Calculation of the Capital Requirement Using the Monte Carlo Simulation for Non-life Insurance[1]

*Vladimír MUCHA – Michal PÁLEŠ – Katarína SAKÁLOVÁ**

Abstract

The aim of the paper is to demonstrate the possibility of using the Monte Carlo method within the field of risk reduction within the framework of a developed model by applying a particular form of insurance. It is focused on the area of non-life insurance in which the collective risk model is suitable for describing the total claims in a given portfolio of insurance contracts. The Monte Carlo simulation method is the starting point, from which one can generate values of the total claim amount and their statistical treatment for the needs of measuring the value of the capital required to ensure solvency. As a final result the paper presents simulations as an effective problem solving tool, by enabling the development of interactive studies in the risk management process. The methodology presented makes use of Visual Basic for Applications under Microsoft Excel. This opens up the potential of developing actuarial software for solving risk reduction problems by applying various forms of insurance. Given the ability of the method to react flexibly to changes in the given form of insurance or its parameters it can be used also to optimise the choice of suitable scenarios.

Keywords: *Monte Carlo simulation, types of insurance, Solvency II, solvency capital requirement, internal model*

JEL Classification: G22, C63

Introduction

The Solvency II project sets out a conception for the future regulation of solvency in the insurance sector within the European Union. Its implementation requires the systematic and complex management of risk. From this viewpoint

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[1] The paper arose with the support of the VEGA research project No. 1/1/0806/14 *The calculation of the SCR for covering non-life insurance risks taking account of the needs of practice*.

what is important is an integrated approach to all types of identifiable risks linked to increased demands on an insurers internal control system. One of the main priorities of the Solvency II project is to ensure that the capital requirements for solvency are adequate taking into account the underlying risks and to develop internal models for insurers. The current issue in insurance is therefore to focus insurers' attention on their own assessment of risks and solvency (ORSA).[2] As part of this process insurers should focus on analysing and capturing all their current and future risks. If an insurer decides to develop its own internal model it must have at its disposal in particular adequate material, personnel, technical and technological capacity and resources. It is necessary to mention that all the official material published under the auspices of the European Union relating to the issue of internal models contain only general principles, whereas the creation of such own models is entirely a matter for those insurers, who decide to implement an internal model.[3] An insurer, which decides on an internal model, must ensure that the model is capable of generating outputs, which are sufficiently detailed to allow the insurer's management to use them so as to take appropriate decisions. Apart from a complete internal model an insurer can decide only for a partial internal model, provided it is approved by the supervisory authority, for example in respect of one or more modules, respectively sub-modules, of the Basic Solvency Capital Requirement (BSCR). As illustration in the case of the module non-life underwriting risk we can mention the submodule the premium and technical reserve risk in non-life insurance. A key component of every internal model is the so-called prediction of the probability distribution. The methods used by an actuary to calculate the prediction of the probability distribution are based on up-to-date information and the current advances made in insurance mathematics and are adapted to the data containing historical information about the risks under consideration (Cipra, 2015). The volume and nature of the data must ensure that the estimates obtained from an internal model based on these data do not contain any significant error of estimation. Not only that the insurer should update the data files which it uses to calculate the prediction of the probability distribution. As far as diversification effects are concerned the insurer can treat them within each separate risk category, provided the system used to measure such effects is considered appropriate. Based on the methodology behind the measure of risk, respectively a related method,

[2] The aim of the ORSA is the ensure that the insurer identifies and quantifies all the risks to which it is currently exposed as well as those to which it may be exposed, maintains adequate capital to cope with such risks and develops and better uses techniques for the identification and management of these risks.

[3] An insurer which decides to use an internal model must ensure that the modelling approaches used reflect the nature, extent and complexity of the risks related to its activities.

the insurer will derive the capital requirement for ensuring solvency directly from the prediction of the probability distribution gained from the internal model with a 0.995 credibility over a one year time horizon. Demonstrating that the policyholders and other interested persons are provided with an adequate level of protection is not a pure formality, but must include evidence, that the model does not cause a significant error in setting the solvency capital requirement (SCR). In banking supervision Basel III regulation specifically aims to improve the quantity of capital which have to hold by providing additional stability through new capital buffers (Matejašák, 2015). We will use Visual Basic for Applications to carry out simulations and we will assume a model portfolio of non-life insurance contracts which we set up in respect of one particular product, in this case car accident insurance.

