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Misspecified Demand and Oligopolistic Competition: An Evolutionary Analysis

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Abstract

This paper proposes a simple oligopoly model in which firms can use the real demand function or a misspecified version of it to maximize their profits. While in cases of demand underestimation it is always more convenient for the firms not to distort this estimate, in cases of demand overestimation the possible configurations of equilibria can include any type of outcome based on the level of distortion to the demand. This result is made more robust by considering an evolutionary version of the game, in which the previous static game is played over time by firms randomly drawn from a population of companies. The last part of the paper provides some important considerations in terms of welfare, which underline the effect of demand distortion from the consumers' point of view.

Keywords

Evolutionary dynamics, misspecified demand, oligopoly games.

JEL Classification: L13, C73, D83

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1. Introduction

Evolutionary games provide a very flexible tool for modelling the long-term dynamic effects related to the strategic interaction of agents not necessarily endowed with the highest degree of rationality. In these models, agents follow strategies and evaluate their goodness in relation to the payoffs obtained over time. The success of each available strategy, therefore, follows a Darwinian evolution process and has as a dynamic prototype the *replicator equation*; see Weibull (1985) for an extensive overview. This dynamic approach has been employed in various economic contexts, including behavioural finance (Brock and Hommes, 1997) and industrial organization. In the latter field, Droste et al. (2002) propose a simple example of duopoly to evaluate the fitness of using expensive Nash strategies with respect to more myopic (and cheaper) strategies. Interestingly, Schaffer (1989) shows that in an evolutionary setting, profit-maximizing firms are not always the best survivors, provided that firms have effective market power (in a more general setting this can also be deduced from the results in Heifetz et al., 2007). In analogous models, Bischi et al. (2015) and Cerboni Baiardi et al. (2015) elaborate on the use of different behavioural rules and their adoption according to an evolutionary approach and with different information costs. We refer the reader to Bischi et al. (2010) for a complete overview of dynamic oligopoly models.

These models were also used to study the use of alternative production technologies with backward-looking agents (Lamantia et al., 2018), forward-looking agents (Lamantia and Radi, 2018) or in contexts of technologies with different environmental impacts (Lamantia and Radi, 2015; Lamantia, 2017). Another relevant example concerns the competition in mixed-type oligopolies, in which some companies can also add to their objective part of the well-being of the community through Corporate Social Responsible (CSR) practices; Kopel et al. (2014) has shown that if the level of internalization of CSR in the company's objective function is not too high, then it might be convenient from a strategic point of view for the company to maximize an objective function that considers both profits and part of the firm's CSR. These results are then extended in Kopel and Lamantia (2018), which proposes an evolutionary setup with more than two firms to consider the effect of increasing competitive pressure in long-run outcomes of the model.

In this paper, we consider a different perspective related to information held by companies. Léonard and Nishimura (1999) have shown how mistaken beliefs can persist and create dynamic outcomes that are different from the standard outcomes observed in similar games with full information. Their approach assumes that at least one firm distorts the demand function and has full information on the other quantities of the game. Similarly, Chiarella et al. (2002) adopt a related setup to investigate the different possible equilibria in a continuous-time model where firms may adopt misspecified demands. Bischi et al. (2007) propose an analogous model with misspecified demands and learning with heterogeneous levels of distortion. Jin (2001) has proposed a related model, in which monopolistic competition under bounded rationality is addressed by assuming that firms may assess (often incorrectly) the slope of the demand function.

In this paper, we first show what happens in the one-shot competition between companies that may or may not be informed about market demand in an extremely simple setup, that of a duopoly with linear demand and linear costs. Note that firms may purposely disregard information on market demand – that is, they can distort demand for strategic reasons. Although it may seem counterintuitive, it is not always the case that maximizing profit by assuming full knowledge of market demand leads to competitive advantage for a company. In fact, in the case of an overestimation of the demand, this distortion creates a commitment effect for the company that in the competition can lead to greater profits for the company that distorts the demand compared to the firm that maximizes based on effective demand. This type of effect is similar to what happens in delegation models in which the manager has a compensation scheme different from own profits (e.g. market share, revenues), which creates a commitment effect for more aggressive market behaviour (Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987). In an evolutionary setup, this phenomenon has recently been studied by De Giovanni and Lamantia (2016).

