JEL Classification: G11, Q02
Keywords: minimum-variance portfolio, energy, precious metals, risk-adjusted ratios

# Energy Commodity Price Risk Minimization with Precious Metals in a Multivariate Portfolio

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*Abstract*

*This paper constructs four minimum-variance multivariate portfolios, combining energy commodities (Brent oil, WTI oil, gasoline and natural gas) with precious metals (gold, silver, platinum and palladium). In order to reflect different situations of market participants, we impose constraints of minimum energy share in portfolio in amount of 30% and 70%. Portfolio optimization indicates that highest share in all portfolios have gold, while only in two cases some tiny percentage goes to palladium. Silver and platinum do not have share in portfolios, whatsoever. We find more risk reduction in 30% portfolios, than in 70% portfolios, which means that investors who want to pursue less risky energy-portfolio should include more gold in portfolio. Examining some characteristics of portfolios, we find that portfolio with natural gas has the lowest downside risk, in both 30% and 70% energy-portfolios. According to various hedge effectiveness indices, in most cases, the most effective risk-reduction we find in portfolio with natural gas. Brent portfolio has the highest Sharpe and Sortino ratio, but when mCVaR is taken into account, then natural gas has the best return-risk results.*

## 1. Introduction

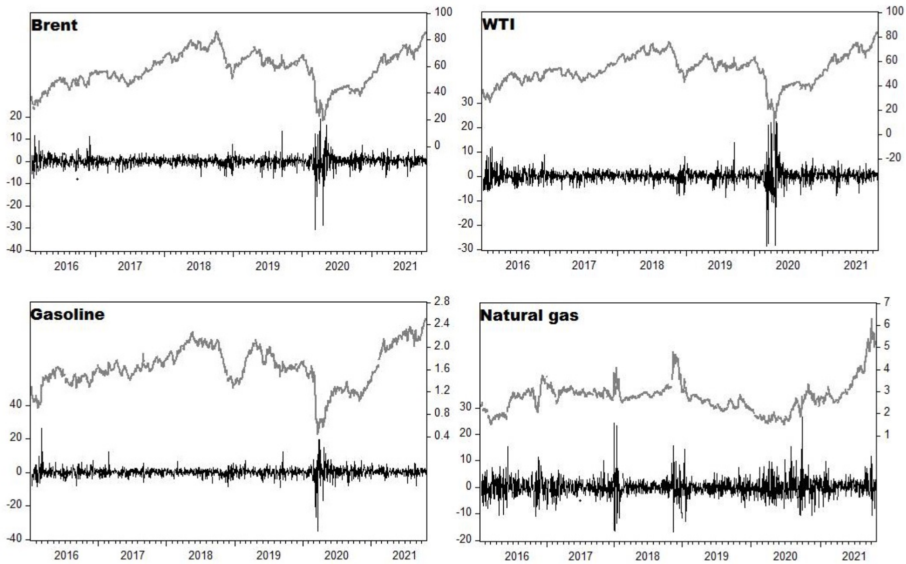
Development of today's world relies heavily on fossil fuels, while frequent and significant price oscillations of these commodities have considerable impact on the socioeconomic activities worldwide. Alvarez-Ramirez et al. (2015) asserted that global rising demand of fossil fuels, backed by diversity of geopolitical concerns, induces high price volatilities of these assets. Prices of energy commodities have exhibited large swings in last two decades, which are caused mainly by current and expected global economic activity and uncertainties in supply and demand. For instance, Salisu et al. (2021) contended that ongoing COVID19 pandemic has led to global economic slowdown, which caused a drop of West Texas intermediate oil price below zero in April 2020. Ali et al. (2020) claimed that huge crude oil price swings happened partly due to the COVID19 pandemic and partly due to political manoeuvres among oil producers during the period. Very intense price oscillations are also recorded in Brent and gasoline markets in the first half of 2020 when COVID19 crisis came to the fore, as Figure 1 depicts. Obvious price fluctuations of crucial energy prices, imposes a concern for various market participants, such as

https://doi.org/10.32065/CJEF.2022.01.03

The authors thank two anonymous referees for their helpful suggestions and comments.

producers, traders, investors, policy makers, and raises them a question how to find a best way to protect themselves against energy price uncertainties.

**Figure 1 Price and Return Dynamics of Four Major Energy Markets**




*Notes:* Greyed line indicate price dynamics of four energy assets, while black line denotes returns dynamics. Right (left) Y axis represent prices (rate of returns) of energy commodities. Prices of Brent oil and WTI oil are expressed in U.S. dollars per barrel. Gasoline is in US dollars per gallon, while natural gas is in US dollars per million metric British thermal unit (mmBtu).

The best-known risk-reducing asset in the world is gold, as many recent papers documented and confirmed this assertion (see e.g. Dutta et al., 2020; Alkhazalia et al., 2020; Trabelsi et al., 2021; Živkov et al., 2021). First reason why gold is chosen by many researchers stems from the fact that gold has relatively low volatility, comparing to other financial and commodity assets (see e.g. Narayan et al., 2010; Jin et al., 2019). The paper of Ji et al. (2020) found strong hedging characteristics of gold during COVID19 pandemic, and this is the second reason why many recent papers took into account gold, when they researched portfolio construction during pandemic period. Third, and probably the strongest reason why gold is an appropriate instrument in portfolio with energy assets, is derived from the fact that energy and gold markets behave intrinsically different in crisis and tranquil periods. In other words, energy markets are strongly linked with global economic activity, which price is dominantly determined by global demand, while gold is more regarded as safe haven and store of value, which price is not susceptible to strong price movements (see Beckmann et al, 2019). This means that when global economy is in downturn, prices of fuel also fall due to falling fuel demand. On the other hand, price of gold in these occasions starts to rise, because gold is perceived as a safe asset that has relatively stable value, which prompts investors to start buying gold in order to avoid losses. In periods of economic prosperity, reverse happens, i.e. prices of fuel rise, because economic activity intensifies, while prices of gold stagnate or fall, because investors move their funds in more lucrative investments, such as stock markets. Therefore, gold can serve as a good hedging instrument for fuel markets.

Unlike most of the papers that hedge energy only with gold (see e.g. Das et al., 2020, AlKhazali et al. 2021, Salisu et al., 2021), this paper makes a multi-asset minimum variance portfolio (MVP) of Markowitz between four spot energy commodities – Brent oil, WTI oil, gasoline and natural gas and four precious metals futures – gold, silver, platinum and palladium. In other words, we design four five-asset MVPs, consisting of one energy commodity and four precious metals, with aim to find optimal weights of every asset in a portfolio that will have minimum variance. In such portfolios, the fuels are perceived as primary asset in a portfolio, while precious metals are auxiliary assets. The main reason why we broaden analysis to all precious metals lies in the fact that these metals have very low correlation with all energy commodities, as Table 1 shows, which is an important precondition for successful portfolio optimization. According to Table 1, gold has the lowest correlation with all selected fuels, which hardly exceed zero. This also applies for natural gas, while other precious metals have correlations between 10-20% with the selected fuels.

