Modelling of Returns and Volatility Co-movements of Central European Currencies

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Abstract **– This paper studies the returns and volatility co-movements of the selected Central European currencies, namely the Czech koruna, Hungarian forint and Polish zloty against the European euro. The research uses the daily data covering the 15 years' period of membership of these countries in the EU (May 2004 – April 2019). The preliminary analyses based on calculation of the Pearson's unconditional correlations and of cross-sectional standard deviation of exchange rate returns are followed by estimation of symmetric diagonal BEKK-GARCH and asymmetric diagonal BEKK-GARCH models to assess both the dynamics of conditional volatility and the volatility co-movements of analysed Central European currencies.**

Keywords **– Central Europe, co-movement, exchange rate returns, volatility, diagonal BEKK-GARCH model.**

1. Introduction

Nowadays, the world is facing a relatively high degree of international economic integration, coupled with deepening of trade relations and intensifying flows of goods and services as well as financial flows between countries.

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Despite the expanding areas using the same currency, the exchange rate still plays the significant role in the globalization process. The process of European integration lasting several decades culminated in January 1999 with the adoption of the euro, thus creating the second economically largest region of the same currency after the US. Today, the European Union (EU) is one of the biggest economies in the world, comprising 27 Member States, 19 of which use the euro [4]. Another important milestone in the history of the EU was its enlargement in 2004. The Central European (CE) countries (the Czech Republic, Poland and Hungary) joining the EU in 2004, with the future goal of adopting the euro, using still their own national currencies, attract the attention of international investors. Studying of the volatility co-movements of CE currencies is of great relevance concerning the possible diversification advantages from allocation to the Central European assets [10]. Since the issue of common volatility movements and the volatility spillover in the international stock market is quite often addressed in the literature (see e.g., [5], [7], [14], [21], [39], [42]) confirming the stronger co-movements during the crises' periods, the joint behavior of the exchange rate volatilities is considerably less addressed.

In general, there are several options and approaches how to analyse the co-movement of exchange rates, e.g., assessing the evolution of selected currencies against global currencies (EUR, USD) using the "currency beta" tool [18], investigating long-term relationships using the concept of co-integration or short-term relationships using Granger causality [40], conditional Granger-causality approach [31], calculation of unconditional correlation coefficients, using *sigma*-convergence indicator based on the cross-sectional dispersion of exchange rate returns [1], [38], up to modelling volatility using the multivariate models of conditional heteroscedasticity, MGARCH (Multivariate Generalised Autoregressive Conditional Heteroscedasticity), which enable consideration of multivariate analysis of the time-varying covariance/correlation movement (see e.g., [37]).

In recent years, various types of MGARCH models have been developed (see e.g., [6]). Bollerslev et al. [9] presented the multivariate version of the univariate GARCH model denoted as VECH (Vectorized GARCH), very popular are also the Baba-Engle-Kraft-Kroner (BEKK) model defined in [19], CCC (Constant Conditional Correlation) model [8] and DCC (Dynamic Conditional Correlation) model [20].

Papers dealing with the volatility spillovers between exchange rates are beginning to emerge in the 1990s. Bollerslev [8] used the CCC model to the five European currency exchange rates against the US dollar and detected a common movement of volatility of the analysed exchange rates. Kearney and Patton [28] analysed the volatility co-movement between the selected European currencies and the US dollar based on the BEKK model. Antonakakis [2] confirmed the common volatility movement as well as volatility spillovers among the selected exchange rates using the BEKK and DCC models. The study of Výrost [41] presented the use of the DCC model to exchange rates of Slovak, Czech, Hungarian and Polish currencies against the euro and confirmed the highest correlations between the Polish zloty and Hungarian forint against the European euro. Orlowski [36] examined the common movements of the Polish, Czech and Hungarian currencies and the euro over the period 2000–2015. Kočenda and Moravcová [29] investigated the exchange rate co-movements and volatility spillovers between the Czech, Polish and Hungarian currency against the euro and the USD/EUR exchange rate from 1999 to 2016. Using the DCC model of Engle, they captured the development of currency linkages and volatility spillovers over calm as well as crises periods (global financial crisis and sovereign debt crisis in euro area). Mittal et al. [34] examined the dynamic currency interdependencies for Brazil, Russia, India and South Africa and 15 other economies (including Hungary and Poland) via asymmetric dynamic conditional correlation model based on weekly data from January 2001 to September 2018. Katsiampa et al. [27] used both the symmetric and asymmetric diagonal BEKK framework to the high-frequency data for selected cryptocurrencies in order to investigate the conditional volatility dynamics as well as the volatility co-movements of major cryptocurrencies. Niyitegeka and Tewari [35] studied the volatility spillover and dynamic conditional correlation between the South African rand and the euro based on the use of BEKK-GARCH(1,1) and DCC-GARCH(1,1) approaches. Gorman, Orlowski and Roessler [25] analysed dynamic interactions between the Czech, Polish, Hungarian and the euro currency (expressed in U.S. dollars) during the period January 2000 - April 2019 using the multiple break-point regression and the Markov regime switching model. Das and Roy [17] investigated the linkages and volatility analysing the Brazilian, Russian, Indian, Chinese and South African currencies during the period 2006–2019. They proved significant co-movements of returns as well as volatility linkages between various foreign exchange markets. Baklaci and Yelkenci [3] provided the investigation of volatility spillovers in global currency markets utilizing the multivariate VAR-BEKK-GARCH model.

The aim of this paper is to analyse the returns and volatility co-movements of the selected CE currencies, namely the Czech koruna (CZK), Hungarian forint (HUF) and Polish zloty (PLN) against the EUR. Calculation of the Pearson's unconditional correlations and of cross-sectional standard deviation of exchange rate returns is followed by estimation of the symmetric diagonal BEKK-GARCH[1] and asymmetric diagonal BEKK-GARCH model. The analysis is based on the daily data covering the 15 years' period of membership of these countries in the EU (May 2004 – April 2019).

