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Structure of Final Consumption in Light of GDP Dynamics[1]

Luboš Marek [2] | *University of Economics, Prague, Czech Republic*
Stanislava Hronová [3] | *University of Economics, Prague, Czech Republic*
Richard Hindls [4] | *University of Economics, Prague, Czech Republic*

Abstract

This paper studies the evolution of the relationship between households' and general government's expenditure on final consumption in the most recent 15 years; our data covers more than 30 countries. The most recent 15-year period has been characterised by variability in the worldwide economic development; hence this paper mainly focuses on the relationship to the rate of GDP growth. The mentioned relationship has also been assessed from the viewpoints of both the economic policy and the concepts that may be utilised by political representatives as the tools for controlling such economic policy.

This text aims at pointing out the importance of utilising the indicated issues, as related to the evolution of the relationship between the structure of the final consumption and the rate of economic growth, as well as the necessity of a proactive approach to the cognitive and behavioural activities, rather than in the modelling area, which has been visibly prevailing lately.

Keywords	***JEL code***
Final consumption expenditure of households, government final consumption expenditure, GDP, economic crisis	*E21, C82*

INTRODUCTION

The turbulent development in the worldwide economy we have seen in the past 15 years has brought about a number of issues related to the causes of crises and methods to alleviate the crises' consequences. Analytical studies have focused on both purely economic aspects and the degree of influence the behaviour of political elites has on the economic troubles. The very nature of national accounts implies that a great deal of attention has been given to the aspects and factors of the expenditure approach to estimating the Gross Domestic Product (hereinafter "GDP").

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[2] Faculty of Informatics and Statistics, Department of Statistics and Probability, W. Churchill Square 4, 130 67 Prague 3, Czech Republic. E-mail: marek@vse.cz.

[3] Faculty of Informatics and Statistics, Department of Economic Statistics, W. Churchill Square 4, 130 67 Prague 3, Czech Republic. E-mail: hronova@vse.cz.

[4] Faculty of Informatics and Statistics, Department of Statistics and Probability, W. Churchill Square 4, 130 67 Prague 3, Czech Republic. E-mail: hindls@vse.cz.

Within all such views on the economy, an important role is played by the key factor of the economic universe – the human factor as both an architect of investment and production plans and a user of the newly created wealth. The created resources are mainly realised in the form of individual consumption, represented by the actual final consumption by households, i.e., final consumption expenditure by households increased by social transfers in kind, paid to households by the general government and non-profit institutions serving households.[5] The actual collective consumption, i.e., the collective consumption expenditure by the general government, mainly includes the expenses incurred on defence, security, and administration, that is, in favour of society as a whole (not just households).

It is therefore clear that the main proportion of the final consumption, i.e., individual consumption expenditure, directly goes to households. If, however, we would like to view households as a factor influencing the dynamics of the economic development (i.e., from the viewpoint of the expenditure approach to estimating the GDP), we will be interested in households' "spending", expressed as the final consumption expenditure by households. In analogy, the expenses by general government incurred on final consumption represent the proportion influenced by their decision-making and determined by their economic behaviour.

The scope of the final expenditure by the general government depends on the level to which the role of the state in the economy is fulfilled, in other words, to what extent the state is trying, with the aid of its social and cultural policies,[6] with inequalities implied by households' different income levels and, at the same time, to what extent the state ensures its own functions as the centre of power and administration.[7] This second component, of course, depends on not only the fulfilment of this role of the state,[8] but also on the size of the general government' sector.

The above-mentioned considerations imply that it will be interesting to observe the relationship between the final consumption expenditure by households and that by general government and the evolution of this relationship in time and space. The ratio of these two indices reflects the relationship between the roles of households and general government in their effects on economic growth. This paper should find an answer to the question of to what extent the evolution of this ratio is connected with specific conditions prevailing in a given country, and with different stages of the economic cycle.[9]

1 CONSUMPTION SMOOTHING AS A REFLECTION OF ECONOMIC DYNAMICS

Consumption has a general character of both the cause and the effect in economic development; moreover, it is often considered from the viewpoint of a certain corrective role in this development. In times of recession, the effect on households' consumption is delayed after the worsening economic conditions. In other words, households' spending stereotypes (the trend to spend money or not reduce consumption) survives for a certain short period of time – this feature slows down the occurrence of recession and alleviates it in its beginning. And the other way around: with arising economic recovery, the consumption grows more slowly than the whole, households' spend their money cautiously, putting off the overall growth. Economists often use the term "consumption smoothing" to describe this phenomenon.

5 The amount of social transfers in kind paid to households by the government and the non-profit institutions is equal to that of the individual final consumption by the general government and non-profit institutions.

6 In particular, via the expenses incurred on health care, education, culture, etc., corresponding to the social transfers in kind in favour of households.

7 Via the collective consumption expenditure.

8 Informally, they are sometimes called "royal services".

9 The final consumption expenditure by non-profit institutions serving households is usually (due to its small value) added to the final consumption expenditure by households. We will follow this principle in this paper.

Consumption smoothing is an economic feature which expresses households' desire to have at their disposal a stable access to consumption. That is why households utilise their consumption in times of higher income to save for the times of recession to achieve a higher degree of economic stability and predictability. On the other hand, households reduce (or put off) consumption in times of uncertainty and unfavourable economic conditions to reduce uncertainty and prevent future problems. This tendency again survives even the beginning of economic recovery. This is the way in which households slow down the coming recovery.

The final consumption by households accounts for about 50% of the GDP (this proportion is even higher in some countries), it plays a dominant role in the economic climate. Such issues were addressed by Modigliani and Brumberg (1954) or Friedman (1956) shortly after World War II, in the time of the after-war economic boom; this interest was probably caused by reminiscences of the big crisis of the late 1920s and early 1930s. Another model of consumption smoothing was formulated 20 years later; it was Hall's model, inspired by Milton Friedman, cf. Hall (1978). Hall's work, to a certain extent, went against the then prevailing idea that households' tendency to consumption is marginal, and consequently their consumption is tightly correlated with the current income. On the contrary, he claimed that, assuming purposeful behaviour, households endeavour to optimise and keep the consumption stable, from which the smoothing effect ensues.

As already mentioned above, the final consumption by households is not a sole factor of wealth use; this use also includes the final consumption by the general government. A problem arises here on the borderline between economics and politics, namely, the ratio of these two types of expenditure in the respective economy. Political representations create and approve legislative measures (regarding taxes, budget, control, etc.), which may, even though to a limited extent, influence the value of this ratio. In other words, it is a form of "command economy", in which the state intervenes in the economy, or rather the creation of the economic policies. Within a suitable setting, such interventions may be an important tool of protection from recession. On the other hand, they may also accelerate the recession (especially in the social area, characterised by a high degree of redistribution) if the measures are accepted hastily and without a deeper concept.

The data from the period of 2002 through 2015 is analysed for evaluating the relationship between the final consumption expenditure by households and that by the general government. A prevailing majority of European countries, the U.S.A., and Japan are included in this analysis. First of all, the ratio of the final consumption expenditure by households (hereinafter "*HFCE*") with respect to the final consumption expenditure by general government (hereinafter "*GFCE*") was calculated in each of the years 2002 through 2015. For each of the 33 countries the arithmetic mean (non-weighted) was calculated over the entire period and the maximum and minimum values were identified, together with the years in which the extreme values were taken on. That is:

$$\overline{k}_i = \frac{\sum_{t=1}^{T} \frac{HFCE_t^i}{GFCE_t^i}}{T}, \tag{1}$$

where:

$T = 14$ is the number of the years in which the $HFCE_t/GFCE_t$ ratio was observed, $t = 1, 2, \ldots, 14$;
$n = 33$ is the number of the countries in which the $HFCE^i/GFCE^i$ ratio was observed, $i = 1, 2, \ldots, n$.

The *HFCE*/*GFCE* ratio reflects many long-term relationships: the scope of redistribution of newly created income; the degree of individual solidarity in society; cultural, historic and social conventions; differences between countries; tax (in)stability; etc. Undoubtedly, this ratio thus expresses the long-term concept of the respective country's economic policies. The results are shown in Table 1 and Figure 1.

Table 1 *HFCE/GFCE* ratio (average, minimum, and maximum)

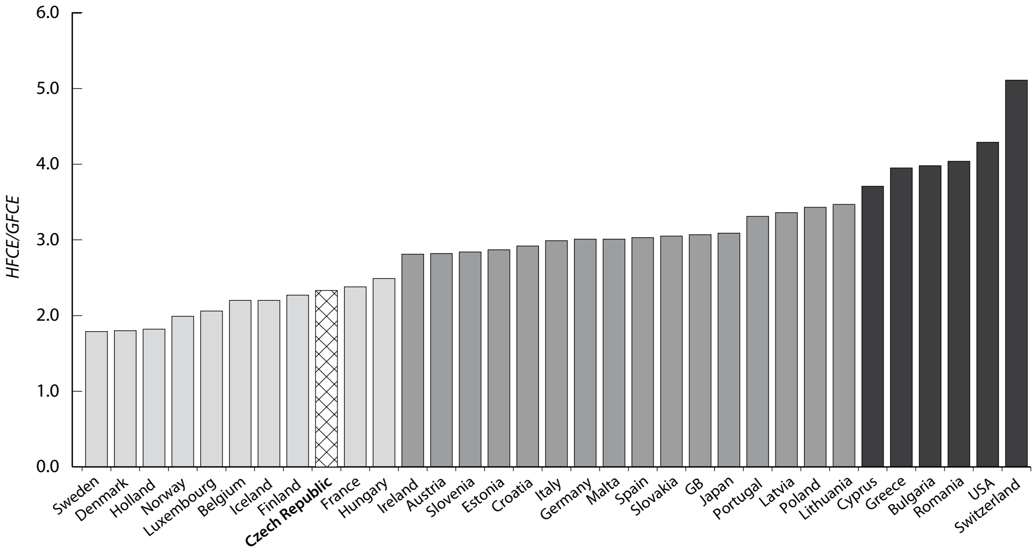
Country *i*	$\bar{k}_i$ = average *HFCE/GFCE*	Minimum/year	Maximum/year
Sweden	1.79	1.73/2015	1.84/2005
Denmark	1.80	1.66/2009	1.87/2007
Netherlands	1.82	1.61/2010	2.11/2002
Norway	1.99	1.84/2015	2.14/2006
Luxembourg	2.06	1.77/2015	2.54/2002
Belgium	2.20	2.11/2010	2.34/2002
Island	2.20	1.98/2010	2.41/2005
Finland	2.27	2.20/2010	2.35/2002
Czech Republic	2.33	2.21/2003	2.45/2013
France	2.38	2.30/2014	2.47/2006
Hungary	2.49	2.36/2006	2.67/2012
Ireland	2.81	2.54/2012	3.14/2002
Austria	2.82	2.65/2015	2.97/2005
Slovenia	2.84	2.70/2010	3.05/2007
Estonia	2.87	2.38/2009	3.40/2006
Croatia	2.92	2.38/2010	3.40/2003
Italy	2.99	2.77/2009	3.22/2015
Germany	3.01	2.78/2015	3.19/2006
Malta	3.01	2.71/2014	3.31/2005
Spain	3.03	2.64/2010	3.39/2002
Slovakia	3.05	2.95/2015	3.26/2008
United Kingdom	3.07	2.70/2010	3.37/2015
Japan	3.09	2.99/2010	3.21/2002
Portugal	3.31	3.11/2005	3.64/2015
Latvia	3.36	3.07/2002	3.61/2007
Poland	3.43	3.26/2015	3.74/2002
Lithuania	3.47	2.92/2003	3.93/2006
Cyprus	3.71	3.25/2003	4.46/2014
Greece	3.95	3.22/2012	4.45/2006
Bulgaria	3.98	3.70/2003	4.27/2007
Romania	4.04	3.43/2003	4.56/2012
USA	4.29	4.09/2011	4.54/2002
Switzerland	5.11	4.93/2014	5.28/2007

Source: EUROSTAT, authors' own calculations

Table 1 shows that the average values of the *HFCE*/*GFCE* ratio have been rather variable in the recent 14 years, ranging from 1.79 to 5.11. We have classified the countries into three categories: up to 2.50, between 2.51 and 3.50, and 3.50 plus values of the ratio (of course, a different categorisation would be admissible).

It is not surprising that all North European countries fall into the category with the smallest value of the *HFCE*/*GFCE* ratio – these countries have traditionally had a high degree of regulated redistribution, as well as social cohesion and solidarity. Neither is surprising the fact that countries such as Cyprus and Greece have a large value of this ratio, being countries in the south of Europe undergoing crises and high indebtedness of the general government in consequence of extraordinarily generous budget policies; this aspect has led to a drastic reduction of the final consumption expenditure by the general government, with a subsequent "transfer of full responsibility" for the consumption onto households. Another instance of a high level of the *HFCE*/*GFCE* ratio is the U.S.A., in which – as compared to Europe – completely different concepts of social solidarity, health and social security insurance prevail, with a high degree of tax liberalism. Another example of an unusually high level of this ratio is Switzerland, for which the value is implied by a low volume of other non-market output by the general government[10] and the social transfers in kind in favour of households.

Figure 1 Average values of the *HFCE/GFCE* ratio



Source: EUROSTAT, authors' own calculations

Regarding the maximum values, it is worth mentioning that 24 out of 33 countries (i.e., nearly three-quarters) achieved the *HFCE*/*GFCE* maximum values either in the years 2002 through 2007, that is, prior to the mortgage and subsequent fiscal crisis, or later, in the years 2014 and 2015, after the crises faded away. There is an exception, Slovakia, in which the maximum occurred as late as 2008, after the beginning of the mortgage crisis; this country suffered badly during the crisis (with about a five-per-cent decrease of the GDP growth in 2009).

The minimum values of the *HFCE*/*GFCE* ratio, on the contrary, were taken on in the mid-crisis, mainly in the years 2010 and 2011. However, the distribution of the *HFCE*/*GFCE* minimum values is, regarding the years they occur, more variable than that of the maximum values.

2 RELATIONSHIP BETWEEN HFCE AND GFCE AS A TOOL OF ECONOMIC POLICY?

No less interesting is the time evolution of the average *HFCE*/*GFCE* ratio over all 33 countries in the 14-year period from 2002 to 2015. Namely,

$$\overline{k}_t = \frac{\sum_{i=1}^{n} \frac{HFCE_t^i}{GFCE_t^i}}{n}, \tag{2}$$

[10] The proportion of non-market output by general government in the overall national economy production amounts to less than 6% in Switzerland, but to about 8% in the Czech Republic and more than 10% in France.

where:

$t = 1, 2, \ldots, 14$ stands for the years in which the $HFCE^i/GFCE^i$ is observed in the $n = 33$ countries, that is, $i = 1, 2, \ldots, 33$.

The results are shown in Table 2.

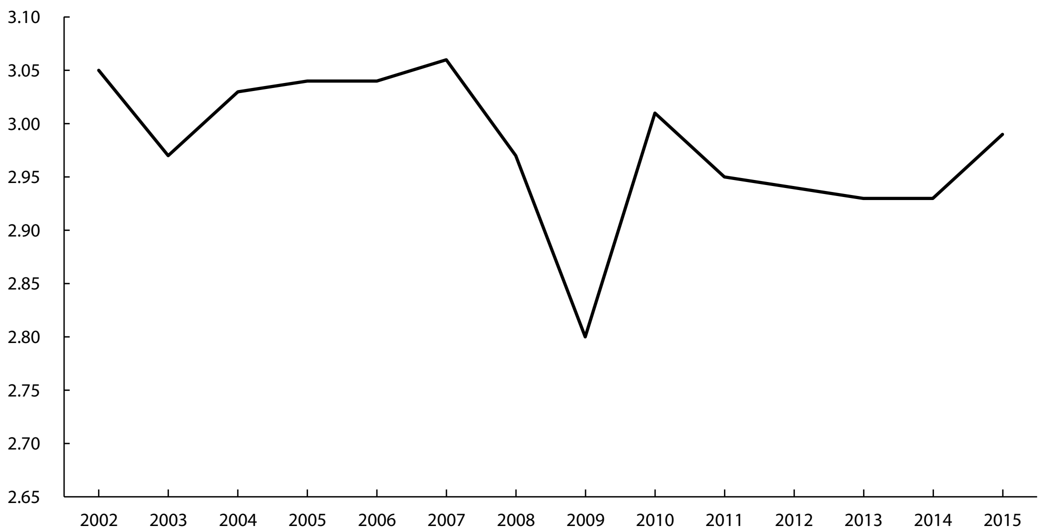
Table 2 Average values of the *HFCE/GFCE* ratio

Year t	$\bar{k}_t$ = average *HFCE/GFCE*	Year t	$\bar{k}_t$ = average *HFCE/GFCE*
2002	3.05	2009	2.80
2003	2.97	2010	3.01
2004	3.03	2011	2.95
2005	3.04	2012	2.94
2006	3.04	2013	2.93
2007	3.06	2014	2.93
2008	2.97	2015	2.99

Source: EUROSTAT, authors' own calculations

Plotting the values from Table 2 into a chart (Figure 2), we can see that in the years of the mortgage and subsequent fiscal crisis (say, from 2008 to 2013, with a temporary and insignificant recovery in 2010 and 2011) the *HFCE*/*GFCE* ratio value tended to be decreasing, and to be increasing (that is, the consumption by households getting higher) again after the crises fading out and a vague indication of recovery in 2014 and later. An exception is represented by the year 2010, in which the ratio temporarily returned to a higher level (3.01) at the end of the mortgage crisis, but the quickly coming fiscal crisis sent the *HFCE*/*GFCE* ratio's trend back to decreasing.

Figure 2 *HFCE/GFCE* ratio values (averages over 33 countries, years 2002 through 2015)

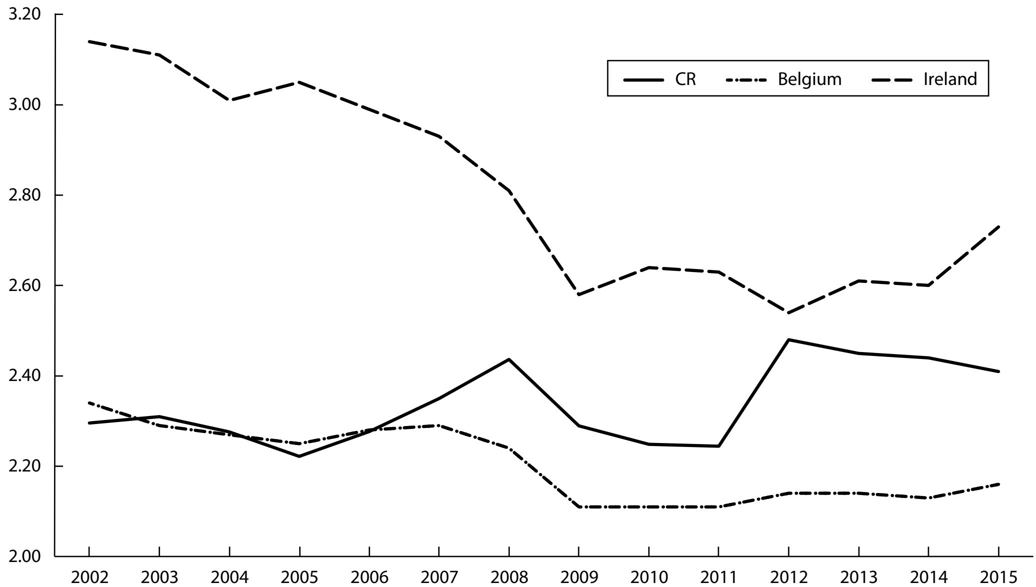


Source: EUROSTAT, authors' own calculations

In Figure 3 we compare the *HFCE*/*GFCE* ratio values for the Czech Republic with those of two more countries (Belgium and Ireland) whose sizes are similar to that of the Czech Republic, even though, on a long-term basis, their economic conditions and concepts of economic policies a\re somewhat different.

What is interesting is Ireland – considered an economic tiger some time ago – and the significant decrease of its final-consumption ratio in the time of crisis. This decrease is not so distinct, even if it is present, for the Czech Republic and Belgium. The ratio began to grow after the crises ended.

Figure 3 *HFCE/GFCE* ratio values (Czech Republic, Belgium and Ireland, years 2002 through 2015)



Source: EUROSTAT, authors' own calculations

Let us now have a look at a phenomenon that is not often analysed: the relationship between the *HFCE/GFCE* ratio and the economic growth, expressed as the GDP growth rate. As we will see below, it may be generally expected and proven that, in the time of imminent recession, political elites and representations endeavour to increase the level of redistribution and economic regulation, while they aim at liberalisation of consumption in the time of recovery by releasing the temporary regulation measures, thus lowering the level of restrictions on the behaviour of consumers and investors.

Table 1A in the Appendix shows data of the *HFCE* and *GFCE* values, and of the GDP growth rate values in 33 countries and the years 2002 through 2015. The data of the *HFCE/GFCE* ratio says by what proportion the *HFCE* is larger than the *GFCE* in the respective country and year. The economic growth rates (in percentages) express the dynamics of the GDP as a top-level aggregate index of the national accounts (again for the same 33 counties and years 2002 through 2015). Figure 4 sums up this data taken from the national accounts.

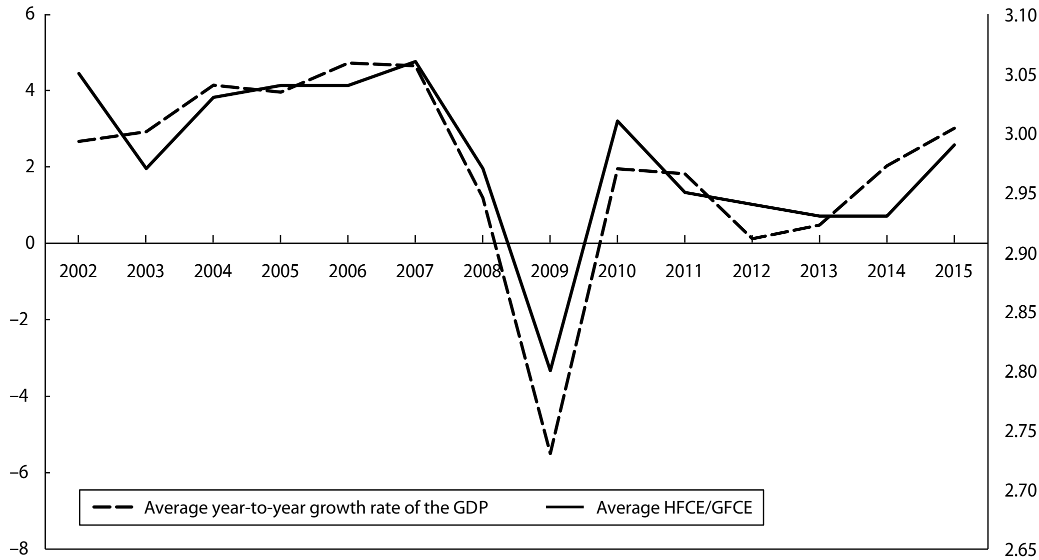
Figure 4 clearly shows that the *HFCE/GFCE* index of final consumption and the GDP growth rate are very tightly correlated. In order to calculate the value of this correlation, we first have to remove the linear trends from both series. For the average growth rate values, the trend function is:

$$T_t = 419{,}0731 - 0{,}2076 \cdot t, \quad t = 2002, 2003, \ldots, 2015,$$

and for the HFCE/GFCE ratio it is:

$$T_t = 18{,}8266 - 0{,}0079 \cdot t, \quad t = 2002, 2003, \ldots, 2015.$$

Figure 4 The *HFCE/GFCE* ratio vs. GDP growth rates (years 2002 through 2015, averages for 33 countries)



Source: EUROSTAT, authors' own calculations

The residuals after this data cleaning lead to values of 2.063 for the consumption and 1.556 for the growth rates in the Durbin-Watson test, which means that the autocorrelation has been successfully removed from both series. The correlation coefficient for the residuals of both series amounts to 0.9233, which confirms the strong mutual dependence between these series, already deduced from Figure 4. We can therefore observe that the time evolution of the *HFCE/GFCE* ratio is nearly a carbon copy of the GDP evolution in the years of both recession and recovery worldwide, regardless of their actual levels.

The above-mentioned considerations imply that the final consumption is highly sensitive with respect to the aggregate dynamics of the economy. However, utilisation of tools aimed at increasing consumption to alleviate the effects of crises is rather limited. The relationship between both indices is obvious (cf. Figure 4), but political representations have a rather small chance to make actual use of the *HFCE/GFCE* ratio. Hence the changing character of the final consumption during recessions is a consequence of the above-described *consumption smoothing* more than a result of purposeful decision-making by political elites at times of a coming crisis. There are several reasons:

- In nearly all countries, even if to a different extent, the proportion of mandatory and quasi-mandatory expenses is high (up to about 85% in some countries); this fact does not allow for continuous regulation of consumption by households and restricts the options in the area of expenditure by the general government;
- Political representations are unable to modify the structure of the final consumption, i.e., the *HFCE/GFCE* ratio, in a sufficiently operative way (whether up or down) because legislative processes are traditionally rigid, consensus is hard to achieve, and the negotiation procedures (such as about pension reforms) are dragging;
- The growth rates of wages (which represent a decisive factor for the growth of the final consumption by households) is more or less independent of the actual measures taken by the decision-makers – the wages hence grow with a time lag after occurrence of economic recovery (in consequence of careful employers unwilling to let wages grow too quickly after the recession has faded out, lengthy collective bargaining processes, etc.), and vice versa;

- In the economy, the prevailing tendencies are to liberalisation and it is not popular to regulate the final consumption in the liberally oriented Euro-Atlantic environment, not even if the recession is coming or has already arrived. The conflict between liberalism and economic interventions from the centres of influence of power has been predetermined by the traditions prevailing in the respective society and such stereotypes are hard to overcome, even in times of economic need;
- It is very difficult to predict occurrences of crises – estimates of turn-points lack in efficiency, as we saw in connection with the two most recent crises. Both economic theory and mathematical-statistical modelling fail as a rule, and political elites do not have sufficient support from economic research to help them carry through their – often unpopular – regulatory measures;
- From the formal viewpoint, the variations in the *HFCE*/*GFCE* ratio values – measured by the variation coefficient for each country in the years 2002 through 2015 – are many times lower than those in the GDP growth rates. The variation coefficient values range from approx. 2% (1.7% in Sweden) to approx. 12% (12.21% in Croatia); the variation coefficient values for the GDP growth rates are as high as tens or even hundreds of per cent. For both of these indices it is true that stronger economies are characterised by much lower variability compared with weaker, more fluctuating ones, in particular those in the south of Europe or certain East-European economies (of the former socialistic block). A low variability in the values of the *HFCE*/*GFCE* ratio, in formal expression, indicates smaller changes in the structure of consumption and, consequently, a lower potential to purposefully influence this ratio.

Nonetheless, we can say that, despite the above-listed objective restrictions, the extent might be higher to which the final consumption is utilised as a corrective measure to influence the economic development. Achievement of this goal should mainly be facilitated by a deeper analysis of the causes for the changes in the *HFCE*/*GFCE* ratio values in times of recession and recovery and by giving enough attention to the consumption smoothing. Other aspects of this problem include utilisation of behavioural and institutional economics.

CONCLUSIONS

The relationship between the final consumption expenditure and the growth rate, as well as the possible utilisation of this mechanism for alleviating economic recessions, is a slightly explored area. That is also why it is a seldom utilised tool of active economic policy. However, one aspect of this relationship, namely, the consumption smoothing, is better known, especially in the model-descriptive context. Motivation aspects of economic subjects have, however, not been studied so well. Analysing this relationship and, above all, its time evolution in dependence on the current stage of the economic cycle, which is also a topic of the present paper, aims directly at the focus of economic recessions and recoveries.

It is turning out that utilisation of the relationship between the individual and collective consumption on the one hand and the aggregate growth rate on the other hand is, as also shown above, rather difficult and meets a number of more or less objective obstacles. Hence the effort to overcome these obstacles, in particular, in the legislative, cognitive and behavioural areas, might gradually lead to a strong tool of regulatory protection against recessions and stalling of economic growth.

It is the analysis of the above-mentioned issues, and efficient action in the cognitive and behavioural areas, rather than in model-forming, which seems to have been prevailing recently, which might contribute to our deeper knowledge of motivations driving economic subjects towards positive activation in their economic behaviour in the current environment.

Analytical work, as well as possible recommendations, should go in this direction: the utilisation of this "behavioural ratio" for aiming at the mechanisms governing the arrivals and departures of economic peaks and saddles, thus facilitating alleviation of the socio-economic consequences of such cycles in combination with households' consumer habits and traditional behaviour.

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APPENDIX

Table 1A HFCE/GFCE ratio (ratio) and year-to-year growth rate (g.r.) of the GDP

	2002		2003		2004		2005		2006		2007		2008		2009		2010		2011		2012		2013		2014		2015		Ø	Ø
	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %	ratio	g.r. %
Czech Rep.	2.30	2.1	2.21	3.8	2.28	4.7	2.22	6.8	2.28	7.0	2.35	5.7	2.44	3.1	2.29	-4.5	2.25	2.5	2.24	1.8	2.48	-1.0	2.45	-0.9	2.44	2.7	2.41	4.5	2.33	2.74
Belgium	2.34	1.4	2.29	0.8	2.27	3.3	2.25	1.8	2.28	2.7	2.29	2.9	2.24	1.0	2.12	-2.8	2.11	2.3	2.11	1.8	2.14	-0.1	2.14	0.2	2.12	1.3	2.13	1.4	2.20	1.29
Bulgaria	3.93	4.7	3.70	5.5	3.77	6.7	3.90	6.4	4.24	6.5	4.27	6.4	4.18	6.2	4.06	-5.5	3.94	0.4	4.08	1.8	4.27	0.6	3.77	0.9	3.82	1.5	3.74	3.0	3.98	3.22
Danmark	1.81	0.5	1.80	0.4	1.82	2.3	1.85	2.4	1.86	3.4	1.87	1.6	1.82	-0.8	1.66	-5.7	1.69	1.4	1.74	1.1	1.82	-0.4	1.81	0.4	1.81	1.3	1.84	1.0	1.80	0.64
Estonia	3.08	6.6	3.06	7.8	3.20	6.3	3.23	8.9	3.40	10.1	3.27	7.5	2.88	-4.2	2.38	-14.1	2.40	2.6	2.49	9.6	2.75	3.9	2.74	0.8	2.65	2.9	2.61	1.1	2.87	3.56
Finland	2.35	1.8	2.34	2.0	2.31	4.1	2.29	2.9	2.33	4.4	2.35	5.3	2.29	0.3	2.20	-8.5	2.20	3.4	2.21	2.8	2.24	-1.0	2.21	-1.4	2.24	-0.7	2.27	0.2	2.27	1.11
France	2.39	0.9	2.38	0.9	2.39	2.5	2.40	1.8	2.43	2.5	2.47	2.3	2.46	-0.1	2.37	-3.1	2.35	1.7	2.37	2.0	2.34	0.0	2.32	0.2	2.30	0.6	2.31	1.3	2.38	0.96
Croatia	3.40	4.9	3.40	5.4	3.32	4.1	3.36	4.3	2.73	4.9	2.73	5.1	2.68	2.1	2.41	-6.9	2.38	-2.3	2.41	-0.2	3.00	-2.2	3.00	-0.9	3.03	-0.4	3.01	1.6	2.92	1.39
Ireland	3.14	5.4	3.11	3.7	3.01	4.2	3.05	6.1	2.99	5.5	2.93	5.0	2.81	-2.2	2.58	-6.4	2.64	-1.1	2.63	2.2	2.54	0.2	2.61	-0.3	2.60	8.5	2.73	23.6	2.81	3.89
Island	2.16	0.1	2.20	2.4	2.27	7.8	2.41	7.2	2.39	4.7	2.37	6.0	2.14	1.2	1.98	-6.6	2.10	-4.1	2.19	2.7	2.17	1.5	2.15	3.3	2.16	2.0	2.12	4.0	2.20	2.30
Italy	3.06	0.5	3.00	0.0	2.94	1.7	2.91	0.9	2.94	2.2	2.98	1.7	2.94	-1.2	2.77	-5.5	2.81	1.7	2.88	0.4	3.14	-2.4	3.10	-1.9	3.16	-0.3	3.22	0.8	2.99	-0.10
Japan	3.21	0.3	3.18	1.7	3.17	2.4	3.15	1.3	3.19	1.7	3.17	2.2	3.12	-1.0	3.03	-5.5	2.99	4.7	2.99	-0.5	3.00	1.4	3.01	1.2	2.99	1.4	3.02	1.5	3.09	0.91
Cyprus	3.55	2.1	3.25	1.9	3.62	4.2	3.59	3.9	3.46	4.1	3.82	5.1	3.88	3.6	3.45	-1.9	3.36	1.3	3.38	0.4	3.81	-2.4	3.90	-5.4	4.46	-2.5	4.44	1.6	3.71	1.14
Lithuania	2.96	6.8	2.92	10.3	3.24	7.4	3.59	7.8	3.93	7.8	3.57	9.8	3.21	2.9	3.29	-14.8	3.45	16.0	3.76	6.0	3.62	3.7	3.74	3.3	3.72	3.0	3.58	1.6	3.47	5.11
Latvia	3.07	7.1	3.22	7.7	3.35	8.8	3.45	10.1	3.35	11.0	3.61	10.0	3.37	-2.8	3.15	-17.7	3.24	-1.3	3.38	5.3	3.50	5.2	3.47	4.1	3.48	4.0	3.41	3.2	3.36	3.91
Luxembourg	2.54	4.1	2.30	1.7	2.20	4.4	2.15	5.3	2.16	4.9	2.14	6.6	2.15	-0.7	2.00	-5.6	1.94	3.1	1.91	1.9	1.95	-0.2	1.89	2.1	1.82	4.1	1.77	4.8	2.06	2.61
Hungary	2.46	4.5	2.40	3.9	2.45	4.8	2.46	4.0	2.36	3.9	2.53	0.1	2.50	0.9	2.47	-6.8	2.45	1.1	2.50	1.6	2.67	-1.7	2.65	1.1	2.49	3.7	2.49	2.9	2.49	1.71
Malta	3.17	2.4	3.14	0.7	3.18	-0.3	3.31	3.6	3.17	2.6	3.21	4.1	2.96	3.9	2.94	-2.8	2.93	4.2	2.99	1.5	2.84	0.8	2.88	2.6	2.71	3.5	2.73	6.4	3.01	2.37
Germany	3.07	0.0	3.08	-0.4	3.13	1.2	3.16	0.7	3.19	3.7	3.16	3.3	3.12	1.1	2.97	-5.1	2.91	4.0	2.91	3.3	2.93	0.7	2.89	0.4	2.83	1.6	2.78	1.7	3.01	1.16
Netherlands	2.11	0.1	2.04	0.3	2.04	2.2	2.06	2.0	1.88	3.4	1.83	3.9	1.79	1.8	1.64	-3.7	1.61	1.5	1.62	0.9	1.71	-1.2	1.72	-0.8	1.73	1.4	1.76	2.0	1.82	0.99
Norway	2.06	1.5	2.06	1.0	2.11	4.0	2.14	2.6	2.14	2.3	2.10	2.7	2.01	0.1	1.90	-1.6	1.91	0.5	1.92	1.3	1.90	2.9	1.89	0.6	1.85	2.2	1.84	1.6	1.99	1.55
Poland	3.74	1.4	3.64	3.9	3.68	5.3	3.50	3.6	3.42	6.2	3.38	6.8	3.32	5.1	3.36	1.6	3.30	3.9	3.41	4.5	3.44	2.0	3.35	1.6	3.28	3.3	3.26	3.6	3.43	3.77
Portugal	3.25	0.8	3.21	-0.9	3.19	1.6	3.11	0.8	3.24	1.4	3.35	2.4	3.42	0.0	3.17	-2.9	3.14	1.9	3.14	-1.3	3.58	-3.2	3.42	-1.4	3.56	0.9	3.64	1.5	3.31	0.11
Austria	2.93	1.7	2.90	0.9	2.95	2.6	2.97	2.4	2.94	3.7	2.90	3.7	2.84	1.4	2.76	-3.8	2.75	1.8	2.74	2.8	2.71	0.9	2.72	0.3	2.70	0.6	2.65	1.0	2.82	1.43
Romania	4.54	5.1	3.43	5.2	4.23	8.5	3.99	4.2	4.13	7.9	4.18	6.3	3.83	7.3	3.47	-6.6	3.59	-1.1	3.67	2.3	4.21	0.6	4.30	3.5	4.40	3.0	4.55	3.8	4.04	3.57
Greece	3.98	3.4	4.13	5.9	4.07	4.4	4.23	2.3	4.45	5.5	4.23	3.5	4.29	-0.2	3.82	-3.1	4.13	-4.9	4.24	-7.1	3.22	-7.0	3.48	-3.9	3.54	0.7	3.52	-0.2	3.95	-0.05
Slovenia	2.93	3.8	2.95	2.9	2.91	4.4	2.85	4.0	2.81	5.8	3.05	7.0	2.91	3.4	2.71	-7.9	2.70	1.3	2.71	0.7	2.79	-2.5	2.78	-1.1	2.79	3.0	2.81	2.9	2.84	1.98
Slovakia	2.85	4.6	2.78	4.8	3.02	5.1	3.14	6.7	3.02	8.3	3.24	10.5	3.26	5.8	3.09	-4.9	2.98	4.4	2.99	3.0	3.21	1.8	3.11	0.9	3.01	2.5	2.95	3.6	3.05	4.08
Spain	3.39	2.7	3.31	3.1	3.25	3.3	3.21	3.6	3.19	4.1	3.12	3.5	2.95	0.9	2.64	-3.8	2.64	-0.2	2.73	0.1	2.97	-1.6	2.96	-1.2	3.01	1.4	2.98	3.2	3.03	1.36
Sweden	1.81	2.5	1.79	2.3	1.82	4.2	1.84	3.2	1.82	4.3	1.83	3.3	1.80	-0.6	1.76	-5.0	1.76	6.6	1.79	2.9	1.80	0.9	1.78	1.6	1.76	2.3	1.73	4.2	1.79	2.34
Switzerland	5.17	0.2	5.10	0.0	5.15	2.4	5.13	2.7	5.17	3.8	5.28	3.8	5.25	2.2	5.15	-1.9	5.23	3.0	5.19	1.8	4.94	1.0	4.94	2.0	4.93	1.9	4.94	1.7	5.11	1.76
USA	4.54	1.8	4.43	2.8	4.45	3.8	4.47	3.3	4.46	2.7	4.45	1.8	4.25	-0.3	4.16	-2.8	4.17	2.5	4.09	1.6	4.15	2.3	4.22	2.2	4.13	2.4	4.13	2.1	4.29	1.87
UK	3.32	2.3	3.19	3.9	3.10	4.09	3.04	3.2	2.98	2.8	3.02	3.4	2.97	-0.8	2.77	-5.2	2.70	1.7	2.74	1.1	3.16	0.3	3.26	1.7	3.29	3.1	3.37	2.2	3.07	1.70
Average	3.05	2.67	2.97	2.92	3.03	4.14	3.04	3.96	3.04	4.72	3.06	4.65	2.97	1.19	2.80	-5.50	2.81	1.95	2.85	1.82	2.94	0.12	2.93	0.48	2.93	2.03	2.93	3.01	2.95	2.01

Source: EUROSTAT, authors' own calculations

Selected Factors Determining Inward of Foreign Direct Investment in the Czech Regions in Years 2002 to 2012

Ondřej Babuněk [1] | *University of West Bohemia, Pilsen, Czech Republic*

Abstract

This study aims at the research of the influence of unemployment rate (Un), exchange rate of CZK/USD (Ex), inflation rate (Inf), expenditures on research and development (RaD), size of wages and environmental pollution on inward foreign direct investment (FDI) into regions in the Czech Republic in the period 2002–2012. The study dealt with the whole period, then the pre-crisis and crisis period, altogether with the inclusion or elimination of Prague in or out of the group of Czech regions. Models without and with dynamic parameter were checked. For estimation of influence of the above mentioned parameters the fixed effects model, random effects model and pooled ordinary least squares (POLS) were used. For dynamic model the generalized method of moments and POLS were applied. The results showed that Wage, appreciation of Ex and RaD positively determined the inflow of FDI to Czech regions and no negative determinant of inward FDI has been found. On the other hand, results of dynamic model imply that inward FDI in preceding year, appreciation of Ex, RaD have positive impact on inward FDI in current year. However, negative impact of Un and Inf on inward FDI were detected. Results of this research enable the policy makers or decision makers try to focus their attention on specific factors and eliminate to consume scarce funding.

