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Part I.

Part II.

Comparison of Selected Methods for American Option Pricing

Lun Gao, Miroslav Čulík[1]

Abstract

An American option is an option that can be exercised anytime during its life. American options allow option holders to exercise the option at any time prior to and including its maturity date, thus increasing the value of the option to the holder relative to European options, which can only be exercised at maturity. The article focuses on the calculation of American option prices. The calculation speed and calculation accuracy of implicit finite different method, explicit finite different method and least square Monte Carlo method are compared using C++ programming. At the same time, the theoretical results are compared with the actual results through the data of the Hong Kong Stock Exchange.

Key words

Option pricing, Black-Scholes Model, Stochastic distribution, Gaussian distribution, Finite different method, Least square Monte Carlo model.

JEL Classification: Here write the JEL classification code(s)

1. Analysis of financial derivatives and their pricing

A derivative is a financial contract that derives its value from an underlying asset. The buyer agrees to purchase the asset on a specific date at a specific price.

Derivatives are often used for commodities, such as oil, gasoline or gold. Another asset class is currencies, often the U.S. dollar. There are derivatives based on stocks or bonds. Still others use interest rates, such as the yield on the 10-year Treasury note.

The contract's seller doesn't have to own the underlying asset. He can fulfill the contract by giving the buyer enough money to buy the asset at the prevailing price. He can also give the buyer another derivative contract that offsets the value of the first. This makes derivatives much easier to trade than the asset itself.

Many different types of derivatives have different pricing mechanisms. The most common derivative types are futures contracts, forward contracts, options and swaps. More exotic derivatives can be based on factors such as weather or carbon emissions. A derivative is a financial contract with a value based on an underlying asset.

Options on stocks and exchange-traded funds are also common derivative contracts. Options give the buyer the right, as opposed to the obligation, to buy or sell 100 shares of a stock at a strike price for a predetermined amount of time. The best-known pricing model for options is the Black-Scholes method. This method considers the underlying stock price, option strike price, time until the option expires, underlying stock volatility and risk-free interest rate to provide a value for the option.

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An American option is an option that can be exercised anytime during its life. American options allow option holders to exercise the option at any time prior to and including its maturity date, thus increasing the value of the option to the holder relative to European options, which can only be exercised at maturity.

We often use two different methods in the pricing of options: series of methods derived from the hypothetical distribution of the underlying asset prices (e.g Black-Scholes model and Normal Inverse Gaussian distribution model) and method derived from the maximum entropy principle to derive and solve the probability density distribution of the underlying asset, and then to find the option pricing model then calculate option price. This paper will only assume that the underlying asset price changes are based on normal distribution.

Classically, the factors affecting the pricing of options are price of underlying asset, exercise price, expiry date, volatility of underlying asset price, risk free rate and size of the proposed dividend. These factors have different influence on different kinds of options. In fact, price of underlying asset is the key variable influence the option price.

2. Description of selected methods for option pricing

According to option theory because American options have the problem of early exercise, it is difficult to obtain the option value through the numerical solution of the partial differential equation. In this paper, Finite-difference method and the least square Monte Carlo method will be applied to calculate the option price and compared.

2.1 Finite Difference Method

In most cases, it is almost impossible to require an exact solution of a partial differential equation. At this time it is necessary to use the finite difference approximation. The basic idea of the difference method is to replace the partial derivative in a partial differential equation by Taylor expansion in a certain point. According to the definition:

$$\frac{\partial u}{\partial t}(x,t) = \lim_{dt \to 0} \frac{u(x,t+dt)}{dt}$$

Now dt is not regarded as a variable that tends to 0, but as a small amount which greater than 0, an approximate. We can obtain an approximate

$$\frac{\partial u}{\partial t}(x,t) \approx \frac{u(x,t+dt) - u(x,t)}{dt} + O(dt) \tag{2.1.1}$$

This is called the finite difference approximation of $\frac{\partial u}{\partial t}$. The smaller the time interval, the more accurate the approximation is. What considered here is the time change from t to $t+dt$, often referred to as the forward difference.

If do the following approximation:

$$\frac{\partial u}{\partial t}(x,t) \approx \frac{u(x,t) - u(x,t-dt)}{dt} + O(dt) \tag{2.1.2}$$

then it is called the backward difference.

Also, the central difference can be defined as

$$\frac{\partial u}{\partial t}(x,t) \approx \frac{u(x,t+dt) - u(x,t-dt)}{2dt} + O((dt)^2). \tag{2.1.3}$$

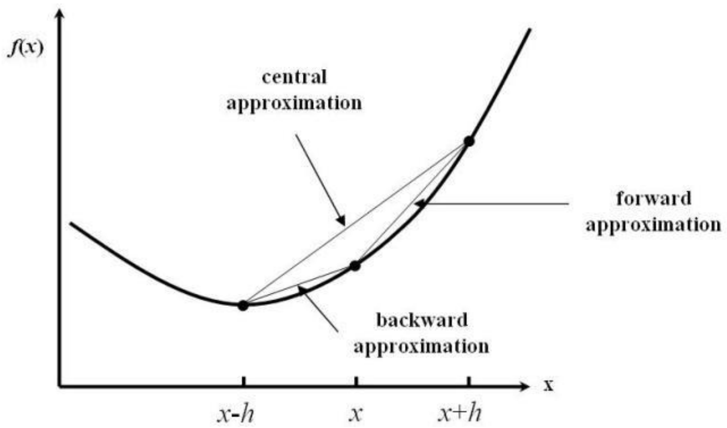

Figure 3.2.3 Forward, backward and central difference

When applied to the diffusion equation, the forward difference approximation leads to the explicit difference method, and the backward difference approximation leads to the full implicit difference method. The center difference approximation is rarely used because it often causes bad behavior in the solution process. In the commonly used Crank-Nicolson difference method, the central difference defined by the next formula

$$\frac{\partial u}{\partial t}(x,t) \approx \frac{u(x,t+\delta t/2)-u(x,t-\delta t/2)}{2\delta t}+O((\delta t)^2). \tag{2.1.4}$$

The difference method is equivalent to dividing the x axis into a space segment with an equidistance of δx and the t axis into a time interval with δt as an equidistance. Thus, the (x,t) plane is divided into a grid.

By performing differential processing on the Black-Scholes partial differential equation, we can derive the expression of the explicit finite difference method.

$$a_j f_{i+1,j-1} + b_j f_{i+1,j} + c_j f_{i+1,j+1} = f_{ij} \tag{2.1.5}$$

where

$$a_j = \frac{1}{1+r\Delta t}\left(-\frac{1}{2}rj\Delta t+\frac{1}{2}\sigma^2 j^2\Delta t\right),$$

$$b_j = \frac{1}{1+r\Delta t}\left(1-\sigma^2 j^2\Delta t\right),$$

$$c_j = \frac{1}{1+r\Delta t}\left(\frac{1}{2}rj\Delta t+\frac{1}{2}\sigma^2 j^2\Delta t\right).$$

Next we will use the explicit finite difference method to solve the American option. First of all we need to set the key boundary conditions.

For the American call option:

The value of the option at expiration date is $\max\{S_T - X, 0)$, in which S_T is the stock price at the time of T. So

$$f_{Nj} = \max\left\{ j\Delta S - X, 0 \right\}.$$

When the stock price is 0, the price of the call option is 0. So

$$f_{i0} = 0, i = 0,1,2,\ldots,N;$$

When the stock price is $S = S_{\max}$, the price of the call option is $S_{\max}$. So

$$f_{iM} = S_{\max}, i = 0,1,2,\ldots,N.$$

For the American put option:

The value of the option at expiration date is $\max\{X - S_T, 0\}$, in which S_T is the stock price at the time of T. So

$$f_{Nj} = \max\left\{X - j\Delta S, 0\right\}.$$

When the stock price is 0, the price of the call option is X. So

$$f_{i0} = X, i = 0,1,2,...,N;$$

When the price of stock tends to infinity, the price of the put option is 0. So

$$f_{iM} = 0, i = 0,1,2,...,N.$$

From the key boundary conditions of American option the program could be apply. Because of the existence of the rounding error in the explicit finite difference method, so sometimes the solution of the difference equation does not converge to the solution of the partial differential equation. To solve this problem we will introduce the implicit finite difference method. It can solve more $\}x$ nodes at the same time step. Through the differential treatment of the Black Scholes partial differential equation, we can get the expression of the implicit finite difference method.

$$a_j f_{i,j-1} + b_j f_{ij} + c_j f_{i,j+1} = f_{i+1,j}, \tag{2.1.6}$$

where

$$a_j = \frac{1}{2} r j \Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t,$$

$$b_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t,$$

$$c_j = \frac{1}{1 + r\Delta t}\left(\frac{1}{2} r j \Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right).$$

In the implicit finite difference method, the calculation of $f_{i,j}$ by $f_{i+1,j}$ needs to solve $M+1$ equations at the same time the amount of calculation is very large. So the matrix library needs to be introduced in the program.

2.2 Least Squares Monte Carlo Method

The most widely used is the Least Squares Monte Carlo simulation proposed by Longstaff and Schwartz. The basic principle is: at a limited number of discrete time points, according to the cross-sectional data of the simulated sample path of the target asset price at each moment, use least squares regression to find the expected return on continued holding options. And compare it with the proceeds that were immediately exercised at that moment. If the immediately exercise is greater than continued holding, it will immediately exercise or it will continue to hold. Suppose the option expiration date is T, the exercise time is T^e. The basic steps of Least Squares Monte Carlo simulation are similar as the European option. But should notice that the European option can only be exercised at expiry date that is $T = T^e$ but for American option $T^e \in [0,T]$, that is, the option can be exercise at any time before the expiration date. As the proceeds at the time of exercise are not only affected by the asset price, but also affected by the path taken by the asset price from the issue date to the maturity date. For European options, it has been mentioned how to calculate. But for American options, we need to compare the instantaneous income (intrinsic value) immediately exercise at that moment and the expected return to continue holding when determining the optimal exercise time. What needs to be established is the value to continued holding the option $F(\omega, t_k)$. According to no arbitrage principle:

$$F(\omega, t_k) = E_Q\left[\sum_{j=k+1}^{K} \exp\left(-\int_{t_k}^{t_j} r(\omega, s)\,ds\right) C(\omega, t_j; t_k, T) \middle| F_{tk}\right] \tag{2.2.1}$$

Where $r(w,s)$ is riskless discount rate, the expectation is taken conditional on the information set. F_{t_k} at time t_k,. With this representation, the problem of optimal exercise reduces to comparing the immediate exercise value with this conditional expectation, and then exercising as soon as the immediate exercise value is positive and greater than or equal to the conditional expectation. The LSM method is to calculate the expected condition in formula (2.2.1). For example, if this conditional expectation function belongs to the Hilbert space L^2, the value of continuing to hold the option $F(w,t_{k-1})$ can be expressed as follows:

$$F(w,t_{k-1}) = \sum_{j=0}^{\infty} a_j L_j(X) \tag{2.2.2}$$

Where is X a Markov process, a_j is a constant and L_j is a set of basic functions. In practical applications, the infinite-dimensional space will not be discussed. The usual choice is based on the previous M basis functions to calculate $F_M(w,t_{k-1})$ instead of $F(w,t_{k-1})$The statistical estimate $\breve{F}_M(w,t_{k-1})$ can be calculated by the $C(w,s;t_{k-1},T)$ through a mapping or regression. In the following use, weighted Lagrange polynomials will be used as regression basis functions.

$$L_0(X) = \exp(-\frac{X}{2})$$

$$L_1(X) = \exp(-\frac{X}{2})(1-X)$$

$$L_2(X) = \exp(-\frac{X}{2})(1-2X+\frac{X^2}{2})$$

$$\cdots$$

$$L_n(X) = \exp(-\frac{X}{2})\frac{e^X}{n!}\frac{d^n}{dX^n}(X^n e^{-X}). \tag{2.2.3}$$

Once the function $F_M(w,t_{k-1})$ is determined, the coefficients before each basis function are determined accordingly. From this, the value of $\breve{F}_M(w,t_{k-1})$ can be calculated and compared immediately with the gain of the execution of the option and make the decision on whether to exercise American options here. Then continue to iterate until the initial moment to find an optimal execution moment. Then discount it to get the value of the option.

The LSM algorithm provides a simple and elegant way of approximating the optimal early exercise strategy for an American-style option. While the ultimate test of the algorithm is how well it performs using a realistic number of paths and basic functions, it is also useful to examine what can be said about the theoretical convergence of the algorithm to the true value $V(X)$ of the American option.

The first convergence result addresses the bias of the LSM algorithm and is applicable even when the American option is continuously exercisable.

Proposition 1. For any finite choice of M, K and $vector\, q \in R^{M \times (k-1)}$ representing the coefficients for the M basis functions at each of the $K-1$ early exercise dates, let $LSM(w;M,K)$ denote the discounted cash flow resulting from following the LSM rule of exercising when the immediate exercise value is positive and greater than or equal to $\hat{F}_M(w,t_{k-1})$ as defined by q. Then the following inequality holds almost surely,

$$V(X) \geq \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} LSM(w_i;M,K). \tag{2.2.4}$$

The intuition for this result is easily understood. The LSM algorithm results in a stopping rule for an American-style option. The value of an American-style option, however, is based on the stopping rule that maximizes the value of the option; all other stopping rules, including the stopping rule implied by the LSM algorithm, result in values less than or equal to that implied by the optimal stopping rule.

This result is particularly useful since it provides an objective criterion for convergence. For example, this criterion provides guidance in determining the number of basic functions needed to obtain an accurate approximation; simply increase W until the value implied by the LSM algorithm no longer increases. This useful and important property is not shared by algorithms that simply discount back functions based on the estimated continuation value.

By its nature, providing a general convergence result for the LSM algorithm is difficult since we need to consider limits as the number of discretization points K, the number of basic functions M, and the number of paths N go to infinity. In addition, we need to consider the effects of propagating the estimating stopping rule backwards through time from t_{k-1}, to t_1. In the case where the American option can only be exercised at $K=2$ discrete points in time, however, convergence of the algorithm is more easily demonstrated. As an example, consider the following proposition.

Proposition 2. Assume that the value of an American option depends on a single state variable X with support on $(0,\infty)$ which follows a Markov process. Assume further that the option can only be exercised at times t_1, and t_2, and that the conditional expectation function $F(w,t_1)$ which is absolutely continuous and

$$\int_0^\infty e^{-X} F^2(w;t_1)\,dX < \infty$$

$$\int_0^\infty e^{-X} F_X{}^2(w;t_1)\,dX < \infty$$

Then for any $\epsilon > 0$, there exists an $M < \infty$ such that

$$\lim_{N\to\infty} \Pr\left[\left|V(X) - \frac{1}{N}\sum_{i=1}^{N} LSM(w_i; M, K)\right| > \epsilon\right] = 0 \tag{2.2.5}$$

Intuitively this result means that by selecting M large enough and letting $N\to\infty$, the LSM algorithm results in a value for the American option within ϵ of the true value. Thus the LSM algorithm converges to any desired degree of accuracy since ϵ is arbitrary. The key to this result is that the convergence of $F_M(w,t_1)$ to $F(w,t_1)$ is uniform on $(0,\infty)$ when the indicated integrability conditions are met. In summary the American put option can be written as

$$PutAme(0,S_{0,}K,T) = PutEur(0,S_{0,}K,T)E[K - S_{t^*}e^{-rt^*} - PutEur(t^*,S_{t^*},K,T)e^{-rt^*}] \tag{2.2.6}$$

3. Examples study and result

The following is written in C++ program to calculate the results of American option pricing. Assume that when the number of simulations of the binomial tree method is infinite, the numerical solution of the partial differential equation is infinitely close to the analytical solution. This paper choose the date example from the CBOT to calculate the American put option price. The following convergence graph reflects the degree of deviation of the numerical solution from the analytical solution for the same number of simulations under the two methods.

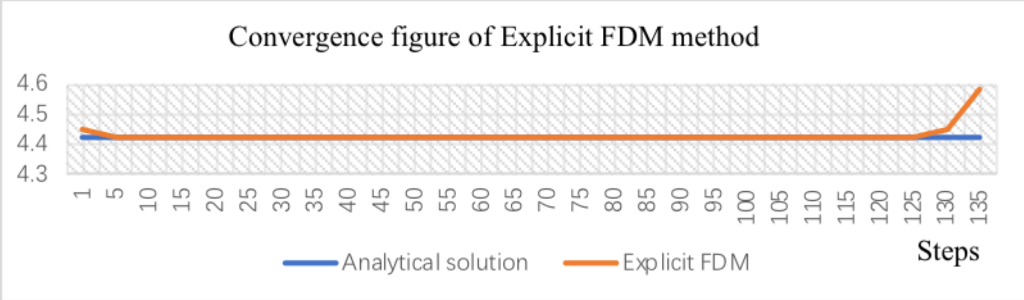

Figure 3.1 Convergence figure of Explicit FDM method in American option

It can be clearly seen that due to the accumulation of errors caused by the overflow effect, when the number of simulations is about 125 steps, the numerical solution and the analytical solution are greatly different.

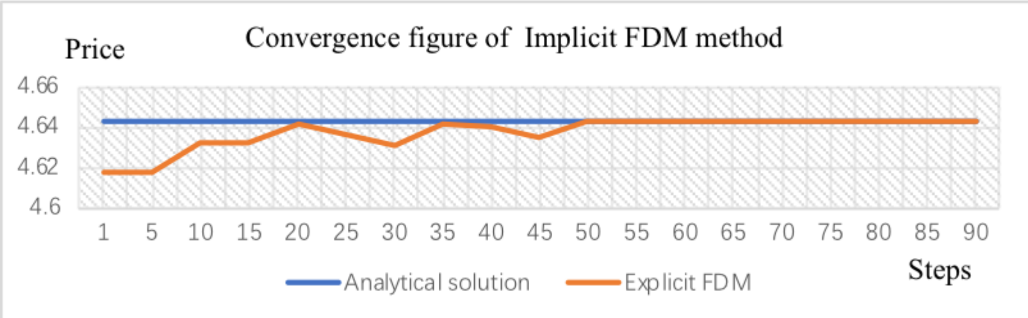

Figure 3.2 Convergence figure of Implicit FDM method in American option

Figure 3.2 is a convergence graph of the implicit finite different method. It can be clearly seen that there is no error as the number of simulations increases. However, due to the models need to refer to the external matrix library during programming, the system is extremely unstable, and it is difficult to debug frequently.

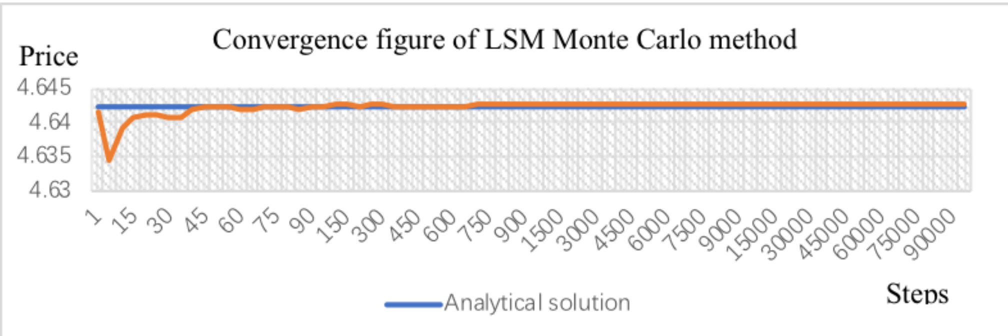

Figure 3.2 Convergence figure of LSM Monte Carlo method

Figure 3.3 shows the LSM Monte Carlo method. Originally, when the program was implemented, the American option was considered as a Bermuda option, and the corresponding exercise date was set artificially.
Although it does not affect the results, it is impossible to perform large-scale simulations. Break through this limitation after switching programming ideas. It can be seen from the figure that the numerical solution results are very stable and approximate equal to the analytical solution in the case of the maximum number of simulations. Prominently high precision.

In the empirical analysis, several stock options of HSBC Holdings (HK.0005) will selected for research.

T	Strike price	Real price	LSM MC	Im FDM
t=0.41918	65	0.91	0.5037872	0.51127
	67.5	1.44	0.8369717	0.847237
	70	2.07	1.36331	1.36591
	72.5	3.11	2.231524	2.22645
	75	4.47	3.384697	3.40199
	77.5	6.11	4.897231	4.90949
	85	12.35	11.07543	11.1254

Table 3.1 American Put option of HSBC Holding

Chart 3.4 Put option price of HSBC Holding

It can be seen from Tables 3.1 and 3.4 that the actual value of HSBC Options is higher than the theoretical value predicted by the algorithm, but the actual value is close to the theoretical value, which means that the algorithm is real and effective. Meanwhile the real market value and the approximate solution of the least square Monte Carlo Simulation and the finite difference method is mostly negative. This also shows that the market price is high than the theoretical price. The market price is overestimate to some extent. We think the reasons may have the following:

1. In recent years, the stock prices of China Construction Bank and HSBC Holdings have been at a high level and the return on net assets has risen (Construction Bank (HK.0939): 13.16% and HSBC Holdings (HK.0005): 5.68% in 2017). Investors are optimistic about the stock's outlook and the option price is overvalued.
2. According to modern financial theory, when the price of financial assets seriously deviates from the theoretical price, the market will have opposite expectations. If the issuer or market makers are allowed to sell short, it may create a constraint on the price trend of the financial asset. The model of option pricing itself assumes that there is a short selling mechanism in the market. However, virtually no short-selling mechanism exists in the securities markets of the Chinese mainland or the Hong Kong Special Administrative Region. The unilateral nature of the market mechanism has made it difficult for the securities market to

form an effective spontaneous restraint mechanism. As a result, effective suppression of market prices has not been applied.

3. Even though the Hong Kong Special Administrative Region is an international financial center with an extremely sophisticated financial system, many individual investors still have a lot of blinding and following trends in investment behavior, the ability to identify and judge risks is insufficient, so there is inevitable blindness in the investment process.

Due to the limited ability of author, there are still many places that are worth discussing. It can be perfected through deeper research. First, the thesis considers only Black-Scholes partial differential equations. However, with the development of mathematical sciences, more and more studies tend to think that financial asset price distribution has fat tails and skewness, which are not suitable for normal distribution. Due to the programming technique reasons, the article did not include a variance gamma distribution model (VG) or a normal inverse Gaussian distribution model (NIG) for comparison studies. Finally, this thesis does not correct some of the limitations of the Monte Carlo simulation method.

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Risk measures and their practical consequences

Jiří Málek[1], Tran van Quang[2]

Abstrakt

In this article, we focus on the most well-known and most widely used (downside) risk measures Var and Expected shortfall and recently more promoted Expectiles. In the first part, we refer to types of risk measures (coherent, spectral, elicitable). We also discuss the advantages and disadvantages of individual types of risk measures and give directions of further research

Key words

Risk measures, Value at Risk, Expected Shortfall, Expectiles

JEL Classification: C13, C51, G11, G32

1 Introduction

Risk management is the foundation of long-term existence of financial institutions such as banks, insurance companies, investment funds and others. But in order to manage the risks, we need to measure them. The risk can be expressed through the distribution function, but it is often necessary to express it with one number.

Artzner et al. in 1999 desirable properties of risk measures have been formalized in a set of axioms. Because VaR does not satisfy property of subadditivity, it has been replaced by Expected Shortfall (ES) in many financial institutions for risk management and, in particular, for capital allocation. The BIS also recommends replacing VaR by ES in internal market risk models. Recently, there has pointed out that there is a problem with direct backtesting of Expected Shortfall estimates because Expected Shortfall is not elicitable. Therefore, with a view on the feasibility of backtesting, Expectiles have been suggested as coherent and elicitable alternatives to Expected Shortfall.

In this paper, we discuss and compare the properties of VaR, ES and Expectiles from the view of coherence, elicitability, comonotonicity and spectral measures.

2 Classification of risk measures

In this part of the paper we present the basic types of risk measures: coherent, spectral and elicitable. We will only interest in risk measures that depend only on the distributions of the losses because their values can be estimated from loss observations only (i.e. no additional information like stress scenarios is needed) .

2.1 Coherent risk measure

Artzner et al. (1999) state four axioms which risk measure should satisfy. Such risk measures are then said to be coherent.

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Definition

Let X, Y are loss random variable (profits are negative, loss positive). Risk measure ρ is said be coherent if it satisfied

1) Monotonous: $X \geq Y \ \Rightarrow \ \varrho(X) \geq \varrho(Y)$
2) Positively homogeneous: $h > 0 \ \Rightarrow \ \varrho(hX) = h\varrho(X)$
3) Subadditivity: $\varrho(X+Y) \leq \varrho(X) + \varrho(Y)$
4) Translation invariant: $c \in R, \varrho(X+c) = \varrho(X) - c$

2.2 Comonotonicity

Real valued random variables Y_1, Y_2 are said comonotonic if there exists random variabĺe X and non-decreasing functions f_1, f_2 such that

$$Y_1 = f_1(X), \quad Y_2 = f_1(X).$$

A risk measure ρ is said comonotonicaly additive if for any comonotonic random variables Y_1, Y_2 holds that

$$\rho(Y_1 + Y_2) = \rho(Y_1) + \rho(Y_2)$$

Hence, if a risk measure is both comonotonically additive and subadditive , then it rewards diversification (via subadditivity) but does not attribute any diversification benefits to comonotonic risks.

2.3 Spectral risk measure

Spectral risk measures have certain advantages especially of possibility adapting this measure directly to the risk aversion of investors (or to their utility function)

Let random variable loss function X with cumulative distribution function $F(p)$. Spectral risk measure is given by

$$M_\phi(X) = \int_0^1 \phi(p) F_X^{-1}(p) dp$$

where $\phi(p)$ is nonnegative, nonincreasing function (called spectral density) such that

$$\int_0^1 \phi(p) dp = 1.$$

All spectral risk measures are coherent, but the opposite may not be hold.

Examples[3]

The following examples show the determination of spectral risk measures adapted to the exponential and power utility function.

1. Exponential utility function

$$U(x) = 1 - e^{-\alpha x}$$

[3] According Dowd et al. (2008). The authors concede that their determination of spectral density is rather heuristic.

$$\phi(p) = \frac{\alpha e^{-\alpha(1-p)}}{1 - e^{-\alpha}}$$

$$M_\phi(X) = \frac{\alpha}{1 - e^{-\alpha}} \int_0^1 e^{-\alpha(1-p)} F_X^{-1}(p) dp$$

2. Power utility function

$$U(x) = \frac{x^{1-\gamma} - 1}{1 - \gamma}$$

$$\phi(p) = \gamma(1-p)^{\gamma-1}$$

$$M_\phi(X) = \int_0^1 \gamma(1-p)^{\gamma-1} F_X^{-1}(p) dp$$

2.4 Elicitable risk measures

Simply, elicitable risk measure is funcional T on a set of probability measures M (which can be determined by set of random variables) if it can be defined as the minimizer of a suitable scoring function.

Definition

Let $T: M \longrightarrow 2^R$ (where 2^R is power set on real line) be set–valuable functional. Functional T is elicitable relative to the class M if there exist scoring function

$S(x, y): R^2 \longrightarrow [0, +\infty)$ such that

$$T(F) = \arg\min_x \int S(x, y) dF(y) \text{ for } \forall F \in M$$

Scoring functions may be understand as combinations forecasts and realizations . For example,let us denote with y the stock price realization at time t and x with its forecast, suitable scoring functions may be

the mean square error: $(y - x)^2$,

the relative mean square error: $\left(\frac{y-x}{x}\right)^2$.

Elicitable risk measures are coherent but not spectral.

3 Value at Risk, Expected Shortfall and Expectiles

VaR has become a standard risk measure in finance. But it has been criticised for lacking subadditivity which means that the (diversified) portfolio may have a higher risk (VaR) than the sum of its individual parts.

Value at Risk (VaR) at the level $\alpha \in (0,1)$ is defined by

$$VaR_\alpha(Y) = inf\{x \in \mathbb{R} | F_Y(x) \geq \alpha\}$$

where Y is loss random variable with cumulative distribution function $F_Y(x)$.

VaR_α is not spectral risk measure (but it is comonotonic) however it can be expressed using Dirac delta function (that gives the outcome $p = \alpha$ infinite weight and zero weight to others) as weighting function $\Phi(p)$. But this function is not nonincreasing. VaR is also elicitable as

$$VaR_\alpha(X) = \underset{x}{\operatorname{argmin}}\ \alpha\, E\big[(X-x)_+ + (1-\alpha)\, E[(X-x)_-]\big]$$

Expected shortfall (ES), has been proposed as an alternative to VaR risk because it has a subaddition property. However, it is often criticized for computational difficulty, limited backtesting capabilities and high sensitivity to extreme data (lack of robustness. Expected shortfall sometimes called conditonal or average value at risk (CVaR)[4] is defined by

$$ES_\alpha(Y) = \frac{1}{1-\alpha}\int_\alpha^1 VaR_s(Y)ds$$

ES is comonotonic and spectral measure, with the spectral function

$\phi(p) = p\ \ for\ \ p \in (0,\alpha)$ and $\phi(p) = 0\ \ for\ \ p \in [\alpha,\infty)$.

ES is not elicitable, which explains difficulties with robustness and backtesting[5].

Expectiles

Definition

$$e_q(X) = \underset{x}{\operatorname{argmin}}\, q\, E[(X-x)_+^2] + (1-q)E[(X-x)_-^2]$$

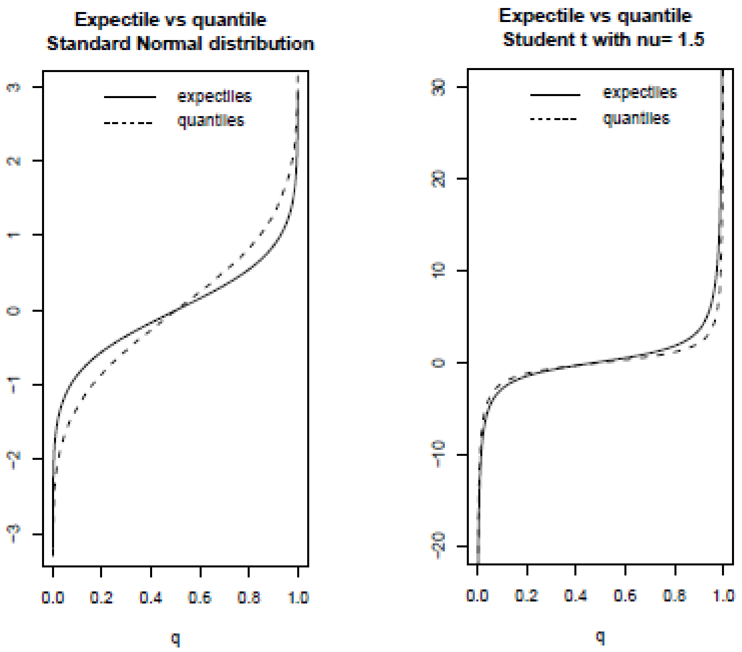
where $x_+ = \max(0,x), x_- = \max(0,-x)$ and $\alpha \in (0,1)$.

Expectiles are similar to quantils and for alpha = 1/2 the given expectile is equal to the expected value (see Figure 1). Ziegel (2013), showed that expectiles are the only elicitable coherent risk measure. However, is not comonotonic.

[4] These two terms (ES and CVaR) are not exactly the same but they are identical for continuous distributions.

[5] I tis possible to find procedures for evaluating ES forecasts and backtesting; see for example McNeil and Frey (2000); Christoffersen (2003). But these methods do not allow for a direct comparison and ranking of competing forecasting methods (Gneiting, 2011).

Fig. 1: Comparison between quantiles and expectiles for some distributions (Source Bellini and Bernardino 2015)

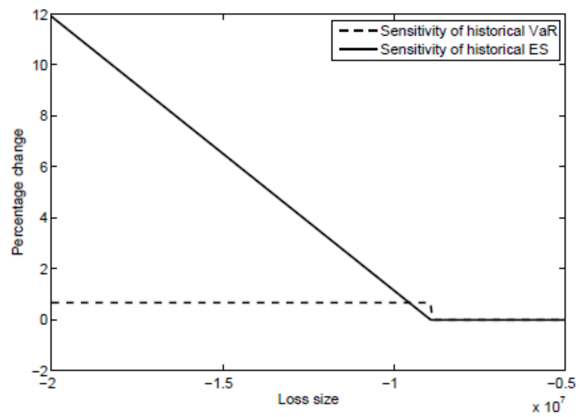


4 Robustness

Important issue when estimating risk measures is robustness. Without robustness,results are not meaningful, since then small measurement errors can have a big impact on the estimated value of risk.

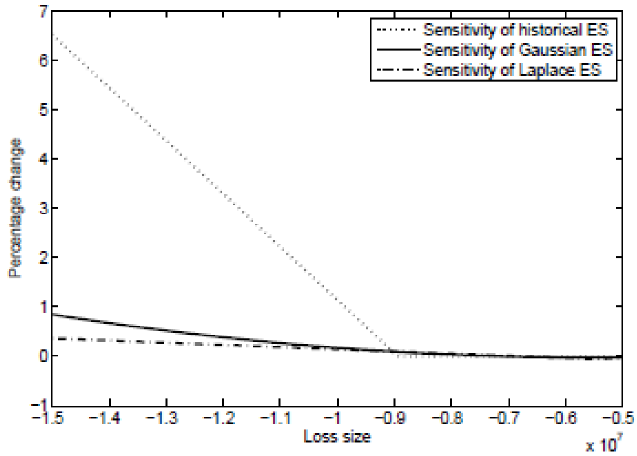
There are two possibilities how to estimate risk measure The first one is to estimate it directly from the empirical distribution, the second option is based on the approximation of empirical distribution by some known distribution, and then to calculate the risk measure using this estimated distribution. Cont et al. (2007) examined the robustness of VaR and ES (see Figure 2)

Figure. 2: Empirical sensitivity (in percentage) of the historical VaR99% and historical ES 99%. (Source Cont at al. 2007)



Further, it turns out that for the same risk measure the estimation method can have a significant impact on the sensitivity. The estimation of the risk measure can react in a different way on an additional data if we use approximate distribution instead of the empirical distribution (see Figure 3)

Figure 3: Comparison sensitivity ES using different distribution (Source Cont at al. 2007)



Cont et al. (2007 also show that there is a conflict between the subadditivity and robustness of a risk measure. These attributes cannot coexist together[6].

5 Discussion: VAR or ES or Expectiles

Danielsson et al. (2005) report that for most practical applications VaR is sub-additive and there is no reason to choose a more complicated risk measure than VaR, solely for reasons of subadditivity. Ibragimov and Walden (2007) showed that for very heavy-tailed risks diversification does not necessarily decrease tail risk and sometimes can increase it which makes the subadditivity requirement unnatural. According these authors (and others) these objections to the subadditivity deserve to be considered, and support the choice of robust risk measurement.

According to other authors, the main deficiency of Var (in addition to lack of subaditivity) is that it does not cover tail risks beyond VaR although it makes VaRa risk measure more robust than the other risk measures. This deficiency can be particularly serious when one faces choices of various risks with different tails.

ES makes good for the lack of subadditivity of VaR but it is not elicitable. This means that backtesting of ES is less straightforward than backtesting of VaR.. But there feasible approaches to the backtesting[7] of ES although to reach the same level of certainty more validation data is required for ES than for VaR.

[6] The concept of Cont et al. allows to distinguish between different degrees of robustness. However, this concept makes it hard to decide whether or not a risk measure is still reasonably risk sensitive or no longer robust with respect to data outliers in the estimation sample (according Emmer et al. 2013)

[7] One of the possible backtests (also recommended by BIS) is based on a subsequent approximation:

$ES_\alpha(X) \approx \frac{1}{4}\left[q_\alpha(X) + q_{0.75\alpha+0.25}(X) + q_{0.5\alpha+0.5}(X) + q_{0.)25\alpha+0.75}(X)\right]$

where $q_\alpha(X), q_{0.75\alpha+0.25}(X), q_{0.5\alpha+0.5}(X), q_{0.)25\alpha+0.75}(X)$ are corresponding quantiles. So we have to backtest them just like we do at VaR (see Acerbi 2002)

Expectiles have a number of attractive features but their underlying concept is less intuitive[8] than the concepts for VaR or ES. In addition, Expectiles are not comonotonically additive which implies that in applications they may fail to detect risk concentrations due to non-linear dependencies.

6 Conclusion

In our paper, we assumed that risk rates are determined by probability distribution. But the axiomatic approach based on coherence includes a much broader range of risk measures. Since, as shown by Cont et al., robustness and subadditivity are inconsistent with distribution based risk measures, it is advisable to focus on other types of risk measures. An example may be the SPAN[9] method used by many exchanges.

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Extracting Knowledge Problems from Economic Databases – an Application of SV Machine for Inflation Modelling

Dušan Marček[1]

Abstract

In Support Vector Machines (SVM's), a non-linear model is estimated. We investigate the quantifying of statistical structural model parameters of inflation in Slovak economics. Dynamic and SVM's modelling approaches are used for automated specification a functional form of the model. Based on dynamic modelling, we provide the fit of inflation over the period 1993-2003 in the Slovak Republic, and use them as a tool to compare their forecasting abilities with those obtained using SVM's method. Some methodological contributions are made to dynamic and SVM's modelling approaches in economics and to their use in data mining systems. The study discusses, analytically and numerically demonstrates the quality and interpretability of the obtained results. The SVM's methodology is extended to predict the time series models.

Key words

Support vector machines, data mining, learning machines, time series analysis and forecasting, dynamic modelling.

JEL Classification: C13, G32

1. Itroduction

In economics there are many situations where quantities under investigation are functionally related or related in a more obscure manner. Very often it is difficult, or even impossible, to distinguish the independent variables. Traditionally, the formulation of multivariate models based on the statistical approach requires the following steps: (1) the choice of independent variables; (2) specification of a functional form of the model; (3) analysis of a data set appropriate to the theoretical and functional form of the model; (4) model estimation, statistical testing and evaluation of the model´s forecasting capabilities. Input selection and functional specification of the model are of crucial importance to the successful development of multivariate models. Any multivariate statistical (regression) model must first be based on a theoretical foundation for the multivariate relationship. We are of the opinion that humans find it easy to say what causes what, but hard to put exact the measure of relationships among variables.

In contemporary statistical data mining systems, potential inputs are mainly chosen based on traditional statistical analysis. These include descriptive statistics, data transformations and testing. Input selection relies mostly on correlation and partial autocorrelation, cross-correlation analysis, cluster analysis, classification techniques, statistical tests, methods for finding the dependency structures among variables and other statistical tools (Jensen, Cohen,

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2002). Although all these tools are in reality linear, they are deemed to provide a useful tool for the determination of the input lag structure and the selection of inputs.

In economic and finance, where our understanding of real phenomena is very poor and incomplete, it seems to be more realistic and more useful instead of making unrealistic mathematical assumptions about functional dependency, to take the data as they are and try to represent the relationships among them in such a way that as much information as possible would be preserved. Very frequently, in such cases more sophisticated approaches are considered. These approaches are based on the human experience knowledge. They consist of series linguistic expressions each of which takes the form of an ´if ... then ...´ fuzzy rule and they are well known under the common name fuzzy controllers. But also, an expert is usually unable linguistically describe the behavior of economic processes in particular situations. Hence, most recent researches in the fuzzy controllers design for deriving of linguistically interpreted fuzzy rules have been centered on developing automatic methods to build these fuzzy rules using a set of numerical input-output data. For applying these methods it is supposed that a database describing previous input-output behavior of a system is available. Majority of these models and data-driven techniques relies on the use of Takagi-Sugeno type controllers and fuzzy/non-fuzzy neural networks clustering/fuzzy-clustering and genetic algorithm approaches In (Marcek, 2000), it is shown how to use and how to incorporate both fuzzy sets theory and computational networks to determine the fuzzy relational equations from the database for inflation modelling.

2. Automated Modelling

The strategy for selecting an appropriate model is based on so called a "specific to general" methodology (Holden, 1997). This strategy is well known under the common name Dynamic Modelling in Economics (DME). The DME methodology leads to two stage modelling procedure. In the first stage the researcher use simple economic theory or can incorporate some prior knowledge which might be used to formulate and estimate a model and, if found to be unsatisfactory, in the second phase is generalized until it is acceptable.

Next, we will demonstrate these phases for modelling economic time series, say inflation which may be explained by the behavior of another variables. According to the inflation theory (Phillips, 1957), the variable inflation is explained by the unemployment rates and wages. In this section we will present the DME approach in the modelling and investigating of the relationship between the dependent variable of inflation measured by *CPI* (Consumption Price Index) and two independent variables the unemployment rate (*U*), and aggregate wages (*W*) in the Slovak Republic. In the next section the SV (Supper Vector) regression (SVM´s method) is applied. Finally, the results are compared between a dynamic model based on statistical modelling approaches and the SV regression model.

To study the modelling problem of the inflation quantitatively, the quarterly data from 1993Q1 to 2003Q4 was collected concerning the consumption price index *CPI*, aggregate wages *W* and unemployment *U*. These variables are measured in logarithm, among others for the reason that the original data exhibit considerable inequalities of the variance over time, and the log transformation stabilizes this behavior. Fig. 1a illustrates the time plot of the *CPI* time series. This time series shows a slight decreasing trend without apparent periodic structure. Using simple economic inflation theory the model formulation may be

$$CPI_t = \beta_0 + \beta_1 W_t + \beta_2 U_t + u_t \qquad (1)$$

where u_t is a white noise disturbance term, $\beta_0, \beta_1, \beta_2$ are the model parameters (regression coefficients). Using time series data the model is estimated as

$$C\hat{P}I_t = 11.3302 - 1.355 W_t + 1.168 U_t \qquad R^2 = 0.374, \quad \mathrm{DW} = 0.511 \tag{2}$$

Model (2) is not satisfactory. The estimated coefficients do not have the correct signs, there is evidence of first order positive autocorrelation. The quantity R^2 is called the coefficient of determination. It is often used to judge the adequacy of a regression model ($R^2 \in \langle 0,1 \rangle$). The model does not well fit the data inside the estimation period. R^2 is often referred loosely as the amount of variability in the data explained or accounted for by the regression model (only 37 percent of the variance in CPI_t is explained by the model).

There are various methods and criteria for automated selecting the lag structure of dynamic models from a database. As we have mentioned above, auto-regression, partial auto-regression and cross-correlation functions can provide powerful tools to determine the relevant structure of a dynamic model. So after the data are transformed, the first thing to do is to perform a suitable differencing of the input and output series, and to analyze autocorrelation, partial autocorrelation and cross-correlation function of the series to produce an appropriate model of the input-output series. In this procedure the orders of AR processes are usually determined within the procedure itself using an information criterion, e.g. Bayesian or Akaike information criterion (Akaike, 1974).

The AR model is also known as the pole-zero model in signal processing. In practice, input-output signals are often non-stationary, i.e. the statistical autocorrelation and cross-correlation sequences and hence their estimates vary with time. In this case, the process of calculating AR parameters is repeated as each new data point becomes available to track the changing statistics. Adaptive estimation methods for AR models have been developed as a result of adaptive filtering applications. Detailed knowledge can be found in (Bayhan, 2000) .

Experimenting with these methods (Adamca, Marcek, Pancikova, 2004), the following reasonable model formulation was found

$$C\hat{P}I_t = 0.5941 - 0.0295 W_{t-1} - 0.00359 U_{t-1} + 0.84524 CPI_{t-1},$$
$$(0.229) \qquad (0.3387) \qquad (0.1035) \qquad R^2 = 0.7762 \tag{3}$$

where the standard deviations of the model parameters are presented in parentheses.

Fig. 1a: Natural logarithm of quarterly inflation January 1993 to December 2003

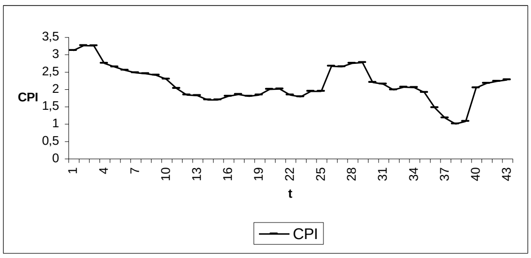
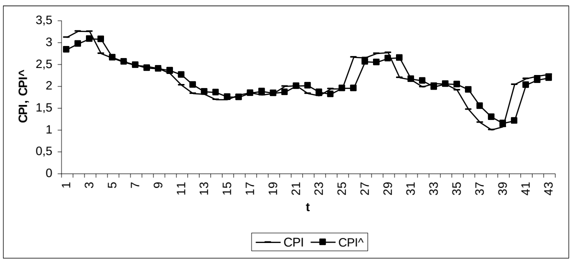


Fig. 1b: Natural logarithm of actual and fitted from inflation values (models (3), (4))



The model specification (3) is the lagged dependent variable model in which the dependent variables, lagged one period, appear as independent explanatory variables. This model is based on a distributed lag model (Banerjee et al., 1993). One of the primary reasons for using this functional form is to determine the long-run response of dependent variable to change in each of the independent variables. In model (3), the standard deviation values 0.229 and 0.3387 show that the parameters β_1, β_2 are statistically insignificant at five percent level. In addition they have not the appropriate magnitude. In all probability the lagged dependent variable CPI_{t-1} substitutes for the inclusion of other lagged independent variables (W_{t-1}, U_{t-1}). The inclusion of CPI_{t-1} and W_{t-1}, U_{t-1} is redundant and over-specified the model. A graph of the historical and the fitted values for inflation is presented in Fig. 1b. The model follows the pattern of the actual very closely.

Finally another attempt was made supposing a more sophisticated dependence of current inflation on the previous observation performed with the help of SV regression. This approach is presented in the next section. As it is well known, we can not have a model in which the coefficients are statistically insignificant. We made an arbitrary decision which deleted "insignificant" explanatory variables. Then the equation (3) becomes the first-order autoregressive process, i.e.

$$C\hat{P}I_t = \beta_0 + \beta_1\, CPI_{t-1} = \underset{(0.158)}{0.292} + \underset{(0.072)}{0.856}\, CPI_{t-1}. \qquad R^2 = 0.776 \tag{4}$$

Statistical modelling approach based on dynamic models have found extensive practical application. These models naturally arise in areas where either a correlative or causal structure exists between variables that are temporally or spatially related. These models are also useful in many types of process and quality-control problems and everywhere, where the value of the dependent variable at time *t* is related to the adjustment to the controllable process variables at previous time periods *t -1*.

3. Causal Models, Experimenting with Non-linear SV Regression

In this section we will discuss the problem of selecting the appropriate functional form of the SV regression model (Vapnik, 1998). Given a training set of *n* data points $\{y_i, x_i\}_{i=1}^{n}$ where $x_i \in R^n$ denotes *i*th input, and $y_i \in R$ is the *i*th output. The support vector approach aims at constructing a function of the form ¨

$$f(\mathbf{x}) = \sum_{\substack{i=1\\ j=1}}^{n} (\alpha_i - \alpha_i^*)\psi(\mathbf{x}_i\mathbf{x}_j) + b, \tag{5}$$

where $\alpha_i - \alpha_i^*$ are positive real constants and *b* is real constant, $\psi(./.)$ is so called kernel function.

As a solution in a dual space by applying the Karush-Kuhn-Tucker condition, the real constants are obtained from the solution of the following quadratic programming (QP) problem

$$L_d(\boldsymbol{\alpha}, \boldsymbol{\alpha}^*)\ \max_{\alpha, \alpha_i^*} \quad -\frac{1}{2}\sum_{i,j=1}^{n}(\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*)\psi(\mathbf{x}_i^T\mathbf{x}_j) - \varepsilon\sum_{i=1}^{n}(\alpha_i + \alpha_i^*) + \sum_{i=1}^{n} y_i(\alpha_i - \alpha_i^*) \tag{6}$$

subject to constrains

$$\begin{cases} \dfrac{\partial L_p}{\partial w} = 0 & \rightarrow \mathbf{w} = \sum_{\mathrm{i=1}}^{\mathrm{n}} (\alpha_i^* - \alpha_i)\psi(\mathbf{x}_i), \\ \dfrac{\partial L_p}{\partial b} = 0 & \rightarrow \sum_{\mathrm{i=1}}^{\mathrm{n}} (\alpha_i^* - \alpha_i) = 0, \\ \dfrac{\partial L_p}{\partial \xi_i} = 0, \dfrac{\partial L_p}{\partial \beta_i} = 0 & \rightarrow 0 \le \alpha_i \le C, \ \ i = 1,...,n, \\ \dfrac{\partial L_p}{\partial \xi_i^*} = 0, \dfrac{\partial L_p}{\partial \beta_i^*} = 0 & \rightarrow 0 \le \alpha_i^* \le C, \ \ i = 1,...,n \end{cases} \tag{7}$$

where $L_d(\boldsymbol{\alpha}, \boldsymbol{\alpha}^*) \max_{\alpha, \alpha_i^*}$ is the Lagrangian with Lagrange multipliers given by $\alpha_i, \alpha_i^* \ge 0; \beta_i, \beta_i^* \ge 0$; ξ_i, ξ_i^* are so called slack variables(for details see (Suikens, 2000).

After calculating Lagrange multipliers pairs $(\boldsymbol{\alpha}, \boldsymbol{\alpha}^*)$, can be find an optimal desired weights vector of the regression hyperplane as $\mathbf{w} = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*)\mathbf{x}_i$ and an optimal bias b of the regression hyperplane as $b = \frac{1}{n}\left(\sum_{i=1}^{n} (y_i - \mathbf{x}_i^T \mathbf{w}) \right)$.

In the SV regression, to estimate its parameters, the user must further choose some attributes that affect their estimates. These are the following attributes: measure of error approximation (Loss Function) (ε), the regularization and weights vector norm, kernel function ψ and its degree (for further details see (Vapnik, 1998). In Table 1 some attributes od SV regression and the corresponding approximation rates (RMSE) are calculated. In Figure 2 the quarterly nominal wages of the SR population for the period 1991-2005 by using different types of kernel functions are graphically depicted with forecasting performance measures.

Table 1: SV regression results of three different choice of the kernels and the results of the dynamic model on the training set (1993Q1 to 2003Q4). In two last columns the fit to the data and forecasting performance measures are analyzed. See text for details.

Fig.	MODEL	KERNEL	σ	DEGREE-*d*	LOSS FUNCTION	R^2	*RMSE*
2a	causal (5)	Exp. RBF	1		ε - insensitive	0.9711	0.0915
2b	causal (5)	RBF	1		ε - insensitive	0.8525	0.0179
2c	causal (5)	RBF	0.52		ε - insensitive	0.9011	0.0995
2d	causal (5)	Polynomial		2	ε - insensitive	0.7806	0.0382
2e	causal (5)	Polynomial		3	ε - insensitive	0.7860	0.0359
2f	time series (6)	RBF	0.52		quadratic	0.9999	1.1132
1b	dynamic (4)					0.7760	0.0187

The use of an SV machine is a powerful tool to the solution many economic problems. It can provide extremely accurate approximating functions for time series models, the solution to the problem is global and unique. However, these approaches have several limitations. In general, as can be seen from equations (6), (7), the size of the matrix involved in the quadratic programming problem is directly proportional to the number of observations inside the estimation period. For this reason, they are many computing problems in which general quadratic programs become intractable in their memory and time requirements. To solve these problems they have been introduced many modified versions of SVM´s (Suykens, 2000).

4 Forecasting with SV Regression Models

Unfortunately, the SVM´s method does not explicitly define how the forecast is determined, the point estimates of the fitted model are simple values without any degree of confidence for the results.

Despite this fact, the point estimates for the large data sets can be calculated. The entire data set is partitioned into two distinct data sets: the training data set, i.e. the sample period for analysis, and the validation data set as the time period from the first observation after the end of the sample period to the most recent observation. The parameters *C*, σ, ε must be tuned as follows.

First an SV machine is estimated on a training set by solving (6) and (7). Secondly, the performance is evaluated on a validation set. The parameter set with the best performance on the validation set is chosen. With these parameters, the point estimates of the ex-post forecast may be calculated by simply putting the value of the vectors $\mathbf{x}_j^v$ and $\mathbf{x}_i$ into the following SV regression

$$\hat{f}(\mathbf{x}) = \sum_{\substack{i=1 \\ j=1}}^{n} (\alpha_i - \alpha_i^*) \psi(\mathbf{x}_i \mathbf{x}_j) + b \tag{8}$$

where *T* denotes the sample training period ends, τ is the forecasting horizon, i.e, the number of data points on the validation set, α_i, α_i^* are known real constants (Lagrange multipliers), *b* is known parameter (bias), $\mathbf{x}_i$ is a *T*-dimensional vector of the inputs, $f\hat{(\mathbf{x})}$ are the point estimates or forecasts of the series of y_t predicted at the point $\mathbf{x}_j^v = (x_1^v, x_2^v, ..., x_m^v)$, and $\psi(./.)$ denotes the admissible kernel function used in the fitting phase of the SV regression model.

An obvious limitation to the use of causal models is the requirement that independent variables must be known at the time the forecast is made. In our case, the new *CPI* value is correlated with the *CPI* value one quarter previous. This fact may be used by obtaining one-step ahead forecast of the *CPI* value operating on a moving horizon basis. Generally, we denote the current period by *T*, and suppose that we will forecast the series y_t in period *T* +1 (*T* = *n*, τ = 1). The forecast for future observation $C\hat{P}I_t$ is generated successively from the Eq. (10) by replacing the vector of the independent variable $\mathbf{x}_j^v$ with CPI_T. As a new observation become available, we may set the new current period *T*+1 to *T* and compute the next forecast again according to the Eq. (10). In situations where the independent variables are mathematical functions of time, the point estimates or forecasts of $CPI_{T+\tau}$ are just values of the Eq. (8) at the points $\mathbf{x}_j^v = (T+1, T+2, ..., T+\tau)$.

As three new observations become available, the summary measure the ex ante forecast summary statistics (RMSE) may be calculated. The RMSE statistics generated by each SVM´s and dynamic model (4) respectively are given in Tab. 1. As illustrated in Tab. 1, a curve fitted with many parameters follows all fluctuations (the R^2´s values increase) but is poor for extrapolation (the RMSE´s values increase, too, i.e. the forecast accuracy decreases). The model in Fig. 2b gives best predictions outside the estimation period and clearly dominates the other models. It should be pointed out that we are ranking the seven models within one category of the forecast summary statistics. This is not a statistical test between models, but one way of trying to determine subjectively which of the models generates best the data of the inflation process.

Figure 2: Training results for different kernels, loss functions and σ of the SV regression (see Table 1). The original functions (plus points), the estimated functions (full line), the ε-tube (dotted lines) are shown. Fig. 2a, 2c, 2d, 2e, 2f correspond to a good choice of the parameters, Fig. 2b corresponds to a bad choice

Fig.: 2a

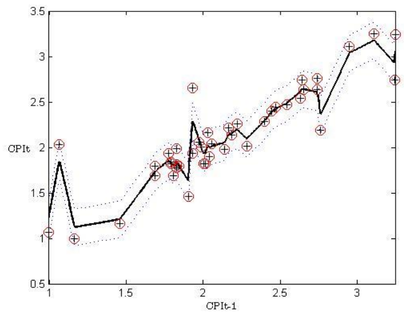


Fig.: 2b

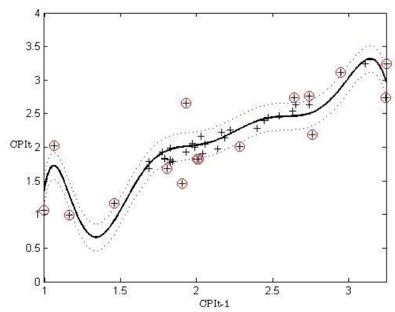


Fig.: 2c

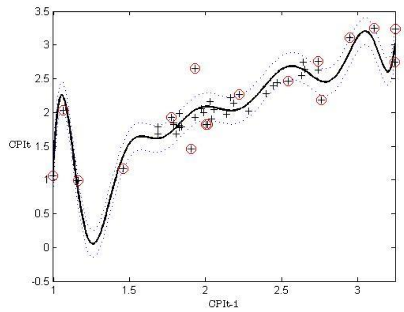


Fig.: 2d

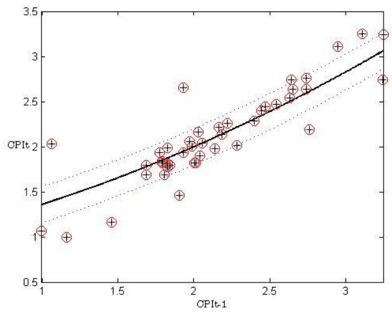


Fig.: 2e

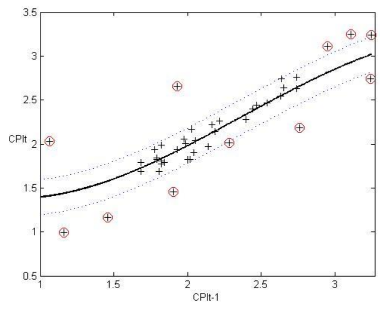
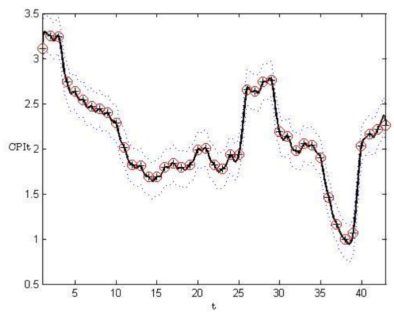


Fig.: 2f



5 Conclusion

In this paper, we have made some methodological contribution for SVM´s implementations to the causal statistical modelling and extended the SVM´s methodology for time series forecasting problems. We have shown that too many model parameters results in overfitting, i.e. a curve fitted with too many parameters follows all the small fluctuations, but is poor for generalization. Our experience shows that SV regression models deserve to be integrated in the range of methodologies used by data mining techniques, particularly for control applications or short-term forecasting where they can advantageously replace traditional techniques. Finally, with the fine tuning of so many SVM´s parameters so crucial to the final outcome, the successful use of SVMs for economic modelling requires a great deal of experience.

