
EVALUATING THE PERFORMANCE OF VARIANTS OF THE LEE-CARTER METHOD FOR MORTALITY FORECASTING IN THE POPULATION PROJECTION OF THE CZECH REPUBLIC FROM THE CZECH STATISTICAL OFFICE 2023–2100

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Abstract

This article discusses the use of the Lee-Carter model, one of the most widely used methods for mortality forecasting. It also discusses its modified versions, namely the cohort and coherent variants, which were developed to improve the forecast accuracy of the model. This article compares them in terms of their ability to predict long-term mortality trends in the case of the Czech Republic. The projection of mortality was based on historical data on age-specific mortality rates from 1980 to 2019 for the so-called European mortality potential, which was constructed from the lowest observed mortality rates in developed European countries. On the basis of the results, the coherent Lee-Carter model was selected as the most appropriate model, which is described by the literature as more robust and suitable for projections with a long-term horizon.

Keywords: mortality forecasting, Lee-Carter method, European mortality potential, Czech Republic

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INTRODUCTION

Rising life expectancy beyond previously established limits underlines the continuing importance of mortality forecasting (*Booth – Tickle*, 2008). The

Lee-Carter stochastic model, introduced in 1992 by Ronald Lee and Lawrence Carter for mortality projections in the US, marked a breakthrough in mortality forecasting. The Lee-Carter model provided the basis for the development of other

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mortality models. Its framework has inspired many subsequent studies and models that have extended and improved upon the original approach. The Lee-Carter model remains relevant and important today.

This article focuses on an evaluation of the model with two of its main variants – a cohort variant based on the Booth-Maindonald-Smith methodology (Booth *et al.*, 2002) and a coherent variant based on the Li-Lee methodology (Li – Lee, 2005) – and their application to mortality data for the Czech Republic. The cohort variant of the model was developed to improve the accuracy of forecasts by capturing both short- and long-term mortality trends. The coherent variant of the model was developed for multi-population mortality projection to capture common trends in different populations, e.g. between sexes or countries, assuming convergence of mortality rates. The aim of this article was to assess which of the considered Lee-Carter models provides the most reliable long-term mortality projection, which was produced as part of the Population Projection of the Czech Republic in 2023 by the Czech Statistical Office.

The article is divided into several sections. First, the methodology of mortality projection within the population projection of the Czech Republic prepared by the Czech Statistical Office is briefly described. The next section of the article is devoted to the description of the Lee-Carter method, including its considered modified variants. Subsequently, the forecast accuracy of the projections is evaluated and, based on this analysis, the most appropriate model is selected. The last part of the article presents the results and compares them with the other official forecasts.

PROJECTION METHODOLOGY

In 2023, the Czech Statistical Office published the Population Projection of the Czech Republic up to 2100 (CZSO, 2023a). The basis for estimating the future development in mortality rates was the introduction of the so-called European mortality potential (hereafter referred to as the potential) as a set of the lowest specific mortality rates by age and sex achieved between 1980 and 2019 in 17 demographically developed countries. The period analysed starts in 1980, when mortality in Western Europe was already relatively stable, and ends in 2019

so that long-term trends are not distorted by the effects of the pandemic (Hulíková Tesárková *et al.*, 2024). The population projection for the Czech Republic was prepared in three basic variants: medium, low, and high. The medium variant represents the most likely scenario of future population development, while the low and high variants place certain limits on the expected development of the population and its age distribution.

In order to eliminate random fluctuations in mortality rates and to extrapolate to the age of 110, the age-specific mortality rates of the potential were smoothed and modelled using the methodology for calculating life tables by the Czech Statistical Office (CZSO, 2024). The calculation of the potential life tables required data on the number of deaths and exposed population. The number of deaths was calculated by multiplying the observed age-specific potential mortality rates (up to 100+) by the population of the Czech Republic as at 1 July 2019. These standardized numbers of deaths and the population of the Czech Republic in 2019 were used as input data for the calculation (and smoothing) of the life tables. The mortality rates were smoothed using the method of generalized adaptive models combined with the so-called P-splines, sometimes referred to as the P-GAM method (Eilers – Marx, 1996). At the oldest ages, the model chosen was based on a logistic curve that takes into accounts the slowing pace of mortality decline with age. Specifically, this is the model first used by Kannisto (Thatcher *et al.*, 1998).

The mortality projection for the Czech Republic was based on the potential mortality development from the lowest observed mortality rates in developed European countries by age and sex, using the Lee-Carter method (Lee – Carter, 1992). Specifically, a coherent version of the Lee-Carter model by Li and Lee (Li – Lee, 2005), which is considered to be more robust and particularly suitable for projections with a long-term horizon, was chosen for the mortality forecast. The coherent version of the Lee-Carter model ensures the coherence of mortality trends between males and females. It is assumed that the differences between male and female mortality rates will not change significantly in the future and that mortality trends will be similar for both sexes. The input data

used to estimate the parameters of the Lee-Carter model were potential mortality rates by age (up to 110+) and sex for the period 1980–2019.