Keywords

Foreign direct investment, unemployment, inflation, expenditure on research and development, pollution, panel data

JEL code

E22, E24, F21

INTRODUCTION

Foreign direct investment (FDI) is one of the driving forces of the economy and has always been a widely-discussed topic by politicians, economists but also by the public. The Czech Republic as the recipient of FDI used the investment to transform the economy. After the transformation period Czech got fully integrated in the global economy and now it is strongly interlinked with the neighbouring

[1] Department of Economics and Quantitative Methods, Faculty of Economics University of West Bohemia in Pilsen, Univerzitni 22, 306 14 Pilsen, Czech Republic. E-mail: babunekon@gmail.com, phone: (+420)296854943. Author is also working at the General Financial Directorate, Lazarska 15/7, 117 22 Prague 1, Czech Republic.

countries and, as was proved by comparison, the Visegrad Group countries and their main business partners account for the identical inward FDI per capita (Babuněk, 2012).

Almost all governments have an enormous interest in attracting FDI as it can create new job opportunities, new technologies and, in a broader sense, it can support the growth of competitiveness. FDI positively influences domestic competitiveness by introducing new technologies and improving human capital (skills) by raising wages of employees (Feenstra, 1997). In Czech, inward FDI is supported not only by domestic institutions (e.g. CZECHINVEST and others), but also by the geographical location of Czech within Europe. The location is also one aspect of the so called Dunning Eclectic Paradigm (Dunning, 2001), which was worked out by Dunning in the year 1988.

The inward FDI in Czech regions is uneven. The financial and later the economic crises that hit the Czech economy in the years 2008–2012 highlighted these disproportions even further. The Olomouc, Karlovy Vary and Zlin regions can be classified as the regions with the smallest inward FDI. On the contrary, regions like Prague, Central Bohemia, Moravia, Silesia and South Moravia received more than four times as much investment as the "worst" regions. More than half of all FDI that "flowed" into the Czech Republic headed for Prague (see Figure 1 to Figure 4). The privileged position of Prague is caused not only by the agglomeration effect (Budd, 2004) but mainly by its central location within the geographical structure of the Czech Republic. The policy of towns or regions that is inclined towards the FDI inflow has a better chance of attracting the investment if the town or region is economically (and administratively) near other urban or regional localities (Blanc-Brude, 2014).

The aim of the research is an analysis of the chosen determinants influencing the FDI flow into Czech regions, and later, their comparison in both the pre-crisis and crisis period, together with the inclusion or elimination of Prague in or out of the set of regions. An analysis of aspects influencing the FDI flow into the Czech Republic has already been worked out many times, nevertheless, the research of determinants of the FDI flow based on the panel data of Czech regions in the period 2002–2012 has not been done yet.

The study has the following structure: In the next part the existing literature dealing with determinants influencing inward FDI is described. The third part contains methodology and collected data necessary for analysis. The fourth part deals with the results and the last one is devoted to the conclusion.

Figure 1 FDI stock in 2000 in Czech regions

Source: CNB, own construction

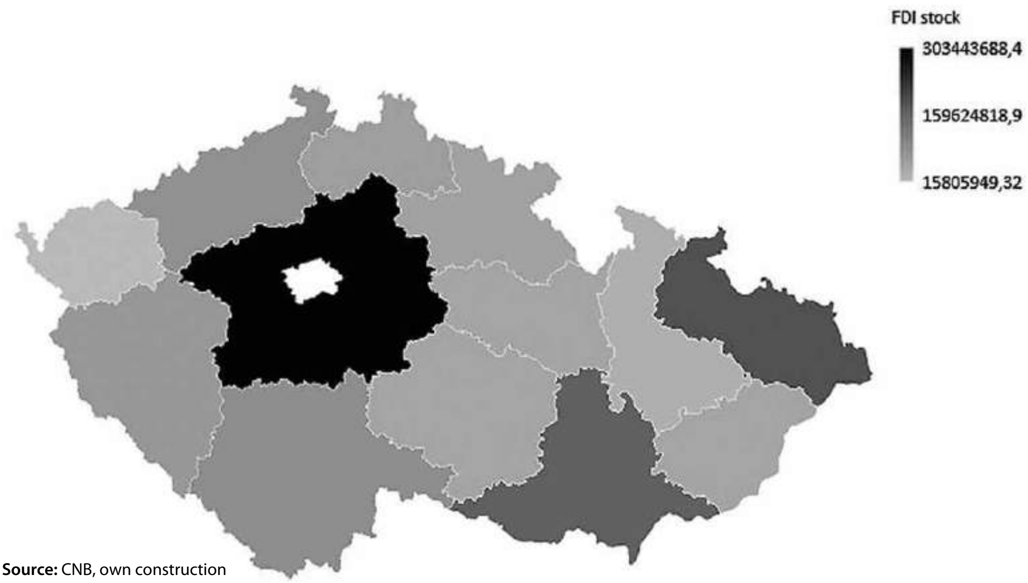
Figure 2 FDI stock in 2012 in Czech regions

Source: CNB, own construction

Figure 3 FDI stock in 2000 in Czech regions without Prague

Source: CNB, own construction

Figure 4 FDI stock in 2012 in Czech regions without Prague



Source: CNB, own construction

1 LITERATURE REVIEW

Large numbers of factors determining the inward FDI flow can be found anywhere but the study deals with aspects that should be significant.

1.1 Wages and unemployment

By the FDI flow into regions the demand for skilled workforce grows and in the regions where inward FDI is concentrated wages grow roughly by 50% faster than in the regions recording basically no FDI flow (Feenstra, 1997). This fact also deepens the structural differences in earnings (Nakamura, 2013). As seen from the global point of view, it can be said that inward FDI increases wage disproportions in the business sector (Chen, 2011), nevertheless, trade unions often make governments support inward FDI as much as they can. This pressure results in the fact that the governments offer foreign companies a tax discount (or a subsidy bonus) and go beyond the usual routine consisting in stipulation what should be done to attract investors because of higher wages demanded by trade unions (Haufler, 2011). Higher wages raise the price of the workforce and can lead to the growth of unemployment as the domestic companies usually show lower level of competitiveness. Higher wages of companies with foreign capital can cause internal tension in some companies without foreign capital (employees basically demand the same wage level as in the companies with foreign capital).

Özkan-Günay (2011) dealt with the identification of the basic factors necessary for obtaining FDI within the EU countries and two candidate countries (Croatia and Turkey) in the period 1998–2008. According to his study the unemployment rate is not a significant factor for inward FDI in the EU-15 countries but after the entry of the new member states from Central and Eastern Europe and the two candidate countries it became obvious that the unemployment rate has a positive impact on inward FDI (in this sense see also Lessmann, 2013; Long, 2015). When taking a complex look at the influence of the unemployment rate on the FDI inflow one cannot neglect the studies claiming that the unemployment rate has a negative impact on inward FDI (Boateng, 2015). The studies of Chen (2011), Olney (2013) and Huang (2013) also found out that the size of wages and the protection of employment by legal measures have a negative impact on inward FDI.

1.2 Exchange rate

Schiavo (2007) verified that the long-term fixed exchange rates have a positive impact on inward FDI, because in the fixed rate mode the volatility is usually eliminated. The empirical influence of the exchange rate in case of the FDI flow into the developing countries indicates that these countries de facto accept fixed exchange rate which substantially improves inward FDI, rather than those countries which opted for the floating exchange rates (Abbott, 2012). Kiyota (2004) pointed out that the level of exchange rate had a negative impact on inward FDI and he inferred that the depreciation of the currency of the host country contributed to the FDI inflow into its economy. The devaluation (depreciation) has a robust positive impact on the flows of FDI, while the average devaluation (depreciation) and its volatility do not have these impacts (Chakrabarti, 2002). When looking at the Eurozone it appears that the single currency and stability of the exchange rate are the main factors which helped the FDI inflow into the Eurozone countries (Usman, 2012).

1.3 Inflation

Boateng et al. (2015) also examined factors influencing inward FDI. He found out that the factors, such as money supply, inflation and interest rate seemed to have a negative impact on inward FDI. Kolstad (2008) and Li (2005) found out that the negative effect of inflation is statistically significant only in the developing countries, nevertheless, in the developed countries it is not significant. This may be caused by the fact that in the developed countries the economic development is lower than in the developing countries. Sánchez-Martin (2014) et al. also came to the conclusion of the insignificance of the factor of inflation and nominal interest rate, nevertheless, the results showed that it was not possible to determine negative or positive effects of inflation on inward FDI unambiguously. The inflation rate is often used as a proxy variable of the internal economic stability.

1.4 Research and development (RaD)

Even the business environment and infrastructure can be included in the set of key determinants influencing the attractiveness of inward FDI (Groh, 2012; Castiglione, 2012). The factor of infrastructure holds possible perils. There are several studies dealing with the influence of infrastructure on inward FDI. These studies found that the size or density of infrastructure had a positive effect on inward FDI (Yu, 2011; Castiglione, 2012; Long, 2015), but they were focused on the density of the network of motorways or railways. If the density of the network of motorways or railways was also included in the chosen determinants, this paper would more or less only follow the above studies. This article is going, instead of the generally included infrastructure variables, to try and explain the influence of expenditures on RaD. The RaD expenditures can be considered to be proxy variable of the technological infrastructure (Özkan-Günay, 2011). It has been found that the RaD expenditures are one of the main factors that helped the FDI inflow into the Eurozone (Usman, 2012). The growth of the public spending on RaD, and on education in general, influences the FDI inflow positively (Ramirez, 2013), but if the FDI inflow into the host economy increases, the innovation activities of the domestic companies are usually attenuated, which can be seen as a negative element.

1.5 Environment

It has been found that the long-term limitation of the environment in one country, which is caused by making standards of the environment protection stricter, may cause that the environmental pollution moves to the countries which do not try to reduce the environmental pollution (Kayalica, 2005). Low (1992) came to the conclusion that during the 70s and 80s a lot of multinational corporations moved their production capacities in the form of FDI into low income countries with no strict standards concerning the environmental protection. The production of highly polluting waste, such as pesticides and heavy metals (e.g. copper and zinc) was also moved to areas with mild environment protection (Anderson, 1995), nevertheless, Javorcik (2004) did not confirm the results of Anderson´s study. Rezza (2013) pointed out the fact that a high rate of the environment protection and its enforcement did not seem to be a significant factor influencing the FDI inflow. The environment protection can be mainly observed in the effort to reduce the production of greenhouse gases, e.g. CO_2, but not in the production of community waste. The influence of the greenhouse gases production on inward FDI has been a subject of a number of research studies (see especially Pao, 2011; Omri, 2014a, 2014b), nevertheless, it has not been processed yet as a proxy variable of the environment protection in the form of community waste production.

1.6 Tested hypotheses

Six hypotheses are tested in this article. Tested hypotheses are mentioned below and all hypotheses are tested at α = 5% significant level. Hypothesis is rejected if parameter does not show an assumed impact on inward FDI and, contemporaneously, if parameter shows an assumed impact, but is significant only at α = 1% significant level.

H1: Increasing the regional unemployment rate shows a positive influence on the FDI flow into Czech regions.
H2: Increasing regional wages influence inward FDI into Czech regions negatively.
H3: Appreciation of the rate CZK/USD has a positive impact on the FDI flow into Czech regions.
H4: The inflation rate influences the FDI flow into Czech regions negatively.
H5: Growth of the RaD expenditures in regions has a positive impact on inward FDI into Czech regions.
H6: Inward FDI into Czech regions is negatively influenced by the growth of pollution of the environment in Czech regions.

2 METHODS A METHODOLOGY

2.1 Data

The data verifying the hypotheses were collected from the statistics of the Czech National Bank (CNB) and the Czech Statistical Office (CZSO). The years 2002 up to 2012 are the period under review.

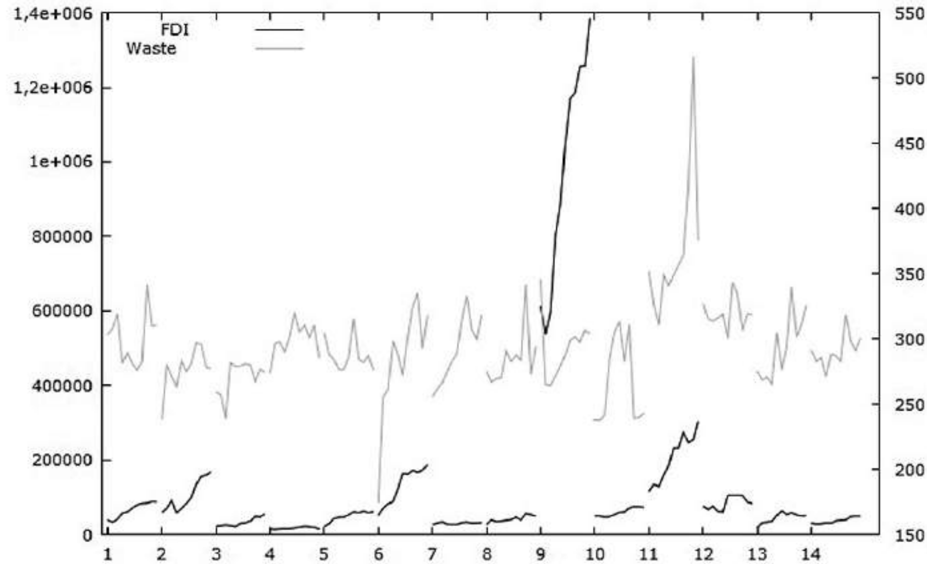
Regarding the situation when the Czech Republic was hit by the financial or economic crisis in the year 2008, it is, according to the development of individually included variables, necessary to examine not only the period 2002–2012 but also the pre-crisis period, i.e. the years 2002 up to 2007 and the crisis period, i.e. the years 2008 up to 2012.

The Prague region is a problematic part of the research as the FDI inflow into Prague includes roughly one half of all FDIs in the Czech Republic. At the initial stage of the research the congruence of inward FDI per capita for the whole Czech Republic was analysed. As became obvious from the analysis (the results are fully available from the author upon request), this inflow is not identical in all regions and is not identical even after combining the Prague region and the Central Bohemian region into one "natural central Bohemian metropolitan area" (Viturka, 2010; Hampl, 2011). It is therefore necessary to eliminate Prague from the set of regions, nevertheless, the aim of the study is to examine all Czech regions, and that is why the research has to be divided into several parts.

The first area includes all the regions including Prague in the years 2002–2012. The second area includes all Czech regions with the exception of Prague. The third area includes all Czech regions in the pre-crisis years (i.e. 2002–2007). The fourth area includes all Czech regions with the exception of Prague in the pre-crisis years (i.e. 2002–2007). The fifth area includes all Czech regions in the crisis period (i.e. 2008–2012) and the last area includes all Czech regions with the exception of Prague in the crisis period (i.e. 2008–2012).

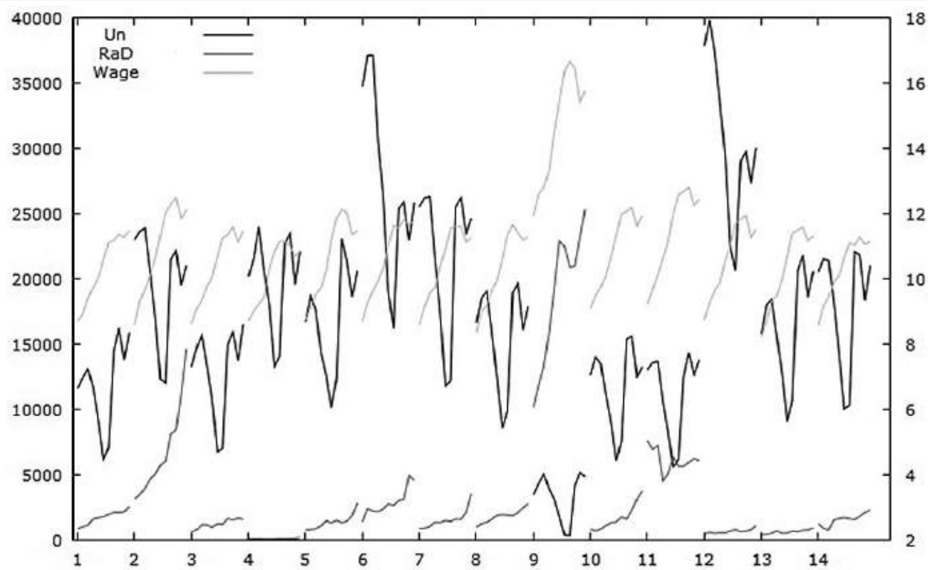
The collected data from CNB about inward FDI into the individual regions have a structure of timelines (see Figure 5 and Figure 6); nevertheless, in accordance with the aims of the research the data have to be transformed into panel data. With regard to more suitable application and in accordance with the generally accepted paradigm the data have been transformed logarithmically. After the logarithmical transformation, the estimated parameters have a form of elasticities which indicate by how much the value of the explained variable changes in case of 1% change of the explanatory variable (Greene, 2003).

Figure 5 Time series of FDI and Waste



Note: FDI in millions, waste on the right axis, kg per capita.
Source: CNB and CZSO, own construction

Figure 6 Time series of Un, RaD and Wage



Note: RaD in millions; Un on the right axis.
Source: CNB and CZSO, own construction

2.2 Determinants

In accordance with the studies of Abbott (2012), Huang (2013) and Lessman (2013) the unemployment rate (Un) has been included in the determinants influencing the FDI inflow. The unemployment rate is considered to be a proxy variable of the workforce availability. The wage size works, among other things, against the workforce availability. In the model the average brutto wage in the region has been applied as a proxy variable of wage size (Wage).

Moreover, another variable has been included which is considered to be a proxy variable of the external economic stability, i.e. the exchange rate (Ex). For a more complex view, it was appropriate to include one more variable which is considered to be a proxy variable of the internal economic stability, i.e. inflation rate (Inf).

For the purposes of the paradigm of inclusion of at least one infrastructure variable, a variable of government spending on research and development (RaD) has been applied as it is considered, among other things, to be a technological infrastructure variable.

Contrary to the above studies including the CO_2 emissions, this study applies the size of communal waste production per capita as the variable for the environmental protection (Waste).

2.3 Model

In the preliminary stage of the research other variables were included in the model than those above mentioned, such as crime rate, motorway network density, the size of the region population and the number of newly reported cases of inability to work. When including all these variables the model contained collinearity. Gradual elimination of the variables obtained the final shape of the model. In some models´ heteroscedasticity and autocorrelation of the residua were detected and in this case the robust co-variation matrix – Heteroscedasticity and Autocorrelation Consistent (HAC) was applied to remove them. The application of this method does not change the estimated values of parameters themselves and enables to remove the distortion of the test statistics in case of the presence of heteroscedasticity and autocorrelation in the model.

The applied model has the below general form:

$$FDI_{it} = \beta_0 + \beta_1 Un_{it} + \beta_2 Wage_{it} + \beta_3 Ex_{it} + \beta_4 Inf_{it} + \beta_5 RaD_{it} + \beta_6 Waste_{it} + e_{it}\,, \tag{1}$$

where: $i = 1, ..., 14$, β_j are unknown (non-random) parameters, $j = 1, 2, \dots, 6$, e_{it} is a random mistake in the i-th observation in year t, t is the year 2002, 2003, … , 2012 (Lind 2005).

Regarding the fact that the obtained data have the panel structure, Fixed Effects Model (FEM) (see Formula 2) and Random Effects Model (REM) (see Formula 3) were used. FEM, unlike Pooled Ordinary Least Squares (POLS), contains α_i, which is a unit specific and time independent term, while α_i is also considered a fixed parameter that is estimated. This may be implemented by including a dummy variable for each cross-section unit (and each suppressing global constant). Sometimes this model is called Least Squares Dummy Variables (LSDV). REM contains a random mistake $u_{it} = v_i + \varepsilon_{it}$, and unlike FEM in the REM there is the term v_i, which cannot be considered a fixed parameter but a random component, and terms v_i and ε_{it} are mutually independent (Nerlove, 1971; Cipra, 2013).

$$y_{it} = \alpha_0 + X_{it}\beta_i + \alpha_i + \varepsilon_{it} \tag{2}$$

$$y_{it} = \alpha_0 + X_{it}\beta_i + \text{v}_i + \varepsilon_{it} \tag{3}$$

It should be considered that in economics many of phenomena have a dynamic character. Hence, apart from the above stated form, for checking the above mentioned hypotheses dynamic panel data model was used (see generalized form of dynamic panel data model Formula 4). For the estimation of the dynamic panel data model Generalized Method of Moments (GMM) (Arellano, 1995, Blundell, 1998) and POLS were used. For the estimation of the dynamic panel data model a 2-step system GMM was applied including an asymptotic standard error.

$$y_{it} = \alpha + X_{it}\beta_i + \delta y_{i,t-1} + \nu_i + \varepsilon_{it} \quad (4)$$

For estimating of models were used econometric software Gretl.

3 RESULTS

3.1 General form of the model

From the results of the general form emerged for the whole-time, both including and excluding Prague, that the Ex, RaD, Wage and Waste were statistically significant determinants. Results showed that no factor which statistically negatively influenced inward FDI for the whole-time was found.

Similar results were identified for the pre-crises period but with one exception that was the Waste. Waste was not significantly positively influenced by inward FDI in comparison for the whole-time.

The results of the general form covering the crises period, both including and excluding Prague, show that the RaD, Wage and Waste were statistically significant determinants. Results exposed that only one factor was found showing significantly negative impact on inward FDI for the crises period. The Ex was this factor but only in REM including Prague.

The statistical verification speaks about the suitability of the applied models and their high information value (the adjusted coefficients of determination or LSDV are in the range 0.70 up to 0.88), nevertheless, in case of the founded results it is necessary to consider the applied methods.

In accordance with premises of the tested hypotheses it was stated that the Ex and RaD had positive impact on the FDI inflow. Hypotheses H3 was not rejected for the whole time, both including and excluding Prague, and for the pre-crises period. Hypotheses H5 was not rejected for the whole time, both including and excluding Prague, for the pre-crises period, both including and excluding Prague, and for the crises period, both including and excluding Prague. On the other hand, no other factor was found showing negative impact on inward FDI. Other hypotheses were rejected.

Results of Hausmann´s tests found out that GLS estimates were not available but if HAC were eliminated, results of Hausmann´s tests indicated that GLS estimates were inconsistent. Hence, results of REM were not desirable for later treatment. Results of FEM should be preferred to REM.

3.2 Dynamic panel data model

It was obvious from results for the whole-time that inward FDI is determined positively significantly by the variables of inward FDI in the preceding period, Ex, RaD and Wage. If Prague was eliminated from the examined sample of regions, it was evident that inward FDI is determined positively significantly by the variables of inward FDI in the preceding period, Ex and Wage but negatively significantly determined by the Un.

The results for the pre-crises period, both including and excluding Prague, show that inward FDI is positively significantly determined only by inward FDI in the preceding period but negatively significantly determined by the Inf.

The results for the crises period show that inward FDI is positively significantly determined by the variables of inward FDI in the preceding period, RaD, Wage and Waste. However, results were ambiguous about the Ex and Inf, because the results may be influenced by the estimated methods. If Prague was eliminated from the examined sample of regions, it was evident that inward FDI is positively significantly determined by the variables of inward FDI in the preceding period, Ex and Wage but Inf was determined negatively significantly by inward FDI. However, Ex results were ambiguous, because they should be influenced by the estimated methods.

Table 1 Results of general form in whole time

	Whole time			Whole time without Prague		
	FEM	REM	POLS	FEM	REM	POLS
	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)
const	−9.0638** (−2.4800)	−19.4892*** (−3.7220)	−56.4779*** (−6.2990)	−9.3201* (−2.0980)	−12.3068** (−2.4420)	−41.0331*** (−3.3350)
l_Un	0.1170 (1.1240)	−0.0525 (−0.5620)	0.1324 (0.4953)	0.1008 (0.8121)	0.0437 (0.3856)	0.2388 (0.7670)
l_Ex	0.0941 (0.5848)	0.8052*** (2.6710)	2.6931*** (6.1620)	0.1199 (0.6298)	0.3551 (1.4920)	1.8313** (2.6130)
l_Inf	0.0003 (0.0347)	−0.0100 (−1.0370)	0.0179 (0.6707)	−0.0045 (−0.4463)	−0.0082 (−0.7746)	0.0212 (0.7052)
l_RaD	0.0793 (0.9394)	0.2638*** (3.5010)	0.3483*** (6.6850)	0.0715 (0.8172)	0.1924*** (2.9680)	0.3668*** (6.5640)
l_Wage	1.8494*** (6.1230)	2.5283*** (5.8390)	5.0544*** (7.8350)	1.8846*** (5.5590)	1.9690*** (4.8790)	3.5875*** (3.5640)
l_Waste	0.0989 (0.3031)	0.1766 (0.6279)	1.0367** (2.8580)	0.0592 (0.1646)	0.1776 (0.5671)	1.2937** (2.9240)
n	154	154	154	143	143	143
Adj. R-squared			0.8576			0.6993
F			122.7565			41.9741
P-value(F)			<0.0001			<0.0001
LSDV R-squared	0.9759			0.9483		
Within R-squared	0.7161			0.7008		
Hausman test (p-value)		NA			NA	

Note: NA – not available; *** significant level α = 10%, ** significant level α = 5%, * significant level α = 1%.
Source: Own construction

Table 2 Results of general form in pre-crises period

	Pre-crisis period			Pre-crisis period without Prague		
	FEM	REM	POLS	FEM	REM	POLS
	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)
const	–11.4045 (–0.7861)	–48.9235*** (–4.6070)	–66.4082*** (–6.8530)	–8.5548 (–0.5043)	–35.3360** (–2.0000)	–65.7924*** (–3.2040)
l_Un	0.0847 (0.4316)	0.3472*** (2.9910)	0.5069* (2.0820)	0.0647 (0.2979)	0.3192*** (2.9230)	0.5065* (2.1290)
l_Ex	0.2879 (0.4292)	2.1355*** (4.0130)	2.7846*** (5.8770)	0.1507 (0.1974)	1.4232* (1.6970)	2.7498** (2.5710)
l_Inf	0.0009 (0.0826)	–0.0057 (–0.4567)	–0.0066 (–0.4009)	–0.0036 (–0.3338)	–0.0071 (–0.4976)	–0.0085 (–0.4632)
l_RaD	0.0141 (0.1273)	0.3023*** (4.1190)	0.3419*** (6.6260)	–0.0098 (–0.0883)	0.2613*** (3.5050)	0.3412*** (6.4950)
l_Wage	2.1611 (1.6290)	5.0604*** (5.6130)	6.0821*** (8.4420)	1.9597 (1.2620)	3.9627*** (2.6350)	6.0006*** (3.4080)
l_Waste	–0.0492 (–0.1072)	–0.0064 (–0.0165)	0.8187 (1.7700)	–0.1111 (–0.1967)	–0.300 (–0.0679)	0.8729 (1.6230)
n	84	84	84	78	78	78
Adj. R-squared			0.8778			0.7053
F			238.4391			18.4479
P-value(F)			<0.0001			<0.0001
LSDV R-squared	0.9718			0.9338		
Within R-squared	0.4986			0.4740		
Hausman test (p-value)		NA			NA	

Note: NA – not available; *** significant level α = 10%, ** significant level α = 5%, * significant level α = 1%.
Source: Own construction

Table 3 Results of general form in crisis period

	Crisis			Crisis without Prague		
	FEM	REM	POLS	FEM	REM	POLS
	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)
const	−7.2863 (−0.7015)	−32.4417*** (−4.3290)	−59.7786*** (−4.9340)	−7.4156 (−0.5796)	−24.6682*** (−3.3930)	−58.7216*** (−3.1860)
l_Un	0.3865** (2.1790)	0.3437* (1.9180)	0.3633 (0.7879)	0.5699 (1.4650)	0.3688 (1.0250)	0.3443 (0.6285)
l_Ex	−0.7355 (−1.2880)	−1.4564*** (−3.2300)	−1.5472 (−1.4270)	−1.1301 (−1.6960)	−1.3682* (−1.9430)	−1.6028 (−1.2820)
l_Inf	0.0219 (0.7287)	−0.0063 (−0.1896)	0.0314 (0.4804)	0.0486 (0.6983)	0.0005 (0.0089)	0.0259 (0.3266)
l_RaD	0.1312 (0.9742)	0.3756*** (5.9850)	0.3205*** (5.3920)	0.1068 (0.6263)	0.3075*** (6.0630)	0.3185*** (4.0780)
l_Wage	1.8608 (1.7480)	4.2432*** (5.3210)	6.3852*** (4.6410)	1.9775 (1.6100)	3.5156*** (4.1610)	6.2990*** (3.0950)
l_Waste	0.0192 (0.0946)	0.2660 (1.4160)	1.3661*** (7.5400)	−0.0334 (−0.1265)	0.2144 (1.2570)	1.3722*** (4.6720)
n	70	70	70	65	65	65
Adj. R-squared			0.8765			0.7227
F			691.8692			277.0459
P-value(F)			<0.0001			<0.0001
LSDV R-squared	0.9918			0.9822		
Within R-squared	0.2606			0.2758		
Hausman test (p-value)		NA			NA	

Note: NA – not available; *** significant level α = 10%, ** significant level α = 5%, * significant level α = 1%.
Source: Own construction

Table 4 Dynamic panel data model in whole time

	Whole time		Whole time without Prague	
	GMM	POLS	GMM	POLS
	Coefficient (z)	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)
l_FDI_{t-1}	0.8006*** (6.9010)	0.9276*** (21.3500)	1.5066*** (3.0140)	0.9360*** (21.3700)
const	−3.1726 (−1.4250)	−2.8487 (−0.9375)	−8.8849** (−2.4390)	−4.3262 (−1.2780)
l_Un	−0.0871* (−1.9000)	−0.0526 (−1.0930)	−0.3041** (−2.1310)	−0.0719 (−1.3330)
l_Ex	0.2881*** (3.6990)	0.2579* (1.9780)	0.8914** (2.2410)	0.3409* (2.1690)
l_Inf	−0.0184 (−1.5450)	0.0019 (0.1100)	−0.0158 (−1.4350)	−0.0058 (−0.3463)
l_RaD	0.0608 (0.9296)	0.0408** (2.7970)	−0.2398 (−1.1430)	0.0340* (2.0420)
l_Wage	0.8412** (2.3670)	0.1907 (0.7450)	0.7477*** (2.9920)	0.3387 (1.0600)
l_Waste	−0.7252 (−0.9848)	0.1457 (1.0750)	−0.7641 (−1.4150)	0.1019 (0.5789)
n	140	140	130	130
Sargans test (p-value)	Over 0.9999		Over 0.9999	
Walds test (p-value)	<0.0001		<0.0001	
Adj. R-squared		0.9787		0.9541
F		9 937.9770		1 489.1710
P-value(F)		<0.0001		<0.0001

Note: NA – not available; *** significant level α = 10%, ** significant level α = 5%, * significant level α = 1%.
Source: Own construction

Table 5 Dynamic panel data model in pre-crises period

	Pre-crisis period		Pre-crisis without Prague	
	GMM	POLS	GMM	POLS
	Coefficient (z)	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)
l_FDI_{t-1}	1.0458*** (17.0900)	0.9726*** (11.6200)	1.0471*** (10.0700)	0.9738*** (11.5400)
const	3.8851 (0.4064)	0.5811 (0.0837)	3.6495 (0.1478)	3.4306 (0.3069)
l_Un	0.0128 (0.0967)	0.0128 (0.1375)	0.0157 (0.2711)	0.0186 (0.2087)
l_Ex	–0.6524 (–1.5890)	–0.4261 (–0.8732)	–0.6537 (–0.4195)	–0.6476 (–0.8750)
l_Inf	–0.0386** (–2.3710)	–0.0274 (–1.0230)	–0.0483*** (–2.6090)	–0.0375 (–1.3340)
l_RaD	0.0031 (0.1126)	0.0242 (0.7560)	–0.0059 (–0.1233)	0.0260 (0.8524)
l_Wage	–0.3183 (–0.4144)	0.0549 (0.1019)	–0.2507 (–0.1218)	–0.1970 (–0.2016)
l_Waste	0.1607 (0.7757)	0.0763 (0.2843)	0.0926 (0.2868)	0.1313 (0.4176)
n	70	70	65	65
Sargans test (p-value)	0.7963		0.8326	
Walds test (p-value)	<0.0001		<0.0001	
Adj. R-squared		0.9642		0.9170
F		1 078.2940		232.0862
P-value(F)		<0.0001		<0.0001

Note: NA – not available; *** significant level α = 10%, ** significant level α = 5%, * significant level α = 1%.
Source: Own construction

Table 6 Dynamic panel data model in crises period

	Crisis		Crisis without PHA	
	GMM	POLS	GMM	POLS
	Coefficient (z)	Coefficient (t-ratio)	Coefficient (z)	Coefficient (t-ratio)
l_FDI_{t-1}	0.3363* (1.7870)	0.9385*** (18.6800)	0.4151** (2.2800)	0.9346*** (20.5200)
const	−38.9526*** (−3.3260)	−2.5799 (−0.6678)	−34.6576*** (−2.9660)	−4.8017 (−1.2320)
l_Un	0.1779 (1.3760)	−0.0332 (−0.4899)	0.0924 (0.5528)	−0.0829 (−1.1660)
l_Ex	−1.5420* (−1.9590)	0.6896** (2.3670)	−1.2055 (−1.6150)	0.6597** (2.3020)
l_Inf	0.0889** (2.0240)	−0.0741** (−2.6180)	0.0339 (0.9358)	−0.0777*** (−3.1780)
l_RaD	0.1993*** (4.8430)	0.0608*** (3.7250)	0.1916*** (4.2270)	0.0542*** (3.4390)
l_Wage	4.3758*** (3.1200)	0.0296 (0.0722)	3.8215*** (2.7220)	0.2881 (0.6798)
l_Waste	0.8385*** (3.1810)	0.1103 (0.8176)	0.7907*** (3.1730)	0.0949 (0.6126)
n	56	56	52	52
Sargans test (p-value)	0.9248		0.9391	
Walds test (p-value)	<0.0001		<0.0001	
Adj. R-squared		0.9901		0.9787
F		9 701.5690		904.1397
P-value(F)		<0.0001		<0.0001

Note: NA – not available; *** significant level α = 10%, ** significant level α = 5%, * significant level α = 1%.
Source: Own construction

The statistical verification speaks about the suitability of the applied models and their high information value (the Sargans test (p-value) up to 0.05, Walds test (p-value) low to 0.05 and the adjusted coefficients of determination up to 0.91), but it is necessary to consider the applied estimation methods.

In accordance with premises of the tested hypotheses it was found out that the Ex, RaD and inward FDI for the preceding period had positive impact on inward FDI. Hypotheses H3 was not rejected for the whole time, both including and excluding Prague. Hypotheses H5 was not rejected for the whole time including Prague, and for crises period, both including and excluding Prague. On the other hand, it was stated that the Un and Inf had negative impact on inward FDI. Hypotheses H1 was not rejected only for the whole-time excluding Prague. Hypotheses H4 was not rejected for the pre-crises period, both including and excluding Prague, and for the crises period excluding Prague. Other hypotheses were rejected.

CONCLUSION

The aim of this study was to provide an analysis of some chosen determinants influencing inward FDI into Czech regions in the monitored period in the years 2002 up to 2012, in the pre-crisis period in the years 2002 up to 2007 and in the crisis period in the years 2008 up to 2012, with or without the inclusion of Prague in the set of regions. The chosen aspects were the following: unemployment rate (Un), exchange rate (Ex), inflation rate (Inf), expenditures on research and development (RaD), average brutto wages (Wage) and environmental pollution (Waste). The analysis of the aspects influencing inward FDI was based on the panel data from Czech regions. The following were opted: Fixed Effects Model (FEM), Random Effects Model (REM) and Pooled Ordinary Least Squares (POLS) for checking of hypotheses. There were opted 2-step system Generalized Method of Moments (GMM) including an asymptotic standard error and POLS for estimation of dynamic model. The comparison of results was carried out after the completion of the analysis.

Results of general form imply that the appreciation of Ex, RaD and Wage determined inward FDI to Czech regions positively for the whole-time, pre-crises and crises period, both including and excluding Prague. Ex and RaD results are consistent with defined premises and articles of Usman (2012) and Ramirez (2013). However, results of Wages were not confirmed presumption of negative influence on inward FDI that were inferred from the studies of Chen (2011), Olney (2013) and Huang (2013). Certain "paradox" was detected in the Czech environment showing that inward FDI was positively determined by Wages. Wages results may indicate that the Czech government should give support to increasing of Wages for inward FDI. However, high level of Wages increases the price of labour force and therefore goods and services should be more expensive. On the other hand, high level of Wages should contribute to higher consumption in Czech regions. Some question remains what is beyond this finding. The above mentioned question has several explanations. One of them is that inward FDI (with elimination of Prague) goes to regions with lower Wages and with higher support of national institutions, for example the Moravia-Silesia region. Other explanation is that development of average Wages in Czech has showed slightly slow increase in selected term with one exception which is crises period when average Wages were decreased. Other explanation should be that the average of Wages in the Czech Republic is one third of the average of Wages in OECD countries. The crisis began in 2008 and now nine years passed after the start of crises. The Czech Republic should be in front of a gate of new crises and the Czech government need to reflect this situation. No aspect which has had negative impact on inward FDI for the whole-time, pre-crises and crises period, both including and excluding Prague was found.

