measures taken to build climate resilience. Due to small population of most of these municipalities, the indicators we propose are compatible with analysis of k-anonymous input data (for $k = 5$), which was used in the analysis.

Keywords: green transformation, heat stress, mortality, risk indicators, climate resilience

1 INTRODUCTION

As part of raising awareness of the impact of climate change on everyday life, municipalities are preparing their strategies and related action plans for climate neutrality and resilience. Municipalities can significantly take into account their specific's in these strategies and thus reduce their vulnerability to the effects of climate change by implementing effective measures. Small municipalities want to know the purpose of their investments, on the basis of which they can measure the impact of these investments. For this, they need a statistically significant measurement of climate neutrality in order to justify the costs of investments in adaptation to climate change. It is for this reason that we study a population of municipalities in the Styrian region and identify several feasible and several likely candidates for such statistically significant indicators of heat exposure.

Adaptation to climate change represents a set of measures and policies for planned reduction of vulnerability and increase of resistance to the perceived and expected impacts of climate change. An effective response would contribute to greater safety and well-being of people, the protection of nature and a more sustainable economy. In the paper, we draw on the current regulation in the field of climate change, which is described below.

In December 2019, the European Commission presented the EU Adaptation Strategy [6], a comprehensive plan of measure for the transition to a green, sustainable economy, with the aim of overcoming climate and environmental challenges and achieving the set goals of climate neutrality. In February 2021, the European Commission adopted a new EU Climate Change Adaptation Strategy, which builds on the previous one. It is based on the long-term vision of the EU becoming a climate change resistant society by 2050 and fully adapted to the inevitable consequences of climate change. The strategy has four main goals: to make adaptation smarter, faster and more systematic, and to strengthen international action to adapt to climate change. The strategy thus supports the further development and implementation of adaptation strategies and plans at all levels of management.

Based on the described regulation, we defined epistemology of the strategy of climate neutrality and resilience and its methodological elements and indicators on the other hand, that enable the abstract modeling of these indicators and the study of their mutual relations. It also enables the collection of data for the calculation of these indicators and the data quality verification. Based on this, it is possible to implement selected measures aimed at reducing exposure and sensitivity, and thus also vulnerability, in a given climate situation and its dynamics. Indicators of the implementation and effectiveness of the measures help us monitor the predictability of consequences of the measures, the creation and exchange of good practices between municipalities and the other carriers of the implementation of climate neutrality and resilience strategies.

In this paper, we summarize the unified methodology that we developed in the previous article for the purpose of a data-based climate neutrality and resilience strategy, we describe the calculations of heat stress indicators, which are addressed by indicators of daily heat wave intensity, heat wave length and number of heat waves, and mortality indicators, which are addressed by the number of deaths and the number of deaths due to stroke by different demographic data. We established a connection between the indicators of heat stress and mortality, according to the date of occurrence. Based on that, we calculated the number of deaths during heat waves and estimated the probability of deaths during a heat wave. The data is shown on the example of the smaller municipalities in the Styrian region. We discuss the main challenges in data collection and data processing and also in the actual use of data and calculations on our way to a resilient society. Based on the proven risk of heat stress on mortality, in conclusion we suggest possible further steps and measures to adapt to the changes.

2 METHODOLOGY

The methodology is based on a previously developed methodology in [5] where we, in the context of raising awareness of the impact of climate change on everyday life and as part of the preparation of strategies and action plans, presented a model that enables the development of a unified methodology and an agile process that leads to its adaptation. To effectively prepare for the adaptation and its consequences, we first connected the methodology with the Cynefin taxonomy of decision-making context by Snowden [16], adapted by French [8]. In this frame of reference, every decision is embedded in the context of perceiving and understanding the circumstances in which it is made, as well as understanding and anticipating the consequences that alternative decisions lead to. We tie in this complexity of the aforementioned decision-making processes, which we developed in previous publications as a universal process model [4]. We take in the model of an agent in a stochastic decision process, which Hoffman et al. upgraded to the interface theory of perceptions [10]. The methodology, which is applicable to all sectors considered in the strategy of climate neutrality and resilience of municipalities, is illustrated on the indicators of the health sector, which measure the effect of climate change on health.

2.1 Heat stress and mortality indicators and calculations

In this paper, we make an upgrade to this methodology and apply it to the obtained data. For this, we first had to define data sets and calculations of heat stress and mortality. It should be noted that there is no unified universal definition of heat waves [13]. The World

Meteorological Organization (WMO) proposes general guidelines for defining and monitoring heat waves [12]. The IPCC states that Heatwaves and warm spells have various and, in some cases, overlapping definitions, including heat index, heat stress and marine heatwave [11]. In this paper we take Heat-wave Magnitude Index (HWMI) as the main key indicator to define heat stress. To calculate this indicator, we need a 30-year comparative period, in our case this period was the years 1981-2010. The sample of daily temperature data that is compared with the reference period is based on the period 2012 - 2019. The threshold value for above-average heat is calculated for each day of the year separately. For the entire comparison period, we consider the same part of the year (15 days before and 15 days after the day in question) and calculate the 90th centile of the daily maximum temperature. A heat wave is defined as periods of at least three consecutive days when the limit is exceeded. Heat waves were determined at two weather stations in the Styrian region, more precisely in the municipality of Maribor. These weather stations are EDVARD RUSJAN AIRPORT MARIBOR (ERA-MB) and MARIBOR - VRBANSKI PLATO (MB-VP).

The data model of heat stress is based on public data from the Slovenian Environment Agency [1]. On the other hand, we have a data model of the health state and mortality, which is derived from the data of the National Institute of Public Health [14, 15]. This data defines the number of deaths in a given day and year, according to the cause of death, municipality of residence and activity status. The cause of death is defined according to the main categories of cause, and in our case we focus mainly on deaths that occurred as a result of a heart attack The municipalities of residence are focused on around 40 municipalities in the Styrian region, with a focus on the municipality of Maribor, but also on the entire Styrian region. The activity status defines 11 different types of activity of residents, depending on their involvement in work, i.e. whether it is and unemployed person, a child, a student, a retired person or and employed person.

The presence of heat waves was determined in four ways: the presence of a heat wave at the ERA-MB weather station, the presence of a heat wave at the MB-VP weather station, the presence of a heat wave at one or the other weather station, and the presence of a heat wave at both weather stations at the same time. We determined days where heat waves were present and compared them with days where heat waves were not present. Heat waves in this way define two populations, the first when the heat wave condition is met and the second when the heat wave condition is not met. Based on the sample data, we first calculated the proportion of days with (at least one) death separately for each group. To test the hypothesis whether the probability of death during heat waves is statistically significantly higher than the probability of death outside heat waves, we then performed a one-sided independent samples t-test at the 0.05 significance level. Prior to determining the appropriate t-statistic value, we conducted a test of homogeneity of variance. The t-statistic provides a measure to quantify the difference between the population proportions, while the p-value is the probability of obtaining a t-value at least as large as what has been observed, under the assumption that the population proportions are equal. Hence, a low p-value indicates statistical significance differences between groups. In our case, the hypothesis was confirmed if the calculated significance value (p-value) was less than 0.05.

3 RESULTS

In this section, we present the results of the risk calculations for the occurrence of death during heat waves. The tables below show the results for the municipalities with a statistically significant higher probability of death during a heat waves. The tables are separated according to the method of determining the heat wave by station. The tables below show 13 values by column that we invesigated. Column municipality defines the municipality in the Styrian region, column SumTrue defines the number of death occurrences during a heat wave, column

SumFalse defines the number of death occurrences during a non-heat wave, column AvgTrue defines the probability of a death occurring during a heat wave, column AvgFalse defines the probability of a death occurrence during a non-heat wave on a day, column AvgDiff define the difference between the probability that death will occur during a heat wave and that death will occur outside of a heat wave, Ratio define the ratio of the probability that death will occur during a heat wave and the probability that death will occur outside of a heat wave, column sTrue defines the standard deviation of the occurrence of death during a heat wave, column sFalse defines the standard deviation of the occurrence of death during a non-heat wave, column sAlmostEqual defines whether the standard deviations of sTrue and sFalse are almost the same, column t-Statistic gives the calculated value of the t-statistic (appropriately determined depending on whether or not equal standard deviation in both groups can be assumed), calculated p-values for the independent samples t-test are displayed in the column p-value. When the p-value is lower than 0.05, it confirms that the probability of death during heat waves is statistically significantly higher than the probability of death outside heat waves. In such instances, the column Significant is marked as True; otherwise, it is marked as False and not displayed.

Table 1 shows data of municipalities on the probability of death, in the case of the presence of a heat wave at both weather stations at the same time. The municipalities that show a statistically significant difference in probability of at least one death occuring in a given day during heat waves compared to non-heat wave days are Cirkulane, Ptuj, Sveta Trojica v Slovenskih Goricah and Trnovska vas. Cirkulane has an 10.0 % probability of death occurring during a heat wave, which is 2.9 % more than the probability of a death occurring outside of a heat wave, and has a ratio between the probability of death occurring during a heat wave and the probability of death occurring outside a heat wave equal to 1.41 with a statistical significance of 0.046. Ptuj has an 68.1 % probability of death occurring during a heat wave, which is 7.8 % more than the probability of a death occurring outside of a heat wave, and has a ratio between the probability of death occurring during a heat wave and the probability of death occurring outside a heat wave equal to 1.13 with a statistical significance of 0.048. Sveta Trojica v Slovenskih Goricah has an 8.1 % probability of death occurring during a heat wave, which is 3 % more than the probability of a death occurring outside of a heat wave, and has a ratio between the probability of death occurring during a heat wave and the probability of death occurring outside of a heat wave equal to 1.60 with a statistical significance of 0.019. Trnovska vas has an 6.3 % probability of death occurring during a heat wave, which is 2.7 % more than the probability of a death occurring outside of a heat wave, and has a ratio between the probability of death occurring during a heat wave and the probability of death occurring outside a heat wave equal to 1.74 with a statistical significance of 0.016.

Municipality	SumTrue	SumFalse	AvgTrue	AvgFalse	AvgDiff	Ratio	sTrue	sFalse	sAlmostEqual	t-Statistic	p-value	Significant
Cirkulane	27	188	0.1	0.07089	0.02911	1.410638	0.090335	0.07117	True	1.687337	0.045823	True
Ptuj	184	1600	0.681481	0.603318	0.078163	1.129556	0.574749	0.538171	True	1.6662706	0.048239	True
Sveta Trojica v Slov. Goricah	22	135	0.081481	0.050905	0.030577	1.600658	0.07512	0.05135	True	2.068614	0.019335	True
Trnovska vas	17	96	0.062963	0.036199	0.026764	1.739352	0.066653	0.035656	True	2.134914	0.016426	True

Table 1: Probability of death in municipalities, in the case of the presence of a heat wave at both weather stations at the same time.

Table 2 shows data of municipalities on the probability of death, in the case of the presence of a heat wave at one or the other weather station. The municipalities that show a statistically significant higher probability of death during heat waves are Cirkulane (probability of death during a heat wave equals to 10.7 %; ratio = 1.55; $p = 0.006$), Juršinci (probability of death during a heat wave equals to 9.3 %; ratio = 1.50; $p = 0.015$) and Trnovska vas (probability of death during a heat wave equals to 6.3 %; ratio = 1.79; $p = 0.006$).

We also checked the statistically significant causes of death during heat waves and the statistically significant activities of the population at the time of death during heat waves. In the case of the presence of a heat wave at both weather stations at the same time, there are

Municipality	SumTrue	SumFalse	AvgTrue	AvgFalse	AvgDiff	Ratio	sTrue	sFalse	sAlmostEqual	t-Statistic	p-value	Significant
Cirkulane	39	176	0.106849	0.068831	0.038019	1.552351	0.101189	0.068813	True	2.517428	0.005937	True
Juršinci	34	159	0.093151	0.062182	0.030968	1.498027	0.0902	0.061468	True	2.170039	0.015042	True
Trnovska vas	23	90	0.063014	0.035197	0.027816	1.790289	0.0647	0.034754	True	2.534024	0.005664	True

Table 2: Probability of death in municipalities, in the case of the presence at one or the other weather station.

no statistically significant causes of death during heat waves. In the case of the presence of a heat wave at one or the other weather station or at one weather station, statistically significant causes of death during heat waves occur in four cases. This is in the municipality of Cirkulane, Juršinci and Destrnik where neoplasms are a statistically significant cause of death during a heat wave and in the municipality of Ptuj, where a statistically significant cause of death during a heat wave is heart attack. The activity of the population that turns out to be the only statistically significant activity of the population are retired people.

3.1 Result summary

As we described in the previous section, among the municipalities that have the highest probability of death during heat waves are the municipalities of Cirkulane, Ptuj, Sveta Trojica v Slovenskih Goricah, Trnovska vas, and Juršinci as we can see in the Figure 1, where the mentioned municipalities are colored in orange. The other municipalities of the Styrian region did not show a statistically significant increased probability of death and are therefore colored green. In the picture we can see the two weather stations marked with a white and fuchsia cross sign, on the basis of which we defined heat waves. We can see a band where deaths occur with greater probability during heat waves. This belt geographically represents the border between the Pohorje and Pannonian parts of Slovenia. We used data from the Global Administrative Areas 2015 (v2.8) dataset to plot the municipalities [9].

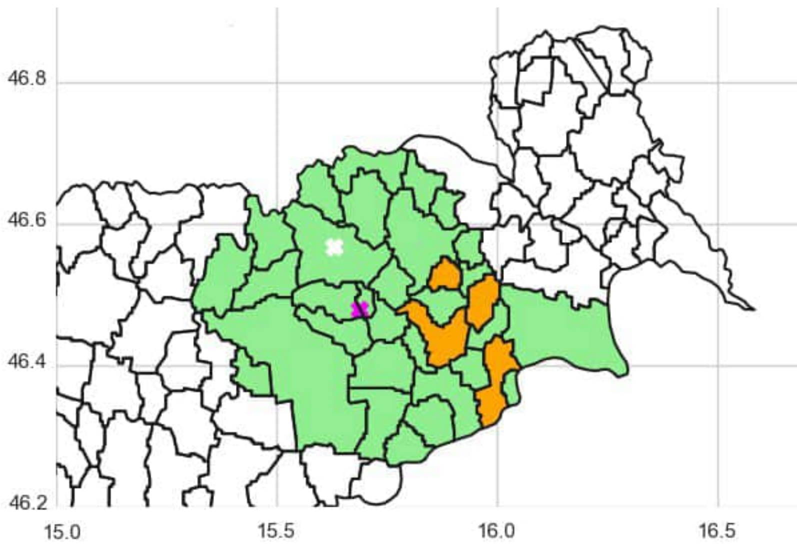

Figure 1: The figure shows the municipalities according to the increase in the probability of death during heat waves.

4 DISCUSSION

The conducted research is the basis for data support of exposure to heat waves. With the help of the performed analysis, we can identify municipalities that are statistically more exposed and sensitive to effects of heat waves. This indicates the need to define and implement measures to adapt to climate change in municipalities. The research opens up many possibilities for further study. In the near future, we will also check the changing effect of heat waves on mortality when changing the included periods. We also want to check the effects of heat waves on hospital treatment and sick leave. It would also be interesting to expand the field of research to the whole of Slovenia. In the long term, however, we want to establish a way to check data-supported trends in the field of climate change in real time and establish a process for defining adaptation measures and regularly monitor the effects of the designated adaptation measures.

5 Acknowledgements

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On code quality and code relevance metrics

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Abstract: Following a review of some developer analysis bibliography, we propose two developer efficiency metrics that would stimulate developers to write more tested and higher quality code. The first relies on defining the wake period of a new functionality commit as the interval in which the LOC committed and the number of issues reported are highly correlated, while the second relies upon counting the number of tests that cover specific lines of code committed by a developer and the variability of these tests. The second metric is merely a proposal as the data for it is not available in current coverage measuring solutions. Additionally, we test the first metric using the GitLab data from a year of development in a medium sized software development company.

Keywords: Software metrics, developer quality, bug reports analysis.

1 INTRODUCTION

The importance of objectively assessing the quality and improvements in software development has always been an important aspect. As projects expand in their complexity and the number of developers increases, it is important to monitor their development efficiency in order to facilitate on-time and high quality delivery of results. Without the presence of the software testing methods, projects become increasingly exposed to bugs. Releasing faulty software reflects negatively on the customer's experience. It is estimated that every year around $20 billion could be saved if better testing was done before releasing new software [6], which shows market potential for solutions improving the code delivery process. Therefore, it is essential for software companies to stimulate and support developers to cover their code with automated testing.

In this contribution, we review some used metrics of the software development quality and relevance, and propose a novel metric, which objectively indicates the relevance of given parts of program code. The main motivation for the metric is based on how the untested code affects the probability of producing malfunctioned software. We analyzed how publishing the untested code affects the increase of bug reports in a private company. Fixing the issues requires a developer to change the focus from developing the features of a software to delivering solutions to bugs or to even operate more projects in parallel, which consequently affects their productivity and enthusiasm. Yin et al. estimated that about 20% of bug fixes become the reason for post-release faults [14]. Karampatsis and Sutton evaluated a project, for which about 33% of the bugs tend to be single-statement bugs, whose fixing commit contains only single-statement changes, excluding stylistic differences and comments [9]. By introducing higher test coverage of the code, we could prevent these two issues and thus massively alleviate the amount of faulty software releases. For this reason, our next aim is to present a metric, which measures the reliability of a developer's contribution. As a consequence, the developers having higher code reliability might most likely produce less faulty software.

In summary, this paper provides the following contributions:

- It describes a metric for assessing code relevance.
- It conveys the importance of software testing through analyzing the effect of untested code on bug reports.

- It represents a strategy of stimulating diverse software testing development.

2 RELATED WORK

The requirement of software metrics arised already in the previous century. One of the pioneering and straight-forward metrics is the source lines of code (SLOC) [10], which simply counts the lines of code having changed. As a consequence of its simplicity, SLOC does not provide a sufficient image of the productivity, because of programming language selection dependency, the lack of cohesion with functionality, programmer's previous experience and how the metric influences its style of writing the code [3].

Consequently, researchers focused on metrics, which describe the complexity of the software. Among the most known are the McCabe cyclomatic complexity [11], which is inferred by measuring the number of linearly independent paths through the program and the Halstead complexity measure [7], which evaluates the effort invested in the development by the developer. However, even the described metrics contain shortcomings, such as ignoring the complexity of nesting the code and applying too strict assumptions, which have been alleviated and detailed in [16].

With the evolution of the open source software, the metrics focused not only on changes in lines of code, but also to commits. The most intuitive metric based on commits is the number of commits over time [1]. Similarly to SLOC, the aforementioned metric does not yield an adequate measure for developer's contribution [13]. To ease the drawbacks, Gousios et al. [5] presented a method, which measures developer's contribution as a weighted sum of actions that might have a positive or negative effect on the project. Furthermore, character-based evaluation is proposed in [13] to differ from the complexity of line changes. Additionally, project ownership has been defined as the ratio between the number of commits in a project published by the developer and the number of all commits in the project [4].

3 DATA

In the following section, data acquisition and processing for analyzing the effect of releasing untested features on the software quality is described. The metrics used are the SLOC and the number of commits per date of all projects categorized by the file types. Although the metrics are one of the most straightforward, they provide enough details to assist in the assessment. File type might be one of the following: production file, test file or documentation file. Based on the proportion of the file types, we distinguish:

1. functionality introduction - numerous production files and test files updated or created,

2. fixes - several test file updates versus fewer production files,

3. untested code edits - many production files opposed to little test file changes.

To gather the data, Gitlab REST API is utilized [12]. It provides multiple endpoints for retrieving the analytics of all commits per project. For every project in the company's Gitlab repository, the commits of each branch are fetched including the files being updated in a commit. Each file is categorized by a predefined classifier, which determines the file type based on its path in the filesystem. Moreover, the number of additions, deletions and changes is calculated from the diff of a commit. The data is then saved to a local database for less time-consuming post-processing. The process is repeated every hour to reflect the most recent state.

Having the data stored locally allows us to aggregate it on request. Upon aggregation request, the data is grouped by committed date and file type. In addition, the SLOC and the number of commits per day per file type is returned.

Bug report assembly is performed using the YouTrack REST API [15]. Bug reports were published on a specific YouTrack project, which allowed us to filter the number of opened issues per date. The data used for analysis was collected in a period of a year starting with 01/01/2022 and ending on 31/12/2022. The number of projects updated throughout the year was 349 and the number of active developers was 59.

From figures 1 and 2, we might observe how the developers mostly introduced untested code edits, which manifested in an abundance of open issues throughout the year. Issues were evenly distributed with the minimum in the summer period and peaks in the beginning of the year. Due to sensitive information, the actual numbers of the charts have been suppressed.

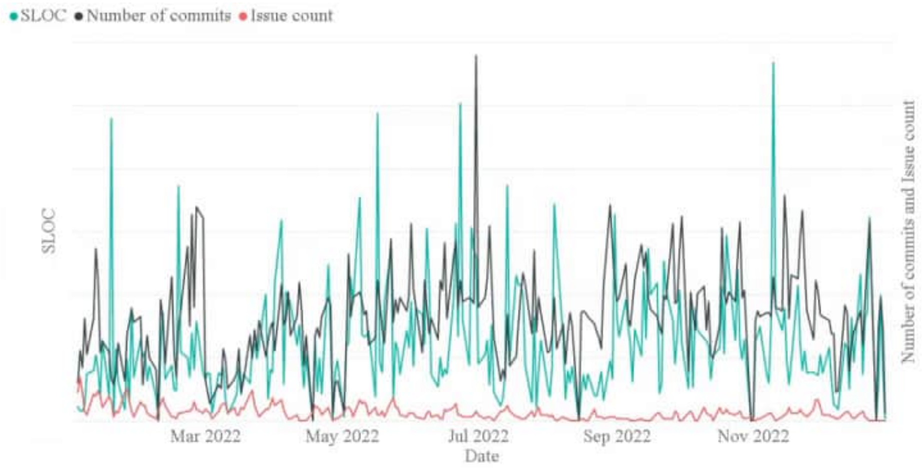

Figure 1: SLOC and the number of commits by date including opened issues count by date.

4 METHODOLOGY

Using the data from Figure 1 as three time series, one might also note the trend of bug report increase as the untested code edits were presented and vice versa. The growth occurred mostly with a slight delay in days. For this reason, we searched for the most frequent delay in days past a new release before the clients posted a bug report. To calculate it, the Pearson correlation coefficient [2] is computed between the SLOC or the number of commits metric and the number of bug reports. The Pearson correlation coefficient signals how the number of open issues and the selected metrics are correlated, where the issue counts were shifted for d days.

The above metrics identifies the number of days after a major new functionality commit on which there are most new issue reports, as well as the day following this peak when the number of new issue reports is the smallest. We define the period of this interval to be the wake period of a functionality commit and propose that the issues reported in this period be the core responsibility of the commit. Additionally, we define the quality of the commit to be the SLOC of the commit divided by the number of issues reported in the commit's wake period, and the quality of the developer to be the average quality of developer's new functionality commit.

As the developers might utilize the code generation tools to initialize the projects and thus increase their SLOC metric without contributing much effort, we analyzed two scenarios. Firstly, the developer quality is computed for the commits having SLOC less than 1000. We define 4 categories of the developer qualities. Category C are the developers having the quality in the range from 0 to 4, category B developers have the quality in the range from 4 to 12, category A from 12 to 30 and category A+ are the developers acquiring the quality greater than 30. Similarly, we evaluated the developer qualities for unfiltered commits and labeled the ranges 0-10, 10-250, 250-1000 and 1000+ with the same categories.

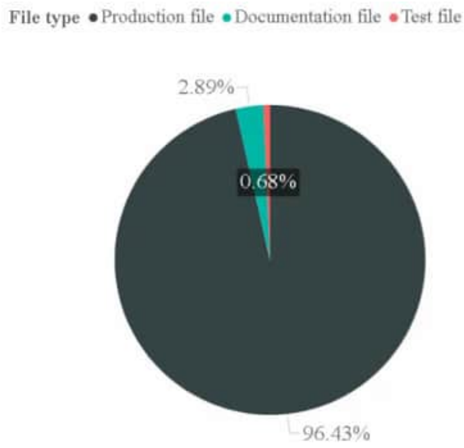

Figure 2: The distribution of commit file types.

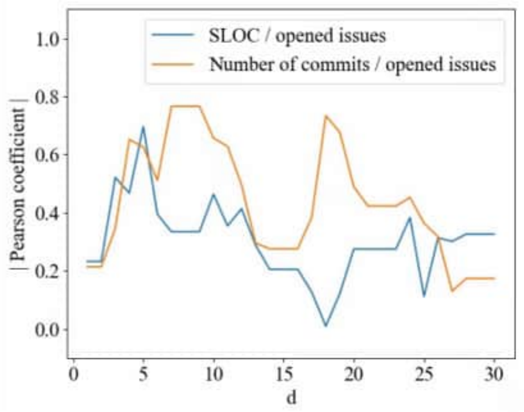

Figure 3: Values of Pearson correlation coefficient between the selected metrics and the number of bug reports, delayed by d days.

5 RESULTS

Inspecting the figure 3, we notice the peak in the Pearson correlation coefficient value mainly after the 4 to 6 day delay, which is consistent with the fact of planning the work for a week in advance, while the valley is visible at 18 day delay, making the wake period of a commit last for about 2 weeks. In addition, the peak in the number of commits per opened issue count after 18 days is expected as the developers are fixing the issues and therefore producing numerous commits having less changes. The value of the Pearson correlation coefficient in the peak ranges from 0.7 to 0.8, which is another sign of high correlation between mostly untested code edits and bug reports.

Examining the figures 4 and 5 one might perceive the presence of category C developers in both distributions of developers by assigned quality as the category ranges only slightly differ. The visualization is beneficial in exploring the lead developers, who are responsible for creating new applications as they are associated with categories above A in a distribution considering all commits and categories below A in a distribution ignoring the commits having more than 1000 SLOC. Furthermore, it assists in assessing the urge to trigger the process to motivate developers below category A in achieving it and especially category C developers to reach category B.

6 CODE RELEVANCE METRIC

To alleviate the number of open issues, we propose a code relevance metric as a tool to quantify how relevant the published code is. As a result, developers having higher metric values could be motivated to write test code.

The code relevance metric is highly associated with code coverage tools, which represent the main factor of the assessment. Code coverage is a measure of the degree to which the system is tested [8]. By close examination of the data used to compute the metric, we can obtain the lines of code being covered by tests and by how many they are covered. Having this information, we define the line relevance as the number of tests that cover the line. Similarly, the commit relevance is specified as the fraction of the sum of relevances of lines in a commit in the total relevance of all lines of code. Finally, we define the developer relevance as the fraction

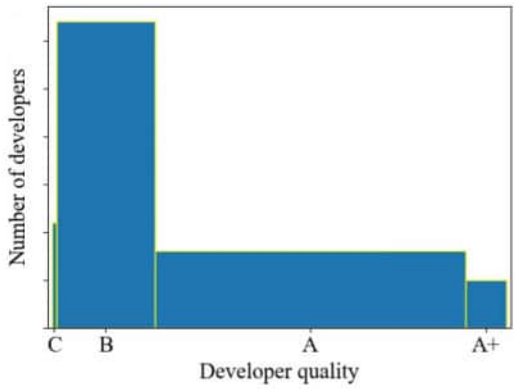

Figure 4: Distribution of developers by assigned quality considering all commits.

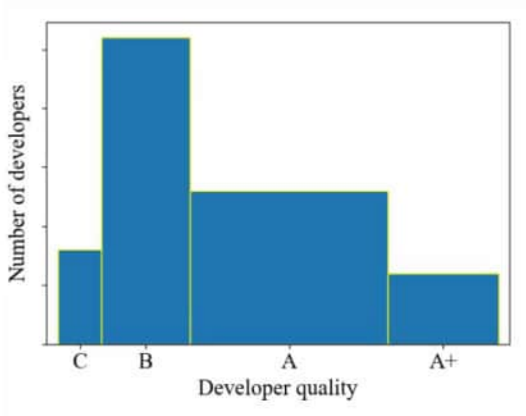

Figure 5: Distribution of developers by assigned quality ignoring the commits having more than 1000 LOC.

of the total of all lines produced or modified by the developer in the total relevance of all lines of code.

The metrics defined as above would motivate developers to overtest the lines of code using similar tests that predominantly test their lines of code. To minimize such strategic testing, the line relevance can be weighted by a factor per each test, resulting in the contribution of two tests testing the same lines of code to line relevance being 1. By employing such factors, we signal how spread the tests that occupy the line are. Whether the tests covering the line are similar, the relevance of the line is reduced as a consequence of the weights.

7 CONCLUSIONS AND FURTHER RESEARCH

In the above, we discuss various metrics that can be used in organizing the developer effort and stimulating their efficiency of coding and balancing between contributions of new functionalities, contributions to a test code, and contributing bug fixes. It is evident that the new functionalities are the primary goal of code development, and that bug fixes are the primary goal of code maintenance. Also, efficiency of the development is attained at a suitable optimal spot between functional development and code testing. Testing competes for time with functionality development as it saves time by alleviating the possibility of bug production when updating the code with improvements and adjustments. However, the test code may present a maintenance burden and can be costly. On the other hand, developing unit tests forces developers into better understanding of the code and can aid new developers in mastering that code.

The metrics we have introduced allow for measurement of various aspects of the relevance of both functional code as well as the test code. In our future research, we plan to connect these metrics with time measurement and thus offer the developers an insight into how their efficiency develops through time in the project. In addition, should they work in a trustful environment that fosters sharing such information, they would be able to compare their efficiency and share best practices in the team, thus emphasizing intra-team collaboration over competition. Furthermore, we propose future work to analyze whether the code relevance metric is consistent with producing less faulty software.

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CROATIA VS EU FROM THE PERSPECTIVE OF DIGITAL SKILLS

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Abstract: This paper deals with statistical analysis of digital skills in Croatia using modern methodological procedures of official institutions. Empirical research using two-step random sample with post stratification on the basis of sex, age, education level and working status proves statistically significant difference only in digital skills depending on the age and level of education of the Croatian respondents. The results of several comparative analyses of Croatia vs EU countries are presented with general conclusion that positions Croatia as the country with an intermediate level of digital skills and efforts in developing digitally empowered society.

Keywords: comparative analysis, digital skills in Croatia, hypothesis testing, public opinion survey

1 INTRODUCTION

The contemporary era is marked by the proliferation of Information and Communication Technology (ICT) appliances, leading to a significant increase in the demand for digital skills across various domains [13]. This work centers around the pivotal role of digital skills in driving scientific, technological, educational, and broader socioeconomic advancement. Moreover, raising the general level of digital skills is an unavoidable prerequisite for Croatia's inclusion in the modern European environment. For this reason, Croatia offers numerous opportunities to its population, especially the younger generations, through all levels of the education process. Employees recognize these efforts and take advantage of state incentives to open numerous newly founded, especially IT, companies. Even people from other parts of Europe choose Croatia and as "online nomads" live and work here, appreciating the Croatian climates, more favourable working conditions and characteristics of the mentality of Croatian natives.

Researches, the results of which are presented in this paper, follow the guidelines of the European Commission’s Digital Education Action Plan (2021-2027) [7] which sets out a vision for quality, inclusive and affordable digital education in Europe. Besides, measurement procedures within the scope of these researches are based on [4] and [12]. After introduction, the second part of this paper is dedicated to the empirical findings interpretation. After interpretation of the public opinion survey results about Croatian digital skills, comparative analysis of Croatia vs EU countries from the same perspective was carried out. The third part of the paper contains concluding remarks and implications for further research. Cited literary units are at the paper end.

2 EMPIRICAL RESEARCHES

2.1 Public opinion survey results

In July and August, a survey was conducted in Croatia using a questionnaire to analyze the impact of age, sex, education level, and employment status on the digital skills levels of respondents. The survey targeted individuals between 18 and 65 years of age, representing the working-age population. Prior to developing the questionnaire, extensive research was conducted to ensure its alignment with the relevant standards and guidelines established in statistical literature and practices [10]. The survey design was based on the European Digital Competence Framework for Citizens and the Eurostat Digital Skills Indicator, ensuring consistency and comparability with European standards.

The questionnaire used in this study gathered sociodemographic data from the respondents, followed by fifteen closed-ended questions about various digital activities. Each question focused on a specific activity and offered three answer options representing different levels of digital skills. The questionnaire encompassed five competence areas: information and data literacy, communication and collaboration, digital content creation, digital security, and digital problem solving. For each competence area, there were three questions. The answer options represented skill levels as basic, intermediate, or advanced.

To determine the overall level of digital skills for each respondent, the quantified answers from the questionnaire were used to calculate the arithmetic mean. This mean value represented the individual's overall digital skills level. Respondents with an arithmetic mean ranging from 1 to 1.5 were classified as having a basic level of digital skills. Those with an arithmetic mean ranging from 1.5 to 2.5 were considered to have an intermediate level, while respondents with an arithmetic mean ranging from 2.5 to 3 were classified as having an advanced level of digital skills. The statistical analysis of the data collected from the questionnaire was performed using the TIBCO Statistica 14.0.0 software in 2020.

In terms of methodology, the study initially conducted the Kolmogorov-Smirnov test to determine whether the data followed a normal distribution. This test is essential for subsequent statistical procedures that rely on this assumption. In the second step of the statistical analysis, the results indicated that the overall arithmetic mean for digital skills in Croatia was 1.94, positioning the country at an intermediate level of digital skills. To examine the relationship between digital skills and sex, hypothesis testing was conducted using an independent samples t-test. This test was used to compare the means of digital skills between different sex groups. The results of the t-test showed that there was no statistically significant difference in the overall level of digital skills based on sex. This conclusion is supported by the findings presented in Table 1, which provides the final results of the statistical analysis.

Table 1: Statistical analysis results of the digital skills dependence on the sex of the respondents

Variable	T- test: Grouping: Gender (Total digital skills) Group 1: Female Group 2: Male										
	Mean Female	Mean Male	t- value	df	p	Valid N Female	Valid N Male	Standard deviation Female	Standard deviation Male	F- ratio Variances	P Variances
Digital skills	1,9259	1,9791	-1,0244	427	0,3062	241	188	0,51092	0,5619	1,2097	0,1649

The analysis of digital skills by age groups, completed level of education, and working status involved the use of One-Way Analysis of Variance (ANOVA). One-Way ANOVA is a statistical technique used to compare means across multiple groups. The results of the One-Way ANOVA analysis revealed a statistically significant difference in the overall level of digital skills based on the completed level of education. Similarly, the results for the age groups of the respondents also led to the same conclusion, indicating a significant difference in digital skills proficiency among different age groups.

Table 2 presents the final results of the One-Way ANOVA analysis, specifically focusing on the digital skills categorized by the completed level of education. This table provides the relevant statistical values and outcomes of the analysis, offering insights into the variations in digital skills across different educational backgrounds.

Table 2: Statistical analysis results of the digital skills by the completed level of the respondents ‘education

Effect	Univariate results Sigma- restricted parameterization Effective hypothesis decomposition				
	Degrees of Freedom	Digital skills High school	Digital skills Bachelor’s degree	Digital skills Master’s degree (and above)	Digital skills p
Intercept	1	795,5601	795,5601	3903,339	0,00
Completed level of education	3	35,3774	11,7925	57,859	0,00
Error	425	86,6215	0,2038		
Total	428	121,9989			

2.2 Comparative analysis of Croatia vs EU from the Perspective of Digital Skills

As a pertinent depiction of digital skills presence in the life of the average Croatian citizen, the authors also conducted a Eurostat based [8] secondary data analysis of the purposes for which individuals utilize the Internet such as energy management in the households [9], intelligent security solution [2], intelligent household appliances [6], virtual assistant [1], Smart TV [11], cloud computing [3] and e-public/e government services [5]. The findings of this analysis are presented Table 3 and Table 4, which provide valuable insights into the diverse ways in which digital skills are employed in various aspects of life.

Table 3: Intelligent internet solutions used by EU countries

	Energy management in the household		*Intelligent security solutions*		*Intelligent household appliances*		*Virtual assistant*		*Smart TV*	
	2020	2022	2020	2022	2020	2022	2020	2022	2020	2022
Austria	5,46	14,57	4,28	7,33	4,80	9,33	17,49	18,67	45,91	60,03
Belgium	10,25	14,13	12,14	12,67	3,57	9,56	8,84	6,46	:	51,16
Bulgaria	2,08	1,15	2,06	2,26	1,06	4,29	0,83	0,82	23,19	30,12
Croatia	**3,52**	**5,58**	**3,90**	**5,87**	**4,36**	**7,46**	**8,23**	**2,98**	**44,96**	**34,71**
Cyprus	1,07	6,59	11,11	10,62	0,85	10,25	4,31	5,44	48,14	65,67
Czechia	2,85	5,13	4,78	7,81	2,39	6,75	3,86	5,99	36,20	47,85
Denmark	11,45	14,57	14,53	16,61	12,05	19,72	19,35	21,68	62,50	65,51
Estonia	14,79	16,07	9,21	15,72	8,45	18,02	6,18	6,35	41,37	51,76
Finland	7,78	12,35	10,27	11,97	4,09	8,55	17,41	15,02	53,41	64,17
France	:	10,88	:	11,39	:	13,79	:	15,69	:	53,84
Germany	8,06	7,11	3,87	4,32	5,25	7,08	16,56	14,94	50,71	49,15
Greece	1,70	3,65	4,43	5,85	2,27	5,84	0,75	1,11	29,37	38,62
Hungary	4,44	5,18	5,67	6,54	3,86	6,53	3,82	3,02	37,71	49,04
Ireland	13,63	21,41	12,96	15,96	3,95	5,78	17,74	38,89	37,58	67,64
Italy	1,63	4,38	5,37	9,13	1,93	4,19	11,80	15,09	29,54	49,05
Latvia	3,44	4,69	4,27	5,40	3,39	8,60	4,02	2,33	46,43	48,59
Lithuania	2,45	5,31	4,86	8,08	4,50	12,12	1,70	3,03	31,24	42,66
Luxembou	12,39	10,74	13,15	14,81	7,99	12,19	12,12	14,59	57,02	60,98
Malta	8,25	8,67	13,11	13,95	11,34	19,80	16,79	16,92	72,19	77,76
Netherland	68,72	65,69	11,52	22,68	5,72	15,05	19,51	24,85	56,57	64,13
Poland	2,27	3,33	2,42	3,01	3,26	5,73	1,75	2,09	30,87	37,55
Portugal	4,00	6,34	5,19	5,85	4,42	8,71	7,62	7,33	43,85	52,43
Romania	0,66	2,09	1,65	3,30	0,93	3,79	0,90	1,41	24,92	39,75
Slovakia	2,53	3,94	4,24	6,26	4,06	11,89	3,56	4,32	42,96	59,37
Slovenia	10,03	9,28	6,01	8,45	13,91	17,54	14,32	13,88	40,86	49,29
Spain	7,85	11,77	8,88	11,48	10,09	16,34	16,86	23,51	66,39	69,34
Sweden	13,30	15,94	16,81	18,23	6,53	12,65	17,85	18,93	62,27	67,96

According visualized data (Table 3) related to the intelligent internet solutions, Croatia is statistically significantly behind Belgium, Denmark, Germany, Estonia, Ireland, Spain, France, Luxemburg, Slovakia, Finland and Sweden. On the other hand, Croatia is positioned as medium developed in relation to use intelligent internet solutions, since the mentioned solutions are used twice as small in Greece and five times less in Romania.

In the observed period (2020-2022) grows only the use of energy management in the households, intelligent security solutions and intelligent household appliances while virtual assistants and smart TV record a decline. While the level of adoption varies compared Croatia to other EU countries, Croatia evidently makes significant efforts to promote available energy management in households. More specifically, in Croatia, several initiatives have been implemented to promote energy efficiency, such as encouraging energy-efficient renovations, utilizing renewable energy sources, deploying smart meters for better energy consumption management, and implementing Energy Performance Certificates for residential buildings. There is also a growing adoption of Energy Management Systems and Smart Home Technologies in the country, enabling homeowners to monitor and control energy usage, optimize heating and cooling, and automate energy-saving measures.

Notwithstanding the implementing of intelligent security solutions vary in terms of the scale and sophistication in Croatia compared to other EU countries, Croatia has made specific efforts in the following areas: (i) expanding video surveillance systems in public spaces and transportation hubs, (ii) utilizing surveillance technologies, biometrics, and border control systems to enhance border security and manage the flow of people and goods, (iii) establishing national cybersecurity strategies and initiatives to enhance cyber resilience, (iv) implementing, intelligent emergency response systems to improve public safety and response capabilities and (v) using intelligent security solutions, including access control systems, intrusion detection systems, and surveillance technologies, to protect critical infrastructure such as power plants, transportation networks, and telecommunications systems.

Furthermore, although the market is in developing phase compared to some larger EU countries, Croatia evident increase in the adoption of intelligent household appliances.The main drivers behind Croatia's progress include: (i) growing interest among homeowners in integrating smart devices for enhanced comfort, convenience, and energy efficiency, (ii) increasing demand for appliances with high energy ratings and smart features that optimize energy consumption and enable better control of energy usage, (iii) expanded availability of connected appliances, allowing remote control and monitoring, (iv) rising popularity of intelligent household appliances for home security and safety and (v) continual expansion of the availability and variety of intelligent household appliances in the Croatian market.

According data in Table 3, in terms of virtual assistants the market is still in developing phase compared to some larger EU countries. Insights into Croatian society reveal the following: (i) citizens frequently adopt virtual assistants for tasks such as voice commands, information retrieval, and home automation control, (ii) availability of Croatian language support slowly becomes crucial factor for virtual assistant adoption, (iii) Croatian homeowners increasingly use virtual assistants to control connected devices like lights, thermostats, and appliances through voice commands, (iv) virtual assistants in Croatia offer various services and skills, including general information, weather updates, news briefings, and basic tasks like setting reminders and timers, (v) concerns about personal data collection, storage, and usage are prevalent among Croatian users, leading to increased attention to data privacy regulations and user consent.

Onwards, Smart TV technology markets is also still in developing phase compared to some larger EU countries. Despite positive trends, the key points regarding Smart TVs in Croatia are: (i) increasing consumer interest in Smart TVs due to advanced features and internet connectivity, (ii) Smart TVs in Croatia provide access to streaming services, social media platforms, and online content, (iii) many Smart TVs in Croatia support the Croatian language, allowing users to access localized content and apps, (iv) Smart TVs offer interactive features such as web browsing, gaming, and social media integration, and (v) pricing and availability of Smart TVs in Croatia may vary based on brand, model, and screen size.

In addition, Table 4 presents cloud computing, public administration and usage of e-government by EU countries. Croatia in these digital skills types of use also stably maintains a middle positon in front of Bulgaria, Greece, Spain, Latvia and Hungary. It is interesting that in the most EU countries in 2022 in comparison with 2021 year, usage of internet in public administration as well as in e- government records a decline. The mentioned types of intelligent internet solutions during the period 2015 – 2021 records growth in all EU countries.

Table 4: Cloud computing, public administration / e-government usage and ICT experts by sex, by EU countries

	Cloud computing usage			*Public administration / e-government*			*ICT experts bx sex*					
							Man			*Women*		
	2015	2020	2021	2015	2020	2021	2015	2020	2021	2015	2020	2021
Austria	:	38,4	37,1	51,04	61,58	62,67	85,7	79,7	81,0	14,3	20,3	19,0
Belgium	:	:	:	41,96	46,22	58,55	84,9	82,6	80,4	15,1	17,4	19,6
Bulgaria	4,4	8,7	9,7	16,04	19,02	19,13	69,4	71,8	71,8	30,6	28,2	28,2
Croatia	**19,4**	**40,0**	**34,8**	**32,24**	**35,72**	**42,49**	**82,8**	**81,9**	**79,2**	**17,2**	**18,1**	**20,8**
Cyprus	6,0	27,1	43,9	31,00	48,01	55,87	79,2	81,8	80,7	20,8	18,2	19,3
Czechia	:	26,8	38,9	30,62	53,02	57,94	90,2	89,7	90,0	9,8	10,3	10,0
Denmark	34,5	66,6	62,8	85,51	88,63	90,60	79,6	77,0	77,1	20,4	23,0	22,9
Estonia	:	54,9	58,2	70,56	66,67	68,73	77,6	77,3	77,4	22,4	22,7	22,6
Finland	52,3	83,0	85,2	72,96	85,30	85,76	77,7	76,7	76,1	22,3	23,3	23,9
France	:	25,1	27,7	44,31	:	51,49	84,2	79,9	79,1	15,8	20,1	20,9
Germany	:	30,8	38,8	52,35	64,83	46,30	83,7	82,4	81,0	16,3	17,6	19,0
Greece	6,7	:	18,7	42,22	52,08	52,27	81,8	72,3	79,0	18,2	27,7	21,0
Hungary	9,4	23,3	24,8	38,56	59,57	72,14	87,9	87,7	86,0	12,1	12,3	14,0
Ireland	35,9	60,5	67,4	40,96	37,43	67,40	79,8	79,3	80,0	20,2	20,7	20,0
Italy	:	59,4	61,9	20,29	21,70	26,20	85,3	84,3	83,9	14,7	15,7	16,1
Latvia	7,7	19,1	26,7	51,76	68,19	67,78	73,4	77,2	77,4	26,6	22,8	22,6
Lithuania	14,5	30,6	35,3	42,31	53,71	56,84	79,5	76,4	76,3	20,5	23,6	23,7
Luxembourg	:	26,4	36,2	46,92	30,35	48,35	86,3	80,0	80,3	13,7	20,0	19,7
Malta	19,9	:	53,4	38,11	45,53	51,23	83,0	89,3	74,3	17,0	10,7	25,7
Netherlands	:	48,7	62,0	70,72	81,40	81,86	86,1	82,4	82,5	13,9	17,6	17,5
Poland	6,3	23,3	28,2	19,07	27,22	29,36	86,4	85,0	84,5	13,6	15,0	15,5
Portugal	:	24,2	29,4	39,61	39,13	41,80	82,4	78,8	79,3	17,6	21,2	20,7
Romania	6,0	15,7	11,2	9,01	10,05	10,81	72,7	73,8	74,0	27,3	26,2	26,0
Slovakia	19,4	24,5	33,8	43,71	51,39	51,84	88,1	84,2	85,1	11,9	15,8	14,9
Slovenia	16,2	38,9	42,9	41,12	56,50	60,79	83,8	82,8	83,4	16,2	17,2	16,6
Spain	12,7	24,6	27,9	45,25	54,10	56,10	81,9	80,7	80,6	18,1	19,3	19,4
Sweden	:	75,4	76,9	69,21	79,21	85,46	81,1	78,7	78,1	18,9	21,3	21,9

Although cloud computing adoption in Croatia is still in the early stages compared to larger EU countries, the current status in Croatia are: (i) global cloud service providers like Amazon Web Services (AWS), Microsoft Azure, and Google Cloud Platform are present,(ii) many organizations in Croatia utilize cloud services for storage, data backup, SaaS applications, and IaaS solutions, (iii) the Croatian government promotes cloud computing adoption in the public sector, (iv) data protection and compliance with EU regulations, particularly GDPR, are essential in Croatian cloud computing and (v) Croatia is increasingly focusing on developing local cloud capabilities, including national cloud infrastructure and data centers.

Croatia makes efforts to advance e-government services and digitize public administration too. Regarding the digitization of public services in Croatia: (i) the Croatian government has focused on providing online access to government services through the e-Citizens portal, (ii) the e-Citizens system offers a secure digital identity for citizens to access e-government services, (iii) efforts have been made to enhance interoperability among government systems and improve data exchange between agencies, and (iv) the government has encouraged citizen participation and engagement through online channels. Croatia's digital maturity level in public administration seems at an earlier stage compared to some EU countries. The pace of adoption and breadth of digital services may vary across different government agencies and regions.

As far as sex representation in ICT is considered, data in Table 4, in the observed period, every EU country records more than three or four times less women than men in ICT sector. Historically, the representation of women in the ICT sector has been relatively low in Croatia, as is the case in many countries worldwide. However efforts have been made in Croatia to promote sex diversity in ICT careers through educational programs, scholarships, and initiatives [14]. The government, along with organizations and industry stakeholders, implements initiatives to encourage women's participation in ICT education and careers, address sex biases and provide mentorship and networking opportunities.

3 CONCLUSION REMARKS

This paper presents a comparative analysis of digital skills between Croatia and other EU countries. It is part of an ongoing research project that aims to address various scientific questions related to digital skills in Croatia. The research conducted by the authors includes statistical analysis and interpretations of empirical research results. Croatia's participation in EU initiatives, along with the anticipated expansion of intelligent household appliances, virtual assistants, Smart TVs, cloud computing, and e-government capabilities, demonstrates the country's potential for further digital advancement. By embracing these trends and aligning with best practices, Croatia can foster a digitally empowered society and drive socio-economic growth.

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Improvement in Continuous Black-box Setting Search Performance by Tuning L-SHADE Differential Evolution Historical Memory Size Parameter

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Abstract: In this work, using Modular Differential Evolution (Modular DE), we investigate the impact of parameter memory size (MS) on the search performance of Success-history based parameter adaptation (L-SHADE) for Differential Evolution. To benchmark it, we use the blackbox optimization benchmarking (**BBOB**) test suite from the COmparing Continuous Optimizers in a black-box setting (COCO) platform. We run the algorithm on all 24 functions in **BBOB** and observe results separately, on each of those function. Experiment and assessment performance are done with Iterative Optimization Heuristics Profiler (IOHprofiler).

Keywords: Differential Evolution, L-SHADE, Memory Size, COCO, IOHprofiler.

1 INTRODUCTION

Operations research [1, 2] is an important enabling component in digital transformation and Society 5.0 [3] and includes smart methods, like optimization algorithms that we want to improve upon in this paper. It is challenging to make comparisons between variants [4], which refer to the same algorithm. In our comparisons, we decide to use a modular algorithm [5], [6], [7] and therefore take advantage of implementation of the algorithmic modifications into a common framework. In this work, we investigate the impact of parameter MS on the search performance of L-SHADE algorithm [8] implemented within the Modular DE framework [4]. The tuned parameter of this algorithm L-SHADE, MS, is a size of historical memory for storing good control parameters and reusing them later as necessary [9]. This mechanism in the algorithm is available also in other L-SHADE algorithm variants and applications [2, 10].

Next section is dedicated to the experiment. Its results we find out in Section 3, where the experiment analysis of the output from IOHAnalyzer are explained. Finally, in the last section, the contribution concludes.

2 EXPERIMENT

We run the experiment using Modular DE framework [4] with IOHExperimenter, which generates the running time data. The results are analyzed with IOHAnalyzer, which produces the summarizing comparisons and statistical evaluations. Both, IOHExperimenter and IOHAnalyzer are parts of IOHprofiler, a tool for analyz / compare iterative optimization heuristics [11].

The experimental part is built on the COCO software, which was adjusted in [12] to cope with optimization problems that are formulated as functions:

$$f : S^D \to R, \tag{1}$$

where $S = \Pi_{j=1}^{D}[x^{\min_j}, x_j^{\max}]$ and each $x_j^{\min}$ to $x_j^{\max}$ are bounds of the search space.

For the post-processing, IOHprofiler enables evaluation of running time data of arbitrary benchmark problems. We run the experiment with L-SHADE algorithm from [4] on 24 functions available in **BBOB** COCO test suite (version 2.6.3). **BBOB** COCO test suite contains 5 groups of functions, which are similar in certain properties. Results, displayed in next section are made on one function from each of those groups, on one instance, on dimension 5 and on a set of 81 targets, logarithmically scaled between 10^8 and 10^{-8}. We test different values to tune the parameter MS, namely 1, 2, 3, 5, 50, 100, and 300. The historical memory size of 100 would denote, that the algorithm is designed to store and retain information about the 100 most promising or best control parameters. These solutions can include individuals from previous generations or iterations that have shown desirable characteristics or high fitness values. We optimize the functions with `ioh.Experiment`, which is a part of ioh library in Python. The framework of Modular DE, we apply, is also publicly available at Zenodo repository [13].

The optimization algorithm that we call with `ioh.Experiment`, is L-SHADE_interface('saturate') [8]. L-SHADE is one out of 11 known versions of Differential Evolution, that are implemented in the Modular DE [4]. There, authors use modularity to create a framework where a full combinatorial range of modules is available for each algorithm component.

We assess the search performance of the L-SHADE algorithm with different MS and aggregate measured runtimes and expected runtimes. The runtime is the count of evaluations, needed to solve the problem. In other words, it is about the number of evaluations, used to reach or surpass the target value for the first time [12].

There are several ways to aggregate the measured runtimes. We used averaging, as an estimator of the expected runtime from [14]. The estimated expected runtime of the restarted solver (ERT), is calculated according to the following equation:

$$E(RT) = E(RT^{\mathrm{s}}) + \frac{1 - p_{\mathrm{s}}}{p_{\mathrm{s}}} E(RT^{\mathrm{us}}), \tag{2}$$

where label s denotes a success and us, a failure. p_{s} denotes the probability of algorithm success.

3 RESULTS

Figure 1 shows ERT (in number of objective function evaluations to reach quality indicator target values), depending on the MS for the functions: Sphere Function - f1, Attractive Sector Function - f6, Ellipsoidal Function - f10, Rastrigin Function - f15 and Schwefel Function - f20.

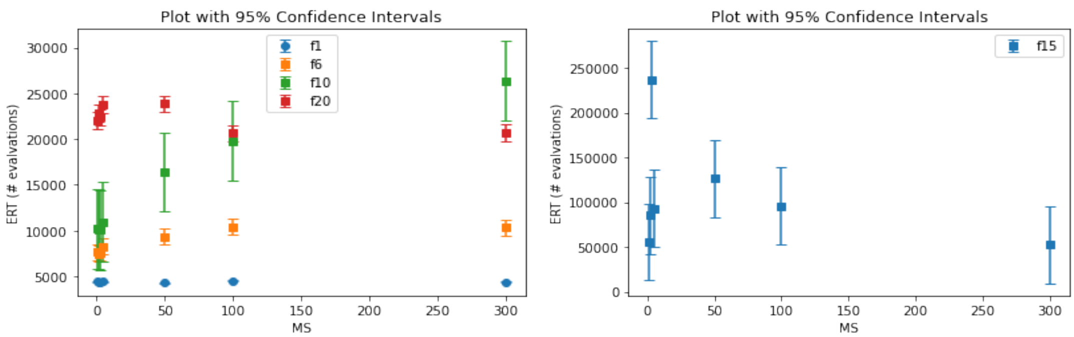

Figure 1: ERT depending on the MS for the function f1, f6, f10, f20 (left) and f15 (right).

Results of Fridman test [15] for these comparisons show small p-value, namely 1.143416e-08. Since it is less than the conventional significance level of 0.05, we can reject the null hypothesis. Null hypothesis says, that there is no difference in the sample. This rejection implies that there are significant differences among the performances using parameters MS that is compared. The Friedman chi-square statistic, which measures the overall difference among the groups of MS, is 48.071429. A larger value indicates a larger difference between the groups with MS.

Kendall's W [16] is 0.333829 and indicates a fair (moderate) agreement effect size (degree of difference). The value of Kendall's W varies between 0 and 1, where a value of 0 indicates complete discordance between raters, while a value of 1 indicates complete concordance.

	MS 1	MS 2	MS 3	MS 5	MS 50	MS 100	MS 300
MS 1	1.0	0.661328	0.595409	0.264624	0.008197	0.000869	0.000869
MS 2	0.661328	1.0	0.939935	0.107813	0.001657	0.000175	0.000175
MS 3	0.595409	0.939935	1.0	0.086959	0.001195	0.000175	0.000175
MS 5	0.264624	0.107813	0.086959	1.0	0.151822	0.030298	0.030298
MS 50	0.008197	0.001657	0.001195	0.151822	1.0	0.530827	0.530827
MS 100	0.000869	0.000175	0.000175	0.030298	0.530827	1.0	0.0
MS 300	0.000869	0.000175	0.000175	0.030298	0.530827	1.0	1.0

Table 1: Reported values of Conover-Iman post-hoc test for ERT between MS pairs.

For post hoc analysis, following a Friedman test, we used Conover-Iman test [17], because our experiment fits on all three Conover-Iman following scenarios (Non-normal data, Unequal variances, Ordinal data).

Post hoc tests is performed to determine which specific groups differ significantly from each other. In Table 1 we can see, that the significantly difference on ERT are between MS pars (1, 100), (1, 300), (2, 100), (2, 300), (3, 100) and (3, 300). The lower value in the table, means the greater difference between MS in a pair.

ERT improved its value on MS=3 by 19 BBOB functions and achieved worse results on 5 those functions, of total 24 functions, compering to the default value MS=5.

Based on the above reported results of post-hoc tests, it is demonstrated that ERT performance of L-SHADE, depends on the historical memory size parameter MS for **BBOB** COCO test suite.

4 CONCLUSION

In this work we observed the impact of parameter MS from L-SHADE with Modular DE, on ERT performance in case of 24 blackbox optimization benchmarking (**BBOB**) COCO test functions. We run the experiment of different MS with L-SHADE algorithm, as one of 11 variants of DE, included in Modular DE. For the experiment we used COCO platform and IOHprofiler. As explained in previous section, results show, that ERT performance of L-SHADE with Modular DE depends on the memory size MS, based on Fridman and Kendall's test for **BBOB** functions. Results of Friedman test show small p-value, namely 1.143416e-08, so we can reject the null hypothesis. This demonstrates that there are significant differences among the parameters MS being compared. The Friedman chi-square statistic, which measures the overall difference among the groups of MS, is 48.071429. Kendall's W is 0.333829 and that indicates a moderate agreement effect size. We upgraded the Fridman test with Conover-Iman post hoc tests. In Table 1 we reported the results, which show the significant difference on ERT between MS pars: (MS 1, MS 100), (MS 1, MS 300), (MS 2, MS 100), (MS 2, MS 300), (MS 3, MS 100) and (MS 3, MS 300). ERT improved its value on MS=3 by 19 BBOB functions and achieved worse results on 5 functions.

In future work, we want to speed up the process of gathering the results with the help of parallelization. In such a way we could implement more complex experiments and measure hypothetical improvements to algorithms, quicker.

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THE MATURITY MODEL FOR CLIMATE NEUTRALITY AND BUSINESS PROCESS OPTIMIZATION IN SLOVENIAN COMPANIES OF THE FUTURE

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Abstract: Our Climate Neutrality Model is motivated by the commitment of the European Union Member States to achieve climate neutrality by 2050. The 5-stage maturity model presented in our paper aims to give companies an insight into its maturity on the path of green and digital transformation. The model enables companies to identify their level of maturity and define strategies to enhance certain areas. We conducted 35 interviews with companies and, based on the answers received, classified each company to one of the stages in the maturity model.

Keywords: Climate Neutrality, Maturity Models, Business Process Optimization, Environmental, Social, and Governance standards.

1 INTRODUCTION

Transitioning into a zero-emission society is crucial to reduce long term impacts of climate change and extreme weather events [8]. These are caused by observed changing weather patterns, which are, as established by meteorological models, consequences of increasing atmospheric heat absorption caused by greenhouse gas emission [6]. Medium to large businesses involved in various sectors can contribute significantly to greenhouse gas emissions, which highlights the importance of optimizing processes to achieve climate neutrality. Upgrading and continually updating technologies, using raw materials, semi-finished products, and final products that minimize or eliminate greenhouse gas emissions, are key requirements for the transition to climate neutrality [8].

The SIQ standard [7] covers regulatory requirements, cost control, resource efficiency, pollution prevention, and addressing the expectations of stakeholders and the public [7]. The model also incorporates ISO 14040 and ISO 14044, which are based on Life Cycle Assessment (or LCA) analysis and provide guidelines for evaluating the life cycle of a product or service. The assessment encompasses the environmental impact throughout the entire life cycle, including raw material extraction, production, use, waste management, and final recycling [3]. The model also considers Environmental, Social and Governance (ESG) guidelines, which assess environmental attitudes (E), social aspects related to employees (S), and overall company management (G). The environmental perspective encompasses a comprehensive evaluation of a company’s environmental impacts, including resilience to climate risks, carbon footprint, chemical usage in processes, air, water, and soil impacts, as well as the environmental impact of suppliers [2]. The Maturity Model for Climate Neutrality is based on various standards and environmental indicators. Maturity models utilize levels to describe growth of subjects. The models are based on the assumption of predictable growth patterns within the observed model, and they describe an assumed, desired or expected stage-by-stage growth of subjects [4]. An early example is the Technology Readiness Level (or TRL) scale, first introduced by NASA in the 1970s and adopted by the European Commission for H2020 projects and the program Horizon Europe [5]. Maturity models refer to classes of entities [4]. In the context of our study, these entities refer to companies. They are assigned to specific classes based on their growth levels within the maturity model, indicating their level of maturity determined by the indicators used as classifiers in the model. Maturity models have been used to monitor progress in various aspects of life, including companies, individuals, and processes [4]. In our paper, we establish a maturity model for climate neutrality of companies, where we assess the maturity level of each company based on their structure, processes, and other indicators.

2 METHODOLOGY

We developed a maturity model for climate neutrality consisting of five stages. Each stage is assigned specific characteristics that determine the placement of companies within those stages. The stages are labelled as follows: 1. - Beginner, 2. - Follower, 3. - Expert, 4. - Leader and 5. - Influencer.

Table 1: Description of the stages of climate neutrality maturity model.

Category/ Level name	Beginner	Follower	Expert	Leader	Influencer
Strategies	no business, digital or HR strategy, no need for it, does not process data to support decision-making processes, no analysis	no business, digital or HR strategy, aware they will need it occasionally process data to support decision-making processes	business, HR and digital strategy in development, only use some collected data (superficially) for decision making process	well-defined business, digital and HR strategy, actively and regularly processes data to support decision-making processes	well-defined and comprehensive business, digital and HR strategy, collected data is optimized precisely and consistently to meet the needs of business processes
Digitalization	not familiar with ERP	are familiar with ERP but don't use it	are familiar with ERP	good understanding of ERP, extensive integration in the operations	employs ERP for digitalization and consistently identifies opportunities for improvement and upgrading
Environment	no environmental report, does not collect environmental badges, does not follow any ISO standards	no environmental report, aware of environmental standards and badges, does not apply them	no environmental report, some aspects in the annual report, aware of the importance of environmental standards and badges but don't use either	annual report includes an environmental report, they also have basic legal provisions with various environmental standards and guidelines, utilizes environmental badges, not consistently	environmental report references relevant ISO environmental standards, environmental badges required from both suppliers and customer

Data analysis	no performance monitoring, only collect legally required data, meets minimum standards	collects data that meets minimum standards, rarely do analysis	regularly process data for records and statistics, process the collected data in real-time only enough to meet minimum standards, monitors its own performance, but not at all stages or processes	business process optimization is through and consistent, but results not fully taken into account, Data analysis tools are utilized	collect diverse data and regularly process and analyse it, closely monitors its prices throughout the entire process
LCA analysis	not familiar with LCA analysis	are familiar but don't use LCA analysis	LCA analysis partly done on some of the products, not connected to environmental standards or badges	LCA analyses made for most products, there is still room for improvement	conducts LCA analyses for all products

Based on a five-stage model, we created a questionnaire that served as the basis for conducting structured interviews with the companies. The questionnaire consisted of 20 questions, divided into five sets. The first set, the general part, aimed to define the company's scope of work, size, business strategy, digital strategy, and HR strategy. This was followed by the digitalization section, where we assessed the company's level of digital transformation. Since ERP is considered a key digitalization system, due to the storage and processing of most data, we paid special attention to it in our model. There are other ways of measuring digitalization, such as measuring the number of users relative to the number of licenses purchased or analysing the breadth and limitations of usability. In the future, as an extension of our work, we will also take other measurement approaches. Next was the environmental section, which covered aspects such as environmental reporting, compliance with standards, and environmental certification and the data analysis section with business process optimization. The final section is focused on LCA analysis and calculating production price.

Our maturity model can be compared with the Digital Maturity Assessment methodology since both models consist of five maturity levels. They differ from each other based on the number of sets or competencies. As mentioned before, our model is divided into five parts, while the Digital Maturity Assessment methodology contains 50 competencies [1]. There are parallels with our sets in project management, technology, data, and reporting. The key difference is that our model is upgraded with an environmental aspect.

We conducted 35 interviews with companies from various industries, considering factors such as their years of operation, size in terms of employees, and net profit. Based on these interviews, we determined the maturity level of each company. Our objective in developing this model was to make a significant contribution to accelerating the achievement of climate neutrality goals and fostering the overall maturity of companies in the areas of business, digitalization, and environmental practices.

3 RESULTS

Among 35 companies, 22 defined themselves as micro companies, 4 as small and medium-sized enterprises (SMEs), 2 as large companies, and the size classification was unspecified/unknown for 3 companies. The figure shows distribution of levels of maturity in the five sections of analysis: Business, digital and HR strategies and decision-making process; Digitalisation; Environmental characteristics; Data analysis; LCA analysis.

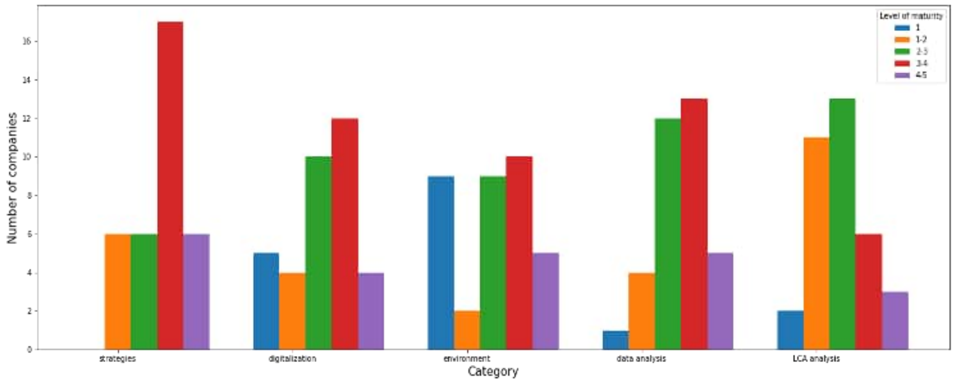

Figure 1: Distribution of interviewed companies in levels of maturity for each observed category.

The following are the average maturity levels for the observed categories: Business, digital and HR strategies, and decision-making process: 3.337; Digitalization: 2.900; Environmental characteristics: 2.762; Data analysis: 3.057; and LCA analysis: 2.571.

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ON SOLVING THE STABLE SET PROBLEM BY GRAPH PARTITIONING AND QUANTUM ANNEALERS

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Abstract
Quantum annealers have become more popular and advanced in recent years. However, their capabilities are still very limited when we try to solve optimization problems. This short paper aims to present an approach to reduce the complexity of the problem of finding the maximum stable set of a graph. This involves breaking down the problem into multiple smaller ones, making it feasible to solve the entire larger graph using a Quantum Processing Unit (QPU).

Keywords: The stable set problem, Quantum annealing, D-Wave, CH-partition

1 INTRODUCTION

Combinatorial optimization problems appear in various fields, such as network design, logistics, scheduling, and bioinformatics. These problems require the determination of the best solution from a set of options, often with high combinatorial complexity. Such problems are typically very hard, theoretically and practically, and usually belong to the class of NP-hard problems, therefore efficient algorithms for precise solutions within polynomial time are lacking for these problems, unless P = NP. Despite this challenge, significant research has been directed towards developing effective algorithms to tackle these problems for small or medium size.
In this paper, we build upon the research presented in the paper [1], which was about solving the stable set problem using D-Wave's Quantum Processing Units (QPUs). The referenced paper specifically focuses on graphs with up to 125 vertices. In contrast, the aim of this paper is to introduce a technique for partitioning larger and denser graphs into smaller subgraphs to solve the stable set problem. This approach enables us to perform the necessary calculations on these subgraphs using the QPUs, allowing us to fully address the stable set problem on QPUs, even for larger graphs.
This paper is organized as follows. In Section 2, we define the stable set problem, provide a QUBO formulation for it and then show a method on how to divide a stable set problem of a large graph into multiple smaller subgraphs. The information regarding our implementation on D-Wave, considered instances as well as computational results are presented in Section 3. Finally, we conclude with a short discussion of our results and possible future work in Section 4.

2 STABLE SET PROBLEM AND GRAPH PARTITIONING

Let $G = (V, E)$ be a simple undirected graph with $|V| = n$ vertices and $|E| = m$ edges. A set $S \subseteq V$ is called a stable set if the vertices in S are pairwise nonadjacent. The stability number of G is the cardinality of the largest stable set in G and is denoted by $\alpha(G)$. The stable set problem asks to determine $\alpha(G)$.
As stated in [1], the QUBO formulation of the stable set problem, which is a suitable formulation

for D-Wave solvers, is:

$$\min\ \{x^T Q x \mid x_i \in \{0,1\}\} \tag{1}$$

where Q is the input matrix defined as

$$Q = -I + \beta A, \tag{2}$$

where A is the adjacency matrix of the underlying graph, I is the identity matrix and $\beta \geq 1$.

It is also worth noting the fact stated in [1] that the solution is neither guaranteed to be optimal nor feasible. Although certain penalties β were imposed in (2), this does not ensure that they will always be sufficient to yield a stable set. The feasibility of the solution can be easily checked using equality [1]:

$$|e^T x| = |-e^T x + \beta x^T A x|.$$

If the equality holds, then the solution is feasible; otherwise, it is not.

As we will see, some graphs are too dense and/or too large to be embedded on the QPU. In some cases, we can use CH-partitioning [4] to split a graph into multiple smaller subgraphs, which can then be embedded on the QPU.

In *CH-partitioning*, there are two levels of dividing the vertices of G into subsets. In the *core partitioning*, the set V of vertices is divided into nonempty *core* sets $C_1, \ldots, C_s$, such that $\bigcup_i C_i = V$ and $C_i \cap C_j$ for $i \neq j$. There is one *complementary* set H_i of vertices for each core set C_i, defined as the set of neighbor vertices of C_i that are not from C_i. Recall that a vertex w is a neighbor of a vertex v iff there is an edge between v and w. We define the cost of the CH-partitioning $\mathcal{P} = \{(C_i, H_i) \mid 1 \leq i \leq s\}$ as

$$\mathrm{cost}(\mathcal{P}) = \max_{1 \leq i \leq s} (|C_i| + |H_i|).$$

The *CH-partitioning problem* is finding a CH-partitioning of G with minimum cost for some s.

The following proposition gives us the desired property, which is proved in [4]:

Proposition 1 *Given a CH-partitioning $\{(C_i, H_i) \mid 1 \leq i \leq s\}$ of a graph G for some s, the size of the maximum clique of G is equal to* $\max_i\{k_i\}$*, where k_i is the size of a maximum clique of the subgraph of G, induced by $C_i \cup H_i$.*

Let us denote a subgraph of G, induced by X, as $G[X]$. Recall that a clique of a graph G is a subset of vertices $S \subseteq V(G)$, where every two vertices from S are adjacent in graph G. The clique number $\omega(G)$ is the cardinality of the maximum (the largest) such set in the graph. Also recall that X is a (maximum) clique in $\overline{G}$ iff X is a (maximum) stable set in G, where $\overline{G}$ denotes the complement of a graph G, i.e. a graph with its edges inverted. Also note that $G = \overline{\overline{G}}$.

This yields the following algorithm: Let us suppose we want to find the maximum stable set in graph G. We first take the graph $\overline{G}$ and we try to partition it by finding the best CH-partition

$$\mathcal{P} = \{(C_i, H_i) \mid 1 \leq i \leq s\}$$

for some desired s (for this, we can use METIS algorithm as it is proposed in [3]). Then we use Proposition 1 and we get that

$$\max_i \{\omega(\overline{G}[C_i \cup H_i])\} = \omega(\overline{G}) = \alpha(G).$$

If $X_i \subseteq C_i \cup H_i$ is a maximum clique in graph $\overline{G}[C_i \cup H_i]$, then we know that

$$X_i \text{ is a maximum clique in } \overline{G}[C_i \cup H_i] \Leftrightarrow X \text{ is a maximum stable set in } \overline{\overline{G}[C_i \cup H_i]}.$$

Combining all this, we get

$$\max_i\{\alpha(\overline{\overline{G}[C_i \cup H_i]})\} = \max_i\{\omega(\overline{G}[C_i \cup H_i])\} = \omega(\overline{G}) = \alpha(G).$$

It's easy to show (by using definitions) that $\overline{\overline{G}[X]} \cong G[X]$ which gives us:

$$\max_i\{\alpha(G[C_i \cup H_i])\} = \alpha(G).$$

This means that we can just calculate maximum stable set in graphs $G[C_i \cup H_i]$ and take the best result. If $C_i \cup H_i \subset V(G)$ for $1 \leq i \leq s$, we found a family of smaller graphs which we might be able to embed on the QPU.

The algorithm described works especially well on dense graphs G, because when we take the complement of a graph, we obtain $\overline{G}$, which is a sparser graph and can be partitioned more effectively using the method described above.

3 COMPUTATIONAL RESULTS

In this section, we applied the method described above to perform calculations on graphs from [5]. As we will observe, the provided method works effectively with denser graphs, while it struggles with even smaller, sparser ones. Note that all codes and data are available on github[1].

We perform computations on Leap quantum cloud service by using the class of Binary Constraint Models (BQM). We use D-Wave's Advantage System, which uses the Pegasus topology and contains more than 5000 qubits [2]. All computations are done with default parameters and number of reads on QPU is set to be 100.

The graphs we will be working with are the same ones used in [1], along with the graphs *c_fat_200_2* and *c_fat_500_5*, two of which are too large and dense to be directly embedded onto the QPU. Here, "dense" refers to their high edge density, which is defined as the ratio between the number of edges and the number of all possible edges. In Table 1, you can observe the results obtained using the partitioning method described above. As you can see, not all the graphs from [1] are included. This is because not all the graphs could be partitioned in a way that satisfies $\text{cost}(\mathcal{P}) < n$, making these partitions not worth investigating. But please note that $\text{cost}(\mathcal{P}) = n$ doesn't necessarily mean that every subgraph is equal to the graph, but rather that at least one is. This means we could still utilize the partition if we limit ourselves to elements $(C_i, H_i) \in \mathcal{P}$ such that $|C_i| + |H_i| < n$. However, we have chosen not to pursue this in the current work and leave it for future research.

The columns 1-4 contain general information about the instances: the instance name, the number of vertices n, the number of edges m, as well as the stability number $\alpha(G)$. For each instance, we perform calculations for $\beta = 1$ in (2), as it showed the best results in [1]. Columns 5 and 6 contain information about the cardinalities of the solutions for $\beta = 1$, obtained from the solver in the paper [1] and in this paper (using CH-partitioning), respectively. As previously mentioned, the solution returned by the solver is not necessarily a stable set. Therefore, we check whether the solution is a stable set or not. If the solver returned a solution that is not a stable set, we mark the cell as dark gray. If the solver returned a solution that is not optimal, we mark the cell as gray.

As we can observe, the results of the two *c_fat* graphs come remarkably close to the optimal solution, and partitioning significantly reduces the maximum graph size. Unfortunately, this method proves itself useful only when dealing with denser graphs, as we can see from the rest of the graphs. As density decreases, the positive effect that CH-partitioning has on the graph

[1] `https://github.com/aljazkrpan/stable-set-partitioning.git`

Table 1: Result for some graphs in [1] plus results for two bigger, denser *c_fat* graphs, calculated on QPU using CH-partitioning

Graph	n	m	$\alpha(G)$	$\beta = 1$ in [1]	$\beta = 1$	$\mathrm{cost}(\mathcal{P})$	$\lvert\mathcal{P}\rvert = s$	Edge density
c_fat_200_2	200	16665	24	-	24	46	34	0.84
c_fat_500_5	500	101559	64	-	64	127	57	0.81
dsjc125.5	125	3891	10	4	6	115	42	0.50
hamming6_4	64	1312	4	4	4	48	30	0.65
johnson8_2_4	28	168	4	4	4	26	12	0.44
johnson16_2_4	120	1680	8	8	8	118	57	0.24
paley61	61	915	5	5	5	54	27	0.50
paley73	73	1314	5	5	5	65	25	0.50
paley89	89	1958	5	4	5	79	40	0.50
paley97	97	2328	6	5	5	87	33	0.50
paley101	101	2525	5	5	4	90	34	0.50

tends to decrease as well. but with all that said, we still managed to improve the solution from the last time on graphs *dsjc125.5*, *johnson16_2_4*, *paley89*. Table 2 displays all the graphs addressed in [1], as well as the two *c_fat* graphs addressed today, ordered by edge density. The partitioning results were calculated using the METIS algorithm mentioned earlier. For each graph, we calculated the partition with the minimal $\mathrm{cost}(\mathcal{P})$ function for $1 \leq s \leq n$, and then selected the value of s with the smallest minimal cost. If $\mathrm{cost}(\mathcal{P}) = n$, then no value of s is provided, as the cost is the same for every s. As we can see, the denser the graph, the better the results.

4 CONCLUSIONS AND FUTURE WORK

The goal of this work was to provide a way to divide a graph into multiple smaller subgraphs, on which we can then calculate the stable set problem on a QPU. The method provided was successful with denser graphs (density above 0.8), it showed some improvement on medium-dense graphs (density around 0.5) but little to no improvement on sparse graphs (density below 0.3). But even when we can't significantly reduce the graph size, the partition can still help us improve the solution compared to the standard approach, as we have seen in graph *dsjc125.5*, *johnson16_2_4* and *paley89*. However, this comes at the cost of additional computational complexity.

For further investigation, the first suggestion would be to attempt to utilize only specific elements of partitions where $\mathrm{cost}(\mathcal{P}) = n$ as explained in Section 3. As for the second suggestion, it would be interesting to determine whether the root problem lies in graph density alone or if the METIS algorithm might not be the most suitable approach for this purpose. If it is not, exploring alternative algorithms could be worthwhile.

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Table 2: Best partitions found with METIS algorithm which have the lowest cost function across all sensible s

Graph	n	m	cost($\mathcal{P}$)	$\lvert\mathcal{P}\rvert = s$	Edge density
c_fat_200_2	200	16665	46	34	0.84
c_fat_500_5	500	101559	127	57	0.81
hamming6_4	64	1312	48	30	0.65
dsjc125.5	125	3891	115	42	0.50
paley61	61	915	54	27	0.50
paley73	73	1314	65	25	0.50
paley89	89	1958	79	40	0.50
paley97	97	2328	87	33	0.50
paley101	101	2525	90	34	0.50
johnson8_2_4	28	168	26	12	0.44
johnson16_2_4	120	1680	118	57	0.24
johnson8_4_4	70	560	26	-	0.23
C125.9	125	787	125	-	0.10
hamming6_2	64	192	64	-	0.10
dsjc125.1	125	736	125	-	0.09
MANN_a9	45	72	45	-	0.07
spin5	125	375	125	-	0.05
torus11	121	242	121	-	0.05

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ADVANCING STABLE SET PROBLEM SOLUTIONS THROUGH QUANTUM ANNEALERS

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Abstract
We assess the performance of D-wave quantum solvers for solving the stable set problem in a graph, one of the most studied NP-hard problems. We perform computations on some instances from the literature with up to 125 vertices and compare the quality of the obtained solutions with known optimum solutions. It turns out that the hybrid solver gives very good results, while the Quantum Processing Unit solver shows rather modest performance overall.

Keywords: The stable set problem, Quantum annealing, hybrid solvers, D-Wave

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1 INTRODUCTION

Combinatorial optimization problems arise in diverse fields such as network design, logistics, scheduling, and bioinformatics. These problems involve finding the optimal solution from a finite set of possibilities and are often NP-hard; hence, unless P = NP, there are no polynomial time algorithms which can solve these problems exactly. Nevertheless, a considerable effort has been dedicated to the development of efficient algorithms for solving these problems.

Exact algorithms for combinatorial optimization problems are algorithms that guarantee finding the optimal solution. However, exact solvers are often computationally expensive, and solving larger problems becomes challenging. An alternative to exact solvers are heuristics. They provide good quality solutions significantly faster than exact methods, but do not guarantee finding the optimal solution.

In recent years, a new approach for solving combinatorial optimization problems has emerged based on quantum annealing, a computing paradigm that exploits the principles of quantum mechanics to solve optimization problems. Quantum annealing can be practically implemented using quantum computers. One of the leading companies in this area is D-Wave, which has developed commercial quantum annealing systems called D-Wave Quantum Processing Units (QPUs). However, the size of problems that can be solved using D-Wave QPUs is limited by the number of qubits in their quantum processor. To handle larger problems, D-Wave also offers hybrid solvers that combine classical with quantum computing.

This work deals with the stable set problem, one of the fundamental NP-hard problems [7] in combinatorial optimization. The stable set problem asks to determine the cardinality of the largest set of pairwise nonadjacent vertices in a graph. By formulating the problem as a quadratic unconstrained binary optimization (QUBO) problem, it can be solved with D-Wave's QPU and hybrid solvers. The goal of this work is to assess and compare the performance of respective solvers in terms of quality of the solutions. For this purpose, we conduct experiments on instances from the literature with up to 125 vertices and report computational results.

This paper is organized as follows. In Section 2 we first define the stable set problem and formulate it as a QUBO problem. In Section 3 we give a short overview of related work that aims

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to solve the stable set problem with D-Wave. The information regarding our implementation on D-Wave, considered instances as well as computational results are presented in Section 4. Finally, we conclude with a short discussion of our results and possible future work in Section 5.

2 THE STABLE SET PROBLEM

Let $G = (V, E)$ be a simple undirected graph with $|V| = n$ vertices and $|E| = m$ edges. A set $S \subseteq V$ is called a stable set if the vertices in S are pairwise nonadjacent. The stability number of G is the cardinality of the largest stable set in G and is denoted by $\alpha(G)$. The stable set problem asks to determine $\alpha(G)$.
A standard method in combinatorial optimization is to write an NP-hard problem as an integer program. Let x_i be a binary variable indicating whether the vertex i is in the stable set or not. Then the stability number of a graph G is the optimal value of the following integer optimization problem with linear constraints

$$\alpha(G) \;=\; \max\{e^T x \mid x_i + x_j \leq 1 \;\forall \{i,j\} \in E,\; x_i \in \{0,1\} \;\forall i \in V\}, \tag{1}$$

where e denotes the all-ones vector. Linear constraints in the formulation (1) can be replaced by quadratic ones. Let x_i be a binary variable as defined in (1). Then the edge constraints can be written as $x_i x_j = 0$ for all $\{i,j\} \in E$, and therefore

$$\alpha(G) \;=\; \max\{e^T x \mid x_i x_j = 0 \;\forall \{i,j\} \in E,\; x_i \in \{0,1\} \;\forall i \in V\}. \tag{2}$$

The formulations (1) and (2) have n binary variables and m constraints. The number of constraints can be reduced in the following way. Let A be the adjacency matrix of a graph G. Since x is a binary variable we have that $x_i^2 = x_i$, so the edge constraints $x_i x_j = 0$ can be written as $\frac{1}{2} x^T A x = 0$. Hence,

$$\alpha(G) \;=\; \max\{e^T x \mid \frac{1}{2} x^T A x = 0,\; x_i \in \{0,1\} \;\forall i \in V\}. \tag{3}$$

In order to use D-Wave solvers, we need a QUBO formulation for the stable set problem. Generally, a QUBO is a problem of finding

$$z^* \;=\; \min\{f^T x + x^T Q x \mid x \in \{0,1\}^n\}.$$

In the formulation (3) we use that $\max e^T x = -\min -e^T x$. Furthermore, we have that the edge constraints can be written as $\frac{1}{2} x^T A x = 0$. Nevertheless, the penalty for an edge should be greater than the contribution of a vertex. That is why we impose the penalties as $\beta x^T A x$ for $\beta \geq 1$, and get to the following formulation

$$\alpha(G) \;=\; \min\{-e^T x + \beta x^T A x \mid x \in \{0,1\}^n\}. \tag{4}$$

Analogous formulations with different values of the parameter β have already been introduced. For instance, in [3] the parameter was set as $\beta = 1$, while $\beta = \frac{1}{2}$ was considered in [1]. An alternative approach is to reward the contribution of vertices instead of penalizing the edges. Such formulation was given in [8].
D-Wave solvers are designed to minimize a QUBO formulated as

$$\min \quad x^T Q x,$$

where x is a binary variable. Due to the fact that $x_i^2 = x_i$, we can write $-e^T x$ as $x^T(-I)x$, where I denotes the identity matrix. Thus, we define for $\beta \geq 1$ the input matrix Q as

$$Q = -I + \beta A. \tag{5}$$

It is important to note that the solution obtained from D-Wave is not necessary the optimal solution. But also, we would like to emphasize that the solution returned by D-Wave is not necessarily a stable set, either. Although we impose certain penalties in (4), it does not mean that the sum of penalties in the final solution will be zero. The correctness of the solution can be easily checked. If

$$|e^T x| \neq | - e^T x + \beta x^T A x|,$$

then the underlying solution is clearly not a stable set. We will discuss this issue in Section 4 in more detail.

3 RELATED WORK

The previous work focused on solving the stable set problem with the D-Wave QPU solver. For this purpose, several authors considered instances which fit the architecture of D-Wave, while for other arbitrary instances partitioning methods were proposed. The performance was compared with exact methods or heuristics. The authors focused mostly on computational times and investigated whether a speed-up can be realized or not. In the following text we give more details regarding the respective research.

One of the first practical results for solving the stable set problem with quantum annealing in D-Wave was shown in [8]. The results were obtained on D-Wave Two AQO with 512 qubits in Chimera graph architecture. The experiments were done on Chimera graphs, which fit the underlying architecture. However, it turned out that Selby's exact method [11] outerperformed the quantum annealing approach.

Further numerical experiments were conducted in [3], and that on D-Wave 2X with about 1000 qubits in the Chimera graph architecture. The first set of experiments was done on random graphs with 45 vertices. It turned out that classical solvers available are usually faster for such small instances. The second set of experiments was performed with subgraphs of D-wave chimera graphs. For large instances with 800 or more vertices, D-Wave provided the best solutions. The last set of experiments was performed with instances that generally do not fit the D-Wave architecture. A decomposition method was proposed for these instances. However, it turned out that the proposed method was effective only for relatively thin graphs. The method in question was further improved in[9].

Experiments on D-Wave 2000Q were done in [12]. The obtained results for random graphs with up to 60 vertices were compared to some classical algorithms. It was shown that D-Wave QPU was outerperformed by simulated thermal annealing.

Recently, the algorithm introduced in [9] was combined with the method of parallel quantum annealing [10]. The suggested approach was applied on graphs with up to 120 vertices. All computations were performed on the D-Wave Advantage System 4.1. Among others, it was shown that the proposed algorithm can compute solutions for certain instances up to around two to three orders of magnitude faster than a classical solver.

4 COMPUTATIONAL RESULTS

In this section we report computational experiences and compare the solutions obtained by D-Wave with the exact ones, known from the literature or recomputed by BiqBin solver [5]. Note that all codes and data are available as supplementary files on the arXiv page of this paper and can also be found on github[2].