This paper contributes to novelty of research by application of the modern econometric techniques in order to analyse return and volatility co-movements of the CE currencies. Although several research studies dealing with co-movements of different exchange rates have been published, interactions among the CE currencies are not so heavily examined. This paper assesses the returns' and volatility dynamics and brings the complex analysis of returns and volatility co-movement. Based on the methodologies under consideration we were able to identify periods of convergence and divergence of the analysed currency markets. Furthermore, the estimation of the asymmetric diagonal BEKK-GARCH model enabled to investigate the asymmetries in the Czech and Polish market.

The paper is organised as follows: section 2 deals with the methodological issues, section 3 comprises the data and empirical results, concluding remarks are gathered in section 4.

2. Methodology

It is well known, that the most straightforward way to analyse the returns co-movement is calculation of the Pearson's unconditional correlation coefficient. Its values can vary between -1 and +1, the strength of linkages between the return series (see e.g., [26]) can be characterized as: weak (absolute value of

[1] Selected results of this analysis were presented in [15].

correlation coefficient 0.1 – 0.29), middle (absolute value of correlation coefficient 0.3 – 0.69) or strong (absolute value of coefficient 0.7 – 1.0).

To analyse if the exchange rate returns tend to become more similar over the analysed period, the concept of *sigma*-convergence, originally developed in economic growth literature, can be used [1], [13]. Declining values of cross-sectional standard deviations confirm the convergence of analysed currency markets, on the other hand, the increasing values indicate the divergence.

With regard to the MGARCH methodology, modelling of currency co-movements consists of two parts – specification of conditional mean equation and specification of appropriate MGARCH model. From the wide variety of MGARCH models we will briefly present the trivariate symmetric diagonal BEKK-GARCH model and asymmetric diagonal BEKK-GARCH model since both will be estimated in the empirical part of this paper (for more information about advantages of these models see e.g., [33]).

In general, the conditional mean equation capturing the dynamic relationship in returns can be specified as a vector autoregressive (VAR) model with *k* lags [15]:

$$\mathbf{r_t} = \boldsymbol{\omega} + \sum_{i=1}^{k} \boldsymbol{\Gamma}_\mathbf{i} \mathbf{r_{t-i}} + \boldsymbol{\varepsilon}_\mathbf{t} \tag{1}$$

where $\mathbf{r_t}$ is a vector of daily exchange rate returns, $\boldsymbol{\omega}$ is a vector of constants, $\boldsymbol{\Gamma}_\mathbf{i}\ (i = 1, 2, \dots k)$ are matrices of parameters of dimension (3×3) and $\boldsymbol{\varepsilon}_t | \Omega_{t-1} \sim N(0, \mathbf{H}_t)$ is a vector of disturbances (Ω_{t-1} denotes the information set at time $t-1$).

The equations of the VAR model (1) enable to capture and assess the significance of the own and cross-mean spillovers [11]. The diagonal elements of matrices $\boldsymbol{\Gamma}_\mathbf{i}$ $(i = 1, 2, \dots k)$ measure the impact of own past returns, the off-diagonal elements reflect the return spillovers, i.e. the cross-mean interactions.

The specification of the conditional variance-covariance matrix $\mathbf{H_t}$ (3×3) in the symmetric BEKK-GARCH(1,1) model of Engle and Kroner [19] is defined as:

$$\mathbf{H_t} = \mathbf{C'C} + \mathbf{A'}\boldsymbol{\varepsilon}_\mathbf{t-1}\boldsymbol{\varepsilon}'_\mathbf{t-1}\mathbf{A} + \mathbf{B'H_{t-1}B} \tag{2}$$

Symbol **C** denotes a positive definite (3×3) dimensional upper triangular matrix of parameters, **A** and **B** are (3×3) matrices of parameters and $\boldsymbol{\varepsilon}_\mathbf{t}$ denotes the three-dimensional vector of shocks from the VAR(*k*) model (1)[2]. The elements of the matrices **A** and **B** reflect the impact of shocks in individual markets and the persistence of shocks in each individual market, respectively [2], [24]. Elements of matrix **A** provide us with information about "direct" shock spillovers (the conditional variance in individual market reacts to its own lagged shocks and/or lagged shocks in other markets) and "indirect" shock spillovers (the conditional variance in individual market responds to any combination of the lagged shocks cross-terms). Matrix **B** enables to capture the "direct" volatility spillovers (the conditional variance in individual market responds to its own lagged volatility and/or to lagged volatility in the other markets) and "indirect" spillovers (the conditional variance in individual market responds to any lagged covariance)[3].

Model specification (2) needs estimation of $N(5N+1)/2$ parameters, i.e. with $N=3$ it is necessary to estimate 24 parameters. In the diagonal BEKK model the matrices **A** and **B** are diagonal and the number of parameters to be estimated reduces to 12, i.e. model (2) can be written as indicated by (4).

