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Measuring income inequalities beyond Gini coefficient [1]

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Abstract

Growing interest in the analysis of interrelationships between income distribution and economic growth has recently stimulated new theoretical as well as empirical research. Since existing theoretical models propose inequality is detrimental to growth, while others point at income inequality as an essential determinant supporting economic growth. Measures such as head-count ratio for poverty index or widely used Gini coefficient are aggregated indicators without deeper insight into income distribution among the poor or the households. To derive an indicator accounting for income distribution among the income groups, we propose output oriented DEA model with inputs equal unit and weights restrictions imposed so as to favour higher income share in lower quantiles. We demonstrate the merit of this approach on the quintile income breakdown data of the European countries. Prioritizing lower income groups´ welfare, countries – e.g. Slovenia and Slovakia – can be equally favoured by the new proposed indicator while assessed differently by Gini index. Intertemporal analysis reveals a slight deterioration of income distribution over the period of 2007 – 2017 in a Rawlsian sense.

Keywords: Income distribution, Rawlsian utility, data envelopment analysis, weights restriction, Malmquist index

JEL codes: I31, C61, O15

1 Introduction

In recent years there has been increasing volume of research examining how inequality affects economic growth. This growing interest has recently motivated both theoretical and empirical investigations. Efficiency-equity trade-off is shaping policy discussions in most countries around the world. Policy measures are justified by welfare improvement considerations, the quest for solid theoretical background presents a lasting challenge. Assessing welfare quantitatively amounts to the aggregation of the individual welfare functions. These are for the most part associated or

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derived from the level of individual income or consumption. The issue of income distribution arises as one of the most important determinant of total social welfare. Empirical analysis of welfare and economic performance involves accounting for multiple socio-economic characteristics. With its capacity to account for multiple inputs and outputs, data envelopment analysis (DEA) proved an appropriate tool in this domain. Prasada Rao and Coelli (2010) embodied inequality measures in the technology. Lábaj et.al (2014) derived measure of social welfare assessing simultaneously economic, ecological, and social dimensions. Use of DEA allows intertemporal analysis. Shown by Färe et al. (1994), the overall change in productivity described by the Malmquist index can be decomposed to expose catch-up and frontier-shift effects.

We further proceed as follows. In Section 2 we describe DEA output oriented model with fixed input equal to one. As the evaluation of income distribution encompasses descriptive as well as prescriptive issues (Sen, 2000), some preferences need to be involved in the prospective method. Proposing data on income distribution as outputs we introduce Rawlsian weighting of quantile income shares. A method of assessment of productivity change over time using Malmquist index is presented afterwards. In Section 3 empirical result are presented to demonstrate how the proposed measure performs compared to Gini index providing deeper insight into individual distributions of 29 European countries. Intertemporal analysis spans the period of 2007 – 2016.Section 4 concludes and outlines further research.

2 Rawlsian measure of income distribution

Over the years, a multitude of indices has been developed to facilitate designing and evaluating policies with respect to social development. Most of them aim to capture aspects *beyond GDP* utilizing a bulk of indicators, famously articulated in Stiglitz et al. (2009). Letting aside environmental issues, the most discussed aspect of the well-being is social justice and distribution of wealth. There is a number of indicators capturing distribution of wealth or income on an aggregate level comprising Gini or Theil indices as a class of statistically based indicators. However, when redistributive policies are ethically founded it gives them more credibility and strength. As well, measures such as head-count ratio for poverty index or widely used Gini coefficient do not provide deeper insight into income distribution among the poor or the households. Indices are closely linked to social welfare functions. Quantities characterizing health or education are closely correlated with the income. This is why income distribution analyses dominate the domain of social aspect of well-being research.

