Non-Stochastic Argumentation in Predicting Economic Indices

Luboš Marek[1] | *Prague University of Economics and Business, Prague, Czech Republic*
Richard Hindls[2] | *Prague University of Economics and Business, Prague, Czech Republic*
Stanislava Hronová[3] | *Prague University of Economics and Business, Prague, Czech Republic*

Abstract

This paper studies the use of statistical prognostics in predictions of short-term year-to-year evolution of GDP and other aggregate indices of the national accounts. It shows the utilisation of a non-stochastic prediction range to be used as a prediction tool that, to a certain extent, overcomes the validity of the *ceteris paribus* principle, on which most of the currently used stochastic approaches are based. The non-stochastic range is a resultant outcome of a wide assortment of time-series models; at the same time, a point forecast for short-term evolution is derived from the said assortment of models. We illustrate our methodology on a year-to-year evolution of GDP indices in France as a time series with a sufficiently large number of observations.

Keywords	***JEL code***
Statistical prognostics, non-stochastic point forecast, non-stochastic prediction range, GDP	*C10, C53*

INTRODUCTION

The most significant economic indices that sensitively respond to the prevailing economic climate include, first and foremost, the gross domestic product (hereinafter GDP), but also other related aggregate indices of the national accounts; in particular, final consumption expenditure, gross capital formation, and exports and imports of goods and services. Monitoring these values statistically not only provides information on the current situation of the national economy, but can also be used in analysing its evolution as a basis for deriving short- and medium-term predictions.

Regarding the subsequent utilisation of such data for the purposes of estimating the performance of the economy, as well as providing a basis for creating the state budget, particular importance are predictions of short- and medium-term evolution of the said indices. The usual methods utilised in economically developed countries when estimating the GDP evolution for two to three years ahead are mainly based on predictions put forth by relevant expert groups, combined with econometric and statistical models. When creating state budgets, regression model approaches are also employed; either the concept of extrapolating the prevailing trend, or deriving the national accounts' aggregates

[1] Prague University of Economics and Business, Faculty of Informatics and Statistics, Department of Statistics and Probability, W. Churchill Sq. 4, 130 67 Prague, Czech Republic. E-mail: marek@vse.cz.
[2] Prague University of Economics and Business, Faculty of Informatics and Statistics, Department of Statistics and Probability, W. Churchill Sq. 4, 130 67 Prague, Czech Republic. E-mail: hindls@vse.cz.
[3] Prague University of Economics and Business, Faculty of Informatics and Statistics, Department of Economic Statistics, W. Churchill Sq. 4, 130 67 Prague, Czech Republic. E-mail: hronova@vse.cz.

from a tightly related index that is already known, is closely monitored by the respective statistical office, and can be viewed as a suitable "prompt" to reveal the anticipated behaviour of the aggregates to be estimated.

The results achieved in this area entitle us to conjure that, under such circumstances, certain statistical methods become very important; namely those which are capable of providing the necessary information about the expected GDP evolution. The techniques for obtaining such predictions undoubtedly include methods used in time series analysis. Developments in statistical forecasting and its applications in economics extended the range of tools that can be used in macroeconomic decision-making. In the present paper, we study one of such approaches, which can be distinguished by a high success rate in short- and medium-term predictions.

1 METHODS OF TIME SERIES ANALYSIS AND PREDICTION PROCEDURES STUDIED IN THE LITERATURE

Approaches that employ time series models in predicting a given index occur quite frequently in the literature. Some of these approaches have found their ways to being used in the practice of economics, for example, in the procedures used by governments and other authorities; others have not proved their worth. To a large extent, the success rate of a prediction model is implied by that model's ability to overcome difficulties related to the *ceteris paribus* principle (saying that the future will, under unchanged circumstances, be a continuation of the past). In the reality we encounter in economics, the *ceteris paribus* principle can hardly be maintained; or rather, it is nearly completely unmaintainable in practice. As a matter of fact, economic processes are subject to many different types of interventions, whether legislative (therefore non-stochastic, such as amendments to tax and other laws) or natural (such as the stages of the economic cycle, etc.). Frequently we encounter combinations and even mutual interactions between stochastic and non-stochastic interventions. All such aspects de facto deny the *ceteris paribus* principle. That is why we often encounter in the literature routines and tools aimed at overcoming that extrapolation principle, which is too unrealistic in practice.

Of course, we can find in the literature a great number of suggestions for utilising statistical techniques in predictions. The foundations for extrapolation techniques were laid by many authors and, even though some of their concepts date back quite a few years, they have not been abandoned. Let us mention, as just a few of many possible examples, the monographs by Theil (1966), Granger and Newbold (1986), and Pankratz (1991); in the Czech Republic it was Cipra (1986). We can generally observe that the use of non-stochastic deterministic approaches is less frequent than those based on probability theory and entropy. Even if a non-stochastic concept is employed, it often represents a certain modification of brainstorming methods. This approach is not new, as a matter of fact; its first occurrence dates back to the post-World-Word-II decade; for example, Osborn (1963) described a creative group technique as a specific method for predicting and decision-making. We will go in this direction below, but we will mutually confront not personal opinions, but results of potentially acceptable models of time series.

The developments go on; newer – and, in sense, perhaps more efficient – methods are sought to mutually intertwine the deterministic and stochastic principles. The present paper is also an attempt at an original combination of deterministic and stochastic concepts. We can take lessons from the literature, in which different non-traditional approaches can be encountered. Let us recall a recent concept published in Lui (2020), which provides an in-detail study of the relationship between deterministic and stochastic-based interval predictions and attempts at bridging the gap between them with the aid of a certain hybrid approach. An Israeli geophysicist Eppelbaum (2013) derived estimates in the predictive model so that they were, predominantly, highly correlated with historic data, i.e., a reference variable with correlated evolution in time. Yachao et al. (2016) brought deterministic and probabilistic interval predictions (namely, for short-term prediction of electricity generation from wind) based on a decomposition of the variation mode.

A list of examples found in the literature indicates that the applications of the proposed techniques are, to a great extent, illustrated on data coming from the natural sciences. Economics is, however, a social science; its phenomenology is therefore completely different from that of the natural sciences. Moreover, it is marked with the existence of extensive behavioural elements. In this respect, the range of data-based experiments found in the literature is rather less abundant.

