Application of the Lifecycle Theory in Slovak Pension System¹

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Abstract

In the wake of population ageing and increasing public pension liabilities, many countries have reformed their pension systems, moving away (at least partially) from defined benefit (DB) pension schemes and towards defined contribution (DC) pension schemes. Slovakia's reformed its pension system into two main pillars – I. pillar PAYG scheme and II. pillar DC scheme. Slovak pension reform brings up a lot of questions regarding the optimality of two main pillars pension system set up. In this paper, we seek to investigate the effects of different Slovak pension system set up alternatives and their implications on lifecycle savings and individual welfare using own stochastic life cycle model of partial equilibrium. Results could serve as a basis for further discussion on improving the legislature on parameters' set-up of both pension pillars in Slovakia as well as abroad.

Keywords: pension system, life cycle theory, retirement, individual welfare

JEL Classification: D12, D14, D81, E21, G18

Introduction

To undertake the task of analysing the Slovak pension system alternatives, we develop a stochastic lifecycle model with five income types of individuals (i.e., income quintiles with different wage-age profile based on the United States Census Bureau data of earnings related to education and calibrated to Slovak conditions) making optimal decisions about their consumption/savings to maximise

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lifetime utility. Savings for retirement are made during the working years and consist of ordinary (liquid) savings and tax-exempt mandatory pension (illiquid) contributions. Upon exogenous retirement at age 62, individual receives pension depending on the given alternative.

We create for different alternatives: (I) with no pension system, (II) with I. pillar only, (III) with II. pillar only and (IV) with both I. and II. Pillar (current Slovak setup). We then examine the effects of these pension system setups on lifecycle behaviour and welfare. The model aims to approximate lifecycle behaviour of Slovak household. It incorporates the main features of Slovakia's tax and pension policy settings in year 2016, including personal income taxes and the two publicly stipulated pension pillars – large pay-as-you-go (further only PAYG) pillar with pensions based mainly on past earnings and a smaller pre-funded DC pillar with four percent contributions.

The paper is organized in order to present research findings on the life cycle analysis of Slovak pension system alternatives. Following chapter presents quick literature review then we present information on current Slovak pension system setup and regulation and thus defining the limitations for the research methodology. Then we present the methodology of our research and data for stochastic simulation. Last chapter discusses findings. In conclusion we summarize our findings and discuss recommendations for further research.

1. Literature Review

Lifecycle theory (or hypothesis) was developed in 1954 by latter Nobel Prize winner Franco Modigliani and his student Richard Brumberg (Modigliani and Brumberg, 1980). The lifecycle theory implies that individuals plan their consumption and savings behaviour over their entire lifecycle. This theory is then trying to explain individual's consumption patterns. Individuals intend to even out their consumption in the best possible manner over their entire lifetimes, doing so by accumulating when they earn and dis-saving when they are retired. The key assumption (in model with no risks) is that all individuals choose to maintain stable lifestyles. This implies that they usually do not save up a lot in one period to spend furiously in the next period, but keep their consumption levels approximately the same in every period.

There is a lot of recent literature and studies on the topic of lifecycle framework and retirement income products. Scholars use various approaches and assumptions (e.g. different risks involved, different utility functions) while looking on set up of individual's position in lifecycle framework. Main focus is put on the modelling of asset allocations and selection of retirement income products.

Although annuities are in economic theory seen as an optimal retirement product in reality demand for them is very low (Butler and Teppa, 2007). First and ground founding work studying lifecycle theory and its implications combined with retirement products was done by Yaari (1965). He concludes that optimising or rationally-behaving agent with no bequest motive will fully annuitize his accumulated wealth in order to cover longevity risk arising from uncertain life span. The recent work of Davidoff, Brown and Diamond (2005) extends Yaari's work and concludes that assumptions for full annuitization being optimal are not that strict. They also show that even with incomplete annuity markets (set of annuities is highly limited), consumers will generally want to annuitize a substantial portion of their wealth. Mitchell et al. (1999) show that the fees and expenses associated with annuities are not large enough to explain the lack of annuitization and Harrison, Byrne and Blake (2009) summarize issues of current UK annuity industry settings, with recommendations for improvement (e.g. information provided to employees, comparative table of all annuity rate). Benartzi, Previtero and Thaler (2011) favour annuities as optimal retirement product and give various reasons why annuities are inevitable for every pensioner (e.g. People do not know how fast they should spend their retirement wealth). The authors suggest several reasons of poor demand for annuity products, including regulatory framework and behavioural aspects (i.e., framing, mental accounting). They conclude, from comparing types of pension schemes, that DB plans' annuity is behaviourally more preferred as DC pension schemes are seen as an investment resulting in final cash balance (lump sum payment). Authors offer solution in variable retirement age (from 62 to 70) even though vast majority of people retire by rule of thumb at 65 (custom or accepted practice).

