

Impact Of The Terms-Of-Trade On Business Cycles In Slovak And Czech Economies

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Abstract

Using Slovak and Czech data and the empirical structural vector autoregressive model, the observed impact of the terms-of-trade on business cycles is very small. Furthermore we observe Obstfeld-Svensson-Razin effect of terms-of-trade in both countries. The trade balance negatively reacts on the changes in terms-of-trade. However the theoretical model with import-able, export-able and non-tradable goods calibrated with the empirical observations in Slovakia and Czech Republic does not suit to the empirical model. The reactions of consumption, output and real exchange rate on the terms-of-trade shocks are overestimated and the trade balance reacts positively on the terms-of-trade shocks in the theoretical models for both countries. Moreover, the theoretical model calibrated using Slovak and Czech data predicts Harberger-Laursen-Metzler effect. The trade balance positively reacts on the changes in terms-of-trade. The theoretical and empirical mismatch in literature dealing with the terms of trade influence is confirmed. The paper is divided into two parts. Firstly, structural vector autoregressive model is introduced and estimated. Secondly, the theoretical model with import-able, export-able and non-tradable goods is presented and calibrated. Data of Slovak and Czech economy are used.

Keywords: Terms of Trade, Business Cycle, Trade Balance

Introduction

Terms-of-trade is theoretically significant source of business cycles and it causes shifts in trade balance. However different theoretical and empirical studies lead to different results of the short-run terms-of-trade impact on output and on trade balance. There are two theoretical effects of terms-of-trade impact on trade balance. Harberger (1950) and Laursen and Metzler (1950) used traditional Keynesian model to show that trade balance grows with terms-of-trade. On the contrary, dynamic optimizing models of

Obstfeld (1982) and Svensson and Razin (1983) leads to a conclusion that positive effect of terms-of-trade on the trade balance is weaker the more persistent a terms-of-trade shock is. Uribe and Schmitt-Grohe (2016) showed that in small open economy real business cycle model (or dynamic stochastic general equilibrium model) with capital costs sufficiently permanent terms-of-trade shocks have negative impact on the trade balance. Empirical studies of Aguirre (2011), Broda (2004) and Uribe and Schmitt-Grohe (2016) surprisingly do not support statistically significant impact of term-of-trade on output in poor and emerging countries. In general authors can confirm an intuition that the more open the economy is the higher effect of terms-on-trade on trade balance is. This result may not be achieved in theoretical general equilibrium models even if non-tradable goods are considered. Uribe and Schmitt-Grohe (2016) developed MXN model with tradable and non-tradable goods to show that an existence of non-tradable goods “reduce the importance of terms-of-trade shocks.” However authors state that the theoretical model still overestimates the signification of the terms-of-trade impact on the business cycles.

In this paper we confirm this theoretical and empirical mismatch. In the first part of the paper we present and estimate the empirical SVAR model of the terms-of-trade impact on the Slovak and Czech business cycles to state that the influence of the terms-of-trade is very small. In the second part we present and calibrate MXN model of Uribe and Schmitt-Grohe (2016) using Slovak and Czech observations to state that the theoretical model predicts relatively high impact of the terms-of-trade on business cycles.

Empirical Model

First, we used vector autoregressive (VAR) models for our analysis. Every endogenous variable is a function of all lagged endogenous variables in the system in VAR models. See Lutkepohl (2005) for more details about them. The mathematical representation of the VAR model of order p is:

$$\mathbf{y}_t = \mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{A}_2 \mathbf{y}_{t-2} + \dots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{e}_t \quad (1)$$

where $\mathbf{y}_t$ is a k vector of endogenous variables; $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_p$ are matrices of coefficients to be estimated; and $\mathbf{e}_t$ is a vector of innovations that may be contemporaneously correlated but are uncorrelated with their own lagged values. Due to problem of over-parametrisation we can use the Bayesian approach to estimation.

The VAR model (1) can be interpreted as a reduced form model. A structural vector auto-regressive (SVAR) model is structural form of VAR model and is defined as:

$$\mathbf{A} \mathbf{y}_t = \mathbf{B}_1 \mathbf{y}_{t-1} + \mathbf{B}_2 \mathbf{y}_{t-2} + \dots + \mathbf{B}_p \mathbf{y}_{t-p} + \mathbf{B} \mathbf{u}_t \quad (2)$$

A SVAR model can be used to identify shocks and trace these out by employing impulse response analysis and forecast error variance decomposition through imposing restrictions on used matrices.

