# Long-run Relations in a Small Open Economy of the Czech Republic and the Slovak Republic[1]

*Jana HANČLOVÁ* – Jana JURIOVÁ** – Jana ZÁVACKÁ**

**Abstract**

*A small open economy is highly dependent on foreign environments. This article investigates equilibrium relations between a small open economy and its foreign trade partners. Based on long-run relationships developed by Garratt et al. (2003) a structural model for the Czech Republic (CR) and Slovak Republic (SR) is constructed for period 2002Q1 to 2015Q4. As most of the macroeconomic variables are nonstationary, the Cointegrated Vector Autoregressive Approach (CVAR) is used for empirical analysis. The following five long-run equilibrium relations are examined: relative purchasing power parity, uncovered interest rate parity, Fisher inflation parity, money market equilibrium, and output relation. The estimation results of the long-run relations confirmed similarities between these economies.*

**Keywords:** *cointegrated VAR, long-run relationships, small open economy*

**JEL Classification:** C32, F41, F42

## Introduction

The goal of this article is to investigate long-run relationships in a small open economy. For this purpose two macroeconometric models for small open economies of the Czech Republic (CR) and the Slovak Republic (SR) are constructed.

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They both have a common history as they were parts of one country until 1993. Therefore, there were many similarities in both economies. Later on, in 2009, Slovakia chose a different path and adopted euro. Thereat, it is interesting to follow their recent developments and to compare their estimated long-run equilibrium relations.

The CR and the SR have become increasingly integrated with the rest of the world during the last decade, and their international trade has expanded. To determine the degree of openness of their economies we use the ratio (Exports + Imports)/GDP at current prices.[2] Nowadays, the SR with its openness of approximately 185% and the CR with about 163% (according to data from 2015) belong among the most open economies in the world.

Several attempts to construct a long-run macroeconometric models for the CR and the SR are reported in Hančlová, Lukáčik and Szomolányi (2010), updated for the Czech macroeconomy in Hančlová (2011) and for the Slovak economy in Juriová (2013). In comparison with previous papers, our paper includes also longer, after crisis period, and comparison of results for both economies.

This article uses a cointegration structural model developed by Garratt et al. (2003) for the economy of the United Kingdom, later verified by Schneider, Chen and Frohn (2008) for Germany and by Assenmacher-Wesche and Pesaran (2008) for Switzerland. Their model is a macroeconometric model with transparent theoretical foundations, providing insights into the behavioural relationships that underlie the functioning of macroeconomy. In this modelling approach there is an inherent belief that economic theory is revealing of the long-run relationships, whereas the short-run restrictions are more disputable (Garratt et al., 2006).

The article is organized as follows. The Section 1 describes our theoretical concept – a derivation of long-run relations for a small open economy. The Section 2 describes used econometric methodology. The empirical results are included in the Section 3 – data, estimated models and comparison of the long-run relations. The last section concludes with main findings.

## 1. The Theoretical Concept

As the framework for modelling a small open economy, we used a theoretical approach based on the derivations of Garratt et al. (2003) and the descriptions in Krugman, Obstfeld and Melitz (2015). Both approaches set the conditions in accordance with the economic theory for domestic equilibrium in a country and

[2] Also other measures of openness of economy can be used, e.g. only exports as a percentage of GDP or only imports as a percentage of GDP (Mankiw, 2010). We have decided to use the most common measurement.

the conditions for setting equilibrium towards the external environment. The relations developed by Garratt et al. (2003) are derived from the economic theory based on a macroeconomic framework for an open economy. They include the following five long-term equilibrium relationships: purchasing power parity, uncovered interest rate parity, Fisher inflation parity, equilibrium relation at money market, and equilibrium relation of production functions. We used these relationships as benchmarks to construct the cointegrated VAR models for the Czech and Slovak economies. Some of the relations we revisited and derived in accordance with theory given in Krugman, Obstfeld and Melitz (2015). The relationships were verified for the case of small open economies with their foreign trade partners approximated by the economy of Euro Area (EA) as a whole. The reason is that the EA is the most important trading partner of both countries.[3]

The theoretical concept emphasises arbitrage conditions. In the international market the exchange rates compensate different price levels and interest rates in various countries. The basic theoretical mechanisms that operate in the international market are the law of one price and the concept of arbitrage. The law of one price states that identical goods or assets are sold at the same price in all world markets. The arbitrage concept means that if prices in the event of changes in the exchange rate are not the same everywhere, the arbitrage of goods or assets will ensure that they will even out. The equilibrium in the asset market, therefore, requires fulfilment of two international parities – purchasing power parity and uncovered interest parity conditions, which we defined on the basis of traditional economic theory in Krugman, Obstfeld and Melitz (2015).

