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GAS SWING OPTIONS: INTRODUCTION AND PRICING USING MONTE CARLO METHODS*

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Abstract

Motivated by the changing nature of the natural gas industry in the European Union, driven by the liberalisation process, we focus on the introduction and pricing of gas swing options. These options are embedded in typical gas sales agreements in the form of offtake flexibility concerning volume and time. The gas swing option is actually a set of several American puts on a spread between prices of two or more energy commodities. This fact, together with the fact that the energy markets are fundamentally different from traditional financial security markets, is important for our choice of valuation technique. Due to the specific features of the energy markets, the existing analytic approximations for spread option pricing are hardly applicable to our framework. That is why we employ Monte Carlo methods to model the spot price dynamics of the underlying commodities. The price of an arbitrarily chosen gas swing option is then computed in accordance with the concept of risk-neutral expectations. Finally, our result is compared with the real payoff from the option realised at the time of the option execution and the maximum ex-post payoff that the buyer could generate in case he knew the future, discounted to the original time of the option pricing.

Keywords: Energy markets, gas sales agreement, gas swing option, Monte Carlo simulations, spread option pricing

JEL Classification: C63, G12, G13

Introduction to Gas Sales Agreement and Gas Swing Options

A contract for the purchase and sale of natural gas providing an offtake flexibility concerning volume and time, generally called a "gas sales agreement" (GSA), has been commonplace in the natural gas industry for many years. For more information on gas sales agreements, see [e.g., Asche et al., 2002a; 2002b; Creti and Villeneuve, 2003; Neumann and Hirschhausen, 2005]. Most of today's European natural gas contracts are long-term contracts including a so-called "take-or-pay" clause. This provision obliges the buyer of gas to pay for the minimum contract quantity whether or not this is actually offtaken by him. However, the take-or-pay obligation very often refers only to a certain part of the annual contractual quantity. The remaining part, usually defined as a downward quantity

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tolerance, can, but does not have to, be offtaken. It constitutes an option, called a "swing option", for the buyer to offtake this part of the annual quantity of gas or not. And once there is a market where gas is liquidly traded spot (or day-ahead) and forward, the gas sales agreements need to be valued mark-to-market in order to avoid arbitrage opportunities. Even if these contracts are widely exercised, the techniques used for pricing them have not yet been fully developed.

Gas swing options can be defined as spread options on energy commodities. There are two main groups of models trying to find a fair value price of a spread option: numerical methods and analytic approximations. For the two-factor analytic models see [Kirk, 2005; Poitras, 1998; Carmona and Durrleman, 2003; Alexander and Venkatramanan, 2007]. The preferred numerical method due to calculation time is the Monte Carlo simulations (MCS). Boyle was among the first to propose applying the MCS to the option pricing problem [1977]. Since then, many researchers [e.g., Hull and White, 1987; Johnson and Shanno, 1987; Boyle et al., 1997] have engaged the MCS for analysing options markets. The markets of energy commodities are characterised by the limited ability of market players to arbitrage since the players miss a sufficient level of storage capacity or there is even no economical way of storing the energy commodity, therefore the calculation mechanism is different from traditional markets.

We employ numerical methods to derive the tool for the pricing of gas swing options. Specifically, we choose the Monte Carlo simulations to model the spot price dynamics of the underlying commodities. The price of an arbitrarily chosen gas swing option is then computed in accordance with the concept of risk-neutral expectations. Finally, our result is compared with the real payoff from the option realised at the time of the option execution and the maximum ex-post payoff the buyer could generate in case he knew the future, discounted to the original time of the option pricing.

The gas swing option is actually a set of several American put options on a spread between the prices of natural gas and other energy commodities, expiring on the last days of the delivery period defined by the gas sales agreement. See the following explanation.

1) Several put options: The Downward Quantity Tolerance provision[1] constitutes the buyer's right to not offtake a certain amount of the agreed quantity of gas, without incurring any sanctions. It can be seen as the buyer's right (the option) to sell (put option) a part of the annual contract quantity back to the seller for the contractual price [see Hull, 2006]. Furthermore, as the GSA defines the Maximum Daily Quantity,[2] the total amount of gas under the option cannot be exercised at once. The option amount is thus divided into several sub-options whose number N and volume (hence "option volume") depend on the daily minimum and maximum offtake constraints.

1 The Downward Quantity Tolerance (DQT) is the amount by which a buyer may fall short of its full Annual Contract Quantity (ACQ) in a Take-or-Pay gas sales contract without incurring sanctions. If there is no provision requiring the buyer to take supplementary volumes in subsequent years to make good for the deficiency, the ACQ becomes, in effect, the ACQ minus the DQT (Source: http://www.gasstrategies.com/industry-glossary).

2 The Maximum Daily Quantity (MDQ) is the rate at which the Seller's facilities must be capable of delivering gas, expressed as a volume of gas per day, or as a multiple of the Daily Contract Quantity. Also known as the Daily Delivery Rate (Source: http://www.gasstrategies.com/industry-glossary).