Uribe and Schmitt-Grohe (2016) proposed a specification of the SVAR, through which we can determine responses on terms-of-trade impulse:

$$\mathbf{A} \begin{pmatrix} f_t \\ tb_t \\ y_t \\ c_t \\ i_t \\ rer_t \end{pmatrix} = \sum_{i=1}^p \mathbf{B}_i \begin{pmatrix} f_{t-i} \\ tb_{t-i} \\ y_{t-i} \\ c_{t-i} \\ i_{t-i} \\ rer_{t-i} \end{pmatrix} + \mathbf{B} \begin{pmatrix} u_t^f \\ u_t^{tb} \\ u_t^y \\ u_t^c \\ u_t^i \\ u_t^{rer} \end{pmatrix} \quad (3)$$

where f is relative cyclical component of the terms of trade, tb is relative cyclical component of the trade balance to output ratio, y is relative cyclical component of output, c is relative cyclical component of consumption, i is relative cyclical component of investment and rer is relative cyclical component of real exchange rate.

The u_t^f , u_t^{tb} , u_t^y , u_t^c , u_t^i and u_t^{rer} are structural shocks of given variables. We estimated the parameters of the SVAR specification (3) using Amisano and Giannini (1997) approach. The class of models may be written as:

$$\mathbf{A}\mathbf{e}_t = \mathbf{B}\mathbf{u}_t \quad (4)$$

The structural innovations $\mathbf{u}_t$ are assumed to be orthonormal, i.e. its covariance matrix is an identity matrix. The assumption of orthonormal innovations imposes the following identifying restrictions on $\mathbf{A}$ and $\mathbf{B}$:

$$\mathbf{A}\Sigma_e\mathbf{A}^T = \mathbf{B}\mathbf{B}^T \quad (5)$$

Noting that the expressions on both sides of (5) are symmetric, this imposes $k(k+1)/2 = 21$ restrictions on the $2k^2 = 72$ unknown elements in $\mathbf{A}$ and $\mathbf{B}$. Therefore, in order to identify $\mathbf{A}$ and $\mathbf{B}$, we need to impose $(3k^2-k)/2 = 51$ additional restrictions. The matrix $\mathbf{A}$ of unrestricted specification is a lower triangular matrix with unit diagonal (15 zero and 6 unity restrictions) and matrix $\mathbf{B}$ is a diagonal matrix (30 zero restrictions) in this just-identified specification. Other tested restrictions are imposed on elements of matrix $\mathbf{A}$ (matrix of contemporary effects between endogenous variables), which means that our specification becomes over-identified and also testable.

The selected lag of model (3) is validated by sequential modified likelihood ratio test statistic and information criteria and by the LM test for autocorrelations. Significant values of serial correlation for lower lags could be a reason to increase the lag order of an unrestricted VAR, but this is not

our case. We verified the stability of a VAR model (i.e. whether all roots have modulus less than one and lie inside the unit circle). We estimated the parameters of restricted and unrestricted specifications. Using the logarithm of the maximum likelihood functions of both specifications we calculated the likelihood ratio statistics and verified the significance of restrictions. All tests are explained in Lutkepohl (2005) for example.

Using matrix polynomial in lag operator $A(L) = B_1L + B_2L^2 + \dots + B_pL^p$ we can rewrite (2) as structural moving averages (SMA) representation:

$$y_t = [A - A(L)]^{-1} B u_t = C(L) u_t = C(0) u_t + C(1) u_{t-1} + \dots + C(h) u_{t-h} + \dots \quad (6)$$

Hence, $C(0)$ is the coefficient matrix on impact, $C(1)$ at a one period lag, and so on. Generally, $C_{ij}(h)$ element is the impulse response of variable i to shock j at horizon h . The forecast error of y at horizon s is:

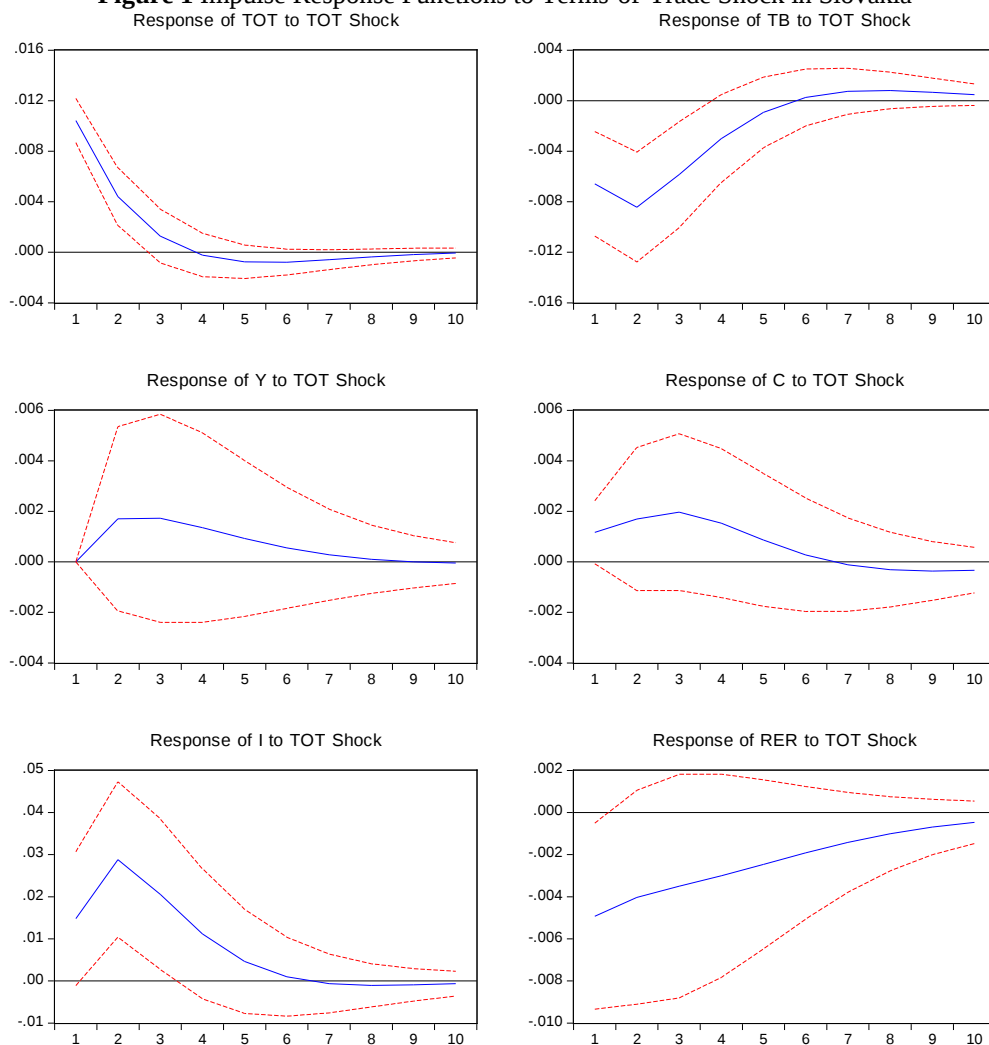
$$y_{t+h} - \hat{y}_{t+h} = C(0) u_{t+h} + C(1) u_{t+h-1} + C(2) u_{t+h-2} + \dots + C(h) u_t \quad (7)$$

Variance of the forecast error (assuming orthogonality) is expressed as sum of the individual variances of shocks:

$$\text{var}(y_{t+h} - \hat{y}_{t+h}) = C(0)IC(0)^T + C(1)IC(1)^T + \dots + C(h)IC(h)^T \quad (8)$$