1. Types of Insurance and Risk Analysis

The application to one particular type of insurance is one of the possibilities for risk management in the developed model of insurance contracts and the consequent prediction in relation to ensuring solvency. In this paper we will be concerned with the simulation process when modelling insurances at their full value in combination with excess deductions using the Monte Carlo method. We will use the method to generate values of the total claim amount in Microsoft Excel.[4] After processing them statistically we will get their distribution or the required indicators needed to determine the level of risk. We will present the methodology which allows us through simulation to predict, within the studied contracts, the value of the solvency capital requirement for insurance with an average clause with an excess. To visualise this we will therefore state and define various types of loss insurance and excesses. These are characterised by the fact that the total claim amount depends on the size of the loss arisen and we can differentiate these types (Cipra, 2010):

- *Pure indemnity insurance (also Insurance without sum insured)*
- *Insurance with an average clause (also Full value insurance)*
- *First loss insurance,*
- *Quota insurance.*

These types of insurance can be combined with supplementary forms of insurance:

- *with an excess*
- *without an excess*

[4] By making use of Visual Basic for Applications (VBA) in Microsoft Excel.

According to the type of insurance and excess for the given portfolio of contracts we assume that we know the insured amount S, the insurable value H,[5] the amount of the franchise F and the distribution of the number of claims and the amount of each claim. We will now present the various types that we will deal with in this paper.

Pure indemnity insurance (*PII*) is an insurance where the amount paid on a claim equals the amount of the loss. The claim amounts can be described by the following relationship

$$^{P}g_{PII}\left(x\right) = x \quad x \in \left\langle 0;\ H \right\rangle \tag{1}$$

The insurer has a 100% participation in the amount of the claim.

Insurance with an average clause (*IWA*) is an insurance where the amount paid in respect of a claim depends on the amount of the loss incurred according to the ratio[6] of the insured amount S to the insurable value H. The relationship is defined as follows

$$^{P}g_{IWA}\left(x\right) = \frac{S}{H}x \quad x \in \left\langle 0;\ H \right\rangle,\ S \le H \tag{2}$$

The insurer's participation in covering the incurred loss is 100 × $\frac{S}{H}$%. In the case where $S = H$ we have pure indemnity insurance insured (Fecenko, 2012).

With excess (*E*) is a supplementary form of cover where the insurer does not participate up to the amount of a franchise F and where it is exceeded the insured's share of the loss incurred is limited to the amount of the franchise F. The amount paid by the insurer can then be represented by the following relationship defined as follows

$$^{P}g_{E}\left(x\right) = \begin{cases} 0 & x \le F \\ x - F & x \in \left(F;\ H\right\rangle \end{cases} \tag{3}$$

The separate types of insurance can be combined with an excess. We could thus implement the methodology based on simulation for various forms of insurance cover. In this paper we will focus on a combination of insurance with an average clause with an excess. We are comparing the situation before and after the application of an average clause and an excess in the context of solvency. We compare the situation with regard to ensuring solvency before applying insurance with an average clause with an excess and then after their application. One

[5] H is the actual value of the item insured, whereas the insured amount S is the amount for which it is insured, which is usually lower.

[6] We can define this ratio as the intensity of insurance cover $I = \frac{S}{H}$

of the modern ways of measuring risk is by using the Conditional Value at Risk by means of which it is possible to state the solvency capital requirement with a level of probability given in advance. The application of an average clause or an excess to the claim amount represents an alternative form of risk management and the possibility of its measurement. By its use we show how simulations are capable of providing the information needed for solvency assessment purposes. It is obvious that participation of the insured in the claim amount paid manifests itself by a reduction in the solvency capital requirement. We need to emphasise that the presented methodology also allows us to measure the reduction quantitatively. This approach to investigating the effect on the solvency capital requirement of different forms of insurance has a great potential which can be exploited in developing actuarial analytical software. The fact that within the Monte Carlo method we can vary the type of insurance gives it the character of an optimalisation process (Korn, Korn and Kroisandt, 2010).