For our computations we consider instances from the literature with up to 125 vertices. More precisely, we consider some instances from the Second DIMACS Implementation Challenge [6].

[2] `https://github.com/DunjaPucher/Stable_set_experiments.`

Table 1: Results for computations done with QPU solver

Graph	n	m	$\alpha(G)$	$\beta = 1$	$\beta = 10$	$\beta = 100$
C125.9	125	787	34	24	22	20
DSJC125.5	125	3859	10	4	4	6
DSJC125.9	125	789	34	26	21	19
hamming6_2	64	192	32	32	29	25
hamming6_4	64	1312	4	4	3	4
johnson8_2_4	28	168	4	4	4	4
johnson8_4_4	70	560	14	11	9	9
johnson16_2_4	120	1680	8	8	6	7
MANN_a9	45	72	16	16	15	15
paley61	61	915	5	5	5	5
paley73	73	1314	5	5	5	4
paley89	89	1958	5	4	5	5
paley97	97	2328	6	5	5	4
paley101	101	2525	5	5	5	5
spin5	125	375	50	46	35	34
torus11	121	242	55	55	43	40

Furthermore, we consider some small Paley graphs, torus graphs as well as spin graphs. The stability numbers of these instances were previously considered in [4].
We perform computations on Leap quantum cloud service by using the class of Binary Constraint Models (BQM), since the problems in this class can be solved with both the QPU as well as the hybrid solver. We use D-Wave's Advantage System, which uses the Pegasus topology and contains more than 5000 qubits [2]. Finally, we would like to note that we perform all computations with default parameters, and for QPU computations we set the number of reads to be 1000.
The computational results obtained by QPU and hybrid solvers are gathered in Tables 1 and 2. The columns $1-4$ contain general information about the instances: name of the instance, number of vertices n, number of edges m, as well as the stability number $\alpha(G)$. For each instance we perform several experiments for different values of the parameter β in (5). We set $\beta = 1$, $\beta = 10$ and $\beta = 100$. The columns $5-7$ contain the information about the cardinalities of the solutions obtained from the solver, i.e. about the stability numbers of the considered instance. As previously mentioned, the solution returned by the solver is not necessarily a stable set. Therefore, we check whether the solution is a stable set or not. If the solver returned a solution which is not a stable set, we mark the cell as dark gray. If the solver returned a solution which is not optimal, we mark the cell as gray.
Based on the results in Table 1 we make several observations. First, not every solution returned by QPU is a stable set, see results for *dsjc125.5*, *dsjc125.9* and *johnson16_2_4*. Second, obtained solutions are not always optimal; some of the solutions are quite far from the optimal values, see for example results for *C125.9* and *dsjc* graphs. The smallest instance for which we got a non-optimal value for parameter $\beta = 10$ is *hamming6_4* with 64 vertices. Finally, we observe that imposing higher penalties on edges, i.e. setting the value of the parameter β to 10 or 100 does not necessarily yield better solutions than setting $\beta = 1$, see for instance results for *dsjc125.9*, *spin5* and *torus11*.
On the other side, from the results in Table 2 we note that for all instances and for all considered values of the parameter β we obtained optimal solutions with the hybrid solver.
We now examine the solutions obtained from the QPU solver in more detail. For that purpose, we check for all gathered samples whether the solution is a stable set or not. As previously mentioned, we perform 1000 runs per instance. Table 3 contains information about the percentage of samples which are stable sets, i.e. solutions obtained from the QPU solver for different

Table 2: Results for computations done with hybrid solver

Graph	n	m	$\alpha(G)$	$\beta = 1$	$\beta = 10$	$\beta = 100$
C125.9	125	787	34	34	34	34
DSJC125.5	125	3859	10	10	10	10
DSJC125.9	125	789	34	34	34	34
hamming6_2	64	192	32	32	32	32
hamming6_4	64	1312	4	4	4	4
johnson8_2_4	28	168	4	4	4	4
johnson8_4_4	70	560	14	14	14	14
johnson16_2_4	120	1680	8	8	8	8
MANN_a9	45	72	16	16	16	16
paley61	61	915	5	5	5	5
paley73	73	1314	5	5	5	5
paley89	89	1958	5	5	5	5
paley97	97	2328	6	6	6	6
paley101	101	2525	5	5	5	5
spin5	125	375	50	50	50	50
torus11	121	242	55	55	55	55

Table 3: Percentage of samples which are stable sets obtained from QPU solver

Graph	$\beta = 1$	$\beta = 10$	$\beta = 100$	Graph	$\beta = 1$	$\beta = 10$	$\beta = 100$
C125.9	2.80	20.10	3.30	MANN_a9	85.31	95.80	97.20
DSJC125.5	3.30	0.80	0.00	paley61	2.00	7.40	8.70
DSJC125.9	2.50	17.20	19.30	paley73	2.20	5.20	1.70
hamming6_2	99.77	90.50	94.90	paley89	1.30	3.70	4.90
hamming6_4	2.30	8.80	12.40	paley97	1.70	5.10	1.60
johnson8_2_4	30.90	49.94	60.96	paley101	4.10	3.70	3.00
johnson8_4_4	4.10	16.30	19.90	spin5	32.70	68.00	61.60
johnson16_2_4	0.50	4.30	3.80	torus11	85.60	94.10	96.00

values of the parameter β.

The results presented in Table 3 show that, for some instances and in that for all considered values of the parameter β, the percentage of samples which are stable sets is quite low. For instance, for $\beta = 1$ the percentage is under 5% for 11 out of 16 considered instances. The situation is slightly better for $\beta = 10$ and $\beta = 100$, where this is the case for 4, i.e. 7 instances. Finally, we note again that imposing higher values of β does not guarantee a better quality of solutions, see for instance results for *dsjc125.5* and *hamming6_2*.

5 CONCLUSIONS AND FUTURE WORK

The goal of this work is to analyze the performance of D-Wave solvers for solving the stable set problem in a graph. We therefore used the class of BQM problems, for which a QUBO model of the problem is used. Since a stable set is a subset of vertices which are pairwise not adjacent, a QUBO formulation for the stable set problem requires imposing certain penalties on the edges. We considered different levels of penalties and analyzed the quality of the solutions returned by QPU and hybrid solvers.

We note that a general drawback of using a QUBO formulation is that basically every binary vector is a feasible solution. Hence, the obtained solution may not be a stable set at all. This fact was observed for several solutions returned by the QPU solver. Also, it appears that

imposing higher penalties on the edges does not necessarily improve the quality of the solutions obtained with the QPU solver. Overall, we note that the QPU solver shows a rather modest performance, since the returned solutions are in some cases quite far from the optimum. On the other side, all results obtained by the hybrid solver were optimal.
In this work we focused on the instances which can be solved with both the QPU and the hybrid solver. Since the hybrid solver showed very good results, it would be interesting to investigate in detail how it performs on larger instances from the literature.

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Solving 100-Digit Challenge with Score 100 by Extended Running Time and Parallel Benchmarking

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Abstract: This paper assesses Distance-based Success History Differential Evolution for 100-Digit Challenge and Numerical Optimization Scenarios (DISHchain3e+12). DISHchain3e+12 algorithm is based on DISH algorithm, using a Differential Evolution (DE) parameters control mechanism based on Euclidean distance between trial and parent in a search space to weigh Lehmer means in scaling and crossover historical parameter memories. In this paper, the maximum number of fitness evaluations and initial population size parameters are tailored in the DISHchain3e+12 algorithm per problem, since the benchmark used allows tuning up to two of the same parameters independently for each problem.

Keywords: Differential Evolution, 100-digit challenge, DISHchain3e+12

1 Introduction

Operations research [4, 16, 19] is an important enabling component in digital transformation and Society 5.0 [3] and includes smart methods, like optimization algorithms that are improved upon in this paper. This paper hence assesses and reports an improvement in a population-based optimization algorithm for the 100-Digit Challenge numerical optimization competition case that was organized by P. N. Suganthan, Ken V. Price, Kalyanmoy Deb, Noor Awad, Mostafa Z Ali, Guohua Wu, Rammohan Mallipeddi, and Hui Li [5]. The optimization algorithm applied is the recent DISH algorithm, Distance-based Success History Differential Evolution [10], which was already previously tailored for the associated competition benchmark [12] and now further improved to the perfect score of 100 by extending running time, hence, named DISHchain3e+12. This paper also addresses of speeding up attaining perfect scores at 100-Digit Challenge through running in parallel, as this is an important opportunity to use with High-Performance Computing [13].

Related work is presented in the following Section. Section 3 presents the applied algorithm. Section 4 reports results as defined for the associated competition. Section 5 provides conclusions with suggestions for future work.

2 Related Work

In this section, the works from the applied population-based optimization algorithm are covered in the next subsection, then the 100-Digit Challenge is explained further in the second subsection.

2.1 Population-based Optimization

The base algorithm of DISHchain3e+12 algorithm is the DISH algorithm [10], which already includes the computational mechanisms, like those from the basic architecture of Differential Evolution (DE) [6] (population-based optimization, mutation, crossover, and selection over floating-point vectors), Success-History based Adaptive Differential Evolution (SHADE) algorithm [7] (parameter adaptation with external archive and historical control parameter memory), and its updates L-SHADE [8] (linear decrease in population size).

The basic DE [6] consists of an evolutionary loop, within which are evolved new population D-dimensional population vectors $\mathbf{x}_i$, $\forall i \in \{1, 2, ..., NP\}$. During each generation step number $g \in \{1, 2, ..., G\}$, on the population, computational operators are performed like mutation, crossover, and selection, until a termination criterion is satisfied, like a fixed number of maximum fitness evaluations (MAX_FES).

2.2 100-Digit Challenge

As motivated within the technical report [5], "research on single objective optimization algorithms often forms the foundation for more complex scenarios, such as niching algorithms and both multi-objective and constrained optimization algorithms."

As indicated further for [5], "Traditionally, single objective benchmark problems are also the first test for new evolutionary and swarm algorithms.", this can be demonstrated over the plethora of algorithms, including the recently widely applied SHADE algorithm [7].

The goal of benchmark [5] and hence this paper is to understand better "the behavior of swarm and evolutionary algorithms as single objective optimizers" [5] and, hence, this paper introduces the DISH algorithm to the 100-Digit Challenge.

Further motivated in [5], "The SIAM 100-Digit Challenge was developed in 2002 by Nick Trefethen in conjunction with the Society for Industrial and Applied Mathematics (SIAM) as a test for high-accuracy computing" [9, 1], specifically, "the challenge was to solve 10 hard problems to 10 digits of accuracy" and, in a similar vein, the 100-Digit Challenge is proposed in [5]. These 10 problems described in [5], are implemented as C and Matlab functions to be optimized with an optimization algorithm and the functions all have the optimum at 1, with from 9 upto 18 search parameters, and a continuous floating-point search range (see Table 1).

The name of the SIAM 100-Digit Challenge is due to, as [5]: "One point was awarded for each correct digit, making the maximum score 100". The new "100-Digit Challenge asks contestants to solve all ten problems with one algorithm, although limited control parameter tuning for each function is permitted to restore some of the original contest's flexibility',' and it defines that the "difference is that the score for a given function is the average number of correct digits in the best 25 out of 50 trials.".

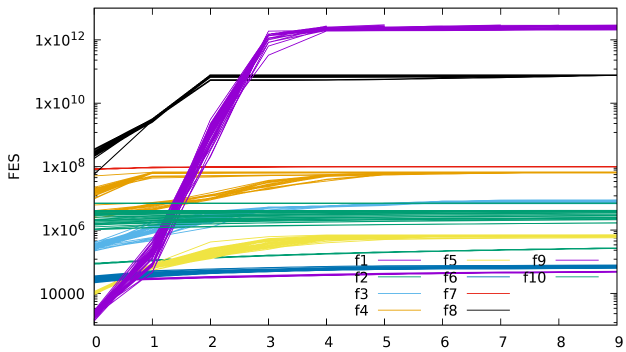

Figure 1: Function evaluations to reach accuracy up to certain digit (combined on all functions 1–10, with same line type per function), using logscale axis for FES.

3 DISHchain3e+12 Algorithm

This section reports the required components of a paper prepared for [5], which, mandates the paper to contain 5 components, respectively: 1) description of the algorithm, 2) listing dynamic ranges for tuned parameters, 3) providing guidelines on how to adjust the tuned parameters, 4) reporting the default values of other important parameters of the algorithm that were held constant (over different test problems), and 5) describing the termination criterion when a trial

Table 1: Fifty runs for each function sorted by the number of correct digits (for DISHchain3e+12 algorithm). $\mathbf{X}^*$ denotes an optimum (transformed to 1 for all functions).

					Num. correct digits											
No.	Problem name	$\mathbf{X}^*$	D	Search Range	0	1	2	3	4	5	6	7	8	9	10	Score
1	Storn's Chebyshev Polynomial Fitting Problem	1	9	[-8192,8192]	0	0	0	0	0	0	0	0	0	0	50	**10**
2	Inverse Hilbert Matrix Problem	1	16	[-16384,16384]	0	0	0	0	0	0	0	0	0	0	50	**10**
3	Lennard-Jones Minimum Energy Cluster	1	18	[-4,4]	0	0	0	0	0	0	0	0	0	0	50	**10**
4	Rastrigin's Function	1	10	[-100,100]	0	0	0	0	0	0	0	0	0	0	50	**10**
5	Griewangk's Function	1	10	[-100,100]	0	0	0	0	0	0	0	0	0	0	50	**10**
6	Weierstrass Function	1	10	[-100,100]	0	0	0	0	0	0	0	0	0	0	50	**10**
7	Modified Schwefel's Function	1	10	[-100,100]	0	0	0	0	0	0	0	0	0	0	50	**10**
8	Expanded Schaffer's F6 Function	1	10	[-100,100]	0	0	0	0	0	0	0	0	0	0	50	**10**
9	Happy Cat Function	1	10	[-100,100]	0	0	0	0	0	3	5	1	6	1	34	**10**
10	Ackley Function	1	10	[-100,100]	0	0	0	0	0	0	0	0	0	0	50	**10**
															Score (total):)	**100**

does not reach the 10-digit level of accuracy. These 5 are presented in the following subsections, respectively.

3.1 Description of the algorithm

The description of the DISHchain3e+12 algorithm is based on changes as it is reformed from DISH, jSO, and SHADE over different mechanisms and test functions. Namely, in this paper, the maximum number of fitness evaluations MAX_FES and initial population size NP_0 parameters are tailored per problem in the DISHchain3e+12 algorithm, as the competition allows tuning up to two of the same parameters independently for each problem. The tuned parameter MAX_FES is, hence, used for the 10 functions, as follows in the next section. The DISHchain3e+12 algorithm includes the termination criterion as described in the previous sentence and utilizes the 100-Digit Challenge benchmark [5]. The algorithm was benchmarked in parallel, which enabled to extend of the running time for each of individual runs in same wall time. Other than this, the algorithm is the same as the original DISH algorithm [10].

3.2 Listing dynamic ranges for tuned parameters

The dynamic ranges of the MAX_FES parameters are, within the initial population size, all to maximum countable integer value, i.e. in C++ for a 64-bit machine this is $MAX\_FES \in [NP_0, 2^{63} - 1]$; NP_0 is the initial population size. NP_0 ranges within $NP_{\min}$ and MAX_FES.

3.3 Providing guidelines on how to adjust the tuned parameters

The guidelines on tuning the MAX_FES parameter are motivated by [17, 19], describing the important real-time limitations in scenario re-planning over real cases and missions: The most important is the decision deadline, and the use of High-Performance Computing (HPC) is suggested to increase the speed of yielding results over full benchmarks limited by MAX_FES [14]. In the case to export the DISHchain3e+12 algorithm on other domains such as discrete optimization, the surveys like [15] also point out the suggestions for measuring the amounts of reflectivity of known information about a problem, which can many times be an important unallowed trick, as also pointed out in [5] as "No tricks (since you already know the answers). You may not enlist special knowledge. Algorithm performance must not depend on the function number. These problems should be treated as black-box problems. You may not reference the explicit equations of the functions in your scheme.". Regarding tuning of NP_0, the values in multiples of default value are suggested.

Table 2: Tuned parameter values for DISHchain3e+12 algorithm.

Function	*MAX_FES*	NP_0
1	1e+5	$25\sqrt{D}\log D$
2	1e+6	$25\sqrt{D}\log D$
3	1e+7	$25\sqrt{D}\log D$
4	1e+8	$250\sqrt{D}\log D$
5	1e+6	$25\sqrt{D}\log D$
6	1e+5	$25\sqrt{D}\log D$
7	1e+8	$2500\sqrt{D}\log D$
8	1e+11	$10000\sqrt{D}\log D$
9	3e+12	$25\sqrt{D}\log D$
10	1e+7	$25\sqrt{D}\log D$

3.4 Reporting the default values of other important parameters of the algorithm that were held constant (over different test problems)

The default values of other important parameters of the DISHchain3e+12 algorithm held constant over all problems, are same as in jSO [2]. These are: historical memory size $H = 5$, archive size $A = NP$, initial population size $NP_0 = 25\sqrt{D}\log D$ and minimum population size $NP_{\min} = 4$, for pBest mutation $p = 0.25$ and $p_{\min} = p\max/2$, with initialization of all but one memory values at $\mathbf{M}_{\mathrm{F}} = 0.5$ and $\mathbf{M}_{\mathrm{CR}} = 0.8$ and the one memory entry with $\mathbf{M}_{\mathrm{F}} = \mathbf{M}_{\mathrm{CR}} = 0.9$, and pBest-w strategy with weight value limits F_w at $0.7F$, $0.8F$, and $1.2F$ for one mutation using parameter F as defined in [2].

3.5 Describing the termination criterion when a trial does not reach the 10-digit level of accuracy

The termination criterion when a trial does not reach the 10-digit level of accuracy is the *MAX_FES*: The algorithm is run until the limit of *MAX_FES* is reached. Although the report [5] defines to "apply any method to any problem and take as long as needed to solve it", the report also defines the requirement that a paper is submitted, while "all participants are allowed to improve their algorithms further after submitting the initial version of their papers", with the CEC 2019 Timeline (`http://cec2019.org/#Dates`) defining "`Camera ready paper due: 31 March, 2019`", or associated SEMCCO 2019 conference [18]. Therefore, some details of the (data) mining mission are worthwhile to be discussed in the context of termination criterion. The mission can be re-planned in some real challenges for e.g. 20 minutes[19], but in this 100-Digit Challenge, this time-span is much longer.

4 Results

To conform to the Section 3.2 of presenting results for [5], the "Tables 3 and 4" from [5] are reported for this paper in this section, following the presentation of other required template content [5] in the previous section. The fifty runs for each function sorted by the Number of Correct Digits Table is provided as Table 1 and Tuned Parameter Values Table as Table 2. The algorithm has also been evaluated under 2019 100-Digit Challenge competition and these results were also submitted as a competition entry for SEMCCO 2019 (https://github.com/P-N-Suganthan/CEC2019/blob/master/Codes of DISHchain3e+12 paper.zip), under 100-Digit Challenge numerical optimization competition case, achieving the top rank (winner) scores. The associated codes were published at same GitHub directory, at https://github.com/P-N-Suganthan/CEC2019/blob/master/PaperCodes-DISHchain.zip, associated also with the paper for DISHchain1e+12 [12] that has been reported at GECCO 2019 as a winning entry among GECCO 2019 submissions for the associated competition.

4.1 Scores

As required in [5], Table 1 "lists for each function the number of trials in a run of 50 that found n correct digits, where $n = 1, 2, ..., 10$. In the final column is entered, the average number of correct digits in the best 25 runs, i.e. the score for that function. The total score is entered (the sum of the scores for all 10 functions) in the bottom right-hand cell." The reported DISHchain3e+12 algorithm score is, hence, 100. A combined plot shows all functions 1–10 together in Figure 1 with a logscale axis for FES to more distinctly highlight clusters of specific values across functions and the nearly linear correlations. The improvement in score from 96 (DISHchain1e+12 [12]) to 100 (DISHchain3e+12) was due to extending running time (FES) and hence deploying sufficient HPC resources provided a capability to solve this challenge.

5 Conclusions

This paper assessed the DISHchain3e+12 algorithm on 100-Digit Challenge on the Single Objective Numerical Optimization. The DISHchain3e+12 algorithm results were presented according to the specification of the benchmark. The currently reported obtained score for the DISHchain3e+12 algorithm is 100, which is the maximum value (100% out of 100).

In future work, the DISH-based algorithms can be further applied to other domains, like discrete optimization or real world applications like deep ocean underwater glider path planning [13]. Another interesting aspect of algorithm implementation on HPC systems is also evaluation of parallelization strategies over a full given time limit and distributed pipelines [11].

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Special Session 7:

Social Innovations in Ageing Studies Supported by OR Models

THE INFLUENCE OF RESIDENTIAL DISPERSION ON THE OPTIMAL LONG-TERM CARE OF SENIOR CITIZENS

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Abstract: Most seniors want to grow old in their homes or near the members of their social network. European Semesters on Long-Term Care (LTC) and the new Slovenian Act on LTC also support this idea. However, the spatial dispersion of housing units and the low population density in rural areas significantly impact logistical costs (transport of materials, labor, older people to day programs, etc.). Based on the age structure of residents in each building, we estimate the expected logistic costs in the structure of LTC costs in determined municipalities/regions. We then optimize the routing and scheduling of care needs by combining them with the depreciation costs of a new care unit for scattered elderly within the community. We simulated the routes of care workers based on the Multiple Travelling Salesman Problem (mTSP), where the solutions indicate the amount of work and time required to provide the services. We present the case for the local level.

Keywords: dispersed building, mTSP, goal programming, long-term care, LTC, social gerontology.

1 INTRODUCTION

Many older people in Slovenia, as well as in other EU member States, want to stay in their family home or close to their social networks. Bogataj et al. [3, 4] found that people aged 81 or older would only move to a community or nursing home if it were close enough (less than or equal to 5 km) to their family home. However, in the Slovenian NUTS 3 regions, the proportion of those living more than 5 km from the nearest nursing home is high (in eight out of twelve regions, more than 1/3 of older adults aged 65 or more live far away). For more than half of them, home-based care with more frequent visits by caregivers or community-based care close to their current residence should be provided now or soon.

In rural areas, the challenges are travel costs in case of home care [2, 14], spatial plans and financial sources for building facilities for those whose home environment poses a threat to life safety [2]. Also, the distances between LTC users reduce the frequency of visits and the quality of the LTC. This paper addresses how these challenges can be considered in the optimal choice of care networks and housing for long-term care.

2 METHOD

We start from the results of Bogataj et al. [3, 4], who studied the impact of dispersion (a measure of average distances traveled between two LTC users) on the required optimal number of care providers. We obtain the results by developing mTSP (multiple traveling salesman problem) simulations, where we considered an 8-hour constraint on the daily workload of caregivers. More sustainable approaches to long-term care are needed to provide adequate care and other affordable services, especially in remote rural areas. We showed that the needs category (service time) also influences the proportion of travel time. In the micro-level case study of the municipality of Krško, it averaged 22% (with a coefficient of variation of over 42%) if the caregivers' journeys to and from home were not considered.

Since we study housing for older adults in rural areas, where overcoming distances between users of home-based long-term care (homecare) is very important, accounting for a large part of the cost of care when clients remain in the home environment of their families and friends, we looked for works that address these challenges. This issue is also important because after moving into adapted housing for older adults, the social networks of those needing care are significantly disrupted. This phenomenon intensifies with the distance of the adapted social housing to which the older adults move from their parental homes.

We wish to compare the cost of travel with the depreciation of community facilities (sheltered housing units), from which the optimal structure of LTC in a community can be calculated. We study how long-term planning of social housing for older adults can be addressed when the distance between users' family homes and community care units (CCUs) in LTC plays a role.

3 MODEL AT THE MACRO LEVEL OF SLOVENIA

At the macro level of Slovenia, we focus on finding the lowest number of caregivers needed that affects the cost of care and compare it to the cost of constructing low-energy buildings (1500 €/m^2) and depreciating them in 80 years. Bogataj et al. [5] calculated the required number of caregivers for twelve NUTS 3 regions in Slovenia in the three years considered (2023, 2033 and 2043).

We adopt a limit of 430 minutes of daily workload and travel time between LTC users by assuming a 90% workload and the remaining 10% for contingencies. In previous research [1, 9], where the priority matrix was also considered, as in the algorithm of Patterson and Albracht, we obtained the data on travel time and standard deviation of time distances between users living in the rural areas of Slovenia (we excluded those living in the central towns of NUTS3 regions and municipalities), which affect the cost of human resources and their transport. For computing the average travel time between the houses of residents aged 65 or over, we took the travel speed from the micro-level of Krško municipality (42km/h) data. Using goal programming to achieve the required number of studios close to LTC users in each municipality and region, we compared the construction costs of 10 studio units with the travel costs of home care. With the average travel time in Slovenia between care users, we lose 22% of the labor costs for traveling the required number of caregivers. The numbers of caregivers by NUTS 3 regions in Slovenia who are lost for service because they need time for travel are in [5].

The numbers of caregivers lost to care due to their travel between LTC users in homecare in Slovenia at an average of 22% of travel time in a workload yearly in 2023, 2033, and 2043 are 258, 342, and 477, respectively. Let us assume that the gross salary of a caregiver is €1,500 per month or €18,000 per year (average in 2022 [8]). In this case, the annual personnel costs for the employer come to €21,000 per caregiver. Then the total costs for Slovenia amount to €5.5 million in 2023, €7.2 million in 2033, and €10 million in 2043. In between years, we can interpolate the annual costs.

At the cost of €1,500 per m^2 for constructing low-energy care units, we can build over 2350 studios with 18 m^2 per inhabitant in Slovenia in the first decade and over 3180 studios in the second decade. Accordingly, given the forecasts of new demand each year, also giving available plots we use goal programming for constructing community housing units with care allocated on these plots.

4 A NUMERICAL EXAMPLE AT THE MICRO-LEVEL OF KRŠKO MUNICIPALITY

At the local level of analysis, we considered the projected number of possible housings with care and homes to be visited in case of homecare in the municipality of Krško. We considered the locations of buildings where older adults (65 years or older) living alone and without family members could provide informal care, the existing nursing home (NH) in the town of Krško and five potential community care units in the municipality (CCUs). Only 52% of older adults live within 5 km of NH and five CCUs. Krško is one of the ten most dispersed municipalities in Slovenia. Let us assume that five plots of land with the appropriate locations are available to build a municipal care facility. These could be built in blocks of 10 units. The needs to be fully met in 2023, 2033 and 2043 are 104, 137 and 197 older adults living alone respectively. The goal programming procedure is described below (cf. [6], where different dynamics were implemented with a non-linear model).

Criterion f.: $\boldsymbol{min}\sum_{\boldsymbol{t}}\left\{KG(t)\cdot p(s)\cdot\sum_i\sum_{j\in\boldsymbol{B_{j(t)}}}c(l(i,j,t)\cdot x_{i,j})+\sum_{j\in B_{j(t)}}\nu_j {d_j}^+(t)+\mu_j h(t)\right\}$;

$$j\in\boldsymbol{B_j}(t);\quad r\in\boldsymbol{B_r}(t)\ ;\quad \boldsymbol{B_j}(t)\cup\boldsymbol{B_r}(t)=\boldsymbol{B}(t);\quad \boldsymbol{B_j}(t)\cap\boldsymbol{B_r}(t)=\emptyset, \tag{1}$$

where $\boldsymbol{B_j}(t)$ is a set of opened $CCUs$, $\boldsymbol{B}(\mathrm{t})$ is the total set of $CCUs$ in t; $d_j^+\in\{0,1\}$; $\boldsymbol{x_{i,j}}\in\{0,1\}$; $if\ \sum_{s=0}^{t}d_j^+(s)=1\ \rightarrow\ h(t)\geq 0$; if $\sum_{s=0}^{t}d_j^+(s)=0\ \rightarrow\ h(t)=0$; $\nu_j=10{,}000$; $\mu_j=1{,}000$; p(s) is probability of needing one of the facilities; and $KG(t)$ is data from geo-gerontological projections [5].

Here ν_j are the investment costs in a new CCU and μ_j the starting investments in blocks of 10 housing units (studios) each. The potential clients wish to move less than $a(l)$ far from the family home. If they must go further from home than the critical distance, each kilometer is weighted by a factor of M (M was chosen to be 5 and can be varied). The values of $a(k,t)$ are given in [4], showing the critical distances k [0.2; 0.34; 0.28; 0.18; 0.18]. Therefore,

$$\mathrm{c(l(i,j,t)}=\sum_k a(k,t)[l+M(l-k)_+]. \tag{2}$$

Constraints 1: Each potential user ($i=1,2,\ldots 614$) goes to exactly one available facility:

$$\sum_{j\in B_j}x_{i,j}=1\ ;\text{ for each } i;\ \ \boldsymbol{x_{i,j}}\in\{0,1\} \tag{3}$$

The probability of needing one of the facilities $p(s)$ is derived from the geo-gerontological projections [6]. It gives the expected need for 2023 (104) and 2033 (137), while the values for the in-between years are linearly interpolated.

Constraints 2: The annual depreciation of capital investments, investment maintenance, and material costs per year should not exceed travel costs in homecare.

Constraint 3: We may add modules with exactly ten studios to the base building each year if the CCU is already open, but only up to the plot constraints.

5 CONCLUSION

Table 1 shows that every year, the human resources cost arising from the caregiver trips between the LTC users is higher than the depreciation of new studios for the same LTC users living very close to each other (under one roof). Of course, it is vital to realize that in addition to depreciation, material and service costs may be higher in CCUs than in-home care. By contrast, there are also additional costs for in-home care due to mileage allowance or the use of a company car. By goal programming, we achieve or exceed all stepwise goals in the period (2023-2033).

Table 1: Dynamics of constructing the studios in blocks of 10 units, reducing travel (source: own calculations)

Year	Cumulative[1] of the effective demand	Reduction[2] of yearly travel [1000€]	Investment costs [1000€][3]	Depreciation [€][4]	Total costs per year of care [€][5]	CCU1	CCU2	CCU3	CCU4
						Cumulative number of constructed studios			
	A	B	C	D	E	F1	F2	F3	F4
2023	104	63,000	2970	49,500	711,750	50	30	30	0
2025	111	67,240	270	54,000	759,656	50	30	30	10
2027	118	71,481	0	54,000	807,563	50	30	30	10
2029	125	75,721	270	58,500	855,469	60	30	30	10
2031	131	79,356	0	58,500	896,531	60	30	30	10
2033	137	82,990	810	72,000	937,594	70	30	30	30

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1 Goal in goal programming. In the case of two visits per day, there are 208 visits per 104 patients, and according to the norms there are 104/7.5=14 caregivers.

2 The travel time is equal to workload of 14*0.22= 3 caregivers. Their yearly salary would be €63,000.

3 $18\text{m}^2 * €1{,}500 * \sum_i [F_i(t+1) - F_i(t)]$; $F_i(2022) = 0$

4 A 60-year depreciation is taken into account.

5 Emphasis from analyses of institutional care for the older adults and special groups of adults in 2021 from [7], without food or medical supplies. Multiplied by A.

GENERATIONAL DISTINCTIONS IN MIGRATIONS BETWEEN SLOVENIAN MUNICIPALITIES

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Abstract: The age structure in municipalities and regions is changing and the number of older adults is increasing due to ageing and migration. The age structure of future citizens in a municipality may change not only because of the ageing of the overall population, but also because different factors affect migrants of different ages differently (Drobne and Bogataj, 2022). But there could also be a dynamic of these influences. Therefore, in this study, we compared inter-municipal internal migration data in Slovenia for 2020/2021 with inter-municipal migration data from 2018/2019 to improve a cohort-based modelling approach for spatial interactions that can be used to estimate future migration between municipalities in the country. The modelling approach addresses the dynamics of differences in attractiveness and retentivity between municipalities and seeks to answer the question of how to support decisions about investing in age-friendly communities.

Keywords: spatial interactions, gravity model, age friendly environment, intergenerational distinctions, built environment, social gerontology.

1 INTRODUCTION

In order to keep residents in the community, we need to know what factors influence migration between communities and what generations value in a local environment as a suitable place to live. Drobne and Bogataj [5] and also Mizuno and Fujimoto [6] have shown that the attractiveness of communities is influenced by several factors and that these differ between generations. The question is whether these factors are constantly strong or whether their strength changes over time, even over a shorter period of time. We try to answer this question in this article by using a gravity model with relative variable values.

2 SPATIAL INTERACTION MODEL OF RELATIVE VARIABLES

In order to estimate the influence of the analysed factors and their changes between 2019 and 2021, the spatial interaction model was introduced:

$$M_{ij}^{(y,c)} = k\,K(d_{ij})^{\beta} \prod_r K(r)_i^{\gamma(r)}\,K(r)_j^{\alpha(r)}, \tag{1}$$

where $M_{ij}^{(c,y)}$ is the estimated intensity of migration flows of age cohort c for year y from a municipality of origin i to a municipality of destination j; age cohorts were defined $c =$ 0–65, 66–74, 75+; analysis was conducted for years 2018/2019 [4,5] and 2020/2021, $y =$ 2019, 2021; k is the proportionality constant, $K(d_{ij})$ is the coefficient of the fastest time-spending distance on the state road network between the centre of origin municipality i and the centre of destination municipality j; $K(r)_i$ and $K(r)_j$ are the coefficients of the factors r in origin municipality i and destination municipality j, defined as the value of the factor in municipality i and municipality j, respectively, divided by the average value of this factor in Slovenia. Taking into account the factors analysed in the NSIM (Normalised Spatial Interaction Model) for three cohorts of migrants, model (1) can be written as follows:

$$\begin{aligned} M_{ij}^{(y,c)} = k\,K\left(d_{ij}\right)^{\beta} & K(POP)_i^{\gamma(POP)}\,K(POP)_j^{\alpha(POP)}\,K(GUEMP)_i^{\gamma(UEMP)}\,K(UEMP)_j^{\alpha(UEMP)} \\ & \cdot K(GEAR)_i^{\gamma(GEAR)}\,K(GEAR)_j^{\alpha(GEAR)}\,K(NDWE)_i^{\gamma(NDWE)}\,K(NDWE)_j^{\alpha(NDWE)} \\ & \cdot K(PDM2)_i^{\gamma(PDM2)}\,K(PDM2)_j^{\alpha(PDM2)}\,K(MREV)_i^{\gamma(MREV)}\,K(MREV)_j^{\alpha(MREV)} \\ & \cdot K(AGEI)_i^{\gamma(AGEI)}\,K(AGEI)_j^{\alpha(AGEI)}\,K(HELD)_i^{\gamma(HELD)}\,K(HELD)_j^{\alpha(HELD)} \end{aligned} \tag{2}$$

Table 1: Factors in NSIM (1) and (2).

Notation	Factor value	Additional description	Sources
$M_{ij}^{\prime(c,y)}$	Number of migrants in age cohort c for year y from municipality of origin i to municipality of destination j	Average of yearly values for 2018/2019 respectively 2020/2021	SORS and authors' calculation
$M_{ij}^{(y,c)}$	Estimation of the number of migrants in age cohort c for year y from municipality i to destination j	The estimation of the real value regarding model (1) or (2)	Authors' calculation
$K(d_{ij})$	Coefficient of the fastest time-spending distance between municipal centre of origin i and destination j	The ratio between the value for a pair of municipal centres and the average for Slovenia for 2019 respectively 2021	SIR and authors' calculation
$K(POP_{\circ})$	Coefficient of the number of inhabitants in the municipality	The ratio between the factor value for the municipality and the average value for Slovenia for 2019 respectively 2021	SORS and authors' calculation
$K(UEMP_{\circ})$	Coefficient of registered unemployment rate in the municipality	The ratio between the factor value for the municipality and the average for Slovenia for 2019 respectively 2021	SORS and authors' calculation
$K(GEAR_{\circ})$	Coefficient of gross earning per capita in the municipality	The ratio between the factor value for the municipality and the average for Slovenia for 2019 respectively 2021	SORS and authors' calculation
$K(NDWE_{\circ})$	Coefficient of number of dwellings per 1000 inhabitants in the municipality	The ratio between the factor value for the municipality and the average for Slovenia for 2018 respectively 2021	SORS and authors' calculation
$K(PDM2_{\circ})$	Coefficient of average price per m^2 of dwelling in the municipality	The ratio between the factor value for the municipality and the average factor value for Slovenia for 2018/2019 respectively 2020/2021	SMARS and authors' calculation
$K(MREV_{\circ})$	Coefficient of municipal revenue per capita	The ratio between the factor value for the municipality and the average factor value for Slovenia for 2019 respectively 2021	MFRS and authors' calculation
$K(AGEI_{\circ})$	Coefficient of ageing index in municipality	The ratio between the factor value for the municipality and the average factor value for Slovenia for 2019 respectively 2021	SORS and authors' calculation
$K(HELD_{\circ})$	Coefficient of capacity of care homes in the municipality	The ratio between the factor value for the municipality and the average factor value for Slovenia for 2019 respectively 2021	Breznik et all. [3], Bogataj et al. [7] and authors' calculation

Notes: SORS - Statistical Office of the Republic of Slovenia (https://pxweb.stat.si/SiStat/en), SIR - Slovenian Infrastructure Agency (https://www.gov.si/en/state-authorities/bodies-within-ministries/slovenian-infrastructure-agency/), SMARS - Surveying and Mapping Authority of the Republic of Slovenia (https://www.gov.si/en/state-authorities/bodies-within-ministries/surveying-and-mapping-authority/), MFRS - Ministry of Finance of the Republic of Slovenia (https://www.gov.si/en/state-authorities/ministries/ministry-of-finance/).

Model (2) was linearized and solved by IBM SPSS using Ordinary Least Squares (OLS) regression analysis for 2019 and 2021 and the significant values of $\alpha, \beta,$ and γ were compared.

3 RESULTS

In Table 2, the regression coefficients of the linearized model (2) are given according to the age cohorts of 0–65, 66–74, and 75+ year-olds and for years 2019 and 2021 (the values of the regression coefficients where the p-value is higher than 0.05 are in parentheses).

4 DISCUSSIONS

- For the generations up to 74 years, **distance** has an increasingly weaker influence on the choice of a longer residence, while the oldest residents increasingly want to stay close to their former residence, which is also evident in the survey by Rogelj et al. [7], where older adults fear the loss of their social networks.

Table 2: Statistics of standardized coefficients $\alpha, \beta,$ and γ for cohorts 0–65, 66–74, and 75+ year-olds (inter-municipal migration in Slovenia in 2019 respectively 2021; ANOVA p-value <0.0001 for all).

Age cohort	**0–65 years**		**66–74 years**		**75+ years**	
Year	**2019**	**2021**	**2019**	**2021**	**2019**	**2021**
Adjusted R^2	0.617	0.628	0.354	0.426	0.383	0.414
SE	0.776	0.771	0.531	0.560	0.666	0.673
No. of observations	14,096	16,292	1920	3401	2481	2914
ANOVA stat. F	1336.89	1622.22	62,84	149,31	91,46	122,04
β	-.649	-.641	-.475	-.456	-.560	-.572
$\gamma(POP)$	.464	.476	.401	.460	.445	.450
$\alpha(POP)$	.448	.437	.181	.249	(.030)	.104
$\gamma(UEMP)$	.016	-.016	(.001)	(.015)	-.054	(-.014)
$\alpha(UEMP)$	.025	(.003)	.056	(-.002)	(.022)	(-.027)
$\gamma(GEAR)$	.057	.043	(.011)	.088	(.003)	.064
$\alpha(GEAR)$	.043	.041	(.026)	(.015)	(.036)	(-.006)
$\gamma(NDEW)$	.130	.135	.122	.156	.068	.107
$\alpha(NDEW)$	.147	.177	.128	.215	.060	.090
$\gamma(PDM2)$	-.027	(.004)	(.012)	.060	(-.027)	(-.015)
$\alpha(PDM2)$	(-.007)	.021	(.049)	.048	(-.003)	(-.018)
$\gamma(MREV)$	.037	.035	.054	(.018)	.074	(.033)
$\alpha(MREV)$	.016	.022	(-.002)	(.028)	(.020)	(.026)
$\gamma(AGEI)$	.029	.025	(.023)	(.034)	.086	(.039)
$\alpha(AGEI)$	(.015)	.029	(.017)	(-.011)	(.038)	(.017)
$\gamma(HELD)$	(.015)	(.010)	(.005)	(-.044)	-.060	-.069
$\alpha(HELD)$	.018	.019	.131	.095	.282	.290

Note: The values of the regression coefficient where p-value > 0.05 are in parentheses.

- The larger size of the municipal **population** has the effect of lower retention power for all generations. This effect is higher in 2021 than in 2019. The same is true for destination attractiveness, although the effect of size on attractiveness is weaker than the effect on retention. This is especially true for the oldest generations, for whom the effect was not significant at all in 2019.
- **Unemployment** at the origin of migration flows increased migration only among the younger population and only in 2018/2019, later this influence was no longer significant or even had the opposite effect. So, it looks like we will not be able to retain workers only with the increasing employment opportunities, while the older ones are understandably not interested. Even the higher unemployment in the destination municipality in 2018/2019 had only a small influence on the increase in immigration of the younger generations, while this influence is no longer significant from the beginning of 2020/2021.
- A higher coefficient of **gross earning per capita** in the municipality of origin has a weaker effect on out-migration of younger generations than two years earlier. In 2020/2021, they proved to be significant for the 66+ generations, while no effect was observed for the older generations before. The effect of higher wages in the destination municipality on more intensive immigration is weaker than in the municipality of origin, which surprises us. This effect is not observed for the 66+ generations, neither before nor now, which is understandable since these people are retired.
- In 2018/2019, higher **dwelling prices** in the municipality of origin had a retentive effect and influenced the out-migration of the cohort up to 65 years from the municipality; today, however, higher prices attract two cohorts under 75 years, while older residents do not respond to dwelling prices.
- Higher coefficients of **municipal revenue per capita** have a stronger impact on the origin than on the destination of migration flows, especially when considering the youngest cohorts.

- The **age index** has a small impact on out-migration of the youngest age cohort, but the impact on the oldest group, which was positive in 2018/19, is no longer significant in 2021, but has a positive impact on in-migration of the younger cohorts today.
- The impact of relatively available **care homes** in the destination community increases in-migration and also reduces out-migration for the oldest group of citizens. The impact of in-migration was slightly decreasing in the 66-74 age cohort, but other changes are not significant.

5 CONCLUSIONS

For the older generations that are the focus of our study, we can see that (a) they are less and less likely to want to move far from their old homes, as Bogataj et al. [1,2] have already shown; (b) interestingly, their migration is increasingly influenced by the greater availability of housing in the municipality of origin than in the municipality of destination; (c) the importance of municipal revenues is becoming less significant or even unimportant for migration; (d) the ageing index, which two years ago was slightly significant in the municipality of origin and pushed the oldest cohort to other municipalities, has now become insignificant; (e) the existence of care homes for elderly citizens has the effect of preventing them from moving out of the previous municipality and attracting all generations at the destination to migrate to the municipality that has more care home beds available. Building a new adapted homes for the elderly population will attract all generations to such communities. In general, the age-friendly environment will keep communities from shrinking.

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THE IMPACT OF HIERARCHICAL SPATIAL LEVELS ON INTERNAL MIGRATION BY AGE COHORTS IN SLOVENIA

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Abstract: The shrinking and ageing of individual communities is also influenced by internal and external migration. In this article, we examine how individual factors of population retention or attraction in different age groups influence migration to other regions at the NUTS 3 and NUTS 2 levels in Slovenia. Distance has the least influence for the 65-74 generation at all spatial levels and that higher wages attract only the under-65 cohort. The coefficient for the capacity of care homes in the municipality has a significant influence on the attractiveness for the oldest cohort (75 years or more) to move to another region, while younger cohorts (under 65 and 65-74) are influenced by the number of dwellings served by the care provider – mainly at the NUTS 3 level. However, the rank correlation is highest between the municipality and NUTS 3 level factors for the population aged 75 years and above. There is also a high correlation between the strength of the NUTS 2 and NUTS 3 level factors for residents under 65.

Keywords: migration, older adults, age cohorts, gravity model, hierarchical spatial levels, gerontology.

1 INTRODUCTION

The challenges posed by the ageing of the European population and the related issues of migration of older citizens can be traced, especially in the last decade, when we began to recognise the fact that as populations age, their preferred environments change. Recently, Drobne and Bogataj analysed the differences in community attractiveness and stickiness for different cohorts [2] or the differences in these effects between two years [3]. However, there are only three articles in the Web of Science Core Collection so far that deal with the migration of the oldest citizens if we select the topics "gravity model", "older adults" and "migration", namely: [1, 4, 5]. Therefore, we studied the impact of various factors on the migration of citizens from Slovenian municipalities to other regions at two different hierarchical spatial levels, namely at the level of statistical (NUTS 3) and cohesion regions (NUTS 2).

2 METHODOLOGY

The influence of the analysed factors on Slovenian internal intermunicipal migrants by different age cohorts and at different spatial levels for Slovenia was analysed using the normalised spatial interaction model (NSIM, [2]):

$$M_{ij}^{(c,s)} = k\, K(d_{ij})^{\beta} \prod_r K(r)_i^{\gamma(r)}\, K(r)_j^{\alpha(r)}, \qquad (1)$$

where $M_{ij}^{(c,s)}$ is the estimated intensity of migration flows of age cohort c at spatial level s from a municipality of origin i to a municipality of destination j; age cohorts were defined $c =$ 0–65, 66–74, 75+; analyses was conducted for three spatial levels, $s =$ municipal level, NUTS 3 level, NUTS2 level; $K(r)_i$ and $K(r)_j$ are coefficients of factors r in origin i or destination j, defined as the value of factor in municipality i and municipality j, respectively, divided by the average value of this factor in Slovenia; $K(d_{ij})$ is the coefficient of the shortest distance by state road network between the centre of origin municipality i and the centre of destination municipality j; $K(POP_\circ)$ is the coefficient of the number of inhabitants; $K(UEMP_\circ)$ is the coefficient of registered unemployment rate; $K(GEAR_\circ)$ is the coefficient of gross earning per capita; $K(NDWE_{\circ})$ is the coefficient of number of dwellings per 1000 inhabitants; $K(PDM2_{\circ})$ is the coefficient of average price per m^2 of dwelling; $K(MREV_{\circ})$ is the coefficient of municipal revenue per capita; $K(AGEI_{\circ})$ is the coefficient of ageing index; $K(HELD_{\circ})$ is the coefficient of capacity of care homes; $K(SR_{\circ})$ is the coefficient of number of single rooms in care homes; $K(TAP_{\circ})$ is the coefficient of number of temporary accommodation places; $K(DCP_{\circ})$ is the coefficient of number of day-care places; $K(SDWE_{\circ})$ is the coefficient of number of dwellings serviced by the care provider; $K(COVCH_{\circ})$ is the coefficient of the index of coverage of the care needs of elderly people in care homes; and k is the constant of proportionality. The coefficients $K(HELD_{\circ})$, $K(SR_{\circ})$, $K(TAP_{\circ})$, $K(DCP_{\circ})$, $K(SDWE_{\circ})$ and $K(COVCH_{\circ})$ are understood as the social infrastructure coefficients for long-term care for older population.

Data for the analyses come from the Statistical Office of the Republic of Slovenia (data on internal inter-municipal migrants by age cohorts for 2020/2021, data on the municipality population for 2021, data on registered unemployment rate in the municipality for 2021, data on number of dwellings per 1000 inhabitants in the municipality for 2021 and data on ageing index in the municipality for 2021), Slovenian Infrastructure Agency (data on state roads for 2021), Surveying and Mapping Authority of the Republic of Slovenia (data on hierarchical spatial units for 2021 and data on average price per m^2 of dwelling in the municipality for 2020/2021), Ministry of Finance of the Republic of Slovenia (data on municipal revenues for 2021), and the Community of Slovenian Social Institutions (data on the capacity of care homes in the municipality, data on the number of single rooms in care homes, data on the number of temporary accommodation places, data on the number of day-care places, data on the number of dwellings serviced by the care provider and data on the index of coverage of the care needs of elderly people in care homes, all these data for the end of 2021).

At the NUTS 2 level, we analysed migration flows between municipalities of 2 different NUTS 2 regions, at the NUTS 3 level, we analysed inter-municipal migration flows between 12 NUTS 3 regions but in the same NUTS 2 region; and at the municipal level, we analysed migration flows between 212 municipalities in the same NUTS 3 region. We analysed migration flows between municipalities in Slovenia as an average for 2020/2021. Model (1) was linearised and solved using IBM SPSS using OLS regression analysis for three age cohorts and three spatial levels, and the significant values of α, β and γ were compared. We also summarised the powers of an analysed factor in immigration and emigration and calculated ranks of sums of the powers. The degree of similarity between pairs of rankings was analysed with the use of the Pearson correlation coefficient.

3 RESULTS

In Table 1, the regression coefficients of the linearised model (1) are given as a result of the NSIM calibration according to three different hierarchical spatial levels and according to three age cohorts.

4 DISCUSSIONS

The size of the standardised coefficients in Table 1 reflects the magnitude of the factor's impact on migration. We see that the 65-74 age cohort is least affected by distance at all spatial levels, that only the under-65 cohort is significantly attracted to regions with higher wages, that care homes significantly attract the oldest cohort (75 years or more), and that the number of sheltered housing units at the NUTS 3 level mainly affects the younger cohorts (under 65 and 65-74).

To better illustrate the relationships between factors influencing different cohorts for different hierarchical levels between origin and destination of migration flows, we summarised the power of a factor in immigration and emigration. The ranks of the sum of the powers are shown in Table 2. The correlation coefficient was calculated, measuring the degree of similarity between two rankings (two columns in Table 2). The results are available in Table 3.

We can find significant correlation in ranking of factors between population 66–74 ages and 75+ on the municipal and NUTS 3 level. There is also significantly similar ranking between municipal and NUTS 3 level migrations for cohort 75+ and between NUTS 3 and NUTS 2 level migration for cohort 0–65 years old citizens.

Table 1: Statistics of standardised coefficients α, β and γ for cohorts 0–65, 66–74, and 75+ year-olds at municipal, NUTS 3 and NUTS 2 hierarchical spatial levels (internal inter-municipal migration in Slovenia in 2020/2021; ANOVA p-value <0.001 for all).

Spatial level	**Municipal level**			**NUTS 3 level**			**NUTS 2 level**		
Age cohort	**0–65**	**66–74**	**75+**	**0–65**	**66–74**	**75+**	**0–65**	**66–74**	**75+**
R	0.881	0.734	0.727	0.751	0.645	0.618	0.779	0.660	0.618
R^2	0.776	0.539	0.528	0.563	0.416	0.382	0.607	0.436	0.382
Adjusted R^2	0.775	0.528	0.518	0.561	0.402	0.361	0.605	0.422	0.362
SE	0.691	0.603	0.693	0.749	0.562	0.556	0.707	0.545	0.555
No. of obs.	3,697	1,154	1,255	6,424	1,153	833	6,171	1,094	826
ANOVA stat. F	471.249	48.782	50.904	305.553	29.667	18.393	351.654	30.545	18.300
β	-.575	-.371	-.452	-.378	-.206	-.374	-.338	-.310	-.402
$\gamma(POP)$	.473	.595	.571	.446	.464	.398	.583	.536	.586
$\alpha(POP)$	.422	.301	.194	.434	.258	.264	.563	.343	(.167)
$\gamma(UEMP)$	-.025	(.034)	(-.048)	(-.007)	(.038)	(-.032)	(.003)	(-.011)	(.074)
$\alpha(UEMP)$	(-.015)	(-.017)	(-.053)	(.006)	(.001)	(-.016)	.024	(.017)	(.040)
$\gamma(GEAR)$	.018	(.038)	.048	.043	.106	.078	.054	.122	(.060)
$\alpha(GEAR)$	.017	(.006)	(.003)	.027	(.021)	(.024)	.071	(.027)	(.033)
$\gamma(NDEW)$	.097	.170	.092	.105	.146	.124	.144	.108	(.096)
$\alpha(NDEW)$	.154	.176	.103	.152	.227	.130	.189	.214	(.030)
$\gamma(PDM2)$	-.038	(-.034)	-.064	.031	.076	(.026)	.024	(.007)	-.133
$\alpha(PDM2)$	(-.014)	(.029)	-.078	.065	.097	(.053)	.027	(-.006)	-.109
$\gamma(MREV)$	.068	.052	.072	(.002)	(-.038)	(.010)	(-.003)	(-.034)	(-.011)
$\alpha(MREV)$	.023	(.039)	(.005)	(-.001)	(-.016)	.063	(-.012)	(-.004)	(.031)
$\gamma(AGEI)$	(-.002)	(.026)	.059	(.014)	(.057)	.087	(.013)	.104	(.020)
$\alpha(AGEI)$	(-.016)	(-.009)	(-.015)	.032	(.024)	(.075)	(.020)	(-.009)	(.045)
$\gamma(HELD)$	(.015)	(-.069)	(-.064)	(-.027)	(-.069)	(-.070)	-.079	(.011)	(.131)
$\alpha(HELD)$	(.042)	.178	.588	(-.052)	(-.049)	.165	-.051	(.077)	.291
$\gamma(SR)$	(-.019)	(.036)	(.024)	(.042)	(.099)	(.035)	.063	(-.003)	(-.118)
$\alpha(SR)$	(-.032)	(.002)	-.156	.075	(.124)	(-.031)	.061	(-.036)	(-.078)
$\gamma(TAP)$	.055	(.007)	(.020)	.020	-.123	(-.060)	-.032	-.078	(-.009)
$\alpha(TAP)$	.035	(.010)	(.013)	.023	-.113	-.106	-.032	(-.023)	(-.031)
$\gamma(DCP)$	(.021)	(.001)	(.002)	(.014)	(-.044)	(.014)	(.015)	(-.052)	(-.098)
$\alpha(DCP)$	(.025)	(.032)	(-.034)	(.000)	(-.050)	(-.082)	(-.020)	(-.063)	(-.064)
$\gamma(SDWE)$	.021	.060	(.035)	.099	.058	.104	.070	(.065)	(.060)
$\alpha(SDWE)$	(.013)	(-.007)	(.022)	.090	.070	(.033)	.071	(.039)	(.035)
$\gamma(COVCH)$	.038	-.066	-.081	-.062	-.130	(-.073)	-.069	-.104	-.115
$\alpha(COVCH)$	.050	(-.003)	(-.022)	-.072	(-.043)	(-.024)	-.082	(-.016)	(-.030)

Note: The values of the regression coefficient where p-value > 0.05 are in parentheses.

Table 2: A rank of sum of powers of factors (in-flows and out-flows) for internal inter-municipal migration in Slovenia in 2020/2021 for different hierarchical spatial levels and different cohorts.

Spatial level	**Municipal level**			**NUTS 3 level**			**NUTS 2 level**		
Age cohort	**0–65**	**66–74**	**75+**	**0–65**	**66–74**	**75+**	**0–65**	**66–74**	**75+**
d_{ij}	1	1	1	1	3	1	1	1	1
POP	14	14	14	14	14	14	14	14	14
$UEMP$	3	5	8	4	6	6	8	6	5
$GEAR$	9	3	9	10	10	10	11	11	7
$NDEW$	13	13	12	13	13	13	13	13	3
$PDM2$	2	7	3	11	12	3	9	7	2
$MREV$	12	10	11	5	7	9	7	8	9
$AGEI$	4	8	10	7	5	8	5	12	8
$HELD$	4	12	13	6	7	12	3	4	13
SR	4	4	2	8	2	4	10	8	10
TAP	11	9	6	9	1	2	4	5	10
DCP	4	6	7	3	7	5	6	8	10
$SDWE$	8	11	5	12	11	11	12	2	6
$COVCH$	10	2	4	2	4	6	2	3	4

Table 3: The rank correlation of intensity of factors influencing migration between cohorts at the same spatial level (upper part) and the rank correlation of intensity of factors influencing migration of the same cohorts between two spatially designed level (lower part).

0–65 and 66–74			**66–74 and 75+**			**0–65 and 75+**		
Municipal level	NUTS 3 level	NUTS 2 level	Municipal level	NUTS 3 level	NUTS 2 level	Municipal level	NUTS 3 level	NUTS 2 level
0.53	0.69	0.62	0.73	0.70	0.32	0.53	0.55	0.13
Municipal level			**NUTS 3 level**			**NUTS 2 level**		
0–65	66–74	75+	0–65	66–74	75+	0–65	66–74	75+
0.49	0.57	0.83	0.83	0.50	0.39	0.44	0.40	0.56

4 CONCLUSIONS

In this study, we investigated the influence of various factors on the migration patterns of three different age cohorts among Slovenian municipalities and regions at two regional spatial levels. Our findings contribute to the understanding of migration dynamics and can inform policy and planning efforts to meet the needs of different age groups.

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THE APPLICATION OF AGE MANAGEMENT PRACTICES IN ORGANISATIONS: DOES THE ECONOMIC SECTOR, SIZE, AND FAMILY BUSINESS STATUS MATTER?

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Abstract: The ageing of the population has become an issue in many countries. It influences many activities. One of them is the human resource management of organisations. Utilising age management practices, the organisation tries to deal with this issue. Within the framework of age management, different practices can be selected to maintain employee performance. The aim of the paper is to assess whether the application of age management practices differs depending on the economic sector, the size of the organisation, and the family business status. By means of a statistical test, differences in some areas were statistically confirmed.

Keywords: age management, age management practices, economic sector, size, family business

1 INTRODUCTION

The ageing of the population makes changes in an organisational environment, see [5], [8]. At the same time, the behaviour of employees, particularly among members of the so-called "generations Y and Z", is changing. Organisations need to take actions to prolong the working careers of employees and to harness the potential of all employee groups in the company to achieve employment satisfaction. An approach that incorporates diverse employee requirements across different age groups to optimise their contributions to the organisation is termed age management and is part of diversity management.

In age management, age-related factors should be considered in human resource management, including work arrangements and individual work tasks, so that everyone, regardless of age, feels empowered to achieve their own and corporate goals [6]. Initially, age management was mainly associated with older workers, but today it is considered a tool to exploit the potential of workers of all ages. The main goal is to maintain the long-term work ability of each individual, from which benefits not only the employee but also the organisation and the entire society.

At the organisational level, age management is important for retaining experienced and highly qualified employees, responding to organisational changes, and promoting socially responsible behaviour. Human resources managers can implement practices that improve the organisation's image, promote a positive internal climate, and support employee development [4]. This leads to better relationships with employees, greater satisfaction and loyalty, and a desire for career growth. Satisfied and healthy employees contribute to the organisation's prosperity through work productivity and efficiency, as well as through reduced work disability and presenteeism.

At the societal level, the application of age management practices is crucial for addressing macroeconomic issues related to labour market policy. Demographic developments require the use of pension and social policy instruments to maximise the contribution of ageing workers to economic and social development [3]. The public interest in the retention and development of ageing workers is aligned with the benefits to economic and social development. Age management practices support older workers to remain active in the labour market, resulting in increased productivity and economic growth, as well as social benefits.

The aim of the paper is to determine whether the economic sector, the size of the organisation, or the family business status affect age management practices.

2 METHODOLOGY

The research was carried out in March 2023 to determine what age management practices are being used in organisations. A questionnaire consisted of 20 questions on age management application, and it was intended to identify if age management is or isn't used in organisations, how long it has been applied, what would force organisations to address this, which specific practices are used, and which benefits are observed. The questionnaire contained both closed and open questions. Respondents could select the practices applied and the possible benefits observed from the options offered. These options were defined based on the available literature [9], [1], [2], [3], and our previous research [7]. Respondents were also able to add their own answers. As part of the identification questions, organisations filled in which sector of the economy they belong to, how many employees they have, and if they are the family businesses. Czech organisations across sectors were contacted. The selection was performed by random sampling from a database[1]. In total, 1,300 organisations were contacted, and 192 completed the questionnaire. After verifying the data, 189 correctly completed questionnaires were received.

From the responses, we identified the most commonly applied practices by the organisations and analysed whether the applied practices differed depending on the economic sector, the size of the enterprise, or the family business status. For each practice, a Chi-square test of independence of variables in a contingency table with Yates' correction for continuity [11] was performed to determine whether any of the factors were likely to affect its application.

3 RESULTS

Findings on if the applied practices differ depending on the economic sector, the size of the organisation, and the family business status will be presented. A total of 189 organisations participated in the survey, 5 of them from the primary sector, 123 from the secondary sector, and 61 from the tertiary sector. In terms of organisation size, 71 respondents belong to small organisations, 78 respondents belong to medium organisations, and 40 respondents belong to large organisations. In terms of family business status, 100 organisations are family businesses, and 89 organisations are not.

Results are presented in Table 1. First of all, it can be seen which practices are most commonly applied by the organisations. Subsequently, it is examined whether the economic sector, size of the organisation, and family business status have an impact. Statistical significance was determined at 10%, 5%, and 1% significance levels. From the results, the factors of economic sector and size influence the application of age management practices more often than family business status.

[1] The database of the organisations was obtained from https://www.databazefiremcr.cz/.

Table 1: Applied age management practices and influencing factors

Age management practices	***Freq.***	***Factors***		
		Economy sector	*Organisational size*	*Family bus. status*
Adaptation of work programs to suit different age groups	113	More often applied in the secondary sector*	Not significant	Not significant
Reassignment of staff to the more suitable position	104	More often applied in the secondary sector*	Not significant	Not significant
Care for employee health, promotion of physical fitness, and healthy eating	97	More often applied in the tertiary sector**	Increases with the size of the organisation***	More often applied in family businesses*
Support for personal and career development through skills training at every stage of an employee's career	93	More often applied in the tertiary sector**	Increases with the size of the organisation***	Not significant
Development of health and safety practices	63	Not significant	Increases with the size of the organisation***	Not significant
Workforce planning for age diversity and promotion of a positive age policy	55	Not significant	Not significant	Not significant
Creating tools to support intergenerational learning	29	More often applied in the tertiary sector*	Increases with the size of the organisation***	More often applied in non-family businesses**
Motivational programmes according to the needs of different generations	27	Not significant	Decreases with the size of the organisation ***	Not significant
Tailored further training for older workers	18	Not significant	Not significant	Not significant
Use of mentoring and reverse mentoring	11	More often applied in the secondary sector***	More often applied in small and large organisations**	Not significant
Special forms of recruitment for different age groups	8	Not significant	Not significant	Not significant

* 10% level of significance, ** 5% level of significance, *** 1% level of significance

4 DISCUSSION

It can be concluded that factors such as economic sector, organization size, and family business status influence the application of age management practices. For the most frequently applied practices, the influence of factors is low. The economic sector and the size of the organisation most often influence the application. In the literature review, there wasn’t found any prior research that examined the application of specific age management practices in relation to these or other factors. However, it was found [10], that the economic sector and the organization size do not influence whether the organisations apply an age management strategy or not.

In the context of further research, it would be appropriate to focus in detail on individual factors and their impact on the implementation of specific age management practices. Furthermore, it is possible to identify whether these factors influence the benefits observed by organisations applying age management. The reasons for their influence can also be examined. It is possible to look at the links between practices and observed benefits both overall and depending on sector, firm size, or family business status.

5 CONCLUSION

Age management is a set of organisational strategies and practices designed to optimise the performance contributions of all age groups of employees. The survey focused on the application of age management in organisations was conducted. The aim of the paper was to determine whether the sector of the economy, the size of the organisation, and the family business status play a role in determining age management practices.

The contribution of the paper could be seen in two ways. First, the paper presents practices that are used in age management. Second, it can serve as an inspiration for other organisations in choosing appropriate practices based on their economic sector, size, or status. At the same time, it provides a basic starting point for further in-depth analyses, either in relation to the factors mentioned or to the possible exploration of the links between the practices applied and the observed benefits.

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COVID-19 PANDEMIC AND PROFITABILITY DETERMINANTS IN ELDERLY CARE HOMES: EVIDENCE FROM CROATIA

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Abstract: The paper focuses on determinants of profitability of privately-owned for-profit elderly care homes. Multivariate regression analysis for the 2019-2022 period has been made on companies operating in Croatia. ROA has been used as a measure of profitability, while liabilities/asset ratio, net asset turnover, labour productivity, age and size of a company have been used as explanatory variables. Obtained results indicate that, in times of distress, revenues seem to be more important than assets, and increasing liabilities to overcome unexpected circumstances positively affect profitability. The importance of labour productivity also changed since it became evident that understaffing elderly care homes is not beneficial to profitability.

Keywords: elderly care homes, long-term care, profitability, COVID-19 pandemic, Croatia

1 INTRODUCTION

Elderly care homes provide residential and, in many cases, medical services, and the ageing of society, along with various socio-economic changes, increase the demand for this type of care. Therefore, besides publicly-owned elderly care homes, there is a growing number of privately-owned homes. Even though elderly care homes provide services that should not be observed strictly from an economic perspective, it is necessary for them to operate efficiently by providing high-quality service while earning profits. Further, this sector has particularities that mainly relate to its labour intensity since many activities related to elderly care demand human labour. High capital investments are also necessary since residential service demand large residential areas, whether rented or owned by elderly care homes. Additionally, residents and their families strive for high-quality service that requires space and employees' time and knowledge, i.e. demand-driven innovations are inevitable in this sector. In other words, managing an elderly care home, like in any other industry, requires balancing between: 1) high costs that should enable a company to provide quality service, 2) price setting that enables sufficient revenue while being competitive on the market and 3) investments that should provide necessary inputs for the company without jeopardising its profitability.

Along with business particularities, activities in elderly care homes have been under the strong impact of the COVID-19 pandemic. Without diminishing the devastating effect it had on residents' health, the pandemic increased both costs and complexity of this type of business. Hence, this paper aims to answer two following research questions:

1. What are the profitability determinants for elderly care homes?
2. Did the outbreak of the COVID-19 pandemic change the profitability determinants of elderly care homes?

Determinants of a company's profitability have been analysed for decades, and regardless of the analysed industry, they can be classified into those related to the company (internal determinants) or those related to external factors. Internal determinants that are analysed in the context of profitability can be classified into financial and nonfinancial variables. Financial variables include various indicators of the company's indebtedness and liquidity, activity, productivity etc. [23; 7; 18]. Nonfinancial indicators include variables such as the company's age and size [23; 10], market share [20], ownership [3], and gender diversity of corporate boards [21]. Regarding external determinants of a company's profitability, they include market, i.e. industry-specific and location-specific variables. Industry-specific determinants often include variables such as market concentration [11; 18] or capital intensity [23; 18]. Location-specific determinants are less often used, and they include variables such as inflation rate and growth rate [16; 18]. On the other hand, profitability as a dependant variable has been measured by various indicators such as EBIT or EBITDA (earnings before interest and taxes, depreciation, and amortization [14; 17], return on equity, i.e. ROE [12], net profit margin [11]. However, due to its characteristics and advantages over other measures [16], many authors, while analysing profitability determinants, use return on assets (ROA) as a dependent variable [20; 2; 9; etc.].

There are papers theoretically analysing COVID-19 and profitability in senior care [23], papers dealing with characteristics of the for-profit nursing home industry [4] or trends in nursing home chains [8], predicting nursing homes' financial distress [13], or those exploring profitability determinants but with variables reflecting strategy, market and organizational characteristics [5]. However, when it comes to the financial determinants of elderly care homes, there is an evident literature gap. This is even more emphasised considering the impact of the COVID-19 pandemic, and to the author's best knowledge, this is the only paper addressing these two issues simultaneously. Besides providing new insights into the company's profitability determinants and their characteristics during strong external factors such as the pandemic, this paper will hopefully add to the field by analysing Croatian elderly care homes. Namely, Croatia is a rather small country, but with the severe problem of an ageing society. Hence, gathering information on the profitability of elderly care homes in Croatia might be beneficial when analyzing similar markets for scientific or professional, i.e. entrepreneurial purposes. In that sense, a multivariate regression analysis for each year in the period 2019-2022 has been performed on privately owned companies operating in Croatia, focusing on determinants of profitability for firms operating under code 87.3 Residential care activities for the elderly and disabled.

The remainder of the paper is structured in the following way. Chapter 2 provides a data description and the methodology used. It is followed by Chapter 3, which presents results and discussion while concluding remarks are presented in the last chapter.

2 DATA DESCRIPTION AND THE METHOD

Empirical analysis has been made on 108 Croatian companies that during the period 2019-2022 operated under code 87.30 Residential care activities for the elderly and disabled according to NACE Rev. 2 classification of economic activities in the European Community. Data on each business subject registered for this activity have been retrieved from Croatian Financial Agency (FINA) database named info.BIZ. The following criteria have been used to filter companies: a) the company is active during the whole period, b) total revenue for each observed year is higher than 0 euros, c) the number of employees in each observed year is higher than 0 and d) company is privately owned.

Since the research focuses on detecting profitability determinants in elderly care homes and analysing whether they have changed during and/or after the COVID-19 outbreak in 2020, a multivariate linear regression, instead of a panel regression model, has been used for each of the observed years. In that manner, analysing the same 108 companies separately for each year allows us to detect if there are any changes in determinants of their profitability and if yes, to detect when they have occurred. The following linear regression has been used:

$$ROA_{it} = \beta_0 + \sum_{c=1}^{C} \beta_c X_{it}^{c} + \varepsilon_{it} \quad (1)$$

where i=1, 2,…, N represents the index for individuals (companies), t is the index for periods (years, namely 2019, 2020, 2021, 2022), c is the index for each of the five independent company-specific variables (c=1, 2,…C).

Return on assets before tax (ROA) has been used as a standard measure of profitability, while liabilities/asset ratio (LAR), net asset turnover (ATO), labour productivity (Productivity), company age (Age) and firm size (Size) are internal determinants of profitability that have been selected as explanatory variables and their definitions are presented in Table 1. Since LAR has been selected as a measure of indebtedness, companies with lower levels of indebtedness are expected to have higher levels of profitability. On the other hand, higher profitability is expected in more active companies, i.e., companies with higher net asset turnover. Labour productivity is expected to have a positive impact on profitability as well. When it comes to the age of the company and its size, both positive and negative impacts on profitability can be expected. Namely, larger companies can benefit from economies of scale, and easier access to financing, while older companies can benefit from accumulated learning and incumbent effect. On the other hand, younger and smaller companies can be more profitable since they are expected to be more prone to technology and can more easily adapt to changes in demand, i.e. their residents' needs.

Table 1: Description of variables

Variable	***Definition***
ROA	(EBIT/Total asset)*100
LAR - liabilities to asset ratio	Total liabilities/total asset
ATO - net asset turnover	Operating revenue/total asset
Productivity - labour productivity	Operating revenues/number of employees
Age - company age	Number of years since incorporation
Size - firm size	Natural logarithm of total asset

Source: Author based on information retrieved from FINA database info.BIZ.

3 RESULTS AND DISCUSSION

Eight regression models (presented in Table 2) have been tested to analyse profitability determinants in elderly care homes. In order to test the potential problem of multicollinearity, the matrix of Pearson coefficients has been used, and a high degree of statistically significant correlation between the pair of independent variables (LAR and ATO) has been detected in each of the observed years. Consequently, to mitigate the problem of multicollinearity, the aforementioned variables have been alternately used in models, resulting in two models (A and B) for each of the four observed years.

Results for the measure of indebtedness (LAR) indicate that in a year before the outbreak of the COVID-19 pandemic (2019) and in the following two years (2021, 2022), this variable, as expected, had a statistically significant negative impact on profitability, as in [19; 22]. However, in 2020, the impact of this variable changed, and according to the estimated models, it seems that companies with higher levels of indebtedness had higher profitability, as in [15]. The outbreak year has been a challenging year in many aspects, and obviously, in a financial way as well, signalling that in times of strong distress, it might be beneficial to increase the level of liabilities in order to surpass the crises.

Unexpected results are related to net asset turnover (ATO) in terms of its impact on profitability. In all estimated models, it has a statistically significant impact, yet the nature of this impact changes in 2020. Before and after the outbreak of the pandemic, this variable had a negative impact, while in 2020, it had a positive impact, as in [7; 19]. These results might seem contradictory to theoretical assumptions on profitability determinants, but when observed together with results for liabilities to asset ratio, they indicate that in this industry, in times of distress (such as in 2020), different segments of a company's financial characteristics become more important. Namely, it appears that in times of distress, liquidity and revenues become more important, while in other years, assets play a more significant role since this industry is highly capital-intensive and requires residential premises.

Table 2: Profitability determinants in elderly care homes in Croatia in the 2019-2022 period

Year	***2019***		***2020***		***2021***		***2022***	
Dependent variables	***model A***	***model B***	***model A***	***model B***	***model A***	***model B***	***model A***	***model B***
LAR	-2.898***		1.634***		-3.155***		-6.594***	
	(0.389)		(0.267)		(0.209)		(0.450)	
ATO		-4.369***		1.971***		-4.208***		-14.188***
		(0.695)		(0.499)		(0.510)		(0.693)
Productivity	0.001**	0.001**	0.001*	0.001*	0.000	0.000	0.000	0.000
	(0.001)	(0.001)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
Age	0.563	-0.005	0.389	0.677	1.025	1.017	1.243	1.948
	(1.057)	(1.111)	(0.567)	(0.615)	(0.873)	(1.222)	(2.266)	(1.771)
Size	-0.018	-0.823	-0.888	-0.551	-13.570***	-17.057***	20.078**	-18.822**
	(4.779)	(5.039)	(2.625)	(2.856)	(3.857)	(5.878)	(10.083)	(8.656)
Constant	-37.816**	-18.018	-17.953	-27.568**	152.358***	202.161***	-243.810**	241.032**
	(17.683)	(19.231)	(10.922)	(12.504)	(41.989)	(64.483)	(112.226)	(97.609)
R-squared	0.361	0.290	0.252	0.114	0.684	0.389	0.731	0.836
n	108	108	108	108	108	108	108	108

Source: Author's calculations.

Notes: *** indicate significance at the 1% level, ** indicate significance at the 5% level, * indicates significance at the 10% level. Numbers in parentheses below unstandardized coefficients are standard errors. According to the Durbin-Watson test, there is no autocorrelation in respective models since its value for all models is within the acceptable range (1,5-2,5). Further, values for Tolerance are greater than 0,2, and all Variance Inflation Factors (VIF) are less than 5, indicating no multicollinearity problem. Heteroscedasticity has been checked by plotting a graph of the residuals. All estimated regression models are statistically significant (1%). IBM Statistics SPSS 23 has been used for the analysis, and respective results can be obtained at request.

Labour productivity, as expected, had a statistically significant and positive impact on profitability in 2019 and 2020, as in [7]. However, for the following years, presented results indicate that this variable stopped having a statistically significant impact. A possible explanation might be the very nature of the industry. Namely, labour productivity is calculated by the ratio of operating revenues and number of employees. When observed strictly from an economic perspective, a lower number of employees with the same revenues is expected to result in higher productivity, which will lead to higher profitability. However, the COVID-19 pandemic strongly impacted elderly care homes in a way that revealed numerous cases of understaffed homes, unfortunately often resulting in fatal consequences for their residents [1; 6]. Therefore, it is understandable that the observed labour productivity stopped being statistically significant in the context of profitability since the policy of minimizing the number of employees turned out to be an inadequate business policy in this industry. This became even more evident after the outbreak of the pandemic, which emphasized the importance of quality of care provided in elderly care homes that is highly correlated with the number of employees.

Even though company age was expected to influence profitability, none of the presented models confirms this connection. Even though some authors [7; 18] reported the significance of this variable, the lack of similar research in this industry makes it hard to make strong assumptions about this variable without further analysis. However, a possible explanation might be correlated to the type of service provided in elderly care homes. Activities related to taking care of the elderly at its core do not change much, new knowledge might be implemented, but obviously, the impact of accumulated learning is not significant in the context of profitability.

When it comes to the size of the company, there are obvious changes after the outbreak of the COVID-19 pandemic. In 2019 and 2020, the size of a company did not have a statistically significant impact on profitability, while after the outbreak of the pandemic, it started to have a statistically significant negative impact in 2021 and in model B in 2022. However, this variable has a statistically significant positive impact on profitability in model A in 2022, as in [7; 17], which makes it hard to make strong assumptions about the impact of this variable on profitability. Nevertheless, it is obvious that external changes such as the pandemic have made its impact more significant.

4 CONCLUSION

When results of all estimated models are observed together, managing elderly care homes obviously has its peculiarities. In other words, in times of distress, such as an outbreak of a pandemic, increasing liabilities to overcome unexpected circumstances positively affects profitability. Further, revenues seem to be more important than assets in times of distress, while assets seem to be more important in other years, meaning that high levels of assets are necessary for operating with higher profits. The COVID-19 pandemic obviously changed the importance of labour productivity observed from a microeconomic perspective. Namely, after 2020 it became evident that understaffing elderly care homes and an obvious decrease in the quality of provided service are not beneficial to profitability. In short, while managing elderly care homes, it is evident that one needs to be prepared that factors determining profitability might change in their significance and type of their impact.

However, the presented analysis has certain shortcomings. Namely, due to limitations in data availability, data on the internal organisation (e.g. number of medical and non-medical staff, number and type of residents, occupancy rate etc.) have not been included. These data and additional data on residents' satisfaction might have provided additional insight into the field. Further, another downside of this analysis is the focus solely on Croatia, and if more countries had been included in the analysis, they could have been compared from the macroeconomic perspective as well. In that sense, further analysis will include expanding the sample to include more countries and, according to data availability, will include additional nonfinancial internal data on elderly care homes.

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PANEL DATA ANALYSIS IN PREDICTING DEMAND FOR INSTITUTIONAL LONG-TERM CARE FOR OLDER ADULTS

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Abstract: Panel data analysis has been performed in order to forecast demand for institutional long-term care (LTC) for older adults in 18 selected OECD countries in the period from 2013 to 2019. Results of the formed random effects model indicate that the demand, measured by the number of LTC recipients in institutions, will be higher in countries with a higher percentage of the population that is 65 years and older, longer life expectancy, a higher portion of the population aged 65 and more that perceive their health status as good or very good and in countries with a higher number of available beds in residential LTC facilities.

Keywords: demand, institutional long-term care, older adults, OECD countries, panel data analysis

1 INTRODUCTION

Demand for institutional long-term care (LTC) for older adults continuously grows due to multiple demographic and sociological factors. Hence, projecting demand for this type of service, besides the scientific aspect of studying the respective issue, has practical value. Observed from a national level, gathering information on factors that influence demand for institutional LTC is valuable to policymakers in terms of planning how to provide this service efficiently [11]. At the same time, estimates of demand for this type of care enable public sector stakeholders to steer developing substitutes such as non-institutional [10; 15] and informal LTC services [2] since the trend in highly developed countries goes towards deinstitutionalising care for older adults [15]. On the other hand, entrepreneurs interested in providing LTC service also benefit from gathering insight into the determinants of institutional LTC on a country level. Namely, providing this type of service requires high investments in residential areas as well as employing carers that have as least some medical training, keeping in mind labour shortages that are highly correlated to unfavourable and, in some cases, precarious working conditions often causing staff turnover and employee migrations [1; 13]. Therefore, analysing determinants that affect demand in the observed sector enables entrepreneurs to reallocate their resources to countries that are more suitable to their available resources and business policy, and it also provides valuable information for those dealing with complementary products such as long-term care insurance [5].

This paper is focused on the institutional LTC sector observed on a country level, and in this sense, its main objective is to detect determinants that affect demand. In other words, it will add to the field by providing new knowledge that might help further develop and adjust this sector since providing LTC service should transcend purely economic reasoning, i.e. as [4] stated, the LTC system should better meet the needs of workers and users of the service.

The rest of the paper is structured in the following way. The data description and the methodology are presented in Chapter 2. Chapter 3 presents results and discussion, while concluding remarks are presented in Chapter 4.

2 DATA DESCRIPTION AND THE METHOD

The analysis is focused on selected OECD countries (Canada, Denmark, Estonia, Finland, Hungary, Ireland, Israel, Korea, Latvia, Lithuania, Luxembourg, Netherlands, Norway, Slovak Republic, Slovenia, Spain, Sweden, and Switzerland) observed in the period from 2013 to 2019. The number of LTC recipients in institutions represents the demand for institutional LTC on a country level. As stated in Table 1, six explanatory variables have been selected to forecast the respective demand.

According to the basic microeconomic principle, countries with a higher number and a portion of the older population are expected to have a higher level of demand for observed service, unlike the results presented in [6]. On the other hand, perceived health status, measured by a portion of the population that is 65 and older, is expected to have a negative impact, as in [8]. Namely, older adults prefer to maintain their independence and to remain at their own homes; therefore, if more individuals perceive their health as good or very good, they are unlikely to be admitted to an LTC institution. Regarding GDP growth, the impact on demand for institutional LTC can be twofold. In countries with higher GDP growth rates older adults can benefit from favourable economic conditions because they can afford substitutes in terms of non-institutional care, or their family members can afford to stay away from the labour market and provide them informal care at home. Following the presented rationale, higher values of these variables decrease demand for institutional LTC. However, countries with favourable economic conditions are more likely to have resources invested in the LTC sector, meaning that they are more likely to provide this service to a high share of those in need, resulting in a higher demand for respective services. Similar logic applies to the number of beds in residential long-term care facilities, i.e. a positive effect is expected. Life expectancy is another determinant that is expected to positively impact demand for institutional LTC. The ageing process inevitably decreases an individual's ability to perform everyday tasks, and with higher age and longer life expectancy, more older adults will eventually need institutional LTC. Even though frontier technologies and older adults care at first seem to have little connection, a country's readiness to use and adopt frontier technologies is expected to decrease demand for institutional LTC. Namely, a higher ability to use advanced technologies will provide plenty of substitute products and services to older adults that will enable them to remain at home and still receive the care they need (e.g. increased use of social robots might keep away from older adults homes those older adults that want to be admitted to LTC institutions mainly because of loneliness or just need help with certain, but not all everyday activities) [3; 9].

Table 1: Description of variables

Variable (code)	***Definition***	***Source***
lnLTC	LTC recipients in institutions (other than hospitals); ln of a total number	OECD
y_ptot	Population 65 years old and over; % of total population	OECD
good_health_tot	Perceived health status "Good/very good health"; % of total population aged 65 years and over	OECD
GDP_g	GDP growth; annual; %	World Bank
Bed_65y	Beds in residential long-term care facilities; number per 1 000 population aged 65 years and over	OECD
Life_exp	Life expectancy at birth; number of years	World Bank
FTRI	Frontier technology readiness index; range 0 to 1	UNCTAD

Source: Authors based on information retrieved from OECD, UNCTAD and World Bank.

The empirical analysis is performed using static panel data analysis. Pooled Ordinary Least Squares (POLS) is the simplest panel model:

$$y_{it} = \beta_0 + \beta_1 x_{it1} + \beta_2 x_{it2} + \cdots + \beta_K x_{itK} + u_{it};\ \ i = 1, \ldots n,\ t = 1, \ldots T \qquad (1)$$

Where y_{it} is the outcome variable (natural logarithm of a total number LTC recipients in institutions other than hospitals) for observation unit *i* in period *t*, n denotes the number of observation units (countries), and T denotes the number of periods, while x_{it} represents explanatory variables of observation unit *i* in period *t*. The parameter β_0 is a constant term that is the same for all observation units and does not change over time, while $\beta_1,\ldots\beta_k$ are parameters estimated by the model. In addition, the error term u_{it} of the *i*-th observation unit in period *t* is assumed to be independent and identically distributed by the observation unit and time $Cov(u_{it}, u_{js}) = 0$. This estimator has the most limitations because it assumes that observations from the same unit (country) are independent (uncorrelated random variables). It is unrealistic to expect panel data to satisfy this assumption as they contain data on the same observation unit over several time periods $Cov(y_{it}, y_{js}) \neq 0$; $Cov(u_{it}, u_{js}) \neq 0$. Similarly, it is not possible to expect constant variance between observation units (homoscedasticity). As a result, the POLS parameter estimates are biased and inconsistent.

Omitted variable bias is common in the panel data framework due to data heterogeneity. The fixed effects estimator (FE) deals with heterogeneity by controlling for variables that cannot be observed (because they are not available or cannot be measured), but they are correlated with the regressors:

$$y_{it} = \alpha_i + \beta_1 x_{it1} + \beta_2 x_{it2} + \cdots + \beta_K x_{itK} + u_{it};\ \ i = 1, \ldots n,\ t = 1, \ldots T \qquad (2)$$

Where α_i is the unknown intercept for each observation unit (*n*-specific intercepts). The FE model assumes the correlation between the observation unit error term and explanatory variables. The estimated parameters of this model are not biased because the FE model controls for all time-invariant differences between individuals. However, the main drawback is that the FE estimator cannot be used to examine the effect of time-invariant variables [14].

Unlike the fixed effects model (FE), the random effects model (RE) assumes that variation among units (countries, individuals) is random and uncorrelated with the independent variables.

$$y_{it} = \beta_0 + \beta_1 x_{it1} + \beta_2 x_{it2} + \cdots + \beta_K x_{itK} + \mu_{it};\ \ i = 1, \ldots n,\ t = 1, \ldots T \qquad (3)$$

Where $\mu_{it} = \alpha_i + u_{it}$ is the error term with two components – individual α_i and one for regression u_{it}. The main difference between FE and RE is whether the unobserved individual effects are correlated with the explanatory variables. In RE, the individual's error term is assumed to be uncorrelated with the regressors. In addition, estimates of RE are obtained using the Generalized Least Square (GLS), while estimates of FE are obtained using the OLS estimator, implying that random effect estimates have a lower variance for large samples. The choice between fixed-effect and random-effect models is based on the results of the Hausman test (1978), which tests whether the difference between the estimated parameters of the FE and RE models is significant. Both estimators are consistent if there is no correlation between individual characteristics α_i and explanatory variables x_{it}. In this case, the estimates of FE are not significantly different from the estimates of RE, and RE should be used as it has a lower variance. On the other hand, if the random error component α_i is correlated with one or more explanatory variables, the RE estimator is inconsistent. The fixed effect estimator (FE) is still consistent, and then FE should be used. In this case, the difference between the estimated parameters of FE and RE will be significant. The reason for the difference between the parameters of the two estimators is the existence of the correlation (x_{it}, α_i).

3 RESULTS AND DISCUSSION

As stated, three models have been formed (Table 2). According to the Hausman test, the RE model is consistent and efficient, as the null hypothesis that there is no correlation between individual characteristics and explanatory variables is accepted. Therefore, the results obtained in this model are the focus of here presented research.

