THE ANALYSIS OF ADVANTAGES OF USING STOCK EXCHANGES IN NUMERICAL MODEL WITHIN THE COMMODITY PRICE FORECASTING

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Abstract: *In the paper the numerical model for forecasting prices on commodity exchanges which is based on the exponential approximation of commodity stock exchanges was derived. The price prognoses of aluminium on the London Metal Exchange were determined as numerical solution of the Cauchy initial problem for the 1st order ordinary differential equation. To make the numerical model more accurate the idea of the modification of the initial condition value by the aluminium price (stock exchange) was realized. The derived numerical models were observed to determine the influence of the modification of the initial condition value by stock exchange to the accuracy of the obtained prognoses. The causes of inappropriate use of chosen stock exchanges according to evolution of aluminium price were studied. By having analyzed the forecasting success of the chosen initial condition drift types, it was found out that the initial condition drifts provided more accurate prognoses mostly for all chosen types of the initial condition drifts. The suggested modification of the original model made the commodity price prognoses more accurate.*

Keywords: Exponential approximation; numerical modeling; price forecasting; commodity exchange

1 INTRODUCTION

Observing trends and forecasting movements of metal prices is still a current problem. There are a lot of approaches to forecasting price movements. Some of them are based on mathematical models [1], [2], [3], [4], [5], [6], [7], [9]. Forecasting prices on commodity exchanges often uses the statistical methods that need to process a large number of historical market data [1], [4], [9]. The quantity of needed market data can sometimes be a disadvantage. In our prognostic models numerical methods were used. Derived numerical models for forecasting prices were based on the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equations [5], [6], [7].

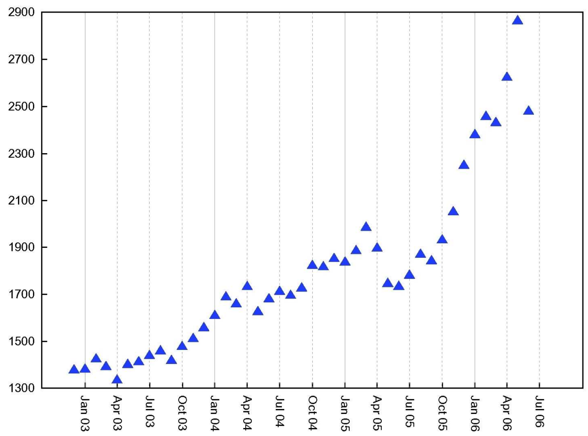

Fig. 1 Course of the aluminium prices on LME in the years 2003 – 2006.

The aluminium prices presented on the London Metal Exchange (LME) were worked on. We dealt with the monthly averages of the daily closing aluminium prices "Cash Seller&Settlement price" in the period from December 2002 to June 2006. The market data were obtained from the official web page of the London Metal Exchange [10]. The course of the aluminium prices on LME (in US dollars per tonne) within the observing period is presented in Figure 1.

2 MATHEMATICAL MODEL

We considered the Cauchy initial problem in the form:

$$y' = a_1 y, \;\; y(x_0) = y_0 \,. \tag{1}$$

The particular solution of the problem (1) is in the form $y = k\, e^{a_1 x}$, where $k = y_0\, e^{-a_1 x_0}$. The considered exponential trend was chosen according to the test criterion of the time series' trend suitability. The values $\ln(Y_{i+1}) - \ln(Y_i)$, for $i = 0, 1, \ldots, 42$ have approximate-ly constant course, where Y_i is the aluminium price (stock exchange) on LME in the month x_i.

The price prognoses were created by the following steps:

The 1st step: Approximation of the values – the values of the approximation term were approximated by the least squares method. The exponential function in the form $\tilde{y} = a_0 e^{a_1 x}$ was used. When observing the influence of the approximation term length on the prognoses accuracy, we found out that the prognoses obtained by longer approximation terms are more accurate [5]. Let us consider two different variants.