In order to assess the differences between the Czech mortality rates and the potential, relative differences represented by excess mortality indices have been chosen, which give the age-specific mortality rates for the Czech Republic and the potential separately for each sex and each calendar year. These indices then express how many times the specific mortality rate of the Czech Republic is higher than the potential. The course of the indices as a function of age was analysed using joint-point regression and a function was proposed to model the course of the indices across ages and calendar years. The medium variant of the mortality forecast was based on the assumption that the level of the excess mortality indices will be kept constant, which is based on the average level in the period 2015–2019. Despite the constant level of the indices, the difference in life expectancy between the Czech Republic and the potential decreases, which is caused by a change in the mortality pattern of the potential and a shift in mortality towards age groups in which the level of the index is lower. In contrast, the low variant assumes a constant difference in life expectancy between the Czech Republic and the potential, as it was in the base year of the projection of 2023. The high variant assumes that the specific mortality rates of the Czech Republic will converge towards the potential, so that convergence leads to a higher life expectancy at birth than in the medium variant. A more detailed explanation of the excess mortality indices can be found in *Hulíková Tesárková et al.* (2024).

THE LEE-CARTER METHOD

One of the most commonly used mortality forecasting models is the Lee-Carter model (*Lee – Carter, 1992*). This model, which is based on the analysis of historical mortality data, allows long-term trends in age-specific mortality rates to be captured and extrapolated. Lee and Carter first used this model in 1992 to project US mortality rates from 1990 to 2065. Since then, the model has been used in many demographic studies. According to *Basellini et al.* (2023), the success and widespread use of the model is mainly due to its

simplicity and performance. The advantage of the model is its stochastic nature, which allows confidence intervals to be calculated. In addition, the approach is purely extrapolative, i.e. does not require any subjective expert judgement other than the choice of the length of the input data period to estimate the model parameters.

The basic idea of the Lee-Carter model (*Lee – Carter, 1992*) is to decompose the logarithm of the mortality rate $\ln m_{x,t}$ at age x and time t into age- and time-specific factors (or parameters) as follows:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t},$$

where α_x is the average of the observed logarithms of the mortality rates at age x , $\beta_x \kappa_t$ is the coefficient of relative change in mortality at age x (the parameter of β_x) and the time coefficient κ_t at time t , $\varepsilon_{x,t}$ is the error term of a random event at age x and time t .

It is not possible to use an ordinary regression model to estimate the model parameters because the equation in its basic form has an infinite number of solutions. The Singular Value Decomposition (SVD) method is used to find the beta and kappa parameters. To predict the time-varying coefficient κ_t at time t , Lee and Carter in 1992 used a random walk model with drift, which assumes that the parameter decreases in a constant linear. Therefore, each age-specific mortality rate is assumed to decrease with its constant exponential, which is determined by the individual beta parameter. The model has the following two constraints, which imply that the coefficient α_x is equal to the average of the logarithms of the mortality rates over time (*Lee – Carter, 1992*):

$$\sum_x \beta_x = 1 \text{ and } \sum_t \kappa_t = 0.$$

Estimates of age- and time-specific factors for the Czech Republic and the European mortality potential based on the Lee-Carter model for 1980–2019 are shown in Figure 1 for males and females.

The alpha expresses the average dependence of mortality on age. For males, it initially decreases with age until about the age of 10, when mortality starts to increase again. An accelerated increase in mortality can be observed up to the age about 20,

followed by a period of stagnation up to the age of about 30. In males, the mortality is more pronounced during adolescence and young adulthood, which is reflected in higher values of the alpha compared to females for these age groups. For females, the increase is less pronounced and the average mortality rate decreases with age earlier than for males. In the literature, this period of this increased mortality is referred to as the “adult mortality hump” and can be understood as excess mortality above the prevailing senescent mortality level (Remund *et al.*, 2017). After this period, mortality increases almost continuously with age and the increase slows down towards the end of life.

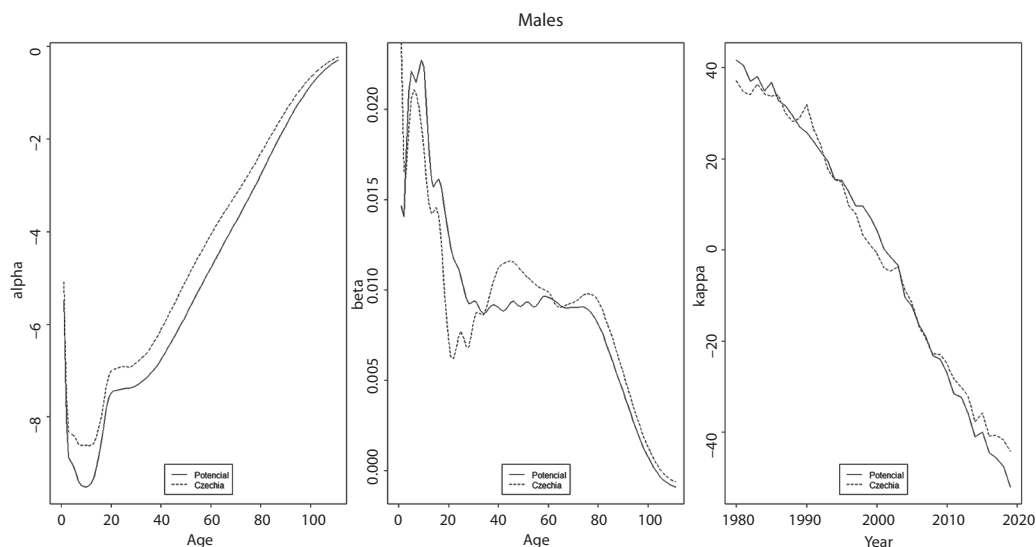
Comparing the average age dependency of the Czech male mortality rate with the potential mortality rate, the potential mortality rate initially falls faster at younger ages (up to 10 years). At later ages, mortality follows a similar pattern, with potential (from about age 90) declining later than Czech mortality (from about age 80). The average age dependency of potential mortality is lower than that in the Czech Republic at all ages, and this is true for both males and females.

