SOLUTION OF SOCIO-ECONOMIC EFFICIENCY OF PUBLIC PROJECTS UNDER THE CONDITIONS OF VARIABLE AMOUNT OF SUBSIDY

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Abstract. *The paper will deal with modelling cash-flows of public investment projects under the conditions of variable amount of subsidy from public funds. A deterministic model based on a rational fractional function will be used to assessment of socio-economic efficiency of public projects subsidized from the European funds. In the model, we will examine the influence of the amount of the subsidy and critical changes of socio-economic cash-flows generated by a project on assessing public investments. We will use the European Union methodology for assessing socio-economic evaluation of project utility by means of cost-benefit analysis. The results of the crisis scenario analysis of the model in the Maple program created on the basis of the concept of Economic net present value (ENPV) and Economic internal rate of return (ERR) will be presented in 2D projection and evaluated by means of the ENPV function depending on the project parameters. The paper will also present using the model for stress test of socio-economic efficiency of innovation project from sphere digitalization of public government, and broadens so far published research.*

Keywords: *Public Projects, Socio-economic Efficiency, the ENPV Function, ERR, Subsidy from European Funds, Cost-benefit Analysis.*

JEL Classification: *C20, H43.*

Introduction

The principles of socio-economic efficiency of public projects are provided in frame of *welfare economics*. Public project efficiency evaluation uses in its mathematical basis commercial methods – the net present value method, and the internal rate of return method (IRR). The commercial net present value method (NPV) is after applying the theoretical principles of *welfare economics* (cash-flows are the quantification of utility) available for evaluation public projects of investment character for its stress on expressing time-rated utility. It corresponds to utilitarian bipolar concept of utility as the difference between the benefit and harm (loss) to the welfare for a social entity. The adjusted method of economic net present value ($ENPV$) is generally used as the preferred method for evaluating investment oriented projects from European funds. These projects are characteristic by a certain amount of subsidy to the supported subjects determined according to the objectives of the European Union cohesion policy in case of European funds. The second in order preferred method for evaluating investment character projects subsidized from the European funds is the so called method of economic internal rate of return (ERR). This is based on finding a real root of the $ENPV$ function from the parameters of the proposed project, which corresponds with the condition of indifferent evaluation of public project efficiency from the investor's perspective. The social discount rate of a public project is then compared to the ERR that has been found. (European Commission, 2015)

1. Statement of a problem

The order of preferences for using both methods for public projects corresponds also with the approach for evaluating commercial investment projects financed from private sources (Levi, Sarnat, 1994, Brealey et al., 2011). The recommendation for evaluating public projects by the commercial method of IRR reflects the fact that the IRR method has, in comparison with the NPV method, several significant disadvantages. These are connected namely with the so called non-conventional cash-flows generated by some projects, when more than one change of cash-flow polarity occur. The definition of the conventional type of investment is mentioned by (Bussey, Eschenbach, 1992) as an investment that contains one or more negative cash outflows, followed by one or more positive cash inflows. A non-conventional investment is defined as an investment that intersperses the positive and negative cash flows. Problems of the IRR method which are generally valid also for the ERR modification for public projects were completely dealt with in the past e.g. by (Teichroew et al., 1965) in compliance with solving the polynomial by the so called Descartes' rule. (Hazen, 2003) has solved the problem of multiple real IRRs by linking the present value of the outstanding capital expenditure of a project with the difference between any IRRs and the cost of capital. Derivation mathematical rules for the construction of the NPV function and solution for nonconventional stream of cash-flow is in (Marek, Radová, 2006). (The topic of using IRR and NPV criteria has been summarised by (Magni, 2011). The generalized net present value (GNPV) method for nonconventional project evaluation is dealt with (Kulakov, Kulakova 2013). The generalized internal rate of return (GIRR) and generalized external rate of return (GERR) indices based on the generalized net present value (GNPV) are presented by (Kulakov, Kastro 2015).