2) American options: As a result of the Take-or-Pay clause[3] defining the minimum offtake obligation under the GSA, the buyer can exercise his right not to offtake gas from the seller only to some extent. The number N of days when the buyer can exercise his right is thus limited and is based on the amount of the Downward Quantity Tolerance and daily minimum and maximum offtake constraints. The buyer has the right to exercise his options at any time from the first day until the last day of the contract period (year).
3) Spread options: Since the buyer can trade on the spot (or day-ahead)[4] and forward gas market, he compares the price under the gas sales agreement with the market price of gas to find the cheaper source for his purchase. The value of his savings realised by the cheaper purchase (compared to the purchase under the GSA) depends on the amount of a spread between these two prices. Since contractual prices under typical gas sales agreements have been linked to the price changes of competing fuels from natural gas, as described in [Asche et al., 2002a; 2002b], we actually investigate the option whose value is derived from a spread between prices of natural gas and its competing fuels. Such a characteristic is typical for so-called "spread options" falling into a group of "correlation options".

The payoff from a general put spread option at the time of Exercise T is then given as:

$$\left[q-\left(bS_{2T}-aS_{1T}\right)\right]^{+}, \tag{1}$$

where S_{1T} and S_{2T} are spot prices of two various assets present at the time T, a and b are constants and q is a strike price[Carmona and Durrleman, 2003]. Equation (1) represents GSA which is linked to only one competing fuel. In case the price formula includes more than one commodity, generally we speak about a multi-factor option and spot prices of other commodities need to be added.

Pricing of Spread Options using Monte Carlo Simulations

Because the payoff usually depends on the spread between the prices of two or more commodities and the pricing represents a multidimensional valuation problem, analytical models are hardly applicable, Monte Carlo methods are the best solution as they *tend to be used when it is infeasible or impossible to compute an exact result with a deterministic algorithm* [Hammersley and Handscomb, 1964].

Using these methods under risk-neutral expectations we try to find the fair value prices of the put spread options (hence also "option premiums"). The put spread option premium is considered as an expectation of discounted future cash flows for a probability

3 Take-or-Pay (ToP) is a common provision in gas contracts under which, if the Buyer's annual purchased volume is less than the ACQ minus any shortfall in the Seller's deliveries, minus any DQT, the Buyer pays for such a shortfall as if the gas had been received (Source: http://www.gasstrategies.com/industry-glossary).

4 Hence the terms "spot market" and "spot price" actually denote a day-ahead market and a day-ahead price.

structure called risk-neutral. The price of such an option is given by the risk-neutral expectation:

$$p = e^{-rT} E\left\{\left[q - \left(bS_{2T} - aS_{1T}\right)\right]^{+}\right\}, \tag{2}$$

where r is a risk-free interest rate. This finding is very useful in practice as it provides the natural generalisation to more complex problems [Carmona and Durrleman, 2003].

To get the expectation of the future option payoff $E\left\{\left[q - \left(bS_{2T} - aS_{1T}\right)\right]^{+}\right\}$ it is necessary to model the future spot prices of the underlying commodities S_{1T} and S_{2T}. The most widespread method is the application of one of the models focused on the spot price dynamics (*spot price models*) [see Sobczyk, 1991].

Such models can produce expectations which are not consistent with the actual forward prices shaped by the market. This is mainly the case of commodity markets where seasonality and mean reversion features are usually observed [see Gibson and Schwartz, 1990; Cortazar and Schwartz, 1994; Schwartz, 1997; Hilliard and Reis, 1998; Miltersen, 2003]. Miltersen tried to capture such features and synchronise spot price dynamics with the forward prices of commodities by adding other factors such as convenience yields and the cost of storage to the stochastic factors driving the models. Pilipovic [1997] was then the first to introduce the two-factor mean-reverting model. The inconsistency problem is an important finding since the market value of the option should be in line with the observed forward prices of the underlying commodities. In case the price of the option is derived from expectations other than an equal to the forward prices, the option owner is endowed by an arbitrage potential or it makes the option unmarketable. On the energy markets, it is likely that the inconsistency problem will arise because they are characterised by the limited ability of market players to arbitrage due to insufficient or even no storage capacities in the players' portfolios. Therefore it is necessary to calibrate spot price dynamics of commodities for which the no-arbitrage theory fails with the observed forward prices [see Carmona and Durrleman, 2003; Hikspoors and Jaimungal, 2007].

Another set of models is based on the stochastic time evolution of the entire forward curve and that is why these models are called *forward curve models*. These models have used ideas from the Heath–Jarrow–Morton theory (HJM) developed for fixed-income markets [Heath, Jarrow and Morton, 1992]. This theory represents a deviation from the models focused on spot price dynamics solving the inconsistency with the existing forward curves only secondarily. Heath's theory focusing on the forward curve dynamics has proved useful in the analysis of energy markets. An example could be [Alexander, 2001] who applied the theory to crude oil.

That is why we also focus on the forward prices of the underlying commodities, thinking of them as they are the market's expectations of the future spot prices. We assume that the forward prices are the best estimates of the future monthly averages of daily spot price quotations of the underlying commodities. However, a few adjustments need to be made to such deterministic processes.

- As the real values of the future monthly averages depend on a resolution of uncertain parameters, we must add randomness to the underlying price processes given by the forward curves.
- We also assume that the daily quotations can randomly deviate from the average monthly values.
- Since the underlying commodities are energy commodities showing seasonality and mean-reversion features, it seems to be appropriate to assume that price volatilities of these commodities differ across various calendar months.
- Next, it is reasonable to assume that both spot and forward prices of natural gas and its competing fuels entering into the price formula under the GSA are positively correlated.
- Finally, since we are interested in modelling commodities prices that cannot take on negative values, specific distributions or truncated distributions should be applied. We assume the log-normal distribution, the most commonly used in finance.