The beta expresses the change in mortality over time at a given age compared to other age groups.

The beta shows a very similar pattern for the potential and Czech mortality, both for males and females. The potential mortality for males has a slightly higher variability than for the Czech Republic, while the opposite is true for females. The youngest age groups contribute most to the change in mortality over time. With increasing age, the influence of age groups on the change in mortality gradually decreases and becomes almost constant. Later on, however, the influence on the change in mortality increases again for some age groups and decreases from around age 80.

The kappa reflects the level of mortality over time for all age groups. Over the period 1980–2019, the overall level of mortality showed a decreasing trend. The potential mortality rate for males decreased faster over time than in the Czech Republic. Similarly, the mortality for females in the Czech Republic decreased less, indicating a slower improvement in mortality than in the potential mortality. Overall, the decline in mortality in the Czech Republic has been more stable and gradual in the Czech Republic than in the potential mortality. The development of mortality over time is more variable for females than males, both for the Czech Republic and for the potential mortality.

Figure 1 Estimates of age- and time-specific factors of the Lee-Carter model, Czech Republic and European mortality potential, 1980–2019

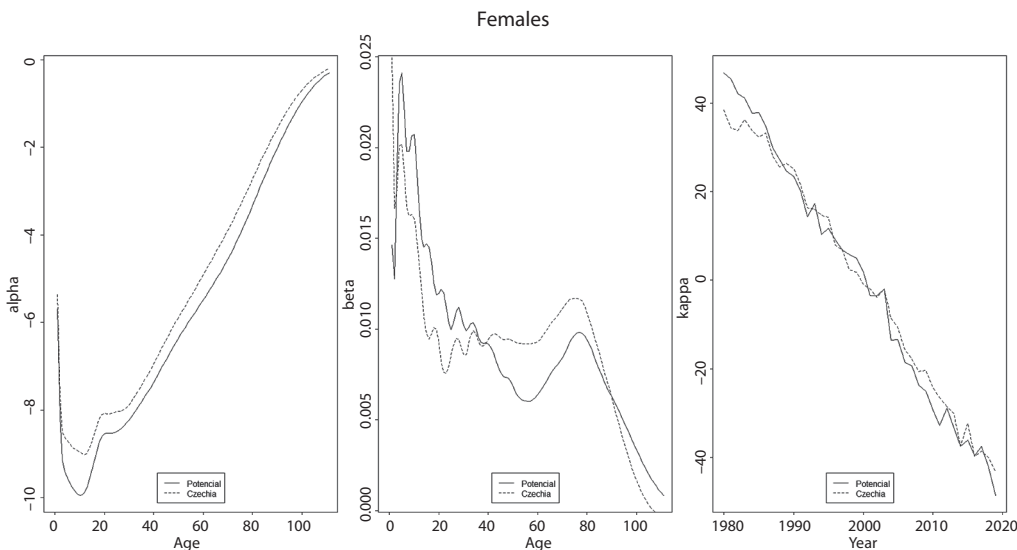


Note: From left – alpha, beta, kappa parameters of the Lee-Carter model.

Source: Eurostat 2023a; Eurostat 2023b; author's calculation.

Figure 1

(continuation)



Note: From left – alpha, beta, kappa parameters of the Lee-Carter model.
Source: Eurostat 2023a; Eurostat 2023b; author's calculation.

MODIFICATIONS OF THE LEE-CARTER METHOD

In some cases, the Lee-Carter model is not always be appropriate for given historical data (Renshaw – Haberman, 2003b; Cairns *et al.*, 2006). A major shortcoming of the model is considered to be “the lack of age coherence in forecasting”, which means that “forecasts produced by the Lee-Carter model may differ infinitely in the long run and are not biologically reasonable” (Shi, 2024; Chang – Shi, 2023). Due to the shortcomings of the Lee-Carter model, several modified versions of the model were later developed. These modifications were primarily designed to increase the accuracy and flexibility of the model with respect to different specific needs and applications (Renshaw – Haberman, 2003a; Booth *et al.*, 2002). In particular, cohort and coherent versions of the original Lee-Carter model have been developed. For a comparison of different approaches to mortality forecasting, discussing the advantages and disadvantages of the Lee-Carter model, see for example Basellini *et al.* (2023) or Lee – Miller (2001).