2. Generation of the Total Claim Amount Using the Monte Carlo Method

Rapid technological progress and with it the software capabilities of computers has brought to the forefront new problem solving techniques based on qualitatively new approaches to using computational technology. As an illustration we can mention one of the successful and current research approaches, namely that based on the use of the R language, which is freely available and used mainly by academics and other researchers.[7] The standard version and supporting packages implement a large number of advanced functions which actuaries can use in their analyses, see for example (Páleš, 2014). The development of such functions is expanding. A no less important role is played here by its very precisely elaborated approach in particular its intuitive graphical support. In this paper however we will focus on the use of another programming language namely Visual Basic for Applications as used in Microsoft Excel. We will use this language to carry out simulations, which represent for researchers a very effective tool for the analysis and investigation of complicated processes. Simulations permit the construction of models of the real world and by making use of experiments with them we can obtain a better understanding of how such systems function as well as providing a check on their analytical solutions. Their use can be justified mainly in the case where a developed model of the real world cannot be solved using the mentioned analytical approaches, as to do so would be at the

[7] R is a programming language aimed particularly at statistical calculations.

least extremely onerous. Simulation allows us to study how a system will behave based on its models in speeded-up time, which allows us to develop predictions (Hušek and Lauber, 1987). On the basis of the developed scenarios it is possible to proceed to their analysis. The non-existent or extremely onerous analytical solutions can be replaced by the flexible and interactive simulation approach as a tool that actuaries can use to solve problems in the area of risk theory.

Simulations are closely associated with the Monte Carlo method, which solves numerically probability problems with the help of organised statistical trials. Using this method we gain the necessary solutions to problems using artificially generated random processes. These are generated so that certain characteristics or functions which are defined by the given process are at the same time solutions to the original problem. The basic idea of the Monte Carlo method is the search for the relationship between the parameters which are the solution of the studied problem and the characteristics of the random processes modelled on the computer. The output from the simulations does not have to be concrete information providing an answer to the solved problem, in which case the results of the simulations have to be worked on further statistically or optimised (Rubinstein, 1981). A great potential is thus opened up for the implementation of new actuarial and database software for risk management. Simulations can also be carried out within modern forms of risk management, based on the *Value at Risk (VaR), Conditional Value at Risk (CVaR)* or applications of the form insurance with excesses. That would then permit the meeting of one of the pillars of the Solvency II project in the area of risk management, whose aim is to determine the solvency capital requirement, which an insurer must hold so as to be capable of meeting its liabilities. For these calculations it is necessary to predict the distribution of total claims in a given model portfolio of insurance contracts using the Loss Distribution Approach (LDA) (Panjer, 2006). From this point of view it is important to perform the collection and control of the data and the construction of a statistical database with the data relating to the number and size of claims.

We assume a model portfolio of non-life insurance contracts which we set up in respect of one particular product, in this case car accident insurance. The role of the actuary is to develop an analysed portfolio of cars such that their individual parameters correspond to the insurable value of the car and its assumed insured amount. The random variable describing the distribution of the total claim amount has the form of a compound distribution which we will denote by S, i.e.

$$S \sim Co\left(p_N\left(n\right);\ F_X\left(x\right)\right)$$

where the random variable N describes the number of claims and X the distribution of the individual claim amounts (Horáková and Mucha, 2006). So as to get the distribution of the total amount paid by the insurer ${}^{P}S$, after allowing for insurance with an average clause with an excess, we define the following

F – the franchise,
S – the insured amount,
H – the insurable value.