In conclusion, we refer to the knowledge about data mining written by Ken Holden (Holden, 1997). Economic theory gives little guidance to dynamic behavior. Whatever statistical approach is used is always open to the criticism of "data mining" – that is, finding relationships which are essentially spurious rather than structural. As the amount of data observed increases, so does the opportunity for adopting more objective procedures based on statistical testing. However, it is not possible to "let the data are speak for itself", since some theoretical structure is needed in order to decide what data are relevant and to eliminate some of the large number of alternative models which are available. The way forward must involve economic theory, data and statistical tests.

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Using Early Warning System in Small and Medium Sized Enterprises

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Abstract

At present, the importance of social responsibility is growing. It is part of it to act as both its partners and outsiders responsibly and adhere to individual business attributes. Companies are increasingly measuring and evaluating their performance, using various tools and methods of financial diagnostics. It is very important to choose the right and key measures that have the required degree of verbal value and can bring relevant results about the financial health of the company. Financial analysis is most often done using a number of analytical methods and tools that can reliably detect both weaknesses and strengths in the business. In order to simplify the monitoring of risk factors, it is possible to use semi-automated financial diagnostics models, which, in regular evaluations, also reveal soft signals of changes in business activity. The submitted article deals with and specifies the individual architecture of the created model in MS Excel, which serves not only to identify and work with financial risks, but also perform the function of a database, which can lead to individual risk data. As an example of the possibilities of using the model, the particular enterprise to which the system has been applied was chosen.

Key words

Financial risks, Enterprise Risk Management, Early Warning System

JEL Classification: E22

1. Introduction

The importance of small and medium-sized enterprises in the national economy of Slovakia is significant. Small and medium-sized enterprises represented 99.9% of the total number of business entities in the territory of the Slovak Republic in the year 2016 [1,2]. For the economy of the Slovak Republic they are a very important elements providing job opportunities for more than three quarters of the active labour force.

The competitiveness of small and medium-sized enterprises is determined mainly by the environment in which they carry out their business activities. It is therefore essential to identify the changes on a regular and timely basis, as their impact will be reflected in the environment with time. For small and medium-sized enterprises, the need to analyze these changes, respond appropriately to them and prepare measures that enable them to overcome barriers and take advantage of opportunities to improve the business environment grows. In rural areas, businesses often face many difficulties, such as a lower average level of population literacy, higher average age of population and decreasing demographic trends, a poorly developed economic base, reliance on job attendance, low information rates, conservative population thinking, and underdeveloped services and infrastructure [3].

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Support for small and medium-sized enterprises has long been resonant in the European institutions. We are currently witnesses of various debates and attempts to support small and medium-sized businesses. Both the Government of the Slovak Republic and the Ministry of Economy periodically issue several calls for granting a non-repayable financial contribution in the form of subsidies. However, in order to obtain the subsidy, it is necessary to assess the environment and to find suitable gaps in which the business activities of these enterprises in Slovakia can be developed.

The current business environment is characterised by numerous changes, dynamics and constantly changing individual determinants of the business environment. These changes are increasingly affecting business activities and it is therefore necessary to measure and evaluate as well as the performance of the company as well as its financial health [4]. Just measuring and evaluating the performance of an enterprise is currently considered one of the biggest problems within the financial management of the small businesses. There are many problems in measuring and evaluating financial performance. One of them is the correct application of the business performance assessment and the analysis of the company's financial health.

Measuring enterprise performance doesn't mean end in itself. The goal is not to measure but to enable performance improvement. Measurement using certain indicators must be part of the ongoing program of analysis, assessment and improvement of operational performance [5].

The article highlights the importance of measuring and evaluating business performance and using financial diagnostics to identify individual risks in an enterprise. The main target of the article is to describe the proposed architecture of the model created in the MS Excel environment, which doesn't only identify individual financial risks, but also it can save them in the form of databases.

2. Theoretical background

Business performance is a multilateral term in various economic and social areas defined as the way or course in which the investigated entity performs a certain activity on the basis of similarity to the reference manner of performing this activity. Interpretation of enterprise performance characteristics assumes the ability to compare the survey and reference phenomenon with respecting the specified scale. If we applied this definition to a business and entrepreneurial position, business performance can be defined as a way of doing business on the basis of similarity with the reference framework of other business entities that engage in business. This is the comparison of the surveyed (compared) enterprise with the compared enterprises on the basis of the selected reference elements.

Performance can be evaluated by the customer, manager or contractor. From the customer's point of view, an enterprise's performance assessment is oriented to whether it can meet its individual needs and whether a company can flexibly respond to individual customer needs. But it's quite different from the managerial point of view. Manager examines the performance of the business in terms of its prosperity. The measure of enterprise prosperity represents the ability to adapt to changes in the environment and to achieve revenue (profit). From the supplier view, it measures the performance of the partner's business in terms of securing future relationships. In most cases, they are focused on the past, and the focus of their assessment can be seen in particular in the financial criteria. We consider the primary objective of traditional approaches to identification of performance indicators that allow comparison and using knowledge of trends, or serve to reveal reserves [7,8].

Business performance assessment is most often processed through ex-post analysis, while there is important to take into account also aspects of ex-ante analysis that can predict future business development and identify individual risks to individual business objectives. Assessing the performance of an enterprise becomes a risk management element that can be used to identify risks individually, or to properly identify the individual activation risks and symptoms that underlie the crisis in an enterprise. Based on well-defined upper and lower boundaries of selected indicators, it is possible to identify individual symptoms in advance and to ensure an early warning system in the company.

Financial analysis is most often used to measure business performance. Analysis of business health is often considered to be the primary tool of financial management in the enterprise. It provides a certain connection between the expected goals and the actual status. Financial analysis is closely linked to company accounting since individual financial statements represent an important source of information for financial decision-making. Financial Analysis "includes a review of past, present and future. It compares the results of several periods and, on that basis, evaluates the development trend of the enterprise. Its goal is to identify the financial health of the business, identify weaknesses and highlight strengths. Financial analysis serves for comparison in time, with other businesses or industry. The information it provides serves for creditors, shareholders, managers, investors, business partners, state institutions, employees, auditors and the public. These are mainly financial information, including financial statements, financial analysts and senior management forecasts, stock market intelligence, and other information such as official economic statistics, expert press reviews, independent evaluations, and forecasts." [9]

Kislingerova [9] distinguishes the business performance evaluation in three ways:

• Evaluation based on selected sets of indicators by assessing five areas, namely liquidity, activity, capital structure, profitability and market value (the advantage of applying the indicators in these areas is, in particular, their independence and the parallelism within the of the indicators)

• Evaluation of indicators sets that are organized into individual schemes based on key pyramid decompositions (e.g. DuPont's ROE decomposition)

• Evaluation of performance based on the use of an aggregate indicator, which can be simply described as a synthesis of partial indicators (predictive model).

3. Methodology

The assessment procedure can be applied when analyzing business performance and identifying individual risks, which can be identified from the point of view of the individual financial statements of an enterprise, defined by many authors differently. Hudymačová [7] considers important steps in particular:

• specification of individual business objectives,
• selection of individual financial and non-financial indicators,
• setting target values,
• analysis results.

Based on a number of authors [2, 6, 7, 8, 9, 10, 12, 14, 15, 17], it is possible to define a business performance measurement analysis approach in financial risks identification (Fig. 1)

• selection and quick definition of the compared object,
• selection of objects to which the subject can be compared,
• selection of individual indicators or sets of indicators,
• determination the scale of acceptability of indicators values,
• evaluation of analysis results and identification of individual risks.

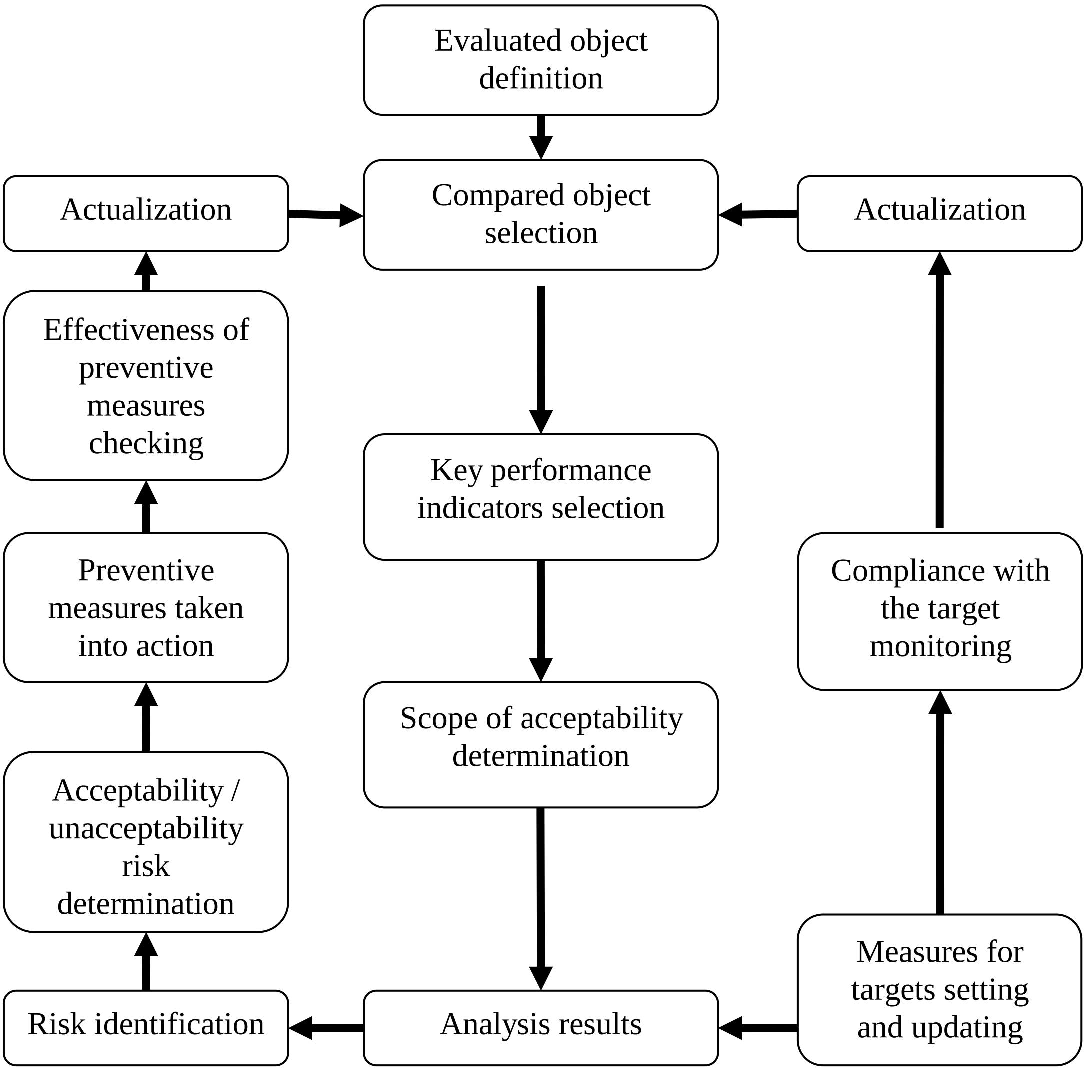

Fig.1 Proposed procedure of enterprise performance management and risk identification

4. Results

It is possible to divide the architecture of the proposed model into three main parts:

- Basic data is the starting point of the entire proposed model. It is necessary to enter input data into prepared fields, reflecting the architecture of balance sheet, profit and loss account according to the Decree of the Ministry of Finance of the Slovak Republic.
- By the using financial indicators, the model adequately evaluates individual financial indicators, based on pre-loaded data. However, it is also necessary to enter the range of values within the resulting company values are compared.
- The third section, Enterprise Risk Management, is focused on risk management and provides the user with a wide range of risk management functions.

  Part Basic data is divided into smaller parts, namely:
- Brief Business Characteristics contains items according to assessed enterprise can be identified. This section contains information such as: Business name, Company ID, VAT number, SK NACE classification and other important business details.

- Entry data provides input data and must be entered to ensure program functionality. The content of the templates reflect the Balance Sheet and the Profit and Loss Account issued by the Ministry of Finance of the Slovak Republic on 1.1.2017.

The Financial Indicators section consists of 8 parts:

- Liquidity and Earnings Calculation using Financial Liquidity Indicators (instant, current and total liquidity), and Earnings Indicators (EBIT, EAT, NOPAT, EBITDA). The IS will automatically highlight the green acceptability range and the red colour acceptability.
- Rentability, Debt Management, Asset and Liability Management - this section calculates financial assets from asset management and liabilities management. All these indicators are automatically compared based on industry-specific values.
- Horizontal and vertical financial analysis - in this section, the individual items of the balance sheet as both assets and liabilities are expressed in both vertical and horizontal analysis in the individual time periods entered into the given IS.
- Financial health of the company & creditworthiness - his section contains overview to the financial health of the company. It is evaluated through complex models such as: Altman's Enterprise Valuation Model, Kralik's Fast Test, Golden Balance Rule 1, Golden Balance Rule II, Gold Ribbon and the Credit Score.
- Cash flow of the enterprise - this section is directly devoted to the analysis of the cash flow of the enterprise (so-called Cash-Flow analysis). The cash flow is divided into three parts.
- Du Pont's decomposition of ROE - the pyramid decomposition of the ROE indicator. This decomposition is considered as the basic pyramid structure of the ratios.
- Report of Financial Indicators - deals with the summary as a final part of the whole chapter is the most important indicators that are needed in the decision making process of financial management as well as other managerial structures. The section includes: Company cash flow, Company liquidity, Business performance development, Asset and liability management tables, Company profitability, Company indebtedness, Altman model, Kralik's test, Gold balance rules, Evaluation index and selected items analysed over time (Horizontal and Vertical Analysis).
- Overview - assesses an enterprise based on individual parameters that are important for the financial health of the enterprise. In its calculations, it also reflects the individual characteristics of the given financial indicators and thus generates an overall evaluation of the company, as it relates to its overall success and financial performance as well as its profitability and liquidity. This model is based on the Creditwortnes model.

The Enterprise Risk Management section consists of 4 secondary sections:

- Enterprise risk management explains the basic terms and relationships between them. The user of the program will get closer to the meaning and application possibilities of the program. In this section, the different types of risks are further specified, suitably arranged and described. For the best and most effective work with the product, individual scales and risk matrices are predefined. The only thing the user has to do is to determine the percentage of the occurrence and the impact of the risks on the enterprise. Based on the elements, they will determine the individual boundaries of the items. The advantage is also to alert the user in case of inappropriately defined margin.

- Analysed Financial Risks - this part can be classified as the primary risk database. The user has 50 predefined risks in the database that can occur. User is also allowed to write additional risks into database. Thus, the user works at a complex risk by choosing or typing in the given cell field: the type of risk, the date of writing, the area of risk, the risk owner, the warning indicator, the probability label, the verbal description of the probability (generated automatically), the consequence, the value of the risk, the current risk management measures, the proposed measures, the re-estimation of the probability and the consequence of the risk, the estimated financial losses due to the risk, the date of the update of the status during the process of working with the risk.

- Problems encountered section is designed to work with problems within the enterprise. Keeps an overview of individual issues. Offers a selection to user or inserts information about the problem: type of problem, date of registration, area of occurrence, description, current effects, effect of the last edit date when the status of the problem was terminated.

- Risk Report provides the results of risk analysis to a user with wide range of information regarding individual risks and problems. It's the end of the whole model. Reports, figures and tables clearly show the individual risks and problems based on their multiplicity, type, current status, and so on.

5. Discussion

The issue of financial diagnostics is quite extensive and currently more and more preferred in the business area. This fact also arises from the requirements of company's executives to achieve individual results and goals in their business activities.

The benefits of the proposed model are based on easy accessibility, fast and very simple modifications, rapid response, 99% support for compatibility with IT.

The assumption of using the model is considerable. It can be used either in a business to assess individual strengths or weaknesses in terms of financial health, enterprise success and stability, but it also reveals the hidden locations and problems of partner's weaknesses to enable the better awareness in a company for possible financial problems of the business partner. Due to the fact that works exclusively with the items that are found in the Financial Statements Database of the Ministry of Finance of the Slovak Republic and are public, many partners need not have the knowledge that their financial health and the stability of the company are being checked.

The model itself works reliably, and it can be based on individual results calculated by the model that is reliable, easy to use and meets individual requirements that have been applied to the model.

In the future, it is expected to expand to a higher level where not only will be calculated the current items but it will expand the system processes. This prerequisite model to be able to analyze individual deviations in time and to become an effective early warning tool in an enterprise. In order for the user to be able properly combine the elements in the upcoming version, it is necessary to master this part, to be able to use it in a water way, as well as to interpret the individual items of analysis.

On the basis of the above, it can be stated that the model fulfilled both as primary and secondary objectives. In addition, it can be said that the model has been designed is a great foundation for further work and development, both in individual business areas and in public administration, to better control individual public finances and budget areas.

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Methodology for Measuring the Economic Potential of Financial Institutions

Adrianna Mastalerz-Kodzis [1]

Abstract

The concept of potential comes from physics, but it also applies to economics, spatial econometrics. In economic analyzes, inter alia, the spatial potential of territorial units is considered, in particular the potential of population and income. Then the size of the potential depends on the distance of individual territorial units, for example, geographical coordinates are taken into account in the research. The result of the analysis depends on the adopted metric. The aim of the article is to transfer the methodology taken from the spatial econometrics to the analysis of the organization's potential, showing the method of calculating the economic potential of the organization, including financial institutions. The article consists of two parts; the first discusses the research methodology, the second part contains the results of empirical analyzes.

Key words

economic potential, method of potential, organization potential.

JEL Classification: C1, C58, G20,

1. Introduction

Potential is defined in economics as a resource of capabilities, capabilities or performance that can be assigned to a given organization. The potential of both an organization, a company, as well as a region or area is a multidimensional concept, because it depends on many different, often dependent on each other factors. The potential determines the individual's capabilities based on the ability, competence and efficiency that the organization has, and measures the ability to use these resources and from the resources of the environment.

The economic potential has been defined, inter alia, in the works of Dutton (1970), Paelinck, Klaassen (1982), Rich (1980) and Suchecki (2010). In the work Mastalerz-Kodzis (2017), the use of the quotient of potential to support decision-making concerning the labour market, both by employers and employees, was discussed. In the article Mastalerz-Kodzis (2018) the potential of territorial units was analyzed on the example of Polish voivodships and simulation examples of organizational potential.

This article considers the application of a modified potential method for analyzes regarding the activities of various types of organizations (including enterprises, companies, wholesalers and stores, but also financial institutions). Each organization functions in a specific economic environment. Its mode of operation, efficiency, and the ability to generate profits depends both on the possibilities it has itself and on the relationship with the environment with which it cooperates. The concept of distance between organizations will then have economic significance. Its values are, for example, the turnover between organizations, the percentage of

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transactions, the size of financial flows, or another type of cooperation intensity meter. The purpose of the work is to construct the measure of the organization's potential and show its ownership, as well as to show the application of the potential method for the organization.

The work consists of two parts, the first of them presents the method of potential and the method of measuring the potential of the organization, the second part is empirical, contains the results of simulation analysis of calculating the economic potential of the organization.

2. The definition of potential

In physics, the potential is the auxiliary quantity used to describe the physical field. The field of physical quantities is a combination of functions characterizing space points and other parameters (eg time). For a given system of physical units $\left(i, j = 1, \ldots, n; n \in N\right)$, for example, the partial potential P_{ij} is used to analyze the interactions of units. For two arbitrary units, it is designated in accordance with the formula:

$$P_{ij} = \alpha_0 \frac{P_i P_j}{d_{ij}} \tag{1}$$

The value of the partial potential is interpreted as the energy quantity between units i and j per unit of mass P_i, P_j, then the mass of units, d_{ij} the distance between units, and the α_0 parameter is.

The organization's potential depends on many factors. In a simple simplification, it can be noted that, among other things, it depends on material resources, labor resources and financial capital as well as on the ability to manage these factors. The measurement of the organization's potential should therefore be made at various levels of resources and should constitute a synthetic variable of the discussed components. In addition, the organization always functions in a specific economic surrounding. Therefore, its potential is also dependent on this surrounding. However, there is a certain difficulty of computational nature associated with determining the distance between organizations. The article adopted the distance of economic nature, namely the intensity of cooperation measured by the amount of cash flows between organizations. The size of the potential (own and surrounding) depends on the distance taken.

The following signs are assumed in a given unit of time t (year, quarter, month):

$ZP_i(t)$ - the total potential of employees in the *i-th* organization,

$DP_i(t)$ - total profit potential in the *i-th* organization,

$P_i(t)$ - the quotient of potentials in the *i-th* organization,

$D_i(t)$ - the profit of the *i-th* organization,

$Z_i(t)$ - number of employees in the organization,

$d_{ij}(t)$ - intensity of cooperation between organizations *i* and *j*.

We can define the potential of employees (employment) and the potential of the organization's profit as the sum of own potentials and the surrounding:

$$ZP_i(t) = Z_i(t) + \sum_{j=1}^{n} Z_j(t) d_{ij}(t) \tag{2}$$

$$DP_i(t) = D_i(t) + \sum_{j=1}^{n} D_j(t) d_{ij}(t) \tag{3}$$

as well as the quotient of the potential for the *i-th* organization at the moment *t*:

$$P_i(t) = \frac{DP_i(t)}{ZP_i(t)} \tag{4}$$

In the spatial econometrics approach in the equivalents of formulas (2) and (3) there is a quotient of income and employment and distance, because according to Tobler's first right of geography, the smaller the distance between individuals, the stronger the mutual interaction. In this work, in formulas (2) and (3), it was proposed to calculate the product of profit and the number of employees and the intensity of cooperation, the higher the intensity of cooperation, the stronger the impact on other organizations. On the basis of the value of the potentials of total profit and employment and the quotient of the potential, organizations can be evaluated, rankings constructed for them, and the changes in the potential value over time can be assessed. In addition, the model of cooperation potential between organizations gives the opportunity to analyze the sensitivity to changes in the size characterized, it takes into account the impact of choosing the intensity of cooperation on the amount of profit.

3. The empirical analysis

Analyzing the method of potentials for the group of organizations, simulation examples for various intensity of cooperation were considered. It is assumed that the organizations cooperate with each other and the results of this cooperation are presented in tables 1-6. The considered organizations may be various financial institutions, but also their clients.

It was assumed as (d_1) the intensity of cooperation between organizations, calculated as (turnover) / 1 000 000 (the intensity of cooperation is shown in table 1).

Table 1: Intensity of cooperation between organizations– d_1.

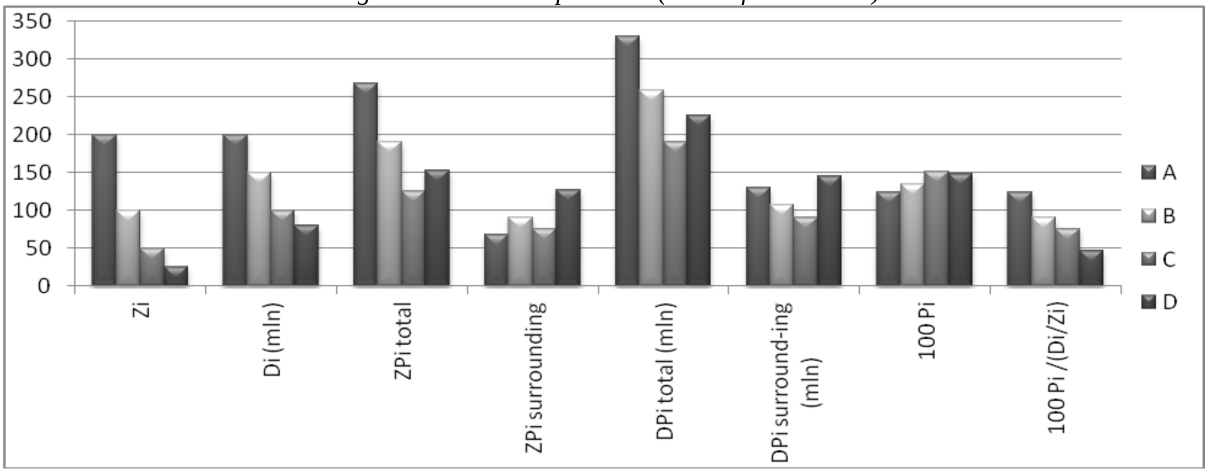
Intensity of cooperation	A	B	C	D
A	0	0.4	0.3	0.5
B	0.4	0	0.12	0.2
C	0.3	0.12	0	0.15
D	0.5	0.2	0.15	0

It turned out that the largest total potential is held by organization C, the greatest potential of employees and income is held by organization A, the greatest employee potential and environmental profit was noted for the organization B. The most dependent on the environment is organization A (table 2, value 1.2336), the most independent organization is D (table 2, value 0.4611).

Table 2: Profit and employment potential - data and calculation results for d_1, example 1.

Organization	Z_i	D_i *(mln)*	ZP_i *total*	ZP_i *surround ing*	DP_i *total (mln)*	DP_i *surround -ing (mln)*	P_i	P_i / (D_i /Z_i)
A	200	200	268	68	330	130	1.2336	1.2336
B	100	150	191	91	258	108	1.3508	0.9005
C	50	100	126	76	190	90	1.5109	0.7555
D	25	80	153	128	225	145	1.4754	0.4611

Figure 1: Economic potential (values from table 2)

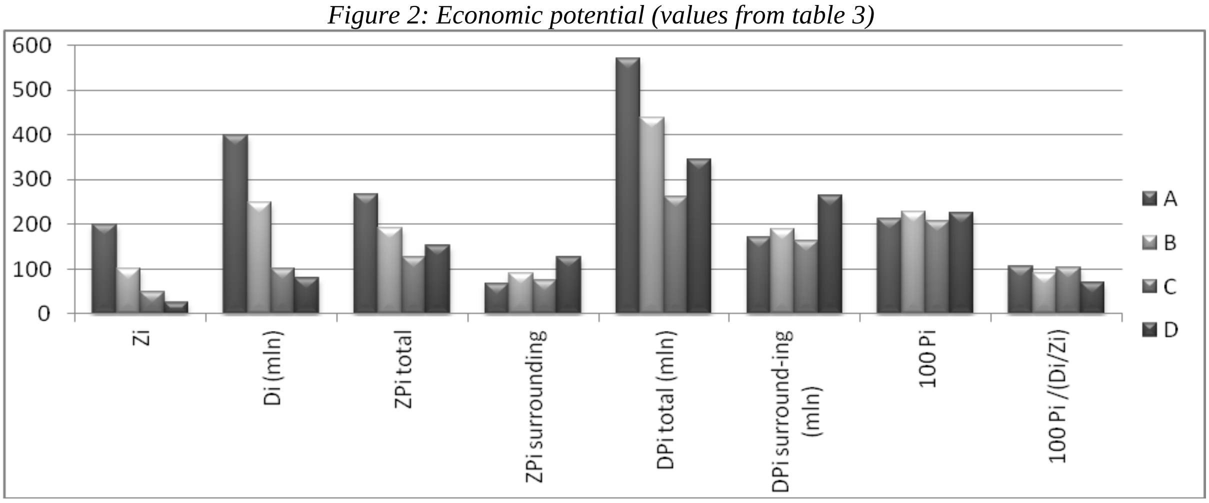


When the profit of the A and B organization changes with unchanged employment parameters, then these organizations will not be so much dependent on the other organizations. The situation is shown in table 3 (values in the last column).

Table 3 Profit and employment potential - data and calculation results for d_1, example 2.

Organization	Z_i	D_i *(mln)*	ZP_i *total*	ZP_i *surrounding*	DP_i *total (mln)*	DP_i *surround-ing (mln)*	P_i	$P_i / (D_i / Z_i)$
A	200	400	268	68	570	170	2.1269	1.0654
B	100	250	191	91	438	188	2.2932	0.9173
C	50	100	126	76	262	162	2.0794	1.0417
D	25	80	153	128	345	265	2.2549	0.7070

Figure 2: Economic potential (values from table 3)



However, if there is a change in the intensity of cooperation, for example, we assume that organization A cooperates strongly with other organizations, and organization B cooperates to a very small extent (d_2), then the analysis result is also changed (table 4, 5 and 6, figure 3 and 4).

Table 4 Intensity of cooperation between organizations - d_2.

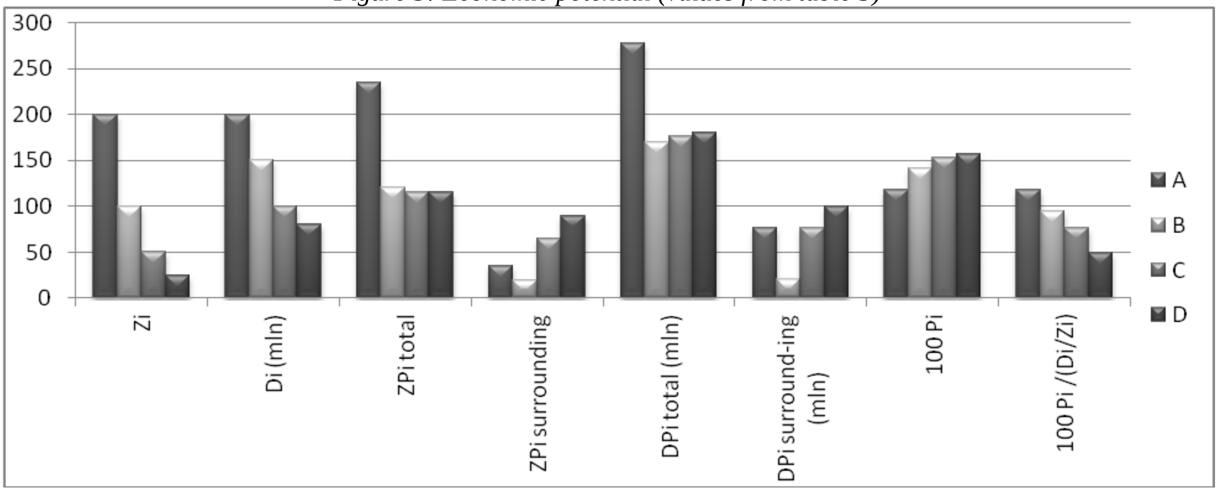
Intensity of cooperation	A	B	C	D
A	0	0.1	0.3	0.4
B	0.1	0	0.001	0.001
C	0.3	0.001	0	0.2
D	0.4	0.001	0.2	0

If there is a reduction in employment in organizations A and B with unchanged profit parameters, then these organizations will become more independent of other economic entities, their independence will increase. The situation can be observed by comparing the results in tables 5 and 6.

Table 5 Profit and employment potential - data and calculation results for d_2, example 3.

Organization	Z_i	D_i (mln)	ZP_i total	ZP_i surrounding	DP_i total (mln)	DP_i surrounding (mln)	P_i	$P_i/(D_i/Z_i)$
A	200	200	235	35	277	77	1.1787	1.1787
B	100	150	120	20	170.18	20.18	1.4182	0.9449
C	50	100	115	65	176.15	76.15	1.5317	0.7652
D	25	80	115	90	180.15	100.15	1.5665	0.4891

Figure 3: Economic potential (values from table 5)

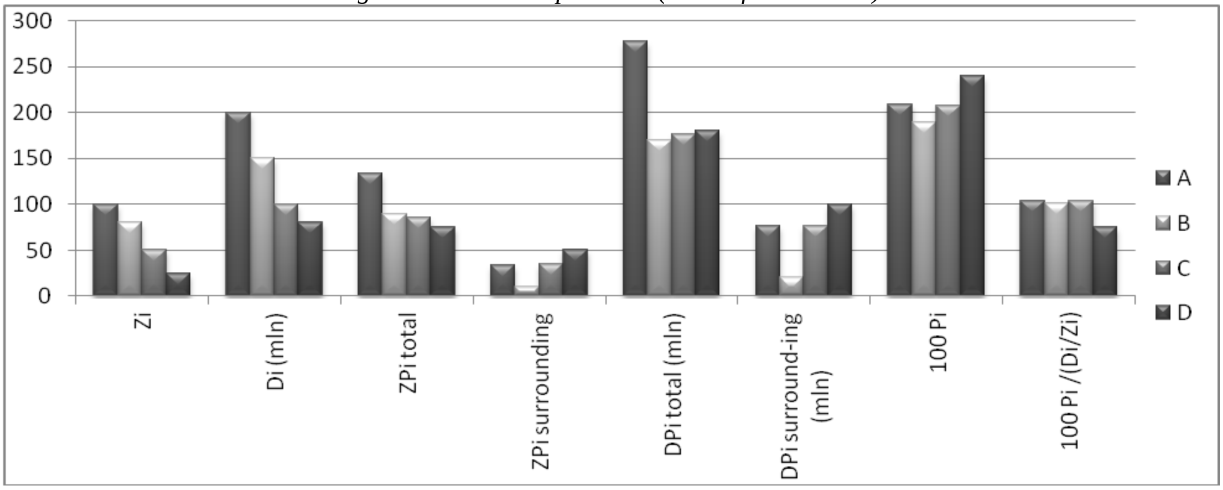


In the case of cooperation intensity d_2, C and D organizations have the greatest potential, the smallest organization A. Organizations C and D are least dependent on the surrounding (0.7652 and 0.4891), other organizations (considered in the simulation example) least affect their potential and economic success.

Table 6 Profit and employment potential - data and calculation results for d_2, example 4.

Organization	Z_i	D_i (mln)	ZP_i total	ZP_i surrounding	DP_i total (mln)	DP_i surrounding (mln)	P_i	$P_i/(D_i/Z_i)$
A	100	200	133	33	277	77	2.0827	1.0414
B	80	150	90	10	170.18	20.18	1.8909	1.0076
C	50	100	85	35	176.15	76.15	2.0724	1.0352
D	25	80	75	50	180.15	100.15	2.4020	0.7498

Figure 4: Economic potential (values from table 6)



It can therefore be concluded that the greater the economic potential, the less the organization is dependent on other organizations. This is evidenced by the results contained in the last two columns in tables 2, 3 and 5 and 6.

Analysis of the effects of cooperation between organizations from a given system is important from the point of view of profitability and efficiency of business operations. The method of measuring the economic potential of an organization is therefore an interesting and practically applicable instrument that can be a component of the management process of organizations, including financial institutions.

4. Summary

In this article, based on the proposed method of economic potential of the organization, an attempt was made to determine the possibility of the organization functioning in a specific economic environment. The use of the intensity of cooperation between organizations in the analysis process allows for large application possibilities of the proposed methodology, but also requires further empirical and simulation analyzes. It seems, however, that the proposed approach is interesting and has a wide range of applications.

The financial success of a business depends on its economic potential. The distribution of economic activity in a given territory is significantly influenced by the level of profit of individual entrepreneurs. For example, when considering a network of financial institutions in a given area, you can identify the potential of an institution as a turnover and refer to the number of clients. Depending on the metric used, the comparison of potentials gives different results. In the metric, you can include, in addition to the distance between the institutions, the online accessibility of the connected services and certain customer preferences.

Analysis of concentration and dispersion of economic activity using the potential method can be used to more effectively use the available forms of activity. It happens that excessive concentration of economic activity with a given profile has a negative effect on the achieved profits, while excessive dispersion prevents some types of activity from running. The analysis of the potential of the individual against the background, using the appropriate metrics, can increase the effectiveness of economic activity.

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Alternative measurement of elasticity

Jiří Mihola, Petr Wawrosz[1]

Abstract

The article investigates the relation of an alternative expression of elasticity by means of dynamic parameters of intensity and extensity. In the area of changes in prices and in the quantity, this method of measurement ascribes normalised shares of the effects in the interval -100% to +100%. This enables a much better economic interpretation than the commonly used elasticity which takes values from 0 to∞.

Key words

dynamic parameter, efficiency, intensity, extensity, elasticity

JEL Classification: L21, O12, O31, O33

1. Introduction

Complicated analytical tasks of a successful company include determination of effect of quantitative (extensive) and qualitative (intensive) development factors. The major problem of methodology consists in the fact that to express a share of effect of some factor on the development of a certain aggregate quantity requires an additive link at disposal. If the total revenue is a sum of achieved profit and total costs, then it is easy to calculate how the profit and the total costs contribute to the total revenue in a given period of time. However, the indicators of a qualitative change are highly varied. The change in quality often shows in a change in some intensive quantity such as density, speed or in economic efficiency or price. Historically, this topic has been developing especially at the national economic level as an equation of growth accounting which is not universal enough and its use is conditional on a number of serious prerequisites. The analysis of qualitative phenomena is usually made at the company level as well, but it is not based on exact algorithms. The usual basis is an additive relation among growth rates in which the multiplicative member is ignored. This methodology enables to distinguish:

- purely extensive development which is determined by a change in the scale of production at keeping the existing method of production by the company or the country or
- purely intensive development which is determined by intensive development factors such as technological progress, innovations, more efficient management and organisation of work, better utilisation of people's capabilities, etc.

In reality, a purely intensive development or a purely extensive one occurs only rarely. The development is mostly mixed, where both of the above-mentioned factors play their role in a varying extent. Also, it is not always a growth of the output, since the above factors can have an effect on an increase as well as on a decrease. In the article we present dynamic parameters which are universal in terms of potential changes and cover all the possibilities. In this text we deal with their application in the development analysis of production revenues with regard to price development and changes in the quantity sold. Traditionally, sensitivity of price changes or a change in the quantity sold is expressed by "elasticity" which represents sensitivity of a

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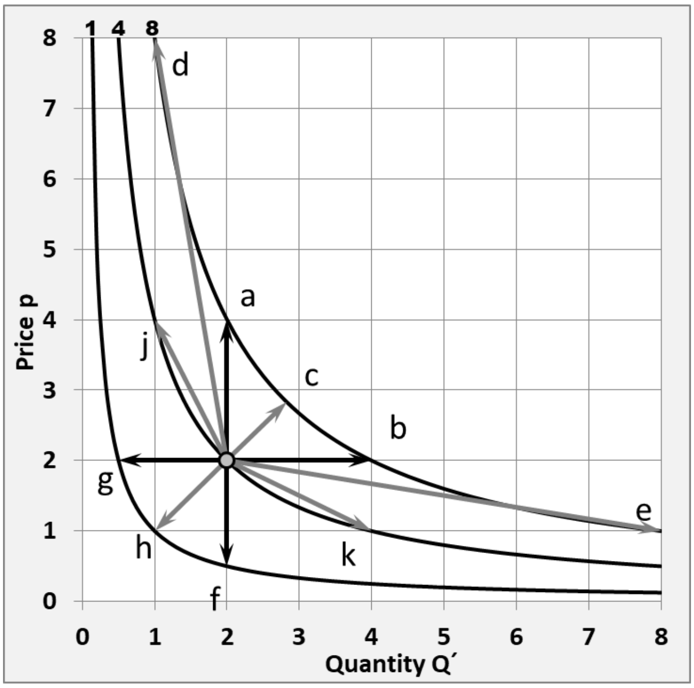
change in the demanded quantity of certain goods to changes in their price or, in other words, a relation between a percentage change in the demanded quantity of some goods and a percentage change in the price of demanded goods, and it is expressed as a ratio of a percentage change in the quantity of demanded goods to a percentage change in the price. The coefficient of price elasticity of the demand says by how many percent the demanded quantity will increase or decrease when the price decreases or increases by one percent. The aim of this paper is to compare the traditional relation for elasticity calculation with the relation for calculation of dynamic parameters and show why it is better to use the dynamic parameters proposed by us that enable a clear and unambiguous interpretation.

2. Share of price effect on revenues development

A revenues development analysis is a task that every entrepreneur faces. Despite that, neither a uniform terminology of all types of developments nor well interpreted parameters exist here which would unambiguously express the effect of changes in prices and the effect of changes in sold quantity on sales development. In the simplest example, revenues can grow either solely as a result of price growth, at a steady quantity sold or they can grow only as a result of growth of the quantity sold at steady prices. These developments when one of the considered factors is employed at 100%, whereas the other not at all are “pure basic developments”. In reality, revenues mostly develop under a simultaneous effect of both prices and sold quantity.

For illustration purposes, each basic development can be plotted on a market area where the horizontal axis shows the sold quantity Q´ and the vertical axis the price p. The starting point plotted here [2; 2] represents a sale of 2 pieces at 2 PJs (PJ = monetary unit) a piece. The revenues from the sale were 4 PJs. These revenues can be also obtained at different combinations of quantity and price. All the combinations are illustrated by a hyperbolic isoquant representing the revenues Tr = 4 PJs running through the starting point [2; 2]. A larger network of permanent revenues isoquants can be also defined. Graph 1 shows isoquants plotted for Tr = 1 and Tr = 8.

Graph 1 – Illustration of the basic types of revenues development



Source: Author

The isoquants of regular revenues from the sale are equiaxed hyperbolas, since

$$p = Tr / Q' \tag{1}$$

The formula follows from the trivial relation expressing revenues from the sale as a product of the sold quantity and the corresponding price. The arrows from the starting point [2; 2] stand for all the key, typical developments of revenues in terms of effect of changes in sold quantity Q´ and prices p. Graph 1 shows a pure revenue growth marked with the letter a. From the point [2; 2] we can get to the point [2; 4], which means that in the second period of time we sold 2 pieces again, however, this time at 4 PJs a piece, so we earned 8 PJs. Analogically, the pure quantity development of sales, marked with the letter b, illustrates that in the second period of time we sold 4 pieces at 2 PJs a piece, so we earned 8 PJs again. The pure developments also include some sales decreases, e.g. to Tr = 1. The decrease in the sales in the example f is caused by a decrease in the price at a steady sold quantity or in the example g by a decrease in the sold quantity at a steady price. The example c represents an increase in the revenues to a double at the same growth of the price change and at the growth of the sold quantity. The revenues growth can be also achieved at opposing developments of both factors, either by a partial offset of the sold quantity by a growth of price, i.e. the example d where we go from the point [2; 2] to the point [1; 8], or the price decrease partially compensated for by a growth of the sold quantity, which is the case of example e, where we go from the starting point [2; 2] to the point [8; 1]. The example h is analogical to the example c, since both factors have the same effect (decreasing to a half), this time, however, both on the revenues decrease. The last two examples represent full compensations at which the revenues do not change. The example j represents a full offset of the decrease in the sold quantity by a price increase. Similarly in the example k, the price decrease is fully compensated for by the growth of the sold quantity.

Table 1 – Typology of sales developments

	Name of the change Tr	Specification	Relation for indexes	Share of effect of Price v(p)	Share of effect of Quantity v(Q)	Elasticity e(p)
a	Purely price growth	Pure growth	I(p)>1, I(Q‘)=1	100%	0%	0
b	Purely quantitative growth	Pure growth	I(p)=1, I(Q‘)>1	0%	100%	∞
c	Price and quantitative growth	Mixed growth	I(p)=I(Q‘)	50%	50%	1
d	Price compensation growth	Partial compensation	I(p)<1, I(Q‘)<1	-33%	77%	-0.56
e	Quantity compensation growth	Partial compensation	I(p)<1, I(Q‘)>1	77%	-33%	-1.80
f	Purely price growth	Pure decrease	I(p)<1, I(Q‘)=1	-100%	0%	0
g	Price compensation	Pure decrease	I(p)=1, I(Q‘)<1	0%	-100%	∞
h	Price and quantity compensation	Mixed decrease	I(p)=I(Q‘)	-50%	-50%	1
j	Price compensation	Full compensation	I(p)=1/I(Q‘)	50%	-50%	-1
k	Quantity compensation	Full compensation	I(Q‘)=1/I(p)	-50%	50%	-1

Source: Author

All the described basic developments of revenues are in Table 1 which also lists the corresponding shares of effect of prices and those of the sold quantity on each basic development. The sum of absolute values of the shares of effect always equals 100%. If only one factor has an effect, it is logical that the share of this effect is 100%. If both factors have the same effect, the shares of such effect are the same figures. If both factors compensate for each other fully, the shares of effect are of the same value, but logically they have different signs, so their sum is 0%. The last column lists the calculated price elasticity which we will still discuss later. The relations used for calculating shares of effect will be construed to reach the values stated in Table 1 at the basic developments.

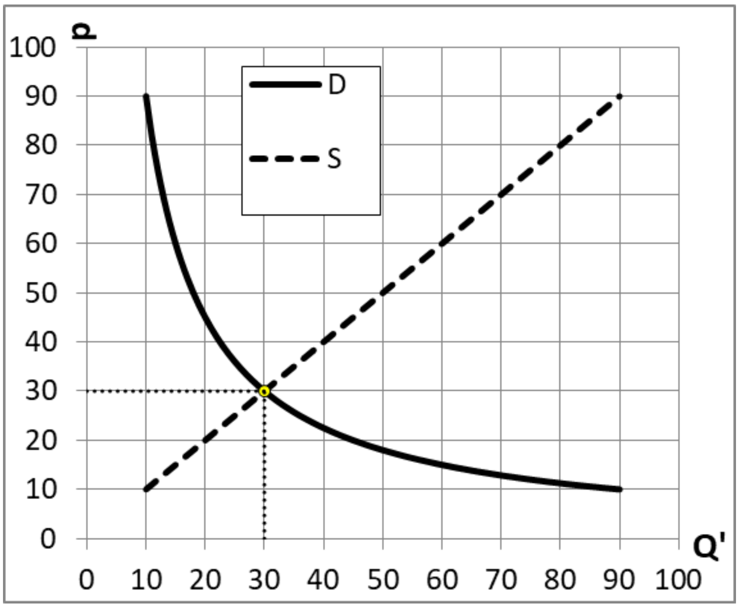
3. Price elasticity of demand and supply

Demand and supply meet in every market. Mathematically, we can simulate a market by a demand and supply curve. Both these curves are plotted on Graph 2, where the axis x shows the quantity Q' and the axis y the price p. We usually express the demand function by a decreasing function, since more consumers will buy cheaper goods. We usually express the supply function by an increasing function, since we presume that more entrepreneurs (companies) want to produce more expensive goods.

An example of the market simulation where we represent the demand curve by a part of hyperbola and the supply curve by a line segment is shown in Graph 2. Specifically here we have the demand curve for regular sales of 900 PJs

$$p = 900/Q' \tag{2}$$

Graph 2 – The market model with a hyperbolic demand and a linear supply curve.



Source: Author

The supply is a function of the direct proportion, in this case being represented by the identity $p = Q'$. Selected for both functions were the same domain $10 \leq Q' \leq 90$ and the range of the function $10 \leq p \leq 90$. The purpose of this selection is to employ such functions which will have constant price elasticity. The market balance occurs in the point [30; 30], i.e. for 30 units of quantity and the price amounting to 30 monetary units.

As a standard, sensitivity of a relative change of quantity to a relative change of price is expressed by price elasticity of demand or supply (hereinafter “elasticity”). Price elasticity is defined as a share of a relative change of quantity in a relative change of price, i.e. what percentage change in the sold quantity is caused by the change in price by 1%.

Demand elasticity was dealt with by e.g. Brons, Nijkamp, Pels, Rietveld (2008) and Dahl, (2012) who had been using various methods of examination in different parts of the world, e.g. North America (Lau, Ogucu, Suvankulov, 2012), South America (Hofstetter, Tovar, 2008), Middle East (Ben Sita, Marrouch, Abosedra, 2012) or in Europe (Pock, 2010), examining whether the revenues and the price elasticity itself differ among nations.

Relative change is usually defined as an increment of a given quantity divided by the value of the respective quantity at the initial moment. At elasticity calculation the increment of a given quantity is divided by the arithmetical mean of the preceding and the following values. The index *j* represents a serial number of the price or quantity in the sequence and the index *j*-1 is the preceding figure in this sequence. This special relative change *g* of the quantity and price is represented by these expressions

$$g(Q') = \Delta(Q')/((Q'j + Q'j\text{-}1) / 2) \quad \text{and} \quad g(p) = \Delta(p)/((pj + pj\text{-}1) / 2) \tag{3}$$

Elasticity is defined as follows

$$e(p) = g(Q') / g(p) \tag{4}$$

It is highly interesting that the values of elasticities and below-proposed dynamic parameters have the same isoquants, but their values will differ in the two methods under comparison. That means that, for example, the compensation hyperbolic isoquants in Graph 1 and in Graph 2 include constant elasticity, namely 1. Elasticity for pure developments caused solely by a change in the prices is 0, whereas it is ∞ for pure developments caused solely by a change in the quantity. The demand or even the supply curve with zero elasticity is part of a line which is in parallel to the axis y. As this demand or supply *Q'* does not respond to a change in prices at all, since it is independent of it, we call it perfectly inelastic. By contrast, the demand or the supply curve with infinite elasticity is part of a line which is in parallel to the axis x. Since such demand or supply can be of any size at a given price *p*, within the bounds of its domain, we describe it as perfectly elastic, because in this case the quantity is independent of the given price. Table 1 contains calculated values of elasticity for all the cases mentioned therein.

4. Dynamic parameters for price effect and effect of changes in the sold quantity on revenues development

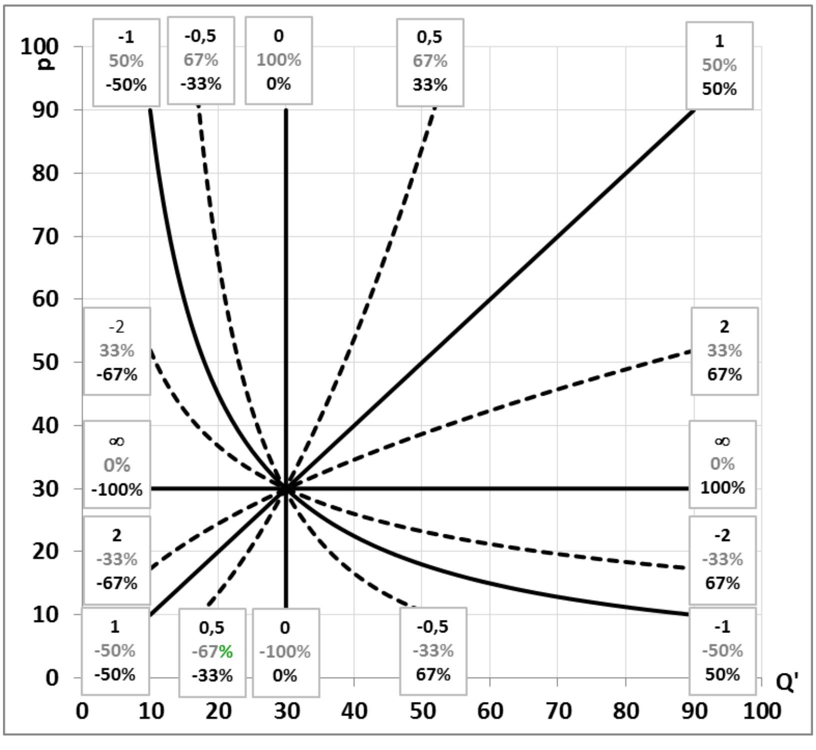
Furthermore, the relation between a relative change in prices p and a relative change in the quantity Q‘ is systematically dealt with by dynamic parameters that have been inferred by the authors of this article. In their universality these dynamic parameters express what kind of relation is between a relative change in prices and a relative change in the quantity along a given curve, e.g. a demand or supply curve. Their construction is the same as in case of an analysis of revenues from the perspective of effect of changes in prices and sold quantity or dynamic parameters expressing the share of effect of intensive and extensive factors on the development of outputs (or of the total income of the company TR or GDP). These parameters cover all the examples of potential movement in prices and quantity determined by Graph 1. The below-mentioned dynamic parameters of effect of changes in prices v(p) (5) and the share of effect of changes in the sold quantity v(Q´) (6) take values listed in Table 1 for the basic developments.

$$v(p) = \frac{\ln\ I(p)}{|\ln I(p)| + |\ln I(Q´)|} \tag{5}$$

$$\mathrm{v}(Q´) = \frac{\ln\ I(Q´)}{|\ln I(p)| + |\ln I(Q´)|} \tag{6}$$

The symbol *I(p)* means an index (change coefficient) of prices and *I(Q´)* means an index of the sold quantity. To compare values of dynamic parameters of effect of prices and effect of the quantity with values of elasticity, we intersect the starting point of the market balance defined in the previous section (i.e. the point [30; 30]) such curves which includes both elasticities and our dynamic parameters constant. That means that we complete Graph 2, already containing a hyperbolic demand curve and a linear supply curve, with a line of absolute flexibility in parallel to the axis x and a line of absolute inflexibility in in parallel to the axis y. These 4 curves divide the surroundings of the point [30; 30] into 8 segments. For better illustration of distribution of elasticity near the starting point as well as for better understanding the differences between elasticity and dynamic parameters, we put an additional, dashed isoquant of permanent elasticity in each segment.

Graph 3 – A model of a market by a demand and supply curve



Source: Our own calculation

All the curves are plotted on Graph 3. The end point of the isoquants includes three figures:

- Elasticity *e(p)* according to the relation (4)
- Share of effect of the change in the price *v(p)* according to the relation (5)
- Share of effect of the change in the *v(Q‘)* according to the relation (6)

To determine the sign for infinitive elasticity is very difficult, since it is true for an absolutely flexible elasticity line in parallel to the axis x that the values coming from left and right are very high positive and negative values. Therefore, the question is whether we calculate the limit on the right, or on the left.

It is evident from comparison of the values of dynamic parameters (5) and (6) with the results which are provided by elasticity (4) according to Table 1 and Graph 3 that the shares of effect $v(p)$ and $v(Q')$ have a very good economic interpretation in contrast to elasticity $e(p)$. Determining the percentage share of effect of changes in prices and effect of changes in the sold quantity, since these values complement each other and are standardised in the interval from -100% to 100%, whereas elasticity ranges in the interval from $-\infty$ to $+\infty$. The values of classical elasticities are not universal, since they neither distinguish whether both factors have a compatible effect, i.e. whether prices and quantity have an effect on a growth, or on a decrease, nor what kind of compensation it is, i.e. whether the change in the price offsets the change in the quantity or vice versa. With regard to elasticity it is also difficult to determine the sign of ∞ on absolute elasticity curves. All of the above-mentioned constitutes compelling practical arguments for introducing the calculation of elasticity of demand and supply curves using the relations for dynamic parameters (5) instead of the commonly used relation (4). The dynamic parameters are presented in detail by e.g. Mihola (2007), Hájek and Mihola (2009), Mihola, Kotěšovcová, Wawrosz (2015, 2017), Mihola and Wawrosz (2014).

5. Conclusion

This article shows that the systematic solution of the problem of evaluating quantitative and qualitative aspects of development by means of dynamic parameters can be used not only when intensity and extensity are measured at the level of the state or the company, but also in component analyses, e.g. in the analysis of revenues development from the viewpoint of effect of two factors, namely the effect of development and effect of sold quantity. It enables a very clear illustration of unambiguous, well-interpretable results and any reasonable time intervals can be selected for relevant indicators. For the purpose of analysing developments of both demand and supply curves, the above-mentioned dynamic parameters are much better interpretable than the commonly used elasticity. Where the so-far conventionally measured elasticity shows 0, i.e. a perfectly inelastic line in parallel to the axis x, the dynamic parameter we have introduced for the effect of quantity (extensity) is also 0 and the effect of price (intensity) is 100%. For the parallel line to the axis x, i.e. the perfectly elastic line where elasticity is∞, the dynamic parameter of effect of the quantity (extensity) shows 100% and effect of the price (intensity) is 0%.