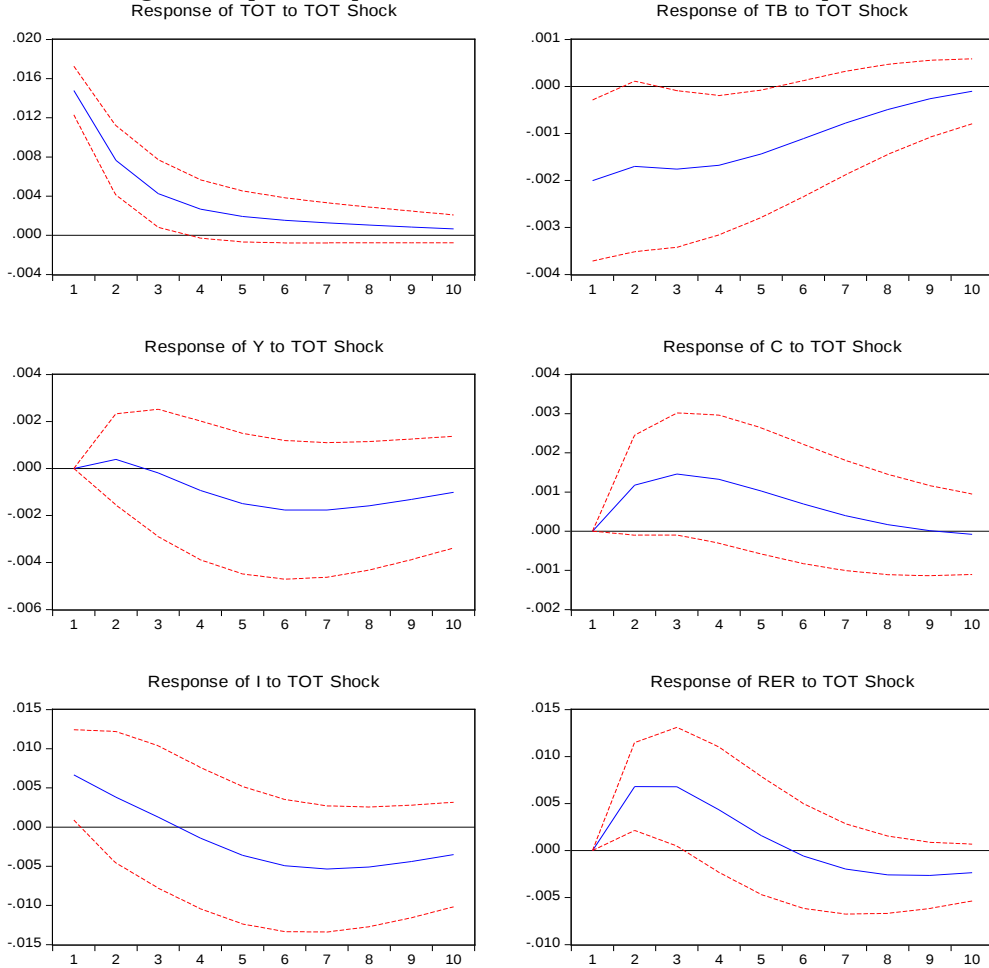
The fraction of the forecast error variance of variable i due to shock j at horizon h , is then the (i,j) element of expression (8) divided by the total forecast error variance and is expressed as a percentage. We calculated the impulse response functions. Generally, the impulse response function traces the effect of a one-time shock in one of the innovations on current and future values of the endogenous variables.

Data for Czech and Slovak economies are gathered from the Eurostat portal. The responses to the terms-of-trade shock in Slovakia are in the Figure 1. As output shock elasticity coefficient is not statistically significant, the improvement in terms-of-trade has no impact on the aggregate activity and the one-quarter delayed output expansion is statistically insignificant. The same result applies to the consumption. Investment displays a somewhat larger expansion, albeit with a one-quarter delay. Real exchange rate falls immediately. On the other hand, the impact of the terms-of-trade shock on trade balance is clearly statistically significant. The 10 % increase in the terms of trade causes a decrease of 6.7 % in trade balance. Furthermore a huger contraction is delayed by one quarter.

Figure 1 Impulse Response Functions to Terms-of-Trade Shock in Slovakia

Source: Own processing

The responses to the terms-of-trade shock in Czech Republic are in the Figure 2. As output shock elasticity coefficient is not statistically significant, the improvement in terms-of-trade has no impact on the aggregate activity and the one-quarter delayed output expansion is statistically insignificant. The same result applies to the consumption and real exchange rate. Investment displays a small expansion. On the other hand, the impact of the terms-of-trade shock on trade balance is statistically significant. The 10 % increase in the terms of trade causes a decrease about 2 % in trade balance. Again, the result suggests confirmation of Obstfeld-Svensson-Razin effect rather than Harberger-Laursen-Metzler effect of the terms-of-trade in both Slovakia and Czech Republic

Figure 2 Impulse Response Functions to TOT Shock in Czech Republic

Source: Own processing

Theoretical model:

Uribe and Schmitt-Grohe (2016) presented the model with import-able (m), export-able (x) and non-tradable (n) sectors. The presence of non-tradable goods should reduce the importance of terms-of-trade shock.

We consider a large number of identical households with preferences described by the utility function

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t - \frac{(h_t^m)^{\omega_m}}{\omega_m} - \frac{(h_t^x)^{\omega_x}}{\omega_x} - \frac{(h_t^n)^{\omega_n}}{\omega_n}}{1-\sigma} \right]^{1-\sigma} - 1 \quad (9)$$

where c_t denotes consumption, for sector $j \in \{m, x, n\}$, h_t^j denotes hours worked in the sector j . Sectoral labour supplies are wealth inelastic and

parameters ω_j denote wage elasticity in the sector j . The symbol E_0 denotes the expectations operator conditional on information available in initial period 0. The parameter σ measures the degree of relative risk aversion.