Even though the gas price under the GSA can be linked to more than one energy commodity, we focus for the sake of simplicity, but without a loss of generality, only on the pricing of two-factor spread options. Based on our assumptions, we will define the price dynamics of the underlying commoditiesusing two stochastic processes.

Let us have in pairs uncorrelated random variables ε_{1m}, ε_{2m}, ε_{1dm} and ε_{2dm} drawing from the standard normal $N(0,1)$ distribution and dependent variables $\ln S_{1dm}$ and $\ln S_{2dm}$ given as:

$$\ln S_{1dm} = \ln F_{1m} + \sigma_{1m}^{M}\left(\rho^{M}\varepsilon_{2m} + \sqrt{1-\rho^{M2}}\,\varepsilon_{1m}\right) + \sigma_{1m}^{D}\left(\rho^{D}\varepsilon_{2dm} + \sqrt{1-\rho^{D2}}\,\varepsilon_{1dm}\right), \tag{3}$$

$$\ln S_{2dm} = \ln F_{2m} + \sigma_{2m}^{M}\varepsilon_{2m} + \sigma_{2m}^{D}\varepsilon_{2dm},$$

Where S_{1dm} and S_{2dm} denote the spot prices of commodities 1 and 2, respectively. On day d of a month m, F_{1m} and F_{2m} are the forward prices of these commodities in the month m shaped by the market at the time of the option pricing. σ_{1m}^{M} and σ_{2m}^{M} stand for standard deviations of monthly averages of daily quotations from the formerly shaped forward prices F_{1m} and F_{2m} in the month m. σ_{1m}^{D} and σ_{1m}^{D} refer to standard deviations (distribution) of the daily quotations from (around) the monthly average values in the month m. ρ^M is a monthly correlation coefficient and ρ^D is a daily correlation coefficient [for similar example see Carmona and Durrleman, 2003].

Furthermore, for those holds that:

$$E\left(\ln S_{1dm}\right) = \ln F_{1m}, \quad E\left(\ln S_{2dm}\right) = \ln F_{2m},$$

and:

$$\operatorname{var}\left(\ln S_{1dm}\right) = \sigma_{1m}^{M2} + \sigma_{1m}^{D2}, \quad \operatorname{var}\left(\ln S_{2dm}\right) = \sigma_{2m}^{M2} + \sigma_{2m}^{D2}.$$

Then, the following explicit solutions for the log-normally distributed random variables S_{1dm} and S_{2dm} from (3) describe the price dynamics of the underlying commodities:

$$S_{1dm} = F_{1m} \exp\left[\sigma_{1m}^{M}\left(\rho^{M}\varepsilon_{2m} + \sqrt{1-\rho^{M2}}\varepsilon_{1m}\right) + \sigma_{1m}^{D}\left(\rho^{D}\varepsilon_{2dm} + \sqrt{1-\rho^{D2}}\varepsilon_{1dm}\right)\right], \tag{4}$$

$$S_{2dm} = F_{2m} \exp\left[\sigma_{2m}^{M}\varepsilon_{2m} + \sigma_{2m}^{D}\varepsilon_{2dm}\right].$$

Using the Monte Carlo methods, the idea is to generate a large number of random paths n of the underlying process ln S_{1dm}, and the same number of paths of the underlying process ln S_{2dm}, from the standardised normal distribution N(0,1), over the delivery period under the GSA. We can then use these paths to get estimates of the risk-neutral expectation defined as $E_{dm}\left\{\left[q-\left(bS_{2dm}-aS_{1dm}\right)\right]^{+}\right\}$. Specifically, the estimate for day d of month m is given as:

$$\hat{E}_{dm}\left\{\left[q-\left(bS_{2dm}-aS_{1dm}\right)\right]^{+}\right\} = \frac{1}{n}\sum_{i=1}^{n}\left[q-\left(bS_{2dmi}-aS_{1dmi}\right)\right]^{+} \frac{-b \pm \sqrt{b^2-4ac}}{2a}. \tag{5}$$

See Boyle's pioneering paper [1977] on a Monte Carlo simulation method for solving option valuation problems.

The final step to get the fair value prices of the spread options as given by (2) is to discount the estimates $\hat{E}_{dm}\{...\}$ at the risk-free interest rate. Doing so, we get an option value for each day of the contract (or delivery) period:

$$\hat{p}_{dm} = e^{-r_{dm}T_{dm}}\hat{E}_{dm}\left\{\left[q-\left(bS_{2dm}-aS_{1dm}\right)\right]^{+}\right\}. \tag{6}$$

However, as mentioned in section about Gas Swing options, the number of the spread options N is limited by the amount of the DQT, daily minimum and maximum constraint. Based on this, the buyer can exercise his right to not offtake gas from the seller only on N days of the contract year. That is why the next step is to disclose these N days on which the spread options are supposed to be exercised. As noted previously, the buyer in fact owns N put spread options of an American style. It means these options can be exercised by their owner at any time up to their expiration dates [see Hull, 2006].

There are some American-style spread options for which it holds that it is never optimal to exercise them early. Specifically, American calls on non-dividend paying stocks and American calls or puts on forward contracts are two examples [James, 2003]. Prices of such options are thus equal to the prices of the corresponding European calls or puts. Regarding American spread options on commodities, there are specific features such as seasonality and mean reversion due to which the pricing of these options is more complicated.

