

# An Empirical Small Labor Market Model for the Czech Economy\*

Jan BRŮHA – Czech National Bank (jan.bruha@cnb.cz; jan.bruha@yahoo.co.uk)

## Dedication

At the beginning of this paper, I would like to render homage to Osvald Vašíček, one of the most distinguished scholars I met during my studies. I was a student at Masaryk University during the 1990s, and Osvald was one of my most inspiring professors. I learnt a lot from him. He introduced me to the realm of stochastic processes, stochastic filtering, and dynamic macroeconomics. He helped me in my first steps when I was trying to understand filtering and its computer implementation, and pointed me toward interesting applications in macroeconomics. This paper is therefore dedicated to Osvald.

During my studies at Masaryk University, Osvald was head of the Department of Applied Mathematics. This was one of the most inspiring environments I ever encountered. There were other inspiring scholars, but let me mention just two of them: Pavel Osecký and Miloslav Mikulík. Their lectures were inspiring and invited me into the difficult but exciting fields of statistics and numerical techniques. I wish to express my deepest respect and homage to these two great personages as well, and to applaud Osvald for creating such an inspiring environment, which significantly influenced my subsequent career.

## Abstract

*An empirical small labor market model for the Czech Republic is estimated in the state-space framework. Its purpose is joint modeling of the labor force, employment, wages, hours worked, output, and the GDP deflator in a consistent “structural” framework suitable for short-run forecasting. The model entails, in the long run, five driving forces: a trend labor force component, a trend labor productivity component, a long-run inflation rate, an unemployment trend, and a trend hours worked component.*

*In the short run, the dynamics are governed by a VAR model. The model aims at describing co-movements in the labor-market variables, provides a model-based decomposition into the trend and cyclical components of the underlying series, and outperforms unrestricted VARs in forecasting. The paper also describes the second moments of labor market data at various frequencies and discusses to what extent these properties can be replicated by the data.*

## 1. Introduction

The main objective of this paper is to contribute to the understanding of the dynamics of key variables of the Czech labor market in the consistent framework of structural<sup>1</sup> multivariate time series. For these purposes, a small labor market model, containing the labor force, output, employment, hours worked, wages, and inflation, is proposed and estimated in the state-space framework.

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<sup>1</sup> The term “structural” is understood here in the sense of time series econometricians (see Harvey, 1989, p. 2: “A structural time series model is one which is set up in terms of components which have a direct interpretation.”) and not in the sense of the Cowles Commission.

The unobserved states possess a straightforward interpretation and the model variables can be decomposed into various frequency components (short-run movements versus movements in trends), which implies *inter alia* that observed movements in time series can be decomposed into trend and cyclical components without the need to apply ad hoc statistical approaches.

The model entails long-run dynamics and short-run fluctuations. The long-run dynamics are derived from five primitive trends (labor force, labor productivity, inflation rate, unemployment, and hours worked). These primitive trends then span the trends in all observable variables using theoretical restrictions. The long-run restrictions are consistent with a frictionless economy where a neoclassical production function is used to derive the desired level of employment by firms. The short-run dynamics are based on an empirical VAR, which aims at replicating the second moments in the labor market variables. The model is cast in the state-space form and is estimated on Czech data from 1996Q1–2010Q1.

The proposed framework can be useful for the following purposes:

- as a benchmark against trend-cycle decomposition based on purely statistical filtering methods, especially against the Hodrick-Prescott filter (henceforth the HP filter<sup>2</sup>);
- as a means of learning about the statistical properties of labor-market data at various frequencies;
- finally, as a tool for short-term forecasting.

Since the estimation technique used allows us to distinguish between cycles and the trend, there is no need to detrend the data before estimation: the long and short-run dynamics of the model are estimated jointly, hence we can avoid the unfortunate practice of detrending data by purely statistical methods prior model estimation.<sup>3</sup> Moreover, as I will discuss, the careful treatment of trends alleviates “end-of-sample” bias, which is due to the statistical properties of trends. I show on actual data that the output gap based on the HP filter is subject to substantial revisions, which is not the case of the filter based on the presented model. Moreover, I show that there are periods when the assessment of the cyclical position of the economy significantly differs between the presented model and the HP filter; the leading example of such a discrepancy is the recent recession.

The model is cast in the state-space framework, which is very convenient for shock decomposition, for incorporating expert judgment, and for running conditional projections. For “real-time” forecasting, two properties are especially important. First, state-space models can easily deal with missing data, which means that if some series is available sooner than other series, this earlier piece of information can be incorporated into the model without the need to wait until all series are available. This is especially useful for extending the model to a data-rich environment, as some series, the dynamics of which can provide useful pieces of information about the core series,

<sup>2</sup> The reader may be interested to learn that the HP filter, popular in empirical macroeconomics, was actually introduced in science by Leser (1961) about two decades before the “discovery” of the filter by macroeconomists.

<sup>3</sup> Canova and Ferroni (2010), who adopted a non-structural approach to detrending using a parsimonious econometric specification, recently confirmed by simulations that incorrect specification of trends distorts the estimation of the parameters of the cyclical part of macroeconomic models.

such as sentiment indicators or inflation measurements, are available sooner than the national accounts data.

Second, the model can be calibrated so that the measurement noise variances are increased for the latest observations. This can be useful if a significant revision of the core series is expected.

Finally, the paper characterizes the second moments of the Czech data and confronts the moments implied by the model with that of reality. The extent to which a model with long-run neoclassical features is able to replicate the spectral properties of observed time series is discussed.