Income distribution measures derived from Social welfare function utilize mapping of individual incomes $(y_1, y_2, \ldots, y_n) \mapsto W(y_1, y_2, \ldots, y_n)$. The function *W* could take a variety of forms comprising

$$W(y_1, y_2, \ldots, y_n) = \prod_{j=1}^{n} y_j$$

$$W(y_1, y_2, \ldots, y_n) = \prod_{j=1}^{n} y_j^{\theta}, \quad 0 < \theta < 1$$

$$W(y_1, y_2, ..., y_n) = \prod_{j=1}^{n} y_j^{\theta_j}, \quad 0 < \theta_j < 1,$$

as summarized in a compact way in Nicola (2013). In each of the expressions a progressively greater effect of parameters ascribed to individual values (incomes) is present. Contributions of different incomes to the overall value vary across individuals in the latter formula. The question of assigning parameters to individual incomes arises.

Adopting the statement of justice as "… the basic structure of society" (Rawls, 1971), we would value overall welfare with respect to the lowest income. We deviate from the strict max-min principle, maintaining however the spirit of greater lower-incomes´ contribution to the total value of *W*. Since the detailed data on income brackets are hardly available for a study across European countries, we make use of income distribution data where income shares y_r of *s* population quantiles are accessible. Rawlsian social welfare would be the higher the more income would be received by poorest.

2.1 AR model

For quantitative evaluation on the country level we propose to employ data envelopment analysis model with countries (distributions) under consideration acting as decision making units (DMUs). We suggest that the performance index $RI = \sum_{r=1}^{s} u_r y_r$ is determined as a weighted sum of income shares over *s* quantiles of the population with weights optimized by the linear program. While the sum of the shares always equals unity, we aim to distinguish between varied distributions by choosing the multipliers u_r. Imposing the Rawlsian criterion we introduce weights constraint

$u_1 \geq u_2 \geq \ ... \geq u_{s-1} \geq u_s$.

Letting a distribution (country) under evaluation choose optimal weights so as to maximize its score given the fixed input normalized to unity leads to DEA Assurance Region (AR) model pioneered by Thompson et al. (1986). We exploit the fact that under constant returns to scale input and output oriented radial models yield the same scores (values of the proposed indicator) and resort to radial output oriented AR-O model which would conveniently generate projections of the income shares adding up to 1 (Appendix D). Sticking henceforth to the notation of Cooper et al. (2007), the model takes the form of

$$
\begin{array}{llll}
(RI) & \min\limits_{\mathbf{u},\mathbf{v}} & \mathbf{v}^{\mathrm{T}}\mathbf{x}_0 & (1) \\
 & \text{s.t.} & \mathbf{u}^{\mathrm{T}}\mathbf{y}_0 = 1 & (2) \\
 & & -\mathbf{v}^{\mathrm{T}}\mathbf{X} + \mathbf{u}^{\mathrm{T}}\mathbf{Y} \leq \mathbf{0} & \\
 & & \mathbf{u}^{\mathrm{T}}\mathbf{Q} \leq \mathbf{0} & (3) \\
 & & \mathbf{u} \geq \mathbf{0},\ \mathbf{v} \geq \mathbf{0}, &
\end{array}
$$

where **X** and **Y** are respective input and output data matrices, **u** and **v** respective multipliers while **Q** is a matrix of bounds for output weights. Output oriented model is proposed with quantile shares as outputs and fixed unit input for each DMU.

2.2 Projections

From the point of view of decision makers benchmarks providing theoretical support for redistributive politics are of the most interest. The proposed model projects observed data onto the efficiency frontier utilizing optimal solutions from the dual *envelope* program

$$\max_{\theta,\boldsymbol{\lambda},\boldsymbol{\tau}} \quad \varphi \tag{4}$$

$$\text{s.t.} \quad \mathbf{x}_0 \geq \mathbf{X}\boldsymbol{\lambda} \tag{5}$$

$$\varphi \mathbf{y}_0 \leq \mathbf{Y}\boldsymbol{\lambda} + \mathbf{Q}\tau \tag{6}$$

$$\boldsymbol{\lambda} \geq \mathbf{0},\ \boldsymbol{\tau} \geq \mathbf{0},$$

where $\boldsymbol{\tau}$ are dual variables associated with weight restrictions in (3). Output projections of interest are then calculated by means of optimal values of $\boldsymbol{\lambda}$ as $\hat{\mathbf{y}}_0 = \mathbf{Y}\boldsymbol{\lambda}^*$.