Further we assume that the largest possible individual claim amount corresponds to the insurable value H. If the individual loss amount does not exceed the franchise F the insurer will not participate in covering this loss. On the other hand if the individual loss amount is higher than the franchise F the insurer will participate in covering the loss, in terms of the loss amount less the franchise multiplied by the intensity of insurance cover. The values of the variable representing the amount paid by the insurer are as follows

$$ {}^{P}g_{IWA+E}(x) = \begin{cases} 0 & x \le F \\ (x-F)\dfrac{S}{H} & x \in \left(F;\, H\right\rangle \end{cases} \tag{4} $$

If we denote $P(X > F) = p_F$, then the random variable describing the number of claim payments made by the insurer and its characteristics satisfy (Horáková, Páleš and Slaninka, 2015)

$$ {}^{P}N_{IWA+E} \sim Co\left(N;\, A\left(p_F\right)\right) \tag{5} $$

$$ E\left({}^{P}N_{IWA+E}\right) = p_F \cdot E(N) \tag{6} $$

$$ D\left({}^{P}N_{IWA+E}\right) = p_F \cdot \left(1 - p_F\right) \cdot E(N) + p_F^2 \cdot D(N) \tag{7} $$

On the assumption that the individual claim amount exceeds the franchise F the random variable representing the individual claim payments made ${}^{P}\overline{X}_{IWA+E}$ satisfies the following

$$ {}^{P}\overline{X}_{IWA+E} = {}^{P}X_{IWA+E} \,\middle|\, X > F \tag{8} $$

or

$$ {}^{P}\overline{X}_{IWA+E} = {}^{P}X_{IWA+E} \,\middle|\, {}^{P}X_{IWA+E} > 0 \tag{9} $$

Using the random variables ${}^{P}N_{IWA+E}$ a ${}^{P}\overline{X}_{IWA+E}$ we can, on the basis of the assumptions meeting the conditions of the collective risk model, specify the characteristics of the total claim amount paid ${}^{P}S$ [8]

$$E\left({}^{P}S\right) = E\left({}^{P}N_{IWA+E}\right)\cdot E\left({}^{P}\overline{X}_{IWA+E}\right) \tag{10}$$

$$D\left({}^{P}S\right) = E\left({}^{P}N_{IWA+E}\right)\cdot D\left({}^{P}\overline{X}_{IWA+E}\right) + E^{2}\left({}^{P}\overline{X}_{IWA+E}\right)\cdot D\left({}^{P}N_{IWA+E}\right) \tag{11}$$

Experience with generating values of the total loss amount S by means of the values of the random variable representing the number of claims N and the individual loss amounts X using the Monte Carlo method led us to the thought of generating values of the total claim amount paid by the insurer. In this connection however it was necessary to answer the following questions.

If we adjust the generated values of the individual loss amounts as per equation (4) will we then get for the non-zero values so transformed generated values for the random variable ${}^{P}\overline{X}_{IWA+E}$?

Will their number represent the generated value of the random variable ${}^{P}N_{IWA+E}$ describing the number of claim amounts paid by the insurer?

Will we obtain by summing them the value of the random variable ${}^{P}S$ representing the total amount of claim amounts paid by the insurer?

If the answers to these questions are yes, we would thus have obtained an effective algorithm for generating the values of the random variables ${}^{P}N_{IWA+E}$ and ${}^{P}\overline{X}_{IWA+E}$, respectively the values of the total claim amounts paid by the insurer ${}^{P}S$ without needing to specify the distribution of the random variables ${}^{P}N_{IWA+E}$, ${}^{P}N_{IWA+E}$ and ${}^{P}\overline{X}_{IWA+E}$.