The *purchasing power parity (PPP)* equation expresses the equilibrium between external and internal price levels, and is based on the idea that the current price of a basket of goods expressed in the same currency is the same in all international markets. The fulfilment of this condition is ensured just by the arbitrage in the goods market. Specifically, if the goods are fully mobile between countries, arbitrage ensures that prices of goods expressed in the same currency equilibrate through exchange rate adjustment. The exchange rate is adjusted to account for the changes in price levels between different countries. In practice, however, often not all goods and services are traded in both countries (are not completely mobile). According to Krugman, Obstfeld and Melitz (2015), the existence of non-tradable goods can lead to a permanent long-term deviation from the equilibrium state, especially in less developed countries, where the marginal product of labour is lower. For that reason, we adjust the possible difference due

[3] Though the Slovak Republic is a member of the Euro Area since January 2009, we used the whole EA as a proxy for the most important foreign partners because in the terms of GDP the SR represents a very small part of EA (below 1%).

to non-tradable goods by introducing a parameter $\lambda$ in the equation (1). In the ideal case, when the goods are fully mobile between the countries, this parameter would be equal to 1. In a hypothetical situation, if goods are not mobile at all between countries, there would be no foreign trade and no exchange rate as well and $\lambda$ would be equal to 0. In a realistic case the parameter $\lambda$ would attain values between 0 and 1. Then we express PPP as

$$\frac{P_{t+1}}{PS_{t+1}} = \lambda E_{t+1}\, exp\left(\eta_{ppp,t+1}\right) \tag{1}$$

where

$E_t$ – the nominal exchange rate,

$P_t$ – represents the domestic price index,

$PS_t$ – foreign price index.

The term $\eta_{ppp,t+1}$ represents a stationary or trend-stationary process. Another equilibrium relationship stems from the arbitrage between holding domestic and foreign assets. It is based on the international Fisher effect, which says that the nominal exchange rate adjusts to the difference in the interest rates between two countries. We use an expectation hypothesis – the *uncovered interest parity (UIP):*

$$\left(1+R_t\right) = \left(1+RS_t\right)\cdot\left(1+\frac{E^e_{t+1}-E_t}{E_t}\right)\cdot exp\left(\eta_{uip,t+1}\right) \tag{2}$$

where

$R_t$ – denotes the domestic nominal interest rate,

$RS_t$ – denotes the foreign interest rate,

$E^e_{t+1}$ – the expected nominal exchange rate in the next period.

The term $\eta_{uip,t+1}$ captures the short-run deviations from UIP which can be connected with effects of bonds and foreign exchange uncertainties. Although, a vast literature demonstrates the empirical failure of the UIP hypothesis (for example Mylonidis and Semertzidou, 2010), in our analysis we assume that this term is stationary. We argue that we estimate all five relationships together as one structural model and according to some empirical studies, if UIP does not hold individually, it can hold together with PPP (Johansen and Juselius, 1992, Juselius, 1995). Moreover, there is a recent work of Lothian (2016) which suggests that UIP can hold in the long run.

The equilibrium conditions in the international assets market are also closely connected with the equilibrium in the internal money market. Hence, the Fisher inflation parity and money market equilibrium conditions should be met as well.

The third equilibrium relationship included in our model is the *Fisher inflation parity (FIP)* which describes the relationship between the domestic interest rate and domestic inflation:

$$\left(1+R_t\right)=\left(1+\rho^e_{t+1}\right)\cdot\left(1+\frac{P^e_{t+1}-P_t}{P_t}\right)\cdot exp\left(\eta_{fip,t+1}\right) \tag{3}$$

where

$\rho^e_{t+1}$ – denotes the expected real interest rate,

$P^e_{t+1}$ – the expected price index in the next period.

The term $\eta_{fip,t+1}$ is the risk premium connected with the effects of money and goods uncertainties. This condition was developed, in more detail, by Garratt et al. (2003) and captures the equilibrium outcome of the arbitrage between holding bonds and investing in physical assets at internal money market.

The *money market equilibrium (MME)* represents the equilibrium in the internal money market. According to Krugman, Obstfeld and Melitz (2015) the money market is in equilibrium when the money supply in the economy is equal to the aggregate demand for money. Money supply in the economy is partly influenced by the central bank. The central bank directly regulates the amount of money in circulation and also has indirect control over the amount of check deposits issued by commercial banks. The nominal money supply is then given by decision of the central bank and commercial banks as well. For the sake of simplicity, we assume that the central bank will determine the amount of nominal money supply $M_t$ (expressed as monetary aggregate M0) at a desired level and thus, the money supply is an exogenous variable. The real domestic money supply is determined as a ratio of the nominal money supply $M_t$ and the price level $P_t$. The aggregate money demand is the total demand of all households and firms in the economy, e.g. of individual subjects' demands for money. The individual demand for money depends on the amount of income and interest rates. An increase in interest rates causes for each economic entity a decrease in demand for quickly liquid money, as money becomes more expensive. Aggregate demand for liquid money therefore decreases when interest rates rise. Then the real money demand is a function of real domestic product $Y_t$ and domestic nominal interest rate $R_t$:

$$\frac{M_{t+1}}{P_t}=f\left(Y_t,R_t\right)exp\left(\eta_{mme,t+1}\right) \tag{4}$$

where the value of the function $f(Y_t,R_t)$ falls when $R_t$ rises, and rises when $Y_t$ rises.[4] The term $\eta_{mme,t+1}$ is a stationary process which captures the effects of various

[4] Naturally, $f(Y_t, R_t)$ rises when $R_t$ falls, and falls when $Y_t$ falls.

factors that contribute to the short-run deviations of real money supply from its long-run determinants.