After describing the static model, we consider its dynamic extension through an evolutionary game employing the setup proposed in Droste et al. (2002). At each time period, two firms are extracted from a large population of firms and interact by deciding the amounts to be played according to the previous model of quantity competition. Firms, even if identical, can use real market demand or its distorted version; however, the validity of each strategy (whether informed or not) is established using the real profit obtained, following a so-called *indirect evolutionary approach* (Königstein and Müller, 2000; Alger and Weibull, 2013). In this way, the fraction of companies either distorting information or not is updated dynamically based on the results of this competition game, which is then iterated over time. While in the case of underestimation it is always convenient for companies to use real market demand, in the case of overestimation this effect depends on the level of distortion to the demand. In fact, it is possible that it is always more convenient to overestimate the demand (if the level of overestimation is not too high) or vice versa, to use the real demand (if the level of overestimation is very high). For intermediate distortion levels, mixed-type cases arise, in which the game assumes an anti-coordination form. More generally, such behavioural heterogeneity can be observed when a pre-commitment stage takes place in which firms may select a given behavioural rule; see Chirco et al. (2013) for a discussion on this point.

The last part of the analysis sheds light on welfare resulting from these results. We show that, not surprisingly, the aggregate profit of the industry is always greater if all companies do not distort demand. However, total welfare, understood as the sum of total profits and consumer surplus, is always maximized when companies overestimate the demand. This effect is due to the more aggressive behaviour of firms that, although reducing total profits, allow consumers to purchase goods at a lower market price.

This paper is organized as follows. Section 2 states the basic model assuming a static perspective. Section 3 extends this setup to an indirect evolutionary setting by introducing replicator dynamics. The main results for market outcomes are fully studied as a function of the parameters of the model. Section 4 provides a welfare analysis of the outcomes of the model. Section 5 provides the conclusions.

2. Model

Consider the following *one-shot* duopoly game. Suppose two firms producing homogeneous goods. Linear demand is postulated so that the selling price p is given by the following linear inverse demand function:

$$\mathrm{p} = A - q_1 - q_2, \qquad (1)$$

where q_i denotes the quantity delivered by firm *i* and $A > 0$ is referred to as the choke price (maximum selling price of the good). For the sake of simplicity, the same linear technology is employed so that production cost is simply $c(q) = cq$, with $A > c \geq 0$ and fixed costs are disregarded. Firm's *i* profit is thus given by

$$\pi_i = q_i(A - c - q_1 - q_2). \qquad (2)$$

Through the standard arguments, the *Cournot–Nash* equilibrium of the game is easily obtained as the production plan where each of the two firms produces and delivers to market the following quantity

$$q_{SS} = \frac{A-c}{3},$$

thus obtaining profits

$$\pi_{SS} = \frac{1}{9}(A-c)^2.$$

Now, following Léonard and Nishimura (1999), suppose that both firms distort true demand by underestimating or overestimating the true inverse demand function. Following their suggestion, we assume that firms select the perceived demand function from a one-parameter family of demand functions such that firms are able to assess the specific shape of inverse demand but they are not able to know its scale exactly. In particular, the expected price is given by the following *distorted* inverse demand

$$\varepsilon \mathrm{p} = \varepsilon(A - q_1 - q_2), \text{ with } \varepsilon > 0. \qquad (3)$$