Dedication

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Literature

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Predicting the Risk of Employee´s Long Term Absenteeism

Ondřej Mikulec [1]

Abstract

Absenteeism is becoming an issue with high importance for all companies increasing risks of decreased production, increased overtime work or higher costs for replacements. This work aims to present the analysis of factors which are influencing for absenteeism and estimate a predictive model that tells a likelihood of an employee to leave for a long-term absenteeism event for each employee within an organization. Logistic regression is used to predict individual risk of employee´s long term absenteeism leave using eight selected factors as predictors. It is concluded that the model has significant predictive ability with good proportion of variability explained by the model but lower specificity with correct classification of long term absent employees from the whole sample due to large number of unpredictable events possible to cause long term absenteeism.

Key words

Absenteeism, Bradford factor, logistic regression, sick leave prediction

JEL Classification: C53, M1, M54

1. Introduction

Proper work with quantitative data is important for each segment of a company including Human Resource Management (HRM). People analytics represents a data-driven approach to HRM and implementation of quantitative models in decision processing about company´s workforce planning (Armstrong Taylor, 2014). Absenteeism is becoming an issue with high importance for all companies and especially production companies. High costs following increasing absenteeism relate to decreased production, increased overtime work or recruitment for replacement and learning curve of the replacement and companies feel the necessity to set action steps and try to influence the outcomes instead of being just a passive player as is highlighted also in Mikulec (2017). People analytics already dealt with models predicting the likelihood of employee's leaving / turnover rate and this article aims to use similar method to predict the risk of employee´s long term absenteeism (LTA) leave. LTA is each event causing absence longer than three months. LTA is assumed to be binary variable, so an employee either is or was LTA or not.

Binary logistic regression is used to predict individual risk of employee´s long term absenteeism leave using selected factors as predictors. Data from 7387 employees from five production companies with information about their absenteeism are used. Data included in the predictive model are following:

1) Total amount of sick leave absenteeism events during last five years 2013-2017 (min = 0, max = 13),

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2) Bradford factor score for twelve consecutive months begging in January 2017 (min = 0, max = 2336). Bradford factor score is a managerial tool to evaluate each employee with a score based on the amount of sick leave events and total amount of absent days,
3) Age (min = 18, max = 65),
4) Seniority (min = 0, max = 48),
5) Category (0 = blue collar, 1 = white collar),
6) Gender (male = 0, female = 1),
7) Average performance evaluation for last five years (min = 1, max = 4,1),
8) Distance to work (0 = employee does not come from the same municipality as in which is his or her workplace, 1 = employee comes from the same municipality as in which is his or her workplace),
9) Long term absenteeism (0 = employee has not reached LTA status during 2012 – 2017, 1 = employee has reached at least once LTA status during 2012 – 2017).

LTA status is going to be predicted using other variables as independent in the model. Variable were chosen based on studies Georgios, Evangelos (2015), Hoonakkera, Duivenboodenb (2012), Ivancevich (1985), Schouteten (2017) or similar study focused on predicting turnover with same method as in Breukelen, Vlist, Steensma (2004). Having the estimation of one's risk of becoming LTA can be particularly useful in workforce planning. It can be also useful for being a step ahead and prevent possible production losses and consequently save considerable amount of money.

2. Methodology

Logistic regression will be used to estimate a model for LTA prediction. We assume LTA as binary event: either someone has been or currently is on a long-term sick leave or not. Bradford factor score is a managerial tool to assess one's absenteeism risk. It represents aggregated value combining the number of sick leave events and total length of sick leave in twelve consecutive months and is proposed to be used as one of independent variables to help explaining the variance in the independent variable LTA.

1.1 Logistic regression model

Logistic regression is a special form of regression analysis which is closely related to discriminant function analysis. The regression can have binary, ordinal or categorial outcome. Primary goal of logistic regression model is to estimate independent variables contributing to explanation of dependent variable in Meloun, Militký, Hill (2012).

Dependent variable in binary or categorial expression may acquire a limited amount of values. If the feature which is followed occurs than the expected value comes with probability π. The opposite value equals to $1 - \pi$. The value of chance can be expressed as $\frac{\pi}{1-\pi}$, where π represents the probability that given event or feature occurs. Logarithm of previous equation is equal to the value from interval (-∞; +∞). Logistic model can be expressed as

$$\pi(x) = \frac{e^{\beta_0+\beta_1 \cdot x}}{1 + e^{\beta_0+\beta_1 \cdot x}}. \quad (1)$$

where β_i is a parameter of each independent variable. $\pi(x)$ can be transformed by logit transformation with the domain (-∞; +∞) as following

$$g(x) = \ln\left[\frac{\pi(x)}{1-\pi(x)}\right] = \beta_0 + \beta_1 \cdot x \quad (2)$$

So called “logit” is created by this transformation as a result of weighted sum of independent variables which can have value from interval (-∞; +∞).

Hosmer, Lemeshow and Sturdivant (2000) and (2013) propose optimal maximum likelihood function which can be under assumption of independent observations in case of binomial distribution assuming independent observations can be described as following

$$l(\beta) = \prod_{i=1}^{n} \pi(x_i)^{y_i}[1-\pi(x_i)]^{1-y_i} \tag{3}$$

The objective of maximum likelihood method while estimating each parameter is to maximize the likelihood function. This function expresses a probability of obtaining a specific random selection. Logarithm of previous function can be expressed as following

$$L(\beta) = \ln[l(\beta)] = \sum_{i=1}^{n}\{y_i \ln[\pi(x_i)] + (1-y_i)\cdot \ln[1-\pi(x_i)]\}. \tag{4}$$

To find a value β maximizing $l(\beta)$, it requires to differ between β_0 and β_1 and sum up the equations to zero. These equations known also as likelihood equations can be expressed as

$$\sum[y_i - \pi(x_i)] = 0 \qquad \sum x_i \cdot [y_i - \pi(x_i)] = 0 \tag{5}$$

B value is given by solving equations (8) and (9). It is called maximum likelihood estimation and final logistic model is defined as following

$$\ln\left(\frac{L_{(1)}}{L_{(0)}}\right) = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \cdots + b_p \cdot x_p \tag{6}$$

Where $\ln\left(\frac{L_{(1)}}{L_{(0)}}\right)$ represents the logarithm likelihood ratio (logit) and b_i are individual regression coefficients.

1.2 Bradford factor score analysis

Bradford factor is represented by a simple formula that allows firms to apply a relative weighting to employee’s sick leave absence. Many companies use this factor as a trigger point to alert them if an employee’s absenteeism level is reach to worse or unacceptable level. Bradford factor score is well defined in People HR Blog online (cit. 2018). Bradford factor calculation is a combination of absence frequency and duration for giving and individual Bradford factor score. Bradford factor score is calculated as following

$$B = \left(\sum_{i=1}^{n} D_i\right)\cdot E^2, \tag{7}$$

where B represents Bradford factor score, D represents amount of absent work days in i-th absenteeism event and E represents an amount of sick leave absenteeism events during observed period.

Action steps which triggers when a certain limit of Bradford factor score can be set to create a clear sick leave policy. Online source of Employment law clinic (cit. 2018) presents certain action steps based on the level of Bradford factor score which were adjusted based on empirical evidence to set easier orientation. Flat limit levels described in Mikulec, Špačková (2017) are easier to monitor and grasp while they keep the value in identification of absenteeism risky employees.

Bradford factor score below 100 is acceptable and requires no action. This includes most of the employees, usually around 90%. Bradford factor score 100-249 represents a group of employees which already have a few sick leave absenteeism events or long-term illness and it is necessary to keep track of them to monitor whether they move up or down. Bradford factor score 250-499 already represents a potentially risky employee with necessity to issue a verbal warning. Bradford factor score 500-999 is very high and requires to be actively solved, we recommend issuing a written warning and discuss the situation of the employee with the supervisor. Bradford factor score over 1000 can be reached only by frequent repetitive absenteeism and it is recommended to terminate a contract with and employees with such score.

3. Prediction of risk of LTA

Logistic regression is used to predict individual risk of employee´s LTA leave using selected factors described in first chapter as predictors. Maximum likelihood method as described in chapter 2 is used to receive the best fit model. Independent variable LTA is binary, so estimated model is calculating the probability that LTA = 1.

Model is estimated with 7387 observations as in Tab. 1. In Tab. 1 we can also see that the LR chi2 as likelihood chi-square test statistic is high enough to confirm the amount of variability in LTA is significantly explained by used variables and Pseudo R-squared indirectly implies that the proportion of the total variability of the outcome that is accounted for by the model equals to 0.344. This represents a substantial amount of explained variability considering all possibilities that can lead to employee´s LTA event. Z-value for each variable lead to identification of which values contribute significantly and which do not. In Tab. 2 we see that variables Age, Work distance and also closely Gender are factors which do not contribute significantly to explain LTA on alpha level = 0.05 while all other variables account significantly.

Table 1: Logistic regression model estimation

Stat	Value
Observations	7387
LR Chi2 (9)	2126.92
Prob > chi2	0.000
Pseudo R2	0.344

LTA	Coef.	Std. Err.	Z	P > | Z |
Total events	0.701	0.028	25.19	0.000
BFS 2017	0.004	0.001	6.13	0.000
Age	0.004	0.006	0.69	0.490
Seniority	0.066	0.005	12.71	0.000
Category	-0.804	0.121	-6.62	0.000
Gender	0.237	0.121	1.96	0.051
Avg performance	-0.889	0.124	-7.19	0.000
Work distance	-0.042	0,082	-0.52	0.606
Constant	-1.728	0.439	-3.94	0.000

Tab. 2 with summary of the goodness of fit is obtained by running the in-sample prediction using the model estimated in Tab. 1. In Tab. 2 we see that 408 out of 1093 were classified well representing specificity 37.33% and giving total positive predictive value 408 out so 623 as 65.49% of correctly classified LTA employees. Only 215 out of 6294 were identified as false positive representing only 3.42%. On the other hand, 685 out of 1093 employees who should be identified as LTA = 1 were incorrectly classified as negative representing 10.13% of all who should be classified as negative, i.e. not LTA. Total amount of correctly classified is 6487 out of 7387 representing 87.82% which is a very good result.

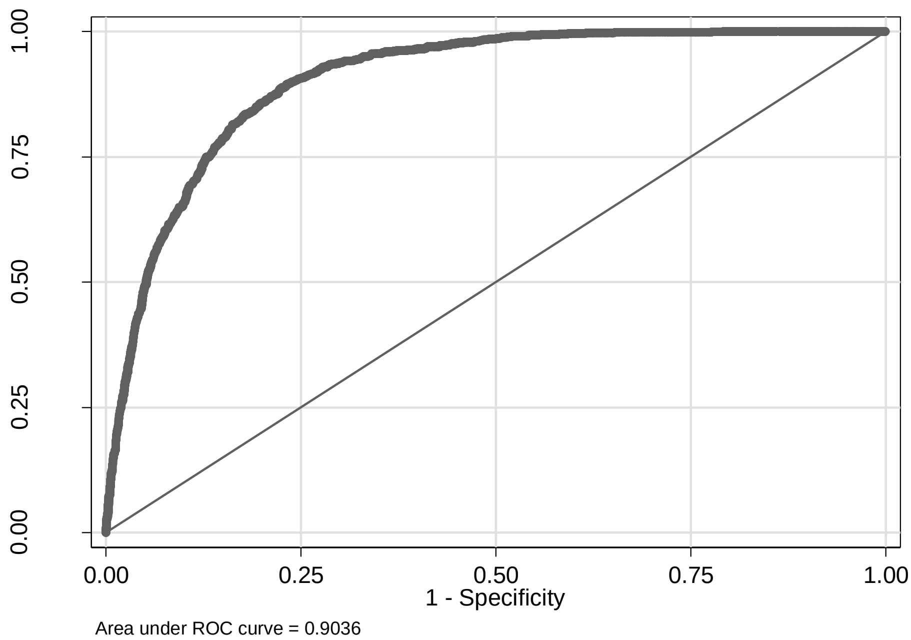
Table 2: Goodness of fit after logistic regression and classification statistics

Classified	True (D)	False (~D)	Total
True (+)	408	215	623
False (-)	685	6079	6764
Total	1093	6294	7387

Sensitivity	Pr (+ | D)	37.33%
Specificity	Pr (- | ~D)	96.58%
Positive predictive value	Pr (D | +)	65.49%
Negative predictive value	Pr (~D | -)	89.87%
False + rate for true ~D	Pr (+ | ~D)	3.42%
False - rate for true D	Pr (- | D)	62.67%
False + rate for classified +	Pr (~D | +)	34.51%
False + rate for classified -	Pr (D | -)	10.13%
Correctly classified		87.82%

Results of estimated model in Tab. 1 and classification statistics in Tab. 2 help us to create a ROC curve and visualize the predictive power of the model. Each point on the ROC curve represents a sensitivity/specificity pair corresponding to a particular decision threshold thus having a large sample as in this study gives us smooth curve circumscribing correctly classified values below the curve.

Figure 1: ROC curve



4. Conclusion

This study investigated the topic of employees' absenteeism with focus on long term sick leave absenteeism. Predictive model was estimated using logistic regression for binary variable LTA with the proportion of the total variability explained by other variables that were accounted in the model equaled to 0.344 giving a great insight in what is influencing for LTA. It is concluded that the model has significant predictive ability with good proportion of variability explained by the model but lower specificity with correct classification of long term absent employees from the whole sample due to large number of unpredictable events possible to cause long term absenteeism as visible in Tab. 2. Odds ratio analysis of Tab. 1 is telling us that the probability of an employee becoming TLA is increasing with increasing amount of absenteeism events, Bradford factor score, age, seniority and decreasing with higher average performance. Coefficient for variable Category shows that Blue Collar employees are much more likely to become LTA. Gender is insignificant to the model, but including the variable says that women are slightly more likely to become LTA than man, knowing that more than 80% of employees in the data sample are men. Work distance variable is also insignificant to the model saying that necessity to travel to work or having a work closer to home does not affect a risk of becoming LTA. It is concluded that the model be can be particularly useful in workforce planning as well as for being a step ahead of events and prevent possible production losses and consequently save considerable amount of money to the company.

Acknowledgements

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Applying simulations in the individual risk model using R.

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Abstract

The aim of the paper is to demonstrate the application of simulations within the individual risk model using R. By generating aggregate claim amount values in a portfolio of life insurance contracts during a given period and treating them statistically, it is possible to obtain information for actuarial risk management calculations. The presented simulation tool in comparison with the standard methods in this area provides flexibility and simplifies numerical computations. This stochastic approach using the Monte Carlo method permits the analysis of portfolios with a large number of insurance contracts.

Key words

Individual risk model, life insurance on death, portfolio modelling, simulation, Monte Carlo method, total claim amount, Value at Risk

JEL Classification: C15, G22

1. Introduction

IT developments in the context of computer software have led also in the actuarial fields to the appearance of modern stochastic approaches, based on simulation tools for solving problems. Given the particular characteristics of a portfolio of life insurance contracts it is possible to make use of the individual risk model. Determination of the distribution of the total claims plays an important role in those actuarial calculations aimed at ensuring solvency and reducing any potential loss for the insurer. In this context the approximation by the normal distribution, derived from the central-limit Lindeberg-Feller theorem, is very often used. Because of the characteristics of the normal distribution particularly in the tail of the distribution of total claims the results of this approximation can be imprecise and its values are often undervalued, even more so if application of the approximation requires a large number of insurance contracts (Kaas, Goovaerts, Dhaene and Denuit, 2008). Another approach for example is to determine the total claim distribution with the help of Laplace's transformation or the Fourier transform. Where there are a large number of contracts both approaches can be, even using computers, both numerically onerous and time consuming. An alternative approach to solving this problem is provided by simulation in the framework of the Monte Carlo method, which can be performed using the *R* programming language. The data required for ensuring solvency can be gained by processing statistically the generated total claim values. The paper deals with the modelling of life insurance contracts payable in the event of death within a given period, whereby an agreed sum will then be paid on a claim arising. The lives insured will be divided into various groups based on given criteria.

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2. Individual risk model

Let us consider a portfolio of *n* independent insurance contracts, whereby there number does not change during the period observed and at most one claim amount can be paid in respect of each contract. Denoting the random variable for the total claim amount by S^{IND}, we then have (Horáková, Páleš and Slaninka, 2015)

$$S^{IND} = X_1 + X_2 + ... + X_n, \tag{1}$$

where the random variable X_i describes the claim amount for the ith contract, $i = 1, 2, ..., n$, whereby it may be that $x_i = 0$, given that not every contract may lead to a claim during the observed period. In this case we can talk about a null claim amount. We denote by q_i the probability a claim arises in respect of the ith contract.

We can express the random variables X_i (they need not be identically distributed) with the help of two further variables Y_i, N_i by the relationship

$$X_i = N_i \cdot Y_i, \tag{2}$$

where

- the random variables $Y_1, ..., Y_n$, $N_1, ..., N_n$ are independent.
- the random variable Y_i represents the claim value from the ith contract given that a claim has arisen.
- the random variable N_i describes the number of claims from the ith contract and based on the assumptions has the alternative distribution $N_i \sim A(q_i)$, thereby taking the value 0 with probability $1 - q_i$ and the value 1 with probability q_i.

For the basic characteristics of the total claim amount $E(S^{IND})$ a $D(S^{IND})$, and their distribution function $F_{S^{IND}}(x)$ we have

$$E(S^{IND}) = E(X_1) + E(X_2) + ... + E(X_n) = \sum_{i=1}^{n} E(N_i) \cdot E(Y_i) \tag{3}$$

$$D(S^{IND}) = D(X_1) + D(X_2) + ... + D(X_n) = \sum_{i=1}^{n} D(Y_i) \cdot E(N_i) + E(Y_i)^2 \cdot D(N_i) \tag{4}$$

$$F_{S^{IND}}(x) = P(S^{IND} \le x) = P(X_1 + X_2 + ... + X_n \le x) \tag{5}$$

The individual risk model is suitable for life insurance, for example for insurances payable on death during a given period, where for the ith contract the probability of death is q_i and a fixed amount b_i is paid on a claim arising. In this paper we will only deal with the so-called degenerative random variable Y_i, which takes with non-zero probability only one value namely the insured sum b_i. In the case we are considering the distributions of the random variables Y_i, X_i are then given by the probability functions $p_{Y_i}(x)$, $p_{Y_i}(x)$ (Klugman, Panjer and Willmot, 2012)

$$p_{Y_i}(x) = \begin{cases} 0 & x = 0 \\ 1 & x = b_i \\ 0 & x \ne 0; b_i \end{cases} \tag{6}$$

$$p_{X_i}(x) = \begin{cases} 1 - q_i & x = 0 \\ q_i & x = b_i \\ 0 & x \ne 0; b_i \end{cases}. \tag{7}$$

In the case of the degenerate variable Y_i above the characteristics of the total claim amount $E\left(S^{IND}\right)$ and $D\left(S^{IND}\right)$ are then given by

$$E\left(S^{IND}\right)=\sum_{i=1}^{n} q_i \cdot b_i \quad (8) \qquad\qquad D\left(S^{IND}\right)=\sum_{i=1}^{n} q_i \cdot \left(1-q_i\right)\cdot b_i^2 . \quad (9)$$

3. Simulation of the total claim values S^{IND} using R

In this section we zoom in on the possibility of generating the total claim values S^{IND} using the programming language *R* (Venables, Smith and the R Development Core Team, 2018). The total claim amount is the sum of the values of the random variables $X_1, X_2, ..., X_n,$ where the random variable X_i has the compound alternative distribution which we can write as

$$X_i \sim CoA\left(q_i; p_{Y_i}(x)\right). \tag{10}$$

We can generate the value of the random variable X_i, representing the claim amount for the *i*th contract based on the following steps:

- first, we generate the value n_i of the random variable $N_i \sim A\left(q_i\right)$
- then we generate n_i values of the random variable Y_i .

To generate these values, we use the *R* function `sample`, which allows us to realise the generation of values from a given discrete distribution on the basis of repeated random choice. The value so generated then represents the value of the random variable X_i.

We use the function `sample` to generate single values of the random variable $N_i \sim A\left(q_i\right)$ and in the case we are considering to generate a single value of the random variable Y_i. If the generated value in the first step is given by $n_i = 0,$ then a null claim value will be generated for the *i*th contract, i.e. no claim amount will be paid, and we are talking about the null claim amount mentioned earlier, i.e. where $x_i = 0.$ For our purposes we state the basic syntax of the function `sample` (Páleš, 2017).

```
sample(x, size, replace, prob)
```

`x` – is a vector whose values are the values of the given discrete random variable, or more generally the vector of values from which random samples will be taken

`size` – is the number of generated values of the given random variable, or more generally the extent of the sample produced after the random sampling from the given vector of values

`replace` – in the case of the value `True` the individual values chosen allow for replacement and in the case of the value `False` they do not.

`prob` – is the vector, whose elements are the values of the probability function of the random variable. These values are in the same order as the values in the vector `x` and determine the realisation of the random sample of its individual values on the basis of the relevant probability.

Remark 1: It would also be possible to use the method of simulation shown in the case where the random variable Y_i takes more than one value.

On the assumption that $n_i = 1$ the value of the random Y_i represents in the presented life insurance model the insured amount b_i. By summing the n generated values of the random variable X_i we obtain with the help of the *R* function `sum` the first simulated value of the total claim amount S^{IND} in the portfolio of n insurance contracts, which we write as

$$s_1 = x_1 + x_2 + ... + x_n$$

Remark 2: It would be possible to perform simulations of the total claim amounts using the function `sample` also directly by means of the random variable X_i. We have however on purpose chosen to generate the values with the help of the random variables N_i and Y_i, so that we can emphasise the behaviour of the individual risk model.

People taking out insurance contracts payable on death can be divided into groups (sub-portfolios) according to state of health, or according to age categories. In this case within each group the same conditions apply with regard to determining the probability of death q_i in the followed period and the level of insured benefit b_i. In the following we illustrate by means of simulation the modelling of various scenarios formed from individual sub-portfolios.

So we will consider a portfolio of n independent insurance contracts made up from three sub-portfolios:

$$S^{IND} = \underbrace{X_1 + ... + X_k}_{I.} + \underbrace{X_{k+1} + ... + X_l}_{II.} + \underbrace{X_{l+1} + ... + X_n}_{III.} \tag{11}$$

Table 1: Data for the portfolio of life insurance contracts.

group (sub-portfolio)	Number of contracts in group	Probability of death	Insured sum
I.	k	$q_{I.}$	$b_{I.}$
II.	$l-k$	$q_{II.}$	$b_{II.}$
III.	$n-l$	$q_{III.}$	$b_{III.}$

Using the described approach we will generate the claim amounts for each contract:

- k values of the random variable X_k, where $X_k \sim CoA\left(q_{I.}; p_{Y_{I.}}(x)\right)$, given that $X_1 = ... = X_k$.
- $(l-k)$ values of the random variable X_l, where $X_l \sim CoA\left(q_{II.}; p_{Y_{II.}}(x)\right)$, given that $X_{k+1} = ... = X_l$.
- $(n-l)$ values of the random variable X_n, where $X_n \sim CoA\left(q_{III.}; p_{Y_{III.}}(x)\right)$, given that $X_{l+1} = ... = X_n$.

For repeated generations of the random variable X_i we use the function `replicate`. For our purpose we present the basic syntax of this function (Páleš, 2017).

```
replicate(n, expr, simplify)
```

`n` – the number of generated values of the random variable X_i, respectively the number of repeated generations of claim amounts from the given group.
`expr` – the expression which is replicated, in our case it stands in for the generation of the values of the random variable X_i

`simplify` – the default value `True` means that the function `replicate` returns a value in the form of a vector or matrix..

Summing the values k, $(l-k)$, $(n-l)$ we obtain the first sample s_1 of the total claim values S^{IND}, which we can write as

$$s_1 = \underbrace{x_1 + \ldots + x_k}_{I.} + \underbrace{x_{k+1} + \ldots + x_l}_{II.} + \underbrace{x_{l+1} + \ldots + x_n}_{III.} .$$

We perform the described algorithm *m* times using *R* and thus get *m* simulated values of the total claim values S^{IND} (Mucha, 2012)

$$s_1, s_2, \ldots, s_m .$$

We can then analyse statistically the obtained values and get the information about the given portfolio of insurance contracts needed for actuarial analyses. As an example we show the determination of the quantile risk measure *VaR* (*Value at Risk*), which represents the estimate of the maximum loss, which can arise with confidence $1-\varepsilon$ in a given future period (Cipra, 2015).

$VaR_{1-\varepsilon}(S^{IND})$ of the random variable S^{IND} is the $(100\cdot(1-\varepsilon))\%$ quantile, denoted as $x_{1-\varepsilon}$, $0<(1-\varepsilon)<1$, for which we have

$$VaR_{1-\varepsilon}\left(S^{IND}\right) = x_{1-\varepsilon} = \inf\left\{x \in R : F_{S^{IND}}(x) \geq (1-\varepsilon)\right\} . \tag{12}$$

We can interpret $VaR_{1-\varepsilon}(S^{IND})$ as the maximum loss which can arise with probability $1-\varepsilon$ in a certain period. That means that with probability $1-\varepsilon$ the random variable S^{IND} will take values less than or equal to the value $VaR_{1-\varepsilon}(S^{IND})$, which we can write as

$$P(\, S^{IND} \leq VaR_{1-\varepsilon}(S^{IND})\,) = 1-\varepsilon \tag{13}$$

which we can also use to determine the risk premium $RP_{S^{IND}}$ on the basis of the relationship

$$P(\, S^{IND} \leq RP_{S^{IND}}\,) = 1-\varepsilon \tag{14}$$

respectively

$$F_{S^{IND}}\left(RP_{S^{IND}}\right) = 1-\varepsilon \tag{15}$$

By analysing statistically the generated values of the total claim amounts S^{IND} we can determine values of $VaR_{1-\varepsilon}\left(S^{IND}\right) = x_{1-\varepsilon}$ and $F_{S^{IND}}(x)$ with the help of the *R* functions `quantile` and `subset`.

With the help of the function subset we can determine the values of the distribution function of the total claim amount $F_{S^{IND}}(x)$ by extracting *sub* values from the *m* values of the total claim amounts S^{IND} according to the condition $S^{IND} \leq x$ on the basis of the relation

$$F_{S^{IND}}(x) \approx \frac{sub}{m} \tag{16}$$

The function `subset` has the following syntax (Páleš, 2017).

```
subset(x, subset)
```

`x-` a vector of values from which we want to get a subset on the basis of fulfilling a given condition `subset-` condition which will determine the subset

With the help of the function `quantile` we can determine the quantiles $x_{1-\varepsilon}$ of the random variable S^{IND}, respectively the risk measure values $VaR_{1-\varepsilon}\left(S^{IND}\right)$. The function `quantile` has the following syntax.

```
quantile(x, probs)
```

`x` – the vector of the generated total claim amounts S^{IND}, from which we want to calculate the required quantile.
`probs` – the probability vector which specifies the estimated quantile.

Remark 3: In connection with the calculation of the values of the distribution function $F_{S^{IND}}(x)$ we have compared the results obtained using the presented simulation approach for a small number of contracts with the results obtained using a Laplace transform. The distribution function values obtained from each approach are comparable.

4. Practical example

This part of the paper is concerned with modelling a portfolio of life insurance contracts with benefit payable only on death using the *R* programming language. The insured lives are divided into three groups. In the event of a claim arising the appropriate insured sum is paid. The following parameters have been chosen only for the purpose of presenting how simulations can be used within the individual risk model. Let us consider then that in the first group there are 5,000 insured lives whose probability of dying in the given period is 0.0015 and the insured amount is 6. In the second group there are 3,000 insured lives with probability of dying 0.002 in the given period and insured amount 5. In the third group there are 2,000 insured lives with probability 0.0025 of dying in the given period and insured amount 4. The probabilities of dying q_i, insured amounts b_i and number of insured lives p in each group are summarised in Table 2.

$$S^{IND} = \underbrace{X_1 + \ldots + X_{5000}}_{I.} + \underbrace{X_{5001} + \ldots + X_{8000}}_{II.} + \underbrace{X_{8001} + \ldots + X_{10000}}_{III.}$$

Table 2: Probabilities of dying and insured amounts for each group.

group	p	q_i	b_i
$I.$	5,000	0.0015	6
$II.$	3,000	0.0020	5
$III.$	2,000	0.0025	4

Using equations (3) a (4) we can calculate the basic characteristics of the total claim amounts $E\left(S^{IND}\right)$ and $D\left(S^{IND}\right)$

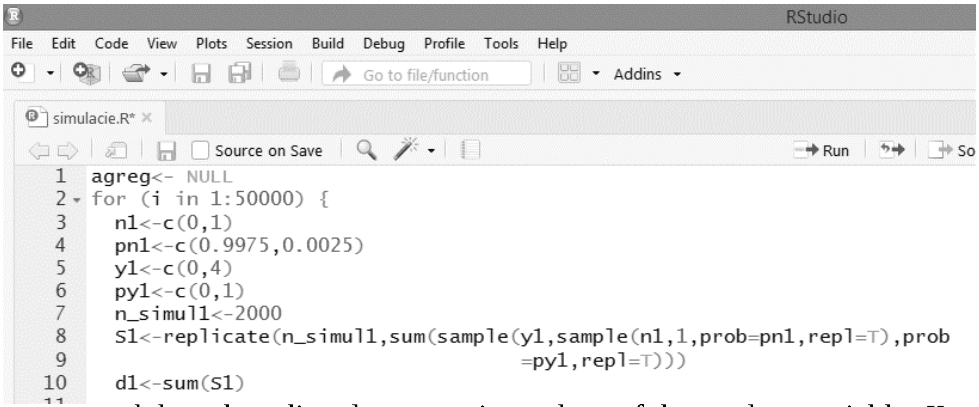
$$E\left(S^{IND}\right) = 5000 \cdot E\left(X_1\right) + 3000 \cdot E\left(X_{5001}\right) + 2000 \cdot E\left(X_{8001}\right) = 95$$
$$D\left(S^{IND}\right) = 5000 \cdot D\left(X_1\right) + 3000 \cdot D\left(X_{5001}\right) + 2000 \cdot D\left(X_{8001}\right) = 499.095$$

To generate the values of the total claim amounts S^{IND} in *R* one can use:

- an approach based on generating values of the random variables N_i a Y_i,

  part of the coding in *R* Studio is shown in Figure 1.

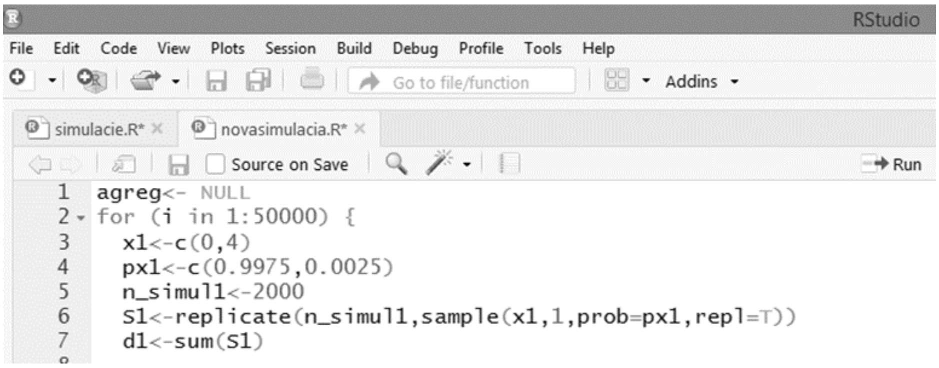
Figure 1: Part of the coding in R Studio for generating 2000 claim amounts in one of the groups with the help of the random variables N_i and Y_i.




```
agreg<- NULL
for (i in 1:50000) {
  n1<-c(0,1)
  pn1<-c(0.9975,0.0025)
  y1<-c(0,4)
  py1<-c(0,1)
  n_simul1<-2000
  S1<-replicate(n_simul1,sum(sample(y1,sample(n1,1,prob=pn1,repl=T),prob
                                    =py1,repl=T)))
  d1<-sum(S1)
```

- an approach based on directly generating values of the random variable X_i, part of the relevant coding in *R* Studio is shown in Figure 2.

Figure 2: Part of the coding in R Studio for generating 2000 claim amounts in one of the groups using directly the random variable X_i.




```
agreg<- NULL
for (i in 1:50000) {
  x1<-c(0,4)
  px1<-c(0.9975,0.0025)
  n_simul1<-2000
  S1<-replicate(n_simul1,sample(x1,1,prob=px1,repl=T))
  d1<-sum(S1)
```

On the basis of the described algorithm and with the help of simulations we generated *m* values of the total claim amounts S^{IND}, which were then analysed statistically to estimate $E\left(S^{IND}\right), D\left(S^{IND}\right)$ and their respective quantiles $x_{1-\varepsilon}$, see Figure 3. The values of the estimated characteristics are shown in Table 3.

Figure 3: Statistical analysis of the generated values S^{IND} using the functions mean $\left(E\left(S^{IND}\right)\right)$, var $\left(D\left(S^{IND}\right)\right)$, quantile $\left(x_{1-\varepsilon}\right)$, subset and length ($F_{S^{IND}}(x)$) in R Studio.

```
str<-mean(agreg)
str
disper<-var(agreg)
disper
```

```
q<-quantile(agreg,probs=c(0.9,0.95,0.99))
q
df<-subset(agreg, agreg < 130)
hdf<-length(df)/50000
hdf
```

Table 3: Estimates of $F_{S^{IND}}(x)$ and the characteristics $E\left(S^{IND}\right), D\left(S^{IND}\right), x_{1-\varepsilon}$ of the total claim amounts S^{IND} for various numbers of simulations m.

m	$E(S^{IND})$	$D(S^{IND})$	$F_{S^{IND}}$ 130	$F_{S^{IND}}$ 140	$F_{S^{IND}}$ 150	$x_{0.90}$	$x_{0.95}$	$x_{0.99}$
1,000	96.0970	521.9335	0.9150	0.9630	0.9910	125	134	158
5,000	95.3724	508.4712	0.9348	0.9690	0.9876	125	134	153
10,000	95.1534	504.4557	0.9301	0.9725	0.9892	124	133	150
30,000	95.0147	497.6746	0.9325	0.9701	0.9891	124	133	150
50,000	95.0116	498.3546	0.9330	0.9721	0.9888	124	133	150
100,000	95.0614	498.9041	0.9328	0.9713	0.9895	124	133	150

We can illustrate the interpretation of some of these achieved results thus for example:

- the value $F_{S^{IND}}(150) \approx 0.9888$ implies that there is a probability of 0.9888 that the total claim amount S^{IND} in the given portfolio over the given time period will be less than or equal to 150.
- the value $VaR_{0.90}(S^{IND}) = x_{0.90} \approx 124$ implies that the maximum total claim amount S^{IND}, which can arise with probability 0.90 in the given portfolio over the given time period.

We compare in Table 4 the values of the quantiles $x_{1-\varepsilon}$ and the distribution function $F_{S^{IND}}(x)$ which we have estimated using 50,000 simulations, with the results obtained on the basis of approximating the distribution of the total claim amount S^{IND} by the normal distribution on the basis of the following relationship

$$F_{S^{IND}}(x) = P\left(S^{IND} \le x\right) = P\left(\frac{S^{IND} - E\ S^{IND}}{\sqrt{D\ S^{IND}}} \le \frac{x - E\ S^{IND}}{\sqrt{D\ S^{IND}}}\right) \approx \Phi\left(\frac{x - E\ S^{IND}}{\sqrt{D\ S^{IND}}}\right) = F_N(x)$$

to get

$$S^{IND} \approx N\left(E\left(S^{IND}\right); D\left(S^{IND}\right)\right)$$

Table 4: Comparison of the values $x_{1-\varepsilon}$ obtained by simulation (sim) with the results obtained by approximation by the normal distribution (aprox).

$x_{1-\varepsilon}$	*sim*	*aprox*	$x_{1-\varepsilon}$	*sim*	*aprox*
$x_{0.15}$	72	71.8456	$x_{0.90}$	124	123.6304
$x_{0.50}$	94	95.0000	$x_{0.95}$	133	131.7467
$x_{0.75}$	110	110.0684	$x_{0.99}$	150	146.9716

Table 5: Comparison of the values of $F_{S^{IND}}(x)$ obtained by simulation (sim) with the results obtained by approximation by the normal distribution (aprox).

$F_{S^{IND}}$ x	*sim*	*aprox*	$F_{S^{IND}}$ x	*sim*	*aprox*
$F_{S^{IND}}(55)$	0.0265	0.0367	$F_{S^{IND}}(110)$	0.7491	0.7490
$F_{S^{IND}}(70)$	0.1246	0.1316	$F_{S^{IND}}(135)$	0.9561	0.9633
$F_{S^{IND}}(95)$	0.5063	0.5000	$F_{S^{IND}}(150)$	0.9888	0.9931

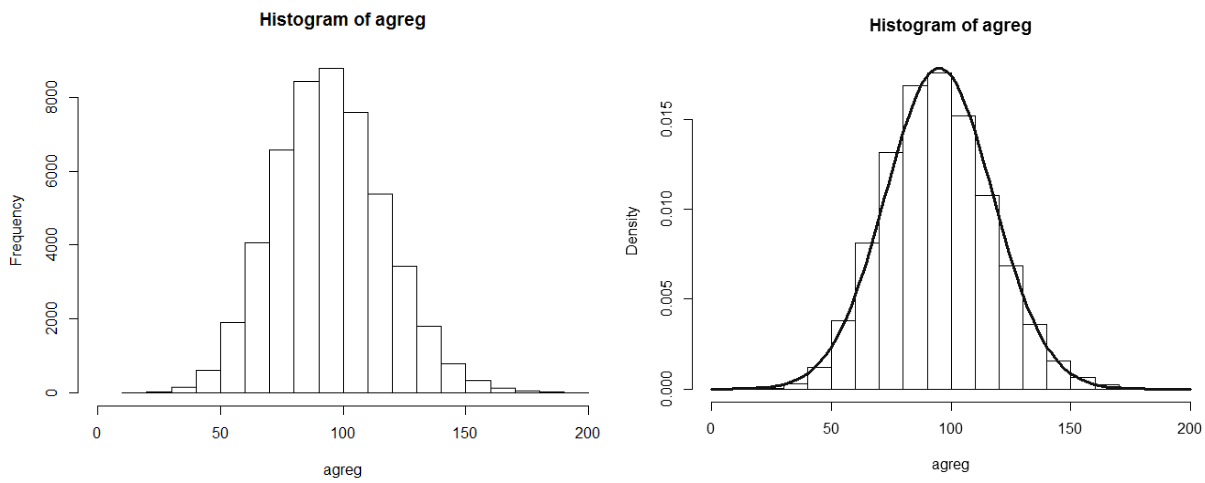
The differences between the values using simulation and results obtained by approximation by the normal distribution are to be expected given the symmetry of the latter. In the case where the end of the distribution of the total claim amount converges more slowly towards the *x* axis as the graph of the density of the normal distribution then the values of the quantiles determined using this approximation are underestimated and the values of the distribution function overestimated. This is clear when we compare the results shown in Tables 4 and 5 and their visualisation in Figure 5 right-hand diagram. Histograms of the generated results for the total claim amounts were created in *R* Studio using the function `hist`, Figure 4.

Figure 4: Creation of histograms from the generated values of S^{IND} and using the density function of the normal distribution in R Studio.

```
hist(agreg,breaks=20,xlim=c(0,200),freq=T )
curve(dnorm(x,mean=95,sd=22.34043419),add=T,col="blue",lwd=3)
```

The left-hand side of Figure 5 shows the classical histogram on the basis of the presentation of the absolute numbers of generated values in each interval. On the right-hand side is shown the histogram as a representation of the relative frequency of the generated values together with a graph of the density function of the given normal distribution $N(95;499.095)$.

Figure 5: Histogram of the generated values of S^{IND} in R Studio.



5. Conclusion

This paper has presented the possibility of using the Monte Carlo method in modelling a portfolio of life insurance contracts with benefits payable only on death in the context of the *R* programming language. By simulating insurance claim amounts for each contract and then analysing statistically the generated values of the total claim amounts S^{IND} it is possible to obtain the data needed for actuarial calculations in connection with risk management and ensuring solvency. With regard to simulations the paper offers two approaches to generate the values of the total claim amount S^{IND}, where the first generates the values with the help of the random variables N_i a Y_i arising from the base itself of the collective risk model. Thanks to the second approach based on the direct generation of the values of the random variable X_i, the time onerousness of the computations is in comparison with the first approach reduced in the case where there are a large number of contracts and hence of simulations. Compared to the approximate approach which requires a large number of contracts and which never ensures an adequate accuracy of the obtained results, simulations are a suitable for solving problems in this field. The same applies in comparison with the numerical and time onerous approaches mentioned in the introduction also in the case where computers are used.

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Risk controlling of traditional and Islamic banking systems

Hussam Musa, Viacheslav Natorin[1]

Abstract

This article is focused on credit and operational risks in the traditional banking system and in the Islamic one. Apart from that, the risk of opportunistic behavior among bank employees is emphasized as the separate issue. This type of risks is not researched enough, thus there are no recommendations in the non-fiction about how to minimize it in practice. The main indicators of risks in the banking sector of Slovak Republic, as the representative sample of traditional banking, were researched by authors in this article. Also, the analyzed principles of Islamic financing became the basis for authors' recommendations about banking risks minimization. Moreover, authors proposed general methods and instruments to regulate credit risk in the traditional banking system as well as in the Islamic one.

Key words

Islamic banking system, Shariah supervisory board, Risk, credit risk, operational risk, liquidity risk, interest rate, consumer credit

JEL Classification: E14, G21

1. Introduction

The main incentive for this research became rapid increase of household arrears. According to NBS data, consumer credit in Slovakia exceeded 38% of GDP at the end of previous year. During the last years growth of this type of credits definitely was the quickest in the EU. Back to 2009 year, Slovak household indebtedness was the second smallest by size in Central and Eastern Europe. Since that time it's increased more than in two times and reached the highest level in the region. In 2014, household credit share of GDP in Slovakia exceeded the level of Czech Republic and fast growing Poland in 2017. Together with credit indebtedness there is the increasing level of bad debts, thus search of ways of credit risk minimization is priority task for us.

Methodological basis of this research consists of general scientific methods. The Using of general scientific methods of research was carried out by application of methods of induction, deduction, synthesis within the framework of logical approach.

The importance of evaluation and analysis of the borrowers' creditworthiness can not be questionable. As a rule, the correct decisions in the process of lending should be taken in 99.5% of cases. The level of allowable error is so small that an increase in loan losses can not only make the loan portfolio unprofitable, but it often affects the profitability of the financial organization as a whole (Morsman, 1997).

Credit risk is the most common type of financial risk and it is an element of uncertainty when the counterparty fulfills its obligations under certain agreements related to the

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repayment of borrowed funds. Thus, credit risk is the possibility of loss of a financial asset, as the result of inability of borrower to fulfill the contractual obligations, which is imposed on him. For a creditor, failure to fulfill these obligations entails a loss of principal, plus unpaid interest, minus the amount of the compensation received by selling collateral, executing guarantees or sureties, for instance.

Traditionally, credit risk is one of the most important in terms of its main role in the volume and profitability of the bank's active operations.

According to the source of credit risk, it can be divided into two types:

1. External (counterparty risk), due to the solvency, reliability of the counterparty, the probability of declaring a default and potential losses in case of default;

2. Internal (credit product risk), connected with the characteristics of the loan product and the possible losses due to the counterparty's failure to fulfill his obligations (Evdokimova, 2016)

Among the foreign researchers, who deals with the problems of risk management, the works of such authors as K. Barltrop, E. Dolan, T. Koch, KD Campbell, RJ Campbell, D. MakNoton, J. Matouk, J. Rivoire, J Sinki are the most interesting. In the published scientific works, authors investigates the essence of credit risk and methods of its estimation in the process of granting credits.

2. Analysis of credit risks in Slovakia

Every commercial or state bank has its own credit policy and the corresponding division of risk management, but even more important measures to reduce bank risks are taken at the government level by regulatory bodies. We would like to remind you that in December 2010 two important documents were adopted: "Basel III: A global regulatory framework for more resilient banks and banking systems" and "Basel III: International framework for liquidity risk measurement, standards and monitoring". According to the second one, the calculation of the capital to cover the credit risk of the counterparty (Counterparty credit risk CCR) was revised mainly with the changes based on the calculation methodology connected with the internal models (in particular, VaR). Apart from the cover of credit risk of a default by capital based on a standardized or IRB approach, the bank must also maintain capital to cover losses from the market revaluation of derivatives traded on the OTC market, which are related to the counterparty credit risk (for example,credit default swap CDS). Such losses are known as credit valuation adjustments (CVA). The bank must conduct stress testing of the counterparty's credit risk (at least monthly) using market factors.

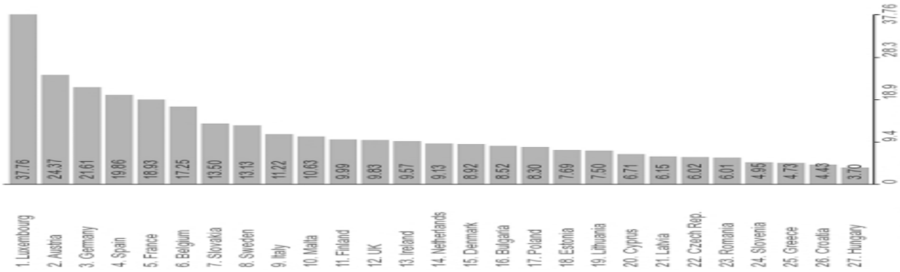
The National Bank of Slovakia also takes steps to reduce credit risk at the state level. Let's consider some examples in the mortgage field. In July of this year, Restrictions of the DTI index (debt to income), which establishes the client's credit limit in the amount of its net annual income multiplied by 8, came into force. In addition to mortgage, this limit includes consumer loans, credit cards and overdrafts. Also, a ceiling is set for loans with LTV between 80% and 90%, 100% of mortgages are now banned at all.

From the point of view of the IMF, which conducted a study of the growth of household debt in 2017, the steps taken by NBS coincide with the recommendations of specialists. At the same time, such a rapid growth of household debt cannot be ignored. The IMF study found that in a sample of 80 countries, the ratio of household debt to gross domestic product has increased since 2008. Among the developed economies, the median debt ratio of last year was 63% compared to 52% in 2008. Among emergent market countries, it rose from 15% to 21%. As the IMF study shows, an increase in the household debt ratio can lead to higher economic growth and employment in the short run. But after three - five years, this influence will become opposite, the growth will be lower than it would be in case of the absence of this factor and the likelihood of a financial crisis will increase. More specifically, the conclusion of the experts is: "The increase in the household debt ratio by five percentage points of GDP over a three-year period indicates that in three years the inflation-adjusted growth rate will decrease by 1.25 percentage points. A higher level of debt is connected with significantly higher unemployment in the period up to four years in the future. And raising the debt by 1 percentage point increases the likelihood of a future banking crisis by 1 percentage point. This is a significant increase, considering that the probability of a crisis is 3.5 percent even without any increase in debt. "

What is the current situation in the banking area in Slovakia?

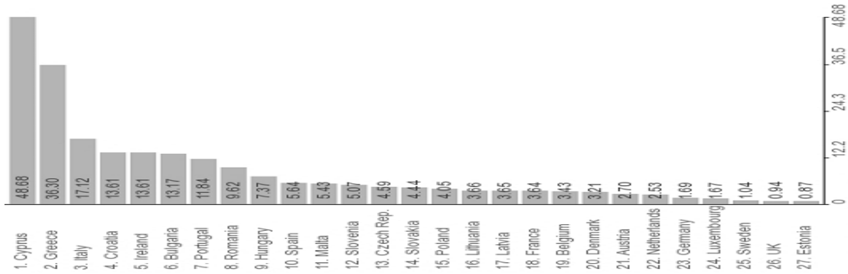
The general stability of the Slovak banking system can be considered in comparison with other EU countries using the Z-score index.

Figure 1: Banking system Z-scores, 2015



According to Bankscope data for 2015, Slovakia was on the 7th place with the index Z-score 13.5. Compared to other EU countries, this index is relatively large and is closely related not only to the value of ROA (Return On Assets), but also to the level of NPLs (Bouvatier, 2017).

Figure 2: Non-performing loans as percent of all bank loans, 2016



According to the data of The World Bank, among all the European Union countries Slovakia was on the 14th place between the Czech Republic and Poland in terms of amount of NPLs (bad loans) in 2016.

But the situation, that we have been observing since 2014, tells us that the Slovak banks use effective credit policy and the level of NPLs decreases annually: 2015. - 4.97%, 2016. - 4.52%, 2017 - 3.75%.

Figure 3: Share of NPLs on the total amount of loans to clients, 2017

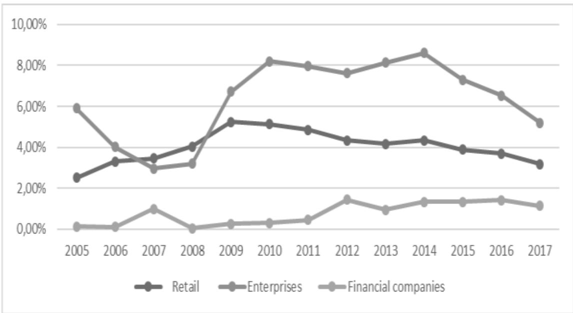
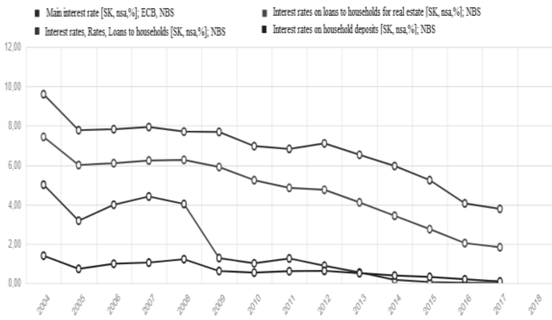


Figure 4: Interest rates in Slovakia, 2017



Interest rate risks in Slovakia don't play a significant role any longer, since the main interest rate as well as the rates of household deposits tend to zero (Figure 4), namely household deposits are 67.32% of all deposits according to NBS. But it is noteworthy that the volume of loans issued and the volume of deposits are almost equal, what did not happen before, or at least since 2005[2].

The decreasing of interest rates and the rapid growth of lending have prompted NBS to take action, but the result will be apparent only next year. From the point of view of authors, for decades the growing partnership funding or Islamic financing has been applying principles, that can help risk management in the traditional banking.

3. Implementation of the principles of Islamic finance in the risk management of traditional banks

The most important principle applied in Islamic banking is the ban on interest, so-called "riba". "Islamic banks developed their own, specific, interest-free system that arose by applying the rules of the Qur'an, the Holy Book of Muslims. All Islamic banks all over the world work with the basic principle of banning interest from the Koran "(Bellalah, 2014, p. 4). "Riba" means "increase," "growth". Interest is forbidden in the Islamic law, because it creates injustice in society, thus Islam and social justice are inseparable.

The basic principles of Islamic funding are trust, honesty, justice, clarity and transparency among business participants. A particular feature of Islamic funding is the existence of the Islamic Finance Commission (the Sharia Commission) at every Islamic financial institution and the conduct of Islamic financial operations. It is the function of the Sharia Commission to decide whether the activities and operations of Islamic financial institutions follows the principles of Islamic funding or not. The principles of Islamic funding are based on transparency of governance. This means that there is an ethical type of business, in which there are less speculative operations than in traditional ways of financing (Kalimullina, 2014).

Within the framework of this article, we will not characterize all Islamic banking products. But using the example of the Murabaha agreement we will describe the alternative financing principles. In addition, the Murabaha agreement is very similar to traditional consumer lending, it can be implemented into traditional banking without significant changes in legislation and, according to the experts in the field of Islamic finance Habib Ahmed and Tariqullah Khan (2007), this type of financing has the lowest credit, operational, trade and liquidity risks.

[2] Deposit data on the NBS website is available since 2005.

According to the international standards of AAOIFI, murabaha is a transaction, in which a bank or other financial intermediary acquires an asset, that client needs, from a seller, and sells it to the customer with a deferred payment. This transaction involves the sale of goods at the price at which the goods were purchased by financial intermediary, with a certain agreed mark-up. This mark-up can be a percantage share of the purchase price or it could be a certain amount of money. Traditionally, this kind of transactions is used in financing trade operations. It is also possible to use Murabaha to finance private acquisitions in a similar manner to consumer lending. A fundamental difference from traditional lending is that the financial intermediary should become a full-fledged proprietor of resale property with all the resulting risks. Moreover, the client, as a rule, cannot be forced to buy the asset acquired by the financial intermediary, although it is possible to demand reimbursement of expenses, which are incurred, as a result of selling the asset to another person.

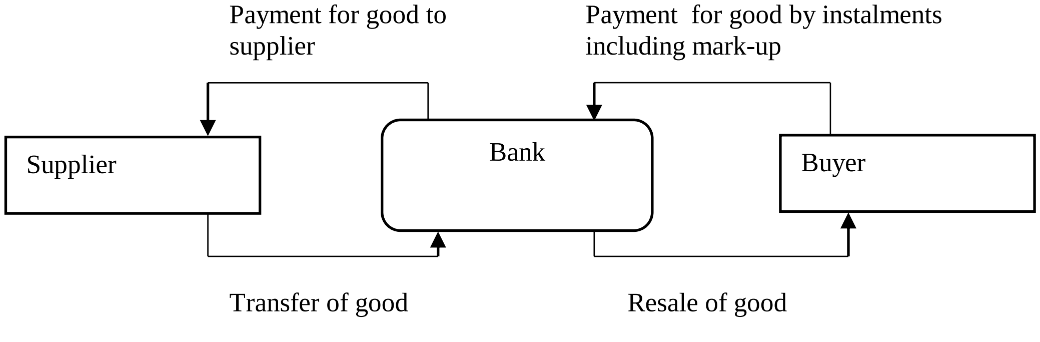
The deal can be concluded without a preliminary promise of a ransom, in this case it is called "ordinary murabaha", or with a preliminary promise of repurchase made by a person interested in acquiring assets through the intermediary agency, in this case it is called "trust Murabaha," that is, murabaha for the ultimate buyer.

The conditions for the validity of the Murabaha deal are following:

1. The subject of the sale must be an existing good authorized by the Shariah, and also a tangible asset, whose quality and quantity can be easily determined.
2. The financial institution has to receive the good as a physical or constructive property.
3. The client must be aware of the cost of good and the mark-up. Size of the mark-up must be established by mutual agreement of the parties.
4. The price of the murabaha contract can be paid at a time from the beginning of validity of the contract, the payment can also be postponed and repaid by parts or it could be completely paid at the end of the validity of contract.
5. It is prohibited to change the price of goods during the validity of contract. Change of payment terms is allowed in order to facilitate the position of debtor.

In general, the Murabaha contract can be represented by the following scheme. At the first stage, the purchase by the financial intermediary, which is necessary to the customer, is carried out and the second stage is the sale of the goods to the customer with a deferred payment.

Figure 5: Scheme of Murabaha agreement



Thus, the application of a new contract from the Islamic finance product line would allow to accomplish several tasks. On the side of the bank, this is excess liquidity, as the bank needs to acquire a real asset and possibly bear the costs of delivery and storage. On the client side, the benefits of such a product are even greater. The bank, before buying an asset, examines the conditions of the suppliers and bank is interested in the delivery, components and everything that may affect the price of the good, which brings additional quality control for the client. The next benefit for the client is the absence of interest, the client needs to pay a pre-agreed amount of money and this is not affected neither by the discount rate of the central bank (even if it starts to increase dramatically), nor by the cost of resources in the market. And also, as the financial crisis showed, when client has difficulties with repayment of loan, banks can defer only payment for the principal of loan, which means that the client "runs in place" - he pays interest, and the principal of loan does not decrease. And the next important benefit is the absence of penalties and fines for non-compliance with the repayment schedule. Such sanctions are simply prohibited in the Islamic financing. This does not mean that the Murabaha agreement can be violated without consequences, because, in case of willful violation, client has to transfer funds for charity, according to conditions in Murabaha agreement.

4. Risk of opportunistic behavior

Among the risks listed above, such as credit, operational, trading and liquidity risk, we would like to discuss the risk of opportunistic behavior of clients and bank employees. This risk can be associated with credit and operational risk as well. Because in each case it is a violation of the terms of the banking product and of the entire regulatory documents adopted in the financial institution.

We examine in detail the types of offenses related to unfair behavior. The first possibility arises when the offense acquires the character of a customer-natural person's fraud.