2.3 Intertemporal analysis

The *RI* index derived in the section 2.1 assesses inequality in a given period of time. This approach can be further utilized to analyse intertemporal changes in individual countries and/or the shift of the frontier. For this purpose Malmquist productivity index (7) is employed. Based on the pioneering work of Caves (1982), Färe and Grosskopf (1992) defined the index in terms of distance functions (7) triggering a mass of studies employing the approach in the variety of applications up until now.

$$M = C \times F = \frac{d^2(\mathbf{x}_0,\mathbf{y}_0)^2}{d^1(\mathbf{x}_0,\mathbf{y}_0)^1}\left[\frac{d^1(\mathbf{x}_0,\mathbf{y}_0)^1}{d^2(\mathbf{x}_0,\mathbf{y}_0)^1} \times \frac{d^1(\mathbf{x}_0,\mathbf{y}_0)^2}{d^2(\mathbf{x}_0,\mathbf{y}_0)^2}\right]^{1/2} \tag{7}$$

The expression (7) shows that the overall change in performance, in the case of the fixed input driven effectively by the changes in outputs, can be decomposed into two factors – catch-up effect (C) and frontier-shift effect (F).

A DMU_0 represented by the activities $(\mathbf{x}_0, \mathbf{y}_0)^t$, ($t = 1, 2$) is assessed in two time periods with respect to two technology frontiers of the period 1 and 2 by distance functions d^T, ($T = 1, 2$). Thus we compute the terms involved in (7) by solving linear programs of two types. *Within* scores are obtained by optimization

$$d^T(\mathbf{x}_0,\mathbf{y}_0)^T = \max_{\varphi,\boldsymbol{\lambda}} \varphi \tag{8}$$

$$\text{s.t.} \quad \mathbf{x}_0^T \geq \mathbf{X}^T\boldsymbol{\lambda}$$

$$\varphi \mathbf{y}_0^T \leq \mathbf{Y}^T\boldsymbol{\lambda} + \mathbf{Q}\tau$$

$$\boldsymbol{\lambda} \geq 0,\ \boldsymbol{\tau} \geq 0,$$

where $\mathbf{X}^T = (\mathbf{x}_1^T, \dots, \mathbf{x}_m^T)$ and $\mathbf{Y}^T = (\mathbf{y}_1^T, \dots, \mathbf{y}_s^T)$ are respectively input and output data matrices for the period *T*.

Intertemporal scores come from the program of the form

$$
\begin{aligned}
d^T(\mathbf{x}_0, \mathbf{y}_0)^t \quad &= \max_{\varphi, \lambda} \varphi \\
\text{s.t.} \quad & \mathbf{x}_0^t \geq \mathbf{X}^T \boldsymbol{\lambda} \\
& \varphi \mathbf{y}_0^t \leq \mathbf{Y}^T \boldsymbol{\lambda} + \mathbf{Q}\tau \\
& \boldsymbol{\lambda} \geq 0, \ \boldsymbol{\tau} \geq 0,
\end{aligned} \tag{9}
$$

We thus employ the "exclusive" scheme (Cooper et al., 2007, p.) allowing the value of *d* in (9) take on values lower than unit in the sense of super-efficiency first introduced by Andersen and Petersen (1993).

3 Empirical results

For empirical demonstration of the *RI* index performance we consider 29 European countries acting as DMUs. Data are sourced from Eurostat comprising Gini index and income shares of disposable income of five quintile population groups based on EU-SILC survey, the latter entering our model as five outputs. For intertemporal analysis we collected data from two periods – 2007 and 2016. Concentrating on outputs we fix input to unit value. Input data matrix thus collapses to unit vector rendering the model robust to the returns to scale assumption. To underscore the discriminating power of the model, we can set the Rawlsian weights constraints in a more strict way letting the ratio of the successive multipliers be bounded by a specified number *l*. Matrix **Q** from (3) will then take the form

$$
\mathbf{Q} = \begin{pmatrix} l_{1,2} & 0 & 0 & 0 \\ -1 & l_{2,3} & 0 & 0 \\ 0 & -1 & l_{3,4} & 0 \\ 0 & 0 & -1 & l_{4,5} \\ 0 & 0 & 0 & -1 \end{pmatrix}.
$$