Note that for $0 < \varepsilon < 1$ the expected price is lower than the actual price – that is, the firms underestimate the true price. Instead, for $\varepsilon > 1$ the demand is overestimated. For ε=1, firms use the true inverse demand given in (1). As in this case, firm *i* believes that its profit is given by

$$\pi_i = q_i(\epsilon(A - q_1 - q_2) - c). \qquad (4)$$

The Cournot–Nash equilibrium of the game when both firms distort the inverse demand is then given by

$$q_{DD} = \frac{1}{3}\left(A - \frac{c}{\epsilon}\right),$$

which is the quantity that both firms deliver to the market. Notice that when $\epsilon > 1$ it always holds that $q_{DD} > 0$. When $0 < \varepsilon < 1$, it is $q_{DD} > 0$ provided that $\frac{c}{A} < \varepsilon < 1$. As for profits, because the *actual* selling price is determined through the *true* inverse demand (1), firms end up gaining the following profits:

$$\pi_{DD} = q_{DD}(A - c - 2q_{DD}) = \\ = \frac{(A\epsilon - c)(A\epsilon + c(2 - 3\epsilon))}{9\epsilon^2}.$$

Now suppose that just one firm distorts true demand, while the opponent uses the true linear form of the inverse demand. By following the previous arguments, the firm that distorts demand sets the quantity by maximizing (4), whereas the competitor that employs the true demand maximizes (2). The (asymmetric) Cournot–Nash equilibrium that arises, in this case, is given by the following quantities

$$q_{SD} = \frac{A\epsilon + c - 2c\epsilon}{3\epsilon}$$

and

$$q_{DS} = \frac{A\epsilon + \epsilon c - 2c}{3\epsilon},$$

where q_{ij} denotes the quantity delivered to the market by a firm of type *i* when its competitor is of type j, $i, j \in \{S, D\}$. Here S means a *standard* firm (a firm that uses the actual inverse demand function (1)), whereas D stands for a *distorting* firm (a firm that uses the distorted inverse demand function (3)). In the following, we refer to a firm that does [not] distort the (inverse) demand function as a D-firm [S-firm].

With the consequent meaning of the symbols, in the mixed duopolistic case, a standard firm (S-firm) gains *profits* given by

$$\pi_{SD} = \frac{(A\epsilon + c - 2c\epsilon)^2}{9\epsilon^2},$$

when playing against a distorting firm (D-firm), and a D-firms obtains

$$\pi_{DS} = \frac{(A\epsilon - c(2-\epsilon))(A\epsilon - c(2\epsilon - 1))}{9\epsilon^2},$$

when its competitor is an S-firm.

Notice that for $\varepsilon = 1$ the various expressions reduce to the standard textbook case without distortion.

It is then easy to prove the following proposition, which provides the economically meaningful ranges of parameters that we will then assume in the following.

Proposition 1 [Nonnegativity of equilibrium production].

When:

- *at least one firm underestimates inverse demand, that is, when* $0 < \epsilon < 1$, *then all Cournot–Nash equilibrium quantities are positive provided that* $\frac{2c}{A+c} < \epsilon < 1$;
- *at least one firm overestimates inverse demand, that is, when* $\epsilon > 1$, *and:*
  - $A < 2c$, *then the Cournot–Nash equilibrium quantities are positive provided that* $1 < \epsilon < \frac{c}{2c-A}$;
  - $A \geq 2c$, *then the Cournot–Nash equilibrium quantities are always positive.*

Table 1 collects the relevant equilibrium quantities of the game at hand when the first player is a row-player.

Table 1 Cournot–Nash Equilibrium quantities

	S	D
S	$q_{SS} = \frac{A-c}{3}$	$q_{SD} = \frac{A\epsilon + \epsilon c - 2c}{3\epsilon}$
D	$q_{DS} = \frac{A\epsilon + \epsilon c - 2c}{3\epsilon}$	$q_{DD} = \frac{1}{3}\left(A - \frac{c}{\epsilon}\right)$

The corresponding profits are summed up in the following matrix represented in Table 2, which has the following form: $\pi = \begin{bmatrix} \pi_{SS} & \pi_{SD} \\ \pi_{DS} & \pi_{DD} \end{bmatrix}$.