Variant B: The values from the period January 2003 – June 2003 were approximated. The next approximation terms were created by sequential extension of this period by 3 months. Thus the duration of the approximation terms was extended (the n^{th} approximation term has $6+3(n-1)$ stock exchanges) (see Figure 2).

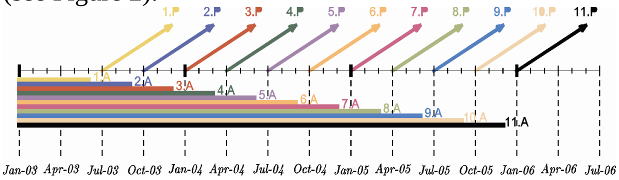

Fig. 2 Variant B
(A – approximation term, P – forecasting term)

Variant E: We approximated values within 12 months and each term was shifted by 1 month (see Figure 3). (The first approximation term was January 2003 – December 2003.)

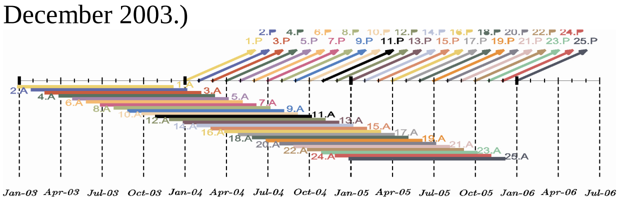

Fig. 3 Variant E
(A – approximation term, P – forecasting term)

36 forecasting terms of the original model in both variants B and E were observed. From among all forecasting terms, 11 of them belonged to variant B and 25 ones were part of variant E, whereby 9 forecasting terms were common for both variants.

The 2nd step: Formulating the Cauchy initial problem – according to the acquired approximation function $\tilde{y}$, the Cauchy initial problem (1) was written in the form :

$$y' = a_1 y, \;\; y(x_i) = Y_i, \qquad (2)$$

where $x_i = i$ and Y_i is the aluminium price on LME in the month x_i, which is the last month of the approximation term.

The 3rd step: Computing the prognoses – the formulated Cauchy initial problem (2) was solved by the numerical method based on the exponential approximation of the solution. A detailed solution method is seen in [8]. The method uses the following numerical formulae:

$$x_{i+1} = x_i + h,$$

$$y_{i+1} = y_i + bh + Qe^{vx_i}\left(e^{vh} - 1\right),$$

for $i = 1, 2, 3, \ldots$, where $h = x_{i+1} - x_i$ is the constant size step. The unknown coefficients are calculated by means of these formulae:

$$v = \frac{f''(x_i, y_i)}{f'(x_i, y_i)}, \;\; Q = \frac{f'(x_i, y_i) - f''(x_i, y_i)}{(1-v)\,v^2 e^{vx_i}},$$

$$b = f(x_i, y_i) - \frac{f'(x_i, y_i)}{v}.$$

If we consider the Cauchy initial problem (2), the function $f(x_i, y_i)$ has the form $f(x_i, y_i) = a_1 y_i$ and then $f'(x_i, y_i) = a_1 y'(x_i) = a_1{}^2 y_i$, $f''(x_i, y_i) = = a_1{}^2 y'(x_i) = a_1{}^3 y_i$.

We calculated the prognoses within six months that follow the end of the approximation term in this way.