That is why we will apply our approach to an economic time series. Conceptual inspiration can be found even here: in order to gradually improve the quality of prediction, Tribbia and Baumhefner (2013) recommend that the facts of the situation be examined in general at first, and then set up the goal of the particular prediction from the phenomenological point of view. Later on, the phenomenological and non-deterministic aspects of the prediction should be intertwined at the given time horizon. We will try below to follow a similar line of thought: we will employ an assortment of models, thus introducing into our prediction uncertainty pertaining to each of those models. Subsequently we will reduce the uncertainty by deriving a compound solution based on all of the primarily used models. One of the main characteristics of such a prediction will be the fact that the uncertainty pertaining to each of the models will be reduced in the compound prediction range.

As can also be seen in the literature, the common denominator of such approaches is to derive a plausible prediction and, at the same time, to eliminate to the maximum possible extent the non-realistic *ceteris paribus* assumption. The main point here is to apply stochastic modelling in as wide a sense as possible, directed towards reflecting the effects of intervention attacks of diverse origin and nature. The weakest point of a similar approach is that the resulting prediction interval is usually too broad; consequently, its usefulness for the decision-making processes is dubious. Because of that, we will suggest a procedure below that should provide a more useful interval of the prediction.

2 TRADITIONAL REGRESSION APPROACH

Now we will derive the statistical prediction for the year-to-year evolution of any index. This technique is based on statistical tools inherent to analysis of one-dimensional time series. Let us begin with an overview of necessary basic notions.

The traditional regression-based approach to extrapolations of a time series y_t, $t = 1, 2, \ldots, n$, where n is the number of the (past) observations of the time series, is generally formulated as a requirement to begin the prediction with a suitable estimate for the future values y_{n+i}, $i = 1, 2, ..., N$, where N is a selected positive integer characterising the length of the prediction. Of course, we can resolve this prediction problem with the aid of a model supposedly governing the behaviour of the relevant time series, y_t. This is the so-called *point prediction*.

It is, however, a well-known fact that a point prediction is too authoritative about the future evolution of the index to be predicted. It is difficult to find a specific reason why a particular model should be singled out (incidentally, good results in interpolation – i.e., modelling the past evolution of the time series – need not be a sufficient guarantee for a good prediction). Moreover, the point prediction is utterly incapable of overcoming the *ceteris paribus* condition (of the future being a continuation of the past under circumstances that otherwise remain unchanged). This problem is viewed as very serious in economics. From the factual viewpoint, economic indices are very unstable variables; assuming that factors affecting their future evolution remain unchanged is in its substance absurd and stands in a deep contradiction with the substance of economics as a social and political science.

There are techniques which try to cope with the authoritative character of the point prediction; such techniques lead to interval predictions of future developments. In a vast majority of instances, such techniques are based on stochastically formulated and interpreted predictions with respect to a pre-set level of confidence. Another problem arises at this moment. In addition to the fact that our prediction interval stems from a point prediction, encumbered with all its weaknesses (as mentioned above), two additional requirements must be met: the confidence level of the prediction must be sufficiently high (in practice not smaller than 90%); and, at the same time, the prediction interval must be reasonably narrow. Unless both these requirements are met, the prediction interval will be more or less useless in economic practice. It is well known that requirements for a high confidence level and a narrow prediction interval are in a mutual conflict. Not even an interval prediction is able to overcome the *ceteris paribus* principle (which will also be seen from our application below). Setting up an interval, we create a funnel trough which additional possibilities are "drawn" into the prediction. The usual price to pay for this aspect is an excessive breadth of possible values, which is difficult to interpret.

If we construct an estimate for the future value y_{n+i} at time $(n + i)$ as:

$$P_{n+i} - \Delta < y_{n+i} < P_{n+i} + \Delta \,, \qquad (1)$$

where P_{n+i} is the point prediction for the time period $n + i$ as estimated by any model, and Δ is the admissible error of the prediction; the latter depends on the selected confidence level of the prediction interval, as well as on the number and variability of the real observations from the past, y_t. Inequality (1) is valid for the pre-set level of confidence (that is, with a certain – sufficiently large – value of probability); that is why we call it *stochastic interval prediction*. An interval prediction defined in this way is symmetric. Since the stochastic (probabilistic) interval prediction stems from three point predictions for which inequality (1) holds, the conditions determining the success of the interval prediction is the quality of the original point prediction P_{n+i}. The actual value of the index to be predicted for time $n + i$, i.e., y_{n+i}, is "enveloped" by the stochastic prediction interval.

The particular form of the admissible error (that is, the stochastic confidence interval) of the prediction can, for example, be described by the following Formula for the linear trend model – cf. Cipra (1986):

$$\Delta = T_{1-\alpha/2}[n - 2] \cdot s \cdot f_i \,, \qquad (1a)$$

where T is the quantile of Student's distribution with $n - 2$ degrees of freedom, n is the number of the past observations in the time series; the length (horizon) of the prediction is $i = 1, 2, ..., N$ (we set $N = 2$ for the purposes of macroeconomic prediction in our case for practical reasons),

$$s = \sqrt{\frac{\sum_{t=1}^{n} y_t^2 - \sum_{t=1}^{n} \hat{y}_t^2}{n - 2}} \,,$$

$$f_i = \sqrt{1 + \frac{1}{n} + \frac{(N - \overline{t})^2}{\sum_{t=1}^{n} t^2 - n \cdot \overline{t}^2}}$$

and $\hat{y}_t$ = the model estimate for the value of the time series y_t, $t = 1, 2, \ldots, n$.

There is, however, one significant drawback encumbering the construction of a stochastically argued prediction interval: even if the number of observations is sufficient and the selected model has a good quality (from the interpolation point of view), the prediction interval on an adequate level of confidence is too broad. This is bad news regarding the practical requirements we have put on predictions.

3 CONSTRUCTION OF NON-STOCHASTICALLY ARGUED PREDICTION RANGE

Experience with predicting values of economic indices thus generally indicates that the resolution of the prediction problem needs more than a purely "regression-like" formulated short-term prediction. We have already mentioned several reasons for misgivings pertaining to a traditional regression concept to be applied in predicting macroeconomic evolution indices: a large (and, consequently, conflicting) assortment of point predictions for the same index; a practically useless breadth of the stochastically constructed interval predictions according to Formula (1) (or (1a) for the linear trend); and the traditional statistical assumptions are usually not valid for real economic data; etc.