Thus, for example, product of the vector of weights $\mathbf{u}^{\mathrm{T}} = (u_1, \dots, u_5)$ with the 3rd column of matrix **Q** produces lower bound for the ratio $u_4 / u_3 \leq l_{3,4}$. In this way a sequentially constrained weights are embodied in the DEA model. We exemplify our approach by choosing the concrete values for $l_{i,j}$ from the matrix **Q** equal to 0,95. This implies the following decreasing geometric sequence of weights

$$
u_1 \geq 0{,}95 u_2 \geq 0{,}95^2 u_3 \geq 0{,}95^3 u_4 \geq 0{,}95^4 u_5.
$$

Solving AR model for each DMU we calculate RI indicator for 2016 as well optimal values for weights and dual variables. In Table 1 data and values of RI index are displayed along with the corresponding value of Gini index. Complete results for all countries can be viewed in Table A.1 (Appendix A).

Table 1: Data, RI scores, and Gini index for selected countries

	Q1	Q2	Q3	Q4	Q5	RI	Gini
Slovenia	9,5	14,9	18,7	22,9	34,0	**1**	24,4
Slovakia	9,3	15,2	18,8	23,0	33,7	**1**	24,3
Ireland	8,6	13,1	17,6	22,9	37,8	**0,994**	29,5
Czech Republic	10,1	14,6	17,9	22,0	35,4	**1**	25,1
Hungary	8,6	13,9	17,8	22,9	36,8	**0,995**	28,2
Germany	8,2	13,5	17,7	22,8	37,8	**0,994**	29,5

Source: Authors´ calculations

The model determines four efficient DMUs with the unit score – Czech Republic, Slovenia, Slovakia, and Norway. Noticeably, Czech Republic gained its RI-efficiency from the massive income share of the first quintile income group (the poorest 20%). Slovenia and Slovakia (both relative efficient according to RI) slightly differ in Gini index – 24,4 and 24,3 respectively, with Slovenia offsetting an advantage in Q1 (9,5,1 vs 9,3) by a poorer value of Q2 (14,9 vs 15,2). Germany and Ireland with the same value of Gini (29,5) are indistinguishable with the chosen ratio of weights though one can observe. When compared to Hungary, Ireland´s share in Q2 and Q3 is lower (while Q1 are equal) which results in a lower RI score.

For robustness check, we alternatively employed the additive variant of the DEA model (1)-(3) denoted AD-AR. Weights restriction (3) is imposed on the core based on Andersen – Petersen (1993) approach. As Table A4 in Annex demonstrates, ranking based on the AD-AR scores is merely identical to that of AR employed previously (rank correlation equal to 0,999). AD-AR generates the same set of three efficient DMUs as the radial AR model.

Having demonstrated the sensibleness of the proposed measure, we derive results potentially useful in policy making. Projections for outputs suggest desirable income share adjustment in individual quintile groups needed to perform at the *best practice* level. An example of selected countries is given in Tab. 2. Adjustments in the table are computed as difference between the projection and the data, so positive values indicate need for increasing the share in particular income group.

Table 2: Income share adjustment

	score	adjustment				
		Q1	Q2	Q3	Q4	Q5
Belgium	**0,997**	0,20	1,30	0,40	-0,50	-1,40
Bulgaria	**0,984**	3,70	3,80	2,60	0,40	-10,50
Czech Republic	**1**	0,00	0,00	0,00	0,00	0,00
Denmark	**0,997**	0,30	1,00	0,90	0,80	-3,00
Germany	**0,994**	1,10	1,70	1,10	0,20	-4,10

Source: Authors´ calculations

Naturally, in the Rawlsian sense the most massive redistribution would be needed in the "richest" quantiles indicated by negative values of suggested adjustment. Clearly, for efficient DMUs like Czech Republic no changes are needed and adjustments are zero. Since projections sum to unity, adjustments add up to zero (proof in Appendix B). An extreme value for Bulgaria suggests need for a extensive redistribution from Q5 (-10,5). Complete results are given in Table A.2 (Appendix A).

