

METAECONOMICS¹

AN APPROACH TO ANALYSIS AND COMPARISON OF PARALLEL ECONOMIC

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Metaekonómia: Prístup k analýze a porovnávaniu paralelných ekonómií

Abstract: *There are two parallel Economics in economic science as a whole. The first one is seeking to introduce oneself as Economics (Economics I), directly questioning of the economy in objective reality. The other one openly declared oneself as Economics (Economics II), investigating the object that is Economics I as one but virtual too, mastered by subjects upon the economy in objective reality. There is, however, a third approach too, which is seeking to analyse all these three/four objects integrative as a complex object (vicarious method of study). This analysing approach mentioned before is Metaeconomics. In this way, the double-unique, mutually intertwined objects of Metaeconomics may be the Economy as existing in objective reality and together with the Economics as the subjective product created above the former. The Economy in objective reality³ is an organismic modus vivendi of human society. Unfortunately, that entity is unobservable in contrast to social insects as e.g. honey bees, termites, ants, paper wasps, and so on. The common feature of human and insects societies is a similar modus for creating containers for materials, energies, information (knowledge), skills, spaces, etc. not only in bodies structure (network) of society members but communal too, that is, extracorporeal ones. For these double-unique scientific purposes and/or subjects, the Metaeconomics creates creating ontological, epistemological and methodological foundations. In this paper,*

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³ Basically meaning, the *Objective Reality* is a reality that completely exists independent of any conscious entity to observe it, and the *Subjective Reality* is what we construct above Objective Reality, also the reconstruction on the base of what we observe, perceive, read, etc. The *Economy* is the Objective Reality and an *Economics* is the Subjective One.

the author seeks to use nonconventional approaches in Economics, namely from the aspect of ontological ideas about qualitative aspects of objective economic reality and applies advanced methods based on mathematics and mainly on computational intelligence assisted virtual experimentations. To avoid any misunderstanding, the author emphasises that the paper is not based on economic data from objective economic reality. The majority of theories, models and concepts mentioned in the paper are well-known among economists; several of them are part of textbooks, so the author decided to omit direct citations.

Keywords: *Agent-based computational economics, Agent-oriented programming, Artificial Neural Network, Behavioural uncertainty, Complex adaptive systems, Computational experimentations, Economies as Artificial (Computational) Worlds, Economics, Economy, Learning, Metaeconomics, Emergent hierarchical organizations, Endogenous interactions, Decentralized market processes, Financial Modelling, Institutions, Mainstream Economics, Ontological and methodical mistakes, Parallel Economics and Economies, Political Economy, Strategic rivalry, Virtual laboratories.*

JEL Classification: B 4, C 6, C 53, C 62, C 69, C 7, C 90, D 4, D 5, D 58, D 6, D 8, G 12, L 1.

1 Introduction

There is no unique exact notion what either economic science or Economics is. Another difficult open question is: “*What is really Economy in objective reality?*” In the literature, one can find several more or less close-fitting explications of these matters. As we know, the more often used explication owes to Lionel Robbins in his famous book *An Essay on the Nature and Significance of Economic Science*[11]: “*Economics is the science which studies human behaviour as a relationship between given ends and scarce means which have alternative uses.*”... Several years before him, Alfred Marshall wrote in his *Principle of Economics* (Chapter One) [7], quoting ad largum, that *Political Economy or Economics is a study of mankind in the ordinary business of life; it examines that part of individual and social action that is most closely connected with the attainment and with the use of the material requisites of well-being.* On the other hand, it is known that the Economics and/or the Political Economy have from their birth-hour to our time several ontological,

epistemological and methodological misunderstandings about objective economic reality, which causes a lot of theoretical and practical sufferings not only for creators but for users too. Those sufferings are concern specially the so called Mainstream Economics. Such criticisms bear elements of faults due to positive and/or normative function of Economics. We are convinced that the suggested Meta-economics approach allows deeper ploughing into ontological, epistemological and methodological misunderstanding connected with ideas treated by mathematical formalisms.

The double-unique object of a Metaeconomics is the Economy as existing in the objective reality and together with the Economics as the subjective product created above the former entity by economists, for everybody their own unique product, i.e. Economics as canonised against economics that exists in the consciousness of each entity. The Economy in the objective reality is an organismic base and modus vivendi of human society. Metaphorically speaking, this complex social organism is a self-organizing network of spatially distributed and qualitatively differentiated containers assuring survival of mankind which is nested into holistic social organism perpetually created by them in the planet Gaia. Unfortunately, that entity is unobservable by contrast to social insects like honey bees, termites, ants, paper wasps, and so on. The common feature of human and insects societies is similar modus for the creation of containers for materials, energies, information (knowledge), skills, spaces, etc. not only in bodies of society members but communal too, that is, extracorporeal ones. For these double-unique scientific purposes and/or subjects, the Metaeconomics creates ontological, epistemological and methodological foundations. In this essay, the author seeks to use the approaches nonconventional in the Economics, namely from the aspect of ontological imaginings of objective economic reality and applies advanced methods based on mathematics and mainly on computational intelligence assisted virtual experimentations. In other words, it is very important what primordial metaphorical image is behind the economist's mental model, if any, as the first step of grasping the chosen object separating his/her subject from reality. Elaborating the singular mental model constructed upon economist metaphor by appropriate mathematics is the second and more important step to blowing off from *very genuine economic reality*. This kind of elaborating is the approach and methods, too, which we denote with the term *creative modus from up to down*. In our opinion, *Dynamic systems theory* in general, (among other mathematical theories) is very interesting for studying formalized economic mental models constructed via up-to-down

mode as an idea in Economics to solve a class of complex untouchable tasks. In that context using *mathematical topological methods* for deeper analysis of interesting trouble is very important. Such approaches allowed broader community of economists better understanding of experimental simulation in virtual laboratories assisted by *ICT* and *CI* (Computational Intelligence).

The paper continues as follows. In the paragraph, two we are analyses of object and subject matter of the possible various products of general economic science.