- **The first month prognosis** was determined by solving the Cauchy initial problem in the form (2). The interval $\langle x_i, x_{i+1} \rangle$ of the length $h = 1$ month was divided into n parts, where n is the number of trading days on LME in the month x_{i+1}. We got the sequence of the division points $x_{i0} = x_i$, $x_{ij} = x_i + \frac{h}{n} j$, for $j = 1, 2, \ldots, n$, where $x_{in} = x_{i+1}$. For each point of the subdivision of the interval, the Cauchy initial problem in the form (2) was solved by the chosen numerical method. In this way we obtained the prognoses of the aluminium prices on single trading days y_{ij}. By calculating the arithmetic mean of the daily prognoses we obtained the monthly prognosis of the aluminium price in the month x_{i+1}. So, $y_{i+1} = \frac{\sum_{j=1}^{n} y_{ij}}{n}$.
- **The prognoses for the following five months** were calculated after modification of the initial condition value. The initial condition value in the month x_{i+s}, $s = 1, 2, 3, 4, 5$ was replaced either by the calculated monthly prognosis y_{i+s} (the original model) or in case of higher absolute percentage error of given monthly prognosis y_{i+s} by some aluminium stock exchange (the modified model). The Cauchy initial problem $y' = a_1 y,\, y(x_{i+s}) = y_{i+s}$, or $y' = a_1 y,\, y(x_{i+s}) = Y_p$ (where Y_p is chosen aluminium stock exchange) was used for calculating daily prognoses and their arithmetic mean served to define the monthly price prognosis y_{i+s+1} for the month x_{i+s+1}.

By comparing the calculated prognosis y_s in the month x_s with the real stock exchange Y_s, the absolute percentage error $|p_s| = \frac{|y_s - Y_s|}{Y_s}.100\,\%$ was determined. The price prognosis y_s in the month x_s is acceptable in practice, if $|p_s| < 10\,\%$. Otherwise, it is called the critical forecasting value of. To compare the accuracy of the forecasting of all forecasting terms, the mean absolute percentage error (MAPE) $\bar{p} = \frac{\sum_{s=1}^{t} |p_s|}{t}$ was determined, where, in our case, $t = 6$.

The modification of the initial condition value by the real aluminium stock exchange price was called the initial condition drift. Let us name the selected minimal absolute percentage error of the prognosis, causing the initial condition drift, the limiting value error. The month in which the absolute percentage error of the prognosis had at least the limiting value error was considered as the limiting month.

3 RESULTS

3.1 The influence of the initial condition drift on the forecasting accuracy

We started from the original model calculating the prognoses within six months following the approximation term after modification of the initial condition value by the obtained monthly price prognoses [5]. Based on the prognosis accuracy analysis of the original model, forecasting terms were classified into three classes [6]:

I. trouble free forecasting terms (18 terms)
All absolute percentage errors of the monthly prognoses were less than $10\,\%$.

II. forecasting terms with a small error (10 terms)
The mean absolute percentage error of the forecasting term was less than $10\,\%$, but the absolute percentage errors of some monthly prognoses were at least $10\,\%$.

III. forecasting terms with a big error (8 terms)
The mean absolute percentage error of the forecasting term was at least $10\,\%$.

Within the studied group of 36 forecasting terms, the forecasting within 14 of them was so accurate that the initial condition drift did not occur. The initial condition values were replaced just by calculated monthly prognoses. Since in the remaining 22 forecasting terms the forecasting was less accurate, some of the prognoses gained the absolute percentage error higher than chosen limiting value error, and the initial condition drift occurred. Therefore the forecasting results differ from the original model.

Three types of the initial condition drift with regard to their length were considered, namely one-month drift, drift before the limiting month and drift to the limiting month. One-month drift was the shortest chosen initial condition drift, where the initial condition value was replaced by the stock exchange Y_{i+p}, $p = 1, 2, 3, 4, 5$ in the month x_{i+p}, where x_i was the last month of the approximation term and p was the initial condition drift order in the forecasting term. Using drift before the limiting month, the initial condition value was replaced by the stock exchange Y_{L-1} in the month x_{L-1}, where x_L was the limiting month. By means of drift to the limiting month the stock exchange Y_L in the month x_L changed the initial condition value. The limiting value errors of the size 7 % and 8 % were chosen. Thus, within each forecasting term in the variants B and E, six different types of the initial condition drift were taken into account (for three different lengths of the initial condition drift two different sizes of the limiting value error were chosen).

Table 1 The influence of the initial condition drifts on the forecasting accuracy – variant B.