Several tricks have already been used by other authors to reconcile the commodity market models with their equity relatives. By adding other factors such as the convenience yields and cost of storage to the stochastic factors driving the models, researchers have tried to achieve the consistency with the no-arbitrage theory. And, in case the convenience yields of the underlying commodities are unequal, the price of an American-style spread option before its expiration is always higher than the price of its European equivalent. Focused on energy markets, the analysis is even more complicated due to limited arbitrage ability as mentioned previously. See e.g. Meinshausen and Hambly [2004] who focused on the pricing of American-style spread options on the energy markets.

It is thus reasonable to assume that it can be optimal to exercise some of the analysed spread options early. We assume that the buyer's decision is a result of a dynamic optimisation. On each day of the delivery period the buyer is supposed to make a decision on the source of his purchase based on a spot value of the spread and his expectation of the future spot spreads. Every time the spread is greater than the strike price the buyer is not supposed to exercise any of the options. In case the spread is less than the strike price, it is profitable to exercise one of the options. However, as the number of options N is less than the number of days within the contractual year, it could happen that it is not optimal to exercise one of the options despite the fact that it is profitable. This is given by the fact that the buyer wants to maximise his payoff from the gas swing option. In our valuation problem it means that every time the buyer expects there is going to be enough (*N minus the number of already exercised options*) days with a greater payoff than the one present on the decision-making day, he is not supposed to exercise any of the options. Regarding his expectation-making, we assume that he relies on estimates shaped by the market, i.e., on forward prices.

Following such optimisation criteria with respect to the observed forward prices means to choose N days of the contract year that offer the greatest estimated option values $\hat{p}_{dm}$. The final step is then to add these N greatest option values and multiply them by the option volume. Doing so, we get the total value of the swing option premium.

Application to an arbitrarily chosen gas sales agreement

In this section, we apply the above-mentioned steps to the pricing of an option embedded in one arbitrarily chosen GSA and compare our results with the ex-post value of this swing option.

Firstly, we must define the option parameters. We choose the swing option that consists of fifty-five spread options with parameters summarised in the following table.

Table 1 | Option Parameters*

Date of evaluation	October 1, 2007	
No. of spread options	55	
Underlying assets		
commodity 1	fuel oil 1PCT FOB Barge Rotterdam (in $ per ton), reference period 4-0**	
commodity 2	high-calorific natural gas with delivery at TTF (in € per MWh)	
Exercise period	1 January 2008 - 31 December 2008	
Strike price	-1.2666	€/MWh
Option volume	240	MWh
Coefficients		
a =	0.072117	
b =	1	

* The parameters refer to one of 55 spread options the buyer owns and are the same for all the options.
** See [Asche 2002a and 2002b] for the detailed description of the typical long-term Take-or-Pay contract and its reference period.

Source: authors' own calculation

Although we focus only on two commodities, we are actually exposed to a larger than two-factor valuation problem. The number of dimensions increases due to exchange and interest rates entering into the evaluation. However, for simplicity, but without loss of generality, we will not model these factors and simplify the problem into two factors.

The remaining part of the text then summarises the results of the application.

For an estimation of the parameters of the underlying processes (3), we use spot and forward prices of the underlying commodities observed in the gas years 2005/06 and 2006/07 (in-sample data). See Tables 2, 3 and 4 for the estimated values of these parameters.

Table 2 | Estimates of σ_{1m}^D and σ_{2m}^D

m	$\hat{\sigma}_{1m}^D$	$\hat{\sigma}_{2m}^D$
1	0.039	0.066
2	0.059	0.275
3	0.064	0.302
4	0.127	0.316
5	0.148	0.263
6	0.155	0.283
7	0.211	0.216
8	0.240	0.215
9	0.262	0.381
10	0.105	0.380
11	0.139	0.296
12	0.162	0.444

Source: authors' own calculation

We found out that the prices of Commodity 2 were more volatile than the prices of Commodity 1. Next, the prices of Commodity 1 were more volatile in the summer months (m = 4, 5, ...,9) than in the winter months (m = 1,2,3,10,11,12). In the case of Commodity 2, there is no such straightforward feature. We have observed larger deviations in the months representing the start and end of the summer and winter seasons (m = 3,4,9,10). However, it is necessary to emphasise that our estimates result just from a small set of observations (GY 2005/06 and GY 2006/07) which makes a deeper analysis difficult. More data is not available as it is difficult to obtain these data sets.

In Table 3, we can see the estimates $\hat{\sigma}_{1m}^M$ and $\hat{\sigma}_{2m}^M$. Even though the parameters are not annualised, it is already visible that for both underlying commodities there is a break-even point (month) where the estimate shifts from a growing to decreasing trend. However, to make the estimates comparable among the months, it is necessary to annualise them.