2 The Object and Subject Matter of Economics

There are several definition of objects and subject matters of Economics known from history of economic science, because various economists have different views about these subjects. The problem is with the object of Economics because not every economist clearly specifies of the object of his/her scientific interest. Very frequently there are in their mind in fact something as Economy in objective reality, but is staying in shadow or foggy stage, without clearly specification of their characteristic features. That is just it what is leading to several misunderstanding from the point of view of ontology, epistemology and/or methodical (toolbox) approaches. By Epistemology we understand the study or a theory of the nature and grounds of economic knowledge in dependency on ontological imagination and especially with reference to limits and validity of knowledge achieved with such a way. In the context with former by Ontology we are mean clear specifying the object matter of the directly and/or hidden used entity named Economy. Because of low consequentialness in specification object and subject matter of an Economics there arising several heavy misunderstanding not only among economists but among political parties and persons and in business too. We bring in lower item some very typical specification of the subject matter of Economics and/or a Political Economy, while deriving from famous scholars.

The very founder of the Political Economy A. Smith in his “Wealth of Nations” [12] defined Economics as an enquiry into the nature and causes of wealth of Nations. It lays importance on wealth rather than welfare of human beings. It points to man’s usage of wealth produces wealth and how wealth is exchanged and distributed in the economy. From several Smith’s ideas (namely from the labour theory of value) and methods also coming from

K. Marx [8], who defined Economics, exactly speaking *Political Economy (PE)*, starting from the relations between people and classes, and tried to understand the economy not as a perfect clockwork mechanism but as a *dynamic system full of contradictions* and pre-arranged to be replaced. In his view, *PE* is not about the relationship between commodities, prices, supply and demand: it is first and foremost about people and the socio-economic relationships between them – about the owners of wealth and how they use it to exploit others; about what is produced and how. That is he was working with concealed name of object that we name *complex social network*, in our time. In that sense, economics is both political and social and historical. Marx does agree with contemporary academics with their artificial divisions in the world, who tend to obscure how things are really interconnected. Among other things, he strictly differentiated between Labour (that is realised potency) and Labour Force (*LF* ... that is only potency)⁴. This means that the producer buying the potency can use it in production process with alternative results as different value against *LF*. It is according to the very respectful 19th century economist A. Marshall “Economics is the study of mankind in the ordinary business of life. It examines that part of individual and social action, which is most closely connected with the attainment and with the use of material requisites of well-being. ... It is on the one side a study of wealth and on the other and more important side is a part of the study of man”. [7, Chapter 1, Introduction]. He has shifted the emphasis from wealth to man, that is, he gave priority to human beings and places wealth at secondary level. He accentuated from the ontological and methodical points of view that the Mecca of economics is Biology. In this circumstance, we must also to emphasize that Marshall preferred economics dynamic rather than the static one. He used the tools of classical mechanics, including the concept of optimization. With these tools he, like neoclassical economists who have followed in his footsteps, took as given technology, market institutions, and people’s preferences. But Marshall was not satisfied with his approach. Yet, he once really wrote that “*the Mecca of the economist lies in economic biology rather than in economic dynamics*”. In other words, Marshall was arguing that the economy was an *evolutionary process* in which technology, market institutions, and people’s preferences evolve along with people’s behaviour. Other famous economist Lionel Robbins [11] defined economics in his landmark essay on the nature of economics, as “the science which studies human behaviour as a relationship

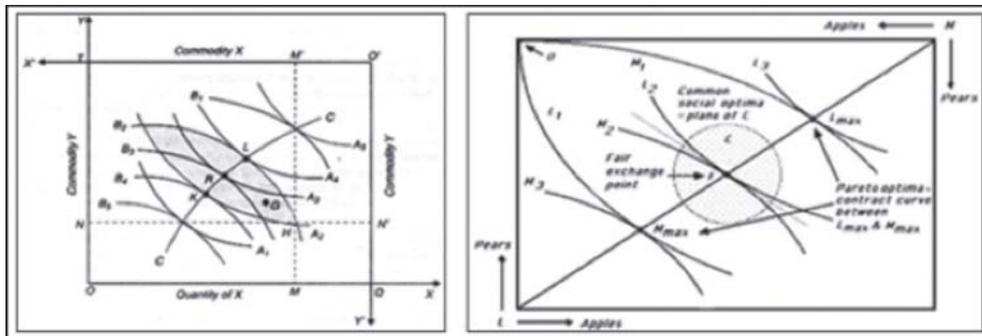
⁴ In fact it is not the *Labour* that is a commodity being bought in the wage transaction: more accurately it was the ability to work: what he called *Labour Power*.

between ends and scarce means which have alternative uses". [10, p. 16] J. M. Keynes, who realised great macroeconomic revolution in Economics, completely resigns on dynamics and long-run analysis by his famous claim that "*In the long run we are all dead*". Keynes said that to distinguish his views from the *laissez faire* views that were and still are popular. He was implying that we ought to do now whatever we can to help (so called normative function of Economics). He believes that a well-functioning and flourishing economy may be created with a combination of private sector and government help. By government help, Keynes meant an active monetary and fiscal policy, which works to control the money supply, and adjusts Federal Reserve interest rates in accordance with changing economic conditions. By government help, Keynes meant an active monetary and fiscal policy, which works to control the money supply, and adjust Federal Reserve interest rates in accordance with changing economic conditions. By contrast to him, the *free-market economists*, alternately called *laissez-faire economists*, advocate a government "hands off" policy, rejecting the theory that government intervention in the economy is beneficial (that is, they fulfil positive function of Economics).