When expressing the function, it is clear, that it is a real aggregate demand for liquidity expressed as the demand for the possession of a certain amount of real purchasing power in liquid form. Keynes's liquidity preference theory also takes into account variations in the velocity of money. It means that the velocity of money is also dependent on the factors of money demand.

In the case of a small open economy it is reasonable to suppose that, in the long run, a domestic output is determined also by the foreign technological progress. Following the neoclassical growth model, the growth of output depends on the level of technological progress and capital and labour endowment in the country. We assume that in the long run the optimal capital endowment as well as technological progress of a small open economy converge to the capital endowment and technological progress of its foreign trade partners, similarly also their output growth. However, due to the different savings rates, government policies or local environments, there may be a difference between the domestic and foreign outputs. Then we use the *output relation (OR)* as derived in Garratt et al. (2003):

$$y_t - ys_t = ln(\gamma) + \eta_{or,t} \tag{5}$$

where

$y_t = ln(Y_t / P_t)$ – the logarithm of real domestic output,

$ys_t = ln(YS_t / PS_t)$ – the logarithm of real foreign output,

$\gamma$ – captures the productivity differentials, capital stocks differentials and labour differentials,

$\eta_{or,t}$ – represents stationary, mean zero disturbances capturing the effects of information lags due to technology flows across different countries.

For empirical purposes, we used a log-linear approximation of the long-run equilibrium relationships (equations (1) – (4)). In accordance with Garratt et al. (2003), we assume that expectations errors $\eta^e_{e,t+1}$, $\eta^e_{p,t+1}$, $\eta^e_{\rho,t+1}$ follow stationary processes, and expectations are formed as follows:

$$E^e_{t+1} = E_t exp\left(\eta^e_{e,t+1}\right) \tag{6}$$

$$P^e_{t+1} = P_t\, exp\left(\eta^e_{p,t+1}\right) \tag{7}$$

$$\rho^e_{t+1} = \rho_t\, exp\left(\eta^e_{\rho,t+1}\right) \tag{8}$$

The long-run relationships then have the following form:

PPP: $p_t - ps_t - e_t = b_{ppp} + \varepsilon_{ppp,t+1}$ (9)

UIP: $r_t - rs_t = b_{uip} + \varepsilon_{uip,t+1}$ (10)

FIP: $r_t - \Delta p_t = b_{fip} + \varepsilon_{fip,t+1}$ (11)

MME: $m_t = b_{mme} + \beta_{43} r_t + \beta_{45} y_t + \varepsilon_{mme,t+1}$ (12)

OR: $y_t - ys_t = b_{or} + \varepsilon_{or,t+1}$ (13)

where

$p_t = ln(P_t)$, $ps_t = ln(PS_t)$, $e_t = ln(E_t)$, $m_t = ln(M_t / P_t)$, $y_t = ln(Y_t / P_t)$, $ys_t = ln(YS_t / P_t)$, $r_t = ln(1+R_t)$, $rs_t = ln(1+RS_t)$, $\Delta p_t = ln(P_t) - ln(P_{t-1})$, $b_{ppp} = ln(\lambda), b_{fip} = ln(1+\rho), b_{or} = ln(\gamma)$.

We have allowed for intercepts $b_{uip}$ and $b_{mme}$ in equations UIP and MME to ensure that long-run reduced-form disturbances $\varepsilon_{i,t+1}, i = ppp,\ uip,\ fip,\ mme,\ or$ have zero means. These disturbances are related to the (long-run) structural disturbances $\eta_i$ from each equation (1) to (5) and expectation errors in the following manner:

$$\begin{aligned}
\varepsilon_{ppp,t+1} &= \eta_{ppp,t} \\
\varepsilon_{uip,t+1} &= \eta_{uip,t+1} + \eta^{e}_{e,t+1} + \eta_{\Delta e,t+1} - b_{uip} \\
\varepsilon_{fip,t+1} &= \eta_{fip,t+1} + \eta_{\rho,t+1} + \eta^{e}_{p,t+1} + \eta^{e}_{\rho,t+1} + \eta_{\Delta\Delta p,t+1} \\
\varepsilon_{mme,t+1} &= \eta_{mme,t+1} \\
\varepsilon_{or,t+1} &= \eta_{or,t}
\end{aligned} \tag{14}$$

where $\eta_{\Delta e,t+1} = \Delta ln(E_{t+1})$, $\eta_{\Delta p,t+1} = \Delta ln(P_{t+1})$.

Structural disturbances included in (14) represent different factors that could be responsible for disequilibria between the variables included in the particular long-run relationships (9) – (13).