Table 3 | Estimates of σ_{1m}^{M} and σ_{2m}^{M}

m	$\hat{\sigma}_{1m}^{M}$	$\hat{\sigma}_{2m}^{M}$
1	0.141	0.065
2	0.161	0.140
3	0.195	0.218
4	0.230	0.273
5	0.263	0.401
6	0.281	0.575
7	0.290	0.652
8	0.282	0.670
9	0.280	0.679
10	0.273	0.731
11	0.263	0.671
12	0.243	0.642

Source: authors' own calculation

Table 4 | Annualised estimates of σ_{1m}^{M} and σ_{2m}^{M}

m	annualized $\hat{\sigma}_{1m}^{M}$	annualized $\hat{\sigma}_{2m}^{M}$
1	0.243	0.113
2	0.250	0.217
3	0.275	0.309
4	0.302	0.357
5	0.322	0.491
6	0.324	0.664
7	0.318	0.715
8	0.295	0.700
9	0.280	0.679
10	0.263	0.702
11	0.244	0.621
12	0.218	0.574

Source: authors' own calculation

Calculating annualised values of $\hat{\sigma}_{1m}^{M}$ and $\hat{\sigma}_{2m}^{M}$ (Table 4), we find out that the annualised deviation increases with forwardness until month 7 in the case of Commodity 2 and until month 6 in the case of Commodity 1.

This can be explained by a mean-reversion process observable on the commodity markets. The variance (i.e., the square power of a standard deviation) of a variable following a mean-reverting process does not grow proportionally to the time interval. It grows at the beginning and after some time it stabilises at a certain value.

Table 5 | Estimates of ρ^D and ρ^M

$\hat{\rho}^D$	$\hat{\rho}^M$
0.136	0.616

Source: authors' own calculation

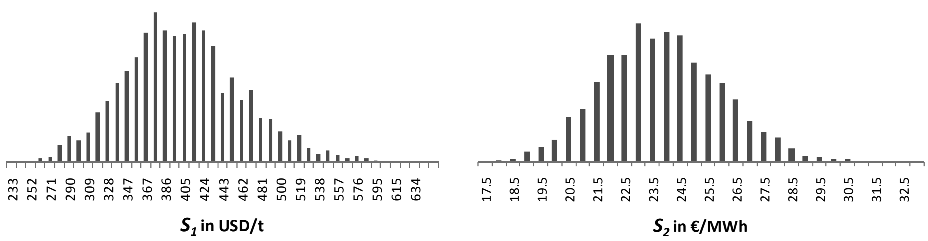
The value of $\hat{\rho}^D$ is lower compared to the value of $\hat{\rho}^M$ which indicates that monthly values of commodity prices tend to behave more similarly than the daily values do. As the value of $\hat{\rho}^D$ (0.135) is less than 0.3 we can talk about a very weak correlation. Since the value of $\hat{\rho}^M$ (0.616) is higher than 0.6 we can talk about a strong correlation. That gives us an interesting result: While the monthly values of commodities are strongly correlated and tend towards the same trend, the daily values on these different markets are almost independent and do not follow each other.

Monte Carlo simulation of option premium

We generate *n* random paths (n = 2,000) of the underlying process ln S_{1dm}, and the same number of paths of the underlying process ln S_{2dm} (see 3), from the standardised normal distribution *N(0,1)* and over the interval starting at the time of option pricing (Oct 1, 2007) and ending at the maturity of the GSA (Dec 31, 2008).

Using the generated random paths it is easy to construct the probability distributions of the underlying variables for every day of the observed period (see 5) and also the distributions of their spreads $\left(bS_{2T} - aS_{1T}\right)$.

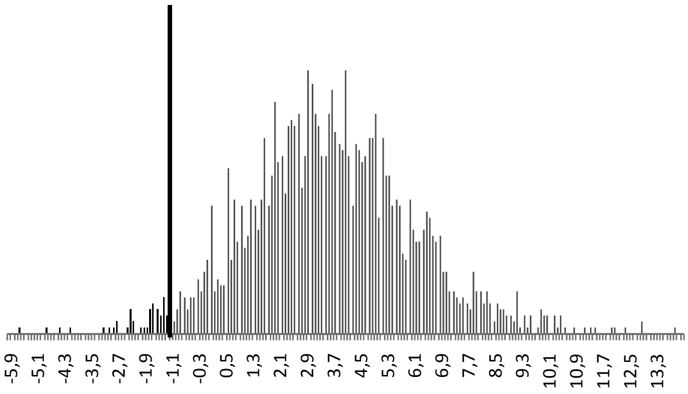
Graphs 1, 2 | Histograms of S1 and S2 as of 1 January 2008



Source: authors' own calculation

As an example, Graph 1 shows the probability distribution of the underlying S1 in USD/t for the first day of the delivery (1 January 2008) calculated on the basis of 2,000 random paths generated by the Monte Carlo simulations. Analogically, calculated on the same basis, in Graph 2 we can see the probability distribution of the underlying S2 in EUR/MWh.

Graph 3 | Histogram of spread as of 1 January 2008



Source: authors' own calculation

Graph 3 shows the probability distribution of the spread $\left(bS_{2T} - aS_{1T}\right)$ for the first day of the delivery (1 January 2008) created using the differences between the random values of S_2 and S_1 variables.

On that day, the spread will be with a 2.1% probability (bold area of the histogram) less than the strike price $q = -1.2666$ EUR/MWh. This means that with the 2.1% probability the market price will be lower than the contractual price, making the option with an expiration date on 1 January 2008 in-the-money.

Actually, the values of the random paths are enough to calculate the estimates of the risk-neutral expectation given by Equation (5). Discounting the estimates at the risk-free interest rate (see 6), we get an option value for each day of the contract period. See Appendix A for the results.

The second step is to disclose the days on which the options are supposed to be exercised. This means we choose the fifty-five days of the contract year that offer the greatest estimated option values. For our results see Appendix A and the bold-marked values.