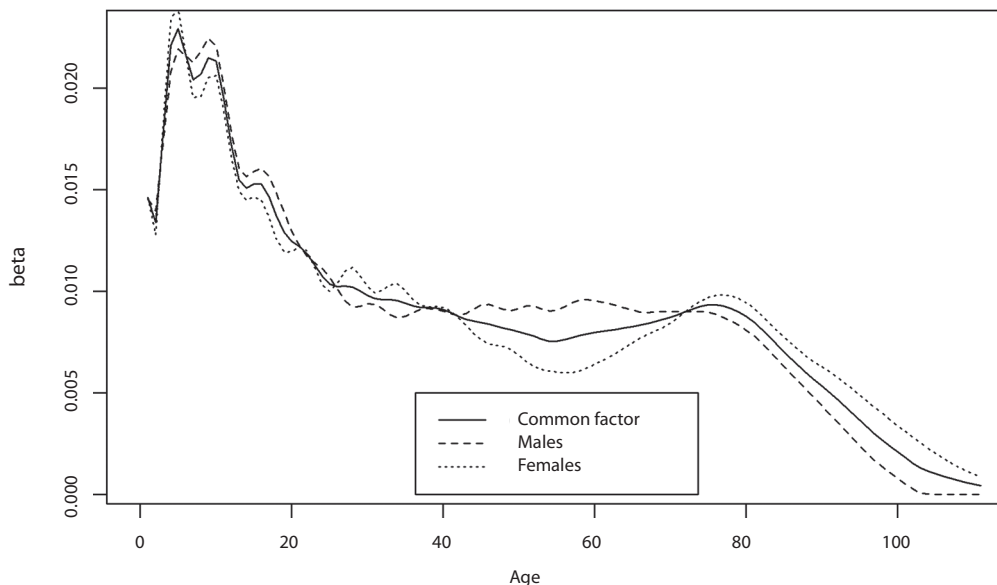
The cohort version of the Lee-Carter model was introduced by Booth-Maindonald-Smith (BMS) with several improvements (Booth *et al.*, 2005), using the

same formal model notation as in the original model. The first is the selection of the fit period based on statistical goodness-of-fit criteria assuming linearity of the time-varying parameter κ_t . The second improvement is the adjustment of κ_t , which involves fitting to the age distribution of deaths rather than the total number of deaths using the Poisson distribution to model the death process and the deviance statistic to measure goodness of fit. Finally, the jump-off rates are taken as the fitted rates used in the adjustment described above.

A further extension of the Lee-Carter model is the Li-Lee (LL) coherent variant. This variant of the model assumes that mortality patterns for similar populations will not differ much and that differences in mortality between populations are unlikely to increase in the long term (Li – Lee, 2005). That is, this variant focuses on keeping coherence between mortality projections for different populations (e.g. between males and females or between countries). The coherent Lee-Carter model according to Li – Lee (2005) is as follows:

$$\ln m_{x,t}^{(i)} = \alpha_x^{(i)} + \beta_x^{(i)} \kappa_t^{(i)} + b_x k_t + \varepsilon_{x,t}^{(i)},$$

Figure 2 Estimates of the common factor, male and female values of beta parameters for the coherent Lee Carter model for the European mortality potential, 1980–2019



Source: Eurostat 2023a; Eurostat 2023b; author's calculation.

where $\alpha_x^{(i)}$ is the average of the observed logarithms of the mortality rates for population i at age x , $\beta_x^{(i)}\kappa_t^{(i)}$ is the coefficient of relative change in mortality for population i at age x (the parameter β_x) and the time coefficient κ_t at time t , $b_{x,t}$, describes the change in mortality over time (the common factor for each population in the group), $\varepsilon_{x,t}^{(i)}$ is the error term of a random event for population i at age x and time t .

In addition, it is possible to calculate, the ultimate b_x^u as defined in Li *et al.* (2013). Based on empirical evidence and theoretical discussion, the ultimate is obtained by assuming a decline in mortality that decelerates at younger ages and accelerates at older ages.

In this article, using the coherent Lee-Carter model, the estimated beta parameters for male and female mortality rates were assumed to have the same value, which is the average of the age-specific beta parameters (see Figure 2). The estimated alpha and kappa parameters were based on the original Lee-Carter model (Lee – Carter, 1992) as described above for the European mortality potential for the period of 1980–2019 (Figure 1).

FORECAST ACCURACY

In order to assess the forecast accuracy of the different variants of the Lee-Carter model, the error measures averaged across ages and calendar years according to the variants of the Lee-Carter model were calculated for the European male and female mortality potentials for the period 1980–2019 (Table 1).

Based on the error averages across ages MPE (Mean Percentage Error) and MAPE (Mean Absolute Percentage Error), the coherent variant of the Lee-Carter model appears to be the most appropriate for mortality forecasting. The MPE indicates that the model has the lowest systematic forecast error. This is similar to the MAPE indicator, which measures the average absolute size of the percentage errors.