Households maximize the lifetime utility function (9) subject to the budget constraint

$$\begin{aligned} c_t + i_t^m + i_t^x + i_t^n + \phi_m (k_{t+1}^m - k_t^m)^2 + \phi_x (k_{t+1}^x - k_t^x)^2 + \phi_n (k_{t+1}^n - k_t^n)^2 + p_t^\tau d_t = \\ = \frac{p_{t+1}^\tau d_{t+1}}{1+r_t} + w_t^m h_t^m + w_t^x h_t^x + w_t^n h_t^n + u_t^m k_t^m + u_t^x k_t^x + u_t^n k_t^n \end{aligned} \quad (10)$$

where for sector $j \in \{m, x, n\}$, i_t^j denotes gross investment, k_t^j denotes capital, w_t^j denotes real wage rate and u_t^j is the rental rate of capital in the sector j . Quadratic terms of the budget constraint (10) are capital adjustment costs, where ϕ_j denotes capital adjustment cost parameter in the sector j . The variable p_t^τ denotes the relative price of the tradable composite good in terms of final goods, d_t denotes the stock of debt in period t denominated in units of the tradable composite good and r_t denotes the interest rate on debt held from period t to $t + 1$. Consumption, investment, wages, rental rates, debt, and capital adjustment costs are all in units of final goods.

The capital stocks accumulation is given by

$$k_{t+1}^j = (1 - \delta) k_t^j + i_t^j; \quad \forall j \in \{x, m, n\} \quad (11)$$

where δ denotes constant depreciation rate.

There are 5 types of large number of identical firms in the economy which differ according to their output: firms producing final goods, tradable composite goods, import-able goods, export-able goods and non-tradable goods.

Final goods are produced using non-tradable goods and a composite of tradable goods via the CES technology

$$B(a_t^\tau, a_t^n) = \left[\chi_\tau (a_t^\tau)^{1-\frac{1}{\mu_{\tau n}}} + (1 - \chi_\tau) (a_t^n)^{1-\frac{1}{\mu_{\tau n}}} \right]^{\frac{1}{1-\frac{1}{\mu_{\tau n}}}} \quad (12)$$

where a_t^τ denotes the tradable composite good and a_t^n the non-tradable good, $0 < \chi_\tau < 1$ denotes distribution parameter and $\mu_{\tau n} > 0$ is the elasticity of substitution between tradable composite good and non-tradable good.

The tradable composite goods is produced using importable and exportable goods as intermediate inputs via the CES technology

$$a_t^\tau = A(a_t^m, a_t^x) = \left[\chi_m (a_t^m)^{1-\frac{1}{\mu_{mx}}} + (1 - \chi_m) (a_t^x)^{1-\frac{1}{\mu_{mx}}} \right]^{\frac{1}{1-\frac{1}{\mu_{mx}}}} \quad (13)$$

where a_t^m denotes import-able good and a_t^x the export-able good, $0 < \chi_m < 1$ denotes distribution parameter and $\mu_{mx} > 0$ denotes the elasticity of substitution between import-able and export-able goods. Import-able, export-able and non-tradable goods are produced with capital and labour via the Cobb-Douglas technologies

$$y_t^j = A_t^j (k_t^j)^{\alpha_j} (h_t^j)^{1-\alpha_j}; \forall j \in \{x, m, n\} \quad (14)$$

where sector $j \in \{m, x, n\}$, y_t^j denotes output and A_t^j denotes total factor productivity in the in sector j .

To ensure a stationary equilibrium process for external debt, we assume that the country interest-rate premium is debt elastic

$$r_t = r^* + \psi (e^{d_{t+1} - \bar{d}} - 1) \quad (15)$$

where r^* denotes the sum of world interest rate and the constant component of the interest-rate premium, the last term of (15) is the debt-elastic component of the country interest-rate premium and we assume the parameter debt-elastic $\psi > 0$.

Model implied terms-of-trade f_t is assumed to follow AR(1) process

$$\log \frac{f_t}{\bar{f}} = \rho \log \frac{f_{t-1}}{\bar{f}} + \pi \varepsilon_t \quad (16)$$

where ε_t is a white noise with mean zero and unit variance, and $\bar{f} > 0$. The serial correlation parameter is $0 < \rho < 1$ and terms-of-trade standard error is $\pi > 0$.