According to our best knowledge, we are familiar only with two papers which combined all four precious metals with oil (Salisu et al., 2021; Mensi et al., 2021). However, we do not know any paper that combine four different fuels with all precious metals in a multi-asset Markowitz portfolio. This gives us a lot of space to fill up a void in the international literature, and this is the reason why we find a motivation to carry out this research.

**Table 1 Mutual Pearson Correlations between the Energy Commodities and Precious Metals**

| | *Gold* | *Silver* | *Platinum* | *Palladium* |
|---|---|---|---|---|
| *Brent* | 0.043 | 0.124 | 0.198 | 0.143 |
| *WTI* | 0.025 | 0.117 | 0.164 | 0.158 |
| *Gasoline* | 0.006 | 0.114 | 0.179 | 0.171 |
| *Natural gas* | -0.004 | 0.010 | 0.037 | 0.016 |

In the process of minimum-variance portfolio optimization, we want to accentuate two different risk-minimizing strategies that reflect different situations of market participants. Namely, not all market agents that work with energy commodities are in a position to conduct wide-range diversification of the energy assets, i.e. to hold large amounts of precious metals futures contracts. This group of market participant could be fuel producers or energy traders, who have large quantities of energy in their possession, and they could not make a large-scale diversification, because they would have to invest a lot of funds in auxiliary assets, which they simply cannot do. In order to portray this situation, we restrict weight of energy in multivariate portfolio to minimum of 70%[1]. Another group of market participants could involve hedge funds, individual investors or portfolio managers, who want to invest in energy commodities, but not too much, which leaves them

[1] The way of how we constrain the minimum share of energy in portfolios is totally arbitrary, and it is set up only to reflect two diametrically different situations.

much of a space to invest in some other assets as well. For this type of market agents, we restrict weight of energy in a multi-asset portfolio to minimum 30%.

In order to be thorough in the analysis, after construction of four multi-variate portfolios, we evaluate their downside risk and mean-variance characteristics. Downside risk is important to measure, because it takes into account the only risk that is important for market participants, and that is the risk of potential losses. Parametric Value-at-Risk is a basic measure of downside risk, which was originally introduced by J.P. Morgan in 1994, and regarding a long-position investor, a left tail VaR shows minimum loss within a given time period, assuming pre-specified probability level. VaR essentially observes one particular quantile in left tail of the Gaussian distribution, which can be insufficient when actual loss exceeds VaR. Therefore, we also consider parametric conditional Value-at-Risk (CVaR), also known as Expected Shortfall (ES), which measures an average expected loss of the particular level of probability. However, both parametric downside risk measures consider in their calculations only first two moments, neglecting skewness and kurtosis of distribution. From this regard, parametric risk measures are perceived as inferior risk measures to semiparametric risk measures, which are also known as modified VaR (mVaR) and modified CVaR (mCVaR). Both mVaR and mCVaR are based on so called Cornish-Fisher expansion, whereby modified VaR was firstly introduced by Favre and Galeano (2002).

Knowing a downside risk level of multi-asset portfolio is very important. However, this is only half of a story, because every portfolio designer wants to know how much he can earn by holding particular portfolio. In this regard, we calculate four different return-to-risk ratios for every created MVP. The first indicator is a traditional Sharpe ratio, which observes relation between excess returns and common standard deviation. The basic deficiency of Sharpe ratio is that it uses standard deviation as a measure of risk, and standard deviation is biased risk measure since it gives equal weight to positive and negative returns. Therefore, we also calculate Sortino ratio and modified Sharpe ratio, which correct this potentially serious drawback of a common Sharpe ration. Sortino ratio uses as denominator only negative returns, which is known as downside deviation. This means that Sortino ratio calculates level of earnings per unit of average downside deviation. Modified Sharpe ratio is even stricter indicator comparing to Sortino ration, which means that it uses as denominator only specific set of negative returns under certain level of probability. We gauge these negative returns by the strictest downside risk measure, which is mCVaR. The last indicator is Treynor ratio, which observes sensitivity of a portfolio to systemic risk. In other words, Treynor ratio observes a relation between excess returns and beta (β), as a measure of systemic risk in denominator. In other words, this ratio tells portfolio holder what is the level of earnings per unit of market risk or beta. Beta alone indicates a reaction of a portfolio to systemic risk. If beta is higher (lower) than 1, portfolio reacts stronger (weaker) to external shocks than the whole market. If beta is equal to one, then portfolio reacts in the same way as whole market. All four indicators are better if their values are higher.

Besides introduction, the rest of the paper has following form. Second section presents literature review. Third section explains used methodologies – multivariate portfolio construction and the way how we calculate downside risk measurements and four return-to-risk ratios. Fourth section contains dataset. Fifth section has two subsections – minimum-variance portfolio construction and characteristics of created portfolios. The last section concludes.

## 2. Review of the Literature

Number of papers researched construction of portfolios in order to reduce volatility of energy assets, but these papers mostly refer to oil. For instance, Das et al. (2020) examined the hedging and safe-haven properties of Bitcoin against crude oil implied volatility (OVX) and structural shocks. Applying a dummy variable GARCH and quantile regression model, they compared the hedging and safe-haven performance of Bitcoin with gold, commodity and US Dollar. They found that Bitcoin is not the superior asset over others to hedge oil-related uncertainties, whereby hedging capacity of different assets is conditional upon the nature of oil risks and market situation. AlKhazali et al (2021) used the stochastic dominance approach, in order to examine whether the gold-oil portfolio return stochastically dominates the oil portfolio return. They reported that the gold-oil portfolio stochastically dominates the one without gold. They also indicated that portfolio risk decreases as more gold is added into the oil portfolios. They concluded that risk-averse investors in the oil market should include gold in their portfolios to maximize their expected utilities. Mensi et al. (2021) studied the volatility transmission between crude oil and four precious metals and investigated whether oil can be considered as a hedge or safe-haven asset against four precious metals. They concluded that Brent oil is a diversifier, but a weak safe haven for precious metals. They contended that combined portfolio composed of Brent oil and precious-metals futures could yield better hedging effectiveness. Hammoudeha et al. (2013) used Value-at-Risk to analyze the downside market risk associated with four precious metals, oil and the S&P 500 index. They contended that optimal portfolios should have more gold than any of the other assets under study over the sample period, which contradicts the conventional wisdom that about 10% of a diversified portfolio should be in gold. They reported that most efficient VaR-based portfolio consists of gold, oil and the S&P500, while pure precious metals portfolio is the least efficient.