Let us point out complaisance on some mistakes arising from inconsequent specification of object and subject matter of Economics. In the 19th century, several economists were enchanted by accomplishments of mathematics in Natural, Chemical and Technical Sciences. In such a way, they intended to add exactness to Economics, too. Their expectations however strike against the fact that Economy in objective reality is not as simple as for example mechanical systems. Because they forbore from clearly defining the object matter of mathematized mental models such approach was seemingly correct. Unfortunately, Economy as we claimed earlier is not a simple system but a complex evolving social organism and using mathematical formulas for partial economic phenomena is not identical with Economy in its wholeness. It is known that such mathematized mental models were constructed by using the up-to-down approach, whereas contemporary ICT and Computational Intelligence (CI) allow us to let evolving computational economies without direct participation of human subjects. We can bring a few convincing examples of such primordial formalized mental models beginning from *Duopoly* of A. A. Cournot; several mathematical formulas of W. S. Jevons, F. Y. Edgeworth's *Box Diagram*, see Fig. 1 for visualisation of two-sided exchange, up to M. Ezekiel *Cobweb Theorem* [5] and so on. A few such but a little more sophisticated models we will analyse by means of CI assistance in a subsequent paragraph.

Figure 1

Region of Mutual Advantage and Gain from trade of two Commodities by F. Y. Edgeworth



3 Analysis of Simple Mental Models Assisted by Computational Intelligence (CI)

3.1 Using routines of iDMC for analysis mental models created from up to down

As a CI device for analysing earlier quoted models, we are using very suited for those purposes routines of iDMC⁵ [15]:

1 Trajectory

This set of routines form an efficient and flexible way of viewing trajectories and orbits.

1.1. Plot

There are options for viewing the trajectory.

a. State space

This is the default choice for systems of more than one variable. The user must simply provide values and then click on Start. Other options available: *zoom*, *Big dots* and *Connect dots* (click redraw).

b. Time plot

The plot represents the time evolution of the variable chosen for the Range axis.

c. Variation Multiple trajectories

This can be displayed in a single plot by using this option. The user can increase or decrease the value of a parameter or initial condition by specifying the amount to change at each variation in the second column of input fields that appears when the Variation routine is clicked on.

⁵ Device iDMC the interactive Dynamical Model Calculator simulates and performs graphical and numerical analysis of systems of differential and difference equations. The authors of software iDMC are Marji Lines (Department of Statistics, University of Udine, Italy) and Alfredo Medio, (Universita Ca'Foscari, Venezia, Italy).

2 Shifted and cobweb

These two routines are available only for one-dimensional maps, that is, a single difference equation.

a. Shifted

The default is the Shifted plot, which permits to plot the values of the variable, shifted forward k periods, on the ordinate axis against the current values on the abscissa.

b. Cobweb animation

This routine draws the k^{th} iterate of the map in the $(x_n; x_{n+k})$ plane as in the *Shifted plot*. The user supplies the Initial value and should avoid critical values for the map; also chooses to view transient behaviour by assigning 0 to transients or chooses to view asymptotic behaviour by assigning transients a high value. The motion is slowed down and the speed can be adjusting by dragging on the arrow above the plot.

3 Bifurcation

This routine represents the limit sets of a dynamical system as one or two parameters are varied. In the current version the algorithm is available for maps and a rudimentary version is available for 3-dimensional differential systems. Differences in applying the routine to systems of differential equations are also present. Fixed initial point option should be ignored.

a. Single parameter

b. Double parameter

c. Bifurcation for differential systems in 3-d

4 Cycles

This algorithm calculates all k -cycles of a map for a given period and all values of the k periodic points. The algorithm uses an iterative Newton method to solve the system $G^k(x) = (x)$. Once a periodic point is found the others are determined by iteration.

5 Basin of attraction

This routine allows representing the basins of attraction for attracting limit sets over a user-specified region of the state space. This routine is available only for maps, that is, systems of difference equations with set of different starting points.

a. Basin of infinity

b. All basins

c. Algorithm attractor

6 Lyapunov exponents

a. Parameter

This is the default routine. The user selects which parameter vary and gives the min and max values to be considered. The Vertical range refers to the values of the Lyapunov exponents, which typically have fairly large negative exponents and small positive exponents. However most interest centres on zero and positive exponents for which very small ranges can be used. The Lyapunov exponent is a time-averaged value and sufficient iterations should be specified.

b. Time

c. Parameter space

7 Manifold

The manifold routines are available only for maps, that is, systems of difference equations, for which the manifold of interest is a curve in the plane. In the current version, only manifolds of fixed points are calculated and in some cases, the repositioning of the points lying in the lockout region seems to be malfunctioning.

a. Unstable

b. Right Left Both

c. Node approximation

d. Algorithm input fields

e. Lockout region ranges

8 Absorbing area

This routine allows the user to plot the absorbing area of an iterated map using the method of *Critical Lines* (*LC ... Linea Critique*). There are a number of steps, which should be followed in the order given below. The input fields will be explained in the step descriptions. The Jacobian matrix must be provided in the model file and currently the maximum is 2 dimensions.

a. Plot critical set

b. Plot attractor

c. Choose segments

d. Clear

e. Iterate chosen segments.

Based on upper description it is understandably the iDMC is promising useful assistant for economists (scholars and students too) for deeper analysing of complex economic phenomena. The economists who is deeper interesting on methods how to use iDMC for economic investigations can study those routines from freely accessible in Internet "*iDMC: interactive Dynamical Model Calculator; user's guide*"[6].

3.2 Financial market model analysis

Because of shortage of space, we are choosing the *financial market model with imitators*⁶ as one of a little more sophisticated mental model constructed via *up to down*, which is very suited for our purpose. In their model A. Agliari and I. Foroni with fundamentalists and imitators are being considered. By using several results published in their paper, we can study and demonstrate some *global bifurcations* arising in a heterogeneous financial model with fundamentalists and imitators. Such bifurcations which cause the appearance and disappearance of *closed invariant curves* (CIC) – attracting or repelling – involve the stable and unstable sets of a *saddle cycle* with consequent changes in their dynamic behaviour. Numerical investigations with their model realised in virtual laboratory made in iDMC⁷ can among other show that the transition between two qualitatively different regimes is characterized by the occurrence of homoclinic tangles with chaotic dynamics. In order to describe the price dynamics of the heterogeneous stock market, the authors used a synergetic approach and on these bases we can study some global bifurcations arising in their model by using routines of iDMC, too. It is also shown that the fundamental equilibrium point P^* may be destabilized through a subcritical Neimark – Sacker bifurcation and that two invariant closed curves, one attracting and one repelling, appear when P^* is still stable. This particular bifurcation scenario allows us to show some noticeable features of the market that emerge when the imitation effect is emphasized. Among these features are, for instance, the volatility clusters associated with the presence of multistability (i.e. coexistence of attractors) and the hysteresis phenomenon. The moderate progress in comparison to primordial models is the using of population approach in the difference to single or few agents. The model describes a single asset stock market with two colour population of agents where: a) n is the number of traders; b) x_t^i ($i = 1, \dots, n$), is the *investment attitude* of agent i s.t.; c) first subpopulation $x_t^i = 1$, if agent i wants to buy an asset's unit at t (as *optimistic investor*); d) second subpopulation $x_t^i = -1$, if agent i wants to sell an asset's unit at t (called *pessimistic investor*).