Fraud is, according to the definition of the bank, forgery of personal and private data, documents and data about the employer; manipulation or attempted manipulation of data (by employees or applicants) in order to obtain goods or money; misuse of powers, as well as knowledge of processes and procedures to obtain money, goods, and personal advantage (tangible or intangible); any further steps that lead to a bank's loss or damage to the bank's reputation. An example is to provide false information in the receipt, tax return, using false documents such as passports, third party credit, and so on. There are a number of varied variations of machinations, which are being constantly improved in the context of the Bank's risk management progress. For example, in the area of income confirmation, the following situations can occur: 1. Employee has an increased salary, after a secret agreement with the employer, from a minimum to exactly 3 months, then the salary is reduced again (under the conditions of banks it is necessary to prove the salary for 3 months, after which no one checks it); 2. Although the bank checks wage payments through a social insurance company, only employees of the bank know that when client applies for a loan in a domestic bank, where the applicant already has a current account, it doesn't need to send application form to the social insurance company. This means that a person can receive money even though he is no longer employed, he can receive a wage even after being fired and ask for a loan; 3. a self-employed person or tradesman may deliberately increases his income or pay taxes in such way that the benefit of the following credit fraud is higher than the cost. In the area of falsification of documents, it is no longer a popular falsification of special cards, but in the case of a legal person, the falsification of accounting documents is very frequent. Often, documents, which confirms purposeful use are forged; for example, when funds were used to buy securities or foreign currencies instead of investing into the project. In the area of third-party use, we can name a common promise to repay a loan instead of a borrower, to register loans on name of homeless persons or dead men, to use documents that have not been properly destroyed, or to approve a credit via the human resources department for employees, whom they plan to fire.

The biggest damage to the credit institution is caused by internal fraud (Natorin, 2017), involving employees of the credit institution, because apart from the economic loss, there is the damage of reputation of the bank in such a situation. When we are talking about criminal offense involving employees of a credit institution, two ways can be considered: 1. the theft of funds using software and knowledge of internal technologies, 2. the provision of untrue information to acquire tangible goods or cash. The first group will include cases, in which every week or month bank technicians can take, for example, 1 cent from client's cards, which may be a large amount in a big bank, or, for example, knowledge of the rating allows the bank's employees to obtain a higher credit score for a familiar client whith changed education or family status, because these data do not require confirmation such as a diploma, marriage certificate, etc. In the second group, we include, for example, employees who illegally find out PIN code of the bank's clients. To prevent crime (fraud) from the first area, the credit institution diversifies the decision-making steps of granting credit with the assignment of the password to each employee at each decision-making stage, changing the password is done according to the set deadline. The use of anti-fraud (integrated) systems allows the credit institution to perform transaction monitoring, control and analysis of the activity.

We would like to emphasize the unfair behavior of third parties, such as experts and credit brokers. In bank practice, there are expert opinions, in which the important information is concealed by expert, such as a crack of foundations, which would substantially reduce the value of the building. Among credit brokers, it is currently very common practice that if the client hasn't enough its own funds and the LTV is set at 70%, the bank gets a purchase contract with an exorbitant property price, which is different in comparison with that one signed by the notary.

To address these frauds, and as one of the ways to minimize transaction costs and risk of opportunistic behavior, we propose a motivation system for bank employees.

Table 1: Motivation system for bank employees

	Regulations	Basis for the calculation of remuneration	Liability
Front office manager	The number of loan applications for the accounting period	Content of exhausted loans	Share of rejected requests in the total number of requests
Security service	Request processing time	Number of deferred payments	
Risk manager	Request processing time	Content of exhausted loans	Number of deferred payments
Legal Service	Request processing time	The success of court sessions	
Underwriting service	Request processing time	Number of deferred payments	

Source: Own processing

There are some methods that, in our opinion, affect the prevention of opportunistic behavior:

- to constantly update databases of non-paying debtors (currently performed in Slovakia by Slovak Banking Credit Bureau, p. o.);
- inform participants about the responsibility for the offence and the inevitability of the punishment;
- Perform rotation of employees.

Looking for ways of minimizing the risk of unfair behavior, we learned the foundations of Islamic banking. During the investigation, it was found that Islamic banking is influenced by the institute of religion: investors do not expect profits from deposits, banks help their partners with delays, and when the client deliberately delay payments, the fine is transferred to charity by bank. It may seem that ethical finance doesn't bear the risk of unfair behavior, but the situation is opposite.

In Islamic Banking, it is primarily necessary to carry out audits of funded projects in order to ensure proper management and adequate risk assessment. We consider it is necessary to note that when the Islamic Bank provides funds to the entrepreneur to realize his project only on the basis of the entrepreneur's specific abilities and project perspectives, often requiring no control, the entrepreneur, as well as the bank, has incentives for opportunistic behavior. For example, as part of the project realization a client may try to conceal real revenue from the bank. In addition, in case of the Mudaraba (capital connection), the Bank has the right to require repayment of a principal only if the project has made a profit. If the project was unprofitable, the bank cannot get the money back. This means that the risks of an Islamic bank are far higher than the risks of a traditional bank.

Islamic banks apply effective monitoring and control of the allocated funds. Without proper control, an entrepreneur can misuse his financial situation and engage in unlawful business practices that ultimately lead to losses - both for entrepreneur and for the bank. It is useful to apply these tools in traditional banking, and it is also important to increase the financial literacy of the population, to change the attitude of the participants to each other in the credit process, to propagandise the moral qualities in order to maintain social justice, social development, and to support helpless people to maximize their income at all costs.

5. Conclusion

According to the IMF experts' assessment of the rapid growth of household debt in Europe and the Slovak Republic in particular, the authors think that NBS measures are not enough. There is a need for alternative ways to reduce risks in the banking sector, mainly credit risk. In addition to general recommendations on working with credit risk, the authors gave arguments in favor of Islamic methods of financing. One of these arguments is the fair and more ethical nature of Islamic banking. Based on the results of observations of opportunistic behavior of clients and employees of the bank, recommendations were proposed to reduce this type of risk in the Islamic and traditional banking as well. Commercial banks should pay attention to:

1. Improvement of the motivation system for employees - participants in the credit process - with emphasis on the share of delays and default credits in the loan portfolio of each employee;
2. development of the disciplinary influence in case of fraud in the process of decision-making and negotiation of a loan agreement with legal entities.

In addition, it is useful to increase financial literacy, to minimize customer and employee contact, to verify standards and credit procedures with regard to already detected frauds, and to carry out effective monitoring and control over the allocated funds using methods of the Islamic banking.

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Analysis of the Effective Rate of Income Tax in the Banking Sector of the Czech Republic

Josef Novotný[1], David Ohanka [2]

Abstract

In this paper it is analyzed the determinants of the Effective Tax Rate (ETR) of the Income tax in the banking sector of the Czech Republic using a panel data of 45 Czech banks over the period 2005 – 2016. The results show that the ETR of analyzed sample was below the Statutory Tax Rate (STR) during almost whole examined period. However, as the analysis showed, there was significant deviation between STR and ETR during the period of financial crisis, mainly caused by higher degree of allowance for bad debts creation due to credit defaults in retail segment. More specifically, it is noted that banks that are larger and more profitable in terms of calculated profit per one employee have relatively lower tax burden than the smaller and less profitable entities.

Key words

Effective Tax Rate, Czech Banking Sector, Tax Burden

JEL Classification: H20, G20, K34, H25, G21, G29

1 Introduction

The process of globalization that the international economy has experienced throughout of the last decades has greatly conditioned the evolution of the national tax systems. This is particularly true for taxes that fall on capital and commercial profits.

In this new context, the tax on profits is a fundamental tool within the tax system, not only because of its tax collection capacity, but also because of the influence it may have on the economic decisions of the entities subject to its assessment.

Throughout the world, this tax has been characterized by having experienced a gradual decrease in the legal Statutory Tax Rate (STR) in recent years. In the last annual report of the audit company KPMG (2018), it is shown that for the 116 countries analyzed, the STR has fallen 4,95 percentage points in the last ten years, going from 28,95 % in 2004 to 24,00 % in 2018, which represents a reduction of almost 17 %. Czech Republic have not been spared out of this process as well as STR of 28 % in 2004 have been reduced to 19 % in 2018.

However, according to the Government Accountability Office (GAO, 2008), STR does not provide an adequate measure of corporate fiscal pressure because it does not contemplate a series of aspects such as temporary differences, compensation of negative tax bases and other incentives, while the Effective Tax Rate (ETR) is a good indicator of the fiscal pressure when considering all the variables that intervene in the determination of the tax. In this regard, it is worth highlighting the obligation established by the Securities and Exchange Commission (SEC) since the 1970s for American companies quoted on the Stock Exchange to provide their ETRs in the annual accounts.

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The aim of this paper is to analyze Effective Tax Rate on Income tax in the Czech banking sector during the period 2005 – 2016 and to investigate what variables are the most important drivers in the determination of the Income Tax.

2 Literature review

The brief review of related studies together with theoretical background of Effective Tax Rate approaches to calculation are presented within following chapter.

2.1 Related studies

The analysis of the main conditioning factors of the ETR supported by companies has been addressed, both theoretically and empirically, by numerous works primarily at international level, among which Quinn and Shapiro (1991), Wilkie and Limberg (1993), Gupta and Newberry (1997), Buijink et al. (1999), and Feeny et al. (2006). More recently the subject has begun to be treated in Czech Republic in recent years by works such as those of Koštuříková (2011), Pelouch (2016) or Lisztwanová and Ratmanová (2016). However, in these previous studies, financial entities have always been excluded. For example, Buijink et al (1999) exclude the financial services sector firms from their study based on financial reporting requirements differences which make comparison with other companies difficult. The exclusion by Quinn and Shapiro (1991) is based on the low capital intensiveness of the financial services sector.

What has come to our notice, that is close to this respect, is the study of Diaz, Rodriguez and Arios (2011) that examines ETR in the banking sector of the Spanish economy using extensive data set of 55 banks and 51 savings banks over the period 1993-2004. Specifically, Diaz et al were examining determinants of ETR and came to conclusion that entities that are larger, less capital intensive and possess of higher levels of fixed assets, burden a lower tax obligations whereas entities with higher profitability have higher ETRs. They also found that links between ETR and its determinants differ depending on the type of entity. Another article by Salaudeen (2017) analyses corporate ETRs in the Nigerian financial services sector. Results in this study shows that profitability, capital intensiveness and firm leverage are determinants of ETR in Nigerian financial services sector and therefore have the largest impact on resulting tax burden.

2.2 Approaches to Effective Tax Rate calculations

There are three basic methods for calculating the effective tax rate - the micro-backward looking, the macro-backward looking and micro-forward looking approaches. The difference between them lays in the use of different data on which each method is based. Macro approach essentially uses macro-economic data; i.e. national income or product accounts. On the other hand, micro approach is based on company level data extracted from financial statements either from individual companies or aggregate industrial sectors (Nicodeme, 2001). While backward looking approach is using ex-post data, forward looking approach can be derived based on utilization of ex-ante data as this approach deals with future gains of current investment.

As we find micro-backward approach the most suitable one to apply within our paper, further description of approach is in place. For this method, corporate financial statements are the main source of data. The effective tax rate is defined as the ratio between corporate income tax paid and the tax base. Three approaches to determining the tax base are used:

- profit before tax (taxable profit from ordinary activities before tax);
- net turnover (i.e. total operating income);
- gross operating profit (i.e. total operating revenue less operating costs less other operating charges and taxes less personnel costs).

The first of the above mentioned approaches is appropriate when comparing the effective corporate tax rate with statutory rates. However, comparisons at international level may be problematic, mainly due to different national accounting legislation. In the second option, it is necessary to take into account that the low value of the calculated effective tax rate may be due to the high costs of the company that must be covered by operating revenues. The result is therefore low, as well as the tax paid, even though the statutory tax rate may be high. There is a distortion because the cost information is missing in this calculation. In the last of the variants, the profit denominator is before deducting write-offs. This is advantageous for international comparisons since it eliminates the problem of different methodology and rules for asset depreciation.

3 Methodology

In order to analyze determinants of Effective Tax Rate on Income Tax in Czech banking sector, explanatory variables were selected based on review of related studies and regression model was estimated and tested.

3.1 Data and sample selection

Due to chosen approach of micro-backward looking study, unconsolidated financial statements data from banks operating in the Czech banking sector were used. The financial statements were obtained from www.justice.cz. Unconsolidated data better captures the specifics of the firm and enables to compare data over the years more effectively (Sebastian, 2012). As some of the banks have not published its financial statements for year 2017 as at 30. 6. 2018, we have restricted period only for years 2005 – 2016.

There are forty-five banks operating in the Czech banking sector as at 30.6.2018 (according to Czech National Bank) To be included in the sample, bank must have complete financial statements for the selected period, must have been profitable during the period (i.e. not made loss in any of selected years) and the financial statements must include unconsolidated Czech statutory financial statement that are freely accessible. Only 19 banks meet these conditions and therefore they represent the sample of the study.

3.2 Model specification

Following variables were analyzed as a possible determinants of ETR on the basis of a review of various related studies. Diaz, Rodriguez and Arios (2011) in their study examined possible relationship between ETR and bank size, profitability measured as return on assets, degree of own funds and proportion of bad loans to total assets of a bank. Salaudeen (2017) added to this matter variables such as proportion of loans to clients or total productivity measured as EBIT per one employee. Based on those studies, following variables were selected and used in our model:

- banks size (SIZE) - bank size is measured as a value of net total assets;
- return on assets (ROA) - as a profitability measure, ROA is defined as the ratio of earnings before tax to total assets;
- capital intensiveness (CAPIN) - capital intensiveness is calculated as ratio of fixed assets to total assets;
- financial leverage (LEV) - leverage ratio is measured as Shareholders' capital divided by total assets;
- loans to clients on total assets (LOA) - this ratio show proportion of loans to clients to total assets of bank;
- total productivity (PROD) - defined as earnings after tax per one employee;
- bad debts ratio (BAD) - ratio of allowances for bad debts on loans to clients.

In order to establish the possible multivariate relationship between selected variables and dependent variable ETR, the following model is estimated:

$$ETR_{it} = \beta_0 + \beta_1 SIZEit + \beta_2 ROA_{it} + \beta_3 CAPIN_{it} + \beta_4 LEV_{it} + \beta_5 LOA_{it} + \beta_6 PROD_{it} + \beta_7 BAD_{it}, \quad [1]$$

where *ETR* means effective tax rate, β_0 is the constant, β_1 ,β_2….. β_7 are coefficients, *SIZE* represents bank size, *ROA* is return on assets, *CAPIN* is capital intensiveness, *LEV* is firm leverage, *LOA* is loans to clients on total assets, *PROD* is total productivity, *BAD* is bad debts ratio.

3.3 Correlation analysis

Correlation is a linear relationship between variables. The correlation coefficient (*r*) can take values in interval <-1; + 1>. The greater the absolute value of coefficient, the greater the correlation between the two variables. The positive correlation coefficient expresses the positive relationship between the variables, the negative correlation coefficient expresses the negative relationship between the two variables. If the correlation coefficient value is zero, there is no correlation between the variables. The correlation coefficient *r* = +1 expresses the full (linear) direct dependence of the variables, the correlation coefficient *r* = -1 indicates the full (linear) indirect dependence of the variables.

3.3.1 Pearson correlation coefficient

Pearson's correlation coefficient gives us the degree of interdependence between variables. According to Hančlová (2012), it can be calculated as:

$$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}}, \qquad [2]$$

where *n* is the sample size, x_i and y_i are the individual sample points indexed with i, $\bar{x}$ and $\bar{y}$ are the sample means.

4 Application part

In this chapter, we will analyze Effective Tax Rate of banks within Czech banking sector and attempt to identify its drivers.

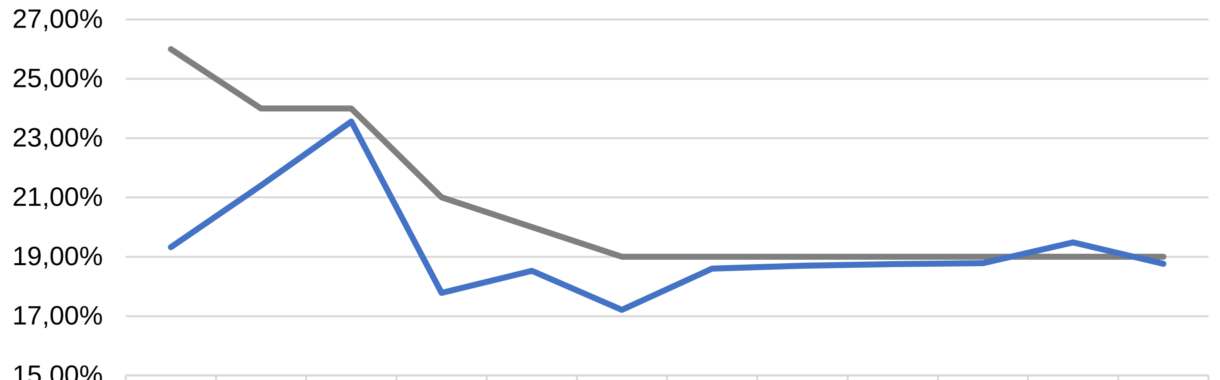
Firstly, the comparison of Statutory Tax Rate and Effective Tax rate will be held. Then, the correlation analysis will be conducted in order to identify determinants of Effective tax rate. Finally, the results of regression analysis will be presented.

4.1 Empirical results

To address the objective of analyzing the tax burden of banks within the Czech banking sector, Effective Tax Rate (ETR) of selected sample was calculated and compared with the respective Statutory Tax Rate (STR) for period of study, 2005 – 2016. Extreme values of ETR over 50 % and below 0 % were excluded from calculation to avoid biased results.

Comparison of STRs and ETRs in given period is presented in Figure 1. Annual average of ETRs ranged from 17,21 % in 2010 to 23,56 % in 2007.

Figure 1: Comparison of STR and ETR



	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
STR	26,00%	24,00%	24,00%	21,00%	20,00%	19,00%	19,00%	19,00%	19,00%	19,00%	19,00%	19,00%
ETR	19,32%	21,40%	23,56%	17,78%	18,53%	17,21%	18,60%	18,70%	18,75%	18,78%	19,48%	18,76%

In the period of 2011 – 2016, calculated Effective Tax Rate of Czech banks is stable with values similar to Statutory Tax Rate. However, there is a noticeable deviation between ETR and STR in the period 2008 – 2010. The decrease in ETR in those year is mainly attributed to higher proportion of allowance on bad debts accounted for by banks. During the financial crisis, banks responded by increasing the creation of the allowances for bad debts as a result of reduced solvency of their clients who failed to meet their obligations (Petrášová 2016).

As the allowances reduce the tax base (if the conditions set out in the Act on Income Tax and Act on Reserves are fulfilled), and their level has almost tripled over the period under review, it is likely to have an impact on the bank's total taxation (Petrášová 2016). This could explain, for example, the drop in the effective tax rate between 2007 and 2008, when the allowances has increased significantly. However, it is a question of how big proportion of the allowances were tax deductible.

4.2 Descriptive statistics

The descriptive statistics of ETRs used in the study is presented in Table 1.

Table 1: Descriptive statistics of dependent variable ETR

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance
ETR	228	1,1418	0,0414	1,1832	0,2219	0,1253	,016

The mean for ETR is 22,19 % with 4,14 % as a minimum and 118,32% as a maximum value. The standard deviation do not vary significantly between ETR of banks in the sample.

The descriptive statistics of analyzed determinants are to be shown in Table 2.

Table 2: Descriptive statistics of explanatory variables

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance
SIZE	228	1066422,9160	103,0840	1066526,0000	201331,9723	273473,4013	74787701235,841
ROA	228	15,3559	,0005	15,3564	,1953	1,4349	2,059
CAPIN	228	,9729	,0002	,9731	,0230	,1035	,011
LEV	228	71,5235	,0006	71,5241	1,1210	7,9351	62,966
LOA	228	299,4713	,0001	299,4714	4,8010	32,5965	1062,535
PROD	228	8,1760	,01773	8,1937	1,6596	1,6496	2,721
BAD	207	,38772	,0003	,3880	,03740	,0540	,003

As observable from results, mean of ETR determinants' values and its standard deviations vary widely.

4.3 Correlation between Effective Tax Rate and Selected Determinants

The results of correlation level between banks characteristics and the Effective Tax Rate are provided in Table 3.

Table 3: Pearson's correlation coefficients

		SIZE	ROA	CAPIN	LEV	LOA	PROD	BAD
ETR	Pearson Correlation	-0,220**	-0,019	-0,002	-0,015	-0,004	-0,171**	0,051
	Sig. (2-tailed)	0,001	0,775	0,971	0,825	0,954	0,010	0,465
	N	228	228	228	228	228	228	207
**. Correlation is significant at the 0.01 level (2-tailed).								

As observable from the results, statistically significant correlations are to be confirmed only between ETR and the size of bank expressed as total value of its assets and between ETR and productivity expressed as a profit per one employee. Otherwise, we cannot confirm that rest of selected variables have any impact on the level of effective tax rate of banks within Czech banking sector.

The variable size of entity is statistically significant at the level of 1 % with coefficient of -0,220 suggesting negative relationship between ETR and size of the bank. The larger size of bank therefore leads to lower level of ETR. This finding could be attributed to more efficient tax optimization and fiscal planning which is characteristic for large banks.

Productivity per one employee proved to be another determinant of ETR with coefficient of -0,171. The negative coefficient suggests that the more profitable a bank is, relatively less is its tax burden.

4.4 Regression model

Regression analysis was performed on the above mentioned data sample. Regression analysis is a statistical method by which we estimate the value of the dependent variable (ETR) based on the knowledge of the independent variables (SIZE, ROA, CAPIN, LEV, LOA, PROD, BAD). Three variables are important in this analysis, namely R^2, Sig. of the whole model used in F-test, and Sig. of individual beta coefficients used in T-test. The first is the determination coefficient, or R^2, which evaluates the suitability of a given regression function. It takes values from 0 to 1, the greater the coefficient, the better the predictive power of the model. It is also the Sig. of the whole model, which evaluates the statistical significance of the model. The last significant variables are the Sig. of the level constant and the individual factors that can be used to assess their statistical significance. The regression was performed for following model:

$$ETR_{it} = \beta_0 + \beta_1 SIZE_{it} + \beta_2 ROA_{it} + \beta_3 CAPIN_{it} + \beta_4 LEV_{it} + \beta_5 LOA_{it} + \beta_6 PROD_{it} + \beta_7 BAD_{it}$$

It is clear from the table below that regression analysis results are not very satisfactory.

Table 4: Model summary

R	R Square	Adjusted R Square	Std. Error of the Estimate	F	Sig.
0,347[a]	0,120	0,092	0,124	4,189	0,000

The coefficient of determination, R^2, is only 12,0 - which means that 12 % of the variability of the dependent variable is explained by the regression model and 88 % is a random component. To evaluate the statistical significance of the whole model, it is important to look at Sig. value. Sig. value in estimated model is lower than 0,01 significance level and therefore we can conclude that estimated model is statistically significant.

To determine statistical significance of each explanatory variable, the comparison of individual Sig. and significance level (1 %) needs to be performed.

Table 5: Coefficients of estimated model

	Unstandardized Coefficients		Standardized Coefficients		
Variable	B	Std. Error	Beta	t	Sig.
(Constant)	0,281	0,018		15,542	0,000
SIZE	-1,259E-7	0,000	-0,267	-3,939	0,000
ROA	0,011	0,044	0,123	0,240	0,811
CAPIN	-0,689	0,810	-0,573	-0,850	0,396
LEV	-0,004	0,012	-0,281	-0,382	0,703
LOA	0,003	0,003	0,757	0,923	0,357
PROD	-0,021	0,006	-0,269	-3,499	0,001
BAD	0,279	0,165	0,116	01,690	0,093

As the results suggest, the statistical significance was proven only for level constant, variable SIZE and variable PROD at the level of 1 %.

5 Conclusion

The aim of our findings was to analyze Effective Rate of Income Tax in Czech banking sector and to find determinants of Effective Tax Rate during the period 2005 – 2016. ETR, calculated using micro-backward approach, was examined together with seven potential conditioning factors.

Within the comparison of statutory rates of Income tax and calculated Effective tax rate of analyzed sample of bank, it was shown that ETR was significantly lower than STR in the period 2007 - 2010 due to higher degree of allowance for bad debts creation during the years of Financial crisis connected to high amount of credit defaults in retail segment.

The results of correlation analysis suggest that ETR of banks is conditioned by its size and profitability per one employee. Both determinant proved to be statistically significant at the level of 1 %, both indicating negative relationship between them and ETR. The result of regression analysis was not satisfactory as the coefficient of determination was only 12 %. However, both F-test and T-test proved that both, whole model and estimated coefficients, were statistically significant.

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Implications of conditional expectation in portfolio theory

Sergio Ortobelli [1], Noureddine Kouaissah [2]

Abstract

This paper examines and explores the implications of using conditional expectation estimators in portfolio theory. In particular, we focus on two financial applications – (i) approximation of the conditional expectation within large-scale portfolio selection problems, and (ii) performance valuation considering the heavy tails of returns. The aim is to examine the extent to which bandwidth selection and kernel functions impact portfolio returns estimation. Thus, we compare the ex-post wealth obtained from applying the portfolio strategies, which use alternative performance measures based on a conditional expectation.

Keywords

Conditional expectation, large-scale portfolio selection, performance measures, bandwidth selection.

JEL Classification: G11, G12.

1. Introduction

In financial literature, it is well-known that asset returns are not normally distributed and show heavy tails, for a survey of criticism see Mandelbrot [9], Fama [5] and Rachev and Mittnik [11]. Empirical research has established few stylized facts about asset returns: (a) clustering of volatility, (b) skewness, and (c) fat tails (see for instance Kim et al. [8]). Over the years, a significant number of studies on portfolio selection problems have been published. Most of these have suggested different formulations based on operations research models that try to overcome the mean-variance shortcomings.

In this paper, we assess the impact of non-parametric techniques that use conditional expectation estimators in portfolio theory. Conditional expectation is a fundamental concept in probability and statistics and is extremely useful in financial modeling. It plays a significant role in portfolio theory and various pricing and risk management problems. In particular, we examine the impact of bandwidth selection and kernel function choices in two financial applications: *i*) approximation problems within large-scale portfolio selection problems, and *ii*) performance valuation considering the heavy tails of returns.

The first contribution of this paper is to investigate the impact of an alternative return approximation method that depends on *k*-factors in large-scale portfolio problems (e.g., the *k*-fund separation model of Ross [13]). In particular, we examine a non-parametric approximation of returns based on factors obtained using a principal components analysis (PCA). The most common approach to estimate the relationship between returns and *k*-

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factors is the linear approximation based on the ordinary least squares (OLS) estimator (see Ross [13]). This approximation appears to be good enough when the returns are normally distributed. However, Ortobelli et al. [15] found that the non-parametric regression outperforms its parametric counterpart. Moreover, we believe there is substantial evidence of non-linearity in the financial returns (see, e.g., Rachev et al. [12]). For this reason, we use a non-parametric regression analysis to approximate the returns. This approach relaxes the assumptions of linearity and is suitable even for non-Gaussian distributions. The empirical analysis is provided using a portfolio consisting of S&P 500 components assuming that no short sales are allowed since in general the markets admit only limited short sales and during a crisis period the short sales are strictly limited or even not allowed.

The second contribution of this paper is to properly evaluate portfolio choices that take into account the tails of a portfolio distribution. Portfolio distributional tails play a crucial role in finance because they represent the probabilities to obtain losses or gains. In this regard, we consider the behavior of the portfolio returns when some particular events have an impact on the portfolio or on the market (positive or negative trends, crisis, etc.). In particular, we forecast the conditional expected portfolio returns with respect to a given σ-algebra of events (generated either by possible profits or by possible losses) using a new alternative conditional expectation estimator proposed by Ortobelli et al. [14]. Thus, we empirically test sound portfolio performance measures recently proposed by Ortobelli et al. [15] when the most significant events (for investors) happen. Moreover, we illustrate how the new performance measures are impacted by the choice of the kernel function and bandwidth selection. Here, we use an ex-post empirical analysis to show their greater capacity to produce wealth in the US market considering some different rules.

The rest of the paper is organized as follows. In Section 2, we discuss and examine the impact of the approximation method within large-scale portfolio selection problems. In Section 3, we introduce and empirically test two performance measures based on a conditional expectation. Finally, our conclusions are summarized in Section 4.

2. Practical and theoretical aspects of return approximation in portfolio selection problems

In this section, we focus on approximation methods within large-scale portfolio selection problems. In practice, there are many different ways to reduce the dimensionality dependence of a large-scale portfolio selection problem see Ortobelli et al. [16]. In this section, we widely discuss a non-parametric regression model to reduce the dimensionality of a large-scale portfolio problem. In particular, we reduce the dimensionality of the problem and approximate the return series using a multifactor model that depends on a proper number (i.e., not too large) of factors. Following Ortobelli et al. [15], we determine the principal components by applying the PCA to a proper Pearson linear correlation matrix of returns.

We consider n risky assets defined on a probability space $(\Omega,\mathcal{F},Pr)$, where $\mathcal{F}$ is a σ-algebra of events on (i.e., a collection of subsets of closed under all countable set operations: union, intersection, and complement). We point out the portfolio gross returns $x'z$, where $x = [x_1, \dots, x_n]'$ is the vector of nonnegative allocations among n risky limited liability investments, with gross returns $z = [z_1, \dots, z_n]'$. According to the portfolio literature, a portfolio gross return admits a finite mean and belongs to $L^p(\Omega,\mathcal{F},Pr)$, (the set of all $\mathcal{F}$ measurable real random variables $X : \Omega \to R$; such that the p-th absolute moment is finite i.e., $E(|X|^p) < 1$) for some $p \geq 1$.

With parametric regression models, we can replace the original n correlated time series $\{z_i\}_{i=1}^{n}$ with the n uncorrelated time series $\{R_i\}_{i=1}^{n}$ (obtained by the PCA), assuming that each z_i is a linear combination of R_i. In particular, the dimensionality reduction is obtained by choosing only the first factors that sufficiently summarize a large part of the variability. In this setting, we call portfolio factors f_i the first s principal components R_i with significant variability, while the remaining $n-s$ principal components with smaller variability are summarized by an error, ϵ. Typically, the OLS estimator is widely used to approximate the returns using the following linear relation:

$$z_i = b_{i,0} + \sum_{j=1}^{s} b_{i,j} f_j + \sum_{j=s+1}^{n} b_{i,j} R_j = b_{i,0} + \sum_{j=1}^{S} b_{i,j} f_j + \epsilon_i \quad i = 1, \dots, n \tag{1}$$

where z_i is the gross return for the asset i, $b_{i,0}$ is the fixed intercept for the asset i, $b_{i,j}$ is the coefficient for factor f_j , s is the number of factors, ϵ_i is the error term for asset i and n is the number of assets.

In general, the OLS estimator is a well-established and very useful procedure for solving regression problems when the returns are normally distributed. However, the returns are often characterized by a heavy-tailed distribution (see, e.g., Rachev and Mittnik [11]). Therefore, we cannot assume that the dependence between the returns and the principal components is linear. For this reason, we use a non-parametric regression analysis as an alternative to the classic parametric approach (1). In several financial models (APT, CAPM, etc.) the returns are assumed to be elliptically distributed (e.g., Chamberlain [4] and Ingersoll [7]), and the large-scale portfolio problem is solved by approximating the returns using a regression model on some uncorrelated market factors. Differently, we reduce the complexity of the large portfolio model using a non-parametric regression model, where s factors are determined by applying a PCA to a linear correlation measure and the $(s+1)-$th factor M_{s+1} is a market index, that is,

$$z_i = E(z_i | f_1, f_2, \dots, f_s, M_{s+1}) + \epsilon_i = m(f) + \epsilon \tag{2}$$

One of the most used estimators for $m(\cdot)$ is the multivariate version of the Nadaraya{Watson kernel estimator. However, this estimator presents certain disadvantages. In particular, it corresponds to the local constant fit and may be biased, depending on the marginal density of the design. To overcome these drawbacks, a general class of non-parametric regression estimator based on locally weighted least squares has been proposed (see, e.g., Ruppert and Wand [17]). In this setting, an estimate of the regression function $m(f)$ is readily obtained by estimating the parameter a as the argument in the following minimization problem:

$$\min_{a,b} \sum_{i=1}^{T} \{z_i - a - b'(f_{(i)} - f)\}^2 K_H(f_{(i)} - f) \tag{3}$$

where H is an $s \times s$ symmetric positive definite matrix that depends on the number of observations T, $f_{(i)}$ is the i-th observation of vector f, and $K_H(\cdot)$ is a multivariate kernel estimator. Several kernel functions exist in the literature, see among others Scott [18] for their review. We provide some possible choices of the kernel function in the univariate setting (multivariate kernel function could be seen as the product of the univariate kernel):

- Uniform kernel $\mathrm{K(u)} = \frac{1}{2} 1(|\mathrm{u}| \leq 1)$;
- Epanechnikov kernel $\mathrm{K(u)} = \frac{3}{4} (1 - \mathrm{u}^2) 1(|\mathrm{u}| \leq 1)$;
- Quadratic kernel $K(u) = \frac{15}{16} (1 - \mathrm{u}^2)^2 \mathbf{1}(|u| \leq 1)$;

- Gaussian kernel $K(u) = (2\pi)^{-1/2} \exp(\frac{-u^2}{2})$;

Several researchers have shown that the choice of kernel is not critical, while the performance of the smoothed regression function is more a question of bandwidth choice. Fan and Gijbels [6] give a survey on bandwidth selection for the univariate local polynomial smoothing technique, which contains the Nadaraya–Watson estimator as a special case. However, there is little guidance in the literature on bandwidth selection for multivariate kernel density estimation, which certainly remains an important issue in empirical studies. The most widely used bandwidth selection methods are the rule-of-thumb and the plug-in bandwidth selections. In particular, the former is the normal reference rule for kernel density estimation presented in Bowman and Azzalini [2]. For general multivariate kernel estimators, Scott [18] suggests the following bandwidth selectors:

$$\text{Scott's rule in } \mathbb{R}^d\text{: } \hat{h}_i = \hat{\sigma}_i n^{-1/(d+4)}, \tag{4}$$

where $\hat{\sigma}_i$ is the usual estimate of the standard deviation of each variable x_i , and n is the sample size.

To emphasize the implications of non-parametric approach, we examine recent performance measures, proposed by Ortobelli et al. [15], based on the conditional expectation that takes into account the portfolio distributional behavior on the tails. More specifically, the first suggested performance measure is based on two different σ-algebras (the σ-algebra generated by the portfolio losses, and the σ-algebra generated by the portfolio profits). The second performance measure considers σ-algebras generated by the joint losses and gains of all assets in the market. These σ-algebras are approximated using σ-algebras generated by proper partitions (of losses or of gains). Due to space limitation, for theoretical and practical discussion on the new performance measures, we refer to Ortobelli et al. [15]. The TOK ratio (see Ortobelli et al. [15]) is a highly flexible performance measure that considers the expected portfolio returns, given the σ-algebras generated by the portfolio profits and the σ-algebra generated by the portfolio losses. Moreover, in view of this approach, the second performance measure JTOK considers an important feature of the market, namely, the joint losses and gains among the assets. This performance measure takes into account the heavy tails of all returns jointly (i.e., the joint losses and gains among the assets) that should be considered by portfolio managers.

The aim of this paper is not to discuss or to define the new performance measures, which are well presented in Ortobelli et al. [15], but to examine the extent to which these performance measures are impacted by estimating the conditional expectation. In particular, we investigate the implications of bandwidth selection and kernel function choice. The following section as illustration presents the differences among two well-known reference rules and distinguishes two defined kernel functions.

3. Empirical analysis

In this section, we compare the optimal portfolio approaches solved according to the performance measures (i.e., TOK and JTOK ratios) with different parameter choices, see Ortobelli et al. [15] for more details. In other words, the ex-post wealth of the optimal portfolios is evaluated considering the new performance measures and different approximation methods. We use all active stocks on the S&P 500 index from October 28, 2003, to June 10, 2016. The data set is obtained from the Thomson Reuters DataStream database and consists of 3177 daily observations. In particular, we employ the proposed techniques to reduce the dimensionality of large-scale portfolio problems (see Section 2).

Specifically, we perform a PCA on a Pearson correlation matrix of the returns. Then, we approximate the portfolio returns using the RW regression model. Moreover, in order to evaluate the conditional expected value with respect to a given σ-algebra, we use the OLP estimator as suggested by Ortobelli el al. [14].

We recalibrate the portfolio every month (20 trading days), respectively. To guarantee sufficient diversification, which is known to lead to greater long-run portfolio performance (see, e.g., Statman [19]), we set the upper bound of investment to each asset to 20% (i.e., $x_i \leq 0.2$). We use a moving average window of 500 trading days to compute each optimal portfolio. Following Biglova et al. [3], we use a larger number of observations (namely 500) to compute and estimate the JTOK ratio, since we have to account the joint risk of several assets. Starting with an initial wealth $W_0 = 1$, which we invest on October 28, 2003, we evaluate the ex-post wealth sample paths obtained by maximizing the new performance measures. Therefore, at the *k*-th optimization, three steps are performed to compute the ex-post final wealth:

Step 1 Apply the PCA to the Pearson correlation matrix. Then, approximate the portfolio returns as suggested in Section 2 (via the RW estimator). Here, we investigate the implications of bandwidth choices and kernel functions. In particular, due to space limitations, we use two methods for bandwidth selection (namely Scott's rule and Bowman and Azzalini rule (for more details on this rule see Bowman and Azzalini [2]) and two different kernel functions (namely, Gaussian and Epanechnikov). An extended version of this paper considers various rules and different kernel functions to shed some light on the impact of their choices.

Step 2 Determine the market portfolio that maximizes either TOK or JTOK performance measures applied to the approximated returns. Observe that when we apply the new performance measures, the computational complexity must be analyzed since the conditional expected portfolio is not a linear function of portfolio weights. Consequently, we may have many local maxima and an increased computational complexity. Clearly, standard optimization algorithms may be not adequately suited to optimize portfolio problems. Thus, in order to optimize performance measures in an acceptable computational time, we use as a starting point the optimal solution obtained using a heuristic algorithm for overall optimization [the ones proposed by Angelelli and Ortobelli [1] for portfolio optimization]. Then, we improve this solution by applying the Matlab heuristic function pattern search implemented in Matlab 2015 to solve global optimization problems.

Step 3 Compute the ex-post final wealth. In this step, we consider proportional transaction costs of 20 basis points

We apply the algorithm until the observations are available. The results of this analysis are summarized in Tables 1-2. In both Tables, we present summary statistics (mean, standard deviation, VaR5%, CVaR5%, final wealth) for the ex-post log returns obtained by maximizing the TOK and JTOK ratios with Pearson correlation matrix and using two estimators of the conditional expectation (i.e., OLP and RW). Table 1 illustrates the results of the Gaussian kernel function and two bandwidth rules. Table 2 represents the case of the Epanechnikov kernel function and the two reference rules.

Table 1: Statistics of the ex-post returns obtained by maximizing the TOK and JTOK ratios using the Gaussian kernel function and two reference rules

	Mean	St.dev.	VaR5%	CVaR5%	Final W
Scott's rule					
TOK(5)	0.134	2.839	4.008	6.485	18.2577
TOK(7)	0.112	2.721	3.838	6.433	10.9141

TOK(10)	0.141	3.414	4.907	7.834	13.9400
JTOK(3)	0.099	2.199	3.401	5.379	10.6968
JTOK(5)	0.075	2.208	3.430	5.450	5.0778
JTOK(10)	0.073	2.167	3.335	5.365	4.8858
S&P 500	0.029	1.210	1.780	2.964	2.0209
Bowman and Azzalini rule					
TOK(5)	0.128	2.734	3.398	6.315	17.4251
TOK(7)	0.109	2.524	3.232	6.246	10.0125
TOK(10)	0.132	3.125	4.521	7.254	12.8469
JTOK(3)	0.097	2.124	3.376	5.246	10.3215
JTOK(5)	0.072	2.107	3.235	5.124	4.9125
JTOK(10)	0.069	2.032	3.198	5.213	4.4129

Table 2: Statistics of the ex-post returns obtained by maximizing the TOK and JTOK ratios using the Epanechnikov kernel function and two reference rules

	Mean	St.dev.	VaR5%	CVaR5%	Final W
Scott's rule					
TOK(5)	0.137	2.828	4.011	6.315	18.2497
TOK(7)	0.114	2.732	3.829	6.458	10.8438
TOK(10)	0.142	3.424	4.911	7.837	13.8911
JTOK(3)	0.102	2.201	3.404	5.381	10.7215
JTOK(5)	0.077	2.211	3.442	5.461	5.0864
JTOK(10)	0.074	2.169	3.339	5.371	4.9124
Bowman and Azzalini rule					
TOK(5)	0.129	2.739	3.401	6.317	17.4242
TOK(7)	0.110	2.531	3.239	6.250	10.1245
TOK(10)	0.133	3.129	4.528	7.260	12.9015
JTOK(3)	0.099	2.127	3.381	5.250	10.3321
JTOK(5)	0.074	2.109	3.238	5.129	4.9891
JTOK(10)	0.071	2.037	3.201	5.217	4.4369

From Tables 1-2, we observe that:

- The impact of bandwidth choice is slightly greater than the choice of the kernel function. The results are slightly different from Scott's rule to the Bowman and Azzalini rule even maintaining the same evolution. The differences impacted by kernel function is minimal and this in line with previous research that has shown that the choice of kernel is not critical, while the performance of the smoothed regression function is more a question of bandwidth choice.
- The TOK strategies present the greatest average, final wealth, Sharpe ratio (mean/St.dev.). However, they show the highest risk (standard deviation, VaR5%, CVaR5%).
- The strategies based on TOK and JTOK performance measures perform much better than the S&P 500 benchmark, which shows the worst results in terms of mean, final wealth, and Sharpe ratio. However, it achieves the lowest risk (standard deviation, VaR5%, CV aR5%) among the strategies.

Overall, these results confirm and provide strong support for the proposed performance measures, and the proposed methodology to reduce the dimensionality of the problem, taking into consideration the bandwidth selection problem.

4. Conclusion

Portfolio selection problems often involve unknown parameters that have to be properly approximated from the data. Therefore, in this paper, we consider the implications of conditional expectation estimators, especially the bandwidth selection and kernel function choice, within the portfolio theory. In particular, we focus on two financial problems, namely, approximation problems within large-scale portfolio selection problems and optimal portfolio choices with consistent estimations of the conditional expected returns. Thus, we test new performance measures that account for the heavy-tailed distribution of the returns and their joint risk. In this context, we stressed the important role of bandwidth selection with respect to the kernel function choice. In addition, we confirm that the proposed portfolio selection models yield the best performance and outperform the market benchmark. Therefore, the proposed empirical analysis supports the significance of the conditional expectation estimators within the portfolio theory.

Further research could involve theoretical and empirical studies. On the one hand, a natural extension of this research would take into account several rules and parameters. On the other hand, the use of semi-nonparametric approaches could be considered. Future research will investigate these aspects.

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Health and Economic Risks of Longevity in European Countries

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Abstract

The article deals with the health risks of longevity and their economic consequences in the European countries. It uses various international comparisons to identify longevity factors, the healthy life expectancy and the most common causes of illness and mortality among the elderly. It also focuses on the economic consequences of population aging, especially in relation to long-term care funding and their prognosis. The results of factor and cluster analysis allow a comprehensive comparison of European countries according to the quality of life, health and serious illnesses of people age 65 and over.

Key words

Elderly people, causes of death, health expenditures, comparisons, multidimensional analysis.

JEL Classification: C38, I14, I15, J14

1 Introduction

Statistics concerning causes of death among persons aged 65 or over (the elderly) are of increasing interest. A dramatic change in the nature and delivery of healthcare over the past century has resulted in much longer life spans and a greater prevalence of chronic illnesses. This in turn has led to increased demand on healthcare systems, particularly for long-term care. Public health programmes throughout the EU are often targeted at reducing mortality among people aged less than 65 through preventive measures, for example, the promotion of healthier lifestyles through improved nutrition, lower tobacco and alcohol consumption, an increase in physical activity or a reduction of professional risk (Swiss Re, 2014).

The percentage of the population aged 65 or over in the EU-28 is projected to increase, on average, from 18.9% of the total population in 2015 to 29.0% of the total by 2058, thereafter dipping slightly before rising again to reach 29.2% in 2081 (European Commission, 2015).

The financing and provision of effective care services for the elderly will be one of the most challenging issues facing society, driven by various demographic and societal trends. Public spending on long-term care (LTC) will increase significantly in the coming decades, straining government budgets in advanced economies. In emerging markets, public finances are less stressed but the low starting point of LTC spending and the immense growth of funds needed will still be a huge challenge (Eurostat Statistics Explained, 2017; Eurostat, 2016).

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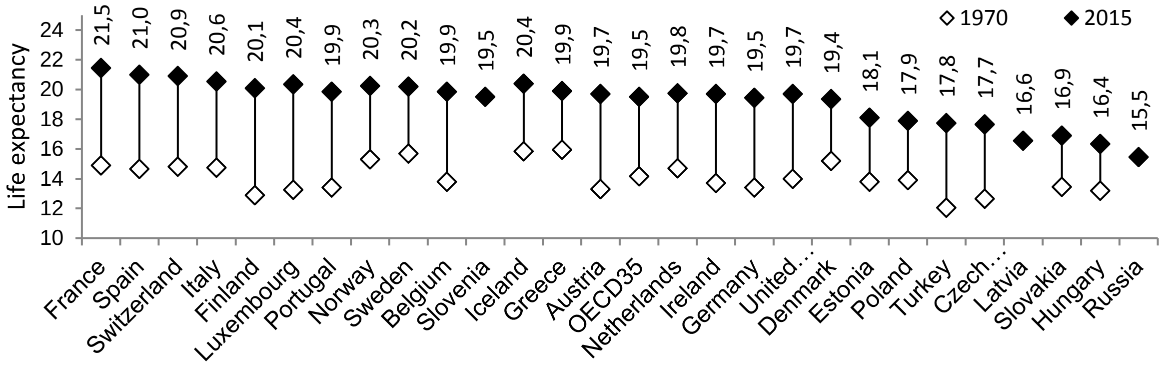
2 Description health risks of elderly people

Life expectancy at age 65 has increased significantly for both men and women over the past few decades in European countries (Figure 1). Some of the factors explaining these gains in life expectancy at age 65 include advances in medical care combined with greater access to health care, healthier lifestyles and improved living conditions before and after people reach age 65.

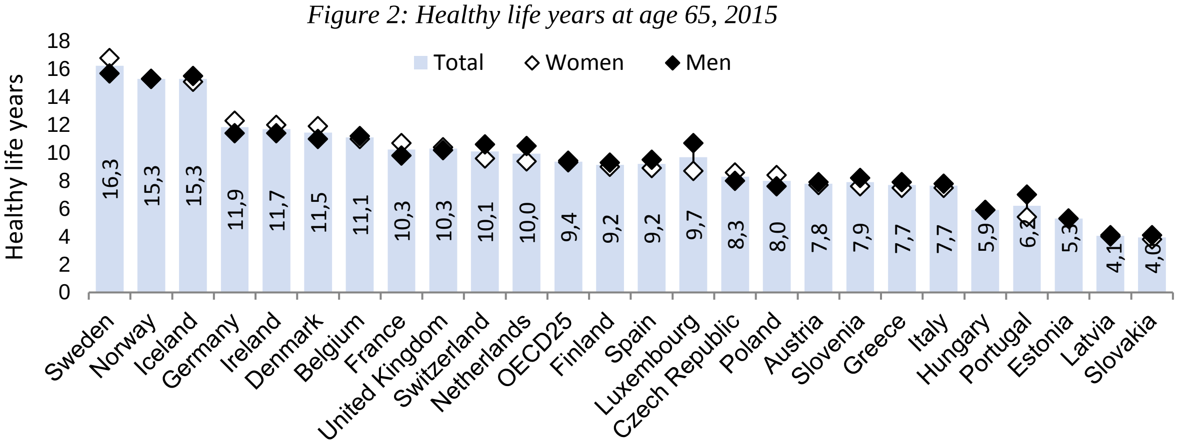
Figure 1: Life expectancy at age 65, 1970 and 2015



Source: Eurostat Statistics Explained 2017, self-processed in Excel

Increased life expectancy at age 65 does not necessarily mean that the extra years lived are in good health. In Europe, an indicator of disability-free life expectancy known as "healthy life years" is calculated regularly. Among European countries the average number of healthy life years at age 65 was almost the same for women and men, at 9.3 years for women and 9.4 years for men in 2015 (Figure 2). Life expectancy and healthy life expectancy vary by European countries. Nordic countries had the highest number of healthy life years at age 65 in 2015. In Sweden, women could expect to live an average of an additional 17 years, and men 16 years, free of disability.

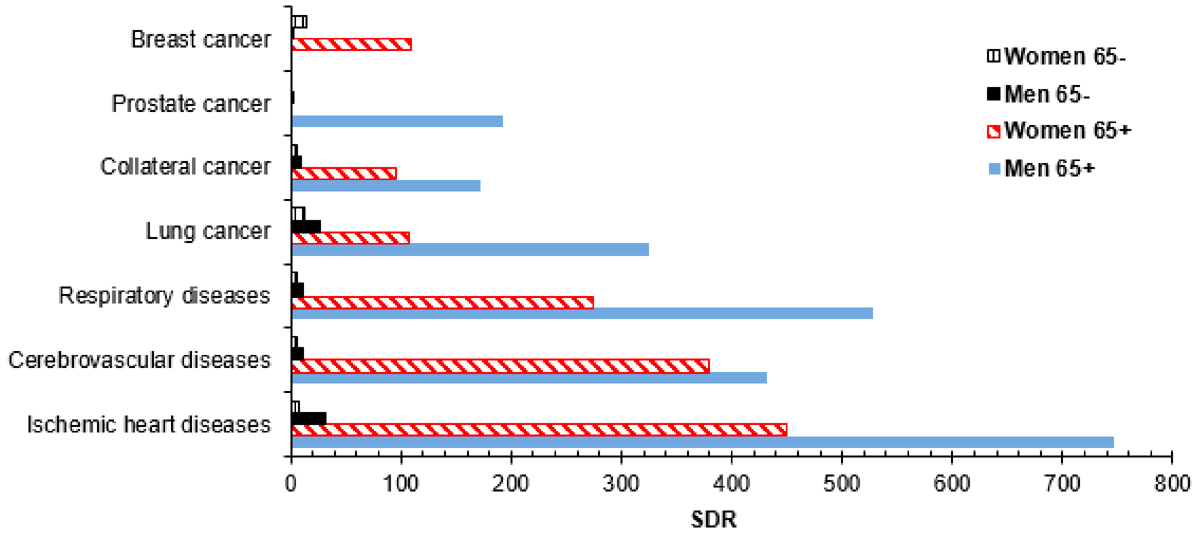
Figure 2: Healthy life years at age 65, 2015



Source: Eurostat Statistics Explained 2017, self-processed in Excel

The latest information for EU-28 countries relating to causes of death is available for the 2014. The diseases of the circulatory system and cancer (malignant neoplasms) were the leading causes of death in the EU. Respiratory diseases were the third most common cause of death in the EU-28. Significant differences in mortality for serious diseases exist across European countries and also by demographic dimensions, including sex and age. To assess the inequalities in EU-28 members according to demographic characteristics with respect to mortality for serious diseases we have compared the standardised death rate per 100 000 inhabitants for leading causes of death in EU. Mortality rates were monitored separately for men and women and for persons aged less than 65 years and aged 65 and over. Visualized standardized mortality rates for serious diseases make it possible to compare for two age categories - persons aged less than 65 years and persons aged 65 and over and for both gender groups (Figure 3).

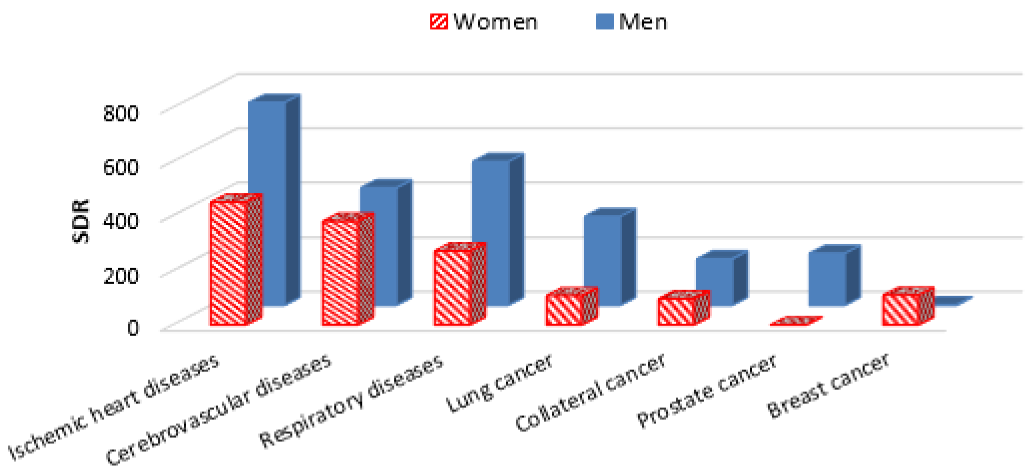
Figure 3: Comparison standardised death rate per 100 000 inhabitants by gender and age groups, EU-28, 2014



Source: Eurostat Statistics Explained 2017, self-processed in Excel

Standardized mortality rates for leading serious illnesses are significantly higher for people 65 years and over in EU-28. Differences in mortality rates by gender are also significant in both age groups, with higher rates of male mortality. Inequalities in mortality rates by leading causes of death at age 65 and over and by gender are also clear (Figure 4).

Figure 4: Comparison standardised death rate per 100 000 inhabitants by main causes of death at age 65+, EU-28, 2014

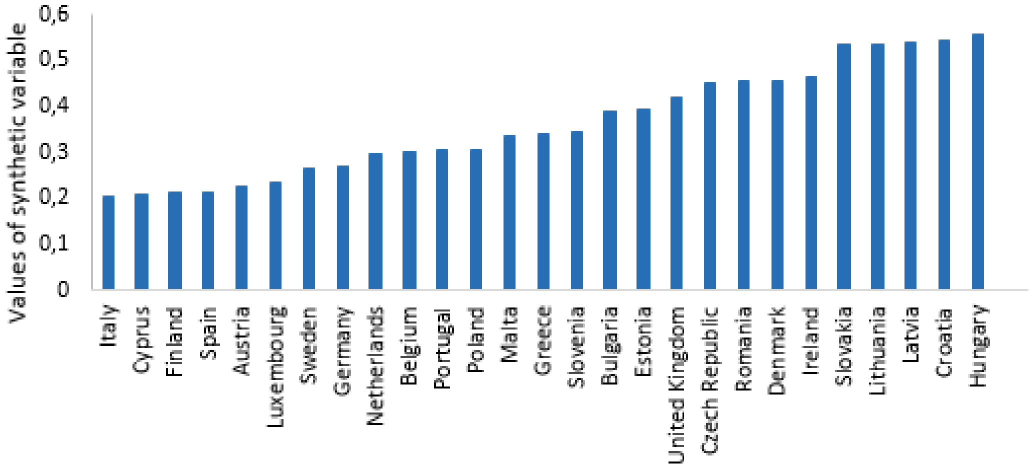


Source: Eurostat Statistics Explained 2017, self-processed in Excel

The population in the European Region is ageing rapidly: its median age is already the highest in the world, and the proportion of people aged 65 and older is forecast to increase from 14% in 2010 to 25% in 2050. People in nearly every part of the Region are living longer, but their chances of spending these later years in good health and well-being vary within and between countries. To assess the inequalities in EU-28 members according to mortality for serious diseases synthetic mortality indicator for causes of death at Figure 5 has been constructed by standardization formula (1) using weights for the standardized values, taking into account the mortality rate for each of the main causes of death in EU-28. In this way, the different severity of the diseases was taken into account. For details see (Stankovičová and Vojtková, 2007).

$$u_{ij} = \frac{x_{ij} - min\{x_{ij}\}}{max\{x_{ij}\} - min\{x_{ij}\}} \tag{1}$$

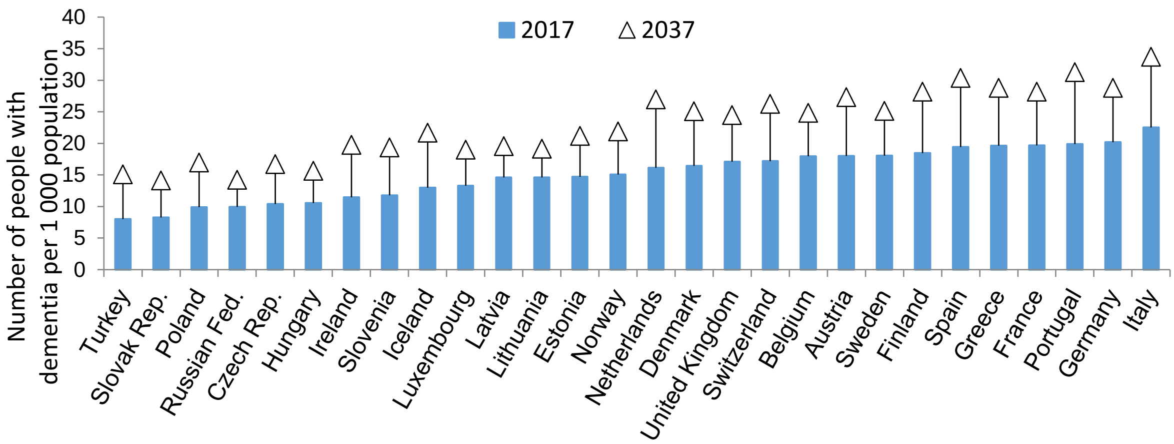
Figure 5: Ordering of EU-28 countries by the synthetic indicator of leading serious deceases for persons age 65 years and over



Source: Eurostat Statistics Explained, 2017, authors' calculations in Excel

Ageing populations mean that dementia will become more common in the future, and the most rapidly ageing countries will see prevalence more than double in the next 20 years (Figure 6). The dementia is one of the diseases that require long-term care and in the future it will entail considerable financial costs.