Table 2 Profits at Cournot–Nash Equilibrium

	S	D
S	$\pi_{SS} = \frac{1}{9}(A-c)^2$	$\pi_{SD} = \frac{(A\epsilon + c - 2c\epsilon)^2}{9\epsilon^2}$
D	$\pi_{DS} = \frac{(A\epsilon - c(2-\epsilon))(A\epsilon - c(2\epsilon-1))}{9\epsilon^2}$	$\pi_{DD} = \frac{(A\epsilon - c)(A\epsilon + c(2-3\epsilon))}{9\epsilon^2}$

In the following section, we extend this one-shot game to a repeated setup, in which firms playing the game are drawn from a population of firms where the share of D-firms and S-firms is dynamically updated according to an indirect evolutionary process whose fitness is represented by the accrued profits in Table 2.

3. Evolutionary setting

Consider a large population of firms. At any point in time, two firms are drawn at random to play the game discussed in the previous section, thus obtaining profits summarized in matrix π reported in Table 2. Given a specific parameter setting, the conditions for the nonnegativity of the Cournot–Nash equilibria are assumed to be satisfied, as specified in Proposition 1. Let *r(t)* denote the actual share of firms that do not distort inverse demand (clearly the complementary fraction *1-r(t)* represents the share of firms distorting inverse demand). Assuming that the dynamics of the share of S-firms follow a *replicator equation* (for details, see Weibull, 1995; Cressman, 2003), we can obtain the following differential equation that models the evolutionary competition between S-firms and D-firms:

$$\dot{r} = f(r) = r\left(\begin{bmatrix}1\\0\end{bmatrix}^T \pi \begin{bmatrix} r \\ 1-r \end{bmatrix} - \begin{bmatrix} r \\ 1-r \end{bmatrix}^T \pi \begin{bmatrix} r \\ 1-r \end{bmatrix}\right) = r\,\frac{c(1-r)(1-\epsilon)(c(3+r(\epsilon-1)-4\epsilon)+A\epsilon)}{9\epsilon^2}. \quad (5)$$

Note that even if firms distort true demand, evolutionary pressure follows the accrued profit by either strategy – that is, an *indirect* evolutionary approach is here adopted (for details, see Königstein and Müller, 2000; Alger and Weibull, 2013). Consider the evolutionary model described by the replicator equation (5). The next proposition collects all of the main results of its dynamics.

Proposition 2 [Dynamics of the Replicator Equation for the D-S game].

Underestimation. *Assume that firms either underestimate inverse demand using* (*3*) *with* $0 < \epsilon < 1$ *or employ the true inverse demand* (*1*). *Moreover, the nonnegativity constraints on Cournot–Nash equilibrium quantities are satisfied as indicated in Proposition 1. Strategy S then always dominates strategy D. In this case, replicator equation* (*5*) *admits only the two boundary equilibria:* $r_0=0$ (*unstable*) *and* $r_1=1$ (*locally asymptotically stable*).

Overestimation. *Assume that firms either overestimate inverse demand using* (*3*) *with* $\epsilon > 1$ *or employ the true inverse demand* (*1*). *Moreover, the nonnegativity constraints on Cournot–Nash equilibrium quantities are satisfied as indicated in Proposition 1.*