Besides the above stated topic of nonconventional cash-flows, for the area of public projects another significant problem is determining the discount rate of public projects, which cannot be derived on the basis of market principles. (Marglin, 1963) deals with criteria for evaluating public projects by the cost-benefit analysis method and the social discount rate of public projects and with. For evaluating the efficiency of public projects, he uses the *Optimal Rate of Investment* (Marglin, 1967). Similarly, (Feldstein, 1964) uses the term *Net Social Benefit* for evaluating public projects utility. (Dasgupta et al. 1972) create the methodology of development projects evaluation for UNIDO. Economic analysis of public projects is dealt with by (Squire, Van der Tak, 1975). The present methodology of the European Union issues namely from works of (Florio 2006, 2007) and distinguishes two phases in public investment projects evaluation (European Commission, 2015) In the first phase projects are evaluated on a purely commercial basis of financial cash-flows by means of the so called FNPV (*financial net present value*) and FRR (*financial rate of return*), which corresponds to the above mentioned IRR. In the other phase of the economic analysis of a project, economic cash-flows are aggregated by means of the so called fiscal corrections of calculated financial cash-flows and by adding quantified negative and positive externalities. The resulting evaluation of social efficiency of public projects is carried out on the basis of discounting economic cash-flows by means of the $ENPV$ and ERR (European Commission, 2015).

Projects are supported from the EU funds in the situation when a project generates negative FRR and FNPV values and so it is inefficient from purely commercial perspective. By adding positive cash-flows from positive externalities of proposed projects we then arrive, in the stage of project economic analysis, from negative values of FRR and FNPV at positive values on the basis of ERR and $ENPV$. A project built like this can obtain a subsidy from European funds because it shows positive utility and contributes to increasing social welfare and natural resources protection. In the real economic situation, however, there may be scenarios which can generate negative cash-flows in a project, usually proposed as a conventional one. Those will cause a nonconventional nature of the project, i.e. there may not be exactly one positive real-valued root of the $ENPV$ function (i.e. the value of $ERR > 0$). Then it is not possible to decide conclusively about a project on the basis of the ERR method.

The presented paper aims at analysing the impacts of the change in the amount of a subsidy of a public project in selected economically justified scenarios of evaluating a project by means of the ERR, which is based on a modification of the commercial IRR method. This application contains both advantages and disadvantages of the commercial IRR method, described in the above mentioned specialist sources and summarized in (Magni 2011). For the description of public project behaviour under the conditions of variable amount of subsidy from European funds, we will use determinist model of the $ENPV$ function within numeric solution of the model by the Maple program. These model scenarios will capture changes of behaviour of the $ENPV$ function and its roots with a view of the amount of subsidy depending also on changes of further parameters, i.e. cash-flows generated by an investment in the building, operational and liquidation phases.

2. Methods

2.1 Notations

For construction and analysing the ENPV model, the following notation and assumptions are used (Dvořáková, Jiříček 2013).

ERR	Economic rate of return
$ENPV = f(k)$	Economic net present value displayed by means of the investment curve of project
k	Social discount rate
t	Years of project lifetime
n	Investment project lifetime
$CF = [CF_0, \ldots, CF_n]$	Stream of cash flows

2.2 Assumptions

Conventional projects – they are characterized by only one change of their polarity in the sequence of the stream of cash-flows generated within the project (e.g. $[-,--++++]$).

Nonconventional projects – they are characteristic by more than one change of project generated cash-flows in the sequence of the CF stream (e.g. $[-,-++++-]$).

Investment curve of the project – defined by the rationally fractional function

$$ENPV = \sum_{t=0}^{n} \frac{CF_t}{(1+k)^t} = CF_0 + \frac{CF_1}{1+k} + \frac{CF_2}{(1+k)^2} + \cdots + \frac{CF_n}{(1+k)^n} \ . \tag{1}$$

It expresses the relation between economic net present value $ENPV$ of the project (dependent variable) and the social discount rate k of a public project (independent variable). The investment curve will be analysed with a view of solving real-valued roots.