For details of households' and firms' problem first-order conditions, market clearing and competitive equilibrium derivation and definitions see Uribe and Schmitt-Grohe (2016).

Calibrating the model we follow Uribe and Schmitt-Grohe (2016) process. The calibrated values of the model parameters are in the Table 1 for Slovak economy and in the Table 2 for Czech economy. We assume the values of σ , δ , r^* , ω_m , ω_x and ω_n from the small open economy real business cycle model calibrated for Slovak data by Jurkovicova (2015). We assume that wage elasticity is same in all three sectors. Toroj (2012) calibrated Slovak elasticity of substitution between tradable composite good and non-tradable good, μ_{tn} , to be 0.76. Uribe and Schmitt-Grohe (2016) provide a rich discussion with literature references on calibrating the elasticity of substitution between import-able and export-able goods. Further we adopt Uribe and Schmitt-Grohe (2016) ideas to calibrate α_m , α_x , α_n in the Slovakia. We assume that Czech and Slovak characteristics are similar to calibrate same values of σ , δ , r^* , in both Slovak and Czech economies. The values of ω_m , ω_x , ω_n , μ_{tn} , α_m , α_x , α_n , are gathered from Ambrisko (2015). Considering high-frequently (i.e. quarterly) data it is assumed that $\mu_{mx} = 0.8$ in both

economies. Calibrating $\bar{f}$, A^m and A^n in both countries we adopt values Uribe and Schmitt-Grohe (2016). The values of terms-of-trade serial correlation, ρ , and standard error, π , correspond to the data characteristics used in empirical models. To calibrate χ_m , χ_τ and A^x we follow a process of Uribe and Schmitt-Grohe (2016) and implied moment restrictions of average share of value-added exports in GDP , s_x , average trade balance-to- GDP ratio, s_{tb} , and average share of non-tradable goods in GDP , s_n . Likewise Uribe and Schmitt-Grohe (2016) we use OECD Trade in Value-Added (TiVA) and UNCTAD statistical databases to find values of these moment restriction. The values of the rest implied structural parameters, $\bar{d}$ and β come from the values of calibrated ones. We fail to reach a negative reaction of the trade balance to a terms-of-trade shock in the theoretical MXN model to follow empirical facts observed in the Figures 1 and 2. After substituting big values for the parameter ψ , the response of the trade balance is close to 0 – positive using Slovak values and negative using Czech values. Therefore we calibrate ϕ_j , $j \in \{m, x, n\}$ and ψ to capture moments observed in the empirical model. From the Figure 1 it follows that there is no statistically significant reaction of investment to the terms-of-trade shock in Slovakia. On the other hand the Czech SVAR model implies that that investment-terms-of-trade volatility ratio conditional on terms-of-trade shocks equals approximately to 0.45. As Uribe and Schmitt-Grohe (2016) pointed out, the standard deviation conditional on terms-of-trade shock of investment in the trade sector is 1.5 times as large as its counterpart in the non-traded sector.

Table 1 Calibration of the MXN Model: Slovakia

Calibrated Structural Parameters			Moment restrictions		
σ	2	Jurkovicova (2015)	s_n	0.27	UNCTAD
δ	0.1		s_x	0.37	OECD
r^*	0.04		s_{tb}	-0.015	
ω_m	2.7		$p^m y^m / (p^x y^x)$	1	Uribe and Schmitt-Grohe (2016)
ω_x	2.7		$\sigma_{im+ix} / \sigma_{in}$	1.5	
ω_n	2.7		no reaction of investment		
μ_{tn}	0.76	Toroj (2012)	Implied Structural Parameter Values		
μ_{mx}	0.8	Uribe and Schmitt-Grohe (2016)	χ_m	0.875	
α_m	0.35		χ_τ	0.78	
α_x	0.35		$\bar{d}$	−0.509	
α_n	0.25		A^x	1.374	
$\overline{f}$	1		β	0.962	
A^m	1		ϕ_m	0	
A^n	1	Empirical model	ϕ_x	0.159	
π	0.013		ϕ_n	0	
ρ	0.464		ψ	1.5017×10^{-5}	