In the case of averages across years, the coherent version of the Lee-Carter model has the lowest error values for the Integrated Percentage Error (IPE), which reflects the percentage error integrated across all age groups. The IAPE (Integrated Absolute Percentage Error) indicator shows the lowest values for males in the original Lee-Carter model. For females, however, the values are comparable between the models, with the lowest values for the coherent Lee-Carter model.

Table 1 Error measures of forecast accuracy averaged across years and ages according to variants of the Lee-Carter model for European male and female mortality potential, 1980–2019

Indicator	Lee-Carter model (LC)		Cohort Lee-Carter model (Booth-Maindonald-Smith)		Coherent Lee-Carter model (Li-Lee)	
	Males	Females	Males	Females	Males	Females
Averages across ages						
MPE	0.003848	0.006016	0.004058	0.006060	0.002574	0.004418
MAPE	0.043668	0.058997	0.043696	0.058942	0.045404	0.057752
Averages across years						
IPE	0.427074	0.667756	0.450393	0.672676	0.285757	0.490409
IAPE	4.847112	6.548670	4.850231	6.542611	5.039825	6.410431

Source: Eurostat 2023a; Eurostat 2023b; author's calculation.

Validation of the Lee-Carter model using the prediction error Δ_t is essential to assess its accuracy in forecasting mortality (Duerst *et al.*, 2023):

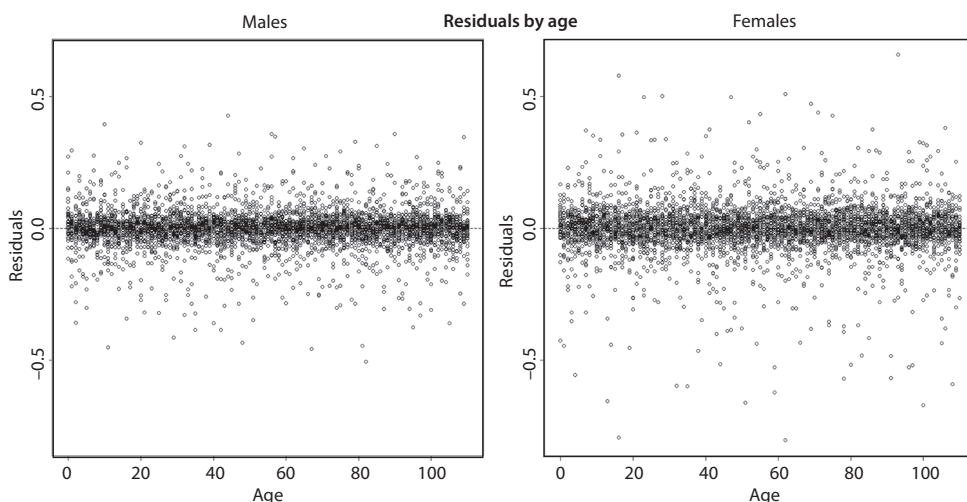
$$\Delta_t = F_t - Y_t,$$

where F_t are the forecast (fitted) values and Y_t are the observed values.

The results show that the coherent variant of the Lee-Carter model provides the most accurate forecast. An estimate of the difference (residuals)

between fitted and observed values of mortality rates by calendar year and age for the coherent Lee-Carter model for the European male and female mortality potential for the period of 1980–2019 is shown in Figure 4. Randomly distributed residuals around zero indicate that the model fits the data well and that there is no systematic bias presence. This is true both when evaluating the residuals across ages and across years. The variance of the residuals is constant and does not change with increasing age or time.

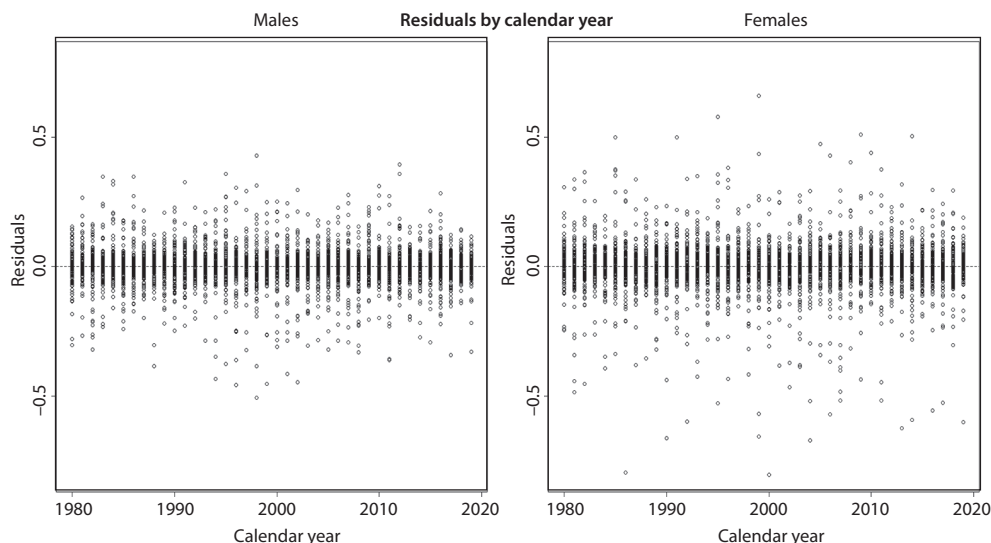
Figure 3 Estimates of the difference (residuals) between observed and fitted values of mortality rates by calendar year and age for the coherent Lee Carter model for the European male and female mortality potential, 1980–2019



Source: Eurostat 2023a; Eurostat 2023b; author's calculation.