Adding these N = 55 option values and multiplying them by the option volume, we get a resulting value of the swing option premium: it is equal to 77,249 euros.

Comparison with the real and maximum ex-post value

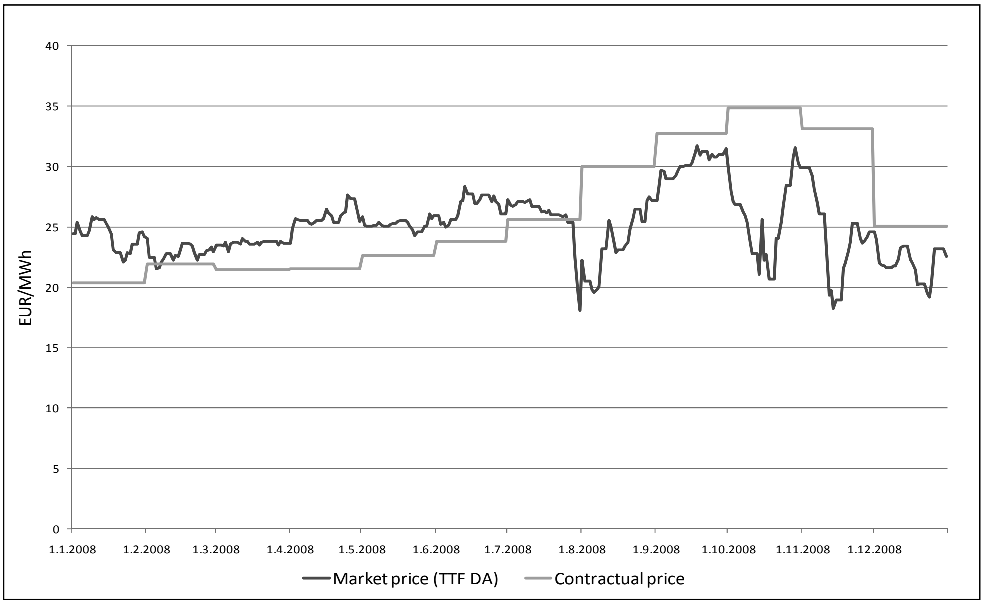
As the gas swing option is not a tradable instrument, it is difficult to test the model efficiency. That is why we focus on the calculation of the real and maximum ex-post value of the option.

Using the daily values of spot and forward prices of the underlying commodities observed in the calendar year 2008 (out-of-sample data), we can calculate the profit the buyer could realise by exercising his right stemming from the downward quantity tolerance provision. Following the decision-making criteria described above, we calculate the real payoffs from the spread options realised at the time of the options execution, and discount them to the original time of the option pricing.

All the spot payoff values are shown in Appendix B. Those of them which are results of the application of the described strategy are marked in bold. Discounting the realised payoffs to the time of the option valuation (using the same discount rates as in case of the ex-ante option pricing), we get the real value of the gas swing option in the amount of 57,044 euros.

For comparison, we also calculate the maximum value the buyer could generate from the option if he knew the future, which consists of three steps. Firstly, we compute the discounted payoff values for all days of the contract period using the respective spot prices of the underlying commodities and the same discount rates as in the case of the ex-ante option pricing. Secondly, we choose the fifty-five largest values. See Appendix C for the results. The ideal time for the options execution is also demonstrated in Graph 6 where the spot (day-ahead) market prices and the prices under the GSA in 2008 are shown. The highest spreads between them, in the months of August, October and November, correspond to the bold-marked days in Appendix C. Finally, we add these discounted payoffs and multiply them by the option volume. Doing so, we get the maximum ex-post value of the gas swing option which is equal to 129,186 euros.

Graph 4 | Spot market (TTF DA) vs. contractual prices



Source: authors' own calculation

The reason for the ex-post value to be much higher than the calculated ex-ante option premium is given by the specific nature of the year 2008, influenced by the global financial crisis. The crisis induced more volatile prices of energy commodities. Specifically, prices of natural gas and its competing fuels, such as Brent crude oil, gas oil or fuel oil, rose steeply during the first three quarters of the year 2008, and subsequently started falling even steeper.

Though the prices of both underlying commodities (natural gas and fuel oil) followed such a trend, the value of the spread between the market and contractual price of gas changed rapidly, due to the fact that fuel oil has entered into the price formula under the GSA as an average of its prices observed in the four preceding months. This resulted in a situation where the market price of gas was falling and, at the same time, the contractual price was growing. Such a situation that the market price is lower than the contractual price in winter months (October – December) was not expected by the option buyer during most of the time of the contract period. That is why the buyer decided to exercise his options earlier and was not able to profit from the nonstandard situation of the end of the year.

Conclusion

In this paper, we first gave an introduction to the Gas Sales Agreement and Gas Swing Options to better introduce the background and motivation. Then we applied the Monte Carlo simulations to find a fair value price of an option embedded in an arbitrarily chosen gas sales agreement, i.e., a gas swing option. The gas swing option is a set of several spread options on energy commodities. More precisely, it consists of several American put options on a spread between the market price of gas and the market prices of two or more competing fuels. Pricing of the gas swing option thus represents a multifactor valuation problem. For the sake of simplicity, but without a loss of generality, we focused on a two-factor option pricing, i.e., the case when the puts are written on natural gas and only one competing fuel.