The degree (n) of the $ENPV$ investment (1) – it represents the period of economic lifespan of a public project (in the model, n will equal 6 years).

The constant term (CF_0) – it represents capital expenditures of the project in the model. It will acquire non-positive values, which corresponds to a negative cash-flow reflecting project investment costs or as the case may be various (i.e. up to 100%) amount of subsidy from the donor.

Coefficients ($CF_1,\ldots,CF_n$) – in the model represent cash-flows caused by the investment and they can generally acquire both positive and negative values, while $CF_1,\ldots,CF_{n-1}$ represent the operational phase of the project and CF_n the liquidation phase of the project.

Years of project's lifetime (t) – the model is built on the initial prerequisite of the possibility to separate the investment phase (year 0), the operational phase (years 1 to $n-1$) and the liquidation phase (year n) of the project.

Stream of CFs – the sequence of cash-flows generated by the project in the form of $CF = [CF_0, CF_1, \ldots, CF_n]$, in the model in the basic form $CF = [-6,1,1,1,1,1,1]$, which corresponds to realistically predictable flows of proposed public projects.

Economic internal rate of return of the project (ERR) – the root of the investment curve (1) of the project, i.e.

$$ENPV = \sum_{t=0}^{n} \frac{CF_t}{(1+ERR)^t} = 0 \ . \tag{2}$$

3. Problem solving

3.1 Modelling the cash-flow of a public project

For the proposed model from a set of variants of public investment projects, this paper will only consider some chosen economically justified scenarios when an originally conventional project may change into a project of non-conventional character on the basis of social evaluation of utility. For the purpose of comparing modelling results, we analyse also two types of conventional projects – a classical conventional model of a public project whose stream of cash-flow contains a negative investment expenditure as well as positive utilities from the project, and a type when after a negative investment expenditure at the beginning of the operational phase of the investment there is a negative cash-flow (e.g. due to delay of the project implementation). We also analyse two types of non-conventional projects.

We display the course of the project investment curve $ENPV = f(k)$ (see the definition (1)), i.e. the relation of the utility from the project to the social discount rate. We use 2D projection and seek the dependence of the curve course and root position on the amount of the subsidy from public sources, manifesting in the change of CF_0 for the defined cases of (Dvořáková, Jiříček 2013):

Classical conventional project $CF = [-,++++++]$

Conventional project with a crisis in the building phase $CF = [-,-+++++]$

Nonconventional project with a crisis in the operational phase $CF = [-,++-+++]$

Nonconventional project with a crisis in the liquidation phase $CF = [-,+++++-]$

For these alternatives we chose basic scenario with sequence of cash flow in shape $CF = [CF_0, CF_1, CF_2, CF_3, CF_4, CF_5, CF_6] = [-6, 1, 1, 1, 1, 1, 1]$

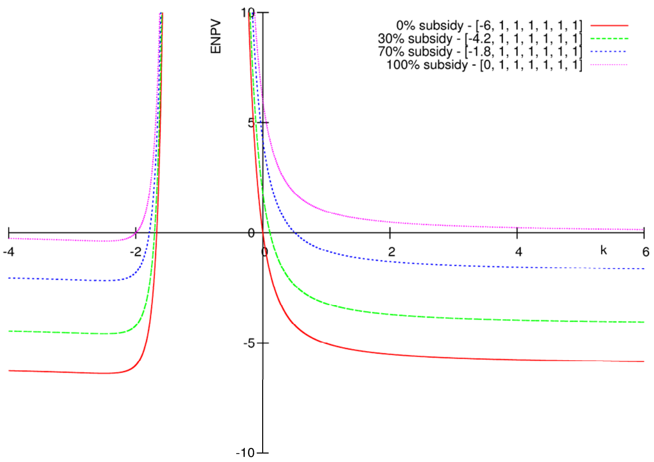
In each of these alternatives we will simulate situation, when the public investment project will be supported in level 0%, 30%, 70% and 100% subsidy from European funds.