The rest of the article is organized as follows. The rest of this section reviews related literature. Section 2 presents the model and discusses the data and estimation. Section 3 summarizes the properties of the model. Section 4 concludes.

### **1.1 Related Literature**

The model presented here is related to Proietti and Musso (2007), who apply the framework of structural multivariate time series to the euro area. They specify a model with a production function and two Phillips curves and identify potential output and the output gap by specifying the permanent and transitory components in factor inputs. The main difference between the two papers is in the specifications of the models spanning the trends and cycles.

Hjelm and Jönsson (2010) review various approaches to filtering the trend component from economic time series, including multivariate model-based approaches. The presented model can be considered an example of multivariate, model-based filtering.

Andrle (2008) discusses the role of stochastic trends in macroeconomic modeling and the effects of detrending. He argues for joint modeling of trends and cycles, as permanent shocks may spill over the whole frequency range. The purpose of this paper is different, as it does not adopt a structural approach and the decomposition into trends and cycles here is taken from the statistical (not economic) perspective. However, the papers have a similar emphasis on multivariate filtering, which respects selected economic relations.

The paper is also related to a growing literature on the use of real-time data in economic forecasting. For example, Beneš et al. (2010) illustrate how to incorporate real-time data into the Small Quarterly Structural Model for the United States with real-financial linkage (see Carabenciov et al., 2008, for a description of the model). Their main concern is a theoretically coherent approach to the asynchronous release of data (financial data are usually available in real time, while national accounts are available with a lag of about two quarters).

The research here is also related to papers studying the relations between low-frequency movements in employment, productivity, and possibly other variables. For example, Ball (2009) argues that low-frequency movements in unemployment are caused by unemployment hysteresis. Farmer (2010) documents the low-frequency correlation between output dynamics and unemployment and explains this correlation in a model where self-fulfilling beliefs select the equilibrium from a set of possible equilibria. For Farmer (2010), this is the preferred interpretation of Keynes' original ideas. Finally, King (2005) reviews the literature on the low-frequency cor-

relation between productivity and unemployment and investigates the correlation of productivity with job matching and job destruction. He concludes that the trend movements in unemployment cannot be explained by productivity alone.

Over the past five years, there has been some interesting research on the Czech labor market.<sup>4</sup> Some papers deal with issues related to institutional features and hence to what New Keynesians would call the natural unemployment rate. Galuščák and Pavel (2007) investigate the effect of net replacement rates on work incentives, contributing to the understanding of the equilibrium level of unemployment. Bičáková et al. (2006) investigate the employment effects of changes in taxes and net benefits.

Other studies deal with labor-market rigidities, hence they have implications for fluctuations of labor-market variables over the cycle. For example, Babecký et al. (2008) use a firm-level survey to investigate the determinants of wage and price formation in Czech firms. They find efficiency wage models to be relevant for wage setting.

Finally, some studies try to distinguish between structural and cyclical factors. Hurník and Navrátil (2005) estimate the time-varying NAIRU to distinguish between the two factors. Galuščák and München (2007) address the issue of structural and cyclical unemployment using movements in the Beveridge curve parameters.

## 2. The Model

It is assumed that any observable variable  $x_t$  is given as the sum of the trend component and the cyclical component:

$$x_t = \bar{x}_t + \tilde{x}_t$$

where  $\bar{x}_t$  is the trend component and  $\tilde{x}_t$  is the cyclical component. These two components are not directly observable, and the model can be used to filter them. The dynamics of the cyclical components are modeled simply as a VAR process. The dynamics of the long-run component are described in the following sub-section.

### 2.1 Long-Run Dynamics

The building blocks of the long-run dynamics are the production function and the labor demand equation. The production function links the long-run trend in the log<sup>5</sup> of employment  $\bar{e}_t$  and hours per employee  $\bar{h}_t$  to the trend component of the log of real output  $\bar{y}_t$  using a log-linear specification:

$$\bar{y}_t = \bar{e}_t + \bar{h}_t + \theta_t^y \quad (1)$$

where  $\theta_t^y$  is long-run labor productivity.

The labor demand equation links the trend log real wage  $(\bar{w}_t - \bar{p}_t)$  to the trend marginal product of labor:

$$\bar{e}_t + \bar{h}_t = \bar{y}_t - (\bar{w}_t - \bar{p}_t) \quad (2)$$

<sup>4</sup> The reader can consult the papers quoted in the following paragraphs for references to older studies.

<sup>5</sup> All variables are in logs unless otherwise stated.

The other trends are the long-run growth in the GDP deflator  $\theta_t^p$ , the trend component in the labor force  $\theta_t^l$ , the trend component in hours per employee  $\theta_t^h$ , and the trend unemployment  $\theta_t^u$ . The first-mentioned trend should be pinned down by monetary policy, while the three latter trends reflect institutional issues of the labor market and demographic factors, which are outside the scope of this paper. Therefore, the long-run trend in the log of the GDP deflator  $\bar{p}_t = \theta_t^p$ , and the long-run trend in the log of the labor force is given simply as  $\bar{l}_t = \theta_t^l$ .