There exists a certain generally positive outcome regarding these user-unfriendly situations; they, to a considerable extent, determine the acceptance of statistical predictions in the areas of macroeconomic studies, conjunctural surveys, etc. We must take into account a non-traditional approach to setting up our predictions, an approach within which the prediction interval (1) is argued not in a probabilistic, traditionally regression-driven way, but as follows:

$$P_{n+i} - \delta_1 < y_{n+i} < P_{n+i} + \delta_2, \qquad (2)$$

where the prediction range $P_{n+i} - \delta_1$ and $P_{n+i} + \delta_2$, $i = 1, 2, ..., N$, is understood as a resultant outcome of different point predictions on the basis of a large number of factually admissible models for the past behaviour of the respective time series (see Formulae 3 and 4 for the construction of deviations δ_1 and δ_2). An interval prediction defined in this way will not be symmetric with respect to y_{n+i}.

Determination of the prediction according to general Formula (2) will be called a **non-stochastic prediction range**. Such a range is based on the idea that we can set up several (or many) models for a given time series, which may all properly describe the past behaviour of the respective series and be admissible from the factual and formal points of view. We must still keep in mind this important fact: a model providing a good-quality description of the past behaviour of a time series need not provide a good prediction of its future behaviour, due to possible changes in the conditions to which the time series is subject.

A strong point of the non-stochastic prediction range is its universal nature. In fact, in addition to stochastic models we can also utilise econometric models and combinations of both types for the construction of that range. The only condition is that, for the primary models, factual and formal admissibility should be guaranteed for the underlying problem; and there should be a realistic option to set up a higher number of such models. Extensive use of software enables us to set up many models, compute estimates in them, and compare their prediction outcomes.

Conceptually, such a construction of a non-stochastic prediction range is similar to the usual economic practice, in which different opinions concerning the future evolution are confronted with each other. Here we confront "opinions" ensuing from different models for the underlying time series. The prognosis is then a resultant outcome of all such "components". We will illustrate the construction of a non-stochastic prediction range on an example of year-to-year GDP indices time series in France.

The construction itself goes in two steps; this approach can be understood as an algorithm and programmed as such.

Step 1: *Selection of and estimates in models*

First we set up a certain high number of statistical models of adequate quality and with good factual interpretations (in our case, models for the time evolution of the year-to-year GDP indices time series in France; for the sake of clarity, we sum up these models in Table A2 in the Appendix, where the estimated model shapes, denoted by M1, M2, …, through M22, are also shown). Our basic idea here is that each model represents an opinion ("winnowing our facts") – in an analogy to the "normal human thinking under uncertainty", which may take into account several admissible variants. Our prediction task can certainly be classified as such a situation. The selection of the models is the key stage of the prediction process, because these models lay the foundations for the prediction range. Hence we must responsibly consider factual and formal statistical viewpoints during the selection.

In our experience, it is purposeful to select between 10 and 30 models from different categories, such as smooth analytical trend functions for different types of exponential fitting, from the Box-Jenkins methodology, etc.; our criterion should be based on the quality and interpretation of the models. The character of the index to be predicted should also be reflected. No other restrictions should be considered. Of course, we must always keep in mind that a primary model that is "interpolation-good" is no guarantee for a good quality of prediction due to the unrealistic assumptions hidden in the *ceteris paribus* principle.

Step 2: *Construction of prediction range and point prediction*
Let us derive three values for each year in each of the models set up and tested according to Step 1 (in our case, M1, M2, …, and M22: namely, point prediction P_{n+i} (that is, a year-to-year indices of the GDP growth for the years 2018 and 2019); the stochastic lower bound for the prediction, Lower 95.0% Limit; and the stochastic upper bound for the prediction, Upper 95.0% Limit, both bounds at the 95% confidence level. For example, the mentioned three values will look as follows for Model M1:

Table 1 Model M1. Random walk with a drift

Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
2018	1.022 060	0.984 296	1.059 830
2019	1.021 130	0.967 717	1.074 540

Note: Similarly for the remaining models, M2, M3, …, through M22, cf. Table A2 in the Appendix.
Source: Authors' own calculations

Having selected 22 models, we can see that we would get 22 point predictions P_{n+i} for each of the years to be predicted, i.e., 2018 and 2019; and 22 stochastic interval predictions at the same time. Each of these predictions can be viewed as relevant, but they are numerically different from each other. This aspect is rather indeterminate with regard to subsequent considerations. That is why we will now show a way to arrive at non-stochastic predictions, while adequately using the specific information provided by each of the models we computed.

As regards point predictions P_{n+i}, we will derive a sole aggregated value of the point prediction based on all 22 models M1, M2, …, M22; namely, we take the average of those 22 values. In a way, this approach is an analogy to colloquia, in which opinions of relevant members are comprised. Here the 22 models stand for such members, and the result is described in the part 4.2.

When setting up the interval prediction corresponding to Formula (2), we will first derive a sole aggregated lower bound δ_1 from the Lower 95.0% Limit values of all models M1, M2, …, M22; namely, we will take their maximum, that is,

$$\delta_1 = \max\{\text{Lower 95.0\% Limit of M1, M2, …, M22}\}. \quad (3)$$

In a similar way, we then derive a sole aggregated upper bound δ_2 from the Upper 95.0% Limit values as their minimum:

$$\delta_2 = \min\{\text{Upper 95.0\% Limit of M1, M2, …, M22}\}. \quad (4)$$

In other words, this particular application of Formula (2) is based on the maximum stochastic lower bound and the minimum stochastic upper bound of the traditional prediction intervals (our result is again described in the part 4.2). As already mentioned above, an interval prediction defined in this way can no longer be symmetric, in contrast with the stochastic interval prediction.

4 APPLICATION OF THE MODEL

In order to verify our model of non-stochastic prediction range, we have chosen the time series of year-to-year GDP indices in France in the period from 1950 to 2019. This is a highly aggregated index occurring on French national accounts. Our main reason for this selection is the long time series we can use to illustrate our approach.

The developments in the French national economy over the course of nearly 70 years represent a diverse mixture of various influences (post-war recovery, cold war, adoption of euro, etc.), internal political decisions (taxation, monetary/fiscal interventions, changes in the play of political forces), as well as international economic interventions (oil crisis, local military conflicts, etc.).

4.1 Economic developments in France since the end of World War II

As already explained, we will make use of the French GDP data to illustrate our method of setting up the prediction.[4] A factual description of the economic developments in France since the late 1940s until this date is important to help us provide effective interpretations and utilise the derived estimates and predictions.

The post-war period of the so-called French Fourth Republic (1945–1958) was characterised by relatively high economic growth, a high inflation rate and low unemployment rate. In the beginning of the post-war period, the Marshall Plan played the main role in the economic recovery. On the other hand, the Fourth Republic was politically rather volatile (the Prime Minister was changed 28 times in 12 years); this aspect did not contribute to the country's economic stability.