Type of forecasting term	Positive influence of drifts		Negative influence of all types of drift
	For all types of drift	Only for some types of drift	
Trouble free forecasting terms		January 2004 – June 2004 (drift to the limiting month, 7%, 8%)	
Forecasting terms with a small error	October 2003 – March 2004	July 2005 – December 2005 (one-month drift, 8%)	
	January 2005 – June 2005		
	January 2006 – June 2006		
Forecasting terms with a big error	April 2005 – September 2005		
	October 2005 – March 2006		

Table 2. The influence of the initial condition drifts on the forecasting accuracy – variant E.

Type of forecasting term	Positive influence of drifts		Negative influence of all types of drift
	For all types of drift	Only for some types of drift	
Trouble free forecasting terms	May 2004 – October 2004	January 2004 – June 2004 (drift to the limiting month, 7%, 8%)	
		February 2005 – July 2005 (one-month drift,7%)	
Forecasting terms with a small error	January 2005 – June 2005	March 2005 – August 2005 (one-month drift, 7%)	
	May 2005 – October 2005	June 2005 – November 2005 (one-month drift, 7%)	
	July 2005 – December 2005		
	January 2006 – June 2006		
Forecasting terms with a big error	April 2005 – September 2005		
	August 2005 – January 2006		
	September 2005 – February 2006		
	October 2005 – March 2006		
	November 2005 – April 2006		
	December 2005 – May 2006		

For each forecasting term, in which the initial condition drift was occurred, the success rate of the initial condition drift was determined by observing the change of MAPE. We were interested in whether the forecasting improvement occurred for all chosen types of the initial condition drift. The success rate of the forecasting within each group of the forecasting terms with different error rate of the original model was studied (trouble free forecasting terms, forecasting terms with a small error and forecasting terms with a big error). Table 1 and Table 2 show the distribution of the forecasting terms according to the influence of chosen types of the initial condition drift on the forecasting accuracy in the variants B and E. When the influence of the forecasting by means of the initial condition drift was partial, the type of the drift in which the forecasting by the original model was more accurate is written in the brackets, which means that this type of the initial condition drift was not suitable within the observed forecasting term.

The tables clearly show that within all forecasting terms in which the initial condition drift occurred the forecast using the chosen strategy became more accurate. No worse results of the original forecast at all types of the initial condition drift were observed. In spite of a positive influence of the initial condition drift on the prognosis accuracy there were the forecasting terms in which the improvement of the forecasting results occurred only for some types of the drift. Table 3 and Table 4 show the number of the forecasting terms in which the forecasting by the initial condition drift was either totally accurate (for all chosen types of the initial condition drift) or only partially accurate (for some of the chosen types of the initial condition drift).

Table 3 Positive influence of the initial condition drifts on the forecasting accuracy – variant B.

Positive influence of drifts	Trouble free forecasting terms	Forecasting terms with a small error	Forecasting terms with a big error
Partial	1	1	0
Total	0	3	2

Table 4 Positive influence of the initial condition drifts on the forecasting accuracy – variant E.

Positive influence of drifts	Trouble free forecasting terms	Forecasting terms with a small error	Forecasting terms with a big error
Partial	2	2	0
Total	1	4	6

In both variants B and E the forecasting by the initial condition drift was so advantageous that the forecasting improvement was noticed for all chosen types of the initial condition drift. From among 7 forecasting terms in the variant B, in which the initial condition drift was used, the total positive influence of the forecasting by the initial condition drift was observed within 5 terms. In the variant E from among 15 forecasting terms, the decline of the mean absolute percentage error for all drift types occurred within 11 of them.

By observing the positive influence of the initial condition drift within the forecasting terms with different error rate of the original forecasting, it was found out that the strategy of replacing the initial condition value by a chosen stock exchange significantly improved the forecasting within the forecasting terms with a big error, within which the forecasting by the original model had failed. Within such terms the use of any types of the initial condition drift did not worsen the prognosis results obtained by the original model. Within the forecasting terms with a small error, the forecasting by the initial condition drift was more often accurate for all types of chosen drifts. On the contrary, within the trouble free forecasting terms, the forecasting by the initial condition drift was mostly suitable just for some types of the drift.