This model uniquely spans the trends in all observed variables. Let us denote the matrix that maps the long-run trends  $[\theta_t^y \ \theta_t^l \ \theta_t^u \ \theta_t^h \ \theta_t^p]^T$  into the trends in observable variables  $\bar{x}_t = [\bar{y}_t \ \bar{e}_t \ \bar{h}_t \ \bar{w}_t \ \bar{l}_t \ \bar{p}_t]^T$  as  $\mathbf{T}$ . The matrix is given as follows:

$$\begin{bmatrix} \bar{y}_t \\ \bar{e}_t \\ \bar{h}_t \\ \bar{w}_t \\ \bar{l}_t \\ \bar{p}_t \end{bmatrix} = \bar{x}_t = \mathbf{T}\theta_t = \begin{bmatrix} 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_t^y \\ \theta_t^l \\ \theta_t^u \\ \theta_t^h \\ \theta_t^p \end{bmatrix} \quad (3)$$

Equation (3) is basically a reduced-form version of the long-run part of the model.

The trends  $\theta_t^x$  ( $x \in \{y, l, h, p, u\}$ ) are modeled as random walks with drift processes:

$$\theta_t^x = \theta_{t-1}^x + \gamma_{t-1}^x + \sigma_{\theta}^x \varepsilon_{1t}^x$$

where the drifts  $\gamma_t^x$  follow stationary AR processes:

$$\gamma_t^x - \mu^x = \rho^x (\gamma_{t-1}^x - \mu^x) + \sigma_p^x \varepsilon_{2t}^x$$

where  $\{\varepsilon_{1t}^x\}_{t=0}^{\infty}$  and  $\{\varepsilon_{2t}^x\}_{t=0}^{\infty}$  are i.i.d. white-noise processes. Hence, the trends  $\theta_t^x$  can be represented as ARIMA(1,1,0) processes.

The interpretation of the model for the permanent components is the following:  $\gamma_t^x$  can be considered the growth rate of the potential in the corresponding variable  $x_t$  and this growth rate moves around  $\mu^x$ , which is the steady-state growth. This formulation is identical to the one used in Carabenciov et al. (2008).

Note that the Harvey-Jaeger (1993) process would be obtained for  $\mu^x = 0$  and  $\rho^x = 1$  (and thus the trend component would be given as an ARIMA(0,2,0)). It is worth explaining why I depart from the original Harvey-Jaeger formulation.

First, for some processes, it is desirable to require the drift to fluctuate around a certain value. For example, most people would expect the growth in labor productivity to fluctuate around some positive constant, which is given by  $\mu^y$ . Therefore,

even if during a severe recession the growth of trend productivity is perceived to be negative, if we expect a growth recovery in the long run,<sup>6</sup> we would want the drift  $\gamma_t^y$  to return to  $\mu^y$ . But this is something which does not happen under the Harvey-Jaeger framework: under it, if labor productivity growth becomes negative, all its future expected values will remain negative.<sup>7</sup>

Similarly, the drift in the price trend  $\gamma_t^p$  is the trend inflation, and the Harvey-Jaeger ARIMA(0,2,0) model would suggest that it is a random walk. This feature is not plausible in an economy with monetary policy aiming at bringing inflation back to the target. The formulation suggested here avoids such features; in fact, the coefficient  $\mu^p$  corresponds to what is implied by the inflation target.

## 2.2 State-Space Formulation of the Model

The model is formulated and estimated jointly in the state-space framework.

The state equation is given as:

$$\begin{bmatrix} \theta_t \\ \gamma_t \\ \tilde{x}_t \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Omega} \end{bmatrix} \begin{bmatrix} \theta_{t-1} \\ \gamma_{t-1} \\ \tilde{x}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ (\mathbf{I} - \mathbf{P})\mathbf{M} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{\Sigma}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_\Omega \end{bmatrix} \varepsilon_t \quad (4)$$

where

$\theta_t = [\theta_t^y \ \theta_t^l \ \theta_t^m \ \theta_t^h \ \theta_t^p]^T$  is the column vector of trends,  $\gamma_t = [\gamma_t^y \ \gamma_t^l \ \gamma_t^m \ \gamma_t^h \ \gamma_t^p]^T$  is the column vector of drifts,  $\tilde{x}_t$  is the column vector of cyclical components,  $\mathbf{I}$  is the identity matrix and  $\mathbf{0}$  is the zero matrix of an appropriate dimension,  $\mathbf{\Omega}$  is the VAR matrix, which determines the cyclical dynamics of the model,<sup>8</sup>  $\mathbf{P}$  is the diagonal matrix containing  $\rho^x$ ,  $\mathbf{M}$  is the column vector containing  $\mu^x$ ,  $\mathbf{\Sigma}_1, \mathbf{\Sigma}_2, \mathbf{\Sigma}_\Omega$  are diagonal matrices of standard deviations, and  $\{\varepsilon_t\}_{t=0}^\infty$  is the multivariate white noise process with  $E[\varepsilon_t] = 0$  and  $E[\varepsilon_t \varepsilon_s^T] = \delta_{st} I$ .

The observation equation is given as:

$$\mathbf{x}_t = \begin{bmatrix} \mathbf{T} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \theta_t \\ \gamma_t \\ \tilde{x}_t \end{bmatrix} + \mathbf{\Sigma}_v \mathbf{v}_t \quad (5)$$

<sup>6</sup> The relevant question is obviously how long the long run is, but this can be influenced by a sensible choice of  $\rho^y$ .

<sup>7</sup> Indeed, under the Harvey-Jaeger formulation, if  $\gamma_t^y < 0$ , then  $E_t \gamma_{t+k}^y < 0$  for arbitrary large  $k > 0$ , and hence  $E_t \theta_{t+k}^y < 0$  would exponentially decrease as a function of  $k$ . On the other hand, under the formulation suggested here,  $\lim_{k \rightarrow \infty} E_t \gamma_{t+k}^y \rightarrow \mu^y$  even if  $\gamma_t^y < 0$ , and therefore a recovery of productivity is expected after a negative productivity shock.