Table 2: POLS, FE and RE estimates with robust option (period from 2013 to 2019)

Variables ***Dependent variable: lnLTC***	***(1)*** ***POLS_robust***	***(2)*** ***FE_robust***	***(3)*** ***RE_robust***
y_ptot	0.087**	0.048***	0.047***
	[1.982]	[3.438]	[3.689]
	(0.044)	(0.014)	(0.013)
good_health_tot	-0.005	0.004**	0.004***
	[-0.507]	[2.708]	[2.805]
	(0.010)	(0.001)	(0.001)
GDP_g	-0.039	-0.001	-0.001
	[-1.655]	[-1.343]	[-1.350]
	(0.023)	(0.001)	(0.001)
Bed_65y	0.001	0.014***	0.014***
	[0.051]	[3.386]	[3.391]
	(0.010)	(0.004)	(0.004)
Life_exp	0.102	0.051**	0.053***
	[1.534]	[2.247]	[2.659]
	(0.066)	(0.023)	(0.020)
FTRI	4.299***	0.172	0.194
	[3.517]	[0.416]	[0.476]
	(1.222)	(0.413)	(0.408)
Constant	-2.374	4.663**	4.462***
	[-0.467]	[2.534]	[2.868]
	(5.084)	(1.840)	(1.556)
Observations	125	125	125
R-squared	0.282	0.569	
Number of countries		18	18
POLS vs FE (F-test and Prob > F)		3562.9 (0.00)	
POLS vs RE (chibar2 and Prob > chibar2 – BP test)			340 (0.00)
Hausman test (chibar2 and Prob > chibar2)		2.19 (0.90)	

Note: ***, **, * denote significance at the 1%, 5%, and 10% significance level. Robust standard errors are in parenthesis, while numbers in brackets are z-statistics. F test that all u_i=0: F(17, 101) = 3562.86.
POLS vs FE - The overall F-test for 17 individual differences (F-test that all α_i=0) shows that there are significant differences between individuals and that the FE model is more appropriate.
The Breusch-Pagan test (BP) is used to test for the presence of random effects (POLS vs RE): We reject the null hypothesis that there are no random effects and conclude that random effects are present.
The Hausman test fails to reject the null hypothesis meaning that the individual characteristics are uncorrelated with the regressors, and the random effect is a consistent and efficient estimator.
We did not reject the null hypothesis that the coefficients for the years are jointly equal to zero (Prob > F is 0.91).
Source: Authors' calculations.

Presented results of the RE model indicate that, as expected, countries with a higher portion of the population 65 years old and over will have a higher demand for institutional LTC. Similar logic applies to the variable related to life expectancy. Namely, obtained results confirmed that in countries with a longer life expectancy, demand for this service would increase. In other words, the longer older adults live, the probability for them to need institutional LTC due to the nature of ageing increases, consequently leading to a rise in demand for respective services. The number of available beds in residential LTC facilities is a precondition for the older adults to get the care they need, and its positive impact on demand is statistically significant. In other words, if there is no sufficient number of available beds, the older adults and their families will use substitutes for this type of care, e.g. informal LTC care or non-residential institutional care [16].

According to obtained results, the GDP growth rate and the ability of a country to use and adopt frontier technologies do not have a statistically significant impact on demand for institutional LTC. Regarding GDP growth, as previously stated, its impact can be both positive and negative, so obtained results are in line with expectations since it seems that these two contrasting impacts annulated each other, resulting in a non-significant impact on respective demand. Regarding frontier technologies, a statistically insignificant impact on demand for institutional LTC can be explained by, at the observed period, still insufficiently developed innovations that are applicable in LTC sectors of observed countries. Finally, the percentage of the population of those aged 65 years and older that find their health good or very good is the only variable whose impact is statistically significant but to a certain extent unexpected, e.g. in [6], similar variable turn out to be insignificant. Namely, results indicate that a higher portion of these individuals will increase demand for institutional LTC. At first, these results might seem controversial, but when we observe the sample, it is evident that it includes countries with developed LTC systems. In other words, older adults that have a good and very good health status will increase demand for institutional LTC since it enables them to get the high-quality care they need due to their age and at the same time to deal with a problem of loneliness and high residential costs if they continue to live independently at home [7]. Additionally, in countries with highly developed LTC systems, LTC institutions provide a variety of services [12] that attract older adults without severe health issues, and there is also a sufficient number of institutions for those in need of palliative care so they can receive more adjusted types of care.

4 CONCLUSION

Results of the formed random effects model indicate that the demand for institutional LTC will be higher in countries with a higher percentage of the population that is 65 years and older, longer life expectancy, a higher portion of the population aged 65 and older that perceive their health status as good or very good and in countries with a higher number of available beds in residential LTC facilities. GDP growth rate and the country's readiness to use and adopt frontier technology do not have a statistically significant effect. To a large extent, obtained results confirm the expectations and appropriateness of the methodology used to predict demand for institutional LTC on a country level. However, this study has its limitations. Namely, due to the unavailability of data, it includes the period until 2019 and countries with developed LTC systems. Including newer data, especially in the context of the rapid speed of technological innovations, might alter obtained results regarding frontier technologies. Further, expanding the sample to include countries with lower economic standards and undeveloped LTC systems would enable a comparison of determinants of demand for institutional LTC between groups of countries. In that sense, future research in this field will include expanding the observed period, providing additional explanatory variables, and enlarging the sample.

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ELDERLY POPULATION ACTIVITY IN CROATIA: GENDER COMPARISON BY TEXT ANALYSIS APPROACH

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Abstract: Due to retirement and old age elderly population needs to change their habits and activities. The paper aims to compare the main activities between elderly males and females in Croatia. The computer-assisted telephone interview survey was conducted on a sample of 273 elderly males and 428 elderly females. The respondents' answers to selected survey questions were merged for each respondent separately and they are observed as corpora. The text analyses of those corpora have shown that elderly males and females share their main activities but the difference in the order of activities is present.

Keywords: senior population, activity, text analysis, gender, Croatia.

1 INTRODUCTION

According to [5] physical activity is "bodily movement produced by skeletal muscles that require energy expenditure". [3] emphasizes that regular physical activity is one of the most important keys to good health. The importance of physical activity becomes even more important in older age [4]. However, in older age is not important only to do some physical activity but it is important to stay active even after retirement and stay in touch with their surroundings. Therefore, in the paper under activities, a broader term than just physical activities will be used. So, under activities here will be included, in addition to pure physical activities, activities like Internet use, media following, different ways of communication with family and friends, different ways of socializing, etc.

The paper aims to investigate whether the main activities of elderly males and females in Croatia are different. Therefore, the research hypothesis is that elderly males and females have the same main activities. The research in the paper will be based on the text analysis approach.

The paper is organized as follows. After the brief introduction, in the second chapter, the conducted survey and methodology are described. In the third chapter, text analysis results are presented and briefly discussed. The final, fourth, chapter concludes the paper.

2 DATA AND METHODS

In the paper, the data from a survey conducted among the elderly population in Croatia will be analysed. The target population was individuals who are living in Croatia and are of age 65 years or more. The individuals were randomly selected but quota sampling was applied as well. Quota sampling was applied to obtain the structure in the sample as it is in the population according to two variables: gender and place of residence according to the NUTS 2 regions in Croatia. When the survey was conducted, in January 2022, the census information from the census conducted in 2021 was not available yet. Therefore, the structure for quota sampling was followed by observing census 2011 data. The applied survey mode was a computer-assisted telephone interview (CATI). In the survey participated overall 701 respondents which structure is shown in Table 1.

According to Table 1, in the survey participated 273 (39%) males and 428 (61%) females. In Table 1 the structure of respondents is presented according to their place of residence according to the NUTS 2 regions in Croatia, their age and the settlement type in which respondents live. According to the results, most respondents are living in Adriatic Croatia (238 respondents), have from 65 to 69 years (248 respondents) and live in an urban settlement (399 respondents). If the structure of respondents is observed according to the gender and those three variables, it can be concluded that the share of females in each variable item tends to be above 60% (up to 66%). The lowest share of females in the sample appears in the age group of respondents from 80 to 84 years where the share of female respondents is 55%.

Table 1: Respondents' structure in the sample

Variable	***Item***	***Gender***				***Sample total***
		Male	***Male %***	***Female***	***Female %***	
Total		273	39%	428	61%	701
NUTS 2 region	*City of Zagreb*	49	39%	77	61%	126
	Adriatic Croatia	99	41%	140	59%	239
	Pannonian Croatia	76	37%	128	63%	204
	Northern Croatia	49	37%	83	63%	132
Age	*65-69*	102	41%	146	59%	248
	70-74	66	34%	131	66%	197
	75-79	56	39%	86	61%	142
	80-84	38	45%	47	55%	85
	85 and more	11	38%	18	62%	29
Settlement type	*Urban settlement*	150	38%	249	62%	399
	Rural settlement	123	41%	179	59%	302

Table 2: List of observed variables

Question code	***Question***	***No. of responses***		
		Male	***Female***	***Total***
B1	What type of physical activity have you been doing in the last five years?	272	428	700
B3	What types of activities and hobbies are you interested in?	273	428	701
B7	Do you think that you are sufficiently informed about recreation or physical activity in terms of the prevention of some diseases?	273	428	701
C5	How often do you use the internet?	273	428	701
C6	Which device do you most often use to access the Internet?	273	428	701
C7	For what content did you use the Internet in the last three months?	182	224	406
D1	Are you still working or were you still working after turning 65?	273	428	701
D2	If you are working now or have been working after turning 65, what working form was it?	62	61	123
D4	Do you want to become active and work again?	211	367	578
D5	Why do you want to become active and work again?	44	37	81
D6	What could you as a person offer or do you offer on the current labour market?	39	36	75
E1	Can you independently perform all your basic life needs (buying food, paying bills, etc.)?	273	428	701
E3	Do you use a technological device that improves your health and quality of life (for movement, hearing, speaking, typing, etc.)?	273	428	701
F2	Where most often do you have contact and socialise with people who are not your family?	273	428	701
F6	How do you find out (where do you get information) about your civil rights and the health and social programs intended for you?	273	428	701
F7	What media do you regularly follow?	273	428	701
F9	How do you usually get to the place where all the public institutions you need are located (doctor, bank, post office, pharmacy, etc.)?	55	84	139

The questionnaire consisted of questions set that covered different life aspects of the elderly population. In that way, the following topics were covered in the survey: physical activity; digital skills and use of digital services; participation in the labour market; using technologies to improve the quality of life; social inclusion and media monitoring; and financial needs and sufficiency of funds. For the purpose of the analysis, overall 17 questions were selected. The list of selected questions is provided in Table 2.

The selection of questions was based on two principles. The first principle was that the question should be related somehow to the activity of the elder population. A good example of activity is the B1 question which is related to the types of physical activities done by the elder population. Of course, the selected questions do not cover just physical activities-related questions but here questions about working after 65 years, using the Internet, places visited for socializing, etc. are included as well. The second principle, which was followed, is that on selected questions qualitative answers can be given. Almost all questions were close-ended questions. In that way, it could be possible to easier recognize respondents with the same opinions and activity level. However, the most chosen question had an option, in addition to given answers, for respondents to provide additional answers and descriptions. Furthermore, the respondents could provide just one answer to some questions and more than just one answer to some other questions. Also, in the survey filter questions were used and therefore not all 701 respondents were asked all questions. Therefore, for some questions, the number of respondents is lower than 701.

To conduct the text analysis, the data had to be prepared. In the first step, respondents' responses to the observed 17 questions are merged into one. In that way, corpora were made for each respondent separately. In the next step, those corpora are edited so that any possible problems during conducting the text analysis will be avoided. Because the observed corpora are given in the Croatian language, letters that appear in the Croatian alphabet but not in the English one are adjusted. For example, the letters "č" and "ć" are transformed into "c", the letter "ž" become the letter "z", etc. Also, all punctuation marks, like commas or dots, are removed from the corpora. In addition, all words were transformed into lowercase written words. The conjunctions and words "and", "in", "for", "as" and "with" (in Croatian "i", "u", "za", "kao", and "s") are left from the corpora as well.

In the analyses, bigrams will be observed separately for males and females as well. Bigrams have two words. By observing bigrams it would be possible to account for negations and similar collocations. The top 20 collocations will be emphasized by applying the lambda criteria. According to [2] the coefficient lambda presents the n-way interaction term in a log-linear model which also contains all lower-order interaction terms. The calculation of the coefficient lambda is explained in detail by [1].

3 ANALYSIS AND DISCUSSION

The analysis of corpora starts with an analysis of the most frequent words or features in the corpora. Because the focus is given to the comparison of the results between different genders, the results are provided separately for males and females analogously.

In Table 3 the document feature table for male respondents along with additional information provided. According to Table 3, the most frequent word that appears in male corpora is the word "no". It appears over a thousand times in corpora of 265 respondents. So, on average, the word "no" appears 4.4 times per male corpus. In addition, the word "no" appears in 97% of male corpora. However, it has to be emphasized that the rank of the word "no" has much with characteristics of the Croatian language. Namely, the word "no" can appear twice in case of negation like "ne, ne koristim internet" (no, I do not use the Internet). With the word “yes” is a similar situation but here is the case that this word in Croatian can also have another meaning. Also, the word “me” itself does not provide much information about elderly male activities. However, there are pointed out some useful words which can lead to the recognition of activities conducted by males: Internet, television, walking, friends, the newspaper and portals.

Table 3: Document feature table for male respondents, n=273

Rank	***Feature (Croatian)***	***Feature (English)***	***Frequency***	***Respondents***	***Average frequency***	***Share of male respondents***
1	ne	no	1,174	265	4.4	97%
2	internet	Internet	857	273	3.1	100%
3	koristim	I use	648	273	2.4	100%
4	da	yes	637	271	2.4	99%
5	televizija	television	475	271	1.8	99%
6	setnja	walking	455	235	1.9	86%
7	prijateljima	friends	329	190	1.7	70%
8	novine	the newspaper	302	233	1.3	85%
9	informacija	information	293	242	1.2	89%
10	portali	portals	287	159	1.8	58%
11	mogu	I can	283	273	1.0	100%
12	glavni	main	238	238	1.0	87%
12	izvor	source	238	238	1.0	87%
14	zelim	I want to	227	211	1.1	77%
14	treba	need	227	227	1.0	83%
14	mi	me	227	227	1.0	83%
17	obitelji	families	224	155	1.4	57%
18	mediji	media	221	221	1.0	81%
19	casopisi	magazines	201	181	1.1	66%
20	komunikacija	communication	195	135	1.4	49%

Table 4: Document feature table for female respondents, n=428

Rank	***Feature (Croatian)***	***Feature (English)***	***Frequency***	***Respondents***	***Average frequency***	***Share of female respondents***
1	ne	no	2,193	421	5.2	98%
2	internet	Internet	1,232	428	2.9	100%
3	koristim	I use	1,071	428	2.5	100%
4	da	yes	888	418	2.1	98%
5	setnja	walking	731	353	2.1	82%
6	televizija	television	715	419	1.7	98%
7	prijateljima	friends	457	286	1.6	67%
8	mogu	I can	450	428	1.1	100%
9	novine	the newspaper	443	344	1.3	80%
10	informacija	information	418	356	1.2	83%
11	zelim	I want to	374	367	1.0	86%
12	glavni	main	354	354	1.0	83%
12	izvor	source	354	354	1.0	83%
14	treba	need	343	343	1.0	80%
14	mi	me	343	343	1.0	80%
14	radno	working	339	339	1.0	79%
17	aktivan	active	336	336	1.0	79%
18	portali	portals	331	188	1.8	44%
19	biti	be	329	329	1.0	77%
20	obitelji	families	315	239	1.3	56%

Document features table for female respondents, given in Table 4, revealed the same words that are repeated the most time as in the case of corpora of male respondents. However, the order is somewhat different in comparison to the male corpora. So, according to the female corpora, the top five female activities are related to the Internet, walking, television, friends and the newspaper.

In Tables 3 and 4, can be recognized collocations like “I use the Internet” or “main source”. Therefore, the collocations analyses have to be done as well. In Table 5 bigram analysis results for elderly male respondents are presented whereas the bigram results for elderly female respondents are given in Table 6.

Table 5: Collocation analysis, bigrams, male respondents, n=273

Rank	***Collocation (Croatian)***	***Collocation (English)***	***Count***	***lambda***	***z***
1	glavni izvor	the main source	238	16.72	8.36
2	treba mi	I need	227	16.67	8.33
3	rad vrtu	work in the garden	159	16.32	8.16
4	svakodnevnim dogadanjima	everyday events	159	16.32	8.16
5	odlazak prirodu	going to nature	158	16.32	8.15
6	drustvene mreze	social networks	153	16.29	8.14
7	mreze facebook	Facebook networks	153	16.29	8.14
8	kod kuce	at home	148	16.25	8.12
9	aplikacijama slanje	applications sending	122	16.06	8.02
10	elektronicke poste	electronic mail	122	16.06	8.02
11	poruka viber	viber message	122	16.06	8.02
12	poste aplikacijama	post applications	122	16.06	8.02
12	slanje poruka	sending messages	122	16.06	8.02
14	viber whatsapp	viber whatsapp	122	16.06	8.02
14	whatsapp messenger	whatsapp messenger	122	16.06	8.02
14	prostorima trznica	market spaces	117	16.02	8.00
17	biti radno	to be busy	167	13.81	9.39
18	javnim prostorima	public spaces	117	13.08	8.99
19	vise puta	multiple times	132	12.58	8.76
20	komunikacija obitelji	family communication	195	12.45	8.72

Table 6: Collocation analysis, bigrams, female respondents, n=428

Rank	***Collocation (Croatian)***	***Collocation (English)***	***Count***	***lambda***	***z***
1	glavni izvor	the main source	354	17.48	8.74
2	treba mi	I need	343	17.45	8.72
3	odlazak prirodu	going to nature	261	17.18	8.58
4	kod kuce	at home	246	17.12	8.55
5	svakodnevnim dogadanjima	everyday events	188	16.85	8.42
6	prostorima trznica	market spaces	173	16.77	8.38
7	drustvene mreze	social networks	158	16.68	8.33
8	mreze facebook	Facebook networks	158	16.68	8.33
9	aplikacijama slanje	applications sending	157	16.67	8.33
10	elektronicke poste	electronic mail	157	16.67	8.33
11	poruka viber	viber message	157	16.67	8.33
12	poste aplikacijama	post applications	157	16.67	8.33
12	slanje poruka	sending messages	157	16.67	8.33
14	viber whatsapp	viber whatsapp	157	16.67	8.33
14	whatsapp messenger	whatsapp messenger	157	16.67	8.33
14	djeca rodbina	children relatives	148	16.61	8.30
17	unuci djeca	grandchildren children	148	16.61	8.30
18	poziva zoom	calls zoom	102	16.25	8.11
19	skype teams	skype teams	102	16.25	8.11
20	zoom skype	zoom skype	102	16.25	8.11

According to Table 5, the main activities of elderly males are working in the garden, going to nature, use of social networks, being at home and use of email. On the other hand, the results from Table 6 are suggesting that the main activities of elderly females are going to nature, being at home, going to market spaces, use of social networks and use of email. So, the bigrams analyses have also shown that elderly males and females have about the same activities in their old age. However, the order of main activities is slightly different.

4 CONCLUSIONS

Physical activity is very important for people no matter their age. The activity becomes even more important when people come to their old age (65 years and above). However, due to their ages, some older people cannot be as physically active or physically active in the same way as before. Also, during their working period, they travelled to work and back, meet other people, communicate with co-workers, etc. Therefore, in the paper under activity, a broader term than just physical activity was inspected.

In order to identify the main activities of the elderly population in Croatia, responses to questions related to the activity of the elderly population were inspected. Responses were observed in a textual form and therefore text analysis approach could be applied. In the first - step, unigrams from male and female corpora were analysed. Even though corpora omitted conjunctions and similar short words, unigram analyses resulted in emphasized words like “no”, “yes” or “me” that do not have any meaningful value themselves. That happened due to the characteristics of the Croatian language. One solution would be to go to the beginning of the analysis and omit such words as well. However, those words could be important in the case of the collocation analysis by giving meaning to the collocation in the case of bigrams. Therefore, such words were just skipped in the unigram analyses and the focus was given to activity-related unigrams.

The results have shown that elderly males and females share the same activities but in different order. Therefore, ageing policies need to take into account those differences between genders and improve elderly males’ and females’ activity levels by using different approaches. The main limitation of this research is that the focus is given only to the identification of main activities. To better understand elderly population activity, in future analyses, the importance and frequency of doing activities population should be investigated further on.

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Unravelling the Business Models of Sharing Economy by Applying Methods of OR and Statistics

TIME SERIES ANALYSIS OF AIRBNB HOUSE RENTALS PRICE IN THE BALKAN REGION

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Abstract: Shared accommodation is one of the most recognizable business models of sharing economy. Shared accommodation enables residents to temporarily rent out their properties to others through online platforms for a predetermined price and for a defined period of time. One of the factors which impact the success of the business model is the daily rental price. The research question we raise in this study is related to daily housing rentals price prediction of Airbnb properties. In our case study, we used ARIMA modelling to model and predict housing rentals prices of the properties listed on the Airbnb platform in Ljubljana, Slovenia and Zagreb, Croatia.

Keywords: Sharing economy, Shared accommodation, Airbnb, Time series analysis, Balkan region.

1 INTRODUCTION

First mentioned in 2008, sharing economy is defined as „collaborative consumption made by the activities of sharing, exchanging, and rental of resources without owning the goods" [1]. Over time, the sharing economy has attracted the interest of various scientific disciplines, such as economics, law and business administration. For example, the subjects of macroeconomic research are the reasons and motivation of people to participate in the new way of consuming products and services [2]. The popularity of the sharing economy phenomenon is constantly increasing among consumers and various other stakeholders. The rise of the Internet and Web 3.0 significantly contributed to the digital transformation that has enabled new forms of consumption. As a result, the relationship between the consumer and the product has changed, and the traditional way of consumption, which implies ownership of the product, has been overcome [3]. Looking at the four distinct categories of the sharing economy in tourism – tour guide services, transportation, dining, and accommodation [4], and with the last being the most prominent sector in the economy of tourism in general [5], it is important to discuss the economic and social implications of the quick rise of a new sharing model – peer to peer accommodation or shared accommodation. Shared accommodation is predominantly realised on platforms such as Airbnb, Couchsurfing, HomeAway, Home Exchange and others.

The research question we raise in this study is related to daily housing rentals price prediction of Airbnb properties. As the data on housing rentals prices are available over time, time series (TS) time series analysis deemed suitable for implementation. In our case study, we used ARIMA modelling [6] to model and predict housing rentals prices of the properties listed on the Airbnb platform in Ljubljana, Slovenia and Zagreb, Croatia. In section two, we outline a brief literature review on the issue of pricing and price predictions of Airbnb listing. In the following section, we present our case study and study results. Finally, we finish the paper with a discussion and concluding remarks.

2 LITERATURE REVIEW

Airbnb operates as a multi-sided platform, where hosts and guests communicate and trade directly, and the platform earns a commission from both sides of the market. For hosts, the value Airbnb offers lies in lowering potential risks by identifying adequate guests, as well as helping in the operational and promotional aspects of renting spaces, while simultaneously offering value to the potential guests – Airbnb filters acceptable accommodation, lowering the risk in that regard, as well as delivering an enhanced travel experience [7]. The numbers associated with Airbnb's business are impressive. Currently, there are over 6 million active listings, over 4 million hosts and a base of over 150 million users on the platform. Airbnb is growing its business worldwide, and currently operates in 191 countries [8], including Croatia, and Slovenia, on which this work is based on. One of the crucial things for Airbnb hosts is setting the right prices for their listings, in order to achieve their own financial goals. This is something that the company itself recognises, which is why they constantly put an effort to provide a suitable help in the forms of specific pricing tools and tips. To do so, it is important to identify the price determinants and power of their influence. To explore the price determinants of accommodations in the sharing economy, in their research Wang and Nicolau [9] used ordinary least squares (OLS) and quantile regression (QR). The results show that attributes such as superhost status, more listings, and verification, all influence the price increase. Setting the right price is not an easy task, so there have been a lot of researchers trying to find a solution through modelling and predicting price. In one of the recent studies, a group of authors [10] used time series analysis for such a purpose as well.

3 TIME SERIES ANALYSIS OF THE RENTAL PRICES ON THE AIRBNB PLATFORM

A time series analysis was conducted to analyse and predict the rental prices on the Airbnb platform in the Balkan region. Two cities were chosen for the analysis: Ljubljana, Slovenia and Zagreb, Croatia. The data was retrieved from the website Alltherooms.com [11] for the period from 19th December 2022 until 1st May 2023. The available data were weekly average daily price rentals. The prediction period was from 8th May to 5th June 2023. To model the time series (TS) ARIMA (p,d,q) models and Box Jenkins methodology to ARIMA models were used, while the best model selection criterion was the Akaike Information Criterion (AIC) [6]. To explore the stationarity of the TS Augmented Dickey Fuller test was used (ADF). The presence of autocorrelation was assessed using the Durbin Watson test (DW), while the presence of heteroskedasticity was assessed using ARCH test.

3.1 Time series of the rental prices on the Airbnb platform in Ljubljana

The collected data is presented in orange in Figure 2. From it, we can see that there was a sharp rise in the rental price around New Year’s Eve to 167 euros. After that period, the prices dropped sharply to 80 euros and slowly increased to 119 euros per night until the end of the observed period. First, it was necessary to determine whether the Ljubljana TS needed integration. ADF test was performed. In the level, the value of the ADF statistics is -2.286 with p>0.05. Therefore, the conclusion is that the TS should be integrated. In the 1st difference, the value of the ADF statistics is -7.323 with p<0.05. The result in the 1st difference indicates that there is no need to further integrate the TS. In the next steps, several ARIMA models have been created. The model with the lowest AIC was ARIMA (1,1,0) with AIC of 8.425. The obtained R squared was 0.1071. The value of the Durbin Watson statistics is 1.339 which is in the region where the presence of autocorrelation cannot be determined. The value of the ARCH test statistics is 0.707, with p>0.05. Therefore, we accept the null hypothesis and conclude that homoskedasticity is present in the model. Considering all the performed tests, the model can be used for predictions. The Ljubljana TS forecasting is presented in Figure 1.

In the future period, in May and June 2023, according to our model, the predicted values of average daily rent on the weekly level in Ljubljana will be from 118.628 Euros to 120.581 Euros. We can see a slight increase in the rental prices, but the increase is minimal. Therefore, no major price change can be expected.

Figure 1: TS modelling results for Airbnb price rentals in Ljubljana, Slovenia: collected and predicted data

3.2 Time series of the rental prices on the Airbnb platform in Zagreb

The collected data is presented in orange in Figure 2. From it, we can see a sharp rise in the renting price around New Year's Eve to 167 euros. After that period, the prices dropped sharply and oscillated around 80 euros per night until the end of the observed period. First, it was necessary to determine whether the Zagreb TS needed integration. ADF test was performed. In the level, the value of the ADF statistics is -8.678 with p<0.05. Therefore, the TS should not be integrated, and there is no trend in the data. In the next steps, several ARMA models have been created. The model with the lowest AIC and model elements which were statistically significant was ARIMA (1,0,0) with AIC of 9.425. The obtained R squared was 0.357. The value of the DW statistics is 1.726 which is in the region where the presence of autocorrelation cannot be determined. The value of the ARCH test statistics is 0.090, with p>0.05. Therefore, we conclude that homoskedasticity is present in the model. Taking into account all the performed tests, the model can be used for predictions. The Zagreb TS forecasting is presented in Figure 2

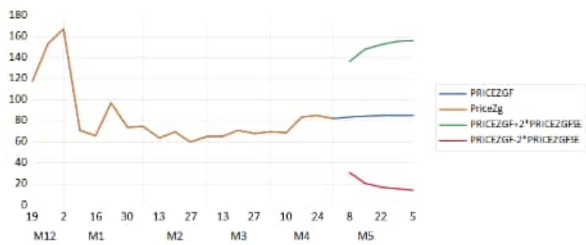

Figure 2: TS modelling results for Airbnb price rentals in Zagreb, Croatia: collected and predicted data

In the future period, in May and June 2023, according to our model, the predicted values of average daily rent on a weekly level in Zagreb will be from 83.537 Euros to 85.486 Euros. We can see a slight increase in the rental prices, but the increase is minimal. Therefore, no major price change can be expected. However, seasonality should be considered, as the main touristic season in Croatia is in the summer period.

4 DISCUSSION AND CONCLUSION

The results of our study show that TS analysis can be used with success to model the prices of Airbnb listings in the Balkan region. In Zagreb, Croatia, the data shows a stable pricing market with low volatility. On the other hand, in Ljubljana, Slovenia, the data demonstrate a positive trend, signalling that the price rentals are expected to grow at a stable pace in the upcoming period. The limitation of the conducted research is the relatively short time series which was available. The study could be extended to other cities and countries in emerging markets in the Balkan region [12]. We hope that this study might act as an impetus for further research on the topic of sharing economy in the Balkan region.

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SHARED ACCOMMODATION IN EUROPE: CONSUMER BEHAVIOUR ANALYSIS

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Abstract: Sharing economy is described as an economic model in which individuals and groups share goods and resources for a defined period for a predefined price. To better understand the mechanisms of sharing economy business models, it is important not only to observe the platforms which allow the interactions between individuals offering assets (providers) and individuals in search for goods (users), whereas to explore the behaviour of participants in the sharing economy business model. In our research, we will focus on the behaviour of the users of a particular form of sharing economy, of shared accommodation. Our research explores did and how the behaviour of consumers changed in the last couple of years and there are some differences in the behaviour of consumers based on their income level and residence regarding the usage of shared accommodation.

Keywords: Sharing economy, Shared accommodation, Behavioural change, Consumer behaviour, paired samples tests.

1 INTRODUCTION

Sharing economy is described as an economic model in which goods and resources are shared by individuals and groups for a defined period for a predefined price. This business model would not be possible without modern technologies such as Web 3.0, the Internet, Blockchain, Social networks and ICT platforms. ICT platforms facilitate the transaction between the provider and the customer. Platforms enable a large volume of users to interact at the same time using an understandable user interface with low transaction costs [1]. These platforms allow the sharing of accommodation, mobility, skills, money, and other goods/services.

Peer-to-peer (P2P) accommodation or shared accommodation allows hosts to rent out parts of their property – or the property itself – to other people, for a short time, mainly through ICT platforms [2]. This model of sharing economy is projected to grow at a rate six times greater than its more traditional counterparts [3]. As such, it significantly affects the profitability and structure of tourist accommodation in general. With this in mind, it is important to understand consumer behaviour related to usage of shared accommodation.

From the tourist's perspective, the main motivation behind opting for a shared accommodation is the economic benefit, because shared accommodation is usually lower priced than other available types of accommodation [4]. Next to that, the other main reason why shared accommodation is attractive to tourists is more intrinsic, and it roots in the tourist's desire to experience a more authentic, deeper and even home-like environment [5] when travelling to a foreign place.

The research gap which emerges is related to a better understanding of behaviour of the users of shared accommodation. This question has been raised by several authors so far [6-7]. Our research explores did and how the behaviour of consumers changed in the last couple of years and are there some differences in the behaviour of consumers based on their income level and residence regarding the use of shared accommodation. The structure of the research is following. After the introduction, we provide an explanation of the data collection procedure and the indicators provided by Eurostat, which we used in the study. The results are outlined in section three. We finish the paper with the discussion and concluding remarks.

2 RESEARCH METHODOLOGY

Collecting data on sharing economy is no easy task for several reasons. The main obstacle is the fact that the transactions are made on platforms, which are not obliged to share their data with official statistical offices. Therefore, most of the available data is related to reports of consulting agencies, platforms, or results of academic research. As the European Commission is aware of the importance of the sharing economy for the overall economy, additional efforts have been made to collected sharing economy related data. Within the Eurostat database and its survey on the use of information and communication technologies (ICT) in households and by individuals (*ICT usage in households and by individuals (isoc_i)*) there are indicators related to the use of sharing economy [8]. The main survey respondents are individuals aged 16 to 74, whereas in some countries there are data for participants who are younger than 16 and older than 74 [9]. The annual ICT survey is conducted in all EU member states, United Kingdom (UK), Iceland, Norway, Switzerland, candidate countries and potential candidate countries. The data collected for this study encompassed 36 European countries. The data provided by the Eurostat is the percentage of individuals that used shared accommodation. Besides, it is possible to generate categorised data as the percentage of individuals that used shared accommodation per gender, educational attainment, income group, other socio-demographic characteristics and their combinations. The most recent available data is for the year 2019.

3 RESEARCH RESULTS

The first research question was how the percentage of users of shared accommodation changed from 2017 to 2019. To answer the raised question, we used the paired samples t-test and conducted pair-wise comparisons. The results are presented in Tab.1. The results indicate that there has been a statistically significant increase in usage of shared accommodation between 2017 when it was 13.7% on average and 2018 when it was 16.67% on average ($t=-3.343$, $p=0.002$). The same pattern was observed between 2018 and 2019, when it was 19.20% on average ($t=-6.669$, $p=0.000$). Therefore, we can observe a steady increase in the usage of shared accommodation in Europe.

Table 1: Pair-wise comparisons of the usage of shared accommodation in Europe, 2017-2019

Pairs	*Descriptive statistics*		*Paired-samples t-test*	
	Mean difference	*Std of the difference*	*Statistics*	*p value*
2017-2018	-2.931	4.802	-3.343	0.002
2018-2019	-2.509	2.194	-6.669	0.000

The next research question was to explore whether there are differences in the usage of shared accommodation based on the individuals' income. Eurostat provides a categorisation of respondents regarding income on four quartiles. Therefore, the most recent data was used, for the year 2019, and analysis of variance was used to explore the differences between the four categories. The results indicate that there are statistically significant differences ($F=13.170$, $p<0.001$). To detect between which pairs there are statistically significant differences, Independent samples t-test has been performed. The obtained results are graphically presented in Figure 1. The results indicate that the usage level is the same among those with income in Q1 and Q2 ($t=-1.084$, $p=0.284$), as well as those with Q2 and Q3 ($t=-1.978$, $p=0.053$). There is a statistically significant difference in the usage between those with Q1 and Q3 ($t=-2.939$, $p=0.005$), whereas the individuals with higher income use the shared accommodation more. Interestingly, those with income in Q4 statistically significant use shared accommodation more than individuals with Q1 ($t=-5.783$, $p=0.000$), Q2 ($t=-4.744$, $p=0.000$), and Q3 ($t=-2.486$, $p=0.016$). The results clearly indicate that those with higher income use shared accommodation more.

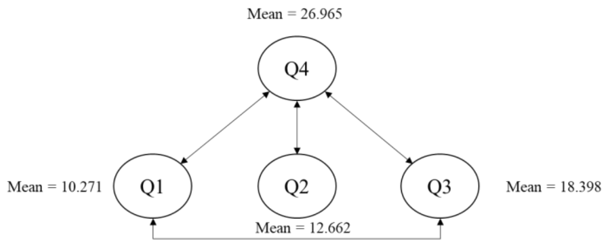

Figure 1: Detected patterns in differences in usage of shared accommodation based on the individual's income.

The final research question was to explore whether there are differences in the usage of shared accommodation based on the individual's residence. Eurostat provides a categorisation of respondents regarding residence on three categories: people living in cities, towns and suburbs, and rural areas. Therefore, the most recent data was used, for the year 2019, and analysis of variance was used to explore the differences between the three categories. The results indicate that there are statistically significant differences ($F=6.218$, $p<0.001$). To detect between which pairs there are statistically significant differences, independent samples t-test has been performed. The results indicate that the level of usage is the same among those from rural areas and towns and suburbs ($t=-1.383$, $p=0.172$). Interestingly, those from cities statistically significant use shared accommodation more than individuals from rural areas ($t=-3.463$, $p=0.001$) and from towns and suburbs ($t=-2.125$, $p=0.038$). The results clearly indicate that those from urban areas use shared accommodation more than those from rural areas.

4 DISCUSSION AND CONCLUSION

The sharing economy market is expected to grow in the upcoming period. Predictions are favourable for both shared accommodation and mobility, as well as gig economy. Therefore, to better understand the market, consumer behaviour studies in the field should be conducted. Herein, we strived to tackle the change in consumer behaviour from 2017 to 2019 in Europe regarding the usage of shared accommodation. We also took into consideration the effect of two socio-demographic factors: income and residence. Both factors proved to have a statistically significant impact on the usage of shared accommodation. Those with higher income and residing in cities use shared accommodation more. Our results suggest that the concept of shared accommodation should be promoted more in rural areas and among those with lower income. Also, demonstrations and tutorials on using and participating in shared accommodation could be organized to raise awareness and educate individuals. Future research deriving from the study could be towards conducting agent-based simulation to observe how and when will a particular individual start to use or stop using shared accommodation. A similar study was done in the sphere of shared mobility [10]. Also, it would be of interest to conduct conjoint analysis of the individuals' preferences regarding shared accommodation to better understand their true needs. The idea for such a study came from the work of Bojkovic et al. [11] who analysed the preferences for car sharing service attributes among university students. Also, segmentation analysis of users, or even countries, can be done using clustering and biclustering methods [12, 13].

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DETECTING TRENDING TOPICS CAPTIVATING CIRCULAR ECONOMY: A BIBLIOMETRIC-BASED APPROACH

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Abstract: Circular economy is a modern and innovative business model based on the regenerative utilisation of outputs from one process as inputs to another. Since it has many practical implementations, a few distinguished terms essentially mean the same – sharing resources is novel value creation. This paper aims to examine the trend topics corresponding to a circular economy in scientific production. Supported by the available *R Studio Bibliometrix* package for bibliometric analysis, the study's results, based on publications from three different groups of countries, suggest that a few distinguished terms are the most frequently used in all publications observed. Moreover, the study concludes that the three groups' immense difference in scientific contribution is notable.

Keywords: Sharing economy, Bibliometrix analysis, gig economy, circular economy, European region.

1 INTRODUCTION

Sustainable business and socially responsible management have come to the position of second to none when considering the strategically essential elements of a modern company. Therefore, numerous business models are proposed as a potential remedy. Currently, the most popular one is noted as the *circular economy*. Briefly described as a regenerative system consisting of input elements such as waste, emission and energy leakage, and core business processes referenced as maintenance, repair or reuse, thus all participating in creating a new customer value [1].

The ingenious idea of circular economy is its capacity to transform goods and services at the end of their business lifecycle into resources for other processes [2]. Therefore, various tech-based companies have turned their attention to the possibility of enhancing their operations by incorporating the new business model. Nowadays, the most popular sharing economy initiatives are those managing the business of accommodation sharing and vehicle sharing, followed by little book sharing [3]. However, there is a slight difference between sharing economy and circular economy. The first is usually related to *business-to-customers or customers-to-customers* business models; its global representatives are *AirBnb, Uber, CarGo, ShareNow, CouchSurfing* etc. The latter is mainly associated with *business-to-business* alternatives, which depend on resource-sharing processes. Currently, the most popular companies devoted to a circular economy are *IKEA, Unilever, Adidas etc.*

To demystify as many ambiguities following the concept of circular economy as possible, we intend to outline trending topics in academic literature connected with the central theme. Furthermore, we aim for three different groups of European countries based on their similar level of economic growth. Analysing trending topics is generally applied in social media content analysis [4], but we aim to propagate its usage in new research fields. More to the point, the results of this study are generated by the *R* package *Bibliometrix*. It is a practical and user-friendly software tool for bibliometric analysis widely utilised in any scientometric research [5–7]. Therefore, a brief literature review is presented in section 2, followed by an in-depth *Bibliometrix* analysis report. Furthermore, in section 4, feasible ameliorations are proposed for future similarly based analysis as a firm and concise discussion chapter. Finally, we summarise the study with closing words considering future research aspirations.

2 LITERATURE REVIEW

One of the most frequent approaches in bibliometric analysis is examining a term over an extended period. From that aspect, many parameters of scientific production could be assessed, such as authors' production over time, affiliations' production over time and co-citation authors' network [8]. Nonetheless, isolating one term as a significant representative of an evaluated topic is indicatively fallible. Sometimes there are a few similar terms in use for the same topic, but they are not interchangeable in all aspects of the current study context – they should all be considered in the analysis.

On the other hand, some studies examine various terms with the same meaning [9] but need more quantitative capacity to assess broader conclusions. Also, that pattern of using slight and dwarf sample sizes of publications whilst scrutinising essential topics such as circular economy is nothing but rare. That implication might be observed as rational since most authors are utilising efficient but not scalable software tools for bibliometric analysis [10,11], which disables them from examining greater samples of publications. Finally, a remarkable study on connections between circular economy and business and management [12] has recently shown that theoretical conceptualisations and technological solutions are the foundation of modern circular economy manifestations.

3 BIBLIOMETRIC ANALYSIS OF TREND TOPICS IN THE CIRCULAR ECONOMY

Bibliometric analysis was conducted in a statistical program called *R Studio*, explicitly focusing on *Bibliometrix* online library tool [7]. Firstly, necessary data was retrieved from the *Web of Science* online database, including the period from January 1st 2017 to December 31st 2022. More to the point, the query included eight similar terms for circular economy defined at [3], found only in article-based documents with authors based in three diverse groups of countries. Group number one consisted of Germany, France and Netherlands. Group number two occupied Hungary, Slovakia and Czechia, and the final group were Slovenia, Croatia and Serbia. Thus all sums up in a number of 5607 article-type documents.

Amongst many available functionalities, the three were particularly interesting to this paper, considering its essential idea of finding trending topics. The first indicates overall scientific production in publications in a particular group of countries. The second indicates the most frequently used words in publications for a specific group of countries. Finally, the third is the graph of trend topics in publications in a particular group of countries.

3.1 Trend topics in a group of Germany, France and the Netherlands

The corresponding group evaluated 4668 documents with an average citation rate of 20.27. The top three most productive affiliations are Delft University, University of Utrecht and Leiden University. A detailed preview of top trending topics is given in *Figure 1*.

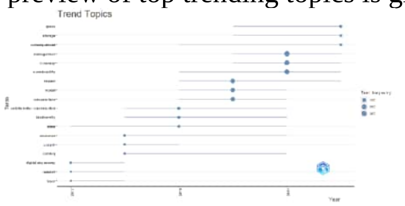

Figure 1: Trend topics in the group of documents from authors based in Germany, France and the Netherlands

As seen in *Figure 1*, the words: management, economy, and sustainability are the most frequent. However, there are topics with continuous presence over the timespan, such as green, storage and biodiversity. This could infer a direction of future terms used when describing a field of the circular economy.

3.2 Trend topics in a group of: Hungary, Slovakia and Czechia

544 documents with an average citation rate of 20.89 were evaluated within this central European group of countries. The top three most productive affiliations are Brno University, the University of Kosice and Charles University in Prague. An in-depth preview of top trending topics is given in *Figure 2*.

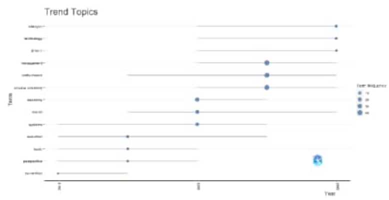

Figure 2: Trend topics in the group of documents from authors based in Hungary, Slovakia and Czechia

According to *Figure 2*, the words: management, performance, and circular economy are the most frequent. Nevertheless, topics which stand out with their continuous applications in literature are drivers, technology and nitrogen. Therefore, potential ideas for further examination of the topic might be established on these often-used terms.

3.3 Trend topics in a group of Slovenia, Croatia and Serbia

The final group evaluated 395 documents with an average citation rate of 27.32. The top three most productive affiliations are the University of Zagreb, the University of Ljubljana and the University of Belgrade. A detailed preview of top trending topics is shown in *Figure 3*.

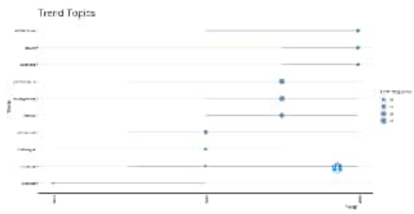

Figure 3: Trend topics in the group of documents from authors based in Slovenia, Croatia and Serbia

As *Figure 3.* suggests, words: management, performance and impact are the most frequent ones. However, a few terms on the list have continuously been used in the recent period, such as optimisation, model, pyrolysis, recovery and absorption. Notably, further alterations in scientific research in this field might follow the consumption of the latter terms.

4 DISCUSSION AND CONCLUSION

The results of our study show that circular economy is mainly described with popular terms such as sustainability, recovery, performance, management and sharing economy. Moreover, these are all examples of the trend topics in different countries which have a distinguishable impact on the research field. The first group significantly influences the field more than the other two groups combined. However, a slight drawback of the study might be an undersized number of examined publications and countries. The study could be a firm and practical instance of the *Bibliometrix* package and trend topics analysis usage in other bibliometric domains [13,14].

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The 17th International Symposium on
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Session 1:

Econometric Models and Statistics

FROM LINEAR TO SPATIAL REGRESSION: PARAMETRIC VERSUS NONPARAMETRIC FUNCTIONAL FORM

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Abstract: In the econometric regression framework, it is one of the primary importance to correctly specify the functional form of the model that ensures that the model's hypothesis tests are valid, and its prediction accuracy is satisfactory. The inherent correlation in economic data, which is a well-known phenomenon in econometrics, between observations at a specific location can be affected by observations from neighbouring locations, can cause the instability in the estimated model. The main objective of this paper is to outline a theoretical framework of parametric and nonparametric econometric models, both in the context of classical linear and spatial regressions, of which the goodness-of-fit are evaluated empirically. In the case study, we use a data set from the Eurostat regional statistical database, with an empirical model that attempts to explain the dynamics of the European regional unemployment rate.

Keywords: regional unemployment, semiparametric regression, spline functions, spatial regression.

1 INTRODUCTION

The main objective of this paper is to compare the application of parametric vs nonparametric regression models in the context of the econometric modelling. We commence our study by reviewing a classical linear regression model, where the model's parameters are estimated through the method of ordinary least squares. The study of linear models is now part of any standard textbook on the introduction to multivariate statistical analysis, e.g., Fahrmeir et al. (2021). To extend the linear model to more flexible framework, i.e., in the case when the relationship between the response and covariates exhibits local nonlinearities, we transpose covariates into spline functions, which are piece-wise polynomials, joined at break points, also called knots. A theory and application of spline functions is provided by De Boor (2001).

It is a well-known phenomenon that economic data that are observed in specific locations are affected by observations from neighbouring locations, which is called a spill-over effects. We outline a theoretical framework of the spatial autoregressive (SAR) model that incorporates spatial spill-over effects. A comprehensive review of spatial regression models is provided by Chi and Zhu (2019). Similarly, we follow by extending the spatial parametric model into more flexible nonparametric approach, which should capture the nonlinearities between the response and covariates. The paper by Basile at al. (2014) demonstrates the estimation technique for the spatial semiparametric model, which is carried out by using a 2-step "control function" approach since the two-stage least squares method might lead to inconsistent estimates of the regression parameters.

In the empirical study, we demonstrate the application of regression models outlined in the previous section and compare its suitability by examining the visual diagnostic tests of estimated models.

2 THEORETICAL FRAMEWORKS

This chapter defines data used in the empirical part of the paper and briefly introduces the most important theoretical aspects.

The paper uses a set of data from the Eurostat regional statistical database (Eurostat, 2023) to model the European unemployment problem. Database contains 209 European regions at NUTS 2 level (NUTS - Nomenclature of territorial units for statistics). Figure 1 provides an overview of the study area. This figure shows a real spatial distribution of Unemployment rates across the EU regions.

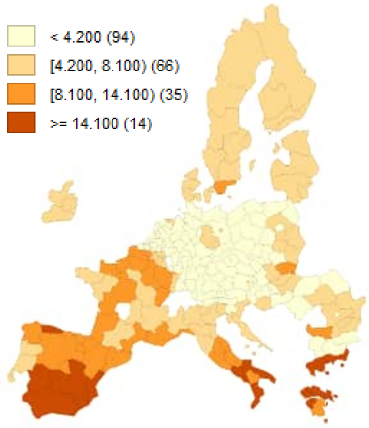

Figure 1: Spatial distribution of Unemployment rates 2019 – natural breaks map[1]
Source: authors' elaboration

In order to explain the dynamics of total unemployment rate (in percentage), we considered its key determining factors: *GDP* – gross domestic product (Euro per inhabitant), *HTC - employment in technology and knowledge-intensive sectors – high-technology sectors* (percentage of total employment), *SERV - employment in technology and knowledge-intensive sectors - services* (percentage of total employment) and *DEN - population density* (persons per square kilometre). *GDP* and *DEN* variables have logarithmically transformed forms. Empirical part of this paper consists of estimation of four econometric models to determine the factors that influence regional unemployment:

- Non-spatial parametric (linear) model - Ordinary Least Squares (OLS) regression:

$$y_i = \sum_{k=1}^{K} \beta_k x_{k,i} + \varepsilon_i, \quad i = 1, 2, \ldots, N \qquad \varepsilon_i \sim i.i.d.\left(0, \sigma_\varepsilon^2\right) \tag{1}$$

- Non-spatial nonlinear (semiparametric) model – spline regression:

$$y_i = \sum_{k=1}^{K} \beta_k x_{k,i} + \sum_{\delta=1}^{\Delta} g_\delta\left(x_{\delta,i}\right) + \varepsilon_i, \quad i = 1, 2, \ldots, N \qquad \varepsilon_i \sim i.i.d.\left(0, \sigma_\varepsilon^2\right) \tag{2}$$

- Spatial Autoregressive model (SAR) parametric (linear) model:

$$y_i = \rho \sum_{j=1}^{N} w_{ij} y_j + \sum_{k=1}^{K} \beta_k x_{k,i} + \varepsilon_i, \quad i = 1, 2, \ldots, N \qquad \varepsilon_i \sim i.i.d.\left(0, \sigma_\varepsilon^2\right) \tag{3}$$

- Semiparametric (nonlinear) SAR model – spline regression:

[1] Compared to the quartile map, the natural breaks criterion is better at grouping extreme observations. Interestingly, unlike quantile maps, the number of observations in each category can be highly unequal.

$$y_i = \rho \sum_{j=1}^{N} w_{ij} y_j + \sum_{k=1}^{K} \beta_k x_{k,i} + \sum_{\delta=1}^{\Delta} g_\delta \left(x_{\delta,i} \right) + \varepsilon_i, \quad i = 1, 2, \ldots, N \qquad \varepsilon_i \sim i.i.d.\left(0, \sigma_\varepsilon^2\right) \tag{4}$$

where y_i is the i-th response variable, β_k is a parameter of parametric function, x is a predictor, $g(.)$ is nonparametric function, ρ is spatial autoregressive parameter, w_{ij} are the elements of spatial weights matrix **W** describing the structure and intensity of spatial effects, ε_i is a random error term, and σ_ε^2 is random error variance.

Nonparametric regression is a more flexible modelling of the effects of continuous covariates on the dependent variable since classical linear model might not sufficiently capture local nonlinearities. Nonparametric functions $g(.)$ in models (2) and (4) are formed by transposing a covariate to spline functions. The spline functions have high flexibility and are able to handle data that changes in subintervals. The spatial regression models defined in (3) and (4) contain a spatial lag of the dependant variable. This means that the expected value of unemployment in the *i*th region is no longer influenced only by exogenous regional characteristics, but also by the exogenous characteristics of all other regions through a spatial multiplier. The estimation of the spatial econometric model is based on the queen contiguity weights (matrix **W**) – these binary weights indicate whether regions share a boundary or not. For detailed descritptions of the underlying theory and estimation procedures see, e.g., Fahrmeir et al. (2021) and Chi and Zhu (2019).

3 RESULTS

This chapter shows the results for diagnostic tests of econometric models (1-4)[2].

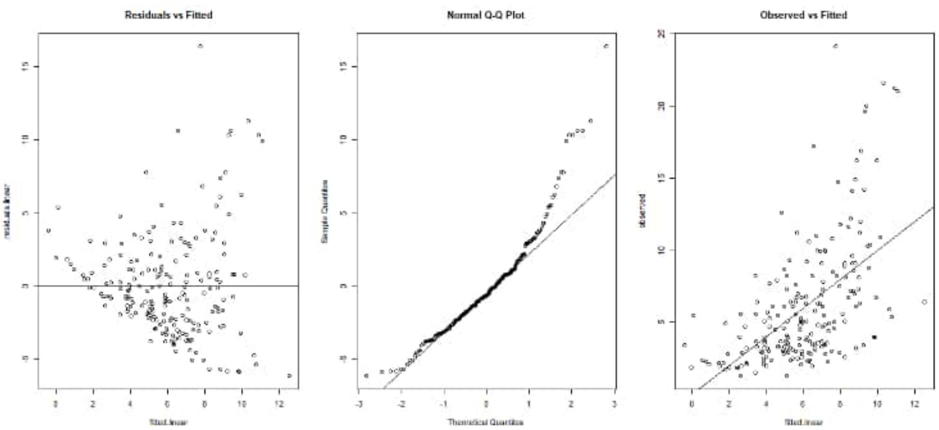

Figure 2: Non-spatial parametric (linear) model
Source: authors' elaboration

Fig. 2 shows that the plot of residuals vs fitted values have a non-random distribution, exhibiting a 'fan-like' shape, which indicates a correlation in random errors and a non-constant variance. This applies that the classical linear functional form for the fitted model is not appropriate. Moreover, the tails of the Q-Q plot further show that the assumption of the normal distribution of random errors is violated. The third plot of observed vs fitted values shows that

[2] In models (2) and (4) we considered non-parametric functions $g(.)$ for all covariates.

fitted values are much smaller than observed values, i.e., the fitted model severely underestimates the observed values.

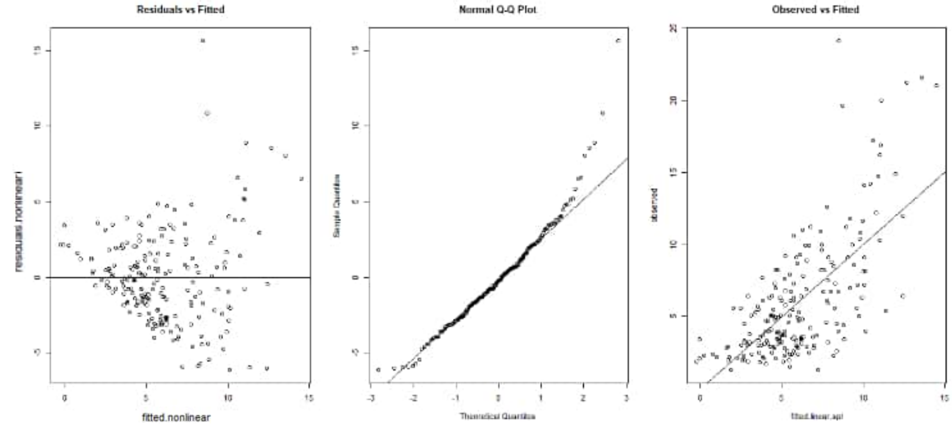

Figure 3: Non-spatial nonlinear (semiparametric) model – with 10 splines
Source: authors' elaboration

Fig. 3 shows that using basis spline functions in the estimator of the linear regression improves the fitted model. In particular, it can be seen that a deviation of tails in the Q-Q plot is less severe than in the linear model. The plot of observed vs fitted values shows that the prediction accuracy has significantly improved. However, a presence of the correlation among random errors remains relatively high as can be noted from the plot of residuals vs fitted values.

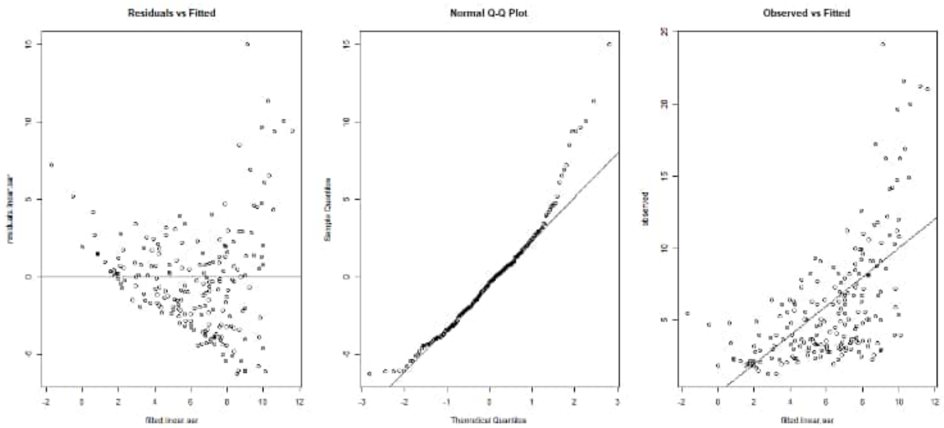

Figure 4: Spatial Autoregressive model (SAR) parametric (linear) model
Source: authors' elaboration

Fig. 4 shows that the spatial regression model does not contribute to any improvement in the fitted model. The plot of residual vs fitted values shows a non-random distribution and a correlation among errors. The digression of tails in the Q-Q plot indicates a violation of the normality and the plot of observed vs fitted values clearly shows that the predictive accuracy decreases compared to the non-spatial nonlinear model with splines.

Fig. 5 shows that in the semiparametric SAR model, having covariates in the form of basis spline functions does not contribute to any improvement in the fitted model's diagnostics, which can be read from the first two plots. The plot of observed vs fitted shows a slight improvement in the prediction accuracy when compared to the classical SAR.

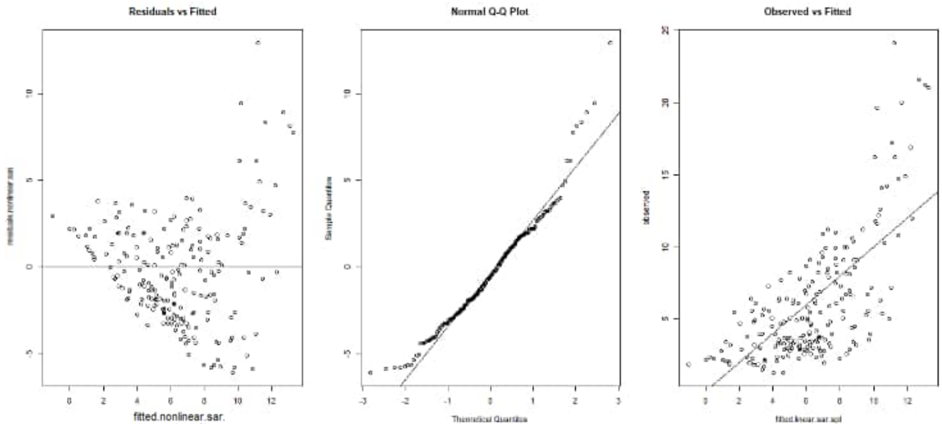

Figure 5: Semiparametric (nonlinear) SAR model – with 10 splines
Source: authors' elaboration

4 CONCLUSIONS

The main objective of this paper is to compare the application of parametric and nonparametric regression models for both classical linear, nonlinear and spatial framework. The advantage of spatial regressions is that these models incorporate economic spill-over effects, which occur between neighbouring locations, into the covariate structure. Moreover, a nonparametric model, defined by covariates extrapolated by basis spline functions, is more flexible in capturing the local nonlinearities between the response and covariates.

In the empirical study, the results show that using spline functions in the regression model lead to an improvement in the models' prediction accuracy. The presence of the correlation among random errors remain for all models that might be due to the model's simplistic structure, i.e., missing explanatory covariates or non-modelled error term. The best predictive accuracy is attained by the non-spatial nonlinear (semiparametric) model – spline regression. Our research can be expanded to encompass further extensions of nonparametric models such as generalised additive models and a more complex spatial regression with the correlated error term. Since the shape of the spline estimator is influenced by the location and the number of knots, it will be necessary to pay more attention to this issue.

Acknowledgement
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HIERARCHICAL CLUSTERING OF CEE COUNTRIES ACCORDING TO EDUCATIONAL AND LABOUR MARKET INDICATORS

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Abstract: This paper classifies Central and Eastern European (CEE) countries according to time-series data on wage ratios, tertiary education, and NEET rates from 2008 to 2021. Principal component analysis (PCA) and hierarchical cluster analysis (HCA) were used, first the combination of PCA and HCA for all three variables together, and then HCA for each variable separately. Multivariate analysis revealed a diverse labour market picture in CEE countries during two crisis periods.

Keywords: time series clustering, complete linkage, minimum wage, tertiary education, NEET, CEE countries

1 INTRODUCTION

The youth labour market is an eternal research topic that never gets old. Economies constantly undergo changing business cycles that shape market opportunities. The global financial crisis (2008-2013) hit the young workforce hard. After several recoveries, a new COVID-19 crisis placed renewed pressure on young people. The subject of this article is the youth labour market in the period from 2008-2021, measured by three indicators: the wage ratio, tertiary education and NEET (Not in Employment, Education or Training) rate. We investigate the dynamics and dissimilarities of the labour markets in 11 CEE (Central and Eastern Europe) countries. Before presenting our work, we offer a brief overview of recent research on this topic.

Speckesser et al. [15] combined educational, demographical and ALMP (Active Labor Market Policy) variables in a dynamic panel data model for European Union (EU) countries between 1999 and 2012. They found that employment incentives and job creation measures are effective tools for reducing youth unemployment in the EU. While youth between ages 15 and 19 experienced more positives from wage subsidies, job creation polices resulted with a higher benefit for the age group 20-24.

Maynou et al. [12] studied the convergence rates and determinants of youth NEET population at the EU`s NUTS 2 level, for 2000 through 2019. The NUTS (Nomenclature of territorial units for statistics) classification was used in order to identify different convergence patterns across EU regions. Their mixture of cluster analysis with panel data models identified the unemployment rate and ELET (Early Leavers from Education and Training) rate as the main drivers of NEET rates in all EU regions.

Vodă et al. [17] investigated the impact of minimum wage increases on youth employment in various EU NUTS 2 regions. Panel data analysis for the period 2008-2014 revealed a negative nexus between relative minimum wage (as measured by the Kaitz index; or the ratio of the minimum to the average wage) and youth employment. A higher minimum wage reduced the employment of young people ages 15 through 24.

Sturn [16] examined the effect of minimum wages on low-skilled and youth employment by using six different static and dynamic estimators (such as two-way fixed-effects (FE), LASSO – Least Absolute Shrinkage and Selection Operator, difference and system GMM – Generalized Method of Moments) on 19 OECD-Member countries. Two samples of different sizes and unbalanced panels showed little evidence supporting the negative effect for low-skilled and young workers between 1983 and 2013.

Fialová and Mysíková [8] analysed the youth employment effects of minimum wages in the so-called Vysegrád Group countries. Their panel data model for 2003 through 2016 indicated a decrease in regional youth employment in Hungary (2008-2011) and Czechia (2003-2007). Changes in relative minimum wage had a non-negative effect overall in all observed countries.

Pennoni et al. [13] applied maximum likelihood estimation with a hidden Markov model and cluster classification to EU countries. During the period 2004-2019, a three-cluster solution arose, forming groups from the best to the worst performing countries. In terms of NEET rates, Czechia showed the best results. Poland and Slovakia lowered the overall youth unemployment rate. On the other hand, Italy performed the worst on both youth labour indicators.

Mamucevska Bojadjieva et al. [11] analyzed the effect of different educational attainment levels on youth employability in SEE (southeastern European) countries from 2009 to 2019. They discovered a statistically significant and negative correlation between NEET rate and educational level in Croatia and North Macedonia. Panel data with multiple regression models confirmed dependence of the NEET rate on the velocity of the school-to-work transition.

This paper is divided into four parts. After this introduction and literature review, a brief explanation of this article's multivariate approach follows. This paper performs hierarchical time-series clustering with complete linkage method. Data sources and empirical results are closely described and presented in the third part. Emphasis falls on the graphical representation of results through heatmaps and dendrograms. The main focus is on 11 CEE countries and their performance in terms of educational and labour market indicators, from 2008 to 2021. This timespan covers the events of the global financial crisis (2008-2013), the post-crisis phase, and the most recent COVID-19 crisis. To the best of our knowledge, this is the first such study in the literature. Concluding remarks on the observed dynamics and recommendations for further research are presented in the final part.

2 METHODOLOGY

This paper classifies entire time series in 11 CEE countries in order to identify different patterns in the youth labour market. The basic idea is to perform a time series clustering with three variables: the wage ratios, tertiary education and NEET rates. For this purpose, we decided to use multivariate statistical methods. Our methodological approach combines hierarchical clustering with principal component analysis (PCA). First, we standardized all data points prior to multivariate analysis in order to improve data quality and secure internal consistency. In the next step, we performed PCA. PCA reduces the dimensionality of the data, contributes to a better classification of objects and interpretability by transforming the original variables into new mutually uncorrelated variables. [7]

Finally, we applied hierarchical clustering with complete linkage and Euclidean distances to whole time series. This exercise yielded four heatmaps and four dendrograms. Therefore, we used a combination of PCA and cluster analysis for all three variables together, and then cluster analysis for each variable separately. The entire multivariate analysis was carried out in Python. Key methodological terms are explained below.

The complete linkage (CL) algorithm calculates the maximum distance between two objects, a and b, in two different clusters, A and B, according to the next formula [3]:

$$\boldsymbol{\delta_{CL}(A,B) = \max_{i\epsilon A,\, j\epsilon B} \{d_{ij}\}} \tag{1}$$

Euclidean distance represents the shortest straight-line distance between two observations in Euclidean space. [9]. It is often used as the similarity measure between two data points. In the N −dimensional space the distance between objects a and b is usually calculated as follows:

$$\boldsymbol{d_{ij} = \|a - b\|_2 = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \ldots + (a_n - b_n)^2}} \tag{2}$$

The advantages of this approach are the compactness of cluster solutions, better performance with clusters of different sizes, and computational speed. On the other hand, the inseparability of previously created clusters, the inappropriateness of analysing large samples and the sensitivity to outliers are considered weaknesses. Euclidean distance is preferred for low-dimensional data, since there is no danger of falling into the curse of (high) dimensionality, that some authors in the existing literature warn against [1, 6]. We dealt with only three dimensions in our data (country, time and three economic indicators), which argues for the chosen distance measure.

Hierarchical clustering results are usually interpreted through heatmaps and dendrograms. Heatmaps are used for visualizing large quantities of multidimensional data. They represent a matrix combination of rows and columns with a colored grid displaying a pattern. In cluster analysis, heatmaps help to identify rows (objects) with similar values, since they are displayed as regions with similar colors. Dendrograms serve as a graphical tool for determining between possible clusters and the final cluster solution. They show the order of cluster pairing process. Both these approaches together make an excellent visual support in exploring the similarities between the observations used.

3 DATA AND RESULTS

Our analysis relies on the annual data taken from Eurostat databases. The area of interest consists of 11 CEE countries: Bulgaria (BG), Croatia (HR), Czechia (CZ), Estonia (EE), Hungary (HU), Latvia (LV), Lithuania (LT), Poland (PL), Romania (RO), Slovakia (SK) and Slovenia (SI). We investigate labour market dynamics over a 14-year period (2008-2021) is investigated. For comparability reasons, we standardized the data before generating heatmaps and dendrograms. We aligned all three variables in this study in the same direction by reversing NEET rate and expressing it as $100 - NEET$. This transformation enabled rising values in each variable to express social attributes in a monotonically positive way. Higher values of all three indicators are desirable. We therefore report NEET as the percentage of young individuals with some sort of employment, education or training. The acronym SEET might be apt.

Minimum wage is an indispensable tool in the fight against income inequality and poverty. The ratio of minimum to average wages, also known as the Kaitz index, was chosen for the analysis. The Kaitz index is often used by researchers as a relative measure of the minimum wage in relation to the national wage structure [2, 18].

The tertiary education indicator refers to young people aged 25-34 who have gained a certain level of college education according to ISCED 2011 (International Standard Classification of Education). For this purpose, we use ISCED levels 5-8, which include short-cycle tertiary education, and three college-level degrees: bachelor's, master's and doctoral degrees.

The NEET rate stands for the young people aged 25-34 who are not in employment, education or training. It can be disaggregated into NEET rate for unemployed youth and NEET rate for inactive youth. Eurostat provides this indicator for various age groups (such as 15-17, 15-19, 15-24, 15-29, 15-34, 18-24, 20-24, 20-34, 25-29). As explained earlier, we used a reversed version of NEET rate so that higher values of this variable can be interpreted as having a positive social impact.

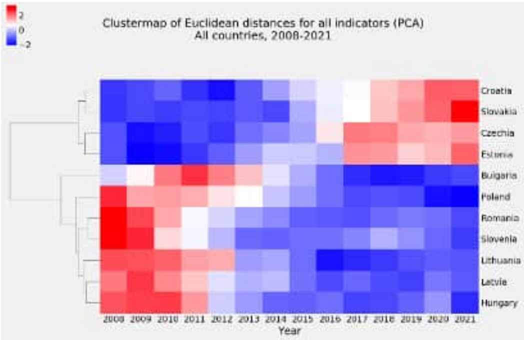

Figure 1: Heatmap of Euclidean distances for all educational and labour market indicators (PCA)

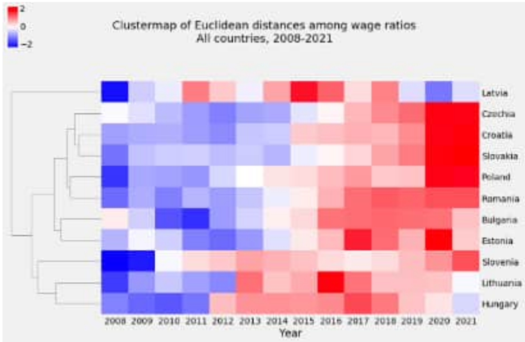

Figure 2: Heatmap of Euclidean distances among minimum-to-average wage ratios

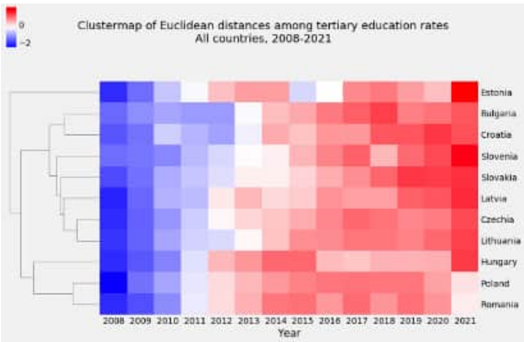

Figure 3: Heatmap of Euclidean distances among tertiary education rates

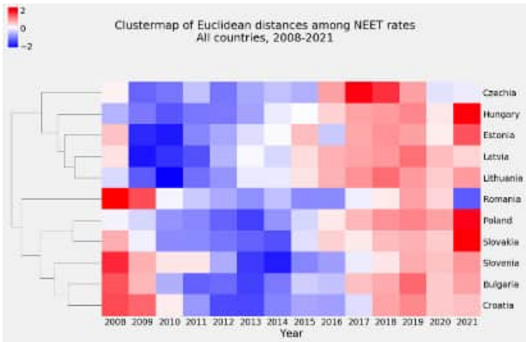

Figure 4: Heatmap of Euclidean distances among NEET rates

Cluster heatmaps and dendrograms visualize the hierarchical clustering results (Figures 1-8). Warmer heatmap colors indicate higher standardized Euclidean distances, while colder heatmap colors indicate lower standardized Euclidean distances. Observed countries are displayed in rows while years are in columns. Patterns in the heatmap indicate the association between the rows and columns.

The first heatmap and dendrogram (Figures 1 and 5) suggest a choice between two and three clusters. We opted for a two-cluster solution due to a very clear change of color pattern in the heatmap (from blue to red and vice versa). HR, SK, CZ and EE are classified into one group, and the rest of the countries are in the other. The first group shows progress through the overall positive trend in labour market and educational indicators. At the opposite pole, the members of the "setback group" (BG, RO, SI, PL, LT, LV and HU) experienced greater difficulty during the global recession and the COVID-19 pandemic. In response to these events, labour market moved in the opposite direction.

Single variable analysis is less clear. The second heatmap and dendrogram (Figures 2 and 6) point to different clustering options. The heatmap suggests a visually striking four-cluster solution. Both LV and SI stand out as single clusters, LT and HU are grouped together. All other CEE countries (from CZ to EE) form one major cluster. On the other hand, the dendrogram clearly suggests a two-cluster solution; a bloc of seven countries (from BG to RO) on one side, and a group of four (from LV to HU) on the other.

Wage ratios move in various directions, depending on crises and government reforms. LV ("the Baltic in-betweener") recorded oscillating wage ratios in the observed period, with a peak in 2015 and a downward trend with the onset of the COVID-19 crisis. SI ("CEE's top wage performer") has seen tremendous progress in wage ratios. Slovenia adopted two giant minimum wage increases in 2010 and 2018 [14], making it the country with the smallest gap between minimum and average wage among all CEE countries. These are the main reasons behind that country's assignment to a single-member cluster. LT and HU displayed a similar pattern after the COVID-19 crisis, which also justifies their joint linkage. Therefore, a four-cluster solution in the heatmap appears to be more appropriate than a two-fold division in the dendrogram.

The third heatmap and dendrogram (Figures 3 and 7) are complementary. It is possible to pick between 3-5 clusters. We opted for a three-cluster solution. EE had a positive trend until it experienced a slump in higher education enrollment in 2015, which can be attributed to that country's inherent demographical challenges. [10]. Nevertheless, EE continued to strive for new heights in this department since then. HU, PL and RO constitute a group of educationally consistent countries, attaining a steady state in tertiary education.

The biggest grouping gathers together the rest of the countries, from BG to SK. These educationally prosperous countries have high rates of tertiary education. The value of education increases in times of crisis and investment in tertiary education is the best substitute for a lack of work experience. At the same time, the opportunity costs of higher education are considerably lower due to the lack of employment opportunities for youth population. [4] Therefore, it is actually beneficial to use the time of recession for education.

The fourth heatmap and dendrogram (Figures 4 and 8) have opposite graphical results. The heatmap pattern signals 2-5 clusters, with a three-cluster solution as a reasonable pick. This way of grouping puts the Baltic countries (EE, LV and LT) alongside CZ and HU. RO is distinguished as a single cluster. The rest of the CEE countries (from PL to HR) form the third circle of NEET (SEET) performers.

The dendrogram provides multiple clustering options ranging between two and three clusters. In this case, the better solution may consist of two clusters. For geographical and geopolitical reasons, one group may consist of the four Vysegrád countries and the three Baltic countries. The other cluster contains SI, HR, BG and RO. The former cluster is characterized by similar time series for the NEET (SEET) indicator. These countries (from PL to LT) have struggled to contain the growth of the NEET population. Moreover, each one of them (except CZ) achieved a lower rate of young people lacking education, employment, or training, as between the first and last time points.

By contrast, it is interesting to see SI in the company of HR, BG and RO. SI is arguably the best at dealing with NEETs (rate around 10%), while the other trio faced double the NEET rates (above 20%) in the observed period. The reason for combining these countries into a single cluster can be found in the growing trend during the impact of the financial crisis (2008-2013). HR, BG and RO in 2021 recorded a higher NEET rate compared to 2008. SI achieved the opposite. These results suggest that we can accept the solution obtained by the dendrogram.

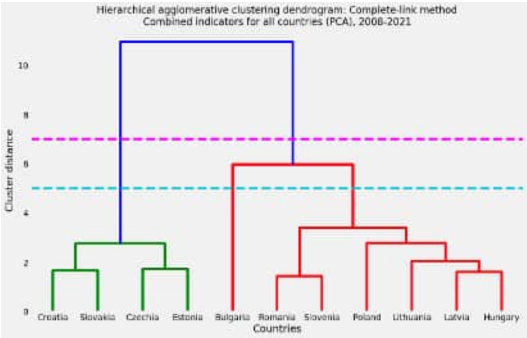

Figure 5: Dendrogram of combined educational and labour market indicators (PCA)

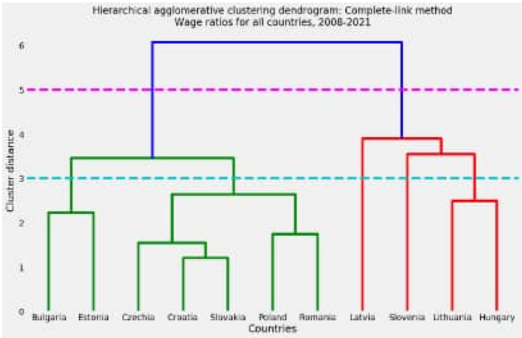

Figure 6: Dendrogram of minimum-to-average wage ratios

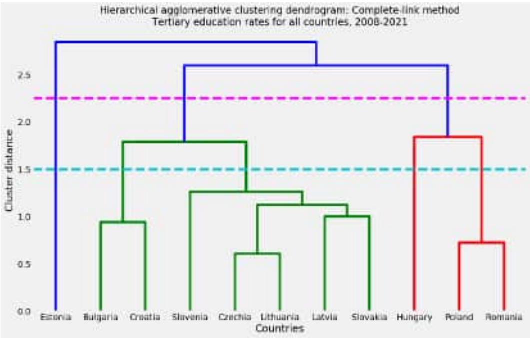

Figure 7: Dendrogram of tertiary education rates

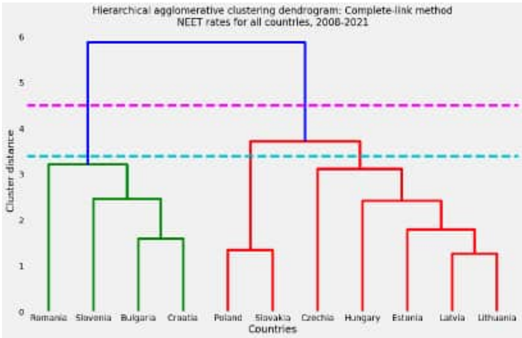

Figure 8: Dendrogram of NEET rates

Analysis of the labour market in CEE countries in the observed period shows pronounced dynamics and heterogeneity. The two crisis periods reveal that certain countries (such as SI) are able to withstand the crisis more easily, while others (such as BG) experience a stronger impact. This is precisely why it is worth investigating the total and individual effects (of variables) on different grouping of countries.

4 CONCLUSION

We conducted time-series cluster analysis of 11 CEE countries. The outcomes are very intriguing. The complete linkage algorithm produced clusters of uneven sizes. The results depend on the variables included in the clustering algorithm. Croatia, Slovakia, Czechia and Estonia made the most progress from 2008 to 2021. The rest of the CEE region lost ground.

To the best of our knowledge, this paper marks the first application of time-series clustering on CEE countries across the two most recent crisis periods. It illuminates the impact of minimum wage legislation and educational attainment on the European youth labour market. In addition to the selected ones, there are many other variables that also shape the image of CEE countries, such as other levels of education and labour market indicators. Furthermore, due to multiple missing values, the original Kaitz index (the ratio of minimum to median wages, [5]) was not used in the analysis. Instead, we chose a modified version of the same index (the ratio of minimum to average wages). This places some limitation on the results, as the median wage is generally considered more representative than the average wage. Overcoming the aforementioned could provide a more accurate assessment of the youth labour market. Future research should apply other clustering methods such as *k*-means, DBSCAN

(Density-Based Spatial Clustering of Applications with Noise) and spectral clustering. Different clustering methods will provide additional insight.

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USING THE STOCHASTIC FRONTIER APPROACH TO DETERMINE THE GENDER WAGE GAP AND LABOUR MARKET EFFICIENCY

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Abstract: This paper analyzes the gender wage gap, as this issue has been insufficiently studied in the labor market of the Republic of Serbia. Using data from a panel study derived from a survey on income and living conditions, the paper examines whether the crisis caused by the COVID-19 pandemic has affected the extent of the wage gap in the Republic of Serbia. The paper employs the stochastic frontier model, which is used to determine the discrepancy between men's and women's wages and the differences in efficiency in the labor market. The results of applying this approach confirmed that there is a statistically significant difference between men's and women's wages and that participants in the labor market differ in their efficiency, which is measured by the difference between realized and potential wages of workers.

Keywords: gender wage gap, potential wage, Serbia, stochastic frontier, survey data.

1 INTRODUCTION

The extent and structure of gender differences in wages are observed considering paid work at the main job. The analysis of this practice is important for every society because it can have wider economic and social consequences on the formation of the total labor supply. In post-transition European countries, the issue of gender differences in wages becomes even more significant because it was assumed that the transition to a market affects the worse position of women in the labor market, and, thus deepening of gender differences in wages. However, the results of empirical studies do not lead to a single conclusion and differ from country to country [13, 14]. In most post-transition countries, the economic position of women was worse during the first years of transition. Thereafter, the relative position of women improved, not only because the market prevented discriminatory behaviour, but also because of the structural change that accompanied privatization and the opening of economies. Although there is a general trend toward wage convergence due to women's improved characteristics, the gender wage gap remains deeply rooted [5, 6, 10].

Numerous studies dealing with this problem present results showing that the gender wage gap is due to the greater participation of women in certain economic and property sectors, educational segregation, segregation by occupation with less access to high-paying occupations, which, combined with greater income inequality due to more difficult access to top positions, leads to a deepening of the wage gap [2, 4, 9]. On the other hand, the low participation of women with worse human capital attributes in the labour market leads to a narrowing of the wage gap, reflecting an unrealistic situation.

Therefore, the research presented in this paper aims to show how big the gender gap in wages was during the pandemic caused by the COVID-19 crisis. Basic indicators of the labor market indicated slight movements in the labor force, which led to imbalances. The experiences of European countries with the impact of the COVID-19 pandemic on the labor market are different. In the most developed European economy, imbalances in the labor market that arose under the influence of the pandemic, viewed through the prism of working conditions, are not the most important factor explaining the subjective feeling of differences in well-being among women [15]. Studies dealing with the impact of government measures on monetary poverty and income inequality have confirmed their effectiveness [1]. On the other hand, studies that dealt with the impact of COVID-19 on the position of women in the family indicated a deepening of gender differences regarding unpaid care of dependent persons, such as children, the elderly, and helpless persons [7, 11]. Those studies confirmed that COVID-19 had an additional impact on existing labor market gaps.

2 METHODOLOGY

The stochastic frontier model is applied in estimating the discrepancy between men's and women's wages. This approach is recently used in [3] to establish the empirical relationship between wage determination and the potential impact of market discrimination in European countries. The stochastic frontier approach enables market discrimination to be embedded in a female dummy included in the model of realized wages. Model (1) represents the wage frontier:

$$\ln{(wage)_{it}} = c + aHCC_{it} + bJRC_{it} + e_{it} - u_i \qquad (1)$$

The dependent variable measures hourly net wages from the main job transformed using the natural logarithm, while human capital characteristics (HCC) and job-related characteristics (JRC) are a set of explanatory variables included in the wage model (1) following the theoretical constructs. This part of the model (1) is a mathematical representation of the realized wages, with the first error component e_{it} being a random disturbance term. The second term $-u_i$ is an inefficiency component and, in the empirical model (1), represents the discrepancy between the realized and potential market wages. Each of these error components has a different distributional assumption. Based on the distribution of the composite error term [8] showed that the maximum likelihood (ML) is a consistent estimator of the stochastic frontier model (1).

To perform the empirical exercise, a pooled sample of individuals who were employed over 2019-2020 and self-reported positive wages and hours worked were included in (1). The data come from the Serbian Survey on Income and Living Conditions for 2019-2020 [12].

3 RESULTS AND DISCUSSION

Table 1 reports the ML estimates of the stochastic frontier model for wage earners in Serbia. Both individual attributes and job-related characteristics, including those that characterize the employer, are employed in the empirical model estimation. This approach identifies the potential market wage an employee may earn given the attributes mentioned (human capital, job- and employer-related). The difference between the realized and the potential wage measures the inefficiency an employee meets to achieve full market earnings. The estimates of the characteristics in the stochastic frontier model explain employee inefficiency in achieving potential earnings. Since the labor market in Serbia is characterized by persistent differences in wages of men and women, it was of particular interest to investigate whether this difference was significant during the COVID-19 pandemic. Results reported in Table 1 (Model 1, the left-hand side panel) show that being a woman statistically significantly reduces earnings potential in the Serbian labour market given the same attributes as men. Statistical interpretation of this finding is that employed women have a log hourly net wage lower by 0.11 compared to men, given all other characteristics held at the same level. This result leads to the conclusion that there is gender wage discrimination in the Serbian labor market.

The obstacle for women to achieve total market wages is reflected in the negative impact of health conditions, regional dislocation of jobs observed through the level of urbanization, the private sector ownership, and the associations with particular sectors and occupations. These characteristics reduce women's earnings, i.e., increase the wage disparity between men and women. The fact that women have better educational attainment and somewhat less work experience is insufficient to reconcile the observed wage differences.

A parsimonious stochastic frontier model is estimated on a sample of those who made the transition in the labor market, moving from the status of unemployed to the status of an employed person. Selected variables are included (unreported job-related characteristics significantly reduce the initial sample). Results in Table 1 (Model 2, the right-hand side panel) show that women employed during the pandemic may expect log wages lower by 0.09 compared to men.

Table 1: ML Estimates of the Wage Model
(Dependent variable log hourly net wage)

Variable	***Model 1***			***Model 2***		
	Estimate			***Estimate***		
	Coeffi-cient	*95%* CI_{lower}	*95%* CI_{upper}	*Coeffi-cient*	*95%* CI_{lower}	*95%* CI_{upper}
Age	0.02	0.00	0.03	0.01	-0.02	0.04
Age squared	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
Experience	0.01	0.00	0.01	0.00	-0.01	0.02
Experience squared	0.00	-0.00	0.00	-0.00	-0.00	0.00
Medium education	0.09	0.04	0.14	0.07	-0.06	0.19
High education	0.34	0.27	0.40	0.40	0.24	0.56
Married	0.00	-0.03	0.03			
Permanent employment	0.03	-0.01	0.06			
Responsibility	0.17	0.13	0.21			
Health status	-0.12	-0.22	-0.02			
Region	0.15	0.13	0.18			
Degree of urbanization (intermediate)	-0.00	-0.03	0.03			
Degree of urbanization (densely)	-0.00	-0.04	0.03			
Small enterprise	0.09	0.06	0.12			
Medium and large enterprise	0.12	0.08	0.16			
Privately owned	-0.12	-0.15	-0.09			
Female	-0.11	-0.14	-0.09	-0.09	-0.17	-0.00
Intercept	5.10	4.83	5.37	5.01	4.47	5.55
Model fit statistics						
No. of obs.	2,840			432		
LL	-998.47			-172.32		
$\boldsymbol{\mu}$	-1.99			-1.19		
$\boldsymbol{\chi^2(k)}$	1495.62			38.31		

Note: Sectoral and occupational dummies are included. Model 1 includes k=22, and model 2 k=7 (k=variable).
Source: Author based on SILC 2019-2020 data.

4 CONCLUSION

In this paper, the stochastic frontier model is used to determine the discrepancy between men's and women's wages and the differences in efficiency in achieving total market wages. The results of applying this approach confirmed a statistically significant difference between men's and women's wages and that participants in the labor market differ in their efficiency, which is measured by the difference between realized and potential wages of workers. These results have implications for public policies, not only in the domain of social policy affairs but the overall vulnerability of women in the labor market.

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CONFIRMATORY FACTOR ANALYSIS OF THE SPORTS TEAM REPUTATION SCALE: A CASE STUDY IN CROATIA

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Abstract: The sports industry is among the most developed industries in the world. Despite the absolute importance of reputation in the business, there was no psychometrically sound scale to measure the spectator-based sports team reputation. This empirical research attempted to confirm the validity of the SSTR on a convenient sample of the most hot-blooded football fans. Based on exploratory and confirmatory factor analysis, results showed that factor structure is confirmed and that the SSTR is a valid instrument for measuring the sports team reputation. Financial soundness did not prove to be a key factor in reputation.