2. Slovak Pension System Design Overview

Slovak pension system is based predominantly on mandatory PAYG scheme. Contributions to the scheme are made from gross salaries and account for 18% of gross salary. After the reform in 2005, mandated system features also 1bis pillar constituted as private funded defined contribution (further only FDC) scheme. This pillar is currently (after several legislative changes) voluntary (with opt-in). From 2005 till 2012, the level of contributions was set at 9% for PAYG scheme and 9% towards FDC scheme. Since September 2012, if and individual opts for the 1bis pillar, social insurance contributions account for 14% of his gross salary and 4% are contributed into 1bis pillar. Starting 2017, the contributions towards 1bis FDC scheme started to increase gradually by 0.25 p.p. annually until they reach 6% of a gross salary in 2024.

3. Research Methodology

In this section, we provide a description of our model, which is used to investigate the effects of different Slovak pension system alternatives on lifecycle behaviour and individual welfare of five representative individuals. We stylised the model to the Slovak economy and incorporate key aspects of the Slovak retirement income policy.

With current knowledge in this discipline, optimization problems of complex life cycle models cannot be solved analytically (see Carroll, 1997; Hubbard, Skinner and Zeldes, 1994). For this reason is our model, similarly to all other models in the life cycle model literature, solved numerically.

Our model is a pure lifecycle model as the original life cycle model proposed in 1954 by Modigliani and Brumberg (1980) in the sense that the individual enters the model with no assets and has no bequest motive. We consider a single individual who enters the model at age 20 (period 1) and lives at most until the maximum age of 100 (period 80). In each period, the individual faces the uncertainty about his/her length of life described by exogenously given survival probability calculated from Slovak data (INFOSTAT, 2016). We assume that survival rate to age 101 equals 0. We assume exogenous retirement, at the age 62, which is minimum retirement age in Slovakia in 2016.

The individual makes consumption/savings decisions to maximize his/her lifetime expected utility subject to budget constraints. Individual in the model faces also uncertainty concerning the amount of money received from the labour income. We rule out unemployment from the model to investigate the life cycle savings motive (savings for the retirement). The wages also vary over the lifecycle due to adjustment factor that accounts for different wage-education dependency productivity of individuals in different ages. This productivity profile does not exhibit the standard hump-shape, but has rather concave profile (what is empirically tested by Casanova, 2013). We study five representative individuals differenced by education, and therefore the wage they get for their work (Guvenen, 2009). Different wage-age profiles are based on Julian and Kominski (2011) work on estimation of synthetic work-life earnings. To estimate regression coefficients and develop work-life earnings by age and education attainment for Slovakia using US Multiyear American Community Survey (ACS) dataset presented by Julian and Kominski (2011), we calibrated the data by the minimum, median and average earnings of full-time employed persons in Slovakia.

² We base our assumption on Harrison, Byrne and Blake (2006, p. 106), who argue that "bequests are usually satisfied outside of a pension savings framework and pension wealth is typically not bequeathable".

Estimating work-life earnings using Slovak data would increase the risk of error due to high volatility of labour market during the transition period of the economy that overestimates the growth coefficients and in general makes the life-cycle earnings curve more concave (especially for older workers). Additional logical arguments for applying US data are tied to the usage of financial markets and macroeconomic data to create one unique dataset of interconnected date for each month. Julian and Kominski (2011) ACS dataset obtains 9 different educational cohorts. We created 5 different educational cohorts to better fit with Slovak statistical data on average earnings and for each educational cohort, we profiled respective economic agent: (A) with elementary education, (B) with high school (with graduation) education, (C) with some college education, (D) with master's degree and (E) with doctoral degree.