<sup>8</sup> The third-order VAR was chosen on empirical grounds. The matrix  $\mathbf{\Omega}$  and the vector  $\tilde{x}_t$  are rewritten into the first-order form using the obvious transformation.

where  $x_t$  is the vector of observable variables,  $v_t$  is the measurement noise and  $\Sigma_v \Sigma_v^T$  is its covariance matrix, and the matrix  $\mathbf{T}$  is given in (3).

For later use in the paper, I introduce the following notation compactly describing the model. The system (4) and (5) is written in the compact form:

$$\chi_{t+1} = \mathbf{A}\chi_t + \mathbf{B} + \Sigma_x \varepsilon_{t+1} \quad (6)$$

$$x_t = \mathbf{C}\chi_t + \Sigma_v v_t \quad (7)$$

where  $\chi_t = \begin{bmatrix} \theta_t^T & \gamma_t^T & \tilde{x}_t^T \end{bmatrix}^T$  is the stacked vector of all states, and the matrices  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ , and  $\Sigma_x$  refer to the matrices from the state-space system, i.e.:

$$\mathbf{A} = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Omega} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ (\mathbf{I} - \mathbf{P})\mathbf{M} \\ \mathbf{0} \end{bmatrix}, \Sigma_x = \begin{bmatrix} \Sigma_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Sigma_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Sigma_\Omega \end{bmatrix}, \mathbf{C} = [\mathbf{T} \quad \mathbf{0} \quad \mathbf{I}]$$

### 2.3 Data and Estimation

I use quarterly national accounts from 1996Q1 to 2010Q1. I use seasonally adjusted data on the labor force, output, employment, the nominal wage, and the GDP deflator. The model is estimated using the prediction-error minimization approach. The parameters of the model are constrained so that the growth rates are stationary, which means simply that  $|\rho^x| < 1$  for  $x \in \{l, u, h, p, y\}$  and that the modulus of the eigenvalues of  $\mathbf{\Omega}$  is less than one.

## 3. Model Properties

In this section, I briefly describe the properties of the model. These are: (i) filtering data, (ii) replicating moments, and (iii) forecasting.

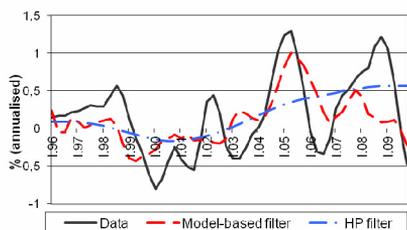
### 3.1 Multivariate Filtering

The model can be used for multivariate filtering<sup>9</sup> in the main model variables (output, employment, hours, and wages) so that the main restrictions (mainly the production function) are satisfied. *Figure 1* compares the growth rates in the observed variables (solid line), the growth rates in the model-based trends (dashed line), and the growth rates in the HP trends (dot-dashed lines). One can see that the model-based filter yields a somewhat more volatile trend in the labor force and hence in employment and output than the HP-based trends. The hours, inflation, and productivity trends are comparable to their HP counterparts.

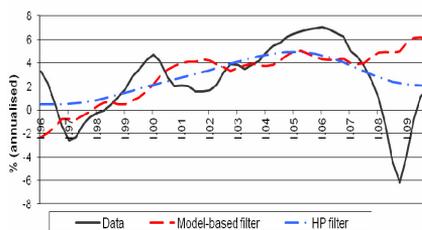
Despite being consistent across intra-temporal restrictions, the model-based filtering seems to avoid the problem of what is called the end-point bias of the HP filter. *Figure 2* displays growth rates in the trends for the model filter and the HP filter. It contrasts the ‘real-time’ estimates (the filtered trend at time  $t$  is computed with data up to this date only), with estimates based on the whole sample. It contrasts

<sup>9</sup> Throughout this paper, when I report filtering results, those results are based on the Kalman smoother. I implement the Rauch-Tung-Striebel smoother.

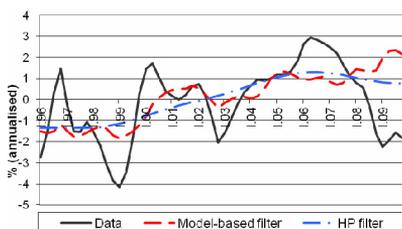
**Figure 1a Growth Rates: Labor Force**



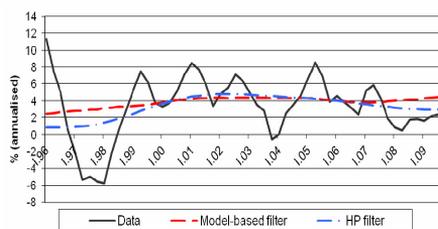
**Figure 1b Growth Rates: Output**



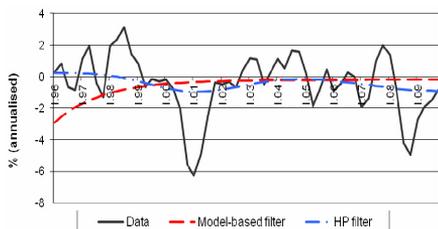
**Figure 1c Growth Rates: Employment**



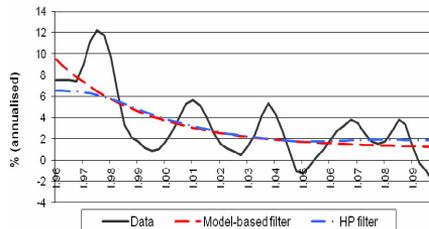
**Figure 1d Growth Rates: Real Wage**



**Figure 1e Growth Rates: Hours per Employee**



**Figure 1f Growth Rates: GDP Deflator**



the ‘real-time’ estimates (the filtered trend at time  $t$  is computed with data up to this date only), with estimates based on the whole sample.<sup>10</sup> Two trends are shown: the trend in real output and the trend in the GDP deflator. The results reveal that the HP-based trend is subject to significant revisions. It is interesting to note that the HP trend growth in the year 2008 was revised first up and then down, so the revisions need not even be monotonous. On the other hand, the model-based filter exhibits only slow revisions.