The investors' average opinion is, then, $x_t = \sum_{i=1}^n \frac{x_t^i}{n} x_t \in [-1, 1]$. Let

⁶ The author of that model is A. Agliari, with I. Foroni (see [1, pp. 30-45], [2]).

⁷ Of course, a straightforward computation allows us to verify some of characters that mental model formalized as map T too. For example, the unique equilibrium point of the map T is $P^* = (x^*, y^*) = (0, 0)$. That was verified by T. Asada, et al, see [3], but for map T in model of cyclical growth. Also I. Foroni with R. Grassi studied in general approaches, the local stability of P^* , see [6].

n_b and n_s be the totality of buyers and sellers with $n_b + n_s = n$. Thus, we get $x_t = \sum_{i=1}^n \frac{x_t^i}{n} = \frac{n_b + n_s}{n}$. Transition rates in a unit time period, is the variations: a) in the buyers' group are $n_s g_{bs} - n_b g_{sb}$; b) in the sellers' group are $n_b g_{sb} - n_s g_{bs}$. Transition rate from buyers to sellers let be $g_{bs}(k - p_t, x_t) = \nu \left(\mu + e^{-(\beta(k-p_t)+\gamma x_t)} \right)$ and transition rate from sellers to buyers similarly $g_{sb}(k - p_t, x_t) = \nu \left(\mu + e^{\beta(k-p_t)+\gamma x_t} \right)$. In those formulas β is the strength of the fundamentalist reaction (difference between the market price and the fundamental value k); γ is the imitation effect; ν is the speed of opinion change; and μ measures the remaining factors affecting the asset price in traders opinion. The model of evolution over time of the average investment attitude can be expressed as $x_{t+1} - x_t = \frac{(n_s g_{bs} - n_b g_{sb}) - (n_b g_{sb} - n_s g_{bs})}{n}$. The

$$= (1 - x_t) g_{bs} - (1 + x_t) g_{sb}$$

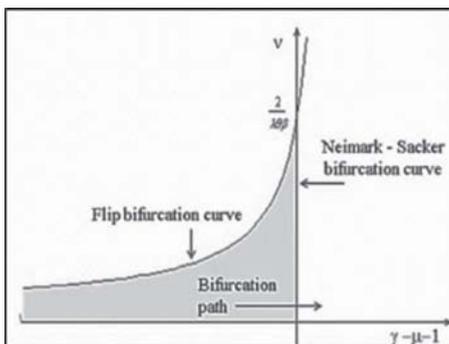
adjustment price mechanism is then $p(t+1) = p(t) + \lambda \arctan(\vartheta x_{t+1})$, where $\lambda > 0$ is the speed of adjustment and $\vartheta > 0$ is the volume of trading per agent. The dynamics of both the average investment attitude x_t and the asset price p_t are described by the following two dimensional nonlinear map

$$T : \begin{cases} x_{t+1} = (1 - 2\nu\mu) x_t + \nu \left(e^{\gamma x_t + \beta(k-p_t)} (1 - x_t) - e^{-(\gamma x_t + \beta(k-p_t))} (1 + x_t) \right) \\ p_{t+1} = p_t + \lambda \arctan(\vartheta x_{t+1}) \end{cases}$$

By means of an axis translation we can express the price p_t as the deviation from the fundamental value k , obtaining alternatively to former map

Figure 2

Riemann foliation (Bifurcation diagram - fundamental equilibrium point in 2D plane)



$$T : \begin{cases} x' = (1 - 2\nu\mu)x + \nu \left(e^{\gamma x + \beta(k-p)} (1 - x_t) - e^{-(\gamma x + \beta(k-p))} (1 + x) \right) \\ p' = p + \lambda \arctan(\vartheta x') \end{cases}$$

The author of that model *I. Foroni & A. Agliari* get for the situation of *Local Stability Region* very instructive schematic picture in region (variable plane of x', p') ($\gamma - \mu - 1, \nu$) with *Fixed point* $P^* = (0, 0)$ and quotation that the *Neimark-Sacker bifurcation*⁸ may be either of *super-* or *subcritical type*. She gets picture of Riemann foliation for *Noninvertible Map* with Critical

Curves *LC* too (Fig. 2) $LC_{-1} = LC_{-1a} \cup LC_{-1b} = \{(x, p) : \det J_T = 0\}$ and

$LC = LC_a \cup LC_b = T(LC_{-1a}) \cup T(LC_{-1b})$. We are using the former map for constructing virtual laboratory in device *iDMC*. For convenience with *iDMC* device we have to use latin letters for *parameter* symbols: ν, m, g, b, l , and o ; script of model will be

```
--@@@
name = "Foroni"
description = "See essay Andrášik"
type = "D"
parameters = {"v", "m", "g", "b", "l", "o"}
variables = {"x", "y"}
function f(v, m, g, b, l, o, x, y)
    x1 = (1 - 2*v*m)*x + v*(math.exp(g*x - b*y)*(1 - x) - math.exp(-g*x + b*y)*(1 + x))
    y1 = y + l*math.atan(o*((1 - 2*v*m)*x + v*(math.exp(g*x - b*y)*(1 - x) - math.exp(-g*x + b*y)*(1 + x))))
    return x1, y1
end
function Jf(v, m, g, b, l, o, x, y)
    return
    (1 - 2*v*m) + v*(-math.exp(g*x) - math.exp(-g*x)), v*(math.exp(-b*y) - math.exp(b*y)),
    l/(1 + o*((1 - 2*v*m) + v*(-math.exp(g*x) - math.exp(-g*x))))^2, 1 + l/
```

⁸ *Neimark-Sacker bifurcation*, (J.I. Neimark; R. J. Sacker) is interpreted in literature as the birth of a *closed invariant curve* from a fixed point in dynamical systems with discrete time (iterated maps with variety of starting points of trajectories in very dense matrix), when the fixed point changes stability via a pair of complex eigenvalues with unit modulus, that is the alternative of *Hopf bifurcation* in case of *ordinal differential equations (ODE)*.