The described approach was checked by comparing the results of the more onerous simulation principle on the basis of the inverse transformation method and the Monte Carlo method. As part of this check a comparison was also made of the characteristics of the total claim amount paid by the insurer gained by simulation with those gained by the exact approach, as well as the values of the distribution function as gained by simulation with the values obtained using an approximate approach (Mucha, 2012b). On the basis of these checks we obtained positive answers to our questions. We must once again remember that in the case of compound distributions in the collective risk model there are many cases

[8] The paper Mucha (2014) gives a derivation of the characteristics for the given random variables for verification purposes. It should though be noted that in many cases of compound distributions it is not possible to express these exactly.

where an exact approach is in practice not available due to its extreme onerousness. Precisely for this reason the approach we present for getting the distribution of the total claim amounts paid by the insurer using simulation is such a significant research tool. For the purpose of the analysis and checking of the presented methodology the generated values were first entered into the cells of a Microsoft Excel spread sheet, which from a programming point-of-view significantly affected the time factor in their realisation given the number of operations involved. By entering the values of the total claim amounts paid by the insurer without intermediate results we obtained a significant reduction in the time taken to carry out the simulations.

To generate the values of the total claim amount paid by the insurer we will therefore make use of the principle which we will set out as an algorithm in the following steps.

1. First we generate the value of the random variable N describing the number of claims in the given portfolio, which we will denote as n_1

2. Secondly we generate n_1 values of the random variable X, representing the individual loss amounts

$$x_{11},\ x_{12},\ x_{13},\ \ldots,\ x_{1n_1}$$

3. Thirdly we apply to these values insurance with an average clause with an excess, i.e. we transform the values of the individual loss amounts X generated in second step on the basis of the following

$$x \leq F \quad \Rightarrow \quad {}^{P}g_{IWA+E}(x) \ = \ 0 \tag{12}$$

$$x \in \left(F;\ H\right\rangle \quad \Rightarrow \quad {}^{P}g_{IWA+E}(x) \ = \ (x - F)\cdot\frac{S}{H} \tag{13}$$

The values which are different from zero are the values of the random variable ${}^{P}\overline{X}_{IWA+E}$ and their number, which we denote as ${}^{P}n_1$, represents the generated value of the random variable ${}^{P}N_{IWA+E}$.

4. By summing these values $\left(x_{1j} - F\right)\cdot\frac{S}{H},\ \left(x_{1k} - F\right)\cdot\frac{S}{H},\ \ldots,\ \left(x_{1l} - F\right)\cdot\frac{S}{H}$ we obtain the first value of the total claim amounts paid by the insurer ${}^{P}S$ i.e. ${}^{P}s_1$

$${}^{P}s_1 \ = \ \left(x_{1j} - F\right)\cdot\frac{S}{H} \ + \ \left(x_{1k} - F\right)\cdot\frac{S}{H} \ + \ldots + \ \left(x_{1l} - F\right)\cdot\frac{S}{H} \tag{14}$$

By repeating[9] this algorithm m times we get m values of the random variable ${}^{P}S$ i.e.

$$ {}^{P}s_1,\ {}^{P}s_2,\ {}^{P}s_3,\ \ldots,\ {}^{P}s_m \tag{15} $$

Statistical manipulation of the generated total claim amounts paid by the insurer allows us to derive the characteristics which enable us to measure risk and to reduce the capital requirement.

3. Determining the Capital Requirement before and after Applying Insurance with an Average Clause with an Excess

This part of the paper is based on the stochastic Monte Carlo method using the statistically manipulated generated values in order to determine the capital requirement. This represents the amount of capital which the insurer must have at its disposal so that with a probability p it will ensure its solvency. We will denote this as $K_p^{CVaR}({}^{P}S)$ which we define as follows (Horáková, 2012; Horáková, Páleš and Slaninka, 2015)

$$ K_p^{CVaR}({}^{P}S) = CVaR_p({}^{P}S) - E({}^{P}S) \tag{16} $$

where we define the value $CVaR_p({}^{P}S)$ as the expected total claim amount paid by the insurer on the assumption that it exceeds the value of the quantile ${}^{P}s_p$ of the distribution of the total claim amounts paid by the insurer ${}^{P}S$

$$ CVaR_p({}^{P}S) = E({}^{P}S \,|\, {}^{P}S > {}^{P}s_p) \tag{17} $$

where for ${}^{P}s_p$ of the random variable ${}^{P}S$ we have

$$ {}^{P}s_p = F_{{}^{P}S}^{-1}(p) = \inf\left\{s \in R, F_{{}^{P}S}(s) \geq p\right\}, \quad 0 < p < 1 \tag{18} $$