Figure 1 Year-to-year GDP indices, France

Source: <*www.insee.fr*>

[4] The input data – year-to-year French GDP indices – is depicted in Figure 1 and numerically listed in Table A1 in the Appendix.

Due to the destruction prevailing after the war, the government undertook the task to recover the French economy. Electric power plants, the coal industry, big bank and insurance companies, Renault, aerospace industry, etc., were nationalised. The nationalised sector was an important tool for implementing the government's economic policies. The first Monnet Plan was begun in 1947 (a programme for investments into the key industries and reduction of the economic dependency on abroad). The second Monnet Plan (1954–1957) was focused on public investments and ensuring higher productivity in material and human resources. A higher GDP growth rate in 1950 brought about a higher inflation rate. The subsequent slowdown in the economic performance after 1950 meant a gradual return to the normal production potential; adoption of a deflation policy also contributed to the slowdown. From 1954, the GDP regular growth by more than 5% a year prevailed. Another slowdown and an onerous economic situation in the late 1950s were mainly caused by the colonial war in Algeria, the imminent civil war, and growing expenses on nuclear armament. Under the aggravated conditions, a new Constitution was written, the Fifth Republic was born and Charles de Gaulle was elected President of France for seven years in the office. In addition to the war in Algeria, another reason for the economic slowdown was the overall decline of the conjuncture.

After the new Constitution was approved by a referendum, a new plan was developed to put the French economy back on its feet, and substantial savings reduced the budget deficit but also the expenses incurred on the social care. The economic measure brought the French economy to a recovery in the early 1960s and the economic unbalance was eliminated. A faster economic growth stated in 1960, also in consequence of franc devaluation, which favoured sales of French goods on foreign markets.

In the mid-1960s, the French GDP dynamics went down because of decreasing wages and consumption. This situation later (in 1968) caused extensive strikes because employees' economic standing was worsening. A strike lasted several weeks and the economy was paralysed; the consequent drop in the production led to the smallest GDP growth rate value in the entire 1960s (4.5% year-to-year in 1968). Charles de Gaulle resigned his presidency at noon, April 28, 1969. The new political elite, headed by President Pompidou, adopted a new plan to stabilise the economy – the primary aspects included the support to exports and restriction on imports, reduction of the state budget deficit, and a substantial increase in taxes. Nonetheless, France did not avoid the financial crisis connected with the termination of the Bretton Woods system, which caused a monetary crisis of the franc.

The GDP growth had a descending trend after 1973. The French economy was hit by a recession and went to the bottom in 1975 (a deep drop caused by the Yom Kippur War and the subsequent oil crisis was accompanied by a high level of inflation and unemployment rates). In 1976, a new plan aimed at stabilising the franc and recovering the budget balance was adopted. The plan worked as expected and the inflation was temporarily stopped. The balance was recovered, but the high unemployment rate continued to prevail. The second oil crisis and a return of the high inflation rate (nearly 14% in 1980) were negative factors. All these facts and other economic-crisis phenomena affected the presidential elections in 1981. The political establishment increased the minimum wages, the lowest pensions, and the family benefits. Nearly all banks, insurance companies and key industries were nationalised. In the late 1980s, the state-intervention policies were being abandoned, with a continuing liberalisation of prices, and decreasing taxes. The average economic growth (measured by the GDP growth rate) achieved more than 3% per year.

The international situation (the Gulf War) with the world trade cooling off, growing oil prices, as well as procrastination of the necessary reforms, were the factors that caused another slowdown in the French growth rate. The economic developments in France after 1990 are characterised by stable year-to-year GDP growth, at an average rate of about 1.5%, a low inflation rate and consistently high unemployment rate. A critical point occurred at the 1992/1993 year break (the recession began in autumn of 1992 and was relatively short).

The beginning of the new millennium was marked with attenuation of the dynamics, caused by the drop in performance of the American economy and stock markets, while the oil prices were growing. A positive turn came in 2004, but the worldwide economic crisis of the years 2008 and 2009 hit the economies of many countries heavily, including that of France. The French GDP was going down for five consecutive calendar quarters, the general government deficit was growing, and the drop in demand attenuated the price growth (it is called the 2008/2009 deflation). The consequences of the global crisis were not so bad for France as in the majority of big European economies thanks to the growing consumption by households and the fiscal stimuli for exports together with the moderate devaluation of the euro. The French economy recovered from the recession in the third quarter of 2009. The French economy again stagnated in 2012 (the GDP growth rate was 0.3% in that year), and the recovery was coming slowly. The year-to-year GDP growth got above 2% as late as in 2017. But it went back down to 1.7% in 2018 and to 1.3% in 2019.

4.2 Model of non-stochastic prediction range for estimated GDP evolution in France

The French GDP time series is long enough; hence we have been able to set up a large number of models and their variants. We have calculated our estimates of polynomial curves (including exponential ones), moving averages, stochastic models and exponential smoothing models, always verifying their statistical properties; if a model has been found to be statistically or factually unsuitable, it has immediately been discarded. In the end, 22 models have remained.

This collection of a large number of models has enabled us to gather different "statistical opinions" about the series to be predicted, including models that can "discount their memories" (in the sense of older observations having lower weights, such as the previously mentioned exponential smoothing). Table A2 in the Appendix sums up an overview of the models processed, including their parameters and statistical properties, as well as the predictions derived from them for the years 2018 and 2019 (for all models, both the point and interval predictions at the 95% level of confidence).

All these models were identified with the aid of only 68 items in the time series (from 1950 to 2017) – we "stored away" the actual values for the years 2018 and 2019. For each of the models we have, based on 68 observations, predicted the 2018 and 2019 values to compare such prediction results with the actual values (as the subsequent assessments of the predictions). In other words, we have thus "tested" each model's ability to predict.

Table A2 in the Appendix further states each model's estimated (modelled, theoretical) value for the latest actual observation, that is, 2017. The data listed in Table A1 in the Appendix says that the actual value of the year-to-year GDP index in France was $y_{68} = 1.0123$ in 2017; that is, the year-to-year growth value was 1.23%.

A cursory glance at Table A2 in the Appendix reveals a paradox occurring when we use an isolated model from our selection to set up a point prediction for the GDP index in 2018 or 2019. Individual values of point predictions listed in Table A2 show that each model naturally leads to a different prediction for the last "known" period (y_{68}, i.e., 2017; as already stated, we have "stored away" the y_{69} and y_{70} values to be checked later.) Each of the models used pertains to its own dynamics. Judging from past behaviour, it would be very difficult to assess which model is more or less "acceptable" in comparison with others; to put it bluntly: anybody could choose anything.