After the introduction to the theory on spread option pricing, we approached the choice of the valuation method. The widely used numerical method, the Monte Carlo simulations, was the one that we decided to employ when modelling the dynamics of the future spot prices of the underlying commodities. Based on the characteristic features of the underlying energy commodities and their pricing, we defined two stochastic processes representing the price dynamics.

Using the spot and forward prices of the underlying commodities observed in the gas years 2005/06 and 2006/07 (in-sample data) we estimated the values of the parameters of the stochastic processes, such as volatilities and correlations of the underlying prices. The spot and forward prices of the commodities as of the calendar year 2008 (out-of-sample data) were then used to compare our results with the real and maximum ex-post value of the option.

Our model was based on the concept of risk-neutral expectations. More precisely, an option price was considered as an expectation of discounted future cash flows for a probability structure called risk-neutral. Using the Monte Carlo method, we simulated values of the future cash flows for each day of the delivery period (calendar year 2008). After calculating the means of such values and their discounting, it was still necessary to decide on the exercise days since the options were of an American style. We concluded that it could be profitable for the buyer to exercise some of the options early and defined the decision-making criteria. Following all these steps, we found the estimate of the fair value price of the arbitrarily chosen gas swing option.

Our estimate was then compared with the real and maximum ex-post value of the option. As the gas swing option has not been a tradable instrument, it has been difficult to test the model efficiency. However, we explained the reason why the ex-post value was much higher than the calculated ex-ante option premium: It was given by the specific nature of the year 2008 and influenced by the global financial crisis. Prices of natural gas and its competing fuels, such as Brent crude oil, gas oil or fuel oil, rose steeply during the first three quarters of the year 2008, and subsequently started falling even steeper, resulting in a situation when the market price of gas was falling and, at the same time, the contractual price was growing. Such a situation that the market price is lower than the contractual price in winter months (October – December) was not expected by the option buyer during most of time of the contract period. That is why the buyer decided to exercise his options earlier and was not able to profit from the nonstandard situation of the end of the year.

This paper has tried to contribute to a field where evaluation methods have not yet been fully developed and, at the same time, their development has been highly requested. Specifically, the changing nature of energy markets stemming from the process of their liberalisation, has begged for new evaluation approaches. Though this paper has brought some useful contributions it needs to be further extended, mainly by the means of dynamic delta hedging.

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Appendix A

Month/Day	1	2	3	4	5	6	7	8	9	10	11
1	0.019	0.012	0.013	0.014	0.011	0.008	0.011	0.013	0.017	0.011	0.019
2	1.193	1.256	1.221	1.210	1.242	1.172	1.230	1.223	1.299	1.244	1.188
3	2.232	2.366	2.308	2.334	2.180	2.185	2.259	2.201	2.278	2.202	2.173
4	3.506	3.419	3.467	3.500	3.486	3.650	3.557	3.545	3.359	3.520	3.638
5	3.994	3.980	3.974	3.892	3.969	3.947	4.003	4.022	3.871	3.928	4.015
6	5.051	5.123	5.207	5.125	5.124	5.185	5.110	5.109	5.112	5.037	5.142
7	**5.455**	5.404	5.409	**5.504**	**5.478**	**5.418**	**5.420**	5.415	**5.502**	**5.473**	**5.514**
8	5.350	5.411	5.378	5.392	5.360	5.393	5.343	**5.435**	5.287	5.387	**5.430**
9	**6.092**	**6.017**	**5.945**	**6.074**	**6.004**	**6.085**	**6.006**	**6.013**	**6.202**	**6.091**	**6.107**
10	5.413	5.355	5.313	5.351	5.274	5.378	5.344	**5.485**	5.331	5.356	5.366
11	4.575	4.461	4.491	4.561	4.455	4.500	4.485	4.519	4.535	4.446	4.517
12	4.445	4.477	4.391	4.594	4.452	4.413	4.534	4.453	4.493	4.432	4.484

Month/Day	12	13	14	15	16	17	18	19	20	21	22
1	0.015	0.011	0.016	0.011	0.013	0.013	0.009	0.010	0.011	0.017	0.015
2	1.288	1.191	1.196	1.180	1.225	1.221	1.204	1.205	1.245	1.221	1.205
3	2.272	2.217	2.230	2.292	2.236	2.239	2.163	2.141	2.252	2.210	2.074
4	3.437	3.551	3.553	3.468	3.520	3.470	3.445	3.451	3.578	3.389	3.432
5	3.985	3.917	3.943	3.925	3.960	3.875	3.956	3.939	3.987	4.015	3.986
6	5.189	5.160	5.106	5.192	5.141	5.092	5.142	5.144	5.120	5.176	5.148
7	**5.462**	5.394	5.392	5.377	**5.454**	**5.429**	5.412	**5.441**	5.405	5.414	**5.425**
8	5.416	5.366	**5.427**	5.292	5.349	5.401	5.411	5.318	5.346	5.349	5.338
9	**6.078**	**6.056**	**6.055**	**6.063**	**5.998**	**5.987**	**6.006**	**5.933**	**6.023**	**6.050**	**6.067**
10	5.387	5.358	5.386	5.393	5.326	5.258	5.216	5.374	5.300	5.392	5.306
11	4.555	4.468	4.588	4.456	4.612	4.512	4.457	4.573	4.496	4.509	4.543
12	4.404	4.477	4.571	4.356	4.449	4.438	4.291	4.400	4.464	4.559	4.363