The standard symmetric model (2) was extended by Kroner and Ng [30] to capture the asymmetric effects in the variance and covariance, respectively. The asymmetric specification of the BEKK model is as follows:

$$\mathbf{H_t} = \mathbf{C'C} + \mathbf{A'}\boldsymbol{\varepsilon}_\mathbf{t-1}\boldsymbol{\varepsilon}'_\mathbf{t-1}\mathbf{A} + \mathbf{B'H_{t-1}B} + \mathbf{G'}\boldsymbol{\eta}_\mathbf{t-1}\boldsymbol{\eta}'_\mathbf{t-1}\mathbf{G} \tag{3}$$

The last term on the right-hand side denotes additional quadratic form dependent on the outer product of the three-dimensional vector of negative return shocks $\boldsymbol{\eta}_\mathbf{t}$, **G** is thus a corresponding (3×3) matrix of parameters. The asymmetric specification of the conditional variance-covariance matrix (3) enables to capture both the own- and cross-variance asymmetries as well as own- and cross-covariance asymmetries [11]. Since the asymmetric model specification (3) with $N=3$ requires estimation of 33 parameters, its diagonal version (matrices **A, B** and **G** are diagonal matrices) implies estimation of 15 parameters. Diagonal asymmetric specification of the trivariate BEKK model is given by (5).

[2] As pointed out by e.g., [11], [24], since the second and the third term of the right-hand-side in equation (2) are expressed in quadratic forms, the positive definiteness of the matrix $\mathbf{H_t}$ is ensured, so there are no additional constraints for parameter matrices **A** and **B**.

[3] For a more detailed information about interpretation (as well as about some limitations) of the estimated parameters, see e.g. [2], [11], [28].

$$\mathbf{H_t} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{pmatrix} +$$
$$+ \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}' \begin{pmatrix} \varepsilon^2_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{1,t-1}\varepsilon_{3,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon^2_{2,t-1} & \varepsilon_{2,t-1}\varepsilon_{3,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{3,t-1} & \varepsilon_{2,t-1}\varepsilon_{3,t-1} & \varepsilon^2_{3,t-1} \end{pmatrix} \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} + \tag{4}$$
$$+ \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix}' \begin{pmatrix} h_{11,t-1} & h_{12,t-1} & h_{13,t-1} \\ h_{12,t-1} & h_{22,t-1} & h_{23,t-1} \\ h_{13,t-1} & h_{23,t-1} & h_{33,t-1} \end{pmatrix} \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix}$$

$$\mathbf{H_t} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{pmatrix} +$$
$$+ \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}' \begin{pmatrix} \varepsilon^2_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{1,t-1}\varepsilon_{3,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon^2_{2,t-1} & \varepsilon_{2,t-1}\varepsilon_{3,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{3,t-1} & \varepsilon_{2,t-1}\varepsilon_{3,t-1} & \varepsilon^2_{3,t-1} \end{pmatrix} \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} +$$
$$+ \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix}' \begin{pmatrix} h_{11,t-1} & h_{12,t-1} & h_{13,t-1} \\ h_{12,t-1} & h_{22,t-1} & h_{23,t-1} \\ h_{13,t-1} & h_{23,t-1} & h_{33,t-1} \end{pmatrix} \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix} + \tag{5}$$
$$+ \begin{pmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{pmatrix}' \begin{pmatrix} \eta^2_{1,t-1} & \eta_{1,t-1}\eta_{2,t-1} & \eta_{1,t-1}\eta_{3,t-1} \\ \eta_{1,t-1}\eta_{2,t-1} & \eta^2_{2,t-1} & \eta_{2,t-1}\eta_{3,t-1} \\ \eta_{1,t-1}\eta_{3,t-1} & \eta_{2,t-1}\eta_{3,t-1} & \eta^2_{3,t-1} \end{pmatrix} \begin{pmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{pmatrix}$$

The non-linear maximum likelihood (ML) method can be used to estimate the parameters of the above presented models (see e.g., [6]).

3. Data and Empirical Results

The presented analysis uses the daily data of CZK/EUR, HUF/EUR and PLN/EUR exchange rates collected over a 15 years of EU membership between May 3, 2004 and April 30, 2019 (totally 3841 observations). The data were obtained from the web page of the European Central Bank [22]. The analysed exchange rates defined as units of national currency per European euro together with the corresponding continuously compounded returns (calculated as the difference between the natural logarithms of the exchange rate in time *t* and *t*-1)[4] are graphically depicted in Figure 1.

As shown in Figure 1, it is clear that all three currencies (CZK, HUF and PLN) appreciated considerably against the euro before the begin of the financial crisis in 2008, which was followed by their strong depreciation during the last months of 2008 and the first months of 2009. During the next periods, the exchange rates under consideration were more stable with higher fluctuations during the sovereign debt crisis (particularly visible in HUF/EUR and PLN/EUR exchange rates). As for continuously compounded returns, Figure 1 provides a clear evidence of returns' volatility clustering indicating that large/small returns are succeeded by another large/small returns. Especially notable is the extremely high return volatility during the financial crisis period.

[4] Continuously compounded returns = logarithmic return series are denoted with prefix „DL“.