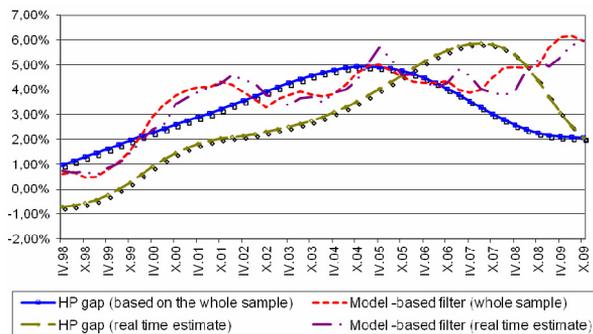
This observation deserves a comment. “End-point bias” is probably a rather confusing term, as it can give the false impression that the HP filter does not adapt at the end of sample.<sup>11</sup> That this is misleading can be easily checked by a simulation study.<sup>12</sup> If the HP filter is used to filter the permanent component for the Harvey-

<sup>10</sup> I also considered the HP filter with end-point bias reduction suggested by Bruchez (2003), but its results are very similar to the standard HP filter. Details are available from the author.

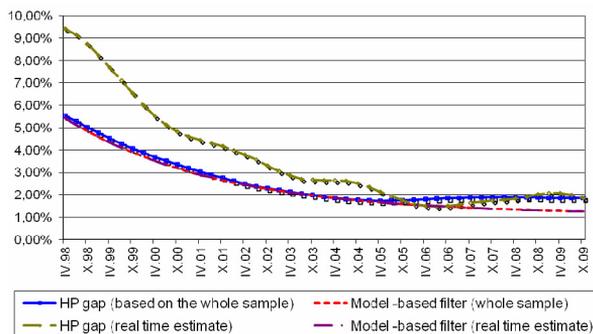
<sup>11</sup> Proietti and Musso (2007) even claim that the state-space model can alleviate the bias *because of the adaptation property of the Kalman smoother at the end of the sample*.

<sup>12</sup> Details available from the author upon request.

**Figure 2a Recursive Identification of Trends – Growth Rates in Output**



**Figure 2b Recursive Identification of Trends – Inflation**



Jaegger (2003) process (henceforth the HJ process), which is a good approximation of the optimal filter, then the end-point revisions are minimal and reflect the new pieces of information. This holds regardless of whether the HP filter is used in the traditional sense (when the trend is obtained by solving a quadratic programming problem) or whether the trend is obtained using the Kalman smoother corresponding to the state-space representation of the HJ process. In fact, numerically, these two implementations give very similar results.

The behavior of the HP filter near the end of the sample is caused by the fact that the spectral characteristics of time series often differ from those implied by the HJ process. Even Kalman smoothing with state-space representation of the HJ process would exhibit the same “end-point” bias as the traditional HP filter. Therefore, contrary to some claims in the literature, it is not the state-space form and Kalman smoothing per se which alleviate the “end-point” bias. The crucial issue is rather whether the filter used is appropriate for the (sometimes implicitly) assumed spectral properties of the investigated economic processes.<sup>13</sup>

Another striking feature is the discrepancy between the implications of the HP filter and of the model filter for the growth in trend output for the most recent quar-

<sup>13</sup> I thank Michal Andrle, who drew my attention to this issue when commenting on an earlier draft of this paper.

ters. The model-based filter indicates a larger and more negative cyclical position than the HP filter.<sup>14</sup> Since labor utilization was still low at the beginning of 2010, as employment and hours had not yet recovered while productivity had partially recovered, the model refuses to see the developments in that period as gap closure. Since the HP filter is univariate, it extracts possibly different non-linear trends from each series independently.<sup>15</sup> Similar discrepancies exist for other years and also for inflation. According to the model, the cyclical inflation position in 1998–2000 was much further below the trend level than the HP filter would imply. The model story can be corroborated by the fact that the evolution of inflation at the time came as a big surprise for economic agents. On the other hand, the tendency of the HP filter “to go through the middle of the series”<sup>16</sup> and to smooth out fluctuations means that the cyclical position, which was initially similar to the model, has been revised up. If the then disinflation really was a surprise for economic agents, then one can argue that the model filtration is more credible than the HP filtration. However, if strong and quick disinflation is followed by a period of stable inflation, then the HP filter would tend to underestimate the trend inflation at the beginning of the disinflation period and overestimate it when the disinflation ends and the economy moves into the steady period. Therefore, here too, economic intuition favors the model-based filter for a more credible story.

The reader may ask whether the model filter is multivariate on trends, on cycles, or on both. This can be answered from the inverse filter<sup>17</sup> for trends and cycles, where the weight of the last observation in the trend component is lower than it would be for the HP filter. I computed the weights implied by the Kalman smoother and it turns out that the filter is indeed multivariate, i.e., that the estimation of the trend (here smoothing) really depends on all the observation variables and not only on the corresponding single variable. Filter weights are available from the author.