Figure 3

(continuation)



Source: Eurostat 2023a; Eurostat 2023b; author's calculation.

LIFE EXPECTANCY TO 2100

To estimate the parameters of the Lee-Carter model, the demography (Hyndman *et al.* 2023) followed by forecast (Hyndman *et al.*, 2024) package can be used to forecast mortality in R. These packages provide functions for modelling and forecasting demographic indicators of mortality, fertility and migration. The R package MortCast (Sevcikova – Li, 2022) is available for estimating the parameters of the Lee-Carter coherent model.

According to the population projection of the Czech Republic prepared by the Czech Statistical Office (CZSO, 2023a), future mortality trends have been projected using a coherent variant of the Lee-Carter

model. The projected mortality rates reflect a continuous increase in life expectancy at birth (Table 2). The annual increase in life expectancy is expected to slow down slightly over time. In 2023, life expectancy at birth is projected to reach 77.0 years for males and 83.0 years for females. By 2050, life expectancy is projected to increase by 5.2 years to 82.2 years for males and by 4.1 years to 87.1 years for females. By the end of the century, life expectancy at birth will increase to 89.4 years for males and 92.6 years for females. The different pace of mortality decline between males and females is expected to further narrow the gap in life expectancy at birth from 5.9 years to 3.2 years.

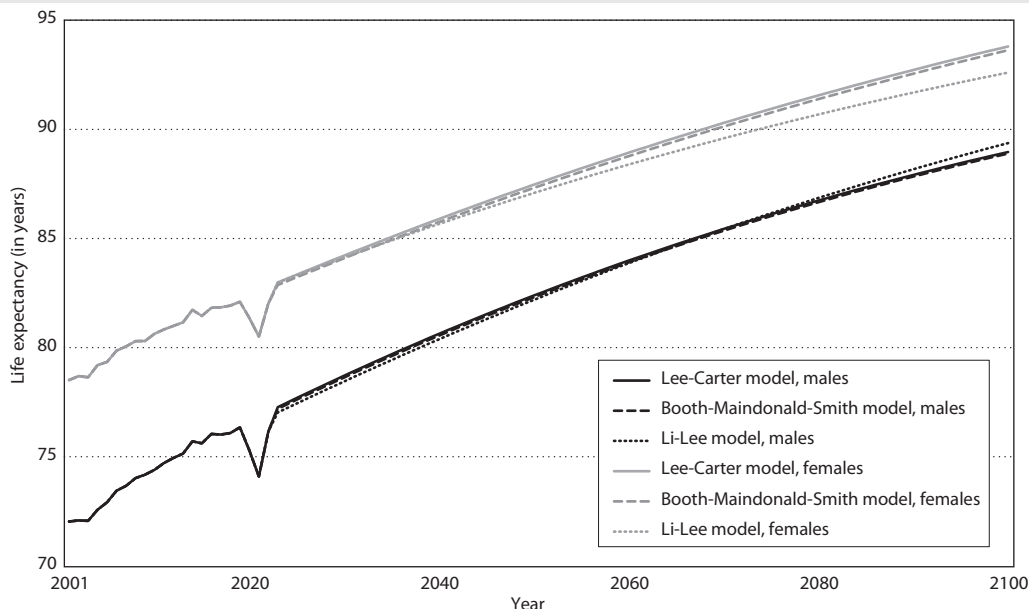
Table 2 Mortality assumptions, a medium variant of Population projection of the Czech Republic up to 2100

Indicator	2019	2022	2023	2030	2040	2050	2060	2070	2080	2090	2100
Male life expectancy at birth	76.3	76.1	77.0	78.5	80.4	82.2	83.9	85.4	86.9	88.2	89.4
Female life expectancy at birth	82.1	82.0	83.0	84.1	85.7	87.1	88.4	89.6	90.7	91.7	92.6
Sex gap in life expectancy	5.8	5.9	6.0	5.7	5.3	4.9	4.5	4.2	3.8	3.5	3.2

Note: The figures for 2019 and 2022 are real.

Source: CZSO 2023a.

Figure 4 Estimated life expectancy at birth for males and females according to variants of the Lee-Carter model, Czech Republic, 2001–2100



Source: CZSO 2023a; author's calculation.

Comparing the estimates of life expectancy at birth up to 2100 in relation to the variants of the Lee-Carter model (Figure 5) for males, a less optimistic increase can be observed based on the original Lee-Carter model and its cohort variant, whose estimation results are very similar to the coherent variant of the model, but towards the end of the projection period they start to diverge more. For females, on the other hand, the estimates of life expectancy at birth are more optimistic in the original model and its cohort versions than in the coherent version of the model.