Let us have a look at Model 18 in Table A2 in the Appendix – ARIMA (2, 1, 1), which was, by software Statgraphics Centurion software, assessed as the best among all of our 22 models. The models' quality levels were verified with the aid of the usual statistics, whose list and more detailed descriptions are given in the Appendix prior to Table A2.

Table 2 Model M18

Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
2018	1.015 570	0.982 584	1.048 560
2019	1.012 110	0.976 179	1.048 050

Source: Authors' own calculations

For the year 2018 or 2019, the 95%-level interval prediction of the year-to-year GDP growth index is approximately between 0.982 584 and 1.048 560, or between 0.976 179 and 1.048 050, respectively; in both instances, the span between the upper and lower bounds amounts to more than 6.6 percentage points. Expressed in absolute volumes, e.g., the French GDP was 3 108.7 billion EUR in 2018, and the 6.6% corresponds to 205 billion EUR. This means nearly 40% of the French gross fixed capital formation (which was 537.9 billion EUR in 2018). It does not make much practical sense to set up a prediction interval whose "uncertainty" amounts to nearly two-fifths of French annul investment volume.

As previously mentioned, we have "stored away" the actual values of the year-to-year French GDP growth index for the years 2018 and 2019. In 2018, this actual value was 1.017, meaning an increase in the GDP of 1.7%. In 2019, the actual value of the year-to-year index was 1.013, i.e., representing an increase of 1.3%. In both instances, the 95%-level stochastic confidence interval we have created is "successful" (and similar observations can be made about other models – cf. Table A2 in the Appendix). However, this interval is too broad for subsequent decision-making.

The non-stochastic prediction range is based on the selected models and their estimated year-to-year French GDP growth indices. Namely, we have the year-to-year indices expressed by the prediction intervals of the 22 models (the Lower 95.0% Limit and Upper 95.0% Limit, values in Table A2 in the Appendix). Let us now look up the *maximum lower bound* and the *minimum upper bound* of the year-to-year index prediction intervals in Table A2 in the Appendix (separately for 2018 and 2019). These values are shown in Table 3 (as well as in Table A2 in the Appendix); they come from Model 5 (Exponential Trend: the minimum value of the Upper 95.0% Limit among all 22 models); and Model 13 (ARIMA (0, 0, 1): the maximum value of the Upper 95.0% Limit among all 22 models):

Table 3 Max lower and min upper bounds

Period	Max (Lower 95.0% Limit)	Min (Upper 95.0% Limit)
2018	0.993 472	1.037 310
2019	0.989 743	1.036 560

Source: Authors' own calculations

Comparing the lower and upper bounds for the prediction ranges in Tables 2 and 3, we can see that the span between them is smaller for the non-stochastically argued prediction range. In fact, this span is just 4.4 percentage points, as compared with 6.6 percentage points (at a 95% confidence level) valid for the best 2018 model, i.e., ARIMA (2, 1, 1). This is an improvement by one-third.

In this way, we have obtained a prediction range (as a difference between the upper and lower bounds) for the expected values of the year-to-year GDP indices in France for the years $n + 1 = 2018$ and $n + 2 = 2019$. Let us denote by P_{69} the prediction for 2018, and by P_{70} that for 2019:

prediction range for P_{69} → minimum upper bound: 1.0373
→ maximum lower bound: 0.9934,

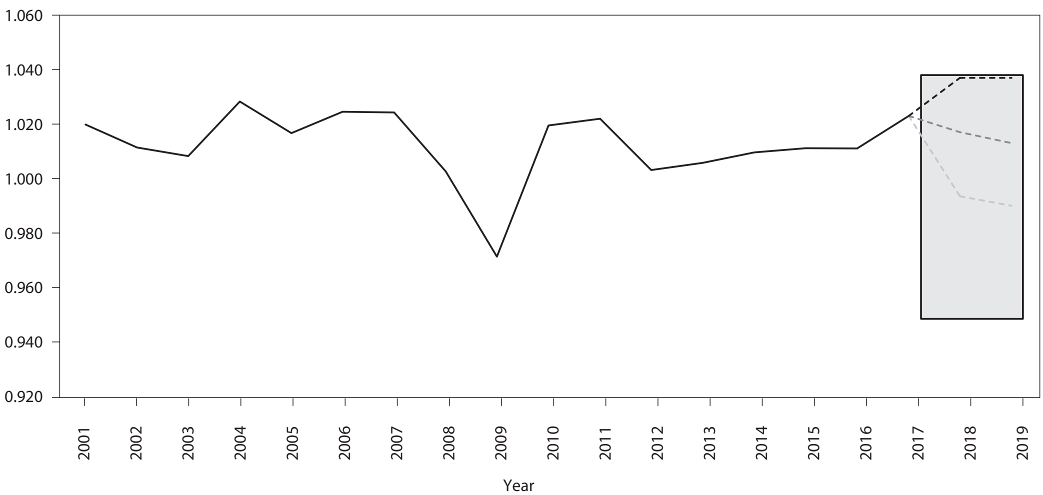
or

prediction range for P_{70} → minimum upper bound: 1.0366
→ maximum lower bound: 0.9897.

We have, for the sake of clarity, graphically enhanced a short end section in Figure 2, which depicts the year-to-year French GDP growth indices from 2005 to 2017; this enhancement helps us see the non-stochastic prediction rage for the years 2018 and 2019 (the lower and upper bounds of the non-stochastic predictions are marked with dashed lines). At the same time, the actual year-to-year evolution of the French GDP indices is easier to see (the solid line).

The 2018 and 2019 data is represented by the actual values (the solid line again) – it is covered by the non-stochastic prediction range (the dashed lines).

Figure 2 Non-stochastic prediction range – a segment of the French GDP time series (year-to-year volume indices from 2001)



Source: Authors' own calculations, <*www.insee.fr*>

From the pragmatic point of view, it is clear that the concept of the prediction range set up and argued in a non-stochastic way is more efficient than the traditional interval predictions, based on an isolated single model, whether best or just "good" – in our case, on the ARIMA (2, 1, 1) process. The non-stochastically interpreted concept sets out the future evolution of the index to be predicted in a band that is much narrower; this reduces uncertainty in the user's decision-making.