## 2. Econometric Methodology

The long-run equilibrium relationships explicitly stated in the previous chapter are included in an unrestricted Vector Autoregression Model (VAR) (Sims, 1980) that suppose that all variables in the system are endogenous. This underlying VAR model serves as the basis to estimate an augmented cointegrated VAR model which incorporates the structural long-run relations. In fact, we use a structural cointegrating VAR approach developed by Garratt et al. (2003). The unrestricted VAR model is then transformed to the unconditional vector error correction (VEC) model which has the following form:

$$\Delta z_t = b_0 + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} - \alpha\beta' z_{t-1} + u_t \qquad (15)$$

where

$t = 1,\ldots,T$ and $z_t = \left( po_t, e_t, r_t, \Delta p_t, y_t, p_t, ps_t, m_t, rs_t, ys_t \right)'$ with $po_t = ln\left(PO_t\right)$ ($PO_t$ is the price of oil),

$b_0$ – a vector of intercepts,

$p$ – the order of underlying VAR model,

$\Gamma_i$ – matrices of short-run coefficients,

$\beta' z_{t-1}$ – the error-correction terms,

$\alpha$ – a matrix of adjustment coefficients,

$u_t$ – a vector of disturbances assumed to be white noise.

The five structural long-run relationships (equations (9) to (13)) imply the following 43 (over)identification restrictions on the cointegration matrix $\beta$ in the unconstrained VAR in equation (15):

$$\beta' = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_{43} & 0 & \beta_{45} & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \qquad (16)$$

The price of oil $po_t$ is used in the model as a long-run forcing variable. A forcing variable means that changes in oil price have a direct influence on the other variables, but it is not affected by the other variables in the model (Garratt et al., 2003). For oil price to be a forcing variable, a necessary condition is that the price of oil is weakly exogenous for the parameter of the conditional process of VAR. This implies restrictions on the adjustment matrix α that insure that the cointegration relations do not have any influence on the forcing variable. This ends up with a conditional Vector Error Correction (VEC) model.

## 3. Empirical Analysis

This section consists of the description and analysis of the data used, estimation of Vector Error Correction models for the CR and the SR and discussion of estimation results.

### *Data*

In this section the data for two small open economies – the CR and the SR are described and analysed. Each model includes 6 domestic variables and 4 foreign variables including the price of oil. A detailed description of all variables is given below. All time series use quarterly data. As the CR and SR were established in January 1993, it is better to use shortened time period for the analysis, as the years before 2000 are connected with extensive structural reforms of both economies and also with resulting structural shocks in some of the variables in the model (in particular the interest rates). In case of CR, the data for monetary aggregate M0 are available from the 1st quarter of 2002. Therefore, the data are used for the period 2002Q1 to 2015Q4 for both countries, thus we have 56 observations available. If the seasonality in the data was statistically significant, the variables were seasonally adjusted by means of Tramo/Seats procedure.

### *The List of Variables, their Description and Data Source*

The domestic variables relate to two small open economies – the Czech Republic and the Slovak Republic:

- $E_t$ – *nominal effective exchange rate (NEER)*[5] *(18 trading partners), index 2010 = 100* (source: Eurostat);
- $R_t$ – *short-term interest rate – 3-month rate, for SR from 2009Q1 Euribor, %* (source: Eurostat);
- $PR_t$ – *harmonized consumer price index (2010 = 100)* (source: Eurostat);
- $Y_t$ – *gross domestic product at market prices, millions of national currency, chain-linked volumes with reference year 2010* (source: Eurostat);
- $P_t$ – *producer prices in industry, total output price index (2010 = 100)* (source: Eurostat);
- $M_t$ – *money aggregate M0, millions of national currency* (source: Czech National Bank, National Bank of Slovakia).

Foreign variables comprise:

- $YS_t$ – *gross domestic product of the EA, millions of national currency, chain-linked volumes with reference year 2010* (source: Eurostat);
- $PS_t$ – *producer prices in industry for EA, total output price index (2010 = 100)* (source: Eurostat);
- $RS_t$ – *short-term interest rate – 3-month rate for EA (Euribor), % (source: Eurostat);*
- $PO_t$ – *price of oil Brent in US Dollars per barrel, price index (2010 = 100)* (source: U.S. Energy Information Administration).

[5] The NEER (or, equivalently, the 'Trade-weighted currency index') of a country aims to track the changes in the value of that country's currency relative to the currencies of its principal trading partners. It is calculated as a weighted geometric average of the bilateral exchange rates against the currencies of competing countries.

Note, that in the empirical analysis, in accordance with Garratt et al. (2003), producer price indices are used to construct deviations between the domestic and foreign price levels in the PPP relationships, and a consumer price index is used to measure domestic inflation in the FIP relationships.

### *Testing of Stationarity*

The Augmented Dickey-Fuller (ADF) test was used to verify the stationarity of the original time series and their first differences, the automatic lag selection was used by means of Schwarz information criterion. The ADF test statistics for the levels and the first differences in the original variables in logarithms are reported in Table 1. If the trend was statistically significant, we used the variant with the trend, otherwise we used the variant with the constant. The results of the test suggest that it is reasonable to treat almost all the variables in consideration as I(1) variables. For these variables the unit root hypothesis is rejected when applied to their first differences at the significance level of 4%. The test suggests also one I(0) variable – differences of consumer price indices for CR. However, all the original variables are I(1).