- *Strategy S dominates strategy D*
  - *for any* $\frac{2c}{3c-A} < \epsilon < \frac{c}{2c-A}$ *if* $A < 2c$;
  - *for any* $\epsilon > \frac{2c}{3c-A}$ *if* $2c \le A < 3c$;
  - *replicator equation* (*5*) *admits two boundary equilibria:* $r_0=0$ (*unstable*) *and* $r_1=1$ (*locally asymptotically stable*).
- *Strategy D dominates strategy S*
  - *for any* $1 < \epsilon < \frac{3c}{4c-A}$ *if* $4c > A$;
  - *for any* $\epsilon > 1$ *if* $4c \le A$;
  - *replicator equation* (*5*) *admits two boundary equilibria:* $r_0=0$ (*unstable*) *and* $r_1=1$ (*locally asymptotically stable*).
- *Neither D nor S dominates the other*
  - *for any* $\frac{3c}{4c-A} < \epsilon < \frac{2c}{3c-A}$ *if* $A < 3c$;
  - *for any* $\epsilon > \frac{3c}{4c-A}$ *if* $3c \le A < 4c$;
  - *replicator equation* (*5*) *admits three equilibria: the two boundary equilibria* $r_0=0$ (*unstable*) *and* $r_1=1$ (*unstable*) *and the inner equilibrium*

$$r^* = \frac{A\epsilon + 3c - 4c\epsilon}{c(1-\epsilon)} \in (0,1)$$

(*locally asymptotically stable*).

Proposition 2 highlights how, in this simple model, overestimation leads to a richer dynamic scenario than underestimation. In fact, underestimation can only be compatible with the standard behaviour of non-distorting the (inverse) demand when firms set their equilibrium quantities: with underestimation, in fact, strategy S always dominates strategy D. Overestimation, however, is compatible with any possible scenario: the dominance of one pure strategy over the other (be it S over D or vice versa) or no dominance at all of a pure strategy over the other. This is because, with overestimation, firms tend to behave more aggressively and this behaviour may spread over the population of firms, as specified in the previous proposition so that, in the long run, we observe a polymorphic configuration share r^* where both strategies are played with positive probability.

When strategy S dominates strategy D, that is, when $\pi_{SS} > \pi_{DS}$ and $\pi_{SD} > \pi_{DD}$, the only fixed point of the replicator equation (5) are r_0=0 (all firms distort inverse demand), which is unstable, and r_1=1 (no firms distort inverse demand), which is locally asymptotically stable. This means that, starting from a generic initial condition $r(0) \in (0,1)$, the (continuous-time) dynamics of replicator equation (5) converge monotonically to r_1. Notice that underestimation of (inverse) demand is never profitable for firms: in fact, if firms underestimate (inverse) demand, they behave less aggressively than standard firms and always end up with fewer profits. Thus, in an evolutionary framework, this kind of behaviour should disappear if the game is played over and over by firms randomly drawn from the population of players.

When strategy D dominates strategy S, that is, when $\pi_{DD} > \pi_{SD}$ and $\pi_{DS} > \pi_{SS}$, replicator equation (5) also has two equilibria: r_0=0 (all firms distort inverse demand), which is locally asymptotically stable, and r_1=1 (no firm distorts inverse demand), which is unstable. In this case, starting from a generic initial condition $r(0) \in (0,1)$, the evolutionary dynamics converge monotonically to r_0: all firms in the population will eventually overestimate (inverse) demand.

When the game is Hawk-Dove (neither D dominates S nor S dominates D), that is, when $\pi_{SD} > \pi_{DD}$ and $\pi_{DS} > \pi_{SS}$, the differential equation (5) admits three equilibria: r_0=0 and r_1=1, which are both unstable, and $r^* \in (0,1)$, which is locally asymptotically stable.[1] The fact that r^* is locally asymptotically stable means that a generic trajectory with initial condition $r(0) \in (0,1)$ will converge to r^*, which is a polymorphic configuration where both S-firms and D-firms coexist in the population. The game classification in the bi-

[1] Equilibrium r^*, is locally asymptotically stable as it holds that $f'(r^*) < 0$, where $f(.)$ is the left-hand-side of replicator equation (4).

dimensional parameter space (A,ε) for c=0.3 is proposed in Figure 1. Finally, Figure 2 presents the analogous information in the three-dimensional parameter space (A,ε,c).