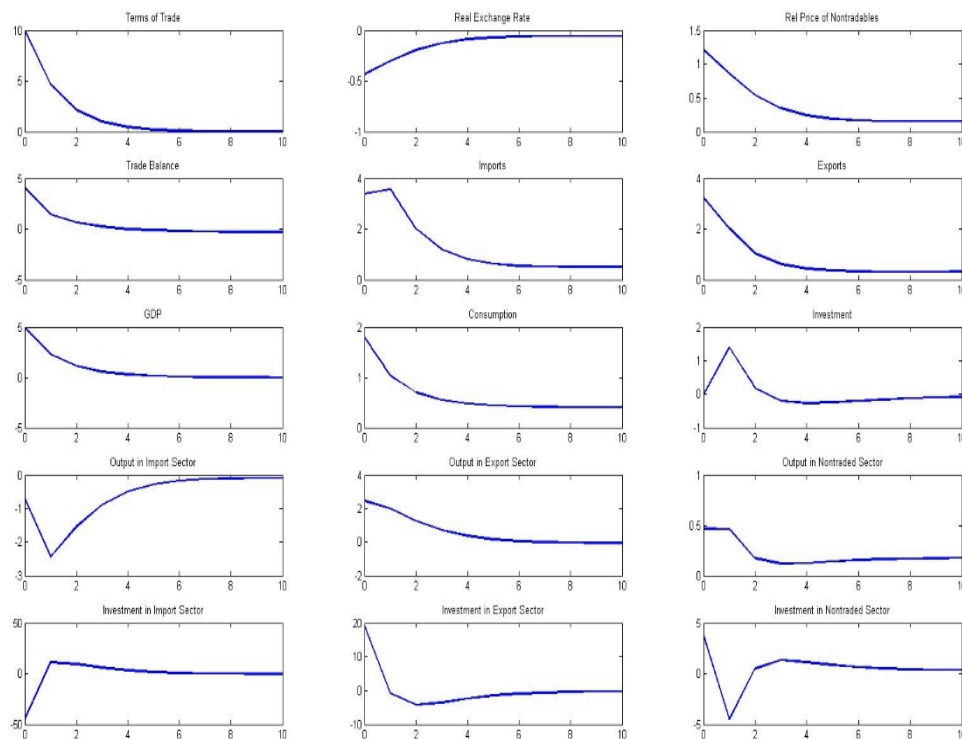
Source: Literature referenced in the table and own processing

Table 2 Calibration of the MXN Model: Czech Republic

Calibrated Structural Parameters			Moment restrictions		
σ	2	Jurkovicova (2015)	s_n	0.27	UNCTAD
δ	0.1		s_x	0.34	OECD
r^*	0.04		s_{tb}	0.08	
ω_m	2.7	Ambrisko (2015)	$p^m y^m / (p^x y^x)$	1	Uribe and Schmitt-Grohe (2016)
ω_x	2.7		$\sigma_{im+ix} / \sigma_{in}$	1.5	
ω_n	2.7		σ_i / σ_{tot}	0.45	Empiric. model
$\mu_{\tau m}$	0.76		Implied Structural Parameter Values		
α_m	0.35		χ_m	0.672	
α_x	0.35		χ_τ	0.979	
α_n	0.25	Uribe and Schmitt-Grohe (2016)	$\bar{d}$	1.800	
μ_{mx}	0.8		A^x	1.436	
$\bar{f}$	1		β	0.962	
A^m	1		ϕ_m	0.0125	
A^n	1		ϕ_x	0.021	
π	0.013	Empirical model	ϕ_n	0	
ρ	0.464		ψ	0.0176	

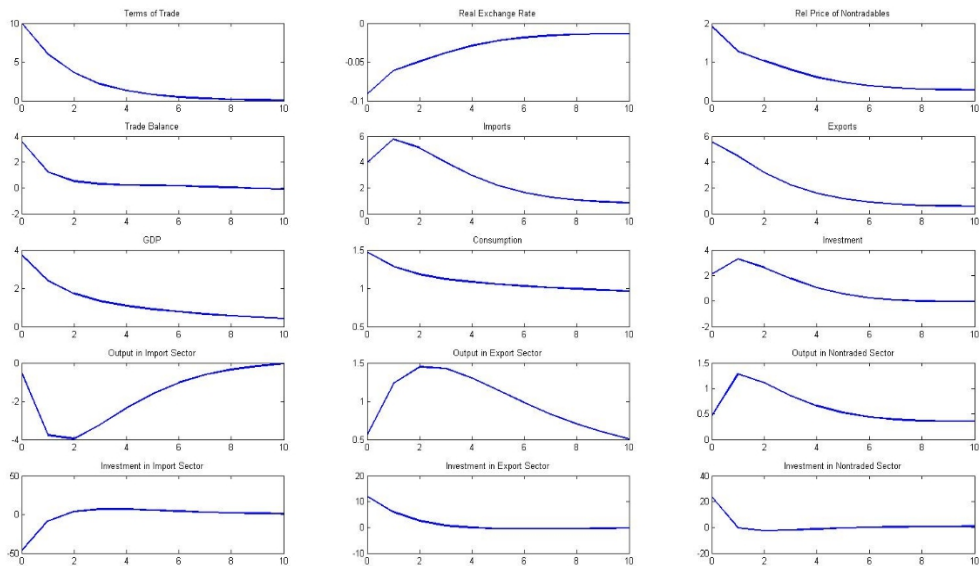
Source: Literature referenced in the table and own processing