In conclusion, let us point out one interesting phenomenon. It is known that many structural relationships are valid among different indices (such as the macroeconomic aggregates). In the case of macro-aggregates, it is of extraordinary importance to consider the relationship corresponding to the expenditure-based method for estimating the GDP:

$$GDP = FCE + GCF + E - I, \tag{5}$$

where FCE stands for the final consumption expenditure, GCF for the gross capital formation, E for exports of goods and services, and I for imports of goods and services.[5] A question arises whether the described method could also be used if we are interested not only in individual indices but also in their sum, e.g., according to Formula (5). It has turned out that the application of the prediction range is also useful for additive relationships. In other words, our approach is also consistent in structural or balance issues, in which aggregation/decomposition of individual indices plays a role.

In the end, let us address a logical question: what is the average value of the point predictions derived within all of the admissible 22 models? The data shown in Table A2 in the Appendix provide the average value for the 2018 prediction as $P_{69} = 1.015$ (an increase in the GDP by 1.5%), and for 2019 it is $P_{70} = 1.014$ (an increase in the GDP by 1.4%). Let us compare these values with the actual values "stored" for the purposes of the prediction assessment: $y_{69} = 1.017$ for 2018, and $y_{70} = 1.013$ for 2019. This result indicates a very good fit; for the sake of interest, the values of the Theil coefficient, cf. Theil (1966), for the estimates in 2018 and 2019 as compared with the actual values equal $T_H = 0.15\%$.

CONCLUSIONS

Having in mind the current empirical results, the utilisation of the prediction range in economics can be viewed as purposeful. General experience with the efficiency of the prediction range based on processing a large number of economic time series has revealed the fact that the success rate of this method is relatively high. Nearly 80% of all the ranges we have set up (mostly time series of financial and macroeconomic indices) were successful when later compared with the actual data. That is why the methodology for the prediction range can become a good corroborative technique in setting up estimates for indices of this type.

Of course, open questions to be addressed in the future remain in the presented outline of the setting up prediction range argued in a non-probabilistic way. In our example we considered a series of annual values. But series with seasonal components may be predicted as well, for example, quarterly aggregates from the national accounts, or completely different time series subject to seasonal fluctuations. Other open question is a methodology for choosing the models on which the non-stochastic prediction range is based. An ideal solution would be the creation of an algorithmic tool to automate, at least to a certain extent, the primary process of model selection and verification. Another important problem to be resolved is the question of evaluating the lower and upper bounds of the non-stochastic prediction range. For more complex traditional approaches, such as those considered by Theil (1966), an obstacle s implied by the non-stochastic character of such bounds; hence simple applications of Theil's processes may be disabled. For the moment, we have to put up with a simple way of evaluation by comparing the results with the actual values when assessing the predictions.

Another option would be to set up the non-stochastically argued prediction, whether a point or interval one, with the aid of the results of the primary models (here M1, M2, etc.) weighted, for example, with the interpolation quantity of individual models (even though we are aware that a suitable description of the past behaviour is only partly reflected in successful predictions).

It will, indisputably, be a great challenge to process the reflection of the COVID-19 pandemic in the 2020 models, as well as the applications to the future years of 2022, 2023, etc., when the economic situation will be getting back to its normal state. It is obvious that it is impossible to predict the economic evolution for 2020. The pandemic intervention in the economic relationships is so extensive that there are no known models which would be able to cope with such predictions. When we get to the economic-recovery stage, it will be interesting to observe to what extent the non-stochastically argued predictions will be capable of estimating the degree of the economic recovery. This paper has been written in the period of massive manifestation of the coronavirus crisis, which is currently the dominant intervention process of the highest intensity.

5 Cf., for example, Hronová et al. (2019).

References

ANDĚL, J. *Statistické metody* [*Statistical methods*]. Prague: MFF UK, 1993.

BOX, G. E. P., JENKINS, G. M., REINSEL, G. C. *Time series analysis, forecasting and control.* 3rd Ed. New Jersey: Prentice-Hall, Inc. Englewood Cliffs, 1994.

BOOTS, J. C. C., FEIBES, W., LISMAN, H. H. C. Further Methods of Derivation of Quarterly Figures from Annual Data. *Applied Statistics*, 1967, 16, pp. 65–75.

BOURNAY, J. AND LAROQUE, G. Réflexions sur la méthode d´élaboration des comptes trimestriels. *Annales de l'INSEE*, 1979, 36, pp. 3–30.

CIPRA, T. *Analýza časových řad s aplikacemi v ekonomii* [*Time series analysis with economic applications*]. SNTL, Prague and ALFA, Bratislava, 1986.

DEMPSTER, A. P., LAIRD, N. M., RUBIN, D. B. Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society, Series B (Methodological)*, 1977, 39, pp. 1–38.

DENTON, F. T. Adjustment of monthly and quarterly series to annual tools: an approach based on quadratic minimization. *Journal of American Statistical Association*, 1991, Vol. 66, No. 333.

DUREAU, G. *Les comptes nationaux trimestriels.* INSEE – Méthodes, 1991, No. 13.

EPPELBAUM, LEV V. Non-stochastic long-term prediction model for US tornado level. *Natural Hazards*, 2013, 69, pp. 2269–2278.

EUROSTAT. *European System of Accounts (ESA 1995).* Luxembourg: Eurostat, 1996.

EUROSTAT. *European System of Accounts (ESA 2010).* Luxembourg: Eurostat, 2010.

FERREIRA, L. C. E. AND CROSS, N. J. G. New Insights into Time Series Analyses. *A&A*, 2017, 604, A121.

FRIEDMAN, M. The Interpolation of Time Series by Related Series. *Journal of American Statistical Association*, 1962, 57, pp. 729–757.

GRANGER, C. W. J. AND NEWBOLD, P. *Forecasting Economic Time Series.* New York: Academic Press, 1986.

HINDLS, R. AND HRONOVA, S. Les prévisions trimestrielles du PIB en République Tchèque. *Bulletin de l´ACN*, 1996, 33, pp. 2–13.

HRONOVA, S., HINDLS, R., MAREK, L. Does the Signal Gap Indicate a Growing Risk of Economic Recession? In: *12th Annual Conference of the EuroMed Academy of Business*, Thessaloniki: EuroMed Press, 2019, pp. 388–399.

HRONOVÁ, S., FISCHER, J., HINDLS, R., SIXTA, J. *Národní účetnictví – od výroby k bohatství* [*National accounting – from production to wealth*]. Prague: C. H. Beck, 2019.

CHOW, G. AND LIN, A. L. Best Linear Unbiased Interpolation, Distribution and Extrapolation of Time Series by Related Series. *Review of Economics and Statistics*, 1971, 43, pp. 372–375.