Results of dynamic model imply that Wages have positive impact on inward FDI for the whole-time and crises period, both including and excluding Prague. The Czech government should push up wages and thus to stimulate the future inward FDI and should also indicate positive effect on economic development and consumption. Conclusions about Wages are not final because it is necessary to reflect economic reality. High Wages mean that labour costs increase and so do the production costs.

The Czech government should be very sensitive to wage increase because the Czech firms may have lost their competitive advantage. Appreciation of Ex have had positive impact on inward FDI for the whole time however appreciation of the Ex generally marks up the costs of the FDI inflow from investor´s country but should be compensated by future profit from proceeded investment and future appreciation of the Ex in host country. Appreciation of the Ex is obviously included to the calculation of net present value, indicating convenience of investment. The RaD determined the FDI positively for the whole time including Prague, and for crises period, both including and excluding Prague. Hence, the Czech government should promote the RaD but it has not been done yet. The RaD continuously grew in selected time periods with one exception which was crises period because the Czech government dramatically cut down the budget of the RaD in crises which appears as a big mistake. These findings are consistent with defined premises and articles of Yu (2011), Castiglione (2012), Usman (2012), Ramirez (2013) and Long (2015). If the RaD budget decreased dramatically foreign investors may notice this situation as an upset of investment atmosphere. On the other hand, negative impacts the Un and Inf on the FDI inflow were identified. It was found out that the Un has negative influence on inward FDI flow in Czech regions excluding Prague only for the whole-time. The Czech government should focus on the Un elimination. The Czech regions excluding Prague have obviously higher Un than Prague. Higher Un should currently promote the FDI inflow but the above mentioned findings evoke that better higher availability of labour force deter foreign investors from investment to Czech regions. One of possible explanations is that foreign investor prefers lower Un because lower Un should indicate better internal situation of Czech economy (for example Czech economy should not experience internal pressure to decrease the wage level). These findings are consistent with defined premises and articles of Chen (2011), Olney (2013), Huang (2013) and Boateng (2015) but they do not comply with articles Lessmann (2013) and Long (2015). The Inf reduced the FDI inflow for the pre-crises period, both including and excluding Prague, and for the crises period excluding Prague. The Inf debase the feasible value of the FDI but sensitivity of the FDI inflow on increase of the Inf is relatively small (values of sensitivity are between 0.03 and 0.07). Foreign investor reflects the Inf, however, the Inf reduces their investment very slowly. Czech Inf is signalized by lower volatility in selected term and it shows that Czech economy is relatively stable even if Czech economy was involved economic crises in 2008. These findings are consistent with articles of Boateng (2015), however, articles of Li (2005) and Kolstad (2008) have not confirmed the above mentioned results because articles of Li (2005) and Kolstad (2008) confirmed negative effect of Inf only in developing countries and no in developed countries where Czech should be subsumed.

It was found out that the applied methods were chosen appropriately but it is always necessary reflect the applied estimation method because results may be influenced by the estimation methods used. For the future research, the number of determinants and time range should be extended by including for example gross domestic product, inflation, law environment, political risk, etc. of Germany, Austria, Slovakia, and Poland, respectively.

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Credit Risk and Regional Economic Disparities

Tomáš Vaněk [1] | *Mendel University in Brno, Czech Republic*
David Hampel | *Mendel University in Brno, Czech Republic*

Abstract

This paper aims to bridge the areas of credit risk and regional economic disparities, and investigates the relationship between credit risk and economic indicators in the Czech Republic at the regional (NUTS 3) level. This relationship is consecutively examined using graphical and correlation analysis, regression techniques, and different types of clustering methods. Regions are then clustered into three groups according to their economic similarities and disparities. Subsequently, it is shown on the real data that region-specific information has the potential to be utilizable in credit scoring and possibly other applications.

Keywords

Credit risk, regional disparities, cluster analysis, Bayesian information criterion, model selection, logistic regression

JEL code

R11, G32, C10, C25, C38

INTRODUCTION

Credit risk is one of the most fundamental and significant risks banks are exposed to, and is generally understood as the potential that a borrower or counterparty will fail to meet its contractual obligations (see BCBS, 2000). With the introduction of the Basel II capital requirements framework in 2004, attention paid to credit risk analysis and management has become even greater. Requirements in this area will be further augmented after January 1st, 2018, when the standard IFRS 9 *Financial Instruments*, introducing a new framework for credit impairment calculation, becomes effective.

Banks evaluate credit risk associated with potential and actual clients within credit scoring, which is a process for prediction of the probability that a client will default (Hand and Henley, 1997). For discussions on the historical context and development of credit scoring see Thomas (2000), or Abdou and Pointon (2011). As suggested above, credit risk evaluation (probability of default estimation) is important not only for internal credit decisions, but also for regulatory purposes – especially quantification of capital requirements within the internal ratings-based approach (see BCBS, 2004; and CRR, 2013) and calculation of credit impairment (loss allowances) within IFRS 9 (see IFRS Foundation, 2015).

Since the 1970s, a quantitative approach to credit scoring has been dominant, with statistical models playing a key role. Over time, logistic regression has become the standard and is usually used as a benchmark when estimating more sophisticated models (e.g. Crook et al., 2007). Li and Zhong (2012), or Lessmann et al. (2015) provide a good overview of the methods and models that have been used in credit scoring.

[1] Mendel University in Brno, Faculty of Business and Economics, Zemědělská 1, 613 00 Brno, Czech Republic. Corresponding author: e-mail: xvanek7@mendelu.cz, phone: (+420)721482751.

This paper aims to bridge credit risk and regional economic disparities representing a persisting development tendency in a majority of countries (e.g. Shankar and Shah, 2003), including the Czech Republic.

In the literature, there are a plenty of studies devoted to the analysis of credit risk in the macroeconomic context from various points of view. Pesaran et al. (2006) model conditional credit loss distributions, Pesaran et al. (2007) explore credit risk diversification, with both studies using 'global' macroeconometric models. In a recent study, Schwaab et al. (2016) also investigated credit risk from a global perspective, using a non-linear state-space model. Studies concerning credit risk and macroeconomy in the Czech Republic have been conducted by Jakubík and Heřmánek (2008), Jakubík (2008), Grešl et al. (2013) or Melecký et al. (2015), who deal with credit risk especially in the context of macroeconomic stress testing.

There are also many studies dealing with regional disparities from various perspectives. OECD (2016) can be considered as an up-to-date and comprehensive study with a global scope. In terms of more recent studies focusing on regional disparities within the Czech Republic, we can mention Kutscherauer et al. (2010), Kahoun (2010), Svatošová and Novotná (2012), Procházková and Radiměřský (2013), Kvíčalová et al. (2014), or Tuleja and Gajdová (2015).

As was outlined above, the primary goal of this paper is to provide in a sense an intersection between credit risk and regional economic disparities and investigate the relationship between credit risk and economic indicators in the Czech Republic at the regional (NUTS 3) level. Banks generally monitor credit risk (e.g. observed default rates) in relation to geographical locations (regions) as a part of their credit concentration risk[2] management, which is also required by Directive 2013/36/EU (CRD, 2013) or CEBS (2010). However, this monitoring is predominantly performed on an individual basis (meaning from an individual bank's or even portfolio's point of view), and to our knowledge, the relationship between credit risk and economic indicators at the regional level has not been paid a great deal of attention on a more comprehensive level. Therefore, this paper aims to address this issue and at the end also demonstrates that region-specific information may be utilizable in credit scoring models or possibly other applications.

1 DATA AND METHODOLOGY

Firstly, credit risk and economic indicators at the regional level were investigated using simple graphical and correlation analysis. After that, the relationship between credit risk and economic variables was analyzed with linear regression models. Subsequently, hierarchical cluster analysis and model-based cluster analysis were performed, the latter within a Gaussian finite mixture modelling framework. Finally, it is demonstrated that region-specific information can be utilized in credit scoring models (using logistic regression). The following data are used in the above-mentioned analyses (at the regional level):

- *Past_due:* a share of population (adult natural persons) with past due obligations in % (source: SOLUS Register – see SOLUS, 2016);
- *Une:* general unemployment rate in % (source: Czech Statistical Office);
- *GDPpc:* gross domestic product per capita in CZK (source: Czech Statistical Office);[3]
- *Wage:* average wage in CZK (source: Czech Statistical Office);
- *Educ:* a share of the population with a university-level education in % (source: Czech Statistical Office – Population and Housing Census 2011).

[2] Concentration risk can be understood to be a sub-risk of credit risk class – see e.g. Holub et al. (2015). It is one of the specific risks subject to supervisory review under Pillar 2 within Basel II, since it is not fully covered by Pillar 1 capital requirements.

[3] For an interesting methodological discussion on GDP per capita see Chlad and Kahoun (2011).

The share of the population with past due obligations of a given region serves as a proxy of credit risk in this paper. It should also be noted that by "regional level", the NUTS 3 level is implied. Therefore, 14 regions are considered: Zlínský (Zlín Region, ZL), Vysočina (Vysočina Region, VY), Jihomoravský (South Moravian Region, SM), Praha (Prague, PR), Pardubický (Pardubice Region, PA), Jihočeský (South Bohemian Region, SB), Olomoucký (Olomouc Region, OL), Královéhradecký (Hradec Králové Region, HK), Středočeský (Central Bohemian Region, CB), Plzeňský (Plzeň Region, PL), Moravskoslezský (Moravian-Silesian Region, MS), Liberecký (Liberec Region, LI), Karlovarský (Karlovy Vary Region, KV), Ústecký (Ústí nad Labem Region, UL).

For the credit scoring model (within the Discussion section), the real data (as of 2014) from a small bank operating in the Czech Republic is used – specifically, a sample of nearly 90 thousand clients (private individuals).

1.1 Model-based cluster analysis

Model-based cluster analysis can be considered as an alternative to traditional clustering methods, such as hierarchical clustering or partitioning clustering (k-means, partitioning around medoids etc.). Since model-based cluster analysis is not used as widely as more traditional methods, it will be briefly described in this section. As Fraley and Raftery (2007) note, together with the development of methods and software tools for model-based clustering, these techniques are becoming increasingly popular and preferred over the heuristic methods mentioned in the beginning of this paragraph. A prevailing statistical approach to clustering is the use of finite mixture models – see e.g. McLachlan and Peel (2000).

In model-based clustering, the data y are treated as coming from a mixture density $f(y) = \sum_{c=1}^{G} \varrho_c f_c(y)$, where f_c represents the probability density function of the observations in group c, and ϱ_c denotes the probability that an observation comes from the c-th mixture component. Therefore, $\varrho_c \in (0,1)$ and $\sum_{c=1}^{G} \varrho_c = 1$. Generally, the individual components (clusters) are modelled using the Gaussian (normal) distribution that is characterized by the mean vector μ_c and the covariance matrix Σ_c. Parametrization of Σ_c allows us to determine various geometric features of the clusters (shape, volume, orientation). The probability density function takes the following form (Fraley and Raftery, 2007):

$$p(y_i| \mu_c, \Sigma_c) = \frac{\exp\left\{-\frac{1}{2}(y_i-\mu_c)^{\mathrm{T}}\Sigma_c^{-1}(y_i-\mu_c)\right\}}{\sqrt{\det(2\pi\Sigma_c)}}, \; i = 1, \ldots, n. \quad (1)$$

A Gaussian mixture model with multivariate mixture components has the likelihood function of the form:

$$\mathcal{L}(\varrho, \mu, \Sigma|y) = \prod_{i=1}^{n} \sum_{c=1}^{G} \varrho_c p(y_i| \mu_c, \Sigma_c). \quad (2)$$

The model parameters ($\varrho_c, \mu_c, \Sigma_c$) are commonly estimated by the expectation-maximization (EM) algorithm initiated by hierarchical model-based clustering. However, alternative approaches for parameter estimation in these kinds of applications exist – for an overview see McNicholas (2011). For further technical details on model-based clustering (including the 'mclust' package in R that is used in this paper), see Fraley and Raftery (2002), Fraley et al. (2012, 2016), or Scrucca et al. (2016).

Model selection strategies can be based on several measures – for an overview see McLachlan and Peel (2000). However, as for example McNicholas (2011) notes, Bayesian Information Criterion (BIC) (Schwarz, 1978) is the most prevalent mixture model selection measure in the literature (and is considered in this paper as well). BIC adds a penalty term to the loglikelihood that takes the complexity of the model (number of parameters) into account. Therefore, the BIC has the form:

$$\mathrm{BIC} = 2\hat{\ell}_{\mathcal{M}}(y|\hat{\theta}) - m_{\mathcal{M}} \log(n), \quad (3)$$

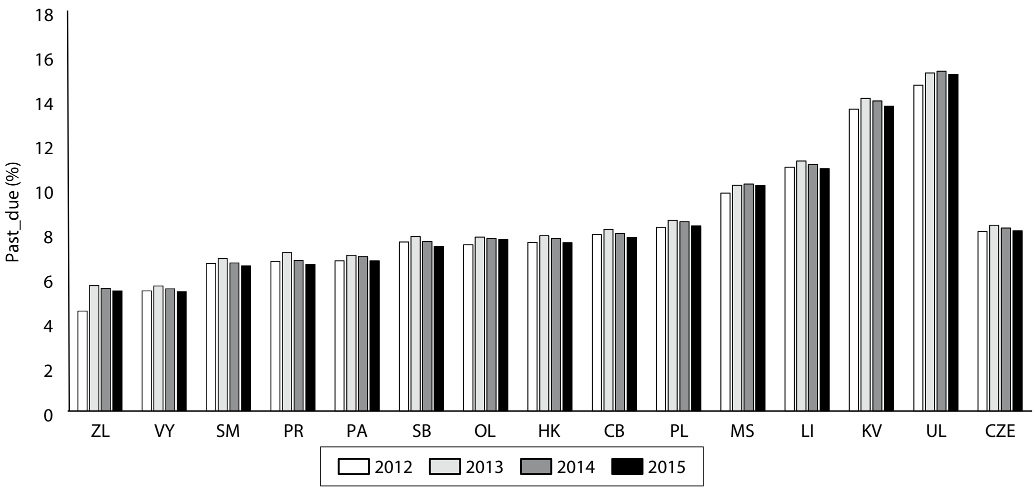
where $\hat{\ell}_{\mathcal{M}}$ is the maximized loglikelihood for model $\mathcal{M}$, $\hat{\theta}$,denotes the corresponding set of estimated parameters, and $m_{\mathcal{M}}$ represents the number of parameters of the model $\mathcal{M}$. Model selection in model-based clustering is discussed in more detail in e.g. Fraley and Raftery (1998), or Raftery and Dean (2006).

2 CREDIT RISK AND ECONOMIC INDICATORS AT THE REGIONAL LEVEL

2.1 Graphical and correlation analysis

Figure 1 depicts a share of the population with past due obligations in all of the considered regions and its development from 2012 to 2015. The last column represents the overall average. As it can be seen, the development of *Past_due* is quite stable over the observed time period.

Figure 1 Development of a share of the past due population in the individual regions from 2012 to 2015



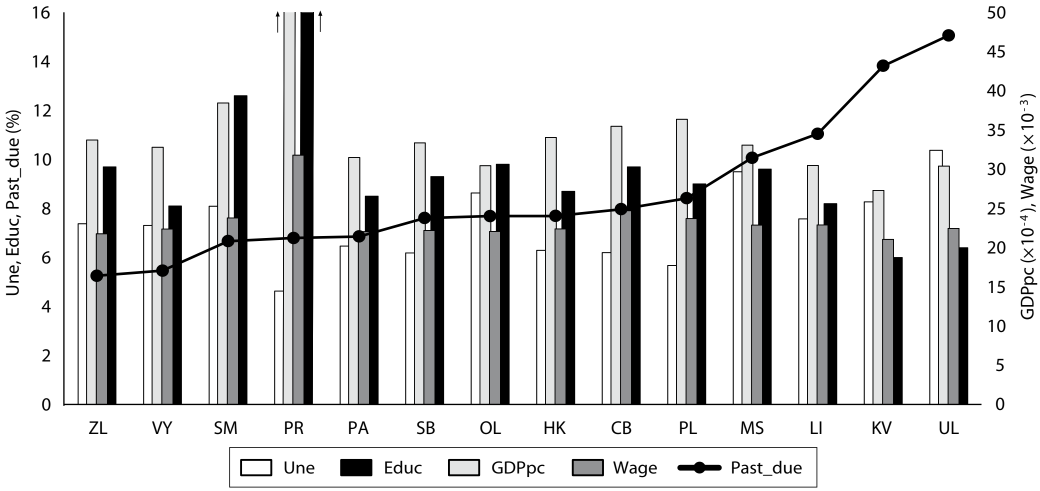
Source: SOLUS Register

The *Past_due* variable in the context of the considered economic indicators is presented in Figure 2. The values are obtained as averages of the variables over the observed period 2012–2015.[4] There is an exception in the case of the variable *Educ* – at the regional level the most current data is from the Population and Housing Census 2011. Even though there are some changes in the development of the variables from an absolute point of view, the proportions within the individual regions remain quite stable. Therefore, by averaging the values over years 2012–2015, no significant loss of information should occur – on the contrary, this procedure should assure that a more "long-term" view is provided by the performed analyses. Given the fact that relatively extreme values can be observed in the case of Prague (PR) – especially *Educ* 20.7% and *GDPpc* 813 thousand CZK – which would make differences among the other regions less visible in the graphical analysis, the scale of the vertical axes is adjusted in a corresponding manner to maintain lucidity.

Certain patterns can be observed from Figure 2. Generally, regions with relatively high shares of the population with past due obligations have higher unemployment rates, lower shares of the population with a university-level education and lower GDP per capita. The relationship between wages and past due rates is inconclusive. These observations are further elaborated below.

[4] *GDPpc* is averaged from 2012 to 2014 since the values of 2015 were not available when writing the paper.

Figure 2 The considered variables in the context of individual regions (averages from 2012 to 2015)



Source: SOLUS Register, Czech Statistical Office

Regarding unemployment, it turned out that it makes no considerable difference whether the general unemployment rate or registered unemployment rate is used in the analyses and estimations. Although there are some absolute changes between these two indicators, the proportions are similar. Moreover, based on the development of the standard deviations of all of the considered variables in the individual years, it can be said that no convergence is observed among the regions. In other words, the regional economic disparities in the discussed context do not become substantially less.

After the graphical analysis, a correlation analysis was performed. The correlations of the considered variables are summarized in Table 1. As above, the averages of the variables over the observed period 2012–2015 are used. Moreover, to avoid distorting the results with extreme values, Prague is excluded from the correlation analysis.

Table 1 Correlation analysis of the considered variables

Past_due	Une	GDPpc	Wage	Educ	
1	0.59*	–0.60*	–0.18	–0.68*	**Past_due**
	1	–0.43	–0.26	–0.21	**Une**
		–1	–0.74*	–0.80*	**GDPpc**
			–1	–0.53*	**Wage**
				–1	**Educ**

Note: Statistically significant correlation coefficients (*p*-value < 0.05) are marked with *.
Source: Own calculations

Focusing on the relationship between credit risk and economic variables, a high negative correlation is observed between *Past_due* and *Educ* (–0.68). This is logical, since it is expected that more educated people would naturally tend to have less problems with repaying their loans (they are expected to be more financial literate etc.). A high negative correlation can also be observed between *Past_due* and *GDPpc* (–0.60), which is also natural given the fact that in regions with higher *GDPpc* (i.e. with higher economic performance) there are higher wages and education rates, which is supported by high positive correlations between *GDPpc* and *Wage* (0.74), and *GDPpc* and *Educ* (0.80).

Nevertheless, the direct relationship between *Past_due* and *Wage* is insignificant. One might expect that regions with higher wages would tend to have smaller shares of the population with past due obligations; however, wages themselves do not provide much useful information in this context, since the value of loans is not taken into account. Therefore, ratio indicators such as debt-to-income (DTI) would be more evidential. The following sections provide a deeper insight into the relationship between *Wage* and *Past_due*.

Furthermore, a relatively high positive correlation between *Past_due* and *Une* can be seen from the analysis (0.59), which is also logical since it is naturally expected that regions with smaller unemployment rates will have smaller shares of population with past due obligations. Therefore, it can be said that the results from the correlation analysis correspond to the prior expectations and in a sense summarize what is depicted in Figure 2.

2.2 Regression analysis

After the graphical and correlation analysis, the relationship between credit risk and economic variables was further investigated using regression analysis. Specifically, cross-sectional and panel data regression analyses were performed. For the purpose of regression and cluster analyses (in Section 2.3), the considered variables are scaled to the same order due to statistical reasons (therefore *Wage* is in tens of thousands of CZK and *GDPpc* is in hundreds of thousands of CZK). Also, Prague is excluded since it can be considered as an extreme case (or outlier) that may distort the results.

As it was noted above, the development of the considered variables is quite stable over the observed time period in the individual regions. This is also shown in Table 2, which summarizes the main results of the five cross-sectional regressions performed (data as of 2012, 2013, 2014, 2015 and averages). Linear cross-sectional regression models were estimated by the ordinary least squares (OLS) method with heteroscedasticity robust standard errors.

Table 2 Cross-sectional regression results

	2012		2013		2014		2015		Averages	
	coef.	*p (t)*	coef.	*p (t)*	coef.	*p (t)*	coef.	*p (t)*	coef.	*p (t)*
const	−22.48	0.1271	−15.46	0.1263	−6.73	0.4970	−4.84	0.5703	−12.64	0.2354
Une	1.11	0.0010	1.05	0.0003	1.04	0.0001	1.19	0.0001	1.11	0.0002
Wage	16.28	0.0155	12.66	0.0042	8.63	0.0493	7.42	0.0554	11.20	0.0172
Educ	−1.51	0.0012	−1.41	0.0004	−1.35	0.0017	−1.29	0.0013	−1.39	0.0009
R^2	0.73		0.79		0.75		0.75		0.76	
p (F)	0.0004		0.0000		0.0000		0.0000		0.0000	

Source: Own calculations

The variable *GDPpc* was excluded from the regressions after the backward elimination procedure. This can be explained by the high correlations between *GDPpc* and *Educ,* and *GDPpc* and *Wage* (see Table 1).[5] However, as it can be seen in Table 2, the signs of *Une* and *Educ* are as expected and described above (in Section 2.1). The sign of the variable *Wage* is positive, nevertheless, as it was noted, in order to create a direct link with credit risk (*Past_due*), an indicator considering both wage and the volume of the loan would have to be used (e.g. DTI). With wage itself, the logic is incomplete. Further comments on this topic are given below.

[5] Based on the values of the calculated variance inflation factors (in all cases under 1.7 after exlusion of *GDPpc*), it can be concluded that no additional multicollinearity problems arised. For details on this topic and other relevant diagnostic tests of linear regression models see e.g. Heij et al. (2004), or Greene (2012).

Furthermore, several panel data regression models were estimated. Table 3 summarizes the final selected one. The null hypothesis that all regions have the same intercept was rejected, hence the individual-specific effects model was preferred. Also, the Hausman test (see Hausman, 1978) proved that the random effects generalized least squares (GLS) estimator is consistent and more efficient than the fixed effects one. Therefore, the random effects model was preferred to the fixed effects model. The random effects panel data regression model was estimated by the Arellano heteroscedasticity and autocorrelation robust GLS estimator (see Arellano, 2003) using the Nerlove method for estimating "within" and "between" variance (see Nerlove, 1971). However, the results were robust to the use of other methods, e.g. Swamy and Arora (1972). The Nerlove method was preferred due to slightly lower standard deviations of the obtained estimates. Although the GDP data at the regional level were not available for 2015, *GDPpc* was not considered in this estimation (similarly as above) – therefore, the data used covers the 'full' time span 2012–2015.

Table 3 Random effects panel regression results

	coef.	***p (t)***
Const	12.23	0.0275
Une	0.26	0.0000
Wage	2.61	0.0021
Educ	–1.27	0.0248
corr$(y, \hat{y})^2$	0.60	
Hausman test *(p)*	0.33	

Source: Own calculations

To a large extent both types of regressions yield corresponding results. After analyzing the relationship between *Wage* and *Past_due*, the results obtained from cross-sectional and panel regressions are also in line – a positive sign can be observed. Since the regression analyses take indirect effects into account, the results obtained in this section are considered to be more plausible compared to the results from the simple correlation analysis (in Section 2.1).[6]

Therefore, the results imply that regions with higher wages tend to have a higher share of the population with past due obligations. The reasoning behind this statement could be that people with higher wages generally tend to take higher loans, and at the same time these people are more sensitive to adverse events, typically losing their job. For people with higher wages it may be more difficult to find a job with corresponding salary in a reasonably short time to cover the relatively high repayments of their loans. Another agrument could be that there are different price levels across regions implying that higher nominal wages do not necessarily mean higher real wages.

2.3 Cluster analysis

So far, we have dealt with the a more "overall" picture. The relationship between credit risk and selected economic indicators was investigated using region-level data. In this section, the individual regions will be directly worked with, in order to cluster them based on various shared economic characteristics. Two types of clustering are performed – hierarchical and model-based.

[6] This reasoning is also supported by the fact that the Pearson partial correlation coefficient (taking other variables into account) of P*ast_due* and *Wage* is positive.

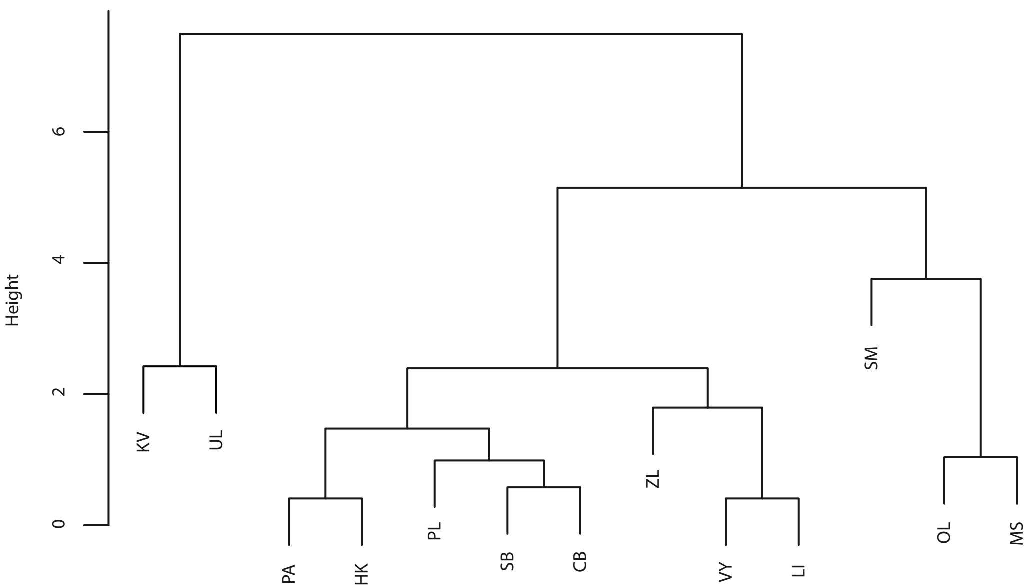
Figure 3 shows a dendrogram obtained from the hierarchical cluster analysis with a Euclidean distance matrix using the complete linkage method. Note, however, that the results yielded by using other methods (e.g. Ward) were very alike. Similarly as above, Prague was excluded from this analysis. Also, scaled averages of variables are used. All of the calculations in this section are performed in the R computational system (R Core Team, 2017).

The results of the model-based cluster analysis are summarized in Table 4. In this case, the analysis was performed in four versions – with 2, 3, 4, and 5 clusters. The mentioned options were set to be reasonable, given the application and subsequent interpretation. In every version, the final model was selected by running an optimization exercise within Gaussian finite mixture models using the 'mclust' package in R (Fraley et al., 2012, 2016). The optimization measure was the Bayesian information criterion (BIC) and the models were estimated using an expectation-maximization algorithm initialized by hierarchical model-based clustering. The selected models for every version have the following features:

- v1 (2 clusters): ellipsoidal distribution, equal volume and orientation;
- v2 (3 clusters): diagonal distribution, equal shape of clusters;
- v3 (4 clusters): diagonal distribution, equal volume and shape of clusters;
- v4 (5 clusters): diagonal distribution, equal volume and shape of clusters.

In this case, the data as of 2014 (most recent and complete) were used, excluding Prague.

Figure 3 Dendrogram of the considered regions



Source: Own calculations

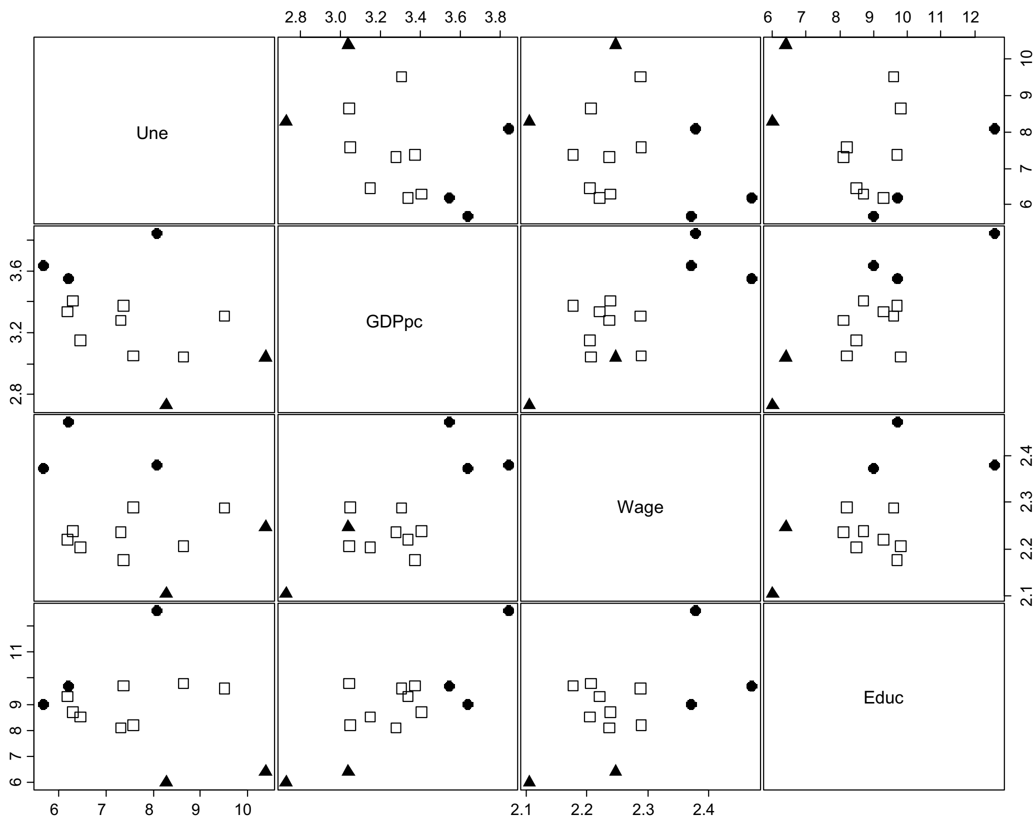
The version with three clusters (illustrated in Figure 4) seems to be the most appropriate. Table 4 suggests that the SM region tends to be left alone as a separate cluster when the number of clusters increases. On the other hand, only two clusters are not sufficient – looking also at Figure 3, it is clear that regions KV and UL (cluster Δ in Table 4 and Figure 4) stand on the side from both an economic and credit risk point of view. Therefore, from the performed analyses it can be seen that the regions are clustered mostly into three groups, and this version is used in the following text.

Table 4 Model-based cluster analysis summary

No.	Region	Past due	2 clusters (v1)	3 clusters (v2)	4 clusters (v3)	5 clusters (v4)
1	ZL	5.50	□	□	□	▪
2	VY	5.48	□	□	□	□
3	SM	6.65	○	○	+	+
4	PA	6.93	□	□	□	□
5	SB	7.61	□	□	□	○
6	OL	7.77	○	□	□	▪
7	HK	7.77	□	□	□	□
8	CB	7.98	□	○	○	○
9	PL	8.50	□	○	○	○
10	MS	10.20	○	□	□	▪
11	LI	11.08	□	□	□	□
12	KV	13.95	○	Δ	Δ	Δ
13	UL	15.28	○	Δ	Δ	Δ
14	PR	6.80	*Excluded*			

Source: Own calculations

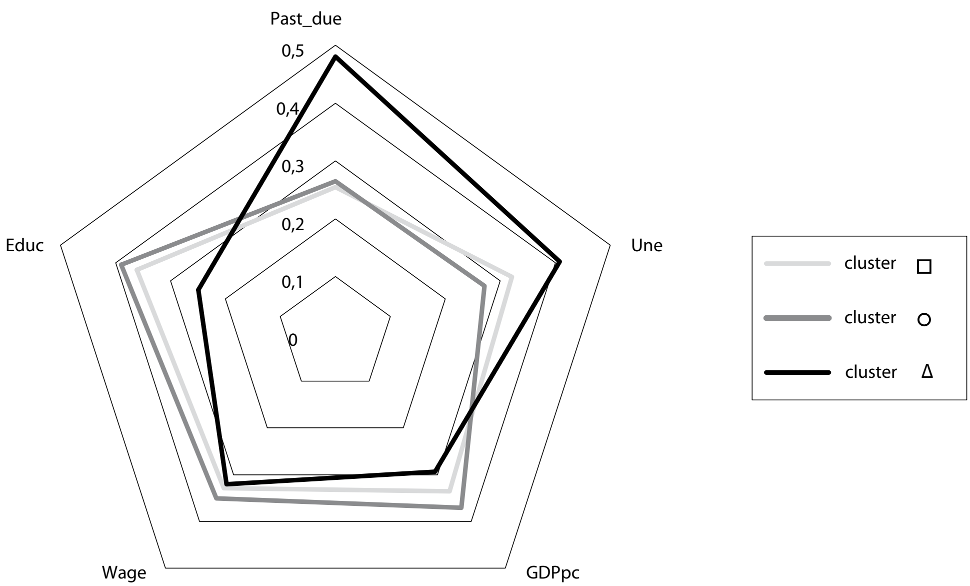
Figure 4 Result of the model-based cluster analysis (three clusters)



Source: Own calculations

The relationship between the considered economic variables and credit risk *(Past_due)* within the individual clusters is further investigated using a spider graph – see Figure 5. Note that the medians of the corresponding variables are used for each cluster of regions. Moreover, for illustrative purposes, the values are normalized across the clusters. The darkest line representing cluster Δ confirms the findings from the previous analyses. Regions in this cluster have the highest unemployment rate, lowest GDP per capita, lowest wages and lowest education, and at the same time the highest share of the population with past due obligations. Regarding the other two clusters, it can be seen that even though the regions in cluster □ (the lightest line) are "worse" than □ in terms of economic performance indicators, their share of the population with past due obligations is comparable (even slightly higher in the case of ○). One may suggest that two clusters could be sufficient due to similarities between clusters □ and ○; however, it should be noted that the scale of the spider graph is in a sense "extended" because of cluster Δ, which optically diminishes the differences. Moreover, the choice of three clusters will also prove to be appropriate in the next section.

Figure 5 Spider graph of individual clusters in the context of considered variables



Source: Own calculations

Therefore, the link between the considered economic indicators and credit risk *(Past_due)* is rather fragmentary and cannot be fully generalized over all of the regions. It can also be noted that there are of course many factors influencing the level of *Past_due*, especially on the individual (client) level. Therefore, it cannot be expected that economic variables to a large extent explain this issue. However, some of the obtained results are quite straightforward and evidential (especially regarding cluster Δ) and the next section investigates whether the region-specific data can be utilized in credit scoring modelling or possibly other applications.

DISCUSSION

In this section, utilization possibilities of region-specific data will be discussed, especially in the context of credit scoring. For this purpose, additional calculations were performed. Firstly, a simple credit scoring model using dummy explanatory variables for each individual region was estimated, with the binary default variable as the dependent variable. Secondly, the findings from Section 2 were utilized. Therefore, the regions were clustered into three groups for which dummy variables were created.[7] The first region/cluster was treated as a reference category. The concept of the model was otherwise the same as in the first case – the credit scoring model had the form of the standard logistic regression and was estimated by the maximum likelihood method. The real data (as of 2014) from a small bank operating in the Czech Republic is used – specifically, a sample of nearly 90 thousand clients (private individuals).

In the first case (with dummy variables for individual regions), only 2 out of 14 regression coefficients were statistically significant at the 5% level. Moreover, one of them was a constant. In the second case (with dummy variables for clusters), all regression coefficients were statistically significant (3 out of 3). The performance of the models was then evaluated using the area under the receiver operating characteristic (ROC) curve – see e.g. Engelmann et al. (2003) or Fawcett (2006). The results for the two models described above were 0.55 and 0.54, respectively. Of course, the values are relatively small and close to 0.5 since the models predict the probability of default of clients only using information about the regions they come from. The main result is that for a relatively little loss of model performance, a much more "compact" model can be obtained, with statistically significant parameters. Therefore, from a statistical point of view, the second model may be preferred.

Khudnitskaya (2010) achieved a better scoring model performance with region-specific (or microenvironment-specific) information using logistic regression with a multilevel structure. Even though American data were used in her study, which are more hierarchically structured compared to the Czech data (given the size of the US), and therefore more comfortable and suitable to use in multilevel models, the multilevel modelling framework seems to be promising in this context.

Apart from Khudnitskaya (2010), this paper cannot be directly compared to the other studies mentioned in the introduction. The first set of studies deals with credit risk in the context of macroeconomy – Pesaran et al. (2006, 2007) use global vector autoregression models, Jakubík and Heřmánek (2008) estimate a vector error correction model (among others), Jakubík (2008) works with Merton-type models, Grešl et al. (2016) provide an interesting overview of the Czech National Bank's stress-testing framework (including time-series and macroeconometric techniques), Melecký et al. (2015) use an autoregressive distributed lag model with instrumental variables, and in a recent study Schwaab et al. (2016) investigate credit risk from a global perspective, using a non-linear state-space model.

On the other hand, there are studies devoted to regional disparities, but without a link to credit risk. Kutscherauer at al. (2010) analyze and evaluate regional disparities especially using integrated indicators and models of the economic power of the regions. Kahoun (2010) studies regional disparities with GDP per capita and net disposable income (using descriptive statistics) and evaluates the suitability of these measures. Svatošová and Novotná (2012) also use descriptive statistics for investigating regional disparities, considering several demographic, social, economic and ecologic variables. Procházková and Radiměřský (2013) study the economic performance of regions using a regression model with socio-demographic and industrial factors. Kvíčalová et al. (2014) identify differences between regions based on economic characteristics using correlation analysis and hierarchical cluster analysis. Tuleja and Gajdová (2015) investigate the economic potential of regions using in particular graphical methods of magic polygons.