3.2 The investment curve course in 2D projection

First, we will analyse the course of the investment curves $ENPV$ for classical conventional projects and for a conventional project with a crisis in the building phase; then the $ENPV$ functions for nonconventional projects will be displayed.

The course of the investment curve of a classical conventional project shows the influence of subsidy of investment costs on the position of the $ENPV$ function roots at a given unit stream of cash-flows in the operational and liquidation phases of the project. The subsidy amount influences the shift of the curves along the y axis.

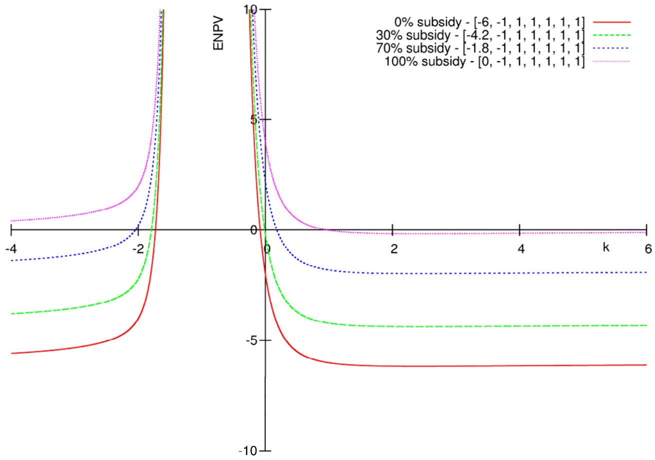
Fig. 1: The course of the investment curve of the project ENPV=f(k) according to the subsidy amount (a classical conventional project)



Source: authors

From the perspective of assessing the project efficiency by the ERR method, we look for positive real-valued roots. According to Descartes' rule of signs these conventional projects have one real-valued root greater than -1. There is a monotonously decreasing function with the asymptote in the value CF_0 for $k \to \infty$. In case of a classical conventional project (Fig. 1) the $ENPV$ function does not have a final positive root at 100% subsidy, but $ERR = \infty$ (see Tab. 1).

Fig. 2: The course of the investment curve of the project ENPV=f(k) according to the subsidy amount (conventional project – crisis in the building phase)



Source: authors

Tab. 1: Roots (ERR) of investment curves at various subsidy amounts (conventional projects)

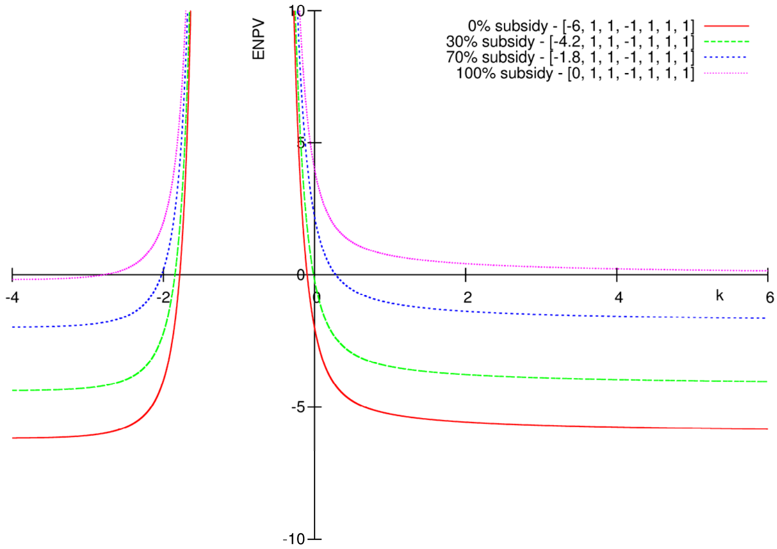
subsidy	parameter CF_0	ERR (classical conventional project) $CF = [CF_0, 1,1,1,1,1,1]$		ERR (crisis in the building phase) $CF = [CF_0, -1,1,1,1,1,1]$	
0%	–6	–1.67	0	–1.729	–0.082
30%	–4.2	–1.705	0.112	–1.793	–0.01
70%	–1.8	–1.787	0.508	–2.045	0.18
100%	0	–2	∞	0.966	∞