During the working years the individual accumulates ordinary savings and mandated (in given alternative) tax-preferred savings at the pre-specified rate in a DC scheme. Ordinary savings can be accessed at any age to fund current consumption, whereas tax-preferred savings accumulated in the DC scheme are preserved until reaching the retirement age. At that age, the individual is given retirement income product – single lifetime annuity.

We used Epstein and Zin (1989) discrete-time recursive utility function and incorporate to this model survival probability according to Blake, Wright and Zhang (2014).

The optimization problem for the individual in our model is:

$$\max U_{t} \tag{1}$$

where

 U_t – expressed as follow:

$$U_{t} = \left((1 - \beta) * C_{t}^{\left(1 - \frac{1}{\varphi}\right)} + \beta S_{t} (E_{t} \left[(U_{t-1})^{1 - \gamma} \right])^{\frac{1 - \frac{1}{\varphi}}{1 - \gamma}} \right)^{\frac{1}{1 - \gamma}}$$
(2)

where

 $y_t > C_t \ge 0$ and U_t – the utility level at time t,

 C_t — the consumption level at time t,

 γ – the coefficient of relative risk aversion (CRRA),

 φ – the elasticity of intertemporal substitution (EIS),

 β — the subjective time preference discount factor,

S, – survival probability.

We assume that the individual entered into labour market as 20 years old (for each education level) and maximum potential age is T = 100).

where

$$x_{t} = w_{t} + y_{t} (p_{t}, g_{t}, v_{t}) + PP_{t} + DP_{t} - T(y_{t})$$

$$w_{0 \le t \le 20} = 0.$$
(3)

Optimization problem has two conditions. First, individual is constrained by budget constraint where x_t is available cash on hand at the beginning of period t, w_t is wealth at time t, y_t is labour income at time t, which is function of p_t – permanent labour income at time t, g_t represents wage growth as a result of labour productivity growth during the time t, v_t – income shock at time t. Income shock is random and has properties of normal distribution, with mean of 1, and standard deviation of 0,05. Individual is in retirement (after period t = 62) also entitled to receive PP_t – first pillar pension, DP_t – second pillar pension. Income is taxed by T – flat rate income tax. Individual is constrained by second restriction (bor-

Model is solved mathematically, using Bellman's (1980) dynamic programming. We use Value function iteration method (Stokey, Lucas and Prescott, 1989) for the simplicity of its implementation and given computational power. We use MS EXCEL 2013 and MATLAB R2015b.

rowing of money) which is not allowed in model at all. Individual in the model

has no initial wealth w at time 0 < t < 20.

We analyse four different pension system alternatives in constructed life cycle model: (I) with no pension system, where an individual accumulates just ordinary savings, with his net income being higher by 18% (otherwise used as pension contributions); (II) with I. pillar (PAYG scheme) only, where all pension contributions (18% of gross earnings) are contributed towards the PAYG scheme. It should be noted that we analyse the utility of an economic agent with different educational attainment and do not discuss the existence of PAYG scheme fiscal imbalance. This area has been analysed in previous work of Virdzek and Šebo (2012); (III) with II. pillar (1bis scheme) only, where all pension contributions (18% of gross earnings) are contributed into private FDC scheme; and (IV) with both I. and II. pillar, where 14% of gross income is contributed into the I. PAYG scheme, and 4% of gross income being contributed into II. pillar with gradual increase of contributions in favour of II. pillar. Fourth alternative represents the actual setup of Slovak pension system for PAYG and 1bis FDC schemes.

We assume that contribution *K* into bond fund which invest only to non-risky bond assets is calculated as follow

$$K_t = y_t * k_t \text{ for } 20 \le t \le 62$$
 (4)

Consider a model with one non-risky asset whose dynamics are given by the traditional continuous-time stochastic Brownian motion (standard Wiener process) and it satisfies the following stochastic differential equation (SDE):

$$r_{b_t} = \frac{dP_t}{P_t} = \mu dt + \sigma dW_t \tag{5}$$

where

 μ ('the percentage drift') and σ ('the percentage volatility') – constants,

dt – represent time changes between two period of time,

P – price of bond asset in time t,

dP – represents change in price between time t and t-1,

 W_t – a Wiener process or Brownian motion (W = N(0, t)).