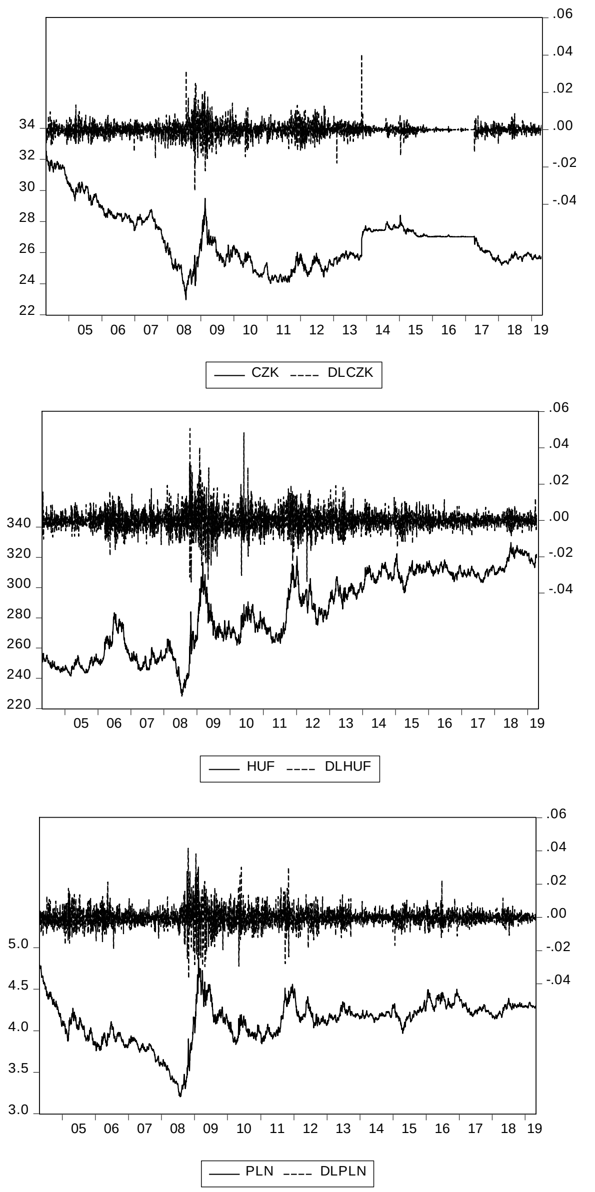


Figure 1. Development of exchange rates (CE currencies against euro) and continuously compounded returns (DLCZK, DLHUF, DLPLN)
Source: author's calculations in EViews

Since the time series of daily exchange rates were non-stationary, the results of the ADF (Augmented Dickey-Fuller) test gathered in Table 1 clearly confirm, that the continuously compounded returns were already stationary. The descriptive statistics presented in Table 1 point out very similar characteristics of all the three return series.

The mean values of all the returns moved around the 0, the highest volatility was detected for the HUF/EUR returns (0.56 %) followed by the PLN/EUR returns (0.54 %) and CZK/EUR returns (0.35 %). All the analysed return series exhibited higher kurtosis than the normal distribution and were positively skewed. The values of the Jarque-Bera test statistics confirmed that the distribution is non-normal.

Table 1. Descriptive statistics of continuously compounded returns and selected test results

	DLCZK	DLHUF	DLPLN
Mean	$-6.19.10^{-5}$	$6.49.10^{-5}$	$-2.90.10^{-5}$
Median	0.0000	-0.0001	-0.0002
Maximum	0.0405	0.0507	0.0416
Minimum	-0.0327	-0.0339	-0.0368
Std. Dev.	0.0035	0.0056	0.0054
Skewness	0.5979	0.4206	0.3374
Kurtosis	16.9473	10.5177	10.4149
Jarque-Bera	31353.28	9155.765	8869.801
Probability	0.0000	0.0000	0.0000
Observations	3840	3840	3840
ADF test	-60.5941	-62.0211	-59.8946
Probability	0.0000	0.0000	0.0000

Source: : author's calculations in EViews

Figure 2 depicts the exchange returns smoothed by the Hodrick-Prescott (HP) Filter[5]. Since till the 2008 the returns seem to fluctuate quite independently, there is a clear strong harmonisation during the 2008-2009 crisis period as well as some specific co-movement during the worsening of the sovereign debt crisis in 2011 and during the next analysed periods.

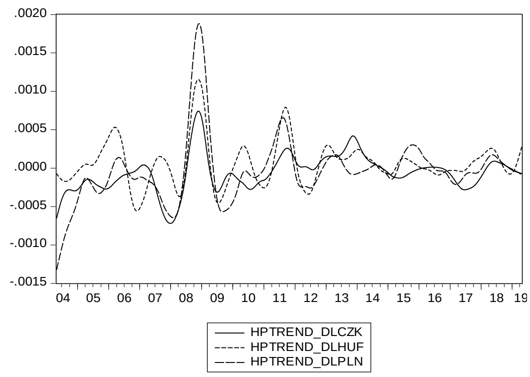


Figure 2. Exchange rate returns smoothed by the HP filter
Source: author's calculations in EViews

One of the approaches to study the linkages between the return series is to calculate the pair-wise unconditional Pearson's correlations. Table 2 shows their values for the analysed pairs of return series. The values indicate the strongest positive linear relationship between the Polish and Hungarian exchange rate returns.

Table 2. Pair-wise unconditional Pearson's correlations, return series

	DLCZK	DLHUF	DLPLN
DLCZK	1.0000	0.4257	0.4614
DLHUF	0.4257	1.000	0.6334
DLPLN	0.4614	0.6334	1.0000

Source: author's calculations in EViews

[5] See e.g., Lütkepohl [32] for more information about HP filter. In this paper the smoothing parameter of 6812100 was used.

The Pearson's correlations calculated as single numbers for the whole period are not able to detect the increasing or decreasing co-movement of the analysed return series. Therefore, the calculations of Pearson's correlations were carried out separately for individual years (Figure 3). As illustrated in Figure 3, the values of Pearson's correlations were in individual years very different without growing or declining tendency during the whole analysed period. Especially varying were the correlations of the CZK/EUR returns and HUF/EUR returns spanning between 0.065 (year 2007) and 0.656 (year 2009) as well as correlations of the CZK/EUR returns and PLN/EUR returns varying between 0.078 (year 2013) and 0.641 (year 2009). The highest correlation values (with exception of the year 2019) were detected for the pair of HUF/EUR and PLN/EUR returns.