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[7] Although Prague was excluded from the cluster analysis, for this application it was included in cluster ○ due to greater similarity with regions inside this cluster. However, the results would not substantially differ in a quantitative way even if Prague was treated as a separate cluster.

CONCLUSION

In this paper the regression techniques and methods of cluster analysis were in a sense combined with the extension to credit scoring application, using region-specific data. This work therefore partially contributes to the both areas of literature (credit risk in the context of economy and regional economic disparities), and its main added value lies in the interconnection of the two.

At this point, it should be noted that a larger set of appropriate economic variables will be analyzed in further research, e.g. factor productivities – see Vltavská and Sixta (2011). Also, considering regional price levels may be an improvement – see Musil et al. (2012) and Kocourek et al. (2016).

Regarding the credit scoring application, banks often categorize the regions in an expert way based on their internal historical experience. Based on the character of the banks' population (character of clients), it may occur that e.g. using solely historical data in application credit scoring could lead to prior incorrect penalization of clients from certain regions if the population is in a sense "biased". On the other hand, the clustering approach used in this paper is independent of the bank-specific population and thus more robust in this context. Using clusters of regions also diminishes the number of parameters to be estimated compared to the case of dummy variables for each region, which may help to minimize potential numerical instability issues. As it was suggested, the clustering approach could be further used in certain types of stress-testing exercises, where augmentation of the analysis dimension by the regional level may help to improve the results (compared to the case where the Czech Republic would be considered as a whole).

To conclude, it can be summarized that this paper bridges the areas of credit risk and regional economic disparities, and investigates the relationship between the credit risk and economic indicators in the Czech Republic at the regional (NUTS 3) level. This relationship was consecutively examined using graphical and correlation analysis, regression techniques, and different types of clustering methods. Regions were then clustered into three groups according to their economic similarities and disparities. Subsequently, it was shown that region-specific information has the potential to be utilizable in credit scoring and possibly other applications. This area will be further investigated in the context of a multilevel modelling framework, which provides the possibility to improve scoring models to a larger extent.

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Evaluation of Subsidies in Recovery of Landscape within the Operational Programme Environment

Michaela Štěpánková [1] | *Czech University of Life Sciences Prague, Prague, Czech Republic*
Miroslav Hájek [2] | *Czech University of Life Sciences Prague, Prague, Czech Republic*
Pavla Kubová [3] | *Czech University of Life Sciences Prague, Prague, Czech Republic*

Abstract

Public expenses are often evaluated in terms of their effectiveness. This is caused by the character of financial resources and by the fact that public activities are dependent on those resources. This paper addresses the evaluation of financial subsidies within the Operational Programme Environment 2007–2013. From the perspective of financial subsidies, the Operational Programme Environment was the second largest programme in the Czech Republic offering almost 4.92 billion EUR. Verifying the effectiveness of the management of these public funds is highly desirable, as the use of public funds is associated with a risk of over-exploitation. The main aim of the paper is to evaluate the supported projects and to offer new indicators. The assessment was based on the 3E survey. The results of the research confirmed that among the evaluated projects the desired outcomes are achieved at greater efficiency.

Keywords

Alley, efficiency of projects, forest ecosystem, landscape-related activities, operational programmes, recovery of landscape

JEL code

H40, H43, H50, O22, Q23

INTRODUCTION

Public spending is often criticised because of low efficiency (Chu et al., 1991). The general approach for assessing the effectiveness is based on the rule that the realised public project makes sense when the benefits obtained exceed its costs (Musgrave et al., 1994). Selecting appropriate methods, particularly in the environmental field (Pearse et al., 2006), is crucial when trying to quantify benefits. Public expenditure

[1] Faculty of Forestry and Wood Sciences, Kamýcká 129, 165 00 Prague 6 – Suchdol, Czech Republic. E-mail: stepankova@ssc.cas.cz.

[2] Faculty of Forestry and Wood Sciences, Kamýcká 129, 165 00 Prague 6 – Suchdol, Czech Republic. E-mail: hajek@fld.czu.cz.

[3] Faculty of Forestry and Wood Sciences, Kamýcká 129, 165 00 Prague 6 – Suchdol, Czech Republic. Corresponding author: e-mail: kubova@fld.czu.cz, phone: (+420)731111149.

programmes are compared to the costs and effects of their goals during economic analysis. This is based on the definition of the expenditure programme and clearly defined objectives and assessment criteria, which include the economy, efficiency and effectiveness.

The programmes discussed were often part of the Operational Programme Environment (OPE), which falls under the programming period 2007–2013, under thematic operational programmes within the Convergence objective. In terms of financial support, it was the second largest Czech operational programme. Projects included in this operational programme were funded by the European Regional Development Fund and the Cohesion Fund. For the period 2007–2013, 4.92 billion EUR was allocated. Projects implemented in the OPE under the National Environmental Policy of the Czech Republic are intended to contribute to the improvement of the state of the environment with a significantly higher efficiency of the implemented measures. Cherry et al. (2012) also emphasise dependence on providing grants for environmental policy whose measures, based on the adopted environmental policy, are much more efficient. Checking the effectiveness of the management of these public funds (Hájek, 2000; Hudon et al., 2009; Cherry et al., 2012; Pukkala, 2011) is highly desirable as the use of public funds is associated with a risk of over-exploitation and corruption (Barone et al., 2015). For European funds, the risk is higher because a larger amount of funds is involved, beyond the normal scope of the national economy.

The main aim of the paper is to evaluate the supported projects and to offer new indicators. The ambition is also to define new methods and discuss its use for evaluating projects in the given area. The research is focused on projects supported by the Operational Programme Environment, particularly on landscape recovery.

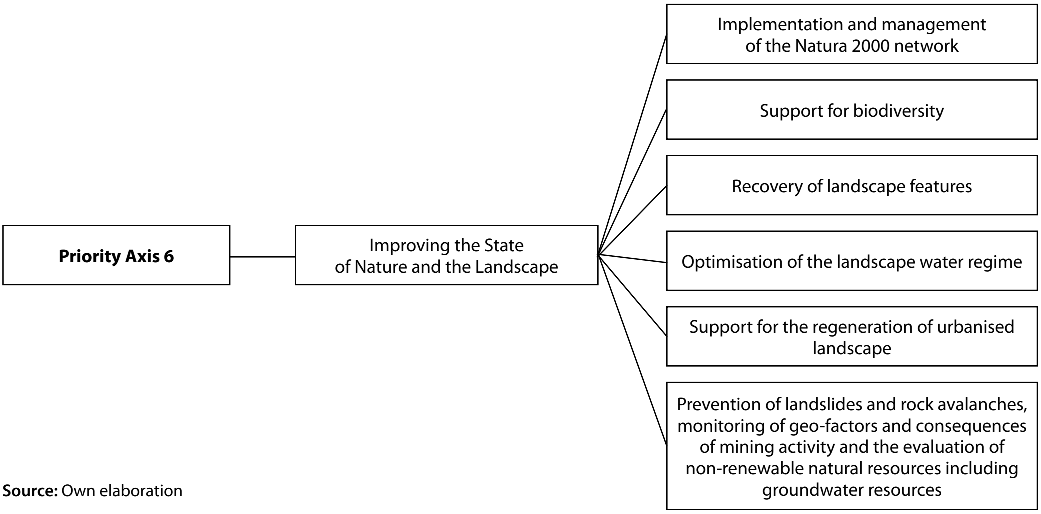
1 SURVEY AND LITERATURE REVIEW

The paper is focused on landscape recovery projects. The processes of landscape change are complex, exhibiting spatial variability (Drummond et al., 2017). These projects are contained in Priority Axis 6 OPE (see Figure 1). The aim of this priority axis is to address the problem as a significant intervention into the landscape structure, which causes excessive fragmentation of the landscape. This is influenced by various life forms, habitats and ecosystem services (Fisher et al., 2009; Mitchell et al., 2015). OPE has therefore focused primarily on the implementation of elements of territorial systems of ecological stability (TSES) and the overall improvement of natural conditions in the open countryside, in the woods or in specially protected areas and Natura 2000. According to Geitzenauer et al. (2017) Natura 2000 is an ambitious and complex venture that requires funding to be successful.

Area of support (6.3 Recovery and landscape) focuses on strengthening ecological stability (Termorshuizen et al., 2007), namely through the creation and restoration of landscape elements, building elements of territorial systems of ecological stability (TSES) and increasing the stability of forest ecosystems (Mandre et al., 2010). It helps to regenerate and improve the age and species composition of forests, increasing the number of landscape features and improving the ecological stability of the landscape. The supported area was carried out in relation to the objectives of the State Environmental Policy 2004–2010, the State Programme of Nature and Landscape, the Biodiversity Strategy of the Czech Republic and the Strategy for Sustainable Development. A complete list of supported areas in Priority Axis 6 is provided in Figure 1 below.

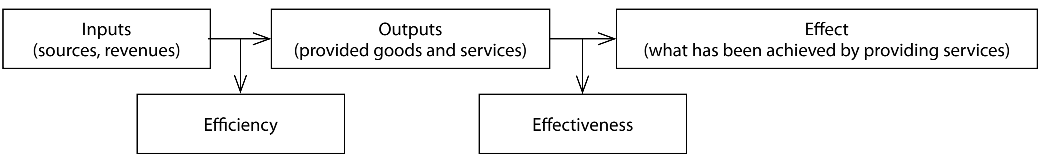
Economic analysis is performed based on the methodology for the evaluation of the expenditure programmes under the so-called 3E approach (Provazníková, 2009; Ochrana et al., 2010). Methods currently in use for generating performance indicators have, according to Liu et al. (2010), limitations, especially when applied to public sector organizations. In general terms a comparison of inputs and outputs was undertaken, where the inputs are investment costs, i.e. the subsidies provided, and the outputs are the benefits described in the project. The financial evaluation was carried out based on the efficiency, which assesses the value of the results relative to the inputs. The higher this ratio is, the greater the efficiency. The effectiveness was also assessed as to what extent and at what quality level the aims were fulfilled. Schematically, the efficiency and effectiveness can be illustrated as follows, see Figure 2 (Provazníková, 2009).

Figure 1 Overview of areas of intervention under Priority Axis 6



Source: Own elaboration

Figure 2 Efficiency and effectiveness



Source: Provazníková (2009), own elaboration

Logically, the project can be evaluated in terms of these three categories of criteria (Behn, 2003), and for each one there will need to be some measures of the indicators as well as some standards. According to Goddard (1989) the public sector has traditionally used the 3E approach, but according to Liu et al. (2010) the 3E approach has a weakness – it ignores efficiency. Does the system actually produce the outputs that it is supposed to? This question is answerable by efficiency (Checkland et al., 1990). Midwinter (1994) argues that economy can be seen as part of efficiency. Johnsen (2005) suggests the 3E approach too, but with analysis of efficiency, effectiveness and equity. Equity can be an explicit requirement of public organizations but not of private ones.

Any assessment based on the 3E survey attains the target of the monitoring indicators. Specific objectives of this area of support, "Recovery of landscape" are as follows: increasing the number and area-based and restored landscape features and elements of territorial systems of ecological stability (TSES), improving natural conditions in forests and improving the soil condition.

2 METHODS AND METHODOLOGY

The basis of the paper is the list of approved OPE projects, issued by the Czech State Environmental Fund (SEF). The specific data was drawn from individual projects in the area of support 6.3 provided for the purposes of this paper. The official monitoring indicator related to the area of intervention 6.3 was determined by the number of projects aimed at improving the state of nature and landscape (target 150 projects).

The monitoring indicators were set for individual projects, which should be met in implementing the projects, by the recipients of the subsidies. In the approved projects, their number in individual regions of the Czech Republic, their status and financial allocation were studied. Given the difficulty of setting benchmarks in this area on one hand and considerable subjectivity on the other hand, unambiguous definitions and proving compliance were assessed in terms of the indicators chosen.

Table 1 shows the classification methodology of the projects into eight categories.

Table 1 Methodology for the inclusion projects of the Priority Axis 6 into categories

No.	Category	Category description
1	Felling	Events with a large amount of logging that were not part of the treatment of landscape plantings or avenues.
2	Landscaping measures	All projects of landscaping measures that directly define the description of the project, including the transfer of biodiversity and USES projects in a wider range.
3	Planting trees outside forests	Categories include the planting of trees outside forests; primarily including planting trees and greenery, without extensive processes eliminating previous vegetation.
4	Restoration of woods	Projects associated with the restoration of forest areas.
5	Restoring alleys and avenues	Classified projects that directly and concretely primarily involve the planting or restoration of alleys and avenues.
6	Treatment of trees and restoration of significant and memorable trees	Only significant and memorable trees.
7	Creation of documentation	Creation and promotion of documentation.
8	Unsorted	No suitable classification found in the above categories.

Source: Own elaboration

The authors divided the grants, according to individual projects awarded, into seven categories and in these intervals the median, the trimmean, a variation range *R* of differences between the estimated and actual expenses were calculated. The median is a set of values divided into two equal parts, it being understood that at least 50% of the values are larger than the median and 50% of the values are smaller than the median (Budikova et al., 2010).

The article also calculated the average excluding the extreme values of the differences between the estimated and actual project costs (trimmean), which are included in the calculations (in this case 5%). The authors determined the average value of the data, so that it separates within 5% of the highest and lowest values of the file, and calculated the average value of the inner part of the plurality of data values. Finally, the variation range *R,* which is a statistical characteristic, expresses the degree of variability of the statistical set. It is the difference between the highest x_{max} and lowest values x_{min} of the quantitative character.

The application of the principles of the 3E Public Procurement Act does not impose or mention. However, this obligation results from a number of other laws, such as the Act on Budgetary Rules, the Act on Municipalities, the Act on Regions, the Act on Property of the Czech Republic, the Act on Financial Control (Pavel, 2008). The Provazníková´s (2009) survey shows that auditing is preferred when auditing the economy and that the 3E method is not always applied. Provazníková (2009) points out that these indicators may conflict with each other. Based on the principle 3E projects were evaluate from three perspectives – efficiency, effectiveness and economy. Economy applies only to inputs, effectiveness refers to inputs and outputs (or result or impact) and efficiency in terms of objectives (and expected outcomes) and the results or impact. Internationally recognized definitions are defined in the Standards of Auditing of the International Organization of Supreme Audit Institutions INTOSAI.

2.1 Efficiency

Efficiency is a comparison between inputs used in a certain activity and resulting outputs (Miguel et al., 2009; Mentzer et al. 1991). Efficiency refers to the extent to which outputs are attained while minimising production costs. Another way to look at efficiency is using the approach of Achabal (Achabal et al., 1984): "*the allocation of resources across alternative uses … [it] is achieved when the marginal productivity per unit of price is equated across all resources that contribute to the firm's output*." Efficiency means the use of public funds to ensure an optimum level of achievement of the objectives in fulfilling the assigned tasks. In other words, the degree of achievement of the objectives and the relationship between the intended and the actual impacts of the activity are understood to be useful. An action that achieves the goals is meaningful without the involvement of other activities and / or undesirable unintended consequences. The principle of efficiency requires the attainment of the objectives of the activity intended effects. The criterion of efficiency examines the economic rationality of the resources used (Ochrana et al., 2010).

The efficiency was assessed as the ratio of the benefits of implementing the project to the costs of doing so (Synek et al., 2015). The efficiency analysis was performed based on the characteristics of the programme, which should enhance the region, especially those with higher fragmentation of the landscape. As a criterion for assessing the number of projects by region, data on the habitat fragmentation is utilised (Andel, 2013). It can deduce the extent of the number of projects related to the regions corresponding to the terms of the focus of the programme.

To determine the tightness of dependence (relative strength) of two variables (the extent of fragmentation – an indicator of effective mesh size in 2005 (Andel, 2010) and the number of approved projects (up to 2015), we measured statistic dependence by using Pearson's correlation coefficient (1):

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{1}$$

The random variable x_i and y_i are quantitative random variables with a common two-dimensional normal distribution.

Effective mesh size – m_{eff} is the numeric indicator of landscape fragmentation barriers broken down into sub-isolated areas. The indicator is based on the calculation of the probability that two individuals at random locations in the studied area will be located in one area, therefore they are not separated by a barrier (Andel, 2010).

From the available data, it was possible to state the achievement of the official objectives of the programme and there was no obvious greater benefit from the implemented projects.

2.2 Effectiveness

Effectiveness refers to the connection between inputs, outputs and more general, secondary type objectives or outcomes (Miguel et al., 2009; Kumar, 2009). We can say that effectiveness is the capability of producing a desired result. The effectiveness corresponded with the extent to which the project's aims were fulfilled and where economy was concerned, the extent to which the public finances provided were spent (Vochozka et al., 2012). Concerning economy, we see the attainment of the objectives at the lowest possible cost, thus minimising the costs while respecting the objectives of the project and still maintaining adequate quality. Effective is an activity that optimizes the use of organizational / program / activity resources to generate outputs, i.e. achieving maximum output from given sources or achieving a given output with minimal resources and maintaining the quality of outputs. The principle of effectiveness requires the best possible relationship between the resources used for the activity and the effects achieved (Pavel, 2008, 2009).

2.3 Economy

Economy in this context means the use of public funds to ensure that tasks are assigned with the lowest possible use of these resources, while respecting the appropriate quality of the tasks to be achieved, i.e. minimizing the cost of resources (inputs) used for the activity with respect to the corresponding quality (Pavel, 2008). Pavel (2008) adds that the principle of economy requires that the resources used by the entity in carrying out its activities are available at the right time, in sufficient quantity, at the appropriate quality and at the most advantageous price.

The ambition is to describe the differences between the estimated and actual costs compared to the grant amount awarded, which corresponds to the economy by the 3E approach. It will determine whether objectives were achieved economically, i.e. with lower costs.

The economy and effectiveness cannot be identical, but there is a close link between these principles under the European legal norms[4] (Sapíková, 2013; Koch, 2013). Therefore, economy and effectiveness will be evaluate under subchapter 3.2 together.

3 RESULTS

The number of approved projects (up to 2015) in support of 6.3 in the database of approved projects under OPE is 837 (see Table 2). The distribution of the projects in the Czech Republic is given in Table 2, in which the approved projects assigned to individual regions of the Czech Republic are shown. It is then clear that the calls lead to the further approval of new projects. The success of individual regions, however, remains the same. Further evaluation of the Priority Axis 6 of the OPE support 6.3 assesses the status of the approved projects. Projects can be in the following states: financing of the project completed, the project is finally closed, project implementation, project completed, approved for financing, project expenditures certified.

Table 2 Number of approved projects in Priority Axis 6 in support of 6.3 up to 2015 disaggregated by region in the Czech Republic

Region	Number of Projects
South Bohemia Region	52
South Moravia Region	238
Karlovy Vary Region	25
Hradec Kralove Region	33
Liberec Region	43
Moravia-Silesia Region	76
Olomouc Region	71
Pardubice Region	23
Pilsen Region	24
Central Bohemia Region	51
Usti Region	49
Vysočina Region	59
Zlín Region	93
Total	837

Source: Own elaboration (Internal database of the State Environmental Fund CR, 2016)

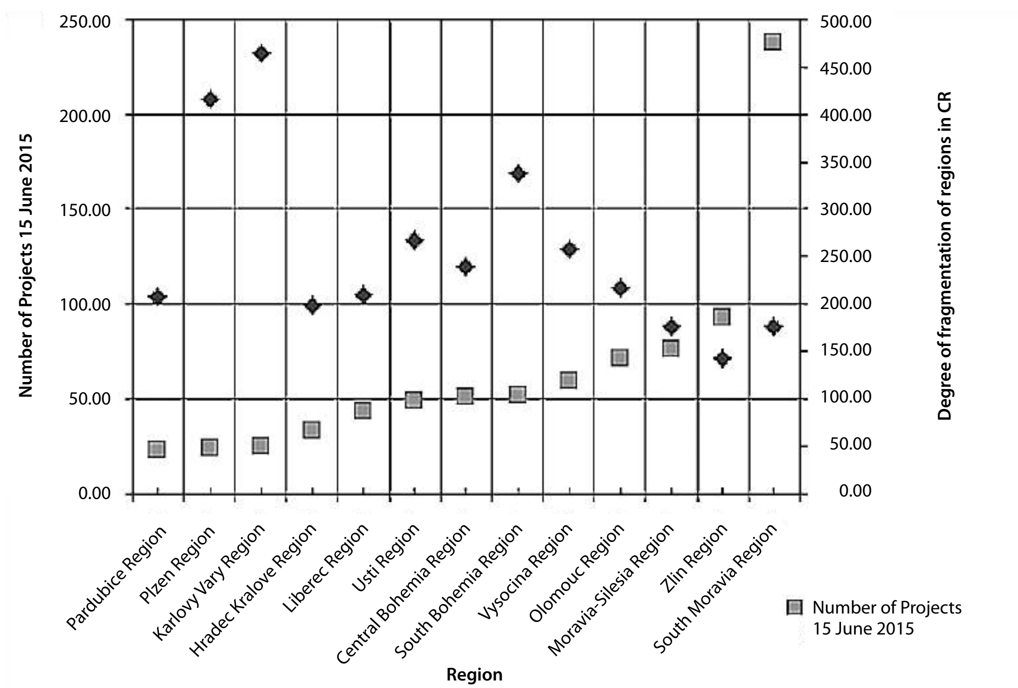
[4] Directive 2004/17/EC of the European Parliament and of the Council of 31st March 2004 coordinating the procurement procedures of entities operating in the water, energy, transport and postal services sectors and Directive 2004/18/EC of the European Parliament and of the Council of 31st March 2004 on the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts.

Significant changes were recorded in 2015 – there were new projects, while there were also significant changes in conditions. There are 50 projects that have been approved for funding, of which 44 were approved in 2015 and show no paid resources. Added to this, there are also finally closed projects and projects under implementation. At the same time, there are a substantial amount of projects awaiting certification of their expenses.

3.1 Efficiency

There are also two newly added projects that are on hold. The following Figure 3 shows the value of the degree of fragmentation of regions in the Czech Republic and the number of projects submitted in the regions of the Czech Republic (excluding the capital Prague) for evaluation of efficiency.

Figure 3 The degree of fragmentation and the number of projects by region in 2015



Source: Own elaboration

Data on habitat fragmentation is utilised as a criterion of efficiency (Andel, 2013). According to Andel (2010), the importance of the issue of fragmentation in the future will continue to increase, not only due to the result of direct pressure from the further construction of roads and settlements, but also in relation to indirect effects such as global climate change. The conclusions of Andel (2010) clearly demonstrate the increasing fragmentation of the landscape in the Czech Republic over the past 25 years and show a pessimistic prognosis for the future due to the decrease in the total area of UAT (Unfragmented Area by Traffic) and continuous reduction in their size and their quality.

These trends are in line with the development of automotive and residential construction and transport infrastructure. In terms of the share of UAT on the areas of the regions in the period 1980–2040, it can be stated that there are huge differences between individual regions in the Czech Republic. As reported by Andel (2010), the South Bohemian Region and Pilsen currently have the highest proportion of unfragmented territories (in 2040, it is predicted that 74.15% of the area of the region will be UAT), whereas the worst situation is in the Moravian-Silesian region, South Moravia and Central Bohemia (in 2040, it is predicted that 37.25% of the area of this region will be UAT).

Pearson's correlation coefficient equals –0.45, which indicates a weak indirect dependence of the number of projects on the degree of fragmentation. That means that the higher the degree of fragmentation, the lower the number of projects in this region.

3.2 Economy and effectiveness

There is a close link between these principles under the European legal norms (Sapíková, 2013). Therefore, economy and effectiveness will be evaluated under this subchapter together.

The key parameter for evaluating projects is primarily the funding. The amount of support is apparent from the beneficiaries, which are local governments and their associations, natural and legal persons (as well as those entrusted with the management of state-owned forests), civic associations and churches, public research institutions, organisations, government (excluding land offices), administrators of watercourses and catchment areas.

Results on redistributing finance projects support 6.3 (to 2015) are indicated in Table 3.

Table 3 Redistributing finance projects support 6.3 to 2015

Total cost of the operation (CZK)	Total eligible expenses (CZK)	Amount allocated (CZK)	Paid out funds (CZK)
2 241 136 517	2 052 763 630	1 918 814 224	1 387 469 956

Source: Own elaboration (Internal database of the State Environmental Fund CR, 2016)

Redistributing finance projects support have increased negligibly over a period of about half a year with paid resources, by nearly 17 million CZK. The ratio of total aid approved and disbursed funds increased to 72%. Up to 2015, 109 projects were registered for which payment has not been made. Total authorised aid of these projects amounted to 212 302 212 CZK. As stated in the methods, in terms of the 3E, there is a need to monitor the following: effectiveness, economy and efficiency. In terms of the effectiveness it is necessary to assess whether the project objectives have been achieved. According to the fulfilment of the monitoring indicators provided by the SEF (State Environmental Fund) to 2015, the project objectives are met more than 71% of the time. This value is influenced by a large number of projects that are still in progress. Targets (m) in the form of monitoring indicators are shown in Table 4. However, the question remains as to whether the objectives were defined correctly from the outset and whether their performance in such a form is at all correct. The numbers of projects that have committed to meeting the above objectives are also presented in Table 4.

Table 4 Overview of the number of projects and monitoring indicators (up to 2015)

Monitoring indicator	Number of projects
Total length-loaded and regenerated alley	14
Total length of start and reclaimed landscape elements (in the case of linear elements)	422
The total area on which the measures were implemented to promote biodiversity	2
The total number of trees planted and treated	30
The total number of established and reclaimed landscape elements	611
Area revitalised	518
Number of measures taken in connection with the recovery of landscape structures	124
Number of measures taken in connection with the promotion of biodiversity	1

Source: Own elaboration (Internal database of the State Environmental Fund CR, 2016)

From the above, it is clear that the individual projects have to meet more than one objective (*m*). Only 144 of the projects stipulated meet only one objective. It should be noted that these indicators have no explanatory value, do not express how much and what quality was supported by the completed grants. Generally, it can be stated in which area the projects were realised in the area of support 6.3 and what was realised (but not in detail). A more appropriate indicator would be an accurate account, for example in the form of the indicator "the number of trees planted at the prescribed quality" and "the percentage of their death", from which it is clear exactly what has been done and what quality was achieved.

22 projects set out to reach the economy indicator, of which 18 have already met their goal, 17 of which were reimbursed funds. It is, therefore, compared to only 17 projects. All of these projects met their objectives in the form of monitoring indicators, but only one was even slightly increased (with respect to effectiveness). 7 grant recipients behaved economically because their projects exhibit financial savings while meeting their objectives. If a project is financed from one's own resources, the participants act efficiently; such effort is also obvious when the grant recipients are attempting to use up all the funds provided. Two projects did not record any savings, two recorded more than 4%, but the others amounted to almost 23%. Such savings, while meeting the planned targets, are relatively high.

We assess the efficiency ratio of the input and the output efficiency of the resources invested and the benefits they earned. We compare the results and impact of the sources used. Efficiency is often seen as cost efficiency. Achieving high efficiency in a competitive environment is the source of success, which leads to increased profits. In the classical market environment, we can assess the accuracy of the allocation of financial resources according to the ability to compete. The market test proved that public finance resources were insufficient. The evaluation involved only projects with a single objective as this enabled the determination of the cost per unit in CZK. This also includes the projects which have already met their objectives and their required subsidies have therefore been paid. The assessment of efficiency of the projects dealing with the total duration of their initial stages and with reclaimed landscape elements/ TSES (the Territorial system of ecological stability when concerning linear elements) is based on Table 5.

Table 5 Monitoring indicators of their performance, financial summary (up to 2015)

Event title	Target value (*m*)	Real value (*r*)	Total grant approved	Funds reimbursed (CZK)	Saved (CZK)	Price of 1 metre according to reimbursed funds (CZK)
Restoration of vegetation in the municipality Bystřany	940	940	380 109	380 109	0	404.37
Maintenance and restoration of the trees at Pacov	4 000	4 000	638 975	638 975	0	159.74
Regeneration green Klatovy	15 200	15 200	8 474 051	8 420 747	53 303	554
Restoring alley Velešovice	900	900	200 326	200 326	0	222.58
Revitalisation avenue of lime trees Ratibořice	1 020	1 020	354 375	354 375	0	347.43
Treatment of linden alley Milevsko	560	560	208 910	208 910	0	373.05
Alley along the path in the Humňany	2 800	**2 802.79**	3 699 065	3 699 032	32	1 319.77
Planting alley along the road Kazůbkova	1 500	1 500	368 718	358 986	9 732	239.32
Regeneration of alley at Broumov	3 000	3 000	459 073	450 162	8 911	150.05
Regeneration of alley in Lipky	857	857	793 398	793 398	0	925.78
Renewal project road alley road II / 408	25 000	25 000	1 977 477	1 824 786	15 2691	72.99
Restoration of the historic avenue Castle Veveří	81	81	239 661	239 661	0	2 958.78
Stabilisation and treatment Valdštejn linden alley	815	815	937 226	937 226	0	1 149.97
Fruit tree alley Litohoř	475	475	214 575	214 575	0	451.74
Regeneration of dirt track in Sychrov nad Jizerou	1 335	1 335	758 462	713 494	44 969	534.45
Bird alley in Křinice	65	65	102 773	87 879	14 894	1 351.99
Planting alley in rural Dolní Benešov	713	713	287 171	287 171	0	402.76

Source: Own elaboration (Internal database of the State Environmental Fund CR, 2016)

The price of one metre-loaded and regenerated landscape elements in the individual projects is diametrically different. However, it is logical that the restoration of a historic avenue is more expensive than the restoration of a road alley. However, if we compare only the renewal of road alleys, we compare the prices of 159.74 CZK, and 150.05 CZK, and 72.99 CZK per 1 metre. The first two values are comparable, with the third having a significant impact on the number of units (assuming the higher the number of metres, the lower the price per 1 metre). This project also has the highest efficiency. For the indicator "The total number of trees planted and treated" cannot be compared to the performance because there is only one project with such a target indicator (*m*). Data for the indicator "The total number of established and reclaimed landscape elements" is shown in Table 6.

Table 6 Monitoring indicators, their fulfilment, financial summary (up to 2015)

Event title	Target value (*m*)	Real value (*r*)	Total grant approved (CZK)	Funds reimbursed (CZK)	Saved (CZK)	Price of 1 part according to reimbursed funds (CZK)
Restoration and maintenance of dam stands Třeboň	10	10	1 741 420	1 489 427	251 993	148 942.69
Treatment of memorable and significant trees Ralsko	3	3	74 045	74 045	0	24 681.60
Treatment of avenue of lime trees in the village Hořátev	1	1	582 120	582 120	0	582 120
Treatment, retention and completion of oak alley Mimoňsko	4	4	966 740	966 650	90	241 662.60
Telč, Linden alley – treating trees	1	1	508 884	508 884	0	508 883.98
Comprehensive treatment of significant trees Military Domain Boletice	5	5	90 715	84 415	6 300	16 882.92
Revitalisation alleys and avenues KSÚSV	2	2	2 164 959	2 164 959	0	1 082 479.50
Treatment with significant vegetation line Hluboká nad Vltavou	1	1	658 256	658 256	0	658255.50

Source: Own elaboration (Internal database of the State Environmental Fund CR, 2016)

10 projects started with the indicator "total number of established and reclaimed landscape elements", of which 8 have already met their goal. The first of these projects did not use almost 15% of the approved assistance and the savings are, therefore, considerable. This project is, therefore, the most economical and effective, ranking as the third of all projects when considering price per unit. However, it should be noted that the effectiveness of this form is impossible to evaluate. The indicator has no meaningful value, because the price of one "piece" of land-loaded and regenerated element can be up to 64 times higher. The same is evidenced by the indicator "The number of measures implemented in connection with the recovery of landscape structures" in Table 7. The effectiveness again cannot be clearly assessed because the projects are completely different and it would be inappropriate to compare the price. The winner and loser are obvious at first glance, but to compare the price of 25 thousand CZK and 6.1 million CZK is not logically appropriate. This objective was realised in 6 projects, 5 of which have already fulfilled thei objectives.

The "Area revitalised" in hectares, recorded the most projects (105 of them) with this one goal. Of these, 91 projects have already accomplished their goal. To simplify, the authors are presenting an overview of 5 projects only, in Table 8.

4 DISCUSSION

It is clear from the results that the use of the 3E analysis extends the view on the effectiveness of the implemented projects and program 6.3 in the Priority Axis 6 OPE. There are major differences where the economy and efficiency are concerned in the projects and program 6.3 in the Priority Axis 6 OPE for the aforementioned purpose. Savings amount to 33%, which is certainly not negligible. One project not only met the goals set, but at the same time exceeded their performance by 12% while saving more than 25%. Based on that sample, it allows the authors to state that grants recipients have generally behaved

Table 7 Monitoring indicators, their fulfilment, financial summary (up to 2015)

Event title	Target value (*m*)	Real value (*r*)	Total grant approved (CZK)	Funds reimbursed (CZK)	Saved (CZK)	Price of 1 part according to reimbursed funds (CZK)
Developing forest management plan in the National Park Podyjí	1	1	2 827 562	2 704 949	122 613	2 704 949.22
Plan buffer zone Podyjí	1	1	9 425 445	9 425 445	0	942 545
Creation of forest management plans for forest management in the Krkonoše Mts. (Giant Mountains) National Park	3	3	23 877 298	18 417 933	5 459 365	6 139 311.07
Maintenance of memorial trees Palvínov	20	20	599 341	573 524	25 817	28 676.20
Improved care for the memorial of Přílezy	2	2	50 004	50 004	0	25 002

Source: Own elaboration (Internal database of the State Environmental Fund CR, 2016)

Table 8 Monitoring indicators of their performance, financial summary (up to 2015)

Event title	Target value (*m*)	Real value (*r*)	Total grant approved (CZK)	Funds reimbursed (CZK)	Saved (CZK)	Price of 1 ha according to reimbursed funds (CZK)
Reconstruction Sobotovická niva Floodplain	5.2	5.2	152 183	152 183	0	29 265.92
Restoring solitary oaks in Soutok	0.25	0.25	508 031	473 380	34 651	1 893 520.72
Restoring fir Slavkovský forest	3.66	3.66	636 916	636 916	0	174 020.66
Adaptation park castle Sádek	2.97	2.97	2 582 295	2 367 948	214 347	797 289.01
Improving oak, age and spatial composition of forests Vápenice	18.14	18.14	961 356	961 356	0	52 996.16

Source: Own elaboration (Internal database of the State Environmental Fund CR, 2016)

economically. Here it is apparent that assessment of the efficiency cannot be easily performed, since individual projects have quite different objectives and, therefore, costs. Comparable projects dealing with reforestation in the areas of air quality, price per 1 ha then range between 19 and 80 thousand CZK, which at first glance is basically ineffective because the 4 times higher costs are a considerable difference. The database of approved projects up to 2015 was subjected to an elementary statistical analysis. Mean values relating to the financial areas are shown in Table 9.

Table 9 Mean values of approved projects up to 2015

Mean value (CZK)	Total costs of the operation (CZK)	Total eligible expenses (CZK)	Amounts allocated (CZK)	Paid out funds (CZK)
Arithmetic Mean	2 677 582	2 452 525	2 292 490	1 664 754
Median	1 075 898	1 015 688	931 392	696 873
Mode	170 000	382 000	966 000	0

Source: Own elaboration.

The average authorised aid was EUR 2.3 million CZK. The median, a value that divides a series of increasingly aligned results in two equally large halves, with the aid approved of 931 thousand CZK. Modus (the value that frequently occurs in the cohort) is approved for support of 966 thousand CZK. Funds paid to the mode equals 0, since nothing has been paid yet for a considerable number of projects. Table 10 below lists the categories of the projects listed according to the granted aid in CZK and an elementary statistical analysis of the data drawn from the SEF (State Environmental Fund) (the difference between the estimated and actual costs).

Table 10 Elemental statistical analysis of the difference between the estimated and actual costs (CZK)

Category of allocated grants in intervals (CZK)	Arithmetic Mean	Median	Variation Range *R*	Trimmean
<0:300,000)	14 632	6 040	99 348	13 644.53
<300,000:500,000)	28 455	15 274	123 009	27 230.96
<500,000:1,000,000)	51 487	26 400	318 724	47 690.71
<1,000,000:2,000,000)	90 348	28 982	953 196	76 522.36
<2,000,000:4,000,000)	201 604	61 000	1 235 085	189 187.4
<4,000,000:10,000,000)	550 564	145 238	2 847 027	550 564.3
<10,000,000:100,000,000)	3 930 475	1 926 557	29 598 715	3 930 475

Source: Own elaboration

As shown in Table 10, the elementary statistics were calculated from the graduated intervals of the grants. For example, for the projects approved up to 300 000 CZK, the median of the difference between the estimated and actual costs is 6 040 CZK (the trimmean is 13 645 CZK). On the other hand, for projects in the interval from 4 000 000 to 10 000 000 CZK, the median is equal to 145 238 (the trimmean is 550 564 CZK). After rejection of the projects under implementation up to 2015, real costs are, on average, 7.24% lower than estimated. The average value of the differences in the estimated and actual costs of the already implemented projects up to 2015 is 236 266 CZK. Support Area 6.3 is committed to achieving the objective of the indicator in the number of projects aimed at improving the state of nature and landscape. There are 150 projects in the desired state. The monitoring indicator was met. Therefore, it can be stated that in terms of the performance criteria, it is effective to support them (Zoppi and Lai, 2011). Only those projects were relevant, whose financing has already been completed, those that were finally concluded or terminated their implementation and project expenditures were certified. In this case, of course, we get different results, namely in the area of intervention 6.3 for 380 projects which less than 708.5 million CZK had been paid.

The efficiency was also evaluated by the supported projects in various regions. Using the methodology of Andel (2010), regarding the degree of fragmentation of the landscape by region, it was found that the efficiency support in the case of the South Moravian Region was high because most of the projects were supported there (238), and the landscape is the most fragmented. It can negatively evaluate the efficiency in the Central Region, which is also highly fragmented, but only 51 projects were supported there.

The programme can generally be positive in terms of increasing the number and area-based and restored landscape features and elements of territorial systems of ecological stability. As a result of a weak indirect dependence of the number of projects on the degree of fragmentation, inappropriate spatial distribution of the projects that should have primarily been directed to highly fragmented regions can be considered as a negative.

From the perspective of efficiency, the supported projects can be evaluated positively, because an average saving of 7.24% has been achieved in compliance with the planned parameters. It is necessary to assess the allocation of resources within the approved projects. The outstanding 485.6 million CZK represents almost 25% of the budgets of the approved projects amounting to 1 918 million CZK. It is certainly a negligible number. In addition to the specific programming period 2007–2013, the implementation of all projects should have been completed and should also have been financially resolved. Rejecting the funds at such level must be attributed to the fact that this priority axis had not been maintained. As a result, the calls for the projects had to be closed later than in 2013. In 2014, a total of 10 calls were declared, the last of which was entered on the 14th of November 2014. In 2014, there were 170 approved projects within the area of support 6.3, in 2015, there were 44 projects.