Table 1
Values of the Model Parameters

Symbol	Description	Values	Source					
Model Parameters								
_	Date of Birth	Defined						
_	Date of start of work	Defined						
_	Date of retirement	1.1.2016	Defined					
_	Minimal age in the model	20	Defined					
_	Maximal age in the model	100	Defined					
_	Number of simulations	5 000	Defined					
_	Average of wage distribution	1	Defined					
_	Standard deviation of wage distribution	0.05	Defined					
_	Interest rate	4%	Defined					
Demographics								
S_t	Conditional survival probabilities	ı	infostat.sk					
Utility Function								
β	Subjective rate of time preference	0.96	Blake, Wright, Zhang (2014)					
γ	coefficient of relative risk aversion	5	Blake, Wright, Zhang (2014)					
System Parameters								
-	Age of Retirement	62	Current law					
_	Personal income tax	19%	Current law					
k	Contribution rate	4% (gradual increase to 6%)	Current law					

Source: Own work.

Final amount of wealth in the end of saving period is calculated according to Future Value of Annuity formula:

$$w_{62} = K_t * \left[\frac{1 + r_t - 1}{r_t} \right], \text{ where } t \in (20, ..., 62)$$
 (6)

Current legislature allows various pay-out options (life-time nominal or quasi-inflation indexed annuity, programmed withdrawal, temporary annuity) and

optimality of pay-out option has been studied by Šebo and Šebová (2016). However, we consider only one pay-out option – life-time nominal annuity. Annuity from accumulated wealth in II. pillar at the end of saving period (Inkmann, Lopes and Michaelides, 2011) was calculated as follows:

$$DP_{t} = \frac{w_{t}}{AR_{t} * 12}; t = 62$$
 (7)

where

DP - a monthly pension from II. pillar at moment of retirement,

w – amount of savings at the end of saving period,

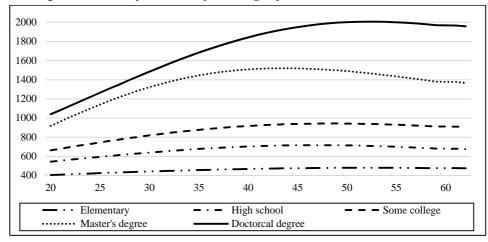
AR – an annuity rate in time t and fixed at rate of 4%.

In our case we do not consider any fees.

Figure 1 shows the trajectory of monthly earnings by educational attainment for five representative individuals chosen for our study.

Figure 1

Development of Life-cycle Monthly Earnings by Educational Attainment



Source: Authors' estimations using Julian and Kominski's ACS dataset (2016).

We use two extreme economic agents. One with elementary education (0-9) grades of elementary school completed) and other extreme one with a doctoral education. Other three profiles are most common in the Slovak population. We use high school graduate representative, whose monthly earnings at the start of the working life are around 550 Euro, representative with some college education and last, representative with Master's degree (in Slovak education system, obtaining Master's degree is very usual, contrary to US/UK practice with most of the students finishing after Bachelor's degree). This representative starts his working career with around 920 Euro in monthly earnings.

4. Results and Discussion

First, from monthly life-cycle earnings we derive annual salaries and pension contributions over the working lifecycle and calculate corresponding pension benefits for all five individuals in four tested alternatives. This calculation is first part of our results. Then, we used this calculated data to simulate individual's life cycle and calculate lifetime utility. This is second part of the results section.

Economic Agent with Elementary Education

First, in Figure 2, we present life cycle development of the income stream coming either in the form of the wage from work or from the given pension benefits for representative individual with elementary education in all four tested pension system alternatives. In all four alternatives, individual accrues his savings (not showed in the Figure 2) in order to maximise his lifetime expected utility and consumes (dis-saves) his wealth in order to finance living costs in the retirement. In case of alternative (I) with no pension systems, savings are the only source of individual's consumption at the retirement. In alternative (I) with no pension system, individual's net annual income is higher on average by 22% comparing to the alternatives II, III and IV, where monthly contributions toward pension schemes are made. Under the alternative I, pension contributions are not transferred to any fund, but stay available for consumption/savings decision of given individual. At moment of retirement, economic agent with elementary education is best off under the alternative III - pension system with II. pillar only. This alternative, quite surprisingly, provides annual pension of 4,564 Euro, compared to 3,939 Euro provided by pension system with first (PAYG) pillar. The explanation could be found in the slope of the life-cycle income as the agent's earnings do not increase significantly and thus return on accumulated wealth within the II. pillar is higher than the annual growth of earnings. However, more in-depth analysis is needed to understand the full consequences, which we leave for further research.