### 3.2 Second Moments

The model aims at replicating the second moments in the data. *Figure 3* compares<sup>18</sup> the correlation of various lags and leads of selected variables (or linear combinations thereof) in data with model implications. The figure shows the sample correlation function (blue solid line) with the correlation implied by the model<sup>19</sup> (dashed red line). I also computed the multivariate spectral density (results available upon request), i.e., the co-spectra, the quadrature spectra, and the coherence.

The first subfigure in *Figure 3* shows the correlation of productivity with the real wage; labor productivity leads the real wage somewhat, but the highest correlation is contemporaneous. The model is able roughly to replicate this feature, although the correlation in the data is somewhat higher. The joint spectral density suggests that

<sup>14</sup> This is due not only to end-point bias, but also to the tendency of the HP filter to smooth large drops; it is a kind of folk theorem: the Great Depression is an unimportant event if it is measured by the HP cycles with the conventional smoothing parameter.

<sup>15</sup> This can be seen as a virtue if one wants to let the data speak (for example to compare the implications of various theoretical approaches). However, it can be seen as a vice if one believes one's theories.

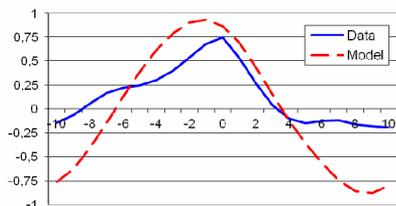
<sup>16</sup> Indeed, by the very construction of the HP filter, a bust must be followed by a boom.

<sup>17</sup> Following Koopman and Harvey (2003), the Kalman smoother is “inverted” to inquire how observations in each series translate to unobserved states (here to trends and cycles).

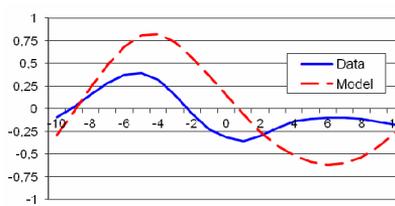
<sup>18</sup> An *Appendix on the web-side of this journal* contains more figures and information.

**Figure 3 Correlation of Selected Variables at Various Lags**

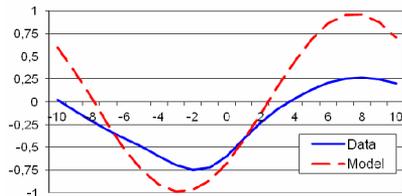
**Figure 3a Productivity – Real Wage**



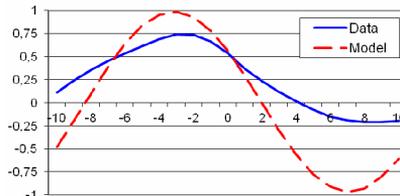
**Figure 3b Real Wage – Inflation**



**Figure 3c Output – Unemployment**



**Figure 3d Output – Employment**



the correlation between the real wage and productivity is due to all frequencies (the coherence is relatively high for all frequencies) and the quadrature spectrum suggests that the lead of productivity is caused on the business cycle frequencies.

The next subfigure shows that there is some slight correlation between the excess of the real wage over productivity and inflation, where the excess leads inflation at about five lags. In other words, the excess of wage growth over productivity growth is corrected using inflation. Both the co-spectra and the coherence suggest that this correlation is mainly caused at frequencies of around five periods. The model tends to overstate this data feature, which is not surprising given that higher-frequency movements are not modeled and the low-frequency components of the relation implied by the model are zero, as the trend inflation is governed by a trend independent of the trend in real wages, i.e., a productivity trend.

The third subfigure shows the correlation between output per capita (here per labor force) and unemployment. There is a negative correlation with some leads (at about 2–3 quarters) of output. The most movements are caused on the business cycle frequencies (especially the lead), with some coherence even at low frequencies. This low-frequency coherence may challenge the view that the natural rate hypothesis is an accurate description of reality. The natural rate is considered to be determined by

<sup>19</sup> The correlations implied by the model are computed from the state-space matrices of (6) and (7) as follows. Denote the population covariance matrices as  $\rho^y$  and  $\Gamma_s^y = E[y_{t+s}y_t^T] - E[y_{t+s}]E[y_t^T]$ . Then

$$\Gamma_s^x = A^s \Gamma_0^x$$

$$\Gamma_s^y = C \Gamma_s^x C^T + \delta_{0s} \Sigma_y \Sigma_y^T$$

where  $\delta_{s0}$  is the Kronecker delta. The matrix  $\Gamma_0^x$  satisfies the equation:

$$\Gamma_0^x = A \Gamma_0^x A^T + \Sigma_x \Sigma_x^T$$

which can be easily solved using the vec operator (Hamilton, 1994):

$$\text{vec} \Gamma_0^x = (I - A \otimes A)^{-1} \text{vec}(\Sigma_x \Sigma_x^T)$$

long-run features, such as labor-market institutions, and is independent of productivity growth and of monetary and other cyclical factors. However, the spectral properties of the investigated time series as well as the anti-cyclical property of the filtered unemployment trend cast some doubts on this view. This may signal the need for alternative views of the labor market. It is worth noting that this low-frequency correlation is not specific to the Czech data. King (2005) reviews studies which document this correlation on the US data. Farmer (2010) also finds it and uses it to support his Keynesian (*NOT new Keynesian!*) interpretation of unemployment dynamics.