Intertemporal analysis was conducted by calculating d-terms described in Section 2.3. labelled d11 and d22 (within scores for 2007 and 2016) along with d21 and d12 (intertemporal scores). Then catch-up (C) and frontier-shift (F) effects as well as the overall Malmquist productivity index (M) were computed. Selection of countries are exhibited in Table 3, the complete results can be seen in Table A.3 (Appendix A).

Tab. 3: Malmquist index, its components and Gini coefficient for selected countries

		d11	d22	d21	d12	C	F	M	G07	G16
1	Belgium	0,996	0,997	0,998	0,996	1,001	0,999	1,000	26,3	26,3
3	Czech Republic	1,000	1,000	1,000	1,000	1,000	1,000	1,000	25,3	25,1
5	Germany	0,992	0,994	0,993	0,993	1,002	0,999	1,001	30,4	29,5
7	Ireland	0,990	0,994	0,992	0,993	1,004	0,999	1,002	31,3	29,5
13	Latvia	0,986	0,988	0,987	0,986	1,002	0,999	1,001	35,4	34,5
16	Hungary	0,997	0,995	0,999	0,994	0,998	0,999	0,997	25,6	28,2
18	Netherlands	0,995	0,997	0,997	0,996	1,002	0,999	1,000	27,6	26,9
20	Poland	0,989	0,993	0,991	0,992	1,004	0,999	1,003	32,2	29,8
21	Portugal	0,984	0,989	0,985	0,987	1,005	0,999	1,003	36,8	33,9
22	Romania	0,982	0,987	0,983	0,986	1,005	0,999	1,004	38,3	34,7
27	United Kingdom	0,989	0,992	0,990	0,990	1,003	0,999	1,001	32,6	31,5
29	Switzerland	0,992	0,994	0,993	0,993	1,002	0,999	1,001	30,4	29,4
	average (total)					***1,0005***	***0,9975***	***0,9980***		

Source: Authors´ calculations

In the table, within score for the period 2 (d22) is identical to RI index (efficiency score) for 2016 analysed above. The discriminating capacity of the model can be seen in the example of Slovakia moving in Gini index from 24,5 to 24,3 i.e. to lower inequality. However, RI approach indicates deteriorating of Rawlsian performance by M value of 0,995 < 1, i.e. the opposite direction. On average, productivity index slightly below unit suggests less egalitarian distributions across Europe. The efficiency frontier-shift effect defined by the most efficient countries reveals deteriorating in the best practice itself. Most individual catch-up is observed in Portugal, Romania, Poland, Ireland and United Kingdom contributing heavily to the general improvement (M >1). Less impressive progress made Germany, Ireland, Latvia, Netherlands, Finland, and Switzerland.

4 Conclusions

We developed a measure of income inequality providing a deeper insight into the distribution than aggregated Gini coefficient. Performance income distribution indicator has been demonstrated to favour more Rawlsian distributions. Projections computed from the model provide policy recommendations as to redistributional adjustments in determined income groups. Intertemporal analysis revealed a slight deterioration of income distribution towards less egalitarian structure. This finding is on average confirmed by increasing value of Gini coefficient. More clear-cut discrimination could be achieved by selecting the more restrictive set of weights. In general, the proposed relative indicator is rather meant to supplement statistically based aggregated indicators as Gini or Theil index with the information applicable in policy making than to fully replace them. The restricted multipliers approach presents a promising avenue for examining the poverty indicators in a similar fashion.