Economic Agent with High School Education

Our second studied individual has high school education (with graduation). We show life cycle development of his income (coming from work or as a pension benefit) in Figure 2. This individual starts his career with yearly salary at around 5,311 Euro, for three alternatives with pension system. Alternative without any pension system provides individual with income being 22% higher, i.e. at the start of working life at 6,487 Euro per year. Figure 2 shows also pension benefits belonging to the individual. With this education, individual has the highest

pension in alternative with only second pillar. First pillar, PAYG scheme, is not able to provide such benefits because of its redistributive mechanisms of intrageneration solidarity.

Economic Agent with Some College Education

Figure 2 shows development of yearly income during life of the individual with some college education in all four studied alternatives. Individual with this education starts his working life with net annual income of 6,306.65 Euro. In alternative without pension system is his net income higher by 22%. Income increases during the life cycle up to 8,661.22 Euro at the age of 48, which is a rise by 37.33%. After retirement this individual receive the highest pension benefits under the "second pillar only" alternative, with an individual replacement ratio being equal to 101.5% of his last pre-retirement salary. Pension system constituted of both pillars provides income compensation of 90% of last pre-retirement salary and is equal to 7,567.08 Euro. Similarly to individual with high school education, this individual receives lowest pension in alternative with "First Pillar only".

Economic Agent with Master's Degree

Figure 2 displays comparison of four studied alternatives for the Individual with Master's degree. This individual enters into the working career with yearly income of 8,455.82 Euro (or 704.65 Euro monthly, what reflects our calibration of income profiles to mimic Slovak economic conditions, where the median monthly salary was at the mark 697.17 Euro in 2016). Yearly income of this individual in the second half of working life is 13,521.83 Euro. This represents increase in income by over 59% compared to the start of the working career. At the time of retirement, individual's yearly net income is 12,320.21 Euro. As with two previously shown figures and individuals, the highest pension benefit comes from "Second Pillar only" alternative. Pension benefit in this alternative is for an individual with Master's degree at 13,418.64 Euro per year. In alternative with both pension pillars individual gets 11,999.52 Euro and under the "First pillar only" alternative, an individual receives 10,052.40 Euro, which is by 25.1% lower comparing to the best alternative.

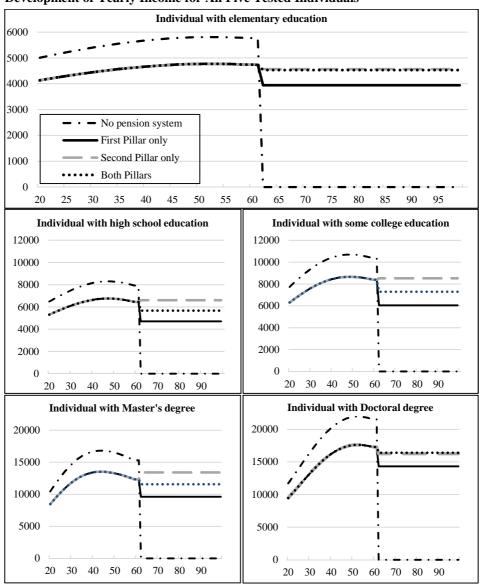
Economic Agent with Doctoral Degree

Yearly income of an individual with doctoral degree is shown in last graph of figure 2. This individual receives the highest yearly earnings among five studied individuals during all tested periods. His initial yearly net income for work accounts for 9,477.44 Euro. Under the "No pension system" alternative, agent's net

income is higher by 23.7%. Hence, individual's income is then 11,723.84 Euro. This individual has also the highest increase in yearly income during the working period. At the age of 52 his salary is 17,622.04 Euro (increase of 85.9%). At moment of retirement, individual's income accounts for 17,286.40 Euro of net yearly salary. He receives pension benefits between 14,968.92 Euro and 17,045.76 Euro, depending on given pension system setup alternative.