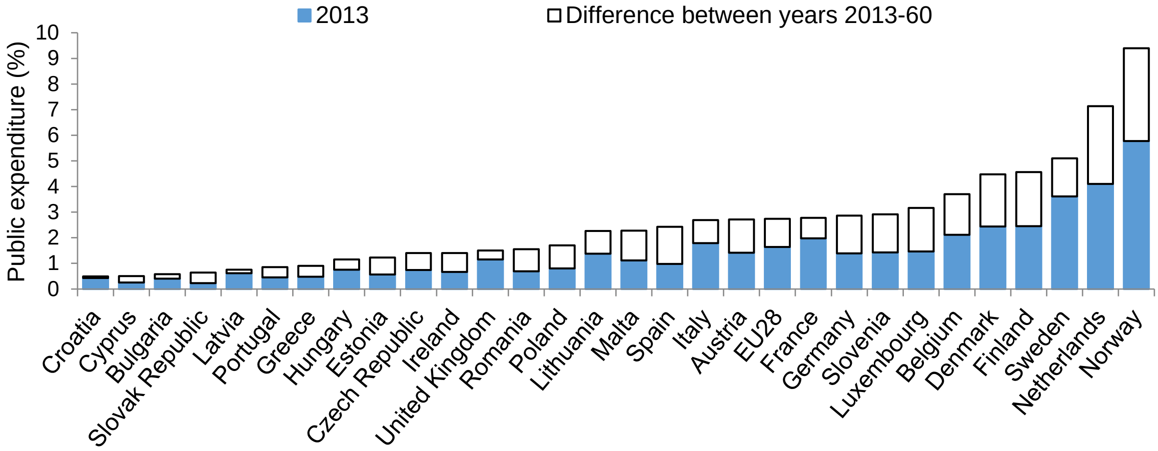
Figure 6: Estimated prevalence of dementia, 2017 and 2037



Source: OECD 2017, self-processed in Excel.

As people age, they are more likely to develop disabilities and need support from family, friends and long-term care (LTC) services. The ranking of countries according to the forecast of LTC expenditure and the differences across European countries is shown in Figure 7.

Figure 7: Prognosis of public expenditure on long-term care as a percentage of GDP, 2013 - 2060



Source: OECD/EU 2016, self-processed in Excel

Long-term care spending is a growing share in many EU countries. The projection model includes a number of factors determining long-term care expenditures. Aging populations may pose a risk to the sustainability of health care financing. It is estimated that the ratio of older people to workers will increase from 27.8% in 2014 to 50.1% in 2060. If life expectancy increases in line with the growing number of years of health, aging may not necessarily occur rising healthcare costs. Empirical research suggests that healthcare technologies are the main driver of growth in healthcare spending.

3 Multidimensional statistical analysis

This chapter summarizes and confirms the conclusions and knowledge of the previous chapters by applying selected multidimensional statistical analysis methods. For details see (Hair et al., 2007; Kopecká & Jindrová, 2017; Pacáková et al., 2016; Pacáková & Kopecká, 2018; Stankovičová & Vojtková, 2007). These methods are applied to a data matrix of nine indicators found in 30 European countries in 2015 from OECD data.

Following indicators were used for thirty European countries:

- GDP – GDP per capita (EUR), 2015,
- E1 – Health expenditure per capita (EUR), 2015,
- E2 – Health expenditure in relation to GDP, 2015,
- E3 – Long-term health expenditure per capita (EUR), 2015,
- E4 – Long-term health expenditure in relation to GDP, 2015,
- H1 – Life expectancy at age 65, 2015,
- H2 – Healthy life years at age 65, 2015,
- M1 – SDR – diseases of circulatory system at age 65+,
- M2 – SDR – malignant neoplasms at age 65+.

The results of the Correlation analysis in Table 1 show the correlation coefficients between each pair of variables. The results indicate a strong dependence of indicators of health status elderly persons in European countries, except malignant neoplasms at age 65+ indicator, on GDP and on health expenditures E1-E4.

Table 1: Results of correlation analysis

	GDP	E1	E2	E3	E4	H1	H2	M1	M2
GDP	1	0,945	0,743	0,926	0,826	0,734	0,775	-0,764	-0,039
E1	0,945	1	0,840	0,903	0,834	0,713	0,781	-0,733	-0,040
E2	0,743	0,840	1	0,761	0,795	0,633	0,759	-0,692	-0,174
E3	0,926	0,903	0,761	1	0,960	0,657	0,781	-0,698	-0,008
E4	0,826	0,834	0,795	0,960	1	0,557	0,779	-0,644	0,022
H1	0,734	0,713	0,633	0,657	0,557	1	0,563	-0,790	-0,417
H2	0,775	0,781	0,759	0,781	0,779	0,563	1	-0,669	-0,112
M1	-0,764	-0,733	-0,692	-0,698	-0,644	-0,790	-0,669	1	0,138
M2	-0,039	-0,040	-0,174	-0,008	0,022	-0,417	-0,112	0,138	1

Source: authors' calculations, output from Statistica 12

By application of factor analysis we try to obtain a small number of common factors which account for most of the variability in the original variables. The MSA = 0.74 statistics show suitability of the origin indicators for factor analysis.

In this case, two factors have been extracted, since two factors have had eigenvalues greater than or equal to 1. Together they account more than 80.701 % of the variability in the original data. *Factor loadings* present the correlation between the original variables and the extracted factors and they are the key to understanding the nature of a particular factor. After *varimax rotation* have been obtained the factor loadings shown in Table 2.

Table 2: Factor loadings after VARIMAX rotation

Proměnná	**HDP**	**E1**	**E2**	**E3**	**E4**	**H1**	**H2**	**M1**	**M2**
F1	0,90	0,95	0,81	0,95	0,91	0,75	0,81	-0,78	0,05
F2	0,03	-0,03	-0,29	0,12	0,06	-0,52	-0,17	0,35	0,91

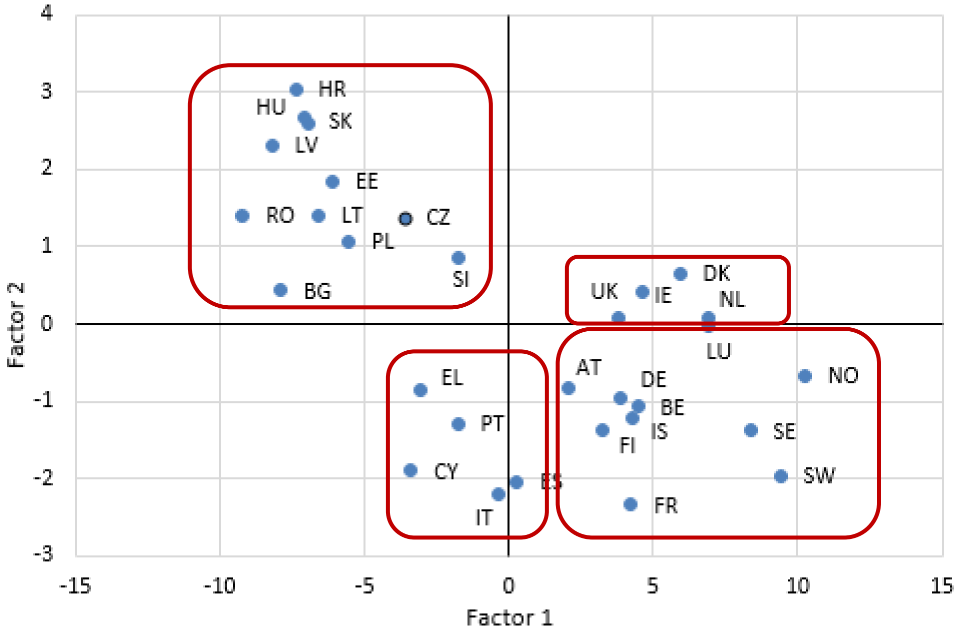
Source: authors' calculations, output from Statistica 12

Based on Factor loadings (Table 2) we can state that the first common factor F1 is strongly positive correlated with all expenditure indicators, even with H1 and H2, and strongly correlates with mortality M1. This factor explains up to 67.358% of the variability of the original 9 indicators and we can call it as a Factor of favourable living and health condition of elderly people. High values of F1 indicate a high level of life and health of seniors in a given country. Factor F2 strongly correlates only with indicator M2, so we can identify it as the Mortality factor for the malignant neoplasms of elderly persons. The higher its value, the higher the mortality rate M2 for malignant neoplasms of the elderly.

The *Factor Scores* present the values of the rotated factor for each country. Graphical display of countries in a two-dimensional coordinate system with the axes of the selected two factors allows us to quickly assess the observed situation in each country and also compare the situation in different countries.

In the coordinate system of the factors F1 and F2 (Figure 8) four groups of countries were created. The first group with high values of factor F1 and low values of factor F2, including all the old EU countries, the second with low to medium values of factor F1 and medium to high values of factor F2, including the new EU members, the third group with the middle level of factor F2 and rather high value the factor F1, including United Kingdom, Ireland, Denmark and Netherlands and fourth group of the south countries of EU with the low values of factor F2 and medium level of the first factor F1.

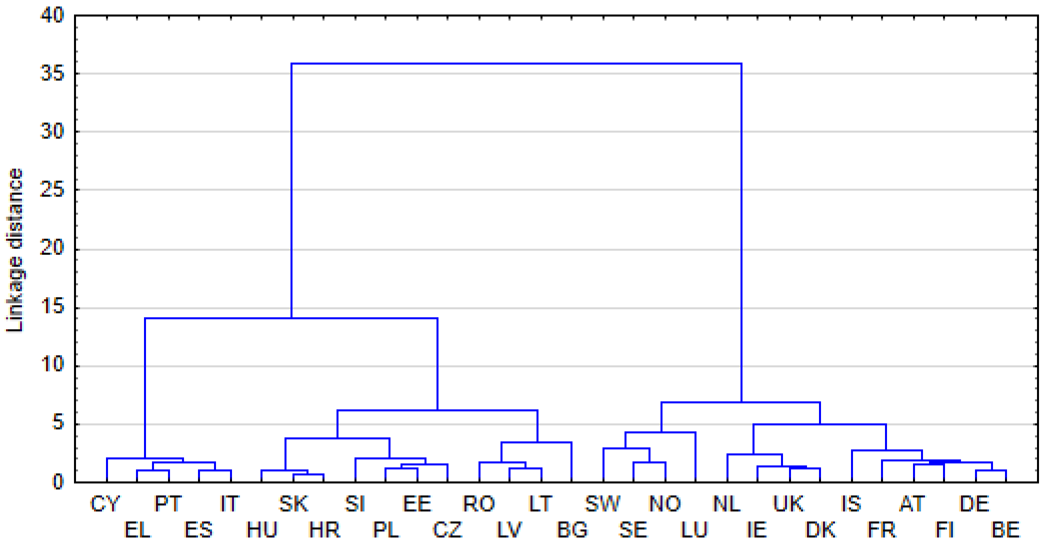
Figure 8: Displaying of the countries in coordinate system of F1 and F2



Source: authors' calculations, output from Statistica 12

Cluster analysis classifies individual countries into groups (clusters) according to their similarity. The results of cluster analysis based 9 variables, the same as in factor analysis, are consistent with the results of factor analysis, as we can see from *dendrogram* on Figure 9 as a results of Ward's Method with Euclidian distance between two different countries. The results of cluster analysis compared to factor analysis even more emphasize the marked differences in the living and health conditions of elderly people in the old and the new EU Member States.

Figure 9: Dendrogram, Ward's method, Euklidean distance



Source: authors' calculations, output from Statistica 12

4 Conclusions

Significant differences in health status of elderly persons exist between European countries. Health inequalities exist along demographic dimensions, including sex, age, geographic area and socio-economic status. Standardized death rates for leading serious deceases are significantly higher for persons age 65 or over and for men in each country and are the highest in the former socialist countries,

Spending on long-term care has increased more than for any other type of health care, but spending varies considerably across countries. Aging populations may pose a risk to the sustainability of health care financing. If life expectancy increases in line with the growing number of years of health, aging may not necessarily occur rising healthcare costs.

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Stress tests in actuarial practice

Michal Páleš, Andrea Kaderová[1]

Abstract

The paper focuses on the need to analyze and measure new types of risks in insurance companies and to re-evaluate the original actuarial methods to assess these risks. An important methodology of risk analysis and measurement at present (also under the Solvency II directive) consists of stress tests and analysis of scenarios. In the paper, we presented these methods in a framework and tried to integrate stochastic models ones in the context of the use of copulas in risk aggregation for risk of liquidity.

Key words

risk management, stress tests, scenario analysis, risk aggregation, copulas

JEL Classification: G22

1. Introduction

Generally, the risks arise mainly as a result of the company's development in technological (cyber risk), social (longevity risk, pandemic risks) and the natural environment (risk of ecological disasters). Their development is consequently enhanced by the increasing complexity and visibility of individual systems. For the insurance sector, this development has a particularly significant impact as risk is its core business. In addition to the known risks posed by the insurance company itself (in particular the risks of insurance, market, credit, operational and liquidity risk) and the risks that the client assumes (the risk corresponds to the insurance product in question), he must also pay attention to the new types of risks the frequency of occurrence and the severity of the impacts are not perfectly known for the insurance company at the time. This condition is due to the nature of the new types of risks as risks associated with either completely new, changing or not yet known phenomena. Their analysis was also enforced by the current Solvency II methodology.

The process of risk assessment, which is part of risk management (risk management) insurance, then is continuous and very complex process as it implies, as we mentioned, both their own risk assessment and, secondly, evaluation of risk-taking, i.e. client risks. The task of risk assessment is to examine the impact of risk on the insurance portfolio and consists, according to [1], mainly of three phases: risk identification, risk analysis and evaluation (measurement) risks. Other methods are available for each phase. Traditional approaches to risk assessment in insurance are mostly quantitative (actuarial methods). However, it is possible that these methods still applicable may not be suitable for assessing new types of risk (for example, the absence of historically comparable data). The aim of the paper is to introduce stress testing and analysis of scenarios in connection with risk aggregation using the copula of analysis functions (also reported in the measurement) of new types of risks.

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2. Techniques to be used in the analysis of new types of risks

In analyzing new types of risk, according to [1], three basic techniques are used: analysis of scenarios, Monte Carlo simulations and Bayesian statistics, which are briefly described below.

Scenario analysis is based on a description of future developmental models and can be used to develop future strategies. Set of scenarios can reflect the best, the worst, and the expected status. The analysis of these scenarios subsequently serves to identify the future state that may occur under certain predetermined circumstances. It also results in the identification of the predicted consequences of the individual scenarios and the likelihood of their occurrence. Since scenario analysis is also focused on identifying causality, it is extremely effective for new types of risk. Scenario creation is primarily based on the definition of the risk or area of interest, as well as the creation of an expert team consisting of experts focusing on both the investigated and the associated areas. A broad-based team helps overcome certain stereotypes of thinking, reduce the limited rationality of a narrowly focused group of professionals, and expand the possibilities of mental models of individual participants. This phase involves identifying risk factors, obtaining the required data or information, and developing a methodology to obtain technical results. After documenting the results obtained, the correctness of the methodology, model and processes used is verified. If this process is successful, the results can be exploited and shifted to the next stage of risk assessment. Deterministically created scenarios are rather intuitive, and their analysis mostly involves a smaller number of scenarios, which in addition may also have a historical character and thus be based on the existence of historical data (historical scenarios). However, given the relative lack or lack of historical data for new types of risk, it is much more appropriate to use hypothetical scenarios that are predominantly based on expert opinions. Specific features have stochastic scenarios that are predominantly based on a large number of simulations created by changing parameters and input variables.

Monte Carlo simulations can be used to solve complex risk systems where random variables are used as inputs and a large number of calculations (so-called simulations) are performed. The output of this modeling can then be a value, probability, or probability distribution, or it can be the identification of the main functions in the model. See also [4].

The Bayesian approach (Bayesian statistics) is based on the subjective interpretation of probability and the use of expert opinions as a robust statistical tool. This approach can be used to the same extent as classical statistical methods with a wide range of outputs. Bayesian statistics are based on Bayes' veto, which is a mathematical formula used to calculate contingent probabilities. It is different from the classical approach in that it does not assume that all partition parameters are fixed, but they understand them as random variables. Bayesian probability perception is based on the measurement of the validity of events with incomplete information. For this reason, these methods may be useful for re-identifying risks, for more information see also e.g. [5].

3. Stress testing and scenario analysis

Stress testing is a risk management technique used to assess potential adverse effects on the current and future financial position of the institution, from the set of changes in risk factors that correspond to unexpected but realizable events. Stress testing is a well-used tool to supplement information on sensitivity to individual types of risk as well as risk profile, on the possible development of risks. The stress tests are based on predefined scenarios describing the future crisis situation. Stress testing is widely used in the banking and insurance sector - by the financial institutions themselves and by the supervisory authorities themselves (the Basel III and Solvency II methodologies, see also [2], [3]). We can divide it into two main categories: analysis of scenarios and sensitivity analysis. In this context, it is also necessary to mention reverse stress testing, which can be considered as an extension of stress testing.

Scenario analysis uses a hypothetical future environment or relevant historical state of the world defined by changes in risk factors affecting the insurer. Typically, it is done within a time horizon that is suitable for business and risk testing. Scenarios in the context of Table 1 are among the deterministic risk measurement tools, and we recognize the stochastic approach as already mentioned.

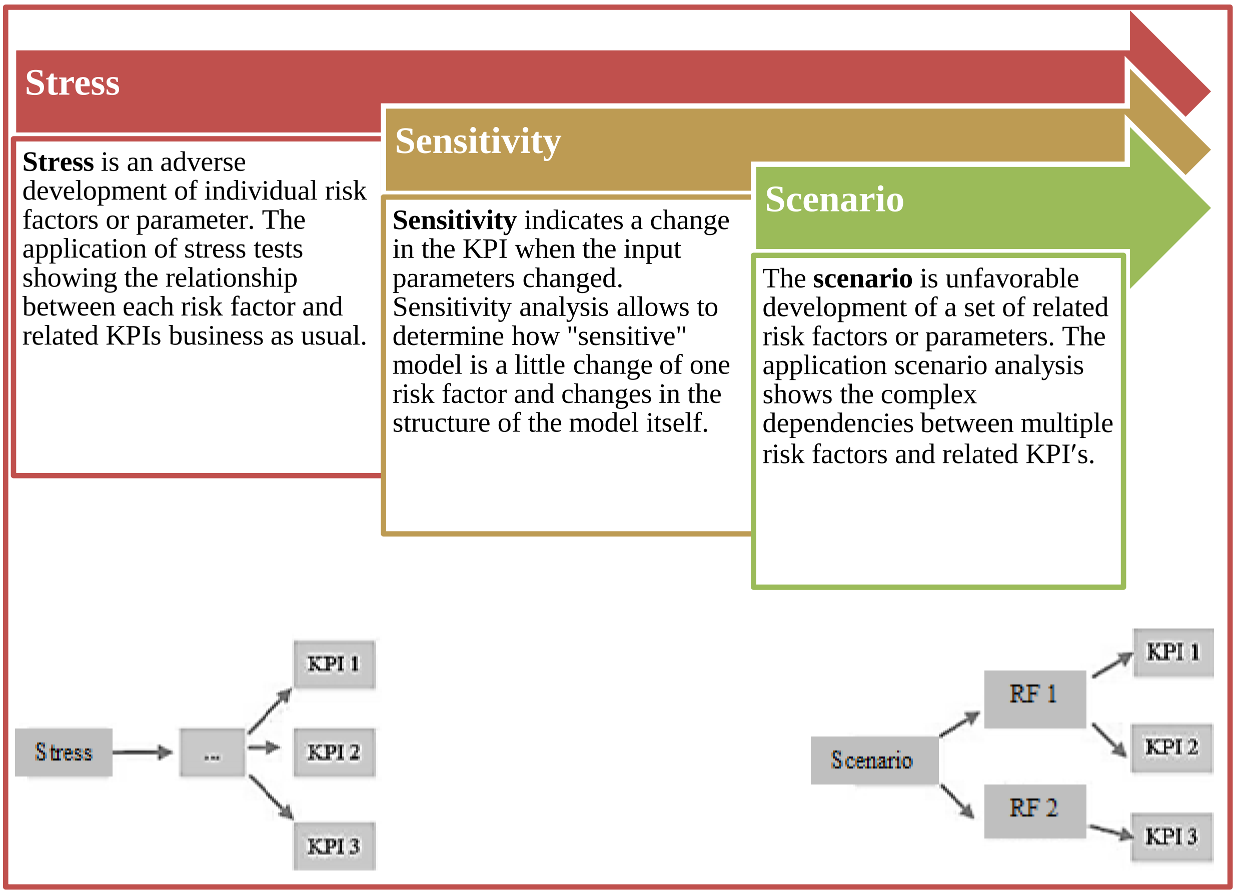
Table 1: Historical and hypothetical scenarios, source: [2], modified according to [6]

Historical scenarios	***Hypothetical scenarios***
– this is a reconstruction of past events with a significant financial impact. Their advantage is that they are based more on reality than on judgment (this is especially true for situations with a high likelihood of their future repetition). On the other hand, the retrospective of the past may not evoke the worst scenarios that can actually happen in the future and, moreover, the relevance of such an approach is undermined by possible market and structural changes in the future. A relevant stress test based on historical scenarios seeks to find an answer to the question of what impact on financial institutions should re-enact a particular crisis situation. *Example.* *2001: Terrorist attack in the USA (conse-quence: for the week, the stock markets were closed in the US and reopened in the phase of a deep decline)* *2011: Earthquake in Japan (consequence: the tsunami caused a nuclear disaster (Fukushima power plant) and severely damaged the Japanese economy, including the fall of stock markets)*	– these are artificially constructed scenarios that can be divided into two groups: 1. on **a real basis**, from events that have occurred in the past (e.g. a massive decline in currency), 2. **only artificial**, "What if" type (e.g. bankruptcy of a major company, unexpected increase in unemployment, hacker software attack, massive cancella-tion of insurance contracts, etc.) – when planning, account must be taken of the following types of risks, risk factors, streaming tests, stress level, effects (second round, dominoes) – the categorization of stress tests and relevant types of hypothetical scenarios is as follows: for market risk, credit risk, liquidity risk, insurance risk. In the area of insurance risk it can be: • increase in insurance costs, • increase in administrative and incidental costs, • changing product ranges, • changing the size of the insurance line, • changing security rates, • enhance antiselection effect. – categorization insurance scenarios EIOPA 2014: • **life insurance** (longevity, mortality, retention scenarios), • **non-life insurance** (storms, hurricanes, earthquakes, floods, airport disasters, lack of technical provisions).

Sensitivity analysis usually includes an alternative to one risk factor (or a limited number of risk factors). It is typically done in a shorter time horizon, an example of which is an immediate shock. Sensitivity analysis requires fewer means than scenario analysis and can be used as a simpler technique for assessing the impact of changes in risk when rapid response or more

frequent results are needed. The magnitude of the change considered is often relatively small, and is often used to estimate the effect of changing the assumption or effect of model parameter errors. Greater quantities can also be used to assess the impact of large changes to one risk factor on an insurer. Sensitivity analyzes are often used as a tool for calculating volatility and other variables based on the use of other assumptions for basic probability distributions, including possible non-linearity and interrelationships between model parameters.
In connection with stress testing, the question arises as to what unit to use to measure the impact of scenarios. One of the options is to select performance indicators - Key Performance Indicators (KPI) quantifying the overall performance of a business in relation to the relevant global target or the critical success factor (having the greatest impact on company performance) and assessing the impact of stress tests on selected indicators. In insurance, the impact of unfavorable scenarios on the solvency of an insurance company may be considered whether it would be the SCR or MCR capital requirements (see [2]). The definition of stress, sensitivity and scenario according to [10] is given in Figure 1.

Figure 1: Key Performance Indicators Properties, source: [10], modified



There are many possible conditions of the insurance company's environment. It is impossible for the set of scenarios to contain all of them. To reduce the burden of selecting a large number of scenarios, you can use the scenario generation model. Different modeling techniques are used in stress testing. The use of a specific risk model will depend on the circumstances of the insurer and the approach to risk assessment and risk management. Common methods are based on static and dynamic modeling and a deterministic or stochastic approach.

In its basic form, static modeling suggests that the analysis of the financial position of the insurer is a fixed point in time, while dynamic modeling considering development within a certain time period.

In the case of *deterministic modeling*, scenarios are defined without any reference to their probability. Using this method, the analyst can assign values for discrete scenarios and track the results of each scenario. Deterministic models use a static set of input variables. The deterministic model answers the question "What if".

Stochastic models are more advanced techniques. They are based on probabilities that predict how key financial parameters interact with each other over time, and create a breakdown of results based on simulations of these parameters in the future. One of the advantages of stochastic modeling is that it provides an indication of the extent and probability of occurrence of different financial results. Access requires advanced modeling expertise and uses parameter estimates based on historical observations.

Stochastic simulations are often integrated into the scenario building process in the scenario analysis. In this case, stochastic simulations are used to generate multiple scenarios. The difference between stochastic modeling and deterministic analysis of scenarios is that the values of the variables in the stochastic simulation scenarios are randomly selected from the probability distribution, while those in the deterministic analysis of the scenarios are predetermined. Scenarios determined when used in random simulation for variable variables are referred to as stochastic scenarios whereas those determined on a predetermined value for variables are referred to as deterministic scenarios. It is worth mentioning that sensitivity analysis and stochastic simulation are in fact a scenario analysis. It depends only on how the selected scenarios are.[10]

Stress testing is today a very topical issue, its importance has increased especially since the 2008 financial crisis. This risk analysis tool is used to quantify losses or risks in a simulated worsening of market conditions. Stress testing of financial systems is a key part of a number of programs designed to ensure early detection of weaknesses in the financial system and the development of appropriate solutions to these deficiencies. In this context, the impact of unfavorable economic and financial scenarios, which are unexpected but still probable, is assessed.

One of the important tasks in the stress testing process here is the design of the script. When designing a scenario, it is necessary to consider:

- *the type and scale of the scenario* – the scenario should capture the major risks and "vulnerability" the portfolio is exposed to, and its occurrence must be possible at least with low probability,
- *dependence between risk factors* – in practice, portfolios consist of a large number of assets and are exposed to a large number of risk factors. When designing these scenarios, account should be taken of existing correlations between the different risk factors. In addition, any risk manager who proposes stress scenarios must be aware that these correlations change over the duration of the turbulence. Correlations tend to be higher during turbulent times, which is at least partially affected by appropriate diversification. We distinguish several approaches that can be applied to the problem of creating scenarios for portfolios with a large number of risk factors, for example:
    - *mixture models* – this solution focuses on solving problems related to correlation in turbulent periods. We know that in volatile times, not only volatility, but also correlation is growing. Mixed models predict, in particular, that the breakdown of change in risk factors is actually a mixture of two distributions: the first is a period without turbulence and the other captures the period of market turbulence (for example, models based on a mixture of two normal distributions are used),
    - *principal component analysis* – this approach addresses the question of the structure of addiction and significantly reduces the number of risk factors. A large number of risk factors are replaced by a relatively small set of uncorrelated major components, with the aim of preserving the greatest possible variability

of original data. Such a set of major components can then be modeled by multidimensional models that could not be used for the original (large) set of risk factors. [11]

For the needs of the application, we will be even closer to the stress tests for liquidity risk only, which are related to the analysis of the inability to realize the assets or otherwise obtain the capital necessary to settle the obligations in force so that it does not imply unreasonable losses or costs to the relevant entity. Corresponding stress tests use, for example, the following specific scenarios, according to [2]:

- *an increase in the spread between assets and liabilities* – scenarios based on a reduction in investment income in the event of an unexpected realization of assets caused by the unexpected application of a larger volume of receivables. In particular, it may be a more massive settlement of contracts and the exercise of options or cancellations due to a certain market situation, catastrophic events, etc.
- *setting an unexpected level of new business* – scenarios based either on the hypothetical growth of a new business, which will result in enormous growth in start-up costs, or vice versa, based on a hypothetical decline that will require an unexpected mobilization of reserves (in both cases, pressures on liquidity).

4. Risk aggregation using copula functions

The European Commission's Solvency II directive focuses on the risk profile of the insurance company, and hence the capital needed to cover it. The increasing number and complexity of insurance products results in the necessity of examining the dependency between individual risks in internal models. Neglect, respectively. an incorrect determination of dependence may cause an under-estimation of the overall risk to which the insurance undertaking is exposed. On the other hand, assuming full dependency between risks may lead to an overestimation of the capital requirement, which is reflected in the high degree of commitment of capital.

Under risk aggregation, we understand that different types of risks are combined into one portfolio, or that each class is linked together under one type of risk. The simplest way to combine the risks is to add them. However, there are other, more sophisticated methods of risk aggregation in practice that can achieve a diversification effect. Several methods can therefore be used to aggregate risks: *adding, assigning fixed percentages, using a variation-covariance matrix, using a copula functions*.

The latter method represents the most sophisticated method of risk aggregation, since copula functions represent a method of modeling dependencies between variables, resulting in a diversification effect. The need to model the development of two factors that are to a certain extent dependent leads to the use of copulas as they allow to capture dependence while preserving the probability distribution of individual risk factors, i.e. marginal distributions. *d*-dimensional copula is defined below, for the wider characteristics of copula functions see below, e.g. [8].

Theorem (*Sklar*). Let $C(u_1, \dots, u_d)$ is d-dimensional copula and $F_1(x_1), \dots F_d(x_d)$ are one-dimensional distribution functions, then a function $F_1(x_1, \dots, x_d)$ defined by the formula

$$F_1(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d))$$

is a conjugated distribution function with marginal distribution functions $F_1, \dots, F_d$.

In practice, copulas of different types are applied. The basic classification is:

- *elementary copulas (independent, comonotatory, contramonotonic copula),*
- *implicit copulas (Gaussian, Student's copula)*
- *Archimedes (explicit) copulas (Gumbel, Clayton, generalized Clayton, Frank's copula).*

Archimedes' copulas are found in practice most often for their relatively light construction and also for a unified mathematical procedure that can be constructed. They are also used most for modeling credit risk (in a bank or other borrowed portfolio, particularly Gumbel's copula, see e.g. [7]).

But we, in connection with our analysis, draw on the Marshall-Olkin Copula, which is not an Archimedes' copula, but it is an explicit copulas. Its advantage is the property of modeling situations where certain random shocks can be dangerous for a particular conglomerate (components, lives, companies, etc.). This copula, however, is expressed not as a combined distribution function *C* as mentioned above but as a combined survival function

$$\bar{C} = 1 - C$$

while generally for the survival function $S(x) = 1 - F(x) = P(X > x)$.

In the context of stress testing, we will also consider the two-dimensional Marshall-Olkin's copula, which models the time to a risk event (failure, failure, crisis, breakdown, ...) as a random variable with an exponential distributions with estimated fault intensities: $\lambda_X, \lambda_Y, \lambda_{XY}$ (for individual or aggregated components). Distribution function F_X component failure $X \sim Exp(\lambda_X + \lambda_{XY})$ and analogical $Y \sim Exp(\lambda_Y + \lambda_{XY})$, while in general we can also work with the Poisson process, which loads the relevant risk events gradually at time *t*. Marhall-Olkin's copula has the following shape:

$$_{MO}\bar{C}(S_X(x), S_Y(x)) = min\left\{\left(S_X(x)\right)^{1-\theta_X} S_Y(y), \left(S_X(x)S_Y(y)\right)^{1-\theta_Y}\right\}$$

and for its parameters

$$\theta_X = \frac{\lambda_{XY}}{\lambda_X+\lambda_{XY}},\ \theta_Y = \frac{\lambda_{XY}}{\lambda_Y+\lambda_{XY}}$$

For more information, see e. g. studies by authors Durante – Girard – Mazo or Embrechts – Lindskog – McNeil.

5. Practical example of using Marhall-Olkin's copula

Let us consider holding two companies X and Y. This holding is repeatedly exposed to the liquidity risk (see Chapter 3). The liquidity crisis in Company X occurs on an average every 2 years, in Company Y every 4 years, and globally across the holding, on average every 8 years. We determine that, with Marhall-Olkin's copula, the likelihood that the company X will not reach the holding company for at least 3 years and the company Y for at least 6 years will not reach the entire holding. We use the R language computing environment for the solution (for the numerical calculation, see [2] or the source code).

```
scen_year_x<-2
scen_year_y<-4
scen_year_xy<-8
hypot_x<-3
hypot_y<-6
lamb_x<-1/scen_year_x
lamb_y<-1/scen_year_y
lamb_xy<-1/scen_year_xy
Sx<-exp(-(lamb_x+lamb_xy)*hypot_x)
Sy<-exp(-(lamb_y+lamb_xy)*hypot_y)
```

```
theta_x<-lamb_xy/(lamb_x+lamb_xy)
theta_y<-lamb_xy/(lamb_y+lamb_xy)
MO<-c((Sx^(1-theta_x))*Sy,(Sy^(1-theta_y))*Sx)
min(MO)
```

and we get the output (probability ≈ 2.3518 %);

```
[1] 0.02351775
```

For comparison, the probability that the entire holding company liquidity crisis will not occur X for at least a year, while company Y for at least two years is ≈ 29 %.

6. Conclusion

The final phase of the risk assessment process is the risk management phase under which risk mitigation measures are implemented. In an insurance company, this phase (for the risks that the insurer takes from clients) also means (in the case of risk acceptability) the transfer of this risk from the risk division to the new product development department. Consequently, the actuarial work is the process of subscribing, valuing and analyzing the excess of assets and liabilities in the context of this assumed risk. Actuary may use various actuarial techniques to assess risk. However, according to some opinions, new types of risks are sometimes beyond the boundaries of current underwriting, valuation and capability of actuator calculations of the height and frequency of the damage caused. The reason why traditional actuarial modeling techniques are difficult to use for new types of risks is in particular the availability or unavailability of data on damage occurring as a result of the manifestations of new types of hazards as well as the likelihood of their occurrence.

Stress testing and scenario analysis is an effective and up-to-date option for actuaries, especially for analyzing and measuring the risks to which the insurance company is exposed. In the paper, we showed an interesting use of the dump of functions in the hypothetical examination of the consequences of liquidity risk. As a computing environment, we have utilized the R language functionality, which also provides other options in the context of these analyzes (simulation, function dump analysis, probability distributions, etc.) in a standard interface (functions `r---` (e.g. `rexp` for generating pseudorandom numbers that control exponential distributions), `sample`, `replicate`, ...) and in additional libraries that a user can download (for example, `copula` library). For more information, see the author's publications ([7] and [9]).

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Securitization and basic risk in agricultural crop insurance

Ľudovít Pinda, Branislav Mišota, Lenka Smažáková[1]

Abstract

Securitization is increasingly being promoted in the insurance sector as an alternative way of risk transfer in those cases, which go beyond the possibilities of reinsurance market. It covers the risks of large claims due to natural disasters by using ILS. In agricultural insurance the systemic risk occurs and causes reduction of the crop in a region because of bad weather conditions. An insurer is looking for catastrophe risk coverage, provided by reinsurer, whose insurance capacity is limited as well. This paper proposes reducing reinsurer's risk of insolvency by issuing - catastrophic bonds. In order to eliminate the basis risk, different soil quality is taken into account, resulting into different yields per hectare. The division of the croppers into four groups dependent on the quality of cultivated land, the examination of their crops each separately, results in a reduction of the basis risk. Through the Epanechnik kernel function the density of the relative loss of production is estimated and used for estimation of catastrophe bond trigger with set threshold. The output is a graph of dependency of the expected bond payoff on the threshold value and a payoff dimishing parameter in each production area.

Key words:

insurance, securitization, CAT bond, production area

JEL Classification: G22

1. Introduction

Securitization is increasingly being promoted in the insurance sector as an alternative way of risk transfer in those cases, which go beyond the possibilities of reinsurance market. The level of insurable risks held by insurer and reinsurer expands with securitization. The reinsurer's instrument for achieving this goal is insurance-linked securities (ILS). ILS transfer risk from insurance industry to investors on capital market. Capital market entities are able to absorb risk beyond possibilities of insurers and reinsurers (Cummins and Weiss (2008)). The structure of ILS products is often a compromise between the requirements of the provider and the beneficiary of the insurance risk. While issuers are demanding compensation for their losses by reduction of payments from ILS, investors expect high yields from them, as well as their independence from the yields of the assets held in their portfolios. An asset with pay-off directly dependent on the losses of reinsurer (issuer) may fulfil the requirements of investors but poses a threat of moral hazard resulting from the possibility of manipulating data that determine the payment. This threat is reduced by adjusting the definition of ILS payment. Another option is that the payment from ILS should be dependent on certain neutral variable, e.g. on the predetermined value of loss index. The loss index is calculated from aggregated losses of policyholders. However, the imperfect positive correlation between individual losses and value

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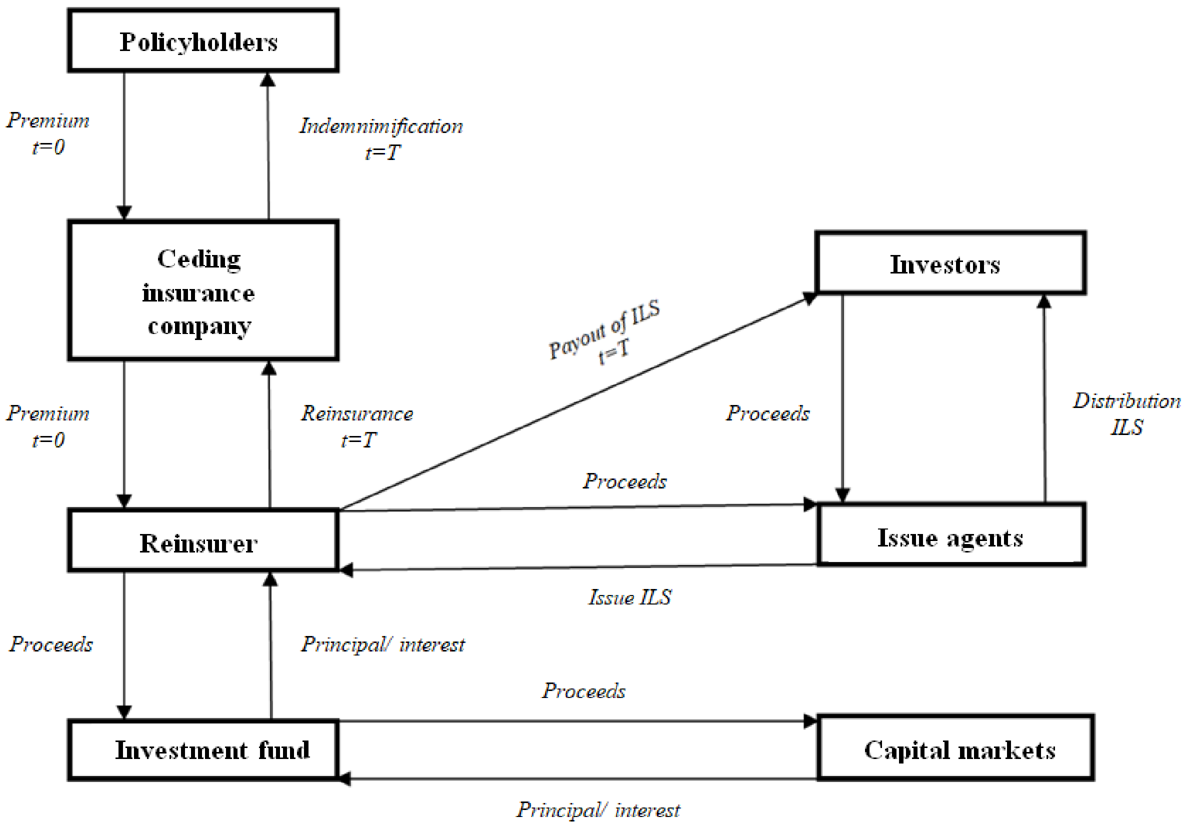
of loss index may be a source of basis risk, which may result into a gap between ILS payment and incurred loss of issuer. A hard-to-reach perfect positive correlation eliminates the basic risk. However, this deficiency can be solved to a certain extent by harmonizing the technical parameters of the insurance contracts with the calculation of the loss index.

2. Cash flows in reinsurance and securitization

The policyholders by premium payments participate on creation of common fund and this fund is made for the loss compensation. So created common fund is managed by insurer. Every participant is naturally interested in as high coverage of the loss with the lowest premium paid as possible. Risk sharing is fair only if a premium of each participant corresponds to exposed risk and the participant receives compensation corresponding to loss explicitly associated with the insured risk. In case that the loss compensation exceeds the capacity of the common fund consisting of premium payments and yields from financial assets of the insurance company, the insolvency threat of insurer arises. This situation is the insurer trying to solve through a reinsurance company by concluding an appropriate contract. Reinsurer has its own insurance capacities as well and if they are not large enough, reinsurer may lose its reputation and position on reinsurance market. For this reason the reinsurer is willing to extend his capacities by an alternative risk transfer. The securitization transfers insurance risk to the capital market participants through ILS. ILS payments are defined to offset the potential low reinsurance capacity of the reinsurer and thereby smooth the fluctuations in costs. The reduction of the risk of critical loss and consequent insolvency is therefore the primary motivation of the reinsurer for securitization. Cash flows in securitization process during time period $\langle 0, T \rangle$ is presented in Figure 1.

Figure 1: Cash flows in securitization process



The reinsurer, besides the premiums paid by insurers, gets cash flow from ILS issuance and its subsequent sale via agents to investors. ILS thus contributes to the diversification of the portfolio of investor. According to Krutov (2010) ILS provides such diversification that cannot be achieved by other securities. The 2008 crisis has shown that although most of securities devalued, including those with the historically lowest correlation, ILS were still developing independently. Secondary motivation is the above-average return on ILS. Another positive property of securitization in comparison to reinsurance is, according to Barrieu and Albertini (2009), the tradability of securitized risk. An investor, as a risk taker, is not tied to an insurer and can sell ILS on the secondary market. In this way, reinsurer will receive capital sources from reinsurance as well as from selling ILS and reinsurer will set up an investment fund, which will be valued on capital markets until maturity at time T.

The long-time most commonly used securitization tool is a catastrophic bond (CAT bond). Contrary to the standard bond its payment depends on the realization of a catastrophic loss of its sponsor which is correlated with the critical amount of indemnification. If a catastrophic event occurs until maturity at time T, payouts from CAT bond are either lowered or eliminated. In this case we say that the bond was triggered. The capital saved from the reduced payout is available for the indemnification of the policyholders. When the bond is not triggered, the investor profits from high return and the reinsurer suffers from the securitization loss.

The catastrophic event triggering a catastrophic bond is quantified by the trigger L_T and its threshold value D. The payout of a zero-coupon CAT bond with maturity at time T and face value F is defined, according to Komadel et al. (2018), as follows

$$V_T = \begin{cases} R_F \cdot F & \text{pre } L_T > D, \\ F & \text{pre } L_T \leq D, \end{cases} \tag{3}$$

where $0 \leq R_F < 1$ is the parameter of reduction of payout. If $R_F = 0$, the payout is eliminated. If $R_F = 1$, there is no payout reduction. The expected payout $E[V_T]$ from (3) is given by

$$E[V_T] = F \cdot P(L_T \leq D) + R_F \cdot F \cdot P(L_T > D). \tag{4}$$

Purchase price V_0 of bond is determined by expected rate of return $\delta(t)$, the height of which, according to the risk, is required by the investor. If the rate of return is constant in time $\delta(t) = \delta$, then the purchase price is given by

$$V_0 = e^{-\delta T} E[V_T]. \tag{5}$$

In this paper we will apply theoretical knowledge on crop insurance. We will also take into account the quality of the soil which affects the amount of crop production.

Let us consider a country with p producers of a particular crop. The amount of crop produced in the year $t + 1$ by a producer i is defined by the per hectare yield ${}^{i}y_{t+1}$, thus the weight of harvest over the area of land, typically measured in tons per hectare (t/ha). Following the approach of Vedenov et al. (2006), the loss of the i-th producer in the year $t + 1$ is defined by the relative loss of the producer's present per hectare yield to the n-year national average $\bar{y}_{n,t}$ gained in year t as

$${}^{i}L_{n,t+1} = \frac{\bar{y}_{n,t} - {}^{i}y_{t+1}}{\bar{y}_{n,t}}. \tag{6}$$

The insurance claim of i-th producer is settled in case that the loss index ${}^{i}L_{n,t+1}$ exceeds the threshold value D, so

$$ {}^{i}L_{n,t+1} > D. \tag{7} $$

From (6) and (7) can be rewritten as the settlement condition for the current yield

$$ {}^{i}y_{t+1} < (1 - D)\bar{y}_{n,t}. \tag{8} $$

For insurer, the situation may arise that the value of aggregate indemnity exceeds the fund created by premiums from producers. In case that reinsurance contract exists, an insurance company expects financial coverage from reinsurer for uncovered losses. The reinsurer expects financial capital from triggered CAT bonds, while trigger depends on the values of per hectare yield y_{t+1} in the year $t+1$ and per hectare yield to the n-year national average $\bar{y}_{n,t}$ gained in year t. So the trigger of CAT bond $L_{n,t+1}$ is defined as

$$ L_{n,t+1} = \frac{\bar{y}_{n,t} - y_{t+1}}{\bar{y}_{n,t}}. \tag{9} $$

Average per hectare yield of i-th producer ${}^{i}y_{t+1}$ in the year $t+1$ does not have to be the same as to national per hectare yield y_{t+1} in the year $t+1$. This possible disproportion can create so-called basis risk. Similar to Lee and Yu (2007) we define

$$ \delta(L_T) = F - V_T. \tag{10} $$

If $\delta({}^{\mathrm{i}}\mathrm{L}_{\mathrm{n,t+1}}) = \delta(\mathrm{L}_{\mathrm{n,t+1}})$ for $\mathrm{i} = 1, 2, \dots, \mathrm{p}$, basis risk does not arise in reinsurance process. Otherwise, if $\delta({}^{\mathrm{i}}\mathrm{L}_{\mathrm{n,t+1}}) \neq \delta(\mathrm{L}_{\mathrm{n,t+1}})$, basis risk arises and makes space for speculation when accepting claims, what means that moral hazard is present. According to (4) we can see that the knowledge of the probability distribution of trigger, loss index $L_{n,t+1}$, will be necessarily needed. For its estimation we will use kernel approach mentioned in Vedenov et al. (2006) in form

$$ \hat{f}(L_{n,t}) = \frac{1}{mh} \sum_{i=1}^{m} K\left(\frac{L_{n,t} - L_{n,i}}{h}\right), \tag{11} $$

where $L_{n,i}$ are relative losses of the national yield for years $i = 1,2, \dots, m$, K is Epanachnik kernel function, and h is the smoothing parameter of K. Using method described in Komadel et al. (2016) the estimation of distribution density $\hat{f}(L_{n,t})$ will be calculated. Then we can determine the expected bond payout from (4) and the purchase price from (5).

3. Crop categorization by agricultural production areas

Better soil quality, if other factors are the same, is a prerequisite for higher crop production. Because of the spatial correlation, the impact of other factors on the producers' harvest is interconnected. . We assume that this correlation is not higher than the spatial correlation of soil with different qualities. Comparing the natural conditions needs of crops with the data stored in the BIS, the natural conditions of the SR according to BPEJ code (Streďanská, A. - Buday, Š (2006)) p. 47-51 were divided into the following agricultural production areas:

- corn production area – 1. area;

- beetroot production area – 2. area;
- potato production area – 3. area;
- mountain production area – 4. area.

The Research Institute of Agricultural and Food Economics in Bratislava is working with the mentioned agricultural production areas. It submits annually a publication about costs and revenues of agricultural products, which contains annual results of the selected set of agricultural companies divided by production areas. We will work with data about national hectare yields by production areas and weighted national wheat yield per hectare for years 1985 - 2015. These data are published the second half of next year for the previous year. This explains why the calculation of CAT bond trigger $L_{n,t+1}$ in (9) is valid.

Due to different soil qualities individual producers earn different per hectare wheat yields. Therefore, with the same premium paid, there are various expectations of loss. Thus, the balance between the level of premium and the size of insurance risk, to which all growers cultivating the same crop are exposed, is violated. The same proportional reduction of per hectare yield for producers with better soil quality does not have to be an insured event, while for producers with worse soil quality it may be. This may cause that producers with better soil quality will not be interested in such insurance.

This deficiency should be eliminated by categorizing producers by soil quality into four categories ($i = 1,2,3,4$) and these categories will correspond to production areas introduced above. After categorization, it is possible to offer producers an insurance coverage at such premiums that reflects better their risk exposure. For each production area the per hectare wheat yields are known and also weighted national per hectare yield, used for comparison, is known. For the calculation of the national average and the average for each area, we use an exponential moving average (EMA) with a three-year period, which showed to be optimal in Komadel et al. (2018). EMA gives bigger weight to the last average per hectare yield values and their values are thus more important in calculations than older ones. According to Kresta (2016), $EMA(n)_t$ will be calculated from recursive formula

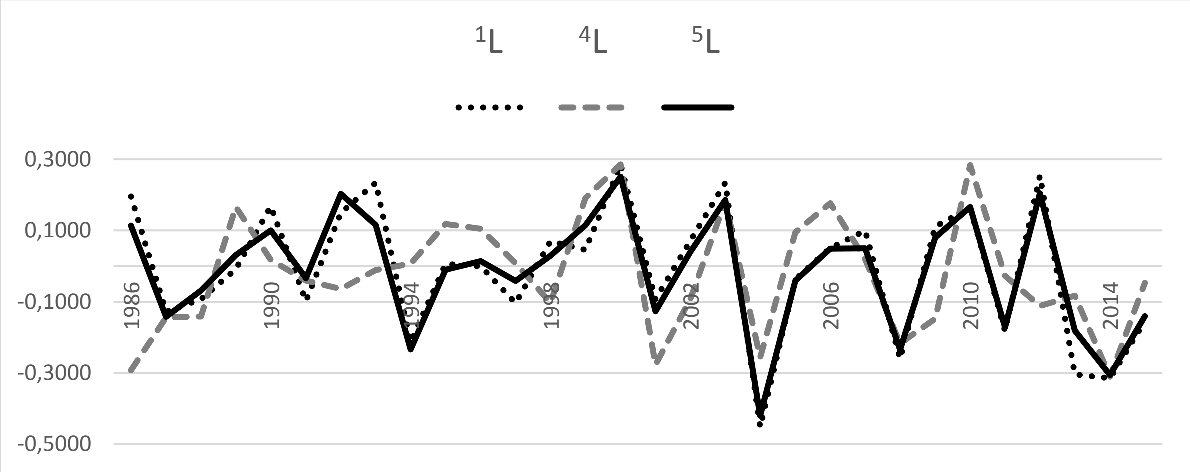
$$EMA(n)_t = EMA(n)_{t-1} + \frac{2}{n+1}(p_t - EMA(n)_{t-1}), \qquad (12)$$

where

$EMA(n)_1 = p_1$
$n = 3$,
p_t – average per hectare whey yields in each production area.

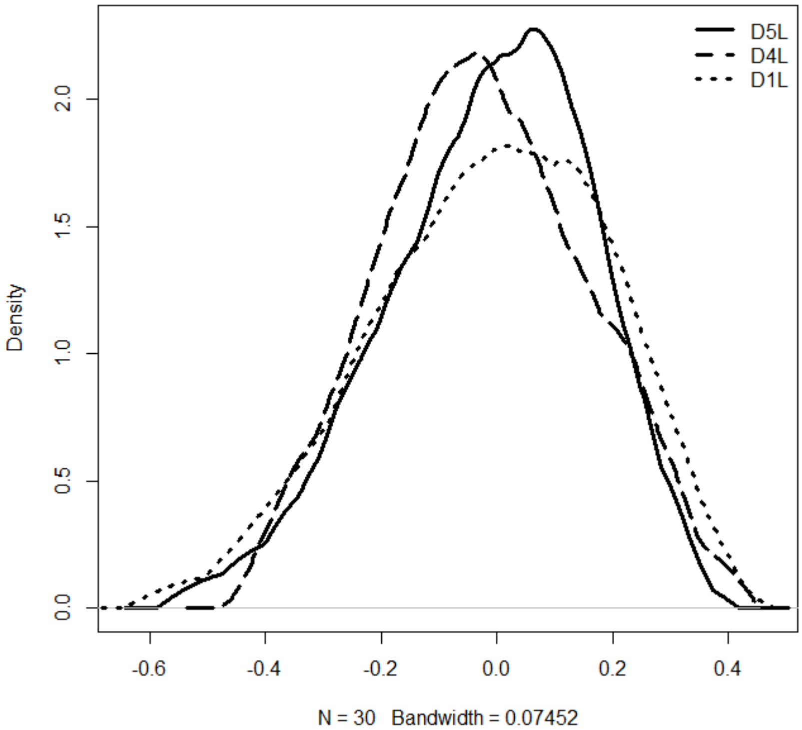
Computed exponential moving averages with three-year period will be used to calculate loss indices for each production area ${}^{i}L_{3,t+1}$, with $i = 1,2,3,4,5$, where index 5 stands for national average at time $t = 1985, \dots, 2014$. These are shown in Figure 2, for illustration only for indices $i = 1,4,5$.

Figure 2: Loss index for first and fourth production area and for national average



Time series $\left\{ {}^{i}L_{3,t+1} \right\}_{t=1985}^{2014}$ for $i = 1,2,3,4,5$ will be used to estimate the distribution of national index of loss and indices for each production area. In Figure 2, where these indices are shown, the differences between production areas are obvious. From construction of loss index in (9) and from (4) can be seen that negative values of index have no influence on reduction of payment from CAT bond. Loss index of the first area copies better the course of national index than the loss index of fourth area. This means that for constant value of trigger $D = 0{,}1$ i.e. 10% in year 2006, the payment from a CAT bond evaluated according to the national index and the index of the first area would not be reduced. While in national and first area loss event in crop does not occur, the fourth area indicates it despite the fact that for calculation of average values were used data of each production area separately. If we did not categorize producers into production area, it would lead to appearance of basis risk.

Figure 3: Estimation of probability distribution of the national loss index and the loss index for first and fourth production area



From the approximations of the probability distributions calculated in language R (Páleš, (2007)), for various values of the trigger D, we calculate a complement to corresponding quantile value for each production area. These are shown in Table 1. If D decreases, the probability that CAT bond will be triggered increases.

Table 1: The triggering probability of CAT bond for various thresholds

D	0%	5%	10%	15%	20%	25%	30%	35%	40%
$P({}^{1}L > D)$	0.5054	0.4150	0.3264	0.2393	0.1612	0.0981	0.0520	0.0216	0.0057
$P({}^{2}L > D)$	0.4964	0.3762	0.2594	0.1549	0.0810	0.0366	0.0127	0.0025	0.0000
$P({}^{3}L > D)$	0.4428	0.3435	0.2504	0.1687	0.1062	0.0580	0.0253	0.0064	0.0002
$P({}^{4}L > D)$	0.4384	0.3405	0.2541	0.1829	0.1234	0.0724	0.0362	0.0150	0.0044
$P({}^{5}L > D)$	0.5071	0.3976	0.2852	0.1847	0.1058	0.0522	0.0204	0.0043	0.0002

In the next two tables, according to (4), the expected payment from CAT bonds for selected values of parameters for the first and fourth production area are calculated.