Keywords: sports industry, sports team reputation, factor analysis, football spectators, dysfunctional fan

1 INTRODUCTION AND THEORETICAL BACKGROUND

The global sports industry is worth 501.43 billion US dollars in 2022 [4]. The previous and expected growth is transforming the sports industry and forcing clubs to apply the most modern business approaches that characterize business in the most profitable global industries. Football is the most popular sport in the world, which also has the largest market share and profit on the global sports market. Most of the big football clubs are organized as business companies, and some of them are listed on stock exchanges around the world. Reyhan [3] points out that football is not just a sport, it is a social phenomenon that greatly affects social life. This especially applies to fans who represent the most important stakeholders in the sports industry. The reputation of sports clubs from the perspective of fans is of exceptional importance for their business in the increasingly strong competition in the sports sector, and in general in the entertainment (leisure) industry. Therefore, marketing managers should continuously examine the reputation and coordinate the club's operations with predetermined goals. This paper builds on prior research [2] in which authors conceptualized and developed the spectator-based sports team reputation scale (SSTR).

There are many problems in national football leagues throughout the EU, such as match-fixing, poor infrastructure, the outflow of young players, violence, etc. The reasons for low attendance at football stadiums stem from all the previously mentioned problems, but also from the insufficient attractiveness of national football league for spectators, sponsors and investors. UEFA set the financial fair play regulations to prevent professional football clubs spending more than they earn and in order not to fall into financial trouble. The absence of result uncertainty, extremely low income from TV broadcasts and modest attendance at stadiums created major problems for clubs in their daily operations. Additionally, hooliganism and increasingly violent behaviour evidence that sports clubs need to use increasingly sophisticated business tools in order to be able to generate income and achieve sports results. Reputation is one such business tool that can help a sports team differentiate itself from the competition and develop a competitive advantage. This concept is extremely important in attracting and binding stakeholders, especially fans. Hunt et al. [1] made the soundest classification of fans. They defined fans as consumers who have a certain level of attachment to a sport-related object. They created a fan typology based on motives and behavior and obtained the following five segments: temporary fan, local fan, devoted fan, fanatical fan and dysfunctional fan. The first four types of fans are characterized by fan-like behavior in contrast to dysfunctional fans. Dysfunctional fans put their favorite club first and neglect their friends and family. Because of their relationship with the sports club, they lose the ability to function in a natural social environment and tend to be socially disruptive, and engage in violent behavior towards other individuals. To our knowledge, in this paper, for the first time, the applicability and validity of SSTR would be tested on a sample of dysfunctional fans.

Marketing managers try to identify factors that can contribute to the club's economic success, and a positive reputation is precisely one of the most valuable intangible assets. It represents an abstract concept that shows how stakeholders perceive the organization. Theoretically, the sports team's reputation origins from connecting the concept of corporate reputation with stakeholder theory. Suchao-in et al. [5] state that it is about the collective perception and define it as "an outcome of a communication process by which individuals develop an emotional reaction and perception to actions associated with a sports team". By creating positive experiences for spectators with the sports team, it is possible to manage their perceptions. Jang et al. [2] define spectator-based sports team reputation as "the overall perception of a sports team based on spectators' reaction to all previous interaction experiences with the sports team's activities". The authors [2] define the following six dimensions of SSTR as the spectator's perception of: (1) team performance – the quality of sports team performance that leads to team success; (2) team tradition – overall achievement and business success in the past; (3) team social responsibility – sports team commitment to building relationship with and contributing to the development of the community; (4) spectator orientation – about the degree to which a sports team puts the spectators at the center of its focus and concern for them; (5) management quality – superior management quality and strategic vision of a sports team in comparison to those of other similar organizations; and (6) financial soundness – sports team's financial solidity and profitability.

In recent years, just a few studies have started to appear in which the validity of SSTR and/or the connection of reputation with related constructs are investigated. Yousaf et al. [8] partially removed doubts about the applicability and lack of generalisability of SSTR. Their version of the scale tested on spectators in a different sports context (cricket) was modified and consists of 17 items extracted into six factors. Takamatsu [6] demonstrated that the SSTR was robust across three different types of stakeholders and proved that it can be applied as a general sports team reputation scale. Yavuz Eroğlu et al. [7] investigated the relationship between sports team reputation and team identification. Their results point to three dimensions of a reputation as extremely important because they increase fan identification - team tradition, team social responsibility, and spectator orientation. On the other hand, team performance, management quality, and financial soundness did not significantly influence fan identification. Suchao-in et al. [5] found that team brand association directly influences sports team reputation and that reputation influences satisfaction and loyalty. However, despite several recent works conducted in different sports contexts and on different samples, doubts still remain about the possibility of the generalisability of SSTR. To our best knowledge, this is the first paper to investigate the validity of the SSTR on a sample of dysfunctional football fans in one of the EU countries.

2 METHODOLOGY AND RESULTS

In the empirical part of the work, descriptive research was conducted. Primary data were collected on a convenient sample among fans of legal age during March and April 2022. The survey questionnaire was created on the Qualtrics Survey website. It was distributed and collected online through three closed Facebook groups that gather the most ardent and aggressive fans of the organized fan group of an extremely popular football club in Croatia. The questionnaire consisted of closed-ended questions. In the demographic part, data related to gender, age, education, and average personal monthly income of the respondents were collected. The measurement scale SSTR [2] was used, which was slightly modified according to the characteristics of the investigated fans. The SSTR scale originally consists of 19 items or six dimensions: team performance (TP), team tradition (TT), team social responsibility (TSR), spectator orientation (SO), management quality (MQ), and financial soundness (FS). A Likert scale from 1 (completely disagree) to 6 (completely agree) was used to measure the attitude of fans regarding the reputation of a sports team. When processing data, the following statistical methods were applied: Kaiser-Meyer-Olkin Measure of Sampling Adequacy, Bartlett's Test of Sphericity, exploratory factor analysis (EFA), confirmatory factor analysis (CFA), and tests of reliability and convergent validity of measurement scale. The statistical program SPSS (version 24) and the SPSS plugin in the form of Amos software were used to help with data processing.

Out of a total of 318 completed surveys, 293 of them were used for further processing. 73% (214) of respondents are male, and 27% (79) are female. The structure of respondents by gender is expected, since football is a dominantly "male" sport in Croatia, and men represent the largest share of the most ardent fans. The largest number of respondents belongs to the age group from 25 to 34 years, 52.6% (154), while the least number of respondents, 6.5%, belongs to the age group over 45 (19). To the age group of 35 to 44 years belong 22.9% of respondents (67), and 18.1% (53) of respondents belong to the age group of 18 to 24 years. It was to be expected that the smallest number of respondents was in the oldest age group because the questionnaire was distributed via the Internet, that is, via fan forums. The largest proportion of respondents, 50.9% (149) have completed high school; 25.3% (74) completed primary school; 17.4% (51) of the respondents have completed and/or are attending undergraduate studies, while 6.5% (19) of the respondents have completed and/or are attending graduate studies. It was to be expected that the structure of respondents would be dominated by fans with a lower level of education (with completed primary or secondary school) and a younger population that has not yet started a family or does not have a permanent job, such as the student population. 33.1% (97) of respondents have an income between 700 - 1000 euros. 20.8% (61) of the respondents have an income between 500 to 700 euros. An income of less than 500 euros has 16.4% (48) of the respondents while 16% (47) have an income in the range from 1001 to 1300 euros. The least percentage of the respondents (13.7% or 40 out of the total) have income greater than 1301 euros.

After the descriptive analysis of the collected data, the suitability of the data for factor analysis was performed, followed by the exploratory factor analysis (EFA) (pilot-testing of the structure of the scale) and confirmatory factor analysis (CFA) (that is, refinement of the measurement scale itself). An initial quantitative evaluation of the structure of the SSTR scale was made, i.e. an exploratory factor analysis (EFA) was performed on the original 19 items. The purpose of the analysis is to clarify which factor structure of the scale best explains the collected data. Examination of the correlation matrix confirmed the suitability of the collected data for factor analysis due to the presence of many coefficients of 0.3 and above. Cronbach alpha results for six components (factors), the same ones initially proposed by Jang et al. [2], were as follows: team performance $r = 0.724$, team tradition $r = 0.732$, team social responsibility $r = 0.711$, spectators orientation $r = 0.829$, management quality $r = 0.853$ and financial soundness $r = 0.663$. Since the internal consistency (reliability) of the financial soundness component is questionable, its items were additionally investigated. The results show that Cronbach's alpha for the financial stability component would increase to 0.785 if item FS1 "the club is doing well financially" is deleted. Since the value of Cronbach's alpha for the SSTR measuring scale is 0.835, and the removal of any initially proposed item would not significantly increase its internal consistency, i.e. the reliability of the measuring scale, we refrained from the exclusion of the mentioned item from the measuring instrument. Based on the results of the analysis, it can be concluded that SSTRs are confirmed as valid instruments for measuring the reputation of sports teams. The Kaiser-Meyer-Olkin measure was 0.787, and Bartlett's test was statistically significant ($p < 0.05$), based on which it can be concluded that there is a significant correlation between the observed components and that the data set is suitable for factor analysis.

Principal component analysis (PCA) was used to investigate the factor structure of the measurement scale. Using the Kaiser criterion, the first six components (extracted factors) recorded eigenvalues above 1. The eigenvalue for the first factor is 5,070, for the second 2,300, for the third 1,887, for the fourth 1,493, for the fifth 1,264 and for the sixth factor 1,033. These six components explain 68,664% of the variance. Inspection of Catell's scree plot revealed a clear break after the sixth component, thus the final decision was to keep all six components for further research. Since the observed sample is larger than 150 respondents, all factor coefficients greater than 0.35 are significant when interpreting the factor structure. Analysing the factor structure, it is evident that there is no single component that does not have at least one significant factor coefficient, which confirms that all components are validly represented by the tested factor model in accordance with the theoretical conceptualization of the measurement scale. From the rotated component matrix analysis, it is evident that in the first item of financial soundness (FS1), "the club is financially sound", there is more significant cross-loadings of the factor coefficients, thus, one should think about removing that variable from the observed model. The EFA results confirmed the identical factor structure obtained by Jang et al. [2] with the exception of item FS1.

A CFA was conducted (using the maximum likelihood method) to determine the validity of the measurement scale and to confirm the factor structure resulting from the EFA. The CFA results for the original SSTR model (model 1) showed that this model meets the minimum requirements but does not adequately fit the data collected for this research; $X^2=425.6$; df=137, RMSEA =0.085, SRMR=0.0879, CFI=0.85, GFI=0.86 and IFI=0.85. Additionally, the ratio of chi-square to degrees of freedom (X2/df) is 3.106, which is above the recommended values and does not suggest the appropriateness of the model. The RMSEA and SRMR values that are above the recommended values, and the values of the other used indices (GFI, CFI and IFI) that are below them, inevitably confirm this. Furthermore, almost all items of the original SSTR showed factor loadings higher than 0.60, except for item FS1 whose loading is 0.33, and the item team social responsibility (TSR4), "the club encourages volunteer work of its players in the local community" with a value of 0.53. The CFA results for the original model indicated that further research should try to reduce the number of items in order to improve the overall suitability of the research model. In order to improve the validity of the model, a second CFA was performed (model 2). At the beginning, based on the value of the modification index and for the better suitability of the future research model, a correlation between errors e1 and e3 within the same factor (team tradition) was added. Furthermore, items FS1 and TSR4 were removed from the original model due to factor loadings that were below the recommended 0.6.

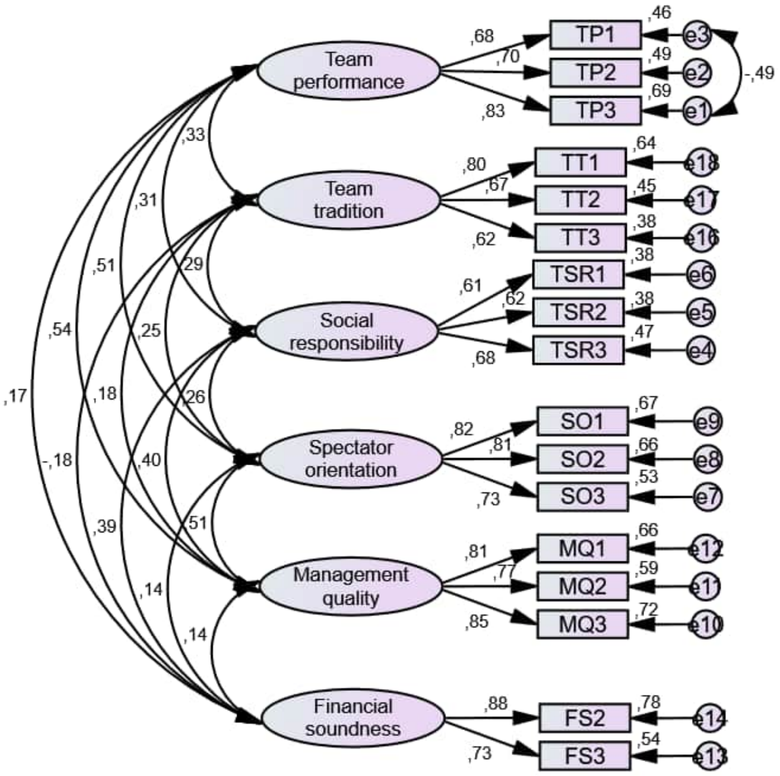

Figure 1: Results of CFA for SSTR (model 2)

The CFA results for model 2 (see Figure 1 and Table 1) shows that the appropriateness of the model that fits the data collected in this research is fully suggested; X^2=263.2; df=103, RMSEA =0.073, SRMR=0.0562, CFI=0.911, GFI=0.902 and IFI=0.912. Since the X2/df ratio is 2.555, and the RMSEA and SRMR suggest the complete suitability of the model, and the values of the used indices (GFI, CFI and IFI) are higher than the recommended values, it can be concluded that the collected data are still more suitable for model 2, which consists of six factors and 17 items, than the original model 1. The composite reliability of the scale was confirmed, since all factors have higher omega values (CR) than the recommended 0.7, except in the case of the team social responsibility factor, where the value is very close to the recommended limit values (0.67). The obtained values of factor loadings and AVE index for all items in the model are higher than the recommended 0.5, which points to the convergent validity of all six dimensions of SSTR.

Table 1: CFA and comparison of fit index values for the original and recommended model

	X^2	*df*	X^2/df	*RMSEA*	*SRMR*	*GFI*	*CFI*	*IFI*
Model 1	425.6	137	3.106	0.085	0.0879	0.862	0.856	0.858
Model 2	263.2	103	2.555	0.073	0.0562	0.902	0.911	0.912
Recommended values	/	/	between 2 - 3	< 0.06	< 0.08	> 0.9	> 0.9	> 0.9

3 CONCLUSION

Sport has become an important economic activity in almost all developed countries in the world, and the reputation of sports clubs is crucial for their successful business in an increasingly competitive environment. The reputation of sports clubs from the perspective of their fans is extremely important because fans represent key stakeholders in the value chain and are still the main sources of income for sports clubs. The CFA results confirmed the factor structure of the original SSTR model, but not a complete fit. Only after items FS1 and TSR4 were removed from the model, the CFA results for the adjusted SSTR model (model 2) showed a fully suggestive model fit to the data collected in this study. The adjusted SSTR model with six factors and 17 items was confirmed, however, financial stability did not prove to be a key dimension of the reputation of sports clubs in Croatia. To sum up, with the added value of this research on the improvement of the original scale, the findings of this paper show the possibility of extending its generalization from the student population, which deals more with positive fan behavior, to a predominantly dysfunctional fan sample characterized by antisocial, disruptive and deviant behavior. The main limitations stem from the research sample, which is not fully representative, and the fact that it is a cross-sectional study. The research was conducted on the most ardent fans of only one football club, and the influence of the social desirability of the answer cannot be ignored. The obtained results can serve as a good starting point for some future longitudinal research on the importance of reputation in sports.

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PREDICTION OF HEART DISEASE USING CLASSIFICATION AND FEATURE SELECTION METHODS

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Abstract: Heart disease, known as cardiovascular disease has been one of the main causes of death worldwide in recent years. It is affected by various risk factors such as high blood pressure, high cholesterol, diabetes, smoking, obesity, physical inactivity, etc. In this study, Cleveland database of UCI repository of heart disease patients is used. The dataset includes 303 instances and 76 variables which of 14 variables are considered. At first, three different feature selection techniques, namely LASSO, Elastic Net and Adaptive LASSO under specific tuning parameters determined by cross-validation, are applied to reduce the number of independent variables used for the prediction. Then, five different classification techniques, K-nearest neighbor, Support Vector Machine, Decision Tree, Random Forest, and Logistic Regression are performed to predict heart disease. Performances of these methods are measured by different metrics, such as accuracy, sensitivity, specificity, and Matthew correlation coefficient.

Keywords: LASSO, Elastic Net, Adaptive LASSO, Classification algorithms, Heart Disease, Performance metrics.

1 INTRODUCTION

Multiple linear regression model is considered as follows:

$$Y_i = \beta_0 + \sum X_{i1}\beta_p + \varepsilon_i,\ i = 1, 2, ..., n. \tag{1}$$

Multicollinearity is occured when there is a close association or interaction between two or more independent variables in the multiple regression. In case of multicollinearity, the signs of the estimated regression coefficients may not conform to the previous expectation; the variances and the standard errors of the regression coefficient estimates may increase, i.e. small t-values can be obtained. Various feature selection methods have been carried out for reducing the number of independent variables. Different techniques, K-nearest neighbor, Support Vector Machine, Decision Tree, Random Forest, and Logistic Regression, have been used for classification. Performances of the classification algorithms are measured by different metrics, accuracy, sensitivity, specificity, and Matthew correlation coefficient. In this study, Cleveland database of UCI repository of heart disease patients is used for feature selection and classification methods.

2 METHODS

2.1 Feature Selection Methods

LASSO (Least Absolute Shrinkage and Selection Operator) proposed by Tibshirani (1996), is suggested as an alternative to Least Squares estimators that loses their feature of being the best linear unbiased estimator. LASSO, one of the popular approaches for studies on big data, is a regression method that performs both coefficient shrinkage and variable selection simultaneously [3,5]. LASSO regression applies a penalization process to the independent variables, reducing some variables to zero, leaving only the most important variables to explain the dependent variable. It is obtained by introducing a constraint that "shrinks" the model parameters towards zero, that sum of the absolute value of the regression coefficients is less than a constant value [3]. In the LASSO, the error term $\sum(Y_i - \beta_0 - \sum X_{ij}\beta_p)^2$ is minimized with respect to β under the constraint $\sum|\beta_p| \le t$. Using the Lagrangian multiplier method and a tuning parameter λ, an unconstrained optimization problem is handled as:

$$\text{Min } L(\beta) = \sum(y_i - \beta_0 - \sum X_{ij}\beta_p)^2 + \lambda \sum|\beta_p|, \ \lambda > 0 \tag{2}$$

The Elastic Net approach introduced by Zou and Hastie (2005) combines the Ridge and LASSO penalties [6]. The Elastic Net estimator is built considering L_2 norm and L_1 norm as follows:

$$\text{Min } L(\beta) = \sum(y_i - \beta_0 - \sum X_{ij}\beta_p)^2 + \lambda_1 \sum|\beta_p| + \lambda_2 \sum \beta_p{}^2 \tag{3}$$

The Adaptive LASSO estimator, which provides consistent coefficient estimation by loading certain weights on the coefficients, is explained as follows. It is defined as $w = 1/\beta_p$ representing the weight vector by following [5]:

$$\text{Min } L(\beta) = \sum(y_i - \beta_0 - \sum X_{ij}\beta_p)^2 + w\sum|\beta_p| \tag{4}$$

2.2 Classification Algorithms

K-nearest neighbor (KNN) is a supervised classification algorithm which is a type of instance-based learning. It is easy to implement without constructing a model or making other assumptions, i.e. non-parametric. Objects are classified based on their nearest neighbors. The Euclidean distance is used to calculate distance of an attribute from its neighbors. Support vector machine (SVM) is one of the supervised learning techniques used for classification. The primary aim of SVM is to separate two classes with a line or plane. It focuses on determining the optimal plane that separates all data points from one class from the other. The better the model, the greater the separation or margin between the two classes. Random Forest (RF) algorithm, is used for classification which is a supervised learning technique A forest is formed by multiple trees in this method. Each tree in the RF represents a class expectation, and the class with the highest votes becomes the prediction of the model. The greater number of trees provides higher accuracy in the RF classifier. The forest RI (random input choice), the forest RC (random blend), and the combination of the forest RI and forest RC are three common methodologies in the RF. Decision tree (DT), another supervised learning technique used for classification, is applied for both categorical and continuous data. It is used to create tree-like structures and it makes analysis based on root node, interior node, and leaf node. Root node implies main node and is based on this all other nodes functions, interior node handles various attributes and leaf node represents the result of each test. Logistic regression (LR), a supervised classification technique, is used for determining the relationship between categorical dependent variable and independent variables. It uses probability to predict the classification of categorical data. The LR has different types, e.g., binary (0 or 1), multinomial (≥3 classifications without ordering), ordinal (≥3 classifications with ordering) [3, 4, 6].

3 RESULTS

The Cleveland dataset of Heart Disease from UC Irvine Machine Learning Repository, which includes 14 variables, one dependent and the others independent, has been studied using various methods to predict heart disease [7]. Missing values are removed and categorical variables are pre-prepared with dummy variable. The feature selection methods, LASSO, Elastic Net and Adaptive LASSO, are applied to dataset which is divided into train and test. The R^2 and MSE values are calculated by performing all feature selection methods (Table 1).

Table 1: R^2 and MSE values and selected independent variables by Feature Selection Methods.

		LASSO		***Elastic Net***		***Adaptive LASSO***	
		Train	***Test***	***Train***	***Test***	***Train***	***Test***
	R^2	0.52	0.57	0.30	0.48	0.52	0.49
	MSE	0.11	0.10	0.17	0.12	0.12	0.15
Dependent variable							
Label	Heart disease status (0=No, 1=Yes)						
Independent variables							
Age	Patients age, in years						
Gender	0=Male, 1=Female	✓		✓		✓	
Cp	Chest pain: (0=typical angina, 1=atypical angina, 2=non-anginal pain, 3=asymptomatic	✓		✓		✓	
Trestbps	Rest blood pressure, Hg/mm						
Chol	Serum cholesterol, mg/dl						
Fbs	Fasting blood sugar>120 mg/dl: (0=False, 1=True)	✓		✓		✓	
Restecg	Rest ecg: (0=normal, 1=ST-T wave abnormality, 2=left ventricular hypertrophy	✓		✓		✓	
Thalach	Maximum heart rate						
Exang	Exercise induced angina: (0=No, 1=Yes)	✓		✓		✓	
Oldpeak	ST depression induced by exercise relative to rest					✓	
Slope	Slope of the pick exercise: (0=upsloping, 1=flat, 2=downsloping)	✓		✓		✓	
Ca	Number of major vessels					✓	
Thal	Thalassemia defects: (0=normal, 1=fixed defect, 2=reversable defect)	✓		✓		✓	

Performance metrics (specificity, sensitivity, accuracy, Matthew correlation coefficient) of the classification methods with and without feature selection methods are shown in Table 2 and Figure 1.

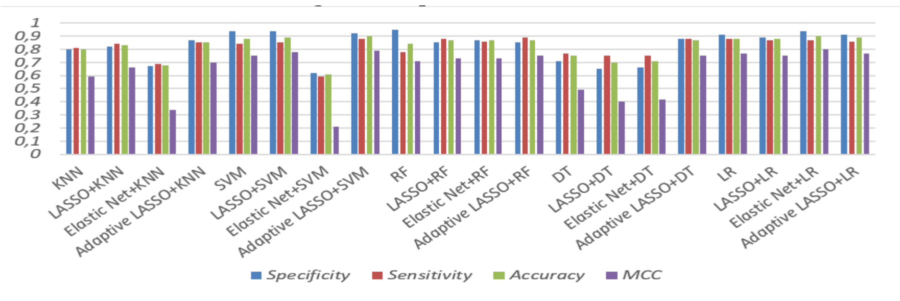

Figure 1: Performance comparisons of all the considered methods.

Table 2: Performance metrics of classification methods with and without feature selection.

Methods	***Specificity***	***Sensitivity***	***Accuracy***	***MCC***
KNN	0,80	0,81	0,80	0,59
LASSO+KNN	0,82	0,84	0,83	0,66
Elastic Net+KNN	0,67	0,69	0,68	0,34
Adaptive LASSO+KNN	0,87	0,85	0,85	0,7
SVM	0,94	0,84	0,88	0,75
LASSO+SVM	0,94	0,85	0,89	0,78
Elastic Net+SVM	0,62	0,59	0,61	0,21
Adaptive LASSO+SVM	0,92	0,88	0,90	0,79
RF	0,95	0,78	0,84	0,71
LASSO+RF	0,85	0,88	0,87	0,73
Elastic Net+RF	0,87	0,86	0,87	0,73
Adaptive LASSO+RF	0,85	0,89	0,87	0,75
DT	0,71	0,77	0,75	0,49
LASSO+DT	0,65	0,75	0,70	0,4
Elastic Net+DT	0,66	0,75	0,71	0,42
Adaptive LASSO+DT	0,88	0,88	0,87	0,75
LR	0,91	0,88	0,88	0,77
LASSO+LR	0,89	0,87	0,88	0,75
Elastic Net+LR	0,94	0,87	0,90	0,8
Adaptive LASSO+LR	0,91	0,86	0,89	0,77

4 DISCUSSION

Heart disease has been one of the main causes of death worldwide. In this study, Cleveland dataset of heart disease is used. According to the results, LASSO outperforms Elastic Net and Adaptive LASSO with respect to R^2 and MSE values. Performance metrics for classification algorithms with and without feature selection, the highest specificity of 0.95 is obtained from RF, the highest sensitivity of 0.89 is from Adaptive LASSO+RF. On the other hand, the highest accuracy (0.90) is provided from Adaptive LASSO+SVM and Elastic Net+LR, the highest MCC (0.80) is from Elastic Net+LR. As a future study, we plan to compare the performance of MARS and its extended models with the results we obtained in this study.

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COMPETENCY MODEL DEVELOPED BY USING A MULTI-CRITERIA APPROACH AND K-MEANS CLUSTERING

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Abstract: This study uses Multi-Attribute Utility Theory (MAUT) and K-means clustering to develop a competency model for wood and furniture industry professionals driving digital and sustainable transformation. MAUT improves impartiality, while K-means clustering categorizes competencies. This approach supports data-driven curriculum decisions and accommodates diverse stakeholder needs. Results show clear competency clusters that prioritize competencies. These methods are versatile for developing competency models for different EQF levels and sectors.

Keywords: competency model; multi-criteria decision making; multi-attribute utility theory (MAUT); K-means clustering; digital; sustainable; wood and furniture industry

1 INTRODUCTION

The increasing challenges to the environment have caused great concern [1–3] and have led nations to formulate several policies. In line with these efforts, the European Commission (EC) has outlined its key objectives for the period 2019-2024 [4]. The bioeconomy, which includes the wood and furniture (W&F) industry [5], plays a critical role in achieving the goals outlined in some of the strategies through the sustainable use of biomass [6]. The efficient use of wood resources to meet the increasing demand for wood products is a core principle of the circular economy [7, 8]. Companies can further advance this principle by embracing digitalization to contribute to sustainable, environmentally friendly progress. However, the successful implementation of sustainable and digital transformation depends heavily on the skills of companies' employees. In the absence of the necessary expertise, companies may face obstacles in integrating sustainable and digital practices [9, 10]. Therefore, it is essential that the education system adapts accordingly. To this end, the creation of a comprehensive competency model is essential and has become standard practice in many organizations [11]. A competency model serves as a descriptive tool that identifies the essential competencies required for competent performance within a particular role, job, profession, organization, or industry [12]. Essentially, it is a behavioural job description that must be defined for each job function and each individual job [13].

To effectively address some of the pressing challenges, it is essential to equip employees with key digital and sustainable competencies in the Slovenian W&F industry as well. Therefore, we have developed an industry-specific competency model that is tailored to the sustainable and digital transformation of the sector and can be adapted to different levels of education. In our pursuit of impartiality within the model, we have used a multi-criteria approach that includes the assessment of key competencies that are critical to the sector's progress. In doing so, we incorporated expert assessments of the importance of the competencies, the industry-specific requirements of the companies, and the relevance of the competency-related content to students in the training programs.

2 METHODOLOGY

The chapter on methodology is divided into two parts. In the first part, we describe the data collection process and go into detail about how the data were collected. The second part focuses on the data processing procedures and the development of the multi-criteria competency model using Multi-Attribute Utility Theory (MAUT) in combination with K-Means clustering.

2.1 Data collection

Based on the European competency frameworks DigComp [14] and GreenComp [15] and the Slovenian W&F industry development document [16], we have developed a set of digital and sustainable competencies that are alternatives in our model. To determine their importance in facilitating the digital and sustainable transformation of the industry, we used a three-fold assessment approach.

First, a 12-member expert panel consisting of VET and HE educational institutions, industry representatives, and consultants rated the importance of these competencies in an online survey using a 4-point Likert scale.

Next, CEOs and HR employees of W&F companies (n=8) used a 9-point scale to rate the competencies that employees with a certain level of education should have to facilitate their company's digital and sustainable transformation. Using a 9-point scale ranging from "1" (lowest) to "8" (highest), where "0" means the competency is not needed.

Finally, we solicited the opinions of students (n=105) who rated the attractiveness of content related to our digital and sustainability competencies using a 5-point Likert scale. This phase provided valuable insights into the relevance of the content to the emerging generation.

Our methodological approach integrates insights from experts, industry professionals, and students to gain a comprehensive understanding of the competencies driving the digital and sustainable transformation of the W&F industry.

2.2 Development of a competency model

In this research, we used Multi-Attribute Utility Theory (MAUT) in combination with K-Means clustering to develop a comprehensive competency model for the sustainable and digital transformation of the W&F industry in Slovenia. In our case, the model was created for the individual education level related to the W&F industry. As an example, for this research, we created a competency model for carpentry students at EQF level 4 in Slovenia.

After successfully obtaining responses from all intended main stakeholders, i.e., companies, experts and students, our next step was to calculate average values for each group, which were used as criteria for our MAUT model. In the case of companies and experts, individual scores for specific competencies were calculated for MAUT. In the case of students, averaging the individual scores represented a cluster of competencies, as can be seen in Table 1.

2.2.1 Multi-Attribute Utility Theory

Multi-Attribute Utility Theory (MAUT) is a part of Multi-Attribute Decision Making (MADM) methods. The development and evolution of MAUT can be largely attributed to Keeney & Raiffa [17]. It involves maximizing a real-valued utility function [18], allows simultaneous comparison of alternatives [19], and assigns criteria weights [18]. Due to its simplicity, it is widely used in various decision problems [20].

We assigned weight values w_i to each criterion, with company ratings of 0.45, expert ratings of 0.45, and student ratings of 0.10. After normalizing all criteria scores $u_i(x_i)$ in the range of 0 to 1, we calculated utility values $U(x)$ using the quadratic utility function in equation (1).

$$U(x) = \sum_{i=1}^{m} \left(u_i(x_i)\right)^2 * w_i \qquad (1)$$

2.2.2 K-Means Clustering

In 1967, MacQueen introduced k-means, an unsupervised learning method for clustering data [21]. It is a partitioning-based technique that iteratively groups data points into different, non-overlapping clusters [22]. The goal is to form clusters with high similarity within and low similarity between them [23].

Assuming that the target object is represented by x, where x_i is the centroid of the cluster C_i, k is the number of clusters (3 in our case), and $|x-x_i|$ denotes the Euclidean distance between data point x and centroid x_i, the criterion function can be defined as follows [24]:

$$Minimize \sum_{i=1}^{k} \sum_{x \in C_i} |x - x_i|^2 \tag{2}$$

3 RESULTS

Table 1 shows the average competence ratings of the different stakeholder groups (experts, companies, and students) using the example of a carpenter with an EQF 4 qualification. It illustrates the different perceptions: experts prefer "sustainable design", "computer aided design" and "wooden constructions" while companies emphasize the highest competence in "promoting nature" and "protecting health and well-being". Students prefer "wood protection" and "constructions" and show a lower interest in sustainability competences.

Table 1: Competence evaluation and clustering for carpenter

A carpenter (EQF 4)		Average scores (Criteria)			Normalized scores (Criteria)			MAUT resluts	K-means clusters
	Weights for MAUT:	45%	45%	10%	45%	45%	10%		
Area of competences	Competences (Attributes)	Experts	Company	Students	Experts	Company	Students	[Quadratic Utility Function]	Number of clusters: 3
Information and data literacy	Browsing, searching and filtering data, information and digital content	2,50	3,25	2,81	0,50	0,41	0,45	0,21	1
	Evaluating data, information and digital content	2,67	2,88		0,56	0,36		0,22	1
	Managing data, information and digital content	2,67	2,88		0,56	0,36		0,22	1
Communication and collaboration	Interacting through digital technologies	3,58	4,00	2,74	0,86	0,50	0,43	0,47	3
	Sharing through digital technologies	3,33	2,50		0,78	0,31		0,34	2
	Engaging citizenship through digital technologies	2,75	2,50		0,58	0,31		0,22	1
	Collaborating through digital technologies	3,42	2,00		0,81	0,25		0,34	2
	Netiquette	3,25	3,63		0,75	0,45		0,36	2
	Managing digital identity	3,25	4,00		0,75	0,50		0,38	2
Digital content creation	Developing digital content	3,00	2,25	2,74	0,67	0,28	0,44	0,25	1
	Integrating and re-elaborating digital content	2,92	1,88		0,64	0,23		0,23	1
	Copyright and licences	3,17	2,25		0,72	0,28		0,29	1
	Programming	3,17	1,38		0,72	0,17		0,27	1
Safety	Protecting devices	3,33	3,88	2,78	0,78	0,48	0,44	0,40	2
	Protecting personal data and privacy	3,50	3,25		0,83	0,41		0,41	2
	Protecting health and well-being	3,42	5,13		0,81	0,64		0,50	3
	Protecting the environment	3,45	3,50		0,82	0,44		0,41	2
Problem solving	Solving technical problems	3,17	3,75	2,53	0,72	0,47	0,38	0,35	2
	Identifying needs and technological responses	3,17	3,38		0,72	0,42		0,33	2
	Creatively using digital technology	3,17	2,25		0,72	0,28		0,29	1
	Identifying digital competence gaps	3,17	1,86		0,72	0,23		0,27	1
Embodying sustainability values	Valuing sustainability	3,27	3,38	2,62	0,76	0,42	0,40	0,35	2
	Supporting fairness	3,33	4,50		0,78	0,56		0,43	3
	Promoting nature	3,50	5,13		0,83	0,64		0,51	3
Embracing complexity in sustainability	Systems thinking	3,33	2,50	2,57	0,78	0,31	0,39	0,33	2
	Critical thinking	3,42	3,13		0,81	0,39		0,38	2
	Problem framing	3,25	2,75		0,75	0,34		0,32	2
Envisioning sustainable futures	Futures literacy	3,25	3,13	2,65	0,75	0,39	0,41	0,34	2
	Adaptability	3,08	2,75		0,69	0,34		0,29	1
	Exploratory thinking	3,33	3,00		0,78	0,38		0,35	2
Acting for sustainability	Political agency	2,73	3,13	2,68	0,58	0,39	0,42	0,24	1
	Collective action	2,92	4,63		0,64	0,58		0,35	2
	Individual initiative	3,25	4,00		0,75	0,50		0,38	2
Design	Sustainable design	3,83	3,50	3,16	0,94	0,44	0,54	0,52	3
	Computer-aided design	3,83	4,63		0,94	0,58		0,58	3
	Smart furniture	3,42	2,63		0,81	0,33		0,37	2
	Restorative environmental design	3,50	2,63		0,83	0,33		0,39	2
Architecture	Energy-efficient and smart houses	3,67	2,00	3,20	0,89	0,25	0,55	0,41	2
	Wooden constructions	3,83	2,13		0,94	0,27		0,46	3
Construction	Mechanical stress simulations	3,25	2,00	3,31	0,75	0,25	0,58	0,31	2
Wood protection	Cultural heritage	3,00	3,13	3,42	0,67	0,39	0,60	0,31	2
	Wood pests and wood protection	3,00	2,00		0,67	0,25		0,26	1
Salvaged timber and wood remnants	Wood waste	3,58	3,75	3,00	0,86	0,47	0,50	0,46	3
	Wood recycling	3,58	3,75		0,86	0,47		0,46	3
Mechanical processing processes	Sustainable consumption and production	3,42	3,63	3,01	0,81	0,45	0,50	0,41	2
	Autonomous and flexible production	3,33	3,57		0,78	0,45		0,39	2
	Human-robot interaction	2,58	1,88		0,53	0,23		0,18	1
	Renewable resources and sustainable energy	3,33	3,63		0,78	0,45		0,39	2
Chemical processing processes	Alternative products based on biomass	2,68	2,13	2,73	0,56	0,27	0,43	0,19	1
Marketing and business operations	Impact of products on the environment	3,33	3,13	2,86	0,78	0,39	0,46	0,36	2
	Circular business model	3,50	2,63		0,83	0,33		0,38	2
	Sustainability of supply chains	3,25	3,13		0,75	0,39		0,34	2
	Industrial symbiosis	2,83	2,25		0,61	0,28		0,23	1
	Legal frameworks for sustainability	2,75	2,25		0,58	0,28		0,21	1
	Digital technology and operations	3,33	2,00		0,78	0,25		0,32	2
	Digital promotion	3,33	2,50		0,78	0,31		0,34	2
	Digitization of consumer behavior monitoring	3,33	2,13		0,78	0,27		0,33	2

Table 1 also shows the results of MAUT and K-means clustering, which provide insights into different stakeholder preferences. These insights are of great value when adapting education and training interventions to different requirements and expectations, in our case for carpentry professionals. Figure 1 visually shows the results of MAUT and the K-Means clustering analysis.

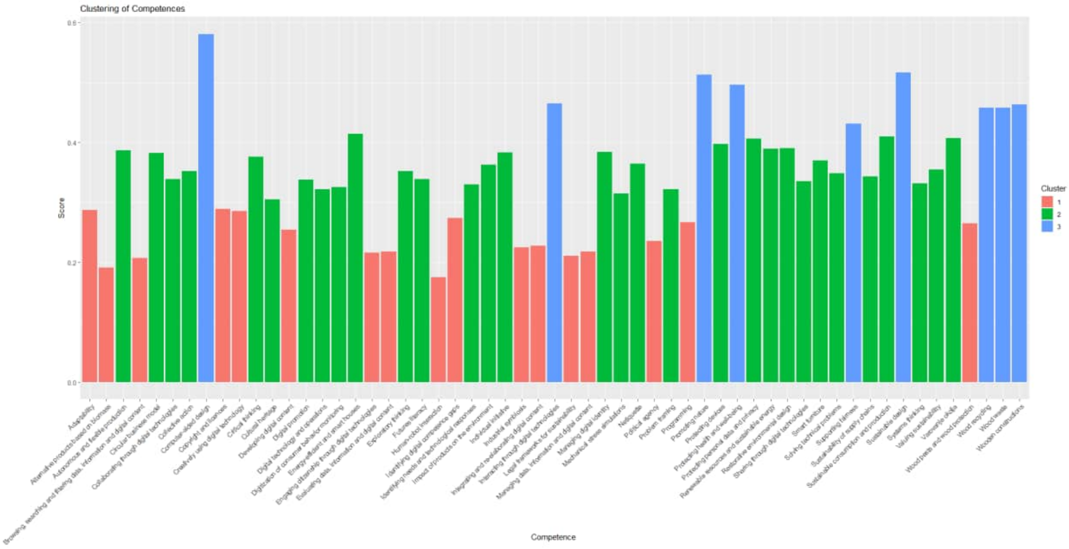

Figure 1: MAUT and K-Means Clustering Analysis Results

The three clusters prioritize competencies (Figure 1). Cluster 3 includes 9 essential competencies that are critical for carpentry professionals, are essential to the industry's digital and sustainable transformation, and should be at the forefront of curriculum reform. Cluster 2 includes 31 competencies that are also important, although to a slightly lesser extent. Finally, Cluster 1 includes 17 competencies that received lower MAUT ratings, suggesting careful consideration of including content from these areas in the carpentry education program.

4 CONCLUSIONS

Our competency analysis for carpenters, which is essential to the digital and sustainable transformation of the industry, has provided valuable insights by using MAUT and K-Means clustering. MAUT allowed us to categorize competencies into distinct groups using K-Means clustering, which enables tailored curriculum development and resource allocation. This research creates a data-driven framework for curriculum decisions that addresses the needs of diverse stakeholders.

In addition, the versatility of the methods used extends their applicability to different EQF levels and sectors, promising the creation of competency models in different contexts. In summary, MAUT and K-means clustering provide a systematic and objective approach to competency analysis.

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THE GENERAL MODEL FOR CURRICULA STRUCTURE PRESENTATION - A MATRIX APPROACH TO MANAGING CURRICULA

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Abstract: This paper presents forming and populating the matrix with connections between learning outcomes as well as proof of characteristics of the formed matrix. This approach employs a matrix to capture the relationships and dependencies between various learning outcomes within the curriculum. In this paper, the proven characteristics of the formed matrix facilitate curriculum alignment in today's fast changes in labour market requirements. Each row and column in the matrix represents a direct relationship between learning outcomes in the curriculum. Overall, the matrix approach provides a comprehensive and visual representation of the curricula structure, assisting educators and administrators in designing, evaluating, and refining educational programs, described in the paper's conclusion.

Keywords: Matrix model, Learning outcomes, Education, Curricula structure

1 INTRODUCTION

Today's labour market requires a broad spectrum of employees' knowledge and skills. Fast-developing information technology enters every kind of job and changes business technology. Accordingly, the requirement for employee knowledge and skills changes is increasingly frequent. Most formal study programs last two (graduated) or three (undergraduate) years. During these periods of study, technology has been changing. So, most students can't recognise their affinity for future jobs they would like to work for two or three years after completing their studies. That way, the education process (of two or three years) is less oriented to the labour market, expensive and has an extended delay in return on investment.

On the other hand, applying information technology in education makes available lots of quality teaching material, seminars and other non-formal education. Those materials usually concentrate on exact problems, methods, and techniques without explaining the background knowledge. In that way, attendants of those courses usually become narrowly specialised and more attractive to the labour market for a short time. But, if they don't finish an adequate formal education, they usually don't have enough background knowledge to solve more significant problems.

One of the solutions to solve the previously explained problems is the combination of lifelong learning concepts and classical high education study programs. That will enable better optimisation of the study program structure to provide faster and better adjustment to the labour market requirements. Starting point is the individual learning path.

1.1 Individual Learning Paths As Solution For Modern Education

Rapid technological advancements are driving an increased need for knowledge and skills at all levels of education [8]. The abundance of curricula and competency models presents significant challenges for primary, secondary school, and university teachers in making curricula. The main problem is obstructing defining optimal or personalised learning paths [2]. As an answer to the previous, providing learners with a more flexible approach to acquiring knowledge has become crucial, allowing them to plan their studies independently. This approach will ensure that the learning process is tailored to the individual needs of each learner, acknowledging that only some have the opportunity to engage in traditional educational methods. By offering a more flexible and personalised learning experience, individuals can access and acquire knowledge that suits their unique circumstances. By representing the curriculum and standards as a graph, we gain insights into the structural characteristics and relationships within the educational content [7].

Learning paths can be identified by identifying learning outcomes and their interdependencies. The approach presented by [7] involves mapping the content of curricula and standards onto a directed graph, establishing connections between their knowledge items through dependency relations. This method facilitates a formalised comparison by leveraging graph theoretical metrics such as highest degrees, numbers of sources and sinks, and connectivity. Another approach presented in [2] incorporates various strategies from natural language processing alongside a refined and simplified version of Bloom's Taxonomy to establish connections between learning outcomes. Software for the Target-Oriented Personal Syllabus system represented the curriculum within the system as a graph of learning outcomes [1]. The software application enables university students to generate customised study schedules while allowing staff members to update and enhance course content and curriculum framework. In the paper [8], the authors also offer a graph-based framework consisting of four graphs to plan and carry out each study process. The paper [9] describes a 3D environment for representing a University of Applied Sciences of Southern Switzerland undergraduate education programme. More specifically, the curriculum data selected for representation are visualised in the 3D environment, and the generation process is detailed. Every 3D graph can be transformed into a 2D graph. The paper [6] is presented CME2 prototype software which also visualises the learning process as a graph. With this tool, students and teachers can easily handle and plan their personal curricula on a larger scale and see the connections between different courses.

On the other hand, ensuring student learning is a significant concern in modern education. Accrediting bodies and government entities now require evidence that students genuinely comprehend classroom material. In response to this requirement, education programs actively integrate assessment procedures into curricula. However, there needs to be more consensus on the best methods for achieving this goal. The approach outlined and developed in the paper [4] helps the curriculum assessment problem using an abstract 2D graph representation. Graph-theoretic metrics are calculated, and visualisation software is utilised to generate a visual representation that aids curriculum assessment.

Visualisation of the learning processes is a powerful way to help students understand their curricula and structure. In addition, concept map-based process visualisation could also be utilised in personnel management and training systems in business companies, adding a flavour of lifelong learning or micro-qualification concepts in these contexts.

Most of the mentioned above graph presentations are partial solutions and use different methods and software to improve the personalisation of curricula. This paper presents a standard method, which can be the theoretical base for most of the previous partial solutions.

1.2 Research problem

The main problem in this paper is the mathematical standardisation of the previous solutions to individualise traditional study programs better and easier. All research results mustn't exclude the traditional realisation of study programs. So, the starting point is a traditional study program. Every traditional study program comprises subjects. Every subject has learning outcomes, i.e. the set of knowledge and skills which students adopt through planned subject activities. Accordingly to the previous, competencies are a set of learning outcomes (LOs). So, all competencies can be put into one series of LOs. Their order is defined only in the subjects and with the order by subject. So, the structure of the classical high-education study program is poorly understandable. In the next paragraph, a previously mentioned standardised model will be presented, including the mathematical proof of it.

2 A Curriculum As A Total Order Set Of Learning Outcomes

Let's forget the subjects and analyse competencies based only on LOs, without the subjects. Since only a set of LOs is observed, it is necessary to arrange them in order. The position between two LOs can be defined according to the following rule: "Learning outcome, prerequisites for adopting other learning outcomes, will be located in front of them. The structure of a study's program can be determined by defining the total order of the set of LOs according to the previous rule. Graphical presentation of differences in traditional and, in this paper, suggested approach shown in Figure 1.

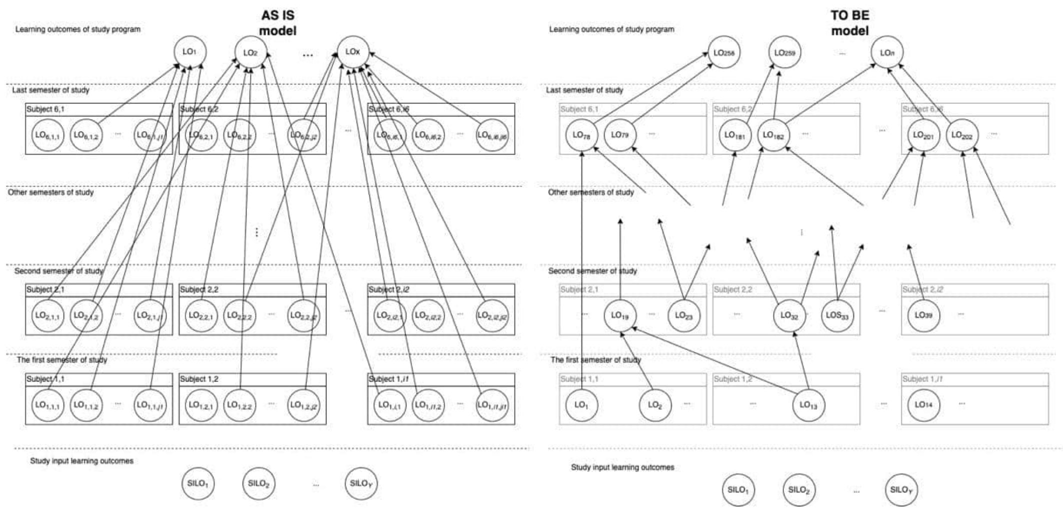

Figure 1. Traditional (AS SI) and matrix approach (TO BE) curriculum structure

In that way, every two LOs are comparable without limitation on the number of LO in a set of LOs. So, let some study program has *n* LOs. It can be presented by equation (1).

$$\tilde{A} = \{LO_1, LO_2, \dots, LO_n,\} \quad (1)$$

According to the previous explanation of the relationship in set $\tilde{A}$, the relation „to be prerequested“ (PR) is determinate. So, the mathematical presentation of the study program’s structure is the matrix *A* (2):

$$A=\begin{bmatrix} 0 & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & 0 & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & 0 & & a_{3n} \\ \vdots & \vdots & & & \vdots \\ a_{n2} & a_{n2} & a_{n3} & \dots & 0 \end{bmatrix} \quad a_{ij} \in \{0,1\} \quad (2)$$

If LO_i is in the *i*-row prerequested to adopting LO_j in the *j*-column, the value of a_{ij}= 1otherwise a_{ij}=0. Every cell in the table is supposed to be filled in this way. In this way, the row in the set LO can be realised using elementary matrix transformations in order to transform matrix *A* on the lower triangle matrix. The following text proves the possibility of transforming the *A* matrix to the lower triangle matrix.

Let it $X, Y, Z \in \tilde{A}$ are randomly chosen from. Based on the nature of the problem, it is easy to conclude that if the element X is a prerequested LO to the LO Y, Y can't be prerequested to LO X, i.e.

$$X \text{ PR } Y \Rightarrow Y \not{\text{PR}} X \quad (3)$$

Therefore, it can be concluded that the relation PR is antisymmetric. For the relation PR defined in such a way, it is possible to determine a property of transitivity as well, i.e.

$$\text{Z PR X and X PR Y} \Rightarrow \text{Z PR Y} \quad (4)$$

Based on (3) and (4) relation PR is called a partial order relation (antisymmetric and transitive). Therefore, the defined relation PR between LOs from set $\tilde{A}$ is the partial order relation.
Another necessary type of relation in the set $\tilde{A}$ is a relation of the total order. The total order relation "<" means that the partial order relation additionally allows comprising every two elements X and Y, i.e.

$$\text{X<Y or Y<X} \quad (5)$$

Every partial order relation can be expanded to the total order relation. The relation PR can expand to the total order relation "<". So, without losing generalisation, it can be assumed:

$$\text{LO1< LO2<...< LOn.} \quad (6)$$

In order to prove that in this order of LO_i, the matrix A is a lower triangular matrix, the opposite is assumed. It can be assumed that $c_{ij}>0$ for some $j>i$. This would result in the fact that the LO_i is prerequested to LO_j, i.e.: LO_i PR LO_j, but $LO_i < LO_j$, which is in contradiction with $LO_j < LO_i$ (as a result from (6)).

It means the mathematical presentation of each hierarchical (graphical) curriculum structure can transform the lower triangular matrix. In this way, curricula management is reduced to elementary matrix calculation. This matrix approach to modelling curricula has many benefits, including all in the previous paragraph mentioned cases about individual learning paths. Presentation of the hierarchical structure of curricula in the form of a matrix allows managing curricula as a whole. Changing the relationship between existing LOs, adding a new or deleting an old LO, and then transforming the new matrix to a lower triangular matrix will result in the updated optimal curricula structure. In that way, optimising curriculum structure reduced to the elementary matrix calculations.

3 CONCLUSION: BENEFIT OF A CURRICULA MATRIX MODELLING

The previous paragraph shows the possibilities of using a matrix calculation to present curriculum structure. Bringing down the curricula structuring problem to matrix algebra contributes to solving the up to dating of high education i.e. general lack of curricula relevance [3]. To make meaningful comparisons of curriculum volumes, it is crucial to model them in a standardised manner. The alignment of competencies with the labour market and employers' requirements is prioritised, enhancing the relevance of education. One approach is establishing a uniform structure model by defining a linear path between Knowledge Areas (KAs) and topics within the curriculum [5].

This paper presented the matrix model that can incorporate all the paths between topics within KA and provides a hierarchical representation of the KA structure. It allows for visualising prerequisite learning outcomes (LOs) necessary for acquiring desired LOs and identifying potential paths for further LO adoption. This visual representation enables a clearer understanding of the relationships between LOs as components of one or more competencies, establishing clear cause-consequence links. This hierarchical structure allows for a systematic and organised analysis of the curriculum volumes, enabling effective comparisons and assessments of the content covered in different curricula. Moreover, it facilitates a comprehensive comprehension of competencies and their multilevel structure, aiding in designing optimal learning paths.

This standardised structure representation enables connections of different KA, resulting in cross-functionary education. It allows extensions vertically, across different degree programs, and horizontally, encompassing different KA. This extensibility allows for integrating competencies and LOs from various disciplines, creating new and in-demand competencies that achieve the cross-functionality and flexibility of education.

The agility of high education curricula is paramount in the context of employment-oriented education. The management of multilevel structures of competencies, presented as short chains of LOs, becomes easily manageable. Any changes in the multilevel structure of curricula result in an automated rebuild of optimal learning paths, ensuring adaptability and responsiveness.

This approach enables the networking of LOs with non-formal courses and micro qualifications as part of the multilevel structure of competencies, additionally expanding the opportunities for skill development and career advancement.

Education becomes a network-based process, navigating through the multilevel structure of LOs and encompassing competencies from all included curricula.

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THE ANALYIS OF EUROVISION SONG CONTEST RESULTS: THE DIFFERENCES BETWEEN PUBLIC AND JURY VOTES

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Abstract: The Eurovision song contest (ESC) is probably the most popular entertainment festival in Europe. It is also a subject of scientific research: the researchers analyzed this topic from different perspectives. One of them is related to analyzing the voting system. This is also the case in this paper. We analyzed the results of last six ESCs from a position of differences between public and jury votes and identified the correlations of votes by public/jury and overall results, using the Spearman rank correlation. The results show that there is a significant variation in results by public and jury in the dataset ($0.22 < |r| < 0.63$). Consequently, when the public/jury correlation is lower, the public/overall and jury/overall correlation is also lower. At the level of country, we identified countries (1) whose public and jury are highly correlated, (2) whose public/jury is the most correlated with results at the European level.

Keywords: Eurovision, correlation, public, jury, voting

1 INTRODUCTION

In 1950, the European Broadcasting Union (EBU) was established with the goal of enhancing radio and television services through cooperation and content sharing. Initially, only content related to news and sports was shared. In order to not be only reliant on only that type of content, they chose to create an event in 1956 where music serves as another way of connecting the citizens of Europe. This is the Eurovision Song Contest (ESC), and the EBU hosts this event every year. This event is a subject of scientific interest. One perspective is related to the voting system. Firstly, the winner was elected by jury only, and further, with development of telephone and ICT technologies, the public was able to participate. For the long time, only public participated in selection of the winner, but in last years, the jury was returned into the system with the goal of decreasing the regional or political favoritism. The purpose of including a jury is to ensure a fair and balanced outcome by considering the opinions of music industry professionals alongside the preferences of the general public. The jury consists of music experts, journalists, and other individuals with knowledge and experience in the music industry. The jury members evaluate the songs based on various criteria, including vocal abilities, composition, and stage performance, among others. On the other hand, many think that now, when the jury is again in the game, that political favoritism will be increased.

This paper is organized as follows: Section 2 brings a short overview of research that are related the ESC from different perspectives. Section 3 describes the methodology of our research. Section 4 presents the results. Section 5 presents conclusion and future work.

2 ESC: PREVIOUS RESEARCHES

The scientific world has not been uninterested in this event, and thus this subject has been studied in many ways by researchers from various nations. In the continuation of the paper authors provide an overview of the various areas of research that have been carried out and are related to subject Eurovision Song Contest. Thus, Yair presents an overview of the broad research literature of ESC in four main areas: (1) research related to imagining a united Europe (2) research focusing on gender (3) research related to political bloc voting and cultural alliances and (4) research conducted based on the analysis of Eurovision data to explain external phenomena involving political and economic disagreements between countries. This research leads to the conclusion that the ideal of unity is largely questioned with regard to national and cultural preferences [1]. Identity and unity with use of quasi-experimental data observes also Coupe and Chaban [2], in this research, they assess the immediate impact of the ESC on whether citizens in Europe consider themselves truly European. The research results clearly showed that there is not a lot of evidence that a competition like the ESC gives a significant immediate boost to the majority of European citizens to feel European or to have a positive image of the European Union as a community of countries. Related to the topic of cultural diversity and the identity of citizens in Europe and whether the ESC fulfils the purpose of the establishment of the EBU the focus of the research [3]. ESC is analyzed as a tool of cultural diplomacy and emphasizing that cultural diplomacy represents the third important pillar of foreign policy, also discuss about the concept of cultural diplomacy and its active practice at ESC [4]. Ibrus, Rohn and Nanì put focus to the problems faced by contemporary public service media (PSM) institutions. The combination of innovation systems theory and public value theory to explore the interrelationships between the production of different forms of values is present [5].

The largest number of searched papers related to the topic ESC refers to voting and research on the impact of various factors on the final outcome of the competition, i.e. which country/song is ranked and wins. In the continuation of the paper, the authors give a brief review of research in the last fifteen years. In 2006 already Gatherer, concludes that the analytical identification of statistically significant trends in voting patterns over observed period of several years is stochastic, and by simulation using the Monte Carlo method presents the reconstruction of the history of this competition. The results showed that in the period from the mid-1990s, large geographical voting blocs appeared and that they decisively influenced the outcome of the competition [6]. Ginsburgh and Noury analyzed the voting behavior and evaluations of judges and the problem of "vote trading". Ultimately, they come to the conclusion that the results depended on the quality of the participants, as well as on the linguistic and cultural connection between the singers and the voting countries, and that there is no reason to connect the results of the competition with political conflicts/friendships [7]. Assessment the influence of common characteristics of jury members or voting colleagues on the outcomes of the voting system and whether these common characteristics lead to voting bias was also analyzed[8]. The research found that there is strong evidence of the presence of voting bias based on geographical, cultural, linguistic, religious and ethnic characteristics. The authors ultimately conclude that these results apply to all cases where groups of jurors or voting colleagues share certain common factors. Similar topic, voting patterns impartiality, friendship-networks and voting behavior was explored by Charron [9], the research results show strong empirical evidence that given that certain blocs show a systematic bias in voting against each other, the bias will be significantly lower among countries with unbiased political institutions as opposed to those with highly biased political institutions. This type of bias was observed also in research conducted by D'Angelo, Murphy and Alfò [10] and Siganos adn Tabner [11]. The investigation of positive or negative bias in votes based on geographical proximity, migration and cultural characteristics of the participating countries through the Bayesian hierarchical model was carried out by Blangiardo and Baio [12]. The result of this research indicates that there is no evidence of negative bias, but that there is a slight positive bias that appears systematically, connecting voters with performers. Bias is also indicated by Budzinski and Pannicke [13] who made a comparative analysis of historical ESC data from 1998 to 2014 and The Bundesvision Song Contest (BSC) data from its inception in 2005 to 2014. The results showed that voting biases are not only present in international competitions, but also appear in similar competitions at the national level with a similar size and quality of organization, regardless of the fact that the culture of participants and voters is more homogeneous. The research by Stockemer, et al. [14] examines the voting behavior of ESC viewers. After analyzing the results of the research carried out in in European countries, conclusions were drawn about the voting patterns. The analysis and evaluation of the rules and results of the ESC are investigated by Ginsburgh and Moreno-Ternero [15], their research was focused on different voting procedures and several alternatives and whether the selected judges are better than the tele-voters. Ultimately, they suggest using the so-called Shapley voting procedure for judges as well as tele-voters.

Nowadays, the results of research indicate that the existence of big data, interaction on social networks and digital globalization have led to changes in the way people make decisions in life or decisions of general importance, as shown also by the analysis of voting data from the final round of ESC song contest data of 2021. This is a consequence of the new theory of social choice, according to which voters judge candidates instead of ranking them. [16]

3 THE METHODOLOGY

The research goals of this paper are: (I) to identify differences between public and jury votes and the correlations of votes by public/jury and overall results; (II) identified countries (1) whose public and jury are highly correlated, (2) whose public/jury is the most correlated with results at the European level. The calculations were implemented in Excel.

The voting system of ESC obtains public and jury votes of each country. The weights of public and jury are equal.

The methodology steps that are implemented to achieve the research goals are:

1. Preparing the data (6 datasets – results of votes from last 6 years 2017-2023; in 2020 ESC was not held). 43 countries in total participated during this time. Most of them are from Europe. The data about the results are overtaken from website https://eurovisionworld.com/.
2. Calculating Spearman rank correlations: PE-JE, PE-FE, JE-FE, PC-JC, PC-PE, JC-JE, PC-FE, JC-FE, FC-FE (P=public, J=jury, C=country, E=ESC, F=P+J). In research, it was needed to calculate 1566 correlation coefficients.
3. Analysis of the results.

4 THE RESULTS

Figure 1 presents correlation between public and jury votes. The results show that there is a significant variation in results by public and jury in the dataset ($0.22 < |r| < 0.63$). Consequently, when the public/jury correlation is lower, the public/overall and jury/overall correlation is also lower.

At the national level, the variation of the public and jury results is even higher ($0.005 < |r| < 0.851$). PC-JC is highly correlated with PE-JE ($r = 0.61$) so we conclude that when jury and public are highly correlated in European level, they will be also highly correlated at the national level. In future research, by including the ESC and music experts, it would be interesting to analyze the songs and see how this correlation can be interpret in the light of the number of the favorite in the competition. The countries whose jury and public are the most correlated are Armenia and Cyprus (|r| > 0.5). The lowest correlations are achieved by Czechia and Bulgaria (|r| < 0.13).

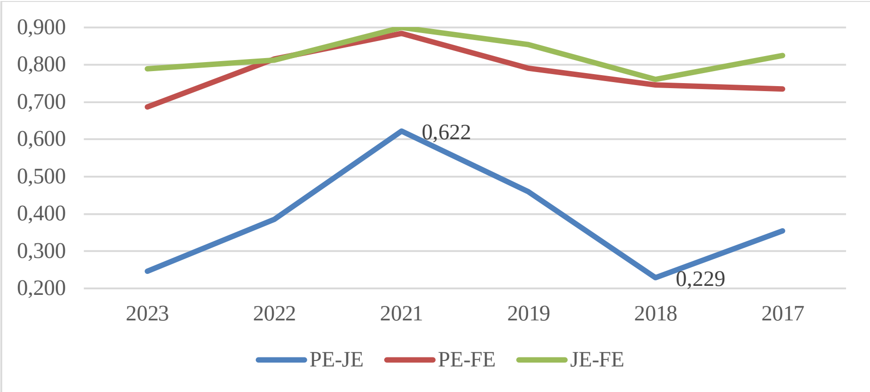

Figure 1: Correlation between public and jury votes per years

We analyzed how national juries and public are correlated with European jury and public (JC-JE and PC-PE). In the case of public votes, the variation of the rank coefficients is high (0.072 < |r| < 0.816); in average, |r| = 0.61. In the case of jury votes, the variation of the rank coefficients is also high (0.025 < |r| < 0.777); in average, |r| = 0.51. Finally, in the case of total votes, the variation of the rank coefficients is also high (0.13 < |r| < 0.82); in average, |r| = 0.51. We can say that in average, public votes among the countries are slightly more correlated with ESC public votes than national jury votes with ESC jury votes.

In Table 1, we can see the lists of TOP 5 countries which are mostly correlated with the ESC results with respect to public, jury and total. In the first column, we can see countries whose public votes are the most correlated with ESC public votes. In the second column, we can see countries whose jury votes are the most correlated with ESC jury votes. In the last column, we can see countries whose total votes are the most correlated with ESC total votes.

Table 1: Countries that are mostly correlated with the ESC results

	Public votes		***Jury votes***		***Total votes***	
	Country	*Averaged 6-year |r|*	*Country*	*Averaged 6-year |r|*	*Country*	*Averaged 6-year |r|*
1	Poland	0.811	Georgia	0.777	Azerbaijan	0.818
2	Netherlands	0.806	Azerbaijan	0.777	Spain	0.789
3	San Marino	0.806	France	0.758	Belgium	0.789
4	Romania	0.800	Czechia	0.757	Israel	0.779
5	Norway	0.798	Iceland	0.753	Belarus	0.756

5 CONCLUSION

In this paper, we analyzed the public and jury votes in the past six ESCs. The main conclusion is that there is a significant variation in results by public and jury (0.22 < |r| < 0.63). In addition, there are many variations in the results at the national level. Since the jury is returned into the competition to ensure objectivity and music quality, our results show that the national juries are less compliance with ESC jury than public. If the quality is to be recognized by juries, why do they think so different?

In future research we plan to use the same dataset and apply the social network analysis to possibly identify substructures of the network: cliques, clans, blockmodels and clusters.

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COMPUTATIONAL SEGMENTATION OF CHILDREN'S MELODY

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Abstract: We present the first results of the comparison of music segmentation between the computational model IDyOM, children of different ages, adolescents, and musical experts. The results have shown that IDyOM is to some extent able to capture differences in music segmentation at different ages and that music segmentation depends (in some cases) on musical structure. The differences in music segmentation found between participants with and without musical knowledge, as well as why and under what circumstances higher and lower-order musical elements are used in music segmentation, need to be further explored.

Keywords: music segmentation, IDyOM, musical elements.

1 INTRODUCTION

In music structure analysis, a piece of music is decomposed (segmented) into its constituent elements in order to determine, from a purely musical theoretical point of view, how a piece of music is constructed, how its structural elements interact, and how they are connected [1]. In a cognitive approach, segmentation focuses on identifying segment boundaries, which can be difficult given the ambiguous nature of music [7]. Even if two listeners interpret a musical composition similarly, there will always be discrepancies in their perceptions of identical structural elements, because it is extremely difficult for a performer to explicitly express how the musical structure should be perceived.

Moreover, musical units themselves can be ambiguous, as they can be altered by changes in rhythm, meter, tempo, and instrumentation [5]. When computational models are used for segmentation, the tasks vary depending on the fields in which they are used, e.g., music information retrieval, audio engineering, generative art, computational musicology, music cognition, and music psychology [6].

This paper presents the first results of the comparison of music segmentation between the computational model IDyOM [9], children of different ages, adolescents and musical experts. Our motivation was that: (i) there is currently no study comparing music segmentation between a computational model, children of different ages, and adolescents, (ii) there are currently no studies addressing how and which musical elements are used independently or in combination in segmenting music across different ages in children using a computational approach, and (iii) IDyOM [1] may be capable of replicating human perception of music and its segmentation across different ages [8].

[1] More about IDyOM on http://www.marcus-pearce.com/idyom/

2 METHODOLOGY

2.1 Data

Three children's songs with normative phrasing[2] determined by musical experts, and 155 Slovenian monophonic songs,[3] consisting of 44 children's folk songs and 111 children's songs were used for training the IDyOM computational model. All the songs were first notated in Sibelius notation software, afterward encoded into MIDI (256 PPQN) with piano timbre, at the same speed, and without any changes in loudness or articulation, and then imported into IDyOM. As MIDI files do not carry explicit information about the phrases, means, that IDyOM has neither been optimized to predict boundaries nor given access to any boundary information.

2.2 Participants

The music segmentation patterns of three songs provided by 106 children and adolescents (with and without musical knowledge), and five musical experts were examined.

2.3 Procedure

The breathing patterns of children and adolescents in three chosen songs was collected during a Breathing Experiment conducted in 2018. Phrasings in selected songs were generated by training the computational model using the 155 monophonic children's songs, and the normative phrasings of three children's songs provided by musical experts. The musical structure was observed with viewpoints, functions implemented into the computational model IDyOM. Peak-picking algorithm, implemented in IDyOM as well, was utilized to segment the musical structure. We used different viewpoints (see Table 1) to capture the differences in music segmentation at various ages: basic viewpoints (`cpitch, dur`), derived viewpoints (`dur_ratio, cpint, cpint_size, ioi, contour, cpitch_class, cpintfip, inscale, cpintfref`), and other (linked) combinations of the chosen viewpoints. Additionally, 12 viewpoints were used for viewpoint selection: `cpitch, cpitch_class, tessitura, cpint, cpint_size, cpcint_size, cpcint, contour, newcontour, cpintfip, inscale, cpintfref`.[4]

Table 1: Viewpoints used in current study.

Viewpoints	Description	Viewpoint	Description
cpitch	chromatic pitch of each event	dur	duration in basic time units
dur_ratio	duration of last/duration of previous	cpint	denotes an interval
cpint_size	absolute value for cpint	ioi	inter onset interval
contour	shape of a melody	newcontour	contour changes
cpitch_class	cpitch modulo 12	cpintfip	chromatic interval from 1st event
cpintfref	chromatic interval from tonic	inscale	"in-key" and "out-key" tones
cpcint	octave equivalent pitch class interval	cpcint_size	absolute value for cpcint

After preliminary testing 30 models[5] using basic, derived, and linked viewpoints, 14 segmentation models out of 30 were chosen to determine if lower- and/or higher-order musical features are dominating in music processing and are utilized to segment musical structure at a specific age. Other 16 segmentation models were excluded either because of their (i) too high

[2]In this paper the term "normative" means the most probable interpretation according to music theory and musical experience. This terminology is used to avoid the implication that there is in general a single correct interpretation.

[3]The data is publicly available on https://github.com/LMihel/LMihelac

[4]Viewpoint selection is a hill-climbing procedure that integrates multiple viewpoints in order to minimize the information content of a dataset.

[5]The term model is used in this paper as "theory and system" and as a component of the computational model.

information content,[6], (ii) because they did not contribute to a better understanding of the utilization of lower/higher order musical features in the segmentation of musical structure, or (iii) because they did not provide any information about boundaries.

A binary indicator was used to indicate a boundary (1 = boundary; 0 = no boundary) for each event in a song and for each participant separately. As each event (note) in the melody can be interpreted and assigned a value of 1 or 0, the boundary occurrences in each melody were independently compared for each participant with the IDyOM boundary occurrences for each event. Boundaries on the first note of each melody were discarded fore the same reason as in the study from de Noijer et al.[4], as there is no information about the context prior to the first event, and including a boundary on the first event would induce anomalies into the statistical analysis.

To analyze the match between each participant and IDyOM, cosine metric was used to determine the similarity of two binary vectors. The greater the cosine similarity, the closer the segmentations match one another. Finally, by averaging the computed cosine similarities across all of the observed (sub)groups, the mean similarity of each observed (sub)group of children to IDyOM's phrasing was calculated.

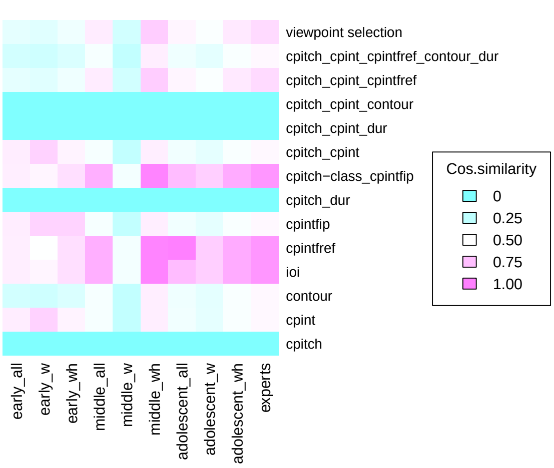
3 RESULTS

The cosine similarity for each group and IDyOM in each song is shown in Figure 3, showing heatmaps for Song 1, Song 2, and Song 3. In Song 1 we can see an increase in matching across the ages in linked viewpoints (the topmost viewpoints), and especially in Song 3 (more complex, linked combinations of viewpoints), indicating that multiple psychological representations of pitch (e.g., pitch height, pitch interval, contour), pitch combined with duration, and higher-order musical features are used in older groups. This is in accordance with the study by Costa-Giomi, in which was found that older children have fewer limitations on their memories and more ways to direct their attention to the pertinent cues of the stimuli, which makes older children better able to process information [3]. When the information stored in the musical content is insufficiently relevant, higher-order musical features are processed and used in the segmentation task, rather than just a few lower-order musical features, however not sequentially from lower to higher order [11], but separately or in combination (depending on the musical structure) and are more effective in older age groups. When viewpoints are linked in various combinations, ranging from two to twelve (as in viewpoint selection), increasing matching outcomes across the ages are noticeable as seen in Figure 3.

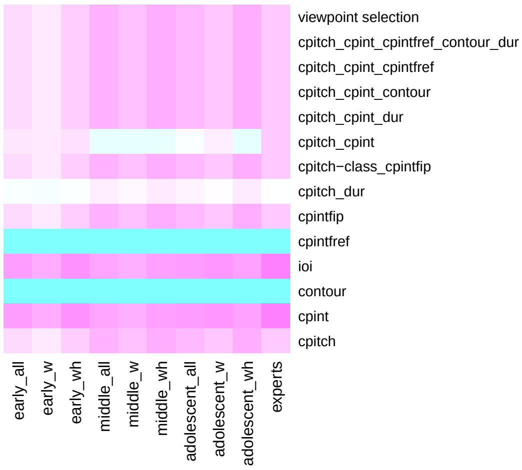
Intriguingly, Song 2 does not demonstrate an increase in age-based matching between groups and IDyOM as Song 1 and Song 3 do when viewpoints are linked and used in more complex combinations. The findings indicate that, regardless of age or musical experience, the use of musical elements in music segmentation is dependent upon (among other factors) the musical structure, which may or may not give unambiguous clues for musical content segmentation.

It is not clear how musical knowledge impacts the segmentation of music and the use of more complex and higher-order musical elements. According to Bigand [2], participants without musical knowledge process music using the same principles as those with musical knowledge. Saari et al. [10], using six musical features representing low-level (timbre) and high-level (rhythm and tonality) aspects of music perception, demonstrated that musical training had the greatest effect on the processing of high-level musical features. As all participants without musical knowledge performed better in Songs 1 and 2, while participants with musical knowledge performed better in Song 3, it is unclear whether musical knowledge or the musical structure itself influences the use of lower- and higher-order musical elements, regardless of age.

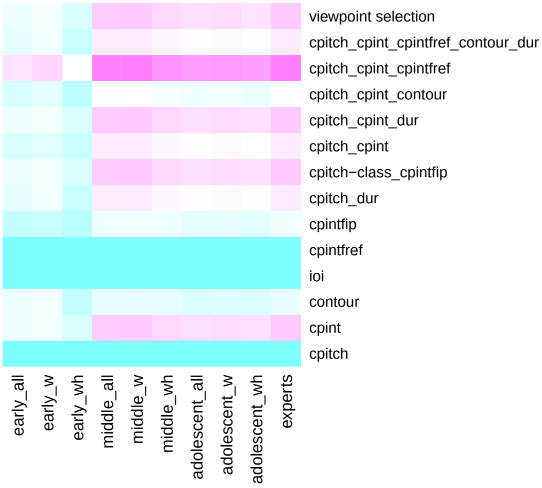
[6]Lower information content of a segmentation model indicates a higher probability of being more similar to the human segmentation of musical structure.



(a) Song 1.



(b) Song 2.



(c) Song 3.

Figure 1: Heatmaps showing cosine similarity in Song 1, Song 2 and Song 3.

4 CONCLUSION AND FUTURE WORK

In this work, the automatic segmentation of IDyOM was compared to human segmentation across different ages. Differences in the perception of musical phrases and phrase boundaries from the perspective of using lower and higher-order musical features in the segmentation of musical content were discovered by using IDyOM to simulate human segmentation and using different viewpoints to observe musical structure. The ability of IDyOM to capture the human perception of musical segments at different ages was demonstrated (to some extent) by increased matching to the normative phrasing from the youngest group (early) to the oldest (musical experts) in similarity measures between human and computational segmentation.

The results suggest that each musical feature may play a different role in phrase identification when used alone or in combination with other musical features, depending on a variety of factors such as experience, age, and combination of features in a particular part of the song. Further research is needed to identify what factors influence how certain events are used in segmenting music across age groups and why some aspects of musical structure are perceived as more important than others.

5 Acknowledgements

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Session 3:

Finance and Investments

RESOURCE PRODUCTIVITY INFLUENCED BY SELECTED CIRCULAR ECONOMY AND DEVELOPMENT LEVEL INDICATORS: PROFILES OF CLUSTERED EU COUNTRIES

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Abstract: In the European Union 27 countries (formation 2020), Resource Productivity, RP, increased from 2000 to 2021 noticeably. In this research, where Eurostat data were analyzed for 2021, RP performed the main study and the dependent variable. In correlation and multiple linear regression analysis, three circular economy, economic development, and environment indicators were used as regressors. The hierarchical clustering of countries was examined, showing that several economically highly developed countries were ranked low for environmental maintenance, while some relatively poor countries were ranked top for environmental indicators, reflecting their circular economy orientation profiles.

Keywords: Domestic Material Consumption, General Government Expenditure for Environmental Protection, Correlation, Mallows' Cp metric, Hierarchical clustering, Euclidean distance, Ward linkage.

1 INTRODUCTION AND LITERATURE REVIEW

According to [4, 5, 14], in 2015, Eurostat developed Resource Productivity (RP) as a new environmental, circular economy (CE), and sustainable development (SD) indicator, which measures the relationship between economic activities, indicating the efficacy of natural resources used with the goal of producing goods and services in an economy. It belongs to Sustainable Development Goals 12 "Responsible Consumption and Production" indicators, [15, 16]. Since RP is defined as Gross Domestic Product (GDP) per unit of Domestic Material Consumption (DMC), as given in [7, 8, 14], measured in EUR per kilogram (€/kg), and a portion of these materials is directly consumed by households, it is not used as an input to production activities. [20] elaborated on measuring material flow and RP, concluding that sustainable resource use contributes to an increase in RP. [21] and [22] considered economic indicators and environmental results effects up to 2060. [23] and [24] focused on measuring the efficiency of EU countries in achieving CE and SD with a nonparametric approach using Data Envelopment Analysis (DEA), for recognizing top and worst-ranked countries. In [19], RP is analyzed as a key green economy element in European countries. In [2] factor and cluster analysis to uncover the differences in the main CE aspects within the EU27, from 2015 to 2019, were applied to show the European gap reducing in the CE, while broadening with respect to sustainability. [25] built a multiple regression model describing the significance of the impact of a number of variables on the resulting RP of the EU member states. In [3], after trend analysis, selected multivariate methods for analyzing RP for selected EU countries in 2020 were applied, resulting in three clusters related to their CE development level. In [18], the relationship between RP and environmental degradation is explored by panel data for EU27 countries from 2000 to 2020.

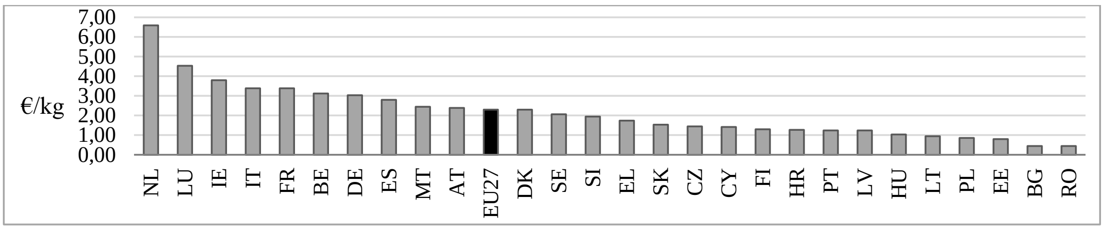
The research hypothesis in this study is that RP and three with it positively correlated CE and development level indicators can, after removing the serious outliers, enable the creation of homogeneous clusters of the EU countries in 2021.

2 DATA AND METHODS

A scatterplot analysis included in [11] presents DMC against GDP levels showing no clear linear relationship between the two values. There are EU countries with low GDPs and high DMC, like Romania, and countries with high GDPs and low DMC, like Ireland. Also, there are countries with low DMC and low GDP, such as Greece, and with high DMC and high GDP, such as Lithuania. At the Member State level, RP moved differently. It increased in almost all countries from 2000 to 2021, most remarkably in Ireland, the Netherlands, and Greece, and doubled in Ireland and Spain. In this research, Eurostat data for RP is used for 27 EU countries (EU27_2020) as the main study and the dependent variable. For 2021, it is denoted as RP_2021, measured in €/kg. To analyze RP_2021, after correlation analysis, an multiple linear regression (MLR) model was built with three regressors: Circular Material Use Rate or Circularity Rate (CMU), which is an environmental CE indicator, defined as the share of material resources used in the EU that came from recycled waste materials, measured in %, according to Eurostat [6, 12]; General Government Expenditure for Environmental Protection, ENVIRGPD, an environmental policy indicator, according to [10], measured as % share of Gross Domestic Product; and, finally, Gross Domestic Product per Capita in Purchasing Power Standards (PPS), expressed in volume indices of real expenditure per capita (EU27_2020=100), [9], denoted GDPpc. A graphical comparison of times series for selected countries was done. Because of the more or less very skewed data distributions for all the variables, after the Exploratory Data Analysis (EDA), a correlation and an MLR analysis with Ordinary Least Squares (OLS) estimated regression coefficients, using three independent variables, were used. The hierarchical clustering, with standardized data, Ward linkage, and Euclidean distance was applied including all four variables providing specific countries' profiles.

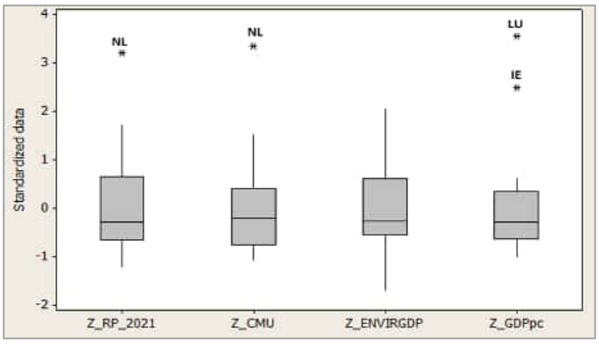
3 RESULTS AND DISCUSSION

The time series for RP, in €/kg, as chain-linked volumes (2015), for the EU27, started in 2000 at 1.55 €/kg, and increased to 2.09 €/kg in 2021, [13]. Figure 1 shows descending ordered bars for RP in €/kg, for 27 EU countries in 2021, ten of them above, and 17 below the EU27 level (2.30 €/kg). The Netherlands (NL) is ranked 1 (6.59 €/kg), and Romania (RO) at the bottom (0.43 €/kg).



Source: Authors' creation, [10].
Figure 1. RP in € per kilogram, in 2021, EU27_2020 countries

Side-by-side boxplots, in Figure 2, for all 27 EU countries in 2021 show that GDPpc has a serious outlier for Luxembourg (LU), with standardized value of 3.6, and a mild outlier for Ireland (IE), while NL has a high extreme outlying value for environmental and sustainable development indicators RP_2021 and CMU, with standardized values higher than 3. No outliers were noted when for the purpose of further analysis EDA was repeated on the data sets reduced for three of the most serious outliers-holding countries, LU, IE, and NL.



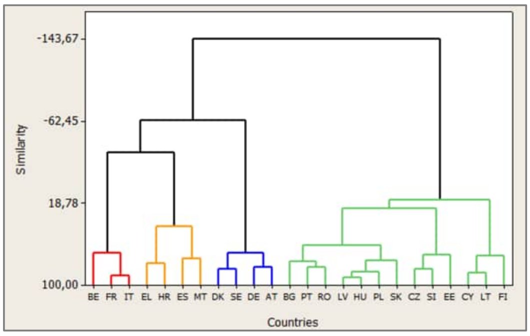
Source: Authors' creation, [6, 9, 10, 11]

Figure 2. Boxplots for standardized data for RP_2021, CMU, ENVIRGDP, and GDPpc in EU27 countries in 2021

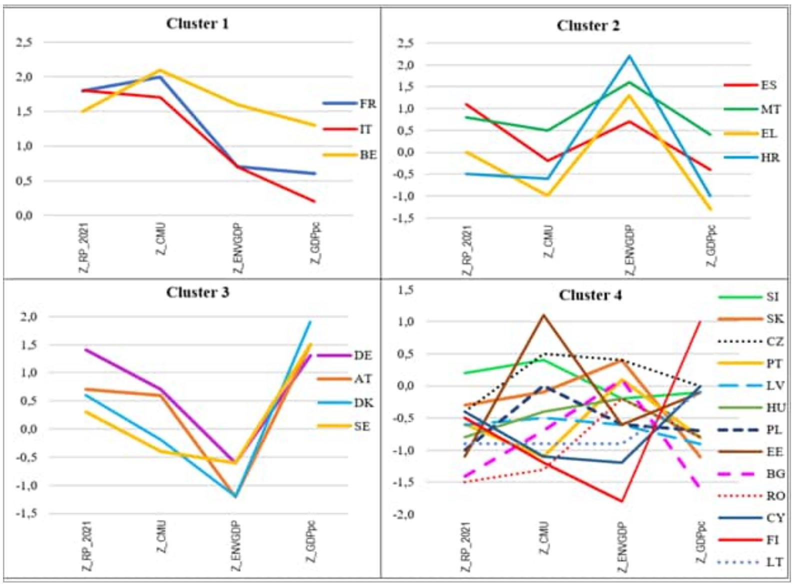
For 27 countries RP_2021 has a positive correlation r with each independent variable: with CMU 0.639, with ENVIRGDP 0.346, and with GDPpc the correlation is the highest, 0.651, still being moderately weak. Between independent variables, CMU and ENVIRGDP, the correlation is 0.482, and between CMU and GDPpc it is only 0.024. Between ENVIRGDP and GDPpc, there is a very weak negative correlation of -0.168. Analyzing only 24 EU countries, the correlation of RP_2021 is stronger with CMU, 0.708, but it is weaker with ENVIRGDP, 0.301, and with GDPpc, it is 0.616, and between the independent variables, correlations are still very weak. After All Possible Regressions analysis for RP_2021 as the dependent variable, and three regressors, for 27 EU countries was applied, an MLR model with the smallest Mallows' Cp metric, including 3 regressors, was built, according to [17]. OLS estimated MLR model is given in equation (1).

$$\widehat{\boldsymbol{RP\_2021}} = -\mathbf{1.59} + \mathbf{0.10} \cdot \mathbf{CMU} + \mathbf{0.84} \cdot \mathbf{ENVIRGDP} + \mathbf{0.02} \cdot \mathbf{GDPpc} \tag{1}$$

The p-values for the estimated regression coefficients at the independent variables are all smaller than 0.05 (at CMU it is <0.0001, at ENVIRGDP it is 0.0416, and at GDPpc it is <0.0001), meaning that each of the regressors is statistically significant at 5% significance level. The overall multiple linear regression model as estimated in (1), is statistically significant at a 1% significance level, with test F(3; 23) of 41.492, a p-value of 1.92e-09, and no model assumption's violation. Model (1) has an adjusted coefficient of determination of 82.37% and a regression coefficient of variation of 27.47%, showing the model's high representativeness, despite outliers discovered in EDA and the leverages at the very same EU countries. The following leverage points appeared: LU (0.645), NL (0.479), IE (0.332), and HR (0.315), which are all in line with EDA results for outliers detected in certain independent variables. The results of hierarchical clustering, of all 27 countries with four standardized variables, Ward linkage, and Euclidean distances showed the three serious outliers-holding countries, NL, LU, and IE, might be omitted from further analysis, [1]. Further clustering of only 24 countries, with standardized data of four variables from 2021, Ward Linkage and Euclidean Distance, gave dendrogram in Figure 3, with Cluster 1, containing three Economically highly developed leading RP and CE countries (BE, FR, and IT), Cluster 2, with four Economically developing countries entering CE society with a high ENVIRGDP indicator (EL, HR, ES, and MT), and Cluster 3, having four Economically developed countries with high RP, that need to improve other CE indicators (DK, SE, DE, and AT). Cluster 4, includes 13 Economically developing or developed countries lagging behind the CE society, BG, PT, RO, LV, HU, PL, SK, CZ, SI, EE, CY, LT, and FI, which have the lowest or moderately low RP, CMU, and ENVIRGDP, and the lowest GDPpc, with the exception of very rich Finland. The countries' profiles within each of the designed four clusters constructed for 24 EU countries in 2021, with all four variables' standardized data, are shown in Figure 4.