Salisu et al. (2021) empirically evaluates the safe haven and hedging properties of gold during oil price crisis. Their results show statistically significant bidirectional returns and volatility spillovers between gold and crude oil returns. They validated the hedging effectiveness of gold against risks associated with crude oil, particularly during the pandemic period. They also investigated whether other precious metals such as palladium, platinum and silver exhibit similar features as gold. They confirmed this assumption, but with lower magnitudes. The paper of Elsayed et al. (2020) analysed the time patterns of volatility spillovers between energy market and stock prices of seven major global financial markets. They calculated optimal weights and hedge ratios for portfolio diversification and risk management. They concluded that returns of World Stock Index and World Energy Index are major transmitters of volatility to clean energy market, whereas they asserted that the optimal portfolio between energy and stock prices are heavily weighted to the stock markets. Ling et al. (2019) researched the risk transmission and hedging strategies between natural gas market and stock markets, using a multivariate GARCH model. The results showed that there exists granger causality from natural gas market to the

Chinese stock markets in crisis regime. As for an optimal portfolio, they asserted that investors in stock markets should have more stocks than natural gas asset in order to reduce their portfolio risk.

Regarding the existing papers, this study contributes to the existing literature in several dimensions. To the best of our knowledge, this is the first paper that combines four different energy assets with four precious metals in Markowitz portfolio. Also, this is the first paper that imposes restrictions to energy assets in portfolio in order to reflect different positions of market participants. Last but not least, every constructed MVP is evaluated from a range of downside risk and mean-variance indicators, which also has never been done before.

## 3. Methodology

### 3.1 Portfolio Optimization Theory

This paper constructs multi-asset MVP, whereby all selected energy commodities are observed as assets that need to be hedged. In other words, all energy assets are combined with four precious metals in a single minimum-variance five-asset portfolio. In this process, we use portfolio optimization procedure that was developed by Markowitz (1952) in modern portfolio theory. The theory graphically presents efficient portfolios *via* efficient frontier line that illustrates all portfolios with maximum returns under particular level of risk, or minimum portfolio risk under particular level of returns. MVP is placed at curvature of efficient frontier line, which means that this portfolio has the smallest risk of all possible portfolios. Horizontal line that divides efficient frontier line, split all possible portfolios into set of efficient portfolios and set of inefficient portfolios. Efficient portfolios are those that have rising risk with rising returns, which is acceptable from the investor’s point of view, whereby the only question is what is the level of investor's risk aversion. Inefficient portfolios have rising risk with the lowering returns, which is bad choice for every investor. All dots within efficient frontier line represent particular assets that have inferior risk-return performances comparing to MVP and efficient portfolios.

Generally speaking, in the process of portfolio weight calculation of minimum-variance portfolio, optimization procedure takes into account variance of all assets in portfolio as well as their pairwise correlations. Basically, an optimization process is achieved by changing weights of assets that are components of a portfolio, with an aim to find the best combination of assets that minimizes portfolio risk (see e.g. Armeanu and Balu, 2008; Cha and Jithendranathan, 2009; Guran et al., 2019).

This paper[2] set up an objective function in a form of minimum portfolio variance, which is presented in equation (1).

$$min\ \sigma_p^2 = min\ \sum_{i=1}^{N} W_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} W_iW_j\sigma_i\sigma_j\rho_{i,j}, \tag{1}$$

where $\sigma_p^2$ is portfolio variance, $\sigma_i^2$ is variance of a particular asset $i$, $W_i$ denotes calculated weight of asset $i$ in a portfolio, while $\rho_{i,j}$ is correlation coefficient between the particular pair of assets ($i$ and $j$).

[2] Portfolio optimization was conducted by ’PortfolioAnalytics’ package in ’R’.

Every portfolio with minimum variance has corresponding rate of returns, i.e. weighted average portfolio return ($r_p$), which can be calculated as in equation (2).

$$r_p = \sum_{i=1}^{N} W_i r_i \tag{2}$$

where $r_i$ is particular rate of return of an asset in portfolio.

In our portfolio optimization procedure, we set up several constraints. Standard constraint is the sum of all asset weights in portfolio must be equal to 1 (see equation (3).

$$\sum_{i=1}^{N} W_i = 1 \tag{3}$$

Our second constraint refers to the the minimum amount of energy assets in portfolios, i.e. one portfolio that has lower weight of energy and higher weight of precious metals, while the other one has higher weight of energy and lower weight of precious metals. The former one should represent a situation where portfolio manager or investor aims to include energy asset in a portfolio, but not too much, leaving a majority of room for other assets. The latter portfolio should depict situation of an energy producer or trader, who has large quantities of energy assets in its possession. They want to mitigate risk of a particular energy asset, but they do not have enough funds to make a wide-scale diversification, which means that the majority of portfolio holdings remains in energy. More specifically, in situation (a), we constrain the minimum weight of energy in portfolio to 30%, while in situation (b), minimum weight of energy in portfolio is not less than 70%. These two, portfolio allocation positions are given in equation (4). At the same time, by calculating two diametrically different portfolio strategies, we can see how much their downside risk and mean-variance characteristics differentiate, which can be an indication which asset should be added (reduced) in a portfolio. In addition, by setting the weight restrictions of energy, we prevent reduction of energy weights to zero or very low percentage, because all energy assets have significantly higher risk than precious metals.

$$a)\ W_{energy} \geq 0.3; \quad b)\ W_{energy} \geq 0.7 \tag{4}$$

The last restriction pertains to the weight of particular precious metal in two portfolios, which is written in expression (5).

$$a)\ 0 \leq W_i \leq 0.7; \quad b)\ 0 \leq W_i \leq 0.3; \;\; i = 1,2,\ldots,N \tag{5}$$

In order to quantitatively estimate how much risk reduction is achieved by construction of minimum-variance portfolio, we use Hedge effectiveness indices (HEI) of variance and two stricter downside risk measures – CVaR and mCVaR. Portfolio HEI risk measure ($HEI_{RM}$) is calculated in the following way:

$$HEI_{RM} = \frac{RM_{unhedged} - RM_{hedged}}{RM_{unhedged}}, \tag{6}$$

where subscript $RM$, in this paper, denotes particular down-side risk measure of a portfolio, i.e. Var, CVaR or mCVaR. Subscript $unhedged$ refers to investment only in energy commodity, whereas the label $hedged$ indicates to investment in a five-asset minimum-variance portfolio. As much as HEI index is closer to 1, the better hedging effectiveness is, and *vice-versa*.

### 3.2 Portfolio Characteristics

After construction of four minimum-variance portfolios, we intend to comparatively check their downside risk and return-to-risk characteristics. In this regard, we can tell which portfolio has the greatest reduction of risk, comparing to unhedged energy asset, the lowest downside risk and the highest various return-to-risk ratios.

#### 3.2.1 Downside Risk Measures

For the purpose of downside risk measurement, we apply several parametric and semiparametric methods, whereby we start with the basic parametric Value-at-Risk. This measure calculates the minimum loss within a single day under certain level of probability. Traditional way of calculating parametric VaR takes into account only first two moments of standard normal distribution, and equation (7) shows how it is done:

$$VaR_{\alpha} = \mu + Z_{\alpha}\sigma \tag{7}$$

where $\mu$ and $\sigma$ are mean and standard deviation of a particular energy-portfolio, respectively, whereas $Z_{\alpha}$ stands for left quantile of the normal standard distribution.