Table 2: Expected payment from CAT bond triggered by loss index ${}^{1}L$

R_F	$D\ {}^{1}L$					
	0%	10%	20%	30%	40%	50%
0,00	0,4946	0,6736	0,83880	0,94800	0,99430	1,00000
0,25	0,62095	0,75520	0,87910	0,96100	0,99573	1,00000
0,50	0,74730	0,83680	0,91940	0,97400	0,99715	1,00000
0,75	0,87365	0,91840	0,95970	0,98700	0,99858	1,00000
1,00	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000

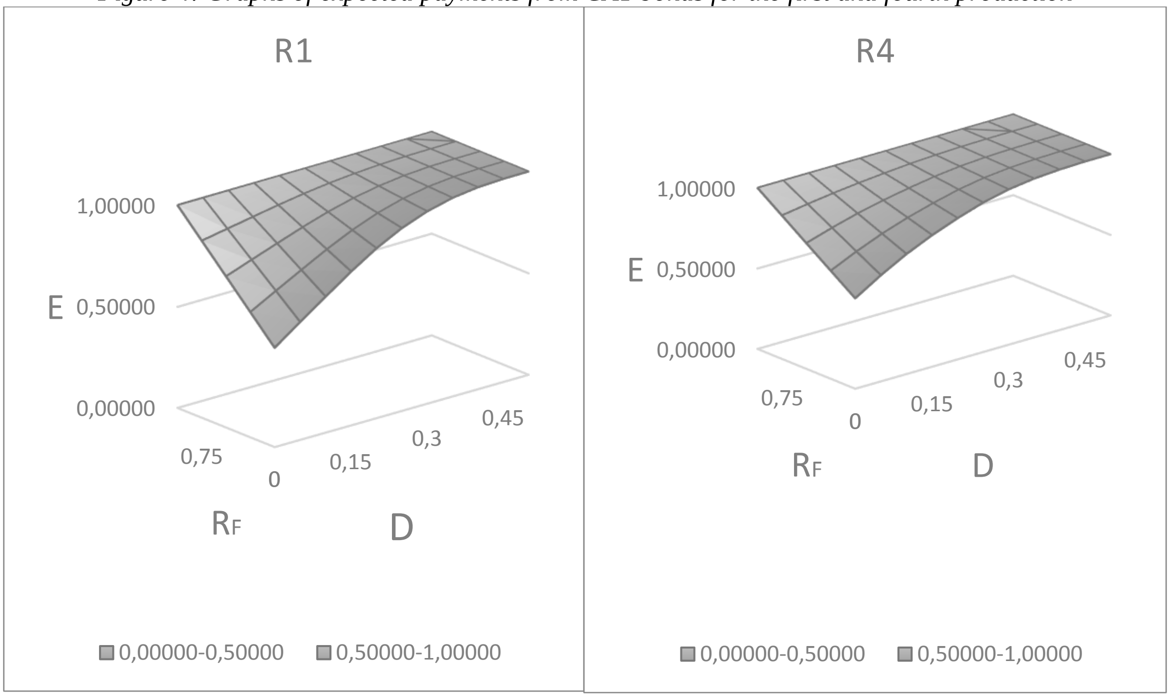
Table 3: Expected payment from CAT bond triggered by loss index ${}^{4}L$

R_F	$D\ {}^{4}L$					
	0%	10%	20%	30%	40%	50%
0,00	0,56160	0,74590	0,87660	0,96380	0,99560	1,00000
0,25	0,67120	0,80943	0,90745	0,97285	0,99670	1,00000
0,50	0,78080	0,87295	0,93830	0,98190	0,99780	1,00000
0,75	0,89040	0,93648	0,96915	0,99095	0,99890	1,00000
1,00	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000

From the comparison of the results it can be seen that the expected payments from bonds do not differ significantly. This conclusion is a result of calculating loss index and exponential moving average of per hectare yield for each production area separately. However, this step has not completely eliminate the basis risk, but it was reduced considerably. There still remains risk that per hectare yield will vary because of e.g. failure to comply with the cultivation procedures and this we are not able to eliminate. If this occurs, insurer does not have to accept a claim.

To make investor interested in CAT bonds, he must be offered above-average earnings. These bonds have one-year maturity resulting from the growth cycle of crop. Therefore, the yield should be annual and effective. If the offered effective yield is 5% p.a., then by discounting with the discount factor $v = e^{-0{,}048790} = \frac{1}{1+0{,}05} = 0{,}95238$ we get purchase prices of bonds from Tables 1 and 2. Graphs of expected payments from CAT bonds for the first and fourth production area are shown in Figure 4.

Figure 4: Graphs of expected payments from CAT bonds for the first and fourth production



The areas of expected payments for selected production areas do not differ significantly, so the basis risk was considerably reduced. This is what we wanted to achieve by classifying producers into production areas.

4. Conclusion

This paper mainly focuses on elimination of risk, that reinsurer, who offers reinsurance to insurer covering risks of low yield, can became insolvent. Reinsurer, in case that his insurance capacities are exhausted, transfers risks of catastrophe events on capital markets via CAT bonds. The bond trigger in a form of a relative loss of this year's national per hectare yield to the n-year national average is a compromise between high moral hazard, when bond is triggered directly by the aggregate loss of reinsurer, and high basis risk, when bond is triggered due to heterogeneous cultivation conditions caused by different soil quality. In order to reduce substantial part of basis risk and a part of moral hazard, the division of producers into four groups according to the production areas, where each area reflects on different soil quality, is justified. For the same reason we introduce four types of CAT bonds according to the production area and value them. In each group we work separately with statistically verified data when calculating the exponential moving average of per hectare yield of wheat in t/ha, then we calculate the loss index which is used to approximate the probability distribution of loss index. By processing data from each production area separately, the differences between expected payouts from various CAT bonds are small.

The risk of catastrophic consequences on reinsurer's reinsurance capacity can be eliminated by suitable set up parameters D and F. Lower (higher) price of a bond does not indicate the quality of its securitization scheme. The quality indicator is the ability of the system of bonds to smooth out year-on-year fluctuations in reinsurer's cash-flows by correlation between the aggregate payouts from all CAT bonds and aggregated compensations of all losses.

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Diversification as a strategy of mergers and acquisitions

Anna Poledňáková [1]

Abstract

The rising dynamics of changes that have a difficult predictable character and, at the same time, crucial to business management, increasing competition intensity, are just some of the things that management has to manage and incorporate into a meaningful whole - a strategy. One of the forms of external enterprise growth strategies is mergers and acquisitions. The contribution focuses on diversification as a significant factor in the growth of enterprises with no direct relationship, ie businesses operating in unrelated industries where the main objective is to mitigate fluctuations in future earnings. Because diversification is a controversial topic in the financial sector, we decided in the paper to decide on the role of diversification in conglomerate mergers and acquisitions.

Key words

mergers and acquisitions, conglomerate mergers and acquisitions, diversification, synergic effect, the value of the company,

JEL Classification: G34

1. Introduction

One of the major motives for mergers and acquisitions is business growth[2]. An enterprise can choose between internal or organic growth and growth through mergers and acquisitions. Internal growth can be a slow and uncertain process. Growth through mergers and acquisitions can be much faster, though it brings with it its own uncertainties. An enterprise can grow in its own industry or expand beyond its business category. Expansion outside the industry means diversification.

Another example of the use of mergers and acquisitions to facilitate growth is when an enterprise expands to another geographic area and tries to leverage its benefits on other markets. In many cases, it may be faster and less risky to geographically expand through acquisitions than through internal strategies. This is particularly true for international expansion where it is necessary to have the characteristics to be successful in the new geographic market.

Given that diversification is a controversial topic in the financial sector, we will try to analyze the positive as the negative aspects of the proces.

Diversification means growth outside the current industry category. The motto of diversification plays an important role in linking businesses, among which there is no direct relationship, ie, that they are enterprises operating in unrelated industries. Many of the businesses that made such a connection were later dealt with through various spin-offs and

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sales. This process raises serious doubts about the value of the diversification-based diversification. Many companies have regretted their attempts to diversify; on the contrary, others can claim to have significantly gained significantly.

One part of the reflection on a successful diversification strategy is the fact that the business is trying to gain benefits in different industries. These benefits can be manifested in a number of ways, including wider consumer awareness on the market as a distribution position. One of the reasons why management can opt for a diversified expansion is its desire to enter into sectors that are more profitable than the current industry of the acquiring company. It could be that the parent company's sector has reached an advanced stage, or that competitive "pressures in the sector exclude the possibility of increasing prices to the level and extra profits".

One problem that some businesses may encounter when trying to expand by entering industries that offer better profits is the lack of certainty that these opportunities will persist for a longer time in the future. Industry that is profitable does not need to be as profitable in the future. Competitive pressures serve to achieve long-term equalization of returns in the sectors. For industries with a higher return, which do not have the impossible barriers to entry, the return will fall until they reach the cross-sectional average.

Economic theory suggests that in the long run, only those sectors that are difficult to enter will have above-average returns. This means that the diversification program will enter the profitable sector, it will not succeed in the long run. An emerging company may not be able to enter into those industries that still exist.

2 Methodology

The analysis of the theme of diversification and its impact on the honesty of the enterprise requires the use of more scientific methods than logical and empirical and accurate. Analysis and synthesis is used to examine the theoretical knowledge as well as to examine the financial statements. In other methods, the method of induction, deduction, comparison is used. In order to assess the financial situation of the merged companies, they will use methods of financial indicators, they will show the indicators of profitability, activity, liquidity and cost.

Problems of diversification and its impact on the value of the business were devoted to the peculiar authors as Berger and Ofek[3], Lang and Stulz[4], Villalonga[5],.

We also have many views on the issues of successful mergers and acquisitions, as well as ways to measure these effects. Some companies begin to analyze the effectiveness of forecasts for future cash flows of the merger and acquisition company. Any increase in revenue or cost reduction that can be attributed to integration depends on the forecasts. There is no such approach in Slovakia to assess the success of the merger due to the lack of a functioning capital market. We will therefore use a financial analysis to assess success and influence on the value of the business. Financial analysis broadly defines K. Zalaí[6] as "a material that summarizes and evaluates the results of the company over the analyzed period, identifies and quantifies the factors (factors) that determine the progress achieved and the results achieved in

[3] Philip Berger and Eli Ofek, „Diversification Effect on Firm Value. Journal of Financial Economics, 37, January 1995, p. 39-65.
[4] Larry Lang and Rene Stulz, Tobins q. Corporate Diversification and Firm Performance, Journal of Political Economy 102, December 1994, p. 1248-1289
[5] Belan Villalonga, Diversification Discount or premium? Journal of Finance59, April 2004, p. 479-506.
[6] Karol Zalai a kolektív:Finančno-ekonomická analýza podniku (9. prepracované a rozšírené vydanie) Sprint dva, 2016.

the future and extends it to the draft measures the implementation of which should ensure achievement of business goals".

To assess the success of transactions, a wide range of financial performance indicators, in particular profitability, activity, liquidity, and indebtedness indicators, can be used.

3 Discussion

Some of the companies that the company has acquired are part of the company's successful diversification strategy[7]. An example is GE. General Electric sought to obtain leadership positions in the various industries in which it owned. These benefits can be manifested in a number of ways, including wider consumer awareness on the market as a distribution position. Corporations in secondary ranges, such as numbers four, five, may sometimes be at a disadvantage that it is difficult for them to get a generic return. Such a sale can then be re-invested in other companies in order to take advantage of this advantage of its dominant position or to use it for the acquisition of leading companies in other sectors.

One of the reasons why management can decide for a diversified expansion is its desire to enter into sectors that are more profitable than the current industry of the acquiring company. One problem that some businesses may encounter when trying to expand by entering industries that offer better profits is the lack of certainty that these opportunities will persist for a longer time in the future. Industry that is profitable does not need to be as profitable in the future. Competitive pressures serve to achieve long-term equalization of returns in the sectors. Obviously, this does not mean that the rate of return in all sectors at any time in the company is the same. For industries with higher returns, which do not have the impossible barriers to market entry, the return will fall.

Business benchmarking analyzes that have diversified with non-diversified firms have confirmed that there is no evidence that diversification has increased corporate values. On the contrary, he found that Tobin's diversified firms were much lower than companies with multiple segments. Other research found that the diversification discount was not limited to the conglomerate era. The study (Berger and Ofek) found that diversification led to a loss of a fixed value that ranged from 13% to 15% on average. This study estimated the imputed value of diversified business segments as if they were separate firms. The results show that the loss of the fixed value was not affected by the size of the firm but was smaller when there was diversification in the related sectors. The loss of company results was supported by the fact that diversified segments showed lower operating profitability than one-off businesses. The results also showed that diversified companies have invested more than one-time businesses in diversified segments. It follows that over-investment may be a partial cause of the loss of value associated with diversification. The study /Lang and Stulz/ also found the effects of diminishing diversification in their study.

Different benefits and costs have been explored, unlike a more focused business structure. A separate question, however, is that typical, diversified, focused companies are better off in the area of mergers and acquisitions. Cihan and Tice[8] analyzed a large sample and found that diversified firms had a 1.5% higher yield than the participants in one segment. They then attempted to find sources for higher value for diversified acquirers. They performed a regression analysis where performance measures reflect the profitability after the

[7] Gaughan ,A.P.,(2010). *Mergers, Acquisitions, and Corporate Restructurings*. 5th ed Willey,ISBN 10-0470561963

[8] Mehmet Cihan and Sheri Tice , Do diversi or Focused Make Better Acquisitionns . Paper Presented the American Finance Associations Annual Meetings, 2015.

concentration and the cost has decreased compared to the candidate's diversification status and the pre-merger performance. The joint ventures in which the bidder was diversified had higher profit margins and lower costs. These results indicate that diversified acquirers are better able to perform post-trade efficiency improvements compared to more targeted bidders.

Our discussion of the problem of control losses in diversified organizations within the above-mentioned economic point of view highlights the challenges in managing complex and heterogeneous organizations. The transition from predominantly stand-alone business organizations to diversified business organizations has been accompanied by new organizational innovations, such as a multi-part (M-form) structure to handle complexity.

However, for a variety of reasons, this structure may be malfunctioning, such as weak monitoring information systems, rival distribution, or weak incentives that misinterpret the distribution of goals with the overall corporate goal of maximizing value. One of the factors that can cause a dysfunctional result is the cost of influence. So the political process of resource allocation in diversified firms can be as important as a seemingly rational economic process. In its model, the extreme diversity of growth opportunities for individual business entities and their profitability means that high profit divisions can see that their surplus profits "capture" the center and redistribute to low-rated divisions. This can act as a disincentive for the first to act in order to maximize the division and hence its value that causes the change of value.

In the case of three of the four conglomerates examined, which we analyzed[9] on the example of Slovak enterprises, the financial situation of the successive enterprises was deteriorated. The deterioration in the financial situation was reflected in the growth of cost, the reduction of the efficiency of the use of the assets and the reduction of the profitability of the enterprises. Out of the three companies, there was only one company growth in market share due to the growth of its sales. However, this positive development may not only be the result of a merger. In other enterprises, the annual decline in sales revenue resulted in a decline in market shares.

4 Conclusion

In the paper, we analyzed various perspectives on diversification in the merger and acquisition process. Different views of this process show that this process is subject to various, often contradictory forces, and the result of the bonding is generally influenced by the combination of external and specific forces. The main motivation of the joint venture is to achieve synergy effect. An enterprise can grow in its own industry or expand beyond its business category. Expansion outside the industry means diversification.

Many of the companies that have grown past into a conglomerate have been discarded through various spin-offs and divestments. This deconglomeration process raises serious doubts about the value of diversification based on extensions. Although many companies have regretted their attempts to diversify, others may claim to have gained considerable diversification.

Business comparison analyzes that have diversified with non-diversified firms have confirmed that there is no evidence that diversification has increased corporate values.

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The Risk of Multi-Criteria Portfolios Taking into Account the Fuzzy Approach

Ewa Pośpiech [1]

Abstract

Portfolio construction can be considered as a multi-criteria problem in which selected fundamental and market indicators are the criteria of evaluation. When building a portfolio that uses elements of fundamental analysis, the appropriate period from which the data will come should be taken into account. At this stage of the portfolio selection, a fuzzy approach can be applied in which the criteria assessments from several years can be treated as fuzzy numbers. In the article, for the constructed effective portfolios in which the multi-criteria fuzzy approach will be used, the main objective is to assess and compare the risk of obtained portfolios.

Key words

TOPSIS method, fuzzy multi-criteria modelling, portfolio selection, portfolio risk

JEL Classification: C44, C61, G11,

1. Introduction

The decision maker investing in shares wants to buy those shares that bring the highest possible profit with the lowest possible risk. These goals are usually the opposite, therefore, when selecting the shares to the portfolio it is worth to use additional information. One can, for example, make an economic and financial assessment of the company on the basis of fundamental and market indicators, which can be treated as criteria for the assessment of the shares. This approach suggests the use of multi-criteria methods. Their application will help select groups of companies on the basis of which portfolios will be built (Trzaskalik, 2006). When using the indicators applied in fundamental analysis there is the question of selecting the period from which the data will be taken. There are suggestions in the literature that they should come from three to five years back (Tarczyński, 2002). Taking into account these suggestions, the values of indicators for the selected period should be determined accordingly. One can also try to approach the issue in a non-standard way using fuzzy numbers (Pośpiech and Mastalerz-Kodzis, 2016). The study proposes using in the analyzes of the three-year period (Pośpiech, 2017; Tarczyński, 2002) and triangular fuzzy numbers, which will be represented by appropriate values taken from the selected three years of the chosen period. An important element of the study will be the analysis of the risk of built portfolios and its confrontation with the profits of these portfolios.

The aim of the article is therefore to compare the grouping results of selected listed companies, analyze the risk of portfolios received on the basis of the proposed approaches and evaluate the profit of portfolios generated on the basis of multi-criteria grouping. The TOPSIS and FTOPSIS (fuzzy TOPSIS) methods were used in the analyzes; they enable calculations to be carried out in the proposed approach (Pośpiech and Mastalerz-Kodzis, 2015; Pośpiech and Mastalerz-Kodzis, 2016).

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The research hypothesis assumes a significant variation in the risk and profit of portfolios suggesting the superiority of the fuzzy variant or other variants that taking into account the selected set of criteria.

2. Research methodology

The analyzes were based on the multi-criteria TOPSIS method in standard and fuzzy approach. Generally, in the TOPSIS method, decision variants are compared with certain reference points called ideal and anti-ideal solutions. As a result of these comparisons, the ranking of variants is obtained – the ordering takes place on the basis of the decreasing value of the relevant indicator. The procedure of building the ranking takes place in stages (Lai, Liu and Hwang, 1994; Trzaskalik, 2014). When determining that m decision variants and n criteria are considered, the evaluation of the variant i in terms of the criterion k will be denoted by the symbol $a_i^{(k)}$, $i = 1, \ldots, m$, $k = 1, \ldots, n$; the set of these elements is a decision matrix with dimensions $m \times n$. In the first step of the procedure, the decision matrix is normalized, which is then weighted[2]. The next stage of the procedure is to determine weighted assessments of the ideal and anti-ideal solutions. Then, the distance between individual variants from the received reference points is calculated – the Minkowski's distance with the parameter p is used (usually $p = 2$). The last step of the procedure is to calculate the relative distance S_i, $S_i \in [0, 1]$, on the basis of which the ranking is built.

In the case of the fuzzy TOPSIS method, the procedure is similar, but the way of building the decision matrix is different (Jahanshahloo, Hosseinzadeh Lotfi and Izadikhah, 2006; Trzaskalik, 2014). The values of criteria assessments are treated as triangular fuzzy numbers. Therefore, assuming that the following number:

$$\tilde{a}_i^{(k)} = (d_{ik}, s_{ik}, g_{ik}), \tag{1}$$

where:

d_{ik} – pessimistic assessment of the variant i within criterion k,
s_{ik} – expected assessment of the variant i within criterion k,
g_{ik} – optimistic assessment of the variant i within criterion k,

is a criterion assessment of a decision-making variant, an appropriate standard decision matrix is determined whose elements are fuzzy numbers[3]. Then, a weighted normalized decision matrix is constructed, on the basis of which the weighted ideal and anti-ideal solutions are determined. The penultimate step of the procedure is to determine the distance of decision variants from the received reference solutions; the distance between two triangular fuzzy numbers is given by the following formula:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(d_a - d_b)^2 + (s_a - s_b)^2 + (g_a - g_b)^2\right]}, \tag{2}$$

where $\tilde{a} = (d_a, s_a, g_a)$ and $\tilde{b} = (d_b, s_b, g_b)$ they are two fuzzy numbers. As in the case of the standard TOPSIS method, the ranking is constructed on the basis of the decreasing value of the S_i coefficient. In the fuzzy version of the TOPSIS method, criterion evaluations were treated as triangular fuzzy numbers (l_k, m_k, u_k), where the following assignment was made: l_k – the smallest value of the criterion evaluation for a given variant (company) from the

[2] A detailed description of the procedure can be found in many works, including in (Lai, Liu and Hwang, 1994; Trzaskalik, 2014).
[3] A description of the method can be found, among others in (Jahanshahloo, Hosseinzadeh Lotfi and Izadikhah, 2006; Trzaskalik, 2014).

selected three years, u_k – the highest criterion evaluation value from three years, m_k – the median of the criterion evaluations from the three years. In the standard version of the TOPSIS method, the values of criterion assessments averaged for three years were determined.

3. Results of the empirical analysis

The analysis covered twenty companies included in the WIG20 index[4] in December 2017. The period from which the data was taken is the years 2015-2017. Data from these years were included for the fundamental and market indicators which were the assessment criteria[5]. The following indicators were selected [6]:

- return of assets ROA (net income/average total assets) – criterion 1,
- return of equity ROE (net income/shareholder equity) – criterion 2,
- P/BV (price-book value) – criterion 3,
- P/E (price-earnings ratio) – criterion 4,
- earnings per share (Z1) – criterion 5.

It has been assumed that each of the criteria will be maximized and all will be considered as equivalent, which means assigning equal weights to the criteria. Six variants were considered: three in the basic (standard) version and three in the fuzzy version. Under variant I (basic one) and IV (fuzzy) criteria 1, 2, 3 and 4 were used. Due to the fact that the P/E ratio, which is often used by investors, has some drawbacks, it can be replaced by the earnings per share indicator (Łuniewska and Tarczyński, 2006; Tarczyński, 2002). In versions II (basic one) and V (fuzzy) the set of criteria consisted of 1, 2, 3 and 5. The last two approaches (III – basic and VI – fuzzy) take into account all five criteria. Using the presented methods and considered variants, the companies of WIG20 index were ordered according to the decreasing value of the S_i measure. The result of ordering together with the companies ranking is presented in Table 1.

Table 1: Values of the S_i indicator and rankings according to individual variants

Co.	Variant I		Variant II		Variant III		Variant IV		Variant V		Variant VI	
	S_i	Rank	S_i	Rank	S_i	Rank	S_i	Rank	S_i	Rank	S_i	Rank
ALR	0.577	15	0.343	14	0.487	15	0.503	14	0.498	11	0.454	11
ACP	0.607	11	0.354	12	0.494	12	0.506	12	0.491	14	0.448	13
BZW	0.627	9	0.389	7	0.524	5	0.531	8	0.528	6	0.481	6
CCC	0.968	1	0.587	2	0.644	2	0.792	1	0.764	2	0.691	2
CPS	0.644	6	0.386	8	0.513	8	0.538	6	0.519	7	0.473	7
ENG	0.600	14	0.349	13	0.488	14	0.497	15	0.479	15	0.437	15
EUR	0.724	3	0.424	4	0.555	4	0.586	4	0.567	4	0.510	4
JSW	0.473	18	0.199	18	0.393	18	0.381	18	0.273	19	0.341	18
KGH	0.416	19	0.035	20	0.357	19	0.290	20	0.270	20	0.250	20
LTS	0.630	8	0.384	9	0.508	10	0.506	13	0.492	13	0.445	14

[4] WIG20 is a stock index of the twenty largest joint-stock companies listed on the Warsaw Stock Exchange.
[5] The data comes from sources indicated in the references.
[6] These are the most frequently considered indicators supporting the company's assessment (Leszczyński, 2004; Łuniewska and Tarczyński, 2006; Tarczyński, 2001; Tarczyński, 2002).

LPP	0.856	2	0.854	1	0.878	1	0.686	2	0.811	1	0.733	1
MBK	0.602	13	0.373	10	0.516	7	0.509	11	0.512	9	0.467	9
OPL	0.476	17	0.188	19	0.400	17	0.377	19	0.370	18	0.334	19
PEO	0.619	10	0.362	11	0.505	11	0.523	9	0.510	10	0.466	10
PGE	0.576	16	0.290	16	0.477	16	0.443	16	0.428	16	0.390	16
PGN	0.643	7	0.390	6	0.513	9	0.534	7	0.515	8	0.469	8
PKN	0.723	4	0.498	3	0.574	3	0.617	3	0.603	3	0.545	3
PKO	0.604	12	0.338	15	0.490	13	0.510	10	0.493	12	0.450	12
PZU	0.663	5	0.407	5	0.524	6	0.574	5	0.554	5	0.503	5
TPE	0.241	20	0.257	17	0.211	20	0.417	17	0.402	17	0.367	17

The received rankings are similar, the largest differences were observed for companies in the middle of the rate. The correlation of the obtained rankings is at a very high level – the values of the Spearman's rank correlation coefficient are in the range [0.923; 0.997], with the strongest correlation between variants V and VI (this suggests that there is a slight difference in ordering companies in a fuzzy variants taking into account four criteria – without P/E, or all five criteria); the weakest correlation, at a level 0.923, is between variant I and VI.

On the basis of the obtained rankings, groups of companies (occupying the highest places in the ranking) were created, from which effective portfolios were built based on the classic Markowitz approach. Groups with five to twelve companies were included. The result of grouping according to the considered variants is presented in Table 2.

Table 2: Results of grouping companies by variants and rankings

No.	Variant – Number of companies	Groups of companies
1	I, II, IV, V, VI – 5	LPP, CCC, PKN, EUR, PZU
2	III – 5	LPP, CCC, PKN, EUR, BZW
3	III, V, VI – 6	LPP, CCC, PKN, EUR, PZU, BZW
4	I, IV – 6	LPP, CCC, PKN, EUR, PZU, CPS
5	II – 6	LPP, CCC, PKN, EUR, PZU, PGN
6	II – 7	LPP, CCC, PKN, EUR, PZU, PGN, BZW
7	I, IV – 7	LPP, CCC, PKN, EUR, PZU, CPS, PGN
8	III – 7	LPP, CCC, PKN, EUR, PZU, BZW, MBK
9	V, VI – 7	LPP, CCC, PKN, EUR, PZU, BZW, CPS
10	II, IV, V, VI – 8	LPP, CCC, PKN, EUR, PZU, PGN, BZW, CPS
11	III – 8	LPP, CCC, PKN, EUR, PZU, BZW, MBK, CPS
12	I – 8	LPP, CCC, PKN, EUR, PZU, CPS, PGN, LTS
13	I, II – 9	LPP, CCC, PKN, EUR, PZU, CPS, PGN, LTS, BZW
14	III, V, VI – 9	LPP, CCC, PKN, EUR, PZU, BZW, MBK, CPS, PGN
15	IV – 9	LPP, CCC, PKN, EUR, PZU, PGN, BZW, CPS, PEO
16	I – 10	LPP, CCC, PKN, EUR, PZU, CPS, PGN, LTS, BZW, PEO
17	II, III – 10	LPP, CCC, PKN, EUR, PZU, CPS, PGN, LTS, BZW, MBK
18	IV – 10	LPP, CCC, PKN, EUR, PZU, PGN, BZW, CPS, PEO, PKO

19	V, VI – 10	LPP, CCC, PKN, EUR, PZU, BZW, MBK, CPS, PGN, PEO
20	I – 11	LPP, CCC, PKN, EUR, PZU, CPS, PGN, LTS, BZW, PEO, ACP
21	II, III – 11	LPP, CCC, PKN, EUR, PZU, CPS, PGN, LTS, BZW, MBK, PEO
22	IV – 11	LPP, CCC, PKN, EUR, PZU, PGN, BZW, CPS, PEO, PKO, MBK
23	V, VI – 11	LPP, CCC, PKN, EUR, PZU, BZW, MBK, CPS, PGN, PEO, ALR
24	I – 12	LPP, CCC, PKN, EUR, PZU, CPS, PGN, LTS, BZW, PEO, ACP, PKO
25	II, III – 12	LPP, CCC, PKN, EUR, PZU, CPS, PGN, LTS, BZW, MBK, PEO, ACP
26	IV – 12	LPP, CCC, PKN, EUR, PZU, PGN, BZW, CPS, PEO, PKO, MBK, ACP
27	V, VI – 12	LPP, CCC, PKN, EUR, PZU, BZW, MBK, CPS, PGN, PEO, ALR, PKO

Because the rankings are similar, the results of grouping are also similar. Groups with the same number of shares usually differ in one or two stock companies. Grouping, especially for sets with fewer elements (containing five to eight elements), did not give clear information suggesting that any of the approaches gives significantly different results. On the basis of the sets received, the portfolios were built using the Markowitz model, in which the variance of the portfolio is minimized at a given rate of return. The following optimization task has been solved:

$$\begin{aligned} & S_p^2 = \sum_{i=1}^{l}\sum_{j=1}^{l} x_i x_j \operatorname{cov}_{ij} \to \min \\ & R_p \geq R_0 \\ & \sum_{i=1}^{l} x_i = 1 \\ & x_i \leq 0{,}3 \qquad i = 1, \ldots, l, \\ & x_i \geq 0, \qquad i = 1, \ldots, l \end{aligned} \tag{3}$$

where:

S_p^2 – portfolio variance,

x_i, x_j – shares of i and j shares in the portfolio,

cov_{ij} – the covariance of return rates of i and j shares,

R_p – portfolio return rate,

R_0 – the value of the portfolio return rate given by the decision maker (average return rate of included companies),

l – the multiplicity of the set from which portfolio was selected.

It was assumed that the share of any listed company should not exceed 30%. Twenty-seven portfolios were obtained (the numbers in the first column of Table 2 indicate the portfolio number). Their structure in most cases varied. The risk and the rates of return of these portfolios are presented in Table 3.

Table 3: Risk (S_p) and rate of return (R_p) of portfolios

Portfolio	P1	P2	P3	P4	P5	P6	P7	P8	P9
S_p	0.868	0.882	0.782	0.718	0.785	0.724	0.688	0.804	0.720
R_p	0.093	0.103	0.106	0.091	0.105	0.121	0.114	0.139	0.086
Portfolio	**P10**	**P11**	**P12**	**P13**	**P14**	**P15**	**P16**	**P17**	**P18**
S_p	0.716	0.734	0.734	0.734	0.774	0.640	0.889	0.924	0.868
R_p	0.084	0.094	0.142	0.142	0.132	0.104	0.092	0.102	0.092
Portfolio	**P19**	**P20**	**P21**	**P22**	**P23**	**P24**	**P25**	**P26**	**P27**
S_p	0.880	0.807	0.857	0.855	0.847	0.814	0.813	0.811	0.851
R_p	0.087	0.079	0.097	0.097	0.095	0.089	0.084	0.085	0.103

Primarily, the risk of portfolios selected from sets of the same size was compared, and then all portfolios were analyzed comparing those with the highest and the lowest risk. The risk of Markowitz's portfolio was also examined, which was generated on the basis of all twenty stock companies. The risk of this portfolio (marked P0) was 0.762 and the corresponding rate of return was at 0.120. For all twenty-eight portfolios, profits (losses) were calculated at the end of the next three months of the first quarter of 2018 (Table 4). For the P0 portfolio, the respective profit/loss values were: 3.48; -3.10; -7.01. The results were confronted with the risk of portfolios.

Table 4: Portfolios profit rates (%)

Rate of profit of portfolio compared to 03.01.2018	P1	P2	P3	P4	P5	P6	P7	P8	P9
31.01.2018	3.44	2.90	3.45	2.45	3.26	4.73	4.25	4.83	3.45
28.02.2018	-7.84	-10.08	-8.06	-7.11	-7.91	-4.47	-3.61	-5.73	-7.91
29.03.2018	-11.26	-14.20	-12.21	-10.29	-12.82	-8.72	-6.63	-10.42	-8.49
Rate of profit of portfolio compared to 03.01.2018	**P10**	**P11**	**P12**	**P13**	**P14**	**P15**	**P16**	**P17**	**P18**
31.01.2018	3.48	3.44	3.92	3.92	4.37	3.61	2.38	2.30	2.25
28.02.2018	-7.98	-7.30	-4.14	-4.14	-4.89	-3.15	-5.33	-5.93	-5.22
29.03.2018	-8.49	-8.29	-6.42	-6.42	-8.40	-6.70	-8.63	-8.92	-8.70
Rate of profit of portfolio compared to 03.01.2018	**P19**	**P20**	**P21**	**P22**	**P23**	**P24**	**P25**	**P26**	**P27**
31.01.2018	2.41	3.91	2.81	2.86	3.20	4.00	3.87	3.98	3.33
28.02.2018	-5.28	-1.61	-4.48	-4.59	-3.98	-1.58	-1.67	-1.65	-3.69
29.03.2018	-8.53	-5.72	-8.09	-7.98	-7.70	-5.76	-5.94	-5.77	-7.70

When comparing P1 and P2 portfolios, the first one seems to be more attractive – it is characterized by lower risk (though a lower rate of return) and higher profits or lower losses, which means that the portfolio determined by the standard approach with five criteria is less attractive. In the next group of portfolios: P3 – P5 (from six companies), P3 and P5 portfolios have similar profit or loss rates and similar level of risk and rate of return, while P4 (standard and fuzzy approach, criteria 1-4) records after the first month lower profits, but in the next two months slightly smaller losses, moreover, it is characterized by lower risk – however, it is difficult to draw unambiguous conclusions on this basis. Among the portfolios selected from the seven-element sets (P6 – P9), one can indicate the P7 portfolio (standard and fuzzy approach, criteria 1-4) with the lowest risk and comparable profit and lowest losses, as well as the P8 portfolio (standard approach, criteria 1-5) with the highest risk, the highest profit at the end of January, but also larger losses at the end of the next two months. The next group of portfolios: P10 − P12, is characterized by a comparable level of risk and profit rate after the first month, however P12 (standard variant, criteria 1-4) has lower losses and slightly higher profits. Among the P13 − P15 portfolios, the last one (P15 − fuzzy version, criteria 1-4) has the lowest risk, slightly lower profit after the first month, lower losses at the end of February and comparable or lower losses at the end of March. Portfolios P16 − P19 are characterized by a fairly high level of risk and very similar profits or losses; it's hard to find any regularities here. The penultimate group of portfolios: P20 − P23, mostly has a high level of risk and similar profits or losses at the end of the next three months. The exception is the P20 portfolio, which is characterized by a slightly lower risk and the highest profits and the lowest losses in this group (standard variant, criteria 1-4). P24 – P27 portfolios are characterized by high risk (the highest for P27) and relatively comparable profits/losses, although for the P27 portfolio the results are the least favourable (fuzzy approach, variant V, VI). The P15 portfolio has the lowest risk (fuzzy variant, criteria 1-4), while the P17 portfolio (the standard variant, criteria 1-3, 5 and 1-5) has the highest risk – one of the lowest profits after the first month, moderate losses after two more. Against this background, the P0 portfolio looks quite good – it has a moderate risk and the potential profits and losses are on an average level; it is similar to P15 (in terms of profits and losses) and P6 (in terms of risk and return rate). One of the most advantageous for the decision maker seems to be the P7 portfolio – one of the lowest risk levels at a moderate rate of return, also in the lead in terms of profits and losses.

4. Conclusion

The analyzes carried out were aimed mainly at assessing the risk of constructed portfolios and their profitability. For the initial selection of companies, the multi-criteria TOPSIS and FTOPSIS methods were applied and five market and fundamental indicators were used (in different configurations), which were the criteria for company assessment. The group of companies obtained on the basis of the multi-criteria analysis were the basis for the construction of effective portfolios using the Markowitz approach.

The results of the companies' ordering were very similar, the correlation between the rankings was not lower than 0.923. Grouping results (in sets with the same cardinality) were also similar. The portfolios received were characterized by different risk levels and different rates of return. Profits or losses were also varied but the discrepancy in terms of losses was greater than in the case of profits. The Markowitz portfolio, which was created on the basis of all the companies included, turned out to be quite good in terms of the properties discussed. On the basis of the conducted research, it seems, it is not possible to obtain an unambiguous recommendation which of the approaches (standard, fuzzy) and which set of criteria will allow to receive (in the given conditions) the most attractive portfolio with acceptable risk for the investor. However, some of the premises resulting from these analyzes may suggest that one of the better results is more often obtained using a standard or fuzzy variant with criteria 1-4 and one of the worse results is obtained using the standard approach with criteria 1-5.

The hypothesis can not be unambiguously verified; the problem requires further research and searching for some new approaches and solutions.

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Application of static and dynamic approach to analyze financial performance of a company

Barbora Ptáčková [1]

Abstract

This paper is dedicated to financial performance evaluation. Evaluation of financial performance belongs to key activities of each company. The importance of the performance concept is widely recognized, it is one of the tools, knowing which helps to manage a company efficiently. Decomposition analysis is also very important. Decomposition analysis helps to analyze the factors affecting selected key measures and quantify their impact on the key measures. In this paper static and dynamic approach is applied to quantify the influence of component ratios to base ratio. The aim of this paper is to analyze financial performance of chosen company, find value drivers of this company and verify if using of static and dynamic approach in case of influence quantifying leads to the similar results.

Key words

Financial performance, economic value added, analysis of deviation, decomposition of variance

JEL Classification: C2, C5, C58, G3, G30

1. Introduction

Financial performance of a company is one of the main goals of the management. Financial performance of a company is usually solved by many authors, see Copeland (2000), Vernimmen (2005), Brealey (2014). From domestic authors, financial performance of a company dealth with authors such as Dluhošová (2004), Mařík (2005), Neumaierová (2005), or Kislingerová (2010). Generally, financial performance of a company is understood as the ability of companies to create some value added.

Meanwhile, in the past financial performance of companies have been analyzed according to traditional indicators based on accounting data, see Vernimmen (2005) or Dluhošová (2010), nowadays modern indicators are more used because of their ability to take into account the market situation as well as costs of capital, factor of time or factor of risk. Economic value added is one of the most used modern indicators. The preference of this approach can be found in many publications, see Ehrbar (1998) Young (2001), Vernimmen (2005), Ross (2013) or Brealey (2014).

Another important task of the financial management is to determine, how the financial performance of a company is influenced. One of the way is to apply pyramidal decomposition method. Then, it is possible to determine the interactions among the indicators and quantify the influence of component indicators to base indicator, Dluhošová (2010). The influence can be quantify using static or dynamic methods.

The aim of this paper is to analyze the financial performance of selected company according to economic value added and compare if using of static and dynamic approach leads to similar results.

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2. Used methods

This part of the paper is divided into three sub-chapters. Firstly, economic value added as a measurement of financial performance of companies is characterized. Then pyramidal decomposition of economic value added is proposed. Then there are characterized static and dynamic approaches to quantify the influence of sub-indicators to base indicator.

2.1 Economic value added

Traditional performance measures such as NOPAT, ROI or ROE have been criticized due to their inability to incorporate costs of capital and therefore accounting revenue is not a consistent predictor of firm value and cannot be used to measure company performance, Vernimmen (2005). One such innovation in the field of financial performance measurement is Economic value added, see Dluhošová (2004).

There are many ways how economic value added can be expressed. It can be distinguished EVA – equity, EVA – entity or relative economic value added. In this paper financial performance of selected company is analyzed according to relative economic value added. Relative economic value added is expressed as

$$EVA/E = (ROE - r_E), \tag{1}$$

Where ROE is return on equity, E is equity and r_E are costs of equity. By using building model of Ministry of industry and trade of the Czech Republic it is possible to express costs of equity as

$$R_e = R_F + RP_1 + RP_2 + RP_3 + RP_4, \tag{2}$$

where R_F is risk free rate, $RP_{1,2,3,4}$ are risk premiums which are determined according to methodology of Ministry of industry and trade of the Czech Republic, mpo.cz.

The difference between ROE and Re is called spread. If this spread is positive, it means that industry or company earns more than the costs of equity are.

2.2 Method of pyramidal decomposition and analysis of deviation

One of the main task of financial analytics is to analyze the deviations of component indicators and to find and quantify the factors that contribute the most to these deviations. Method of pyramidal decomposition is usually used for quantification of the impact of component ratios on the change in the base ratio. This method also allows to determine the interactions and relationships among the component ratios.

There are many ways, how relative economic value added can be decomposed. In this paper, there is proposed possible pyramidal decomposition of relative economic value added as

$$\frac{EVA}{E} = \left[\frac{EAT}{EBT}\cdot\frac{EBT}{EBIT}\cdot\left(\frac{VA}{R}-\frac{W}{R}-\frac{DEP}{R}-\frac{(OR-OC)}{R}\right)\cdot\frac{R}{A}\cdot\frac{A}{E}-(r_f+rp_1+rp_2+rp_3+rp_4)\right], \tag{3}$$

where E is equity, EAT is earnings after taxes, EBIT is earning before interests and taxes, A are assets, R is revenues, OC are operating costs, OR are other revenues, W are wages, DEP is depreciation, VA is value added.

In this case of decomposition, cost of equity is determined according to a build model of Ministry of industry and trade of the Czech Republic. According to this model, cost of equity is calculated as a sum of risk free rate and risk premiums.

2.3 Static approach - Analysis of deviation

The pyramidal decomposition together with the analysis of deviation helps to identify the relationships between the financial ratios and also quantify the impact of selected ratios on the base ratio, Dluhošová (2010).

It is useful to apply the analysis of deviations for in-depth analysis of the impact of component ratios on the base ratio. It is possible to quantify the impact of the changes in the component ratios on the base ratio according to this analysis, Zmeškal (2013).

The influence of component financial indicators can be quantified in one period or within the time series. If one period of financial indicators is analyzed, this is a classic analysis of deviation. Generally, any base indicator x can be expressed as a function of component indicators

$$x = f\left(a_1, a_2 ... a_n\right) \tag{2}$$

The change of the base financial indicator can be determined as a sum of influences of component financial indicators as

$$\Delta y_x = \sum_i \Delta x_{a_i} \tag{3}$$

where x is the base financial indicator, Δy_x is the change in the base financial indicator, a_i is the i-th component ratio, Δx_{a_i} is the impact of i-th component ratio on the change in the base ratio.

Quantification of the impact of component financial ratios on the change in the base financial ratio in pyramidal decomposition can be expressed using two relationships – additive relationship and multiplicative relationship, Zmeškal (2013).

Deviation analysis for additive relationship

According to Zmeškal (2013) additive relationship between financial ratios is expressed as

$$x = \sum_i a_i = a_1 + a_2 + ... + a_n \tag{4}$$

Quantification of the impact under the additive relationship is generally applicable. The total impact is divided in proportion to the changes in the component financial ratios as

$$\Delta x_{a_i} = \frac{\Delta a_i}{\sum_i \Delta a_i} \cdot \Delta y_x \tag{5}$$

where $\Delta a_i = a_{i,1} - a_{i,0}$ is the value of the i-th component financial ratio at the beginning of the analyzed period and a_i is the value of the i-th component financial ratio at the end of analyzed period.

Deviation analysis for multiplicative relationship

Multiplicative relationship among financial ratios is expressed as

$$x = \prod_i a_1 = a_1 \cdot a_2 \cdot ... \cdot a_n \tag{6}$$

According the way in which the multiplicative relationship is handled, it can be distinguished five basic methods: a method of gradual changes, a decomposition method with surplus, a logarithmic method, functional method or the integral method, Zmeškal (2013).

In this paper integral method is used for quantification of influence under the multiplicative relationship. Advantages of integral method are same as for functional method, but it is also possible to analyze non-linear operations and also interpretation can be easier, Zmeškal (2013). Resulted influence quantification according integral method for any component ratio is expressed as

$$\Delta x_{a_i} = \frac{R_{a_i}}{R_{x'}} \cdot \Delta y_x, \tag{7}$$

where $R_{a_i} = \dfrac{\Delta a_i}{a_{i,0}}$ and $R_{x'} = \sum_{i=1}^{N} R_{a_i}, R_x$.

2.4 Dynamic approach – method of decomposition of variance

When a time series of financial indicators is used for analysis, it is a dynamic analysis. Dynamic analysis is based on decomposition of variance. The derivation of method of variance decomposition can be found in Dluhošová, Ptáčková, Zmeškal (2015), Ptáčková (2015) or Ptáčková, Richtarová (2017). Generally, the influence of variance of component indicators is calculated as

$$z_i = a_i^2 \cdot \mathrm{var}(F_i) + \sum_{j \neq i} a_i \cdot a_j \cdot \mathrm{cov}(F_i, F_j) \tag{4}$$

where $a_i = E\left[\dfrac{\partial f(\cdot)}{\partial F_i}\right]$. (5)

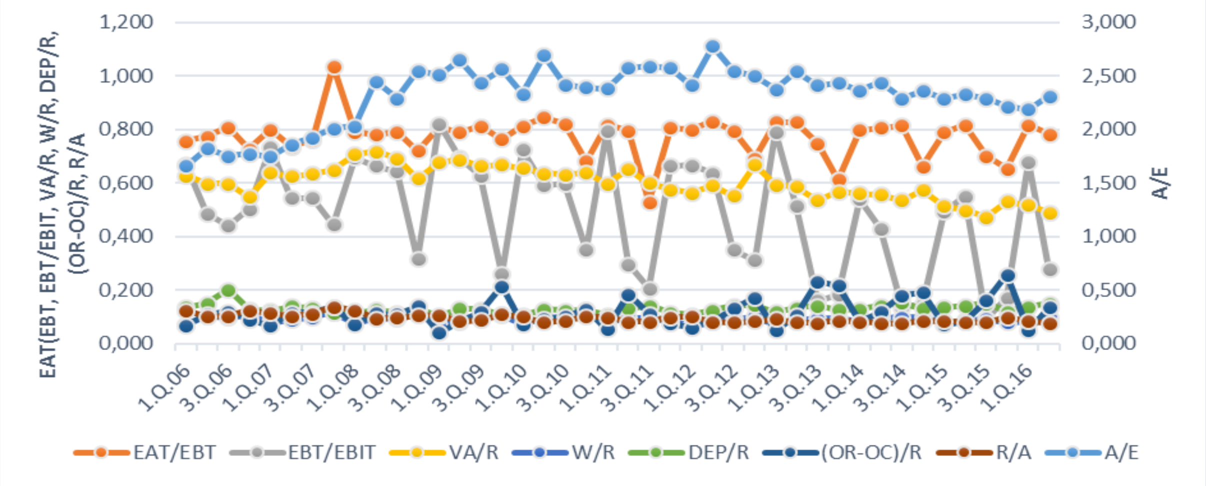
3. Application part

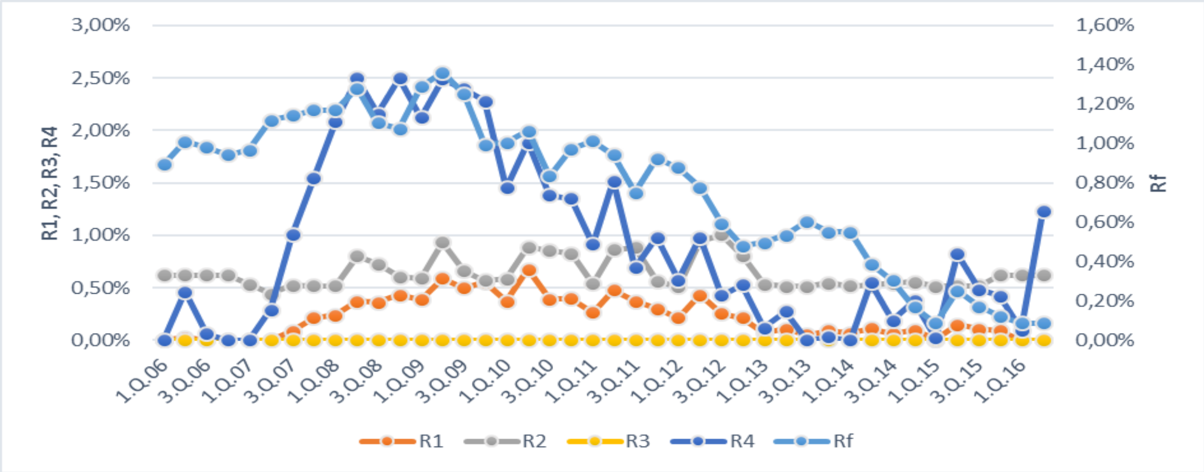
For the analysis company from energy industry of the Czech republic was chosen. Financial performance of selected company is analyzed according to relative economic value added and this indicator is decomposed to component financial indicators using pyramidal decomposition method. Main value drivers of financial performance of this company are found according to integral method (static approach) and method of decomposition of variance (dynamic approach).

3.1 Input data

Quarterly data are used for the analysis. The data were taken from the company´s consolidated financial statements. For a deeper analysis, data in the period 2006 to 2016 were used according to pyramidal decomposition (2). In the Graph 1 there are the amounts of component indicators, risk free rate and risk premiums of analyzed company.

Graph 1: The amounts of component indicators, risk free rate and risk premiums of the analyzed company





Source: own calculation

3.2 Financial performance analysis – static approach

Financial performance of chosen company is firstly analyzed according to classic analysis of deviation. The influence of component indicators under the multiplicative relationship are quantified by integral methods. This is a static analysis. It means that only change between two periods is analyzed. Analysis is made between the 1Q/2006 and 2Q/2016. If this approach is applied the changes that occurred during this period are not taken into account.

Relative economic value added indicator is decomposed according to pyramidal decomposition (3) to component ratios. If there is an additive relationship between the indicators, the influences are quantified according to (5). If there is a multiplicative relationship between ratios, the influences are quantified by integral method as (7). In the Table 1 there are the influences of component indicators and order of the influences.

Table 1 The order of influences of component indicators

	1Q 06/2Q 16	
	influence	**order**
EVA /E	**-4,10%**	
E (v tis. kč)	**0,0000**	
SPREAD (ROE - R_e)	**-4,10%**	
ROE	**-0,0368**	
R_e	**-0,0042**	
EAT/EBT	**0,0016**	
EBT/EBIT	-0,0277	1 (-)
EBIT/R	**-0,0113**	
VA/R	-0,0134	4 (-)
W/R	-0,0027	8 (-)
DEP/R	-0,0016	10 (-)
(OR-OC/R	0,0063	7 (+)

R/A	-0,0179	3 (-)
A/E	0,0184	2 (+)
R_F	0,0081	6 (+)
R_1	0,0000	11 (+)
R_2	0,0000	11 (+)
R_3	0,0000	11 (+)
R_4	-0,0123	5 (-)

Source: own calculation

In the Table 1 there are the amounts of influences of component ratios. It is possible to order these ratios according to the influence. Relative economic value added of analyzed company has decreased by 4.1 % between 1Q/2006 and 2Q/2016. It is clear that the EBT/EBIT indicator was the most influenced indicator. The total impact of this indicator to relative economic value added is negative (-0,0277). Financial leverage is the second most influencing indicator. Total impact of financial leverage is positive (0,0184). The indicator of asset turnover is the third most influential indicator.

3.3 Financial performance analysis – dynamic approach

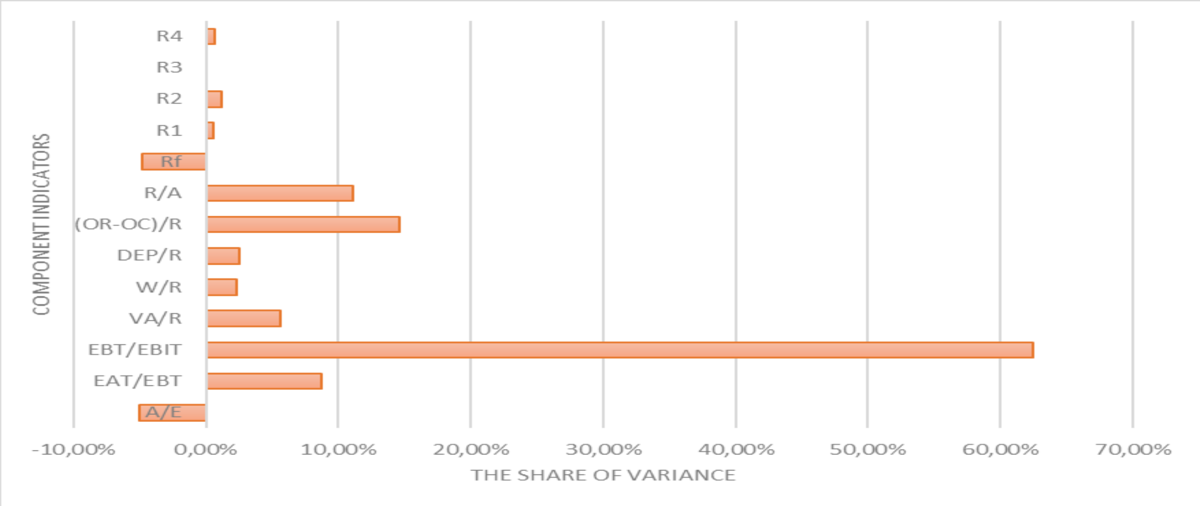
Financial performance of chosen company is analyzed according to dynamic method, too. When a time series of financial indicators is used, then it is a dynamic analysis. The analysis of financial performance of chosen company is made in the period 1Q/2006 to 2/Q2016. If dynamic analysis is applied, changes which occurred during this period are taken into account. Dynamic analysis is based on decomposition of variance of base indicator.
Indicator of relative economic value added is decomposed to component indicators by pyramidal decomposition (3) to component ratios. Generally, the influence of component indicators to base indicator according to method of decomposition of variance is determined as (4). In the applied pyramidal decomposition there are both multiplicative and additive relationship and it is a relation of total thirteen indicators, it is necessary to expand the method of decomposition of variance. After the application of Taylor expansion, it is possible to calculate the influence of component indicators to base indicator as

$$\begin{aligned}
&z_1 = a_1^2 \cdot \mathrm{var}(F_1) + a_1 \cdot a_2 \cdot \mathrm{cov}(F_1, F_2) + a_1 \cdot a_3 \cdot \mathrm{cov}(F_1, F_3) + a_1 \cdot a_4 \cdot \mathrm{cov}(F_1, F_4) \\
&+a_1 \cdot a_5 \cdot \mathrm{cov}(F_1, F_5) + a_1 \cdot a_6 \cdot \mathrm{cov}(F_1, F_6) + a_1 \cdot a_7 \cdot \mathrm{cov}(F_1, F_7) + a_1 \cdot a_8 \cdot \mathrm{cov}(F_1, F_8) \\
&+a_1 \cdot a_9 \cdot \mathrm{cov}(F_1, F_9) + a_1 \cdot a_{10} \cdot \mathrm{cov}(F_1, F_{10}) + a_1 \cdot a_{11} \cdot \mathrm{cov}(F_1, F_{11}) + a_1 \cdot a_{12} \cdot \mathrm{cov}(F_1, F_{12}) \\
&+a_1 \cdot a_{13} \cdot \mathrm{cov}(F_1, F_{13}), \\
&\vdots \\
&z_{13} = a_{13}^2 \cdot \mathrm{var}(F_{13}) + a_{13} \cdot a_1 \cdot \mathrm{cov}(F_{13}, F_1) + a_{13} \cdot a_2 \cdot \mathrm{cov}(F_{13}, F_2) + a_{13} \cdot a_3 \cdot \mathrm{cov}(F_{13}, F_3) \\
&+a_{13} \cdot a_4 \cdot \mathrm{cov}(F_{13}, F_4) + a_{13} \cdot a_5 \cdot \mathrm{cov}(F_{13}, F_5) + a_{13} \cdot a_6 \cdot \mathrm{cov}(F_{13}, F_6) + a_{13} \cdot a_7 \cdot \mathrm{cov}(F_{13}, F_7) \\
&+a_{13} \cdot a_8 \cdot \mathrm{cov}(F_{13}, F_8) + a_{13} \cdot a_9 \cdot \mathrm{cov}(F_{13}, F_9) + a_{13} \cdot a_{10} \cdot \mathrm{cov}(F_{13}, F_{10}) + a_{13} \cdot a_{11} \cdot \mathrm{cov}(F_{13}, F_{11}) \\
&+a_{13} \cdot a_{12} \cdot \mathrm{cov}(F_{13}, F_{12}),
\end{aligned} \tag{6}$$

where the regression coefficient is calculated according to (5).

In the Graph 3 there are shown shares of variance of component indicators to the variance of relative economic value added as a base indicator of analyzed company in the period 2006 to 2016.