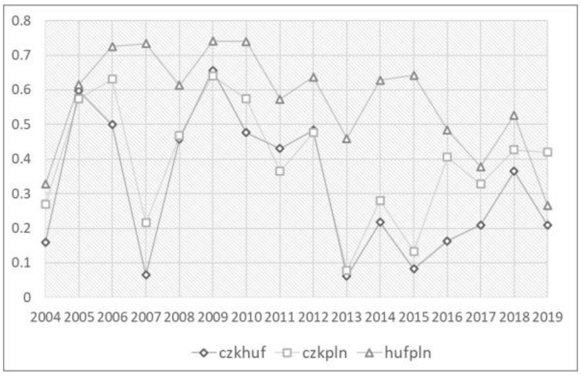


Figure 3. Pair-wise unconditional Pearson's correlations of return series, individual years
Source: author's calculations in MS Excel

Following Adam et al. [1], we can use the concept of *sigma*-convergence to investigate if the analysed exchange rate returns tend to become more similar over the analysed period. Figure 4 displays the cross-sectional standard deviations of the exchange rate returns' pairs as well as the cross-sectional standard deviations among all the three analysed return series. Lower cross-sectional standard deviations correspond to a higher convergence level while growing cross-sectional standard deviations indicate that there are growing differences between/among analysed return series, i.e. divergence. Concerning the graphs in Figure 4, the peaks of the cross-sectional standard deviations were reached during the financial crisis in 2008-2009. There is a clear downward trend especially visible since the beginning of 2016 or in case of CZK/EUR and HUF/EUR return pair even since the beginning of 2015 indicating the strong convergence tendency. Furthermore, our results are in accordance with the results published by the Czech National Bank [16] indicating similar pattern of exchange rate convergence in the Czech Republic, Poland and Hungary till the end of 2019. However, as shown in Czech National Bank [16], since the outbreak of Covid-19 pandemic in March 2020 one can observe an indication of divergence.

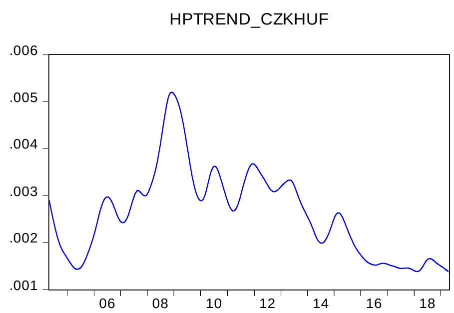
Besides the static Pearson's unconditional correlation coefficients and cross-sectional standard deviations, the dynamic conditional correlations[6] can be calculated. To analyse the behaviour of conditional correlations in time, both the symmetric and asymmetric VAR(k)-BEKK-GARCH(1,1) models were estimated based on non-linear ML method supposing the multivariate normality of disturbances. Based on the information criteria the lags k for the VAR model (1) were specified to be 6 in order to ensure the uncorrelatedness.

The estimated mean equations of the trivariate symmetric VAR(6)-BEKK-GARCH(1,1) model are as follows [15]:

DLCZK = -0.0024*DLCZK(-1) - 0.0007*DLCZK(-2) - **0.0404***DLCZK(-3) + 0.0100*DLCZK(-4) - 0.0116*DLCZK(-5) + 0.0019*DLCZK(-6) - 0.0061*DLHUF(-1) + 0.0006*DLHUF(-2) - 0.0171*DLHUF(-3) + 0.0069*DLHUF(-4) - **0.0194***DLHUF(-5) - **0.0235***DLHUF(-6) + 0.0114*DLPLN(-1) - 0.0102*DLPLN(-2) + 0.0186*DLPLN(-3) - 0.0066*DLPLN(-4) + 0.0119*DLPLN(-5) + **0.0066***DLPLN(-6) - **0.0001**

DLHUF = 0.0362*DLCZK(-1) + 0.0425*DLCZK(-2) - 0.0048*DLCZK(-3) + 0.0311*DLCZK(-4) - 0.0122*DLCZK(-5) + 0.0162*DLCZK(-6) - 0.0060*DLHUF(-1) - **0.0436***DLHUF(-2) + 0.0090*DLHUF(-3) + 0.0076*DLHUF(-4) - **0.0582***DLHUF(-5) - 0.0060*DLHUF(-6) - 0.0219*DLPLN(-1) - 0.0232*DLPLN(-2) - **0.0481***DLPLN(-3) - 0.0111*DLPLN(-4) + 0.0190*DLPLN(-5) + 0.0111*DLPLN(-6) + $3*10^{-5}$

DLPLN = **0.0540***DLCZK(-1) + **0.0552***DLCZK(-2) - 0.0356*DLCZK(-3) + 0.0198*DLCZK(-4) + 0.0127*DLCZK(-5) + 0.0286*DLCZK(-6) - 0.0087*DLHUF(-1) - 0.0102*DLHUF(-2) + **0.0462***DLHUF(-3) + 0.0035*DLHUF(-4) - 0.0137*DLHUF(-5) - 0.0005*DLHUF(-6) + 0.0022*DLPLN(-1) + 0.0011*DLPLN(-2) - **0.0379***DLPLN(-3) - 0.0012*DLPLN(-4) - 0.0131*DLPLN(-5) - 0.0109*DLPLN(-6) – $1*10^{-4}$



[6] See e.g., Forbes and Rigobon [23] who pointed out some limitations connected with calculation of the Pearson's correlation coefficients in the presence of heteroscedasticity in return series.