VaR observes only certain quantile in the left tail of distribution, under particular probability, while all other quantiles (potential estimates of loses) are neglected. This is a serious drawback of common VaR measure, because it cannot recognize loses when actual loss exceeds this level. In order to overcome this issue of basic VaR, we also calculate more strict measure of downside risk – Conditional Value-at-Risk of Rockafellar and Uryasev (2000). Equation for Conditional Value-at-Risk is given as follows:

$$CVaR_{\alpha} = -\frac{1}{\alpha}\int_{0}^{\alpha} VaR(x)dx \tag{8}$$

where $VaR(x)$ is Value-at-Risk of a particular energy-portfolio, while α denotes the left quantile of the standard normal distribution.

However, both VaR and CVaR assume Gaussian distribution of a portfolio, which is very strict assumption, and usually incorrect, which can yield biased parametric downside risk estimates. Due to this reason, we additionally calculate modified VaR and CVaR that are based on a Cornish–Fisher expansion approximation. These measures, besides first two moments, also take into account higher moments, i.e. skewness and kurtosis of the portfolio distribution, which might produce more accurate estimates of a downside risk. mVaR for short position can be calculated as in expression (9):

$$mVaR_{\alpha} = \hat{\mu} + Z_{CF,\alpha}\hat{\sigma}, \tag{9}$$

where $Z_{CF,\alpha}$ is the non-normal distribution percentile adjusted for skewness and kurtosis according to the Cornish–Fisher expansion (10):

$$Z_{CF,\alpha} = Z_{\alpha} + \frac{1}{6}(Z_{\alpha}^2 - 1)S + \frac{1}{24}(Z_{\alpha}^3 - 3Z_{\alpha})K - \frac{1}{36}(2Z_{\alpha}^3 - 5Z_{\alpha})S^2 \tag{10}$$

where $S$ and K are measures of skewness and kurtosis of an energy-portfolio.

Accordingly, mCVaR specification is presented in equation (11):

$$mCVaR_{\alpha} = -\frac{1}{\alpha}\int_0^{\alpha} mVaR(x)dx \tag{11}$$

### 3.2.2 Return-to-Risk Ratios

This subsection explains how we calculate four return-to-risk risk ratios (Sharpe, Sortino, modified Sharpe and Traynor), which can give us a clue what is investor’s level of reward for the taken unit of risk. First and basic risk-adjusted measure is well-known Sharpe ratio of Sharpe (1966), which observes excess returns in relation to common standard deviation ($\sigma$), see equation (12).

$$Sharpe\ ratio = \frac{R - R_f}{\sigma} \tag{12}$$

where $R$ is an average log-return of a particular energy-portfolio, $R_f$ is risk-free rate, and $\sigma$ is standard deviation of a portfolio. Yields of 3M treasury bills denote risk-free rate ($R_f$).

Sharpe ratio is relatively poor return-to-risk indicator, because it gives equal weight to positive and negative deviations from the mean. Next two ratios overcome this problem, and they are Sortino ratio of Sortino and Price (1994) and modifies Sharpe ratio of Gregoriou and Gueyie (2003). The former ratio puts in denominator standard deviation calculated upon only negative portfolio returns ($\sigma_D$), which gives more realistic risk-adjusted returns (see equation 13).

$$Sortino\ ratio = \frac{R - R_f}{\sigma_D} \tag{13}$$

Modified Sharpe ratio is even stricter indicator than Sortino ratio, because it uses as denominator measure of downside risk calculated upon mCVaR metrics. We use absolute value of mCVaR at 97.5% probability, whereas equation (14) shows how it is calculated.

$$modified\ Sharpe\ ratio = \frac{R_p - R_f}{|mCVaR|} \tag{14}$$

The last return-to-risk ratio is Treynor ratio of Treynor (1965), which measures the returns earned per unit of market risk or beta. Beta is calculated by dividing covariance of MVP and whole market[3] $COV(R_P, R_M)$, and variance of a whole market ($\sigma_M^2$), where $R_P$ and $R_M$ are returns of MVP and S&P500, respectively. Equation (15) shows how Treynor ratio and beta are calculated.

$$Treynor\ ratio = \frac{R - R_f}{\beta}; \quad \beta = \frac{COV(R, R_M)}{\sigma_M^2} \tag{15}$$

## 4. Dataset

This paper uses four energy spot commodities – Brent oil, WTI oil, gasoline and natural gas in the process of multivariate portfolio optimization, combining them with four precious metals futures – gold, silver platinum and palladium. In this way, every MVP consists of five assets. We use daily data, covering a data-span of almost five years, i.e. between January 2016 and October 2021. The data-span is long enough to produce reliable shares of assets in a portfolio, while any extension of the sample will not significantly change the structure of portfolio, because up to global financial crisis, all time-series are in relatively tranquil mode. Extension of the sample only makes portfolio optimization process more complex. All data are recovered from stooq.com website[4]. All energy commodities are synchronized with four precious metals, according to the existing observations, and every synchronized asset have 1427 daily observations. Also, all time-series are transformed into log-returns ($r_{i,t}$) according to the expression $r_{i,t} = 100 \times log(P_{i,t}/P_{i,t-1})$, where $P_i$ is price of a particular asset. Descriptive statistics of all time-series is given in Table 2.

**Table 2 Descriptive Statistics of the Assets in MVP**

| | | *Mean* | *St. dev.* | *Skewness* | *Kurtosis* |
|---|---|---|---|---|---|
| *Primary assets in portfolio* | Brent | 0.085 | 2.615 | -0.701 | 23.690 |
| | WTI | 0.074 | 3.050 | -0.566 | 26.749 |
| | Gasoline | 0.061 | 2.962 | -1.226 | 31.669 |
| | Natural gas | 0.055 | 3.598 | 0.672 | 10.144 |
| *Auxiliary assets in portfolio* | Gold | 0.047 | 0.924 | -0.014 | 8.237 |
| | Silver | 0.103 | 1.792 | -0.536 | 10.505 |
| | Platinum | 0.006 | 1.665 | -0.437 | 10.113 |
| | Palladium | 0.089 | 2.052 | -0.632 | 20.633 |

According to Table 2, all assets have positive mean, which means that their prices rise on average in the observed period. It can be noticed that average risk of all energy commodities is higher than risk of all precious metals. This is important, because auxiliary instruments should have lover risk than assets that need to be hedged. All assets, except natural gas, have negative skewness, which means that majority of log-return observations are placed left from the mean, i.e. number of negative log-returns are higher than number of positive log-returns. All time-series have high kurtosis, which is particularly conspicuous in the cases of Brent, WTI and

[3] Whole market is represented by the S&P500 index.
[4] In order to precisely specify which time-series are collected from the website, we list exact accronims of the used commodities: Brent oil cash (CB.C), WTI oil cash (CL.C), gasoline cash (RB.C), natural gas cash (NG.C), gold futures (GC.F), silver futures (SI.F), platinum futures (PL.F) and palladium futures (PA.F).

gasoline. High kurtosis is clear sign of an extreme risk presence, i.e. potentially high daily loses that investors in energy assets might sustain. In other words, high kurtosis means the presence of high downside risk. By calculating downside risk of MVP, we can see how much downside risk is reduced when energy is combined with precious metals.