KOZÁK, J., HINDLS, R., HRONOVÁ, S. Some Remarks to the Methodology of the Allocation of Yearly Observations into Seasons. *Statistics in Transition*, 2000, 4, pp. 815–826.

KOZÁK, J. Možnosti využití expertních informací při konstrukci předpovědí na bázi klasického lineárního modelu časové řady [Possible utilisation of expert information when constructing predictions based on a classical linear model for a time series]. *Acta Oeconomia Pragensia*, 1996, pp. 117–142.

KUHN, M. AND JOHNSON, K. *Applied Predictive Modeling.* New York: Springer, 2013.

LIU, H. *Smart Cities: Big Data Prediction Methods and Applications.* Springer Nature Singapore Pte Ltd. and Science Press, 2020.

NASSE, PH. Le système des comptes nationaux trimestrieles. *Annales de l´INSEE*, 1973, 5, pp. 119–165.

NEUSSER, K. *Time Series Econometrics.* New York: Springer, 2016.

OCDE. *Etudes Economique de l´OCDE: France 2019.* Paris: Editions OCDE, 2019.

PANKRATZ, A. *Forecasting with dynamic regression models.* New York: John Wiley & Sons, 1991.

OSBORN, F. *Applied Imagination: Principles and Procedures of Creative Problem Solving.* 3rd Ed. New York: Charles Scribner's Sons, 1963.

SCA STATISTICAL SYSTEM. *Reference manual for general statistical analysis.* Scientific Associates Corp., Oak Brook, 1991.

THEIL, H. *Applied Economic Forecasting.* Amsterdam – Chicago: North Holland Publishing & R. McNally, 1996.

TRIBBIA, J. AND BAUMHEFNER, D. On the Problem of Prediction beyond the Deterministic Range. In: SHUKLA, J. (eds.). Prediction of Interannual Climate Variations, *NATO ASI Series*, 1993, 6, pp. 251–264.

UN. *System of National Accounts 1993 (SNA 1993).* New York: United Nations, 1993.

WASSENAN, L. *All of Statistics: a Concise Course in Statistical Inference.* New York: Springer, 2004.

WEI, W. W. S. *Time series analysis – Univariate and multivariate methods.* New York: Addison Wesley Publishing, 2006.

WOODWARD, W. A., GRAY, H. L., ELLIOTT, A. C. *Applied Time Series Analysis.* CRC Press, 2012.

YACHAO, Z., KAIPEI, L., LIANG, Q., XUELI, A. Deterministic and probabilistic interval prediction for short-term wind power generation based on variational mode decomposition and machine learning methods. *Energy Conversion and Management*, 2016, 112, pp. 208–219.

APPENDIX

Table A1 GDP France (year-to-year volume indices)

Year	y/y	Year	y/y	Year	y/y	Year	y/y
1950	1.086	1970	1.061	1990	1.029	2010	1.019
1951	1.058	1971	1.053	1991	1.010	2011	1.022
1952	1.031	1972	1.045	1992	1.016	2012	1.003
1953	1.035	1973	1.063	1993	0.994	2013	1.006
1954	1.056	1974	1.043	1994	1.024	2014	1.010
1955	1.053	1975	0.990	1995	1.021	2015	1.011
1956	1.050	1976	1.044	1996	1.014	2016	1.011
1957	1.055	1977	1.035	1997	1.023	2017	1.023
1958	1.027	1978	1.040	1998	1.036	**2018**	**1.017**
1959	1.027	1979	1.036	1999	1.034	**2019**	**1.013**
1960	1.080	1980	1.016	2000	1.039		
1961	1.050	1981	1.011	2001	1.020		
1962	1.068	1982	1.025	2002	1.011		
1963	1.062	1983	1.012	2003	1.008		
1964	1.067	1984	1.015	2004	1.028		
1965	1.049	1985	1.016	2005	1.017		
1966	1.053	1986	1.023	2006	1.024		
1967	1.049	1987	1.026	2007	1.024		
1968	1.045	1988	1.047	2008	1.003		
1969	1.071	1989	1.043	2009	0.971		

Source: *<www.insee.fr>*

Table A2 Overview and comparison of models estimated for prediction purposes

Table A2 in the Appendix lists the estimates within the models we have applied to the given time series, characteristics of their "interpolation quality", and the point prediction P_{n+i} for each model, together with the 95%-level interval prediction of the year-to-year GDP indices (or rather, Lower 95.0% Limit and Upper 95.0% Limit), for the years 2018 and 2019, followed by:

- the root mean squared error (RMSE),
- the mean absolute error (MAE),
- the mean absolute percentage error (MAPE),
- the mean error (ME),
- the mean percentage error (MPE).

M1. Random Walk with Drift
Forecast model selected: Random Walk with Drift = –0.000935166

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0189166	2018	1.02206	0.984296	1.05983
MAE	0.0135242	2019	1.02113	0.967717	1.07454
MAPE	1.31152				
ME	–4.63974E-17				
MPE	–0.0146858				

M2. Constant Mean
Forecast model selected: Constant Mean = 1.03188

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0222124	2018	1.03188	0.987221	1.07654
MAE	0.0181363	2019	1.03188	0.987221	1.07654
MAPE	1.75631				
ME	–3.31434E-16				
MPE	–0.0456116				

M3. Linear Trend
Forecast model selected: Linear Trend = 1.0591 – 0.000788999 · t

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0159303	2018	1.00466	0.971913	1.03741
MAE	0.0118108	2019	1.00387	0.971083	1.03666
MAPE	1.14736				
ME	–2.36739E-16				
MPE	–0.023199				

M4. Quadratic Trend
Forecast model selected: Quadratic Trend = 1.06239 – 0.0010706 · t + 0.00000408112 · t^2

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0159879	2018	1.00795	0.973848	1.04205
MAE	0.0117821	2019	1.00744	0.973088	1.04180
MAPE	1.14392				
ME	–2.72658E-16				
MPE	–0.0230321				

M5. Exponential Trend
Forecast model selected: Exponential Trend = $e^{(0.0574661-0.000762605 \cdot t)}$

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0159219	2018	1.00486	0.973425	1.03731*
MAE	0.0118242	2019	1.00409	0.972644	1.03656*
MAPE	1.14847				
ME	0.000119753				
MPE	–0.011639				

* = the minimum value among Upper 95.0% Limit values of all 22 models

M6. S-Curve Trend = exp (0.0263113 + 0.0685776 / t)

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0200434	2018	1.02768	0.988234	1.06870
MAE	0.0161746	2019	1.02767	0.988219	1.06869
MAPE	1.56663				
ME	0.000188524				
MPE	–0.0183384				