Month/Day	23	24	25	26	27	28	29	30	31
1	0.012	0.018	0.017	0.015	0.014	0.016	0.013	0.013	0.015
2	1.295	1.216	1.150	1.186	1.173	1.163	1.177		
3	2.135	2.349	2.280	2.244	2.207	2.156	2.195	2.218	2.152
4	3.505	3.396	3.483	3.450	3.437	3.451	3.437	3.437	
5	3.928	3.901	3.967	3.869	3.935	3.896	3.938	3.948	3.961
6	5.129	5.140	5.123	5.179	5.079	5.169	5.049	5.069	
7	5.362	**5.444**	**5.454**	5.410	**5.553**	5.339	**5.430**	**5.486**	**5.440**
8	5.350	5.323	5.274	5.342	5.273	5.404	5.350	5.379	5.337
9	**6.177**	**6.039**	**6.110**	**6.088**	**6.012**	**6.056**	**6.112**	**5.980**	
10	5.384	5.290	5.387	5.388	5.337	**5.527**	**5.418**	5.289	5.327
11	4.549	4.460	4.500	4.518	4.476	4.506	4.503	4.528	
12	4.484	4.418	4.411	4.321	4.267	4.457	4.399	4.423	4.506

Appendix B

Month/Day	1	2	3	4	5	6	7	8	9	10	11
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,000	0,000	0,000	0,000	0,469	**0,528**	0,106	0,000	0,000	0,000	0,000
3	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
4	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
5	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
6	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
7	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
8	**6,499**	**8,230**	**8,230**	**8,246**	**9,158**	**9,402**	**9,160**	**9,736**	**6,571**	**6,571**	**6,700**
9	**4,941**	**3,989**	**2,821**	**2,802**	**4,030**	**4,050**	**4,050**	**4,129**	**3,955**	**3,696**	**3,739**
10	2,882	**5,332**	6,382	6,652	6,652	7,162	7,800	7,811	8,469	10,401	11,386
11	1,624	1,624	3,002	2,946	3,576	4,913	6,031	6,986	6,986	6,626	9,661
12	2,252	2,667	4,758	4,944	4,908	5,048	5,048	4,637	4,549	4,331	3,209

Month/Day	12	13	14	15	16	17	18	19	20	21	22
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,016	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
3	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
4	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
5	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
6	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
7	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
8	4,593	5,116	6,371	**7,595**	**7,360**	**7,360**	**7,414**	7,214	**6,783**	**5,521**	**4,670**
9	**3,455**	**3,351**	**3,351**	**3,142**	**2,595**	**1,962**	**0,710**	**2,074**	**1,770**	**1,770**	**0,982**
10	11,386	11,230	12,611	8,442	12,106	11,906	13,971	13,971	13,918	11,161	12,101
11	14,335	13,988	15,010	14,357	14,357	14,398	11,770	11,370	10,703	9,791	8,216
12	2,008	1,848	1,848	1,514	2,299	1,956	1,529	3,972	3,836	3,836	3,767

Month/Day	23	24	25	26	27	28	29	30	31
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,000	0,000	0,000	0,000	0,000	0,000	0,000		
3	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
4	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	
5	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
6	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	
7	0,000	0,000	0,000	0,267	0,267	0,247	**3,244**	**6,328**	**7,766**
8	**3,873**	**3,873**	**3,961**	5,316	4,950	3,270	2,984	3,319	3,319
9	**1,322**	**0,934**	**1,160**	**1,290**	**1,084**	**1,084**	**1,759**	1,390	
10	10,912	10,224	8,479	8,479	8,895	6,364	4,845	5,288	6,544
11	8,216	7,751	8,865	8,931	8,895	8,954	8,566	8,566	
12	4,561	4,814	3,743	0,814	0,814	0,814	0,344	0,647	1,628

Appendix C

Maximum ex-post values of spread options

Month/Day	1	2	3	4	5	6	7	8	9	10	11
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
3	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
4	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
5	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
6	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
7	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
8	**7,701**	**9,432**	**9,432**	**9,432**	**10,191**	**10,418**	**10,162**	**9,942**	0,000	0,000	0,000
9	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
10	0,000	0,000	**7,717**	**7,987**	**7,987**	**7,987**	**8,620**	**8,885**	**9,418**	**11,083**	**12,068**
11	0,000	0,000	0,000	0,000	0,000	0,000	0,000	**7,050**	**7,050**	**7,050**	**9,701**
12	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

Month/Day	12	13	14	15	16	17	18	19	20	21	22
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
3	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
4	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
5	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
6	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
7	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
8	0,000	0,000	0,000	**7,074**	0,000	0,000	0,000	0,000	0,000	0,000	0,000
9	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
10	**12,068**	**12,068**	**13,739**	**9,244**	**12,600**	**12,126**	**14,191**	**14,191**	**14,191**	**10,782**	**10,754**
11	**13,780**	**13,419**	**14,855**	**14,202**	**14,202**	**14,202**	**11,555**	**11,103**	**10,182**	**9,436**	**7,861**
12	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

Month/Day	23	24	25	26	27	28	29	30	31
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
3	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
4	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
5	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
6	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
7	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	**7,493**
8	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
9	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
10	**9,469**	**8,143**	0,000	0,000	0,000	0,000	0,000	0,000	0,000
11	**7,861**	**7,861**	**9,077**	**9,470**	**9,342**	**8,940**	**8,552**	**8,552**	0,000
12	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000