CONCLUSION

The total financial allocation to this area of implementation was 77.925 million EUR. All approved projects in this area were individual projects; no large projects exceeding 25 million EUR in budget were implemented in area 6.3 nor, indeed, in Priority Axis 6. From the perspective of 3E, the OPE projects in the area of support 6.3 were evaluated by researching the efficiency, economy and effectiveness. In terms of the efficiency, we evaluated the achievement of the project objectives. The current performance is equal to 71% and it is expected that by the end of the implementation of the 2007–2013 programming period in 2017, the objectives will be met. A fundamental recommendation is to accurately identify and quantify the programme objectives, so that it is easy to measure their performance and so the explanatory values would be quite obvious. The economy itself helps to solve the attainment of the objectives at the lowest possible cost. A considerable number of projects minimised their expenses, obtained their goals while maintaining the corresponding quality and still showed significant savings. Overall, however, the authors cannot say that all the projects were efficiently dealt with. Economy should be dealt with primarily from the design stage of the project where it is necessary that the overvalued project items are lessened.

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CO_2 Emissions, Real GDP, Renewable Energy and Tourism: Evidence from Panel of the Most-Visited Countries

Eyüp Doğan [1] | *Abdullah Gül University, Kayseri, Turkey*

Abstract

Previous studies on the energy-environment-growth literature overlook the investigation of the most-visited countries. Since these countries do not only belong to the largest economies and the top carbon dioxide (CO_2) emitters in the world but are also listed in renewable energy country attractiveness index, this study analyzes the impacts of real GDP, renewable energy and tourism on the level of CO_2 emissions for the top 10 most-visited countries. Applying several panel econometric approaches, we find out that renewable energy mitigates the pollution whereas real GDP and tourism contribute to the level of emissions. Thus, regulatory policies are necessary to increase the awareness of sustainable tourism. In addition, the use of renewable energy and the adoption of clean technologies in tourism sector as well as in producing goods and services play a significant role in CO_2 mitigation.

Keywords

Carbon dioxide emissions, tourism, renewable energy, real GDP, heterogeneity, cross-sectional dependence

JEL code

C32, C33, L83, O44, Q20, Z32

INTRODUCTION

The world has experienced a tremendous increase in the amount of greenhouse gas emissions over the last several decades. More than 190 parties have signed the Kyoto Protocol and participated in many meetings about the climate change and the environment to fight the pollution.[2] The last United Nations (UN) Climate Change Conference associated with the Kyoto Protocol took place in Paris in November–December 2015. This protocol aims to reduce the level of emissions by targeted rates. In this regard, several potentials have been discussed to keep track of the projected level of emissions. Increases in the share of renewable sources (environmentally-friendly) in energy mix play an important role in this matter. Recent studies Al-Mulali et al. (2015a), Dogan and Seker (2016), Jebli et al. (2016), and Mehdi and Slim (2017) among others show that increases in renewable energy statistically and significantly

[1] Department of Economics, Abdullah Gül University, Kayseri, Turkey. Phone: (+90)3522248800, e-mail: eyup.dogan@agu.edu.tr.

[2] Please refer to the United Nations website at: <www.un.org> for more information.

reduce emissions. The volume of studies investigating the influence of energy by sources on the pollution is still relatively smaller as compared to studies using the aggregate energy consumption in modeling the energy-environment-growth nexus.

Sustainable tourism is also critical for the climate change as it plays a significant role in reaching goals of the Kyoto Protocol referring to the Second International Conference on Climate Change and Tourism on October 2007 jointly organized by the World Tourism Organization (UNWTO), and the UN Environment Programme. Tourism is a rapidly developing sector that grows each year with more arrival points emerging around the world. The 2016 edition of UNWTO reports that the overall number of tourists travelled around the world increased from 0.025 billion in 1950 to more than 1.2 billion in 2015.[3] The large expansion of tourism sector over this period should be connected to the tremendous increase in global carbon emissions. Tourism activities involve energy consumption directly through fossil fuels or indirectly through electricity power in each step from transportation to accommodation. Depending on the source of energy use (e.g. renewable and non-renewable energy), tourism may either mitigate or contribute to emissions. In addition, the influence of tourism on the pollution can alter by supportive policies and government interventions for low level of emissions, and the use of cleaner technologies in the sector activities. Even though tourism is very much related to the environment, only a few studies in the energy-environment-growth literature consider the possible effect of tourism on the pollution and more importantly their findings are mix (Dogan et al., 2015; De Vita et al., 2015).

This study makes several contributions to the existing pool of knowledge: i) this study for the first time analyzes the effects of real gross domestic production (GDP), renewable energy (REN) and tourism (TOUR) on carbon dioxide (CO_2) emissions for the most-visited 10 countries in the world.[4] Because the top 10 touristic (most-visited) countries have recorded about the half of the worldwide tourist arrivals in late years, we focus on a panel study of these countries. Furthermore, these countries are among the top ones in "Renewables Energy Country Attractiveness Index (RECAI)". This RECAI is calculated based on 3 drivers, 16 parameters and 53 datasets;[5] they are among the largest economies and the top emitters in the world; ii) this study uses a source of energy (renewable energy) instead of aggregate energy; iii) the possible presence of issues of cross-sectional dependence and heterogeneity across cross-sections for variables is among main criticisms to panel studies. If both issues appear in the data, a researcher will produce inaccurate and erroneous estimates if the researcher assumes homogeneity and cross-sectional independence in panel time-series data. Accompanying the information that nearly 99% of studies in the energy-environment-growth literature fail to check the mentioned issues and employ first generation econometric approaches, this study first identifies that cross-sectional dependence and heterogeneity exist in the analyzed data for the top 10 economies, and, accordingly, applies second generation econometric techniques. Thus, the results reported herein are more accurate and robust.

This study presents a survey of literature in the next section, provides the model and data in the third section, discusses the issues of heterogeneity and cross-sectional dependence in the fourth section, yields methods and empirical findings in the fifth section, and conclude aims, methods, findings and policy implications in the last section.

1 LITERATURE REVIEW

This section presents and brings together the associated literature in connection with the aim of the current study. The link between tourism and real GDP (or economic growth) is well-established and investigated

[3] Available at: <www.unwto.org>.

[4] The top 10 countries are China, France, Italy, Thailand, the UK, the USA, Turkey, Germany, Russia and Spain.

[5] Please refer to: <http://www.ey.com/GL/en/Industries/Power---Utilities/Renewable-Energy-Country-Attractiveness-Index---Methodology>.

for single country cases and multi-country cases in the literature (Dritsakis, 2004; Oh, 2005; Kim et al., 2006; Santana-Gallego et al., 2010; Arslanturk et al., 2011; Pablo-Romero and Molina, 2013; Balcilar et al., 2014; Bilen et al., 2015; Pérez-Rodríguez et al., 2015; Aslan, 2015; Shahzad et al., 2017). Several studies including Peeters (2007), Gossling et al. (2010), Scott et al. (2010), Scott et al. (2012), Saenz-de-Miera and Rosselló (2014), Gossling and Peeters (2015), Al-Mulali et al. (2015b), Sharif et al. (2017) and Paramati et al. (2017) discuss the emissions-tourism nexus and indicate that tourism adversely impacts the environment. In other words, tourism activities including transportation contribute to the increased level of emissions. Referring to the aforementioned studies, tourism contributes to the output and pollution.

In addition to these nexus, the energy-economic growth nexus is also analyzed for a variety of cases (Ozturk, 2010; Yildirim et al., 2012; Wolde-Rufael, 2014; Bloch et al, 2015; Dogan, 2015). Furthermore, the link between CO_2 emissions, real GDP, aggregate energy consumption is described in a large number of studies including Say and Yucel (2006), Ang (2008), Soytas and Sari (2009), Apergis and Payne (2009), Du et al. (2012), Ozturk and Acaravci (2010), Hossain (2011), Pao and Tsai (2011), Park and Hong (2013), Cowan et al., (2014), Farhani and Ozturk (2015), Baek (2015), Ajmi et al. (2015), Shahbaz et al. (2015), Kasman and Duman (2015), Dogan and Turkekul (2016), Magazzino (2016), and Bekhet et al. (2017). The state-of-the-art reaches a consensus that increases in aggregate energy consumption contribute to the level of emissions. Of those that focus on a panel study mostly employ first generation econometric approaches (e.g. unit root tests by Levin-Lin-Chu (2002), Hadri (2000), Im-Pesaran-Shin (2003), and Pedroni cointegration (Pedroni, 1999; 2004) and Koa cointegration (Kao, 1999)).

As shown in Table 1, several recent studies focus on a type of aggregate energy; namely, renewable energy, and analyze the renewable energy-environment-growth relationship for single country and panel of countries cases. It is worth-noting that the number of studies in this strand is relatively smaller than that in aggregate energy-environment-growth literature. Studies in table 1 (except Farhani and Shahbaz, 2014; Apergis et al., 2010; Boluk and Mert, 2014) yield that increases in renewable energy detract the pollution in a variety of regions and countries by mostly using first generation econometric approaches. In addition, some studies in this group investigate the validity of the EKC hypothesis by including the square of real GDP (GDP^2) into the model as similar to those in aggregate energy-environment-growth literature. However, the aim of the current study is not to show whether or not increases in real GDP lead to environmental improvements; instead, to narrowly focus on how tourism and renewable energy impact the level of emissions by controlling for the income in the model since real GDP is a strong determinant of CO_2 emissions.

The last strand of studies examines the relationship among energy, environment, real GDP and tourism for several cases. As shown in the bottom of Table 1, Lee and Brahmasrene (2013), and Katircioglu (2014a) find that tourism decreases CO_2 emissions for the panel of EU countries, and Singapore, respectively, on the other hand, Katircioglu et al. (2014), Katircioglu (2014b), and De-Vita et al. (2015) indicate that tourism stimulates emissions in Cyprus and Turkey. These studies also suggest that the coefficient estimate on real GDP for CO_2 emissions is positive. There is only one panel study in this strand, and it uses first generation econometric tools in identifying the relationship. Because first generation tests have drawbacks of assuming cross-sectional independence, they may produce inaccurate results. Thus, the results in this study are accurate and reliable since we find the issue of dependence across cross -sections, and accordingly employ second generation approaches.

2 MODEL AND DATA

Inspired by the works of Jebli and Youssef (2015), and Katircioglu (2014b) we propose the following model in which CO_2 emissions are the response variable, and real gross domestic product (GDP), renewable energy (REN) and tourism (TOUR) are the dependent variables:

$$CO_2 = (GDP, REN, TOUR) \tag{1}$$

By including a constant term (β_0) and an error term (e_{it}), we can convert the model in Formula (1) to that in Formula (2) wherein β_k (k = 1, 2, 3) are the coefficients on GDP, REN and TOUR.

$$(CO_2)_{it} = \beta_0 + \beta_1 GDP_{it} + \beta_2 REN_{it} + \beta_3 TOUR_{it} + e_{it} \quad (2)$$

The data used in this study are described as follow. CO_2 emissions is carbon dioxide gas emissions in metric tons; real GDP is the value of real gross domestic product in constant 2005 US$; TOUR is the number of international tourist arrivals at the sample countries; REN is renewable electricity production measured in kilo-watt hours. Following Apergis et al. (2010), Farhani and Shahbaz (2014), Bhattacharya et al. (2016), and Jebli et al. (2016), renewable electricity is used as proxy for renewable energy. The data for CO_2 emissions, real GDP and tourist arrivals are obtained from the "World Development Indicators",[6] and the data for renewable electricity production are drawn from the "US Energy Information Administration".[7] The annual data cover the period 1995–2011. It should be noted that we use the longest available data given the fact that the data for TOUR are not available before 1995, and CO_2 emissions are not available after 2011. Even though using a longer data set is an advantage to produce more robust outcome, we believe that this study is still valuable for the literature as it contributes to the existing pool of knowledge by exposing the importance of tourism as well as renewable energy and real GDP for the environment, and by taking into account the issues of heterogeneity and cross-section dependence. The top 10 countries used in this study are China, France, Italy, Thailand, the UK, the USA, Turkey, Germany, Russia and Spain. Since the panel time-series data are converted into their natural logarithm, β_k (k = 1, 2, 3) can be interpreted as the elasticities of CO_2 emissions with respect to real GDP, REN and TOUR. Referring to the state-of-the-art, the expected sign of β_1 is positive, and β_2 is expected to be negative; β_3 can be positive or negative depending on the net effect of tourism on the environment that we argue in the introduction section.

3 HETEROGENEITY AND CROSS-SECTION DEPENDENCE

The average annual growth of tourism, carbon emissions, real GDP and renewable energy for the analyzed countries are indicated in Table 2. Because of significant variations in the average annual growth of each variable across countries, one can claim the presence of heterogeneity across the top 10 touristic countries for the analyzed variables. In detail, the average annual growth of tourism arrivals is relatively greater for Turkey, China and Thailand as compared to France, the UK and the USA. Moreover, a similar picture is observed for the average annual growth of real GDP and REN. Furthermore, the average annual growth of carbon emissions is negative for Germany, Italy and the UK whereas it is positive for the rest of sample countries. Furthermore, from the point of view of the development level, countries fall into different groups (i.e. developed countries and developing countries). This also suggests a strong heterogeneity within the panel data. Considering the presence of issue of heterogeneity, we should account for it in panel econometric approaches wherein the parameters are allowed to vary across cross sections.

In addition to heterogeneity, the possible presence of cross-sectional dependence in panel time-series data is another potential issue that should be taken into account in panel models. The correlation among the time-series data for the top 10 countries may be exposed because of common shocks (e.g. great recession, global energy and environmental policies, global credit crunch) that potentially have spill-over effects on cross sections. If a researcher assumes no cross-sectional dependence in a panel data but the panel data definitely show cross-section dependence, this incorrect assumption can cause forecasting errors and incorrect estimations. Henceforth, we use the Pesaran's CD-test (Pesaran, 2004) to indicate

[6] <http://data.worldbank.org>.
[7] <www.eia.gov>.

as to whether or not cross-sectional dependence exists within each panel time-series data. The results from the Pesaran's CD-test for testing cross-sectional dependence are posted in Table 3. Referring to the output, we have enough evidence to reject the null hypothesis of cross-section independence in favor of the alternative hypothesis of cross-section dependence for carbon emissions, real GDP, renewable energy and tourism at 1% level of significance. In short, the analyzed variables have cross-sectional dependence.

4 METHODS AND FINDINGS

Since we show the presence of issues of heterogeneity and cross-sectional dependence across the top 10 countries for CO_2 emissions, real GDP, renewable energy and tourism, we should use econometric techniques that account for these problems accordingly.

4.1 Panel unit root tests

The first generation unit root tests (e.g. ADF, IPS, LLC, Hadri unit root tests) do not account for possible existence of cross-sectional dependence in the panel data. Thus, this study employs the second generation unit root tests; namely, the CADF and the CIPS unit root tests (Pesaran, 2007), which consider both heterogeneity and cross-section dependence in identifying stationary process of the panel time-series data. The results from the CADF and CIPS unit root tests are reported in Table 4. The results suggest that although we cannot reject the null hypothesis of unit root at level values, we have enough evidence to reject the null hypothesis of unit root in favor of the alternative hypothesis of no unit root at first-differenced values. In other words, CO_2 emissions, real GDP, renewable energy and tourism contain unit root at their levels but become stationary at their first-differences. Last, we can conclude that the analyzed variables are I (1).

4.2 Panel cointegration tests

The estimation results of non-stationary variables will be economically and statistically unmeaningful and inaccurate unless they are cointegrated and thus show a long-run relationship. Accordingly, this study uses several panel cointegration tests to find whether or not carbon emissions, real GDP, renewable energy and tourism are cointegrated for the sample countries since the analyzed variables are detected to be non-stationary at levels. The Pedroni panel cointegration test (Pedroni, 1999; 2004) is carried out as the first because it is applicable for heterogeneous panels. Pedroni (1999) indicates that there are seven tests statistics shown in Table 5. According to the output posted in table 5, two out of seven tests imply the validity of a long-run relationship among carbon emissions, real GDP, renewable energy and tourism. Although the ADF-statistic has good small sample properties and is more reliable, the outcome is somewhat doubtful. Hence, we need more tests to apply to reach a robust conclusion.

The second panel cointegration test that this study uses is the Kao panel cointegration test (Kao, 1999). This test follows a similar procedure as the Pedroni test but includes cross- homogeneous coefficients on the first-stage regressors. Referring to the results from the Kao panel cointegration test in Table 6, the analyzed variables are cointegrated and have a long-run relationship since we have enough evidence to reject the null hypothesis of no cointegration in favor of the alternative hypothesis of cointegration at 5% level of significance.

Even though the Pedroni and the Kao panel cointegration tests have been frequently used in various literature including the energy-environment-growth nexus, both have drawbacks of assuming cross-section independence, and thus are considered as first generation cointegration tests. Failure of considering the presence of cross-section dependence in panel models has consequences of causing loss of power in the procedure of first generation cointegration tests. Therefore, this study also employs a second generation cointegration test; namely, the LM bootstrap panel cointegration test due to Westerlund and Edgerton (2007) in order to check the verdicts of former tests. The LM bootstrap panel cointegration test accounts for both issues of cross-sectional dependence and heterogeneity in identifying the cointegration relation among the variables, and thus is superior to the first generation cointegration tests. In addition, this test differs from the former tests in that the LM bootstrap cointegration test assumes the null hypothesis of cointegration. The results from the LM bootstrap panel cointegration test are reported in Table 7. Because there is no evidence to reject the null hypothesis of cointegration, this study indicates that CO_2 emissions, real GDP, renewable energy and tourism are cointegrated and have a long-run relationship. The conclusion is that the cointegration relation between the analyzed variables for the top 10 countries become more robust and stronger since the second generation panel cointegration test accounts for heterogeneity and cross-sectional dependence across cross sections for the analyzed variables.

4.3 Long-run estimates

Long-run estimators should produce economically and statistically meaningful, reliable and accurate coefficients on real GDP, renewable energy and tourism for CO_2 emissions since this study in the preceding section confirms that they are cointegrated and moving together in the long-run. The question on which long-run estimator(s) should be used arises from the fact there are many estimators available. This study runs the FMOLS and the DOLS because Lee (2007) suggests that the ordinary least squares (OLS) technique involves invalid standard errors due to second order asymptotic bias. In addition, the weighted DOLS estimator allows for heterogeneity in the long-run variances (Mark and Sul, 2003) and the weighted FMOLS technique is based on heterogeneous cointegrated panels (Kao and Chiang, 2000).[8] Moreover, Herrerias et al. (2013) suggest that the DOLS approach is the least sensitive one to the issue of cross-sectional dependence.

The results from the FMOLS and the DOLS estimators are posted in Table 8. Because the panel time-series data are transformed into their natural logarithm, the coefficient estimates in the table is equivalent to the elasticities of CO_2 emissions with respect to real GDP, renewable energy and tourism. Both approaches produce identical results in terms of sign and significance, but yield a bit different results in terms of magnitudes and goodness of fit of the model (R^2). More precisely, the FMOLS reports that 1% increase in real GDP and tourism raises the pollution by 0.72% and 0.17%, respectively; on the other hand, a 1% increase in renewable energy mitigates carbon emissions by 0.26%. Referring to the DOLS, 1% increases in real GDP and tourism contribute to the amount of carbon emissions by 0.64% and 0.12%, respectively; on the contrary, a 1% increase in renewable energy decreases the pollution by 0.18%. As in line with many studies including Apergis et al. (2010), Lee and Brahmasrene (2013), and Katircioglu (2014b) increase in production leads to bigger carbon emissions. In addition, the adverse of renewable energy on CO_2 emissions is consistent with that of studies including Chiu and Chang (2009), Sulaiman et al. (2013), Shafiei and Salim (2014), Lopez-Menendez et al. (2014), Al-mulali et al. (2015a), Boluk and Mert (2015), Dogan and Seker (2016), and Jebli et al. (2016). The identification of damaging effect of tourism on the environment is in line with that of Katircioglu (2014b), Katircioglu et al. (2014), Solarin (2014), De Vita et al. (2015), Sharif et al. (2017) and Paramati et al. (2017).

Tourism sector in the top 10 most-visited countries boosts the amount of carbon emissions through several links such as transportation, building of touristic facilities, and local and government services. Some policies for the sake of low emissions may be active in sample countries but clearly not sufficient to fight for the environment. One obvious solution to control for the level of carbon emissions is the adoption of the use of more renewable energy and cleaner technologies in not only overall production process but also tourism sector in particular. In this regards, touristic facilities (e.g. hotels) may build their solar panel system for producing energy to meet their needs accompanying the information that solar is a clean

[8] For more information, please refer to the references.

energy and a type of renewable energy. In addition, a bicycle-oriented tourism should be supported and adopted in replacement of motorized and environmentally unfriendly transport. Furthermore, the top 10 countries should aim to increase the share of renewable sources in energy mix, and financially support institutions, universities, researchers to work for the invention of cleaner technologies, particularly, those directly related to tourism sector. Last, policy makers should impose policies in regard to environmental protection and awareness of renewable energy and sustainable tourism.

CONCLUSIONS

This study aims to investigate the relationship of CO_2 emissions, real GDP, renewable energy and tourism for the top 10 most-visited countries for the period 1995–2011. Moreover, we also consider the validity of issues of cross-sectional dependence and heterogeneity in panel data while analyzing stationary properties, cointegration relationship and the long-run estimates. Thus, the results found in the current study are more robust and reliable as compared to those in previous studies. The findings and policy recommendations can be summarized as follow:

- By looking at the average annual growth rates of each variable, and applying the Pesaran's CD test to the panel time-series data, we detect the presence of heterogeneity and cross-sectional dependence across countries for the analyzed data.
- The CADF and the CIPS unit root tests report that the analyzed variables are not stationary at levels but become stationary at first differences.
- The LM bootstrap cointegration test shows that CO_2 emissions, real GDP, renewable energy and tourism are cointegrated and hence have a long-run relationship.
- The DOLS and FMOLS estimators indicate that increases in renewable energy lead to environmental improvements whereas increases in real GDP and tourist arrivals lead to environmental degradation in the top 10 most-visited countries.
- Regulatory policies should be introduced to increase the awareness of renewable energy and sustainable tourism.
- The use of renewable energy and the adoption of renewable energy technologies should be implemented more in production processes and tourism sector in particular.
- Bicycle-oriented tourism should be supported and adopted in replacement of motorized and environmentally-unfriendly transport.
- More projects on the development of environmentally-friendly technologies, especially those in relation with tourism sector, should be sponsored by governments.

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TABLES

Table 1 A survey of literature

Study	Case	Period	Variables	Methodology	Conclusion
Chiu and Chang (2009)	Panel of OECD countries	1996–2005	CO_2, GDP, CPI, REN	Threshold effect	GDP and REN increase CO_2 for lower threshold; REN decreases CO_2 for upper threshold
Sadorsky (2009)	Panel of G7 countries	1980–2005	REN, GDP, CO_2, P(Prices)	Breitung, IPS, LLC, ADF, PP, Pedroni cointegration, FMOLS, DOLS	GDP and CO_2 are major determinants of REN
Iwata et al. (2010)	France	1960–2003	CO_2, GDP, GDP^2, nuclear, URB, TR	PP, ARDL model	Nuclear decreases CO_2
Apergis et al. (2010)	Panel of developed and developing countries	1984–2007	CO_2, GDP, Nuclear, REN	LLC, IPS, ADF, PP, LLL cointegration	Nuclear decreases CO_2, REN and GDP increase CO_2
Bengochea and Faet (2012)	Panel of EU countries	1990–2004	REN, GDP, CO_2, P	OLS with FE and RE, FGLS	GDP and CO_2 increase REN
Sulaiman et al. (2013)	Malaysia	1980–2009	CO_2, GDP, GDP^2, REN, TR	ADF, PP, ARDL model	TR and REN decrease CO_2
Boluk and Mert (2014)	Panel of EU countries	1990–2008	CO_2, GDP, GDP^2, REN, NREN	OLS with FE	REN and NREN contribute to CO_2
Farhani and Shahbaz (2014)	Panel of MENA countries	1980–2009	CO_2, GDP, GDP^2, REN, NREN	Breitung, IPS, Pedroni cointegration, FMOLS, DOLS	REN and NREN increase CO_2;
Lopez-Menendez et al. (2014)	Panel of EU countries	1996–2010	CO2, GDP, GDP^2, GDP^3, REN	OLS with FE and RE	REN decreases CO_2
Shafiei and Salim (2014)	Panel of OECD countries	1980–2011	CO_2, GDP, GDP^2, REN, NREN, POP	ADF, PP, Breitung, Johansen cointegration, Westerlund cointegration, GMM, AMG	REN decreases CO_2; NREN increases CO_2
Baek and Pride (2014)	USA, Japan, France, Korea, Spain, Canada	1970–2007	CO_2, GDP, nuclear	DFGLS, Johansen cointegration	Nuclear decreases CO_2 in all countries, GDP decreases CO_2 in USA, Canada and France
Apergis and Payne (2014)	Panel of Central American countries	1980–2010	CO_2, REN, GDP, P	LLC, IPS, ADF, PP, non-linear panel cointegration, FMOLS	GDP, CO_2 and P increase REN
Al-Mulali et al. (2015c)	Vietnam	1982–2011	CO_2, REN, NREN, GDP, IM, EXP, CA, L	ARDL model	NREN and IMP increase CO_2; REN is insignificant
Boluk and Mert (2015)	Turkey	1961–2010	CO_2, GDP, GDP^2, REN	ADF, KPSS, ARDL model	REN decreases CO_2
Jebli and Youssef (2015)	Panel of North Africa	1971–2008	GDP, CO_2, combustible and waste (CRW)	Breitung, LLC, IPS, Pedroni cointegration, FMOLS, DOLS	CO_2 and CRW increase GDP
Al-mulali et al. (2015a)	Panel of European countries	1990–2013	CO_2, GDP, TR, URB, FD, REN by sources (wind, solar, hydro, nuclear, and CRW)	IPS, ADF, PP, Pedroni cointegration, FMOLS	Five sources of REN decrease CO_2; GDP increases CO_2

Note: IPS (Im-Pesaran-Shin test), LLC (Levin-Lin-Chu test), PP (Phillips-Perron test), ADF (Augmented Dickey-Fuller test), KPSS (Kwiatkowski–Phillips–Schmidt–Shin test), FE (Fixed effects), RE (Random effects), REN (renewable energy), NREN (non-renewable energy), FMOLS (Fully Modified Ordinary Least Squares), DOLS (Dynamic Ordinary Least Squares), TR (trade), FD (financial development), URB (Urbanization).
Source: Own construction

Table 1 A survey of literature (continuation)

Study	Case	Period	Variables	Methodology	Conclusion
Apergis and Payne (2015)	Panel of South America	1980–2010	CO_2, GDP, REN , P	ADF, PP, FMOLS	GDP, CO_2 and P increase REN
Jebli et al. (2016)	Panel of OECD countries	1980–2010	CO_2, GDP, GDP^2,REN, NREN, TR	Breitung, IPS, LLC, ADF, PP, Pedroni cointegration, FMOLS, DOLS	REN decreases CO_2, NREN increases CO_2
Lee and Brahmasrene (2013)	Panel of EU countries	1988–2009	CO_2, GDP, FD, TOUR	Breitung, IPS, LLC, ADF, PP, Johansen cointegration, OLS with FE	GDP increases CO_2, TOUR decreases CO_2
Katircioglu (2014a)	Singapore	1971–2010	CO_2, GDP, GDP^2, EGY, TOUR	Unit root by Carrion-i-Silvestre et al. (2009), Maki cointegration (Maki, 2012), DOLS	EGY increases CO_2, TOUR decreases CO_2.
Katircioglu et al. (2014)	Cyprus	1970–2009	CO_2, EGY, TOUR	KPSS, ARDL model	TOUR and EGY increase CO_2
Katircioglu (2014b)	Turkey	1960–2010	CO_2, GDP, EGY, TOUR	Zivot-Andrews unit root (Zivot and Andrews, 2002), ARDL model	TOUR, GDP and EGY increase CO_2
De-Vita et al. (2015)	Turkey	1960–2009	CO_2, GDP, GDP^2, EGY, TOUR	Unit root by Carrion-i-Silvestre et al. (2009), Maki cointegration (Maki, 2012),	TOUR and EGY increase CO_2

Note: IPS (Im-Pesaran-Shin test), LLC (Levin-Lin-Chu test), PP (Phillips-Perron test), ADF (Augmented Dickey-Fuller test), KPSS (Kwiatkowski–Phillips–Schmidt–Shin test), FE (Fixed effects), RE (Random effects), REN (renewable energy), NREN (non-renewable energy), FMOLS (Fully Modified Ordinary Least Squares), DOLS (Dynamic Ordinary Least Squares), TR (trade), FD (financial development), URB (Urbanization).
Source: Own construction

Table 2 Average annual growth of each variable 1995–2 011 (in %)

Country	TOUR	CO_2	GDP	REN
China	6.39	5.69	8.82	8.83
France	1.61	0.15	1.65	0.25
Germany	3.71	–0.88	1.15	8.89
Italy	2.13	–0.38	0.81	3.91
Russia	4.83	0.40	3.45	–0.32
Spain	2.57	0.71	2.62	8.74
Thailand	5.19	3.60	2.79	1.31
Turkey	9.30	3.44	3.61	2.71
UK	1.65	–0.56	2.3	8.36
USA	2.1	0.32	2.34	0.89

Note: The average annual growth rates are calculated by author.
Source: Own construction

Table 3 Results from cross-sectional independence test

	CO_2	GDP	REN	TOUR
CD-test	4.45**	25.91**	12.36**	22.50**
p-value	0.00	0.00	0.00	0.00

Note: ** denotes the statistical significance at 1% level. The CD-test performs the null hypothesis of cross-section independence.
Source: Own construction

Table 4 Results from panel unit root tests

	CADF		CIPS	
	Level	Δ	Level	Δ
CO_2	–1.54	(–3.14)**	(–1.72)	(–4.60)**
GDP	–1.94	(–2.99)*	(–1.35)	(–3.02)*
REN	–1.98	(–4.37)**	(–2.22)	(–4.37)**
TOUR	–1.69	(–3.32)**	(–2.02)	(–3.54)**

Note: Δ represents the first-differences. **, * denote the statistical significance at 1% level and 5% level, respectively.
Source: Own construction

Table 5 Results from Pedroni cointegration test

Common AR coefficients (within-dimension)

	Weighted statistic	p-value
Panel v-statistic	–1.52	0.93
Panel rho-statistic	2.39	0.99
Panel PP-statistic	–0.29	0.38
Panel ADF-statistic	–0.75	0.22

Individual AR coefficients (between-dimension)

	Statistic	p-value
Group rho-statistic	3.39	0.99
Group PP-statistic	–5.29**	0.00
Group ADF-statistic	–4.69**	0.00

Note: ** denotes the statistical significance at 1% level.
Source: Own construction

Table 6 Results from the Kao panel cointegration test

	t-statistics	p-value
ADF	–1.78*	0.03

Note: * denote the statistical significance at 5% level.
Source: Own construction

Table 7 Results from the LM bootstrap panel cointegration test

Tests	LM statistic	Bootstrap p-value
LM bootstrap	Feb-64	1.00

Note: The LM bootstrap test is calculated using 5 000 replications. The LM bootstrap cointegration approach tests the null hypothesis of cointegration against the alternative of no cointegration.
Source: Own construction

Table 8 Results from the panel DOLS and FMOLS

FMOLS				DOLS			
Regressors	Coeff.	t-stat	p-value	Regressors	Coeff.	t-stat	p-value
GDP	0.72**	74.01	0.00	GDP	0.64**	8.5	0.00
REN	–0.26**	–8.59	0.00	REN	–0.18**	–7.6	0.00
TOUR	0.17**	5.18	0.00	TOUR	0.12**	3.36	0.00
R^2	0.994			R^2	0.997		
Coefficient Diagnostic (Null Hypothesis: β_3=0)				Coefficient Diagnostic (Null Hypothesis: β_3=0)			
Statistic	Value	d.f.	p-value	Statistic	Value	d.f.	p-value
t-statistic	5.18	147	0.00	t-statistic	3.36	150	0.00
Chi-square	26.83	1	0.00	Chi-square	11.29	1	0.00

Note: ** denotes the statistical significance at 1%. Coefficient diagnostic test evaluates the null hypothesis that the coefficient on tourism is equal to zero. It shows that the inclusion of tourism to the model is statistically significant, and thus increases the goodness of fit of the model.
Source: Own construction

Virtues, Limits and Prospects of the National Accounts

Stanislava Hronová [1] | *University of Economics, Prague, Czech Republic*
Richard Hindls [2] | *University of Economics, Prague, Czech Republic*

Abstract

The most important issues that national accounts have to solve today are the changes brought about by the globalization and the activity of multinational corporations on economic life. In June 2017, the 16th Conference of the Association de Comptabilité Nationale was held in Paris to talk over these topical issues. The paper presents the most important discussion topics raised at the conference and reflects their importance for the further development of national accounts.

Keywords	***JEL code***
National accounts, gross domestic product, Association de Comptabilité Nationale	*E21, C82*

VIRTUES, LIMITS AND PROSPECTS OF THE NATIONAL ACCOUNTS

Globalisation, presence of multinational corporations, effort to measure other than economic phenomena, the relationship between economics, and the environment are some of the topics the national statistical offices and their experts have to encounter in the fast developing world. The 16th Conference of the Association de Comptabilité Nationale, held on the premises of the French Ministry of Economy and Finance in Paris from June 7 to 9, 2017, addressed mainly such topics. This Conference (still free of participation fees and still Francophone) provides the grounds for professional discussions and meetings with the most important representatives of not only French accounting authorities but participants from other countries as well. Every two years, it presents new topics in the discussions on the quality, possibilities and contributions of the national accounts; this year the subtitle was *Virtues, Limits and Prospects of the National Accounts*.

More than 150 experts from ten countries participated, representing statistical offices, universities, research institutes and other national and international institutions (Eurostat, International Monetary Fund, and OECD). The topics were divided into six more or less separate thematic units. The Conference was traditionally opened by Jean-Luc Tavernier, Director-General of INSEE, and Georges Haddad, President of University Paris I – Panthéon-Sorbonne. Jean-Luc Tavernier focused on a discussion concerning the problems connected with the undervaluation of the economic growth rate, overvaluation of the price

[1] Faculty of Informatics and Statistics, Department of Economic Statistics, W. Churchill Square 4, 130 67 Prague 3, Czech Republic. E-mail: hronova@vse.cz.
[2] Faculty of Informatics and Statistics, Department of Statistics and Probability, W. Churchill Square 4, 130 67 Prague 3, Czech Republic. E-mail: hindls@vse.cz.

evolution, and the localisation of intellectual wealth in the globalised world. Georges Haddad spoke about the relationship between economics and mathematics and the importance of teaching the national accounts at universities oriented on economics.

Let us recall some of the most important contributions read at the Conference, which was immensely interesting and valuable this year.

André Vanoli presented an introduction to the Conference scope in *Virtues, Limits and Prospects of National Accounts: An Introduction*; this contribution sums up the basic stages in the evolution of the national accounts and pointed out that this topic has been undergoing a latent crisis lately: on the one hand, the national accounts are expected to provide the same data as before (which has been becoming more and more difficult in certain respects); at the same time, new challenges and requirements connected with continuing sustainable development should be respected. A. Vanoli does not deem it suitable (or even possible) to modify the central framework of the national accounts; he raises a question whether satellite accounts will be sufficient to meet the ever growing requirements or if other information systems related to the national accounts will have to be developed in parallel. Claire Plateau (INSEE) presents in her contribution (*Challenges and Criticisms of National Accounts*) an evaluation of the effort aimed at not only the conditions and evolution of the national economy, but also the social and environmental aspects. In principle, the same topics have been recurring:[3] expression of the sustainable economic development, drawing on the exhaustible natural resources, and households' welfare; all of which are viewed within the framework of the national accounts. The adjusted Gross Domestic Product, or rather the Green Gross Domestic Product, is leading to a dead end, because it cannot be properly interpreted. Another option is a scoreboard with a number of indices from the given area (not only environmental, but also social). A problem of the scoreboard is its unclear character for the users if the scoreboard data is not compiled into a single index. The problem of creating the scoreboard itself is that of choosing the proper indices (they are changed rather often, and such changes may be caused by political decisions). Using one succinct index brings an additional problem of setting up the proper weights. However, the last mentioned problem goes beyond the scope of statistics. In expressing the welfare, which in itself is a subjective notion, the only way is to survey households.[4]

Didier Blanchet (INSEE) in his contribution titled *Building synthetic measures of inclusive growth and sustainability* also treats the possibilities and limits of the use and interpretation of a comprehensive index based on the scoreboard. In principle, he sums up the obstacles this approach brings and emphasises that the construction and utilisation of the scoreboard are not in the statisticians' job description. Regarding the welfare expression, he proposes a notion of an equivalent income, based on the indirect monetary expression of aspects that cannot be measured financially (health, life span, social certainties, etc.).

In the following contribution, titled *Are there any substantive changes to the existing national accounts conventions?* Didier Blanchet (INSEE) and Dominique Durant (Banque de France) address the economic basis of the national accounts as related to the notion of sustainable development. Similar to A. Vanoli, they observe that the Green Gross Domestic Product, or other modifications of GDP, is not a solution; a deeper study of sustainability in macroeconomic theory is desirable in the context of quantifying all of its aspects outside of the production area. From this viewpoint, the basic framework of the national accounts need not be modified.

In continuation of this contribution, Nicolas Canry (University of Paris-1) speaks about human resources valuation. He emphasises that discussions about sociality inequality stem from this topic, but such discussions leave aside the issue of valuating human resources. Such valuation may be based

[3] Cf. the conclusions of the Stiglitz's Commission's report <http://ec.europa.eu/eurostat/documents/118025/118123/Fitoussi+Commission+report>.

[4] Cf., e.g., <http://www.oecd.org/statistics/better-life-initiative.htm>.

on inputs, i.e., costs (on education) or outputs, i.e., income (meaning an income level generated by a certain level of education). Both of these approaches bring about problems: on the one hand, households' expenses incurred on education do not grow as quickly as one might expect on the basis of the human-resources growth concepts; on the other hand, wages cannot be used as a proper valuation of human resources (people with the same education and at the same position may be paid quite different wages, women get less than men, etc.).

A very interesting contribution, *Improving the treatment of holding gains and default losses in national accounts* by Dominique Durant (Banque de France), studies the consequences of the fact that profit and loss from possession are recorded in the national accounts outside of the data on production and income. Consequently, the description of income on the national account is different from the expectations and assumptions prevailing in financial institutions. For example, losses from the bad debts in the years of the crisis reduced the banks' ability to provide loans but the national accounts did not record the corresponding decrease in production or income. D. Durant introduced a concept of expressing the profit from possession on the national accounts as income from the services provided (for example, an increase in the price when storing seasonal products, or mediators' exchange-rate profit from securities and foreign currency). He further suggests decreasing banks' FISIM by the amount of receivables if the bank rates include losses expected from those receivables. He explains that such suggestions represent an extension of existing principles of the national accounts and are compliant with the SNA, improve the ability of the national accounts to reflect economic movements and explain the behaviour of economic subjects, especially during financial crises.