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Appendix A Table A.1 Income shares and Gini coefficient data

	2007						2016					
	Q1	Q2	Q3	Q4	Q5	Gini	Q1	Q2	Q3	Q4	Q5	Gini
Belgium	9,1	14,1	18,4	23,1	35,3	**26,3**	9,1	13,9	18,4	23,5	35,1	**26,3**
Bulgaria	5,9	12,3	17,2	23,2	41,4	**35,3**	5,6	11,4	16,2	22,6	44,2	**38,3**
Czech Republic	10,1	14,5	17,7	22,1	35,6	**25,3**	10,1	14,6	17,9	22	35,4	**25,1**
Denmark	9,2	15	18,6	22,6	34,6	**25,2**	9	14,2	17,9	22,2	36,7	**27,7**
Germany	7,8	13,7	17,5	22,5	38,5	**30,4**	8,2	13,5	17,7	22,8	37,8	**29,5**
Estonia	7,4	12,3	16,8	22,5	41	**33,4**	7,1	12,1	17,1	23,9	39,8	**32,7**
Ireland	8,2	12,6	16,8	23,1	39,3	**31,3**	8,6	13,1	17,6	22,9	37,8	**29,5**
Greece	6,9	12,2	16,8	22,8	41,3	**34,3**	6,2	12,5	17,3	23,5	40,5	**34,3**
Spain	7,1	12,8	17,6	23,6	38,9	**31,9**	6,2	12,2	17,3	23,7	40,6	**34,5**
France	9,3	14,2	17,9	22,5	36,1	**26,6**	8,9	13,7	17,2	21,6	38,6	**29,3**
Italy	7,3	12,8	17,5	23,1	39,3	**32,0**	6,3	12,8	17,9	23,5	39,5	**33,1**
Cyprus	8,7	13,4	17,3	22	38,6	**29,8**	8,3	12,4	16,7	22,2	40,4	**32,1**
Latvia	6,6	11,7	16,5	23,4	41,8	**35,4**	6,6	12	16,9	23,3	41,2	**34,5**
Lithuania	7	12,4	16,7	22,7	41,2	**33,8**	6,1	11,4	16,2	22,9	43,4	**37,0**
Luxembourg	9,1	13,9	17,6	22,6	36,8	**27,4**	7,8	13,1	17,3	22,8	39	**31,0**
Hungary	9,6	14,6	18	22,5	35,3	**25,6**	8,6	13,9	17,8	22,9	36,8	**28,2**
Malta	9,1	14,1	18,3	23,1	35,4	**26,3**	8,9	13,3	17,7	22,8	37,3	**28,5**
Netherlands	9,3	14,1	17,6	22	37	**27,6**	9,2	14,1	17,9	22,6	36,2	**26,9**
Austria	9,5	14,4	17,9	22,3	35,9	**26,2**	8,8	14,2	18,2	22,8	36	**27,2**
Poland	7,6	12,8	17	22,5	40,1	**32,2**	7,9	13,4	17,7	23,1	37,9	**29,8**
Portugal	6,9	11,5	15,4	21,8	44,4	**36,8**	7	12,4	16,7	22,6	41,3	**33,9**
Romania	5,4	11,1	16,5	22,9	44,1	**38,3**	5,5	12	17,9	24,6	40	**34,7**
Slovenia	10,1	15,2	18,5	22,8	33,4	**23,2**	9,5	14,9	18,7	22,9	34	**24,4**
Slovakia	10	14,9	18,2	22,3	34,6	**24,5**	9,3	15,2	18,8	23	33,7	**24,3**
Finland	9,7	14,2	18	22,4	35,7	**26,2**	9,9	14,3	18	22,4	35,4	**25,4**
Sweden	10	15,2	18,7	22,7	33,4	**23,4**	8,5	14,1	18,4	23,2	35,8	**27,6**
United Kingdom	7,6	12,6	17	22,5	40,3	**32,6**	7,7	13	17,2	22,9	39,2	**31,5**
Norway	9,4	15,6	19	22,7	33,3	**23,7**	9,4	15,1	18,5	22,4	34,6	**25,0**
Switzerland	8,3	13,3	17,3	22,3	38,8	**30,4**	8,6	13,5	17,4	22,4	38,1	**29,4**