3.2 The influence of the commodity price course on the accuracy of the forecasting by the initial condition drift

Within the analysis of the causes of the acquired results it is necessary to observe the course of the commodity prices. All chosen types of the drift made the forecasting more accurate within the most problematic price movements, such as a steep price increase and rapid changes in the price course. Using the initial condition drift the prognoses approached steeply increasing or decreasing stock exchanges, and forecasting became more accurate.

Within 6 forecasting terms in the variants B and E the forecasting by the initial condition drift failed for some types of the drift. The disadvantages of using these types of the initial condition drift by the commodity price course within the observed forecasting term were analyzed. In the fluctuating price course (*January 2004 – June 2004, variant B and E*) the longest initial condition drift, drift to the limiting month, failed. The initial condition drift within oscillations was caused by local maximal or minimal value. The accuracy of the following calculated prognoses depended on stock exchange that replaced the initial condition value. The disadvantage of the longest drift consisted in the initial condition shift to inappropriate local maximal or minimal value, thus making the forecasting of the next prices less accurate. Thus, within the unstable price course we recommend to use shorter drifts (one-month drift and drift before the limiting month).

By having analyzed the failure of the commodity price forecasting using the initial condition drifts with different length [7], the shortest chosen initial condition drift, the one-month drift, often obtained the least accurate forecasting results. It failed especially within significant long-term price changes. In the forecasting terms, when price decline after price increase occurred (*February 2005 – July 2005, March 2005 – August 2005*) the calculated prognoses were also increasing due to the price increase in the approximation term. The initial condition drift was caused by the prognosis error in the month with a steep price decline. By using one-month drift, the initial condition value was multiple replaced by increasing stock exchanges (higher prices) which were at the beginning of forecasting terms. So, that was a problem for the forecasting of price in decline. Using the limiting value error 7 %, the initial condition drift more often occurred, and thus the forecasting of stock exchanges in decline was more inaccurate than in the original model using lower prices for initial condition values. In case of increase after price decline (*June 2005 – November 2005, July 2005 – December 2005*) the insufficiently long initial condition drift by means of the one-month drift which using the limiting value error 7 % was also repeated within the forecasting term. The initial condition value repeatedly acquired the value of aluminium stock exchange in decline, so the forecasting could not follow a steep price increase. The situation was worsened by moderate increasing or decreasing approximation functions (it was due to previous price decline). Thus, the difference between the real price and prognosis increased and the forecasting using the shortest initial condition drift was worse than the forecasting by the original model.

4 CONCLUSION

The original model forecasted the aluminium price reliably within the stable price course, when the price did not changed rapidly. Within the rapid increase or decrease of stock exchanges, but also in the case of changes in the price course the forecasting failed. Since the variability with rapid and sudden changes is typical of the commodity price course, we judged the possibility of making the forecasting more accurate by using the modification of the initial condition value by aluminium price.

With regard to the results, the strategy of the initial condition drift significantly improved the calculated prognoses. Within the most problematic period of the price movements, all chosen types of the initial condition drift were rather successful. By using them, calculated prognoses were moved closer to the real stock exchanges, so the forecasting became more accurate than the forecasting by the original model. Within this type of the price course, we do not recommend using short types of the initial condition drift, especially not the one-month drift.

When the price fluctuations appeared in the observed period, not any type of the initial condition drift was advantageous. The most accurate was the drift replacing the initial condition value by stock exchange that was the nearest to the next price course. The longest drift usually was not suitable. Therefore we prefer to use shorter initial condition drifts.

Within all forecasting terms, the original forecasting became more accurate when using the most successful type of the initial condition drift. Thus chosen strategy of the initial condition drift was suitable way of the original forecasting improvement.

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