Source: authors

In this case, the ERR method cannot be efficiently used for evaluation projects, in the positive area the root limits to +∞ and the project is thus efficient for any amount of the social discount rate chosen by the investor. The influence of a negative cash-flow due to the project building (Fig. 2) causes a change of the curve course. This curve is still monotonously decreasing. In cases of 30% and 0% subsidy the ERR method cannot be used either, because the roots ERR of the $ENPV$ are negative (see Tab. 2).

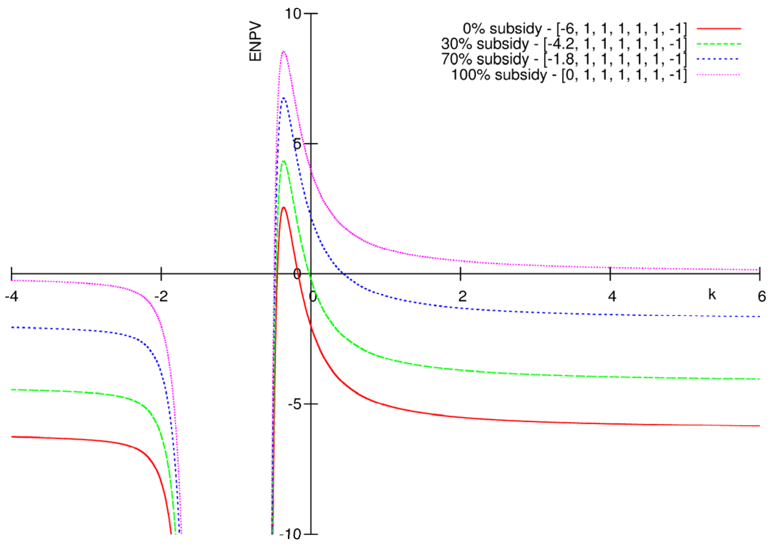
The course of the investment curve of a nonconventional project in the situation of a crisis in the operational phase (change of CF_3 – see Fig. 3) is not different in the right half of the curve from the course of the conventional project (Fig. 1). According to Descartes' rule of signs this project has three or one real-valued root greater than –1. The course of the $ENPV$ function is quite different in case of a crisis in the liquidation project phase (Fig. 4), when the curves "turn over" in $k = -1$ to $-\infty$ on the vertical asymptote. According to Descartes' rule this nonconventional project has two or no root greater than –1.

Fig. 3: The course of the investment curve of the project ENPV=f(k) according to the subsidy amount (nonconventional project – crisis in the operational phase)



Source: authors

Fig. 4: The course of the investment curve of the project ENPV=f(k) according to the subsidy amount (nonconventional project – crisis in the liquidation phase)



Source: authors

Tab. 2: Roots ERR of investment curves at various subsidy amounts (non-conventional projects)

subsidy	parameter CF_0	ERR (crisis in the operational phase) $CF = [CF_0, 1, 1, -1, 1, 1, 1]$		ERR (crisis in the liquidation phase) $CF = [CF_0, 1, 1, 1, 1, 1, -1]$	
0%	–6	–1.784	–0.098	–1.784	–0.098
30%	–4.2	–1.848	–0.013	–1.848	–0.013
70%	–1.8	–2.2024	0.282	–2.2024	0.282
100%	0	–2.792	∞	–2.792	∞

Source: authors

4. Results

Now we will discuss the results of using the proposed 2D model by means of a case study depicting a crisis in the liquidation phase of a public investment project in the conditions of predicted cash-flows (instead of the unit cash-flow in the model). Here, the proposed model testing will be carried out by means of a real-life public project, (see Tab. 3) financed from the European funds from the Integrated Operational Programme from the period of 2007–13.