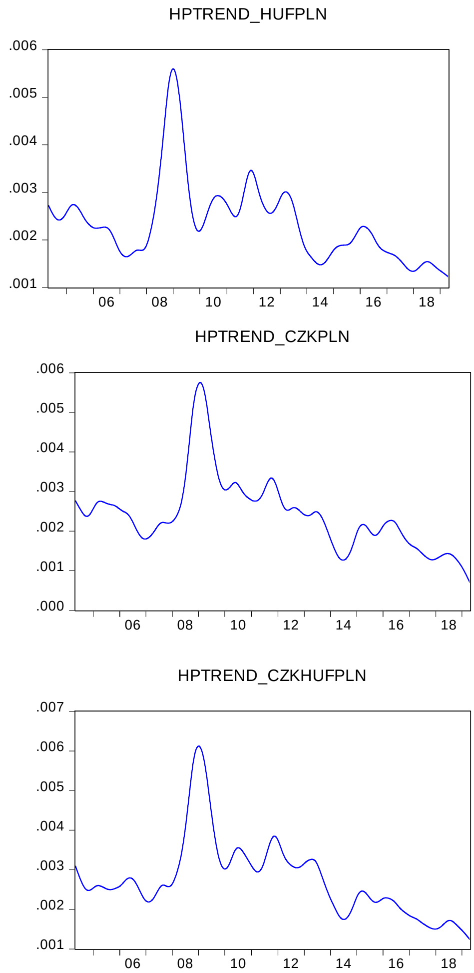


Figure 4. Cross-sectional standard deviations smoothed by HP filter (CZKHUF, HUFPLN, CZKPLN – pairwise calculations, CZKHUFPLN – calculation among all the three analysed return series)
Source: author's calculations in EViews

Concerning the statistically significant parameters from the above specified mean equations (significance level of 5 %, statistically significant parameters denoted in bold), it can be concluded, that the returns of all the analysed CE currency markets depend not only upon some of their own lags, but there is some evidence of returns spillovers, as well. Since in case of the Czech koruna market there exist the return spillovers from both the Hungarian and Polish currency markets and in case of the Polish zloty there is presence of the return spillovers from both the Hungarian and Czech currency markets, the Hungarian forint shows the existence of return spillovers only from the Polish currency market.

As for the mean equations of the trivariate asymmetric VAR(6)-BEKK-GARCH(1,1) model, the estimated parameters are as follows:

$$DLCZK = -0.0020*DLCZK(-1) - 0.0038*DLCZK(-2) - \mathbf{0.0388}*DLCZK(-3) + 0.0105*DLCZK(-4) - 0.0118*DLCZK(-5) + 0.0033*DLCZK(-6) - 0.0050*DLHUF(-1) - 0.0015*DLHUF(-2) - 0.0162*DLHUF(-3) + 0.0083*DLHUF(-4) - \mathbf{0.0199}*DLHUF(-5) - \mathbf{0.0228}*DLHUF(-6) + 0.0092*DLPLN(-1) - 0.0085*DLPLN(-2) + 0.0179*DLPLN(-3) - 0.0066*DLPLN(-4) + 0.0121*DLPLN(-5) + 0.0064*DLPLN(-6) - \mathbf{0.0001}$$

$$DLHUF = 0.0367*DLCZK(-1) + 0.0408*DLCZK(-2) - 0.0046*DLCZK(-3) + 0.0316*DLCZK(-4) - 0.0135*DLCZK(-5) + 0.0169*DLCZK(-6) - 0.0047*DLHUF(-1) - \mathbf{0.0440}*DLHUF(-2) + 0.0091*DLHUF(-3) + 0.0079*DLHUF(-4) - \mathbf{0.0568}*DLHUF(-5) - 0.0058*DLHUF(-6) - 0.0229*DLPLN(-1) - 0.0216*DLPLN(-2) - \mathbf{0.0480}*DLPLN(-3) - 0.0111*DLPLN(-4) + 0.0192*DLPLN(-5) + 0.0116*DLPLN(-6) + 3.10^{-5}$$

$$DLPLN = \mathbf{0.0540}*DLCZK(-1) + \mathbf{0.0547}*DLCZK(-2) - 0.0361*DLCZK(-3) + 0.0188*DLCZK(-4) + 0.0091*DLCZK(-5) + 0.0298*DLCZK(-6) - 0.0066*DLHUF(-1) - 0.0104*DLHUF(-2) + \mathbf{0.0461}*DLHUF(-3) + 0.0024*DLHUF(-4) - 0.0127*DLHUF(-5) - 0.0012*DLHUF(-6) + 0.0025*DLPLN(-1) + 0.0042*DLPLN(-2) - \mathbf{0.0373}*DLPLN(-3) - 0.0003*DLPLN(-4) - 0.0137*DLPLN(-5) - 0.0104*DLPLN(-6) - 0.0001$$

With regard to the statistical significance of the estimated parameters (significance level of 5 %, parameters denoted in bold), the implications presented above for the symmetric model remain true for the Hungarian and Polish currency markets, respectively. In contrast, the Czech koruna market shows besides the dependence on its own lag the presence of return spillovers only from the Hungarian forint market.

The parameter estimates of the variance and covariance equations (2) and (3) are summarised in Table 3. The estimates of the symmetric BEKK-GARCH(1,1) model indicate that all the model's parameters were (with exception of two constant terms from the conditional covariance equations) statistically significant, the highest impact of shocks (given by elements of matrix **A**) was detected for the Czech koruna, followed by the Polish and Hungarian currency markets, respectively. The highest persistence of shocks (given by elements of matrix **B**) was proved for the Hungarian forint, succeeded by the Polish zloty and the Czech koruna. The Czech koruna was indicated to be the most stable currency. Analogically, as pointed out by Katsiampa et al. [27], the conditional covariances are determined both by cross products of past shock terms and by past conditional covariance terms which demonstrates the significant volatility co-movements.

The estimation of the asymmetric BEKK-GARCH(1,1) model furthermore enables to describe the own variance asymmetries in case of the Czech and Polish currency market as the corresponding parameters of matrix **G** representing the market responses to the lagged negative shocks were statistically significant. While the negative shocks in the Czech koruna market have an impact of $a_{11}^2 + g_{11}^2$, i.e. $0.2807^2 + 0.0726^2 = 0.0841$, the impact of positive shocks equals to a_{11}^2, i.e. 0.0788. The effect of negative shocks in the Polish zloty market increases the volatility of the PLN/EUR returns of $a_{33}^2 + g_{33}^2 = 0.0552$ while in case of positive shocks there is a volatility increase of only $a_{33}^2 = 0.0494$. In case of the Hungarian forint market no asymmetry driven by the sign of the shock was proved.