T a b l e 1

**Results of ADF Unit Root Test for Model Variables**

| **Variable** | | ***t*-static (significance level) / lags** | | |
|---|---|---|---|---|
| | Level (trend + constant) | Level (constant) | 1. difference | Order of integration |
| *e_CR* | x | –2.06 (0.26)/1 | –5.32(0.00)/1 | I(1) |
| *e_SR* | x | –1.94 (0.31)/0 | –5.32 (0.00)/0 | I(1) |
| *r_CR* | x | –2.15 (0.23)/1 | –4.46 (0.00)/0 | I(1) |
| *r_SR* | –3.11 (0.12)/1 | x | –4.26 (0.00)/0 | I(1) |
| *Δpr_CR* | x | –4.89 (0.00)/0 | x | I(0) |
| *Δpr_SR* | –6.27 (0.00)/0 | x | –12.45 (0.00)/0 | I(1) |
| *y_CR* | x | –1.86 (0.35)/1 | –2.89 (0.01)/0 | I(1) |
| *y_SR* | x | –1.78 (0.39)/0 | –7.83 (0.00)/0 | I(1) |
| *p_CR* | –2.98 (0.15)/1 | x | –5.13 (0.00)/0 | I(1) |
| *p_SR* | x | –2.76 (0.08)/0 | –5.78 (0.00)/1 | I(1) |
| *ps* | x | –1.73 (0.41)/1 | –3.88 (0.00)/0 | I(1) |
| *m_CR* | x | –2.29 (0.18)/0 | –2.10 (0.04)/1 | I(1) |
| *m_SR* | –3.51 (0.05)/0 | x | –8.89 (0.00)/0 | I(1) |
| *rs* | x | –1.72 (0.42)/1 | –3.81 (0.00)/0 | I(1) |
| *ys* | –2.62 (0.28)/1 | x | –3.22 (0.02)/0 | I(1) |
| *po* | x | –2.22 (0.20)/1 | –5.46 (0.00)/0 | I(1) |

*Source:* Authors' calculation and elaboration.

### *Estimation of the Models*

The variables for the model are $e_t$, $r_t$, $\Delta p_t$, $y_t$, $p_t$, $ps_t$, $m_t$, $rs_t$, $ys_t$ and $po_t$ as an exogenous variable. In the first stage, the order of the unrestricted VAR models for 9 variables was selected. In the selection process we concentrated on estimating stable VAR models without autocorrelation and heteroscedasticity in residuals. Finally, we selected the order of 2 for Czech and also Slovak data based on the autocorrelation tests of VAR models for both countries.

The next step is to test the cointegration rank. The purpose of the cointegration test is to determine whether a group of nonstationary series are cointegrated or not, and what the number of cointegration relations in the VAR models for the Czech and Slovak economies is? The results of the Johansen test are summarized in Table 2 for the Czech model and the Slovak model. The trace test and also the maximum eigenvalue test identify at 5% significance level from 4 to 9 cointegrating relationships among 9 Czech variables and from 3 to 9 cointegrating relationships among the Slovak variables. However, based on the results from stationarity test we should concentrate on the models with linear trend in data as some of the Slovak and also Czech variables had a significant trend. In our theoretical models we considered the intercepts and no trend in both models. If we should be in line with our theoretical concept, we should concentrate on the model with linear data trend and intercept and no trend in the data. In the case of CR there are significant 5 – 6 cointegration relationships for this option, in the case of SR 3 – 4. The 6 cointegration relationships suggested by the trace test are consistent also with the fact that one Czech variable in the model is I(0), which automatically creates an extra cointegration relation. On the basis of these results we can proceed with 5 cointegrating relations in the case of CR. Due to the reason of comparability we used also 5 cointegrating relations for SR.

Table 2

**Johansen Test of Cointegration Rank for the Czech and the Slovak Variables**

| Data trend | Test type | Trace_CR | Max-Eig_CR | Trace_SR | Max-Eig_SR |
|---|---|---|---|---|---|
| None | No Intercept, No Trend | 9 | 5 | 8 | 3 |
| None | Intercept, No Trend | 8 | 6 | 9 | 3 |
| **Linear** | **Intercept, No Trend** | **6** | **5** | **4** | **3** |
| Linear | Intercept, Trend | 6 | 4 | 4 | 4 |
| Quadratic | Intercept, Trend | 6 | 4 | 4 | 4 |

*Source:* Authors' calculation and elaboration.

Then we proceeded in estimating vector error correction models (equations (9) to (13)). To get a reasonable results we had to restrict the parameter $\beta_{45}$ to –1, and then the MME equations, in fact, represent velocity equations. This adjustment is also in accordance with the model estimated in Garratt et al. (2003).

Moreover, in the case of the Czech model we loosened the parameter $\beta_{46}$ for foreign prices (*ps*), because this adjustment helped to solve the problems with the trend in this equation.