Source: Eurostat

Appendix A Table A.2 Projections of the income shares (2016)

	score	adjustment				
		Q1	Q2	Q3	Q4	Q5
Belgium	**0,997**	0,20	1,30	0,40	-0,50	-1,40
Bulgaria	**0,984**	3,70	3,80	2,60	0,40	-10,50
Czech Republic	**1,000**	0,00	0,00	0,00	0,00	0,00
Denmark	**0,997**	0,30	1,00	0,90	0,80	-3,00
Germany	**0,994**	1,10	1,70	1,10	0,20	-4,10
Estonia	**0,989**	2,20	3,10	1,70	-0,90	-6,10
Ireland	**0,994**	0,70	2,10	1,20	0,10	-4,10
Greece	**0,988**	3,10	2,70	1,50	-0,50	-6,80
Spain	**0,988**	3,10	3,00	1,50	-0,70	-6,90
France	**0,995**	0,40	1,50	1,60	1,40	-4,90
Italy	**0,989**	3,00	2,40	0,90	-0,50	-5,80
Cyprus	**0,991**	1,00	2,80	2,10	0,80	-6,70
Latvia	**0,988**	2,70	3,20	1,90	-0,30	-7,50
Lithuania	**0,985**	3,20	3,80	2,60	0,10	-9,70
Luxembourg	**0,992**	1,50	2,10	1,50	0,20	-5,30
Hungary	**0,995**	0,70	1,30	1,00	0,10	-3,10
Malta	**0,995**	0,40	1,90	1,10	0,20	-3,60
Netherlands	**0,997**	0,10	1,10	0,90	0,40	-2,50
Austria	**0,997**	0,50	1,00	0,60	0,20	-2,30
Poland	**0,993**	1,40	1,80	1,10	-0,10	-4,20
Portugal	**0,989**	2,30	2,80	2,10	0,40	-7,60
Romania	**0,987**	3,80	3,20	0,90	-1,60	-6,30
Slovenia	**1,000**	0,00	0,00	0,00	0,00	0,00
Slovakia	**1,000**	0,00	0,00	0,00	0,00	0,00
Finland	**0,999**	0,01	0,44	0,12	-0,16	-0,41
Sweden	**0,996**	0,80	1,10	0,40	-0,20	-2,10
United Kingdom	**0,992**	1,60	2,20	1,60	0,10	-5,50
Norway	**1,000**	0,00	0,02	0,18	0,47	-0,68
Switzerland	**0,994**	0,70	1,70	1,40	0,60	-4,40

Appendix A Table A.3 Malmquist index components and Gini coefficient (2007 and 2016)

		d11	d22	d21	d12	C	F	M	G07	G16
1	Belgium	0,996	0,997	0,998	0,996	1,001	0,999	1,000	26,3	26,3
2	Bulgaria	0,986	0,984	0,987	0,982	0,998	0,999	0,997	35,3	38,3
3	Czech Republic	1,000	1,000	1,000	1,000	1,000	1,000	1,000	25,3	25,1
4	Denmark	0,998	0,997	0,999	0,995	0,999	0,999	0,997	25,2	27,7
5	Germany	0,992	0,994	0,993	0,993	1,002	0,999	1,001	30,4	29,5
6	Estonia	0,988	0,989	0,989	0,988	1,001	0,999	1,000	33,4	32,7
7	Ireland	0,990	0,994	0,992	0,993	1,004	0,999	1,002	31,3	29,5
8	Greece	0,987	0,988	0,988	0,987	1,001	0,999	1,000	34,3	34,3
9	Spain	0,989	0,988	0,991	0,986	0,998	0,999	0,997	31,9	34,5
10	France	0,996	0,995	0,997	0,993	0,999	0,999	0,997	26,6	29,3
11	Italy	0,990	0,989	0,991	0,988	1,000	0,999	0,998	32,0	33,1
12	Cyprus	0,993	0,991	0,994	0,990	0,998	0,999	0,997	29,8	32,1
13	Latvia	0,986	0,988	0,987	0,986	1,002	0,999	1,001	35,4	34,5
14	Lithuania	0,987	0,985	0,989	0,983	0,997	0,999	0,996	33,8	37,0
15	Luxembourg	0,995	0,992	0,996	0,991	0,997	0,999	0,996	27,4	31,0
16	Hungary	0,997	0,995	0,999	0,994	0,998	0,999	0,997	25,6	28,2
17	Malta	0,996	0,995	0,997	0,994	0,999	0,999	0,998	26,3	28,5
18	Netherlands	0,995	0,997	0,997	0,996	1,002	0,999	1,000	27,6	26,9
19	Austria	0,997	0,997	0,998	0,995	1,000	0,999	0,999	26,2	27,2
20	Poland	0,989	0,993	0,991	0,992	1,004	0,999	1,003	32,2	29,8
21	Portugal	0,984	0,989	0,985	0,987	1,005	0,999	1,003	36,8	33,9
22	Romania	0,982	0,987	0,983	0,986	1,005	0,999	1,004	38,3	34,7
23	Slovenia	1,000	1,000	1,026	0,999	1,000	0,987	0,987	23,2	24,4
24	Slovakia	0,999	1,000	1,009	0,999	1,001	0,994	0,995	24,5	24,3
25	Finland	0,997	0,999	0,999	0,998	1,002	0,998	1,001	26,2	25,4
26	Sweden	1,000	0,996	1,024	0,995	0,996	0,988	0,984	23,4	27,6
27	United Kingdom	0,989	0,992	0,990	0,990	1,003	0,999	1,001	32,6	31,5
28	Norway	1,000	1,000	1,019	0,998	1,000	0,990	0,990	23,7	25,0
29	Switzerland	0,992	0,994	0,993	0,993	1,002	0,999	1,001	30,4	29,4
	average					***1,0005***	***0,99749***	***0,99799***		