An Appreciation of Living Standards, Well-Being and Quality of Life by Jérome Accardo (INSEE) returns to the issue of measuring the standard of living, quality of life and welfare, with reference to the conclusions of the Stiglitz's Commission. In his opinion, expressing the standard of living, quality of life and welfare will not be achieved by modifying the Gross National Product, but by the effort to set up an account of a "sole" household (in the sense of one category of households characterised by age, income, social circumstances, etc.). The standard summary account of the household sector does not have the necessary ability to express the quality of life and, moreover, does not contain activities and aspects such as housework, health, habits, free time, satisfaction, happiness, etc. To set up such a "sole household" account we have to combine different data sources and solve the problem of monetary expression of activities and phenomena for which no monetary value has been established. For example, the quantity of free time can be estimated with the aid of time distribution during the whole day for selected people. But how should free time be valued? By the opportunity cost, i.e., net wages? What if the concerned person has not worked at all? Is the contribution of free time to welfare the same for an employed and (long-term) unemployed person? Can the "sole household" accounts be compared on an international level? These are just a few from among the questions J. Accardo is raising. In the end he observes that, even though his own and similar works have enriched the range of the tools for describing certain socio-economic phenomena by means of the national accounts, it must be admitted that scientists keep running up against many obstacles, in many instances insurmountable.

André Vanoli's contribution *Taking into account relationships with Nature* deals with the importance of the so-called environmental accounting. The key point of such considerations is undoubtedly a question whether environmental data can be incorporated into the basic framework of national accounts. A. Vanoli's reply is that such incorporation is impossible. In the original system of environmental accounting, SEEA[5] 1993, the Net Domestic Product was adjusted to the environmental purposes by subtracting the exhaustion of natural resources and depletion of natural assets valued by the costs

5 System of Environmental-Economic Accounting.

on their replacement or renewal. A great deal of attention was then given to discussions about the services provided by nature (with monetary valuation estimated on the basis of gains from specific ecosystems, such as coral or mangrove). A newer system, SEEA 2012, contains two different parts: the first one, Central Framework, has been accepted as an international standard by the United Nations Statistical Commission. It introduces accounting of natural resources (mineral, energy, forest, soil, biological and water – both renewable and exhaustible) normally included in the assets on the national accounts; subsequently, everything is expressed in both natural and monetary units (with methods of valuation identical with those used by the national accounts). The second part of SEEA 2012, Experimental Ecosystem Accounting, has a very ambitious goal – setting up the accounts for ecosystem services in natural and monetary units, or rather, combining the ecosystem accounts with the national (economic) accounts using those monetary units. Unfortunately, this approach leads to exaggerated expectations with regard to possible integration between the natural and national accounts within a global approach to wealth. A. Vanoli accepts that nature and economies are, to a certain extent, intertwined and certain relationships between them should be reflected. On the other hand, they are two separate entities; that is why the national economy accounts and natural ones should not be amalgamated as a whole. This topic is very much alive and remains unresolved; it is clear that it is not to be resolved by statisticians only (valuations in the area of the environment must be carried out by the respective experts, not statisticians). The national accounts are primarily tools for describing economic phenomena and their monetary values are logical. For the environment (ecosystems), A. Vanoli recommends focusing on creating an information system (not accounts), in which everything is expressed in natural units.

Claire Plateau (INSEE) returns to implementation of Stiglitz's Commission's conclusions in her contribution titled *The question of sustainable development*, mainly focusing on the issues of scoreboard indices for the sustainable development. She again pointed out that the indices to be included are influenced politically, and economic theory and statistics are not included in the process of their selection (Stiglitz's Commission's conclusions do not mention any particular ideas in this respect). Understandably, certain theoretical and even practical problems are implied. The only way to eliminate such problems is harmonisation of the indices on an international level. C. Plateau introduces a concept of monitoring the sustainable development accepted at a conference of European statisticians in 2013; this concept outlines a relationship between the basic notions and characteristics of the sustainable development and certain different policies. C. Plateau describes the rather political approach: the United Nations General Assembly accepted the goals of the sustainable development in 2015 and a scoreboard was set up based on those goals. The United Nations Statistical Commission subsequently published a call for the member states to monitor and report on a set of 232(!) indices beginning in September 2017. However, analysis of those indices has revealed that only 37% of them can be currently monitored without additional surveys and costs, 36% indices are lacking any internationally recognised definitions, and 27% of them are simply unavailable in more than a half of the member states. Again it turns out that certain policy goals must be defined with regard to the possibilities of statistics.

Keen interest was reflected in a discussion on Michael Connolly's contribution (Central Statistics Office, Ireland) *Economic Globalization and Global Production*, followed by a highly critical response by François Lequiller (OECD). M. Connolly first introduced certain conceptual problems of the national accounts in the global economy (such as the impact of globalisation on the national accounting standards, consequences of capitalisation of the research and scientific results, role of multinational companies, and intellectual property as related to a residential unit). On example of Ireland he then showed examples of problems such new aspects are capable of bringing, and how Ireland may approach their solutions. In fact, Ireland's GDP year-to-year growth rate jumped up to 26.3%(!) in 2015, while GDP had been more or less stagnating from 2009 to 2013 and the prediction for the growth rate had been at 7.8%. The reason for the high GDP growth was given by income from multinational corporations' licences, which had been administratively transferred to Ireland. The Central Statistics Office of Ireland defended its approach by pointing out compliance with the SNA 2008 principles: income from licences is added to the production index. On the basis of the "Irish peculiarity" a panel of experts was held. It observed that the Central Statistics Office's approach was correct but suggested certain solutions to be accepted within the Irish economy; in particular, the "borderlines" for the intellectual property, and the role of multinational corporations should be resolved. One of the suggestions is to measure the national economy results not by GDP but an adjusted Gross National Income (GNI*); the latter's year-to-year growth was merely 6% in Ireland in 2015. The suggested modification subtracts from the GDP depreciation of fixed assets owned by non-residents and re-invested profit of foreign corporations. This suggestion was officially confirmed by European structures because it better reflects the character of the Irish economy.

F. Lequiller's response gave a critical view of what really happened in the Irish economy, how the Irish economic results were affected, and how similar instances should be resolved within the framework of the national accounts. He admitted that the Central Statistics Office of Ireland proceeded in good faith and within the rules, but claimed that such extreme results based on administrative movements due to tax optimisation by multinational corporations may be dangerous. He showed that the industrial production in Ireland grew by 98% in 2015, and net exports by 102%, but the consumption by households grew by less than 5%, and there was no actual movement of the multinational corporations' localities or their research institutes; neither were any new jobs created in those companies. In other words, mere administrative "movement" of licences doubled the volume of Irish industrial production. Despite the presented explanations and the approval by Eurostat, such results must have stirred uneasiness even in Ireland itself. Economic growth is always connected with growing employment – and how will the Irish government respond to such an extreme growth of GDP and industrial production? What if the same multinational corporations optimise their taxes elsewhere, say, Singapore, next year? A significant decrease in the GDP and industrial production would follow without any real impacts on what was actually produced. How would the Irish government respond to such year-to-year changes?

F. Lequiller further analysed the way taken by the Central Statistics Office of Ireland (which is in compliance with SNA 2008) and possible solutions. The Office's basic argument was the fact that the "economic ownership" of capital was transferred onto the Irish branch office of a multinational corporation. Unfortunately, the definition of the economic ownership by SNA 2008 is vague; hence UNECE[6] recommends using a notion of "legal ownership". This approach is not, however, correct with respect to the national accounts, for which the economic ownership takes priority. With emphasis on the notion of the economic ownership, Irish suggestion of the GNI* must be rejected as well. According to F. Lequiller, this modification is a strange construct interfering with the importance of the economic ownership. Such and similar proposals often initiate discussions whether or not the GDP is obsolete and remains a good indicator of the economic results in the respective territory. The notion of GDP is not obsolete indeed; it is an important indicator of the economic policy, measuring the results of economic activities in a given territory. It is also related to employment rate. Globalisation and the ever-growing importance of the multinational corporations undoubtedly bring new tasks for the national statistical offices, whose effort should be focused on GDP best expressing what it should express.

It is clear that multinational corporations do not care about country borders and put their production where the wages are smallest, taxes where the tax rates are lowest, and research where the concentration of brainpower is highest. If a multinational (American) corporation moves its registration to Ireland without moving its production at the same time, the value of GDP should remain unchanged. The income from licences coming to Ireland is income from ownership, not from production (no value added was

6 United Nations Economic Commission for Europe.

created!). If GDP is a sum of values added in a given economic territory, we always have to ask where the value added was created. Labour force and physical form of capital must participate in such creation, and such participation can be clearly localised. A problem remains with intangible capital, which can be placed anywhere.

The problem that arose in Ireland has a general character; F. Lequiller suggests three possible ways for resolving it:

1. Take the income from licences as property income (not income from production); this was valid in the old SNA 1968 and, essentially, complies with the principles of business accounting.

2. Localise the intangible capital where it was created, i.e., place the licences where the scientific centres are active. The production corresponding to the income from licences will be deemed production by those scientific centres, and the profit will be deemed income from ownership in the country to which it is coming and in which the tax on it is actually paid. This operation will affect the Gross National Income (GNI), but not he Gross Domestic Product.

3. Formulary apportionment of multinational corporations' profit according to the country in which the labour force participating in creating it is located – in reality, multinational corporations move the profit to a locality with the lowest taxation rate. This solution has already been applied to regional accounts when estimating the regional gross domestic products.

In F. Lequiller's opinion, any of these solutions can be implemented. He recalls that, within the BEPS programme,[7] multinational companies will have to consolidate their accounts according to the country. National statistical offices should have such data at their disposal; it would be a real shame if they did not try to make use of such data. The approach taken by the Central Statistics Office of Ireland unfortunately goes against the BESP programme's goals – to make multinational companies pay their taxes where the profit is really created (not in a locality to which it was transferred administratively). This is an expression of a common idea of tax experts and experts in the area of national accounts, which idea stems from an economic basis.

In conclusion of his very interesting contribution, F. Lequiller reiterated the question whether or not the concept of the Gross or Net Domestic Product should be changed. He immediately replied to the question himself: certainly not. The Gross – and also Net – Domestic Product follows a methodology which remains valid regardless of the changes in the corporations‘ organisation. At the same time, he emphasised that the concept of the national accounts must be focused on economic, not legal aspects (in particular with regard to economic ownership), avoiding influence by formal organisation of multinational corporations. Effort should be made to obtain multinational corporations‘ data consolidated by territory within the BEPS project. A working group should be set up for this purpose by OECD.

F. Lequiller‘s contribution was very interesting; its content goes beyond the scope of criticism of the Irish approach. It has outlined new challenges for the national accounts‘ concepts brought about by development in society. He nevertheless underlined that the fundamental idea on which the national accounts are based remains valid and the framework of the national accounts need not be changed.

The closing part of the conference was (non-traditionally) focused on problems in teaching national accounting at French economically-oriented universities. This subject is a usual component of the economic curriculum at various degrees of studies. Unfortunately, often it is presented in a descriptive way without deeper statistical insight and without seminars (only lectures). French colleagues do not deem this teaching method optimal, because it does not enable students to understand the national accounts

[7] Base Erosion and Profit Shifting Action Plan is an OECD programme focused on suppressing strategies whose purpose is to minimise the tax duty via transferring profit to low- or no-tax locations. It was signed by 50 countries.

as data resources and tools of analysis. One of the problems of teaching national accounting at the Bachelor Degree stage is, in our French colleagues‘ opinion, the insufficient knowledge of mathematics obtained at secondary schools.

The 16th Conference of the Association de Comptabilité Nationale held in Paris this year was focused on the key topics in the evolution of the national accounts, providing the participants with new ideas and topics for discussions. Thanks should be expressed to French members of the Association (outstanding personalities of European national accounting) for the excellent preparation and organisation of the event. Support by French Statistical Office INSEE should also be acknowledged.

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Measuring Regional Price Levels in the Czech Republic

Jiří Mrázek [1] | *Czech Statistical Office, Prague, Czech Republic*

Abstract

Regional price level comparison in the Czech Republic was a topic not touched until recently. Then a team of researchers from the University of Economics in Prague pioneered the way for NUTS 3 regions for the year 2007 and repeatedly in 2012. Another step to establish more reliable results came together with a research team of the Technical University in Liberec, which brought qualitatively new and well advanced approaches. This article aims to discuss some aspects of the two attempts to establish regional price level compariso in the Czech Republic.[2]

Keywords

Purchasing power parity, regional price level

JEL code

E31

INTRODUCTION

A price level is a relative quantity existing only in relation to another price level. In economic statistics it is usually the aggregate of a set of prices across a specific spectrum of goods and services produced or consumed, which is compared to an analogous aggregate of prices. A time comparison within the same space results in a price index like inflation, while the comparison over space at the same time gives Price Level Index (PLI). The mathematical essence of the two is the same, yet the practical issues differ.

Inflation is one of the most watched economic indicators in the world. So the corresponding resources for data compilation and processing are superior to those for measuring PLI. The latter is mostly used in the international Purchasing Power Parities (PPP) project. The object of the programme is to compare the price and volume levels of GDP and its component expenditures across participating countries. Before such comparisons can be made, it is necessary to express the volumes, which are in national currencies and valued at national price levels, in a common currency and at a uniform price level. Purchasing power parity (PPP) is used as a conversion bridge here; then the ratio of PPP to exchange rates gives PLI.

PLI is mostly connected with international comparisons, but the concept can also be applied for interregional comparisons within one country. Although Regional Price Level Index (RPLI) provides very valuable and interesting information, it is not yet part of any official statistical output of the Czech Statistical Office or Eurostat. But it is worth mentioning that the international PPP project requires the participating country to provide so called spatial coefficients in order to adjust the average prices obtained in the location of the price survey (usually the capital of the country) to the national average prices.

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[1] Czech Statistical Office, Na padesátém 81, 100 82 Prague 10, Czech Republic. E-mail: jiri.mrazek@czso.cz.

[2] Editor's note: another discussion paper comparing these two articles focused on regional price levels (both published in *Statistika: Statistics and Economy Journal* in 2016) was published in *Statistika* No. 2/2017.

The spatial coefficients are nothing else than a special case of RPLI for the capital. To fulfil requirements of the PPP project the Czech Statistical Office uses rough estimations of the spatial coefficients. They are derived from Consumer Price Index (CPI) collected dataset at the specific detailed level set by the project.

Recently two interesting studies focused on the Czech RPLI emerged. The first one comes from a team working at the Czech Statistical Office and the University of Economics in Prague (UEP), see Kramulová, Musil, Zeman, Michlová (2016). The second one was prepared by a research team of the Technical University in Liberec (TUL), see Kocourek, Šimanová, Šmída (2016). The main source of the price information used in both of the studies came from the CPI database of the Czech Statistical Office.

Next chapter provides some basic evaluation of the two mentioned studies and some difficulties encountered when attempting to measure RPLI. RPLI in this article is a vector of relative price levels for considered regions, where the relative price level for the Czech Republic equals 100%. Only RPLI for the total household final consumption expenditures is considered, breakdowns for detailed COICOP categories are not dealt with.

COMPARISON OF THE TWO PROJECTS

UEP provides comparison of price levels in 2012, an update of similar results from 2007. The regional prices were obtained as averages over NUTS 3 of CPI basket representative items. The study mentions items with centrally collected prices, like energy, transport and internet prices, and their increased share in the basket. These centrally collected data may cause some difficulty or ambiguity to derive RPLI. UEP mentions imputation of some missing prices by an average of the other regions. It should be noted that this method is suitable only for regions where such a procedure can be expected to provide plausible outcomes. Especially for services, highly influenced by the local average wages, a more specific approach is often desirable. Yet, UEP presents a very efficient attempt to establish pilot RPLI in the Czech Republic.

To stay a little bit longer at the theme of the centrally collected prices, we can distinguish several cases. Some prices, like petrol, are strongly regional, but available data sources covering the whole country cannot be regionalized. On the other hand, internet prices are the same for the whole country and do not pose any problem for RPLI calculation. Railway transport is more difficult. It may be considered to have a single country price level, unless we want to raise the local differences in the quality of the service (accessibility, frequency, quality and efficiency of railway connections relevant to a given place or region).

There are difficult cases like an entry ticket to a local football match or a hotel accommodation. The latter is a complex case, as it is usually not only the quality-of-the-shelter component, but also the location, which plays an important role in the price determination. Here a question arises, how to establish a quality factor related to the location and how to abstract from this specific quality in order to obtain pure price level. Moreover, the hotel price is faced mostly by out-of-region customers, which brings about new issues with solutions depending on the intended use of the final RPLI results.

The second study (TUL) provides more advanced methodology, presenting real academic research of the topic, but at the same time also considerably more laborious. The ultimate difference between the two approaches consists in treatment of the quality differences for the compared items. The TUL approach thoroughly differentiates individual narrowly defined varieties within broader CPI representative item description. For some items this may play substantial role.

To explain the issue closer, let us have a simple example of a representative CPI item – white yoghurt, low fat content, standardized weight 150 g. Prices in this example range approximately from 4 CZK to 11 CZK over the Czech Republic (March 2017). In the next step, a further breakdown into individual varieties with different qualities as perceived by consumers is needed, distinguishing producer and mark, like "Olma Klasik, 2.7% fat". It is exactly this level where PPP methodology compares "like with like". Then, we come to the conclusion that the total price variation within representative item is mostly due to the variation between these individual varieties. The average prices of varieties range approximately from 5 CZK to 10 CZK and the dispersion of prices within the specified varieties themselves is quite small and often due only to random discounts.

CPI perception is similar to that of PPP. For CPI monthly change of the white yoghurt price index, it is necessary to keep the same composition of the varieties used for the calculation of the average price in numerator and denominator. This is achieved by a selection of suitable subsets of actually collected data.

The utilization of this detailed information and the price comparison at the level of variety in TUL project is its most valuable aspect. It presents very significant exploitation of the information contained in the CPI collected database. It is no surprise that related TUL data processing was perhaps the most challenging part of the whole project. The Czech Statistical Office also came to the conclusion to use the detailed TUL results as the best estimates for PPP space coefficients in future.

Another ambition of TUL was an attempt to capture price differences between individual districts (former NUTS 4) in spite of the data collection being limited only to 35 of them, presenting barely half of their total number. While estimated RPLI for the districts without original price data may not be always reliable, the aggregated RPLI for the hierarchically higher NUTS 3 is likely to be more accurate than those from the UEP calculation.

Next table shows comparison of the results of RPLI for NUTS 3. Usual alphabetical ordering makes statistics less illustrative than sorting by some important indicator. It suggests itself to use TUL RPLI results as preferred ones, but even they do not seem fully plausible (e.g. high RPLI in Zlínský kraj). Natural expectation says that RPLI should partly depend on an average consumer's income. Therefore the ordering of the Table 1 was determined by an artificial RPLI obtained as a result of the weighted geometric mean of RPLI from TUL with weight of 0.5, RPLI of UEP with weight of 0.25 and the average wage relative level in the respective NUTS 3 in 2012 (source: Czech Statistical Office) with weight of 0.25.

Table 1 RPLI according to UEP and TUL – comparison of results

Region	RPLI % 2012 UEP	RPLI % 2011–13 TUL	Difference p.p.	Average wages % 2012
Hlavní město Praha	122.3	117.1	5.2	135.8
Středočeský kraj	106.3	104.8	1.5	99.6
Jihomoravský kraj	100.6	103.0	–2.4	96.6
Liberecký kraj	100.5	101.4	–0.9	91.6
Plzeňský kraj	100.0	100.1	–0.1	93.3
Královéhradecký kraj	96.7	101.2	–4.5	89.8
Moravskoslezský kraj	97.2	98.9	–1.7	93.5
Zlínský kraj	97.5	101.5	–4.0	86.5
Jihočeský kraj	99.0	99.7	–0.7	87.9
Pardubický kraj	96.2	100.1	–3.9	88.7
Olomoucký kraj	96.9	99.2	–2.3	87.4
Ústecký kraj	96.7	97.4	–0.7	90.7
Kraj Vysočina	93.1	97.7	–4.6	89.4
Karlovarský kraj	99.9	97.7	2.2	83.2
Czech Republic	**100.0**	**100.0**		**100.0**

Source: UEP, TUL, Czech Statistical Office, own calculations

UEP RPLI shows a standard deviation (over individual NUTS 3) of 3.0%, TUL only 2.1%, while for wages it achieves 4.2% (outlying Prague is always excluded). All the three statistics are quite well correlated, as expected. Some part of the variance of RPLI over NUTS 3 in both UEP and TUL can be explained by the fact that the source database for calculation was not primarily designed for the space comparisons and is neither suitably balanced nor sufficiently large for that purpose.

UEP has evidently larger variance. The most likely cause is that in regions with higher wages and consequently higher purchasing power one can expect a composition of varieties within the given representative item shifted towards relatively higher quality products. It suggests that UEP results are therefore overvalued in higher tail and symmetrically undervalued in the lower tail.

Differences between results of UEP and TUL for some NUTS 3 are quite high (see Table 1), which suggest low accuracy of results generally. Obviously, the information beyond decimal point has no meaning.

CONCLUSION

Whatever estimations of RPLI in the Czech Republic are highly appreciated, as official price statistics is concentrated on inflation measurement and resources to price comparison in space dimension are relatively poor. Comparison of price levels over space also reveals many practical problems, hardly perceptible in the domain of the time comparison (price indices). One of them, not mentioned here, is a big sensitivity to the estimations of the relative weights used for the comparisons.

Typically, the quality of achieved outcome is a reward for the corresponding effort. In that sense TUL results are superior. We can take them as a good reference point, perhaps with an exception of Zlínský kraj, where it is more plausible simply to impute relative price level of 100%.

Based on TUL results, it is now interesting to make own, a very rough estimate of RPLI just by assuming the price level being equal to 100% for all the goods in the consumer market for all NUTS 3 (with an assigned relative weight of the goods on total of 60%), and the price level of all the services being equal to the average wage relative level in the respective NUTS 3 (with a relative weight of the services on total of 40%). Then, surprisingly, this somewhat unsophisticated estimate of RPLI would be closer to our reference (TUL) than UEP, where the distance of the RPLI vectors is measured by the sum of the absolute differences of their elements.

What does this finding mean? Simply, the Czech Republic is a small country in terms of economic geography. The price level of consumer goods is rather homogeneous and the variations of RPLI are explicable by the conditions on the local labour market.

Finally, it remains to answer the question whether it makes sense to combine the results of UEP and TUL together. Some kind of such a construction has been proposed in this article to arrange the regions according to their supposed price levels within the Table 1 (statistics not explicitly shown). But, as both TUL and UEP suffer from real data insufficiency and, at the same time, TUL presents better usage of the available data, it is more appropriate to combine only TUL results with the average wage levels. The reason for this approach is that the wage statistics cover the whole region considered and to some extent complement the deficiencies in the full coverage by the price quotations used in both the studies.

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Iteratively Reweighted Least Squares Algorithm for Sparse Principal Component Analysis with Application to Voting Records

Tomáš Masák [1] | *Charles University, Prague, Czech Republic*

Abstract

Principal component analysis (PCA) is a popular dimensionality reduction and data visualization method. Sparse PCA (SPCA) is its extensively studied and NP-hard-to-solve modifcation. In the past decade, many diferent algorithms were proposed to perform SPCA. We build upon the work of Zou et al. (2006) who recast the SPCA problem into the regression framework and proposed to induce sparsity with the l_1 penalty. Instead, we propose to drop the l_1 penalty and promote sparsity by re-weighting the l_2-norm. Our algorithm thus consists mainly of solving weighted ridge regression problems. We show that the algorithm basically attempts to fnd a solution to a penalized least squares problem with a non-convex penalty that resembles the l_0-norm more closely. We also apply the algorithm to analyze the voting records of the Chamber of Deputies of the Parliament of the Czech Republic. We show not only why the SPCA is more appropriate to analyze this type of data, but we also discuss whether the variable selection property can be utilized as an additional piece of information, for example to create voting calculators automatically.

Keywords	***JEL code***
Sparse principal components, IRLS, penalized least squares, roll-calldata	*C38, C55*

INTRODUCTION

Principal component analysis (PCA) is a popular dimensionality reduction technique, which has been successfully applied in virtually all areas of science where multivariate data are encountered. PCA is commonly used in statistics, machine learning, signal processing, genetics, fnance, or meteorology, just to name a few. PCA projects data onto a lower dimensional subspace spanned by the leading eigenvectors of the sample covariance matrix. Sparsity, i.e. restriction on the l_0-norm of the eigenvectors, is often assumed to cope with the issues of poor interpretability and inconsistency of classical PCA in the case of high-dimensional data. Due to the immense growth of data in many disciplines, sparse PCA remains to be area of active research for more than adecade now.

[1] Faculty of Mathematics and Physics, Sokolovská 83, 186 75 Prague 8, Czech Republic. E-mail: tom.masak@gmail.com.

The contribution of our paper is threefold: proposition of a non-convex penalty to be employed in the regression-based PCA setup of Zou et al. (2006), derivation of an iterative algorithm for the resulting non-convex problem together with the proof of its convergence, and application of the new algorithm on a detailed analysis of real-world data. Thus, the paper has three fundamental parts, as outlined in the forthcoming paragraphs.

After a review of (PCA) and the notion of sparsity (Section 1.2), we analyze penalized least squares problem with a non-convex penalty in Section 1.3. We relax the problem introducing additional variables and propose an alternating minimization algorithm to optimize it. The algorithm solves a simple convex optimization problem in every step while the overall algorithm attempts to solve a non-convex problem. We show numerical convergence of the algorithm to a stationary point of the original objective function. Proofs themselves are postponed to the Appendix.

Second, in Section 1.4, we review the S-PCA algorithm of Zou et al. (2006), which possesses several properties that make it attractive in practice, and introduce two of its modifications. First, we describe a very simple modification based on idea of the adaptive lasso (Zou, 2006). This simple modification significantly improves the numerical performance of the S-PCA algorithm, but it also lacks some of the favorable properties of S-PCA. Second, we propose a modification of S-PCA adopting the non-convex penalty function analyzed in Section 1.3. This leads to a new algorithm for sparse PCA. Even though the algorithm is heuristic to some extent, it preserves the favorable properties of the S-PCA algorithm while showing a superior performance.

Finally, we display the differences in performance of the considered algorithms on both real and simulated data in Section 2. We carefully analyze the real data: the voting records of the Chamber of Deputies of the Parliament of the Czech Republic. We examine thoroughly why SPCA is more appropriate than PCA for these data, and what is the value added by the sparsity assumption. The newly proposed algorithm is shown to provide the most easily interpretable results.

The paper is concluded with final remarks in the Discussion.

1 METHODS

1.1 Notation

For a matrix $\mathbf{A} \in \mathbb{R}^{n \times p}$ we denote $\mathbf{a}^{(i)}$ its i-th row, $\mathbf{a}_j$ its j-th column, and a_{ij} its element on the position (i, j). A vector $\mathbf{v} \in \mathbb{R}^p$ consists of entries $v_1, \ldots, v_p$. We also write $\mathbf{v} = [v_i]_{i=1}^p$. All vectors are column-wise, and $\mathbf{I}_k$ stands for the $k \times k$ identity matrix.

For a function $f(\mathbf{x}, \mathbf{y})$ of two vector variables $\mathbf{x} \in \mathbb{R}^p$, $\mathbf{y} \in \mathbb{R}^q$, we denote $\nabla_{\boldsymbol{x}} f(\boldsymbol{x}, \boldsymbol{y})$ the vector of partial derivatives $[\frac{\partial}{\partial x_i} f(\boldsymbol{x}, \boldsymbol{y})]_{i=1}^p$. We abuse the notation slightly when by $\nabla_{\boldsymbol{x}} f(\boldsymbol{x}_0, \boldsymbol{y}_0)$ we denote the part of the gradient evaluated in $(\boldsymbol{x}_0, \boldsymbol{y}_0) \in \mathbb{R}^{p \times q}$.

Finally, note that we abbreviate "sparse principal component analysis" as SPCA while S-PCA refers to a specific algorithm.

1.2 Principal component analysis and sparsity

Suppose we work with a (column-wise) centered data matrix $\mathbf{X} \in \mathbb{R}^{n \times p}$ with the p-dimen-sional observations $\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)}$ in its rows. The objective of PCA is to find linear combinations of the original variables (columns of $\mathbf{X}$) that are uncorrelated and subsequently explain as much variance in the data as possible. Keeping only the first few combinations corresponds to the projection of the original data onto a lower dimensional subspace (called *principal subspace*) best approximating the data in the sum-of-squares sense. One of many equivalent formulations how to find the K-dimensional principal subspace is

$$\underset{\mathbf{X}_\star \in \mathbb{R}^{\mathbf{n}\times\mathbf{p}}}{\arg\min} \quad \|\mathbf{X} - \mathbf{X}_\star\|_F^2 \quad \text{s.t.} \quad \text{rank}(\mathbf{X}_\star) = K, \tag{1}$$

where $\|\cdot\|_F$ is the Frobenius norm, i.e for $\mathbf{A} \in \mathbb{R}^{n\times p}$ it is $\|\mathbf{A}\|_F = \left(\sum_{i=1}^n \sum_{j=1}^p a_{ij}^2\right)^{\frac{1}{2}}$.

Although this problem is non-convex due to the rank constraint, it can be solved explicitly via the singular value decomposition (SVD) of the data matrix $\mathbf{X}$, i.e. $\mathbf{X} = \mathbf{UDV}^\top = \sum_{i=1}^m d_i \mathbf{u}_i \mathbf{v}_i^\top$, where $m = \min(n, p)$, $\mathbf{U} \in \mathbb{R}^{n\times m}$ and $\mathbf{V} \in \mathbb{R}^{p\times m}$ are orthonormal matrices, and $\mathbf{D} \in \mathbb{R}^{m\times m}$ is a diagonal matrix with non-increasing entries $d_1 \geq \ldots d_m \geq 0$ on the diagonal. It follows from the Eckart-Young theorem (Eckart and Young, 1936) that a solution to (1) is obtained by truncating the SVD of $\mathbf{X}$, i.e. $\widehat{\mathbf{X}}_\star = \sum_{i=1}^K d_i \mathbf{u}_i \mathbf{v}_i^\top$.

Recall that $\mathbf{v}_1, \ldots, \mathbf{v}_K$ are called *loadings*, and they form an orthonormal basis of the principal subspace. They also correspond to the directions capturing subsequently the majority of variance in the data, or equivalently, they are eigenvectors of the sample covariance matrix $\widehat{\mathbf{\Sigma}} = \frac{1}{n}\mathbf{X}^\top\mathbf{X}$. Thus $\mathbf{v}_1, \ldots, \mathbf{v}_K$ are estimators of the *population loadings* – the eigenvectors of the population covariance matrix $\mathbf{\Sigma}$. Vectors $\mathbf{u}_1, \ldots, \mathbf{u}_K$ are projections of the data onto the directions of loadings and are usually called *principal components* (PCs).

Suppose we are interested in the first K principal components. Problem (1) can be rewritten in terms of finding an orthonormal basis $\mathbf{V}$ of the principal subspace as

$$\underset{\mathbf{V}_\star \in \mathbb{R}^{p\times K}}{\arg\min} \quad \sum_{i=1}^n \|\mathbf{x}^{(i)} - \mathbf{V}_\star \mathbf{V}_\star^\top \mathbf{x}^{(i)}\|_2^2 \quad \text{s.t.} \quad \mathbf{V}_\star^\top \mathbf{V}_\star = \mathbf{I}_K. \tag{2}$$

This equivalence follows by rewriting $\sum_{i=1}^n \|\mathbf{x}^{(i)} - \mathbf{V}_\star \mathbf{V}_\star^\top \mathbf{x}^{(i)}\|_2^2 = \|\mathbf{X} - \mathbf{X}\mathbf{V}_\star\mathbf{V}_\star^\top\|_F^2$ and considering the SVD of $\mathbf{X}$. Understanding PCA from the perspective of finding a basis of the lower-dimensional subspace best fitting the data points $\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)} \in \mathbb{R}^p$ will become handy in the upcoming sections.

PCA is also connected to *factor analysis*, even though the objectives of PCA and factor analysis often differ (see Jolliffe, 2002, Section 7.3, for a discussion). Suppose that data $\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)}$ are generated from the following factor model:

$$\mathbf{x} = \mathbf{W}\mathbf{f} + \boldsymbol{\epsilon}, \tag{3}$$

where $\mathbf{W} \in \mathbb{R}^{p\times K}$ is a fixed unknown matrix, $\mathbf{f} \in \mathbb{R}^K$ is a random vector (of so-called *factors*), and $\boldsymbol{\epsilon} \in \mathbb{R}^p$ is a random error term, such that $\mathbb{E}\mathbf{f} = \mathbf{0} = \mathbb{E}\boldsymbol{\epsilon}$, $\boldsymbol{\epsilon}$ and $\mathbf{f}$ are independent, $\text{Var}(\mathbf{f}) = \mathbf{I}_K$, and $\text{Var}(\boldsymbol{\epsilon}) = \sigma^2\mathbf{I}_p$ for some $\sigma^2 > 0$. Under this model, it holds $\mathbf{\Sigma} = \text{Var}(\mathbf{x}) = \mathbf{W}\mathbf{W}^\top + \sigma^2\mathbf{I}_p$. Hence, population loadings form an orthonormal basis of the column space of $\mathbf{W}$. Thus PCA can be regarded as an attempt to estimate an orthonormal basis of the subspace spanned by $\mathbf{W}$, i.e. the subspace to which the factors are projected. In other words, PCA can be loosely thought of as describing some hidden factors in the domain of the observed variables.

PCA works well and the theory behind it is well understood in the traditional setting with a fixed, low number of variables p and a higher number of observations n. Nonetheless, PCA encounters both theoretical and practical difficulties when p is comparable to or larger than the sample size n. At various levels of generality, a set of papers showed that eigenvectors of the sample covariance matrix are not consistent estimators of their population counterparts in the joint limit case $p, n \to \infty$, $p/n \to \tau \geq 0$ (e.g. Paul, 2007, Nadler, 2008, and references therein). Essentially, in the high-dimensional case when $p > n$, it is hopeless for loadings to be credible estimates of their population counterparts. Moreover, the interpretability of PCA is problematic even for a moderate p because all the coefficients of the loadings are nonzero.

To regain the lost consistency of PCA in the high-dimensional setup, an additional structure needs to be assumed. The most widely adopted structure assumption is *sparsity* of the loadings $\mathbf{v}_1, \ldots, \mathbf{v}_K$. A vector $\mathbf{v}$ is s-sparse if $\|\mathbf{v}\|_0 \leq s$, where $\|\mathbf{v}\|_0 = \{\#i; v_i \neq 0\}$ is the *cardinality* of $\mathbf{v}$, i.e. the number of nonzero coefficients of $\mathbf{v}$ or the so-called ℓ_0-norm.[1] Under the sparsity assumption, $\mathbf{v}_i$ is presumed to be s_i-sparse for some $s_i < p$. If the values s_i are sufficiently small, it is possible to estimate the loadings consistently even in the high-dimensional case. Sparsity also greatly improves the interpretability of PCA.

However, while standard PCA is an easy task equivalent to SVD of the data matrix, sparse PCA (SPCA) constraining the numbers of nonzero loadings coefficients is known to be an NP-hard problem. SPCA has been a topic of an active research for more then a decade, and many different algorithms for SPCA have been developed. We build upon the procedure proposed by Zou et al. (2006), which enjoys several favorable properties, especially from the practical point of view. Before introducing this algorithm, we discuss a non-convex penalized least squares problem to be utilized later.

1.3 Non-convex sum-of-sguares penalization

The following penalized least squares problem is extensively studied in the literature:

$$\operatorname*{arg\,min}_{\boldsymbol{\beta}\in\mathbb{R}^p} \quad \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \lambda\rho(\boldsymbol{\beta}), \tag{4}$$

where $\lambda > 0$ is a tuning parameter, $\mathbf{Y} \in \mathbb{R}^n$, $\mathbf{X} \in \mathbb{R}^{n\times p}$, and $\rho : \mathbb{R}^p \to \mathbb{R}$ is a penalty function. The most prevalent choices of the penalty function are $\rho(\cdot) \equiv \|\cdot\|_2$, which leads to the *ridge regression* (Hoerl and Kennard, 1970), or $\rho(\cdot) \equiv \|\cdot\|_1$, which leads to the *lasso* (Tibshirani, 1996). Optimally, we would like to solve the problem with $\rho(\cdot) \equiv \|\cdot\|_0$ but this leads to an NP-hard problem. The lasso is often used as a convex relaxation to the ℓ_0-norm.

In this paper, we work with the following penalty:

$$\rho_l(\boldsymbol{\beta}|\delta) = \sum_{j=1}^{p} \log(\beta_j^2 + \delta), \tag{5}$$

where $\delta > 0$ is a regularizing parameter, because it resembles more closely the ℓ_0-norm as shown in Figure 1 and discussed in Example 1. However, the problem

$$\operatorname*{arg\,min}_{\boldsymbol{\beta}\in\mathbb{R}^p} \quad f(\boldsymbol{\beta}|\delta) \quad \equiv \quad \operatorname*{arg\,min}_{\boldsymbol{\beta}\in\mathbb{R}^p} \quad \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{j=1}^{p} \log(\beta_j^2 + \delta). \tag{6}$$

is still non-convex and thus hard to solve. Therefore, we introduce auxiliary variables and propose an alternating minimization algorithm enabling to solve the relaxed problem.