Graph 3: The share of variance of component indicators to variance of relative economic value added of analyzed company



Source: own calculation

Indicator EBT/EBIT is one of the main value drivers of financial performance of analyzed comapny in the period 2006 to 2016. This indicator makes up 62.28 % of variance of variance of relative economic value added. The share of other revenues and costs to revenues is second indicator, which has the main influence. This indicator makes up 14.59 % of variance of relative economic value added. The financial leverage indicator has negative influence. Risk free rate has negative influence, too. The influence of other indicators is negligible due to the following main generators.

3.4 Discussion

One of the important task of the financial management is to determine, how the financial performance of a company is influenced. One of the way is to apply pyramidal decomposition method. The principal of this method is decomposition of base financial ratio to component financial ratios. Then, it is possible to determine the interactions among the indicators and quantify the influence of component indicators to base indicator, Dluhošová (2010). There ae two approaches how the influence of component ratios can be quantified – static approach and dynamic approach. When the static approach is applied the influence of component ratios is quantified only between two periods and changes occurred during this period are not taken into account. This is one of the disadvantage of this approach. Static analysis is based on classic analysis of deviation. When dynamic approach is applied influences are quantified during the all analyzed period and changes of component ratios occurred during this period are taken into account. One of the method of dynamic analysis is variance decomposition method. One of the disadvantage of this method is the fact, that time series of financial ratios is needed. While static approach distinguish the relationship between the ratios, dynamic analysis does not. Based on this analysis, it can be considered that the application of static and dynamic approach in the case of quantification of the influence of component ratios leads to similar results.

4. Conclusion

This paper was dedicated to financial performance of chosen company by static and dynamic methods. Financial performance of selected company was analyzed according to economic value added indicator. Pyramidal decomposition of this indicator was proposed and value drivers were found. The influence of value drivers was quantified according to integral method (static approach) and method of decomposition of variance (dynamic approach).

Indicator EBT/EBIT is considered to be the factor, which has the biggest influence to relative economic value added of analyzed company between the 1Q/2006 and 2Q/2016. When dynamic approach was applied this indicator was found as the most influential factor during the all analyzed period, too.

Based on this analysis, it is possible to use both static and dynamic methods to quantify the influence of component ratios and using of these approaches leads to the similar result.

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Financial regulation evolvement and its expenses impact on banking sector in Czech Republic

Mikuláš Pýcha [1]

Abstract

This paper focuses on continuously evolving financial regulation and its possible implications to output in a country that is very much dependent on effectively working banks that can satisfy the demand for financing. In last year we faced a new banking regulation that brought many new mechanisms promising financial stability and shackled banks in means of accepting too much risk. The main aim of this paper is to point out on banks in Czech Republic that play a key role in providing financing in a country with an undeveloped capital markets. Furthermore the goal is to analyze the evolvement in operating expenses that may be affected by stricter regulatory requirements and combine this with their output. The analysis separates banks into groups by their size and evaluates whether size of a bank plays a role in development of relative amount of expenses and its growth rate from 2005 till 2017 that may be caused by implementation of tougher regulation. Small banks do face same banking regulation as larger banks and therefore corresponding expenses probably constitute larger share of the total operating expenses. This fact indicates that banking regulation favors larger banks and could lead into creation of an oligopoly in banking industry not only in Czech Republic.

Key words

Financial regulation, Basel III, banking industry, oligopoly, crises, Czech Republic

JEL Classification: G21, G28, L13

1. Introduction

When taking a look back, two last decades brought us two major crises and many new rules and requirements that must be fulfilled by financial institutions. The main purpose is pretty clear, developed countries decided to avoid financial crises like those we could witness in 2000 - 2001 and later in 2008. First crisis in 2001 is known as a dotcom crisis and showed us how dangerous the financial bubbles might be in connection with enthusiasm of strong economy growth and new technology evolvements. The debt crisis in 2008 has hit the world economy much harder than the previous one. However both of these crises were of a similar magnitude at the beginning, but the final output differed a lot which is pointed out by Kosuke and Nikolovz (2011) in their research. The actual drop in stock prices during dot-com crisis caused a loss of more than 8 trillion USD for its owners. Despite such a deep losses the following recession was short and mild comparing to the 2008 crises and did not bring major financial instability as reflected by former head of FED B. Bernanke (2012).

In 2008 we have already had an effective banking regulation called Basel II and majority did not believe such risks might be accumulating in financial system before it became clear in 2008. The second crisis of these two was harder not because more value would be lost as mentioned above, but because the value lost was created by debt issuance. When losing debt money, someone else must have solved it, perhaps better to say repay it back. The amount of unpaid debt was extremely high and to keep banking system alive with confidence of its

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creditors, the governments around the globe must have intervened. All economically developed countries financially participated on losses that were created during the crisis and also central banks became involved. For example ECB increased its lending to banks from 400 billion EUR before the crisis to twice as much (Docherty and Viort, 2014). When we start thinking about the possible consequences of such actions, we might easily see a reason why the interest rates plummeted near zero after the crisis and still did not recover globally as debt problems have not been fully solved.

These crises supported regulation makers to keep creating a financial regulation system that helps to avoid crises in the future. It is widely agreed that the precrisis regulation was imperfect and weak with insufficient protection and therefore we had to suffer such crisis. There are many researchers (Hanson, Kashyap and Stein, 2010; Galati and Moessner, 2011) that claim the precrises regulation was way to microprudential and did focus only on each individual bank and their financial stability instead of checking the global financial stability of financial system as a whole. For about the last decade we can see how the regulatory institutions build macroprudential regulation framework that should increase financial stability. Amongst such macroprudential tools may be counted any requirements that work same for the whole financial system, for example bail in mechanism that should remove moral hazard from banks or requirements and more quality capital items as they cover possible losses in time of distress with higher stability. Beginning with 2008 crisis the macroprudential regulation became a reality and it keeps developing and no longer is only an academic theory. Moreover there are also many supporters like De Nicolo, Gamba and Luccheta (2014) that stand for increasing a level of microprudential regulation due to the fact it failed avoiding or reduce the crisis. Microprudential regulation means firstly capital requirements and it is widely agreed that higher capital requirements create less bankruptcies in financial sector, but also must not stay alone and therefore financial regulation needs to be formed together with other requirements like control of deposit rates control, asset restrictions, effective monitoring as highlight Hellmann, Murdock and Stiglitz (2000).

As it seems we have already entered a financial world where we build a perfect system of financial regulation that promises financial stability and no more crises. It really does sound as something demanded, but is it really the most effective way for future economic growth? Are banks, the core institutions of most central Europe economies in means of financing, as effective as we wish them to be with such financial regulation? I am afraid that more and more regulatory requirements, either micro or macroprudential, do suffocate current financial system and decrease not only banks efficiency, but also the aggregate output of economy. As stricter financial regulation provides lower probability of creating a crisis, we must think what crisis actually mean. In general we talk about crisis when GDP stop growing and its growth rate slows down to zero or even become negative. The GDP growth worsens when there are many firms that stop producing and defaults. It is desirable for any economy to have many new projects that may become profitable and will provide services and simultaneously government will benefit from it when collecting taxes. Logically when a new business plan is set up, you can never be sure it will really be working and if the value is not added, respectively the profit is negative, the best and only way is to shut it down. System of financing in Czech Republic, same as it is in other middle European countries with weak capital markets, is dependent on banking industry. Once you let banks to create more credit, there surely will be more businesses starting which boosts the GDP at beginning. Off course it increases the risk of more defaults in future and corresponding drop in GDP. Soft regulatory requirements definitely allow such process to happen and also create prize bubbles that provides misleading information when allocating resources. Hancock and Zahami (2011) mention that bubbles in themselves are not always bad, but they may become disastrous when they leave debt behind. I feel this logical approach can be used same when evaluating crises, because they do not need to be always bad and in many cases they can have purifying impact on the economy.

Once we start applying stricter financial regulation, it is clear less projects receive demanded financing a therefore less of them default. However it works also the other way around, the less financing creates less successful businesses. I believe that more projects we allow to finance, that more of them may turn into a technological evolution, possibly a step forward. We have already witnessed such evolutions in our history, how quickly did automobiles, communication devices, computers, medicine, and internet and off course many more develop. It is true there were crises and they may be understood as a "fee" for growing too fast. On the other hand once we invented something very useful and widely demanded, it stays with us and no crisis takes away, contrarily it helps to overcome the crisis.

After the crisis in 2008 the banking regulation was already ruled by Basel II, which was probably the strictest one in banking history. Nevertheless more regulatory requirements were ahead, because in 2010 was firstly introduced concept of banking regulation called Basel III (Lyngen, 2012). The figure 1 presents how Czech banks change its manner in providing/issuing loans to their clients after 2010.

Table 1: Average growth rate of issued loans before and after Basel III introduced in Czech Republic

average growth rate in issued loans (period 2005 – 2010)	17,1 %
average growth rate in issued loans (period 2011 – 2017)	7,2 %
Difference in average growth rate after Basel III introduced	**- 9,9 %**

note: issued loans corresponds with clients receivables

As the table 1 suggests, banking industry in Czech Republic obviously slowed down on providing new loans after they knew Basel III was going to tighten regulatory requirements. This might have multiple reasoning, such as higher capital costs, liquidity requirements or maybe requirements reducing systemic risk like bail in mechanism. All these newly structured regulatory rules should bring more stability, but they surely bring higher costs (expenses) for banks to fulfill them and for regulatory institutions to prepare them, implement them and control whether they are correctly followed by each regulated institution.

2. Aim of the Paper

The aim of this article is to analyze operating expenses of banks with connection to the stricter regulation implementation. In more detail important banks in Czech Republic are going to be grouped by their size of the balance sheet, then we will analyze possible differences in development of operating expenses in last decade and make appropriate conclusion. This paper is focusing only on expenses of banks and it does not evaluate the amount of expenses that are connected to the preparation and implementation spent by regulatory bodies.

3. Hypothesis

As the financial regulation changing dramatically in last years, we can never be sure what consequences it may bring. It is absolutely vital to validate and keep asking whether there could be threats to current financial system created by relatively newly introduced system of financial regulation. This article targets the impact of this regulation on banks that are different in size and the hypothesis is that corresponding expenses brought by financial regulation have relatively more fixed character, rather than variable course. Basically we assume that the new financial regulation is increasing operating expenses of banks and these expenses do not grow linearly with the size of a bank, but they grow slower due to the structure of operating expenses connected to fulfilling regulatory requirements. The major part of expenses, which are unavoidable for banks, are paychecks of their skilled employees like compliance officers, internal lawyers, internal auditors and many more that work to satisfy regulatory requirements as it is demanded by supervisor (Czech National Bank). As all banks must comply with effective financial regulation, we assume that smaller banks must spend relatively more operating expenses in comparison with larger banks that realize economies of scale.

Larger banks then stand in more comfortable situation and financial regulation puts more cost pressure on smaller banks, which may eventually lead into creation of oligopoly in banking sector. An oligopoly as a one of market failures brings ineffectiveness to banking industry and may negatively effect allocation of scarce resources. This is specifically very risky in banking industry, because most of resource allocation happens here through issuance of loans.

4. Methodology

In this part we describe the approach how were the data extracted and sorted with all relevance information. Firstly it is necessary to split banks into three groups based on their size. For purposes of dividing Czech banks into separate group we use the size of their balance sheets as this methodological approach is used by Czech National bank. The small banks are considered banks with balance sheet not exceeding 50 billion CZK, then medium banks are those with balance sheet over 50 billion CZK and not crossing 250 billion CZK. Finally large banks have their balance sheet over 250 billion CZK (Pýcha, 2014). There are available time series regarding Czech banking sector managed by Czech national bank that were analyzed whether these could be used. These are found in time series system called ARAD. However these data split by balance sheet sizes do not reflect the migration of individual banks between groups. When a small bank becomes a medium one or vice versa it is impossible find which bank was actually migrating and how what figures was adding into aggregate amounts. This issue makes it very hard to use any of these aggregate time series because groups are changing in their size during the analyzed period. Due to this fact, we have extracted data from annual reports of 18 chosen Czech banks. Czech banking sector provides us only with four large banks (Ceska Sporitelna, Komercni banka, CSOB and UniCredit Bank). There are 7 medium banks (Raiffeisenbank, Moneta Money Bank, Hypotecni bank, ING bank, Czech Export Bank, PPF Bank and Sberbank) and 7 small banks (Fio Bank, AirBank, Expobank, Wustenrot, Equa Bank, Commerzbank and J&T Bank) which were analyzed.

Each annual report provides time series starting in 2005 and ending in 2017 and focuses mostly on net interest income, operational expenses with breakdown to administrative expenses, legal expenses or IT expenses contained in income statements for each year. Furthermore the balance sheets provide sum of loans issued and receivables from clients and also receivables from banks. Once the data were obtained, it became clear the time series sample must be shortened due the fact that some banks did not exist before 2010. For this reason the analysis uses only data form 2010 till 2017 as these are comparable amongst defined groups. Analyzing more banks was evaluated as difficult due to lack of annual reports of other banks and more importantly the chosen sample can be considered as adequate because it does contain the most known banks in Czech Republic.

During the process of gathering data from annual reports there were some issues with names of items like operational expenses or administrative expenses because each bank uses different tittles for these items and sometimes these names do change in time. For ensuring the extracted time series are comparable, each change of item´s name was checked in explanatory narrative part of an annual report and also compared with previous reports. Many banks do not report legal expenses and IT expenses in detail rather than aggregately operational expenses that sum all kinds of expenses. Due to these detail missing for many banks, it is not possible to analyze the detailed expenses between defined bank´s groups.

5. Research

This is the main part of this paper and it presents an analysis dealing with changes in banking industry in last decade. After crisis in 2008 the financial regulation strengthened and this analysis brings closer look into the development of expenses and bank´s output and further seeks the connection to current financial regulation. As stated in parts above it is expected that current regulatory framework for banks increases their expenses and pushes them to reduce their loan issuance activity. Increase in expenses and drop in output leads generally into worse effectiveness of any industry. However we may argue that this is the cost of not having financial crises and it is better to grow slowly and steadily, on the other hand using more of banking industry capacity allows economy to grow faster with possible slowdowns we call crises that may really have positive impact of removing inefficient entities. Therefore this article does not present any analysis of costs and benefit arising from having such strict banking regulation. This may be a follow up topic in another article that will continue to examine effects of current financial regulation. Now we can move to analyze our figures for the whole banking industry and then have a look whether the size of a bank plays an important role for their operational expenses and output.

5.1 Aggregate figures for all banks

At the beginning it seems logical to present the analyzed data for all banks aggregately to see how chosen figures developed from 2010. The following table 2 shows how did operational expenses with detail on administrative expenses, net interest income and amount of unpaid loans and receivables evolved during last 8 years.

Table 2: Development of chosen figures for ALL analyzed banks (in million CZK or %, 2010 – 2017)

Year	Operational expenses	Administrative expenses	Net interest income	Unpaid Loans and receivables
2010	63 400	29 506	110 951	2 049 168
2011	66 404	30 146	115 057	2 216 344
2012	68 583	31 397	112 569	2 139 679
2013	70 431	31 576	110 012	2 354 530
2014	70 518	32 654	116 878	2 215 162
2015	72 104	33 750	113 072	2 322 608
2016	73 482	33 384	112 416	3 053 582
2017	75 245	33 970	113 948	3 220 334
Average growth in last 8 years	2,3 %	1,7 %	1,0 %	6,6 %
Average growth in last 4 years	1,7 %	1,9 %	0,9 %	9,0 %
Average inflation	1,5 %			

rate in last 8 years	
Average inflation rate in last 4 years	1,0 %

When looking at the operational expenses there is not any sharp growth during analyzed years, especially when we would add an inflation rate. The growth rate net of inflation in operational expenses is still positive in both analyzed periods, but it is only a very moderate growth. Very similar development is seen in development of administrative expenses that we consider as more addressing figure to measure an impact of new financial regulation requirements than the total operational expenses. This moderate growth can be connected to stricter regulatory requirements which create a higher demand on skilled employees to fulfill them. The weak growth rates in net interest income is of course impacted by low interest rates in last decade (causing lower interest margin), but the reason for it may also be not enough issuance of new loans due to regulatory cost of capital. However when seeing the last column how the unpaid loans and bank´s receivables after clients developed in last 8 years, it is not the issue that banks would not fulfill their core mission of providing financing. Basically the net interest income stagnates due to low interest rates and banks seek another way of raising income like increasing the importance of fees and all kinds of charges. This shows us that interest income as a measure that describes bank´s performance loses its meaning when we try to use it as performance explanatory tool.

The receivables and loans outstanding do grow steadily, but the average growth is 2,4 % weaker in first 4 years which supports the hypothesis that new regulatory framework introduction brought a lot of uncertainty which caused slowdown on new loan issuance. Moreover the average growth rate in 2005 – 2010 period presented in table 1 shows us how significantly did growth rate of new loan issuance plummet. Nevertheless the growth of new loan issuance evinces a pretty decent growth in analyzed years and contrarily with the stated hypothesis is actually not negatively impacted by current financial regulation. It is definitely a question whether it is possible to grow new loans issuance with such speed as it was recorded from 2005 till 2010 under new banking regulatory framework.

5.2 Large banks

Figures presented for all banks aggregately did not sufficiently support the hypothesis that financial regulation slows down banking performance and increases operational expenses, even though slight evidence was achieved. In this part we start analyzing how these figures differ from each other for defined groups of differently sized banks. The analyzed values for large banks can be seen in following table 3. It is important to mention that banking industry in Czech Republic does not provide more than 4 large banks; therefore analyzing these 4 banks makes our sample for this group compact.

Table 3: Development of chosen figures for LARGE banks (in million CZK or %, 2010 – 2017)

Year	Operational expenses	Administrative expenses	Net interest income	Unpaid Loans and receivables
2010	47 088	22 983	82 098	1 408 368
2011	48 534	21 766	84 728	1 530 300
2012	49870	21 779	82 799	1 584 647
2013	50 701	22 595	79 624	1 736 807
2014	50 974	23 065	81 017	1 359 284
2015	52 362	23 177	78 858	1 427 215
2016	52 108	24 282	78 959	2 065 580

2017	52 922	23 226	80 068	2 192 568
Average growth in last 8 years	1,7 %	1,0 %	- 0,3 %	8,0 %
Average growth in last 4 years	1,3 %	0,3 %	- 0,4 %	18,6 %

As the table 3 indicates, large banks have not raised their operational expenses any sharp from 2010. Adding the inflation rate into consideration, the operational expenses same as administrative expenses growth rates become negative. Same as expenses the net interest income slows down and supports the idea that performance of these banks is weaker in analyzed period. However these banks still keep increasing their sum of receivables after their clients, especially from 2015 until 2017 when they grew up this figure by 765 billion CZK. It means that large banks growing up, but do not perform higher interest revenues as result. This means that these banks reach other revenues somewhere else, otherwise it can't be justified they keep around 68 % of the total amount of unpaid loans and receivables out of all analyzed banks. Logically the first place to analyze should be fees and charges that could have a significant impact on banks revenues nowadays and should be also added in future analyses to have more compact view. The very important result for large banks is that they are not witnessing increase in expenses caused by financial regulation rules adopted in analyzed period and simultaneously they still keep closing new business. From 2010 large banks did not lose any significant share of the banking market measured by the sum of issued unpaid loans outstanding amongst all analyzed banks. In last 4 years large banks raised this figure in average by almost 278 billion CZK per year.

5.3 Medium banks

The group of medium banks in Czech Republic is also pretty stable in analyzed period and consists of many respected banks that have a longer history like Raiffeisenbank or Moneta Money Bank previously named GE Money Bank. Nevertheless this group is not as stable as large banks because some banks are migrating between this group and the group of small banks as their balance sheet changes. For purposes of this analysis the groups does not change from the starting point in number of banks in each group. The following table 4 sums up figures for medium banks.

Table 4: Development of chosen figures for MEDIUM banks (in million CZK or %, 2010 – 2017)

Year	**Operational expenses**	**Administrative expenses**	**Net interest income**	**Unpaid Loans and receivables**
2010	13 764	6 496	25 570	535 118
2011	14 536	6 616	26 925	576 189
2012	14 402	6 461	26 003	430 027
2013	15 044	5 878	26 055	450 006
2014	13 803	6 196	27 798	672 458
2015	13 762	5 991	26 411	697 612
2016	15 310	6 502	25 322	749 317
2017	15 792	6 795	24 458	790 571
Average growth in last 8 years	2,1 %	0,8 %	- 0,5 %	7,6 %
Average growth in last 4 years	4,7 %	3,2 %	- 4,2 %	5,6 %

Medium banks surely evince different figures than large banks for the same period. Most importantly we can see a higher growth rate in operational and administrative expenses, which are most probably caused by regulatory requirements that medium banks must have implemented and could not distribute related expenses as easy as large banks with bigger budgets. This development seems to be increasing in last years as many new requirements becoming effective. The overall growth rate in the whole period did not differ a whole lot from large banks, although it was still slightly higher for medium sized banks. The performance based on achieving net interest income was similarly weaker in comparison with large banks. In the end it is clear that medium banks did not grow stronger or performed better than large banks, but reported higher expenses during analyzed years, especially in last four years when the financial regulation sharpened.

5.4 Small banks

This group of banks influenced the sample by the possible starting point that would be comparable for all other banks. Many small and progressive banks were established in 2010 and therefore earlier data could not been used. However for some of these banks there were annul reports available before they gained banking license. For example Fio Bank became a licensed bank in 2010, but we were able to extract data from 2005 as the predecessor company called Fio, burzovni spolecnost, a.s. reported these in their annual reports. A very similar story tells Air Bank, which gained license in 2011 and before that it was named Brusson, a.s. and we could obtain data starting in 2010. This group was selected from banks that are very active on the Czech banking market, which made it easier to obtain necessary data. The following table 5 presents observed values for small banks.

Table 5: Development of chosen figures for SMALL banks (in million CZK or %, 2010 – 2017)

Year	**Operational expenses**	**Administrative expenses**	**Net interest income**	**Unpaid Loans and receivables**
2010	2 548	1 244	3 283	105 682
2011	3 335	1 751	3 404	109 855
2012	4 310	2 340	3 768	125 005
2013	4 686	2 633	4 333	167 718
2014	5 741	3 280	8 063	183 420
2015	5 980	3 477	7 803	197 781
2016	6 064	3 656	8 135	238 684
2017	6 532	3 869	9 421	237 195
Average growth in last 8 years	14,9 %	18,3 %	18,9 %	12,7 %
Average growth in last 4 years	5,6 %	5,7 %	5,6 %	9,3 %

Small banks are very different in means what part of banking sector they serve to. These banks do not offer wide range of services like large banks or bigger medium banks. However the regulatory rules are same for all banks without any adjustments for their size. Furthermore it may work even oppositely, because developing an internal model for calculating capital requirements that help to reduce the amount of required capital is a very expensive process. Having an approved internal model for calculating capital requirements from the regulatory body usually belongs to larger banks that can afford such thing. This may lead into the situation that larger banks have softer requirements because they can convince a regulator about their ability of measuring riskiness of their portfolio and therefore how much core capital is needed.

After analyzing the results for small banks, it becomes obvious they do grow in faster rate than either medium or large banks. Small banks more than doubled the amount of unpaid loans and receivables in their balance sheets. The average growth rate of 12,7 % is almost 60 % higher in comparison with large banks and even more in comparison with medium banks. There is no doubt that small banks in Czech Republic grow very fast and it also explains the high growth in net interest income. Even though low interest rates squeeze bank´s interest margin small banks are increasing a base of issued loans that generate interest income. We can also see that both growth rates of the unpaid loans and receivables and interest income slow down in last four years which indicates some kind of limitation to keep such performance. One of the possible answers may be that such performance generates deeper losses which are undesirable for shareholder in case they last too many years.

Small banks report a very high growth rates of operational expenses and even higher growth rates of administrative expenses. These figures are significantly higher than those observed with large or medium banks. In case we would try to compare an analyzed period, the growth rate of administrative expenses is more than 18 times bigger in comparison with large banks and more than 22 times bigger when comparing with medium banks in means of average growth rate. A very similar development is seen in operational expenses and in its growth rate, which is also incomparably higher than large or medium banks reported. The last four years the extremely high growth in expenses declines and getting closer to average figure that medium banks have, but still slightly higher. In the end the analysis points out that small banks report higher relative growth of expenses in comparison with larger banks and also takes into consideration their performance. The higher growth of expenses for small banks cannot be caused only by their higher activity, but there are some other reasons. This paper suggests that new regulatory framework for banking industry is one of those that play important and significant role.

6. Conclusion

The research shows us that it is very hard to find any evidence of how financial regulation impacts banks aggregately with no segregation by their size. This analysis also clarifies that measuring bank´s performance cannot be done just by looking at their interest income figures, but most probably bank´s fees and charges must be taken into consideration as well to have compact picture. Results of this paper suggest that small banks having significantly higher growth rates of their expenses than large or medium banks, although they grow in size relatively faster as well. However the rate of growing in size can be hardly compared to the growth rate in expenses that is multiple times higher. A similar development is seen between medium and large banks, where medium banks having slightly lower growth rate in expenses with comparable relative performance. These results alert that this may be caused by stricter financial regulation we have never implemented before in such scale and this may very probably favor large banks. Large banks obviously realize economies of scale when fulfilling regulatory requirements which cannot be reached by smaller banks. Furthermore this leads into slow creation of oligopoly structures that are not favorable for consumers. Where could this lead us? Perhaps larger banks will grow larger and cover bigger share of the market, which brings a known issue of “too big to fail”. Market competition is an essence for healthy economy, especially for banking industries in countries where banks play a key role when financing is sought and provided. Financial regulation should never help to support a creation of such oligopoly structure or later a monopoly structure, which may cause more losses to the country´s economy than a temporary decline in GDP that we often call a financial crisis.

This analysis faced many issues like the lack of data available for such research. The data must have been obtained via extracting individual annual reports, which makes this study very time consuming. One of the weaknesses may surely be the amount of analyzed banks, but 18 of them seemed sufficient. The future plan is to enrich this research by adding fees income to see full performance that became obviously needed during this analysis. Another plan for future is to prepare similar research for insurance companies that are also strictly regulated by Solvency II and may face the same market failures as banking industry. The goal would be to have more subjects to analyze to support the hypothesis with stronger data set.

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An Asset Pricing Model with Heterogeneous Beliefs: Stubborn versus Fickle Traders

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Abstract

In this paper, we consider an asset pricing model where traders have heterogeneous beliefs about the future price of a risky asset and update their beliefs chasing the trading strategy that generates the best relative past performance. Traders are distinguished for their beliefs on future prices of the risky asset so that they can be either fundamentalists (the forecasted future price is the fundamental one) or trend followers (the future price is extrapolated from historical price data). Moreover, traders can switch between fundamentalists and trend followers with a different intensity of choice. Specifically, a trader can be stubborn, i.e. his/her propensity to switch strategy is low. Alternatively, he/she can be a fickle trader and switch faster towards the best performing strategy. In this model, traders can change their beliefs but they cannot change their propensity to switch belief. The numerical analysis reveals that market configurations exist such that stubborn traders gain more profits than fickle traders. This is an interesting observation that highlights the need to investigate the role of different intensities of choice in models like the one here considered and to endogenize it.

Keywords

Asset pricing, Heterogeneous beliefs, Brock-Hommes model, Stubborn traders, Fickle traders.

JEL Classification: G12, G17, C62, C73.

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1. Introduction

Asset pricing models with heterogeneous beliefs have flourished in recent years, see, e.g., [9], [3] and [4]. The success of these types of models is due to their ability to explain stylized facts such as fat-tail and volatility clustering that characterize the dynamics of returns of financial risky assets, see, e.g., [7] and [6].

The learning process according to which more profitable trading strategies are selected is a key aspect of these models. This updating mechanism is usually based on social learning: each agent observes the strategy adopted by other traders and selects the strategy that, on average, performed better in the last periods, see, e.g., [2], [5], [1] and [8]. In the framework proposed in [3], for example, traders select beliefs about future prices according to a *random utility model* (or *logit* model), according to which the probability to adopt a trading strategy is proportional to the profits it generated in the past. In this learning process, the propensity to which traders converge towards the best performing strategy or belief depends on a parameter called *intensity of choice*. This parameter is assumed to be the same for all traders and is exogenously given. Increasing the intensity of choice, traders start to update faster their beliefs in order to select the most performing strategy. These types of asset pricing models are mathematically translated into nonlinear dynamical systems and, when the intensity of choice is high enough, are able to generate an entire *zoo* of complex behaviors, from local stability to high-order cycles and chaos.

An interesting question regards the profitability of having a fast switching towards the best relative performance. The intuition would suggest that the higher the intensity of choice, the higher the profits for traders. To question this point, in this paper we propose a generalized version of the asset pricing model in [6] where traders can have different beliefs. They can believe in the fundamental price (*fundamentalists*), or can extrapolate the price trend to forecast next-period price of risky assets (*trend followers*). Moreover, they are grouped into two clusters. One cluster includes stubborn traders, who will update their beliefs according to a small intensity of choice, while the other one is composed of fickle traders, who update beliefs according to a higher intensity of choice. A numerical test shows that stubborn players obtain higher average performance over a one-year trading period. This is an interesting result, which indicates that in a world of boundedly rational agents trying to behave more rationally is not always beneficial.

The road-map of the paper is the following one. Section 2 introduces the setup of the model and Section 3 contains analytical results about the dynamic of the proposed asset pricing model. All numerical results are contained in Section 4. Section 5 concludes. The proofs are collected in the Appendix.

2. Setup of the Model

Let us consider a Brock-Hommes market of one risky asset and one risk-free asset, see, e.g., [3], where:

1. p_t: is the price (ex dividend) at time t of the risky asset;

2. $\widetilde{y}_t = \overline{y} + \varepsilon_t$: is the stochastic dividend process of the risky asset and ε_t is a sequence of independent, identically distributed random variables with zero mean;

3. r : is the (constant) one-period return of the risk-free asset;

4. $W_{h,t}$: is the wealth at time t of an agent of type h;

5. z_{ht} : is the demand of shares at time t of an agent of type h;

6. I : is the set that includes all types of traders;

7. $\mathcal{F}_t = \{p_{t-1}, p_{t-2}, ..., y_{t-1}, y_{t-2}, ...\}$ defines the publicly available information set consisting of past prices and dividends.

Let E_{ht} and V_{ht} denote the expectations (or beliefs) of an investor of type h about expectation $E(\cdot|\,\mathcal{F}_t)$ and variance $V(\cdot|\,\mathcal{F}_t)$ conditional on this information set. Each investor is a myopic mean-variance maximizer, so that the demand of the risky asset at time t by a trader of type h is:

$$z_{ht} = \arg\max_z \left\{ E_{ht}\left[W_{t+1}(z)\right] - \frac{a}{2} V_{ht}\left[W_{t+1}(z)\right] \right\} = \frac{E_{ht}\left[p_{t+1} + y_{t+1} - (1+r)\,p_t\right]}{aV_{ht}\left[p_{t+1} + y_{t+1} - (1+r)\,p_t\right]}$$

where $a > 0$ characterizes traders' risk aversion and $\widetilde{R}_{t+1} = \widetilde{p}_{t+1} + \widetilde{y}_{t+1} - (1+r)\,p_t$ is the excess return per share. Let us normalize to one the total number of traders and let us assume a zero supply of outside shares in each period. Moreover, let us denote by η_{ht} the fraction of traders of type h at time t. Assuming that at each trading section (250 trading sections in one year) the closing price clears the market, we have that the market equilibrium equation writes as follows

$$\sum_{h \in I} \eta_{ht} z_{ht} = 0$$

In the benchmark case, in which all traders are perfectly rational and their expectations on future prices and dividends are given by $E(\cdot|\,\mathcal{F}_t)$ and $V(\cdot|\,\mathcal{F}_t)$, the price equation has a unique solution called fundamental price and is given by

$$p_f = \frac{\overline{y}}{r} = \sum_{k=1}^{+\infty} \frac{E\left(\widetilde{y}_{t+k}|\,\mathcal{F}_t\right)}{(1+r)^k}$$

according to which, today's risky asset price must equal the sum of "discounted" expected tomorrow's prices and dividends.

In the market in which traders have heterogeneous beliefs and update their beliefs according to an adaptive mechanism, the market price of the risk asset can be different from the fundamental price. Specifically, in the following we consider the following types of traders.

Assumption: Traders differ relatively to their beliefs and their propensity to update their beliefs; in particular, traders are categorized into four classes, represented by vectors $h = (i, j) \in I$, where i indicates the trader's belief, either fundamentalist or trend followers, and j indicates the group (propensity to change belief) a trader belongs to, either of *stubborn* or *fickle* traders. Specifically, we assume:

- Investors have homogeneous beliefs about one step ahead conditional means of earned dividends: $E_{ht}\left[\widetilde{y}_t\right] = E\left(\widetilde{y}_{t+k}\middle|\,\mathcal{F}_t\right)$, $\forall h \in I$ and $\forall t$.

- Investors have homogeneous and constant beliefs about conditional variances: $V_{ht}\left[\widetilde{W}_{t+1}\right] = \sigma^2$, $\forall h \in I$ and $\forall t$.

- Beliefs about conditional mean of next period prices are given by $E_{ht}\left[\widetilde{p}_{t+1}\right] = p_f$, if trader h is a fundamentalist, and $E_{ht}\left[\widetilde{p}_{t+1}\right] = p_{t-1} + g\left(p_{t-1} - p_{t-2}\right)$ with $g > 0$, if trader h is a trend follower.

- A fraction $\alpha \in [0, 1]$ of traders are stubborn, while the complementary fraction are fickle traders. Let us index by $I_a \subseteq I$ the subset of stubborns. Traders update their beliefs according to discrete choice Gibbs probabilities: $\eta_{ht} = \frac{e^{\beta_1 U_{h,t-1}}}{\sum_{h\in I_\alpha} e^{\beta_1 U_{h,t-1}}}$ if stubborns and $\eta_{ht} = \frac{e^{\beta_2 U_{h,t-1}}}{\sum_{h\in I|I_\alpha} e^{\beta_2 U_{h,t-1}}}$ if fickle traders, with $\beta_1 < \beta_2$.

In summary, operating the change of variable $x_t = p_t - p_f$, risky asset demands are given by

$$z_{ht} = \begin{cases} \frac{-(1+r)x_t}{a\sigma^2} & \text{if} \quad h \in (1,1) \cup (1,2) \\ \frac{x_{t-1}+g(x_{t-1}-x_{t-2})-(1+r)x_t}{a\sigma^2} & \text{if} \quad h \in (2,1) \cup (2,2) \end{cases}$$

while beliefs update according to

$$\eta_{ht} = \begin{cases} \frac{e^{\beta_1 U_{h,t-1}}}{\sum_{k\in I_\alpha} e^{\beta_1 U_{k,t-1}}} & \text{if} \quad h \in (1,1) \cup (2,1) \\ \frac{e^{\beta_2 U_{h,t-1}}}{\sum_{k\in I_\alpha \setminus I} e^{\beta_2 U_{k,t-1}}} & \text{if} \quad h \in (1,2) \cup (2,2) \end{cases}$$

Plugging the updating belief mechanism and the demand function in the market equilibrium equation, we obtain the following asset-pricing model:

$$\begin{aligned} x_t &= Q\left(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}; \varepsilon_{t-1}\right) \\ \varepsilon_{t-1} &\sim \delta U\left(-1, 1\right) \end{aligned}$$

where

$$Q\left(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}; \varepsilon_{t-1}\right) = \frac{x_{t-1} + g\left(x_{t-1} - x_{t-2}\right)}{(1+r)} q\left(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}; \varepsilon_{t-1}\right) \tag{1}$$

with

$$q\left(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}; \varepsilon_{t-1}\right) = \left(\frac{\alpha}{1 + e^{\beta_1\left(U_{(2,1),t-2} - U_{(1,1),t-2}\right)}} + \frac{1-\alpha}{1 + e^{\beta_2\left(U_{(2,2),t-2} - U_{(1,2),t-2}\right)}}\right)$$

and

$$U_{(2,1),t-2} - U_{(1,1),t-2} = U_{(2,2),t-2} - U_{(1,2),t-2} = \left(x_{t-1} + \varepsilon_{t-1} - (1+r)\, x_{t-2}\right) \frac{x_{t-3} + g\left(x_{t-3} - x_{t-4}\right)}{(1+r)}$$

3. The asset price dynamics

The local properties of the asset price dynamics are synthesized in the following Theorem (see proof in Appendix):

Theorem 1 *Let $\delta = 0$. The fundamental price is the unique equilibrium of the asset pricing map Q. It is globally stable for $g < (1+r)$; it is locally asymptotically stable for $g < 2(1+r)$; it undergoes a bifurcation of complex conjugate eigenvalues of modulus 1 at $g = 2(1+r)$, and it is unstable for $g > 2(1+r)$. At $g = 2(1+r)$, the fundamental equilibrium may undergo a Neimark-Sacker bifurcation.*

4. Stubborn traders can be the best survivals

In this section, we answer our research questions performing numerical analysis. In particular, we choose a parameter constellation for which the fundamental equilibrium is unstable. We simulate the model 25000 times and we plot the last 500 iterations that correspond to 2 years as there are around 250 trading days in a year. Then, we compute the average profit obtained by a stubborn in a year minus the average profit obtained by a fickle trader in the same period. Specifically, we compute

$$\begin{aligned}\overline{U}_{S-F,t} &= \sum_{k=t-250}^{t} \frac{\eta_{1k}U_{(1,1),k} + (1-\eta_{1k})\,U_{(2,1),k} - \eta_{2k}U_{(1,2),k} - (1-\eta_{2k})\,U_{(2,2),k}}{250} \\ &= \sum_{k=t-250}^{t} \frac{(\eta_{1k}-\eta_{2k})\,U_{1k} + (\eta_{2k}-\eta_{1k})\,U_{2k}}{250}\end{aligned}$$

The dynamics of the model is reported in Figure 1. The left-bottom panel shows that the stubborns are the traders that obtain always the higher average performance over the last year, i.e. $\overline{U}_{S-F,t} > 0\ \forall t$. This indicates that, in an asset market model with heterogeneous beliefs and limited forecasting ability, a trader that is anchored to his/her belief can obtain a better economic performance than the one obtained by traders who switch quickly towards the trading strategy that offered the best relative performance in the past. This result opens a question about how traders adjust the intensity of choice over time to obtain the higher performance possible.

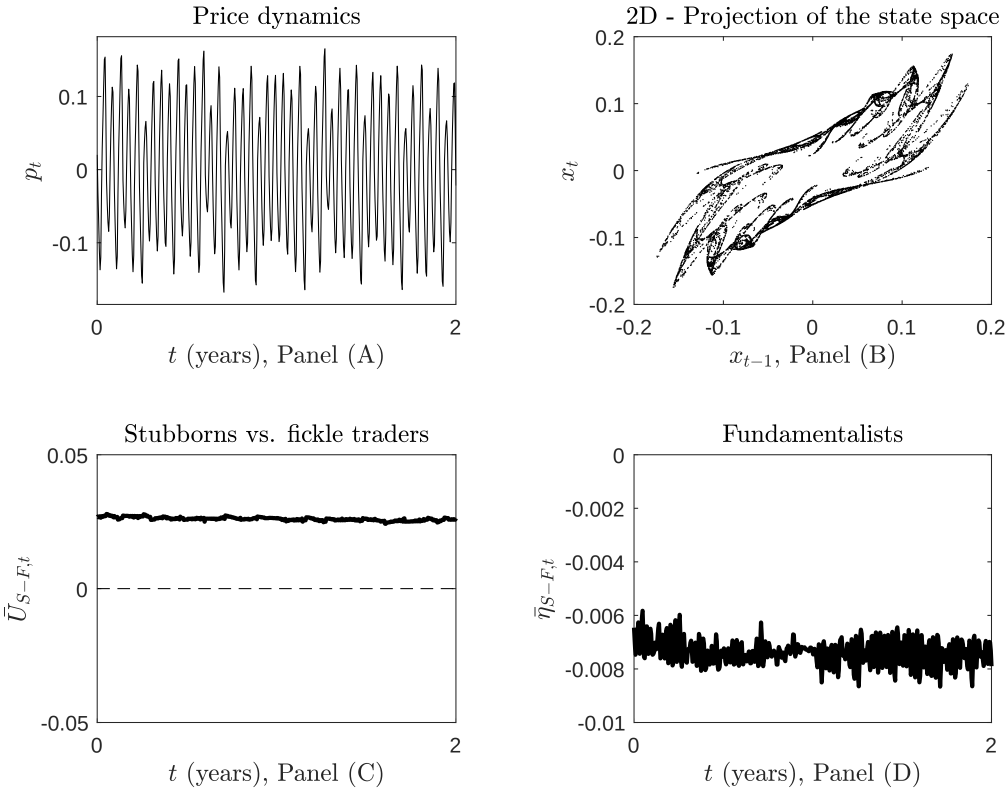

Figure 1: Dynamics of the asset pricing model over 500 trading days. Panel (A), time series of the risky asset price p_t; Panel (B), projection in the $x_{t-1} - x_t$ plane of the complex attractor generated by the asset pricing model; Panel (C), dynamics of the one-year average performance difference between stubborns and fickle traders $\overline{U}_{S-F,t}$, Panel (D), one-year average number of fundamental stubborns minus fundamental-fickle traders. Values of the parameters: $\sigma^2 = 0.01$, $a = 1$, $\beta_1 = 0.5$, $\beta_2 = 1$, $g = 2.5$, $\delta = 0$, $r = 0.01$ and $\alpha = 0.1$.

5. Conclusions

In the last two decades, asset market models with heterogeneous beliefs have offered a theoretical justification to stylized facts that characterize the time series of risky asset returns, such as volatility clustering, fat-tails and autocorrelations of the residuals, and for which the economic models based on the Efficient Market Hypothesis cannot provide a plausible explanation. In the majority of the asset pricing models with heterogeneous beliefs, traders are allowed to update their beliefs over time. The intensity to which traders switch to the best performing trading strategy impacts on the price dynamics. This *intensity of choice* is always assumed exogenous. In this contribution, we assume that two groups of traders exist and update their beliefs at a different pace: stubborn traders update their beliefs slowly while fickle traders are faster to switch to the best relative-performing strategy. A simple numerical investigation underlines that stubborn traders can obtain a one-year average profit that is higher than the one obtained by fickle traders. This observation raises an interesting aspect and underlines that in future research on heterogeneous asset pricing models it may be worth endogenizing the intensity to which traders switch to the most performing strategy.

Appendix

Proof. Since $Q(0,0,0,0)=0$, the fundamental price is an equilibrium of the model. For $\overline{x}\neq 0$, the equilibrium condition $Q(\overline{x},\overline{x},\overline{x},\overline{x})=\overline{x}$ reduces to

$$-\alpha e^{-\beta_2 \frac{r(\overline{x})^2}{a\sigma^2}} - (1-\alpha)\, e^{-\beta_1 \frac{r(\overline{x})^2}{a\sigma^2}} = r\left(1+e^{-\beta_1 \frac{r(\overline{x})^2}{a\sigma^2}}\right)\left(1+e^{-\beta_2 \frac{r(\overline{x})^2}{a\sigma^2}}\right)+1$$

which does not have any solution. Hence, the fundamental price is the unique equilibrium of the model. The local asymptotic stability of the equilibrium can be proved by standard eigenvalues analysis. The Jacobian matrix associated to map Q is given by

$$J(x_{t-1},x_{t-2},x_{t-3},x_{t-4}) = \begin{bmatrix} \frac{\partial Q}{\partial x_{t-1}} & \frac{\partial Q}{\partial x_{t-2}} & \frac{\partial Q}{\partial x_{t-3}} & \frac{\partial Q}{\partial x_{t-4}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where

$$\left.\frac{\partial Q}{\partial x_{t-1}}\right|_{(0,0,0,0)} = \frac{1+g}{2(1+r)};\ \left.\frac{\partial Q}{\partial x_{t-2}}\right|_{(0,0,0,0)} = \frac{-g}{2(1+r)};\ \left.\frac{\partial Q}{\partial x_{t-3}}\right|_{(0,0,0,0)} = 0;\ \left.\frac{\partial Q}{\partial x_{t-4}}\right|_{(0,0,0,0)} = 0$$

Therefore, the characteristic equation at the fundamental equilibrium is

$$\rho(\lambda) = \lambda^2\left(\lambda^2 - \frac{1+g}{2(1+r)}\lambda + \frac{g}{2(1+r)}\right)$$

Solving equation $\rho(\lambda)=0$, the eigenvalues associated to the fundamental price are

$$\lambda_{1,2}=0;\ \lambda_{1,2}=\frac{1}{4(1+r)}\left(1+g\pm\sqrt{1+g^2-(6+8r)\,g}\right)$$

Since, $\rho(1)=\frac{1+2r}{2+2r}>0$ and $\rho(-1)=\frac{2(1+r+g)+1}{2(1+r)}>0$, the fundamental equilibrium can only undergo a bifurcation of complex and conjugate eigenvalues of modulus equal to 1. The two complex conjugate eigenvalues lay in the unit circle if $\lambda_1\lambda_2=1$ and $|\lambda_1+\lambda_2| = 2Re(\lambda_1) = 2Re(\lambda_2) < 2$. Then

$$g=2(1+r)\ \text{ and }\ \frac{(1+g)}{2(1+r)}<2$$

Substituting, the first into the second we have

$$1<2(1+r)$$

which is always satisfied, therefore, at $g=2(1+r)$ a bifurcation of complex and conjugate eigenvalues with modulus equal to 1 occurs. This is the only way in which an eigenvalue associated to the fundamental equilibrium can cross the unit circle in the complex plane and $\lim_{g\to+\infty} Re(\lambda) = +\infty$, which implies that increasing g the fundamental equilibrium becomes unstable. Hence, for $g < 2(1+r)$ the fundamental equilibrium is stable while for $g > 2(1+r)$ it is unstable. To prove that the fundamental equilibrium is globally stable for $g < (1+r)$, let us note that

$$|Q(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4})| \le |H(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4})| \quad \forall (x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}) \in \mathbb{R}^4$$

where

$$H(x_{t-1}, x_{t-2}, x_{t-3}, x_{t-4}) = \frac{(x_{t-1} + g(x_{t-1} - x_{t-2}))}{1+r}$$

Since the map H is a linear one with a unique fixed point in the origin of $\mathbb{R}^4$ which is a locally asymptotically stable (therefore globally stable as the map is linear) if $g < (1+r)$, the global stability of the fundamental price for the map Q as long as $g < (1+r)$ follows. ∎

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Valuation of settlement share in the founding documents of a corporation

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Abstract

The participation of a shareholder in a corporation, including the rights and obligations arising from such participation, represents a share. The amount of the shareholder's share is determined by the ratio of their participation in the company's share capital. The size of this share determines the position of the particular shareholder towards other shareholders. In general, there are three basic types of a share in the company, namely the profit sharing, sharing of liquidation balance, and the settlement share. The subject of this paper was to assess whether the founding documents of the corporations contain instructions on how to solve the settlement share of the leaving shareholder. Memorandums as well as other founding documents of selected companies from the TOP 100 in the Czech Republic for 2018 were analyzed. The research has shown a fairly negative results. From the point of view of a shareholder who voluntarily or involuntarily terminates his/her participation in the company, the lack of description on how the calculate the settlement share in the founding documents is a considerable risk.

Key words

Participation in the company, Ownership interest, Settlement share, Founding documents, Partnership agreement, Limited liability Company, Valuation of settlement share

JEL Classification: K20, G30

1. Introduction

The participation of a shareholder in a corporation, including the rights and obligations arising from such participation, represents a share. The amount of the shareholder's share is determined by the ratio of their participation in the company's share capital. The size of this share determines the position of the particular shareholder towards other shareholders. In general, there are three basic types of a share in the company, namely the profit sharing, sharing of liquidation balance, and the settlement share. The profit sharing takes place when the company successfully operates. On the contrary, the share in the liquidation balance and the settlement share is related to the termination of the shareholder's participation in the company. In case of pay out of the share of the liquidation balance to the shareholders, not only the termination of the shareholder's participation in the company occurs, but also the

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disappearance of the economic and legal entity takes place. On the other hand, the payment of the settlement share to the shareholder is not related to the cancellation and termination of the company. The shareholder is entitled to a share in case their ownership interest ceases to exist (unless it is transferred to a different entity). Rules for calculating the share should be included in the founding documents of the company. The absence of more detailed rules in the case of a pay out of the settlement share to the shareholder leads, in many cases, to litigations in the area of setting the objective (real) value of the settlement share.

2. Ways of terminating ownership interest of a shareholder

There is a number of ways, how can a participation of a shareholder in an entity cease to exist:

1. Transfer of the ownership interest to another party
2. Exit of the shareholder
3. Agreement to terminate the participation of a shareholder
4. Proscription of a shareholder
5. Court ruling to cease shareholder's participation in the company
6. Death of the shareholder

Transferring the ownership interest onto another entity does not mean that a settlement share has to be calculated as the interest is sold (usually for money). This transfer may have to be upon agreement of one the bodies of the entity, as per the partnership agreement (see section 207 to 210 of the Corporation Law

When a shareholder is exiting the company, terminating their ownership interest through an agreement with the other shareholders, is proscribed to continue being a shareholder or in case of a court ruling to cease the shareholder's participation in the company or a death of the shareholder, the settlement share needs to be calculated. This settlement share belongs to the leaving shareholder or their heirs, in case of the death of the shareholder. It is clear from the list above that these situations happen in everyday life on regular basis and therefore it is very important to set the rules on how calculate the settlement share. It can also prevent a litigation.

3. Valuation of settlement share

When the ownership interest in a Limited liability company (LLC) ceases to exist from the above listed reasons (apart from transfer of the interest to a different entity), then we speak about "loosened ownership interest". If the Loosened ownership interest is free to be transferred onto another party, it can be sold for a fair price in the next three months (other shareholder have option to buy the ownership interest first) (Kadlec, 2015). The profit from the sale then becomes the object of the settlement share (Šabránek 2014). All costs related to the sale of the ownership interest as well as any receivables towards the shareholder need to be deducted from the profit before it is paid out to the leaving shareholder. The profit is also decreased by the withholding tax which is calculated as a difference between the purchase price and selling price of the ownership interest.

In case the Loosened ownership interest is not free to be transferred or is not transferred within the next three months (see paragraph above), the settlement share is valuated through the fair value of the assets less liabilities of the company (i.e. net assets or equity) from the last ordinary or extraordinary financial statements. If the value of the settlement share is determined this way, the amount needs to be paid out within one month after the period for sale (3 months, see above) has passed. (Děrgel, 2015)

Determination of the fair value of the settlement share is a time-consuming process as the individual items of the financial statements (i.e. based very often on historical costs) can be very far from the fair value (section 36 of Corporation Law). As an example we can a property purchased in the city center for CZK 1.2m, the fair value of which can nowadays be e.g. CZK 8.6m. Because of this is the fair value of the settlement share determined by a expert opinion. The value of the settlement share is then decreased by costs associated with procuring the expert opinion, withholding tax and any receivables towards the shareholder (Děrgel, 2016)

The value of the settlement share from equity is on the other hand determined as a % of the ownership interest of the particular shareholder on the registered capital of the company. This value is then decreased by the withholding tax and any receivables towards the shareholder before being paid out to the shareholder.

Is it clear from the above mentioned that from the point of view of a shareholder who is voluntarily or involuntarily terminating their participation on the company, and from the point of view of the company itself, insufficient guidelines on how to calculate the settlement share can be very risky. The risk lies above all in the fact that if after the notice of termination, the company decided to apply the equity based valuation, however the shareholder prefers the fair value valuation. If the guidelines on how to calculate the value are not described in the founding documents, the dispute can end up in court and last for many years.

4. Analysis of founding documents of LLCs

4.1 Sample selection and methodology of the research

As basis for this research we have selected the CZECH TOP 100 2018 (list of the largest companies based on their revenues achieved in 2017. The list was divided according to the legal form of the companies into joint-stock companies, LLCs, limited partnership companies, state-owned enterprises and others. The following research is focused solely on LLC entities.

The selected companies were consequently split into companies with a sole shareholder and companies with more than one shareholders. The following research is focused only on companies with more than one shareholders.

Companies that were selected through the above described process were added to the sample list. Their founding documents were retrieved from www.justice.cz and analyzed for the guidelines on how to determine the settlement share in case that one of the shareholders leaves the company.

4.2 Results of the analysis

Companies from the list CZECH TOP 100 were split according to their legal form (see Table 1) and the consequent research has focused only on limited liability companies (in total of 43 companies). These companies were then divided into those with a sole owner and those with more than one owner, as it only makes sense to analyze the rules for calculation of settlement share for companies with more than one owner. Table 2 presents that out of the total number of 43 companies, eleven had more than one owner.

Table 4 shows the results of the analysis of the number of shareholders in the companies. We have tried to split the companies according to the major shareholder type (foreign/ domestic), further according to (i) only Czech shareholders, (ii) only foreign shareholders and (iii) two and more Czech shareholders and one foreign shareholder. This split was important before the analysis of the founding documents. We have been analyzing whether there would be any difference between the quality of the founding documents in case of presence of a foreign shareholder.

Table 1: Overview of legal forms of the companies

	CZECH TOP 100	
Legal form	Joint-stock company	50
	Limited liability company	**43**
	Limited partnership	2
	State owned enterprises	1
	Regional branch of foreign entity	1
	Not available on Justice.cz	3
Total companies		100

Table 2: Split of companies per the number of shareholders

Limited liability company	CZECH TOP 100	
	1 shareholder	32
	2 and more shareholders	**11**
Total companies		43

Table 3: Calculation of the value of settlement share in the founding documents

Limited liability company with 2 and more shareholders from CZECH TOP 100 according to their share on the registered capital					
Company	Czech majority shareholders	Foreign majority shareholder	2 and more Czech shareholders	2 and more Czech shareholders and 1 foreign shareholder	2 and more foreign shareholders
MAKRO Cash & Carry ČR, s.r.o.		X			X
ARMEX GROUP, s.r.o.	X		X		
GoLeft spol. s r.o.[4]		X[5]			X
Kiwi.com s. r. o.	---	---		X	
BOHEMIA ENERGY ENTITY s. r. o.		X		1 Czech and 1 foreign shareholder	
Samsung Electronics Czech and Slovak, s. r. o.		X			X
Gastro City Group, s. r. o.	---	---	X		
Bidfood Czech Republic, s. r. o.		X		6 Czech and 2 foreign shareholders	
Tyco Electronics Czech, s. r. o.		X			X
SAINT-GOBAIN ADRORS CZ, s. r. o.		X			X
HRUŠKA, spol. s r. o.	---		X		

[4] This company is in the list CZECH TOP 100 2018 only by accident. During our analysis of the financial statements of the company GoLeft spol. s.r.o. we have discovered that the company is reporting in CZK and not thousand CZK. The publishers of the CZECH TOP 100 2018 by mistake considered the amounts to be 1000 times higher and attributed 24th position to the company. Same mistake also happened in case of a company named Milja s.r.o. which was listed on the 92nd position. This company has, however, only one shareholder and is therefore excluded from the analysed sample.

[5] Two foreigners, one of them permanently living in the Czech Republic.

Table 4: Information on the way of calculation the settlement share in the founding documents

Information on the transfer of ownership interest and settlement share in the founding documents			
Company	Is approval of the general meeting necessary to transfer the ownership interest?	Is information on settlement share calculation present in the founding documents	
MAKRO Cash & Carry ČR, s.r.o.	No	No	
ARMEX GROUP, s.r.o.	Yes	No	
GoLeft spol. s r.o.	Yes	No	
Kiwi.com s. r. o.	Yes	Yes	Founding documents mention the settlement share but not how it is to be calculated
BOHEMIA ENERGY ENTITY s. r. o.	Yes	Yes	Exit of shareholder is dealt with through a reference to a relevant legislation
Samsung Electronics Czech and Slovak, s. r. o.	No	No	
Gastro City Group, s. r. o.	**Yes**	**Yes**	**Settlement share is calculated based on the value of equity as at the reporting date**
Bidfood Czech Republic, s. r. o.	Yes	No	
Tyco Electronics Czech, s. r. o.	No	Yes	Exit of shareholder is dealt with through a reference to a relevant legislation
SAINT-GOBAIN ADRORS CZ, s. r. o.	No	No	
HRUŠKA, spol. s r. o.	**Yes**	**Yes**	**Founding documents state that the way of calculating the settlement share is either through the value of equity as at the reporting date or the fair value**

5. Conclusion

The result of the research is not a positive one. Out of the total number of analyzed companies (11), only 5 had at least some mention of a settlement share. Out of these, 3 only make a reference to a relevant legislation and only 2 actually describe how to calculate the value of the settlement share. Information on the way of calculation the settlement share can help to quickly resolve any potential disputes when one of the shareholders is leaving the company. It can also help avoiding long lasting court proceedings regarding the determination of the value of the settlement share, as is often the case.