## 5. Research Results

### 5.1 Minimum Variance Portfolio Construction

This subsection presents the results of multivariate minimum-variance portfolios, where the goal is risk-minimization of four energy commodities. As have been said, we restrict weight of energy in MVP, which should reflect different positions of participants in energy markets. Portfolio optimization process ends when optimal weight of every asset in portfolio is calculated, i.e. when minimum-variance goal is achieved.

Table 3 shows the results of a successful portfolio optimization, where calculated weights of five assets are presented. Table 3 contains the results when weight of energy is minimum 30% and 70%. As can be seen, most portfolios constitute of only two assets – energy and gold, while only in two cases, portfolios are made of three assets – energy, gold and palladium. Platinum and silver are not the part of any MVP. Gold has the highest weight in all portfolios, ranging between 65-70%, whereas only in two cases palladium has weight of 1% in Brent portfolio and 5% in natural gas portfolio, when restriction of energy is 30%. On the other hand, when restriction of energy is 70%, only gold is sole constituent with energy, while all other precious metals are left out. The clear reason why gold has dominant role in MVPs is the fact that gold has the lowest variance, as Table 2 indicates. All other precious metals have significantly higher variance, while palladium has the highest one.

**Table 3 Calculated Weights of the Selected Assets in the Minimum Variance Portfolios**

| | *The case when weight of energy is 30%* | | | | *The case when weight of energy is 70%* | | | |
|---|---|---|---|---|---|---|---|---|
| | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** |
| *Energy commodity* | 30% | 30% | 30% | 30% | 70% | 70% | 70% | 70% |
| *Gold* | 69% | 70% | 70% | 65% | 30% | 30% | 30% | 30% |
| *Silver* | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| *Platinum* | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| *Palladium* | 1% | 0% | 0% | 5% | 0% | 0% | 0% | 0% |
| $\Sigma$ | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 100% |

Our results are well in line with some other papers that combined oil and gold in portfolio. For instance, AlKhazali (2021) asserted that gold-oil portfolio stochastically dominates the one without gold, whereby portfolio risk decreases significantly when gold is added to portfolio with oil. Comparing to the results of Salisu et al. (2021), our results mostly coincide with this paper, but they are also different in some aspect. Namely, these authors found that gold is very effective hedge against risks associated with crude oil, which concur very well with our results. However, they also claimed that palladium, platinum and silver exhibit similar features as gold, but at lower magnitudes, which not coincides with our results because we find very low or zero share of other precious metal in portfolios. This discrepancy probably can be explained by the fact that Salisu et al. (2021) constructed two-asset portfolio, whereas we construct five-asset portfolio. This means that hedging capabilities of other precious metals cannot come to the fore in a multi-asset portfolio, because gold takes all the credits due to its minimum variance.

Looking at Table 3, a reasonable question could be raised: why palladium is part of MVPs in the two cases, when palladium has the highest risk of all precious metals, according to Table 2? An answer for this perplexity can be found in Table 4, which contains mutual average correlations between precious metals. In particular, Markowitz theory states that an important aspect of multi-asset portfolio construction is mutual correlation between portfolio's constituent parts, and according to Table 4, palladium has the lowest mutual correlation, comparing to all other precious metals. If we observe average level of correlations between precious metals, then in the case of gold, silver, platinum and palladium it amounts 0.530, 0.587, 0.557 and 0.384, respectively. Probably this fact has an influence in giving some weight to palladium in MVPs. Gold has undoubtedly the highest share in the portfolios, arguably because of its low variance.

**Table 4 Mutual Correlations between the Precious Metals Futures**

| | *Gold* | *Silver* | *Platinum* | *Palladium* |
|---|---|---|---|---|
| *Gold* | 1 | – | – | – |
| *Silver* | 0.765 | 1 | – | – |
| *Platinum* | 0.554 | 0.615 | 1 | – |
| *Palladium* | 0.270 | 0.382 | 0.501 | 1 |

After the construction of four MVPs, we can compare their performance with unhedged investments, which are energy assets. Table 5 contains descriptive statistics of created portfolios, under the assumption of different energy weights in portfolios. According to Table 5, it is obvious that risk of 70% energy portfolios is significantly higher than 30% energy portfolios, because the former ones contain more energy, which is significantly riskier than precious metals. This implies that investor who wants to pursue less risky portfolio should include more gold in a portfolio.

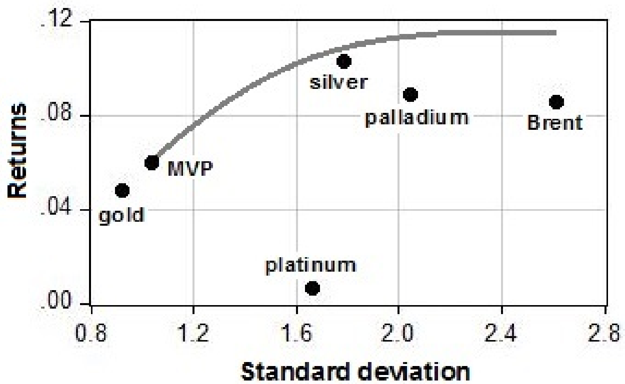
**Table 5 Descriptive Statistics of the Created Minimum Variance Portfolios**

| | *The case when weight of energy is 30%* | | | | *The case when weight of energy is 70%* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Brent* | *WTI* | *Gasoline* | *N. gas* | *Brent* | *WTI* | *Gasoline* | *N. gas* |
| *Mean* | 0.059 | 0.056 | 0.052 | 0.052 | 0.074 | 0.066 | 0.057 | 0.053 |
| *Standard deviation* | 1.038 | 1.134 | 1.102 | 1.252 | 1.863 | 2.160 | 2.093 | 2.533 |
| *Skewness* | -0.437 | -0.537 | -0.369 | 0.488 | -0.713 | -0.602 | -1.037 | 0.682 |
| *Kurtosis* | 13.288 | 18.157 | 16.693 | 7.834 | 23.188 | 27.083 | 29.197 | 10.125 |

Besides, all constructed MVPs have significantly lower risk then sole investment in energy assets, which means that energy risk-minimizing procedure can be successfully implemented. Also, it can be seen that kurtosis of all MVPs is lower, comparing to kurtosis of unhedged energy assets, particularly in those portfolios with 30% energy. This indicates that all MVPs probably have significantly lower downside risk than their unhedged counterparts, which will be checked in the next subsection.