The estimated models (i.e. both the symmetric and the asymmetric) seem to be adequate to describe the first two moments of the analysed series since no multivariate serial correlation was confirmed in the standardized residuals. Diagnostic checking of the standardized residuals for both symmetric and asymmetric model are gathered in Table 3. The Doornik-Hansen multivariate tests with the Jarque-Bera test statistics of 23370.77 and 21343.49, respectively, clearly reject the hypothesis that the standardized residuals are multivariate normal (for more information about normality conditions see e.g., [12]).

Due to the space reasons, we will further present the results concerning the conditional variances, covariances and correlations only for the asymmetric BEKK-GARCH(1,1) model. Figure 5 presents the time-varying conditional variances, covariances and conditional correlations. It is visible, that the volatility persistence is in all three cases generally very high. The highest values of conditional variances and covariances were detected over the financial crisis in 2008 – 2009 characterized by the world economic turndown.

The graphical illustration of the conditional correlations clearly illustrates considerably dynamic character of the conditional correlations' series between the analysed CE currency pairs. The corresponding descriptive statistics for the pair-wise conditional correlations can be found in Table 4.

Table 3. Estimates of variance and covariance equations – symmetric and asymmetric diagonal BEKK-GARCH(1,1) model

	Symmetric model		Asymmetric model	
	Coeff.	Prob.	Coeff.	Prob.
C(1,1)	3.10^{-8}	0.0000	3.10^{-8}	0.0000
C(1,2)	6.10^{-9}	0.3023	7.10^{-9}	0.2981
C(1,3)	9.10^{-9}	0.1116	2.10^{-8}	0.0056
C(2,2)	6.10^{-8}	0.0000	7.10^{-8}	0.0000
C(2,3)	3.10^{-8}	0.0042	4.10^{-8}	0.0010
C(3,3)	7.10^{-8}	0.0000	7.10^{-8}	0.0000
A(1,1)	0.2805	0.0000	0.2807	0.0000
A(2,2)	0.2030	0.0000	0.2035	0.0000
A(3,3)	0.2229	0.0000	0.2223	0.0000
G(1,1)	-	-	-0.0726	0.0037
G(2,2)	-	-	0.0024	0.9291
G(3,3)	-	-	0.0759	0.0017
B(1,1)	0.9643	0.0000	0.9626	0.0000
B(2,2)	0.9791	0.0000	0.9785	0.0000
B(3,3)	0.9740	0.0000	0.9724	0.0000
Diagnostic checking (standardized residuals)				
Q(12)[a]	65.3774	0.9996	84.5460	0.9526
Q(24)[a]	150.6606	0.9997	171.7036	0.9872
Multiv. norm.[b]	23370.8	0.0000	21343.5	0.0000

Note: CZK/EUR=1, HUF/EUR=2 and PLN/EUR=3
a – H_0: no autocorrelation in standardized residuals (Cholesky of covariance)
b – Doornik-Hansen multivariate normality test
Source: author's calculations in EViews

Following the Figure 5 and Table 4 it was proved that similarly as in case of unconditional correlations, the conditional correlations were in average the highest for the pair HUF/EUR and PLN/EUR returns, followed by the pairs CZK/EUR and PLN/EUR, CZK/EUR and HUF/EUR, indicating the strongest relationships between the Hungarian and Polish currency market. These results are in accordance with those of Výrost [41], who based on the DCC model indicated the highest dynamic conditional correlations for the pair Czech koruna and Hungarian forint. Similarly, Mittal et al. [34] confirmed that the highest correlation 0.8065 (calculated as average of time-varying ADCC values) is exhibited by HUF/EUR – PLN/EUR returns. Chmielewska [14] studying the co-movements and volatility of the Polish and Hungarian equity markets proved that the Polish zloty – Hungarian forint correlation significantly strengthened during the turmoil. Wang and Moore [42] analysing the linkages of the CEE stock markets with the Eurozone market using the DCC model presented the close link between the Hungarian and the Czech market. Finally, the results are in line with those of Kočenda and Moravcová [29] who proved the evidence of volatility spillovers among the CE currencies, evidence of the increased volatility during the 2008-2009 crisis period and to some extent during the sovereign debt crisis in 2011, as well.

Furthermore, the descriptive statistics of conditional correlations in Table 4 indicate that the differences between the minimum and maximum were huge especially for the returns' pair CZK/EUR – HUF/EUR, a little bit less volatile were the conditional correlations between CZK/EUR and PLN/EUR returns and the lowest volatility was recorded for HUF/EUR – PLN/EUR returns.

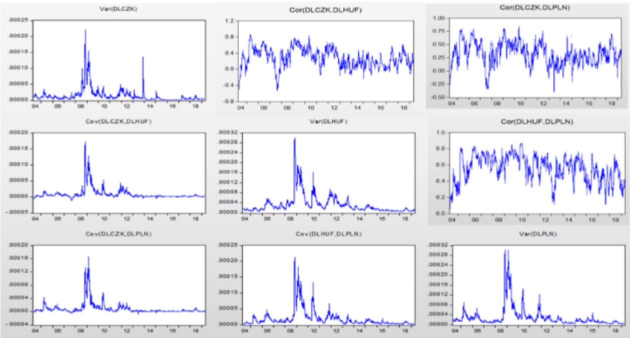


Figure 5. Conditional variances (Var), covariances (Cov) and correlations (Cor) – asymmetric BEKK model
Source: author's calculations in EViews