Source: Authors' creation, [6, 9, 10, 11]
Figure 3. Dendrogram for 24 countries of EU27 in 2021, and four standardized variables



Source: Authors' creation, [6, 9, 10, 11]
Figure 4. Profile charts for each cluster of 24 EU countries in 2021

5 CONCLUSIONS

The presented clustering of European countries only partly proves the research hypothesis that homogeneous clusters of countries regarding CE development level could be created for EU countries in 2021. There are extreme outliers at countries top performing in RP and other CE and development level indicators, like for the NL, accompanied by IE and LU.

There are positive, weak to moderate correlations between RP_2021 and selected CE indicators. Here included CE indicators and poorly correlated to each other, especially to GDPpc. The correlation coefficients are in some cases close to zero, for both n=27 and n=24 countries. The MLR model developed for EU27 shows positive impacts of each regressor to RP_2021 having remaining regressors fixed. Four leverage points were found, for LU (the highest of 0.645), NL, IE, and HR, all related to top-performing countries with a kind of mild outliers in certain independent variables. Clustering the reduced number of only 24 EU countries shows that there are no clear CE policies adopted by a number of EU countries. Cluster 1 includes the highest RP and CMU, high ENVIRGDP, and high GDPpc performing countries. But, some economically highly developed countries, like FI, which is surprisingly low in CMU (similar to economically developed IE and LU) and other CE indicators, have fallen in Cluster 4, close to undeveloped CY and LV, and rather poor RO and BG, which are bottom actors in RP. At the same time, several low-developed economies are high performers for some CE indicators, e.g., HR is ranked top for ENVIRGDP, followed by similarly low economically developed and tourism-oriented Greece. For NL, all here considered indicators are ranked either at the top or very high, and it is not wondering that it should be isolated as an extraordinary example as the most developed CE in 2021. Knowing that GDPpc is in moderately low correlation with CE indicators, additional further research with more indicators might improve the understanding of the features and future trends of RP and other CE indicators in EU countries, as well as their green economy policies.

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DEEP LEARNING FOR PREDICTING CORPORATE FINANCIAL DISTRESS OF CROATIAN IT COMPANIES

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Abstract: The ability to predict the financial distress of companies is important for investors, creditors, and regulators, especially in the IT industry, which is developing strongly in Croatia. Artificial Neural Networks have proven to be very successful in various fields and offer numerous advantages compared to traditional statistical methods. In this paper, Deep Neural Networks were used to build a model for classifying companies according to their liquidity status as one of the first signs of financial distress. The sample consisted of 3866 Croatian IT companies active in 2021, and the input set included 19 company features. The model is evaluated by accuracy, precision, recall, F-score and area under the curve (AUC). The accuracy of 85.5% and AUC of 91.1% show the potential of Deep Learning in predicting financial distress. In addition, the local sensitivity analysis identified important predictors that may be useful for future research.

Keywords: financial distress, liquidity, IT industry, Deep Learning, local sensitivity analysis

1 INTRODUCTION

Financial distress is the state in which a company is in a certain type of financial problems [17]. Financial indicators serve not only as an indicator of a company's performance and growth potential, but also as a warning that the company is experiencing financial problems, which makes them an important decision-making tool for management and potential investors. There are several groups of financial indicators, such as liquidity ratios, activity ratios, profitability ratios, solvency ratios, and efficiency ratios. Financial distress can vary in severity from the mildest sign, namely short-term liquidity difficulties, to the most serious sign, namely a declaration of bankruptcy [9]. This paper focuses on the liquidity indicator as a predictor of financial distress. In simple terms, liquidity can be defined as the ability of a company to meet its immediate financial obligations with its available assets or cash. It illustrates how quickly a company can use its available cash to pay its existing liabilities [9]. Low liquidity makes it difficult for a company to meet due obligations to credit, service, and commodity suppliers and to make timely payments to creditors [12]. For this reason, it is important to identify potential liquidity difficulties before they occur.

This paper examines the liquidity of companies in the Croatian IT industry, for which the government's strategy is to become the second strongest industry after tourism in terms of GDP share in the next 10 years [16]. Therefore, along with policy makers and other interest groups such as creditors and investors, it is important to have a complete picture of the health of companies in this industry. The objective of this paper is to develop a Deep Learning model that is able to identify the liquidity status of the company and, additionally, determine the most important predictors through sensitivity analysis.

2 PREVIOUS RESEARCH

Analysis of previous research in the field of predicting corporate financial distress shows that both traditional statistical techniques such as logistic regression and newer approaches such as ANNs, DNNs and Decision Trees are used. ANNs have certain advantages over traditional techniques, such as the ability to model more complex nonlinear relationships between input and output features, learn from historical data, automatically identify relevant predictors, etc. [2] [19].

Mramor and Valentinčič [10] opted for logit and probit estimation and multivariate discriminant analysis to model liquidity prediction. The average classification accuracy of the logit model, which included only financial ratios, was 87.9%, but they also report a very high percentage of misclassified companies with liquidity problems, which averaged 72% for all three models. The authors attribute the high misclassification to the limited power of financial ratios alone in predicting future liquidity problems. They conclude that information about liquidity problems in the past can contribute significantly to predictive power. Chen and Liu [3] estimated corporate liquidity, showing that ANNs perform better than conventional regression models with an average R^2 value of 0.8. In addition, they report that cash flow, capital expenditures, leverage, market-to-book ratio, and net working capital are significant predictors of liquidity. More recently, Aydin et al. [1] compared the performance of the ANN and DT models in predicting the bankruptcy of manufacturing, service, and trade companies. The input of the model was various financial ratios, including liquidity ratios. Although both models have a very high classification accuracy of up to 95.24%, the ANN model provided a higher rate of correct classifications of bankrupt firms than the DT model when accuracy, precision, and sensitivity measures were evaluated. Other machine learning methods, such as the support vector machine, also achieved high accuracy in classifying companies with financial problems in Mselma et al. [11].

Analysis of previous research concludes that the authors used financial ratios from various reports and databases available to them, so different predictors were found to be significant, and the focus was on the accuracy of the model. In this paper, a predictive model was built, and the most significant predictors were identified from the set of available features.

3 METHODOLOGY AND DATA

3.1 Deep Learning

Deep Learning (DL) represents a new era in Neural Network development. The DL has evolved from machine learning, where methods that learn from examples, such as Artificial Neural Networks (ANNs), tree partitioning methods, support vector machine and other methods aim to approximate the function between input and output through multiple layers or a tree structure [20]. According to LeCun et al. [8] DL enables the creation of computational models with multiple layers that can learn complex data representations at different levels of abstraction. There are different types of DNNs such as: Feedforward Neural Network, Recurrent Neural Networks (RNNs), Convolutional Neural Networks (CNNs), Deep Belief Networks (DBN), etc. [4] [7] [15]. In this paper, a deep feedforward Neural Network is used for modeling. A feedforward Neural Network is called "deep" if it has more than one hidden layer [7].

Learning in classical ANNs is an iterative process that operates through layers of neurons, one of which is an input layer, one or more hidden layers, and an output layer and same process is in DL. The input layer receives data from the input set X with n elements with values $x_i \in R$, $i=1, 2, \ldots, n$ and randomly selected initial weights w_i, typically from the interval [-1,1]. The weighted sum of all x_i values is passed from the input layer to a hidden layer, which generates its output y_c using an activation function [5]. In the output layer, the predicted output is compared to the actual output and the global error E is determined [6]. Each output unit has its own local error e, which is then used to adjust the weights according to a certain learning rule until the global error E is minimized [19]. DNNs allow adding hidden layers and better suited for complex data with a large number of input features and cases [20] [21]. In the case of two hidden layers, the computation of DNN can be expressed [20]:

$$y_c = f_2\left(\sum_{j=1}^{m} f_1\left(\sum_{i=1}^{n} w_i^{(1)} x_i\right) w_j^{(2)}\right) \tag{1}$$

where y_c is the calculated output, x_i are the elements of the input vector, $w_i^{(1)}$ are the elements of the weight vector of the first hidden layer, $w_j^{(2)}$ are the elements of the weight vector of the second hidden layer, f_1 is an activation function used in the first hidden layer, while f_2 is used in the second hidden layer. The initial weights are randomly set from the interval [-1,1] and later adjusted by the error term [20]. Optimization algorithms such as: Adam, Nadam, Adamx, SGD, calculate gradients and try to minimize the loss function [18].

The performance of DNNs is evaluated by calculating the difference between the predicted output y_c and the actual output y_a.

3.2 Data

The real dataset was obtained from the business database of the Croatian Finance Agency, which is responsible for collecting and presenting companies' business, financial and other data. Companies for which financial ratios were not available were removed from the initial dataset. After data cleaning and transformation, the final sample consisted of 3866 IT companies from the Computer programming, consultancy and related activities category that were active in 2021.

The input features used for modeling are listed in the Table 1. The output feature was categorical and consisted of two categories representing liquidity status of company (*Liquid* - *Illiquid*). Companies are considered liquid if their three most common liquidity ratios – the *Cash ratio*, the *Acid test ratio (quick ratio)*, and the *Working capital ratio (current ratio)* – indicate their ability to repay short-term liabilities. In this sample, there were 1972 liquid IT companies and 1894 illiquid IT companies.

Descriptive statistics of input features for each class of liquidity are presented in Table 1. Finally, the total sample was divided into a training sample (80%) and validation sample (20%) to build a Deep Learning model. R software is used for modeling.

Table 1: Descriptive statistics of feature for both liquid and illiquid companies

Feature	***Liquid companies (1972 of them (51.01 %))***	***Illiquid companies (1894 of them (48.99 %))***
Size	Micro: 91.48 %, Small: 7.86 %, Medium: 0.51 %, Large: 0.15 %	Micro: 91.13 %, Small: 7.60 %, Medium: 1.11 %, Large: 0.16 %
Ownership	Private since establishment: 99.65 %, Other: 0.35 %	Private since establishment: 99.52 %, Other: 0.48 %
Percentage of domestic capital	Min.: 0.00, Mean: 92.51, Max.: 100.00, St. dev.: 25.37	Min.: 0.00, Mean: 89.50, Max.: 100.00, St. dev.: 30.31
Years of existence (until 2021)	Min.: 0.00, Mean: 7.47, Max.: 34.00, St. dev.: 8.16	Min.: 0.00, Mean: 9.15, Max.: 41.00, St. dev.: 8.52
Activity	Computer programming: 80.17 %, Computer consulting: 9.23 %, Computer equipment and system management: 2.74 %, Other: 7.86 %	Computer programming: 70.54 %, Computer consulting: 10.03 %, Computer equipment and system management: 5.91 %, Other: %
Subject type	Limited liability company: 88.34 %, Simple private company limited by shares: 10.29 %, Craftsman: 0.86 %, Foreign founder: 0.25 %, Public limited company: 0.15 %, Cooperative: 0.11 %	Limited liability company: 79.14 %, Simple private company limited by shares: 19.38 %, Craftsman: 0.84 %, Foreign founder: 0.48 %, Public limited company: 0.05 %, Cooperative: 0.11 %
Credit rating	A: 55.07 %, B: 43.67 %, C: 0.20 %, D: 0.35 %, Unknown: 0.71 %	A: 13.94 %, B: 71.17 %, C: 8.03 %, D: 2.53 %, Unknown: 4.33 %
EBIT margin 2021	Min.: -2957.71, Mean: 27.96, Max.: 100.00, St. dev.: 111.99	Min.: -4397193.05, Mean: -2695.64, Max.: 100.00, St. dev.: 101167.54
EBITDA margin 2021	Min.: -1988.53, Mean: 32.58, Max.: 100.00, St. dev.: 86.71	Min.: -4397193.05, Mean: -2606.06, Max.: 100.00, St. dev.: 101097.1
EBIT 2021 (€)	Min.: -1666859.51, Mean: 102584.66, Max.: 14671716.90, St. dev.: 484516.48	Min.: -2898897.21, Mean: 44768.58, Max.: 5669439.78, St. dev.: 282384.13
EBITDA 2021 (€)	Min.: -664698.25, Mean: 121581.76, Max.: 20666969.81, St. dev.: 668313.19	Min.: -2452136.17, Mean: 59590.48, Max.: 6626365.25, St. dev.: 335155.45
Financial stability ratio 2021	Min.: 0.00, Mean: 0.14, Max.: 5.44, St. dev.: 0.25	Min.: 0.00, Mean: 1.65, Max.: 824.85, St. dev.: 25.33
Debt ratio 2021	Min.: 0.00, Mean: 0.23, Max.: 48.30, St. dev.: 1.19	Min.: 0.00, Mean: 362.74, Max.: 548687.61, St. dev.: 12679.87
Self-Financing Capacity 2021	Min.: 0.00, Mean: 0.80, Max.: 1.00, St. dev.: 0.20	Min.: 0.00, Mean: 0.37, Max.: 1.00, St. dev.: 0.32
Financial leverage 2021	Min.: 0.00, Mean: 0.53, Max.: 230.44, St. dev.: 5.85	Min.: 0.00, Mean: 4.37, Max.: 1269.60, St. dev.: 44.32
Return on Assets (ROA) 2021 (%)	Min.: -3232.91, Mean: 36.67, Max.: 280.99, St. dev.: 86.39	Min.: -8905488.44, Mean: -4927.82, Max.: 1545032.69, St. dev.: 210509.62
Return on Equity (ROE) 2021 (%)	Min.: 0.00, Mean: 52.95, Max.: 733.50, St. dev.: 43.45	Min.: 0.00, Mean: 33.80, Max.: 3447.00, St. dev.: 132.55
Value added 2021 (€)	Min.: 0.00, Mean: 318097.75, Max.: 134939418.00, St. dev.: 3221191.66	Min.: -1922.09, Mean: 212996.16, Max.: 19137768.93, St. dev.: 981477.50
Profit 2021 (€)	Min.: 0.00, Mean: 95476.47, Max.: 15290595.93, St. dev.: 465884.02	Min.: 0.00, Mean: 45233.53, Max.: 5082004.38, St. dev.: 238624.32

4 RESULTS

To find the optimal DL model, numerous Deep Learning Neural Network architectures were created with different depths of (*two, three and three hidden layers*), different number of neurons in hidden layers (*1 - 100*), activation functions (*sigmoid, hard sigmoid, tangent, ReLu*), optimization algorithms (*Adam, Nadam, Adamax, SGD*) and other parameters and hyperparameters such as *learning rate, dropout, batch size, epoch*, etc. The optimal results for each architecture depending on its depth are shown in Table 2.

Table 2: Descriptive statistics of feature for both liquid and illiquid companies

	DNN – 2 hidden layers	***DNN – 3 hidden layers***	***DNN – 4 hidden layers***
DNN model architecture	35-*30-50*-2	35-*30-50-89*-2	**35-*30-37-49-89*-2**
Activation function	tanh (all hidden layers), sigmoid (output layer)	tanh (all hidden layers), sigmoid (output layer)	ReLU (1st, 2nd and 4th layer) tanh (3rd layer), sigmoid (output layer)
Optimization algorithms	NADAM	ADAM	**ADAM**
Number of parametars and hyperparameters	2861	7345	**8629**
DNN performance parameters			
Average Accuracy	84.9	85.3	**85.5**
Precision	85	83.7	**83.3**
Recall	84.8	87.6	**88.9**
F-score	84.9	85.6	**86**
AUC	91.5	92.1	**91.1**

Table 2 shows that the best performance, i.e., average accuracy rate, was achieved in DNN model with four hidden layers, which is explained below. The DNN architecture "35-30-37-49-89-2" represents a DNN with multiple layers, where each number indicates the number of neurons or units in that layer. The pattern of the architecture is as follows: "input layer - hidden layers - output layer". The input layer consists of 35 neurons that receive the raw input data of the dataset, where in the case of categorical input data, each category represents one neuron. In this architecture, there are four hidden layers with 30, 37, 49, and 89 neurons. These hidden layers are responsible for learning and extracting meaningful representations of the data through non-linear transformations. Different activation functions are used for each hidden layer: Rectified Linear Unit (ReLU) for the second hidden layer, tanh for the third hidden layer, and ReLU for the fourth hidden layer. The output layer consists of 2 neurons (Liquid and Illiquid) that produce the final outputs of the model.

In total, this DNN architecture consists of six layers: an input layer, four hidden layers, and an output layer. The initial weights of the layer were sampled from a uniform distribution. A dropout layer is added to regularize the model and prevent overfitting. The dropout rate was set to 0.2, meaning that 20% of the neurons in the previous layer are randomly set to 0 during training. The model was compiled using the Adam optimizer, an efficient gradient-based optimization algorithm widely used in Deep Learning. Some other parameters are also used, such as batch size, epoch, learning rate, and so on. The batch size is set to 64, which means that the model is updated after processing 64 samples at once. The learning rate is set to 0.02, which adjusts the step size during optimization. The metrics used for evaluation are commonly used for a binary classification model. An average accuracy of 85.5% means that, on average, the model's predictions are correct for about 85.5% of the specific case. A precision of 83.3% means that the model correctly identifies about 83.3% of the predicted positive cases. A recall of 88.9% means that the model correctly identifies about 88.9% of the actual positive cases. An F-score of 86% indicates a balance between precision and recall, which means that the proposed model performs quite well on both aspects. An AUC of 91.1% indicates that the model can discriminate well between the positive and negative classes.

Overall, these metrics indicate that the model performs well in binary classification, with high accuracy, precision, recall, and an acceptable F-score. The AUC, which is close to 1, also suggests that the model's predictions are reliable in distinguishing between the two classes.

A local sensitivity analysis was performed to determine which features were most important at the local level in predicting a single class. Local sensitivity was conducted with Local Interpretable Model-agnostic Explanations (LIME). Ribeiro et al. describe LIME as: “explanation technique that explains the predictions of any classifier in an interpretable and faithful manner, by learning an interpretable model locally around the prediction” [13]**.** This is achieved by learning locally weighted linear models on the neighborhood data of this specific observation to explain its class in an interpretable way [14].

LIME is conducted for each case (company) in test sample. On Figure 1 is shown first case in test sample (first company).

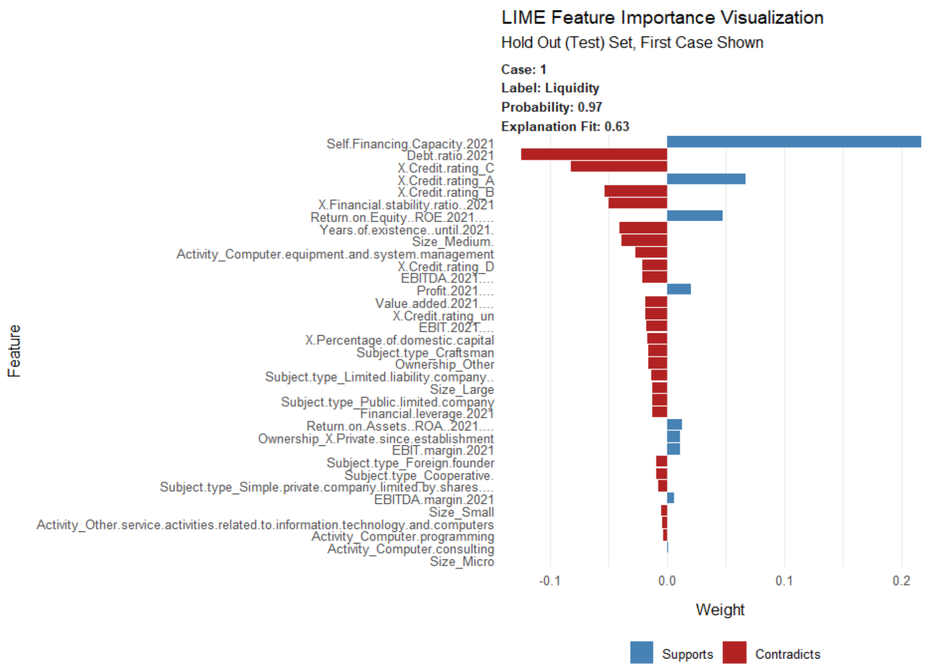

Figure 1: LIME feature importance

Figure 1 shows that the DNN model classified this case as class Liquid with a high confidence level of approximately 93%. An explanatory fit of 0.63 indicates that the interpretable model is a reasonably good approximation, capturing about 63% of the behavior of the DNN model in the local region around the case. The blue "support" line is marked for self-financing capacity, credit rating-A, ROE, profit, ROA, private ownership, EBIT margin, EBITDA margin, and computer consulting activity, indicating that the presence of these features increases the likelihood that this case belongs to the class Liquid. The "Contradicts" for each feature is shown as a red line. It illustrates how the presence of a particular feature lowers the probability that this case falls into a Liquid class. From Figure 1 it can be seen that a higher "debt ratio" suggests increased financial risk, and the DNN model is less likely to predict the case as class Liquid. Same explanation is valid for other features marked with red line.

5 DISCUSSION AND CONCLUSION

The main objective of this paper was to find an optimal Deep Neural Network (DNN) model for predicting liquidity status to prevent financial distress of Croatian IT companies based on financial ratios, ownership, company size and other indicators. The research of financial distress is an interesting research area and for the analysis researchers use traditional statistical techniques such as logistic regression and newer approaches such as Machine Learning. In this paper, different DNN architectures are compared to find the optimal model for predicting liquidity in IT companies. Three DNN architectures with different depths and neurons in hidden layers were tested. The best performance was obtained with a DNN model with four hidden layers: "35-30-37-49-89-2". This architecture had an average accuracy of 85.5%, precision of 83.3%, recall of 88.9%, F-score of 86%, and AUC of 91.1%, indicating good performance in binary classification. Due to the diversity of companies, it is important to identify which features have the greatest impact on liquidity status. For this purpose, Local Interpretable Model-agnostic Explanations (LIME) are used to determine the impact of the features for each of the cases. For example, in the first case, the features self-funding capacity, credit rating, profit, ROE, ROA, EBIT margin, EBITDA margin, ownership, and activity proved to be the most important. This analysis thus makes it possible to identify the main predictors of liquidity for each company, which can be useful for creditors and investors. In future research, it would be useful for researchers to conduct a global sensitivity analysis and identify the most significant features in liquidity modeling. To get a more comprehensive picture of the problem, it would be beneficial to build more models based on Machine Learning methods, such as Decision Trees and Support Vector Machine, which have been shown successful in classification problems, and compare the performance of the models. In this paper, the authors used a limited set of available business and financial data. To gain new insights, it would be necessary to expand the set of features used.

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INTERDEPENDENCE BETWEEN CRYPTOCURRENCY ADOPTION AND FINANCIAL LITERACY: A CROSS-COUNTRY EVIDENCE

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Abstract: This paper investigates the interconnection between cryptocurrency adoption and financial literacy on a cross-country level in the periods before (2016, 2018) and after (2020) COVID. The results for 2016 and 2018 indicate that on average countries where crypto adoption is higher, have higher financial literacy, lower levels of corruption, and more developed stock markets. However, in 2020 the results were different. Financial literacy levels varied within both high and low crypto adoption groups but in total, people were motivated to invest in cryptocurrency during the uncertain COVID period, regardless of financial literacy and previous determinants of crypto trading.

Keywords: crypto-asset, crypto-adoption, Bitcoin, financial literacy, institutional quality, cluster analysis

1 INTRODUCTION

Crypto assets are one of the most significant financial innovations and investing in crypto assets has been rapidly spreading worldwide in the last decade. They became very popular, and their price and trading volume have rapidly grown since their introduction. Today crypto assets have become an important part of institutional investors' portfolios (Huang et al., 2022). Research confirms that cryptocurrencies serve as diversifiers (Kajtazi & Moro, 2019) and can improve portfolio performance when combined with gold (Som & Kayal, 2022). Research before COVID did not confirm macroeconomic and financial variables as determinants of cryptocurrency trade, while it is considered affected by global variables. Previous prices of Bitcoin (Auer et al., 2022; Hakim das Neves, 2020) and VIX are confirmed as significant determinants of Bitcoin trade (Bouri et al., 2017; Goodell & Goutte, 2021). On the country level, institutional quality indicators, especially corruption, seem significant for crypto adoption and Bitcoin exchange trading volume development (Alnasaa et al., 2022; Koziuk, 2022; Škrabić Perić et al., 2023).

The market shocks in crypto markets did not interrupt their further development. On the contrary, the results of Cornelli et al. (2023) show that crypto market activity has grown significantly in the wake of the Terra/Luna collapse and the FTX bankruptcy. However, in that situation on the market, large, informed investors were selling their assets while smaller investors were buying. This behaviour rises the question of whether small investors were sufficiently financially literate to recognize the adverse situations on the market and the investment risks. According to Auer et al. (2022), approximately more than 73-81% of all users from the general population have made losses by investing in Bitcoin from August 2015 to the end of 2022 in 95 world countries.

Despite the losses, crypto digital currencies have become increasingly popular among the general population. It is more adopted in countries with more developed information and communication technologies (Feyen et al., 2022) and higher financial literacy (OECD 2021). On the other hand, according to the Global Adoption Index 2022, acceptance of crypto amongst the general population is rapidly growing in Emerging market countries. Characteristics of all these countries are lower levels of institutional quality, financial development, financial inclusion, and financial literacy (Klapper et al., 2022). These opposite findings impose doubts on the level of financial literacy of countries that exhibit higher crypto adoption. This interconnection is extensively investigated in recent literature.

Panos et al. (2020) investigate the connection between cryptocurrency adoption and financial literacy on microdata for 15 countries and find a negative relationship. Moreover, they concluded that financially literate people were aware of cryptocurrency risk. Yoshino (2020) found that financial literacy is positively correlated with using fetch services and negatively with holding cryptocurrency for Japan's population from 18-78 years. On the other hand, Zhao & Zhang (2021) found a positive relationship between investment experience in financial markets and investment in cryptocurrency for US investors. Moreover, they confirm the positive relationship between financial literacy and crypto investment. Fujiki (2020) explores the characteristics of cryptocurrency investors in Japan, and he found that owners of cryptocurrencies have higher objective financial literacy and experience of financial education at school. Stix (2021) proves that crypto asset owners have more financial knowledge than non-owners on Austrian consumers.

From all mentioned above, it is evident that the relationship between financial literacy and crypto assets adaption is interesting for the scientific community and that recent research does not reach a consensus. Moreover, the different author gives opposite results regardless of the country's development, financial development, and other characteristics. However, most of the mentioned results are conducted on a micro level at different times. It is possible that sample is not representative and that respondents did not give true answers regarding trading on cryptocurrency market. Second, the relationship between financial literacy and crypto adoption may be changing during the time and there might be differences in trading trends between general population and big investors.

Therefore, our research adds to the existing literature in several ways. First, we move focus from individuals to the country level. Second, we consider the relationship in three-time points (2016, 2018, and 2020), and capture the change in the relationship between financial literacy and crypto adoption. By including 2020, we consider the COVID period. Third, we consider three indicators to capture various crypto adoption aspects. Average daily users and downloads of Crypto exchange applications capture crypto adoption amongst non-professional investors, while Bitcoin trading volume captures all crypto activity from one country. By using this approach, we can assess the determinants of crypto adoption of the general population and the whole market. Non-professional investors have lower financial knowledge and lower access to market information. Therefore, the country's financial literacy level could be more critical.

The paper is organized as follows. Section 2 describes the data and methodology. Section 3 gives empirical results and their economic interpretation. Section 4 summarizes the results, limitations, and recommendations for future research.

2 DATA AND METHODOLOGY

In our paper, we examined the yearly values of six variables among 14 world economies. Three directly describe the usage of cryptocurrency in different countries. That are Bitcoin trading volume, average daily users on crypto exchange applications and number of downloads of crypto exchange applications. Additionally, we included two variables, corruption in a country and total equity market trading volumes, that previous studies have shown to significantly influence Bitcoin trading (Alnasaa *et al.*, 2022; Wawrosz and Lansky, 2021; Iyer 2022). Finally, we incorporated the financial literacy data.

To gather information on Bitcoin trading volume by country, we obtained data from the https://coin.dance/ website. We collected the data for 14 countries: Argentina, Australia, Brazil, Canada, China, Indonesia, Japan, Mexico, Russia, South Africa, Saudi Arabia, USA, Turkey, and the United Kingdom. The selection of countries was based on the availability of financial literacy data. To ensure consistency, we converted the data from local currencies to USD using appropriate historical exchange rates. Furthermore, we transformed the weekly into yearly volumes by summing the corresponding weekly values. In addition to Bitcoin trading volume, we also collected data on average daily users of cryptocurrency exchange applications and the number of downloads of crypto exchange applications per 100,000 inhabitants from Sensor Tower BIS (Auer et al. 2022). The corruption index for each country was acquired from the International Country Risk Guide (ICRG) database, while the total value traded on equity markets was collected from https://statistics.world-exchanges.org. The corruption index values are ranged from 100 (Very low risk of corruption) to 0 (Very high risk of corruption) so it could be considered as control of corruption. The data on financial literacy are collected from the S&P's global financial literacy survey (downloaded from https://gflec.org). All the data, for all countries, are collected for three distinct years: 2016, 2018, and 2020. It is worth noting that the year 2020 was marked by the outbreak of the COVID-19 pandemic, potentially impacting the crypto currency trading market and its specifics.

Due to the small dimension and the scarcity of the available data, particularly regarding financial literacy, which is only available for a single year (2014), our options for analysis were limited. Although the Program for International Student Assessment (PISA) provided data on financial literacy for three distinct years, it covered only a limited number of countries so the overlap with the availability of Bitcoin data resulted in only a few countries left.

Our research does not focus on predicting a specific outcome variable but rather aims to determine the potential relationship between a country's Bitcoin usage and its financial literacy. To achieve this, we aim to identify the groups of countries that exhibit similar characteristics in terms of Bitcoin trading and financial literacy. Therefore, we decided to employ cluster analysis. Particularly, we use the k-means clustering algorithm which is widely popular due to its simplicity, ease of implementation and fast convergence. It is a gradient descent technique that aims to partition the data into k clusters by iteratively minimizing the distances between data points and centroids inside a cluster. The centroids are iteratively updated until convergence is reached. The distance measure used in this paper is Euclidean distance, and the starting cluster is chosen randomly by the software. We decide to split the data into two clusters to segregate between financially more and less literate countries. Additionally, after employing k-means clustering on such data (with three variables + financial literacy) we add equity market volumes and corruption in-country data to see if these variables that are previously proven to influence Bitcoin trading, show differences between obtained clusters.

3 RESULTS AND DISCUSSION

Before employing the k-means clustering analysis, all variables were standardized. The results of clustering using the k-means algorithm on the dataset containing Bitcoin trading volume (VOL), average daily users per 100 000 inhabitants (ADU), downloads of Crypto exchange applications per 100 000 inhabitants (DWD), and financial literacy (FINLT) are presented in Table 1.

Table 1: Results of K-means clustering. Values in table are clusters that countries have been assigned. (Cluster 1 or Cluster 2)

	Argentina	***Australia***	***Brazil***	***Canada***	***China***	***Indonesia***	***Japan***	***Mexico***	***Russia***	***South Africa***	***Saudi Arabia***	***USA***	***Turkey***	***UK***
2016	1	2	1	2	1	1	1	1	1	1	1	2	1	2
2018	1	2	1	2	1	1	2	1	1	1	1	2	1	2
2020	1	1	1	1	2	1	1	1	2	1	1	2	1	1

The results for the years 2016 and 2018 reveal that the countries predominantly remained distributed among the same clusters, with one exception being Japan. Literature that investigates relationship between financial literacy and crypto trading in Japan (Yoshino, 2020; Fujiki, 2020) showed both positive and negative relationship which can explain this change. However, the distribution underwent notable changes in 2020, the year of the COVID pandemic. In Table 2, Table 3, and Table 4 we present the means of variables that were calculated for each cluster to get a deeper insight into the changes that occurred. Additionally, we now incorporate the Corruption (COR) and total equity market volumes (EQ) variables to investigate their association with the identified clusters.

Table 2: Means of variables by clusters calculated in Table 1 for year 2016

	2016					
Cluster	***VOL***	***ADU***	***DWD***	***FINLT***	***COR***	***EQ***
1	1.99E+07	4.2134	91.6554	33.3	2.45	2661136
2	1.21E+08	23.4231	287.4251	64	4.75	1.06E+07
Total	4.89E+07	9.7019	147.5896	42.0714	3.1071	4497136

Table 3: Means of variables by clusters calculated in Table 1 for year 2018

	2018					
Cluster	***VOL***	***ADU***	***DWD***	***FINLT***	***COR***	***EQ***
1	1.32E+08	37.3908	476.2065	32.2222	2.2778	1770024
2	1.62E+08	273.9951	1732.031	59.8	4.642	1.15E+07
Total	1.43E+08	121.8923	924.7153	42.0714	3.1221	4771614

Table 4: Means of variables by clusters calculated in Table 1 for year 2020

	2020					
Cluster	***VOL***	***ADU***	***DWD***	***FINLT***	***COR***	***EQ***
1	3.29E+07	185.3993	2211.497	39.8	3.15	1232731
2	2.39E+08	167.024	1563.741	41	2.541667	2.71E+07
Total	8.04E+07	181.1588	2062.015	42.0714	3.00962	7209360

Similar observations can be drawn for the years 2016 and 2018 (Table 2 and Table 3). The findings reveal significant disparities between the two clusters and FINLT results indicate that there are considerable differences in financial literacy. Countries with lower financial literacy values exhibit smaller volumes of Bitcoin trading and usage of crypto trading applications.

Moreover, variables shown to have a significant impact on Bitcoin trading also demonstrate substantial differences between the two clusters, indicating that higher levels of control of corruption and equity market trading volumes are associated with greater Bitcoin adoption.

However, a notable change occurs in 2020. To begin with, the disparities between cluster means have become considerably lower. The small difference between clusters regarding financial literacy alongside the change in countries' distribution among the two clusters from Table 1 indicates that financial literacy no longer determines the cryptocurrency trading in the post-COVID period. Also, it is interesting to note that the total values of the usage of crypto applications (ADU and DWD) have become higher than in the previous years, while the trading volume (VOL) is lower. The crypto exchange applications are assumed to be used by the general population whose volumes of trade are lower while big investors boost the trading volume considerably. Since in 2020 the volume shrunk, while crypto exchange applications usage grew it can be concluded that the big investors have started to withdraw from the Bitcoin market while the general population, in the period of the pandemic, started to enter the market. This is in line with results of Cornelli et al. (2023).

4 CONCLUSION

In this paper, interconnection between financial literacy and crypto adoption has been researched on the basis of 14 countries and three time periods: 2016, 2018 and a COVID influenced 2020. Previous research investigating the relationship between some kinds of crypto adoption and financial literacy has mostly reached opposing findings and thus remained inconclusive. In our paper, we employ clustering analysis to segregate countries into two groups based on financial literacy and the crypto adoption data which includes Bitcoin trading volumes, average daily users and downloads of crypto exchange applications in three different periods to investigate if the relationship between crypto adoption and financial literacy was changing over time. Additionally, corruption and equity market volumes have been added to the analysis as variables that were previously confirmed as determinants of crypto trading. Means of obtained clusters reveal that for the period before COVID countries that have higher financial literacy levels have higher cryptocurrency adoption, lower corruption rates and more developed stock market. On the other hand, year 2020 reveals a shift and the clusters obtained do not show significant differences regarding financial literacy anymore. Moreover, there has been an increase in total crypto adoption determinants with regard to previous years. This imposes that in the time of the COVID pandemic shock, people started trading crypto assets more regardless of their financial literacy.

One limitation of our paper is the dataset that is limited to 14 countries and one year of financial literacy data. By using a more extensive dataset, the relationship between crypto adoption and financial literacy could be investigated more thoroughly. Additionally, apart from financial literacy, some other aspects of literacy such as internet usage, technology adoption etc. could be explored so this remains a potential subject of some further research.

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ON AAII SENTIMENT INDEX INFLUENCE ON S&P 500 STOCKS

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Abstract: This study examines the influence of the AAII sentiment index on S&P 500 stocks. Integrating the sentiment index with the Fama-French 3-factor model, we analyze its impact on stock returns and traded volume. The results indicate limited influence on stock returns, but statistical significance in explaining traded volume. Positive sentiment leads to a decrease in traded volume. These findings highlight the importance of investor sentiment in understanding market dynamics.

Keywords: linear regression, sentiment, stock returns, traded volume

1 INTRODUCTION

Sentiment-driven investors tend to trade based on the AAII Sentiment Survey [2]. The AAII sentiment index not only affects stock price [1], but also has a significant impact on both stock return and volatility [5]. In addition to this, the AAII sentiment index also plays an important role in the performance of initial public offerings [4].

This study explores the relationship between investor sentiment and stock market variables using data from the Investors Intelligence Survey and the Fama-French 3-factor model. By integrating the sentiment index with established factors, such as market risk premium, size premium, and value premium, we aim to assess the impact of sentiment on stock returns and traded volume. The findings contribute to understanding the role of investor sentiment in financial markets and its implications for investment strategies.

2 DATA AND METHODOLOGY

The sentiment index used in this study is obtained from the Investors Intelligence Survey[1], which is conducted by the American Association of Individual Investors (AAII). The sentiment survey collects data from individual investors on their current market outlook and investment decisions. Each week, members of the sentiment survey receive an email asking them to vote for one out of three choices, which represent bullishness, bearishness, or neutral sentiment toward the market. Participants are only able to vote once, and their responses are used to calculate the indices, i.e. the percentage of bullish, bearish, and neutral market outlooks. The sentiment index (AAII sentiment index) is then calculated as a spread between the bullish and bearish percentages of votes, thus ranging between –100% and 100%.

It is important to note that the indices are based on the opinions and investment decisions of individual investors and may not always reflect broader market trends or sentiments. However, the survey provides valuable insights into the opinions of individual investors and can be useful in understanding the general market outlook.

Moreover, as we further work also with the Fama-French 3-factor model, we also obtained the daily data of the factors in this model[2]. For the stocks, the dataset consists of the components S&P 500 index[3] and we downloaded the daily adjusted close values and volumes from Yahoo Finance webpage[4].

[1] https://www.aaii.com/sentimentsurvey
[2] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research
[3] https://en.wikipedia.org/wiki/List_of_S%26P_500_companies
[4] https://finance.yahoo.com

All the data are downloaded for the period from January 1, 2000, to March 1, 2023, thus covering more than 20 years of daily data, however, some of the component stocks have only shorter price histories and thus we work with these shorter periods. In the case of 366 stocks, we utilized the full history, i.e. 5 826 daily observations covering the last 23 years. The boxplot of all time series lengths is shown in Figure 1.

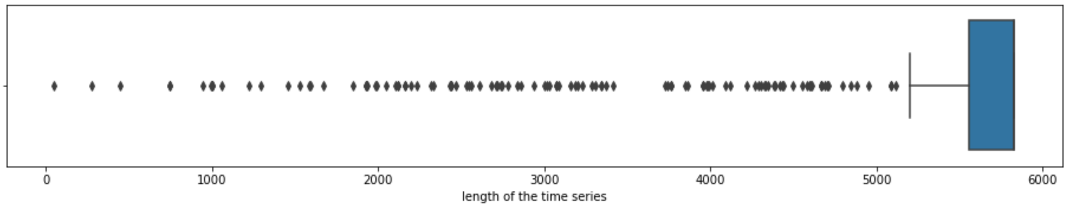

Figure 1: Boxplot of the lengths of time series applied

Further, we assumed several regression models, each estimated via the OLS method. First, we tested whether the AAII index can improve the explanatory power in the Fama-French 3-factor model [3],

$$E(r_i) - r_f = \alpha_i + \beta_{1i}\big(E(r_m) - r_f\big) + \beta_{2i} \cdot SMB + \beta_{3i} \times HML + \gamma_i \cdot AAII, \qquad (1)$$

where $E(r_i)$ is the expected stock return for asset i, r_f is the market risk-free rate, β_1 is a measure of the volatility of a stock compared to the market as a whole, $E(r_m) - r_f$ is the market risk premium, β_2 is the coefficient for the *SMB* factor, *SMB* is size premium (small minus big), β_3 is the coefficient for the *HML* factor, *HML* is value premium (high minus low), γ_i is the coefficient for the sentiment index, and *AAII* is the sentiment index.

We also assumed a simplified version, in which the expected stock return depends solely on the sentiment index,

$$E(r_i) = \alpha_i + \gamma_i \cdot AAII. \qquad (2)$$

Moreover, we also analyzed whether the sentiment influences the traded volume. In a similar manner as in model (2), we assumed the following relationship:

$$E(V_i) = \alpha_i + \gamma_i \cdot AAII, \qquad (3)$$

where $E(V_i)$ is the expected value of the volume traded.

3 RESULTS AND DISCUSSION

In this section, we present the results of the estimated regression models (1), (2), and (3). These models are estimated for 503 stock components of the S&P 500 index. In the results below we always present the number of the stocks for which the parameters are significant at 10%, 5%, and 1% significance levels by means of the t-test.

The results of model (1), i.e. the Fama-French model with added sentiment factor, are shown in Table 1. As can be seen, all the beta parameters for the original factors in Fama-French 3-factor model are usually significant. At a 1% level, for 502 (403 and 447 respectively) stocks out of 503, the returns are significantly dependent on the market risk factor (size factor and value factor respectively). The intercept α_i is not significant most of the time, meaning that over the analyzed period only a small number of the stocks delivered overperformance (in case of positive value) or underperformance (in case of negative value). The distribution of the alpha parameter is shown in Figure 2. As can be seen, more than 75% of the values are positive, meaning that there was usually overperformance compared to the returns predicted by Fama-French 3-factor model, however, this overperformance was usually statistically insignificant.

Most importantly we can see that the gamma parameter, depicting the influence of stock returns on the AAII sentiment index, is most of the time insignificant as well. Only for 14 stocks out of 503 stocks we can consider this factor as statistically significant at a 1% significance level. It must be also noted that due to the high number of statistical tests, some corrections of the p-value, such as e.g. Bonferroni correction, must be made, which is the reason why we consider the 1% significance level instead of the 5% significance level.

Table 1: Number of stocks with significant parameters in the regression model (1)

	α_i	$\beta_{1,i}$	$\beta_{2,i}$	$\beta_{3,i}$	γ_i
at 1% significance level	22	502	403	447	14
at 5% significance level	89	503	422	465	48
at 10% significance level	138	503	432	469	82

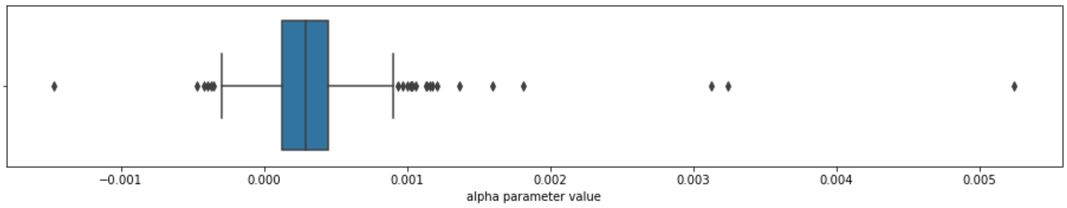

Figure 2: Boxplot of the parameter alpha in the model (1)

The results do not improve even if we simplify the model and omit all the Fama-French factors from the model. In Table 2, the results of model (2), which models the dependence of the stock returns on AAII sentiment index, are shown. As can be seen, the results even worsen as the number of stocks, for which we can statistically accept the AAII sentiment index as statistically significant, lowered compared to the previous model. Considering the total amount of statistical tests (503), the results show that there is no relationship between the sentiment measured by the AAII index and stock returns. To conclude, we found out that the AAII sentiment index cannot be used as a factor explaining the stock returns.

Table 2: Number of stocks with significant parameters in the regression model (2)

	α_i	γ_i
at 1% significance level	138	2
at 5% significance level	300	18
at 10% significance level	385	36

In the last model, linear regression (3), we focus on the question of whether the AAII sentiment index can be considered a factor when modeling the traded volume. The results are shown in Table 3. As can be seen, for a relatively high number of stocks, at a 1% significance level 439 stocks out of 503 stocks, the sentiment index is a statistically significant factor influencing the traded volume. In Figure 3 we show the boxplot of the values of the gamma parameter. As can be seen, more than 75% of the values are negative. This means that when the AAII sentiment index is positive, i.e. bullish opinions outweigh bearish opinions, the traded volume decreases.

Table 3: Number of stocks with significant parameters in the regression model (3)

	α_i	γ_i
at 1% significance level	503	439
at 5% significance level	503	452
at 10% significance level	503	456

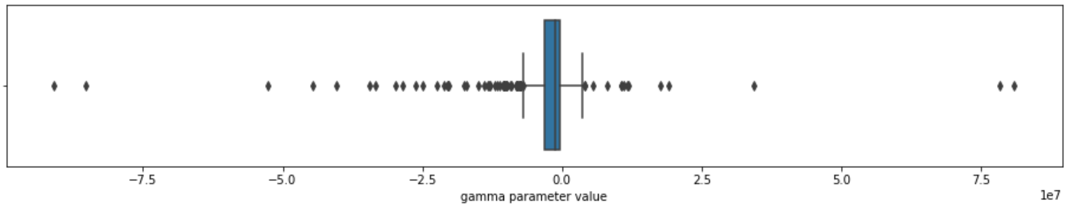

Figure 3: Boxplot of the parameter gamma in the model (3)

4 CONCLUSION

The study analyzed the relationship between investor sentiment and stock market variables. The regression models were applied to examine the impact of investor sentiment on stock returns and traded volume. The results indicated that the Fama-French factors were generally significant in explaining stock returns, while the sentiment index (AAII index) had limited influence. Simplifying the model by considering only the sentiment index did not yield significant improvements in the results, suggesting a limited relationship between sentiment and stock returns.

However, when analyzing the traded volume, the sentiment index showed statistical significance for a significant number of stocks. The results indicated that positive sentiment, with bullish opinions outweighing bearish opinions, leads to a decrease in the traded volume.

In conclusion, the study suggests that while the sentiment index had a limited impact on stock returns, it can be considered a significant factor in explaining traded volume. These findings highlight the importance of investor sentiment in understanding market dynamics, particularly in relation to trading activity.

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MACROECONOMIC FACTS OF THE EU27 COUNTRIES DURING THE COVID-19 PANDEMIC

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Abstract: The analysis calculates moments of the selected time series cyclical components (GDP, consumption, investment, exports, imports, net exports, output export and import prices, terms of trade, labour, the wage rate and the average product of labour) of the countries in the EU during the pandemics. It examines their volatility, cyclical characteristics, and persistence. The results confirm that the COVID-19 pandemic was characterised by high economic volatility, but the investment/GDP volatility ratio was relatively small. Wages and average product of labour were negatively correlated with GDP. The performance of small and large countries differed in the magnitude of investment volatility.

Keywords: COVID-19 pandemics, European Union, macroeconomic cyclical characteristics

1 INTRODUCTION

The outbreak of the COVID-19 pandemic in the period from the second quarter of 2020 to the second quarter of 2021 was presented by the weakening of the workforce due to frequent infections. The economy was affected by political measures limiting social and work-personal contact and crossing borders, which governments took to reduce the pandemic spreading. Dynamic stochastic general equilibrium (DSGE) models are a suitable tool for analysing the economic impacts of the pandemic on the economy. In their project, Dück et al. [2] collected important world papers analysing the economic impacts of the COVID-19 pandemic. Most analyses use models with economic and pandemic parts. Pandemic one is based on the SIR (Suspected, Infected, Recovered) model, in which the population randomly passes through suspected, infected, and recovered states. After the outbreak of the COVID-19 pandemic, Atkeson [1] used the SIR model to forecast the further development of the pandemic over the next 12-18 months. However, the SIR model has been known since the first half of the 20th century when Kermack and McKendrik [6] introduced it.

The goal of this paper is a detailed analysis of macroeconomic indicators development in the European Union (EU27) countries from the second quarter of 2020 to the second quarter of 2021 (pandemic period). Most of the analyses so far have focused on the reactions of macroeconomic indicators in different waves of the pandemic in economies of global importance. The European Union is a union of states of various sizes and openness. Since data on macroeconomic indicators are available for each state in the period under review, we can express the characteristics corresponding to the economic responses to the pandemics and the related policies by analysing them.

The analysis will focus on data across EU27 countries during the pandemic period. We are interested in GDP components, output import and export prices, terms of trade, labour, wage rate and average product of labour. We will express the economic characteristics of the pandemic period by cyclicality, volatility and persistence of indicators [7].

The seasonally adjusted time series can be decomposed into a cyclical and a trend component. By the macroeconomic definitions, the cyclicality of a time series is expressed by a correlation coefficient of its cyclical component with the GDP cyclical component, volatility by the standard deviation of the given cyclical component, and persistence by the autocorrelation coefficient.

2 DATA

In the analysis, we used data series of GDP, consumption, investment, exports, imports, net exports, output export and import prices, terms of trade, labour, the wage rate and the average product of labour. Terms of trade are the share of export and import prices; labour is measured in the number of hours worked; the wage rate is the share of wages and labour deflated by the output price; the average product of labour is the share of GDP and labour. We obtained all available data of the EU27 countries from the first quarter of 1995 to the fourth quarter of 2022 from the EUROSTAT database [4]. In the database, data on the GDP components of the Czech Republic, Italy and the Netherlands are only available from the first quarter of 1996, and data on hours worked are only available from the first quarter of 2009.

Available data of GDP components are seasonally and calendar adjusted; labour is seasonally adjusted. The exception is the wages of Germany and France, which are not seasonally adjusted. We seasonally adjusted them using the well-known X13 procedure used for this purpose. At the time of the paper's completion, hourly labour data in Germany in 2020 was not available. Therefore we used the time series of all working people for the German labour.

We used the Hodrick and Prescott filter [5] with the lambda parameter for quarterly data set to 1600 by default to calculate the cyclic components. We calculated the standard deviations, the correlation coefficient with the series of the cyclical component of GDP and the autocorrelation coefficients of the series.

3 RESULTS AND DISCUSSION

Table 1 calculates the moments of various samples of the examined time series' cyclical components. The results in the first panel from the left correspond to the total data sample, the second to the pandemic period sample, the third to the large countries during the pandemic period sample, and the fourth to the small countries during the pandemic period sample.

The total data sample results (first three numeric columns in Table 1) do not deviate from standard cyclical observations [7]. Consumption, investment, exports and imports are more volatile than GDP. The high consumption volatility corresponds to the fact that the EU27 consists of many small open countries. As the size of the countries does not weigh the data, the importance of small countries in the result is more significant than it is actually. The components of GDP and labour are positively correlated with GDP. Compared with other studies [7], the relatively low value of the correlation coefficient of investment corresponds to the fact that private and public investments are included in investment in the European national accounting system. The value of the correlation coefficient of the average product of labour and GDP is surprisingly low.

The pandemic corresponds to higher economic (GDP) volatility, but the share of investment and GDP volatilities is smaller than in the total data sample. Frequent changes in political and social restrictions can explain high economic volatility. It is also associated with the cyclical components' low to negative serial correlation coefficient. The wage rate was negatively correlated with GDP during the pandemic period. The correlation coefficient between the average product of labour and GDP was lower during the pandemic than in the entire period.

We also performed moment calculations for large and small EU27 countries. We consider Germany, Spain, France, Italy and Poland to be large countries, and Belgium, Bulgaria, the Czech Republic, Denmark, Estonia, Ireland, Greece, Croatia, Cyprus, Lithuania, Latvia, Luxembourg, Hungary, Malta, Austria, Portugal, Slovenia, Slovakia, Finland and Sweden to be small countries. We do not classify the Netherlands and Romania as large or small countries.

Table 1: *Moments of cyclical components of selected time series in EU27 countries for different data samples*

	Total			Pandemic			Large			Small		
	Std. Devs	Cor. w.GDP	Ser. Cor.	Std. Devs	Cor. w.GDP	Ser. Cor.	Std. Devs	Cor. w.GDP	Ser. Cor.	Std. Devs	Cor. w.GDP	Ser. Cor.
GDP	2.69%	1.00	0.62	4.40%	1.00	-0.13	5.44%	1.00	-0.24	4.25%	1.00	-0.09
Cons.	3.03%	0.82	0.55	5.05%	0.79	-0.38	5.63%	0.93	-0.42	5.06%	0.74	-0.35
Inv.	11.94%	0.41	0.22	12.30%	0.33	-0.08	8.44%	0.65	0.18	13.42%	0.27	-0.13
Exp.	5.60%	0.76	0.59	9.95%	0.82	0.07	11.22%	0.94	-0.08	9.62%	0.79	0.14
Imp.	6.27%	0.67	0.53	8.85%	0.71	-0.09	9.12%	0.90	-0.11	8.86%	0.65	-0.08
Exp.price	3.20%	0.41	0.76	2.92%	0.11	0.32	0.84%	-0.05	0.52	3.31%	0.14	0.32
Imp.price	3.84%	0.41	0.82	2.79%	0.11	0.56	1.46%	0.22	0.35	3.07%	0.07	0.59
Out.price	1.79%	0.22	0.65	1.70%	-0.30	0.54	1.14%	-0.54	0.19	1.71%	-0.28	0.58
Terms of tr.	1.65%	-0.16	0.41	2.36%	0.00	0.03	1.57%	-0.23	0.47	2.43%	0.11	-0.10
Net exp.	3.55%	0.05	0.15	3.72%	0.26	-0.07	1.19%	0.64	0.34	4.27%	0.25	-0.07
Wag.	2.85%	-0.21	0.34	5.14%	-0.49	-0.01	4.53%	-0.78	0.21	5.23%	-0.44	-0.07
Lab.	2.93%	0.62	0.29	6.39%	0.75	-0.21	7.94%	0.94	-0.09	6.15%	0.69	-0.28
Aver.prod.lab.	2.19%	0.21	0.27	4.24%	-0.09	0.03	3.42%	-0.59	0.02	4.48%	0.01	0.01

Note: The results in the first panel from the left correspond to the total data sample, the second to the pandemic period sample, the third to the large countries during the pandemic period sample, and the fourth to the small countries during the pandemic period sample.
Source: Eurostat [4] and the author's calculations.

In large countries, investment was even less volatile during the pandemic period. The average product of labour and the wage rate were negatively correlated with GDP. Interestingly, the economic volatility of small countries was lower. Conversely, investment was highly volatile in small countries. The investment/GDP volatility ratio was similar to one in the total period sample. The serial correlation coefficient of investment was the lowest in the sample of small countries during the pandemic period.

A model explaining the economic impacts of the COVID-19 pandemic and the corresponding policy responses in EU27 countries should correspond to the moment values from this analysis. The construction of the model should consider these facts observed in the pandemic period.

The correlation coefficients of average labour product and wage rate with GDP are negative. A likely explanation for this observation is that the shock of the pandemic outbreak is associated with frequent changes in labour demand and supply. The risk of the spread of the pandemic and restrictive political reactions limited the possibilities of carrying out regular business and work activities.

Investment, compared to GDP, is low volatile. The peculiarity of investment behaviour in pandemics was already noticed by Eichenbaum et al. [3], who pointed out that in the first wave of the pandemic, investment fell by about the same amount as GDP and consumption, a shallow fall compared to other economic crises. The authors explained this fact by reducing consumption. The reduction in consumption also results from the high risk of the spread of the pandemic and restrictive political reactions.

Economic performance during the pandemic was different in small and large countries. In small countries, on the other hand, investment was highly volatile. Small countries were exposed to shocks from the pandemic outbreak and trade shocks resulting from price changes on world markets and trade-restrictive policy responses.

4 CONCLUSION

In the paper, we calculated some moments of the cyclical components of important time series of the EU27 countries in the pandemic period. We focused on GDP components and time series characterising the labour market. We examined their volatility (standard deviation of cyclical components), cyclical characteristics (correlation coefficient of cyclical component with GDP) and persistence (autocorrelation coefficient of cyclical component). We used the Hodrick-Prescott filter to calculate the cyclic components.

Most similar analyses of the impacts of the COVID-19 pandemic so far have focused on the reactions of macroeconomic indicators in different waves of the pandemic in economies of global importance. The advantage of examining data in the European Union countries is that each country has separate economic statistics. This contribution brought a systematic statistical view of macroeconomic principles characteristic of the short pandemic period.

Although, the availability and scope of the data still limit the results. We had to seasonally adjust the time series of wages in Germany and France with our processing. We had to replace the time series of hours worked in Germany with the number of workers. In the calculations, we distinguished large and small countries due to the expected different reactions, which resulted in samples with more minor observations. In the sample of large countries during the pandemic, we consider only five countries in 5 periods, representing 25 observations. Special restrictive political measures in various countries also marked the results.

Despite the mentioned data deficiencies, some crucial facts characteristic of the pandemic period in the EU27 countries can be roughly deduced from the results. The pandemic period was manifested in increased economic volatility in the European Union, but the investment was unusually less volatile than in the standard period. The average product of labour and the wage rate were negatively correlated with GDP. The performance of small and large EU27 economies varied. Investment volatility was high in small economies.

These results form an essential starting point for constructing the DSGE model, which can explain the observed facts in the pandemic period in the EU27 economies. In further research, we will focus on constructing and verifying a model that distinguishes sectors with import (iMportable), export (eXportable) and non-tradeable (Nontradeable) products (MXN model) [7], in which pandemic shocks are formulated by a joint shock affecting labour demand and supply, consumption and world price relations. We call the model the MXN model.

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THE IMPACT OF GLOBALIZATION ON GOVERNMENT CONSUMPTION – WORLDWIDE EVIDENCE

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Abstract: This paper investigates the effects of globalization on government consumption in a panel of more than 160 world economies using the period since 1990s. Theoretically the effects of globalization on government consumption may be both positive (the compensation hypothesis) and negative (the efficiency hypothesis). This theoretical ambiguity is also present in empirical studies. The detailed empirical analysis conducted in this paper provides predominantly positive effects of globalization on government consumption supporting the compensation hypothesis.

Keywords: government consumption, globalization, international evidence

1 MOTIVATION AND PAPER BACKGROUND

With globalization pressures mounting, particularly in the era since 1990 which is often referred to as hyper-globalization, national autonomy has been seriously endangered. The conventional view is that globalization decreases substantially the autonomy of a nation-state, resulting in the so-called race to the bottom. As argued by [9] conventionally three globalization mechanisms are listed to suggest that the role of government is expected to decrease with globalization: competitive trade pressures, multinationalization of production and integration of financial markets. Namely, countries with high government consumption and high taxes cannot remain competitive in international markets and consequently will have to retreat. This is referred to as the efficiency (disciplining) hypothesis suggesting that globalization leads to an inevitable decline in government spending due to competitive pressures brought about by openness of world markets. However, despite the strong intensification of globalization processes a number of countries have kept (and increased) their government spending which suggests that the conventional view is overly simplistic. The literature thus provided a competing hypothesis, the so called compensation hypothesis suggesting that the role of government will increase with globalization. Indeed, one of the early explanations provided by [3] and further extended by [13] states that government spending plays a risk-reducing role in economies exposed to a significant amount of external risk. Empirical literature remains also divided on this important issue (see for example [13]; [8]; [2]; [4]; [1]; [14]). The recent meta-analysis by [11] also concludes that the overall empirical evidence is not consistent with strong unidirectional effects of globalization on government spending. Given the ambiguity present in the theoretical and empirical studies additional evidence is needed to shed supplementary light on the link between globalization and government consumption. This important relationship is therefore investigated in this paper in detail through the use of panel data techniques covering the period 1991-2020 in the sample of more than 160 countries.

2 MODELLING STRATEGY AND RESULTS

In order to investigate the relationship between globalization and government consumption we use the broadly used KOF indices of globalization (see for example [6]; [12]; [10]), in particular focusing on its economic subcomponents as we are primarily interested on how economic openness affects government consumption. Following the previous studies (see for example

[13]; [6]; [15]; [11]) in addition to globalization, the model is set quite broadly to allow for a number of possible determinants of government consumption.

In our econometric estimations we use the following model:

$$GOV_{it} = \beta_1 LGDPPC_{it} + \beta_2 GDPg_{it} + \beta_3 KOFGI_{it} + \beta_4 DEP_{it} + \beta_5 URB_{it} + \beta_6\, UNEMPL_{it} + \varepsilon_{it} \quad (1)$$

where *i* refers to a country and *t* to a time period. In this model government consumption as percent of GDP (GOV) is potentially determined by logarithm of GDP per capita as representing the level of development (LGDPPC), GDP growth (GDPg), economic openness (KOFGI) as represented by the overall, economic, trade or financial globalization, age dependency ratio (DEP), percent of urban population in total population (URB) and the rate of unemployment (UNEMPL). While the present model could be further extended with additional determinants, the model in this form is encompassing enough to explore our main research question, but also allowing for a set of control variables that typically figure out as important determinants of government consumption. In addition to the total government consumption as percent of GDP, we also investigate empirically how subcomponents of government consumption are affected by globalization. To that end we substitute the dependent variable government consumption with government consumption on education, military government consumption and government consumption on health, all expressed as percent of GDP. These and other variables used in our estimations are detailed in Table 1 below.

Table 1: Explanation of variables

Variable	***Explanation and definition of variable***	***Source***
GOV	General government final consumption expenditure (% of GDP)	[16]
LGDPpc	Logarithm of GDP per capita	[16]
GDPg	GDP growth rate (%)	[16]
KOFGI	*Overall globalization* - KOF index of overall globalization	[10]
KOFEcGI	*Economic globalization* - KOF index of economic globalization	[10]
KOFTrGI	*Trade globalization* - KOF index of trade globalization	[10]
KOFFiGI	*Financial globalization* KOF index of financial globalization	[10]
DEP	*Age dependency ratio* (% of working-age population)	[16]
URB	*Urban population* (% of total population)	[16]
UNEMPL	*Unemployment, total* (% of total labor force)	[16]

The collected data are annual and cover the period 1991-2020 with the data collected for as many as 167 countries. This broad sample of countries and 30 years of data provide a strong basis to take our findings representative of the whole world. This is further emphasised through the use of modern econometric techniques using panel data. In our estimations we use fixed effects or random effects panel models depending on the calculated Hausman test. Before reporting the results of our econometric estimations we present the descriptive statistics of our variables.

Table 2: Descriptive statistics

Variable	***Obs***	***Mean***	***Std. dev.***	***Min***	***Max***
GOV	5,111	16.870	8.856	0.911	147.719
LGDPpc	6,095	3.599	0.710	1.359	5.370
GDPg	5,967	3.302	6.379	-64.047	149.973
KOFGI	5,720	54.797	16.028	18.770	91.141
KOFEcGI	5,574	54.059	16.037	15.938	94.922
KOFTrGI	5,590	52.821	17.299	13.151	96.770
KOFFiGI	5,608	55.318	17.641	8.667	98.257
DEP	6,355	63.409	19.406	16.172	116.934
URB	6,355	56.452	24.140	5.491	100
UNEMPL	5,850	8.167	6.082	0.095	38.8

Table 2 presents descriptive statistics of our variables. Of particular interest for this paper are government consumption and globalization variables. The mean value of government consumption is 16.87 percent of GDP, with the minimum as low as 0.91 and maximum as high as 147.72 showing a lot of variance in government consumption around the world that should be investigated, among other factors also by globalization. As for globalization variables there seems to be also a lot of variance in globalization indicators with the minimum value around 10 or somewhat above and maximum values above 90 suggesting different levels of globalization reached by countries (the KOF indicators of globalization are measured on a scale from 0 to 100, with higher numbers suggesting higher levels of openness). This paper investigates econometrically the determinants of government consumption with a particular focus on globalization with the main results to follow below.

Table 3: Globalization and government consumption (% of GDP)

	Overall globalization (1)	*Economic globalization (2)*	*Trade globalization (3)*	*Financial globalization (4)*	*Trade and financial globalization (5)*
LGDPPC	-0.281**	-0.362**	-0.337**	-0.288**	-0.272**
	(0.132)	(0.114)	(0.112)	(0.114)	(0.115)
GDPg	-0.117***	-0.118***	-0.120***	-0.116***	-0.119***
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)
KOFGI	0.003				
	(0.011)				
KOFEcGI		0.020***			
		(0.008)			
KOFTrGI			0.025***		0.030***
			(0.007)		(0.007)
KOFFiGI				0.002	-0.010
				(0.006)	(0.007)
DEP	0.027***	0.028***	0.024***	0.026**	0.026***
	(0.009)	(0.008)	(0.008)	(0.008)	(0.008)
URB	0.113***	0.111***	0.102***	0.114***	0.099***
	(0.016)	(0.015)	(0.015)	(0.015)	(0.015)
UNEMPL	0.090***	0.095***	0.090***	0.097***	0.092***
	(0.019)	(0.019)	(0.019)	(0.019)	(0.019)
Countries	167	167	166	167	165
Observations	4537	4507	4490	4510	4447
Hausman test	35.70	50.38	60.35	44.98	67.23
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	Fixed effects	Fixed effects	Fixed effects	Fixed effects	Fixed effects

Standard errors in parentheses; *** - 1 % statistical significance, ** - 5 % statistical significance, * - 10 % statistical significance

Table 3 reports our main econometric findings focusing on total government consumption (% of GDP) as the dependent variable. Five models were estimated and are reported in Columns 1 to 5. The control variables are the same in all models whilst the difference stems from changing the indicator of globalization. As mentioned previously, we use the KOF index of globalization (due to [10]) which comes in different forms and accounts for overall globalization, and within that also measures economic, social and political globalization. As the focus of this study is on economic globalization and its effects on government consumption, across the models 1 – 5 we include as indicators of globalization the overall KOF index of globalization (KOFGI), KOF index of economic globalization (KOFEcGI) and also add two subcomponents of the economic globalization which are the KOF index of trade globalization

(KOFTrGI) and KOF index of financial globalization (KOFFiGI). Before commenting on the estimated coefficients related to globalization, we comment briefly on the coefficients on control variables. All of the control variables are found to be statistically significant and of the same signs across the five models. Logarithm of GDP per capita (as representing the level of development) and GDP growth are statistically significant and negative, while dependency ratio (percent of dependent population in working-age population), urbanization rate (percent of urban population in total population) and the rate of unemployment exert positive and statistically significant effects on government consumption (as percent of GDP). As for our main variables, in model 1 (Column 1) the overall KOF index of globalization is estimated positively but the effect is not statistically significant. As for the other indicators of globalization KOF indices of economic and trade globalization are estimated to have a positive and statistically significant effect on government consumption, while at the same time the KOF index of financial globalization is statistically insignificant. Taken together, these results seem to suggest that economic globalization, and within it in particular trade globalization lead to increased government consumption providing initial support to the compensation hypothesis. This suggests that governments around the world increase their consumption to compensate for the risks that people and companies are exposed to with the increasing (economic) globalization.

Given the very complex interactions that come about with globalization, it should be further investigated how strong and robust the link between globalization and government consumption is. It should be also added that there might be a strong hysteresis in government consumption. In order to check for this and possible endogeneity, we included the lag of government consumption in our estimations. Indeed, there seems to exist a strong hysteresis in government consumption as the effect on the lagged government consumption is strongly statistically significant and positive across all five models estimated previously (the models reported in Table 3). It should be added that even with this inclusion of the lagged dependent variable the relationship between government consumption and globalization remains practically intact. The coefficients on all globalization variables are of the same sign and statistical significance as in our benchmark table (Table 3) reported above. Because of the overall limit on the length of the paper, these results are not reported but are available upon request. Thus, it appears that the estimated models with the lagged dependent variable included confirm our findings related to the positive and statistically significant impact of economic and trade globalization on government consumption. We take this evidence as providing a valuable robustness check further supporting the compensation hypothesis.

In our final step of empirical research we investigate how different components of government consumption are influenced by globalization. Thus, we estimate the same models as in our benchmark Table 3, but instead of total government consumption (as % of GDP) we employ government consumption on education (Panel A in Table 4), government military consumption (Panel B in Table 4) and government consumption on health (Panel C in Table 4).

Table 4 reports how different components of government consumption are affected by globalization. We have the same globalization variables included in the models, as well as the same control variables, but to preserve space we only report the estimated coefficients on the globalization variables. The complete sets of estimations are available upon request. In Panel A of Table 4 we can observe that globalization variables exert positive and statistically significant influence on government consumption on education. The only exception is related to financial globalization whose effect is negative but statistically insignificant. As for government military consumption (Panel B in Table 4), unlike the total government consumption and government consumption on education, globalization variables have negative and statistically significant effects. Finally, in Panel C in Table 4 all globalization variables are found to affect government consumption on health positively, with all of them being statistically significant.

Table 4: Globalization and government consumption – alternative dependent variables: government consumption on education, government military consumption, government consumption on health (% of GDP)

		Overall globalization (1)	*Economic globalization (2)*	*Trade globalization (3)*	*Financial globalization (4)*	*Trade and financial globalization (5)*
		…	…	…	…	…
PANEL A	***KOFGI***	0.015** (0.007)				
Government consumption on education	***KOFEcGI***		0.009** (0.004)			
	KOFTrGI			0.015*** (0.003)		0.018*** (0.004)
	KOFFiGI				-0.001 (0.003)	-0.008** (0.004)
		…	…	…	…	…
PANEL B	***KOFGI***	-0.026*** (0.004)				
Government military consumption	***KOFEcGI***		-0.015*** (0.003)			
	KOFTrGI			-0.014*** (0.002)		-0.013*** (0.003)
	KOFFiGI				-0.009*** (0.002)	-0.004 (0.002)
		…	…	…	…	…
PANEL C	***KOFGI***	0.022*** (0.004)				
Government consumption on health	***KOFEcGI***		0.009*** (0.002)			
	KOFTrGI			0.006*** (0.002)		0.005** (0.002)
	KOFFiGI				0.005*** (0.002)	0.003* (0.002)

Standard errors in parentheses; *** - 1 % statistical significance, ** - 5 % statistical significance, * - 10 % statistical significance

3 CONCLUSIONS AND FURTHER RESEARCH

This paper investigated the relationship between globalization and government consumption using a broad sample of world economies. Through application of panel data techniques this study contributes to empirical literature. Both the theoretical and empirical literature on the link between globalization and government consumption are still lacking of the unison conclusion on this important relationship. Our study appears supportive of the compensation hypothesis with the main findings that economic globalization and trade globalization have positive and statistically significant effects on government consumption. An additional finding raising particular interest are the differing effects on components of government consumption, with globalization exerting positive effects on government consumption on education and health, whilst the government military consumption decreases with globalization. While the conducted empirical investigation is comprehensive and findings robust, there are interesting avenues for further research. In particular this relationship might be investigated across different groupings of countries, by the regional or the reached level of development criteria, as well as through accounting for additional subcomponents of government consumption.

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THE IMPACT OF FINANCIAL LITERACY AND SOCIODEMOGRAPHIC FACTORS ON INVESTOR PREFERENCES

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Abstract: The research highlights the importance of financial literacy and sociodemographic factors in shaping investor preferences across different asset classes. Through survey method, the study sample consists of 96 participants from the EU countries. The findings suggest that self-assessed (i.e. subjective) financial literacy is particularly influential in investment decision-making. Age, gender, education, and income also play significant roles in determining investment choices. The study emphasizes the need for further research in the field of financial literacy, considering the changing investment landscape and the emergence of new asset classes like cryptocurrencies.

Keywords: financial literacy, sociodemographic factors, investor preferences, traditional assets, alternative assets, cryptocurrency.

1 INTRODUCTION

Financial literacy has gained importance and undergone extensive research in the last two decades. A segment of this research area focuses on examining the relationship between sociodemographic factors, financial literacy, and investment decisions, yielding different conclusions. By analyzing the impact of gender, age, education level, and marital status on investments across multiple asset types, researchers concluded that only age and marital status had no bearing on investment choices [3]. In contrast, another study discovered that sociodemographic factors, including gender, age, education level, and income level, significantly correlate with and affect investment decisions [8]. Moreover, a separate investigation found that solely age and education level exert influence on investment choices, while factors like gender, income, occupation, marital status, and investment experience show no impact [19]. A recent study categorizes factors that may influence investment choices as gender, age, marital status, education level, annual income, and prior investment experience [16]. The mentioned research also emphasizes the need for more study regarding the impact of financial literacy on investor behavior. In line with this, a bibliometric analysis of over 2,000 articles in peer-reviewed journals from 2001 to early 2022 concludes that investigating current trends in financial literacy is essential due to its significant impact on individuals' roles in shaping their investment strategies [1].

On account of the general lack of financial literacy in society, most research concerning the influence of financial literacy on individual investment behavior primarily concentrates on conventional investment vehicles. Conversely, driven by a decade of historically low interest rates and new trends of portfolio diversification into alternative assets [7], significant shifts in investments from traditional to alternative forms are taking place [10, 11]. An observation by researchers underscores that financial markets are rapidly changing due to the exponential development of financial technology, creating new and more complex financial products accessible to wider segments of society [12].

A recent phenomenon that emerged as an alternative investment form since its introduction in 2008 and is under research as a new asset class with specific attributes is known as cryptocurrency. Scientific papers span a range from recognizing cryptocurrencies as a distinct investment category [6] to studies enhancing portfolio performance by integrating cryptocurrencies through various models [5, 17]. In the expansion of scientific and professional papers in the field of financial literacy, a smaller number of studies investigate the link between financial literacy and alternative forms of investment, and there are only a few recent ones that explore the relationship between financial literacy and cryptocurrencies. Furthermore, financial literacy is not only comprised of the objective level measured through three combined components of financial knowledge, financial attitudes, and behaviors within the framework for internationally comparable research on financial literacy [15]. The inclusion of self-assessment of financial knowledge through respondents' questionnaires as subjective financial literacy has also demonstrated significance. This extends to the point where subjective financial literacy correlates with more risk-prone investment decisions, while objective measurements are aligned with safer, conventional investments [4]. The potentially different impact of objective and subjective financial literacy on investor behavior and individuals' actions motivates an approach that includes and distinguishes both types of financial literacy.

Due to the significant impact of financial literacy on decision-making and the design of investment strategies, combined with the rapid growth and increased accessibility of financial technology across broader segments of society, as well as the evident surge in investments in alternative assets and the emerging phenomenon of cryptocurrencies, the research goal is to determine the impact of financial literacy and sociodemographic factors on investor preferences across various asset classes. These classes span from traditional assets, through alternatives, and ultimately to the new 'digital' asset class.

Overall, three research hypotheses have been formulated. The first research hypothesis proposes that age, gender, education, and income influence investor preferences. The second research hypothesis suggests that both objective and subjective financial literacy exert distinct influences on investors' preferences. The third research hypothesis posits that financial literacy has a stronger impact on investing in cryptocurrencies than sociodemographic factors.

2 DATA AND METHODOLOGY

The aim of this paper is to assess how the concept of financial literacy and sociodemographic factors influence investor preferences. The main research instrument was a questionnaire, and data were collected through an online survey. The questionnaire included general sociodemographic characteristics of individuals, self-assessment of financial knowledge [14], followed by questions about financial knowledge, attitudes, and behaviors [15], as well as inquiries regarding participation in various forms of investments (bank deposit/money, stocks, bonds, real estate, gold, derivatives), categorized into traditional and alternative investment vehicles [2], which also includes the latest form of investment, cryptocurrencies. The research was conducted from October 15th to December 15th, 2022. A total of 96 participants from 13 EU countries completed and returned the questionnaire. The random non-stratified sampling method was employed. The composition of the sample and the descriptive statistics of the sociodemographic variables are presented in Table 1.

Furthermore, the coefficient of objective financial literacy is calculated from survey questions according to [15]. The average coefficient of objective financial literacy in the sample is 16.04, which is higher than the average of EU countries (13.53). However, the higher coefficient in the sample can be attributed to the fact that 65% of the participants have a college or postgraduate education. High education (some college or above) is positively correlated with higher levels of financial literacy [18]. The coefficient of self-assessed financial literacy is derived by averaging responses from a Likert scale on financial knowledge and understanding, in the areas such as budgeting for day-to-day finances, saving money, managing debt, investing money, planning for financial future, and saving enough money for retirement.

Table 1: Descriptive Statistics

Variables	Freq.	%	Variables	Freq.	%
Age			***Profession/Occupancy***		
18-29	32	33.3	**Other**	9	9.4
30-39	17	17.7	**Unemployed**	3	3.1
40-49	23	24.0	**Full-time student**	17	17.7
50-59	15	15.6	**Self - employed**	12	12.5
60+	9	9.4	**Full-time employment**	55	57.3
Gender			***Annual Income****		
Male	65	67.7	**Low**	20	20.8
Female	31	32.3	**Medium**	46	47.9
Education*			**High**	30	31.3
Primary / Secondary school	24	25	***Financial Assets*****		
Colleague /University student	10	10.4	**Significantly lower**	10	10.4
Faculty / Higher education	45	46.9	**Lower**	31	32.3
Postgraduate study	17	17.7	**Higher**	48	50.0
*highest level of education achieved; **self-assessment; ***self-assessment compared to the financial assets of the average citizen			**Significantly higher**	7	7.3

Source: own calculations on the base of data collected from the questionnaire

Given the collected data, logistic regression proves to be more effective than discriminant analysis, as it doesn't necessitate the fulfillment of basic assumptions such as linearity, normality, or homoscedasticity. Multinomial logistic regression is employed to model nominal outcome variables, wherein the log odds of the outcomes are formulated as a linear combination of predictor variables [13]. In the multinomial logit model,

$$log\frac{Pr(y=k|x)}{Pr(y=K|x)}=\beta_0^{(k)}+\beta_1^{(k)}x_1+\ldots+\beta_p^{(k)}x_p \tag{1}$$

for $k=1,\ldots,K-1$, where $x=(x_1,x_2,\ldots,x_p)$ represents predictor variables, group indicator y response variable and group K (designated as the last group) serves as the baseline category.

3 RESULTS AND DISCUSSION

Initially, a correlation analysis is conducted for sociodemographic variables, the coefficient of objective financial literacy (OFL), and the self-assessed/subjective financial literacy score (SFL), as well as all financial assets, respectively. The results are shown in the Table 2.

The results of the analysis reveal that age is significantly positively correlated with investments in real estate (0.405**), and significantly negatively correlated with investments in cryptocurrencies (-0.286**), which aligns with [9] who highlight two dominant sociodemographic variables influencing investments in cryptocurrencies: age (younger than 35) and gender (males). Male gender is positively correlated with investments in the stock market (0.321**) and bonds (0.261*) and negatively correlated with investments in gold. Higher levels of education imply a preference for real estate investments (0.325**), while being full-time employed or self-employed is associated with investments in real estate (0.220*). Participants with higher annual incomes favor investments in bonds (0.210*) and real estate (0.344**), while participants with greater financial assets prefer derivatives or financial instruments. OFL is positively linked to investments in deposits and stocks, especially with investments in cryptocurrencies (0.455**), and SFL implies similar investment preferences as

OFL, with an expansion into investments in bonds and real estate (0.268**). This similarity in asset preference can be explained by the relationship between OFL and SFL (0.435**). These results encourage further analysis as some fundamental relationships have been confirmed, such as the positive correlations between: age and annual income (0.283**), education level and objective financial literacy (0.220*) which is consistent with [18], and the positive correlation between higher annual income and ownership of greater financial assets (0.424**).

Table 2: Correlation analysis

	Age	Gender	Education	Profession	Income	F.Assets	OFL	SFL
Age	1							
Gender	-0.042	1						
Education	0.316**	-0.196	1					
Profession	-0.077	-0.13	0.206*	1				
Income	0.283**	0.007	0.199	0.126	1			
F.Assets	0.15	0.166	0.131	-0.116	0.424**	1		
OFL	-0.099	0.285**	0.220*	0.157	0.069	0.241*	1	
SFL	-0.098	0.191	0.055	0.004	0.331**	0.422**	0.435**	1
Deposit/Money	-0.028	0.066	0.121	0.07	0.112	0.174	0.299**	0.392**
Stocks	0.017	0.321**	0.039	-0.028	0.059	0.109	0.290**	0.260*
Bonds	0.071	0.261*	0.094	0.036	0.210*	0.183	0.075	0.234*
Real estate	0.405**	-0.083	0.325**	0.220*	0.344**	0.159	0.196	0.268**
Derivatives	0.078	0.144	0.135	-0.027	0.116	0.257*	0.145	0.154
Gold	0.036	-0.270**	0.029	-0.076	0.084	0.031	-0.096	0.171
Cryptocurrencies	-0.286**	0.149	-0.056	0.124	-0.148	0.121	0.455**	0.415**

**. Correlation is significant at the 0.01 level (2-tailed). *. Correlation is significant at the 0.05 level (2-tailed).

For deeper analysis, we conduct multinomial logistic regression for each type of asset (response variable), along with sociodemographic factors: age, gender, education, profession, annual income, and financial assets as predictor variables. Additionally, numerical variables OFL and SFL are included as covariates. An overview of the results is provided in Table 3.

Table 3: Multinomial logistic regression for various investment assets

Investment Asset [a]	Model Fitting Information (Sig.)	Likelihood Ratio Tests (Sig.)		Parameter Estimates Sig.|Exp(B)	
Bank Deposit/Money	0.027	SFL	0.001	0.006	5.065
Stocks	0.026	Gender	0.025	0.038	4.640
Bonds	0.058*	Gender	0.002	-	-
Real Estate	0.000	SFL	0.004	0.009	3.592
Derivatives	0.012	Gender	0.001	-	-
		Income	0.007		
Gold	0.022	Gender	0.004	0.007	**
		SFL	0.039	0.047	2.414
Cryptocurrencies	0.000	OFL	0.003	0.007	1.640
		SFL	0.001	0.005	6.979
		Age	0.002	0.000	**
		Income	0.002	0.004	**

*model is acceptable for prediction (Goodness-of-Fit Pearson/Deviance=0.990/0.999); a. The reference category is: No.

The classification (hit rate) in all models is above 80%, meaning the predicted classification of asset preference corresponds very well to the empirical classification. Notably, the sole variable associated with investments in bank deposits/money is SFL, displaying a substantial and statistically significant odds ratio. Therefore, individuals who highly assess their own financial literacy are more likely to invest in deposits and money. Gender has emerged as a significant determinant of investments in stocks and bonds. The reference category is female, so we can conclude that male investors have a 4.64 times higher chance of investing in the stock market compared to female investors. Despite age and educational level not exhibiting overall significance as variables, parameter estimation reveals that younger investors are more prone to participate in the stock market when contrasted with older investors. Additionally, individuals with higher education levels are more inclined to engage in stock market activities, as opposed to those with lesser education levels**.

While correlation analysis highlighted a positive connection between investing in real estate and variables such as age, education, income, and SFL, logistic regression indicates that only the association with SFL remains significant. In the case of investments in derivatives, gender and income have proven to be significant (overall), however, in parameter estimation, both variables were not statistically significant. In the case of investments in gold, SFL and gender are significant variables, but it is important to note that investment in gold and male gender are inversely related**. Investments in cryptocurrencies are influenced by the most factors. For every unit increase in the coefficient of OFL, probability of investment increases by 64%. SFL has an even greater impact on investments in cryptocurrencies, which is in line with [4] who link higher levels of SFL (and its overestimation) with riskier investments, which is a characteristic of the cryptocurrency market. Furthermore, gender and annual income are also significantly associated with investments in cryptocurrencies, with the highest probability of investing in cryptocurrencies in the age category of 18-29 compared to all others, and with low and average incomes compared to high incomes**.

4 CONCLUSION

Our analysis indicates significant associations between sociodemographic factors, financial literacy, and investor preferences across different asset classes. Sociodemographic factors such as age, gender, education, and income have a substantial impact on investor preferences. Higher age correlates positively with real estate investments and negatively with cryptocurrency investments. Males tend to invest in the stock market and bonds, while higher education levels and greater annual incomes are linked to a preference for real estate. Objective and subjective financial literacy share similar investment preferences only towards investments in cryptocurrencies, both positively linked. However, subjective financial literacy extends its positive impact to investments in deposits, gold, and real estate. The analysis also suggests that both types of financial literacy have a greater influence on investments in cryptocurrency compared to sociodemographic factors. An inverse connection between male investors and gold investments has been noted, and the highest probability of investing in cryptocurrencies is observed among younger individuals and those with lower to average incomes.

These findings underscore the importance of financial literacy and sociodemographic factors in shaping investor preferences across various asset classes, including traditional, alternative, and digital assets. Understanding these relationships provides valuable insights for individuals in designing their investment strategies and for policymakers in promoting financial literacy initiatives.

This study is not exempt from certain limitations. It is crucial to consider the sample size when drawing interpretations from the findings. Undoubtedly, a larger sample would enhance representativeness and thereby afford greater generalizability of the conclusions. Moreover, the study highlights the need for additional exploration into investment preferences, encompassing not just diverse asset classes but also investigating patterns of clustering within traditional and alternative investment options.

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THE ROLE OF THE FINANCING SOURCE IN THE BEHAVIORAL INTENTION TO BUY REAL ESTATE: MULTIGROUP ANALYSIS

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Abstract: The aim of this paper is to determine if there are differences in the behavioural intention of real estate purchase between people who use different primary financing sources. Structural equation modelling multigroup analysis was used to test the theory of planned behaviour in the context of predicting real estate purchase intention. Research was carried out with a survey among 434 respondents in Croatia. The results show that attitudes, subjective norms and perceived behavioural control positively affect purchase intention for people who primarily use loans, while only subjective norms affect the buying intention for people who rather use their own funds.

Keywords: behavioural intention, financing source, multigroup analysis, purchase intention, real estate, structural equation modelling, theory of planned behaviour.