Observing together standard deviations of all four MVPs, portfolio with Brent has the lowest risk, while gasoline follows. This means that market participants that combine Brent oil with precious metals, can enjoy the lowest risk, comparing to all other MVPs. However, when we compare descriptive statistics of calculated MVPs in Table 5 and all empirical assets in Table 2, it is obvious that gold has lower risk than any created multivariate portfolio. This means that created minimum-variance portfolios are not those with the lowest variance, but some room still exists to further reduce risk of portfolios. This happens because restrictions are imposed to energy share in portfolios, and this is why sole investment in gold has lower risk than any MVP. The results in Tables 5 and 2 can be visually presented *via* efficient frontier line, and Figure 2 depicts efficient frontier line of Brent portfolio[5]. In other words, it is clearly visible that gold has lower risk than MVP, and it is also evident that risk of Brent oil is significantly reduced by constructing a portfolio with precious metals. This means that the process of energy risk reduction is successful, but not perfect, because we impose energy restrictions to MVPs.

**Figure 2 Efficient Frontier Line of Brent MVP with Restrictions**




In order to be thorough in the analysis, we intend to find out how perfect minimum-variance portfolio looks like, so we rerun portfolio optimization procedure without restrictions, and results of calculated weights and standard deviation are presented in Table 6. In addition, we also want to see whether gold dominance in portfolio confirms when precious metals are combined with indices, which are connected to energy sensitive economic sectors. In that regard, we consider three well-known American indices – Dow Jones Industrial Average (DJIA), Dow Jones Transportation (DJT) and Dow Jones Automobiles and Parts (DJUSAP).

[5] Due to space parsimony, we present efficient frontier line only for portfolio of 30% Brent, because all other efficient frontier line plots are very similar to this one.

The results of calculated weights in Tables 6 and 3, i.e. without and with constraints, are obviously different. In other words, in portfolios without constraints, weight of gold increases, while weight of energy decreases, while all portfolios also contain some small percent of palladium. Share of gold dominates portfolio because gold has the lowest risk, while some share of energy and palladium can be found in perfect MVPs, probably because all energy assets and palladium have low correlation with gold. In addition, calculated standard deviations of perfect MVPs are all lower than their counterparts in portfolios with constraints. This is definite confirmation that having more energy in portfolio increases risk of portfolio, while an optimal weight of energy in portfolio is 9%, 7%, 8% and 6% for Brent, WTI, gasoline and Natural gas, respectively. In perfect risk-minimizing portfolios, all energy assets achieve relatively similar risk reduction in combination with precious metals, while portfolio with Brent slightly stand out with the lowest risk. Portfolio with Gasoline is the second one, while portfolios with natural gas and WTI follow.

**Table 6 Calculated Weights of Assets and Standard Deviation of the Portfolios without Restrictions**

| | *Energy commodities* | | | | *Indices of energy sensitive sectors* | | |
|---|---|---|---|---|---|---|---|
| | ***Brent*** | ***WTI*** | ***Gasoline*** | ***Natural gas*** | ***DJIA*** | ***DJT*** | ***DJUSAP*** |
| Energy | 9% | 7% | 8% | 6% | 36% | 29% | 13% |
| Gold | 85% | 87% | 86% | 86% | 64% | 71% | 84% |
| Silver | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Platinum | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| Palladium | 6% | 6% | 6% | 8% | 0% | 0% | 3% |
| $\Sigma$ | 100% | 100% | 100% | 100% | 100% | 100% | 100% |
| Port. st. dev. | 0.874 | 0.881 | 0.876 | 0.880 | 0.746 | 0.773 | 0.868 |

*Notes:* DJIA, DJT and DJUSAP acronyms stand for Dow Jones Industrial Average index, Dow Jones Transportation index and Dow Jones Automobiles and Parts index, respectively.

Similar situation is with indices of energy sensitive sectors. In all the cases, gold has very dominant role. More specifically, in two out of three cases, gold is the only precious metal in five-asset portfolio, while only in portfolio with DJUSAP, we also find some tiny trace of palladium, in amount of 3%. These results clearly illustrate that gold is the best auxiliary asset in MVP, not only when we speak about pure energy risk minimization, but also in the case when indices of energy sensitive sectors are at stake.

## 5.2 Some Characteristics of Created Multivariate Energy MVPs

Previous subsection has shown what is an optimal combination of energy and precious metals in a multivariate portfolio, when certain constraints about energy are imposed. On the other hand, this subsection tries to inspect some portfolio characteristics from the aspects of downside risk and return-to-risk ratios. In this way, we can compare performances between MVPs and unhedged energy assets, but also, we can compare performances between the portfolios themselves.

We first focus our attention to downside risk, which is observed *via* four different metrices – parametric and semiparametric VaR and CVaR. All downside risk measures are calculated under 97.5% probability level, which means that only

2.5% of the worst negative returns are observed. This particular probability is considered, because Cavenaile and Lejeune (2012) contended that modified VaR and CVaR are consistent only over a limited interval of confidence level. They claimed that these values should never be calculated under 95.84% probability, while the use of higher confidence levels is limited by the value of skewness. Table 7 shows levels of skewness and corresponding confidence levels. Due to the fact that gasoline has the highest negative skewness of -1.226, according to Table 2, we decide to calculate all downside risk measures at 97.5% probability, regarding both unhedged energy assets and MVPs.

**Table 7 Minimum Skewness for Modified Value-at-Risk Consistency**

| | *Minimum skewness under certain degree of probability* | | | | |
|---|---|---|---|---|---|
| *Confidence level in %* | 96.0 | 97.5 | 99.0 | 99.5 | 99.9 |
| *Minimum skewness* | -3.3 | -1.62 | -0.98 | -0.79 | -0.59 |

Source: Cavenaile and Lejeune (2012)

Table 8 presents downside risk results of unhedged energy assets. These values are basis with which we can compare portfolios' downside risk performances. According to Table 8, all downside risk measures gradually rise when observation is moved from VaR to mCVaR, which is expected, since strictness of the risk measures rise from VaR to mCVaR. Natural gas has the highest parametric risk measures, because natural gas has the highest standard deviation, and parametric risk measures take into account only first two moments. However, situation changes when we speak about semiparametric risk measures. In these situations, natural gas has the lowest mVAR and mCVaR measures, while gasoline ascents to the first place. The reason for these findings lies in the fact that natural gas has positive skewness and the lowest kurtosis, while gasoline has the highest negative skewness and kurtosis (see table 2). Third and fourth moments come to the fore in semiparametric risk measures, and this is the reason why situation reverses diametrically when modified risk measures are at stake.