Source: Authors´ calculations

Appendix A Table A.4 Additive and radial model comparison

	sum(s+)	rank	
		AD-AR	CCR-AR
Belgium	**0,327**	6	7
Bulgaria	**1,679**	25	26
Czech Republic	**0,000**	1	1
Denmark	**0,431**	8	9
Germany	**0,814**	14	15
Estonia	**1,436**	22	23
Greece	**1,374**	21	22
Spain	**1,449**	23	24
France	**0,643**	13	14
Croatia	**0,949**	16	17
Italy	**1,117**	18	19
Cyprus	**1,212**	19	20
Latvia	**1,526**	24	25
Lithuania	**1,823**	27	28
Luxembourg	**0,582**	12	13
Hungary	**0,578**	11	12
Malta	**0,576**	10	11
Netherlands	**0,343**	7	8
Austria	**0,436**	9	10
Poland	**0,885**	15	16
Portugal	**1,310**	20	21
Romania	**1,752**	26	27
Slovenia	**0,097**	3	4
Slovakia	**0,000**	1	1
Finland	**0,153**	4	5
Sweden	**0,201**	5	6
United Kingdom	**1,091**	17	18
Norway	**0,000**	1	1

Appendix B Sum of the projected income shares

Given the optimal solutions for $\boldsymbol{\lambda}$, projections for s quintiles and n DMUs are given by

$$\hat{\mathbf{y}}_0 = \begin{pmatrix} \hat{y}_{10} \\ \hat{y}_{20} \\ \vdots \\ \hat{y}_{s0} \end{pmatrix} = \mathbf{Y}\boldsymbol{\lambda}^* = \begin{pmatrix} \sum_{j=1}^{n} y_{1j}\lambda_j^* \\ \sum_{j=1}^{n} y_{2j}\lambda_j^* \\ \vdots \\ \sum_{j=1}^{n} y_{sj}\lambda_j^* \end{pmatrix},$$

and the sum of the projected shares is $\sum_{r=1}^{s}\sum_{j=1}^{n} y_{rj}\lambda_j^* = \sum_{j=1}^{n}\sum_{r=1}^{s} y_{rj}\lambda_j^* = \sum_{j=1}^{n}\left(\lambda_j^*\sum_{r=1}^{s} y_{rj}\right) = \sum_{j=1}^{n}\lambda_j^*.$

Since the single input is fixed to unit and projected onto itself, for the sum of lambdas we have

$$\hat{x}_0 = \sum_{j=1}^{n} x_j\lambda_j^* = \sum_{j=1}^{n}\lambda_j^* = 1.$$

Thus the adjusted income shares generated by the model add up to unity.