We define a surrogate function

$$g(\boldsymbol{\beta}, \boldsymbol{w}|\delta) = \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{j=1}^{p} w_j\beta_j^2 + \lambda \sum_{j=1}^{p} \big[w_j\delta - \log(w_j) - 1\big], \tag{7}$$

[1] We note here that $\|\cdot\|_0$ is actually not a norm, but the term is traditional due to Donoho (2006).

where $w_j > 0$ for $j = 1, \ldots, p$, and propose to solve (6) by minimizing g over one of the variables $\boldsymbol{\beta}$ and $\mathbf{w}$, keeping the other variable fixed. Starting from some $\mathbf{w}^{(0)}$ (for example $w_j^{(0)} = 1$, $j = 1, \ldots, p$), this leads to the following iterative scheme:

$$\boldsymbol{\beta}^{(l+1)} = \underset{\boldsymbol{\beta}\in\mathbb{R}^p}{\arg\min}\ g(\boldsymbol{\beta}, \boldsymbol{w}^{(l)}|\delta) \tag{8}$$

$$\boldsymbol{w}^{(l+1)} = \underset{\boldsymbol{\beta}\in\mathbb{R}^p}{\arg\min}\ g(\boldsymbol{\beta}^{(l+1)}, \boldsymbol{w}|\delta) \tag{9}$$

Lemma 1. For f of (6) and its surrogate g of (7), the following properties hold:

(a) $\forall\delta > 0, \forall\boldsymbol{\beta} \in \mathbb{R}^p$:

$$f(\boldsymbol{\beta}|\delta) = \min_{\boldsymbol{w}\in\mathbb{R}^p} g(\boldsymbol{\beta}, \boldsymbol{w}|\delta).$$

(b) $\forall l = 1, 2, \ldots \forall\delta > 0 : \boldsymbol{\beta}^{(l+1)}$ of (8) is the unique solution to the weighted ridge regression problem

$$\underset{\boldsymbol{\beta}\in\mathbb{R}^p}{\arg\min}\ \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \lambda \sum_{j=1}^{p} w_j^{(l)} \beta_j^2.$$

(c) $\forall l = 1, 2, \ldots \forall\delta > 0 : \boldsymbol{w}^{(l+1)}$ of (9) is given by

$$w_j^{(l+1)} = \frac{1}{(\beta_j^{(l+1)})^2 + \delta}, \quad j = 1, \ldots, p. \tag{10}$$

The proof consists of simple calculations, and hence we omit it.

Example 1. Let $\mathbf{X} \in \mathbb{R}^{n\times 2}$, $n \in \mathbb{N}$, $\mathbf{Y} \in \mathbb{R}^n$ and suppose $\mathbf{Y} \approx \mathbf{X}\boldsymbol{\beta}$ for an unknown $\boldsymbol{\beta} \in \mathbb{R}^2$. Thus we are in a situation with two regressors. Suppose we want to find an estimate of $\boldsymbol{\beta}$ as a minizimizer of the sum of squares of residuals but having cardinality at most one, i.e. we wish to solve

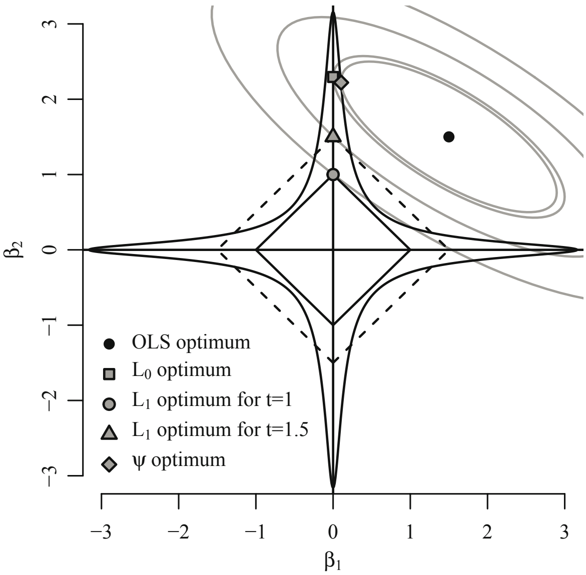
$$\widehat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}\in\mathbb{R}^2}{\arg\min}\ \|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 \quad \text{s.t.} \quad \rho(\boldsymbol{\beta}) \le t, \tag{11}$$

for $\rho(\cdot) \equiv \|\cdot\|_0$ and $t = 1$. Note that the permissible set $\{\boldsymbol{\beta} \in \mathbb{R}^2, \|\boldsymbol{\beta}\|_0 \le 1\}$ corresponds to the whole coordinate axes. Thus, in Figure 1, the estimate corresponds to the point of contact of the coordinate axes with the ellipse (representing a sum-of-squares level set) closest to the ordinary least squares (OLS) estimate. In Figure 1, also solutions to (11) for $\rho(\cdot) \equiv \|\cdot\|_1$ (i.e. lasso estimates) with $t = 1$ and $t = 1.5$ are given. Note that the lasso estimate for $t = 1.5$ is the closest ℓ_1-penalized approximation to the ℓ_0-penalized solution one can get. Increasing t any further results in losing the sparsity and the lasso estimate actually grows away from the desired point closer to the OLS estimate. On the other hand, the solution to (11) for $\rho(\cdot) \equiv \rho_l(\cdot|\delta)$ with appropriate t and δ is able to approximate the ℓ_0 solution to arbitrary precision. In figure 1, the solution with $\delta = 10^{-2}$ and $t = \frac{1}{2}\log(\delta)$ is shown. Note that for a fixed t, it holds for $\delta_1 > \delta_2$ that

$$\Big\{\boldsymbol{\beta} \in \mathbb{R}^p; \sum_{j=1}^{p} \log(\beta_j^2 + \delta_1) \le t\Big\} \subset \Big\{\boldsymbol{\beta} \in \mathbb{R}^p; \sum_{j=1}^{p} \log(\beta_j^2 + \delta_2) \le t\Big\},$$

thus it is necessary to vary both t and δ to approximate the ℓ_0-norm well. This translates to the necessity to both decrease δ and increase λ accordingly in the unconstrained formulation of the problem (6).

Figure 1 Geometry of the ℓ_0, ℓ_1, and p_l penalties in the case of two regressors. Solutions to the constrained problem (11) with various penalties along with the unpenalized OLS solution are shown



Source: Own construction

Lemma 1 (a) shows that successful minimization of g results in the minimization of f. It follows from Lemma 1 (b) that the alternating minimization scheme of (8) and (9) consists of a sequence of weighted ridge regression problems, thus it is in fact an instance of iteratively reweighted least squares (IRLS) algorithm. From Lemma 1 (a) it also follows the algorithm is a majorization-minimization (MM) algorithm (Lange et al., 2000).

Lemma 2. Let $w_j^{(0)} > 0$, $j = 1, \ldots, p$, let $\delta \geq 0$, and let $\{\boldsymbol{\beta}^{(l)}\}_{l=1}^{\infty}$ be the sequence obtained by solving (8) and (9) for $l = 1, 2, \ldots$. Then $\|\boldsymbol{\beta}^{(l+1)} - \boldsymbol{\beta}^{(l)}\|_2 \to 0$ and every limit point of the sequence $\{\boldsymbol{\beta}^{(l)}\}_{l=1}^{\infty}$ is a stationary point of $f(\boldsymbol{\beta}|\delta)$.

The previous lemma can be proved easily by following the general theory of MM algorithms (Lange et al., 2000). We do not show this here since the lemma also follows from more general Proposition 3, which we prove in detail. Nonetheless, Lemma 2 implies numerical convergence of the iterative scheme consisting of (8) and (9). It also implies that the algorithm effectively tries to minimize $f(\boldsymbol{\beta}|\delta)$, although convergence to the true global minimum can not be guaranteed for the non-convex problem.

Remark 2. Weighted ridge regression can be transformed into regular ridge regression by a simple transformation of data. Furthermore, a ridge regression problem can be efficiently solved via the SVD of the data matrix. Thus the iterative scheme (8)-(9) operates by repeatedly executing SVD.

Performance studies of similar algorithms strongly suggest that δ should not stay fixed throughout the iterations (Daubechies et al., 2010). We would prefer to have δ very small to approximate ℓ_0-norm well with the penalty function ρ_l, but the smaller δ the more non-convex problem (6) gets. Subsequently, the greater is the chance the algorithm gets trapped in local minima near the starting point. Thus we would like to start with a relatively large δ to avoid local minima near the

Algorithm 1 Interative algorithm for problem (6) with varying δ

Input: $w_j^{(0)} \geq 0$ (default: $w_j^{(0)} =$), $j = 1, \dots, p$, $\xi > 0$,
a sequence $\{\delta_l\}_{l=0}^{\infty}$ such that $\delta_0 \geq \delta_1 \geq \dots$ and $\delta_l \to \delta > 0$ (default: $\delta_0 = 1$ and $\delta_{l+1} = \max(\delta_l/1.5, \delta_{\min})$, where $\delta_{\min} = 10^{-6}$).
Output: Estimate $\widehat{\boldsymbol{\beta}}$.

begin
Set $l = 0$.
repeat

$$\boldsymbol{\beta}^{(l+1)} = \underset{\boldsymbol{\beta} \in \mathbb{R}^p}{\arg\min} \quad g(\boldsymbol{\beta}, \boldsymbol{w}^{(l)} | \delta_l) \tag{12}$$

$$\boldsymbol{w}^{(l+1)} = \underset{\boldsymbol{\beta} \in \mathbb{R}^p}{\arg\min} \quad g(\boldsymbol{\beta}^{(l+1)}, \boldsymbol{w} | \delta_{l+1}) \tag{13}$$

Set $l = l + 1$.
until $\|\boldsymbol{\beta}^{(l)} - \boldsymbol{\beta}^{(l-1)}\|_2 < \xi$
Set $\widehat{\boldsymbol{\beta}} = \boldsymbol{\beta}^{(l)}$.
end

initialization, and decrease δ gradually during the iterations. This leads to Algorithm 1.

Lemma 1 (b) and (c) still indicate how the solutions of (1) and (1) look like. The only difference is that δ in (10) is replaced by δ_{l+1}. On the other hand, Lemma 1 (a) is useless now. It is not a priori clear which function Algorithm 1 effectively tries to minimize, if any, and Algorithm 1 is no longer an MM algorithm. Despite the fact, we still have numerical convergence to a stationary point of the limit function $f(\boldsymbol{\beta}|\delta)$, where δ is now the limit of the sequence of $\{\delta_l\}_{l=1}^{\infty}$.

Proposition 3. Let $w_j^{(0)} > 0$, $j = 1, \dots, p$, let $\delta_0 \geq \delta_1 \geq \dots$ be a sequence such that $\delta_l \to \delta_\star > 0$ and let $\{\boldsymbol{\beta}^{(l)}\}_{l=1}^{\infty}$ be the sequence generated by Algorithm 1. Then $\|\boldsymbol{\beta}^{(l+1)} - \boldsymbol{\beta}^{(l)}\|_2 \to 0$ and every limit point of the sequence $\{\boldsymbol{\beta}^{(l)}\}_{l=1}^{\infty}$ is a stationary point of $f(\boldsymbol{\beta}|\delta_\star)$.

We defer the proof to the Appendix.

1.4 S-PCA algorithm

Zou et al. (2006) proposed a suitable reformulation of the PCA problem. They showed that for any $\mu > 0$ the solution $(\widehat{\mathbf{A}}, \widehat{\mathbf{B}})$ of

$$\underset{\mathbf{A},\mathbf{B} \in \mathbb{R}^{p \times K}}{\arg\min} \quad \sum_{i=1}^{n} \|\mathbf{x}^{(i)} - \mathbf{A}\mathbf{B}^\top \mathbf{x}^{(i)}\|_2^2 + \mu \sum_{k=1}^{K} \|\mathbf{b_j}\|_2^2 \quad \text{s.t.} \quad \mathbf{A}^\top \mathbf{A} = \mathbf{I}_K \tag{14}$$

satisfies that the normalized columns of $\widehat{\mathbf{B}}$ are exactly the first K loadings $\mathbf{v}_1, \dots, \mathbf{v}_K$ (possibly up to a rotation). We invite the reader to compare (14) to (2).

With the goal to induce sparsity of the loadings, Zou et al. (2006) proposed to incorporate ℓ_1 penalties in (14) leading to

$$\begin{aligned} \underset{\mathbf{A},\mathbf{B} \in \mathbb{R}^{p \times K}}{\arg\min} \quad & \sum_{i=1}^{n} \|\mathbf{x}^{(i)} - \mathbf{A}\mathbf{B}^\top \mathbf{x}^{(i)}\|_2^2 + \mu \sum_{k=1}^{K} \|\mathbf{b}_j\|_2^2 + \sum_{k=1}^{K} \lambda_k \|\mathbf{b}_j\|_1 \\ \text{s.t.} \quad & \mathbf{A}^\top \mathbf{A} = \mathbf{I}_K, \end{aligned} \tag{15}$$

where $\lambda_1, \ldots, \lambda_K > 0$. For a fixed $\mathbf{A}$, (15) can be rewritten as K independent problems of the form

$$\underset{\boldsymbol{\beta} \in \mathbb{R}^p}{\arg\min} \quad \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \mu\|\boldsymbol{\beta}\|_2^2 + \lambda\|\boldsymbol{\beta}\|_1, \tag{16}$$

where $\mu, \lambda > 0$, while, for a fixed $\mathbf{B}$, the solution of (15) is simply given by a suitable rotation.

Zou et al. (2006) proposed an alternating algorithm for problem (15), fixing either $\mathbf{A}$ or $\mathbf{B}$ in every step and solving for the other term. The algorithm is called S-PCA.

Unfortunately, problem (15) is still non-convex, and Zou et al. (2006) did not provide any theoretical guarantees for the convergence of their alternating minimization algorithm. Although numerical convergence to some fixed point could be proven easily (but we do not pursue this due to the space restrictions), the algorithm is not guaranteed to provide a global minimum (Witten et al., 2009). On the other hand, empirical results show that ordinary loadings (obtained via SVD) provide a reliable starting point, and the algorithm does its job.

Remark 3. Even though there are no strong theoretical guarantees for the S-PCA algorithm, the algorithm may very well be the method of choice for many applications because it enjoys several favorable properties lacked by its competitors. These are:

(a) an exact cardinality (i.e. the desired number of nonzero coefficients) can be chosen for each vector of loadings separately,
(b) as the cardinality constraints are lifted, the method gradually reduces to classical PCA,
(c) a chosen number K of sparse PCs is computed at once,
(d) the computed loadings are truly orthonormal, and
(e) the method does not require neither storage nor computation of the covariance matrix (which can be unmanageable for $p > n$).

However, the S-PCA algorithm is outdated in terms of performance, as will be shown in section 2. Hence our goal is to develop a modification that would retain the above stated favorable properties and be competitive at the same time. Since the cornerstone of the S-PCA algorithm is repeatedly solving (16) (called *elastic net* problem, see Zou and Hastie, 2005), a natural way to improve the algorithm's performance is to use the large body of literature dealing with penalization in the regression context. We can substitute the elastic net problem (16) for a different one. This adds up to using different penalty terms in (15).

1.4.1 ADA-S-PCA modification

The most natural way to improve performance of the S-PCA algorithm is to use the *adaptive elastic net* (Zou and Zhang, 2009) instead of (16), i.e. to solve

$$\begin{aligned} \underset{\mathbf{A},\mathbf{B} \in \mathbb{R}^{p \times K}}{\arg\min} \quad & \sum_{i=1}^{n} \|\mathbf{x}^{(i)} - \mathbf{A}\mathbf{B}^\top \mathbf{x}^{(i)}\|_2^2 + \mu \sum_{k=1}^{K} \|\mathbf{b_j}\|_2^2 + \sum_{k=1}^{K} \lambda_k \sum_{j=1}^{p} w_{kj}|b_{kj}| \\ \text{s.t.} \quad & \mathbf{A}^\top \mathbf{A} = \mathbf{I}_K \end{aligned} \tag{17}$$

where $\mathbf{w}_k = (w_{k1}, \ldots, w_{kp})^\top$, $k = 1, \ldots, p$, are vectors of weights. The adaptive elastic net allows different degree of penalization for each coefficient. The weights should be small for nonzero coefficients and large for zero ones to balance out the fact that ℓ_1-norm penalizes all the coefficients equally. Thus, the weights w_{kj} should be taken inversely proportional to the true sparse loadings. In that case, mild penalty would be imposed on nonzero coefficients reducing their shrinkage while more severe penalty would be imposed on zero coefficients forcing them to be estimated as zeros.

Since we do not know the true sparse loadings, it is natural to take $w_{kj} = \frac{1}{v_{kj}}$, that is to take the weights proportional to the ordinary loadings obtained from the SVD of $\mathbf{X}$.

One can transform an adaptive elastic net problem to an elastic net problem by a simple transformation of the data. Moreover, the weights are known in our case because the ordinary loadings are the starting point for S-PCA algorithm. Thus the adaptive version (17) (which we refer to as ADA-S-PCA) of the S-PCA algorithm adds up to a very simple modification requiring negligible additional computation time. Still, the adaptive modification noticeably improves the performance of the S-PCA algorithm, which was observed several times (e.g. Chen, 2011, or Leng and Wang, 2012).

On the other hand, property (b) of Remark 3 of the S-PCA algorithm cease to hold for ADA-S-PCA. This implies, since ordinary PCs explain as much variance as possible, that ADA-S-PCA must be highly suboptimal in terms of explained variance in data whenever the desired cardinality is relatively high. We will observe this in Section 2.1.

1.4.2 IRLS-S-PCA modification

We replace the ℓ_1 penalty in (15) by the penalty function ρ_l of (5). Furthermore, the role of the ℓ_2 penalty in (14) is only to ensure uniqueness of the solution. This would be unnecessary for the numerical algorithm we will propose. Thus we drop the ℓ_2 penalty and propose to solve

$$\underset{\mathbf{A},\mathbf{B}\in\mathbb{R}^{p\times K}}{\arg\min} \quad \sum_{i=1}^{n} \|\mathbf{x}^{(i)} - \mathbf{A}\mathbf{B}^\top\mathbf{x}^{(i)}\|_2^2 + \sum_{k=1}^{K} \lambda_k \sum_{j=1}^{p} \log(b_{kj}^2 + \delta) \quad \text{s.t.} \quad \mathbf{A}^\top\mathbf{A} = \mathbf{I}_K \tag{18}$$

with the help of the alternating minimization algorithm of Zou et al. (2006). That is: for a fixed B, update $\mathbf{A} = \mathbf{U}\mathbf{V}^\top$ where $\mathbf{U}$ and $\mathbf{V}$ are from the SVD $\mathbf{X}^\top\mathbf{X}\mathbf{B} = \mathbf{U}\mathbf{D}\mathbf{V}^\top$, and, for a fixed $\mathbf{A}$, solve K independent penalized least squares problems of the form (6) using Algorithm 1. We implemented the IRLS-S-PCA method in software R.[2]

The motivation behind this proposal is the geometry of the penalty function (5). With SPCA we assume that the true (population) loadings are sparse. Since sparsity is defined in terms of ℓ_0-norm, one should penalize ℓ_0-norms in (14) instead of ℓ_1-norms, which would result in best subset selection problems instead of (16). Best subset selection problems are NP-hard due to the ℓ_0 penalty, and ℓ_1 penalty is commonly used as a convex relaxation of the problem. However, non-convex penalties can naturally resemble ℓ_0 penalty better than ℓ_1 penalty does, see Figure 1. This comes at the expense that the problem becomes non-convex, and must be solved iteratively. Nonetheless, we hope that the obtained solution would still be closer to the ℓ_0-penalized optimum compared to the ℓ_1-penalized optimum.

The reason for the concrete choice of the penalty function ρ_l stems from the algorithm we developed to minimize the resulting penalized least squares problem. The algorithm iterates between two easily and analytically solvable tasks. This comes at the expense that the solution obtained via the algorithm is not itself sparse, see again Figure 1. For a practical implementation of the algorithm, one need to ensure sparsity of the obtained loadings by thresholding. Moreover, we want to ensure our algorithm also allows one to pick the target cardinality for every PC. The simplest way is to threshold the desired number of coefficients in the output $\widehat{\boldsymbol{\beta}}$ of Algorithm 1. Instead, we use a cumulative version of the algorithm, in which the weights actually used in (1) are products of the weights computed from (1) in the last few steps, and we iterate until a prescribed number of coefficients decrease under a fixed threshold.

[2]<http:// www.karlin.mff.cuni.cz/masak/irls_s_pca.R>.

Our IRLS-S-PCA *algorithm possesses the same favorable properties as the* S-PCA *algorithm that were discussed in Remark 3.* We ensure the property (a) by thresholding. Property (b) follows from the fact that with no sparsity constraints the method only attempts to find a solution to (14). But the starting point is exactly this solution (the same as in the case of S-PCA) due to the initialization of the weights. And properties (c)-(e) are inherited from the S-PCA algorithm. Moreover, the algorithm shows a superior numerical performance as displayed in Section 2.

1.5 Connections to previous work

Motivated by the search for an estimating procedure, that would be asymptotically as efficient as the oracle procedure knowing the true support in advance (the famous lasso does not have this *oracle property*), Fan and Li (2001) proposed to work with a concave penalty function. They presented an iterative algorithm using local quadratic approximations to obtain the non-convex estimate (so-called SCAD).

Later, Zou and Li (2008) proposed to optimize another concave penalty function via local linear approximations. These are tight enough so that a one-step estimate can be asymptotically as efficient as a fully iterated one, provided a reliable starting point. The well-known *adaptive lasso* (Zou, 2006) is such a one-step estimate. It has the oracle property while consisting of just two convex optimization tasks, and it is the motivation behind the ADA-S-PCA algorithm presented later.

Our penalty function ρ_l does not fall into the framework of the previously mentioned papers directly, because ρ_l is not concave on $(0, \infty)$. This has two consequences. First, we can not use local linear approximations. Algorithm 1, only with $\delta > 0$ fixed, essentially uses local quadratic approximations, and these are not very tight. Thus a fully iterated estimate is needed. On the other hand, these iterations have much simpler form than local linear approximations. Second, our penalty function is differentiable at zero which implies the resulting estimate is not itself sparse. The differentiability also implies that Algorithm 1 with a fixed $\delta > 0$ is an instance of the Newton-Rhapson algorithm.

Another distinction of our approach is that we let the regularizing parameter δ to vary. The regularizing parameter was introduced by Hunter and Li (2005) to relieve the drawback of the backward variable selection of the SCAD estimator of Fan and Li (2001). The varying regularizing parameter does a similar job, and allows us in addition to avoid local minima near the initialization point at the same time (see Daubechies et al., 2010, and references therein).

2 EXAMPLES

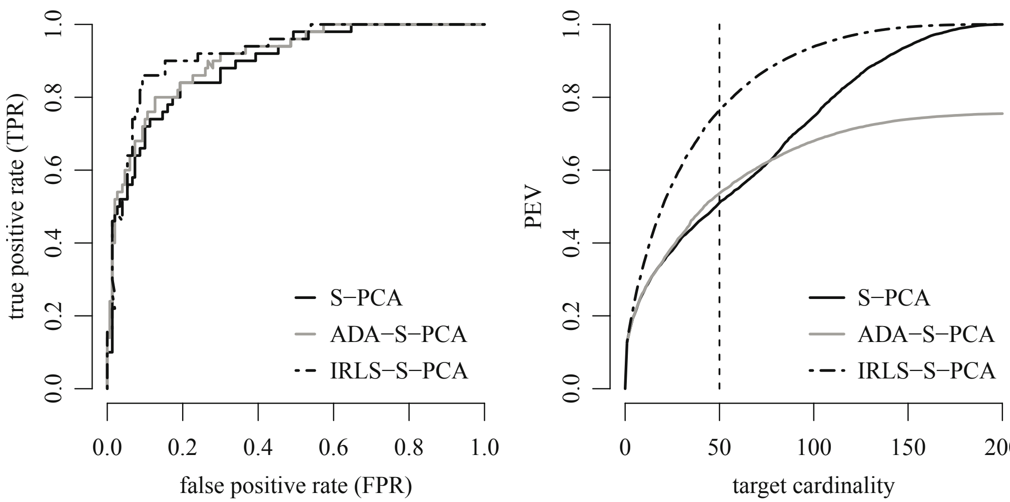
In this section, we first show a simulated example that displays differences between the approaches described above. Then we move to a practical application of SPCA: analysis of voting records of the Chamber of Deputies of the Parliament of the Czech Republic.

2.1 Simulated example

We generate a vector $\widetilde{\mathbf{v}} \in \mathbb{R}^{200}$ with coefficient that are independent, uniformly distributed on the interval (0,1). Then we randomly pick 150 of its coefficients and set them to zero, thus obtaining a 50-sparse vector. We normalize this vector to obtain a 50-sparse unit vector $\mathbf{v}$. Using the procedure described by Shen and Huang (2008), we create a covariance matrix $\boldsymbol{\Sigma}$ with the spectral decomposition

$$\boldsymbol{\Sigma} = \theta \mathbf{v}\mathbf{v}^\top + \sum_{j=2}^{200} \mathbf{v}_j \mathbf{v}_j^\top \in \mathbb{R}^{200\times 200}, \tag{19}$$

Figure 2 *Left:* Sample ROC curves for $\theta = 5$. Higher curve represents better performance. *Right:* Average (over 100 simulation runs) area under the curve as a function of θ.



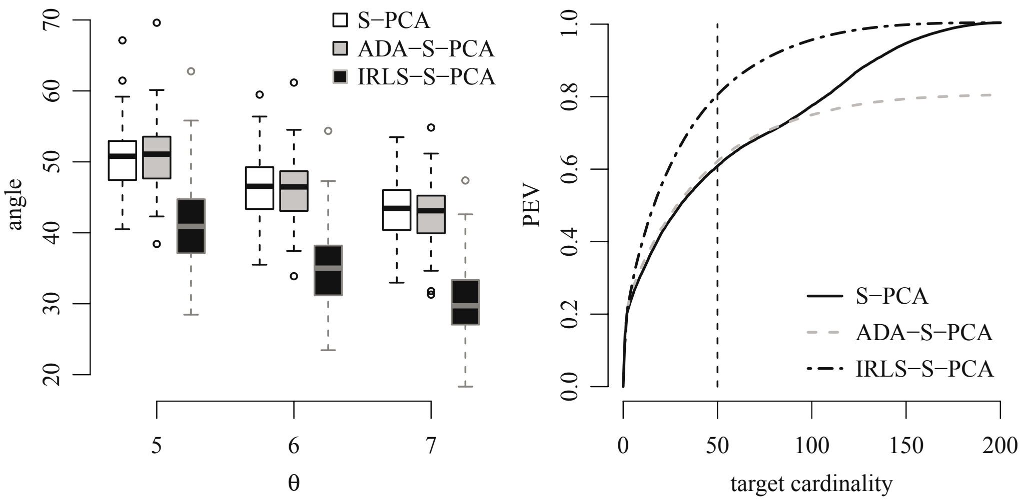
Source: Own construction

where $\theta > 1$, and $\mathbf{v}_2, \ldots, \mathbf{v}_{200} \in \mathbb{R}^{200}$ are orthonormal, all orthogonal to $\mathbf{v}$. Thus the matrix $\mathbf{\Sigma}$ is regular, its leading eigenvector $\mathbf{v}$ is associated with the largest eigenvalue θ, and all the other eigenvalues are equal to one. We generate $n = 100$ independent samples from $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ to obtain the data matrix $\mathbf{X}$. This procedure is repeated 100 times for every value $\theta \in \{3, 4, 5, 6, 7\}$.

Model (19) was introduced by Johnstone (2001). Later, Paul (2007) and Johnstone and Lu (2009) showed that PCA is not consistent under this particular model in the joint limit setting ($p, n \to \infty$ and $p/n \to \tau$) unless $\tau = 0$. The model is called *spiked covariance* as data sampled from this model are "spiked" in the direction of $\mathbf{v}$ and evenly spread in other directions (representing noise). Vector $\mathbf{v}$ can be thought of as a signal and θ as the signal strength. The stronger the signal is (i.e. θ is larger), more easy it is to distinguish it from noise. We are interested in estimating the signal $\mathbf{v}$ from the data, i.e. we are interested in the first principal component.

First, we are concerned with the quality of the considered algorithms with respect to the estimation of the support of $\mathbf{v}$. For a fixed value of s, every considered algorithm identifies exactly s coefficients as nonzero. It is natural to ask whether the correct coefficients were identified. One can compute (for a fixed s) the *true positive rate* (TPR, the proportion of correctly identified nonzero coefficients) and the *false positive rate* (FPR, the proportion of coefficients that were estimated as nonzeros even though they are truly zero). If we plot these pairs for all the values of s, we obtain an analogy to the so-called ROC curve as shown in the left panel of Figure 2. Note that our employment of the ROC curve differs from the typical usage in the context of binary classifiers. But still, higher curve represents a better performance. Thus it is natural to use the area under the curve as a measure of performance in order to have a single number comparing performances of different algorithms. In the right panel of Figure 2, area under the curve is plotted against the signal strength θ. For every $\theta \in \{3, 4, 5, 6, 7\}$, 100 datasets were simulated by the above-described procedure, and the average values are shown. As we can see in the figure, IRLS-S-PCA and ADA-S-PCA both overcome S-PCA in terms of support estimation while there is only a negligible difference between them.

Figure 3 *Left:* Angles between vector **v** and its estimates from our three methods for $\theta = 5, 6, 7$ with $s = 50$. *Right:* Proportion of explained variance as a function of target cardinality s for $\theta = 5$.



Source: Own construction

Second, correct support estimation is not sufficient for reliable estimation of the respective PC. Thus we compute angles between **v** and its estimates obtained by the considered algorithms with the value of target cardinality s set to 50. Results are plotted in the left panel of Figure 3. Note that the knowledge that **v** is 50-sparse is used solely for the purpose of this figure. Even though the performance of ADA-S-PCA and IRLS-S-PCA with respect to support estimation is comparable, IRLS-S-PCA clearly outperforms ADA-S-PCA regarding the angles between the estimated and true vectors of loadings.

Finally, we are also interested in the amount of explained variance. Since no vector can explain more variance in data than the first classical PC, we plot in the right panel of Figure 3 explained variance for different values of target cardinality s as a proportion of the variance explained by classical PCA.[3] We see that IRLS-S-PCA explains higher proportions of variance than S-PCA, and the proportion of variance for both of these methods approach one as s approaches to 200. This is an empirical evidence that both IRLS-S-PCA and ADA-S-PCA reduce to classical PCA as the cardinality constraint is lifted. On the other hand, ADA-S-PCA doesn't have this property and thus it is suboptimal in terms of explained variance.

2.2 Real-world example: analysis of voting records

2.2.1 Data description and objectives

In this section, we show an application of SPCA on a real data set, namely voting records (so-called *roll-call* data) of the Chamber of Deputies of the Parliament of the Czech Republic. The data are

[3] Again, only average values over 100 simulation runs are shown. This is for the sake of clarity and due to the space restrictions. For the right-panel of Figure 2 the standard deviations are of order 10^{-2} with differences of order 10^{-3} across the respective algorithms. For the right-hand panel of Figure 3 the situation is similar, with the exception that standard deviations of IRLS-S-PCA are one order lower than those of other methods.

publicly available on the official website of the Chamber of Deputies.[4] They give information about deputies, their memberships in political parties and different committees, and most importantly their voting records. The data are organized in several tables and date back to year 1993. We are interested mostly in the table "hl_poslanec" with three variables: ID of the deputy, ID of the act, and the individual vote. The only additional information we use comes from the table "poslanec", which enables us to align the deputies to the parties of their membership for the purposes of our outputs. More detailed description of the data set can be found on the website of the Chamber of Deputies.

Note that, from all available data, we are interested only in the voting records of the year 2015, i.e. a one-year period of the current administration.

We have transformed the raw data (table "hl_poslanec") into a data matrix $\mathbf{X}$ with entries

$$x_{ij} = \begin{cases} 1, & \text{if the } i\text{-th deputy voted "yes" in the } j\text{-th act,} \\ -1, & \text{if the } i\text{-th deputy voted "no" in the } j\text{-th act,} \\ 0, & \text{otherwise.} \end{cases}$$

While the original data distinguish several scenarios, why a given deputy did not vote neither "yes" nor "no" in the given act (e.g. he was registered and abstained from voting, he was registered but failed to vote, or he wasn't present), we regard all those scenarios the same as zeros. This is, of course, mainly due to simplicity but also due to the fact that deputies are well aware that the reason of their abstention is irrelevant. Just for information, there are approximately 35% of zeros in the matrix $\mathbf{X}$. Of course, there are no missing data in the sense that a deputy voted and we do not have the information how.

The political situation in the Chamber of Deputies is following. Since the elections in 2014, there is a majority coalition composed of *CSSD*, *ANO*, and *KDU-CSL*. The opposition is formed by *ODS*, *TOP 09*, *Usvit*, and *KSCM*. As for the political positions of the parties, *ODS* is a traditional right-wing party opposed by the communists of *KSCM* on the left-wing of the spectrum. *TOP 09* is centre-right, *CSSD* is centre-left, *ANO* is centre to centre-right and, finally, *KDU-CSL* holds a centre position.

In 2015, 1837 acts took place, and there were no replacements among the deputies (which are 200 in the Chamber of Deputies) during that year. Thus the matrix $\mathbf{X}$ has 200 rows and 1837 columns. The voting record for the i-th deputy, $i = 1, \ldots, 200$, is represented by the vector $\mathbf{x}^{(i)} \in \{-1, 0, 1\}^{1837}$. One deputy thus correspond to a point in a 1837-dimensional space. From now on, let $n = 200$ and $p = 1837$, i.e. $\mathbf{X} \in \mathbb{R}^{n \times p}$.

PCA regards to finding a lower-dimensional subspace best approximating the 1837-dimen-sional data points. If we take the information about the political affiliation of the representatives into account, an interesting underlying structure in the data could be possibly revealed in the lower dimension. Questions such as

- which parties are homogeneous and which are not,
- which deputies are outliers with respect to their parties, and
- which parties stick together, and which do not share the same view,

would be hopefully answered.

[4] <http://www.psp.cz/sqw/hp.sqw?k=1300>.

2.2.2 Assumptions and their justification

For classical PCA, we need to choose a number of PCs to work with. For SPCA, we further need to choose values of target cardinality for the PCs. To justify our choices, recall a data-generative factor model (3).

First, how many PCs should we take into account? Two is the most prevalent option due to the fact that two-dimensional approximation is most easily comprehensible. From the perspective of model (3), dealing with just two PCs means that voting behavior of every respective deputy can be described as a linear combination of two factors, up to a deputy-specific error. This seems appropriate, as one can expect from the very start that one PC would discriminate between coalition and opposition and the other between the right-wing and left-wing parties.

Second, what values of target cardinality should be chosen? Considering model (3) with $K = 2$, there are two unobserved factors, and we observe noisy realizations of those factors in the domain of acts. The question thus translates to how many acts do we need in order to be able to describe the factors reliably. Note that the sparsity assumption seems very reasonable from this perspective. We have chosen 20 acts to describe the first PC and 15 acts to describe the second PC, that is 35 in total. This concrete choice is based on data-non-related considerations. The lists of important acts created by political scientists or journalists often count between 20 and 35 acts per year. We have simply chosen the sum of the desired target cardinalities to be an upper bound to this expert choice because we also believe that 35 acts should be enough to describe the two PCs.

2.2.3 Results

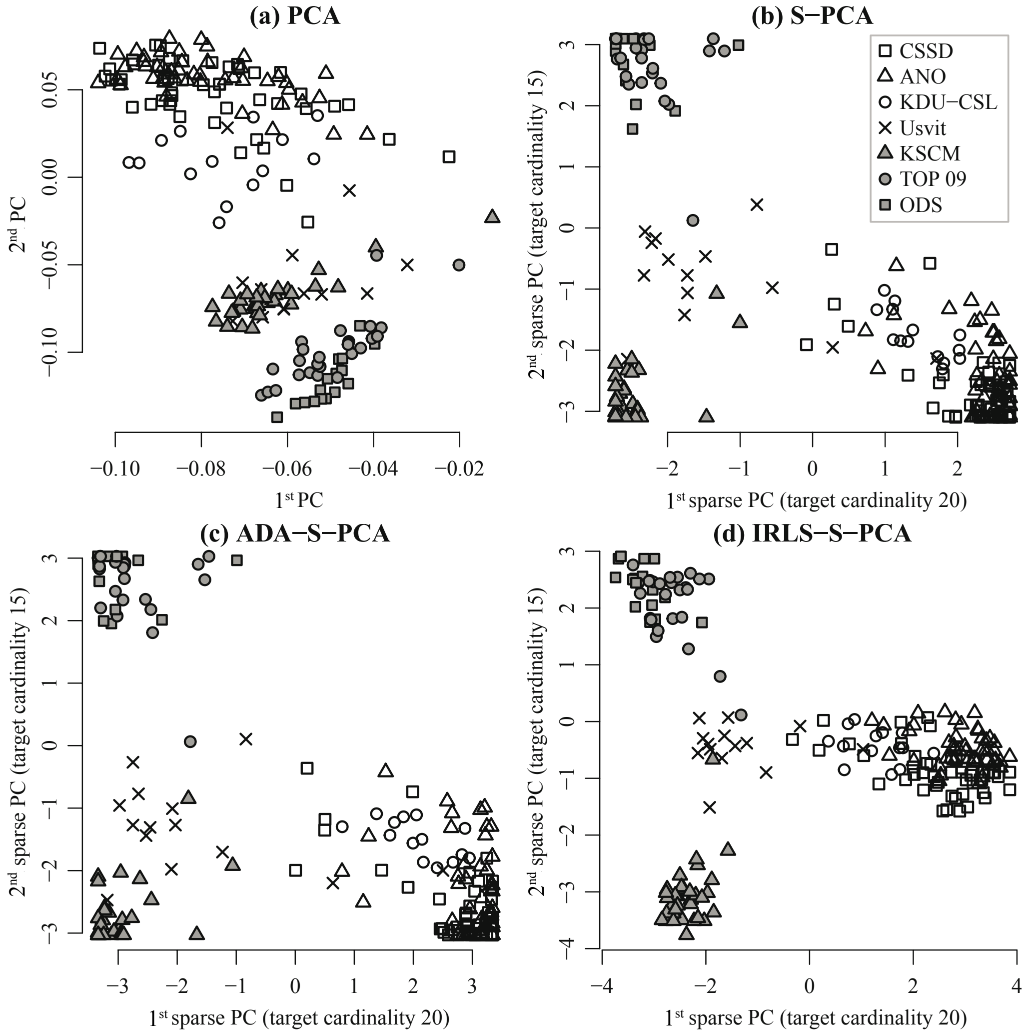
Figure 4 shows results of classical PCA (panel (a)) and SPCA performed by the algorithms presented in Section 1.4 (panels (b)-(d)). All the panels give qualitatively similar results. In all cases, the first sparse PC discriminates between the coalition and the opposition, and the second sparse PC distinguishes between the right and left wings. Even more delicate distinctions can be made. For example, the party *Usvit* is somehow in the middle of the plots, and *KDU-CSL* is the most centered of the coalition parties. Also, one can notice two outliers of *Usvit* in Figure 4 (b): the right-most deputy of the *Usvit* located in between the deputies of *KDU-CSL* and the bottom-left-most deputy surrounded by representatives of *KSCM*. These two outliers can be observed also in (c) and (d) of Figure 4, and they are truly the same deputies.

Nonetheless, the IRLS-S-PCA *algorithm* (Figure 4 (d)) *gives much smoother and more easily interpretable results than the other algorithms*. It sharply discriminates between the coalition/opposition and left-wing/right-wing features, thus the coalition is shown to be in the centre political position. Also, *ANO* is better separated from *CSSD* referring more closely to their expected position. To sum up, Figure 4 (d) captures best the general perception of the Czech political scene.