**Table 8 Downside Risk Measures of the Selected Energy Assets**

| | *Brent* | *WTI* | *Gasoline* | *Natural gas* |
|---|---|---|---|---|
| *VaR* | -5.996 | -7.020 | -6.826 | -8.313 |
| *CVaR* | -6.882 | -8.053 | -7.830 | -9.532 |
| *mVaR* | -19.552 | -24.915 | -27.737 | -11.953 |
| *mCVaR* | -34.438 | -44.739 | -50.080 | -18.489 |

Table 9 contains calculated downside risk measures of constructed MVPs, taking into account both 30% and 70% energy constraints in portfolios, as well as hedge effectiveness indices. Panel A of Table 9 clearly shows that all energy MVPs have significantly lower downside risks in comparison to unhedged energy assets, which indicates that extreme risk of energy assets is notably reduced when energy is combined with precious metals, i.e. mostly of gold. Also, it is obvious that extreme risk of MVPs(70%) is significantly higher than its 30% counterpart, because higher weight of energy aggravates the results of downside risk in MVPs. Portfolios with natural gas has the lowest downside risks, Brent follows, while gasoline and WTI take third and fourth position. This is because portfolio with natural gas has positive skewness and the lowest kurtosis, while WTI has the highest negative skewness and kurtosis (see Table 5).

Panel B of Table 9 contains the results of risk-reduction from the aspects of variance, CVaR and mCVaR. We present HEI results only of CVaR, because it is stricter risk measure than VaR. As can be seen, all MVPs(30%) significantly reduces variance of all energy assets, which goes well above 80%, while for MVPs(70%), this percentage is around 50%. Portfolio with natural gas reduces variance the most, while portfolio with Brent the least. Reduction of CVaR is significantly lower, comparing to variance, and it goes around 53%, 46%, 45% and 38% for natural gas, WTI, gasoline and Brent, respectively. However, from the aspect of mCVaR, an order of the best (worst) portfolios changes. In particular, in MVPs(30%), gasoline portfolio has the best risk reduction with 68%, Brent portfolio follows with 62%. Portfolio with natural gas is the least effective, probably because natural gas has by far the lowest mCVaR. In other words, results undoubtedly indicate that created portfolios are very effective when extreme risk reduction is in the question, and the greatest merit for these results goes to gold, because gold has the lowest kurtosis of all precious metals (see Table 2).

**Table 9 Downside Risk Measures and HEI Indicators of the Constructed MVPs**

| | *30% weight of energy in portfolio* | | | | *70% weight of energy in portfolio* | | | |
|---|---|---|---|---|---|---|---|---|
| | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** |
| ***Panel A: Downside risk results*** | | | | | | | | |
| VaR | -3.723 | -3.800 | -3.779 | -3.902 | -4.484 | -4.833 | -4.764 | -5.309 |
| CVaR | -4.274 | -4.362 | -4.337 | -4.478 | -5.148 | -5.547 | -5.466 | -6.090 |
| mVaR | -8.054 | -10.170 | -9.434 | -5.068 | -14.419 | -17.382 | -18.245 | -7.601 |
| mCVaR | -13.009 | -17.288 | -15.893 | -7.368 | -25.320 | -31.247 | -32.763 | -11.759 |
| ***Panel B: Hedge effectiveness indicators*** | | | | | | | | |
| HEI_Var | 0.842 | 0.862 | 0.861 | 0.879 | 0.492 | 0.499 | 0.500 | 0.504 |
| HEI_CVaR | 0.379 | 0.458 | 0.446 | 0.530 | 0.252 | 0.311 | 0.302 | 0.361 |
| HEI_mCVaR | 0.622 | 0.614 | 0.683 | 0.601 | 0.265 | 0.302 | 0.346 | 0.364 |

On the other hand, significantly lower extreme risk reduction we find for MVPs(70%), because more weight in these portfolios has energy that has very pronounced presence of an extreme risk (see Table 2). Our results, regarding the downside risk of portfolios, coincide very well with the paper of Hammoudeha et al (2013), who used VaR to analyze the downside market risk associated with four precious metals, oil and the S&P500 index, but they also constructed three minimum VaR portfolios. They concluded that optimal minimum VaR portfolios should have more gold than any of the other assets under study, which is perfectly in line with our results.

Last analysis refers to different return-to-risk ratios. All investors care about risk, but they are even more interested to know what is the relation between gains and losses. To this end, we calculate four different return-risk ratios that put different risk measures in denominator. These ratios are Sharpe, Sortino, modified Sharpe and Traynor ratio. Panel A of Table 10 contains the results of calculated four ratios of energy portfolios, while Panel B shows these ratios of energy assets, which serves for comparison purposes. Figure 3 presents portfolio findings illustratively, giving a parallel perspective.

Sharpe ratio is classical and best-known return-to-risk ratio that observes excess returns per unit of standard deviation. According to Table 10, Brent has the highest Sharpe ratio in both 30% and 70% portfolios. The reason is because portfolio with Brent has the highest mean and also the lowest standard deviation (see Table 5). It can be noticed that except for case of Brent, portfolios with 70% of energy have lower Sharpe ratios *vis-à-vis* 30% energy portfolios, which means that adding more energy in portfolio worsens Sharpe ratio. Panel B of Table 10 indicates that all portfolios have better Sharpe ratio than sole investment in energy, which speaks in favour of creating portfolio.

Basic drawback of Sharpe ratio is the fact that it uses standard deviation as risk measure, which can be biased because it observes both positive and negative deviations from the mean. Next two ratios, Sortino and modified Sharpe ratio, tries to resolve this issue. In particular, Sortino ratio uses downside standard deviation as a measure of risk, while modified Sharpe ratio puts mCVaR measure in denominator, which is the strictest way of risk measurement. Portfolio with Brent reports the highest Sortino ratio in both 30% and 70% portfolios. The reason is the same as in the case of Sharpe ratio, i.e. Brent has the highest mean, while it also has relatively low downside deviation (see Table 11). As in the case of Sharpe ratio, all portfolios have higher Sortino ratio than a single investment in energy.

**Table 10 Calculated Return-To-Risk Ratios of the Constructed MVPs and Energy Assets**

| | *30% weight of energy in portfolio* | | | | *70% weight of energy in portfolio* | | | |
|---|---|---|---|---|---|---|---|---|
| ***Panel A: Created portfolios with energy*** | | | | | | | | |
| | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** |
| *Sharpe ratio* | 0.036 | 0.034 | 0.031 | 0.030 | 0.038 | 0.032 | 0.028 | 0.023 |
| *Sortino ratio* | 0.074 | 0.062 | 0.061 | 0.063 | 0.049 | 0.037 | 0.032 | 0.032 |
| *mSharpe ratio* | 0.005 | 0.003 | 0.003 | 0.007 | 0.003 | 0.002 | 0.002 | 0.004 |
| *Traynor ratio* | 0.240 | 0.218 | 0.163 | 0.505 | 0.138 | 0.117 | 0.080 | 0.327 |

| ***Panel B: Sole investment in energy assets*** | | | | |
|---|---|---|---|---|
| | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** |
| *Sharpe ratio* | 0.033 | 0.024 | 0.021 | 0.015 |
| *Sortino ratio* | 0.040 | 0.029 | 0.024 | 0.023 |
| *mSharpe ratio* | 0.002 | 0.002 | 0.001 | 0.003 |
| *Traynor ratio* | 0.079 | 0.098 | 0.043 | 0.253 |