Figure 3 Impulse Response Functions to Terms-of-Trade Shock in Slovakia



Source: Own processing

Figure 4 Impulse Response Functions to TOT Shock in Czech Republic



Source: Own processing

In order of finding model equilibrium the first order linear approximation to the nonlinear solution are applied using algorithms of Uribe and Schmitt- Grohe (2016). Responses to the terms-of-trade impulses and covariance-variance matrix conditional on the terms-of-trade shock is computed using algorithm of Uribe and Schmitt- Grohe (2016).

Conclusion

In empirical models we observe small impact of terms-of-trade on business cycles in Slovakia and Czech Republic. In Slovakia real exchange rate and in Czech Republic investment reacts immediately, while other aggregates do not change (or they change later mostly as reaction of other variables) on terms-of-trade shock. In both countries terms-of-trade has negative effect on the trade balance.

However, theoretical model calibrated to suit empirical observations overestimates the influence of terms-of-trade shocks in both countries. Both output and investment rise after terms-of-trade shock realization in both countries. The theoretical falls in real interest rates are overestimated as well. As we already pointed out, we cannot achieve a negative reaction of the trade balance in the theoretical model as it is in the empirical model.

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References:

- Aguirre, Ezequiel. *Business Cycles in Emerging Markets and Implications for the Real Exchange Rate*. Ph.D. Dissertation, New York: Columbia University, 2011.
- Ambrisko, Robert. *A Small Open Economy with the Balassa-Samuelson Effect*. Working paper series (ISSN 1211-3298), Prague: Charles University CERGE EI, 2015.
- Amisano, Gianni, and Carlo Giannini. *Topics in Structural VAR Econometrics*. 2nd ed. Berlin: Springer-Verlag, 1997.
- Broda, Christian. Terms of Trade and Exchange Rate Regimes in Developing Countries. *Journal of International Economics*, 63(1), 2004, 31-58.
- EUROSTAT portal. Retrieved from <http://ec.europa.eu/eurostat/data/database> [Accessed 4th April 2016]
- Harberger, Arnold C. Currency Depreciation, Income, and the Balance of Trade. *Journal of Political Economy*, 58(1), 1950, 47-60.
- Jurkovicova, Maria. *Model realneho hospodarskeho cyklu v slovenskej ekonomike*. Thesis, Bratislava: University of Economics, 2015.
- Laursen, Svend, and Lloyd A. Metzler. Flexible Exchange Rates and the Theory of Employment. *Review of Economics and Statistics* 32(4), 1950, 281-299.
- Lutkepohl, Helmut. *New Introduction to Multiple Time Series Analysis*. Berlin: Springer, 2005.
- Obsfeld, Maurice. Aggregate Spending and the Terms of Trade: Is There a Laursen-Metzler Effect? *Quarterly Journal of Economics*, 97(2), 1982, 251-270.
- Svensson, Lars E. O., and Assaf Razin. The Terms of Trade and the Current Account: The Harberger-Laursen-Metzler Effect. *Journal of Political Economy* 91(1), 1983, 97-125.
- Torój, Andrzej. *Poland and Slovakia during the crisis: would the euro (non-)adoption matter?* Working paper of Ministry of Finance in the Poland Republic no. 13-2012, Warsaw. 2012.
- Trade in Value Added database (OECD.STAT). Retrieved from: <http://www.oecd.org/sti/ind/measuringtradeinvalue-addedanoecd-wtojointinitiative.htm> [Accessed 4th March 2016]
- UNCTAD database. Retrieved from http://unctadstat.unctad.org/wds/ReportFolders/reportFolders.aspx?IF_ActivePath=P,15912 [Accessed 4th March 2016]
- Uribe, Martin, and Stephanie Schmitt-Grohe. *Open Economy Macroeconomics*. New York: Columbia University. 2016. Retrieved from <http://www.columbia.edu/~mu2166/book/> [Accessed 4th March 2016]