On the basis of this definition it is possible to determine the capital requirement $K_p^{CVaR}(S)$ even without applying insurance with an average clause with an excess. Calculation of the values $CVaR_p({}^{P}S)$ and $CVaR_p(S)$ analytically is in many cases extremely onerous or from an exact point of view impossible. For this reason we will use the Monte Carlo simulation method to determine them. The algorithm

[9] The number of simulations used with the Monte Carlo method was 30,000, which is enough to obtain the required accuracy of the final results. We have dealt with an analysis of the accuracy of the results using the Monte Carlo method according to the number of simulations carried out in the paper (Horáková and Mucha, 2002).

for this we will describe in the next part of this paper. To calculate the values $CVaR_p(^P S)$ we will simulate the values of the total claim amounts paid by the insurer $^P S$. By statistically modifying these values we determine, for a given value of *p*, the relevant quantile $^P s_p$ of the distribution of the total claim amounts paid by the insurer. We can then determine the value of $CVaR_p(^P S)$ using equation (17), whereby we ensure that the algorithm is repeated enough times so as to obtain the required level of accuracy. Out of the generated values of the random variable $^P S$ we select those which meet the required condition, namely

$$^P S > {^P s_p} \tag{19}$$

In this way we select a file containing *k* values

$$^P s_{1_{\left(>^P s_p\right)}},\ {^P s_{2_{\left(>^P s_p\right)}}},\ {^P s_{3\left(>^P s_p\right)}},\ldots,\ {^P s_{k\left(>^P s_p\right)}} \tag{20}$$

After statistically modifying these values we determine the mean $E(^P S \,|\, {^P S} > {^P s_p})$ and value of $CVaR_p(^P S)$ using a point estimate on the basis of the average of the selected values (Mucha, 2012a)

$$CVaR_p(^P S) \approx {^{sim}E}(^P S \,|\, {^P S} > {^P s_p}) = {^P \overline{s}_{\left(>^P s_p\right)}} = \frac{1}{k} \cdot \sum_{i=1}^{k} {^P s_{i_{\left(>^P s_p\right)}}} \tag{21}$$

4. Generation of the Values of the Total Claim Amounts Paid by the Insurer Using an Actual Set of Data

We assume that number of claims and the individual size of claims for the given tariff group of an accident insurance contract have the following distributions

$$N \sim Bi(500;\ 0.2),\ X \sim \Gamma(3;\ 2)$$

with the basic characteristics

$$E(N) = m \cdot q = 100$$

$$E(X) = \alpha \cdot \beta = 6$$

This means that the total insured amount *S* has the compound binomial distribution

$$S \sim CoBi\left(m = 500;\ q = 0.2;\ F_X(x)\right)$$

with the basic characteristics

$$E(S) = E(N) \cdot E(X) = 600$$

We now apply to the given portfolio insurance with an average clause with an excess and we determine, with probability $p = 0.95$, the value capital requirement needed to ensure solvency on the base of the level of risk $CVaR_{0.95}(^{P}S)$. From the distribution of the individual claim amounts X we take the insurable value H (see p. 4, 7) as the value $x_{0.999995}$, i.e.

$$H = x_{0.999995} \approx 35$$

For this portfolio of contracts we set the level of franchise to the value $F = 6$ and the insured amount to the value $S = 15$. For illustration in the paper we compare the characteristics of the total claim amount paid by the insurer ^{P}S obtained by simulation with the values gained with the help of an onerous exact derivation.