SPCA reveals a useful additional piece information as opposed to methods commonly used for analysis of voting data (see Clinton et al., 2004, for an overview) that do not assume sparsity. Namely, the variable selection property indicate which acts are important. Sadly, even though panels (b)-(d) of Figure 4 are qualitatively similar, the acts selected to describe the sparse PCs differ noticeably among the methods. There are only three acts that were selected by all methods. These are:

- act no. 60 403, on the agenda of the meeting, specifically whether a specific amendment should be considered first,
- act no. 61 973, on an amendment on the state budget draft for 2016,
- act no. 62 034, on the state budget draft as a whole.

Figure 4 Voting records of the Chamber of Deputies of the Parliament of the Czech Republic projected onto a plane given by: (a) first two ordinary PCs, (b) first two sparse PCs obtained via S-PCA, (c) first two sparse PCs obtained via ADA-S-PCA, (d) first two sparse PCs obtained via IRLS-S-PCA. For (b)-(d) the cardinality is 20 for the first sparse PC and 15 for the second sparse PC.



Source: Own construction

On one hand, it is convenient that the act on the state budget draft as a whole was picked by all the methods. On the other hand, it is unseemly that behind the act no. 60 403 there is a hidden agenda, and it takes an expert knowledge and going through the stenographic records to discover what the agenda is. The same holds for the act no. 61 973. Nonetheless, this does not change the fact that the sparse representation of the PCs has a practical impact. We give the following example.

Suppose we wish to project another subject onto the plane given by the first two PCs in order to visualize, how the subject's political opinions relate to the deputies and the parties. If the PCs are sparse, the only information that we need is how would the subject vote in the acts that correspond to nonzero loadings coefficients. Thus we can have the subject respond to only several questions, and the final projection will still be punctual. We reflect on similar considerations further in the Discussion.

Finally, we note that IRLS-S-PCA seems fairly robust regarding the choice of target cardinalities. Varying the target cardinalities around the chosen value alters the resulting figure negligibly. However, if we allow one PC to be described only by less than 10 acts, the plots get noticeably distorted. On the other hand, the number of acts describing one PC can be relatively high but, of course, we aim for the most parsimonious and yet credible representation. As an experimental test of robustness of IRLS-S-PCA, we also tried several times to recalculate the two-dimensional projection with 10 percent of deputies left out and projected onto the subspace subsequently. The results were fairly similar. Also, we would like to note that in a three-dimensional projection, the first two PCs remain very much alike as in the two-dimensional projection. Moreover, the clusters made by parties are still well-separated. We do not report these results due to space restrictions.

DISCUSSION AND CONCLUSION

Our algorithm operates with two tuning parameters: λ and δ. At the moment, we update δ based on a rule of thumb (see default setting in Algorithm 1) and enforce the desired target cardinality heuristically. It would be desirable to understand better the relationship between the tuning parameter λ, regularizing parameter δ, and the resulting cardinality. Such a knowledge would possibly lead to a better strategy for choosing the parameters. Another approach to the choice of λ is to find solutions on a grid of values of λ and pick a value of λ leading to the desired cardinality among them. We believe that methods described in Mazumder et al. (2010) can be employed for this purpose. However, we obtain promising results with the rule of thumb and the simple heuristic, which is computationally less demanding.

In analyses similar to our example with voting records, the number of PCs is often chosen based on the famous "elbow plot" (Jolliffe, 2002), and the values of target cardinality for every PC are chosen such that every sparse PC explains some relatively high proportion of variance explained by the respective standard PC (e.g. Zhang, 2011). We do not recommend this procedure because it leads to too many subsequent decisions based solely on the variance in data. Moreover, this approach simply does not work well for complex data such as the voting records. In the case of our voting records, the previous strategy would lead us to choose 2-3 PCs and the values of target cardinality in hundreds for every PC. Thus we would have much more coefficients to estimate in our model than we can hope to estimate reliably from just 200 observations (deputies). This is the reason why we choose the number of PCs and the respective target cardinalities based on additional considerations, not in a data-driven way.

In the analysis of voting records, we show that SPCA produces better interpretable results than classical PCA. The variable selection property can be utilized to choose important acts. This could be potentially used, for example, to automatically create voting calculators – a popular and at the present manually designed tool that help voters to determine their conformity with different parties and representatives. However, we gave examples why appreciable expert knowledge about actions in the parliament is vital in evaluating the usage of SPCA in this way.

For the voting data, one might naturally wonder whether the sum-of-squares criterion we work with throughout the paper is appropriate. The penalization techniques discussed in Section 1.3 could be readily extended to the framework of penalized likelihood. As a probabilistic model for the votes, one could use for example multinomial distribution. Nonetheless, we restrained from probabilistic model formulation and used sum-of-squares criterion. Thus we view the voting records for deputies as points in standard euclidean space. To our perception, this is somehow natural and, again, the bottom line is that we delivered promising results with the simplest-to-study sum-of-squares criterion.

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APPENDIX

Here we prove Proposition 3. We will use the following two lemmas.

Lemma 4. The following properties hold:

(a) Let $\delta > 0$, $\boldsymbol{\beta} \in \mathbb{R}^p$ and let $w_j = \frac{1}{\beta_j^2+\delta}$, $j = 1, \dots, p$. Then we have

$$\nabla_{\boldsymbol{\beta}}\, f(\boldsymbol{\beta}|\delta) = \nabla_{\boldsymbol{\beta}}\, g(\boldsymbol{\beta}, \boldsymbol{w}|\delta). \tag{20}$$

(b) Let $\delta > 0$ and $x, y \in \mathbb{R}$. Then we have

$$\log(x^2+\delta) - \log(y^2+\delta) - 2\frac{y(x-y)}{x^2+\delta} \geq \frac{(x-y)^2}{x^2+\delta}, \tag{21}$$

and equality in (21) holds if and only if $x = y$.

Proof. For (a), simply calculate the gradients

$$\begin{aligned} \nabla_{\boldsymbol{\beta}}\, f(\boldsymbol{\beta}|\delta) &= -2\mathbf{X}^\top\big(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\big) + 2\lambda\Big[\frac{\beta_j}{\beta_j^2+\delta}\Big]_{j=1}^p && (22)\\ \nabla_{\boldsymbol{\beta}}\, g(\boldsymbol{\beta}, \boldsymbol{w}|\delta) &= -2\mathbf{X}^\top\big(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\big) + 2\lambda\big[w_j\beta_j\big]_{j=1}^p && (23) \end{aligned}$$

and use that $w_j = \frac{1}{\beta_j^2+\delta}$. This gives us (a).

For (b), since $2\frac{y(x-y)}{x^2+\delta} + \frac{(x-y)^2}{x^2+\delta} = \frac{x^2-y^2}{x^2+\delta}$, it is enough to show that

$$\log(x^2+\delta) - \log(y^2+\delta) - \frac{x^2-y^2}{x^2+\delta} \geq 0.$$

For a fixed $x > 0$ is is easy to verify that $h(y) = \log(x^2+\delta) - \log(y^2+\delta) - \frac{x^2-y^2}{x^2+\delta}$ has a unique minimum in $y = x$ by differentiation. The same holds for a fixed $x < 0$ and $x = 0$.

Lemma 5. Under the assumptions of Proposition 3 it holds for $l \in \mathbb{N}$:

$$\big(\mathbf{X}\boldsymbol{\beta}^{(l+1)} - \mathbf{Y}\big)^\top\big(\mathbf{X}\boldsymbol{\beta}^{(l)} - \mathbf{X}\boldsymbol{\beta}^{(l+1)}\big) = -\lambda\sum_{j=1}^p \frac{\beta_j^{(l+1)}(\beta_j^{(l)} - \beta_j^{(l+1)})}{(\beta_j^{(l)})^2 + \delta_l}. \tag{24}$$

Proof. Since $\nabla_{\boldsymbol{\beta}}\, g(\boldsymbol{\beta}^{(l+1)}, \boldsymbol{w}^{(l)}|\delta) = 0$ from the first order optimality condition, we have

$$\begin{aligned} 0 &= (\boldsymbol{\beta}^{(l)} - \boldsymbol{\beta}^{(l+1)})^\top \nabla_{\boldsymbol{\beta}}\, g(\boldsymbol{\beta}^{(l+1)}, \boldsymbol{w}^{(l)}|\delta)\\ &= 2\big(\mathbf{X}\boldsymbol{\beta}^{(l+1)} - \mathbf{Y}\big)^\top\big(\mathbf{X}\boldsymbol{\beta}^{(l)} - \mathbf{X}\boldsymbol{\beta}^{(l+1)}\big) + 2\lambda\sum_{j=1}^p \frac{\beta_j^{(l+1)}(\beta_j^{(l)} - \beta_j^{(l+1)})}{(\beta_j^{(l)})^2 + \delta}. \end{aligned}$$

Now we are ready to prove Proposition 3. We calculate for $l \in \mathbb{N}$:

$$\begin{aligned} & f(\boldsymbol{\beta}^{(l)}|\delta_l) - f(\boldsymbol{\beta}^{(l+1)}|\delta_{l+1}) =\\ &= \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}^{(l)}\|_2^2 - \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}^{(l+1)}\|_2^2 + \lambda\sum_{j=1}^p \Big[\log\big((\beta_j^{(l)})^2 + \delta_l\big) - \log\big((\beta_j^{(l+1)})^2 + \delta_{l+1}\big)\Big]\\ &= \|\mathbf{X}\boldsymbol{\beta}^{(l)} - \mathbf{X}\boldsymbol{\beta}^{(l+1)}\|_2^2 + 2\big(\mathbf{X}\boldsymbol{\beta}^{(l+1)} - \mathbf{Y}\big)^\top\big(\mathbf{X}\boldsymbol{\beta}^{(l)} - \mathbf{X}\boldsymbol{\beta}^{(l+1)}\big)\\ &\qquad + \lambda\sum_{j=1}^p \Big[\log\big((\beta_j^{(l)})^2 + \delta_l\big) - \log\big((\beta_j^{(l+1)})^2 + \delta_{l+1}\big)\Big]\\ &\geq \lambda\sum_{j=1}^p \Big[\log\big((\beta_j^{(l)})^2 + \delta_l\big) - \log\big((\beta_j^{(l+1)})^2 + \delta_{l+1}\big) - 2\frac{\beta_j^{(l+1)}(\beta_j^{(l)} - \beta_j^{(l+1)})}{(\beta_j^{(l)})^2 + \delta_l}\Big]\\ &\geq \lambda\sum_{j=1}^p \frac{(\boldsymbol{\beta}^{(l+1)} - \boldsymbol{\beta}^{(l)})^2}{(\boldsymbol{\beta}^{(l)})^2 + \delta_l} \geq \lambda\sum_{j=1}^p \frac{(\boldsymbol{\beta}^{(l+1)} - \boldsymbol{\beta}^{(l)})^2}{(\boldsymbol{\beta}^{(l)})^2 + \delta_0}. \end{aligned}$$

The second equality is just a tedious algebra. In the first inequality, the squared ℓ_2-norm is dropped and Lemma 5 is used. In the second inequality, δ_{l+1} is substituted for greater δ_l reducing the overall value of the expression, and then Lemma 4 (b) is used. In the last inequality, δ_l is substituted again for a greater δ_0.

Note the previous calculations suggest that $f(\boldsymbol{\beta}^{(l)}|\delta_l) \geq f(\boldsymbol{\beta}^{(l+1)}|\delta_{l+1})$ for all $l \in \mathbb{N}$. Thus for all $l \in \mathbb{N}$ we have

$$f(\boldsymbol{\beta}^{(1)}|\delta_1) \geq f(\boldsymbol{\beta}^{(l)}|\delta_l) \geq \lambda \sum_{j=1}^{p} \log\left((\beta_j^{(l)})^2 + \delta_l\right) \geq \lambda \sum_{j=1}^{p} \log\left((\beta_j^{(l)})^2 + \delta_\star\right), \tag{25}$$

from which it follows that there exists a constant c such that for all $l \in \mathbb{N}$ and $j = 1, \ldots, p$ it holds $|\beta_j^{(l)}| \leq c$. It follows that

$$f(\boldsymbol{\beta}^{(l)}|\delta_l) - f(\boldsymbol{\beta}^{(l+1)}|\delta_{l+1}) \geq \frac{\lambda}{c^2 + \delta_0} \sum_{j=1}^{p} (\boldsymbol{\beta}^{(l+1)} - \boldsymbol{\beta}^{(l)})^2 = \frac{\lambda}{c^2 + \delta_0} \|\boldsymbol{\beta}^{(l+1)} - \boldsymbol{\beta}^{(l)}\|_2^2. \tag{26}$$

It also follows from (25) that $\{f(\boldsymbol{\beta}^{(l)}|\delta_l)\}_{l=1}^{\infty}$ converges for $l \to \infty$ monotically to some finite value, let us label it $f^\star$. Thus summing (26) for $l = 1, 2, \ldots$ we obtain

$$f(\boldsymbol{\beta}^{(1)}|\delta_1) - f^\star \geq \frac{\lambda}{c + \delta_0} \sum_{l=1}^{\infty} \|\boldsymbol{\beta}^{(l+1)} - \boldsymbol{\beta}^{(l)}\|_2^2,$$

from which it follows

$$\|\boldsymbol{\beta}^{(l+1)} - \boldsymbol{\beta}^{(l)}\|_2 \to 0. \tag{27}$$

Next, let us have $\{\boldsymbol{\beta}^{(l_m)}\}_{m=1}^{\infty} \subset \{\boldsymbol{\beta}^{(l)}\}_{l=1}^{\infty}$ such that $\boldsymbol{\beta}^{(l_m)} \to \boldsymbol{\beta}^\star$. Existence of this convergent subsequence follows from compactness of the level sets of functions $f(\boldsymbol{\beta}|\delta)$, $\delta > 0$. Then

$$w_j^{(l_m)} = \frac{1}{(\beta_j^{(l_m)})^2 + \delta_{l_m}} \to \frac{1}{(\beta_j^\star)^2 + \delta_\star} = w_j^\star$$

and from (27) also $\boldsymbol{\beta}^{(l_m+1)} \to \boldsymbol{\beta}^\star$. By taking the limit of the first order optimality condition $\nabla_{\boldsymbol{\beta}}\, g(\boldsymbol{\beta}^{(l_m+1)}, \boldsymbol{w}^{(l_m)}|\delta_{l_m}) = 0$ we obtain $\nabla_{\boldsymbol{\beta}}\, g(\boldsymbol{\beta}^\star, \boldsymbol{w}^\star|\delta_\star) = 0$, and by using Lemma 4 (a) we have $\nabla f(\boldsymbol{\beta}^\star|\delta_\star) = 0$, which completes the proof.

Bandwidth Selection Problem in Nonparametric Functional Regression

Daniela Kuruczová [1] | *Masaryk University, Brno, Czech Republic*
Jan Koláček [2] | *Masaryk University, Brno, Czech Republic*

Abstract

The focus of this paper is the nonparametric regression where the predictor is a functional random variable, and the response is a scalar. Functional kernel regression belongs to popular nonparametric methods used for this purpose. The two key problems in functional kernel regression are choosing an optimal smoothing parameter and selecting an appropriate semimetric as a distance measure. The former is the focus of this paper – several data-driven methods for optimal bandwidth selection are described and discussed. The performance of these methods is illustrated in a real data application. A conclusion is drawn that local bandwidth selection methods are more appropriate in the functional setting.

Keywords	***JEL code***
Functional data, nonparametric regression, kernel methods, bandwidth selection	*C14*

INTRODUCTION

Functional data analysis is a relatively recent topic in statistics. It is based on the concept of a functional random variable, which is a random variable taking values in infinite-dimensional space. Natural examples of a functional random variable are random curves and surfaces, but the concept itself also covers more complex objects (Ramsay and Silverman, 1997; Ferraty and Vieu, 2006). Realizations of a functional random variable are called functional data, which are treated as members of an infinite-dimensional space. Typical datasets suitable for this approach are usually high-dimensional and multicollinear, violating assumptions of most traditional regression methods. Therefore, a functional approach brings new options to analyse such data. This paper focuses on a nonparametric functional regression, namely, kernel functional regression with a scalar response. The functional kernel regression is based on kernel smoothing, one of the most popular nonparametric techniques. The main advantages of kernel methods are their simplicity of expression and ease of implementation.

The move from a real random variable to a functional one brings several challenges. One of the most important ones is the choice of a distance measure. In contrast to the finite-dimensional setting, the selection

[1] Department of Mathematics and Statistics, Faculty of Science, Kotlářská 2, 611 37 Brno, Czech Republic. E-mail: xkuruczovad@math.muni.cz.

[2] Department of Mathematics and Statistics, Faculty of Science, Kotlářská 2, 611 37 Brno, Czech Republic. E-mail: kolacek@math.muni.cz.

of distance measure is not straightforward and has a noticeable impact on the properties of the estimates. Ferraty and Vieu (2006) propose the use of semimetrics as a suitable distance measure and introduce three classes of semimetrics: based on derivatives, functional principal component analysis, and partial least squares regression. The use of semimetrics instead of metrics appears to bring good practical results, but poses theoretical challenges as the theory of semimetric spaces is less straightforward compared to the theory of metric spaces. In practice, it also seems that the choice of an appropriate semimetric for the data is crucial. Derivative-based semimetrics appear to be suitable for relatively smooth data whereas the other two kinds of semimetrics work well for rougher or noisy data (Benhenni et al., 2007).

Just as in a finite-dimensional setting, kernel regression estimators depend on a smoothing parameter that controls the smoothness of the estimated curve. The choice of an optimal smoothing parameter is a crucial factor in the quality of kernel estimates. Unlike the finite case, it is not possible to rely on visualization tools when selecting optimal bandwidth. Thus, automatic (data-driven) selection procedures are useful for many practical situations. Successful approaches to bandwidth selection in the finite dimensional setting can be transferred to the functional setting. Most of these procedures are based on the mean squared error (*MSE*) estimation and further minimization of such an estimate. A widely known and commonly used method for bandwidth selection is leave-one-out cross validation (Ferraty and Vieu, 2002). A method of penalizing functions is similar, but lesser known, see e.g. Härdle (1992). It is based on the idea of penalizing the biased estimate of the *MSE* and can be considered a generalization of the leave-one-out cross validation. In this paper, use of the penalizing functions method in the functional setting is proposed. Both approaches mentioned above are global methods, i.e. the value of smoothing parameter is fixed for all data points. However, local methods determine a custom value of the smoothing parameter for each data point. The local version of cross-validation based on the number of nearest neighbors is proposed by Benhenni et al. (2007). One of the latest results in this area is the paper by Chagny and Roche (2016) where an adaptive, data-driven, local bandwidth selection rule was derived.

1 FUNCTIONAL KERNEL REGRESSION

Kernel methods for finite data can be easily extended to functional data. Ferraty and Vieu (2006) demonstrate the application of kernel methods for regression and classification on several datasets from various fields, e.g. economics (electricity consumption dataset), chemistry (spectrometric dataset), and phoneme recognition.

In the case of functional data, the kernel function serves the same purpose as in a finite case – to assign weights to observations. The most notable difference is the use of asymmetrical kernels, because the measure of distance (semimetric) is always non-negative.

If we consider a pair of random variables $(X,Y) \in E \times R$, where *E* is a semimetric space and *R* are real numbers, the functional regression operator with scalar response can be defined as:

$$Y = r(X) + \varepsilon = E(Y \mid X) + \varepsilon, \tag{1}$$

where ε is the random error with an expected value of zero and finite variance, and the regression operator *r* is expressed as the conditional mean of the finite random variable *Y* given the functional random variable *X*.

Let $\{(Y_i, X_i), i = 1,...,n\}$ be an observed data sample with the same distribution as the pair *(X, Y),* then for the observed data the formula is obtained:

$$Y_i = r(X_i) + \varepsilon_i, \tag{2}$$

with independent, identically distributed errors ε_i that are independent of $X_i, i = 1,...,n$.

One of the options for the functional kernel estimate of the regression operator r is a simple extension of the finite case – Nadaraya-Watson estimator (Nadaraya, 1964):

$$\hat{r}(x,h) = \frac{\sum_{i=1}^{n} Y_i K\left(\frac{d(x,X_i)}{h}\right)}{\sum_{i=1}^{n} K\left(\frac{d(x,X_i)}{h}\right)} = \sum_{i=1}^{n} W_{i,h}(x) Y_i. \tag{3}$$

The function K is an asymmetrical kernel function, assuming that:

$$\int_R K(x)dx = 1,$$

where d denotes a semimetric and the parameter $h>0$ is called bandwidth. It can be shown that under certain assumptions (most notably, in the case of continuity of the regression operator r) this estimate converges almost completely to the actual regression operator (Ferraty and Vieu, 2006).

2 BANDWIDTH SELECTION

The functional setting brings several obstacles to the problem of optimal bandwidth selection. One of the very useful tools, visualization, does not transfer well to the infinite-dimensional setting. Another issue is the sparsity of data in certain areas of the functional space (Benhenni et al., 2007).

The problem of bandwidth selection is closely tied to estimation of the mean squared error (*MSE*) of the regression operator estimate in fixed data point x:

$$MSE(\hat{r},h,x) = E[(\hat{r}(x,h) - r(x))^2], \tag{4}$$

where the optimal bandwidth value h can be defined as the value of h that minimizes the mean squared error.

An exact *MSE* formula for the case of separable Banach space was derived by Ferraty et al. (2007) and for semimetric space by Geenens (2015). In both cases the final formula is based on the decomposition of *MSE* into bias and variance terms. The decomposition shows the expected influence of parameter h – with increasing values of h the bias increases and the variance decreases. Compared to the finite case, the influence of h in the variance term is less straightforward. It is moderated through a notion called the small ball probability:

$$\varphi_x(h) = P(d(X,x) \leq h), \tag{5}$$

which expresses the probability of functional random variable occurring within a ball centered around x with a radius of h. As h decreases, the small ball probability decreases, so lower values of h still lead to higher variance, and vice versa.

The minimization problem leads to the usual variance-bias trade-off known from the finite case. However, in the functional setting, the *MSE* formula contains several expressions unknown in practice, so it cannot be directly used for bandwidth selection.

2.1 Global bandwidth selection

One possible approach to bandwidth selection is to select one bandwidth value for the whole dataset. Two methods, both using minimalization of a biased *MSE* estimate obtained through cross-validation, will be presented.

2.1.1 General cross-validation

The general cross-validation function (also called leave-one-out cross-validation) is based on the cross-validation function:

$$CV(h) = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{r}_{-i}(X_i,h))^2, \tag{6}$$

where:

$$\hat{r}_{-i}(X_i,h) = \frac{\sum_{\substack{j=1\\ i\neq j}}^{n} Y_i K\left(\frac{d(X_i,X_j)}{h}\right)}{\sum_{\substack{j=1\\ i\neq j}}^{n} K\left(\frac{d(X_i,X_j)}{h}\right)} \tag{7}$$

is the regression operator estimate at point X_i based on data that does not contain X_i, the well-known leave-one-out regression estimator. This method was proposed in a functional setting by Ferraty and Vieu (2002).

2.1.2 Penalizing function method

The penalizing function method uses another means to avoid the bias caused by using the same dataset for creating the regression operator estimate and estimating response values – a penalizing function Ξ. The penalizing function penalizes small values of *h* to avoid undersmoothing. It was proposed by Härdle (1992) for the case of finite dimension. The same idea can be adapted to an infinite-dimensional setting. Thus, the error function to be minimized takes the form:

$$PF(h) = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{r}(X_i,h))^2\,\Xi(W_{i,h}(X_i)). \tag{8}$$

Several examples of penalizing functions are given in Härdle (1992) and Koláček (2002). One of the best-known examples of a penalizing function is Akaike's information criterion, which is also used in the section concerning practical application of the method.

2.2 Local bandwidth selection

Another option is to select the bandwidth value for each data point separately. Two methods based on different approaches will be presented.

2.2.1 Nearest neighbors method

This method, also known as *k* nearest neighbors method, is based on a local cross-validation (Benhenni et al., 2007) function minimized separately for each data point, where:

$$LCV_x(h) = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{r}_{-i}(X_i,h))^2\, w_{n,x}(X_i). \tag{9}$$

The local cross-validation function uses the leave-one-out regression estimate along with a weight function dependent on data point *x*. If the weight function does not depend on *x*, the function becomes a global cross-validation function.

The weight function for the nearest neighbors method is defined as:

$$w_{n,x}(X_i) = \{^{1 \quad d(X_i,x)<h}_{0 \quad otherwise}. \tag{10}$$

For each value of *h,* the sum of weights:

$$\sum_{i=1}^{n} w_{n,x}(X_i) = k,$$

is an integer that expresses the number of nearest neighbors around the curve *x* that were used to make the estimate. When the local cross-validation function is minimized with respect to *h* for each data point *x*, we get the value of h_x indirectly expressing an optimal number of curves to use for the estimate in *x*.

2.2.2 Chagny-Roche method

Unlike previously mentioned methods, this method presented by Chagny and Roche (2016) is not based on cross-validation, but rather on the actual estimate of the mean squared error. This method was derived in the classical L^2 metric space. Assuming that *K* is a type I Kernel, as defined by Ferraty and Vieu (2006), and the regression operator *r* is Hölder continuous with an exponent of β, then we can determine for $h>0$ the upper bound of the mean squared error as:

$$MSE(\hat{r},x,h) \le C\left(h^{2\beta} + \frac{\sigma^2}{n\varphi_x(h)}\right), \tag{11}$$

where *C* is a positive constant depending only on constants from type I Kernel definition. The first part of the expression corresponds to the bias term of the *MSE* and the second part corresponds to the variance term. Instead of the *MSE,* the upper-bound expression is minimized with respect to *h*.

As all parts of the expression cannot be determined in practice (namely, the parameter β and the small ball probability), an estimate of the expression is used:

$$ChR_x(h) = \hat{A}(h,x) + \hat{V}(h,x), \tag{12}$$

where $\hat{A}(h,x)$ approximates the bias term and $\hat{V}(h,x)$ is an empirical estimate of the variance. See the original paper for exact formulas (Chagny and Roche, 2016).

3 APPLICATION TO REAL DATA

3.1 Quality of estimates

In the context of kernel methods, the empirical version of the *MSE* (also called mean squared prediction error, Ferraty and Vieu, 2006) is the most commonly used tool to compare the quality of various estimates:

$$MSPE(\hat{r},h) = \frac{1}{n}\sum_{i=1}^{n}(\hat{r}(x_i,h) - y_i)^2. \tag{13}$$

Due to the squared difference between the estimated and the actual value, the empirical *MSE* is more sensitive to large differences. This is a desired property as large differences in regression estimates tend to be more problematic than smaller ones. Furthermore, all methods for optimal bandwidth selection are based on the theoretical *MSE*, so using the empirical version of the *MSE* is natural for their comparison.

3.2 Implementation of methods

For practical application of the presented methods, a few issues must be considered. The first is the choice of the set of possible bandwidth values, as it is not feasible to test over the entire theoretical interval. The other is the sparsity of the functional data – a situation where one curve is more distant from all the other curves than the selected optimal bandwidth is much more likely to happen than in finitely dimensional datasets, especially with estimates for the testing dataset. How these issues were resolved in respective implementations will be briefly discussed.

The general cross-validation method is implemented in companion material to the publication by Ferraty and Vieu (2006) and by default uses the 5^{th} to 50^{th} percentiles of distances between curves in a learning dataset (i.e. the dataset with known response values) to determine the optimal bandwidth. If the distance of a curve from all other curves is greater than the optimal bandwidth (and therefore the estimate for this curve cannot be computed), the value of h is increased until all curves can be evaluated.

The penalizing functions method was implemented to correspond to the original general cross-validation function, and the choice of bandwidth interval and distant data treatment therefore remained the same.

The nearest neighbors method was again implemented by Ferraty and Vieu (2006). The method uses the bandwidth interval spanning from the ten nearest curves to 50% of the nearest curves of the training dataset. The step in the sequence is defined as the number of training curves divided by 100 and rounded up to the nearest integer. Since the value of h is mediated through the number of neighbors, the situation in which one curve is too distant is not possible.

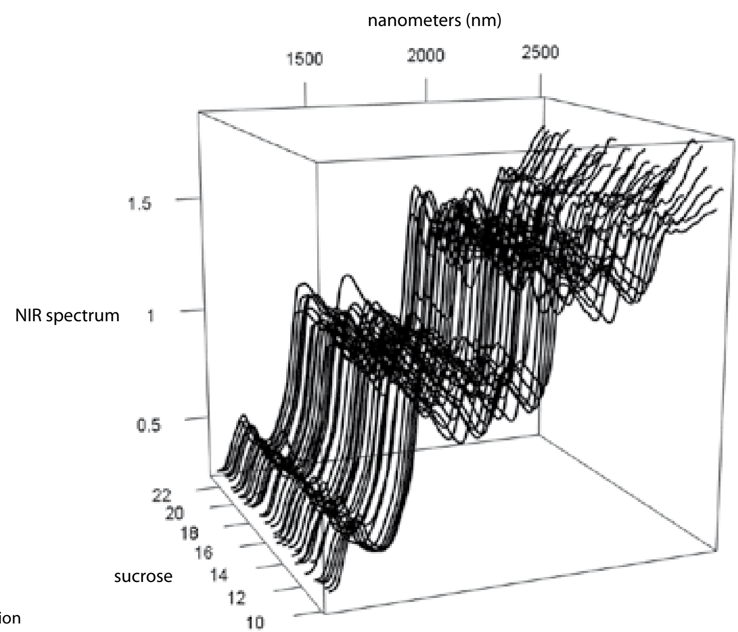
Using the original paper (Chagny and Roche, 2016), the Chagny-Roche method was implemented. The bandwidth interval was chosen in the same way as in the case of the general cross-validation – 5^{th} to 50^{th} percentiles of distances between curves, but separately for each curve. Thanks to this choice, distant curves are not possible.

To provide a fair comparison, a Nadaraya-Watson estimator with quadratic kernel was used for all bandwidth selection methods.

3.3 Spectrometric dataset – fat and sugar content in cookies

To compare the presented bandwidth selection methods to real data, we chose cookie dough spectrometric data (Osborne et al., 1984; Brown et al., 2001). The dataset contains a NIR (near-infrared spectroscopy) reflectance spectrum measured from 1 100 to 2 498 nanometres for each dough piece along with information about fat, sucrose, flour, and water content. Figure 1 illustrates the spectrometric curves that were used as the functional predictor.

Figure 1 NIR reflectance spectra for the cookie dough sorted by their sucrose content



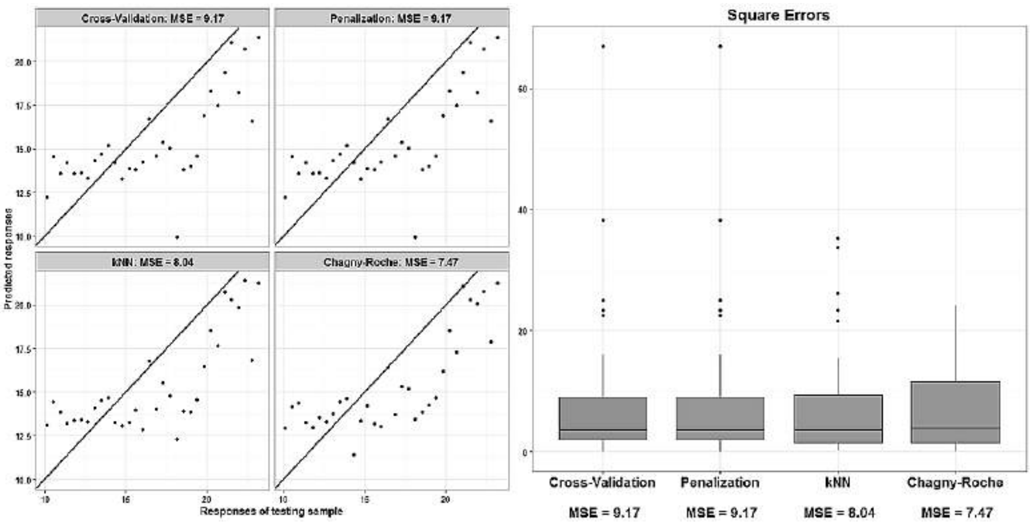
Source: Own construction

The fat and sucrose content were chosen as the two scalar responses. The semimetric based on the second derivative was used to measure the distance between curves as it tested as the most appropriate for this dataset. The dataset was divided into 40 training and 32 testing curves as per the original dataset description.

3.3.1 Sugar content

All four presented methods for bandwidth selection were used to find the optimal bandwidth parameter for the testing dataset, using the information from the training dataset. Afterward, predictions for the testing dataset were compared to the actual values, see Figure 2.

Figure 2 Performance of the four bandwidth selectors



Source: Own construction

The results show that both global selection methods perform the same. The methods based on local selection performed better than the global ones, and the Chagny-Roche method was the best method overall.

Figure 2 also illustrates the distribution of the squared error for all four methods. The boxplots show that prediction using global bandwidth selection methods was quite distant from the actual value in several cases. This phenomenon is less pronounced with the nearest neighbors method and almost non-existent with the Chagny-Roche method. The Chagny-Roche method is the only that does not use the cross-validation function, so the absence of outliers in its case might be attributed to its different mechanics.

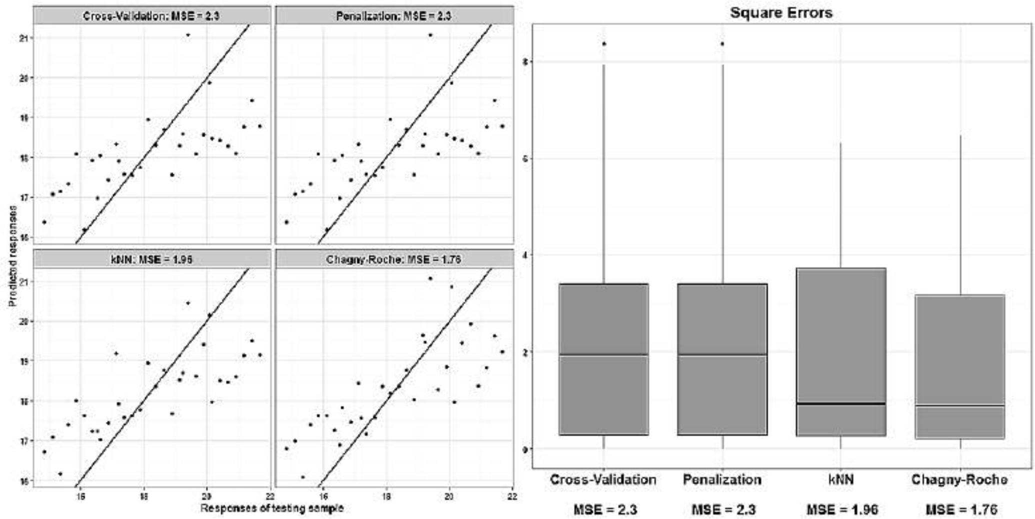
3.3.2 Fat content

The same procedure was applied to the data with fat content as the scalar response, see Figure 3 for the comparison.

The overall *MSE* was smaller, which was expected because the fat content had a lower variance than sugar content in this dataset. The results once again show that both local selection methods performed better than the global ones and that the Chagny-Roche method was the best method overall.

Boxplots on Figure 3 again illustrate that global bandwidth selection methods have more distant predictions than local methods, the superiority of the Chagny-Roche method being even more pronounced in this case.

Figure 3 Performance of the four bandwidth selectors



Source: Own construction

CONCLUSION

Bandwidth selection methods developed for a finite dimensional setting can be successfully transferred to the functional kernel regression. Besides methods already proven to work with an infinite setting (leave-one-out cross-validation, nearest neighbors method), the penalization functions method and a recent result in the field – the Chagny-Roche method – were successfully applied.

A summary of known methods applied to real data and their performance compared in terms of mean squared error for two different scalar responses was presented. According to the results, methods based on local bandwidth selection perform better. This is not a surprising result, as concerns about functional data sparsity appear to be justified. It is also consistent with previous findings by Benhenni et al. (2007), where the leave-one-out cross-validation and local nearest neighbors methods were compared using spectrometric data for predicting fat content in meat. An interesting finding is that the Chagny-Roche method, derived in a metric space setting, demonstrated the best performance even though a semimetric, not a metric, was used. We believe that local bandwidth selection methods should be further developed to obtain better methods for optimal bandwidth selection in an infinite-dimensional setting. We also should point out that finding the optimal bandwidth still leaves us with the problem of the optimal semimetric selection.

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Recent Publications and Events

New publications of the Czech Statistical Office

Expeditures and Consumption of Households Included in the Household Budget Survey in 2016. Prague: CZSO, 2017.
Farm Structure Survey 2016. Prague: CZSO, 2017.
Household Income and Living Conditions in 2016. Prague: CZSO, 2017.
Information Society in Figures 2017. Prague: CZSO, 2017.

Conferences

The ***20th International Scientific Conference Applications of Mathematics and Statistics in Economics (AMSE 2017)*** was held in **Szklarska Poręba, Poland from 30th August to 3rd September 2017.** This scientific conference is organized each year by the Faculty of Informatics and Statistics of the University of Economics in Prague (the Czech Republic), Wrocław University of Economics (Poland) and the Faculty of Economics of Matej Bel University in Banská Bystrica (Slovakia). The conference aims to acquaint its participants with the latest mathematical and statistical methods that can be used in solving theoretical and practical problems and challenges of economics. More information available at: *http://amse.ue.wroc.pl.*

The ***19th Joint Czech-German-Slovak Conference on Mathematical Methods in Economy and Industry (MMEI)*** took place **in Jindřichův Hradec, Czech Republic during 4–6 September 2017**. More information available at: *http://www.karlin.mff.cuni.cz.*

The ***25th Interdisciplinary Information Management Talks (IDIMT 2017)*** was held in **Poděbrady, Czech Republic, from 6th to 8th September 2017**. More information available at: *http://www.idimt.org.*

The ***35th International Conference on Mathematical Methods in Economics (MME 2017)*** took place in **Hradec Králové, Czech Republic, during 13–15 September 2017**. The conference is a traditional meeting of professionals from universities and businesses interested in the theory and applications of operations research and econometrics. More information available at: *http://fim2.uhk.cz/mme.*

The ***11th International Days of Statistics and Economics (MSED 2017)*** was held in **Prague, Czech Republic, during 14–16 September 2017**, organized by the Department of Statistics and Probability and the Department of Microeconomics, University of Economics, Prague, Czech Republic, Faculty of Economics, Technical University of Košice, Slovakia, and Ton Duc Thang University, Vietnam. The aim of the conference is to present and discuss current problems of statistics, demography, economics and management and their mutual interconnection. More information available at: *https://msed.vse.cz.*

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Authors and Contacts
Rudolf Novak*, Institution Name, Street, City, Country
Jonathan Davis, Institution Name, Street, City, Country
* Corresponding author: e-mail: rudolf.novak@domain-name.cz, phone: (+420) 111 222 333

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