The estimated long-run relationships for the *Czech economy*, incorporating the restrictions suggested by the theory, are as follows:

$$\text{PPP: } p_t - ps_t - e_t = -4.4913 + \varepsilon_{ppp,t+1} \tag{17}$$

$$\text{UIP: } r_t - rs_t = -0.0002 + \varepsilon_{uip,t+1} \tag{18}$$

$$\text{FIP: } r_t - \Delta p_t = 0.0140 + \varepsilon_{fip,t+1} \tag{19}$$

$$\text{MME: } m_t - y_t = -13.6799 - 3.8855 r_t + 1.7545\, ps_t + \varepsilon_{mme,t+1} \tag{20}$$

$$\text{OR: } y_t - ys_t = -0.9078 + \varepsilon_{or,t+1} \tag{21}$$

The economic theory of the 5 long-run relations put 18 restrictions over on the matrix $\beta$. The likelihood ratio test statistic for these 18 restrictions is 114.6824. The corresponding p-value based on $\chi^2(18)$ distribution is near zero. However, as argued by Garratt et al. (2003), the distribution of the Log-likelihood Ratio statistic (LR) used to test the validity of the over-identifying restrictions is appropriate only asymptotically. We used a nonparametric bootstrap procedure based on 1 000 replications of the LR statistic testing the 18 restrictions. For each replication, an artificial dataset was generated (with the same length of 53 observations after adjustments) under the assumption that the estimation version of the core model was the true data-generating process, using the observed initial values of each variable, the estimated model and a set of random innovations.

These innovations were obtained by re-sampling with replacement from the estimated residuals. The appropriate 95% critical value of LR for the test of the validity of the over-identifying restrictions is 196.5. Using these bootstrapped critical values, the 18 long-run theory restrictions cannot be rejected at the conventional 5% level.

The estimated long-run relationships for the *Slovak economy* are following:

$$\text{PPP: } p_t - ps_t - e_t = -4.4445 + \varepsilon_{ppp,t+1} \tag{22}$$

$$\text{UIP: } r_t - rs_t = 0.0211 + \varepsilon_{uip,t+1} \tag{23}$$

$$\text{FIP: } r_t - \Delta p_t = 0.0092 + \varepsilon_{fip,t+1} \tag{24}$$

$$\text{MME: } m_t - y_t = -31.0211 - 8.5585 r_t + 5.5826\, ps_t + \varepsilon_{mme,t+1} \tag{25}$$

$$\text{OR: } y_t - ys_t = -1.4980 + \varepsilon_{or,t+1} \tag{26}$$

The likelihood ratio test statistic for 18 restrictions is 143.0935. The bootstrap critical values for joint tests of the 18 over-identifying restriction based on 1 000 replications were 207.8 at the 5% level. The linear restrictions implied by our long-run theory cannot be rejected.

All estimated parameters of both models are statistically significant at 5% significance level. We tested also for the presence of structural breaks in the particular long-run relationships. We used least squares method with breakpoints with estimation of coefficients of covariance matrix by means of HAC (Newey-West). The break due to the change of constant was significant mainly for FIP (period 2010Q1 for CR and 2009Q2 for SR) and partially for PPP in CR and MME in SR. However, these results relate rather to the crisis period.

We tested also the stationarity of all estimated cointegrating relations. We used ADF test statistics for the levels and the test without trend and constant. The testing of stationarity for long-run relations confirms rather stationarity (Table 3) for the significance level of 11% for CR and 3% for SR with exception of PPP.

Table 3

**Results of ADF Unit Root Test for Cointegrating Relationships**

| Cointegration relation | CR | SR |
|---|---|---|
| *PPP* | –1.58 (0.11) | –0.69 (0.41), break –4.54 (0.04) |
| *UIP* | –2.86 (0.01) | –2.19 (0.03) |
| *FIP* | –3.13 (0.00) | –2.46 (0.02) |
| *MME* | –1.66 (0.09) | –2.13 (0.03) |
| *OR* | –1.74 (0.08) | –2.34 (0.02) |

*Source:* Authors' calculation and elaboration.

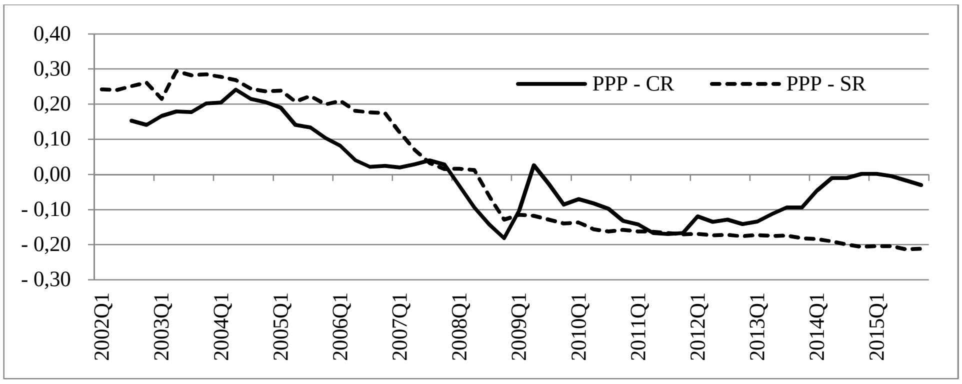
### *Discussion and Comparison of Results*

The estimation results from the first cointegration equations (17) and (22) relate to PPP. According to the PPP theory, the exchange rate and prices may converge towards an equilibrium relationship in the long-run and be cointegrated. Our estimation results for the Czech Republic confirm the existence of a trivariate (including $e_t$, $p_t$ and $ps_t$ in the VAR as separate variables) cointegration relationship at the significance level of 11%. In the case of the Slovak Republic the testing of unit root by means of ADF test indicates non-stationarity and the breakpoint unit root test indicates stationarity with the break in 2006Q4 at the significance level of 4%. Therefore, the existence of equilibrium relations in both countries is questionable. The very similar estimated constant for both countries could represent an expected difference due to the existence of non-tradable goods.