1 INTRODUCTION

Real estate purchase is one of the most complex purchase decisions. Namely, people mostly buy real estate for the purpose of self-living, which is why it is important to choose the real estate wisely. Since this kind of a purchase is a rather large investment, it is expected that financial factors will significantly affect the intention to buy real estate. However, it is also important to analyse the behavioural aspects of purchase intention. Incorporating customer behaviour concepts in real estate research would be beneficial in order to better understand what subconscious factors affect people when buying real estate. It is usual that buying decisions are influenced by financial factors, location and attributes of real estate [7, 8, 11, 14, 17, 18, 23]. However, people are not entirely rational, and their decisions also depend on their state of mind and personality.

According to the theory of planned behaviour (TPB), intention to perform a certain behaviour is determined by the attitude towards behaviour, subjective norm and perceived behavioural control [1, 2]. Attitude represents "the degree to which a person has a favourable or unfavourable evaluation or appraisal of the behaviour in question" [1]. Therefore, it is expected that the more positive attitude towards a specific behaviour will consequently have a positive effect on the behavioural intention [1, 9]. Subjective norm refers to the social pressure to perform a certain behaviour. Its impact on behavioural intention is also expected to be positive, as society members are thought to have a high impact on shaping individuals' opinions and beliefs, thus encouraging them to behave in a certain way [1, 9]. Al-Nahdi [4] even suggested several different aspects of subjective norm in the context of real estate purchase intention. Those aspects are the influence of spouse, children, parents, friends and reference group. Some research suggest that subjective norm has the highest impact in predicting people's behaviour [21]. Another influential variable in TPB is perceived behavioural control. It represents how easy or difficult it is to perform a certain behaviour for an individual, also reflecting their past experience and their expected obstacles. This variable is also expected to positively affect behavioural intention [1, 2, 9]. Previous research has confirmed that all factors of TPB positively impact the intention to buy real estate in various geographical areas [4, 9, 15, 21].

Since real estate purchase is a large investment which requires high amounts of money, it is no surprise that most people use loans as their financing source for the purchase. However, some people do not want to get into a lot of debt, so they choose their own funds instead. It has become usual in developed countries to get a bank loan, i.e. a mortgage, in order to buy real estate. The possibility of getting a loan is highly related to the availability of a home purchase. However, in developing countries loans have a smaller role in this kind of a purchase [25]. People's financial status is an important factor which can determine whether someone can get access to a bank loan. Banks can often create a paradoxical situation, where people with lower income are unable to get access to loans, thus limiting their financing sources. On the other hand, lower income usually means less own funds, so people of this financial status cannot easily buy real estate. As for own funds which can be used for the real estate purchase, some of the most frequent types of funds are personal savings and parental contributions [6, 19, 20]. A common case is the use of both financing sources in a certain proportion. Thus, people often get a loan of an acceptable amount, which they can try to repay as quickly as possible, and use their own funds for the rest of the required price.

Taking the above into consideration, behavioural perspective is very important in predicting one's intentions and actions. In order to explain and predict intentions to buy real estate, in this case in Croatian market, TPB is used as an adequate model. It encompasses the attitudes and perceived behavioural control, as factors which reflect someone's personality traits and behaviour patterns, as well as subjective norm, which reflects the pressure of society. Furthermore, as people use different options as their primary financing source in real estate purchase, it is assumed that there are differences in predicting buying intention between people who primarily rely on bank loans and those who rather use their own funds.

2 DATA AND METHODOLOGY

Data was collected through an online questionnaire [3, 5, 17, 18], which targeted Croatian general population. A combination of snowball and respondent driven sampling was used [10, 13, 22]. The final sample included 434 Croatian residents. The questionnaire was divided into two parts: one with socio-demographic characteristics of the respondents and the other with the questions regarding the factors of TPB. Each of these factors were measured with a set of questions formed as statements on a 5-point Likert scale (1=strongly disagree, 5= strongly agree). Most of the respondents are female (67.74%) compared to male (32.26%), with the median age of 37 years for the whole sample. The majority of the respondents are highly educated, since 85.94% of them have finished graduate studies or a lower education level. As for the primary source of financing that people have used or would want to use in order to purchase real estate, a very high share of respondents used or would primarily use a loan (71.20%) in contrast to those who primarily used or would prefer to use their own funds (28.80%). In order to test whether there are differences in the buying intention among people who primarily use loans and those who rather use their own funds, structural equation modelling (SEM) multigroup analysis was conducted. Namely, the variables in TPB represent certain psychological and behavioural concepts, which cannot be directly measured. Therefore, SEM is a logical method choice, since it simultaneously examines multiple dependence relationships between the unobserved (latent) variables in the model [12, 16]. These latent variables, i.e. TPB factors, are measured with a set of indicators from the survey, as previously mentioned. Groups for each primary financing source was created for the purpose of testing their differences. Data analysis was conducted with Mplus 7.

3 EMPIRICAL RESULTS

The research model follows the logic of TPB, thus including four latent factors in the model: attitude towards behaviour (ATT), subjective norm (SN), perceived behavioural control (PBC) and behavioural intention to buy real estate, i.e. the buying intention (BI). The model was estimated in one step, i.e. both measurement and structural model were simultaneously assessed [12, 26]. After the full model estimation, multigroup analysis was performed. This was done to explore if the loadings and path coefficients are invariant across groups, in this case across groups of people who prefer loans and those who prefer own funds as primary financing source [12, 24, 26]. Firstly, a constrained model was estimated, where the parameters were constrained to be equal in both groups. Afterwards, an unconstrained model with the same structural paths was estimated, but those paths were estimated for each group independently [24, 26]. Before the analysis of the structural models with their paths, it is necessary to check for measurement model validity and reliability [12, 16]. Firstly, Table 1 shows the measurement model results for the full model, as well as for the models by groups. All standardized factor loadings are high and statistically significant at 0.01 level. Convergent validity was further assessed with average variance extracted (AVE). The full model, as well as models by groups, show the acceptable AVE larger than 0.5, indicating that for all models, each factor explains at least 50% of the variance in its indicators [12, 16]. Composite reliability (CR) and Cronbach's alpha (C_α) measures are higher than the threshold of 0.7, implying that the indicators represent their associated constructs very well [12, 16]. The results are given in Table 2. In order to establish good measurement model validity, discriminant validity was also tested, according to the Fornell-Larcker criterion. Table 3 shows the correlation matrix and AVE square root values for all constructs on the diagonal. The diagonal values are higher compared to correlations with other constructs, thus confirming that there is discriminant validity in the model, i.e. the constructs are truly distinct from each other [12].

Table 1: Measurement model results.

			Standardized factor loadings*		
Latent variable	***Code***	***Item***	***Full model***	***Loan***	***Own funds***
ATT	at1	Buying real estate is a beneficial decision.	0.928	0.921	0.942
	at2	Buying real estate is a good idea.	0.963	0.954	0.981
	at3	Buying real estate is a wise decision.	0.887	0.876	0.900
	at4	Buying real estate is an admired decision.	0.785	0.762	0.833
SN	sn1	My family thinks that I should buy real estate.	0.898	0.920	0.867
	sn2	My family would want me to buy real estate.	0.907	0.919	0.900
	sn3	My family agrees with me to buy real estate.	0.878	0.865	0.891
	sn4	My family thinks that buying real estate is a wise decision.	0.820	0.811	0.816
	sn5	I will buy real estate that my family advises me to buy.	0.406	0.405	0.384
	sn6	Before I make a decision, I always collect real estate information from family and friends.	0.409	0.416	0.375
PBC	pbc1	I have enough opportunity (I have easy access to the market) in making a decision to buy real estate.	0.687	0.673	0.709
	pbc3	I have enough money to buy real estate.	0.603	0.598	0.627
	pbc4	I have enough skills and knowledge about real estate to make my own decision.	0.827	0.806	0.867
	pbc5	I have complete control over buying real estate.	0.776	0.750	0.811
BI	bi1	I want to buy real estate.	0.864	0.844	0.904
	bi2	I will try to buy real estate.	0.892	0.888	0.912
	bi3	I plan to buy real estate.	0.909	0.892	0.953
	bi4	I will continue to buy real estate in the future.	0.640	0.630	0.660
	bi5	It is likely that I will buy real estate.	0.871	0.878	0.857

** All standardized factor loadings are significant at 0.01 level*

Table 2: Convergent validity and reliability of the constructs.

Latent variable	***Full model***			***Loan***			***Own funds***		
	AVE	*CR*	C_α	*AVE*	*CR*	C_α	*AVE*	*CR*	C_α
ATT	0.798	0.940	0.936	0.777	0.932	0.927	0.838	0.954	0.953
SN	0.567	0.878	0.872	0.572	0.880	0.872	0.552	0.869	0.871
PBC	0.530	0.817	0.812	0.506	0.802	0.789	0.576	0.843	0.858
BI	0.707	0.924	0.917	0.693	0.918	0.912	0.745	0.935	0.933

Table 3: Discriminant validity of the constructs.

	Full model				***Loan***				***Own funds***			
	ATT	*SN*	*PBC*	*BI*	*ATT*	*SN*	*PBC*	*BI*	*ATT*	*SN*	*PBC*	*BI*
ATT	**0.893**				**0.881**				**0.916**			
SN	0.566	**0.753**			0.528	**0.757**			0.624	**0.743**		
PBC	0.277	0.364	**0.728**		0.194	0.347	**0.711**		0.308	0.405	**0.759**	
BI	0.408	0.569	0.304	**0.841**	0.379	0.549	0.328	**0.832**	0.461	0.618	0.273	**0.863**

Since the measurement model shows good validity, the next step is to analyse the structural model. The model fit statistics for full model and the separate group models is given in Table 4. It can be seen that the full model fits very well, according to all fit indices, except for the Chi-square (χ^2), which is rather large. However, this can often happen due to larger sample size [12, 16]. As for the comparison of the constrained and the unconstrained model, it can be concluded that the unconstrained model gives slightly better fit and it should be preferred over the constrained model. The last step in multigroup analysis is to calculate the χ^2 difference between the constrained and the unconstrained model (Table 5). The χ^2 difference is 10.927, with the difference of 5 degrees of freedom (df). The p-value for the difference test is 0.053, implying a statistically significant difference of the groups in their factor loadings and/or path coefficients, considering the significance level of 0.10. In order to check for the differences, structural model results (Table 6) were examined. It is found that ATT, SN and PBC have a positive significant impact on BI for the people who prefer to use loans as their primary financing source. However, for the group of people who prefer to primarily use their own funds, only SN significantly affects BI in the context of real estate, while ATT and PBC show no significant influence.

Table 4: Model fit statistics.

	group	***$\chi2$/Contribution to $\chi2$***	***RMSEA***	***SRMR***	***CFI***	***TLI***
Full model		599.629	0.085	0.059	0.928	0.915
Unconstrained model	*loan*	543.318	0.091	0.074	0.910	0.904
	own funds	356.860				
Constrained model	*loan*	546.412	0.091	0.077	0.909	0.905
	own funds	364.694				

Table 5: Chi-square difference test.

	Model Chi-square	***df***	***p for difference test***
Constrained	911.106	327	
Unconstrained	900.179	322	
Difference Test	10.927	5	0.053

Table 6: Structural model results.

	full model		***loan***		***own funds***	
path	*path coefficient*	*p-value*	*path coefficient*	*p-value*	*path coefficient*	*p-value*
ATT → BI	0.123	0.019	0.121	0.047	0.121	0.218
SN → BI	0.459	<0.001	0.431	<0.001	0.534	<0.001
PBC → BI	0.109	0.027	0.155	0.008	0.020	0.824

4 CONCLUSION

This paper explores potential differences in the buying intention of real estate between people who use different primary financing sources: bank loans or their own funds. Data was collected with a survey and the final sample consists of 434 Croatian residents. TPB model was used to test which behavioural factors can accurately describe and predict people's buying intentions. SEM multigroup analysis was conducted to test for the differences in the influence of these factors on the purchase intention among two groups. It is concluded that attitude towards behaviour, subjective norm and perceived behavioural control have a positive significant impact on the buying intention for the whole sample of respondents. This is also true for people who prefer to use bank loans as their primary financing source. The result implies that the more positive attitude towards real estate purchase and a higher level of perceived behavioural control lead to a higher purchase intention. Additionally, higher social impact results in higher intention to buy real estate. In contrast, people who prefer to use their own funds in real estate purchase are only influenced by social pressure. Namely, attitude towards behaviour and perceived behavioural control have no significant impact on their buying intention, while subjective norm has a significant positive impact. This leads to a conclusion that their buying intention increases only when there is a higher pressure from family and friends to buy real estate. People who prefer their own funds as the financing source for real estate purchase might be of a higher financial status, they potentially have more self-confidence and risk aversion, which can be some of the underlying causes of this result. That way, their attitude towards buying real estate wouldn't necessarily matter since they would be able to decide for the purchase at any time. Similarly, since it is generally easy for them to perform this kind of a purchase, their perceived behavioural control is not an important predictor of their behaviour. Research findings can be a starting point for testing more differences in general behavioural theories, which might not always accurately explain and predict behaviour. Instead, research should also focus on deeper analyses based on testing differences among groups. Since people are not the same in their personality traits and behavioural patterns, as well as their financial status and other socio-demographic traits, it is important to further analyse how their specific traits affect their behaviour. In this way, real estate marketers and banks can adjust their selling strategies depending on the type of customer.

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Session 4:

Location and Transport, Graphs and their Applications

HOW WOULD THE CONSTRUCTION OF A NEW CONTAINER PORT IN CARTAGENA INFLUENCE THE REDUCTION OF GREENHOUSE GAS POLLUTION

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Abstract: Three years ago, the European Union (EU) started legislative procedures to include an international shipping in the emissions trading system. Large vessels that call at European ports will have to pay for emissions from transport of cargo that enter or go from a European port. We developed a model to evaluate the differences in maritime transport pollution in the case that a new, sufficiently large container port becomes operational and joins the list of the biggest ports at a selected location. Developing approach to the well-connected sequences of gravity models in the intercontinental maritime chains, which we developed for these purposes, we evaluated the differences in routes and quantities of cargo between ports including waiting time in European ports when a new port is open. From the structure of transport modes and their pollutions derived from the case of Qingdao port for which the pollution coefficients are well elaborated, we evaluated the percentage of pollution reduction among 10 the biggest European ports in case that the port of Cartagena would be constructed.

Keywords: gravity model, supply chain, seaport, pollution, intercontinental transport, containers

1 INTRODUCTION

Sea routes and ports have a strong impact on pollution, especially with exhaust gases. Results of [1] demonstrate the contingency between emissions and port infrastructures and show how proper investments can help with reducing pollutant emissions in ports and for maritime transportation in general. Emissions in ports are generated by manoeuvring and waiting in lines including loading and unloading of ships, onshore operations using energy for loading, unloading, and warehousing of cargo, port vehicle traffic, and so on. The exhaust emissions from ships in maritime transport are estimated by two methodologies, namely, activity-based calculation and fuel-based method. In our paper we shall assume that the fuel structure and activity-based structure on the lines from other continents to Europe and from Europe to other continents does not change and that only the length of all sea routes and therefore the time spending on routes and in ports is reduced by investments and operations of a new port. The exhaust emission in the ports will be calculated on the way as in [1], considering the same structure of activities as in this Chinese port. Therefore, the paper consists of four sections. To the short literature review, the method to evaluate the distance advantages of a new port is presented in third section. The method of calculating reductions of CO2 emissions in case of additional port. As the last section the conclusions and some directions to further study are given.

2 LITERATURE REVIEW

Gravity models (GM) is known also in social science since 1963 (see the literature review of Helena, [6], but as a sequence of sub-models in a model of the supply chain was presented first in [8], but as the three-stage sequencing only in [3], also enabling to tie all gravity models together well. The Web of Science Core Collection (WoS) finds the first article linking the topics of »supply chain« and the »gravity model« only at the beginning of the second decade of this millennium and only 41 articles follow to the first publication. Among them, only three papers [4], [5] and [10] include the topic “maritime transport” to the topic “supply chain”. In the basic approach the formulation of GM is as follows:

$$T_{ij} = \alpha Q_i^{\beta} A_j^{\gamma} / d_{ij}^{\delta} \qquad (1)$$

where T_{ij} is the flow between origin j and destination i and d_{ij} is the distance between the origin and destination, Q_i and A_j of are sizes of populations or gross domestic product GDP_i and GDP_j or logistics quality indicator $M_i = \tau_i L_i$ and $M_j = \tau_j L_j$ of hubs; but power is still equal to 1 or at list symmetrical in many papers [13, 14]. In the models of supply chains except [3] and [10] the distance d_{ij} remained geographical. But in papers considering pollution in a supply chain, like in [10], we can find first explanations why it is good to replace geographical distance (road, train, or Euclidean distance) with the time-consuming distance in sea-routs and in waiting lines in a port, like in some GMs of population migrations and commuting [2], [7]. The success of container transport is due to the reduction in the duration of port calls, it means the reduction of waiting and service time in ports. Although spending a shorter duration of time in ports compared with the time season the way, it is still a pollution factor, not only the cost [11]. Randrianarisoa and Gillen [10] studied only one stage of GM, which cannot answer the questions on impact of a new node in a sea chain. Their paper proposes a way to reduce sulphur emissions in international transport. They are focused on the role of logistics activities and vessel size. When transportation intensity increases, pollution is calculated in the case of adding a new number of vessels. The basic GM with the modified factors is extended. There are no solutions about where to locate a new node to reduce pollution. GDPs per capita and not GDPs itself at origin and destination have been two basic variables in the cited paper. The time spending factor is added extra and therefore less fit for our purposes. According to the findings of Nordås and Piermartini [9], the log expression of GM of cargo from j to i (M_{ij}) can be presented as

$$M_{ij} = a_0 \cdot y_i^{a_1} \cdot y_j^{a_2} \cdot d_{ij}^{a_3} \cdot border_{ij}^{a_4} \cdot lang_{ij}^{a_5} \cdot island_{ij}^{a_6} \cdot landlock_{ij}^{a_7} \cdot (1+t)_{ij}^{a_8} \cdot infr_i^{a_9} \cdot infr_j^{a_{10}} \cdot T_i^{a_{11}} \cdot T_j^{*\,a_{12}} \cdot lat_i^{a_{13}} \cdot lat_j^{a_{14}} \qquad (2)$$

where the first three variable gives quite high correlation coefficient and according to our previous study the other variable could be included in the distance if we replace the geographical distance d_{ij} with the time-spending distance τ_{ij} (time assumed in direct proportion with distance). In this case we leave the velocity factor v which is assumed to be equal in all directions, at the intercept: $a = v \cdot a_0$. In this case we may write:

$$M_{ij} = (a_0 v) \cdot y_i^{a_1} \cdot y_j^{a_2} \cdot \tau_{ij}^{a_3} \rightarrow lnM_{ij} = \ln a + a_1 \ln y_i + a_2 \ln y_j + a_3 \ln \tau_{ij} \qquad (3)$$

They proved that the powers (regression coefficients of linearization) in the maritime and road transport are as given in Table 1. Regarding high R^2 for regressions in Table 1, we used these values for evaluation of potential flow of containerised cargo through Cartagena port when it will be constructed.

Table 1: Value of main regression coefficients in the gravity model ($p < 0.01$)

Regression coefficient	Indicator	Road infrastructure	Maritime infrastructure
a_1	GDP importer	0.94	0.80
a_2	GDP exporter	1.12	0.91
a_3	distance between i and j	-1.22	-0.71
Source: Nordås and Piermartini (2004); adjusted R^2		> 0.65	> 0.70

3 POTENCIAL ADVANTAGES OF AN EUROPEAN PORT LOCATION

Continuing the procedure like in the article by [3], in this paper, we added the waiting time for cargo unloading from vessels and loading on truck as τ_k and revised (2) to (4):

$$M_{ki} = a \cdot y_k^{0.80} \cdot y_i^{0.91} \cdot (\tau_{ki} + \beta\tau_k)^{-0.71} \tag{4}$$

and for road transport from port k to EU city l:

$$M_{lk} = b \cdot y_l^{0.94} \cdot y_k^{1.12} \cdot (\tau_{kl} + \gamma\tau_k)^{-0.71} \tag{5}$$

In (4) and (5) β and γ are the proportionality coefficient between the costs of the hour of waiting and unloading or loading in the port and the hours of sailing of the container ship and truck respectively per unit of cargo. The average waiting and service time of ships at ports of some EU and neighbouring countries we may take from UNCTAD [13]. Average waiting and service time of ships at ports of the EU countries is grouped in our paper in three groups: group I: 0.3-0.6 days: Gibraltar, Norway, and Denmark; group II: 0.61-0.8 days: Croatia, Sweden, Lithuania, Slovenia, Spain, Portugal, Poland, Finland, Latvia, France Netherlands; group III: 0.81+ days: others. Ten the biggest ports have been considered. Considering Table 7 of Bogataj et al. (2022) we may calculate the average time-consuming distances of 1 TEU (6.10 x 2.44 x 2.59 m^3) container $(\boldsymbol{\tau_{lk}} + \boldsymbol{\tau_k})$ and their coefficients to the average of EU transport between central locations in EU continental member states (l) and the main 10 EU ports (k) transported by trucks, from where the cargo sharing between ports in 2039 if port will be completed until then, is calculated as given in Table 2.

Table 2: Expansion of the annual cargo handling by 2039 in the main EU ports [in 10^6 TEU].

If Cartagena port is built						
Cartagena	Hamburg	Bremen h.	Antwerp	Zeebrugge	Valencia	Algeciras
7.9	5.5	9.2	-0.6	11.1	3.4	1.6
Barcelona	Le Havre	Marseilles	Rotterdam	Gdansk	Gioia Tauro	
6.1	9.5	9.2	0.1	8.9	5.1	
If Cartagena port will not be completed						
Cartagena	Hamburg	Bremen h.	Antwerp	Zeebrugge	Valencia	Algeciras
0.0	6.1	10.3	0.0	12.4	3.8	1.8
Barcelona	Le Havre	Marseilles	Rotterdam	Gdansk	Gioia Tauro	
6.8	10.6	10.2	0.1	9.9	5.7	

Table 3: Increase of the yearly emissions in the EU biggest ports in 2039 if Cartagena port would be completed and technology does not change [ton].

	Cartagena	**Hamburg**	**Bremen h.**	**Antwerp**	**Zeebrugge**	**Valencia**	**Algeciras**
NO_x	7,798	5,415	9,034	-579	10,888	3,355	1,609
CO	805	559	932	-60	1,124	346	166
HC	331	230	383	-25	462	142	68
CO_2	596,342	414,064	69,0857	-44,257	832,629	256,54	123,019
SO_2	5,556	3,858	6,437	-412	7,757	2,39	1,146
PM_{25}	447	310	518	-33	624	192	92
	Barcelona	**Le Havre**	**Marseilles**	**Rotterdam**	**Gdansk**	**Gioia Tauro**	
NO_x	6,023	9,368	8,995	78	8,76	5,032	
CO	621	967	928	8	904	519	
HC	256	398	382	3	372	214	
CO_2	460,571	716,361	687,857	6,001	669,854	384,81	
SO_2	4,291	6,674	6,409	56	6,241	3,585	
PM_{25}	345	537	516	4	502	288	

Cartagena as potential port was added, and growth of intercontinental cargo will be as forecasted by the World Trade Organization (WTO). Let us take for EU the biggest ports the same structure of emissions of nearly 125,000 ships like in Qingdao in 2016 [12] from where the pollution of container ships is approximately 2.8/5. [NOx, CO, HC, CO2, SO2, PM2,5] = 103*[16.7, 1.7, 0.7, 1,275, 11.9, 0,96], and 1 TEU will be in average equal to nearly 30 metric tons, the yearly emission in 2039, (when Cartagena port would be completed) in EU biggest ports will increase for the values given in Table 3, if the technology will not improve.

4 CONCLUSIONS

The EU started legislative procedures to include an international shipping in the emissions trading system. Large vessels that call at European ports will have to pay for their emissions from transport of cargo that enter or go from a European port. Namely, maritime transport has increased along with increasing global trade. This influences a higher pollution level, including the emissions of greenhouse gas like CO_2 in the structure of pollutants in exhaust emissions. The expected pollution and cost of it can be forecasted if the technology will not change, like presented in table 4. Therefore, we must consider to shortened waiting time and service time in the EU ports, but also support the construction of a new port that would reduce the amount of maritime transport and thus also reduce pollution at sea.

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INTERVAL TRANSPORTATION PROBLEM: THE WORST FINITE OPTIMAL VALUE IS HARD FOR INEQUALITIES

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Abstract: We consider the model of an interval transportation problem, in which the values of supply, demand and the transportation costs are affected by uncertainty and can be independently perturbed within the given bounds. We address the problem of computing the worst (finite) optimal value of the interval model over all possible scenarios. Utilizing a related result from bilevel programming, we prove that computing the value exactly is NP-hard for two commonly used formulations of the interval transportation problem.

Keywords: transportation problem, interval data, uncertainty, optimal value

1 INTRODUCTION

Transportation problems [4] belong to the fundamental models arising in various areas of operations research, such as logistics, inventory control, production planning or human resources management. Since uncertainty often plays an important role in practical optimization, variants of the transportation problem with imprecise data were also considered in the literature.

We study the uncertain transportation problem within the context of interval linear programming. In the model of an interval transportation problem [2] the input data are not known exactly, but can be independently perturbed within the given lower and upper bounds. The basic properties related to feasibility and optimality [3] as well as methods for computing or approximating optimal values [1] of such problems were studied in the recent literature.

One of the tasks in interval programming regarding the optimal values is computing the best and the worst optimal value, also known as the optimal value range problem [7]. Traditionally, this included infinite "optimal" values for infeasible or unbounded programs, however, a modified finite optimal value range problem was also considered in the literature [5].

For the interval transportation problem, a recent result from the area of bilevel programming by Hoppmann–Baum [6] implies that computing the worst finite optimal value of the equation-constrained formulation is NP-hard for interval transshipment problems. In this paper, we extend the result to inequality-constrained interval transportation problems and show that a direct transformation between the formulations preserves the worst finite optimal value.

2 PROBLEM FORMULATION

Let us now formalize the mathematical model of the interval transportation problem considered in this paper. Assume the following data are given:

- $I = \{1, \ldots, m\}$ denotes the set of m *sources* (or supply centers),

- $J = \{1, \ldots, n\}$ denotes the set of n *destinations* (or customers),
- each source $i \in I$ has a limited *supply* s_i available, which is an uncertain quantity varying within the given non-negative interval $\mathbf{s}_i = [\underline{s}_i, \overline{s}_i]$,
- each destination $j \in J$ has an uncertain *demand* d_j to be satisfied, which corresponds to the non-negative interval $\mathbf{d}_j = [\underline{d}_j, \overline{d}_j]$,
- the uncertain *unit cost* of transporting one unit of goods from source i to destination j is represented by the interval $\mathbf{c}_{ij} = [\underline{c}_{ij}, \overline{c}_{ij}]$.

The objective of the transportation problem is to find a minimal-cost transportation plan for shipping goods from the sources to the destinations such that the supply and demand requirements are satisfied. Formally, the *interval transportation problem* can be represented by the interval linear program

$$\begin{aligned} \text{minimize} \quad & \sum_{i \in I} \sum_{j \in J} [\underline{c}_{ij}, \overline{c}_{ij}] x_{ij} \\ \text{subject to} \quad & \sum_{j \in J} x_{ij} = [\underline{s}_i, \overline{s}_i], && \forall i \in I, \\ & \sum_{i \in I} x_{ij} = [\underline{d}_j, \overline{d}_j], && \forall j \in J, \\ & x_{ij} \geq 0, && \forall i \in I, j \in J, \end{aligned} \tag{ITP$^=$}$$

where the non-negative variable x_{ij} corresponds to the amount of goods transported from source i to destination j.

The interval transportation problem can be understood as the set of all linear programs with coefficients from the respective intervals $\mathbf{c} \in \mathbb{IR}^{m \times n}$, $\mathbf{s} \in \mathbb{IR}^m$ and $\mathbf{d} \in \mathbb{IR}^n$ (see [7] for more information on interval linear programs). A particular linear program in the interval transportation problem determined by the cost matrix $C \in \mathbf{C}$, supply vector $s \in \mathbf{s}$ and demand vector $d \in \mathbf{d}$ is called a *scenario*.

Alternatively, we can also consider the inequality-constrained formulation of the interval transportation problem, in which the supplies do not have to depleted and the demands do not have to be satisfied exactly:

$$\begin{aligned} \text{minimize} \quad & \sum_{i \in I} \sum_{j \in J} [\underline{c}_{ij}, \overline{c}_{ij}] x_{ij} \\ \text{subject to} \quad & \sum_{j \in J} x_{ij} \leq [\underline{s}_i, \overline{s}_i], && \forall i \in I, \\ & \sum_{i \in I} x_{ij} \geq [\underline{d}_j, \overline{d}_j], && \forall j \in J, \\ & x_{ij} \geq 0, && \forall i \in I, j \in J. \end{aligned} \tag{ITP$^\leq$}$$

The *worst finite optimal value* of the interval transportation problem, which will be denoted by $\overline{f}$, can then be defined as the maximal optimal value over all feasible scenarios, i.e.

$$\overline{f} = \max\{f(C, s, d) : (C, s, d) \in (\mathbf{C}, \mathbf{s}, \mathbf{d}) \text{ such that } (C, s, d) \text{ is a feasible scenario}\},$$

where $f(C, s, d)$ denotes the optimal value of the scenario with data (C, s, d).

3 THE WORST FINITE OPTIMAL VALUE

An interval transportation problem is said to be *weakly feasible*, if there exists a feasible solution for at least one of its scenarios. Lemma 1 generalizes the classical condition for feasibility of transportation problems with real data to problems with interval-valued supply and demand.

Lemma 1 *An interval transportation problem* (ITP$^=$) *is weakly feasible if and only if*

$$\left[\sum_{i\in I}\underline{s}_i, \sum_{i\in I}\overline{s}_i\right] \cap \left[\sum_{j\in J}\underline{d}_j, \sum_{j\in J}\overline{d}_j\right] \neq \emptyset.$$

Therefore, weak feasibility of (ITP$^=$) can be easily verified. Let us now prove the main result, which shows that using a direct transformation of a weakly feasible problem (ITP$^=$) to (ITP$^\leq$) preserves the worst finite optimal value.

Theorem 2 *Let $\overline{f}_\mathcal{E}$ denote the worst finite optimal value of a given weakly feasible* (ITP$^=$) *model with data* $(\mathbf{C}, \mathbf{s}, \mathbf{d})$ *and let $\overline{f}_\mathcal{I}$ denote the worst finite optimal value of the* (ITP$^\leq$) *model with the same data* $(\mathbf{C}, \mathbf{s}, \mathbf{d})$. *Then $\overline{f}_\mathcal{I} = \overline{f}_\mathcal{E}$ holds.*

Proof. The value $\overline{f}_\mathcal{E}$ is achieved for some feasible scenario $(C^\mathcal{E}, s^\mathcal{E}, d^\mathcal{E}) \in (\mathbf{C}, \mathbf{s}, \mathbf{d})$ of program (ITP$^=$). Since the scenario is feasible, it must be balanced, i.e.

$$\sum_{i\in I} s_i^\mathcal{E} = \sum_{j\in J} d_j^\mathcal{E}$$

holds. Therefore, changing the equation constraints in (ITP$^=$) to inequalities in (ITP$^\leq$) does not allow any new feasible solutions, as all supplies have to be depleted and all demands have to be met exactly to satisfy the constraints in the given scenario. This implies that the value $\overline{f}_\mathcal{E}$ will still be optimal for a scenario of (ITP$^\leq$) and thus $\overline{f}_\mathcal{I} \geq \overline{f}_\mathcal{E}$.

On the other hand, the value $\overline{f}_\mathcal{I}$ is achieved for some feasible scenario $(C^\mathcal{I}, s^\mathcal{I}, d^\mathcal{I}) \in (\mathbf{C}, \mathbf{s}, \mathbf{d})$ of program (ITP$^\leq$). If the scenario is balanced, then its feasible set is the same in (ITP$^=$). This implies that $\overline{f}_\mathcal{I} \leq \overline{f}_\mathcal{E}$ due to the fact that $\overline{f}_\mathcal{I}$ is optimal for some scenario, while $\overline{f}_\mathcal{E}$ is the worst (maximal) optimal value over all scenarios. Otherwise, the scenario $(C^\mathcal{I}, s^\mathcal{I}, d^\mathcal{I})$ is unbalanced and

$$\sum_{i\in I} s_i^\mathcal{I} \geq \sum_{j\in J} d_j^\mathcal{I}$$

holds from the assumption of feasibility. Let us now discuss the three cases that can occur with respect to the relations between the values of $\sum s_i^\mathcal{I}$, $\sum d_j^\mathcal{I}$ and $\sum s_i^\mathcal{E}$, $\sum d_j^\mathcal{E}$ (for the sake of readability, we will omit limits for the sums in inline formulas as no confusion shall arise).

Case I: $\sum s_i^\mathcal{I} \geq \sum s_i^\mathcal{E}$ and $\sum d_j^\mathcal{I} \geq \sum d_j^\mathcal{E}$.
Here, we can decrease the values of $s^\mathcal{I}$ to find a supply vector $s^{\mathcal{I}'} \in \mathbf{s}$ such that $s^{\mathcal{I}'} \leq s^\mathcal{I}$ and $\sum s_i^{\mathcal{I}'} = \sum d_j^\mathcal{I}$. Such a vector is guaranteed to exist, since

$$\sum_{i\in I} s_i^\mathcal{I} \geq \sum_{j\in J} d_j^\mathcal{I} \geq \sum_{j\in J} d_j^\mathcal{E} = \sum_{i\in I} s_i^\mathcal{E}$$

and thus also $\sum d_j^\mathcal{I} \in [\sum s_i^\mathcal{E}, \sum s_i^\mathcal{I}] \subseteq \mathbf{s}$. The optimal value of the scenario $(C^\mathcal{I}, s^{\mathcal{I}'}, d^\mathcal{I})$ is at least $\overline{f}_\mathcal{I}$, because we only reduced the set of feasible solutions by decreasing the available supply (in fact, it is exactly $\overline{f}_\mathcal{I}$, as this is the worst possible optimal value over all scenarios). The obtained scenario is balanced, implying $\overline{f}_\mathcal{I} \leq \overline{f}_\mathcal{E}$ as before.

Case II: $\sum s_i^\mathcal{I} \leq \sum s_i^\mathcal{E}$ and $\sum d_j^\mathcal{I} \leq \sum d_j^\mathcal{E}$.
This case is analogous to the former case. Instead of decreasing the supply vector, we can increase the demands to a suitable value $d^{\mathcal{I}'} \in \mathbf{d}$ with $d^{\mathcal{I}'} \geq d^I$ and $\sum d_j^{\mathcal{I}'} = \sum s_i^\mathcal{I}$ to obtain a balanced scenario with the same optimal value.

Case III: $\sum s_i^\mathcal{I} \geq \sum s_i^\mathcal{E}$ and $\sum d_j^\mathcal{I} \leq \sum d_j^\mathcal{E}$.
In this remaining case, we can both decrease the supply vector from $s^\mathcal{I}$ to one with the total supply equal to $\sum s_i^\mathcal{E}$ and increase the demand vector $d^\mathcal{I}$ to one with the total demand equal

to $\sum d_j^{\mathcal{E}}$. Again, this leads to creating a balanced scenario with a restricted feasible set and the same optimal value. $\square$

Using the property from Theorem 2, we can now show that the problem of computing the worst finite optimal value of (ITP$^{\leq}$) is an NP-hard problem. Here, we utilize a corollary to a result by Hoppmann-Baum [6, corollary of Theorem 3], which is stated in Lemma 3.

Lemma 3 ([6]) *Computing the worst finite optimal value $\overline{f}_{\mathcal{E}}$ of* (ITP$^{=}$) *is NP-hard.*

Corollary 4 *Computing the worst finite optimal value $\overline{f}_{\mathcal{I}}$ of* (ITP$^{\leq}$) *is NP-hard.*

Proof. The proof is by reduction from computing $\overline{f}_{\mathcal{E}}$ for (ITP$^{=}$). If (ITP$^{=}$) is not weakly feasible, then $\overline{f}_{\mathcal{E}}$ does not exist. By Lemma 1, we can easily check whether a problem (ITP$^{=}$) is weakly feasible. Hence the hard cases causing the problem of computing $\overline{f}_{\mathcal{E}}$ to be NP-hard (as Lemma 3 states) are the weakly feasible cases.

By Theorem 2, we know that the weakly feasible cases of computing $\overline{f}_{\mathcal{E}}$ can be solved by computing $\overline{f}_{\mathcal{I}}$ for (ITP$^{\leq}$). Hence computing $\overline{f}_{\mathcal{I}}$ can be (together with the polynomial-time test from Lemma 1) used for solving an NP-hard problem, and thus, it also has to be NP-hard. $\square$

4 CONCLUSION

We studied the task of computing the worst finite optimal value of an interval transportation problem. We have proved that a direct transformation of a weakly feasible equation-constrained model to an inequality-constrained one does not change the worst finite optimal value, thus, an algorithm for computing the value for the latter formulation can also be used to solve the former one. Using this property in combination with a recent result from bilevel programming, which implies NP-hardness of the problem for equations, we have proved that computing the worst finite optimal value is also NP-hard for inequality-constrained transportation problems with interval-valued supply, demand and costs.

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GRAPH-BASED PRIORITIZATION OF RELATED CANCER GENES

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Abstract: This paper introduces Graph-Based Prioritization (GBP), a novel computational approach for gene prioritization in cancer genomics. GBP integrates gene interaction networks and mutation data to identify and rank genes based on their relevance to cancer. By considering gene mutations and the influence from neighboring genes in the network, GBP calculates a mutation score and employs GBP-PR for gene prioritization. Experimental results across six cancer types demonstrate the effectiveness of GBP in identifying known and potential novel cancer genes. Overall, GBP offers a valuable tool for understanding tumor mechanisms and advancing cancer research.

Keywords: Cancer, Gene Prioritization, Ranking, Rating

1 INTRODUCTION

Cancer is a devastating disease caused by genetic mutations in cells, and identifying the key mutations that drive its development is a crucial challenge in cancer research. With the advent of computational methods and the wealth of biological data generated through next-generation sequencing technologies, researchers now have powerful tools to tackle this challenge [1]. By analyzing gene interaction networks and employing statistical and graph theory-based algorithms, these computational methods aim to identify the most significant mutations in cancer genes and shed light on their functional roles in cancer biology. Gene interaction network analysis is a key strategy employed by these computational methods. By studying the interactions between mutated genes and their influence on these networks, researchers gain valuable insights into the functional roles of these genes in cancer[2]. Several methods, including Hierarchical HotNet [3], Dendrix [4], and Multi-Dendrix [5], have been developed to identify driver mutations in gene networks. These methods utilize gene graphs, network diffusion algorithms, and weighted functions to identify relevant gene sets with high mutation frequencies in patients. Statistical methods such as CoMEt[6], MEMo [7], and MEMCover[8] have also emerged as important tools in the identification of mutually exclusive gene sets. These methods employ statistical analysis and network analysis to identify gene modules based on alteration frequency, biological process, and mutual exclusivity. Furthermore, rating algorithms derived from network analysis and graph theory, such as PageRank, Colley, Massey, and Keener[9], have gained prominence in various domains. These algorithms offer valuable tools for analyzing networks and ranking entities, providing insights into the structure and dynamics of complex systems. In the context of cancer research, these algorithms can be applied to prioritize cancer genes based on their mutation data and their influence on gene interaction networks.

This study proposes a graph-based approach that integrates cancer mutation data and gene interaction networks to prioritize related cancer genes. We apply a novel heuristic approaches to build a mutation matrix, calculate mutation scores, create a consensus gene interaction network, generate a gene spreading strength network, extract mutation influence from neighbors, and prioritize genes using a dynamic PageRank algorithm.

2 METHODOLOGY

2.1 Extraction of Gene Spreading Strength (GSS)

Our approach uses multiple gene networks as input, generating an undirected and weighted network (UWN) that preserves original interactions. We then consider gene direct and indirect neighbors to calculate mutations spread from one gene to another in a network by $ss(g_i, g_j) = (1 + r_i \times r_j^{out}) \times p_{ij}$, where $r_j^{out} = \sum_g \in ({}^{N(g_j)}/_{N(g_i)})p(g_i, g_j)$,$r_i$ is the sum of the edge weights of g_i and r_j^{out} is the sum of the edge weights of g_j that are not edges of g_i.

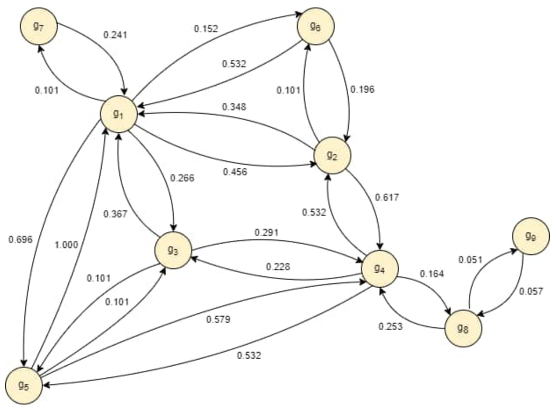

Figure 1: A simple hypothetical graph of extracted GSS

2.2 Extraction of Mutation Neighbors Influence

The spreading strength of genes measures their susceptibility to neighboring mutations and their ability to impact nearby genes. Influence is determined by the function $r(g_i)$ given by $r(g_i) = \sum_{g_k \in N(g_i)} nwm_f(g_k) \times ss(g_k, g_j)$, where $N(g_i)$ are direct neighbors of g_i on GSS. Hence the final gene mutation enrichment score is calculated by $ms(g_i) = wm_f(g_i) + r(g_i)$, where $wm_f(g_i)$ is weighted mutation frequency and $r(g_i)$ is the gene neighbor influence.

2.3 Gene Prioritization

Using the gene mutation enrichment scores obtained in Section 2.2, we generate a rating function to prioritize genes. The mutation enrichment scores matrix of $M \in \mathbb{F}^{g\times g}$ is defined by

$M_{ij} = \#\{ms(g_i) \geq ms(g_j)\}$, where $ms(g_i)$ and $ms(g_j)$ ms score for gene i and j respectively. Next, we prioritize genes using the dynamic PageRank we iteratively calculate gene rating scores by $PR(g_i) = \frac{\lambda}{g} + (1-\lambda)\sum_{g_j \in G^+(g_i)} \frac{PR(g_j)}{ms(g_j)}$, where $G^+(g_i)$ is the set of genes with low mutation score against gene g_i, $ms(g_j)$ is the mutation score of g_j, and $\lambda \in [0,1]$ is a damping factor(usually 0.1 or 0.2) to guarantee convergence. We rewrite the above equation relationship to Markov chains in a vector form as $\mathbf{PR} = \frac{\lambda}{G}[I-(1-\lambda)SD^{-1}]^{-1}\mathbf{1}$, where $\mathbf{PR}$ is the PageRank vector containing values of each gene, D is the diagonal matrix $D = \text{diag}[(D_{ii} = \sum_{\ell=1}^{g} S_{i\ell})_{i=1}^{g})]$, and $\mathbf{I}$ is the $g \times g$ identity matrix.

3 MODEL EVALUATION CRITERIA

3.1 Forward-Looking Approach (FLA)

To evaluate the rating and ranking stability, we applied our previous Forward-Looking Approach with an Expanded Window (FLA-WE)[10]. For rating stability in $S_{EW}^{(K,\Delta k=20)}$ we define Euclidean distance between two rating vectors by $d^2_{EW}(k) = ||\phi_{EW}^{(K\Delta k,k+\Delta k)} - \phi_{EW}^{k})||_2^2$, where EW is window size and $||\cdot||_2$ is the Euclidean norm. Thus, to compute the mean $d^2_{EW}(k)$ for all $top-k$ prioritized genes in EW. For ranking stability we considered the relevance of genes, $\pi_{EW}^{rel_{ct}}$ in consecutive rankings to calculate distance weighted matrix (DWM) Kendall's tau correlation, τ_{EW}^{k} given by

$$\tau_{EW}^{k} = \frac{\sum\limits_{g_i<g_j} w_{g_i,g_j}\left(1 + sgn\left((\pi_{EW}^{1}(g_i) - \pi_{EW}^{2}(g_j))(\pi_{EW}^{rel_{ct}}(g_i) - \pi_{EW}^{rel_{ct}}(g_j))\right)\right)}{2\sum\limits_{g_i<g_j} w_{g_i,g_j}},$$

where w_{g_i,g_j}, is the weight, $\pi_{EW}^{1}(g_i)$ and $\pi_{EW}^{2}(g_j)$ is rank position of genes, g_i and g_j in ranking π_{EW}^{k} and $\pi_{EW}^{(k\Delta k,k+\Delta k)}$, respectively for $top-k$ prioritized genes.

3.2 Ranking Precision and Discounted Cumulative Gain (DCG)

We also evaluated the ranking quality of our proposed model based on precision and DCG. We calculated the precision as the ratio of cancer-related genes that undergo mutation in the top-K predicted set given by $Precision = \frac{TP}{TP+FP}$,where TP is the number of genes prioritized by our method that are in the benchmark. FP is the number of genes prioritized that are not in the benchmark. In addition, we applied DCG to evaluate gene relevance and ranking position, logarithmically decreasing with the position. Hence, DCG score for genes up to position p is calculated by $DCG_p = \sum_{i=1}^{PG_i} \frac{rel_{g_i}^{ct_j}}{\log_2(i+1)}$, where PG_p is the ranking list of p prioritized genes and $rel_{g_i}^{ct_j}$ is the relevance score of gene g_i in cancer type ct_j.

4 RESULTS

Our research compared the GBP-PR method with other peer-rating methods (Colley, Keener, and Massey) and found that GBP-PR demonstrated high rating stability and consistency for gene prioritization across six cancer datasets. The evaluation of gene ranking stability showed that our proposed GBP-PR approach exhibited reliable and robust performance. GBP-PR consistently outperformed other peer methods in precision and Discounted Cumulative Gain metrics for several cancer types. The top 20 prioritized genes by GBP-PR revealed significant genes with crucial roles in cancer development. These findings were validated by benchmark databases and the Cancer Genome Interpreter (CGI) datasets, confirming the biological relevance of our discoveries.

5 CONCLUSION

In conclusion, the GBP-PR method presented in this paper offers a comprehensive and flexible approach to prioritizing significant groups of related genes in cancer. Through the integration of mutation data, gene networks, and asymmetric spreading strength measures, the method effectively identifies potential driver genes and suggests novel genes for further investigation. While future research should focus on incorporating additional biological data and conducting more extensive experimental evaluations, the GBP-PR method has already made a valuable contribution to cancer genomics by providing a robust computational framework for identifying crucial genes involved in cancer.

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LOCATION OF SERVICE DEVICES AT ANY POINT IN TWO-DIMENSIONAL SPACE

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Abstract: Researchers have solved the service devices' location problem for a long time with various modifications and several mathematical programming models that serve to find the best solution to the problem. In the contribution, the authors focused on constructing a model within which service devices can be placed at any point in two-dimensional space. The paper contains the formulation of the problem, the construction of the model, as well as its adjustment to a form that standard mathematical programming tools can solve.

Keywords: location problem, mathematical model, optimization

1 INTRODUCTION

Many technical and logistical location problems are based on the location of service facilities at already existing points, for example, the location of emergency rescue stations, the location of beehives in an orchard, collection points for secondary raw materials, car scrap yards, and charging stations for electric cars. This class of tasks is usually formulated as facility location problems; p-median problems; p-center problems, or covering problems [1], [2], [3], [4], [5], [6] and [7]. In the first part of the paper, the authors present the formulation of the well-known problem of locating service facilities. Subsequently, a location model where the service device can be placed anywhere in space is formulated. In the conclusion, possible ways of solving it are presented.

2 METHODOLOGY: MODELS FOR DETERMINING THE OPTIMAL LOCATION

In classically formulated location problems, the mathematical programming model (1) is standardly used to solve the corresponding location problem of placing the minimum number of service facilities. In the mathematical programming model formulated in this way, the service device (entity providing the service) is assumed to be located in one serviced device (the object to which the service is provided). In case of violation of this assumption, i.e. if the location in the serviced facility is not assumed, the question arises of constructing the model in such a way as to reflect the omission of the mentioned condition. In the first part of the paper, the classical location model of the minimum number of service devices is presented, followed by the formulation of the problem, with enables to locate of the service devices at any point of the two-dimensional space (5).

The problem can be formulated as a binary programming problem with variables $y_{ij} \in \{0,1\}, x_i \in \{0,1\},\ i, j = 1, 2, \ldots n$, where n represents the number of operated service facilities. The variable $y_{ij} \in \{0,1\},\ i, j = 1, 2, \ldots n$ represents whether the i-th service facility is servicing the j-th serviced device (the value is equal to 1); otherwise, the value is equal to 0. The model also deals with a binary variable $x_i \in \{0,1\},\ i = 1, 2, \ldots n$ that models the location of service facilities. Because in this type of problem, the requirement that the service facility can be located only in the places of serviced devices must be met, the value is equal to 1, if the i- th serviced device is also a service facility, otherwise the value is equal to 0. Parameters of the model are represented by the matrix of the shortest distances between individual serviced devices **D**. Next, the parameter K is introduced, which represents the maximum distance of the serviced device from the nearest service facility.

$$
\begin{aligned}
& f_1(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} x_i \to \min \\
& \sum_{i=1}^{n} y_{ij} = 1, \quad j = 1, 2, \ldots n \\
& \sum_{i=1}^{n} d_{ij} y_{ij} \le K, \quad j = 1, 2, \ldots n \\
& y_{ij} - x_i \le 0, \quad i, j = 1, 2, \ldots n \\
& x_i, y_{ij} \in \{0,1\}, \quad i, j = 1, 2, \ldots n
\end{aligned}
\tag{1}
$$

3 MATHEMATICAL PROGRAMMING MODEL FORMULATION

Next, let us consider the case in which it is possible to locate the service facility at any point in the two-dimensional space (therefore, the requirement to locate it in n locations of serviced devices is not reflected). let us assume that the location coordinates of the individual serviced devices (total number n) are known, thus the location of the j-th serviced device is represented by the point $[a_j, b_j]$, where a_i represents the first coordinate of the j-th device and b_j the second coordinate of the j-th device, $j = 1, 2, \ldots n$. Parameter K indicates the value of the permissible maximum distance of the serviced device from the service facility.

Let us further assume that a maximum of m service facilities can be located. The decision variables of the formulated mathematical programming model x_k represent the first coordinate of the location of the k-th service facility (m indicates the maximum number of facilities, and therefore $k = 1, 2, \ldots m$). Other decision variables y_k (k=1, 2,...m) represent the second coordinate of the location of the k-th service facility.

In addition to the mentioned decision variables, the presented model also contains binary variables z_k (k=1, 2,...m), which in the case of acquiring a value of 1 indicate the use of the service facility at point k. In the case of a value of 0, the service facility at point k is not used. The aim is to minimize the number of service facilities. The objective function represents the total number of operated service facilities, while its value is represented by the function sum of variables z_k (k=1, 2,...m):

$$
\min\ f(\mathbf{x}, \mathbf{z}) = \sum_{i=1}^{m} z_i \tag{2}
$$

The aim of the first type of constraints are to ensure that the nearest serviced device is at the maximum distance from the service facility, which must be true for all n served devices (j = 1, 2,...n).

$$\min_{i=1,2,...m}\left\{\sqrt{\left(a_j-x_i\right)^2+\left(b_j-y_i\right)^2}\right\}\le K,\ j=1,2,...n \qquad (3)$$

Another type of constraints implements the calculation of values 1 for binary variables z_k (k=1, 2,...m) for service facility that will be placed at point k.

$$M\cdot\left(\sqrt{\left(a_j-x_k\right)^2+\left(b_j-y_k\right)^2}-\min_{i=1,2,...m}\left\{\sqrt{\left(a_j-x_i\right)^2+\left(b_j-y_i\right)^2}\right\}\right)+z_k\ge 1,\ j=1,2,...n,k=1,2,...m \qquad (4)$$

Based on the above, the problem can be formulated as follows:

$$\begin{aligned}
&\min\ f(\mathbf{x},\mathbf{y},\mathbf{z})=\sum_{i=1}^{m} z_i\\
&\min_{i=1,2,...m}\left\{\sqrt{\left(a_j-x_i\right)^2+\left(b_j-y_i\right)^2}\right\}\le K,\ j=1,2,...n\\
&M\cdot\left(\sqrt{\left(a_j-x_k\right)^2+\left(b_j-y_k\right)^2}-\min_{i=1,2,...m}\left\{\sqrt{\left(a_j-x_i\right)^2+\left(b_j-y_i\right)^2}\right\}\right)+z_k\ge 1,\ j=1,2,...n,k=1,2,...m\\
&x_k,y_k\ge{}^\circ 0,\ k=1,2,...m\\
&z_k\in\{0,1\}
\end{aligned} \qquad (5)$$

While using classical optimization techniques, the formulated problem (5) is unsolvable because it does not meet the condition of linearity of relations. Based on this fact, a metaheuristic approach can be used to solve it [8], [9] and [10].

In formulated problem (5), there is a minimum-finding operation in the structural constraints (3) and (4), which causes significant computational problems. Therefore, from the point of view of the solvability of the task, it is interesting to remove the min function in structural constraints. Next, the transformation of task (5) will be presented, which allows removing the min function from the structural constraints. For such a transformation, new binary variables t_{ij} (i=1,2,...m, j=1,2,...n) must be introduced into the model, the aim of which is to ensure the activity of only boundaries with a minimum value. The constraints $\sum_{i=1}^{m} t_{ij}=1,\ j=1,2,...n$ ensure that the j-th (j=1,2,...n) serviced device is served by exactly one service facilty that is at a specified minimum distance. Therefore, the formulated structural constraints (3) and (4) are modified using the variables t_{ij} (i=1,2,...m, j=1,2,...n), which, for constraints (3), ensures that the maximum distance from the nearest service facility will be a maximum of K. In constraints (4), the variables z_k (k=1,2,...m) will take on the value one only when the service facility is located and operated (the value 0 otherwise). Then the form of the mathematical programming problem is as follows:

$$\begin{aligned}
&\min\ f(\mathbf{x},\mathbf{y},\mathbf{z})=\sum_{i=1}^{m} z_i\\
&\sum_{i=1}^{m} t_{ij}=1,\ j=1,2,...n\\
&\sqrt{\left(a_j-x_i\right)^2+\left(b_j-y_i\right)^2}\cdot t_{ij}\le K,\ j=1,2,...n,i=1,2,...m\\
&M\cdot\left(\sqrt{\left(a_j-x_k\right)^2+\left(b_j-y_k\right)^2}-\sqrt{\left(a_j-x_i\right)^2+\left(b_j-y_i\right)^2}\cdot t_{ij}\right)+z_k\ge 1,\ j=1,2,...n,k=1,2,...m,i=1,2,...m
\end{aligned} \qquad (6)$$

$x_k, y_k \geq °0,\ k = 1, 2, ...m$
$z_k \in \{0,1\},\ k = 1, 2, ...m$
$t_{ij} \in \{0,1\},\ i = 1, 2, ...m,\ j = 1, 2, ...n$

4 CONCLUSION

When solving location problems of any object, one may encounter the problem of its location at any point in space. Standard location models are mainly oriented to the location of service facilities in one of the points of serviced objects. In this case, it is based on the assumption of known distances between devices (road, air, rail distances). However, in the case of location at any point in space, we do not know these distances in advance. The method of construction of the location model is presented in the paper. We assume that the distances are unknown and the calculation is carried out as the shortest air distance. Thus, the problem formulated in this way can be used, for example, in the construction of a network of service devices (transmitters, control stations, etc.), while the maximum distance from the serviced devices is given. The goal is to find the minimum number of service devices. Since standard optimization tools cannot solve the formulated mathematical programming problem, we present the transformation of the problem into a form that is more suitable for its solution. However, the disadvantege of the conversion is a significant increase in the number of variables, which also causes a problem of solvability for real tasks. Two methods are available for solving: tasks of smaller dimensions can be solved with classical tools, and in the case of larger ones, it is necessary to use alternative tools (e.g. evolutionary and genetic tools).

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Estimating node importance in public transport networks

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Abstract: The aim of this paper is first to give a short overview of centrality measures of networks most often used in public transport network (PTN) analysis. Laplacian centrality is introduced and its relation to other centrality measures is examined. Structural properties of the network as reflected in centrality scores are compared to aggregate passenger boarding counts, regarded as a different type of node importance. Pearson correlation coefficients and linear regression fits between passenger counts and centrality scores are calculated. A case study is presented for the bus PTN in Ljubljana, Slovenia.

Keywords: public transport, graph theory, network science, node centrality, GTFS, smart card data, Laplacian centrality

1 INTRODUCTION

Proper evaluation of public transport (PT) systems performance is crucial for their understanding, planning and optimization. The measures of relative importance of stops in public transport networks (PTNs) are of particular interest in connection with ridership demand as well as in analysis of network robustness. Among the main aspects of stop importance is its connectedness to other stops, which can be estimated by determining centrality scores of the nodes in a graph representation of the PT network. The most commonly studied measures in literature are degree, closeness, betweenness and eigenvector centrality (see for instance [8], [1] and [11]).

Several studies have been performed on the network analysis of PTNs. In ([10], [9]) the PTN structure and properties in different graph representations of 14 cities worldwide were studied. A similar analysis was performed for directed networks in [2] where standardized timetable data (GTFS) was used to build the PTN graph representations. While papers mostly examine structural network properties based on its operational features, passenger flow information was incorporated in [3] and a systematic study of correlations between flows and node centrality measures for several PTN representations were presented for tram networks of two Dutch cities.

The stucture of the paper is as follows. In section 2, some preliminaries are presented in 2.1 and the description of the analysis is given in 2.2. Results for the case study are presented in 3. Section 4 presents conclusions and future prospects.

2 NODE CENTRALITY ANALYSIS

2.1 Preliminaries

Definitions of the applied centrality measures are briefly summarized from [4]. Let G be a simple graph with N nodes. The adjacency matrix A of G has entries $a_{ij} = 1$ if there is an edge connecting nodes i and j, and $a_{ij} = 0$ otherwise. Degree of node i is the number of its neighbors and its degree centrality is defined as $C_i^D = \frac{\sum_{j \neq i} a_{ij}}{N-1}$. Eigenvector centrality C_i^E of node i takes into account not only the number of neighbors but also their relative importance and is obtained from the formula $x_i = \frac{1}{\lambda} \sum_{j \neq i} a_{ij} x_j$, where λ is the largest eigenvalue of A and x_i is the $i-$th coordinate of the corresponding eigenvector x. Since x is determined up to a multiplicative constant, only relative eigenvector centrality scores can be obtained, e.g. $C_i^E = \frac{x_i}{\max_j (x_j)}$.

Laplacian centrality

Laplacian centrality has been proposed as a measure of node importance in [6] for weighted networks and in [5] for unweighted networks. Contrary to other measures that are based on the graph adjacency matrix A, it is defined in terms of the graph Laplacian L, where $L = D - A$ and D is a diagonal matrix with elements d_{ii} representing the degree of node i. Laplacian energy E_L of a graph is defined as: $E_L(G) = \sum_{i=1}^{n} \lambda_i^2$ where $\{\lambda_i\}$ is the spectrum of L. $E_L(G)$ is a global network property and a graph invariant [6]. Laplacian centrality of node i has been introduced as the relative difference between $E_L(G)$ of the original network and $E_L(G_i)$ of a modified network G_i, where node i together with all incident edges is removed: $C_i^L = \frac{E_L(G) - E_L(G_i)}{E_L(G)}$. Laplacian centrality is affected by the 2−neighborhood of node i, i.e. nodes that are reachable from i in a walk of distance 2.

2.2 Implementation

In this paper an unweighted undirected graph representation of a PTN is used. Nodes represent stops and edges represent operational connections between them. The representation used is the so-called P-space representation where each route is a clique, i.e. there is an edge between each pair of stops that lie on the same route (at least one) [10]. This is particularly well suited for passenger flow analysis as the direct connections within routes reflect the ability for passengers to travel without transfers. GTFS (General Transit Feed Specification) data were used for network construction. GTFS is a relational database with a set of tables containing information about the network operation in .txt files (e.g. stops.txt, routes.txt). Stops that serve as the same stop in the opposite directions of a route are merged and represented by a single node in the graph. C^D, C^E and C^L of the constructed graph are calculated for all nodes. Automatic fare collection (AFC) data are used to extract aggregate passenger boarding counts at each stop. The AFC dataset includes information on the time, location (stop) and route of boarding. We analyze the dependences between centrality scores and passenger counts. Since C^L has not yet been applied to PTN studies, we also examine its correlation to other centrality measures. The analysis was implemented in Python. Graph construction from data and node centrality calculations were performed using the NetworkX library.

3 RESULTS

The proposed analysis is demonstrated for the bus public transport network in Ljubljana (LPP). GTFS data are publicly accessible [1]. In the undirected P-space representation the Ljubljana network has 370 nodes and 8880 edges. The network is serviced by 43 routes. An AFC dataset for one representative workday (Wednesday, May 11 2022) was obtained from the network operator. The dataset contains records of about 115 000 boardings. LPP is an entry-only system, where passengers only tap the card upon entry and no exit information is available.

Centrality scores were calculated for all stops. This step has been partially studied in [7] where the degree centrality distribution (Figure 1 (a) in the present paper) was presented for the same representation of the LPP network as shown in Figure 4 of that paper. Other results in this paper have not been presented previously. Figure 1 shows distributions for different definitions of node importance. Note that there are a small number of nodes with a significantly higher centrality score in most variables, which also coincides with a large number of boarding passengers. Three nodes are identified as outliers in this respect and have passenger

[1] https://www.transit.land/operators/o-u24q-ljubljanskipotni%C5%A1kipromet

counts of about 5800, 8900 and 12000 while passenger counts for all other nodes fall into the interval between 1 and 2500. Such nodes are known as hubs in many real-world networks and have been observed in PTNs as well (e.g. [2]).

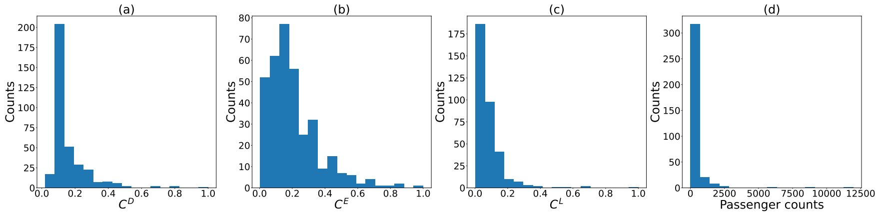

Figure 1: Node centrality distributions: (a) degree, (b) eigenvector and (c) Laplacian centrality. Centralities are scaled to the maximum value. (d) shows the distribution for the number of passenger boardings at each station.

It has been observed in real-world networks that different centralities are highly correlated, with the exact relationship depending on the network under observation ([9], [3]). Here we focus on the relationship between Laplacian centrality and other centrality scores. The results are shown in Figure 2. A positive and non-linear dependence is observed in all cases.

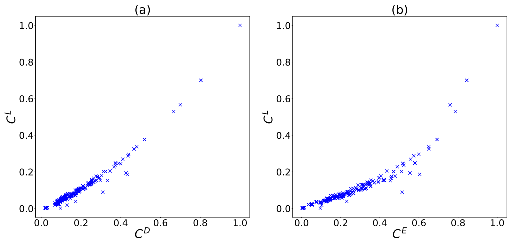

Figure 2: Laplacian centrality w.r.t other centrality measures: (a) degree and (b) eigenvector centrality. Centralities are scaled to the maximum value.

Next, correlations between centrality scores and passenger counts were calculated. The Pearson correlation coefficient was calculated together with the best fit from linear regression for a rough estimate of dependence. This was performed for two cases: first for the set including all nodes and second for a subset of nodes where the three hubs (outliers) were excluded. Results are shown in Figure 3. Significant correlations were observed, especially for degree and Laplacian centrality. It should be noted however, that dependences are nonlinear and the linear correlation coefficients are an approximation.

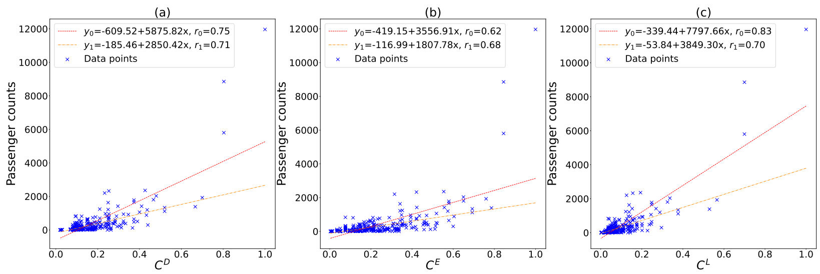

Figure 3: Passenger counts w.r.t. node centrality scores. Line equations for best linear regression fit are included in the legend together with the Pearson correlation coefficient. y_0 and r_0 are results for all nodes, while y_1 and r_1 show results with outliers removed. Results are shown for all centrality measures: (a) degree centrality, (b) eigenvector centrality and (c) Laplacian centrality. Centralities are scaled to the maximum value.

4 CONCLUSION

Node centrality measures were analyzed for a bus PTN. Laplacian centrality has not been used previously in PTN analysis. Its relation to the standard centrality measures was evaluated and positive correlations were observed. Node importance as estimated from node centrality analysis was compared to a measure of importance based on passenger boarding counts for each stop. Positive dependence was observed for all variables with degree and Laplacian centrality exhibiting the strongest correlation. Next, it will be interesting to perform a similar analysis on different graph representations of PTNs, including combinations of the underlying operational representations and their (un)weighted (un)directed variations. Generalization of results to other real-world PTNs will be interesting for comparative assessment.

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DISCRETIZATION-BASED SOLUTION APPROACHES FOR THE CIRCLE PACKING PROBLEM[1]

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Abstract: We consider the problem of packing a set of circles into the smallest containing circle. This problem can be cast as a nonconvex quadratically constrained program, and difficult to solve in general. We provide an iterative solution approach based on a bisection-type algorithm on the radius of the container. Our algorithm is based on discretizing the container into small cells, and solves restricted and relaxed versions of the problem as mixed-integer linear programs. We enhance our algorithm with solution space reduction, bound tightening and variable elimination techniques. We obtain promising results compared to global solvers and heuristic methods from literature.

Keywords: mixed-integer programming, global optimization, continuous location

1 INTRODUCTION

The circle packing problem (CPP) is concerned with packing a given number of different circles into a larger container, such as a square, a rectangle, or a circle, in such a way that circles do not overlap and each circle is entirely in the container. This well-known NP-Hard problem arises in automobile, textile, food, and chemical industries [1]. We primarily consider CPP with the objective of minimizing the radius of the surrounding circle in our paper although our approach can be applied to other versions from the literature. We denote the set of circles by $\mathcal{C}$, and the radius of circle $c \in \mathcal{C}$ by r_c. We assume, without loss of generality, that the center of the surrounding circle is located at the origin, and R is a decision variable denoting its radius. The center of circle c is represented by the decision variables (x_c, y_c). Then, CPP can be modelled as the following non-convex quadratically constrained program:

$$\min R \tag{1a}$$
$$\text{s.t. } (x_c - x_k)^2 + (y_c - y_k)^2 \geq (r_c + r_k)^2 \qquad c, k \in \mathcal{C} : \ c \neq k \tag{1b}$$
$$x_c^2 + y_c^2 \leq (R - r_c)^2 \qquad c \in \mathcal{C} \tag{1c}$$
$$x_c \in \mathbb{R},\ y_c \in \mathbb{R},\ R \geq r_c \qquad c \in \mathcal{C}. \tag{1d}$$

The vast majority of the CPP literature focuses on developing heuristic methods, see, e.g., [3, 2]. The use of systematic global optimization approaches to solve CPP is lacking in the literature. We believe the reason is that it is quite challenging to solve the non-linear formulation (1) with global optimization solvers directly, as confirmed by our preliminary experiments. In this paper, we propose an iterative solution approach based on a bisection-type algorithm considering the minimum radius objective. Our solution procedure relies on discretizing the container into smaller cells, and iteratively solves two mixed-integer linear programming (MILP) formulations designed for a restricted and a relaxed versions of the original problem. This allows us to utilize mature MILP solvers in order to certify lower and upper bounds for the optimal value of problem (1) efficiently. One of the closest works in the literature to ours is [4], which also use discretization but primarily work with LP relaxations enhanced with valid inequalities as opposed to our MILP-based approach. We also propose several enhancements to our algorithm to improve its performances. These enhancements include solution space reduction, bound tightening technique and variable elimination. These enhancements significantly improve the performance of our algorithm, helping us to consistently beat global solvers such as BARON and Gurobi, and heuristic methods from the literature.

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2 FORMULATIONS

In this section, we present the MILP formulations developed for CPP. The idea behind these formulations is to discretize the continuous solution space into small squares. Then, we construct restricted and relaxed versions of CPP, where the corner points of these squares are the guiding points within the MILP formulations. We represent each of these points via binary variables, whose number Θ is logarithmic in the size of the grid [6].

Restricted Version In this version, we define two vectors of binary decision variables corresponding to the candidate points as α_c and β_c for circle c. The binary column vector corresponds to the corner point of cell on x-axis (y-axis) in base 2 where the center of circle c is located is denoted by $\alpha_c = (\alpha_{c,1}, \alpha_{c,2}, \cdots, \alpha_{c,\Theta})$ ($\beta_c = (\beta_{c,1}, \beta_{c,2}, \cdots, \beta_{c,\Theta})$) for the circle $c \in \mathcal{C}$. We also define another decision variable $\psi_{i,j,c}$ which ensures that circle c is totally included by the corresponding circle by stating that the circle's center at (i, j) should be located at one of the admissible points from set $\mathcal{L}_c$. The corresponding formulation is given as follows:

$$\sum_{t=1}^{\Theta} 2^{(t-1)} \, (\alpha_{c,t} - \alpha_{k,t}) \geq \sum_{(u_1,u_2)\in\mathcal{N}_{c,k}} u_1 \, \pi_{u_1,u_2,c,k} \qquad c, k \in \mathcal{C} : c < k \tag{2a}$$

$$\sum_{v=1}^{\Theta} 2^{(v-1)} \, (\beta_{c,v} - \beta_{k,v}) \geq \sum_{(u_1,u_2)\in\mathcal{N}_{c,k}} u_2 \, \pi_{u_1,u_2,c,k} \qquad c, k \in \mathcal{C} : c < k \tag{2b}$$

$$\sum_{(u_1,u_2)\in\mathcal{N}_{c,k}} \pi_{u_1,u_2,c,k} = 1 \qquad c, k \in \mathcal{C} : c < k \tag{2c}$$

$$\left| \sum_{t=1}^{\Theta} 2^{(t-1)} \, \alpha_{c,t} \right| \leq \sum_{i\in\mathcal{I}:\exists j,(i,j)\in\mathcal{L}_c} i \, \psi_{i,j,c} \qquad c \in \mathcal{C} \tag{2d}$$

$$\left| \sum_{v=1}^{\Theta} 2^{(v-1)} \, \beta_{c,v} \right| \leq \sum_{j\in\mathcal{I}:\exists i,(i,j)\in\mathcal{L}_c} j \, \psi_{i,j,c} \qquad c \in \mathcal{C} \tag{2e}$$

$$\sum_{(i,j)\in\mathcal{L}_c} \psi_{i,j,c} = 1 \qquad c \in \mathcal{C} \tag{2f}$$

$$\alpha_c, \beta_c \in \{0,1\}^{\Theta} \qquad c \in \mathcal{C} \tag{2g}$$

$$\pi_{u_1,u_2,c,k} \in \{0,1\} \qquad c, k \in \mathcal{C}, \ (u_1, u_2) \in \mathcal{N}_{c,k} \tag{2h}$$

$$\psi_{i,j,c} \in \{0,1\} \qquad (i, j) \in \mathcal{L}_c, \ c \in \mathcal{C}. \tag{2i}$$

In this model, Constraints (2a)-(2c) satisfy the non-overlapping feasibility rule, and Constraints (2d)-(2f) ensure that each circle is fully contained by the surrounding circle. Here, $\mathcal{N}_{c,k}$ is the pair of minimum required number of cells on x and y axes between the centers of the circles c and k to avoid overlapping of these circles.

Relaxed Version Let decision variable $\gamma_c = (\gamma_{c,1}, \gamma_{c,2}, \cdots, \gamma_{c,\Theta})$ ($\omega_c = (\omega_{c,1}, \omega_{c,2}, \cdots, \omega_{c,\Theta})$) denote the x-axis (y-axis) coordinate of the left-lower corner point of the cell in base 2 where the center of circle c is contained. We also define another variable $\eta_{i,j,c}$ ensuring that circle c is fully contained by the surrounding circle since there is at least one point of the cell whose left-lower point (i, j) is one of the admissible points from set $\mathcal{S}_c$. The formulation is as follows:

$$\sum_{t=1}^{\Theta} 2^{(t-1)} (\gamma_{c,t} - \gamma_{k,t}) \geq \sum_{(u_1,u_2)\in\mathcal{O}_{c,k}} u_1 \, \Pi_{u_1,u_2,c,k} \qquad c, k \in \mathcal{C} : c < k \tag{3a}$$

$$\sum_{v=1}^{\Theta} 2^{(v-1)}(\omega_{c,v} - \omega_{k,v}) \geq \sum_{(u_1,u_2)\in\mathcal{O}_{c,k}} u_2\, \Pi_{u_1,u_2,c,k} \qquad c,k \in \mathcal{C} : c < k \tag{3b}$$

$$\sum_{(u_1,u_2)\in\mathcal{O}_{c,k}} \Pi_{u_1,u_2,c,k} = 1 \qquad c,k \in \mathcal{C} : c < k \tag{3c}$$

$$\left|\sum_{t=1}^{\Theta} 2^{(t-1)}\, \gamma_{c,t}\right| \leq \sum_{(i,j)\in\mathcal{S}_c} i\, \eta_{i,j,c} \qquad c \in \mathcal{C} \tag{3d}$$

$$\left|\sum_{v=1}^{\Theta} 2^{(v-1)}\, \omega_{c,v}\right| \leq \sum_{(i,j)\in\mathcal{S}_c} j\, \eta_{i,j,c} \qquad c \in \mathcal{C} \tag{3e}$$

$$\sum_{(i,j)\in\mathcal{S}_c} \eta_{i,j,c} = 1 \qquad c \in \mathcal{C} \tag{3f}$$

$$\gamma_c, \omega_c \in \{0,1\}^{\Theta} \qquad c \in \mathcal{C} \tag{3g}$$

$$\Pi_{u_1,u_2,c,k} \in \{0,1\} \qquad c,k \in \mathcal{C},\ (u_1,u_2) \in \mathcal{O}_{c,k} \tag{3h}$$

$$\eta_{i,j,c} \in \{0,1\} \qquad (i,j) \in \mathcal{S}_c,\ \ c \in \mathcal{C}. \tag{3i}$$

Similarly, in this model, Constraints (3a)-(3c) satisfy the non-overlapping feasibility condition partially while allowing some intersections. Constraints (3d)-(3f) ensure that there is at least one point of the corresponding cell for locating the circle's center such that it is fully contained by the surrounding circle. Here, $\mathcal{O}_{c,k}$ includes the pairs of the minimum required number of cells where the farthest distance of the cells between the centers of circles c and k are larger than the sum of their radii.

3 METHOD

Algorithm Algorithm 1 aims to progressively improve the upper and lower bounds for the radius of the surrounding circle, and terminates once they are close enough.

Algorithm 1

Require: $\mathcal{C}$; r_c, $\forall c \in \mathcal{C}$; U; L; δ; ε.
Ensure: (x_c, y_c), $\forall c \in \mathcal{C}$; U; L.
1: $R \leftarrow \frac{U+L}{2}$
2: Divide the container with diameter R into square cells with length size δ.
3: Solve the restriction model.
4: **If** the restriction model is feasible, **then** $U \leftarrow R$. Go to Step 7.
5: Solve the relaxation model for the container with diameter R.
6: **If** the relaxation model is feasible, **then** $\delta \leftarrow \frac{\delta}{2}$. Go to Step 2. **Else** $L \leftarrow R$. Go to Step 7.
7: **If** $U - L > \varepsilon U$, go to Step 1. **Else** STOP!

Enhancements Although Algorithm 1 performs better than global solvers applied to formulation (1) directly, we further enhance its performance via three methods. Firstly, we initialize the algorithm with better lower bounds that are derived via geometric arguments. For example, we have some subroutines that estimate a lower bound by relaxing non-overlapping constraint. Secondly, we reduce the solution space by removing symmetric solutions. For example, we can arbitrarily choose the quadrant of a few circles without eliminating any optimal solutions. Finally, we eliminate some binary variables whose values can be fixed due to the size and the shape of the container by considering the full containment restriction.

4 COMPUTATIONS

In this section, we present the results of our experiments demonstrating the efficacy of our approach. All MILPs are solved using Gurobi 9.1.

Comparison with the Global Solvers Baron and Gurobi When we test the performance of global solvers Baron and Gurobi on formulation (1) with the enhancements introduced above, we observe that they are only able to solve 8-circle instances from [5] within the time limit of 30 minutes to a relative optimality gap of 1%. This shows that the global solvers does not provide an efficient approach to solve CPP to global optimality. On the other hand, the improved version of Algorithm 1 is able to solve instances up to 10 circles in less than 10 minutes.

Performance Analysis of Algorithm 1 A critical parameter of Algorithm 1 is the upper bound used to initialize the overall method. We test our algorithm under two settings: i) initialization with the upper bound (UB) algorithm given in [2], ii) best-known values from the literature. We report the results in Table 1 up to 18 circles from [5]. Remarkably, we observe that we are able to solve these relatively large instances to 1% optimality within two hours. We also note that initialization with the UB algorithm does not deteriorate our algorithm.

Table 1: Effects of using best-known values within Algorithm 1

	Initialization with the UB Algorithm [2]				**Initialization with Best-known Values**			
Instance	Upper Bound	Pre-processing Time (m)	Total Sol. Time (m)	GAP (%)	Upper Bound	Pre-processing Time (m)	Total Sol. Time (m)	GAP (%)
$Zimm-12$	31.546	4.90	20.43	0.988	31.545	3.91	17.26	0.959
$Zimm-15$	42.457	9.94	63.56	0.851	42.457	8.51	61.30	0.838
$Zimm-18$	54.240	19.98	111.51	0.934	54.240	17.19	108.09	0.931

Comparison with an Algorithm Designed for Identical Circles In [3], a heuristic algorithm is developed to solve CPP instances with identical circles. We compare our algorithm against this tailored method in Table 2. Algorithm 1 is able to ensure 1% optimality gap in a reasonable time. However, the heuristic algorithm proposed in [3] in unable to provide provable lower bounds and is only slightly faster than our approach.

Table 2: Results obtained by Algorithm 1 and algorithm in [3]

	Algorithm 1			**Algorithm in [3]**	
No.	Upper Bound	Sol. Time (m)	GAP (%)	Upper Bound	Sol. Time (m)
$Eq-20$	5.122	4.17	0.64	5.122	4.23
$Eq-30$	6.198	9.43	0.62	6.198	5.72
$Eq-40$	7.123	14.42	0.89	7.124	10.59

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On t-rainbow domination number of generalized Petersen graphs $P(ck, k)$

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Abstract: Recently, 2-rainbow and 3-rainbow domination of generalized Petersen graphs $P(ck, k)$ has been well studied. Exact values for some subfamilies and general lower and upper bounds are known. Here we announce extended results to t-rainbow domination for all $t \in \mathbb{N}$. A question regarding the existence of 3-rainbow domination regular graphs is answered.

Keywords: rainbow domination, generalized Petersen graphs, rainbow domination regular graphs

1 Introduction

Graph domination is among very popular in graph theory [10, 11]. It is quite common that in practical applications some additional constraints or desires are taken into account. Inspired by several facility location problems, Brešar, Henning and Rall in 2005 [3, 4] initiated the study of the k-rainbow domination problem. The problem is proved to be NP-complete even if the input graph is a chordal graph or a bipartite graph (see Chang [5]). The problem has already attracted considerable attention, see for example [1] and the references there. Substantial interest has already been devoted to research of rainbow domination of special classes of graphs, for example the generalized Petersen graphs [16, 9]. Besides the Petersen graphs $P(n, k)$ with small k, the Petersen graphs $P(ck, k)$ received considerable attention. In [15], bounds for 3-rainbow domination of the families $P(ck, k)$ were provided, generalizing the results of [8] for $P(6k, k)$ that include exact values for some subfamilies and tight bounds for other subfamilies. For analogous results regarding 2-rainbow domination of $P(ck, k)$, see [7, 2]. Furthermore, note that 1-rainbow domination is equivalent to ordinary domination, which has been addressed in [17]. Hence, for $t = 1, 2, 3$ the t-rainbow domination of $P(ck, k)$ is fairly well studied in the sense that we have exact values for some families and good lower and upper bounds for all other cases.

Using the fact that generalized Petersen graphs are regular graphs, we use results of [14] to extend some known results for 2- and 3-rainbow domination to t-rainbow domination for $t > 3$.

This is an short communication containing a brief report on some ideas and announcement of new results. The details of these proofs will appear in a full paper published elsewhere.

2 Preliminaries

Generalized Petersen graphs. Generalized Petersen graphs were defined in [6]. For $n \geq 3$ and k, $1 \leq k \leq n-1$, the generalized Petersen graph $P(n, k)$, is a graph on $2n$ vertices with $V(P(n, k)) = \{v_i, u_i \mid 0 \leq i \leq n-1\}$ and edges $E(P(n, k)) = \{u_i u_{i+1}, u_i v_i, v_i v_{i+k} \mid 0 \leq i \leq n-1\}$. where all subscripts are taken modulo n. This standard notation was introduced by Watkins [18]. For convenience, throughout the paper, all subscripts will be taken modulo n.

It is well known that the generalized Petersen graphs $P(n, k)$ are 3-regular unless $k = \frac{n}{2}$ and that are higly symmetric. Petersen graphs $P(n, k)$ and $P(n, n-k)$ are isomorphic, so it is natural to restrict our attention only to $P(n, k)$ with $n \geq 3$ and k, $1 \leq k < \frac{n}{2}$. It is convenient to implicitly make use of another symmetry of Petersen graphs. The mapping which maps

$v_i \mapsto v_{i+1}$ and $u_i \mapsto u_{i+1}$ is well-known to be an automorphism, from which it follows that any rotation along the long cycle is an automorphism.

A graph is vertex transitive if, for each pair of vertices u and v, there is an automorphism that maps u to v. For later reference, recall that Petersen graphs $P(ck, k)$ are not vertex transitive. However, $P(5, 2)$ the original Petersen graph is vertex transitive [12] and so are some among generalized Petersen graphs $P(n, k)$. More precisely, $P(n, k)$ is vertex transitive iff $(n, k) = (10, 2)$ or $k^2 \equiv \pm 1 \bmod n$.

Rainbow domination and singleton rainbow domination. Starting with a given graph G and a positive integer t, the goal is to assign a subset of the set $\{1, 2, \ldots, t\}$ of *colors* to every vertex of G such that each vertex with an empty set assigned has all t colors in its neighborhood. Such an assignment of a graph G is called a *t-rainbow dominating function* (shortly, tRD function, $tRDF$) of the graph. The weight of assignment g, a tRD function of a graph G, equals the value $w(g) = \sum_{v \in V(G)} w(g(v))$, where $w(g(v))$ is the number of colors assigned to vertex v. We also say that G is *tRD-colored* (or simply, colored) by g. A vertex is said to be tRD-dominated if either: (1) it is assigned a nonempty set of colors, or, (2) it has all colors in its neighborhood. If $g(v) \neq \emptyset$, a vertex v is said to be *colored*, and is not colored or *uncolored* otherwise. The minimum weight over all tRD functions of G is called the *t-rainbow domination number* $\gamma_{rt}(G)$. A special case when vertices are colored by sets with at most one color is of particular interest. Such functions are called *singleton tRD functions* ($StRD$ functions, $StRDF$) and the minimal weight obtained when considering only $StRD$ functions is called the *singleton t-rainbow domination number*, and is denoted by $\tilde{\gamma}_{rt}$ (see [8]).

Directly from definitions we have, for any graph G and any t,

$$\gamma_{rt}(G) \leq \tilde{\gamma}_{rt}(G). \tag{1}$$

As we are mainly going to work with singleton RDF, we introduce a shorter notation. For a S3RDF f, we write $f(v) = 0$ if v is assigned the empty set, and $f(v) = i$, $i = 1, 2, 3$, means that v is colored by the color set $\{i\}$.

For later reference, let us recall the general bounds,

$$n \leq \gamma_{rt}(P(n, k)) \leq 2n\,. \tag{2}$$

3 Summary of Results

Rainbow domination of $P(ck, k)$ for $r = 1, 2$ and 3 has been studied recently [17, 2, 15]. Here we recall some old and summarize the main new results. To avoid trivialities, we assume $k > 3$. Cases $k = 1, 2, 3$ are left to be treated separately. For example, note that for $k = 1$, $P(ck, k) = P(c, 1)$ is isomorphic to the Cartesian product $C_k \Box K_2$.

First, let us recall the characterization of graphs $P(ck, k)$ for which 3-rainbow domination number is known and equals the general lower bound. This is part of statement of Theorem 3 in [15]

Theorem 3.1 *[15]. Let $k > 3$. Then if $c \equiv 0 \mod 6$, and $k \equiv 1, 5 \mod 6$, $\gamma_{r3}(P(ck, k)) = \tilde{\gamma}_{r3}(P(ck, k)) = ck$.*

In the same paper, lower and upper bounds are provided in Theorems 4 and 5. Here we summarize the two theorems in one statement (omitting some details):

Theorem 3.2 *[15] Let $k > 3$. Then $ck \leq \gamma_{r3}(P(ck, k)) \leq (c + 1)(k + 1) - 3$.*

Regarding the 2-rainbow domination, the next theorem is proved in [2].

Theorem 3.3 *For $c \geq 3$ it holds $\frac{4}{5}ck \leq \gamma_{r2}(P(ck, k)) \leq \frac{4}{5}(c + 1)(k + 1) + 1$.*

This is a short and in some cases not best possible bound, as it summarizes a detailed list of upper bounds given in Theorem 4 in [2]. Let us only recall the case in which exact value is known.

Theorem 3.4 *[2] Let $k > 3$. Then if $c \equiv 0 \mod 5$, and $k \equiv 2, 8 \mod 10$, $\gamma_{r2}(P(ck,k)) = \tilde{\gamma}_{r2}(P(ck,k)) = \frac{4}{5}ck$.*

As 1-rainbow domination is just the ordinary domination, γ, we recall from [13] that

Theorem 3.5 *[13] If $n = ck \equiv 0 \mod 4$, and $k > 3$ odd, $\gamma_{r1}(P(ck,k)) = \gamma(P(ck,k)) = \frac{1}{2}ck$.*

In general case, bounds can be extracted from [17] and summarized as follows.