M7. Simple Moving Average of three terms
Forecast model selected: Simple Moving Average of three terms

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0170942	2018	1.01504	0.976356	1.05373
MAE	0.0129897	2019	1.01504	0.976356	1.05373
MAPE	1.26549				
ME	–0.000988606				
MPE	–0.115514				

M8. Simple Exponential Smoothing
Forecast model selected: Simple Exponential Smoothing with alpha = 0.2456

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0165784	2018	1.01318	0.980931	1.04544
MAE	0.0120414	2019	1.01318	0.979972	1.04640
MAPE	1.17237				
ME	–0.00259263				
MPE	–0.273068				

M9. Brown's Linear Exp. Smoothing
Forecast model selected: Brown's Linear Exp. Smoothing with alpha = 0.1095

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0168608	2018	1.00960	0.976794	1.04240
MAE	0.0123323	2019	1.00911	0.975525	1.04269
MAPE	1.1999				
ME	–0.0014339				
MPE	–0.157938				

M10. Holt's Linear Exp. Smoothing
Forecast model selected: Holt's Linear Exp. Smoothing with alpha = 0.1296 and beta = 0.0413

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0166783	2018	1.00785	0.975643	1.04005
MAE	0.0119991	2019	1.00722	0.974722	1.03972
MAPE	1.16662				
ME	–0.000206347				
MPE	–0.0420116				

M11. Brown's Quadratic Exp. Smoothing
Forecast model selected: Brown's Quadratic Exp. Smoothing with alpha = 0.0764

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.017078	2018	1.00915	0.975922	1.04237
MAE	0.0125613	2019	1.00865	0.974560	1.04273
MAPE	1.22117				
ME	0.000030422				
MPE	–0.0152579				

M12. ARIMA (1, 0, 0)
Forecast model selected: ARIMA (1, 0, 0) with a constant. AR(1) = 0.671835; Constant = 0.338781

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0171491	2018	1.02607	0.991490	1.06065
MAE	0.01233	2019	1.02813	0.986473	1.06979
RMSE	0.0171491				
ME	–0.00059993				
MPE	–0.0837374				

M13. ARIMA (0, 0, 1)
Forecast model selected: ARIMA (0, 0, 1) with a constant. MA(1) = –0.514135; Constant = 1.03217

RMSE	0.0187211	Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
MAE	0.0143691	2018	1.03120	0.993472**	1.06893
MAPE	1.39234	2019	1.03217	0.989743**	1.07459
ME	–0.000302896				
MPE	–0.0638256				
MPE	–0.023199				

** = the maximum value of the Lower 95.0% Limit among all 22 models

M14. ARIMA (1, 0, 1)
Forecast model selected: ARIMA (1, 0, 1) with a constant.
AR(1) = 0.9447; MA(1) = 0.605663; Constant = 0.0573209

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0168699	2018	1.01816	0.984265	1.05206
MAE	0.0120234	2019	1.01918	0.983386	1.05497
MAPE	1.16948				
ME	–0.00177662				
MPE	–0.196771				

M15. ARIMA (1, 1, 1)
Forecast model selected: ARIMA (1, 1, 1) with a constant.
AR(1) = 0.26381; MA(1) = 0.967834; Constant = –0.000579074

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0155851	2018	1.01001	0.978402	1.04162
MAE	0.0114333	2019	1.00601	0.973042	1.03897
MAPE	1.11316				
ME	–0.00149397				
MPE	–0.163421				

M16. ARIMA (1, 1, 0)
Forecast model selected: ARIMA (1, 1, 0) with a constant
AR(1) = –0.296046; Constant = –0.00117087

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0181952	2018	1.01828	0.981894	1.05466
MAE	0.0130038	2019	1.01850	0.974011	1.06300
MAPE	1.26397				
ME	–0.0000630604				
MPE	–0.0224724				

M17. ARIMA (0, 1, 1)
Forecast model selected: ARIMA (0, 1, 1) with a constant.
MA(1) = 0.975448; Constant = –0.000799191

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0160511	2018	1.00513	0.972545	1.03772
MAE	0.0116463	2019	1.00434	0.971736	1.03693
MAPE	1.13506				
ME	–0.00186452				
MPE	–0.200961				

M18. ARIMA (2, 1, 1) – the best model
Forecast model selected: ARIMA (2, 1, 1).
AR(1) = 0.217349; AR(2) = –0.153627; MA(1) = 0.785371

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0161257	2018	1.01557	0.982584	1.04856
MAE	0.0116326	2019	1.01211	0.976179	1.04805
MAPE	1.13346				
ME	–0.0032293				
MPE	–0.332023				

M19. ARIMA (1, 1, 2)
Forecast model selected: ARIMA (1, 1, 2) with a constant.
AR(1) = –0.167067; MA(1) = 0.501519; MA(2) = 0.458159; Constant = –0.000904754

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0155657	2018	1.01042	0.978757	1.04209
MAE	0.0115389	2019	1.00423	0.970864	1.03759
MAPE	1.12252				
ME	–0.00141871				
MPE	–0.156373				

M20. ARIMA (2, 1, 2)
Forecast model selected: ARIMA (2, 1, 2) with a constant.
AR(1) = –0.166424; AR(2) = 0.0439958; MA(1) = 0.495085; MA(2) = 0.462624; Constant = –0.000867036

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.015673	2018	1.01067	0.978763	1.04258
MAE	0.0114815	2019	1.00499	0.971303	1.03867
MAPE	1.11703				
ME	–0.00130787				
MPE	–0.145634				

M21. ARIMA (2, 1, 0)
Forecast model selected: ARIMA (2, 1, 0) with a constant.
AR(1) = –0.404877; AR(2) = –0.336535; Constant = –0.0013624

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0172156	2018	1.01682	0.982146	1.05150
MAE	0.01284	2019	1.01392	0.973570	1.05428
MAPE	1.24743				
ME	–0.000187674				
MPE	–0.0350347				

M22. ARIMA (0, 1, 2)
Forecast model selected: ARIMA (0, 1, 2) with a constant.
MA(1) = 0.610061; MA(2) = 0.351027; Constant = –0.000776596

Statistic		Period	Forecast P_{n+i}	Lower 95.0% Limit	Upper 95.0% Limit
RMSE	0.0154547	2018	1.01136	0.979949	1.04278
MAE	0.0115043	2019	1.00499	0.971277	1.03871
MAPE	1.11889				
ME	–0.00128165				
MPE	–0.142689				

Source: *<www.insee.fr>*