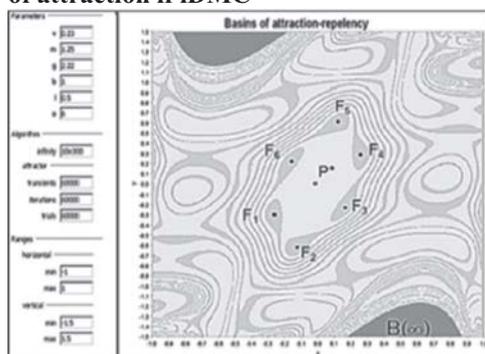
```
(1 + (o*(v*(math.exp(-b*y) - math.exp(b*y))))^2)
end
(The end of script.)
```

In Fig. 3a, Fig. 3b there is an appearance of the repelling closed curve involved in the *subcritical N-S bifurcation*. There exists also an attracting focus cycle of period 6 as well as a saddle cycle of the same period. We look at see six attractive focuses/foci on saddle-focus orbit and at one attractive focus at the central fix point.

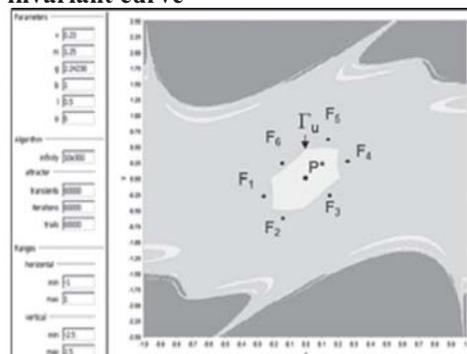
Figure 3a

Figure 3b

Virtual experiment in Basin of attraction if iDMC



Basins of attraction and closed invariant curve



The proof that there are really focuses we can see in Fig. 4. The arrows we used for better orientation of moving the state point drawing branches.

Figure 4

The saddles with repelling branches of state point running in jumps to focuses

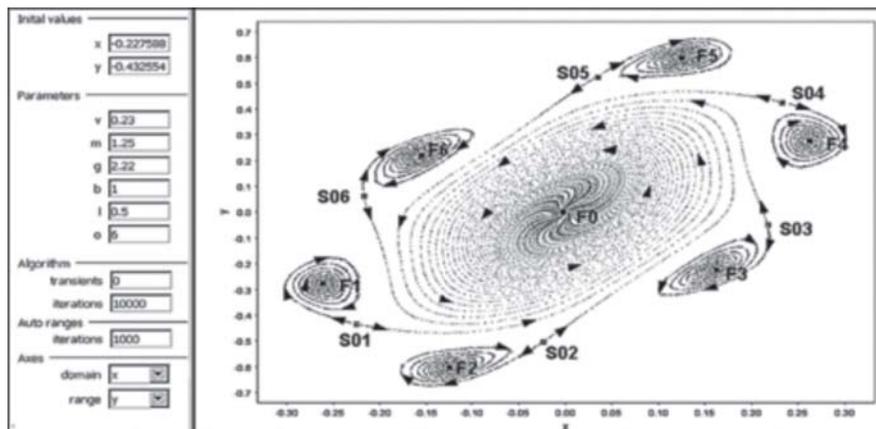


Figure 5

Two enlargement of left snapshot

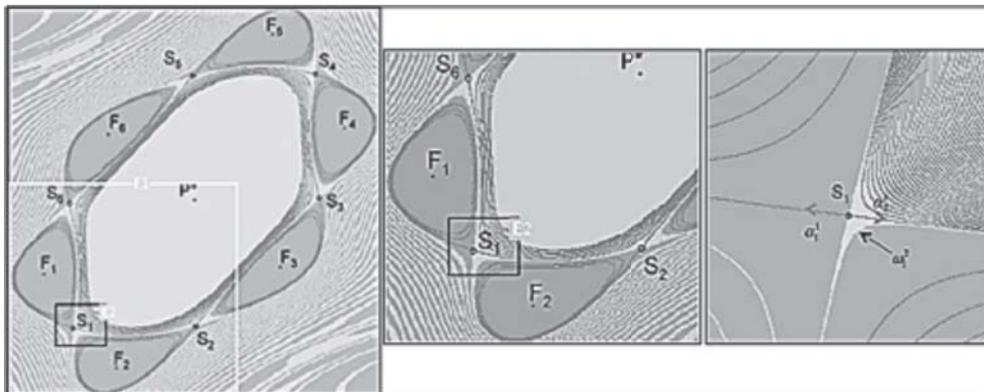


Figure 6

Figure 7

Experiment with attractive branches too (left) Situation after bifurcation to ($g = 2.2424$): Unstable set (right)

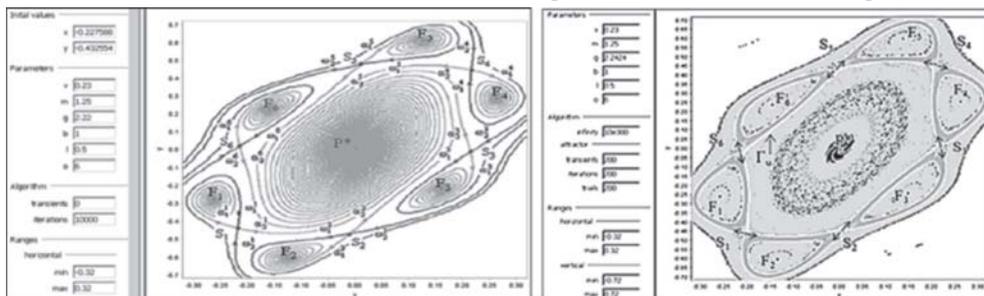


Figure 8

Figure 9

Situation after bifurcation to ($g = 2.2424$): Stable set Basin of attraction under of the manifold