Figure 2

Development of Yearly Income for All Five Tested Individuals



Surprising result of lifetime income of Individual with doctoral degree is alternative which provides the highest pension benefits. Our simulations show that for this individual, favourable alternative is one with "Both Pillars". This result is caused by current setting of Slovak first PAYG pillar. This pillar, by its nature, should have redistributive and solidary mechanisms in favour of those with lower earning. Unfortunately, the formula which should adjust and transform higher pension contributions into future relatively lower pension benefits is not designed properly enough.

Results of Modelled Lifecycle Consumption and Savings

We use computational software MATLAB R2015b to develop and use lifecycle model to calculate periodic consumption and savings of each individual using dynamic programming and value function iteration method. Individual in each period decides how much to consume and save in order to maximize his lifetime expected discounted utility. As a consequence of this maximization, Individual is aware he has to create savings in order to finance his retirement living costs. Figure 3 shows results of our modelling for the individual with elementary education, all other individuals are shown in appendix.

 $Figure\ 3$ Consumption and Savings during Life of the Individual with Elementary Education

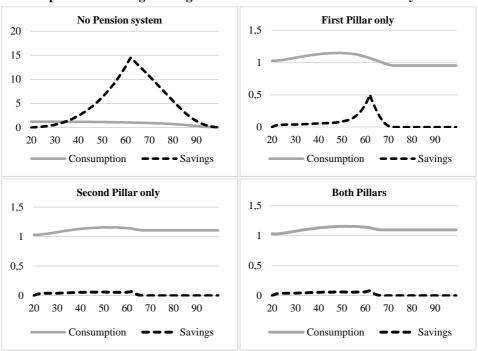


Figure 3 presents results of individual's consumption and savings simulated using constructed life cycle model. Modigliani's Lifecycle theory (Modigliani and Brumberg, 1980) states that individual is trying to maintain steady smooth level of consumption throughout the lifetime. Our results follow this theory. In every pension system alternative, an individual has very smooth consumption, which rises with the rise of one's income. Individual under the alternatives with pension system is aware of this future additional income and has no reason to accrue savings for retirement. On the other hand, an individual under the alternative with no pension system needs to accumulate his savings in order to finance his living costs during retirement.

Results of the Expected Lifetime Utility

The expected utility is a synthetic indicator that expresses the sum of the discounted period utilities of an individual during his life. Each period utility is discounted into the summary indicator by the coefficient β expressing subjective rate of time preference and by the conditional survival rate S. By measuring an individual's expected utility we are able, within a given methodology, to objectively evaluate the various alternatives of the pension system applicable to in the Slovak Republic. We use formula from Kudrna and Woodland (2011) to calculate equivalent variation (EV) of lifetime utility between alternatives, using "No pension system" as a benchmark. Equivalent variation can be interpreted as change in a utility of an alternative to be "paid" in order to get the utility of an original alternative ("no pension system" in our case).

Table 2
Comparison of 4 Pension System Alternatives Using Equivalent Variation

Education Attainment Pension System Arrangement		Elementary education	High school education	Some college education	Master's degree	Doctoral degree
No pension system	Value	-23.5594	-16.7031	-13.3881	-8.9282	-7.5837
	% change	0.00	0.00	0.00	0.00	0.00
First Pillar only	Value	-22.0493	-16.4038	-13.0969	-8.6459	-7.1749
	% change	6.85	1.82	2.22	3.26	5.70
Second Pillar only	Value	-21.6110	-15.6777	-12.5282	-8.2870	-7.0704
	% change	9.02	6.54	6.86	7.74	7.26
Both Pillars	Value	-21.6342	-16.0002	-12.7813	-8.4429	-7.0643
	% change	8.90	4.39	4.75	5.75	7.35

Source: Authors' calculations (2017).