For the characteristics of the random variable ^{P}S as per (10) and (11) we get

$$E\left(^{P}S\right) = 57.60498$$

$$D\left(^{P}S\right) = 135.9409$$

We compare the values so calculated with the values of the characteristics $E\left(^{P}S_{Sim}\right)$, $D\left(^{P}S_{Sim}\right)$ of the random variable ^{P}S, which we obtained by generating 30,000 values of this variable using Visual Basic for Applications in Microsoft Excel. After statistically modifying these values we obtain

$$E\left(^{P}S_{Sim}\right) \approx 57.6718 \quad D\left(^{P}S_{Sim}\right) \approx 135.5415$$

These values are comparable with the values $E\left(^{P}S\right)$, $D\left(^{P}S\right)$, which we considered as the first step when we dealt with checking that using the Monte Carlo method the values generated were those of the random variable ^{P}S representing the total claim amounts paid by the insurer.

5. Risk Reduction Using Insurance with an Average Clause with an Excess

This part of the paper deals with risk reduction by applying insurance with an average clause with an excess to the given portfolio of insurance contracts. We will measure the effect by calculating the capital requirement before and after applying insurance with an average clause with an excess.

We calculate the value $CVaR_{0.95}({}^{P}S)$ based on the statistically adjusted generated values of the total claim amounts paid by the insurer in accordance with equation (21), where ${}^{P}s_{0.95} = 77.66$

$$CVaR_{0.95}({}^{P}S) \approx {}^{sim}E({}^{P}S \,|\, {}^{P}S > {}^{P}s_{0.95}) \;=\; {}^{P}\overline{s}_{\left(> {}^{P}s_{0.95}\right)} \;=\; 83.4593$$

Using equation (16) we calculate the capital requirement, which the insurer must have at its disposal so as to have a 95% probability that it will stay solvency, as follows

$$K_{0.95}^{CVaR}({}^{P}S) \;=\; CVaR_{0.95}({}^{P}S) \;-\; E({}^{P}S) \;=\; 25.78758$$

In the case where we do not make use of insurance with an average clause with an excess, i.e. where the total claim amounts paid by the insurer equal the total insured amounts, we calculate the capital requirement using a similar algorithm.

We determine the value of $CVaR_{0.95}({}^{P}S)$ based on the statistically adjusted generated values of the total insured amounts S where $s_{0.95} = 707.05$.

$$CVaR_{0.95}(S) \approx {}^{sim}E(S \,|\, S > s_{0.95}) \;=\; \overline{s}_{(>s_{0.95})} \;=\; 735.4553$$

We calculate the capital requirement, which the insurer must have at its disposal so as to have a 95% probability that it will stay solvency, in accordance with the following equation

$$K_{0.95}^{CVaR}(S) \;=\; CVaR_{0.95}(S) \;-\; E(S) \;=\; 135.4553$$

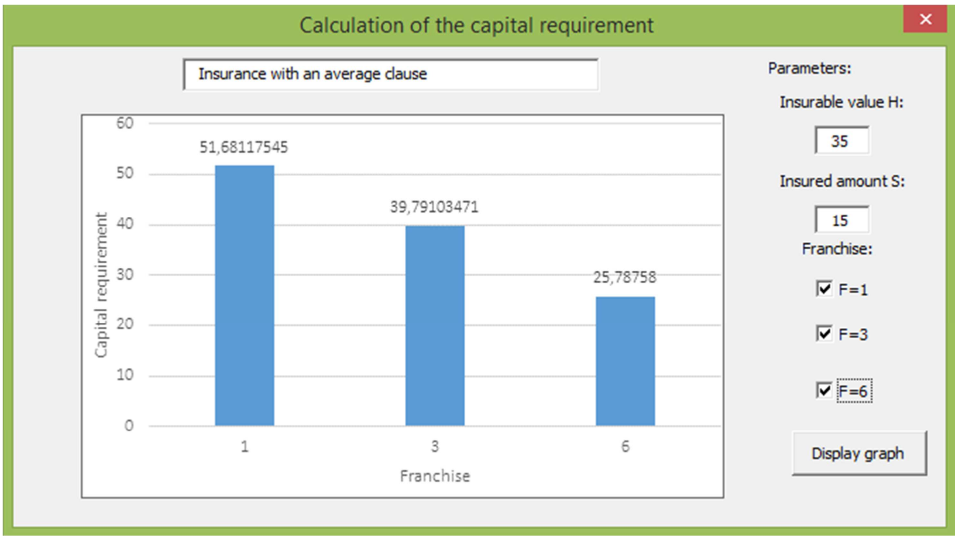
On comparing the values of the capital requirement $K_{0.95}^{CVaR}(S) = 135.4553$ and $K_{0.95}^{CVaR}({}^{P}S) = 25.7876$ before and after applying insurance with an average clause with an excess we can observe a clear reduction in the capital requirement. It is possible to assume a reduction in the risk where the client participates in the claim amount through insurance with an average clause with an excess. It is possible to measure this reduction quantitatively by determining the distribution of the total claim amounts paid by the insurer and the risk measure *CVaR* with the aid of the Monte Carlo simulation method.