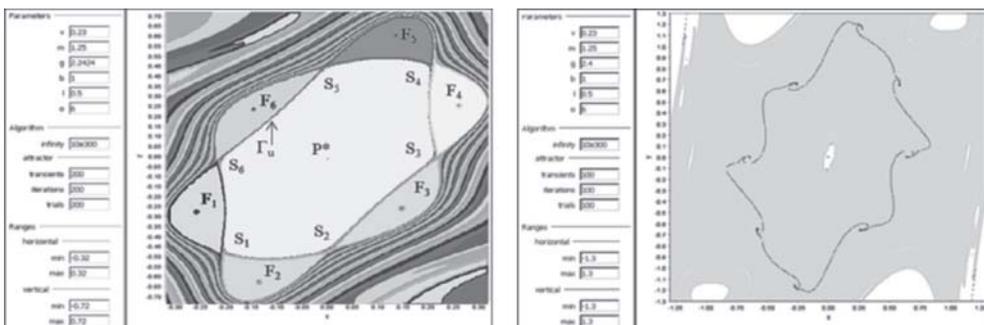
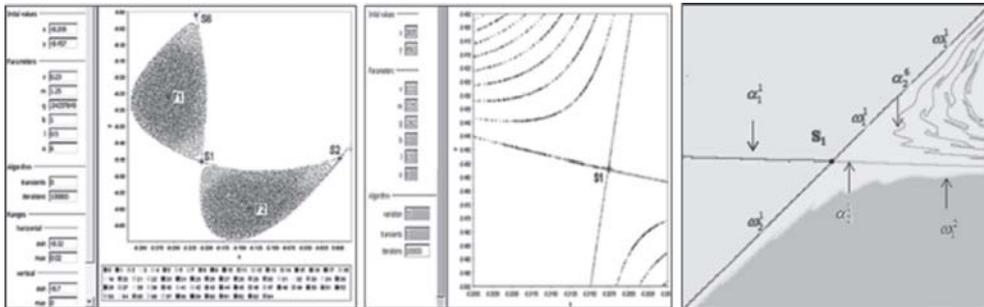


Figure 10

Situation around saddle S1



In right snapshot (visible part of manifold) the boundaries are well defined, but some oscillations are clearly visible too

Figure 11

Figure 12

Orbit with six saddles

Basins and branches

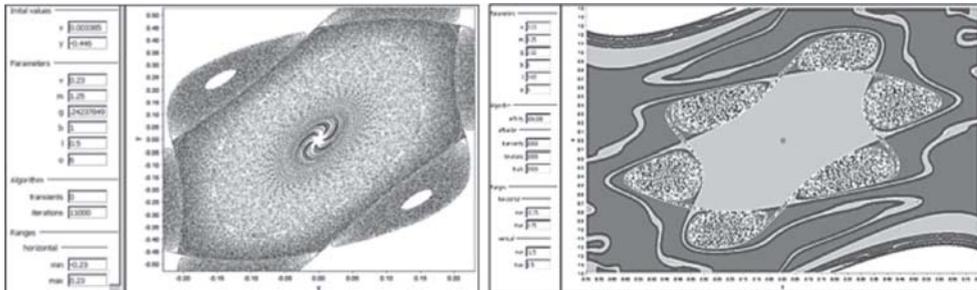
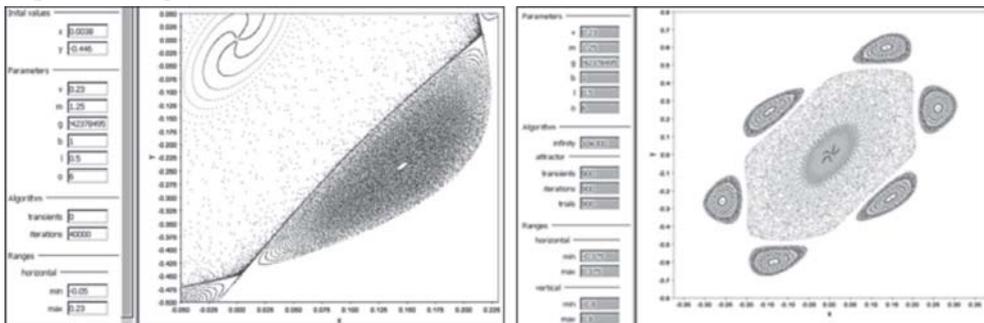


Figure 13

Figure 14

Bifurcation after microscopic change of parameter g

Euclidian spaces as a whole (right)



In order to explain the mechanism giving rise to the appearance of Γ_s , A. Agliari use of the qualitative sketch portrayed in Fig. 16. She consider as a starting point, left sketch where the attracting sets Γ^* and P^* are represented together with the saddle cycle S of period 6 and the dashed curve represents the

repelling closed curve Γ_u , which separates $B_{im}(P^*)$ from the basin $B(\Gamma^*)$. In the drawing it is shown that the trajectories on the branches α_i^1 of the unstable set $W^u(S)$ converge to Γ_i^0 whereas the trajectories on the branches α_i^2 converge to Γ_{i+1}^0 creating a “connection” among the components of the attractor Γ^* .