Table 2 presents comparative analysis of four alternatives examined. Values are shown in absolute and in percentage terms. Individual with primary education is better off by 9.02% of lifetime welfare in "Second Pillar only" alternative compared to "No pension system" alternative. For an individual with doctoral degree,

it is preferred to use "Both pillars" alternative, because of the gain in lifetime utility by 7.35%. Results of our study show that alternative with the "Second Pillar only" (FDC scheme) brings the best increase in lifetime expected utility for four out of five tested educationally differed agents. Increase in the lifetime welfare in alternatives with pension system is caused by presence of additional risk coverage (in the form of insurance type of product providing the pension benefit) otherwise not available to individual in "No pension system" alternative.

Conclusions

Our paper focused on examining four alternatives of the pension system arrangements with features defined for Slovak pension scheme. We examined current pension system setup (combination of PAYG and FDC schemes) and compared it with three theoretical alternatives. We developed and used own stochastic life cycle model of partial equilibrium to examine individual welfare expected from four tested pension system alternatives. Our results showed that current setup with dominating First Pillar PAYG scheme which takes up 77% of person's contributions and auxiliary Second Pillar FDC scheme could be suboptimal, in sense of individual welfare. We found other individual welfare improving pension system setups. For four out of five tested individuals the most welfare improving option is "Second Pillar only" alternative with average improvement in lifetime welfare of 7.54%. Other important finding is about current setup of First Pillar and its redistributive mechanisms. This feature of intra generational solidarity should redistribute relative amount of pension benefits (relative to contributions) from high income earners to low income earners. However, our results showed the opposite. The formula which should adjust and transform higher pension contributions into future relatively lower pension benefits has regressive pattern in higher incomes assessment which is the cause of this distortion. We see several possible extensions for our model, by implementing dynamic asset allocation in accumulation phase (as in Melicherčík, Szüsc and Vilček, 2015) or by introducing innovative deaccumulation phase product – the tontine annuity (Milevsky and Salisbury, 2015).

References

BELLMAN, R. (1980): Analytic Number Theory: An Introduction. Reading, MA: Addison-Wesley, 195 pp. ISBN 9780805303605.

BENARTZI, S. – PREVITERO, A. – THALER, R. H. (2011): Annuitization Puzzles. Journal of Economic Perspectives, 25, No. 4, pp. 143 – 164.

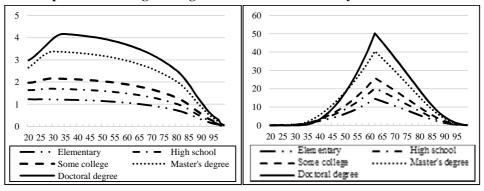
- BLAKE, D. WRIGHT, D. ZHANG, Y. (2014): Age-dependent Investing: Optimal Funding and Investment Strategies in Defined Contribution Pension Plans when Members are Rational Life Cycle Financial Planners. Journal of Economic Dynamics & Control, 38, No. 1, pp. 105 124.
- BUTLER, M. TEPPA, F. (2007): The Choice between an Annuity and a Lump Sum: Results from Swiss Pension Funds. Journal of Public Economics, 91, No. 10, pp. 1944 1966.
- CARROLL, CH. D. (1997): Buffer-stock Saving and the Life Cycle/Permanent Income Hypothesis. The Quarterly Journal of Economics, 112, No. 1, pp. 1 55.
- CASANOVA, M. (2013): Revisiting the Hump-Shaped Wage Profile. [Discussion Paper.] Los Angeles: University of California. Available at: http://www.econ.ucla.edu/casanova/Files/Casanova_wage_older_workers.pdf.
- DAVIDOFF, T. BROWN, J. DIAMOND, P. (2005): Annuities and Individual Welfare. Ameri-
- EPSTEIN, L. G. ZIN, S. E. (1989): Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. Econometrica, *57*, No. 4, pp. 937 969.
- GUVENEN, F. (2009): An Empirical Investigation of Labor Income Processes. Review of Economic Dynamics, 12, No. 1, pp. 58 79.
- HARRISON, D. BYRNE, A. BLAKE, D. (2006): Annuities & Accessibility: How the Industry Can Empower Consumers to Make Rational Choices. [Discussion Papers.] London: Cass Business School & Pension Institute. ISSN: 1367-580X. Available at: http://dx.doi.org/10.2139/ssrn.1344343>.
- HUBBARD, R. G. SKINNER J. ZELDES. S. P. (1994): The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving. [Working Papers 4516.] Cambridge, MA: NBER. Available at: http://www.nber.org/papers/w4516.pdf>.
- INFOSTAT (2016): Slovak Mortality Tables. Available at:

can Economic Review, 95, No. 5, pp. 1573 – 1590.