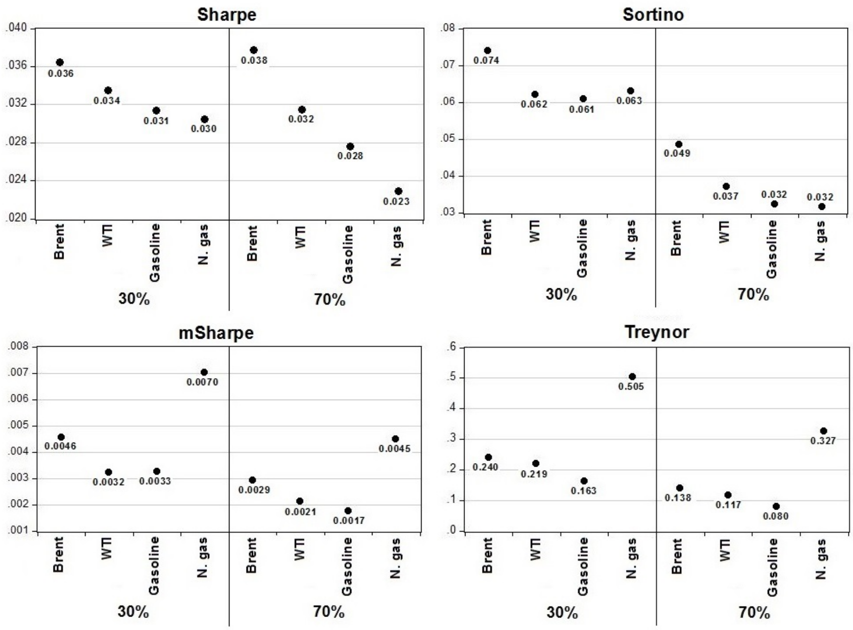
However, when modified Sharpe ratio is calculated, then Brent no longer has an upper hand, but natural gas takes over first position. Although portfolio with Brent has the highest mean, it also has relatively high mCVAR, while mCVaR of natural gas portfolio is significantly lower than all other mCVaRs, and this applies for both 30% and 70% portfolios. This means that portfolio with natural gas has the best ratio between potential gains and extreme losses. As expected, all portfolios have better mSharpe ratio comparing to a single investment in energy, which is the result of significant downside risk reduction (see Table 9).

**Table 11 Calculated Downside Deviation and Beta of the Constructed MVPs and Energy Assets**

| | *30% weight of energy in portfolio* | | | | *70% weight of energy in portfolio* | | | |
|---|---|---|---|---|---|---|---|---|
| ***Panel A: Created portfolios with energy*** | | | | | | | | |
| | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** |
| $\sigma_D$ | 0.653 | 0.666 | 0.661 | 0.645 | 1.523 | 1.789 | 1.777 | 1.679 |
| *Beta* | 0.248 | 0.255 | 0.317 | 0.103 | 0.540 | 0.568 | 0.712 | 0.162 |

| ***Panel B: Sole investment in energy assets*** | | | | |
|---|---|---|---|---|
| | ***Brent*** | ***WTI*** | ***Gasoline*** | ***N. gas*** |
| $\sigma_D$ | 2.153 | 2.545 | 2.557 | 2.395 |
| *Beta* | 0.807 | 0.807 | 1.032 | 0.217 |

**Figure 3 Graphical Illustration of Four Return-Risk Ratios of the Constructed MVPs**




The last indicator is Traynor ratio, which calculates relation between risk-free returns and the level of systemic risk, represented by beta. Beta measures sensitivity of energy-portfolio returns to the movements of the underlying benchmark, which is a whole market, in our case it is S&P500 index. According to Table 11, all portfolio betas are lower than one, which means that all portfolios have lower reaction to global turbulences, comparing to the whole market, i.e. S&P500 index. Also, it is evident that portfolio betas are significantly lower than energy betas, which means that making portfolio has great and positive effect on the reduction of systemic risk. Particularly good resistance to systemic shocks have portfolios with 30% energy, because their betas are very low. Taking into account all portfolios, portfolio with natural gas has by far the lowest beta in both types of portfolios, which means that this portfolio has the lowest reaction to systemic risk. These good beta values transfer to the highest values of Traynor ratio, according to Table 10, which undoubtedly indicates that portfolio with natural gas has the best relation between excess returns and the level of systemic risk, although portfolio with natural gas does not have the highest mean, but the lowest one.

## 6. Conclusion

This paper constructs minimum-variance multivariate portfolios that consist of energy commodities and precious metals. In portfolio optimization procedure, we hedge Brent oil, WTI oil, gasoline and natural gas, combining them with four precious metals – gold, silver, platinum and palladium. In order to portray different positions of market participants, we set up different constraints of energy in a portfolio, i.e. minimum weight of energy in one portfolio is 30%, while in other one, it is 70%.

Based on the results, we have several noteworthy findings to report. First, portfolio optimization suggests that in all energy-MVPs, dominant share has gold, while only in two cases, optimal portfolio includes some tiny portion of palladium. Gold is the most favourable auxiliary asset in a portfolio, because gold has by far the lowest risk of all other precious metals. As a matter of fact, gold alone has lower risk than all constructed portfolios, because energy constraints are imposed. Palladium in rare occasions finds itself in a portfolio, because it has relatively low correlation with other precious metals and energy assets.

Second, all constructed portfolios record significant fall of risk *vis-à-vis* sole energy investment, while 30% energy portfolios have better performance than their 70% counterparts. This means that energy risk-minimization can be very efficient, but it also means that investors who want to pursue less risky energy-MVP should include more gold in a portfolio.

Third, after portfolio construction, we examine their downside risk and return-to-risk performances. We find that portfolio with natural gas has the lowest downside risk, while portfolio with Brent follows in both 30% and 70% portfolios. In 30% energy portfolios, the most effective risk-reduction we find in gasoline portfolio, whereas in 70% portfolio, the most effective risk-reduction reports natural gas portfolio.

The last analysis includes calculation of several risk-adjusted ratios. Our results indicate that Brent portfolio has the highest Sharpe and Sortino ratio, because this portfolio has the highest mean and the lowest standard deviation and downside deviation. However, when we take into account downside risk that really matters – mCVaR, than natural gas has the best results. Traynor ratio suggests that portfolio with natural gas has the best relation between risk-free returns and the level of systemic risk, because this portfolio has by far the lowest beta.

Results from this study can be useful for market participants, who work with energy commodities, because they can construct their minimum-variance portfolio based on these results. Also, these results can show which MVP has the best downside risk characteristics and the best return-to-risk performance.

Future studies can address other risk targets in portfolio optimization procedure, such as VaR or CVaR, Also, it would be interesting to see whether and how portfolio construction differentiates, if distinctively different subperiods are under scrutiny, i.e. crisis and tranquil subperiods.

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