CONTRIBUTIONS TO LIFE EXPECTANCY GAP

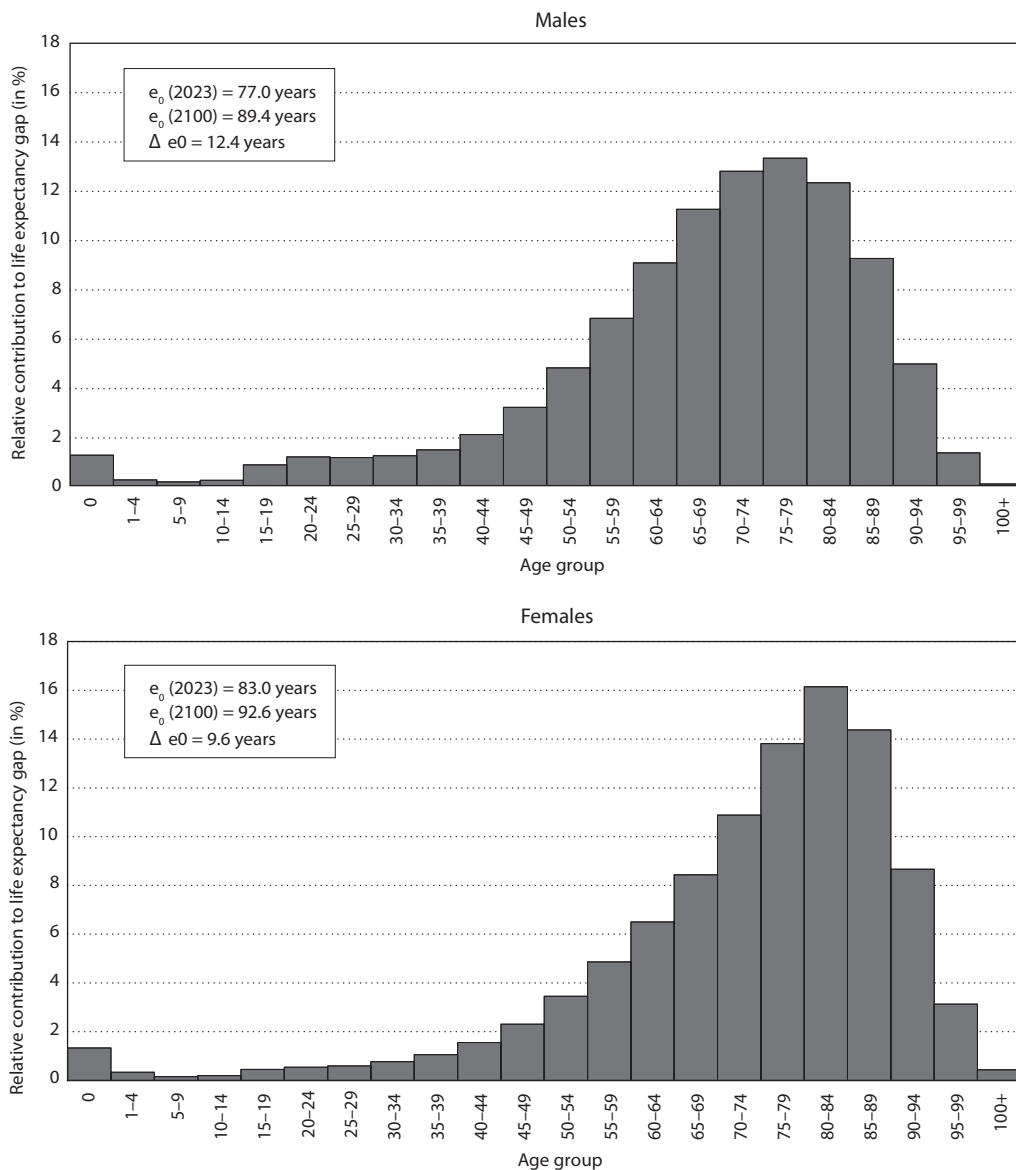
The decomposition of the difference in life expectancy is useful for estimating how differences in mortality rates in a given age group contribute to the overall change in life expectancy at birth (Preston *et al.*, 2001). The decomposition method of Pressat (1985) was used to decompose the difference in life expectancy at birth into age contributions by sex:

$$\Delta(e_0^2 - e_0^1) = \frac{(e_x^2 - e_x^1)(l_x^2 + l_x^1)}{2} - \frac{(e_{x+n}^2 - e_{x+n}^1)(l_{x+n}^2 + l_{x+n}^1)}{2},$$

where l_x^1 , l_{x+n}^1 and l_x^2 , l_{x+n}^2 are the number of survivors at age x , $x + n$ in period 1, 2; e_x^1 , e_{x+n}^1 and e_x^2 , e_{x+n}^2 are the life expectancies at age x , $x + n$ in period 1, 2. It is assumed that $l_0 = 1$.