Figure 15

Compare of the whole and one of six secondary orbits

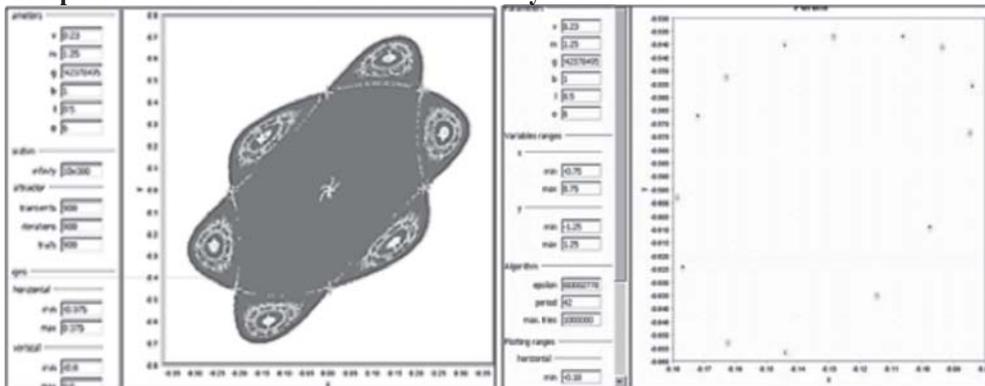
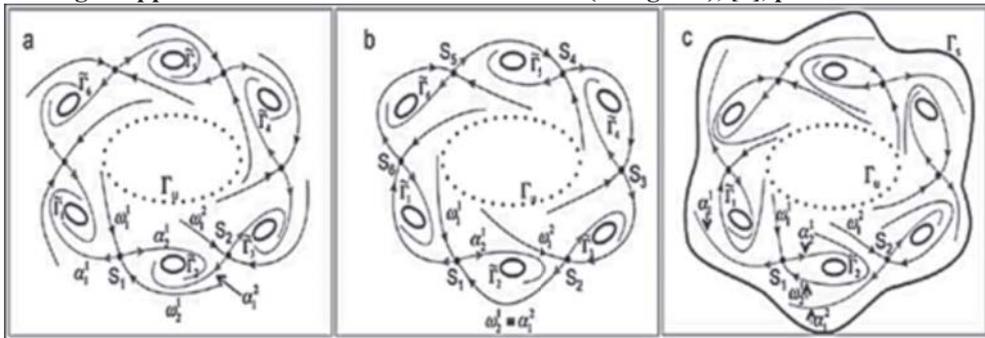


Figure 16

Very instructive for understanding qualitative illustration of the bifurcation mechanism leading to appearance of the stable closed curve Γ_s (A. Agliari), [2], p.1650



In central sketch of Fig. 16 the branch $\omega_1 = \bigcup_{i=1}^6 \omega_1^i$ of $W^s(S)$ issues from the unstable invariant curve Γ_u while the branch $\omega_2 = \bigcup_{i=1}^6 \omega_2^i$ comes from some repelling points probably located on the boundary of the basin $B(\infty)$. Approaching the bifurcation value, the branches α_2^i move towards the branches α_1^{i+1} until they merge creating a structurally unstable situation given by a homoclinic connection of the periodic points of the saddle cycle. In right sketch is final situation where more realistically the homoclinic connection has to be replaced by a homoclinic tangle, at the closure of which an attracting closed curve Γ_s surrounding the cyclical invariant curves appears. As a consequence of the occurred bifurcation, α_2^i and α_1^{i+1} modify their behaviour

since α_1^{i+1} is attracted by the curve Γ_s while α_2^i issue from the unstable closed curve Γ_u . The basins of attraction of the cyclical attractor $\tilde{\Gamma}$ is now bounded by the stable set of the saddle cycle, while Γ_u still bounds the immediate basin of attraction of P^* . The described situations it is possible very instructively realised by virtual experiments using laboratory constructed in iDMC and with such mode clearly visualize for better understanding That is the “reading” of running simulation experiments are more knowledge results generating then merely reading the obvious scientific economic texts.

Figure 17

A global bifurcation occurs when the parameter $g=2.243$; the bifurcation causes the appearance of the attracting curve Γ_s which surrounds $G \sim, P^*$ and Γ_u

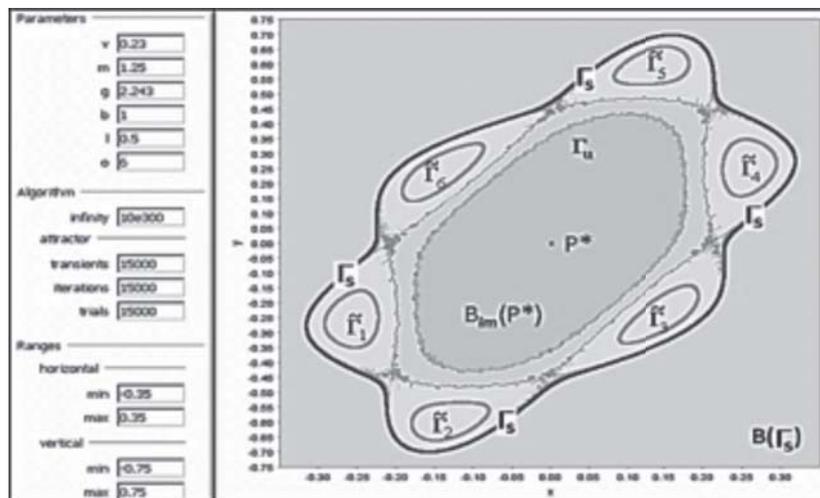


Figure 18

Left snapshot: Simple saddle S1 at $g = 2.2428$; Right snapshot: Complexity around the S1 unstable branches of $W_{(S)}$ join the six closed invariant curves belonging to the attractor $\tilde{\Gamma}$. At the same time the stable branch ω_1^1 issues from the boundary of $B_{(\infty)}$ whereas ω_2^1 comes from Γ_u [2, p.].