Theorem 3.6 *[17] $\frac{1}{2}ck \leq \gamma_{r1}(P(ck,k)) \leq \frac{1}{2}(c+1)k + 1$.*

Rainbow domination regular graphs are defined in [14]. More precisely, a graph G is t-rainbow domination regular (t-RDR graph) if $\gamma_{rt}(G) = \frac{1}{2}|V(G)|$. Two questions are asked:

- **Question 1:** Are there any t-RDR graphs that are not obtained as Cayley graphs over some abelian group?
- **Question 2:** More generaly, are there any t-RDR graphs that are not vertex transitive?

The following facts regarding the rainbow domination of regular graphs from [14] will be used later (Theorem 1.2 and Proposition 3.4).

Theorem 3.7 *[14] Let G be a d-regular graph. Then, for $\ell < 2d$*

$$\gamma_{r\ell}(G) \geq \left\lceil \frac{\ell|V(G)|}{2d} \gamma_{r\ell}(G) \right\rceil \tag{3}$$

and $\gamma_{r\ell}(G) = V(G)$ for $\ell \geq 2d$.

Proposition 3.8 *[14] Let G be a d-regular graph and $d \leq \ell < 2d$. If $\gamma_{r\ell}(G) = \frac{\ell|V(G)|}{2d}$, then $\gamma_{r(\ell+1)}(G) = \frac{\ell+1}{\ell}\gamma_{r\ell}(G) = \frac{(\ell+1)|V(G)|}{2d}$.*

In the full paper, the following five theorems will be proved.

Theorem 3.9 *Let $k > 3$. Then $\frac{4}{3}ck \leq \gamma_{r4}(P(ck,k)) \leq \frac{4}{3}(c+1)(k+1) - 4$.*

Theorem 3.10 *Let $k > 3$. Then $\frac{5}{3}ck \leq \gamma_{r5}(P(ck,k)) \leq \frac{5}{3}(c+1)(k+1) - 5$.*

Theorem 3.11 *If $c \equiv 0 \mod 6$, $k > 3$, and $k \equiv 1, 5 \mod 6$, then $\gamma_{r4}(P(ck,k)) = \frac{4}{3}ck$.*

Theorem 3.12 *If $c \equiv 0 \mod 6$, $k > 3$, and $k \equiv 1, 5 \mod 6$, then $\gamma_{r5}(P(ck,k)) = \frac{5}{3}ck$.*

Theorem 3.13 *A generalized Petersen graph $P(n,k)$ is 3-RDR graph if and only if $n \equiv 0 \mod 6$, k odd and $2k \not\equiv 0 \mod 6$.*

Recall that it is well known that Cayley graphs are vertex transitive. As $P(ck,k)$ are not vertex transitive, hence are also not Cayley graphs, Theorem 3.13 answers both questions of [14].

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Session 5:

Mathematical Programming and Optimization

IMPACT OF THE UNITARIZATION TECHNIQUE ON FINAL RANKINGS BASED ON THE GOAL PROGRAMMING

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Abstract: The goal programming (GP) is a well-known method applied to the continuous and discrete version of multi-criteria optimization. Within this approach the best solution minimizes the weighted sum of deviations between particular criteria performances and their targets. GP can be used in the case of maximized, minimized and neutral criteria. If they are not comparable, it is recommended to unify their values on the basis of the unitarization (standarization or normalization) technique. In the paper the case of neutral objectives with precise and interval tarrgets is mainly investigated. We focus on rankings, which means that only the discrete version of GP is explored. The aim of the research is to examine advantages and limitations of two unitarization procedures. The first one temporarily treats neutral criteria as maximants (or minimants) while the second one treats them as neutral ones during the whole unitarization process. They both seem to be mathematically correct, but a deeper analysis allows us to understand the strengths and weaknesses of each method.

Keywords: goal programming, unitarization, neutral criteria, rankings, targets, estates

1 INTRODUCTION

The goal programming (GP) is one of the methods applied to multi-criteria decision making under certainty (M-DMC). Within this decision situation the decision maker (DM) assesses diverse alternatives (options, decision variants) with the use of more than one criterion and all the parameters of the problem are deterministic.

The original version of GP was formulated by Charnes et al. [1]. Extensions of this approach are presented for example in [2]. The approach may have different forms (e.g. weighted goal programming, lexicographic goal programming, Chebyshew goal programming), but in this research the emphasis is on the weighted GP. In connection with the fact that the paper focuses on rankings, we discuss here only the discrete version of GP.

The essence of the weighted GP is to find the best solution by minimizing the weighted sum of deviations between particular criteria performances and their targets. Weights represent the significance of each objective.

GP can be easily used not only in the case of maximized and minimized, but also for neutral criteria which "consist in reaching a specific value" [3]. Such criteria occur in real decision problems very often. They may concern, for instance, the period of paying off the credit, the duration of the project, the temperature level, the number of rooms in a house, the distance between two places or the level of precipitation [4].

If criteria considered by the DM are not comparable (due to diverse units and scales), it is recommended to unify their values on the basis of the unitarization (standardization or normalization) technique. In this research the first approach is explored. The goal of the unitarization approach is to transform original numbers into values belonging to the interval [0,1]. In the case of maximized and minimized objectives the transformation is quite simple. However, for neutral criteria the choice of a proper equation might be more complicated.

Therefore, the goal of this paper is to describe possible unitarization formulas and identify their characteristics connected with rankings.

The article is organized as follows. Section 2 presents two versions of the weighted GP. Each version applies a different unitarization technique for neutral criteria. Section 3 analyses the features of both approaches. Section 4 describes an example concerning the use of the weighed GP for creating rankings of Poznań estates. Conclusions are gathered in Section 5.

2 WEIGHTED GOAL PROGRAMMING (WGP) – TWO VERSIONS

Generally, the weighted goal programming consists of the following steps:

1. Assign a target value and a weight to each criterion.
2. Compute the deviations from the goals both above and below the target.
3. Minimize the weighted sum of unwanted positive and negative deviations.

Nevertheless, if the problems include varied criteria, the WGP becomes more complex.

The first version referring to unitarization (*weighted GP with two optimization directions – WGP2*) may be described as follows:

1. Assign a target value t_k and a weight w_k to each criterion
2. Make an assumption about the optimization direction of each criterion (max or min)
3. Use a suitable unitarization equation for original data and the target value (Eq.1-2).

$$\boldsymbol{a(max)_{k,j}} = \frac{a_{k,j} - \min_j a_{k,j}}{\max_j a_{k,j} - \min_j a_{k,j}} \tag{1}$$

$$\boldsymbol{a(min)_{k,j}} = \frac{\max_j a_{k,j} - a_{k,j}}{\max_j a_{k,j} - \min_j a_{k,j}} \tag{2}$$

where $a(max)_{k,j}$ is the transformed value for a maximized criterion C_k and alternative A_j; $a_{k,j}$ denotes the original value of criterion C_k for alternative A_j.

4. Compute the deviations $d_{k,j}$ between the unitarized values and targets (Eq. 3).

$$\boldsymbol{d_{k,j}} = \begin{cases} \boldsymbol{0}, & \boldsymbol{if\ a(u)_{k,j} \in \left[t(u)_k^{min}, t(u)_k^{max}\right]} \\ \boldsymbol{a(u)_{k,j} - t(u)_k^{max}}, & \boldsymbol{if\ a(u)_{k,j} > t(u)_k^{max}} \\ \boldsymbol{t(u)_k^{min} - a(u)_{k,j}}, & \boldsymbol{if\ a(u)_{k,j} < t(u)_k^{min}} \end{cases} \tag{3}$$

where $a(u)_{k,j}$ – transformed value for criterion C_k and alternative A_j; $t(u)^{max}{}_k$ – transformed value of the right endpoint of the target interval for criterion C_k.

5. Calculate the weighted sum of unwanted positive and negative deviations (Eq.4).

$$\boldsymbol{GP(u) = \sum_k \left(w_k \cdot d_{k,j}\right)} \tag{4}$$

6. Create the non-decreasing ranking.

As we can observe, the procedure described above assumes that even if a given criterion is neutral, it is temporarily treated as a maximant or minimant! Equation (3) is applied to interval targets. Hence, if the DM declares precise targets, the formula needs to be simplified.

Now, let us make an assumption that each neutral objective must be treated as a neutral one during the whole decision making process. That entails the necessity to use a special unitarization equation for such objectives. If so, the second version (*weighted GP with three optimization directions – WGP3*) may contain the following steps:

1. Assign a target value t_k and a weight w_k to each criterion
2. Assign the real optimization direction of each criterion (max/min/neutral)
3. Use a suitable unitarization equation for original data (Eq.5-7). Targets get always 1.

$$\boldsymbol{a(max)_{k,j}} = \frac{a_{k,j} - \min_j a_{k,j}}{\max_j a_{k,j} - \min_j a_{k,j}} \tag{5}$$

$$\boldsymbol{a(min)_{k,j}} = \frac{\max_j a_{k,j} - a_{k,j}}{\max_j a_{k,j} - \min_j a_{k,j}} \tag{6}$$

$$\boldsymbol{a(neut)_{k,j}} = \begin{cases} 1 & \text{if } a_{k,j} \in \left[t_k^{min}, t_k^{max}\right] \\ \frac{\max_j a_{k,j} - a_{k,j}}{\max_j a_{k,j} - \min_j a_{k,j}}, & \text{if } a_{k,j} > t_k^{max} \\ \frac{a_{k,j} - \min_j a_{k,j}}{\max_j a_{k,j} - \min_j a_{k,j}}, & \text{if } a_{k,j} < t_k^{min} \end{cases} \tag{7}$$

where t^{max}_k and t^{min}_k denote the enpoints of the target interval.

4. Compute the deviations $d_{k,j}$ between unitarized values and targets (Eq. 8).

$$\boldsymbol{d_{k,j}} = \left|\boldsymbol{1 - a(u)_{k,j}}\right| \tag{8}$$

5. Calculate the weighted sum of unwanted positive and negative deviations (Eq.9).

$$\boldsymbol{GP(u)} = \sum_k \left(\boldsymbol{w_k \cdot d_{k,j}}\right) \tag{9}$$

6. Create the non-decreasing ranking.

3 FEATURES OF BOTH APPROACHES (WGP2 AND WGP3)

If the first version is used, we note that a given neutral criterion can be treated as a maximant or minimant. The choice of the optimization direction influences the value of the unitarized target, but not the final deviation (Fig.1)! If the minimal value equals 0, the maximum – 10, and the target – 6.5, the unitarized target will equal 0.35 if the criterion is treated as a minimant, and – 0.65 if this objective is treated as a maximant. But in both cases the unitarized deviation of value 2.5 from 6.5 is equal to 0.4. Hence, the neutral objective may be temporarily assigned to any extreme directions (max or min) – our choice will not affect the final ranking. On the other hand, we can observe that the choice of the approaches *WGP2* or *WGP3* has a significant impact on the final deviation level (Fig.2). In the case of *WGP2* (we still assume that min=0 and max=10) values 4.5 and 8.5 are equally distant from the target (6.5) and after applying *WGP2* the final unitarized deviations are also identical (0.2=0,2). Such a phenomenon occurs very seldom in the case of *WGP3*. If min=0, max=8 and the target value equals 5, then for values 3.5 and 6.5 which are equally distant from 5, the final unitarized deviations are different (0.3 and 0.5, respectively). If the choice of the method (*WGP2* or *WGP3*) affects the transformed deviations, it means that it also influences final rankings which depend on deviation levels.

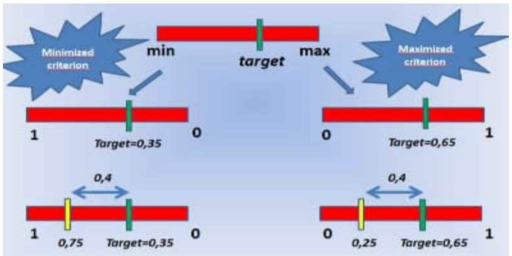

Figure 1: WGP2–unitarized targets and final dev.

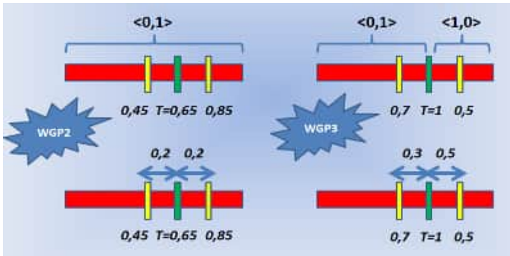

Figure 2: WGP2 and *WGP3* – comparison of final dev.

4 ILLUSTRATIVE EXAMPLE

In order to better illustrate the observed phenomenon and its consequences, rankings of Poznań estates were generated. Poznań is a city located in the western part of Poland. It contains 42 estates. The research included 15 criteria (e.g. number of bus stops, number of kindergartens, distance from the city centre, occurrence of wild animals, surface of green areas and housing prices). Within the survey research (conducted in among young Polish people in 2022), respondents were asked to declare the criteria hierarchy (this information was crucial to estimate weights) and the desired criteria levels given as interval targets. Housing prices turned out to be the most essential and the accessibility to churches was the least important. Figures 3-4 show only the beginning and the end of quite long estate rankings created on the basis of *WGP2* and *WGP3*. Significant discrepancies are visible, but the biggest difference is related to the estate "Grunwald Północ". It obtains the 40th rank in the first ranking (*WGP2*) and the 24th rank when applying *WGP3*.

	RANKING – I APPROACH	RANKING – II APPROACH
1	CHARTOWO	WILDA
2	OGRODY	OGRODY
3	OSTRÓW TUMSKI-ŚRÓDKA-ZAWADY-KOMANDORIA	CHARTOWO
4	GRUNWALD POŁUDNIE	OSTRÓW TUMSKI-ŚRÓDKA-ZAWADY-KOMANDORIA
5	WILDA	GRUNWALD POŁUDNIE
6	SOŁACZ	ŚW. ŁAZARZ
7	WARSZAWSKIE-POMET-MALTAŃSKIE	ŻEGRZE
8	ŻEGRZE	PIĄTKOWO
9	ZIELONY DĘBIEC	RATAJE
10	ŚW. ŁAZARZ	WINIARY

Figure 3: WGP2/3 rankings (ranks 1-10).

33	SZCZEPANKOWO-SPŁAWIE-KRZESINKI	ŁAWICA
34	FABIANOWO-KOTOWO	STARE MIASTO
35	ŁAWICA	ANTONINEK-ZIELINIEC-KOBYLEPOLE
36	MORASKO-RADOJEWO	SZCZEPANKOWO-SPŁAWIE-KRZESINKI
37	KIEKRZ	KRZYŻOWNIKI-SMOCHOWICE
38	NOWE WINOGRADY WSCHÓD	KRZESINY-POKRZYWNO-GARASZEWO
39	KRZYŻOWNIKI-SMOCHOWICE	MORASKO-RADOJEWO
40	GRUNWALD PÓŁNOC	KIEKRZ
41	GŁUSZYNA	GŁUSZYNA
42	STARE MIASTO	FABIANOWO-KOTOWO

Figure 4: WGP2/3 rankings (ranks 33-42).

5 CONCLUSIONS

The goal of this research was to explore two possible unitarization techniques applied to the weighted goal programming with neutral criteria. We noticed that in the case of *WGP2*, if absolute deviations are equal, relative deviations are also equal – proportions are always maintained. For *WGP3*, if the absolute deviations are equal, there is no guarantee that relative deviations will be also equal. There are equal only if the target is in the middle of the range [mix,max]. This time proportions are rarely maintained. Therefore, the choice of the approach should depend on the DM's preferences. If he/she tends to diversify the relative deviations even if absolute deviations are equal, the *WGP3* procedure ought to be implemented.

We admit that the unitarization analysis performed in this paper and the description of the construction of estate rankings are quite short. We are also aware of the fact that the unitarization for neutral criteria can be conducted in other ways (not discussed in the paper) [5]. That is why, we hope that a thorough presentation of the whole topic in a longer article will be possible in the future.

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A GENERAL APPROACH TO HANDLE COMPLEX SENSITIVITY ANALYSIS IN LINEAR PROGRAMMING

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Abstract: The sensitivity analysis in linear programming is a well-known standard, technique to deal with variations of selected entries. Its limitation is that is focuses only on sensitivity of one coefficient or other simple cases. The real life is, however, more complicated. To handle a bit more complex data variations, various approaches were introduced and studied. Herein, we address variations of possibly all input data, controlled by a certain matrix norm. More concretely, the aim is to compute the maximum variation of the data in the norm such that the computed optimal basis remains optimal. First, we present results valid for a general matrix norm. Then we inspect particular norms, such as the spectral and the maximum norm. We also analyse computational complexity to know for which norms the problem is efficiently computable and for which it is NP-hard.

Keywords: linear programming, sensitivity analysis, tolerance analysis, matrix norm, NP-hardness.

1 INTRODUCTION

Sensitivity analysis. The traditional sensitivity analysis in linear or nonlinear programming [3, 4] studies the effect of variations of one coefficient on the optimal value and the optimal solution. Nowadays, it is a standard and widely used technique and many extensions exist. Among them, the tolerance approach to sensitivity analysis [1, 8, 10] was introduced to handle independent and simultaneous variations of certain coefficients. Nevertheless, it is still not clear how to effectively treat variations of possibly all input coefficients and how to perform it by using certain measures (in the data space).

Vector and matrix norms. The vector p-norms are defined for every $p \geq 1$ and $x \in \mathbb{R}^n$ as $\|x\|_p \coloneqq \left(\sum_{i=1}^n |x|_i^p\right)^{1/p}$. In particular, the following three norms are often used: the Manhattan norm (for $p = 1$), the Euclidean norm (for $p = 2$) and the maximum norm $\|x\|_\infty = \max_i |x|_i$ (the limit case $p = \infty$).

For two arbitrary vector norms $\|x\|_\alpha$, $\|x\|_\beta$, the subordinate matrix norm [7] is defined as

$$\|A\|_{\alpha,\beta} \coloneqq \max_{\|x\|_\alpha=1} \|Ax\|_\beta.$$

If we use it for the vector 1-norm and the maximum norm, we obtain the matrix max-norm

$$\|A\|_{\max} \coloneqq \|A\|_{1,\infty} = \max_{i,j} |a_{ij}|.$$

In the opposite order, the resulting subordinate matrix norm has the form

$$\|A\|_{\infty,1} \coloneqq \max_{\|x\|_\infty=1} \|Ax\|_1 = \max_{y,z\in\{\pm1\}^n} y^T Az.$$

In contrast to many other basic subordinate norms, this one is NP-hard to compute [2].

Provided $\alpha = \beta$, the subordinate matrix norm reduces to the standard induced matrix norm. In particular, the spectral norm of A is induced by the Euclidean norm is equal to the maximum singular value, that is $\|A\|_2 = \sigma_{\max}(A)$. The induced 1-norm and ∞-norm have simple explicit expressions $\|A\|_1 = \max_j \sum_i |a_{ij}|$ and $\|A\|_\infty = \|A^T\|_1$. The Frobenius norm $\|A\|_F := \sqrt{\sum_{i,j} a_{ij}^2}$ is also a common matrix norm, however, it is not an induced norm.

Regularity radius. Let $A \in \mathbb{R}^{n\times n}$ be a nonsingular matrix and let $\|\cdot\|_*$ be an arbitrary matrix norm. *The regularity radius* is defined as the distance to the nearest singular matrix and we denote it

$$\mathrm{r}(A) := \min\{\|A - B\|_*;\ B \text{ is singular}\}.$$

For the spectral norm or the Frobenius norm (or other orthogonally invariant matrix norms), the regularity radius is equal to the smallest singular value, that is, $\mathrm{r}(A) = \sigma_{\min}(A)$. More generally, by the Gastinel–Kahan theorem [7], for any subordinate matrix norm $\|\cdot\|_{\alpha,\beta}$, the corresponding regularity radius can be expressed by the formula $\mathrm{r}(A) = \|A^{-1}\|_{\beta,\alpha}^{-1}$.

Particularly for the max-norm, the regularity radius admits an explicit formula [2, 9]

$$\mathrm{r}_{\max}(A) = \|A^{-1}\|_{\infty,1}^{-1} = \frac{1}{\max_{y,z\in\{\pm1\}^n} y^T A^{-1} z}.$$

The formula utilizes exponentially many vectors, but can hardly be avoided because for the max-norm the regularity radius $\mathrm{r}(A)$ is NP-hard. This is the case even on the class of nonnegative symmetric positive definite matrices [9]. Therefore, computationally cheap lower or upper bounds can be useful. Approximations and special matrices were discussed in [6]. In particular, semidefinite programming admits for bounds with one of the best known approximation ratios [5].

Formulation of the problem. Consider the LP problem in the form

$$\min\ c^T x \ \text{ subject to } \ Ax = b,\ x \geq 0, \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$. Let B an optimal basis and $N := \{1,\dots,n\} \setminus B$ the nonbasic indices. We denote by A_B the restriction of A to the basic columns and similarly for vectors and nonbasic indices.

Recall that optimality of basis B is expressed by the following three conditions:

$$\text{(regularity)} \quad A_B \text{ is nonsingular}, \tag{2a}$$

$$\text{(feasibility)} \quad A_B^{-1} b \geq 0, \tag{2b}$$

$$\text{(optimality)} \quad c_N^T - c_B^T A_B^{-1} A_N \geq 0^T. \tag{2c}$$

We can restate the second condition as feasibility of the linear system

$$A_B x_B = b, \quad x_B \geq 0,$$

and the third condition as feasibility of the linear system

$$A_N^T y \leq c_N,\ A_B^T y = c_B.$$

The goal. Given a matrix norm and the input (A, b, c), the problem is to find the nearest data (A', b', c') for which B is not an optimal basis. This *radius of basis stability* is formally defined as

$$\delta^* := \inf\big\{\|(A', b', c') - (A, b, c)\|;\ B \text{ is not an optimal basis for } (A', b', c')\big\}.$$

We write infimum here since the nearest data, for which B is not an optimal basis, may or may not be attained.

2 GENERAL APPROACH

In this section, we present a general approach to computing the radius of basis stability δ^* for an arbitrary matrix norm. To this end, we may write

$$\delta^* = \min\{\delta_1, \delta_2, \delta_3\},$$

where δ_1, δ_2 and δ_3 are the distances to the nearest data violating conditions (2a), (2b) and (2c), respectively. Thus, we will deal with the particular conditions and the corresponding radii of stability separately.

Regularity of B. In this case, we simply have $\delta_1 = \mathrm{r}(A_B)$, where $\mathrm{r}(A_B)$ is the regularity radius of matrix A_B.

Feasibility of B. Let $i \in \{1, \ldots, m\}$ and we focus on the sub-problem of computing δ_2^i, the distance to the nearest data (A', b', c') with $((A'_B)^{-1}b')_i = 0$; we can implicitly assume that $\delta_2^i \leq \delta_1$, so that nonsingularity of A'_B is guaranteed. Then we have

$$\delta_2 = \min_{i=1,\ldots,m} \{\delta_2^i\}.$$

It turns out that the value δ_2^i can again be expressed by means of the regularity radius.

Theorem 1 *We have $\delta_2^i = \mathrm{r}(A_{B,i} \,|\, b)$, where $(A_{B,i} \,|\, b)$ is constructed from A_B by replacing the ith column by b.*

Optimality of B. Regarding the value δ_3, we may proceed similarly as for δ_2. Restricting to the perturbations (A', b', c') no far than δ_1 from (A, b, c), the matrix A'_B is nonsingular. Then we just have to check if the solution $y' := A'^{-T}_B c'_B$ of $A'^T_B y = c'_B$ satisfies the other inequalities $A_N^T y \leq c_N$. Thus we can write

$$\delta_3 = \min_{j=1,\ldots,n-m} \{\delta_3^j\},$$

where δ_3^j is the norm of the minimum perturbation such that $(A'^T_N y')_j = (c'_N)_j$.

Theorem 2 *Define*

$$A_j^c := \begin{pmatrix} A_B^T & c_B \\ (A_N^T)_{j*} & (c_N)_j \end{pmatrix},$$

where $(A_N^T)_{j*}$ denotes the jth row of A_N^T. Then

$$\delta_3 = \min_{j=1,\ldots,n-m} \{\mathrm{r}(A_j^c)\}.$$

Summary. In total, the radius of basis stability δ^* can be expressed as

$$\delta^* = \min\bigl\{\mathrm{r}(A_B), \mathrm{r}(A_{B,1} \,|\, b), \ldots, \mathrm{r}(A_{B,m} \,|\, b), \mathrm{r}(A_1^c), \ldots, \mathrm{r}(A_{n-m}^c)\bigr\}.$$

This means that the problem is reduces to the computation of the regularity radius of $n+1$ matrices of size $m \times m$ or $(m+1) \times (m+1)$.

3 CONCLUSION

We presented a novel approach to deal with sensitivity analysis. Its main benefit is that it is able to handle simultaneous and independent variations of the input data, with respect to a given matrix norm. It turned out that the computation of the radius of (basis) stability can be reduced to a linear number of regularity radii of special matrices.

Of course, there is no free lunch. For some norms, the computation of the maximal allowable tolerances is a hard problem. On the other hand, when the regularity radius is efficiently computable (e.g., for the spectral or Frobenius norm), then the radius of stability is efficiently computable, too.

For the hard cases, we would like to propose some efficiently computable lower and upper bounds. In the future, we would also like to extend the results to some structured variations of the data.

4 Acknowledgements

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COMPUTATIONAL GEOMETRY AND MATHEMATICAL PROGRAMMING: COMBINED TECHNIQUES AND SELECTED APPLICATIONS IN LOGISTICS

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Abstract: We report on two applications in logistics where we have successfully combined methods from computational geometry and operations research: (1) Packing irregular objects and (2) flying drones through obstacles with moving launching and pickup trucks. In both cases we have developed mathematical models that can be solved with commercial software packages because of the integration of computational geometry and operations research techniques. The two applications are briefly described by illustrative examples and key geometric modeling ideas are discussed. It turns out that both fields complement each other neatly.

Keywords: computational geometry, mathematical modeling, cutting and packing, irregular objects, drones, trajectory planning, truck–drone delivery, piggybacking

1 INTRODUCTION: TWO SAMPLE APPLICATIONS

In the recent past we have worked on problems where knowledge from two fields are required: computational geometry and mathematical programming. In this contribution, we mention two applications. The first one is a cutting and packing problem. Cutting and packing has been studied in the operations research literature for decades and many variants of the problem exist (see e.g. [2] and [7]). We are interested in cutting and packing two–dimensional, irregular shapes that can be decomposed into convex polygons. See [6] for a recent literature review on cutting and packing with irregular shapes.

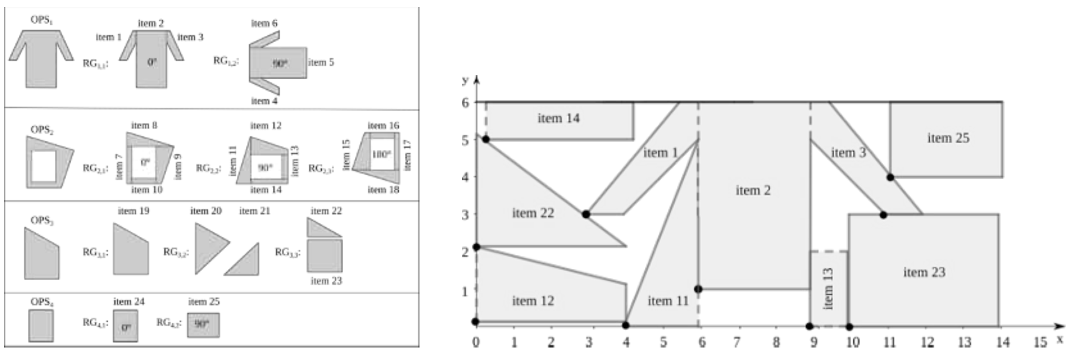

Figure 1: A Cutting and Packing Example with Four Irregular Shapes

Figure 1 illustrates the problem: In this example we assume that four output polygon shapes (OPS) are to be cut from a large object. Each OPS can be rotated (by certain degrees) and partitioned into smaller (convex) items to form a so–called rotation group (RG). For each OPS all the small items from one of the rotation groups must be placed onto the large object in such a way that they do not overlap (and some objective function is optimized). For details we refer to [4].

The second application is a path planning problem variant. Path planning has been discussed intensively especially in robotics, and became even more popular with the advent of drones. Since not only the path but also the timing is important in our context, we prefer the term trajectory planning. An overview of path planning approaches can be found in [3]. Our concern is a trajectory planning problem where a single drone can be transported by some vehicle (e.g. a truck) into a designated area. From this vehicle the drone can be launched and we need to decide when to do this. From this starting point, we have to solve a trajectory planning problem. The drone must fly to a customer avoiding obstacles on its way. After that, the drone must proceed on it's journey to meet up with a second vehicle that picks the drone up.

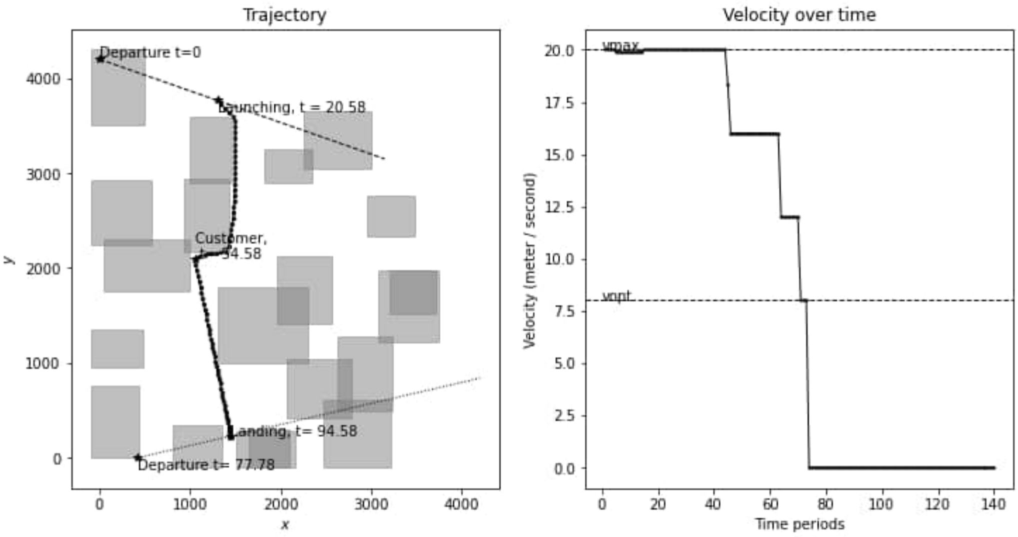

Figure 2: A Trajectory Planning Example with Piggybacking Vehicles

Figure 2 illustrates the problem: A first truck starts at time t=0 and carries a drone. We decide that at time t=20.58 the drone is launched from that truck and flies to a customer without hitting any obstacles. Then the drone moves on to a second truck to be picked up at time t=94.58. In addition to deciding upon when to launch and when to pick up the drone as well as upon which path to take, we decide upon the velocity of the drone that may vary during the course of the flight. The energy consumption of the drone, by the way, is a non–monotone convex function of the velocity, and the drone's battery provides a limited amount of energy. More details can be found in [5].

Both problems clearly have a geometric nature. Therefore, we use computational geometry in combination with mathematical programming to develop models that can be solved with commercial software. A selected modeling aspect is presented in the subsequent section.

2 A SELECTED TOPIC: OBSTACLE AVOIDANCE

A common aspect in both problems is that certain objects must not overlap. In the trajectory planning problem the drone must avoid the obstacles. In the cutting and packing problem items must not overlap. An overview over common approaches to avoid overlapping can be found in [1].

Let's start with the trajectory planning problem: If we assume that obstacles are represented by rectangles the sides of which are parallel to the coordinate axes, things are relatively simple. Let (x_i^l, y_i^d) and (x_i^r, y_i^u), respectively, be the coordinates of the lower left corner and the upper right corner, respectively, of an obstacle i and (X_t, Y_t) be the coordinates of the drone at time t. The drone would be on the right side of the obstacle at time t, if $x_i^r \leq X_t$ holds at time t. Likewise we could check all four directions. At least one of these conditions must hold to make sure that the drone does not collide with the obstacle at time t. Now we can define a binary variable b_{it}^r to indicate if the drone is right of the obstacle i at time t: $b_{it}^r = 0$ ($b_{it}^r = 1$ indicates that the drone is left of the right border of the obstacle). Variables b_{it}^l, b_{it}^u, and b_{it}^d can be defined analogously to cover all four directions. Putting all together, we can model avoiding obstacle i at time t like so (S_x and S_y are the sizes of the considered area along the x– and the y–axis):

$$x_i^r - S_x b_{it}^r \leq X_t \leq x_{it}^l + S_x b_{it}^l$$
$$y_i^u - S_y b_{it}^u \leq Y_t \leq y_{it}^d + S_y b_{it}^d$$
$$b_{it}^r + b_{it}^l + b_{it}^u + b_{it}^d \leq 3$$

When we have convex polygons other than rectangles, things are a bit more sophisticated. To check if a point (X_t, Y_t) is left or right of an edge $(k, k+1)$ with endpoints (x_k, y_k) and (x_{k+1}, y_{k+1}), we can compute $\det\left(\begin{pmatrix} x_{k+1} - x_k & X_t - x_k \\ y_{k+1} - y_k & Y_t - y_k \end{pmatrix}\right)$. The sign of the determinant indicates whether the point is left or right of the edge. The value of the determinant is a measure for the distance of the point (X_t, Y_t) from the edge $(k, k+1)$. For the trajectory planning problem we can thereby check if the position (X_t, Y_t) of a drone is outside of an obstacle: It should be left of at least one of the edges (— edges are considered to have a clockwise orientation).

To check whether or not a convex polygon i overlaps with a convex polygon j we can define for each edge s of one of the polygons a critical vertex P of the other polygon: it is defined to be the vertex that is farthest to the right of the edge according to the determinant defined above (assuming that the reference points of i and j are matched, see Figure 3). The reference point of a polygon is the vertex that has lowest y–value. Among those we select the one with smallest x–value. For each pair (i, j) of polygons we can compute the (edge, critical vertex) pairs in a preprocessing step. While computing the critical vertex, not all edge-vertex combinations must be considered. It is sufficient to consider left edges of one polygon and right vertices of the other as well as right edges of one polygon and left vertices of the other:

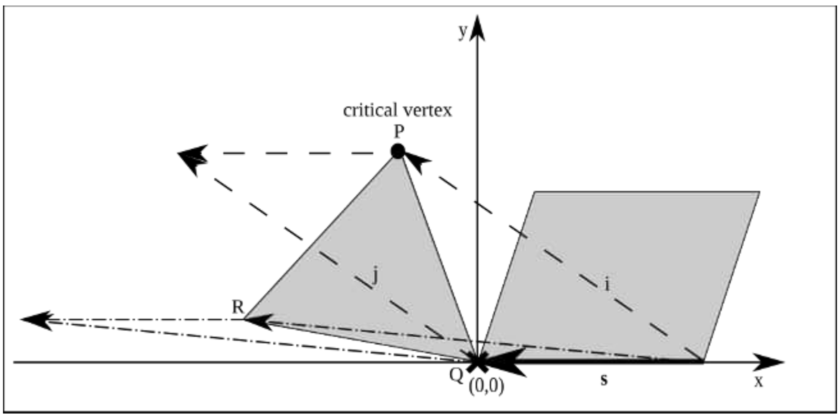

Figure 3: An Edge of Polygon i and the Corresponding Critical Vertex of Another Polygon j

The two polygons do not overlap if there is one edge of one of the polygons such that the corresponding critical vertex of the other polygon is left of it.

3 CONCLUSION

By means of two examples we have shown that computational geometry and mathematical programming sometimes complement each other neatly. The common aspect in our application is that overlapping of objects must be avoided and hence be tested for. This can be done by integrating geometrical insights into mathematical models. For our applications we are able to solve instances with commercial software packages just because such insights were integrated.

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A SIMPLE APPROACH TO SOLVING THE MONOPOLIST'S LONG-RUN PROFIT MAXIMIZATION PROBLEM: THE CASE OF THE HYPERBOLIC INVERSE DEMAND AND COBB-DOUGLAS PRODUCTION FUNCTIONS WITH TWO FACTORS OF PRODUCTION

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Abstract: This paper presents our first research on the profit-maximizing choice of monopoly in the case of hyperbolic inverse demand function and the Cobb-Douglas technology with two factors of production. In the literature, the derivation of the profit function for monopoly for the given demand function, similar to the profit maximization problem under perfect competition, is usually analysed using differential calculus. In contrast to differential calculus, we solve this problem using the weighted AM-GM inequality. We derive the conditions under which the strict global maximum simply exists. In particular, we argue the advantage of our approach over the differential calculus, since the condition under which the weighted AM-GM inequality becomes equality is the only necessary and sufficient condition for strict global extrema. Moreover, from an economic point of view, we point out that the condition under which the weighted AM-GM inequality becomes equality is equivalent to the known condition MR=MC together with MR'<MC'. To our knowledge, this approach is not known in the literature.

Keywords: monopoly, long-run profit maximization, hyperbolic inverse demand function, Cobb-Douglas production function, weighted AM-GM inequality

1 INTRODUCTION

One of the central problems in the theory of the firm is the problem of maximizing long-run profits. This problem is well-studied in the literature, especially under conditions of perfectly competitive markets. Since profit is defined as a difference between total revenue *TR* and total cost *TC*, this problem can generally be formulated as follows

$$\max_{q>0} \ \Pi(q) = TR(q) - TC(q), \tag{1}$$

where q is the output quantity. An almost unavoidable example in textbooks of microeconomics is the profit maximization problem of the perfectly competitive firm with the Cobb-Douglas production function (see, e.g., [1, 4, 5, 7, 12, 13, 14, 15, 16, 18]), solved using differential calculus. However, since the functional form of the Cobb-Douglas production function ([2]) is the weighted geometric mean of its inputs, the long-run profit maximization problem for the perfectly competitive firm can be solved in a complementary way using geometric programming or weighted arithmetic mean - geometric mean (AM-GM) inequality, in addition to calculus (see [8, 9, 10]). This paper studies the long-run profit maximization problem in monopoly under hyperbolic inverse demand and Cobb-Douglas production functions with two factors of production. We aim to show that the monopolist's long-run profit maximization problem can be solved using the weighted AM-GM inequality. Moreover, we derive necessary and sufficient conditions for the (strict global) profit maximum simultaneously, in a simpler way by avoiding the computation of the first and the second derivative. In particular, we compare the advantages and disadvantages between the imputed, weighted AM-GM inequality approaches, and economic conditions when marginal revenue equals marginal cost.

The structure of this paper is as follows. After the introduction, the second section gives the notation and the preliminaries. The third section contains the main results, while the fourth section contains the conclusions, limitations, and ideas for further research.

2 NOTATION AND PRELIMINARIES

When the conditions of perfect competition in a market are satisfied, the profit maximization problem with the Cobb-Douglas production function and two input factors $x_1>0$ and $x_2>0$ (1) can be written as

$$\max_{x_1,x_2>0} TR\left(x_1,x_2\right)-\left(w_1x_1+w_2x_2\right), \tag{2}$$

where the revenue function TR is the product of the fixed price $p>0$ and production quantity $q\left(x_1,x_2\right)=Ax_1^{\alpha_1}x_2^{\alpha_2}$ ($A>0$, $\alpha_1>0$, $\alpha_1>0$). Prices of the first and second factor of production are denoted by $w_1>0$ and $w_2>0$, respectively. Thus, (2) becomes

$$\max_{x_1,x_2>0} pAx_1^{\alpha_1}x_2^{\alpha_2}-w_1x_1-w_2x_2\,. \tag{3}$$

On the other hand, problem (3) can be solved in a two-step procedure ([7, 17]). In the first step, the cost function is derived as a solution to the problem

$$c\left(w_1,w_2,q\right)\equiv\min_{x_1,x_2>0} w_1x_1+w_2x_2 \quad \text{s.t. } Ax_1^{\alpha_1}x_2^{\alpha_2}=q\,. \tag{4}$$

Solving (4), the derived cost function in the case of the Cobb-Douglas production function is equal to

$$c\left(w_1,w_2,q\right)=\omega q^{1/\varepsilon} \tag{5}$$

where $\omega=\varepsilon\rho^{-1/\varepsilon}$, $\varepsilon=\alpha_1+\alpha_2$, $\rho=A\left(\frac{\alpha_1}{w_1}\right)^{\alpha_1}\left(\frac{\alpha_2}{w_2}\right)^{\alpha_2}$. Moreover, optimal x_1 and x_2 from problem (4) define conditional input demand functions given by

$$x_i\left(w_1,w_2,q\right)=\frac{\alpha_i}{w_i}\rho^{-1/\varepsilon}q^{1/\varepsilon},\ i=1,2\,. \tag{6}$$

Now, the second step of the procedure considers solving the profit maximization problem

$$\max_{q>0}\Pi\left(q\right)=pq-c\left(w_1,w_2,q\right)=pq-\omega q^{1/\varepsilon}\,. \tag{7}$$

The solution of (7) gives the supply function of a perfectly competitive firm

$$q=p^{\frac{\varepsilon}{1-\varepsilon}}\rho^{\frac{1}{1-\varepsilon}}, \tag{8}$$

with condition $\varepsilon<1$. The profit function is derived by substituting (8) into (7),

$$\pi\left(p,w_1,w_2\right)=\left(1-\varepsilon\right)\left(p\rho\right)^{1/\varepsilon}. \tag{9}$$

If we substitute (8) into (6), we get the input demand function, or the solution of problem (3):

$$x_i\left(p,w_1,w_2\right)=\frac{\alpha_i}{w_i}\left(p\rho\right)^{\frac{1}{1-\varepsilon}},\ i=1,2. \tag{10}$$

Thus, when conditions of perfectly competitive markets prevail, the profit function of a perfectly competitive firm is given by (9), the input demand functions by (10), and the output supply function by (8). In the next section, we formulate and solve the model for the monopolist's profit-maximizing choice problem in the case of hyperbolic inverse demand and the Cobb-Douglas production function with two inputs.

3 THE MONOPOLIST'S LONG-RUN PROFIT MAXIMIZATION PROBLEM

In a monopoly, the producer faces the market demand curve for his product. Assume that the inverse demand function has a hyperbolic form ([11]), i.e.

$$p(q) = aq^{-b} \ , \tag{11}$$

where a>0 and b>0. The profit-maximizing choice problem for monopoly in the case of hyperbolic inverse demand and the Cobb-Douglas production functions can be written by substituting (11) into (7), i.e.

$$\max_{q>0} \Pi(q) = p(q)q - \omega q^{1/\varepsilon} = aq^{1-b} - \omega q^{1/\varepsilon} \ . \tag{12}$$

3.1 Differential calculus approach

The first derivative of the function Π(Q) from (12) reduces to the known equality between marginal revenue and marginal cost and is equal to

$$\Pi'(q) = a(1-b)q^{-b} - \frac{\omega}{\varepsilon} q^{\frac{1}{\varepsilon}-1} , \tag{13}$$

from which the unique stationary point of Π(q) is derived:

$$q_0 = \left(\frac{a(1-b)\varepsilon}{\omega} \right)^{\frac{\varepsilon}{1-\varepsilon+b\varepsilon}} \ . \tag{14}$$

The relation (14) implies condition 1-b>0, which with b>0 results in

$$b \in \langle 0,1 \rangle \ . \tag{15}$$

Furthermore, the point given by (14) is the global maximizer for (12) if and only if the second derivative of Π(Q) from (12) at the point (14) is less than zero, i.e.

$$\Pi''(q_0) = -ab(1-b)q_0^{-b-1} - \rho^{-\frac{1}{\varepsilon}} \left(\frac{1}{\varepsilon} - 1 \right) q_o^{\frac{1}{\varepsilon}-2} < 0 \ . \tag{16}$$

After tedious algebraic manipulations, it can be shown that (16) is equivalent to the condition

$$\varepsilon < \frac{1}{1-b} \Leftrightarrow 1-b < \frac{1}{\varepsilon} \ . \tag{17}$$

The maximum profit is obtained by inserting (14) into Π(q) from (12):

$$\pi(w_1, w_2) = \frac{1-\varepsilon+b\varepsilon}{1-b} \left[a(1-b) \right]^{\frac{1}{1-\varepsilon+b\varepsilon}} \left[A \left(\frac{\alpha_1}{w_1} \right)^{\alpha_1} \left(\frac{\alpha_2}{w_2} \right)^{\alpha_2} \right]^{\frac{1-b}{1-\varepsilon+b\varepsilon}} \ . \tag{18}$$

Since (14) is the unique global maximizer, the profit function of a monopolist for the given demand function in (18) for this problem is well defined. The monopolist's supply function for the given demand function follows from (14):

$$q(w_1,w_2)=\left[a(1-b)\right]^{\frac{\varepsilon}{1-\varepsilon+b\varepsilon}}\left[A\left(\frac{\alpha_1}{w_1}\right)^{\alpha_1}\left(\frac{\alpha_2}{w_2}\right)^{\alpha_2}\right]^{\frac{1}{1-\varepsilon+b\varepsilon}}. \tag{19}$$

The input demand functions for the given demand function are obtained by substituting (14) into (6):

$$x_i(w_1,w_2)=\frac{\alpha_i}{w_i}\left[a(1-b)\right]^{\frac{1}{1-\varepsilon+b\varepsilon}}\left[A\left(\frac{\alpha_1}{w_1}\right)^{\alpha_1}\left(\frac{\alpha_2}{w_2}\right)^{\alpha_2}\right]^{\frac{1-b}{1-\varepsilon+b\varepsilon}}. \tag{20}$$

3.2 Weighted AM-GM inequality approach

Let us solve problem (12) using weighted AM-GM inequality. It is known (see, for example [3, 6]), that for any real numbers $a_1>0$, $a_2>0$, $\lambda_1>0$, $\lambda_2>0$ the following inequality holds

$$a_1^{\lambda_1}a_2^{\lambda_2}\le\left(\frac{\lambda_1 a_1+\lambda_2 a_2}{\lambda_1+\lambda_2}\right)^{\lambda_1+\lambda_2}, \tag{21}$$

where equality holds if and only if

$$a_1=a_2. \tag{22}$$

First, let us examine some conditions on the parameter b from (12), which must be satisfied in the long run, i.e. by the shape of the function $\Pi(q)$ from (12). Since the long-run profit must be zero for the quantity zero, i.e., $\Pi(0)=0$, we have

$$\Pi(0)=a0^{1-b}-\omega 0^{1/\varepsilon}=0 \iff 1-b>0. \tag{23}$$

On the other hand, when q approaches infinity, profit has to approach negative infinity, i.e., since (23) holds, it must be

$$\lim_{q\to+\infty}\Pi(q)=\lim_{q\to+\infty}q^{1-b}\left(a-\omega q^{\frac{1}{\varepsilon}+b-1}\right)=-\infty \iff \frac{1}{\varepsilon}+b-1>0. \tag{24}$$

The case $1/\varepsilon+b-1=0$ is impossible since it would imply that $\Pi(q)$ is strictly monotone for all $q\geq0$, which makes no sense from an economic point of view. Thus, the following conditions for problem (12) must hold

$$b\in\langle 0,1\rangle \tag{25}$$

and

$$1/\varepsilon>1-b. \tag{26}$$

Now, since we want a positive profit $\Pi(Q)>0$, it follows that

$$\Pi(Q)=aq^{1-b}-\omega q^{1/\varepsilon}>0 \iff q\in\left\langle 0,(a/\omega)^{\frac{\varepsilon}{1-\varepsilon+b\varepsilon}}\right\rangle. \tag{27}$$

Furthermore, let us rewrite $\Pi(Q)$ from (12) in the following way:

$$\Pi(Q)=\omega^{-\frac{\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}}\cdot\frac{1+b\varepsilon-\varepsilon}{\varepsilon(1-b)}\left(\omega q^{\frac{1+b\varepsilon-\varepsilon}{\varepsilon}}\right)^{\frac{\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}}\cdot\left(\frac{a\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}-\frac{\omega\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}q^{\frac{1+b\varepsilon-\varepsilon}{\varepsilon}}\right)^{1}. \tag{28}$$

Let us define

$$a_1=\omega q^{\frac{1+b\varepsilon-\varepsilon}{\varepsilon}},\ \lambda_1=\frac{\varepsilon(1-b)}{1+b\varepsilon-\varepsilon},\ a_2=\frac{a\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}-\frac{\omega\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}q^{\frac{1+b\varepsilon-\varepsilon}{\varepsilon}},\ \lambda_2=1\ . \tag{29}$$

Note that (25), (26), and (27) imply $\frac{1+b\varepsilon-\varepsilon}{\varepsilon(1-b)}>0$, $a_1>0$, $a_2>0$, $\lambda_1>0$ and $\lambda_2>0$. By applying inequality (21) to (28), we obtain the following

$$\begin{aligned}\Pi(q)&\le\omega^{-\frac{\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}}\frac{1+b\varepsilon-\varepsilon}{\varepsilon(1-b)}\left[\frac{\frac{\omega\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}\cdot q^{\frac{1+b\varepsilon-\varepsilon}{\varepsilon}}+1\cdot\left(\frac{a\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}-\frac{\omega\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}q^{\frac{1+b\varepsilon-\varepsilon}{\varepsilon}}\right)}{\frac{\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}+1}\right]^{\frac{\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}+1}\\&=\frac{1-\varepsilon+b\varepsilon}{1-b}\left[a(1-b)\right]^{\frac{1}{1-\varepsilon+b\varepsilon}}\rho^{\frac{1-b}{1-\varepsilon+b\varepsilon}}\\&=\frac{1-\varepsilon+b\varepsilon}{1-b}\left[a(1-b)\right]^{\frac{1}{1-\varepsilon+b\varepsilon}}\left[A\left(\frac{\alpha_1}{w_1}\right)^{\alpha_1}\left(\frac{\alpha_2}{w_2}\right)^{\alpha_2}\right]^{\frac{1-b}{1-\varepsilon+b\varepsilon}}.\end{aligned} \tag{30}$$

By definition, the strict global maximum of the function $\Pi(Q)$ from (12) equals the last term in (30), since it is achieved if and only if $a_1=a_2$ (where a_1 and a_2 are defined by (29)), whence the unique global maximizer follows:

$$\begin{aligned}a_1=a_2\ &\Leftrightarrow\ \omega q^{\frac{1+b\varepsilon-\varepsilon}{\varepsilon}}=\frac{a\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}-\frac{\omega\varepsilon(1-b)}{1+b\varepsilon-\varepsilon}q^{\frac{1+b\varepsilon-\varepsilon}{\varepsilon}}\\&\Leftrightarrow q=\left[a(1-b)\right]^{\frac{\varepsilon}{1-\varepsilon+b\varepsilon}}\left[A\left(\frac{\alpha_1}{w_1}\right)^{\alpha_1}\left(\frac{\alpha_2}{w_2}\right)^{\alpha_2}\right]^{\frac{1}{1-\varepsilon+b\varepsilon}}.\end{aligned} \tag{31}$$

Note that the results in (30) and (31) are equal to (18) and (19), respectively.

Remark 1. Note that for b=0 and a=p, problem (12) becomes (7), i.e., from an analytical point of view, for b=0 and a=p "the monopolist transforms into a perfectly competitive firm", which can be seen from the fact that the results in (30) and (31) transform into (9) and (8), respectively.

4 CONCLUSION, LIMITATIONS, AND FURTHER RESEARCH

Differential calculus is a comprehensive tool used in microeconomic theory. However, it is not always the most elegant tool for solving specific problems, such as the problem studied in this paper - the problem of long-run profit maximization of a monopoly in the case of hyperbolic inverse demand and a Cobb-Douglas productions function with two input factors. We have shown that this problem can be solved elegantly using the weighted AM-GM inequality. The elegance of our approach is reflected in the fact that the verification of the necessary and sufficient conditions for the strict global maximization of the long-run profit, i.e., finding the first and second derivatives, can be replaced by only one step, namely, when the weighted AM-GM inequality becomes equality. From an economic point of view, it is not necessary to solve the equality between marginal revenue and marginal cost (MR =MC) together with the verification of the MR'<MC' condition. For further research, we could analyze the same problem using CES production function as a representation of a firm's technology. Sensitivity analysis (i.e., comparative statics) and some numerical examples could also be performed.

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NUMERICAL COMPARISON OF LONG-STEP INTERIOR POINT ALGORITHMS FOR SOLVING LINEAR COMPLEMENTARITY PROBLEMS

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Abstract: In this paper, we compare the performance of different long-step interior point methods for solving linear complementarity problems. We propose a general long-step algorithmic framework that combines two well-known techniques from the literature: the long-step approach of Ai and Zhang and the algebraically equivalent transformation technique introduced by Darvay.

The transformation function applied in Darvay's technique is part of the input in the algorithmic framework. Therefore, for different functions, we obtain different long-step algorithms. In this study, we compare the numerical performance of six different algorithm variants.

Keywords: Linear complementarity problem, Interior point algorithms, Algebraically equivalent transformation technique

1 INTRODUCTION

Linear complementarity problems (LCPs) have a wide range of applications in different fields, such as engineering [7], game theory [10], and economics [12]. Let $M \in \mathbb{R}^{n\times n}$ and $\mathbf{q} \in \mathbb{R}^n$ be given. The LCP is to determine a vector pair $(\mathbf{x}, \mathbf{s}) \in \mathbb{R}^n \times \mathbb{R}^n$ that satisfies

$$-M\mathbf{x} + \mathbf{s} = \mathbf{q}, \quad \mathbf{xs} = \mathbf{0}, \quad \mathbf{x}, \mathbf{s} \geq \mathbf{0}.$$

Let $\mathcal{F}$ denote the set of feasible solutions, i.e., $\mathcal{F} = \{(\mathbf{x}, \mathbf{s}) : \ -M\mathbf{x} + \mathbf{s} = \mathbf{q}, \mathbf{x} \geq \mathbf{0}, \ \mathbf{s} \geq \mathbf{0}\}$, $\mathcal{F}_+$ the set of strictly feasible solutions, namely $\mathcal{F}_+ = \{(\mathbf{x}, \mathbf{s}) \in \mathcal{F} : \ \mathbf{x} > \mathbf{0}, \ \mathbf{s} > \mathbf{0}\}$.

In general, the LCP is an NP-complete problem [2]; therefore, to be able to introduce efficient algorithms, authors made several different assumptions on the coefficient matrix of the LCP. The class of $\mathcal{P}_*(\kappa)$-matrices were defined by Kojima et al. [9] to analyze interior-point algorithms (IPAs) for solving LCPs.

To be able to introduce IPAs, we need to define the central path problem for LCPs:

$$-M\mathbf{x} + \mathbf{s} = \mathbf{q}, \quad \mathbf{xs} = \tau\mu\mathbf{e}, \quad \mathbf{x}, \mathbf{s} > \mathbf{0},$$

where $0 < \tau < 1$ is a given parameter and $\mu = \frac{\mathbf{x}^T\mathbf{s}}{n}$. According to [8], for $\mathcal{P}_*(\kappa)$-LCPs, if $\mathcal{F}_+ \neq \emptyset$, the central path exists and it is unique. From now on, we assume that M is a $\mathcal{P}_*(\kappa)$ matrix, $\mathcal{F}_+ \neq \emptyset$ and a starting point $(\mathbf{x}_0, \mathbf{s}_0) \in \mathcal{F}_+$ is given.

In our previous work, we combined two well-known techniques from the literature on IPAs; the algebraically equivalent transformation technique proposed by Darvay in 2002 [3] and the method of Ai and Zhang [1].

Darvay's technique applies a continuously differentiable and invertible function $\varphi : (\xi, \infty) \to \mathbb{R}$ to the centering equation of the central path problem ($\xi \in [0, 1)$ is given):

$$-M\mathbf{x}+\mathbf{s}=\mathbf{q}, \quad \varphi\left(\frac{\mathbf{xs}}{\tau\mu}\right)=\varphi\left(\mathbf{e}\right), \quad \mathbf{x},\mathbf{s}>\mathbf{0}.$$

To determine the Newton directions $(\Delta\mathbf{x},\Delta\mathbf{s})$ for IPAs, we apply Newton's method to the transformed system, where $\mathbf{v}=\sqrt{\frac{\mathbf{xs}}{\tau\mu}}$:

$$\left.\begin{aligned} -M\Delta\mathbf{x}+\Delta\mathbf{s}&=\mathbf{0}\\ \mathbf{s}\Delta\mathbf{x}+\mathbf{x}\Delta\mathbf{s}&=\mathbf{a}_\varphi=\sqrt{\tau\mu}\sqrt{xs}\mathbf{p}_\varphi=\sqrt{\tau\mu}\sqrt{xs}\frac{\varphi(\mathbf{e})-\varphi(\mathbf{v}^2)}{\mathbf{v}\varphi'(\mathbf{v}^2)}\end{aligned}\right\}$$

This way, for different functions φ, we can determine different search directions.

In [5, 6], we defined the function p that describes the coordinatewise transformation applied in the right-hand side of the scaled Newton system, i.e. for which $p(v_i)=(p_\varphi)_i$:

$$p:(\xi,\infty)\to\mathbb{R}, \quad p(t)=\frac{\varphi(1)-\varphi(t^2)}{t\varphi'(t^2)}.$$

Based on their step length, IPAs can be categorized into two main groups, long-step and short-step methods. Even though long-step methods are more efficient in practice, for many years, their theoretical complexity was worse than that of short-step IPAs. In 2005, Ai and Zhang [1] proposed a new type of long-step IPA for which they could prove the same theoretical complexity as for short-step algorithms. They decomposed the Newton directions into two components by considering the following two systems, and for the two components, they assigned different step lengths:

$$\left.\begin{aligned} -M\Delta\mathbf{x}_-+\Delta\mathbf{s}_-&=\mathbf{0}\\ \mathbf{s}\Delta\mathbf{x}_-+\mathbf{x}\Delta\mathbf{s}_-&=\mathbf{a}_\varphi^-=\sqrt{\tau\mu}\sqrt{xs}\mathbf{p}_\varphi^-\end{aligned}\right\} \qquad \left.\begin{aligned} -M\Delta\mathbf{x}_++\Delta\mathbf{s}_+&=\mathbf{0}\\ \mathbf{s}\Delta\mathbf{x}_++\mathbf{x}\Delta\mathbf{s}_+&=\mathbf{a}_\varphi^+=\sqrt{\tau\mu}\sqrt{xs}\mathbf{p}_\varphi^+\end{aligned}\right\} \tag{1}$$

The $-$ and $+$ signs in the upper index denote the negative and positive parts of the vectors, respectively. $\Delta\mathbf{x}_+$ denotes the solution of the system with $\mathbf{a}_\varphi^+$ on its right-hand side. The notation is similar in the case of the other solution vectors.

They also defined a new type of wide neighborhood of the central path:

$$\mathcal{W}(\tau,\beta)=\left\{(\mathbf{x},\mathbf{s})\in\mathcal{F}_+:\left\|\mathbf{v}\mathbf{p}_\varphi^+\right\|\le\beta\right\}.$$

2 THE GENERAL ALGORITHMIC FRAMEWORK

In our previous work [6], we defined a general algorithmic framework for linear programming (LP) problems, where the transformation function $p(t)$ is part of the input. To determine the Newton directions, we considered the (decomposed) transformed systems (1) and an Ai-Zhang type wide neighborhood. In our current work, we generalize this algorithmic framework to LCPs. The details are given in Algorithm 1.

3 NUMERICAL EXPERIMENTS

To determine the step lengths α_1 and α_2, we generalized the simplified large-update IPA proposed by Potra [11]. We compared the performance of the general algorithm for six different functions:

$$p_1(t)=\frac{1}{t}-t \quad p_2(t)=\frac{1}{2}\left(\frac{1}{t}-t\right) \quad p_3(t)=\frac{1}{5}\left(\frac{1}{t}-t\right)$$

Input: $M \in \mathbb{R}^{n\times n}$, $\mathbf{q} \in \mathbb{R}^n$, a function $p(t)$, an accuracy parameter $\varepsilon > 0$,
an update parameter $0 < \tau < 1$, a neighborhood parameter $0 < \beta < 1$,
a starting point $(\mathbf{x}_0, \mathbf{s}_0) \in \mathcal{W}(\tau, \beta)$ with $\mu_0 = \frac{\mathbf{x}_0^T \mathbf{s}_0}{n}$.

$\mathbf{x} := \mathbf{x}_0$, $\mathbf{s} := \mathbf{s}_0$ and $\mu := \mu_0$

while $\mathbf{x}^T\mathbf{s} > \epsilon$ **do**

Determine $\Delta\mathbf{x}_+, \Delta\mathbf{s}_+$ and $\Delta\mathbf{x}_-, \Delta\mathbf{s}_-$ from the transformed Ai-Zhang type Newton-systems (1);

$(\alpha_1, \alpha_2) := \operatorname{argmin}\{\mu(\alpha) : (\mathbf{x}(\alpha), \mathbf{s}(\alpha)) \in \mathcal{W}(\tau, \beta)\}$,

where $\mathbf{x}(\alpha) = \mathbf{x} + \alpha_1\Delta\mathbf{x}_- + \alpha_2\Delta\mathbf{x}_+$, $\mathbf{s}(\alpha) = \mathbf{s} + \alpha_1\Delta\mathbf{s}_- + \alpha_2\Delta\mathbf{s}_+$;

$\mathbf{x} := \mathbf{x}(\alpha)$, $\mathbf{s} := \mathbf{s}(\alpha)$, $\mu := \frac{\mathbf{x}^T\mathbf{s}}{n}$;

end

Algorithm 1: General algorithmic framework for LCPs

$$p_4(t) = \begin{cases} \frac{1}{t} - t, & \text{if } t \le 1 \\ 2(1-t), & \text{if } t > 1 \end{cases} \qquad p_5(t) = \begin{cases} \frac{1}{t} - t, & \text{if } t \le 1 \\ \frac{2(t-t^2)}{2t-1}, & \text{if } t > 1 \end{cases} \qquad p_6(t) = \begin{cases} \frac{1}{t} - t, & \text{if } t \le 1 \\ 1 - t^2, & \text{if } t > 1 \end{cases}$$

To generate the test instances, we used the sufficient matrix instances collected on [4]. We used the starting point $\mathbf{x}_0 = \mathbf{e}$ and $\mathbf{s}_0 = \mathbf{e}$, and calculated $\mathbf{q}$ as $\mathbf{q} = -M\mathbf{e} + \mathbf{e}$, where $\mathbf{e}$ is the vector of all ones. We used the parameter settings $\rho_1 = \rho_2 = 0.75$, $\varepsilon = 10^{-5}$, $\beta = 0.5$ and $\tau = 0.1$, where ρ_1 and ρ_2 are parameters required for the procedure determining the step lengths.

The numerical results are summarized in Tables 1-3. In most cases, the best running times and iteration numbers were obtained for the function $p_1(t)$, except for the Csizmadia instances. The smallest average iteration number and running time are highlighted in red in each row. For the Csizmadia instances, the solvable problem size is relatively small as the feasible step length is very small at the beginning for this starting point; see [5].

	$p_1(t)$		$p_2(t)$		$p_3(t)$		$p_4(t)$		$p_5(t)$		$p_6(t)$	
3	5.8	0.0005	21.0	0.0009	63.0	0.0024	5.9	0.0009	7.6	0.0008	7.0	0.0015
4	6.5	0.0005	21.0	0.0009	64.7	0.0024	6.4	0.0010	7.9	0.0007	7.0	0.0015
5	6.3	0.0007	21.7	0.0012	66.0	0.0024	6.1	0.0012	7.4	0.0007	7.6	0.0024
6	5.9	0.0006	22.0	0.0011	66.9	0.0031	7.0	0.0012	7.8	0.0006	7.4	0.0021
7	6.6	0.0005	22.0	0.0010	67.0	0.0024	7.0	0.0012	8.3	0.0008	8.0	0.0021
8	7.1	0.0006	22.2	0.0012	68.0	0.0026	7.8	0.0017	8.7	0.0008	8.3	0.0024
9	7.5	0.0006	22.6	0.0011	68.8	0.0034	7.5	0.0015	8.8	0.0008	8.4	0.0021
10	8.8	0.0084	23.3	0.0018	69.1	0.0035	8.4	0.0040	10.3	0.0030	9.5	0.0045

Table 1: Average iteration numbers and running times for the ENM_SU instances

	$p_1(t)$		$p_2(t)$		$p_3(t)$		$p_4(t)$		$p_5(t)$		$p_6(t)$	
	It.	**T. (s)**	**It.**	**T. (s)**	**It.**	**T. (s)**	**It.**	**T. (s)**	**It.**	**T. (s)**	**It.**	**T. (s)**
10	8.5	0.0006	22.9	0.0011	69.0	0.0027	7.6	0.0016	9.5	0.0011	8.4	0.0025
20	9.0	0.0011	24.1	0.0016	72.7	0.0047	9.8	0.0035	10.7	0.0018	9.7	0.0046
50	7.3	0.0020	25.0	0.0047	77.0	0.0125	8.1	0.0072	9.5	0.0043	8.9	0.0150
100	8.7	0.0090	27.0	0.0167	81.0	0.0460	8.2	0.0282	9.9	0.0181	9.1	0.0411
200	9.2	0.0176	28.0	0.0486	84.0	0.1401	9.5	0.1433	11.3	0.0735	9.5	0.1851
500	10.0	0.1760	29.0	0.5417	89.0	1.6389	10.4	1.6548	11.2	0.6155	11.0	2.2617
700	10.0	0.4681	30.0	1.3360	91.0	3.8912	11.0	4.9945	13.0	1.8751	11.0	6.6644

Table 2: Average iteration numbers and running times for the MGS_SU instances

	$p_1(t)$		$p_2(t)$		$p_3(t)$		$p_4(t)$		$p_5(t)$		$p_6(t)$	
	It.	**T. (s)**	**It.**	**T. (s)**	**It.**	**T. (s)**	**It.**	**T. (s)**	**It.**	**T. (s)**	**It.**	**T. (s)**
10	17	0.0241	25	0.0039	69	0.0052	13	0.0062	15	0.0054	16	0.0080
20	25	0.0263	31	0.0058	76	0.0057	21	0.0351	20	0.0238	23	0.0322
50	50	0.0682	44	0.0560	90	0.0492	130	6.2337	35	0.4007	39	0.6364
100	-	-	-	-	-	-	-	-	69	9.9350	-	-

Table 3: Average iteration numbers and running times for the Csizmadia instances

4 Conclusion

In this paper, we proposed a new long-step algorithmic framework that combines the approach of Ai and Zhang and the AET technique proposed by Darvay. We implemented the general algorithm in Matlab and compared its performance for six different transformation functions. We generalized Potra's method from [11] for determining the step lengths.

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Session 6:

Multi-Criteria Decision-Making

A MCDM METHODOLOGY FOR CYBERATTACK MITIGATION

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Abstract: The implementation of suitable mitigation actions in cybersecurity plays a crucial role in elevating the resilience of ecosystems. As mitigation measures encompass both advantages as well as costs, a proper selection of potential measures is essential. This paper introduces a comprehensive decision-making methodology that helps us assess and choose mitigation measures and strategies. It consists of two phases. We first identify cyber incidents and evaluate their impacts on compromised assets. In the second phase, we identify mitigation measures or strategies and select the most suitable to be implemented. The MCDM model uses the quantitative or qualitative value function. It is based on a standard scoring system and aggregates a veto function to prevent the selection of mitigations with an insufficient improvement in efficiency. It considers criteria from the established frameworks, in particular CIA (Confidentiality, Integrity, and Availability), the Common Criteria for Information Technology Security Evaluation according to the ISO/IEC 15408 standard, and the recommended NESCOR criteria for the assessment of the impacts of infrastructure failures. The MITRE ATT&CK, CIS Critical Security Controls, and ACSC frameworks are applied to identify mitigation measures based on cyber incidents and compromised assets. This process utilizes the mappings between CPE identifiers, CVE vulnerabilities, and MITRE ATT&CK techniques and mitigations. We implemented a decision support system that covers all phases of the multi-criteria decision process and facilitates additional decision analysis concepts, including sensitivity analysis and group decision-making.

Keywords: multiple criteria decision analysis, decision-making processes, decision support systems, value theory, impact and efficiency assessment, cybersecurity.

1 INTRODUCTION

Cybersecurity is becoming an increasingly important concept in IT (*Information Technology*) and OT (*Operational Technology*) systems. Two strategies are available to make IT and OT infrastructures resilient – proactive and reactive. In the first case, we try to foresee and estimate cybersecurity risks, threats, and vulnerabilities. Based on these estimations, we can implement appropriate mitigation measures to prevent that cyber attacks would occur, or if they do, to ensure that they will have the least possible impact. In the latter case, we use systems such as SIEM (*Security Information and Event Management*) and SOAR (*Security Orchestration, Automation, and Response*) to continuously monitor cybersecurity-related information and events. When we identify a cyber incident, we trigger appropriate mitigation measures and responses to remedy the effects of the attack. In both cases, we must select and implement optimal mitigation measures and strategies to ensure business continuity. Because each mitigation comes with a cost and may have a limited effect, it is essential to assess the suitability and efficiency of potential mitigations based on multiple criteria that consider the characteristics of the infrastructure, maturity of available cybersecurity technologies and processes, the severity of cyber incidents, organizational goals and constraints, and several other aspects. A sound and comprehensive MCDM approach is hence of high importance.

Most existing approaches to assessing the impacts of cybersecurity attacks and threats use the risk matrix. It is a simple method to express the relationship between the severity and probability on a qualitative or quantitative scale. More advanced approaches combine the risk matrix with a sequence of questions and a set of decision rules that aggregate the provided responses at multiple hierarchical levels. Such an evaluation usually complies with the ISO 31000 standard, which specifies the risk management process. An alternative option is to use the CVSS (*Common Vulnerability Scoring System*) [1] assessment.

In contrast to existing approaches, we aim to introduce a systematic and comprehensive decision-making methodology that consistently builds upon multi-criteria decision analysis. It addresses the problem of selecting and implementing mitigation measures that allow us to respond to cyber threats and incidents. These measures reduce the impact and severity of incidents in complex IT/OT systems and supply chains. Although we primarily developed the MCDM methodology for the energy sector and its critical infrastructure, we can benefit from it in numerous organizational systems facing cyber threats due to all increasing digitalization. The methodology results from the Horizon 2020 CyberSEAS project aiming to secure energy data services and increase the resilience of modern power grids and energy systems [2].

2 DECISION-MAKING PROCESS

The methodology consists of the decision-making process and the MCDM analysis. Figure 1 depicts the process. It incorporates, connects, and correlates the incident impact assessment MCDM model and the mitigation selection MCDM model. It also integrates external sources and frameworks that support the intelligence phase to obtain decision-making information. A decision support system implements all process phases (not shown due to length limitations).

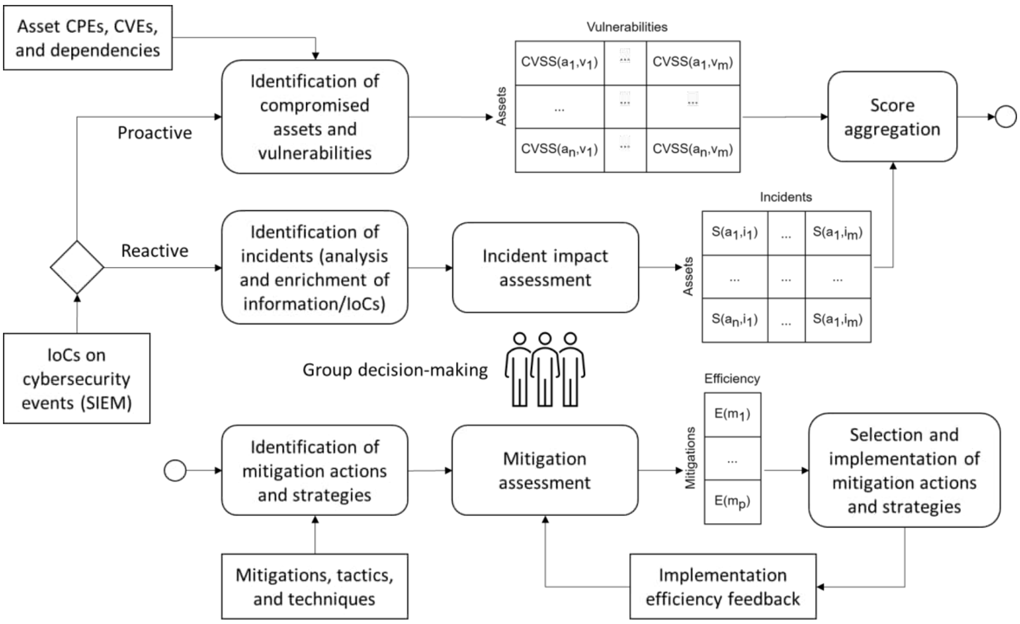

Figure 1: Decision-making process

The primary inputs of the decision-making process are cybersecurity events, where an event refers to one or more cyber-attacks. We use the MITRE ATT&CK framework [3] to identify and characterize a cybersecurity event as a group of techniques T as follows:

$$CSE_i = \{\mathrm{T1087.001}, \mathrm{T1087.002}, \mathrm{T1087.003}, \mathrm{T1087.004}, \mathrm{T1110.002}\} \tag{1}$$

In this example, the cybersecurity event denoted with CSE_i refers to account theft, for which the attacker employs five complementary attack techniques: local, domain, email, and cloud account discovery, as well as password cracking. We must select and implement appropriate mitigation measures to remediate the impact of these techniques. A possible mitigation from the MITRE ATT&CK framework to address the exploited T1110.002 technique would be the multi-factor authentication (M1032), which proposes to “use two or more pieces of evidence to authenticate to a system, such as a username and a password in addition to a token from a physical smart card or token generator." Several mitigations are usually available, but not all are efficient for all cases. Therefore, a decision-making approach is of essential importance. It is also advisable to consider other mitigation frameworks and best practices supplementary to MITRE ATT&CK. These include CIS Critical Security Controls, ACSC (*Australian Cyber Security Centre*) mitigation strategies, vendor recommendations, and internal measures.

The decision-making process utilizes a lot of cybersecurity-related information. The SIEM system provides information on logged events and IoCs (*Indicators of Compromise*). This information may be enriched by further investigation and used to identify cyber-attacks and compromised assets. We denote assets with CPEs (*Common Platform Enumerators*), which can help us to find CVEs (*Common Vulnerabilities and Exposures*) that correlate to standard CVSS severity scores and common attack techniques. An attack may compromise several connected assets, so the decision-makers should consider dependencies between these assets. The impact assessment is thus a multi-criteria problem based on the combinations of detected cyber incidents, employed attack techniques, known vulnerabilities, and the characteristics of compromised assets. All this information also underlies the identification and selection of potential mitigation measures.

3 MCDM ANALYSIS

The MCDM analysis follows the NIST 800-39 recommendations on managing information security risks [4]. Accordingly, decision-making requires collaboration on three levels. At the lowest level of the information system, security experts analyze information on cybersecurity events. At the middle business process level, decision-makers use this information to select the most efficient mitigation measures. The decision must consider business requirements and tolerance constraints of organizational threats. For this reason, senior executive staff at the highest level of the organization approves the decision and provides the resources to implement mitigation measures. These measures are either individual mitigation actions or more complex mitigation strategies consisting of sequences of correlated actions.

Several technical, business, and organizational criteria represent the basis for the proposed decision models. The incident impact assessment model considers the NESCOR criteria for high-risk failure scenarios and the impact analysis in the electric sector [5], asset criticality, and the measured impact according to the CVSS score and SIEM severity magnitude. The NESCOR criteria include the impact on the IT system, the financial impact, ecological concern, safety concern, and system scale. The mitigation selection model primarily utilizes the ISO/IEC 15408 criteria for the security of IT systems [6] and the CIA (confidentiality, integrity, and availability) model. It addresses the required expertise, equipment, cost, and time; the recovery time and point objectives; CIA criteria; scope; financial, reputation, and operational impact; and other relevant criteria.

The assessment is quantitative or qualitative. For this purpose, we introduced a uniform countermeasure scoring system, which puts both approaches on a common denominator. It is aligned with the standard CVSS system, making it appropriate for security experts. It uses a coloring scheme and impact levels of none (0.0), negligible (0.1 to 1.0), low (1.1 to 4.0), medium (4.1 to 7.0), high (7.1 to 9.0), and critical (9.1 to 10.0).

In the quantitative approach, the additive value function is used as a simple aggregation method to make the decision model suitable and comprehensive for security experts without a background in decision theory. The overall impact score of the *l*th incident and the overall efficiency score of the *k*th mitigation, respectively, are calculated with the weighted sum:

$$s^I(A_l) = \sum_{j=1}^{n} w_j s_j^I(A_l) \quad (2)$$

$$s^E(M_k) = \sum_{j=1}^{n} w_j s_j^E(M_k) \quad (3)$$

Here, $s^I(A_l)$ denotes the impact score of the lth incident, $s^E(M_k)$ is the efficiency score of the kth mitigation countermeasure, and w_j represents the weight of the jth criterion. There is a correlation between (2) and (3). If the incident has a high impact, the mitigation measure can successfully remediate it only if it is efficient enough. The veto function is therefore modeled according to our previous methodology [7]:

$$v^E(M_k) = \gamma \begin{cases} 0 & , \quad s^E(M_k) \le \mathrm{MIAT} \cdot s^I(A_l) \\ \frac{s^E(M_k) - \mathrm{MIAT} \cdot s^I(A_l)}{(1-\mathrm{MIAT}) s^I(A_l)} & , \quad \mathrm{MIAT} \cdot s^I(A_l) < s^E(M_k) < s^I(A_l) \\ 1 & , \quad s^E(M_k) \ge s^I(A_l) \end{cases} \tag{4}$$

$$+(1-\gamma) \begin{cases} 0 & , \quad s^E(M_k) \le C_{\max} - C_\delta \\ \frac{s^E(M_k) - (C_{\max} - C_\delta)}{C_\delta} & , \quad C_{\max} - C_\delta < s^E(M_k) < C_{\max} \\ 1 & , \quad s^E(M_k) = C_{\max} \end{cases} \tag{5}$$

It opposes the efficiency of the mitigation measure if at least one of two conditions is met: (1.) the mitigation does not make a sufficient improvement because the impact of a security-related event after its implementation is higher than the initial impact reduced according to the Mitigation Improvement Acceptability Threshold (MIAT), or (2.) it performs very poorly on at least one assessment criterion, such that the scores of $C_{\max} - C_\delta$ or less are acceptable, the scores of $C_{\max}$ or more are unacceptable (strict veto), and the scores in-between result in a linear increase of veto (weak veto). The effective mitigation score is now:

$$S^E(M_k) = s^E(M_k)\big(1 - v^E(M_k)\big) \tag{6}$$

Several types of sensitivity and robustness analyses are applicable. One of them is the LP-based multi-dimensional robustness analysis. It searches for a minimal change of the weight vector for which an observed mitigation countermeasure gets reassigned into a chosen better or worse category according to the standard CMSS limits. The Euclidean distance minimizes the deviation. The definition of LP is as follows:

$$\Delta_w = \min \frac{\left[\sum_{j=1}^{n} |w_j - \tilde{w}_j|^P\right]^{1/P}}{\Delta_w^{\max}} \text{ subject to} \tag{7}$$

$$s^E(M_k) = \sum_{j=1}^{n} w_j s_j^E(M_k) = C_q \tag{8}$$

$$\sum_{j=1}^{n} w_j = 1, 0 \le w_j \le 1, \forall j = 1, \dots, n \tag{9}$$

4 CONCLUSIONS

Cybersecurity in complex systems requires an advanced approach to managing mitigation measures. We introduced a systematic MCDM methodology to justify the costs and resources needed to implement such measures efficiently.

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MEASURING ACCURACY OF APPROXIMATION METHODS FOR PRIORITIES DERIVATION BASED ON PAIRWISE COMPARISONS

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Abstract: This paper aims to present the procedure of measuring the accuracy of the selected approximation methods for priorities derivation based on pairwise comparisons, compared to the priorities obtained by the eigenvalue method. Mean absolute deviation and mean absolute percentage deviation are used as the measures of accuracy. The procedure is illustrated by a numerical example. The results can show potential users which approximation method to use, and lecturers which approximation methods to include in the curriculum portfolio.

Keywords: accuracy, approximation method, mean absolute deviation, mean absolute percentage deviation, pairwise comparisons, priority.

1 INTRODUCTION

The advantages of using pairwise comparisons [5] to express judgments on the importance of criteria and preferences for alternatives are reflected in a number of applications in solving complex problems. However, quality and purpose-developed computer programs are often inaccessible to users due to several factors such as price, incompatibility of computer programs with the operating system, etc. Users can and in these cases also use approximation methods for which they create appropriate spreadsheets with available programs, such as Excel. The question of how to choose the method to calculate the priorities [4] arises. If the matrix of expressed judgments or preferences is inconsistent, different approximation methods give different priorities. When selecting approximation methods, it is therefore appropriate to assess the accuracy of the obtained priorities.

The main aim of this paper is to develop the procedure for measuring the accuracy of the approximation methods for priorities derivation based on pairwise comparisons [3], [7], compared to the priorities obtained by the eigenvalue method. For this purpose, mean absolute deviation and mean absolute percentage deviation are used as the measures of accuracy. This paper aims also to compare the accuracy of the approximation methods on the numerical example characterized by the major differences between the elements of the matrix of judgments or preferences expressed.

2 METHODS

Under the term priority, we understand the weights of criteria and local values of alternatives. When decision-makers cannot determine them directly judgments on criteria importance and preferences for alternatives with respect to a single criterion can be expressed by pairwise comparisons, based on a ratio scale [7].

Let us summarize the basics of the eigenvalue method for the priorities derivation [8]. Judgments on criteria importance, expressed by pairwise comparisons, are the ratios of the criteria weights that indicate that criterion i is a_{ij} times more important than criterion j: $a_{ij} = w_i / w_j$, where w_i is the weight of the i^{th} criterion and w_j is the weight of the j^{th} criterion, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, k$. We can write a square k x k matrix A with the following characteristics: $a_{ij} > 0$, $a_{ij} = 1/a_{ji}$, $a_{ii} = 1$, and a_{im} x $a_{mj} = a_{ij}$. The latter characteristic, the so-called transitivity, applies only in the case of complete consistency. In this case, $Aw = kw$, or

$(A - kE)w = 0$. In practice, the consistency is usually incomplete, so we get the system: $Aw = \lambda w$, where λ is the eigenvalue and w is the eigenvector of matrix A. When determining λ, we obtain a polynomial of the k^{th} level. At λ_{max} we calculate a particular solution so that $\sum_{m=1}^{k} w_m = 1$. Smaller the difference $|\lambda_{max} - k|$, more consistent a decision-maker. Consistency index $CI = \frac{\lambda_{max}-k}{k-1}$ can be used as a measure of inconsistency. However, in this paper, the consistency ratio: $CR = CI / R$, where R is the random index of inconsistency, obtained experimentally considering k [4], [8], is used as a measure of inconsistency.

Preferences on alternatives with respect to each criterion, expressed by pairwise comparisons, are the ratios of the local values that indicate that the alternative A_i is a_{ij} times more preferred than alternative A_j. Let A be the symbol of the matrix of expressed judgments on criteria importance as well as the matrix of expressed preferences to alternatives.

2.1 Approximation methods for priorities derivation

Priorities can be derived using several approximation methods [3]. Ease of use was a fundamental criterion for including the assessment of the accuracy of the following approximation methods in the paper:

I. Divide the sum of the values in each row with the sum of all values in matrix A.
II. Calculate the reciprocal value of the sum of the values in each column in matrix A.
III. Calculate priorities as the average of priorities calculated by I and II.
IV. Add the values in each column in matrix A. Then divide each entry in each column by the total of that column to obtain the normalized matrix. Calculate the average over the rows by adding the values in each row of the normalized matrix and dividing the rows by the number of entries in each. This is an approximative eigenvector method based on normalization [7].
V. Calculate the geometric mean of a row in the pairwise comparison matrix A. That geometric mean is the priority value of the factor indicated by the row. Normalize the priorities by dividing each priority value by the sum of all priorities that is got from the geometric mean. This is the so-called geometric mean method [1], [3].

2.2 Measures of accuracy

We adjusted the selected measures of forecast accuracy [2], [6] to the measures of the accuracy of priorities derived by approximation methods based on pairwise comparisons. To see how accurate the priorities were, the priorities obtained with approximation methods were compared to the priorities obtained with the eigenvalue method, which in this paper is assumed as an exact method. The error is defined as the difference between the priority obtained with the exact method and the priority obtained with an approximation method. The adjusted measures are as follows.

Mean absolute deviation (MAD) is computed by taking the sum of the absolute values of the individual errors and dividing by the number of errors:

$$MAD = \frac{1}{r}\sum_{l=1}^{r}|p_l^e - p_l^a|, \tag{1}$$

where p_l^e is the exact l^{th} priority, p_l^a is the approximate l^{th} priority, and r is the number of simulations regarding CR.

Mean absolute percentage deviation ($MAPD$) is calculated by taking the sum of the absolute values of the individual errors, dividing by the sum of exact priorities, and multiplying by 100:

$$MAPD = \frac{\sum_{l=1}^{r}|p_l^e - p_l^a|}{\sum_{l=1}^{r} p_l^e} \times 100. \tag{2}$$

2.3 Procedure of measuring accuracy of approximation methods for deriving priorities

The procedure of measuring the accuracy of the approximation methods for priorities derivation is based on the simulation. We initiate from the perfectly consistent matrix A with expressed judgments on criteria importance or preferences to alternatives. Then we change a particular element in matrix A so that the inconsistency increased. In this procedure, CR is used as a discrete variable. As we want to measure the priorities' accuracy when a decision-maker is acceptably consistent ($CR \leq 0.1$), the CR's values from 0.01 to 0.1 are considered. First, we obtained the priorities' values with the exact method and then with approximation methods listed in section 2.1. Finally, the accuracy measures MAD and $MAPD$ are calculated by (1) and (2) for each priority and approximation method. For each approximation method, the average of the MAD values is calculated, as well as the average of the $MAPD$ values. The approximation method where the mean MAD and $MAPD$ values are the lowest should be used to prepare the multi-criteria decision-making basis.

3 NUMERICAL EXAMPLE WITH RESULTS

Let us illustrate the procedure of measuring the accuracy of the approximation methods for priorities derivation with a numerical example. The initial A matrix is given as follows:

$$\begin{bmatrix} 1 & 3 & 6 \\ 1/3 & 1 & 2 \\ 1/6 & 1/2 & 1 \end{bmatrix}. \tag{3}$$

$CR = 0$ shows that (3) is a perfectly consistent matrix. Then we changed the element a_{23} so that the inconsistency increased: 2.5 ($CR = 0.01$), 3 ($CR = 0.02$), 3.5 ($CR = 0.03$), 3.75 ($CR = 0.04$), 4 ($CR = 0.05$), 4.25 ($CR = 0.06$), 4.51 ($CR = 0.07$), 4.74 ($CR = 0.08$), 5.02 ($CR = 0.09$), 5.26 ($CR = 0.1$). In (3), the reciprocal values of a_{23} must be calculated for a_{32}, as well.