Tab. 3: E-Government project cash flows

CF item	**2010**	**2011**	**2012**	**2013**	**2014**
Investment costs	–576,150	–57,273,850	–300,000	0	0
Wage costs	0	0	0	–400,000	–400,000
Implementation	0	0	0	–830,000	–830,000
Technical support	0	0	0	–2,600,000	–2,600,000
Maintenance	0	0	0	–800,000	–800,000
Time saving – region	0	0	0	27,871,200	27,871,200
Time saving – state	0	0	0	340,000	340,000
Time saving – clients	0	0	0	12,000,000	12,000,000
Project cash flow	**–576,150**	**–57,273,850**	**–300,000**	**35,581,200**	**35,581,200**

CF item	**2015**	**2016**	**2017**	**2018**	**2019**
Investment costs	0	0	0	0	0
Wage costs	–400,000	–400,000	–400,000	–400,000	–400,000
Implementation	–830,000	–830,000	–830,000	–830,000	–830000
Technical support	–2,600,000	–2,600,000	–2,600,000	–2,600,000	–2,600,000
Maintenance	–800,000	–800,000	–800,000	–20,800,000	–6,800,000
Time saving – region	27,871,200	27,871,200	27,871,200	27,871,200	27,871,200
Time saving – state	340,000	340,000	340,000	340,000	340,000
Time saving – clients	12,000,000	12,000,000	12,000,000	12,000,000	12,000,000
Project cash flow	**35,581,200**	**35,581,200**	**35,581,200**	**15,581,200**	**29,581,200**

CF item	**2020**	**2021**	**2022**
Investment costs	0	0	0
Wage costs	–400,000	–400,000	–400,000
Implementation	–830,000	–830,000	–830,000
Technical support	–2,600,000	–2,600,000	–2,600,000
Maintenance	–6,800,000	–6,800,000	–6,800,000
Time saving – region	27,871,200	27,871,200	27,871,200
Time saving – state	340,000	340,000	340,000
Time saving – clients	12,000,000	12,000,000	12,000,000
Project cash flow	**29,581,200**	**29,581,200**	**29,581,200**

Source: (Olomoucký kraj, 2011)

It is a thirteen-year-long project with the aim to create a technology centre of the region in the system of electronic document and records management system, which should be part of e-Government centres within the CR public administration digitalization. The subsidy of the conventionally proposed project will be 85% of eligible budget costs (Olomoucký kraj, 2011).

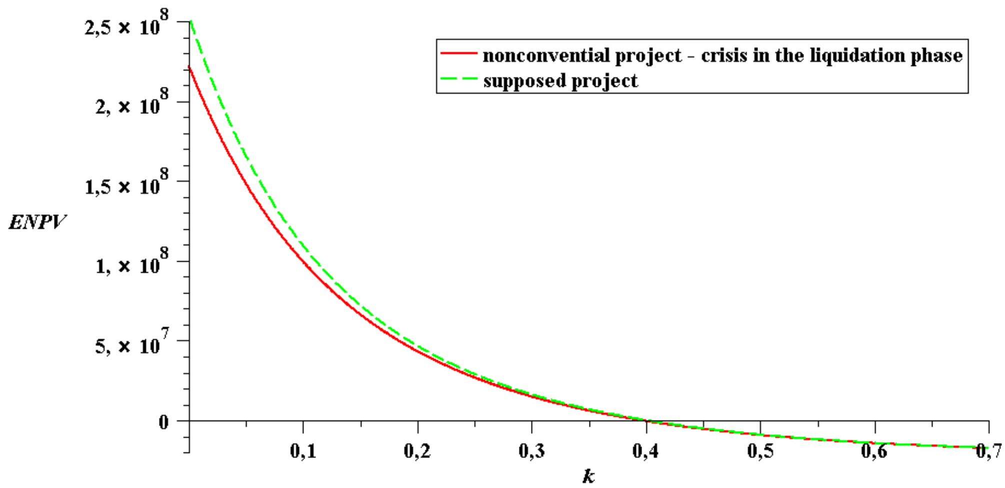
Based on the proposed model, we will now carry out the project socio-economic efficiency evaluation within the scope of one of the model scenarios – the crisis scenario in the liquidation phase due to increased maintenance costs and positive externalities decrease (time saving for project stakeholders) related to the transition of part of the users to newer technologies (see Tab. 4). The year of 2022, i.e. the last year of the project where a decrease of the above-mentioned values is highly probable after twelve years of the project lifetime, was selected as the critical year.