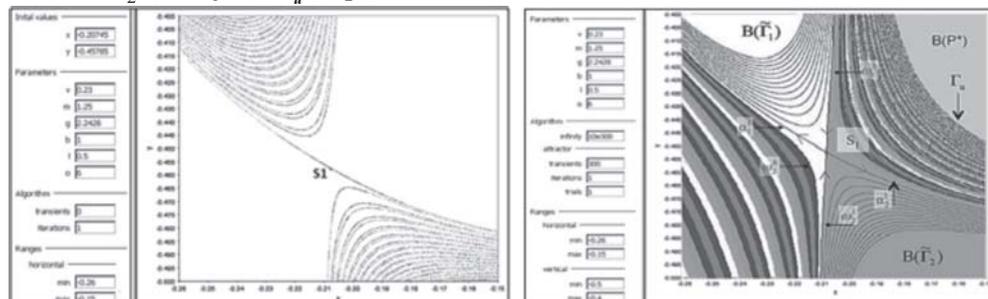


Figure 19
Bifurcation with value of $g = 2.222423785$ (left); the basins of the map T in a neighbourhood of $S1$ are depicted at three slightly different values of $g = 2.222423785$

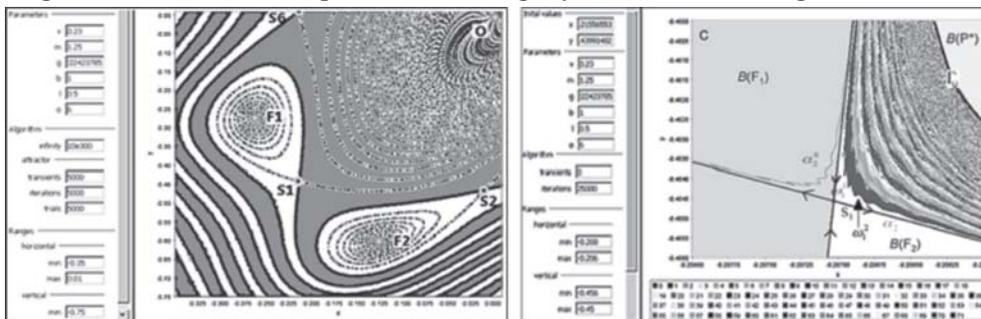
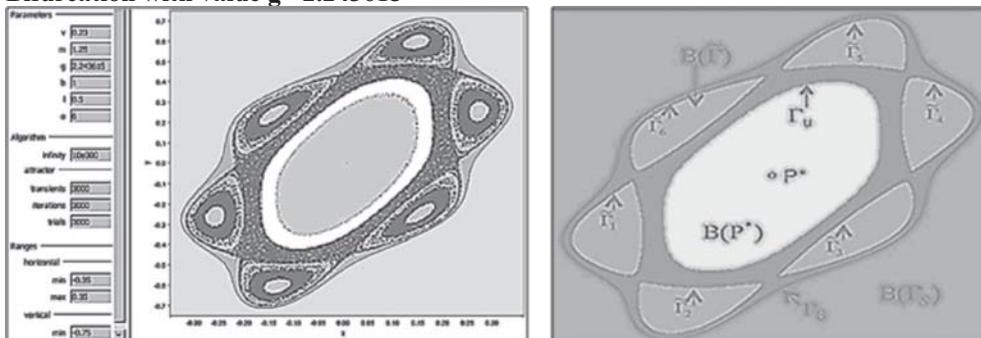


Figure 20

Bifurcation with value $g = 2.243615$



The snapshots of Fig. 20 was made by parameter $g = 2.243615$ the right picture shows by acronyms the existence of three attractors with their basins of attraction after the closure of the homoclinic tangle. However to situation is more complex but for clearer image we show these simpler ones. In situation of model depicted in Fig. 20 the attractor Γ^{\sim} continues to exist even after the closure of the homoclinic tangle. This statement is evidenced by snapshot at right where the attractors Γ^{\sim} , P^* and Γ_s with their basins of attraction are depicted at $g = 2.243615$, value of the parameter at which the basins of the three attractors are well separated meaning that the *chaotic repellor* has disappeared.

4 Conclusions

In this essay we are trying to put elements of Metaeconomics as more precise analysis of some mental models created predominantly by contemporary economists against analysis realised in history of economic thought, but because of limited space in this journal we have chosen only one example. The leading

purpose in this attempt is to demonstrate the useful possibilities of *ICT*, *AI* and namely *CI* in creating virtual laboratories for experimentation bringing results for deeper analysis of nominate mental models. In this way it is possible to create some virtual parallel to economy in objective reality. The first and simple step to such one is building virtual economy by mode from up to down. The second one but more difficult is to allow autonomously in appropriate software to evolve virtual economy from bottom to up. Because of introductory step in building Metaeconomics we remained on the first step, that is we chose for demonstration economic model made from up to down. For these purposes we used some demonstrations to closely clarify possibilities of iDMC routines based on financial model of Foroni&Agliari. These authors explain some bifurcation mechanisms involving the appearance or disappearance of invariant closed curves. For this purposes, they investigated the dynamic behaviour of the map T moving along a bifurcation path leading to a *subcritical N–S bifurcation* of the fundamental equilibrium. This is very purposeful for realization goals in our essay. Not all results of investigations upon this model in iDMC virtual laboratory may implemented to our analysis.

This investigation gave us the opportunity to enhance the results obtained in the existing literature on invariant closed curves. In particular, we verified that, as it has been introduced in [4], such mechanisms may be associated with the existence of a pair of cycles (a saddle and a focus or a cycle and a node) and that the appearance/disappearance of the invariant curves may be related to a homoclinic connection. The originality of our paper lies in the discovery, evidenced by numerically simulations, of a parameter range in which the homoclinic tangle replaces the homoclinic connection in the bifurcation mechanism, this hypothesis being only conjectured in [4]. Moreover, a mechanism associated with the appearance/disappearance of closed curves has been described, similar to the previous mentioned which, instead of an attracting cycle, involves a more complex attractor made up of cyclical invariant closed curves.

Based on our wider metaeconomic investigations described but only partially earlier we can formulate, conditionally, definitions of an economy. From one side, an economy is subjective and/or virtual entity constructed by economists for understanding partial economic phenomena as unitary whole. On the other side, an economy is living inner part of universal environment transforming by mankind for our perpetual survival. Because it is made by society of people it is social organism complexly evolving upon and in universal entropic environment as syntropic wave like Tsunami. In such understanding the genuine Economics is scientific seeking to explain and describe such complex evolving whole.

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