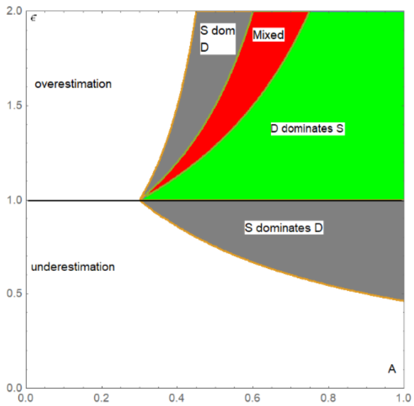

Figure 1 Game classification in the bi-dimensional parameter space (A,ε) for c=0.3. Grey [Green] region represents combinations of parameters (A,ε) such that the replicator dynamics converge to r_1=1 [r_0=0] with all S-firms [D-firms]. In the red region, no pure strategy dominates, and evolutionary dynamics converge to the inner equilibrium with a share of S-firms given by $\boldsymbol{r}^* \in (\mathbf{0},\mathbf{1})$. The white region depicts combinations of parameters (A,ε) such that Cournot–Nash quantities are negative and thus ruled out by Proposition 1.

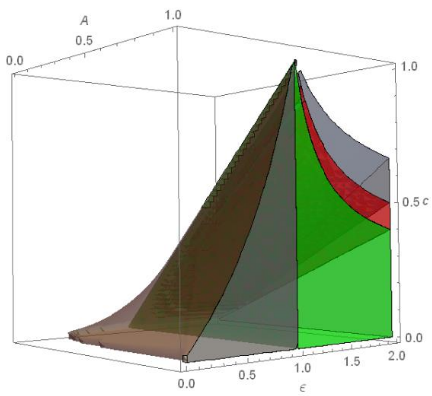

Figure 2 Game classification in the three-dimensional parameter space (A,ε,c). For the meaning of the different colours, see the Figure 1 caption.

4. Welfare analysis

Up to now, we have considered the long-run configuration of the evolutionary system and considered under which conditions the evolutionary oligopoly game will end up in a state where at least one firm may find it useful, for strategic considerations, to distort the knowledge of the (inverse) demand function. Recall that this may only happen when overestimation occurs. Thus, in this section, we will focus on the case of overestimation ($\epsilon > 1$) and briefly analyse the main consequences of this information distortion in terms of welfare.

The first result states, not surprisingly, that a configuration in which both firms distort the knowledge of demand, which is likely to occur as specified in the previous section, leads to an inefficient outcome for the industry. In other words, the system may be trapped in a *prisoner's dilemma*-like situation, in which all firms may consider it beneficial to distort information on market demand (as D dominates S), but in the long run firms are worse-off than in a configuration where no information distortion occurs.[2]

Proposition 3 [Prisoner's Dilemma Trap].

Assume that strategy D dominates strategy S, as specified in Proposition 2. Then aggregate industry profits are lower when both firms distort (inverse) demand than if both firms would employ the correct demand.

The previous proposition can be proven by comparing aggregate industry profits in the two cases and noting that when D dominates S the following inequality holds:

$$\pi_{CC} > \pi_{SS}.$$

Next, we consider and compare consumer surplus

$$CS(Q) = \frac{Q}{2}$$

in the various configurations of the game, where Q is the total output in the market. Direct calculation gives the consumer surplus levels reported in Table 3.

Table 3 Consumer Surplus at Cournot–Nash Equilibrium

$CS_{SS} := CS(2q_{SS}) = \frac{2}{9}(A-c)^2$
$CS_{DD} := CS(2q_{DD}) = \frac{2}{9}(A-\frac{c}{\epsilon})^2$
$CS_{SD} := CS(q_{DD}+q_{SS}) = \frac{(c-2A\epsilon+c\epsilon)^2}{18\epsilon^2}$

It is straightforward to observe that under overestimation it is $CS_{DD} > CS_{SS}$. A graphical comparison of the level of consumer surplus is depicted in Figure 3.