We calculated the values of priorities p_1, p_2, and p_3 with exact and five approximation methods. Based on these values it can be concluded that the higher CR, the more the priorities obtained differ from those at $CR = 0$. This applies to all the methods used, except for II when used for the calculation of p_1, as the first column in (3) does not change in simulations.

MAD and $MAPD$ were calculated by (1) and (2) so that the exact and approximation values of priorities at $CR = 0.01$ to $CR = 0.1$, $r = 10$, were considered. At $CR = 0$, namely, the exact value is equal to the approximation value, regardless of the approximation method used.

Table 1: Values of accuracy measures for priorities obtained with approximation methods.

Approximation method	***MAD***				***MAPD***			
	p_1	p_2	p_3	*Average*	p_1	p_2	p_3	*Average*
I	0.0482	0.0487	0.0003	0.0324	7.482	18.030	0.350	8.62
II	0.0228	0.0354	0.0053	0.0212	3.539	13.106	6.184	7.61
III	0.0128	0.0066	0.0024	0.0073	1.987	2.444	2.801	2.41
IV	0.0051	0.0034	0.0019	0.0035	0.792	1.259	2.217	1.42
V	0	0	0.001	0.0003	0	0	1.167	0.39

The results in Tab. 1 show that on average, approximation method V, the geometric mean method, gives the most accurate priorities' values: mean absolute deviation is 0.0003 (MAD = 0.0003), and the sum of the absolute values of individual errors present 0.39 % of the sum of the exact priorities ($MAPD$ = 0.39). Moreover, this method gives perfectly accurate values of p_1 and p_2 ($MAD = MAPD = 0$), and among the considered approximation methods, the second most accurate value of p_3 (MAD = 0.001, $MAPD$ = 1.167). The second most accurate approximation method is, on average, the approximative eigenvector method based on normalization (IV), followed by the approximation method based on the average of priorities obtained with approximation methods I and II (III), and the approximation method based on the reciprocal value of the sum of the values in each column of *A* (II). The least accurate approximation method is I which is based on the ratio of the sum of the values in each row and the sum of all values in the matrix *A*. The same order of accuracy of the approximation methods also applies to p_1 and p_2. For calculating p_3 values, however, the approximation method I has been shown to be the most accurate, followed by the approximation methods V, IV, III, and II.

4 CONCLUSIONS

The procedure for measuring the accuracy of the approximation methods for deriving priorities is useful for advising on the selection of approximation methods to those users who express their judgments and preferences by pairwise comparisons but do not have access to computer programs in which the calculation of priorities is based on the eigenvalue method.

The results of the considered numerical example show that on average, the most accurate approximation method for priorities derivation, based on pairwise comparisons, is the geometric mean method. This is in line with the summarization [4] that simulations did not identify significant differences between the geometric mean and eigenvalue method.

The numerical example was limited to the matrix of order 3 x 3, i.e., the lowest order in which inconsistencies can arise, and to relatively large ratios between matrix *A* elements. Further research can be oriented toward the matrices of higher order, with different ratios between matrix *A* elements in several simulations.

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RISK MANAGEMENT IN HEALTHCARE: DEX (DECISION EXPERT) EVALUATION MODEL

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Abstract: Quality management is a fundamental effort of modern healthcare organizations. Risk based thinking and risk management are important tools to improve and assure the quality of healthcare services. We present an evaluation model that aims to improve the process of risk evaluation in healthcare organizations. We used Decision Expert (DEX), a qualitative multi-criteria decision method, and software DexiSuite – DexiWin, to design the presented model. The model allows increased transparency of the decision process, addresses uncertainty in risk evaluation and enables in-depth analysis of suggested results in healthcare.

Keywords: Multi-criteria decision making, Risk evaluation, Healthcare, Decision support, DEX

1 INTRODUCTION

Quality management in healthcare aims to provide appropriate systems, policies and processes to minimize patients' harm while improving outcomes [10]. The International Organization for Standardization (ISO) provides precise and specific standards to support healthcare organizations in their efforts to manage and improve quality [2]. The ISO standards define risk as an effect of uncertainty on objectives or a deviation from the expected that can be positive, negative or both and can address, create, or result in opportunities and threats [6]. Risk management is the organisations' effort to manage risks related to the safety of all personnel, and the strategies, operational and financial integrity of the healthcare organization [1]. The focus of this paper is risk evaluation which is one of the main phases of risk assessment (risk assessment includes risk identification, risk analysis and risk evaluation). Its purpose is to evaluate the level of risk in order to compare risks and determine the adequacy of the existing controls or select a set of additional controls [9].

The risk matrix is a risk evaluation tool that uses likelihood and consequence criteria to determine the risk level as the multiplication product between the two criteria. It was first introduced in aviation and has since been widely used in various high-risk industries such as nuclear power, oil, manufacturing and healthcare [9,12]. The risk matrix is widely used but has several weaknesses, such as subjectivity, lack of transparency and resolution, ambiguity of inputs and inadequate management of uncertainties, leading to misleading, inaccurate and non-transparent results that don't allow for in-depth interpretation or comparison of evaluated risks [4,11]. For this purpose, we designed a qualitative evaluation model using the Decision Expert (DEX) method. This approach was chosen since the nature of the decision problem requires qualitative results and must include multiple factors that influence the level of risk for diverse risk in healthcare. Furthermore, the evaluation model must support the evaluator in cases where accurate data is unavailable and expert knowledge is used to evaluate the level of risk, making this approach appropriately suitable for use in our case.

2 METHODS

The DEX method uses qualitative multi-attribute utility theory, where solving several small problems resolves a more complex decision problem. It uses hierarchically arranged qualitative attributes, hereafter referred to as criteria. The model consists of aggregated criteria (hierarchically higher) and basic criteria (hierarchically the lowest). The final result of the decision process (in our case the final evaluation of the risk) is represented by the hierarchically highest criterion. Alternatives are an infinite set of options (in our case risks) that are evaluated according to the basic criteria. The decision maker (in our case the risk evaluator) assesses each individual alternative by selecting the appropriate value for each of the basic criteria. The domain values of each criterion are a finite set of discrete qualitative values ordered from least to most desirable according to the decision problem. Values of hierarchically higher aggregated criteria are computed according to individually predetermined simple if-then rules or utility functions. In contrast to usual weighting sum models, the DEX method enables that the weights of the criteria are not fixed but are dependent on the values of the criteria. For example, a criterion could have little or no impact on the evaluation of the hierarchically higher criteria when its' value is neutral or negative, but a significant or absolute if its value is high [3].

The open access software DexiSuite – DexiWin [7] was used for the model development and alternatives evaluation. Each rule is defined individually; however, the software solution helps the experts by suggesting appropriate rule outcomes and flagging logically mismatched rules.

3 RESULTS

The evaluation model consists of 10 basic and 5 aggregated criteria among which aggregated criteria – 'Likelihood' and 'Consequence' are placed hierarchically highest and being used to calculate the final evaluation of risk. The risk evaluator performs the evaluation by selecting appropriate values for all basic criteria. The presentation of all included criteria and their hierarchical arrangement in the form of a Tree of criteria and the domain values and their descriptions for the basic criterion 'Trend' are presented in Figure 1.

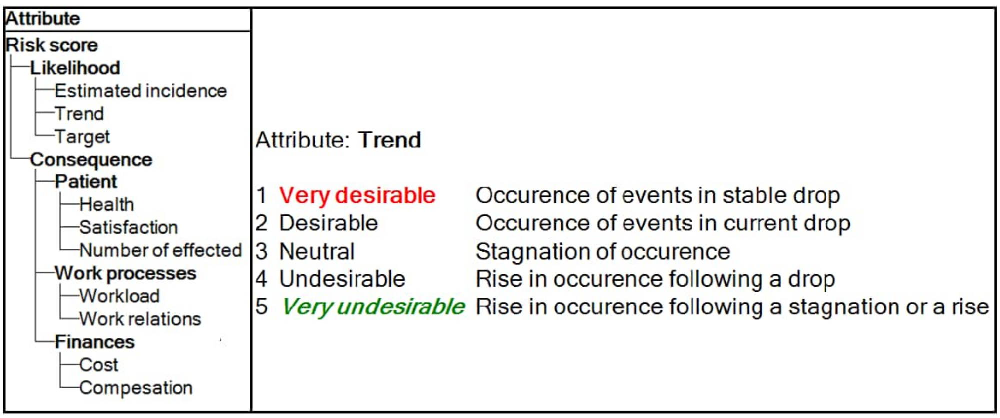

Figure 1: Tree of criteria (left) and domain values of the criterion 'Trend' with descriptions (right)

When the risk evaluator is uncertain about the outcome of the event, the value distribution is used to express this uncertainty within fuzzy set theory. If an event has minor consequences in most cases, is often harmless, but may develop catastrophic consequences in some cases, the evaluator can describe this by evaluating the criterion as Harmless with membership 0.4, Minor with membership 0.6 and Catastrophic with membership 0.1 to express the estimated probability of each outcome. As an example, we present an evaluation for two risks in a healthcare organization – the risk of fire and the risk of water damage due to a burst pipe. Both risks are evaluated identically in criteria 'Likelihood'. The graphical representation of the results can be seen in Figure 2.

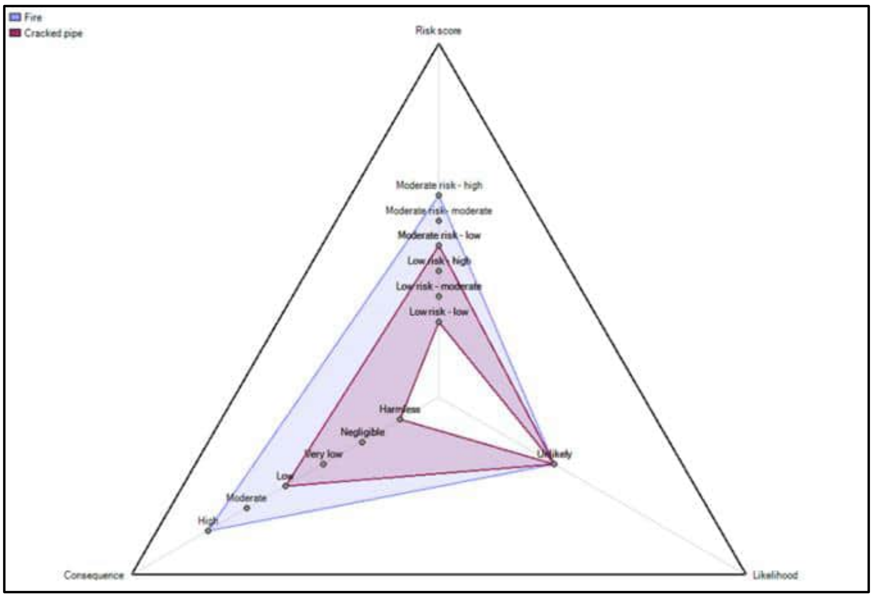

Figure 2: Graphical representation of the two options' evaluation depicting likelihood (right) consequences (left) and final evaluation (up)

The final evaluation is expressed as an interval depicting the uncertainty in the decision process. The variance of the final evaluation is the result of the uncertainty of the consequences, which is in Figure 2 represented as the variance of the 'Consequence' criterion evaluation.

4 DISCUSSION

The presented evaluation model has important implications for an improvement of risk evaluation process in healthcare organizations.

The presented evaluation model enables the risk evaluator to acknowledge that the consequences of an individual event can lead to consequences of unequal severity in different domains or areas of organizational operations. Although the risk matrix can acknowledge diverse consequences of an event and holistic evaluation of risks' consequences is encouraged in healthcare [9], the risk matrix method itself is incapable of acknowledging simultaneous instances of these impacts.

In the presented model is also incorporated in the risk evaluation uncertainty. This greatly improves risk evaluation for two reasons. First, risk management in healthcare is often limited by poor data availability [8]. The second reason why uncertainty should be addressed in healthcare risk evaluation is the uncertainty of the consequences following an event. The consequence of an individual event depend on several factors and circumstances, which is best illustrated in the more complex method of risk analysis – Event Tree Analysis (ETA). ETA traditionally uses crisp probabilities but can utilize fuzzy sets theory to handle incomplete experts' knowledge [5].

Our model incorporates a probabilistic analysis that allows healthcare risk evaluators to account for recognised uncertainties without requiring their full knowledge of the probability distribution.

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AGGREGATION OF INDIVIDUAL LINGUISTIC EVALUATIONS IN SPHERICAL ANALYTIC HIERARCHY PROCESS

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Abstract: Spherical fuzzy sets are a recently developed extension of fuzzy sets that allow uncertainty to be incorporated into multi-criteria decision processes. In the spherical analytic hierarchy process, decision makers can express their judgments using a linguistic scale. To aggregate individual linguistic judgments into linguistic interval-valued group judgments, we propose a new method, called GALJ, based on decision-maker satisfaction. Euclidean distance is used to compare the similarity of individual and group weights in the theoretical example.

Keywords: linguistic evaluations, spherical fuzzy numbers, analytic hierarchy process, group decision making

1 INTRODUCTION

Multi-criteria decision making (MCDM) addresses problems of evaluating alternatives against multiple criteria, and analytic hierarchy process (AHP) is one of the most commonly used methods for solving such problems.

Uncertainty and vagueness are often in the nature of decision making. To deal with the uncertainty of decision makers' (DMs) evaluations and to ensure that the evaluations mimic human reasoning, fuzzy sets and their extensions have been introduced into MCDM. DMs, who are often unfamiliar with decision making methods, evaluate objects using linguistic scales. In a next step, the linguistic evaluations are transformed into fuzzy sets. There are several types of fuzzy sets that can express the degree of membership, non-membership, and hesitancy of each item. In this paper, we use recently developed spherical fuzzy sets (Kutlu Gündoğdu & Kahraman, 2019) based on three-dimensional spherical geometry.

Group decision making is an important part of MCDM. Fuzzy evaluations of DMs are usually aggregated with one of the fuzzy operators and the results are often incomprehensible to the uneducated DMs, which may lead to lower satisfaction with the final compromise decision.

To overcome this drawback, we propose a new method for aggregating linguistic evaluations into linguistic interval group evaluations (GALJ) that takes into account the satisfaction of DMs.

2 SPHERICAL ANALYTIC HIERARCHY PROCESS

Spherical fuzzy sets are an extension of fuzzy sets. Their definition and a brief presentation of spherical AHP are given below (Menekşe & Camgöz Akdağ, 2022).

Spherical fuzzy set $\tilde{A}_S$ of the universe of discourse U is represented by the membership, non-membership and hesitancy membership function:

$$\tilde{A}_S = \left\{ x, \left(\mu_{\tilde{A}_S}(x), \nu_{\tilde{A}_S}(x), \pi_{\tilde{A}_S}(x) \right) \middle| x \in U \right\}, \tag{1}$$

where $\mu_{\tilde{A}_S}(x): U \to [0,1]$, $\nu_{\tilde{A}_S}(x): U \to [0,1]$, $\pi_{\tilde{A}_S}(x): U \to [0,1]$ and $0 \le \mu_{\tilde{A}_S}^2(x) + \nu_{\tilde{A}_S}^2(x) + \pi_{\tilde{A}_S}^2(x) \le 1$ for all $x \in U$, where $\mu_{\tilde{A}_S}(x)$, $\nu_{\tilde{A}_S}(x)$ and $\pi_{\tilde{A}_S}(x)$ are degrees of membership, non-membership and hesitancy of x to $\tilde{A}_S$, respectively.

Let there be n objects to compare. Each DM pairwise compares the objects using a linguistic scale (Table 1). Evaluations are summarised in linguistic pairwise comparison matrices (LPCMs) and converted to spherical fuzzy numbers to obtain spherical fuzzy pairwise comparison matrices (SFPCMs). As in the classical AHP, the consistency of the corresponding matrix of score indices (Table 1) is checked, and a consistency ratio CR of less than 0.1 is considered acceptable. Weights of the objects are derived from SFPCMs using spherical weighted geometric mean operator SWGM:

$$SWGM_w(\tilde{A}_{S1},\ldots,\tilde{A}_{Sn}) = \left(\prod_{i=1}^{n} \mu_{\tilde{A}_{Si}}^{w_i}, \left(1-\prod_{i=1}^{n}\left(1-\nu_{\tilde{A}_{Si}}^{2}\right)^{w_i}\right)^{1/2}, \left(\prod_{i=1}^{n}\left(1-\nu_{\tilde{A}_{Si}}^{2}\right)^{w_i} - \prod_{i=1}^{n}\left(1-\nu_{\tilde{A}_{Si}}^{2}-\pi_{\tilde{A}_{Si}}^{2}\right)^{w_i}\right)^{1/2} \right), \qquad (2)$$

where $w=(w_1,w_1,\ldots,w_n)$, $w_i \in [0,1]$, $\sum_{i=1}^{n} w_i = 1$ are the weights of importance. The same operator is also used to aggregate individual weights into group weights. Finally, the group weights are defuzzified using eq. (3) and then normalized.

$$\tilde{w}_i^S = \sqrt{\left|100\left(\left(3\mu_{\tilde{A}_S} - \frac{\pi_{\tilde{A}_S}}{2}\right)^2 - \left(\frac{\nu_{\tilde{A}_S}}{2} - \pi_{\tilde{A}_S}\right)^2\right)\right|} \qquad (3)$$

Table 1: Linguistic terms and corresponding spherical fuzzy numbers

Linguistic terms	(μ, ν, π)	Score Index
Absolutely more importance (AMI)	(0.9, 0.1, 0.0)	9
Very high importance (VHI)	(0.8, 0.2, 0.1)	7
High importance (HI)	(0.7, 0.3, 0.2)	5
Slightly more importance (SMI)	(0.6, 0.4, 0.3)	3
Equally importance (EI)	(0.5, 0.4, 0.4)	1
Slightly low importance (SLI)	(0.4, 0.6, 0.3)	1/3
Low importance (LI)	(0.3, 0.7, 0.2)	1/5
Very low importance (VLI)	(0.2, 0.8, 0.1)	1/7
Absolutely low importance (ALI)	(0.1, 0.9, 0.0)	1/9

3 GROUP AGGREGATION METHOD BASED ON LINGUISTIC JUDGMENTS (GALJ)

Linguistic evaluation scales are easy for DMs to use, even if they are experts in other fields and unfamiliar with MCDM, because they are intuitive and close to human thinking. Good and widely accepted decisions are the desired outcome of group MCDM. One of the prerequisites for this is a transparent decision-making process, where DMs understand the aggregation process and are satisfied with the aggregated group evaluations. One such possibility is the aggregation of individual linguistic judgments (LJs) into group interval linguistic judgments (GILJs), which we called GALJ method. Interval group judgments allow for higher satisfaction of DMs compared to crisp group judgments (Grošelj & Dolinar, 2023). We define that DM is satisfied with the GILJ if his LJ belongs to the GILJ. A group is satisfied with the GILJ if the percentage of satisfied DMs is higher than the predefined threshold t.

To determine GILJ the following rules should be followed in order:

1. GILJ is the narrowest possible interval that includes more than t individual LJs.
2. If more than t LJs are equal, then GILJ is equal to a crisp LJ.
3. If more than one GILJ satisfies Rule 1, choose the GILJ that contains a higher percentage of DMs' LJs.
4. Define the distance between GILJ and LJ as 0, if LJ belongs to GILJ, and k if LJ does not belong to GILJ and there are k-1 linguistic terms between LJ and the nearest boundary of GILJ. If multiple GILJs satisfy Rule 3, GILJ with the smallest sum of distances between it and individual LJs is chosen.

5. If more than one GILJ satisfies Rule 4, choose the GILJ with the smallest distance to EI.

Examples: Let t=50%.

- Example for Rule 1: SLI, EI, SMI, SMI, SMI, HI, HI, VHI → GILJ=[SMI,HI]
- Example for Rule 2: EI, SMI, SMI, SMI, SMI, HI, VHI → GILJ=SMI
- Example for Rule 3: EI, SMI, SMI, SMI, HI, HI, VHI → GILJ=[SMI,HI]
- Example for Rule 4: EI, SMI, SMI, SMI, HI, VHI →GILJ= [SMI,HI]
  Explanation: We have two possibilities for GILJ: [SMI, HI] and [EI,SMI]. The distance from EI to [SMI,HI] is equal to 1 and the distance from VHI to [SMI,HI] is equal to 1, so their sum is equal to 2. The distance from HI to [EI,SMI] is equal to 1 and the distance from VHI to [EI,SMI] is equal to 2, so their sum is equal to 3. Therefore, we chose [SMI,HI].
- Example for Rule 5: EI, EI, SMI, SMI, SMI, HI, HI → [EI,SMI]

GILJs are then converted to interval-valued spherical fuzzy numbers using Table 1. To derive group weights, equations (1), (2) and (3) are generalized to interval-valued evaluations (Otay, 2023):

Interval-valued spherical fuzzy set is defined as follows:

$$\tilde{A}_S=\left\{x,\left(\left[\mu^L_{\tilde{A}_S}(x),\mu^U_{\tilde{A}_S}(x)\right],\left[\nu^L_{\tilde{A}_S}(x),\nu^U_{\tilde{A}_S}(x)\right],\left[\pi^L_{\tilde{A}_S}(x),\pi^U_{\tilde{A}_S}(x)\right]\right)\middle|x\in U\right\}, \tag{4}$$

where $0\le\mu^L_{\tilde{A}_S}(x)\le\mu^U_{\tilde{A}_S}(x)\le1$, $0\le\nu^L_{\tilde{A}_S}(x)\le\nu^U_{\tilde{A}_S}(x)\le1$ and $0\le\left(\mu^U_{\tilde{A}_S}(x)\right)^2+\left(\nu^U_{\tilde{A}_S}(x)\right)^2+\left(\pi^U_{\tilde{A}_S}(x)\right)^2\le1$, $x\in U$.

$$IVSWGM_w(\tilde{A}_{S1},...,\tilde{A}_{Sn})=$$

$$=\left(\begin{array}{l}\left[\prod_{i=1}^n\left(\mu^L_{\tilde{A}_{Si}}\right)^{w_i},\prod_{i=1}^n\left(\mu^U_{\tilde{A}_{Si}}\right)^{w_i}\right],\left[\left(1-\prod_{i=1}^n\left(1-\left(\nu^L_{\tilde{A}_{Si}}\right)^2\right)^{w_i}\right)^{1/2},\left(1-\prod_{i=1}^n\left(1-\left(\nu^U_{\tilde{A}_{Si}}\right)^2\right)^{w_i}\right)^{1/2}\right],\\ \left[\left(\prod_{i=1}^n\left(1-\left(\nu^L_{\tilde{A}_{Si}}\right)^2\right)^{w_i}-\prod_{i=1}^n\left(1-\left(\nu^L_{\tilde{A}_{Si}}\right)^2-\left(\pi^L_{\tilde{A}_{Si}}\right)^2\right)^{w_i}\right)^{1/2},\left(\prod_{i=1}^n\left(1-\left(\nu^U_{\tilde{A}_{Si}}\right)^2\right)^{w_i}-\prod_{i=1}^n\left(1-\left(\nu^U_{\tilde{A}_{Si}}\right)^2-\left(\pi^U_{\tilde{A}_{Si}}\right)^2\right)^{w_i}\right)^{1/2}\right]\end{array}\right), \tag{5}$$

$$\tilde{w}^S_i=\sqrt{\left|100\left(\left(3\mu^L_{\tilde{A}_S}-\frac{\pi^L_{\tilde{A}_S}}{2}\right)^2+\left(3\mu^U_{\tilde{A}_S}-\frac{\pi^U_{\tilde{A}_S}}{2}\right)^2-\left(\frac{\nu^L_{\tilde{A}_S}}{2}-\pi^L_{\tilde{A}_S}\right)^2-\left(\frac{\nu^U_{\tilde{A}_S}}{2}-\pi^U_{\tilde{A}_S}\right)^2\right)\right|} \tag{6}$$

4 EXAMPLE

Eight DMs of equal importance evaluated four objects. The DMs satisfaction threshold is set at t=50%. Their LPCMs (Table 2) were aggregated into group LPCM using the new GALJ method, which is shown in Table 3. GILJs were transformed into interval-valued spherical fuzzy numbers using Table 1. Group interval-valued spherical weights were derived using eq. (5) and defuzzified using eq. (6). The results are presented in Table 3.

Table 2: DMs linguistic pairwise comparison matrices

EI	SMI,SMI,EI,HI, SLI,HI,HI,SMI	HI,EI,SMI,EI, HI,SMI,SLI,EI	SLI,SLI,LI,VHI, SLI,LI,SLI,SLI
SLI,SLI,EI,LI, SMI,LI,LI,SLI	EI	EI,EI,EI,LI, SMI,SLI,LI,EI	VLI,ALI,VLI,HI, EI,VLI,VLI,ALI
LI,EI,SLI,EI,LI, SLI,SMI,EI	EI,EI,EI,HI, SLI,SMI,HI,EI	EI	LI,LI,LI,VHI, VLI,LI,EI,LI
SMI,SMI,HI,VLI, SMI,HI,SMI,SMI	VHI,AMI,VHI,LI, EI,VHI,VHI,AMI	HI,HI,HI,VLI, VHI,HI,EI,HI	EI

To validate the GALJ method, we compared the defuzzified group weights with the group weights obtained by SWGM (eq. (2)) and defuzzified by eq. (3) (Table 4). We used Euclidean distance to compare the similarity of the defuzzified individual weights (Table 4) and defuzzified group weights.

Table 3: Group LPCM, derived Interval-valued spherical group weights, defuzzified group weights and Euclidean distance for GALJ

Group LPCM				Interval-valued spherical group weights	Defuzzified group weights
EI	[SMI,HI]	[EI,HI]	SLI	([0.495,0.560],[0.428,0.464],[0.296,0.352])	0.274
[LI,SLI]	EI	[SLI,EI]	[ALI,VLI]	([0.278,0.376],[0.604,0.723],[0.243,0.285])	0.169
[LI,EI]	[EI,SMI]	EI	LI	([0.387,0.461],[0.508,0.588],[0.282,0.360])	0.222
SMI	[VHI,AMI]	HI	EI	([0.640,0.659],[0.327,0.338],[0.281,0.283])	0.336
				ED	0.078

Table 4: Individual defuzzified weights, spherical group weights, obtained by SWGM, defuzzified group weights and Euclidean distance for method, using SWGM

DM1	DM2	DM3	DM4	DM5	DM6	DM7	DM8	Spherical group weights	Defuzzified group weights
0.279	0.256	0.234	0.322	0.250	0.259	0.249	0.256	(0.508, 0.469, 0.322)	0.267
0.187	0.159	0.198	0.218	0.279	0.165	0.154	0.159	(0.370, 0.619, 0.280)	0.190
0.194	0.224	0.209	0.322	0.166	0.221	0.293	0.224	(0.446, 0.528, 0.316)	0.232
0.340	0.360	0.358	0.138	0.305	0.355	0.305	0.360	(0.575, 0.420, 0.279)	0.311
								ED	0.084

The comparison of group crisp weights shows that the GALJ method weights the more important objects higher and the less important objects lower than the method using SWGM. The Euclidean distance comparing individual and group crisp weights is lower for the GALJ method, indicating that the GALJ method results are more similar to the individual weights.

5 CONCLUSIONS

The paper introduces the GALJ method, which provides a way to aggregate individual LJs into GILJ. GALJ is presented as part of the spherical AHP method. The result of using GALJ can be a more transparent and understandable aggregation process, which can lead to a more satisfactory group decision process. In future work, GALJ can be extended to other types of fuzzy numbers and other MCDM methods, and the weights of DMs should be included.

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THE WINGS MODEL FOR CHOICE OF INNOVATION STRATEGY

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Abstract:
The paper introduces a novel application of the WINGS method, specifically tailored to address decision problems with an infinite number of alternatives. The efficacy of this approach is exemplified through a practical illustration, showcasing its utility in determining an innovation strategy by allocating funds among resources to achieve desired changes in key performance indicators.

Keywords: innovation strategy, key performance indicator (KPI), MCDA , WINGS

1 INTRODUCTION

The Weighted Influence Non-linear Gauge System (WINGS) model is a powerful framework designed to analyze interrelated components and their influences in complex decision problems [1]. While previous applications of the WINGS method focused on discrete problems with a limited number of alternatives, this model tackles the challenge of allocating funds among resources within an innovation strategy. By incorporating an algorithm capable of handling the high non-linearity of the model and finding optimal solutions, the WINGS method proves to be effective in addressing decision problems with an infinite number of alternatives. In this paper, we introduce the WINGS model and its application in determining the allocation of funds to key resources that drive the success of a firm's innovation efforts. We analyze the system from different perspectives and utilize specific key performance indicators (KPIs) to measure success. Through experiments and comparisons of sample strategies, we demonstrate the model's ability to provide valuable insights for decision-making in innovation strategy. The flexibility of the model allows decision-makers to customize its structure according to their preferences, providing a comprehensive perspective for effective decision-making.

2 THE WINGS MODEL

2.1 Brief Introduction to WINGS

The WINGS method is designed to analyze the system of interrelating components. The power of each component and its potential influences as assessed by experts are expressed using a selected verbal scale (e.g., very weak, weak, moderate, strong, very strong). These estimations are then translated into a suitable numerical scale (e.g., 1, 2, 3, 4, 5; with zero representing no impact). The experts' estimations can be integrated in a desired manner (e.g., mean, geometric mean, etc.). All the numerical values are inserted into the direct strength-influence matrix, denoted as $\mathbf{D}$, which is an $n \times n$ matrix with elements d_{ij} (n refers to the number of components). Here d_{ii} represents a strength of component i, while d_{ij} represents the influence of component i on component j. The matrix $\mathbf{D}$ is scaled according to the following formula:

$$\mathbf{S} = \frac{1}{s}\mathbf{D}, \tag{1}$$

where $\mathbf{S}$ denotes the *scaled strength-influence matrix* and the scaling factor is defined as the sum of all elements of the matrix $\mathbf{D}$, i.e.

$$s = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij} \tag{2}$$

Finally, the effective values, which encompass the internal strength and the effects of all direct and indirect impacts, are calculated using the formula:

$$\mathbf{T} = \mathbf{S} + \mathbf{S}^2 + \mathbf{S}^3 + \ldots = \mathbf{S}(\mathbf{I} - \mathbf{S})^{-1}, \tag{3}$$

The WINGS method yields four measures that characterize the system components. *Total impact* represents the influence of the component i on all other components in the system. It is defined as the row sum $I_i = \sum_{j=1}^{n} t_{ij}$ of matrix $\mathbf{T}$ elements. *Total receptivity* represents the influence of all other components in the system on the component i. It is defined as the column sum $R_j = \sum_{i=1}^{n} t_{ij}$ of matrix $\mathbf{T}$ elements. Total impact and receptivity can be combined to obtain *total involvement* which is $I_i + R_i$ – representing all influences exerted on and received by component i. Additionally, the *role* (position) is calculated as the difference $I_i - R_i$. If the role is positive, component i belongs to the *influencing (cause)* group; while if it is negative the component i belongs to the *influenced (result)* group.

2.2 Model of Innovation Strategy

This paper introduces a framework for determining the allocation of funds to key resources that drive the success of a firm's innovation efforts [2, 3]. The study identifies several crucial resources that play a vital role in innovation, including research and development facilities, technology infrastructure, collaboration and open innovation, strategic alliances, marketing, sales and distribution, intellectual property, and employee education and training. The assessment of innovation strategies is conducted from four different perspectives, with each perspective utilizing specific key performance indicators (KPIs) as measures of success.

The market perspective consists of five KPIs: the number of new products launched, time-to-market, market disruption, market share gain, and access to new markets. The customer perspective identifies four KPIs: customer satisfaction, customer engagement and co-creation, and enhancement of brand equity. The next two KPIs, intellectual property portfolio strength and efficiency improvement, represent the firm's competitive advantage. The final perspective focuses on environmental and social impacts.

The identified resources and KPIs form a complex system of interconnected cause-and-effect relationships. To effectively model and experiment with various decision options, the WINGS method is employed. Expert knowledge is utilized to develop the WINGS map, which includes concepts and their relationships, along with corresponding values characterizing these concepts and relationships.

The resulting network is complex, so it is divided into four sub-networks, each representing a single perspective (see Fig. 1 and 2). The KPIs, highlighted with a light-blue background, occupy the top positions in each sub-network. The bottom section of each sub-network includes the resources that influence the respective KPIs. The numbers in parentheses represent an expert assessment of the state of the KPIs and resources using the scale defined above. Similarly, the labels on arrows represent the assessment of impact. It should be noted that the KPIs may not be independent, as evidenced by the relationship between 'market disruption' and 'access to new markets' (see Fig. 1, left). At the base of the network is the strategy (highlighted with a light-yellow background), which is characterized by the percentage distribution of expenditures on individual resources $(s_1, s_2, \ldots)$. The initial state (without intervention) is defined by the zero distribution of inputs.

2.3 Results of experiments

To assess the viability of the proposed model, two distinct innovation strategies were designed. Within the context of the decision problem, an innovation strategy is represented by the allocation of financial resources among the crucial innovation resources. Table 1 presents two sample strategies. Both strategies emphasize the importance of 'Research and Development

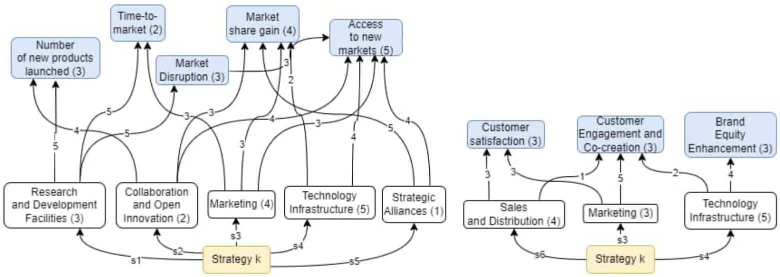

Figure 1: Left: map of 'Market perspective'; Right: map of 'Customer perspective'.

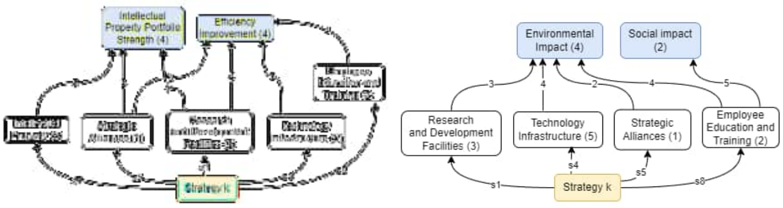

Figure 2: Left: map of 'Competitive advantage perspective'; Right: map of 'Environmental and Social Impact'.

Facilities,' which is currently evaluated as moderate and receives 23% of the funds. Strategy 1 allocates a higher percentage of funds to support 'Strategic Alliances' (assessed as very weak) with 23%, 'Collaboration and Open Innovation' (assessed as weak) with 12%, and 'Employee Education and Training' (assessed as weak) with 11%. In contrast, Strategy 2 allocates more funds to 'Marketing' (assessed as strong) with 23%, at the expense of 'Strategic Alliances,' which receives 17%, and 'Employee Education and Training,' which is supported at a reduced rate of 6%.

Table 1: Percentage allocation of funds for sample innovation strategies. The number in parentheses represents the estimated current state of the resource on a scale of 1 to 5.

Resource	Strategy 1 [%]	Strategy 2 [%]
Research and Development Facilities (3)	23	23
Technology Infrastructure (5)	9	9
Collaboration and Open Innovation (2)	12	6
Strategic Alliances (1)	23	17
Marketing (4)	6	23
Sales and Distribution (4)	7	7
Intellectual Property (3)	9	9
Employee Education and Training (2)	11	6
Sum	100	100

To compare the two sample strategies, we calculated the 'total involvement' metric. Table 2 showcases the relative changes in the positions of KPIs for both strategies compared to the baseline state without intervention. Both strategies demonstrate significant improvements in 'time-to-market' and 'number of new products launched.' The most notable distinction lies in the position of 'customer engagement and co-creation,' which is significantly advanced in Strategy 2, while the ranking of 'Environmental Impact' declines. The final selection and detailed comparison of the strategies heavily rely on the preferences of the decision-maker.

The presented model significantly differs from previous examples of the WINGS method applications. Originally, WINGS was designed to support decision-making in discrete problems with a limited number of alternatives. However, in this case, we encounter an infinite number

Table 2: Comparison of two sample strategies results.

Strategy 1	Rel. change [%]	Strategy 2	Rel. change [%]
Time-to-market	1,23	Time-to-market	2,62
No of new prod. launched	1,18	Customer engag. and co-creat.	0,63
Intellect. prop. portf. Strength	0,78	No of new prod. launched	0,46
Efficiency improvement	0,71	Intellect. prop. portf. strength	0,38
Environmental impact	0,32	Market share gain	0,24
Market share gain	0,22	Efficiency improvement	0,04
Market disruption	0,15	Market disruption	-0,03
Social impact	-0,73	Customer satisfaction	-0,21
Access to new markets	-0,78	Environmental impact	-0,32
Customer engag. and co-creat.	-1,35	Access to new markets	-0,87
Brand equity enhancement	-1,38	Brand equity enhancement	-1,64
Customer satisfaction	-1,60	Social impact	-1,65

of strategies represented by any distribution of funds among resources. Such a problem requires the application of an algorithm capable of finding the optimal solution. Given the high level of non-linearity in the model, a heuristic approach would be the most suitable choice.

The final structure of the model will depend on the preferences of the decision-maker. It could potentially be developed as a multiple criteria model, where the KPIs serve as individual criteria. Alternatively, the KPIs could be aggregated into a smaller set of strategic goals. Eventually, the strategic goals can be integrated into a single overarching goal, resulting in the model with a single criterion.

3 Conclusions

The presented WINGS model offers a unique approach to analyzing interrelated components and their influences, particularly in solving decision problems with an infinite number of alternatives. Unlike previous applications of the WINGS method, which focused on discrete problems with limited number of alternatives, this model addresses the complex challenge of allocating funds among resources within an innovation strategy. By incorporating an algorithm capable of finding optimal solutions and handling the high non-linearity of the model, the WINGS method proves to be suitable for tackling such problems effectively.

Additionally, the flexibility of the model empowers decision-makers to customize the structure of a multiple criteria model according to their preferences. This customization can be achieved by either incorporating the original KPIs or by aggregating them into a smaller set of strategic goals. The integration of strategic goals into a single criterion offers decision-makers a holistic perspective for making informed decisions in innovation strategy. By customizing the model to suit their specific requirements, decision-makers can obtain a comprehensive and focused view, empowering them to make effective decisions in the dynamic landscape of innovation.

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AGGREGATING INDIVIDUAL WEIGHTS INTO GROUP WEIGHTS IN BEST-WORST METHOD

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Abstract: The Best–Worst method is a relatively new multi-criteria decision-making method. To aggregate the individual weights into group weights, the arithmetic mean is usually used. In this paper, the new aggregating method is presented. The importance of the decision makers is based on how similar they select the best/most important criteria. The use of new method is applied on one theoretical and two real case problems. The results show that the proposed method is suitable for application.

Keywords: Best–Worst Method (BWM), group decision making, aggregating method, decision-makers

1 INTRODUCTION

Best–Worst method (BWM) is one of the multi-criteria decision-making methods introduced by Rezaei in 2015 (Rezaei, 2015), as a simplified extension of Analytic Hierarchy Process (AHP) method. BWM is applied to problems where a number of alternatives is evaluated with respect to a number of criteria in order to select the best alternative(s). The method is based on pairwise comparisons. Decision makers (DMs) identify the best (most important, most desirable) and the worst (least important, least desirable) criteria. The DMs make pairwise comparisons between each of these two criteria (the best and the worst criteria) and other criteria (Rezaei, 2015).

Since the method is relatively new, there are not many known ways to aggregate the individual weights into group weights. The most common way to aggregate the weights is to use the arithmetic mean (Wang & Fu, 2019), where all DMs have the same weights. However, DMs are not necessarily equally important. One way is to consider objective weights of DMs importance that take into account the different knowledge and experience of DMs through their different evaluations (Koksalmis & Kabak, 2019). The aim of this paper is to present a new method for determining DMs weights of importance using a similarity-based approach and then using the weighted arithmetic mean to aggregate DMs weights.

2 BEST – WORST METHOD

In this section, the steps of BWM that can be used to derive the weights of the criteria are described.

Step 1: Determine a set of n decision criteria for comparison.

Step 2: DMs identify the best (most important) – B and the worst (least important) – W criterion.

Step 3: DMs determine the preference of the best criterion over all the other criteria using a scale from 1 (criteria are the same important) to 9 (the best criterion is extremely more important). The result is Best-to-Others vector: $A_B = (a_{B1}, a_{B2}, ..., a_{Bn})$, where a_{Bj} indicates the preference of the best criterion B over criterion j.

Step 4: DMs determine the preference of all the criteria over the worst criterion using a scale from 1 and 9. The result is Others-to-Worst vector: $A_W = (a_{W1}, a_{W2}, ..., a_{Wn})^T$, where a_{Wj} indicates the preference of the criterion j over the worst criterion W.

Step 5: The ideal weight for the criterion is the one, where $\frac{w_B}{w_j} = a_{Bj}$ and $\frac{w_j}{w_W} = a_{jW}$ for *j=1,…,n*. To derive the weights following optimization program is used.

$$\begin{aligned}
&\min \xi \\
&\text{s.t. } \left| \frac{w_B}{w_j} - a_{Bj} \right| \le \xi \text{, for all j} \\
&\left| \frac{w_j}{w_W} - a_{jW} \right| \le \xi \text{, for all j} \qquad (1) \\
&\sum_j w_j = 1 \\
&w_j \ge 0 \text{; for all j}
\end{aligned}$$

Step 6: Use arithmetic mean to aggregate individual weights into group weights.

3 A NEW METHOD TO AGGREGATE THE INDIVIDUAL WEIGHTS INTO GROUP WEIGHTS

In this paper, we propose to use the weighted arithmetic mean to aggregate the individual weights using the importance weights of DMs. The weights of DMs are based on how similar they choose the best/most important criterion. The most numerous group of DMs that selected the same criterion as the best/most important, is given the highest weight. In this way, DMs are encouraged to select the same criterion as the best criterion.

Let n be the number of criteria and k be the number of DMs. Let n_i be the number of DMs, that selected criterion i as the best. Then

$$\sum_{i=1}^{n} n_i = k \qquad (2)$$

We define the weight of DM that selected i as the best criterion as:

$$\omega_i = \frac{n_i}{\sum_{i=1}^{n} n_i^2} \qquad (3)$$

The sum of all weights of DMs is equal to 1:

$$n_1\omega_1 + n_2\omega_2 + \ldots + n_n\omega_n = n_1 \frac{n_1}{\sum_{i=1}^{n} n_i^2} + n_2 \frac{n_2}{\sum_{i=1}^{n} n_i^2} + \ldots + n_n \frac{n_n}{\sum_{i=1}^{n} n_i^2} = \frac{n_1^2 + n_2^2 + \ldots + n_n^2}{\sum_{i=1}^{n} n_i^2} = 1 \qquad (4)$$

3.1 Example 1:

The theoretical example with five DMs and three criteria is selected. Let three decision makers DM1, DM2, DM3 selected criterion *a* as the best criterion, DM4 selected criterion *b* as the best criterion, and DM5 selected criterion *c* as the best criterion, then the weights are as follows:

$$\omega_1 = \omega_2 = \omega_3 = \frac{3}{3^2 + 1^2 + 1^2} = \frac{3}{11} = 0,27$$

$$\omega_4 = \omega_5 = \frac{1}{3^2 + 1^2 + 1^2} = \frac{1}{11} = 0,09$$

DM1, DM2 and DM3 have the same weights (0,27) and DM4 and DM5 have the same weights (0,09). The sum of all weights is equal 1.

4 VALIDATION OF THE PROPOSED METHOD

To validate the new method, we presented two examples. We will measure the similarity between individual assessments and group weights with Weighted Total Deviation (WTD) (Hosseini Dehshiri, Emamat, & Amiri, 2022). The smaller the WTD is, the better the results and the method are.

$$WTD = \sum_{i=1}^{k}\sum_{j=1}^{k}\omega_i\left(\left(b_{Bj}^{(i)} - \frac{w_B}{w_j}\right)^2 + \left(b_{jW}^{(i)} - \frac{w_j}{w_W}\right)^2\right) - \sum_{i=1}^{k}\omega_i\left(b_{BW}^{(i)} - \frac{w_B}{w_W}\right)^2 \tag{5}$$

4.1 Example 2:

The new method is presented on a part of DPSIR model for evaluating criteria affecting forest management (Šmidovnik & Grošelj, 2021). Three criteria (timber production, research function and climate function) were evaluated by four DMs. Three DMs selected timber production as the best/most important criterion, while fourth DM selected research function as the best/most important criterion (green coloured fields in Table 1). Table 1 shows the BWM results for each of DMs, the weights of importance of DMs, group weights calculated using arithmetic mean and proposed method, and WTD.

Table 1: The individual results of BWM, criteria weights derived by arithmetic mean and the new proposed method, weights of importance of DMs and WTD

DM	Timber production	Research function	Climate function	Weights of DMs
1	0.650	0.125	0.225	0.286
2	0.663	0.284	0.053	0.286
3	0.257	0.600	0.143	0.143
4	0,744	0.053	0.203	0.286
	Aggregated criteria weights			WTD
Arithmetic mean	0.579	0.266	0.156	0.609
New method	0.624	0.218	0.158	0.161

Results show that DMs that selected timber production as the best/most important criterion have a higher weight than those DM that selected the research function as the best/most important criterion. When the new aggregation method is used, the weight of timber production is higher than when the arithmetic mean is used for aggregation. The results show that the WTD is smaller using our method.

4.2 Example 3:

Seven criteria were evaluated by eight DMs (Hosseini Dehshiri et al., 2022). Four DMs selected C5 as the best/most important criterion, two DMs selected C3 as the best/most important criterion and two DMs selected C6 as the best/most important criterion (green coloured fields in Table 1). Table 1 shows the BWM results for each DMs, the weights of importance of DM, the group weights calculated by arithmetic mean and proposed method, and WTD.

We can see that DMs that selected C5 as the best/most important criterion have a higher weight than those DMs that selected the other criteria as the best/most important criterion.

When the new aggregation method is used, the weight of C5 is higher than when the arithmetic mean is used for aggregation. The results show that the WTD is smaller with our method.

Table 2: The individual results of BWM, criteria weights derived by arithmetic mean and the new proposed method, weights of importance of DMs and WTD

DM	C1	C2	C3	C4	C5	C6	C7	Weights of DMs
1	0.086	0.115	0.173	0.173	0.324	0.086	0.043	0.167
2	0.117	0.176	0.088	0.176	0.333	0.070	0.039	0.167
3	0.171	0.171	0.057	0.171	0.309	0.086	0.034	0.167
4	0.096	0.096	0.345	0.108	0.193	0.128	0.034	0.083
5	0.161	0.107	0.161	0.080	0.161	0.293	0.038	0.083
6	0.199	0.079	0.356	0.132	0.132	0.066	0.035	0.083
7	0.169	0.085	0.169	0.031	0.169	0.292	0.085	0.083
8	0.095	0.126	0.076	0.039	0.348	0.190	0.126	0.167
	Aggregated criteria weights							WTD
Arithmetic mean	0.137	0.120	0.178	0.114	0.246	0.151	0.054	0.082
New method	0.130	0.129	0.151	0.123	0.273	0.137	0.056	0.009

This study shows that the proposed method successfully aggregates the individual weights into group weights. The WTD is smaller in both examples when the aggregation of individual weights into group weights was calculated using the new method.

5 CONCLUSION

In this paper, we discussed how to aggregate the individual weights into group weights in BWM. BWM method is relatively new and aggregation of weights is usually done using the arithmetic mean. We have proposed a new method for aggregation. The weights of DMs are based on how similarly they select the best/most important criterion. To validate and verify the proposed method, the WTD was introduced to compare the proposed method and aggregation with arithmetic mean. For presented examples, the WTD is smaller for the new method.

In summary, the new aggregation method improves the group BWM. The advantage of this method is that it emphasises the weight of the criterion that is selected as the best/most important by most DMs. In summary, the proposed method can be used to aggregate the weights in group BWM.

In the future research, we will upgrade the proposed method to also consider the numerical assessment of the best criterion given by DMs.

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AHP APPLICATION TO MULTISTAGE BIPOLAR METHOD

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Abstract: Multistage Bipolar Method considered in the paper deals with multistage decision processes. Multistage alternatives are not compared directly to each other, but they are confronted to the stage sets of reference objects – desirable and non-acceptable. The original version of the method is based on Electre methodology. The aim of the paper is to consider the possibility of replacement the Electre methodology by a very popular AHP approach.

Keywords: multistage decision process, multiple criteria decision analysis (MCDA), Multistage Bipolar Method, Analytic Hierarchy Process (AHP)

1 INTRODUCTION

The Multistage Bipolar Method (MBM) described in detail in open access papers [4, 5, 6] is an extension of the classical Bipolar procedure, introduced by Konarzewska- Gubała [1]. The multistage alternatives which consist of stage alternatives are not compared directly to each other, but (stage) reference objects are used for comparisons according to Electre methodology [2]. On the basis of these comparisons the position of the stage alternatives in relation to the Bipolar reference system is assigned. At the end multistage alternatives are assigned to predefined classes and the final solution is pointed from the multistage alternatives belonging to the class with the smallest number.

In practice, one of the most frequently used multi-criteria procedures is the AHP method [3]. Its main feature is application of nine-point scale, called the Saaty's scale.

The aim of the paper is to discuss the possibility of including some elements of AHP methodology to the MBM. We will consider two cases. In both of them we will replace Electre comparisons among elements of reference sets and stage alternatives by comparisons applying Saaty's scale. In the first case the rest of the methods is unchanged. In the second one we will replace next steps of original MBM by a simplified procedure.

The paper consists of 5 sections. After introduction in section 1 notation is shortly presented in Section 2 (the full notation is elaborated in [4]). Pairwise comparisons between two elements in AHP method are analysed in Section 3. Inclusion of AHP comparisons to the original MBM is described in Section 4. Simplified version of the method, based on AHP properties is proposed in Section 5.

2 SHORT NOTATION

We will consider a finite, discrete multistage decision process. Let us denote:

- T – number of stages of the process ($t = 1,\ldots,T$),
- $\mathbf{a}_t$ – stage alternative,
- $\mathbf{a} = (\mathbf{a}_1,\ldots,\mathbf{a}_T)$ – multistage alternative,
- K – the number of the all considered criteria ($k=1,\ldots,K$).

The decision maker establishes for $t=1,\ldots T$ reference systems $\mathbf{R}_t = \mathbf{G}_t \cup \mathbf{B}_t$, which consists of the set of „good" objects $\mathbf{G}_t$ and the set of "bad" objects $\mathbf{B}_t$. We assume, that $\mathbf{G}_t \cap \mathbf{B}_t = \varnothing$ for $t = 1,\ldots,T$. We denote:

- $\mathbf{g}_t$ – "good" reference object ($\mathbf{g}_t \in \mathbf{G}_t$),
- $\mathbf{b}_t$ – "bad" reference object ($\mathbf{b}_t \in \mathbf{B}_t$),
- $\mathbf{r}_t$ – reference object ("good" or "bad", $\mathbf{r}_t \in \mathbf{R}_t$).

3 PAIRWISE COMPARISONS OF TWO ITEMS IN AHP

The AHP employs an underlying scale with values from 1 to 9 to rate the relative preferences for two items A and B (Table 1). This synthesis provides the relative priorities for the items A and B with respect to DM's judgment. For example let us assume that the DM evaluates A to be equally to moderately preferred over B. The steps of the numerical procedure are as follows:

Step 1. Sum the values in each column.

	A	B
A	1	2
B	1/2	1

Step 2. Divide each element of the matrix by its column total.

	A	B
A	2/3	2/3
B	1/3	1/3

Step 3. Average the elements in each row.

	A	B	Row Avg
A	0,667	0,667	0,667
B	0,333	0,333	0,333

The relative priorities S(A,B) and S(B,A) for items A and B with respect to all potential DM's judgments are shown in Table 1. The following property holds:

$$S(\mathrm{A,B}) + S(\mathrm{B,A}) = 1$$

Table 1: Pairwise Comparison Scale for the AHP Preferences and *Numerical Ratings for Two Items Comparisons*

Verbal Judgement of Preferences	Numerical Rating A and B	Relative Priorities S(A,B)	Numerical Rating B and A	Relative Priorities S(B,A)
equally preferred	1	0.5	1	0,5
equally to moderately preferred	2	0.667	1/2	0,3333
moderately preferred	3	0.75	1/3	0,25
moderately to strongly preferred	4	0.8	1/4	0,2
strongly preferred	5	0.833	1/5	0,1667
strongly to very strongly preferred	6	0.8571	1/6	0,1429
very strongly preferred	7	0.875	1/7	0,125
very strongly to extremely preferred	8	0.8889	1/8	0,1111
extremely preferred	9	0.9	1/9	0,1

4 INCLUSION OF AHP COMPARISONS TO THE ORIGINAL MBM

W compare a selected stage alternative **a** with a stage reference object $\mathbf{r}_t$, taking into account criterion C_t^k which is essential stage criterion at stage t. We can encounter one of the following situations:

- $\mathbf{a}_t$ is preferred to $\mathbf{r}_t$, hence $S(\mathbf{a}_t, \mathbf{r}_t) \in (0.5, 0.9)$.

- $\mathbf{a}_t$ is equivalent to $\mathbf{r}_t$, hence $S(\mathbf{a}_t, \mathbf{r}_t) = 0.5$.

- $\mathbf{r}_t$ is preferred to $\mathbf{a}_t$, hence $S(\mathbf{a}_t, \mathbf{r}_t) \in (0.1, 0.5)$.

We define stage indicators $\varphi_t^{k+}(\mathbf{a}_t, \mathbf{r}_t)$, $\varphi_t^{k=}(\mathbf{a}_t, \mathbf{r}_t)$, and $\varphi_t^{k-}(\mathbf{a}_t, \mathbf{r}_t)$ as follows:

$$\phi_t^{k+}(\boldsymbol{a_t}, \boldsymbol{r}_t) = \begin{cases} S(\boldsymbol{a_t}, \boldsymbol{r_t}), & \text{if } \boldsymbol{a_t} \text{ is preferred to } \boldsymbol{r_t} \\ 0, & \text{otherwise} \end{cases}$$

$$\phi_t^{k=}(\boldsymbol{a_t},\boldsymbol{r}_t) = \begin{cases} 0.5, & \text{if } \boldsymbol{a_t} \text{ is equivalent to } \boldsymbol{r_t} \\ 0, & \text{otherwise} \end{cases}$$

$$\phi_t^{k-}(\boldsymbol{a_t},\boldsymbol{r}_t) = \begin{cases} S(\boldsymbol{r_t},\boldsymbol{a_t}), & \text{if } \boldsymbol{r_t} \text{ is preferred to } \boldsymbol{a_t} \\ 0, & \text{otherwise} \end{cases}$$

The rest of the MBM, described in [4] remains unchanged.

5 SIMPLIFIED MULTISTAGE AHP/BIPOLAR METHOD

5.1 Stage alternatives

5.1.1 Comparison of stage alternatives with stage reference objects

We apply the formula:

$$d_t(\boldsymbol{a_t},\boldsymbol{r_t}) = \sum_{\boldsymbol{k=1}}^{\boldsymbol{K}} w_t S_k(\boldsymbol{a_t},\boldsymbol{r_t})$$

If $d_t(\mathbf{a}_t, \mathbf{r}_t) > d_t(\mathbf{r}_t, \mathbf{a}_t)$ then $\mathbf{a}_t$ is preferred to $\mathbf{r}_t$. If $d_t(\mathbf{a}_t, \mathbf{r}_t) = d_t(\mathbf{r}_t, \mathbf{a}_t)$ then $\mathbf{a}_t$ is equivalent to $\mathbf{r}_t$. If $d_t(\mathbf{a}_t,\mathbf{r}_t) < d_t(\mathbf{r}_t, \mathbf{a}_t)$ then $\mathbf{r}_t$ is preferred to $\mathbf{a}_t$.

5.1.2 Position of a stage alternative with respect to the bipolar stage reference system

The position of $\mathbf{a}_t$ with respect to $\mathbf{G}_t$ is defined by means of the formula:

$$d_{\boldsymbol{G}(\boldsymbol{a_t})} = \frac{\sum_{g_t \in G_t} d_t(\boldsymbol{a_t},\boldsymbol{g_t})}{card\ \boldsymbol{G_t}}$$

If $d_{\mathbf{G}}(\mathbf{a}_t) > 0{,}5$ $\mathbf{a}_t$ is better than $\mathbf{G}_t$. If $d_{\mathbf{G}}(\mathbf{a}_t) = 0{,}5$, $\mathbf{a}_t$ is equivalent to $\mathbf{G}$. If $d_{\mathbf{G}}(\mathbf{a}_t) < 0{,}5$, $\mathbf{a}_t$ is worse than $\mathbf{G}$

The position of $\mathbf{a}_t$ with respect to $\mathbf{B}_t$ is defined by means of the formula:

$$d_{\boldsymbol{B}(\boldsymbol{a_t})} = \frac{\sum_{b_t \in B} d_t(\boldsymbol{a_t},\boldsymbol{b_t})}{card\ \boldsymbol{B_t}}$$

If $d_{\mathbf{B}}(\mathbf{a}_t) > 0{,}5$ $\mathbf{a}_t$ is better than $\mathbf{B}_t$. If $d_{\mathbf{B}}(\mathbf{a}_t) = 0{,}5$, $\mathbf{a}_t$ is equivalent to $\mathbf{B}_t$. If $d_{\mathbf{B}}(\mathbf{a}_t) < 0{,}5$, $\mathbf{a}_t$ is worse than $\mathbf{B}_t$.

5.2 Relationships in the set of multistage alternatives

For any multistage alternative $\mathbf{a} = (\mathbf{a}_1,...,\mathbf{a}_T)$ we define multistage success achievement degree:

$$d_{\boldsymbol{G}(\boldsymbol{a})} = \frac{1}{T}\sum_{t=1}^{T} d_{\boldsymbol{G}(\boldsymbol{a_t})}$$

If $d_{\mathbf{G}}(\mathbf{a}) > 0.5$, $\mathbf{a}$ is better than $G = \times_{t=1}^{T} G_t$. If $d_{\mathbf{G}}(\mathbf{a}) = 0.5$, $\mathbf{a}$ is equivalent to $\mathbf{G}$. If $d_{\mathbf{G}}(\mathbf{a}) < 0.5$, $\mathbf{a}$ is worth than $\mathbf{G}$.

For any multistage alternative $\mathbf{a} = (\mathbf{a}_1,...,\mathbf{a}_T)$ we define multistage failure avoidance degree:

$$d_{\boldsymbol{B}(a)} = \frac{1}{T}\sum_{t=1}^{T} d_{\boldsymbol{B}(a_t)}$$

If $d_{\mathbf{B}}(\mathbf{a}) > 0.5$, **a** is better than $B = \times_{t=1}^{T} B_t$. If $d_{\mathbf{B}}(\mathbf{a}) = 0.5$, **a** is equivalent to **B.** If $d_{\mathbf{G}}(\mathbf{a}) < 0.5$, **a** is worse than **B.**

5.3 Sorting multistage alternatives

The values of indicators defined above allow to classify the multistage alternatives. For a given multistage alternative **a**, we consider the components of the vector

$$d(\mathbf{a}) = [d_{\mathbf{G}}(\mathbf{a}), d_{\mathbf{B}}(\mathbf{a})]$$

We consider the following classes:

$$\mathbf{A}^{H} = \{\mathbf{a}\in\mathbf{A}: \quad d_{\mathbf{G}}(\mathbf{a}) \geq 0.5,\, d_{\mathbf{B}}(\mathbf{a}) \geq 0.5\}$$
$$\mathbf{A}^{M} = \{\mathbf{a}\in\mathbf{A}: \quad d_{\mathbf{G}}(\mathbf{a}) < 0.5,\, d_{\mathbf{B}}(\mathbf{a}) \geq 0.5\}$$
$$\mathbf{A}^{L} = \{\mathbf{a}\in\mathbf{A}: \quad d_{\mathbf{G}}(\mathbf{a}) < 0.5,\, d_{\mathbf{B}}(\mathbf{a}) < 0.5\}$$
$$\mathbf{A}^{OU} = \{\mathbf{a}\in\mathbf{A}: \quad d_{\mathbf{G}}(\mathbf{a}) \geq 0.5,\, d_{\mathbf{B}}(\mathbf{a}) < 0.5\}$$

Assumptions, described in [4] lead to the conclusion that $\mathbf{A}^{OU} = \varnothing$. As $\mathbf{A}^{H} \cap \mathbf{A}^{M} \cap \mathbf{A}^{L} = \varnothing$ then $\mathbf{A}^{H} \cup \mathbf{A}^{M} \cup \mathbf{A}^{L} = \mathbf{A}$. Each $\mathbf{a}\in\mathbf{A}^{H}$ is better than any $\mathbf{a}\in\mathbf{A}^{M}$. Each $\mathbf{a}\in\mathbf{A}^{M}$ is better than any $\mathbf{a}\in\mathbf{A}^{L}$

5.4 Ranking the multistage alternatives

Let $\delta(\mathbf{a}^{(i)}) = d_{\mathbf{G}}(\mathbf{a}^{(i)}) - d_{\mathbf{B}}(\mathbf{a}^{(i)})$. Within the classes the ordering of the alternatives is defined as follows:

$$\mathbf{a}^{(i)} \text{ is preferred to } \mathbf{a}^{(j)}, \text{ iff} \qquad \delta(\mathbf{a}^{(i)}) > \delta(\mathbf{a}^{(j)})$$
$$\mathbf{a}^{(i)} \text{ is equivalent to } \mathbf{a}^{(j)}, \text{ iff} \qquad \delta(\mathbf{a}^{(i)}) = \delta(\mathbf{a}^{(j)})$$

The best multistage alternative $\mathbf{a}^{**}$ is defined as a multistage alternative which belongs to the non-empty class with the lowest number of level *m* and satisfies the relationship

$$\forall_{\mathbf{a}'\in\mathbf{A}^{m}} \ \delta(\mathbf{a}^{**}) \geq \delta(\mathbf{a}')$$

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The 17th International Symposium on
Operational Research in Slovenia

SOR '23

Bled, SLOVENIA
September 20 - 22, 2023

Appendix

Program of SOR'23

SOR ’23 programme

Slovenian Society INFORMATIKA
Section for Operational Research
- organizer -

University of Maribor
Faculty of Organizational Sciences
- co-organizer -

University of Ljubljana
Faculty of Mechanical Engineering
- co-organizer -

MDPI AG
Mathematics
- sponsor -

EURO
Association of European Operational Research Societies
- supporter -

EuroCC2
National Competence Centres in the framework of EuroHPC
- supporter -

The European Centre of Excellence for Engineering Applications
- supporter -

Rudolfovo
The Scientific and Technological Centre in Novo Mesto
- supporter -

L7-3188

ARIS Research project
Hierarchical Design and Financing of Social Infrastructure of Smart Silver Villages
- supporter -

Wednesday **September 20, 2023**	**Thursday** **September 21, 2023**	**Friday** **September 22, 2023**
8.15 – Registration	8.15 – Registration	8.15 – Registration
9.00 – 9.45 Opening ceremony (Hall I)	9.00 – 9.40 **Plenary lecture (Hall I):** **(PL3) Victor Magron** Institute of Mathematics from Toulouse, France	9.00 – 9.40 **Plenary lecture (Hall I):** **(PL5) Mirjana Pejić Bach** University of Zagreb, Croatia
9.50 – 10.30 **Plenary lecture (Hall I):** **(PL1) Marc Sevaux** **(The EURO plenary)** Université Bretagne Sud, France	9.45 – 11.00 Parallel sessions (Hall I, II and III): **(SS4) DOM&MfRWPD** + **(SS5) GT** + **(S2) HR**	9.40 – 9.55 **Plenary lecture (Hall I):** **(PL6) Janez Povh** Rudolfovo and University of Ljubljana, Slovenia
10.30 – 11.00 Coffee break	11.00 – 11.30 Coffee break	10.00 – 11.00 Parallel sessions (Hall I, II and III): **(SS6) I&S5.0** + **(SS4) DOM&MfRWPD** + **(SS1) AoORiAE**
11.00 – 12.00 Parallel sessions (Hall I, II and III): **(SS2) AoORiI&ME** + **(S1) EM&S** + **(SS8) UtBMoSE**	11.30 – 13.00 Parallel sessions (Hall I and II): **(SS4) DOM&MfRWPD** + **(S5) MP&O**	11.00 – 11.30 Coffee break
12.00 – 12.55 Lunch break	13.00 – 13.55 Lunch break	11.30 – 12.45 Parallel sessions (Hall I and II): **(SS6) AoORiAE** + **(S3) F&I**
13.00 – 13.40 **Plenary lecture (Hall I):** **(PL2) Andrej Kastrin** University of Ljubljana, Slovenia	14.00 – 14.40 **Plenary lecture (Hall I):** **(PL4) Suresh P. Sethi** University of Texas at Dallas, USA	
14.15 – 22.00 Excursion with dinner	14.45 – 15.45 Parallel sessions (Hall I, II and III): **(SS3) AIiB** + **(S3) F&I** + **(SS1) AoORiAE**	12.50 – 13.30 Closing ceremony (Hall I)
	15.45 – 16.15 Coffee break	
	16.15 – 18.00 Parallel sessions (Hall I, II and III): **(S4) L&TG&tA** + **(S6) MCDM** + **(SS7) SIiASSORM**	
	18.15 – 20.00 Social event	

Plenary lectures: 30 minutes for presentation plus 10 minutes for discussion.
Contributed papers: 14 minutes all together for presentation and for discussion.

Wednesday

September 20, 2023

8.15 – Registration

9.00 – 9.45 Opening ceremony (Hall I)

9.50 – 10.30 **Plenary lecture** (Hall I, Chair: Lidija Zadnik Stirn)

PL1 – **Marc Sevaux**: Julia, a programming language for Operations Research (The EURO plenary)

10.30 – 11.00 Coffee break

11.00 – 12.00 Parallel sessions:

(SS2) Chairs: Darja Rupnik Poklukar and Janez Žerovnik **Applications of OR in Industry and Mechanical Engineering** (Hall I)	**(S1)** Chair: Kosovka Ognjenović **Econometric Models and Statistics** (Hall II)	**(SS8)** Chair: Milica Maričić and Veljko Jeremić **Unravelling the Business Models of Sharing Economy by Applying Methods of OR and Statistics** (Hall III)
SS2.1 Simon Brezovnik, Darja Rupnik Poklukar and Janez Žerovnik: Roman and Italian Rainbow Domination Number of Graphs SS2.2 Michaela Chocholatá: Heating with Solid Fuel in Slovak Dwellings: A GWR Approach SS2.3 Igor Reznichenko, Primož Podržaj and Aljoša Peperko: Control Theory and Numerical Analysis of Magnetic Field Involving Mechanical Systems SS2.4 Anita Talaja and Krešimir Samac: The Role of Resource Complementarity and Opportunism in Strategic Alliance Performance	S1.1 Peter Knížat and Andrea Furková: From Linear to Spatial Regression: Parametric Versus Nonparametric Functional Form S1.2 Tomislav Korotaj, James Ming Chen and Nataša Kurnoga: Hierarchical Clustering of CEE Countries According to Educational and Labour Market Indicators S1.3 Kosovka Ognjenović: Using the Stochastic Frontier Approach to Determine the Gender Wage Gap and Labour Market Efficiency S1.4 Mario Pepur: Confirmatory Factor Analysis of the Sports Team Reputation Scale: A Case Study in Croatia	SS8.1 Milica Maricic, Katarina Cvetic, Marina Ignjatovic and Veljko Jeremic: Time Series Analysis of Airbnb House Rentals Price in the Balkan Region SS8.2 Milica Maricic, Andrea Popovic, Katarina Cvetic and Marina Ignjatovic: Shared Accommodation in Europe: Consumer Behaviour Analysis SS8.3 Veljko Uskokovic, Stefan Zdravkovic, Stefan Komazec and Veljko Jeremic: Detecting Trending Topics Captivating Circular Economy: A Bibliometric-Based Approach SS8.4 Marija Vuković: The Role of the Financing Source in the Behavioral Intention to Buy Real Estate: Multigroup Analysis

12.00 – 12.55 Lunch break

13.00 – 13.40 **Plenary lecture** (Hall I, Chair: Janez Žerovnik)

PL2 – **Andrej Kastrin:** Knowledge Discovery by Literature Mining: From Serendipity to Computational Creativity

14.15 – 22.00 **Excursion with dinner**

We will meet at the hotel Astoria lobby, take a bus and go to guided tour to the tunnels under the old town of Kranj (https://www.visitkranj.com/en/sights/tunnels-under-the-old-town-of-kranj/).

After the guided tour we will go to early dinner in the restaurant Lectar in Radovljica where we will take a culinary tour of Slovenian dishes (https://www.lectar.com/).

Thursday

September 21, 2023

8.15 – Registration

9.00 – 9.40 **Plenary lecture** (Hall I, Chair: Janez Povh)

PL3 - **Victor Magron:** Sparse polynomial optimization: theory and practice

9.45 – 11.00 Parallel sessions:

(SS4) Chairs: B. Dávid and M. Krész **Discrete Optimization Methods and Models for Real-World Problem Domain** (Hall I)	**(SS5)** Chair: Tamás Solymosi **Game Theory** (Hall II)	**(S2)** Chair: Sarah Fores **Human Resources** (Hall III)
SS4.1 Daniil Baldouski, Balázs Dávid, György Dósa, Tibor Dulai, Ágnes Werner-Stark and Miklós Krész:Managing and Optimizing Container Flow in Port Logistics SS4.2 József Békési, Gábor Galambos and Imre Papp: Automatic Planning of Vehicle and Driver Schedules for Public Transportation: A Case Study SS4.3 Zuzana Borcinova and Peter Czimmermann; Edges which are Critical for Emergency Service Systems SS4.4 Peter Czimmermann: Detection of Critical Vertices for the Designed Service System SS4.5 Balázs Dávid: Optimization of Tree Bucking with Quality and Volume Requirements	SS5.1 Andrew Clausen and Christopher Stapenhurst: Deterring Bribery with Scotch Hold'em Poker SS5.2 Péter Csóka and P. Jean-Jacques Herings: Non-Cooperative Bargaining on Debt Restructuring SS5.3 Martin Černý: Bounding Solution Concepts of Incomplete Cooperative Games SS5.4 Ziv Hellman and Miklós Pintér: Three Variations on Money Pump, Common Prior, and Trade SS5.5 Tamás Solymosi, Ata Atay and Marina Núnez: On the Core of Many-to-One Assignment Games	S2.1 Özlem Akarçay Pervın, Nimet Yapici Pehlıvan and Gerhard Wilhelm Weber: Prediction of Heart Disease Using Classification and Feature Selection Methods S2.2 Luka Goropečnik, Jože Kropivšek, Matej Jošt and Lidija Zadnik Stirn: Competency Model Developed by Using a Multi-Criteria Approach and K-means Clustering S2.3 Marko Hell: The General Model for Curricula Structure Presentation - A Matrix Approach to Managing Curricula S2.4 Nikola Kadoić, Nikolina Žajdela Hrustek and Maja Gligora Marković: The Analyis of Eurovision Song Contest Results: The Differences between Public and Jury Votes S2.5 L. Mihelač and J. Povh: Computational Segmentation of Children's Melody

11.00 – 11.30 Coffee break

11.30 – 13.00 Parallel sessions:

(SS4) Chairs: B. Dávid and M. Krész **Discrete Optimization Methods and Models for Real-World Problem Domain** (Hall I)	**(S5)** Chair: Vedran Kojić **Mathematical Programming and Optimization** (Hall II)	
SS4.6 Murat Elhüseyni, Balázs Dávid, László Hajdu, and Miklós Krész: Distributed System Based Sensor Networks and the Connected P-Median Problem SS4.7 Zsolt Ercsey, Zoltán Kovács and Tamás Storcz: Optimal Schedule of a Sport Shooting Competition SS4.8 Emrecan Erdem, Ayşe Dilek Maden and Masood Ur Rehman: The Laplacian Energy of some Special Tree Families SS4.9 Jaroslav Horáček: A General Framework for Modelling Opinion Formation SS4.10 Jaroslav Janáček and Marek Kvet: Scatter Search For Bi-Criteria Public Service System Design SS4.11 Marek Kvet and Jaroslav Janáček: Intensification and Diversification for Pareto Front Approximation	S5.1 Helena Gaspars-Wieloch and Katarzyna Wyrębska: Impact of the Unitarization Technique on Final Rankings Based on the Goal Programming S5.2 Milan Hladík: A General Approach to Handle Complex Sensitivity Analysis in Linear Programming S5.3 Alf Kimms, Hédi Király and Christin Münch: Computational Geometry and Mathematical Programming: Combined Techniques and Selected Applications in Logistics S5.4 Vedran Kojić, Mira Krpan and Zrinka Lukač: A Simple Approach to Solving the Monopolist's Long-Run Profit Maximization Problem: The Case of the Hyperbolic Inverse Demand and Cobb-Douglas Production Functions with Two Factors of Production S5.5 Anita Varga and Marianna E.-Nagy: Numerical Comparison of Long-Step Interior Point Algorithms for Solving Linear Complementarity Problems	

Thursday

September 21, 2023

13.00 – 13.55 Lunch break

14.00 – 14.40 **Plenary lecture** (Hall I, Chair: Marija Bogataj)

PL4 - **Suresh P. Sethi:** Promoting Electric Vehicles: Reducing Charging Inconvenience and Price via Station and Consumer Subsidies

14.45 – 15.45 Parallel sessions:

(SS3) Chair: Mirjana Pejić Bach **Artificial Intelligence in Business: Obstacles and Perspectives** (Hall I)	**(S3)** Chair: Ksenija Dumičić **Finance and Investments** (Hall II)	**(SS1)** Chairs: Č. Rozman and K. Pažek **Applications of OR in Agricultural Economics** (Hall III)
SS3.1 Mile Bošnjak and Mirjana Pejić Bach: Factors Affecting COVID-19 Vaccine Uptake of Young Adults: Machine Learning Approaches SS3.2 Aljaž Ferencek and Mirjana Kljajić Borštnar: Topic Modelling of Open Government Data Impact Areas Using GPT 3.5 Model SS3.3 Tea Šestanović and Tea Kalinić Milićević: A MCDM Approach to Machine Learning Model Selection: Bitcoin Return Forecasting SS3.4 Lukáš Veverka: Maximizing Ad Campaign Effectiveness through TV Viewership Analysis: A Machine Learning Investigation	S3.1 Ksenija Dumičić, Mihovil Anđelinović and Blagica Novkovska: Resource Productivity Influenced by Selected Circular Economy and Development Level Indicators: Profiles of Clustered EU Countries S3.2 Aleš Kresta, Bahate Maidiya and Jialei Xiong: On AAII Sentiment Index Influence on S&P 500 Stocks S3.3 Karol Szomolányi, Martin Lukáčik and Adriana Lukáčiková: Macroeconomic Facts of the EU27 Countries During the COVID-19 Pandemic	SS1.1 Živa Alif, Tanja Šumrada and Jaka Žgajnar: Does Economic Situation Cause Land Abandonment? Estimating Economic Viability of Farming in a Sub-Mediterranean Region in Slovenia SS1.2 Maja Borlinič Gačnik, Boris Prevolšek, Antonio Peláez Verdet, Alfonso Cerezo Medina and Črtomir Rozman: Measuring the Efficiency of Spain's Wineries Through Data Envelopment Analysis SS1.3 Jure Brečko and Jaka Žgajnar: Price Volatility and Its Impact on Farm Operations, an Example of Analysis with a Farm Model SS1.4 Gregor Kramberger, Matjaž Glavan and Karmen Pažek: Building Resilient Agricultural Systems: A Multi-Stakeholder Approach for Sustainable Transformation

15.45 – 16.15 Coffee break

16.15 – 18.00 Parallel sessions:

(S4) Chair: Janez Žerovnik **Location and Transport, Graphs and their Applications** (Hall I)	**(S6)** Chair: Vesna Čančer **Multi-Criteria Decision-Making** (Hall II)	**(SS7)** Chairs: M. Bogataj, D. Bogataj and S. Drobne - **Social Innovations in Ageing Studies Supported by OR Models** (Hall III)
S4.1 David Bogataj and Francisco Campuzano-Bolarin: How Would the Construction of a New Container Port in Cartagena Influence the Reduction of Greenhouse Gas Pollution S4.2 Elif Garajová and Miroslav Rada: Interval Transportation Problem: The Worst Finite Optimal Value is Hard for Inequalities S4.3 Peter Juma Ochieng, József Dombi, Tibor Kalmár, András London and Miklós Krész: Graph-Based Prioritization of Related Cancer Genes S4.4 Juraj Pekár, Marian Reiff and Ivan Brezina: Location of Service Devices at any Point in Two-Dimensional Space S4.5 Tina Šfiligoj and Aljoša Peperko: Estimating Node Importance in Public Transport Networks S4.6 Rabia Taspinar and Burak Kocuk: Discretization-Based Solution Approaches for the Circle Packing Problem S4.7 Janez Žerovnik: On *t*-Rainbow Domination Number of Generalized Petersen Graphs *P (ck, k)*	S6.1 Tadeusz Trzaskalik: AHP Application to Multistage Bipolar Method S6.2 Andrej Bregar, Anas Husseis and Jose Luis Flores: A MCDM Methodology for Cyberattack Mitigation S6.3 Vesna Čančer: Measuring Accuracy of Approximation Methods for Priorities Derivation Based on Pairwise Comparisons S6.4 Rok Drnovšek, Marija Milavec Kapun and Uroš Rajkovič: Risk Management in Healthcare: DEX (Decision Expert) Evaluation Model S6.5 Petra Grošelj, Gregor Dolinar and Helena Erika Rojc: Aggregation of Individual Linguistic Evaluations in Spherical Analytic Hierarchy Process S6.6 Jerzy Michnik: The Wings Model for Choice of Innovation Strategy S6.7 Tjaša Šmidovnik and Petra Grošelj: Aggregating Individual Weights into Group Weights in Best-Worst Method	SS7.1 Samo Drobne and Marija Bogataj: Generational Distinctions in Migrations between Slovenian Municipalities SS7.2 Samo Drobne and Marija Bogataj: The Impact of Hierarchical Spatial Levels on Internal Migration by Age Cohorts in Slovenia SS7.3 Terezie Krestová, Aleš Kresta and Lucie Bestová: The Application of Age Management Practices in Organisations: Does the Economic Sector, Size, and Family Business Status Matter? SS7.4 Josipa Višić: COVID-19 Pandemic and Profitability Determinants in Elderly Care Homes: Evidence from Croatia SS7.5 Josipa Višić and Ivana Tomas Žiković: Panel Data Analysis in Predicting Demand for Institutional Long-Term Care for Older Adults SS7.6 Berislav Žmuk: Elderly Population Activity in Croatia: Gender Comparison by Text Analysis Approach SS7.7 Marija Bogataj, David Bogataj, Carmen Rajer, Suresh Sethi and Samo Drobne: The Influence of Residential Dispersion on the Optimal Long-Term Care of Senior Citizens

18.15 – 20.00 **Social event**

We will meet at the hotel Astoria lobby and take a walk to Bled Rose Hotel. We will first listen to a piano concert performed by prof. dr. Tadeusz Trzaskalik and then taste the »kremšnita« (vanilla–custard–cream cake), one of the best known Slovenian desserts that is coming from Bled (http://www.bled.si/en/what-to-see/symbols-of-bled/cream-cake).

SOR'23 - The 17th International Symposium on Operational Research in Slovenia, Bled, SLOVENIA, September 20 - 22, 2023

Friday

September 22, 2023

8.15 – Registration

9.00 – 9.40 **Plenary lecture** (Hall I, Chair: Mirjana Kljajić Borštnar)
PL5 – **Mirjana Pejić Bach**: Operations Research meets Artificial Intelligence: Intersection or Union

9.40 – 9.55 **Plenary lecture** (Hall I, Chair: Mirjana Kljajić Borštnar)
PL6 – **Janez Povh**: European Competence Centre for High Performance Computing

10.00 – 11.00 Parallel sessions:

(SS6) Chairs: D. Bokal and J. Povh **Industry & Society 5.0: Optimization and Learning in Human and Industrial Environments** (Hall I)	**(SS4)** Chairs: B. Dávid and M. Krész **Discrete Optimization Methods and Models for Real-World Problem Domain** (Hall II)	**(SS1)** Chairs: Č. Rozman and K. Pažek **Applications of OR in Agricultural Economics** (Hall III)
SS6.1 Kolos Cs. Ágoston, Marianna E.-Nagy and Janez Povh: Comparing Optimum KMEDIAN and MSS Clustering with Ground Truth Clustering SS6.2 Drago Bokal, Špela Tertinek, Anja Šketa, Janja Jerebic, Robert Repnik, Urška Martinc, Edita Rozina, Metka Zaletel, Branka Mirt and Vlasta Krmelj: Climate Risk Indicators for Small Communities – The Effect of Heat Stress on Mortality SS6.3 Alen Granda and Drago Bokal: On Code Quality and Code Relevance Metrics SS6.4 Elza Jurun, Daniela Garbin Praničević and Valentina Bašić Androja: Croatia Vs EU From the Perspective of Digital Skills	SS4.12 Giuseppe Lancia and Paolo Vidoni: Finding the Best 2-OPT Move on Nearly Random Euclidean TSP Tours in Average Linear Time SS4.13 Maciej Machowiak: The Moldable Tasks in Container Port Terminal SS4.14 Grzegorz Pawlak: Simulation Model for Cyclic Single Track Railway Problem SS4.15 Małgorzata Sterna and Bartłomiej Popielarz: Metaheuristic Methods for TV Advertisement Scheduling Problem	SS1.5 Nikola Obrenović, Maksim Lalić, Dimitrije Stefanović, Marko Panić, Sanja Brdar, Vladimir Crnojević and Oskar Marko: Optimised Routing of the Blueberry Cultivating Unmanned Ground Vehicle SS1.6 Maja Petrač, Krunoslav Zmaić and Jaka Žgajnar: Typical Family Dairy Farms in the Republic of Croatia SS1.7 Boris Prevolšek, Larisa Lorbek, Maja Borlinič Gačnik and Črtomir Rozman: Multi-Criteria Model for Assessment of SPA Service Quality SS1.8 Jaka Žgajnar and Lidija Zadnik Stirn: The Use of Operation Research Methods to Support Agricultural Policy

11.00 – 11.30 Coffee break

11.30 – 12.45 Parallel sessions:

(SS6) Chairs: D. Bokal and J. Povh **Industry & Society 5.0: Optimization and Learning in Human and Industrial Environments** (Hall I)	**(S3)** Chair: Ksenija Dumičić **Finance and Investments** (Hall II)	
SS6.5 Nataša Ošep Ferš and Aleš Zamuda: Improvement in Continuous Black-box Setting Search Performance by Tuning L-SHADE Differential Evolution Historical Memory Size Parameter SS6.6 Melani Potrč, Klemen Tršinar, Špela Kajzer, Špela Tertinek, Urška Martinc and Drago Bokal: The Maturity Model for Climate Neutrality and Business Process Optimization in Slovenian Companies of the Future SS6.7 Janez Povh and Aljaž Krpan: Partitioning Graphs for Advancing Stable Set Problem Solutions through Quantum Annealers SS6.8 Janez Povh and Dunja Pucher: Advancing Stable Set Problem Solutions through Quantum Annealers SS6.9 Aleš Zamuda: Solving 100-Digit Challenge with Score 100 by Extended Running Time and Parallel Benchmarking	S3.5 Ivana Jerković, Ana Rimac Smiljanić and Blanka Škrabić Perić: Interdependence between Cryptocurrency Adoption and Financial Literacy: A Cross-Country Evidence S3.6 Vladimir Šimić: The Impact of Globalization on Government Consumption – Worldwide Evidence S3.7 Ante Toni Vrdoljak and Zdravka Aljinović: The Impact of Financial Literacy and Sociodemographic Factors on Investor Preferences S3.8 Adela Has, Kristina Hodak and Marinela Mokriš: Deep Learning for Predicting Corporate Financial Distress of Croatian IT Companies	

12.50 – 13.30 Closing ceremony (Hall I)

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The 17th International Symposium on
Operational Research in Slovenia

SOR '23

Bled, SLOVENIA
September 20 - 22, 2023

Appendix

Sponsors' Notices

*The 17th International Symposium on Operational Research in Slovenia - **SOR '23***
Bled, SLOVENIA, September 20 - 22, 2023

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EXCELLERAT P2

What is EXCELLERAT?

The EXCELLERAT project is a single point of access for expertise on how data management, data analytics, visualisation, simulation-driven design and Co-design with high-performance computing (HPC) can benefit engineering, especially in the aeronautics, automotive, energy and manufacturing sectors.

What is the goal?

EXCELLERAT aims to tackle the ever-rising complexity of scientific and development endeavours. Thus, Exascale computing is our focus, which will solve highly complex and costly engineering problems, and create enhanced technological solutions even at the development stage.

The goal of EXCELLERAT is to enable the European engineering industry to advance towards Exascale technologies and to create a single entry point to services and knowledge for all stakeholders (industrial end users, ISVs, technology providers, HPC providers, academics, code developers, engineering experts) of HPC for engineering. In order to achieve this goal, EXCELLERAT brings together key players from industry, research and HPC to provide all necessary services.

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Proceedings SOR

Rupnik V. and L. Bogataj (Editors): The 1st Symposium on Operational Research, SOR'93. Proceedings. Ljubljana: Slovenian Society Informatika, Section for Operational Research, 1993, 310 pp.

Rupnik V. and M. Bogataj (Editors): The 2nd International Symposium on Operational Research in Slovenia, SOR'94. Proceedings. Ljubljana: Slovenian Society Informatika, Section for Operational Research, 1994, 275 pp.

Rupnik V. and M. Bogataj (Editors): The 3rd International Symposium on Operational Research in Slovenia, SOR'95. Proceedings. Ljubljana: Slovenian Society Informatika, Section for Operational Research, 1995, 175 pp.

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Zadnik Stirn L., J. Žerovnik, J. Povh, S. Drobne and A. Lisec (Editors): The 12th International Symposium on Operational Research SOR'13, Proceedings. Ljubljana: Slovenian Society Informatika, Section for Operational Research, 2013, 390 pp. ISBN 978-961-6165-40-2.

Zadnik Stirn L., J. Žerovnik, M. Kljajić Borštnar, S. Drobne (Editors): The 13th International Symposium on Operational Research SOR'15, Proceedings. Ljubljana: Slovenian Society Informatika, Section for Operational Research, 2015, 559 pp. ISBN 978-961-6165-45-7.

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