- http://www.infostat.sk/vdc/sk/index.php?option=com_wrapper&view=wrapper&Itemid=50>.
- INKMANN, J. LOPES, P. MICHAELIDES, A. (2011): How Deep is the Annuity Market Participation Puzzle? The Review of Financial Studies, 24, No. 1, pp. 279 319.
- JULIAN, T KOMINSKI, R. (2011): Education and Synthetic Work-Life Earnings Estimates. American Community Survey Reports, ACS-14. Washington, DC: U.S. Census Bureau. Available at: https://www.census.gov/prod/2011pubs/acs-14.pdf>.
- KUDRNA, G. WOODLAND, A. (2011): An Inter-temporal General Equilibrium Analysis of the Australian Age. Journal of Macroeconomics, *33*, No. 1, pp. 61 79.
- MELICHERČÍK, I. SZÜSC, G. VILČEK, I. (2015): Investment Strategies in the Funded Pillar of the Slovak Pension System. Ekonomický časopis/Journal of Economics, 63, No. 2, pp. 133 151.
- MILEVSKY, M. A. SALISBURY, T. S. (2015): Equitable Retirement Income Tontines: Mixing Cohorts without Discriminating. [SSRN Working Paper, No. 2700362.] Available at: http://ssrn.com/abstract=2700362>.
- MITCHELL, O. S. POTERBA, J. M. WARSHAWSKY, M. J. BROWN, J. R. (1999): New Evidence on the Money's Worth of Individual Annuities. American Economic Review, 89, No. 5, pp. 1299 1318.
- MODIGLIANI, F. BRUMBERG, R. (1980): Utility Analysis and the Consumption Function: Interpretation of Cross-Section Data. In: ABEL, A. (ed.): The Collected Papers of Franco Modigliani: The Life Cycle Hypothesis, Vol. 2. Cambridge, MA: MIT Press, pp. 128 – 197.
- ŠEBO, J. ŠEBOVÁ, Ľ. (2016): Suboptimality of Immediate Annuitization in Private Pension Schemes. Ekonomický časopis/Journal of Economics, 64, No. 2, pp. 169 187.
- STOKEY, N. L. LUCAS, R. E. PRESCOTT, E. C. (1989): Recursive Methods in Economic Dynamics. Cambridge, MA: Harvard University Press, 608 p. ISBN 9780674750968.
- VIRDZEK, T. ŠEBO, J. (2012): Applying Automatic Balance Mechanism in the PAYG Pension Scheme in Slovak Republic. Ekonomický časopis/Journal of Economics, 60, No. 5, pp. 482 195.
- YAARI, M. (1965): Uncertain Lifetime, Life Insurance, and the Theory of Consumer. Review of Economic Studies, 32, No. 2, pp. 137 150.

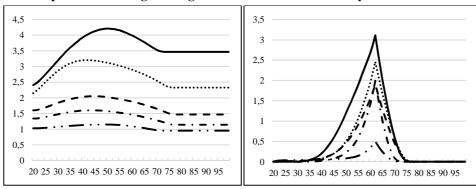
Appendix

Figure 4 Consumption and Savings during Lifetime in "No Pension System" Alternative



Source: Authors' calculations (2017).

Figure 5 Consumption and Savings during Lifetime in "First Pillar Only" Alternative



Source: Authors' calculations (2017).

Figure 6 Consumption and Savings during Lifetime in "Second Pillar Only" Alternative

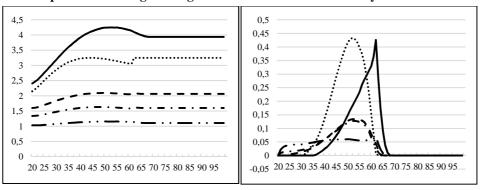


Figure 7
Consumption and Savings during Lifetime in "Both Pillars" Alternative