Comparing the PPP in CR and SR (Figure 1) it is obvious that PPP of the Czech Republic is more volatile after the global financial crisis, while in the Slovak Republic after introducing euro the influence of exchange rate fluctuations was dropped out.

Figure 1

**Cointegration Relations for Purchasing Power Parity**

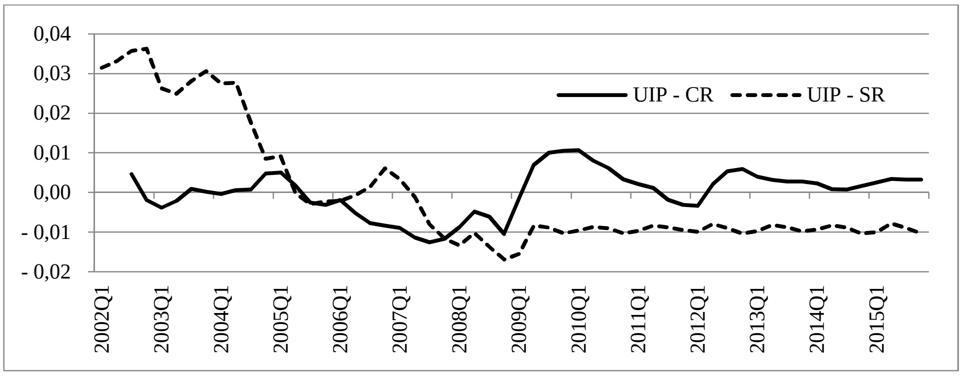



*Source:* Authors' calculation and elaboration.

Considering the second relation (equations (18) and (23)), the estimated UIP relations include the intercept, which can be interpreted as a deterministic component of the risk premium associated with bond and foreign exchange uncertainties. In the case of SR the risk premium is estimated higher than for CR, at approximately 2.11% per quarter compared to CR with –0.02% per quarter. In the Figure 2 we can see a different pattern in the development of UIP equations in both countries. The highest deviations in the CR are in the pre-crisis period of 2007 to 2008 and during the period of financial crisis of 2009 to 2010. These deviations could be explained by higher volatility in the nominal exchange rate. However, the deviations from equilibrium in the CR are very small. For SR we can see a nice picture of convergence of Slovak monetary policy to the policy of the EA, finishing in 2009 by entering Euro Area and accepting the policy of European Central Bank (ECB).

Figure 2

**Cointegration Relations for Interest Rate Parity**

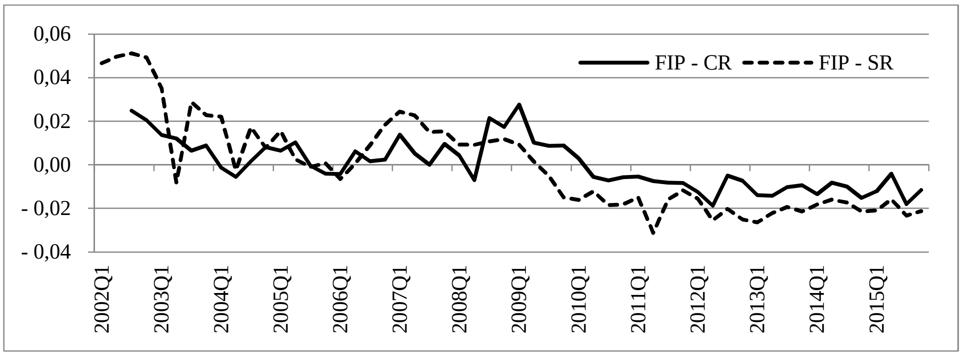



*Source:* Authors' calculation and elaboration.

Concerning the third relation, estimated in equations (19) and (24), the constant in the FIP implies that the average long-run Czech real interest rate is about 1.40% for the period 2002 – 2015 and the average long-run Slovak real interest rate is about 0.92%. The Figure 3 shows the comparison of the estimated values of these equilibria. According to the figures both the empirical FIP oscillate about their long-term equilibrium. The real rate of return close to 0 after 2009 in both countries reflects the monetary policy during the years of economic contraction.