Table 4. Descriptive statistics of conditional correlations – asymmetric BEKK model

	CCORR_ CZKHUF	CCORR_ CZKPLN	CCORR_ HUFPLN
Mean	0.2750	0.3213	0.5562
Median	0.2731	0.3254	0.5765
Maximum	0.8589	0.8463	0.8702
Minimum	-0.5243	-0.4060	0.0567
Std. Dev.	0.2357	0.2137	0.1590
Skewness	-0.1353	-0.2183	-0.4696
Kurtosis	2.8085	3.0643	2.5053
Jarque-Bera	17.5640	31.1182	179.9896
Probability	0.0002	0.0000	0.0000
Observations	3834	3834	3834

Source: author's calculations in EViews

In Figure 6 we can find the pairwise cross-sectional standard deviations and conditional correlations (both smoothed by the HP filter) for analysed pairs of exchange rate returns. Since the lower values of conditional correlations indicate weak co-movement, the low values of cross-sectional standard deviations confirm the convergence of the analysed currency markets.

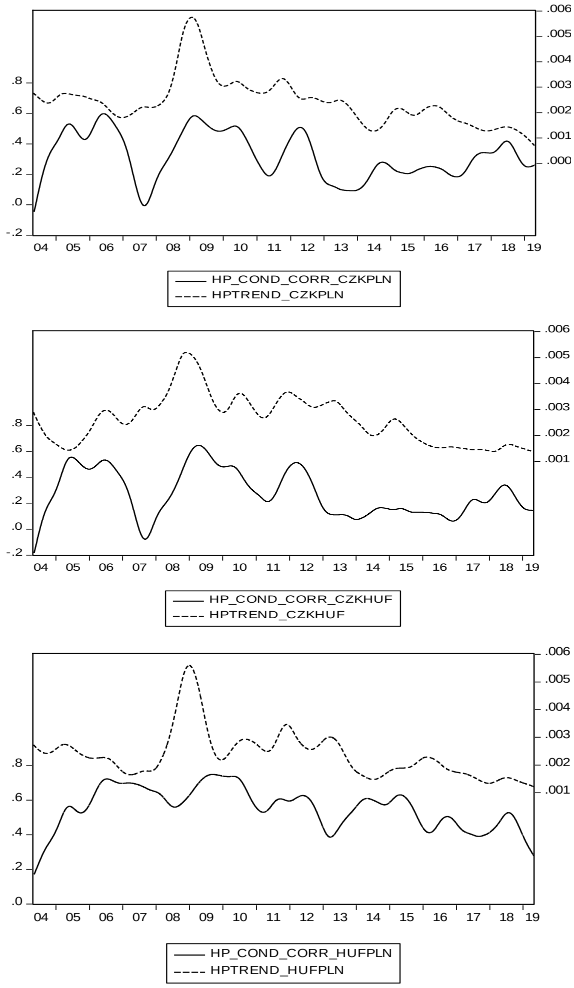


Figure 6. Pairwise cross-sectional standard deviations (dashed line) and conditional correlations (asymmetric BEKK model, solid line) both smoothed by HP filter
Source: author's calculations in EViews

The relationship of the growing/declining cross-sectional standard deviation and simultaneously declining/growing conditional correlation is quite well visible for the pair HUF/EUR and PLN/EUR returns. In case of the remaining two pairs there can be detected some periods where both the measures indicate the co-movement (similar development of return series) and periods in which the markets proved the divergence tendency. As for interpretation, there can be a little bit problem with the period of financial crisis 2008-2009 during which both the measures showed high values indicating simultaneously co-movement and divergence. One of the possible explanations presents Chaloupka [13], who asserts that the financial crisis influenced the behaviour and returns of all markets in the same (downward) direction, but with different magnitudes.

4. Conclusion

This paper analysed the time-varying returns and volatility co-movements for the daily data of CE currencies (expressed against the euro) over May 2004 – April 2019. As initial step, the linkages of middle strength between the pairs of exchange rate returns were identified by the Pearson's unconditional pair-wise correlations the time-varying character of which was clearly evident from the simple calculation for individual years. Strongest linkages were specified between the Hungarian and Polish currency markets. Using the concept of *sigma*-convergence we were able to identify the strong convergence tendency of exchange rate returns since 2008-2009 crisis period.

To investigate returns' and volatility dynamics, both the symmetric and the asymmetric diagonal VAR(6)-BEKK-GARCH(1,1) models were estimated. It was proved that the returns of all the analysed CE currency markets depend not only upon some of their own lags, but there is some evidence of returns spillovers among the analysed CE currency markets. Both the diagonal symmetric and asymmetric BEKK-GARCH(1,1) model revealed that the investors pay the most attention to the news arriving in the Czech koruna market, followed by the news arriving in the Polish zloty and Hungarian forint market, respectively. The highest persistence of shocks was identified for the Hungarian forint, followed by the Polish zloty and the Czech koruna. The Czech koruna was also identified to be the most stable currency. Accordingly, the conditional covariances were affected by past cross-shock terms and past conditional covariances indicating strong linkages between the analysed CE currency markets. The asymmetric version of the BEKK-GARCH(1,1) model furthermore confirmed the own variance asymmetries in case of the Czech and Polish market indicating higher impact of negative shocks on the variances and covariances. Calculated time-varying conditional correlations implied sharp co-movement increases during the 2008-2009 crisis period in case of all three analysed exchange rate return pairs. Comparing the pairwise cross-sectional standard deviations and conditional correlations confirmed a specific status of the crisis period characterized by high values of both these measures indicating simultaneously co-movement and divergence for the pair HUF/EUR and CZK/EUR as well as for the pair CZK/EUR and PLN/EUR.

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