The presented methodology allows one to analyse for a given portfolio of insurance contracts the effect of changing the excess, i.e. the value of the franchise.

In our example we will investigate the effect on the capital requirement of changing the value of the franchise. We will consider three different levels of franchise F = 1, F = 3 and F = 6. Using our developed application we are able to measure interactively the extent of the reduction in risk by calculating the capital requirement, see Figure 1 and Table 1.

F i g u r e 1

Calculation of the Capital Requirement According to the Size of the Franchise in the Case of an Insurance with Insurance with an Average Clause with an Excess Using Visual Basic for Applications[10]



Source: VBA source code is the authors' own.

T a b l e 1

Results of the Interactive Risk Reduction Using the Monte Carlo Method for an Insurance with Insurance with an Average Clause with an Excess where H = 35 and S = 15

Franchise:	1	3	6
Capital requirement:	51.6812	39.7910	25.7876

Source: Authors' own calculations.

With the help of the developed application using Visual Basic for Applications it is possible to determine the value of the capital requirement for each analysed case. We can not only investigate the effect of changing the parameters for a given type of insurance, but also what happens when we change the type of insurance itself. The methodology using simulation which we have described above has a great potential in the area of risk management in non-life insurance. It also makes possible the development of optimisation criteria for ensuring solvency.

[10] The situation analysed is shown graphically in Figure 1 by means of the user dialogue in the form of a graph for the three levels of franchise.

The results are obtained after applying simulation methods within the tents capital requirements in the context of the illustration of the effect of the reduction. Using simulation principle it can be used to create an interactive studies after using other forms of insurance in the examined portfolio. Following the publication of the algorithm described in this paper is perhaps its implementation in the relevant programming language with subsequent verification of the results achieved.

Conclusion

The presented use of simulation, within the field of risk theory and the collective model of risk, in an internal model for non-life insurance, which applies insurance with an average clause with an excess to a portfolio of insurance contracts, can be seen as being very beneficial. What is new here is the use of the simulation methodology to determine the distribution of the total claim amounts paid by the insurer and its characteristics.

At the same time it allows us, on the base of a statistical adjustment of the generated values of the total claim amount paid by the insurer, to determine the value of the capital requirement. It is here that the approach opens up the perspective of an interactive analysis of the results obtained by simulation in terms of the effect of changing the parameters of the contract concerned and even its type. The actuary is then able better to understand the behaviour of the model and develop and analyse various scenarios. Also in this case simulation replaces analytical approaches which are extremely onerous or even in many cases non-existent.

The algorithms for this method were developed using Visual Basic for Applications. The simulation methodology was checked by comparing to results from simulation with those from alternative approaches. The aforementioned elements of simulation were characterised by the stochastic approach, in which the generated value of a random variable were statistically adjusted. On the basis of the results obtained we can identify simulation as a powerful research tool allowing to analyse flexibly an assumed portfolio of insurance contracts in the context of risk management. Actuarial software developed from this would be able to provide interactive answers as to what would happen if the form of the contracts and the extent of any excess is changed within the internal model.

A further benefit of the simulation method, apart from the interactivity already mentioned, is its ability to measure the effect on risk reduction of implementing various types of loss insurance and excesses by looking at capital requirements.

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