[2] It can be shown that when underestimation occurs and P dominates S, then both firms would be better off if they distorted demand.

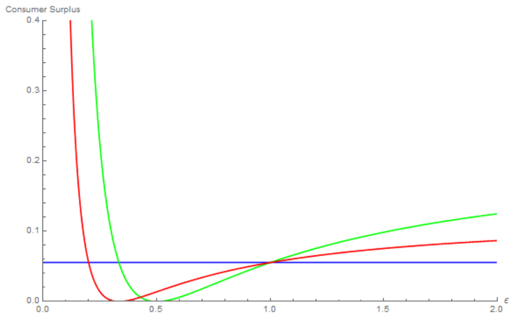

Figure 3 Consumer surplus as a function of the level of distortion ε for A=1 and c=0.3 with two S-firms (blue line), two D-firms (green curve), and with one D-firm and one S-firm (mixed case, red curve).

Finally, let us consider total welfare, measured by summing up total industry profits and consumer surplus. The next proposition shows that welfare is always maximized when both firms distort the inverse demand.

Proposition 4 [Total Welfare].

Assume that $\epsilon > 1$. *Then both in the case in which strategy D dominates strategy S and in the case in which strategy S dominates strategy D* (*see Proposition 2*), *total welfare is always maximized when both firms distort information.*

The previous proposition can be proven by comparing total welfare in the two different scenarios of all S-firms and all D-firms, and noting that in the cases in which a given strategy dominates the other one it always holds that

$$CS_{DD} + 2\pi_{DD} > CS_{SS} + 2\pi_{SS}\,.$$

The last inequality shows the enhancing effect of information distortion on welfare. This result can be explained by noticing that under overestimation of (inverse) demand, information distortion reduces overall industry profits (see Proposition 3). In turn, this effect is beneficial for consumers, as it reduces the market power of the oligopolists, and is reflected in the enhancing effect of information distortion on consumer surplus. As the latter effect prevails over industry profits, total welfare is thus increased by information distortion.

A graphical representation of total welfare in the different scenarios is depicted in Figure 4.

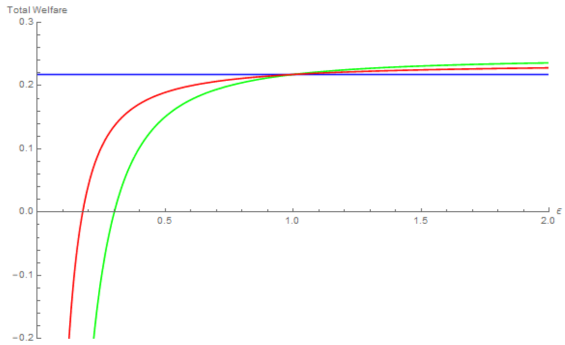

Figure 4 Total welfare as a function of the level of distortion of inverse demand ε for A=1 and c=0.3 with two S-firms (blue line), two D-firms (green curve), with one D-firm and one S-firm (mixed case, red curve).

5. Conclusions

In this paper, we have presented a simple indirect evolutionary model to evaluate the effect of demand distortion in oligopolistic competition. In particular, in the case of an overestimation of (inverse) demand, firms are led to more aggressive market choices that make the competition tougher. This can lead the market to situations that resemble a prisoner's dilemma, as all firms are weakened by the fierce competition, which, in turn, makes the end consumers better off. The model proposed in the paper is deliberately very simple from the point of view of the market structure in order to show that this type of effect can also be found with an extremely simple microeconomic structure. Several aspects of the paper could be subjected to a more in-depth investigation – for example, hypothesizing that firms produce non-homogeneous goods or compete on price. A further open question concerns how the increase in market competition, understood as the number of oligopolists present in the market, impacts the main results of the model.

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