Tab. 4: E-Government project cash flows, crisis in liquidation phase (in CZK)

CF item	**2020**	**2021**	**2022**
Investment costs	0	0	0
Wage costs	400,000	400,000	400,000
Implementation	830,000	830,000	830,000
Technical support	2,600,000	2,600,000	2,600,000
Maintenance	6,800,000	6,800,000	**16,800,000**
Time saving – region	27,871,200	27,871,200	**3,000,000**
Time saving – state	340,000	340,000	**0**
Time saving – clients	12,000,000	12,000,000	**0**
Project cash flow	29,581,200	29,581,200	**–17,630,000**

Source: (authors according to Olomoucký kraj, 2011)

Fig. 5: The course of the ENPV function of the e-Government project



Source: authors

We can see that despite the impact of the crisis situation in the liquidation phase on project efficiency social evaluation, the project is stable even in this scenario (see Tab. 5), which is also reflected by the smooth course of the plane in the following Fig. 10.

As shown in Tab. 5, after modelling a crisis in the liquidation phase of the project there was no significant decrease of the project's internal rate of return. It testifies little sensitivity of the project to the changes of increasing maintenance costs and positive externalities resulting from time savings. The reason may be overstated cash-flow values, especially those from positive externalities in previous years. By further detailed examination of this phenomenon by the sensitivity analysis method in the next stage of modelling, we could find out the sensitivity of the economic internal rate of return of the project to cash-flow changes in individual years and determine the so-called cash breakeven points of the project.

Tab. 5: Results of testing the case study of the e-Government project; evaluation of social efficiency of a nonconventional model (crisis in liquidation phase)

Project social efficiency evaluation	Project proposal	Crisis in liquidation phase
ERR	0.401	0.393

Source: authors

5. Conclusion

Efficiency evaluation of a public project of investment type is carried out on the grounds of cost-benefit analysis which includes, beside evaluating commercial utilities of the project on financial basis (corresponding to commercial projects), also evaluation of social utilities according to theoretical principles of welfare economics. As stated in the article, basic criteria methods for evaluating public projects exploit the same mathematical principles and are based on the same mathematical foundations as commercial methods, i.e. on the grounds of net present value of the project and internal rate of return of the project.

In the area of public investment, the NPV method (according to the EU methodology the ENPV) is considered the preferential method as it reflects the basic principle of welfare economics regarding maximizing the social utility from their implementation, similarly as maximizing the market value of a company is the goal of commercial projects.

Just like in case of commercial projects, the IRR method (according to the EU methodology the ERR) is only the second most preferred method of public investment evaluation; the reason is well-known disadvantages of this method in case of nonconventional projects, mentioned in this article. Yet, some theoretical and practical concepts prefer, as stated in the article, the IRR (ERR) method.

The paper solves nonconventional public projects when negative cash-flows arise due to crisis situations in operational and liquidation phase of a public project. The paper also presents a comparison with public projects of conventional type as usually constructed by applicants for subsidy.

The model is applicable also for commercial projects due to identical mathematical basis of the ERR (IRR) methods. The model application based of cost-benefit analysis principles proves suitable especially for evaluating those commercial projects that cause negative externalities in the form of environmental damage by their implementation.

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