The last subfigure shows the correlations between real output and employment. The correlation between output and total hours is provided in the electronic appendix. There is a positive correlation between real output and the two employment measures with an output lead at about two to three quarters. The quadrature spectrum peaks at the business cycle frequency, which means that this lead is caused by business cycle movements. The correlation is stronger for output and employment than for output and total hours. The intuition of why this is the case is given by the last subfigure in the second row. Although the sample correlation of employment and hours per employee is virtually zero, the strong negative co-spectrum and quadratic spectrum at business cycle frequencies suggest that hours and employees are substitutes at these frequencies. Hence, it appears that firms use the hours margin to manage their labor demand in the short and medium run. The model gives qualitatively the same picture as the data, but overestimates the observed correlations.

The correlations between the real wage and total hours, employment, and hours per employee are displayed in the figure in the electronic appendix. The strongest correlation exists between the real wage and employment with about a four-quarter lead of the real wage. Surprisingly, the sample correlation between the real wage and hours per employee is negative (both in the model and in the data), which is attributable to the discussed issue of substitution between hours and employees at medium and high frequencies. Again, the model tends to overestimate the observed correlations.

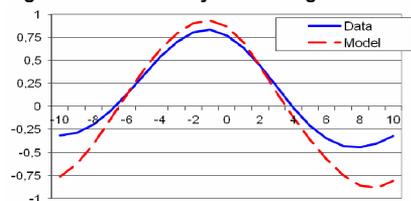
These findings are corroborated by *Figures 4 and 5*. *Figure 4* shows the population cross-correlations, the population cross-correlations for business cycle frequencies<sup>20</sup> (6 to 32 quarters), and the cross-correlations implied by the model. It is apparent that the second moments implied by the model are closer to the second moments at the business cycle than to the sample correlations.

*Figure 5* shows the population cross-correlations, the population cross-correlations for low frequencies (more than 32 quarters), the cross-correlation of the growth rates of HP trends, and the cross-correlations implied by the trend component in the model. Apparently, the spectral properties of some trends in the data are well described by those implied by the model. However, the model fails to replicate other low-frequency movements in the data; this is particularly the case for the relation

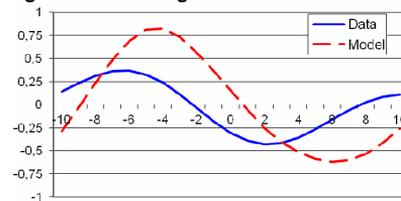
<sup>20</sup> The frequency-specific cross correlations can be derived from the spectral density. Let  $s_{xy}(\omega)$  be the cross-spectral density of variables  $x$  and  $y$  at frequency  $\omega$ . Then the population cross-correlation between  $x$  and  $y$  at lag  $k$  corresponding to frequency band  $[\omega_1 \ \omega_2]$  is given as  $\Gamma_k^{[\omega_1 \ \omega_2]} = \int_{\omega_1}^{\omega_2} s_{xy}(\omega) e^{i\omega k} d\omega$ . To estimate the frequency-specific second moments reported in these two figures, I use the trapezoid rule to approximate the integral, and  $s_{xy}(\omega)$  is given as a linear transformation of the Bartlett estimator of the multivariate spectral density. See Hamilton (1994) or Priestley (1981) for more details.

**Figure 4 Correlation of Selected Variables at Various Lags at the Business Cycle Frequencies**

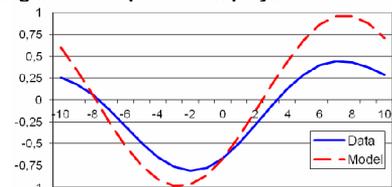
**Figure 4a Productivity – Real Wage**



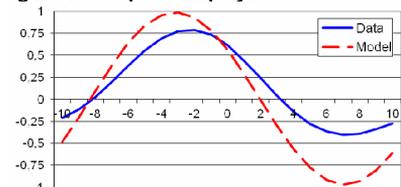
**Figure 4b Real Wage – Inflation**



**Figure 4c Output – Unemployment**

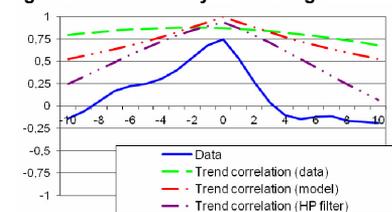


**Figure 4d Output – Employment**

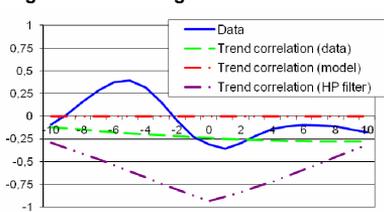


**Figure 5 Correlation of Selected Variables at Various Lags at Low Frequencies**

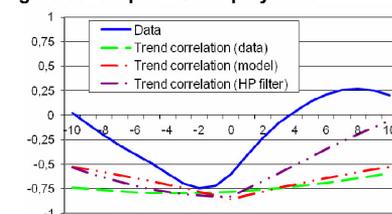
**Figure 5a Productivity – Real Wage**



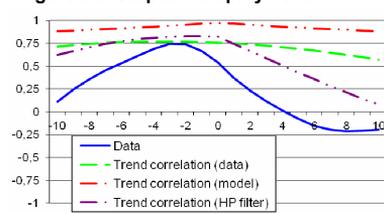
**Figure 5b Real wage – Inflation**



**Figure 5c Output – Unemployment**



**Figure 5d Output – Employment**

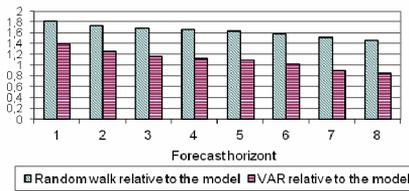


between the real wage and employment and hours. Again, this may indicate that the neoclassical approach to the labor market may fail to account for some interesting long-run movements. If such an interpretation is correct, this would mean that new Keynesian DSGE models, which have the neoclassical long run, may also miss important aspects of the data.