The difference in life expectancy at birth between 2023 and 2100 is 12.4 years for males and 9.6 years for females (Figure 6). Given the very low levels of infant and child mortality, the impact on life expectancy is almost negligible, despite the decline in mortality in these groups. The increase in life expectancy shifts to the age groups 70–74 years for males and 80–84 years for females, and in the last 20 years of the projection to the age groups 80–84 and 85–89, respectively. Thus, the number of deaths is increasingly concentrated around the modal age of death (the most common age), which corresponds to the ongoing process of mortality compression that is evident in the observed long-term trends in all developed countries of the world (CZSO, 2023).

Figure 5 Contributions to the life expectancy gap at birth by sex based on the coherent Lee-Carter model, Czech Republic, 2023–2100



Source: CZSO 2023a; author's calculation.

COMPARISON WITH OTHER PROJECTIONS

This part of the article compares the results of the mortality assumptions and methodology used in the Population projection of the Czech Republic (CZSO,

2023) with other official projections, in particular with an earlier version of 2018 (CZSO, 2018). The Eurostat “EUROPOP” population projections from 2019 (Eurostat, 2019) and 2023 (Eurostat, 2023) and the United Nations “World Population Prospects” are also taken into account.

The CZSO mortality projection from 2018 was based on an analysis of mortality rates in the Czech Republic and developed European countries. The methodology for the calculation of life tables remained unchanged in both the 2018 and 2023 projections of the CZSO. The CZSO assumed that the development in the Czech Republic would follow the trend observed in European countries with high levels of life expectancy. The mortality projection was based on a model of age-specific mortality decline through an indicator called ROMI (Rates of Mortality Improvement). The medium variant was based on a 95% decline in mortality rates observed in 15 developed European countries between 1980 and 2015. The decline in mortality rates was expected to slow down gradually, and to be more pronounced in younger age groups. For a more detailed description, see (Pechholdová, 2018). In the case of Eurostat's mortality projections from 2019 and 2023, the input age-specific mortality rates were smoothed using a weighted B-spline regression. Eurostat assumed that the rates converge to a common life table (the "ultimate" life table), which includes recent mortality trends for selected countries. This interpolation results in a higher rate of mortality decline at the beginning of the period and a slower rate in the long

term, consistent with the assumption of a declining rate of mortality improvement. The United Nations method of projecting mortality from 2024 uses a moving average method to smooth mortality rates and replace inconsistencies in the data with estimates. It uses interpolation across any gaps in the time series using the limited Lee-Carter method (Li *et al.*, 2004). The Lee-Carter method is used to extrapolate age-specific mortality and is adapted for non-sequential or sparse data.

The projections of life expectancy at birth for the Czech Republic up to 2100, broken down by sex and according to different projection sources, are shown in Table 3. The development during the 21st century shows a gradual increase in life expectancy for both sexes. The CZSO 2023 projection predicts the highest increase in life expectancy for males. Eurostat projections predict a steady increase in life expectancy in the Czech Republic for both sexes. The UN projection suggests a slightly more optimistic, but still moderately conservative, development of life expectancy for males compared to the other projections. In all projections, the increase in life expectancy is faster for males than for females. This trend could lead to a narrowing of the sex gap in life expectancy over time.

Table 3 Projections of life expectancy by sex for the Czech Republic, 2019–2100

Projection	Sex	2019	2022	2023	2030	2040	2050	2060	2070	2080	2090	2100
CZSO 2018	Males	76.4	77.1	77.3	78.7	80.5	82.1	83.6	84.8	85.9	86.8	87.7
	Females	82.2	82.7	82.9	84.0	85.5	86.7	87.9	88.9	89.7	90.5	91.2
CZSO 2023	Males	76.3	76.1	77.0	78.5	80.4	82.2	83.9	85.4	86.9	88.2	89.4
	Females	82.1	82.0	83.0	84.1	85.7	87.1	88.4	89.6	90.7	91.7	92.6
EUROPOP 2019	Males	76.5	76.9	77.1	78.4	80.2	81.8	83.4	84.8	86.2	87.4	88.5
	Females	82.3	82.6	82.8	83.9	85.4	86.7	88.0	89.2	90.4	91.4	92.4
EUROPOP 2023	Males	76.4	75.9	76.2	77.9	79.8	81.6	83.3	84.8	86.3	87.6	88.8
	Females	82.2	81.9	82.2	83.5	85.1	86.6	87.9	89.2	90.5	91.6	92.6
UN WPP 2024	Males	76.3	76.2	77.0	78.3	80.0	81.6	83.0	84.3	85.6	86.7	87.8
	Females	82.1	82.1	82.6	83.4	84.5	85.6	86.7	87.8	89.0	90.1	91.2

Source: CZSO 2018; CZSO 2023a; EUROPOP 2019; EUROPOP 2023; UN WPP 2024.

CONCLUSION

The article deals with the forecasting of mortality and the comparison of different variants of the Lee-Carter model for the population of the Czech Republic. The main objective was to find out which model provides the most accurate forecast of long-term mortality trends in the Czech Republic by comparing

different methods of mortality forecasting (namely the original Lee-Carter model and its cohort and coherent variants). The results suggest that while the original Lee-Carter model remains valuable, the coherent variant offers greater accuracy and robustness for projections in populations expected to experience similar long-term mortality trends.

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