Figure 3

**Cointegration Relations for Fisher Inflation Parity**



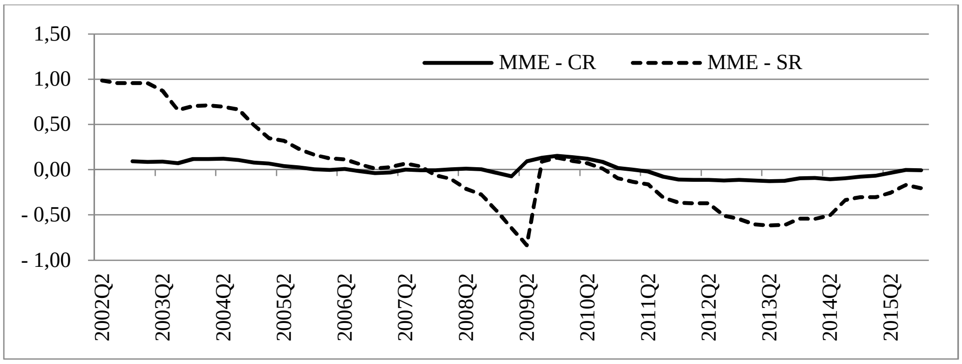

*Source:* Authors' calculation and elaboration.

Considering the fourth relation (equations (20) and (25)), the equilibrium at money market, the long-run elasticity of the influence of the interest rate on the equilibrium relation between the real money supply and the real output is negative in both countries. It means that if interest rates increase, the velocity of the most liquid money M0 decreases. Figure 4 shows that the MME is stationary in both countries, according to the ADF test at the level of 3% for SR and 9% for CR. For SR there is an extreme significant deviation in the 2$^{nd}$ quarter 2009 when the money supply got below its equilibrium. The cause may lie in the beginning of global financial crisis connected with lower amount of liquid money in the circulation and it was also the time when SR switched to euro currency.

The last long-run relationship – output relation – describes the relation between the foreign output and the domestic output. The Figure 5 shows different patterns of the convergence to the output of the external environment in both countries. Even in the past when both countries formed one country, the production potential in both countries was not equal. According to equilibrium relationships (equations (21) and (26)) the output difference between CR and EA was estimated to –0.9078 whereas between SR and the EA –1.4980.

F i g u r e 4

**Cointegration Relations for Money Market Equilibrium**

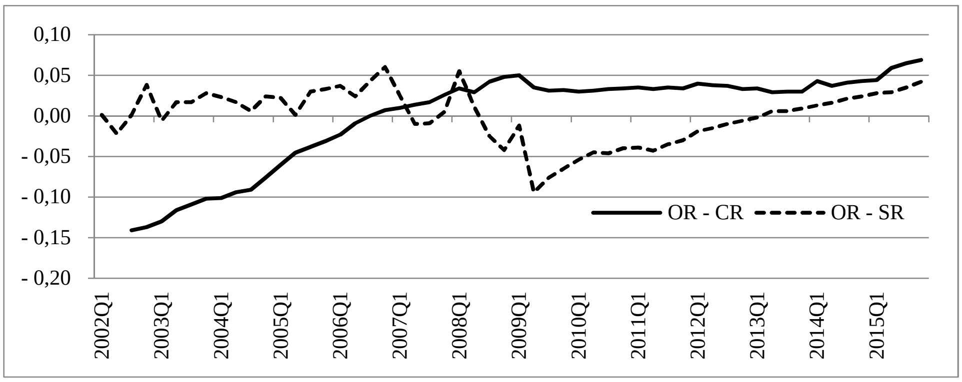



*Source:* Authors' calculation and elaboration.

We can see from Figure 5 and also from the ADF test that both relations are stationary and very close to the equilibrium. During the first years of economic expansion the difference between the output of the Czech and Slovak economy and the output of the EA was decreasing, however, this trend was changed with the crisis triggered at the end of 2008. In CR the output difference got then more or less stabilized. The output difference between the Slovak economy and the EA was much bigger and more volatile. This could be explained by the fact that in the past, the Slovak economy was suffering from low capital endowment. From the beginning of the observed period this was reduced with big foreign direct investments and the output difference began to decline slowly, practically until 2007. The financial crisis increased the output difference between SR and the EA, however, entering the Euro Area in 2009 significantly helped to renew the convergence process to the output of the external environment.

F i g u r e 5

**Cointegration Relations for Output Relation**




*Source:* Authors' calculation and elaboration.

## Conclusion

This article investigates the long-run structural modelling approach for the small open economies of the CR and the SR over the period 2002Q1 to 2015Q4 with ten macroeconomic variables. For this purpose we estimated the VEC models with over-identifying restrictions.

The main focus was placed on the comparison of the estimated long-run relations for both economies. In the past the two states formed one country and thus have a common history. Later on, the similar economies pursued different paths – the SR entered the Euro Area in 2009, while the CR has not done so yet. The constructed VEC(1) model for both countries served, in this way, as a good instrument to evaluate the degree of harmonization of both economies with the economy of the EA – their main trading partner.

The estimation results confirmed a lot of similarities in both economies. According to the results, the long-run relations – money market equilibrium, output relation, Fisher inflation parity and interest rate parity seem to be rather stable with some oscillation about their equilibrium values in both countries. However, the existence of long-run relationships of purchasing power parities remains questionable.

According to the results arising from the output relations we may assume that the Czech economy is closer to the external level of economy than the Slovak economy, in sense of production. If we consider the asset market, we could assume from the results that the Slovak economy, under the policy of ECB nowadays, is more harmonized with the EA than the Czech economy. Big foreign direct investments in the Slovak Republic together with entering the Euro Area seem to have helped the SR to speed up the convergence process.

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