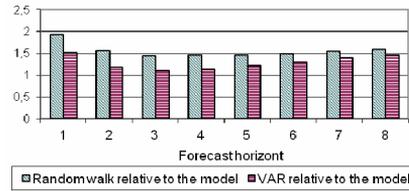
### 3.3 Forecasting

The model can be used for forecasting purposes. *Figure 6* compares the relative root mean square error of the random walk forecast and the VAR forecast<sup>21</sup> to model the forecast for the main variables at various lags (an RMSE higher than one

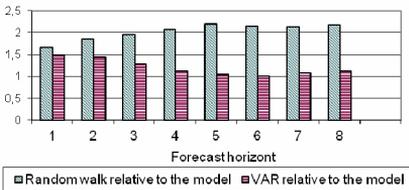
**Figure 6.1 Labor Force**



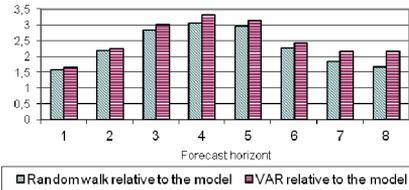
**Figure 6.2 Output**



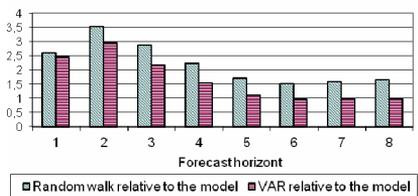
**Figure 6.3 Employment**



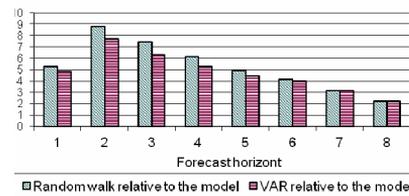
**Figure 6.4 Real Wage**



**Figure 6.5 Hours per Employee**



**Figure 6.6 GDP Deflator**



means that an alternative model forecasts worse than the presented model). It seems that for some variables (especially real wages, hours worked, and inflation) the model does a good job. On the other hand, the labor force and employment forecasts are similar to the unrestricted VAR forecasts. This is not surprising, as the trends driving these variables are identified as being more volatile. For longer horizons, the VAR tends to be better than the model for forecasting hours and employment. This may be caused by the failure of the model to mimic the low-frequency movements in unemployment.

I use the model for a set of experiments. As the model is cast in the state-space framework, the forecasts can be easily conditioned on a variable or on shocks. I run a “pseudo real time” forecast as of the beginning of 2008 as if the decline in real activity due to the global recession had been known. It is interesting that two counter-factual features would emerge: (i) the model would predict a dramatic fall in real wages followed by a rapid recovery, and (ii) the model is accurate in predicting

<sup>21</sup> In the *Figure 6*, I compare the results with VAR(1) forecasts. The higher-order VARs can improve the forecast at short horizons (1–2 quarters), but fail completely at horizons greater than 3 quarters. The lucid paper by Tiao and Xu (1993) provides the intuition of why this may be the case. I also evaluate the forecasts using a set of univariate exponential smoothing methods (I choose a local trend model with dampened trends for each variable – see chapter 2 in Hyndman et al., 2008). For most variables, the exponential smoothing approach is not better than the VAR model, while for the labor force it is much worse than both the presented model and the VAR. This holds irrespective of whether the parameters of the exponential smoothing parameters were optimized with respect to the sum of the squares of the one-period forecasting errors, or whether the objective also contains a higher forecasting horizon.

the total hours worked, but fails in decomposing the total hours into employment and hours per employee. The discrepancy in the first feature is probably caused by the change in the wage distribution (something which the model is not able to capture), as low-skilled and low paid workers were laid off first, which explains why the observed drop in real wages was lower in reality than the model would predict. The discrepancy in the second feature reflects labor hoarding, which is underestimated by the model. These two features can teach us an important lesson: the model should be used as a tool, but one should not rely on it blindly, especially with regard to a severe recession.

#### **4. Conclusion**

In this paper, I propose an empirical small labor-market model for the Czech economy. The model allows for consistent filtering of trends and cycles and for short-run forecasting. I show that the filtered trend differs from that based on the HP filter and propose an explanation for the difference.

The paper then discusses the second moments of the data and assesses the ability of the model to fit these data. I argue that some features of aggregate labor market data may be difficult to rationalize with any model which is neoclassical in the long run, i.e., in which the trend unemployment is given by external (structural) features. Therefore, I think that economists may find it is worth considering alternative theories to the neoclassical approach, which is used to pin down the long-run dynamics even in new Keynesian models.

Despite this, the presented model can be used as a tool for short-run forecasting. I document that the model outperforms VAR models, especially at horizons of two to four quarters. The state-space formulation of the model allows for simple treatment of missing data or asynchronous data release. I have also considered an extension to a data-rich environment (this extension was presented at a conference and the results are available from the author upon request), but such an extension does not bring many benefits. This may be due to short time series (too few cycles have occurred).

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