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Comparison of accounting rules for companies in bankruptcy

Monika Randáková[1], Jan Hospodka [2]

Abstract

Bankruptcy is a common topic in corporate finance as well as accounting study. Most of the time, however, the focus is on prediction or aftermath of the bankruptcy and only rarely the actual process of bankruptcy is discussed and analyzed. The focus of this paper is to analyze accounting guidelines and rules for companies undergoing liquidation bankruptcy in the Czech Republic, the United States and Slovak Republic and compare them with one another. Considering a very limited space given to rules for companies in bankruptcy in the Czech law and accounting standards, the aim of the paper is also to recommend changes to Czech accounting rules in order to improve the clarity of financial statements of such companies.

Key words

Bankruptcy reporting, accounting, liquidation, insolvency, US GAAP

JEL Classification: M41, G33

1. Introduction

Bankruptcies are very common part of market economy and as such they should be researched and analyzed diligently. Nobel Prize winner J.E. Stiglitz mentions that most of the economists do not focus enough on insolvency even though it has massive economic impacts on the economy and modern capitalism would not develop without bankruptcy (Stiglitz 2004). When we think of research related to bankruptcy, in most cases it is devoted to analyzing how well we can predict the nearing financial difficulties (as of course, we all want be able to predict them and avoid them). Because of that, there is numerous amount of papers which focus on analyzing how well we can predict the bankruptcy. We can name the most recent ones such as Lopatta et all (2017), Kovacova & Kliestikova (2017) or Chiaramonte et all (2016). There is of course, many others. What we find rather disappointing, is that there is much less research conducted on firms that are already in the process of bankruptcy, especially with a focus on financial accounting and reporting of such companies.

In our paper, we would like to focus on companies that are no longer going concern. If a substantial doubt about entity's ability to continue as a going concern is present, it usually has a considerable impact on the basis of accounting and reporting requirements of the entity. Because it is very important to firmly establish whether the entity is going concern or is not a going concern, it requires an accurate assessment of the situation in which the company finds itself. We are, however, not going to analyze going concern rules and guidelines as per

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different reporting standards (namely Czech GAAP, US GAAP and Slovak GAAP) as it is not in the scope of this paper. Going concern was analyzed to a large extent by many authors before, such as Venutti (2004), Philipson et all (2016), May (1948) or Kausar & Lennix (2017). In this paper, we take as given that the entity is no longer a going concern and has correctly assessed and classified itself as such. We describe the implications of such situation on the reporting requirements in the Czech Republic, Slovakia and in the United States. We believe that it is very important for an entity to accurately present its financial situation to the readers of financial statements. Because of that, we want to analyze the reporting requirements for companies in bankruptcy liquidation in the United States (which are well known for their extensive set of reporting standards) and Slovakia (country with which the Czech Republic has shared much of its history and economical as well as legislative development) and compare it to the reporting requirements in the Czech Republic.

2. Methodology

We have conducted the research on the basis of the accounting & reporting legislation of the following countries:

- Czech Republic
- Slovakia
- United States

We have analyzed the following pieces of legislation which serve as the basis for reporting requirements in the respective countries:

Table 1: Overview of sources related to reporting requirements

Country	**Legislation/ Standards**
Czech Republic	Act no. 563/1991 Coll. on accounting (the Accounting act)
	Decree no. 500/2002 Coll. on Accounting (Accounting Decree)
	Czech accounting standards (CAS)
Slovakia	Act no. 431/2002 Coll. on Accounting (SK Accounting Act)
	Decree of the Ministry of Finance of Slovakia on accounting (SK Accounting Decree)
United States	US GAAP, ASC 205

Source: Authors' own analysis,

In all the above listed pieces of legislation and reporting standards, we have searched for terms related to bankruptcy liquidation (i.e. "konkurs" in the case of the Czech Republic and Slovakia). In case of the US GAAP, we have analyzed the topic 205 - Presentation of Financial Statements of the US GAAP, Subtopic 30 – Liquidation Basis of Accounting which is applicable when entity no longer meets the going concern assumption.

3. Reporting requirements – bankruptcy liquidation

3.1 Czech Republic

In the Czech Republic, the first mention of bankruptcy liquidation in the Accounting Act is in the section 17 which describes the moments at which the entity is obliged to close or open its books while being in the insolvency process. Section 20 further mentions minor exceptions regarding audit requirements for entities that are undergoing the process of bankruptcy liquidation. Clearly, the Accounting Act does not focus on the bankruptcy accounting very much. Furthermore, the Accounting Decree does not mention bankruptcy liquidation whatsoever.

The only place where there is more information for entities undergoing bankruptcy liquidation, is the Czech Accounting Standard 21. This standard should help entities meet the implicit requirements of the Accounting Act. The standard sets out the following (CAS 21, section 4):

- It provides further instructions on how to open & close books in accordance with section 17 of the Accounting Act
- All accruals, prepayments and deferred items, as well as provisions which will not be verifiably used during the bankruptcy proceedings, should be booked either as (i) receivables or payables or (ii) against income or loss.
- All assets and liabilities should be included in the books, even those that are not part of the bankruptcy proceedings
- New assets which are recognized during the proceedings should be booked against Extraordinary income (please mind that since 2016, Extraordinary income does no longer exist as per Czech GAAP, only the CAS 21 has not been updated)
- Assets which are part of the Bankruptcy estate but are a property of a third party, should be booked as assets and a liability towards these parties. The difference between the selling price and the liability towards the third part, if any, should be booked as Extraordinary income

There are no other guidelines which would give any more clear rules on how to perform accounting and reporting. The CAS 21 itself has not been updated for a long period of time and because of that, it does not match current insolvency legislation (it does not cover e.g. reorganization which has been introduced in 2008) nor the updated Accounting Act (e.g. CAS 21 still refers to Extraordinary income which no longer exists in Czech accounting rules).

It is also important to mention that once the entity enters the proceedings of the bankruptcy liquidation, Insolvency Administrator becomes responsible for the accounting and reporting of the entity instead of its management.

3.2 Slovakia

Slovakian Accounting Act origins in the same piece of legislation as the Czech one. Because of that, many of the section are fairly similar to each other. In section 16, SK Accounting Act also describes the moments at which the entity should open and close its books, some of those moments being related to liquidation bankruptcy. Apart from that, the SK Accounting Act mentions the liquidation bankruptcy in the two following cases:

- Section 2, article 10 describes that entity should not change its classification (micro, small, medium sized or large) after it entered the bankruptcy liquidation proceedings.
- Section 17, article 7 mentions that the entity should open its balance sheet as at the day when the court declares the bankruptcy liquidation

The SK Accounting Decree mentions bankruptcy liquidation in two instances; firstly, it states that an entity should assess its provisions for receivables in case its debtor is entering bankruptcy (i.e. from the creditor point of view). Secondly, it states that entities that are undergoing liquidation bankruptcy, cannot use Type B of accounting treatment of inventory. Apart from the above mentioned, there are no accounting rules for companies that have entered bankruptcy liquidation. To conclude, Slovakian reporting requirements do not tackle the bankruptcy liquidation to a considerable extent.

3.3 The United States

In the United States, reporting rules are described by the US GAAP. US GAAP has a separate section which describes and defines going concern to a considerable detail (ASC 205-40). This section also says that if an entity is not a going concern, it needs to follow standard 205-30 – Liquidation basis of accounting (ASC 205-30).

This topic provides a guidance on how and when an entity should prepare its financial statements using the liquidations basis of accounting and describes the related disclosures that should be made (ASC 205-30-05). It is further mentioned that an entity should follow this standard in case that liquidation is imminent, which means that

- A plan for liquidation has been approved by the person or persons with authority to make such a plan effective
- A plan for liquidation is imposed by other forces (e.g. involuntary bankruptcy)

According to the standard, when applying the liquidation basis of accounting, entity should:

- Recognize other items that were previously not recognized (e.g. trademarks) which are expected to be sold in the liquidation process or used to settle the liabilities
- Recognize new liabilities
- Accrue estimated costs related to the dispose of assets that are expected to be sold during the liquidation process. These costs should be presented separately from other assets or items
- Accrue costs and income that it expects to incur or earn through the end of its liquidation (such as payroll costs or income from preexisting orders that entity expects to fulfill) during the liquidation process).

As per ASC 205-30-30, all assets should be measured at the estimated amount of cash that the entity expects to collect in settling or disposing of these assets during the process of liquidation. Any subsequent change to the carrying value of the assets should be accrued.

The following documents need to be prepared by an entity that applies the liquidation basis of accounting (ASC-205-30-45):

- A Statement of net assets in liquidation
- A Statement of changes in net assets in liquidation

Furthermore, the entity should disclose the following information in its notes to financial statements (i.e. Statement of net assets in liquidation and Statement of changes in net assets in liquidation) (ASC-205-30-50):

- Information about the fact that the financial statements are prepared on the liquidation basis of accounting, including information on facts and circumstances of adoption of the liquidation basis as well as the entity's decision and assessment that liquidation is imminent.
- Description of the liquidation plan, including description of planned disposal of previously not recognized items, planned settlement of its liabilities and expected date of completion of the liquidation process.
- Methods and assumptions used to measure assets and liabilities
- Type and amount of costs and income accrued in the statements

The standard also includes illustrative examples and guidelines (something completely missing in the Czech accounting & reporting requirements).

As per Implementation guidelines of FASB, Statement of Net Assets in Liquidation can look as follows:

Table 1: Example of Statement of net assets in liquidation

Statement of net assets in liquidation (USD 000)	
Cash & Cash equivalents	2,750
Trade receivables	1,280
Property, plant and equipment	5,432
Items previously unrecognized	682
Total assets	**15,140**
Accounts payables	180
Taxes payable	55
Due to affiliates	290
Estimated costs to liquidate	27
Total liabilities	**1,782**
Net Assets	**13,358**

Source: FAF (2013)

The example above shows a statement in which it is easy to see how much should be left available for the shareholders (in this case, net assets are positive as the total carrying value of assets is higher than amount of liabilities, which is very often not the case).

4. Discussion and recommendations

The research has shown the US GAAP has by far the most advanced reporting rules regarding entities facing imminent liquidation, i.e. entities that entered bankruptcy liquidation. Because of that, it could serve as an inspiration to the Czech accounting rules. Marik (1998) states that entities on the verge of bankruptcy or in the bankruptcy process should measure their assets in liquidation values as it is likely that their assets will be sold or liquidated during the liquidation process.

We believe that the Czech Accounting Standard 21 should be updated to reflect the current accounting legislation (i.e. not refer to extraordinary income) and should also describe that any costs and income related to the liquidation process itself should be reported separately from the other items in the P&L, Balance sheet and Cash Flow statement. In order to do that, an adjustment to Accounting decree would also have to be made as this piece of legislation describes the nature of individual items on the balance sheet, P&L and Cash Flow. CAS 21 should also firmly state that entities in bankruptcy liquidation should use liquidation basis of accounting and measure their assets in accordance to this basis (as is the case for US GAAP).

Marsh (2010) concludes that it is necessary to be rich in order to be able to go bankrupt. We realize that it can be counterproductive to apply to many regulations and rules for entities that do not even have means to continue operating. Because of that, we do not think that it is necessary to introduce new type of financial statements (as is the case for US GAAP) or require special information in the disclosures to financial statements. However, it should be clear to the entities which basis of accounting they should use and it should be also clearly visible from the statements which items relate to the insolvency process and which do not.

5. Conclusion

In our paper we have analyzed approach to reporting & accounting requirements for companies that have entered bankruptcy liquidation (which can very well take many years to get completed). We argue that is very important for some readers of the financial statements (e.g. employees, creditors or state institutions) to understand the actual financial situation of the entity. We have compared to reporting requirement in the Czech Republic, Slovakia and the United States and concluded that the US GAAP is by far the most developed set of reporting standards out of these three cases when it comes to bankruptcy liquidation. We believe that US GAAP can server as an inspiration and that there are certain aspects that can be applied to Czech accounting rules to make financial statements more transparent to the reader (as we have indicated in the paper), while at the same time keep the additional costs for the entities as low as possible.

Acknowledgement

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Assessment of Factors Influencing Final Corporate Income Tax of Financial Institutions in the Czech Republic

Iveta Ratmanová, Karolina Lisztwanová[1]

Abstract

Corporate income tax as one kind of direct taxation does not create essential part of governments` tax revenues, in spite of this fact it is animportant part of corporates` financial decision because of influencing of final value of earnings. Moreover, as other taxes it can be used as an important instrument of affecting behaviour of corporates as taxpayers. The paper is concentrated on assessing the impacts of individual stated items determining final corporates tax liability. Main attention is devoted to finance and insurance sector. The pyramidal decomposition of topic indicator is used as methodology for determining main factors affecting the total tax. The analysis of variances is applied for finding out power of influence of determined items. Data provided by Finanční správa of the selected period between years 2005 and 2016 are used as source data.

Key words

Corporate income tax, total tax liability, tax savings, pyramidal decomposition, functional method

JEL Classification: H20, G30, K34, H25, G39

1. Introduction

Taxation is one method of transferring resources from the private to the public sector. Taxes bring money to the government. Except taxation, the government creates source of financing by charging for the goods and services which provides. Another possibility how to obtain money is to borrow it. Taxation has its limits as well, but it is the most important source of government`s revenues.

According to Musgrave, Musgrave (1994), the economic functions of government can be divided into three main categories. The first is to eliminate the inefficiencies of the market allocation of economic sources. The second is the redistribution of income and wealth that it is considered by society as equitable. The third is to influence cyclical fluctuation in the economy and to ensure a high level of employment and price stability.

The tax system should be efficient and should not distort economic decisions, its administrative and compliance costs should not be excessive moreover, it should support willingness to work, save and invest according to Široký (2008), Kubátová (2015). Last, but not least the tax system should be consider fair and should be consistent with macroeconomic policy. In spite of these facts, not every tax system meets all the criteria without problems.

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Regarding the identification of a tax subject there are two groups of taxes – direct and indirect taxes. Corporate income tax, as a part of direct taxes and as the one of the youngest type of taxes is able to create sources of government`s income and moreover it can influence desirable activities and behaviour of corporates by fiscal policy. Shortly said, taxation can supports investment activities, employment benefits, research and development expenditures, willingness to risk and to employ disabled person etc. The real impact of tax policy on corporate income tax is clear from final tax liability of individual corporates, but unfortunately, details are not publicly available. Nevertheless, consequences of corporate tax policy of the Czech Republic respecting certain period can be observed due to the information provided by Finanční správa. The assessment of real impacts of taxable costs, items reducing tax base, tax reliefs and tax rates of the all sectors regarding specific period has been already made according to Lisztwanová, Ratmanová (2017). Notwithstanding, changes of the final tax liability of finance and insurance sector has not yet been carried out and published. In response to this one, main attention and aim of this paper is to identify and to assess impact of tax-deductible costs, changes of tax rate, items reducing tax base and tax reliefs on the final tax liability of enterprises of finance and insurance sector. The pyramidal decomposition of final tax liability as a topic indicator and analysis of variances are used for assessing influence of selected indicators respecting the period between years 2005–2016.

2. Finance and Insurance Sector

The finance and insurance sector is qualified as CZ-NACE section K – Finance and Insurance respecting classification of economics activities. The section contains activities like providing of finance services including insurance and reinsurance activities of pension funds and activities to support financial services. This section also includes asset-holding activities such as holding company activities, funds and similar financial entities.

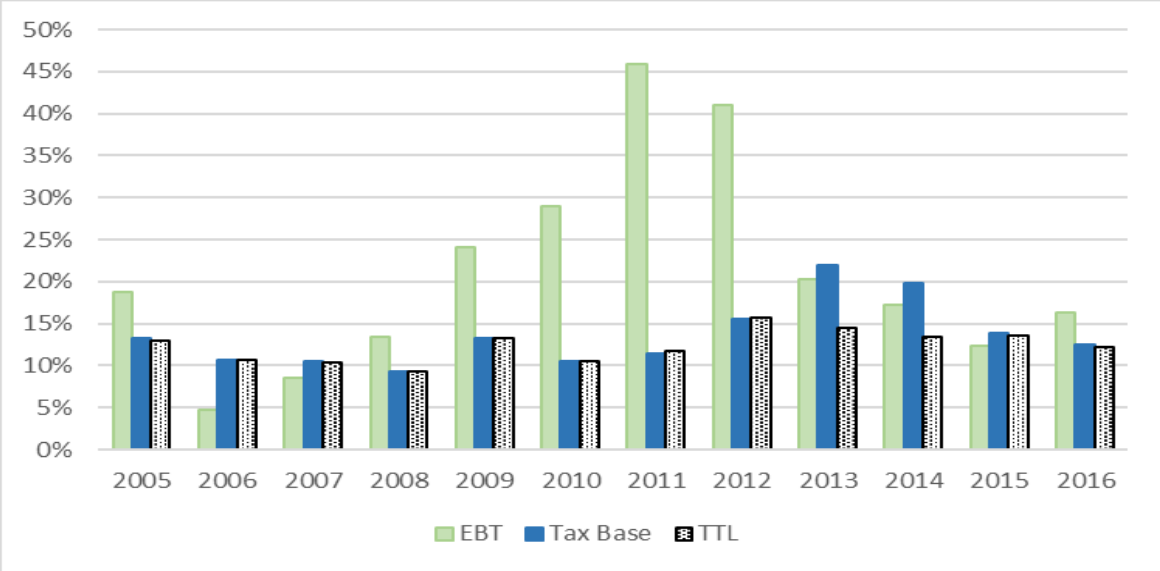
According to MPSV (2015) the section CZ-NACE 64 Financial Intermediation covers the reception and distribution of funds (except for insurance, pension funds and compulsory non-life insurance). The section CZ-NACE 65 Insurance, Reinsurance and Pension Funding, except statutory social security, includes annual rent insurance, insurance contracts, and insurance premiums for the construction of a portfolio of financial assets used to cover claims. This one includes the provision of direct insurance and insurance as well. The section CZ-NACE 66 Other Financial Activities includes the provision of services performed and related to financial intermediation, but does not concern financial intermediation directly.

In comparison with other branches of the Czech Republic referring MPSV (2015), the banking and insurance industries are relatively medium-sized sectors with relatively high performance. The sector`s production is commercial, but it is internationally closed industry whose entire production is home-grown.

Regarding figure 1 ratio between earnings before tax (EBT) of finance and insurance sector and EBT of the all sectors declares big increasing in the year 2011 with gradual decreasing in following years. When it comes to ratio of total tax liability (TTL) of selected sector and the all sectors, we can observe oscillation without extreme values.

The following figure 2 shows development of absolute values of EBT, the tax base and the total tax liability of finance and insurance sector moreover in lights of the numbers of the tax returns. According data of this figure it is clear increasing number of the tax subject of this sector which is reflected in level of 7 000 of the tax returns. Moreover, similar development it can be observed in the case of EBT. Generally can be claimed that the value of EBT increased. It is interesting in spite of the fact that number of the tax returns was growing while EBT suddenly dropped in the year 2015.

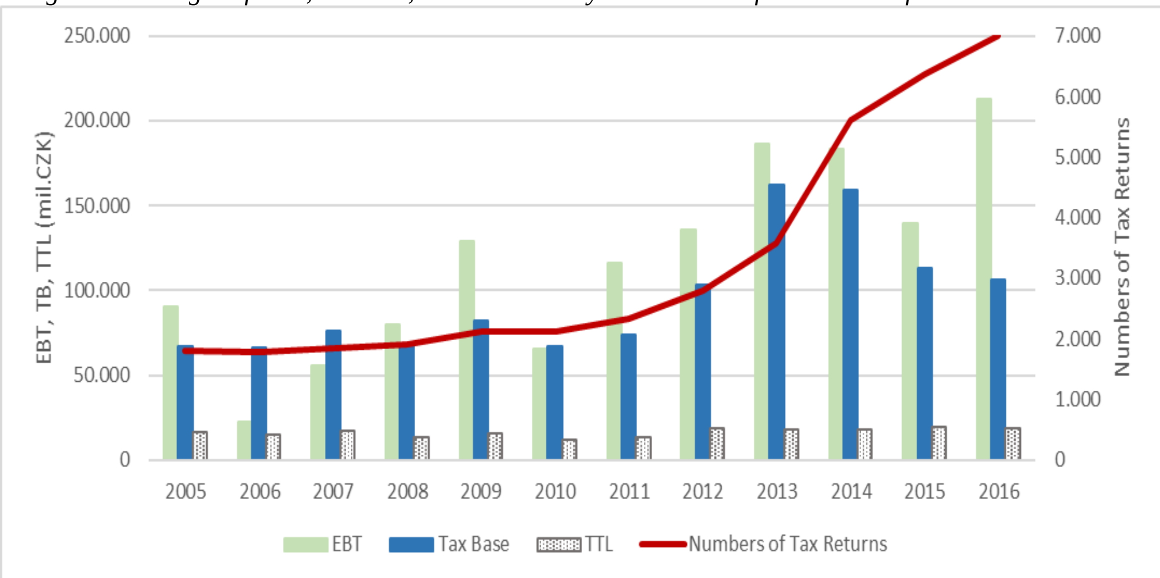
Figure 1: Ratio of EBT, tax base and total tax liability of the selected sector K to the all sectors



Source: Authors` processing according data of Finanční správa

The development of total values of the tax base of finance and insurance sector was similar to changes of EBT. With regard the total tax liability, it can be observed turbulence as well.

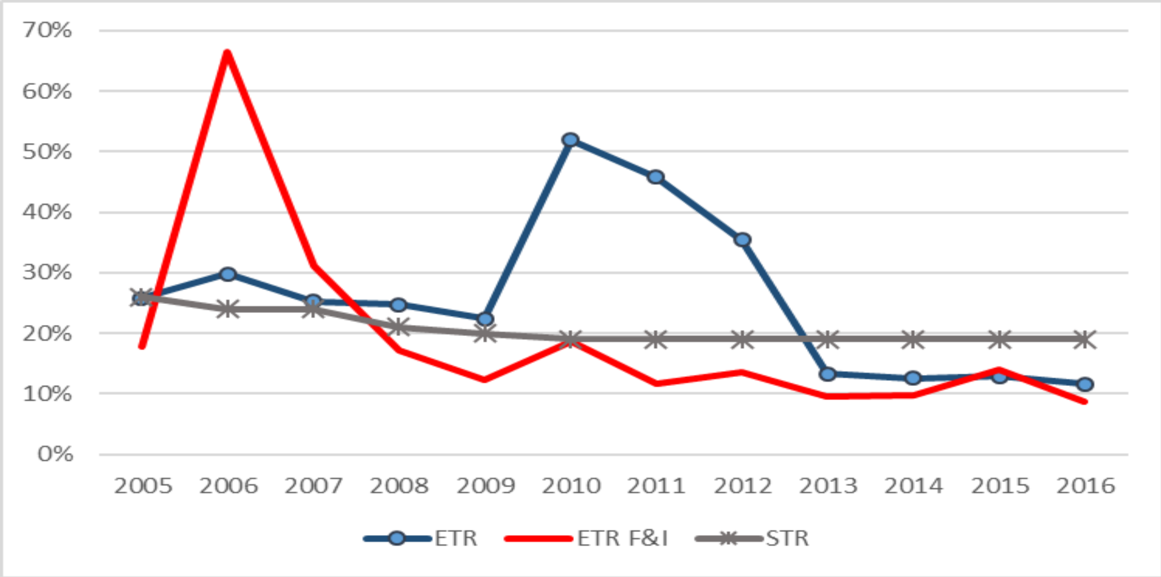
Figure 2: Changes of EBT, tax base, total tax liability and number of tax returns of the selected sector K



Source: Authors` processing according data of Finanční správa

The following figure 3 provides interesting comparison of development of a statutory tax rate (STR) and an effective tax rate (ETR) of finance and insurance sector and of the all sectors. The effective tax rate can be stated according to Lisztwanová, Ratmanová (2017). Its variation of selected sector differs to the variation of effective tax rate of the all sectors. Generally, the effective tax rates of both groups declined in observed period as a result not only eliminating value of the statutory tax rate. When it comes to the assessment of changes of these ones it is necessary to take into account that the results are influenced with summarize data of all finance and nonfinancial companies. Because of it, summarized data works not only with accounting profit as earnings, but moreover with accounting loss. This fact influenced our results.

Figure 3: Changes of STR, ETR of the all sectors and of ETR of the selected sector K



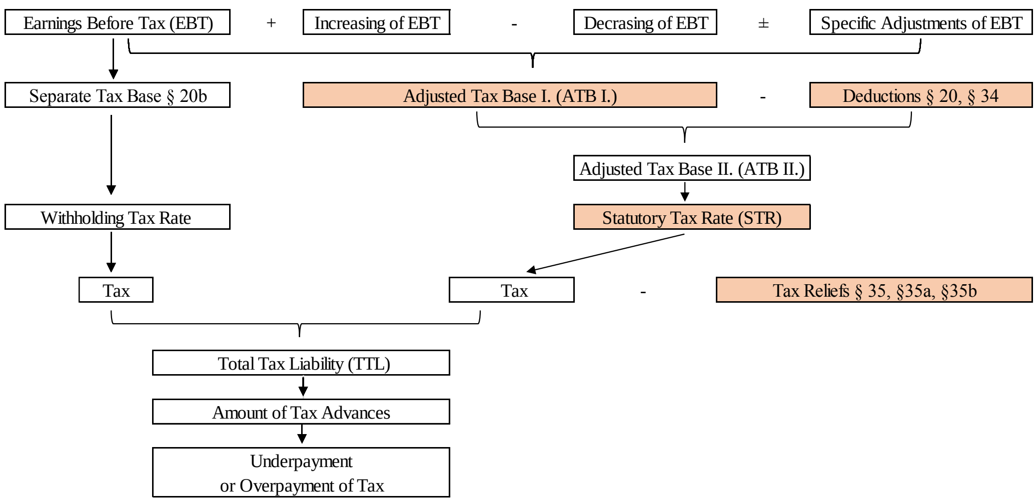
Source: Authors` processing according data of Finanční správa

To sum up the effective tax rate of finance and insurance sector was mostly lower then this one of the all sectors.

3. Methodological Basis of Tax Liability Assessment

Regarding Czech tax legislation assessment of corporates`, tax liability is not a simple process. Accounting profit or accounting loss are input data for determining of final tax. Main individual steps for assessing of this one can be seen in figure 4.

Figure 4: Structure of tax calculation and settlement of total tax liability

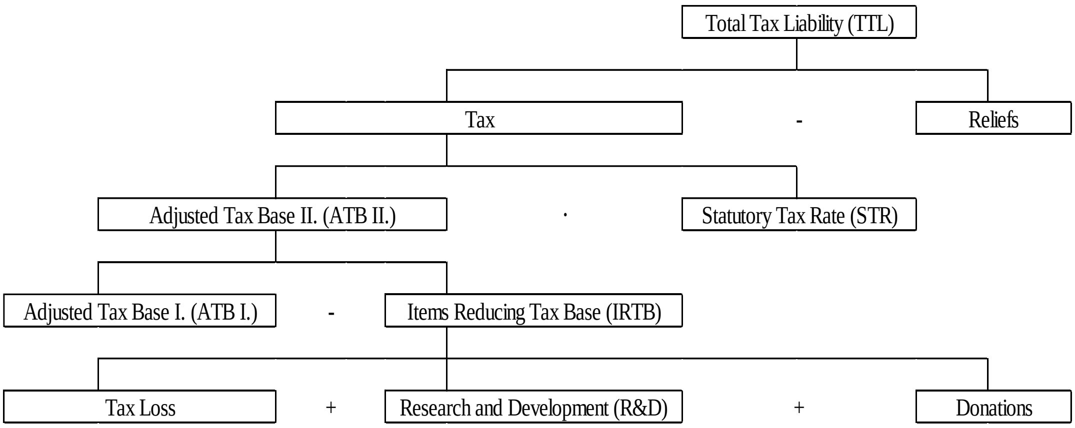


Source: Authors` processing according data of The Act. 586/1992 Coll., on Income Taxes

With respect Figure 4 earnings before tax must be readjusted so that we can identify tax base. A tax base is only affected with tax-deductible costs and taxable incomes. Exact determination of them is stated with The Act. 586/1992 Coll., on Income taxes. Moreover, this one provides information about tax deductions and tax reliefs. Tax loss, research and development expenditures and donations are the most important kind of tax deductions i.e. items reducing tax base. Investment incentives and employment of disables people belong to tax reliefs. Details of above-mentioned corrections are mentioned in Lisztwanová, Ratmanová (2017).

It has been already claimed it is possible to determine impact of individual adjustments of earnings before tax, respectively the power of influence of these ones on changes of the final tax liability. According to Dluhošová (2004), Zalai et al. (2007), Dluhošová et al. (2010), Zmeškal et al. (2013) changes of a top indicator, which can be splitted into its individual items, can be assessed by changes of individual indicators. If there is additional or multiplicative relationship among these indicators, it is not problem mathematically to express impact of the indicators on the topic indicator. In case of identified multiplicative relationship, there are different methods for calculating. We used a functional method. When it comes to the final tax liability, detail decomposition of this one is presented in figure 5.

Figure 5: Pyramidal decomposition of total tax liability



Source: Authors` processing according data of Finanční správa

Concerning Figure 5, it is clear that changes of the total tax liability are influenced by changes of tax loss, research and development expenditures, donations, tax reliefs, tax rates and adjusted tax base I. We are mathematically able to calculate the power of influence of above mentioned indicators on the total tax liability and to identify which of the selected indicators caused changes of the topic indicator the most and which the least.

According to Lisztwanová, Ratmanová (2017) and with regard data of the all sectors during 2005–2016 adjustment of tax base I. has been determined as indicator with the greatest impact on the total tax liability followed with tax loss, changes of statutory tax rates and tax reliefs. On the other hand, research and development expenditures and donations were indicators with the lowest impact on the changes of the final tax liability.

4. Assessment of Influence of Individual Indicators

As could be claimed in introduction our attention was concentrated on describing of impact of the individual items of the total tax liability decomposition concerning the finance and insurance sector. Following two tables provides information about results of our observing. First table, table 1 provides information about year-to-year changes of selected items respecting data of the all sectors. Second table, table 2 includes data of year-to-year changes regarding finance and insurance sector.

With reference to the tables 1 and 2, it is clear value of the total tax liability of the all sectors and finance and insurance sector changed year to year. Nevertheless, in the case of our selected sector we could observe earlier increasing of the total tax liability i.e. already in the year 2011. On the other hand, finance and insurance sector created bigger fall comparing it with the data of the all sectors.

Table 1: Year-to-year changes of selected items within period 2005–2016 of the all sectors in %

	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
TTL	100.0	114.1	116.3	88.5	81.5	97.6	99.0	101.3	104.5	109.1	108.5	105.0
STR	100.0	92.3	100.0	87.5	95.2	95.0	100.0	100.0	100.0	100.0	100.0	100.0
ATB I.	100.0	118.3	113.5	103.8	87.3	102.5	102.9	98.2	113.4	105.6	101.0	104.1
Tax Loss	100.0	81.5	84.9	155.9	98.1	92.6	127.7	67.6	124.9	79.9	101.9	97.0
R&D	100.0	131.7	121.6	92.3	106.3	135.8	143.0	106.6	118.4	97.6	112.1	93.0
Donations	100.0	116.2	100.0	96.3	98.7	103.2	107.5	99.4	140.8	99.4	100.9	105.0
Reliefs	100.0	118.6	117.6	63.6	96.3	126.9	117.2	131.5	247.5	104.2	39.7	101.6

Source: Authors` calculation according data of Finanční správa

When it comes to ATB I., it does not generally mean that its increasing always caused decreasing of the total tax liability. It is understandable, because changes of other items could significantly influence changes of the total tax liability. Comparing changes of the tax loss between the all sectors and our selected sector it is obvious that this one affected more finance and insurance enterprises. Including research and development expenditures, high progressivity of the all sectors is clear in recent four years. Among others, enterprises of finance and insurance sector are not as a big donator as other corporates. Apart from above mentioned, we found out interesting changes of reliefs (i.e. § 35 par. 6, 7 and 8) which importantly influenced the total tax liability of the financial and insurance sector of the year 2013.

Table 2: Year-to-year changes of selected items within period 2005–2016 in % of the selected sector K

	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
TTL	100,0	93,6	114,4	79,3	116,1	76,7	111,2	135,6	95,7	101,6	109,5	94,1
STR	100,0	92,3	100,0	87,5	95,2	95,0	100,0	100,0	100,0	100,0	100,0	100,0
ATB I.	100,0	75,7	110,0	129,8	115,1	62,6	158,9	98,1	158,9	90,5	69,5	95,7
Tax Loss	100,0	15,7	42,8	1.729,0	100,6	15,0	823,2	26,8	136,3	26,3	107,3	139,1
R&D	100,0	348,4	0,0	-	451,4	84,1	863,1	105,3	69,3	90,4	82,6	88,8
Donations	100,0	79,9	112,9	78,7	158,0	79,7	111,6	138,0	349,6	40,2	100,5	88,5
Reliefs	100,0	12,4	146,3	351,1	63,9	82,8	30,2	99,7	108.032,2	89,3	0,1	82,2

Source: Authors` calculation according data of Finanční správa

The following tables 3 and 4 provides our findings of pyramidal decomposition and analysis of variances of finance and insurance branch between years 2005–2016. Results mentioned in the table 3 show absolute value of changes of the total tax liability and absolute value of changes of the individual selected indicators.

Table 3: Power of influences of individual indicators of the selected sector K, absolute changes in mil. CZK

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15	15-16
Δ TTL	-1.043	2.175	-3.579	2.213	-3.723	1.366	4.848	-799	285	1.712	-1.162
Reliefs	96	-6	-50	25	8	26	0	-12.067	1.290	10.773	3
STR	-1.259	0	-2.071	-729	-723	0	0	0	0	0	0
ATB I.	-5.421	1.622	4.988	2.994	-8.098	7.787	-399	12.126	-3.119	-9.024	-894
Tax Loss	5.528	564	-6.454	-39	5.075	-6.335	5.275	-703	1.945	-51	-291
R&D	-2	3	-4	-14	3	-106	-6	39	8	14	7
Donations	15	-7,27	13	-25	13	-6	-21	-192	161	-1	13

Source: Authors` calculation according data of Finanční správa

Results mentioned in the table 4 show the same data but instead of absolute value of change of the total tax liability and absolute value of the change of the individual selected and evaluated indicators we determined percentage value of these changes.

Table 4: Power of influences of individual indicators of the selected sector K, percentage value

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15	15-16
Δ TTL	100	100	100	100	100	100	100	100	100	100	100
Reliefs	-9,24	-0,29	1,40	1,15	-0,21	1,89	0,00	1.511,24	452,69	629,43	-0,24
STR	120,75	0,00	57,86	-32,95	19,42	0,00	0,00	0,00	0,00	0,00	0,00
ATB I.	519,96	74,57	-139,34	135,30	217,53	569,90	-8,23	-1.518,57	-1.094,80	-527,25	76,90
Tax Loss	-530,25	25,92	180,31	-1,75	-136,33	-463,62	108,80	88,09	682,57	-2,95	25,04
R&D	0,20	0,13	0,12	-0,63	-0,07	-7,75	-0,13	-4,85	2,93	0,80	-0,63
Donations	-1,42	-0,33	-0,36	-1,12	-0,35	-0,43	-0,44	24,10	56,61	-0,03	-1,08

Source: Authors` calculation according data of Finanční správa

When it comes to the total tax liability, it is clear that in spite of the fact that total number of tax subjects grew, total tax liability did not. We can at least observe five times declining of this one. Regarding individual indicators, the unchanged tax rate since the year 2010 caused zero value of change of the total tax liability. This means that the constant tax rate did not affect the change of the tax liability. In the case of ATB I., it can be observed that this one influenced total tax liability, but it is not possible to make general simple conclusion that decreasing of ATB I. means increasing of the total tax. The only following fact is irrefutable ATB I. the most impacted the changes of the total tax liabilities as can be seen in last four observed periods. When it comes to the tax loss, generally main idea that increasing of impact of the tax loss means decreasing of the tax liabilities was not confirmed with our data. We can only observed increasing of influence of tax loss on the total tax liability with regard economic crisis and its decreasing impact in the recent evaluated period. Concerning the tax reliefs the changes of these ones reduced changes of the total tax liability obviously between years 2012 and 2013 and importantly decreased the changes of the total tax liability between years 2014 and 2015. Comparing the impact of research and development expenses and its individual development with data of the all sectors, in the case of finance and insurance sector we observed less often decreasing impact of this kind of expenditures on changes of the total tax liability. It can be understand because of lower research activities. Shortly said, finance and insurance sector did not invest significantly in this area.

5. Conclusion

The analysis of variances is able to assess the impact of the indivitual items on the changes of the selected topic indicator assuming that among items multiplicative or additive relationships exist.The following table 5 contains determined order of influence of individual indicators on the changes of the total tax liability. Regarding the finance and insurance sector it can be seen ATB I. was indicator with the most often highest impact on the total tax liability. The tax loss the most influenced topic indicator but its impact was lower comparing it with ATB I. As far as tax reliefs are concered it followed the tax loss. Changes of the statutory tax rate, respective stability of the tax rate was the indicator with the weak impact on the total tax liability. Nevertheless, donation the lowest caused changes of the total tax liability.

Table 5: Order of influences of individual indicators of the selected sector K

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15	15-16
Reliefs	4	4	4	4	5	4	5	2	3	1	5
STR	3	6	3	2	3	6	6	6	6	6	6
ATB I.	2	1	2	1	1	1	2	1	1	2	1
Tax Loss	1	2	1	3	2	2	1	3	2	3	2
R&D	6	5	6	6	6	3	4	5	5	4	4
Donation	5	3	5	5	4	5	3	4	4	5	3

Source: Authors` calculation according data of Finanční správa

Regarding our results, we identified main aspects determining changes of the total tax liability. According data of Finanční správa we identified main aspects concluding ATB I., statutory tax rate, tax loss, donations, research and development expenditures and tax reliefs. Respecting data of the finance and insurance sector with usage of pyramidal decomposition and analysis of variances, we assessed items that the most influenced the total tax liability and items whose impact were the weakness during period between years 2005–2016.

Acknowledgments

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Application of dynamic methods in performance evaluation of selected industries of the Czech Republic

Dagmar Richtarová [1]

Abstract

This paper is focused on the performance valuation of selected industry in the Czech Republic and dynamic approach is applied to quantify the influence of component ratios to economic value added. The aim of this paper is to analyze the financial performance of selected industries of the Czech Republic. The manufacturing and automotive industries were selected for performance evaluation. First, Economic Value Added and variance decomposition as dynamic methods is presented. In the application part, the analysis of the performance of the selected industry is reviewed over the period from 2007 to 2015. For analysis quarterly data are used. Next, factors affecting the Economic Value Added are analysed by using the variance decomposition approach. In the end, comments on the results of the influence quantification are provided.

Key words

Financial performance, economic value added, decomposition of variance

JEL Classification: C1, C5, C58, G3, G30

1. Introduction

Evaluation of financial performance belongs to key activities of each company. Financial performance of the industry or company is a random process, which can be decomposed into the particular indicators. It also can be evaluated according to accounting indicators, economic or market indicators and it is solved by many authors, see Copeland (2000), Maříková, Mařík (2005), Neumaierová (2005), Vernimmen (2005) or Brealey (2014).

Economic value added is one of the most used modern economic indicators which is used because of their ability to take into costs of capital, factor of time or factor of risk. The preference of this approach can be found in many publications, see Ehrbar (1998), Young (2001), Vernimmen (2005) or Ross (2013). Most of the authors solve problems of company performance valuation but only in few publications the valuation of performance of industry can be found, see Dluhošová (2004) or Dluhošová, Ptáčková, Richtarová (2018). In assessing financial performance is also necessary to determinate the factors and the influences that govern it. The influences can be determinated by using static or dynamic methods.

The aim of this paper is to analyze the financial performance of manufacturing and automotive industries according to economic value added and to compare value drivers of selected industries using of dynamic approach.

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2. Methodology

This part of the paper is divided in two subchapters. Firstly, economic value added as a measurement of financial performance of industries is characterized. Then pyramidal decomposition of economic value added is proposed and to quantify the influences according to decomposition of variance as one of the dynamic methods.

2.1 Economic value added

Economic value added is based on the concept of the economic profit. When the economic profit is positive, it means that company earns more than the weighted average costs of capital, which also means that some wealth for the shareholders is created.
There are many ways how economic value added can be expressed, see Ehrbar (1998), Young (2001), Dluhošová (2010) and Mařík (2005). It can be distinguished EVA – Entity, EVA – Equity or relative economic value added. In this paper financial performance of selected industries are analyzed according to relative economic value added. Relative economic value added is expressed as

$$\frac{EVA}{E} = \left(ROE - R_E \right), \tag{1}$$

where ROE is return on equity, E is equity and R_E are costs of equity.

The difference between ROE and Re is called spread. Spread is very important parameter influencing EVA. If this spread is positive, it means that industry or company earns more than the costs of equity are.

2.2 Dynamic approach – method of decomposition of variance

Dynamic analysis is based on a longer time series of the performance indicator. This analysis is based on decomposition of variance. The derivation of method of variance decomposition can be found in Dluhošová, Ptáčková, Zmeškal (2015) or Ptáčková (2015). Generally, the influence of variance of component indicators is calculated as

$$z_i = a_i^2 \cdot \mathrm{var}\left(F_i\right) + \sum_{j \neq i} a_i \cdot a_j \cdot \mathrm{cov}\left(F_i, F_j\right) \tag{2}$$

where $a_i = E\left[\frac{\partial f(\cdot)}{\partial F_i}\right]$, $var(F_i)$ is variance of partial factors and $cov(F_i, F_j)$ is covariance between factors F_i, F_j.

The ratio of the scatter of the indicator to the overall variance of the top indicator can be determined

$$s_i = \frac{z_i}{\sum_i z_i}. \tag{3}$$

3. Application part

For the analysis of financial performance manufacturing industry and automotive industry of the Czech Republic were chosen. Financial performance of selected industries is analyzed according to relative economic value added and this indicator is decomposed to component financial indicators using pyramidal decomposition method.

There are many ways, how relative economic value added can be decomposed. In this paper, there is proposed possible pyramidal decomposition of relative economic value added on the six ratios as

$$\frac{EVA}{E}=\left(\frac{EAT}{EBT}\cdot\frac{EBT}{EBIT}\cdot\frac{EBIT}{R}\cdot\frac{R}{A}\cdot\frac{A}{E}\right)-R_E, \tag{4}$$

where E is equity, EAT is earnings after taxes, $EBIT$ is earnings before interests and taxes, R are revenues, A are assets and R_E are costs of equity which are determined according to a build up model of Ministry of industry and trade of the Czech Republic.

In next level of pyramidal decomposition is ratio R/A distributed as

$$\frac{R}{A}=\frac{1}{\left(\frac{ITFA}{R}+\frac{LTFA}{R}+\frac{RS}{R}+\frac{STFA}{R}+\frac{MS}{R}+\frac{WIP}{R}+\frac{FG}{R}+\frac{M}{R}+\frac{AO}{R}\right)}, \tag{5}$$

where $ITFA$ are intangible and tangible fixed Assets, $LTFA$ are long–term financial assets, RS are reseivables, $STFA$ are short-term financial assets, MS is material, WIP is work in progress, FG are finished goods, M is merchandise and AO are others assets.

And ratio A/E distributed as

$$\frac{A}{E}=\left(\frac{RC}{E}+\frac{OL}{E}+\frac{OE}{E}+\frac{RSS}{E}+\frac{P}{E}+\frac{D}{E}\right), \tag{6}$$

where RC is registered capital, OL are other liabilities, OE is other equity, RSS are reserves, P are payables and D are debts.

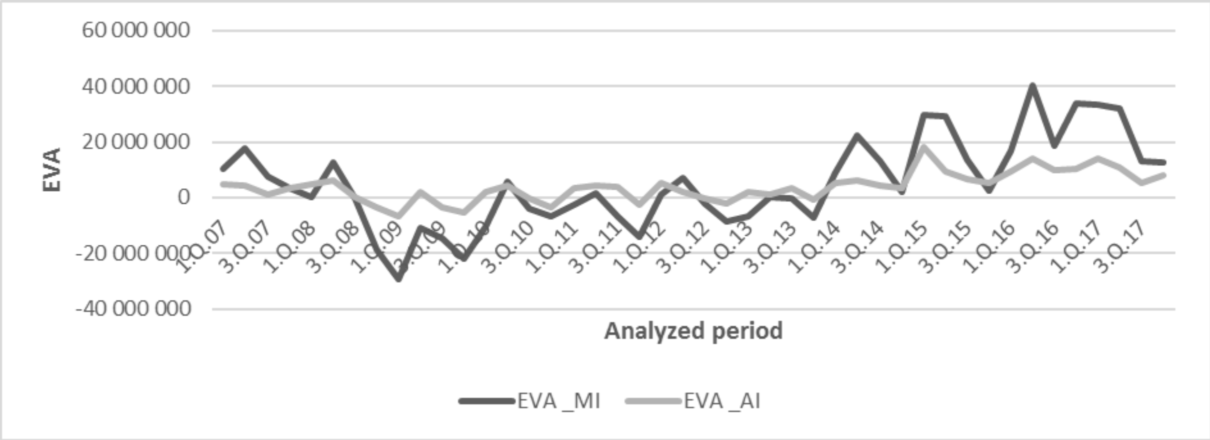
Main value drivers of financial performance of are found according method of decomposition of variance.

3.1 Input data

Financial performance of selected industries of the Czech Republic is analyzed according to economic value added in the period 2007 to 2017. Method of pyramidal decomposition is applied and component indicators are calculated on the base of the input data of the manufacturing industry (MI) and automotive industry (AI) in the Czech Republic. These input data were taken from the website of Ministry of industry and trade (MIT) of the Czech Republic. Quarterly data are used for the analysis.

One of the most important segments of the Czech economy is manufacturing industry. Manufacturing industry contains 24 segments. Automotive industry plays one of the main roles of the manufacturing industry in the Czech Republic. Czech automotive industry belongs to one of the most developed automotive industries in the European Union. According to CZ–NACE automotive industry includes: personal, light utility and freight vehicles, buses and trolleybuses, snowmobiles, golf carts, amphibious vehicles, fire trucks, trailers and semi-trailers, and the manufacture of their parts. Figure 1 shows the evolution of EVA of manufacturing and automotive industries in analyzed period.

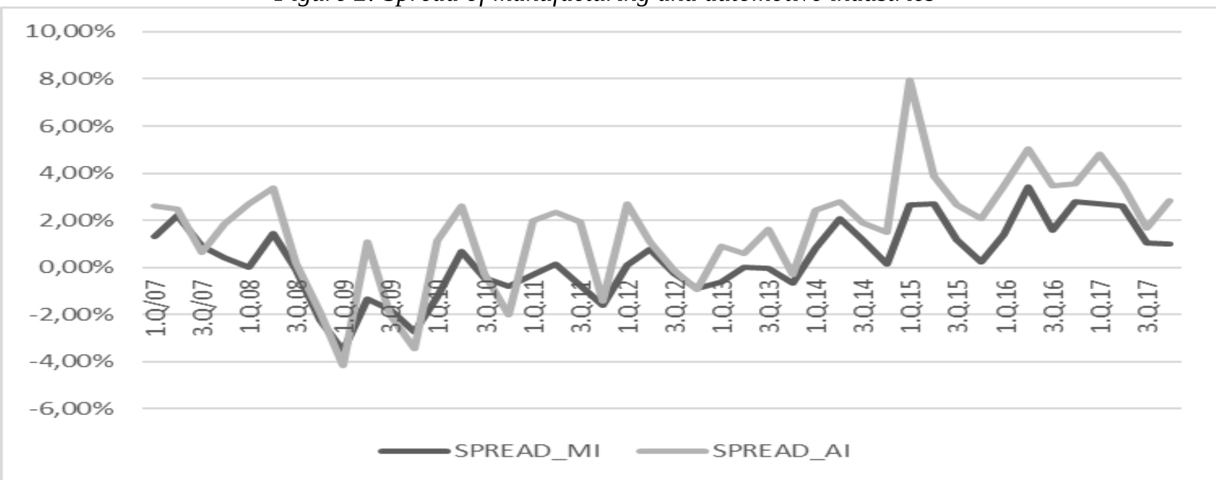
Figure 1: Economic value added (thousand CZK)



Source: own calculation

The economic added value of the manufacturing and automotive industries have archived positive values. Evolution of EVA of automotive industry is copying evolution of EVA of the manufacturing industry. Positive values of EVA are influenced by development of spread. The highest spread was reached in 1Q/2015 in the automotive industry. Figure 2 shows the evolution of spread of manufacturing and automotive industries in analyzed period.

Figure 2: Spread of manufacturing and automotive industries



Source: own calculation

3.2 Dynamic method - analysis of variance

Variance analysis method can be classified as a dynamic analysis, because this method works with longer time series of financial indicators. Each of them was used for analysis 40 periods. Economic value added is used according to formula (2). Main value drivers are calculated by the method of decomposition of variance. After applying pyramidal decomposition on the economic value added is possible to express economic value added by partial indicators as

$$\frac{EVA}{E} = \left[F_1 \cdot F_2 \cdot F_3 \cdot \frac{1}{\left(F_4 + F_5 + F_6 + F_7 + F_8 + F_9 + F_{10} + F_{11} + F_{12}\right)} \cdot \left(F_{13} + F_{14} + F_{15} + F_{16} + F_{17} + F_{18}\right) \right] - F_{19}, \tag{7}$$

where F1 – F19 are component indicators and they are seen in the Table 1.

Table 1: *Component indicators*

F1	F2	F3	F4	F5	F6	
EAT/EBT	EBT/EBIT	EBIT/R	ITFA/R	LTFA/R	Reseivables/R	
F7	**F8**	**F9**	**F10**	**F11**	**F12**	
STFA/R	Materials/R	WIP/R	FG/R	M /RV	Aother/R	
F13	**F14**	**F15**	**F16**	**F17**	**F18**	**F19**
RC/E	OL/E	OE/E	Reserves/E	Payables/E	Debt/E	Re

Variance of partial indicators is determined according to formula (2), while parameter a_i is generally expressed as

$$a_i = E\left(\frac{\partial\left(\frac{EVA}{E}\right)}{F_i}\right). \tag{8}$$

For proposed pyramidal decomposition the formulas of variance influence calculation in coincidence with previous part are expressed as

$$\begin{aligned}
z_1 &= a_1^2 \cdot var(F_1) + a_1 \cdot a_2 \cdot cov(F_1,F_2) + a_1 \cdot a_3 \cdot cov(F_1,F_3) + a_1 \cdot a_4 \cdot cov(F_1,F_4) + \\
&+ a_1 \cdot a_5 \cdot cov(F_1,F_5) + + a_1 \cdot a_6 \cdot cov(F_1,F_6) + a_1 \cdot a_7 \cdot cov(F_1,F_7) + a_1 \cdot a_8 \cdot cov(F_1,F_8) + \\
&+ a_1 \cdot a_9 \cdot cov(F_1,F_9) + a_1 \cdot a_{10} \cdot cov(F_1,F_{10}) + a_1 \cdot a_{11} \cdot cov(F_1,F_{11}) + a_1 \cdot a_{12} \cdot cov(F_1,F_{12}) + \\
&+ a_1 \cdot a_{13} \cdot cov(F_1,F_{13}) + a_1 \cdot a_{14} \cdot cov(F_1,F_{14}) + + a_1 \cdot a_{15} \cdot cov(F_1,F_{15}) + a_1 \cdot a_{16} \cdot cov(F_1,F_{16}) + \\
&+ a_1 \cdot a_{17} \cdot cov(F_1,F_{17}) + a_1 \cdot a_{18} \cdot cov(F_1,F_{18}) + a_1 \cdot a_{19} \cdot cov(F_1,F_{19}), \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
z_{19} &= a_{19}^2 \cdot var(F_{19}) + a_{19} \cdot a_1 \cdot cov(F_{19},F_1) + a_{19} \cdot a_2 \cdot cov(F_{19},F_2) + a_{19} \cdot a_3 \cdot cov(F_{19},F_3) + \\
&+ a_{19} \cdot a_4 \cdot cov(F_{19},F_4) + a_{19} \cdot a_5 \cdot cov(F_{19},F_5) + + a_{19} \cdot a_6 \cdot cov(F_{19},F_6) + a_{19} \cdot a_7 \cdot cov(F_{19},F_7) + \\
&+ a_{19} \cdot a_8 \cdot cov(F_{19},F_8) + a_{19} \cdot a_9 \cdot cov(F_{19},F_9) + a_{19} \cdot a_{10} \cdot cov(F_{19},F_{10}) + a_{19} \cdot a_{11} \cdot cov(F_{19},F_{11}) + \\
&+ a_{19} \cdot a_{12} \cdot cov(F_{19},F_{12}) + a_{19} \cdot a_{13} \cdot cov(F_{19},F_{13}) + a_{19} \cdot a_{14} \cdot cov(F_{19},F_{14}) + + a_{19} \cdot a_{15} \cdot cov(F_{19},F_{15}) + \\
&+ a_{19} \cdot a_{16} \cdot cov(F_{191},F_{16}) + a_{19} \cdot a_{17} \cdot cov(F_{19},F_{17}) + a_{19} \cdot a_{18} \cdot cov(F_{19},F_{18}).
\end{aligned} \tag{9}$$

Relative influences of particular indicators are calculated as (2). In the Table 2 there is mean, variance and value of regression coefficient of all 19 components indicators by manufacturing and automotive industry.

Table 2: Mean, variance and regression coefficient of component indicators of MI and AI

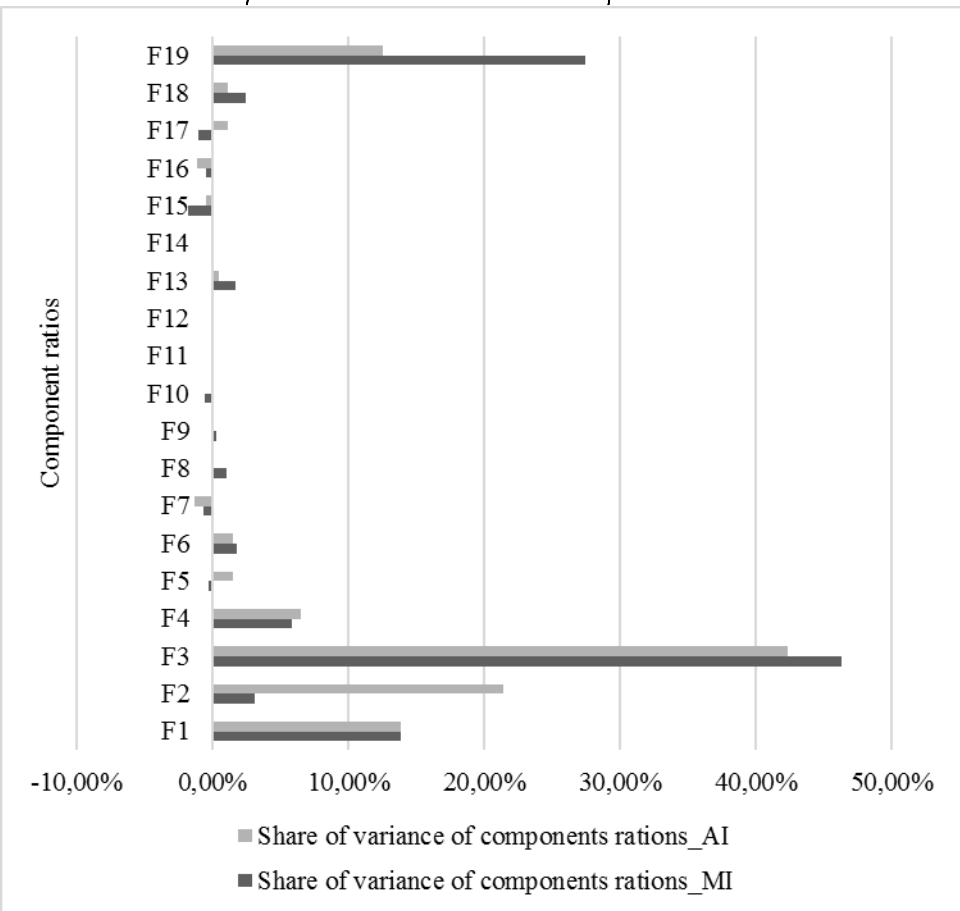
		Mean_MI	Mean_AI	Variance_ MI	Variance AI	Ai_MI	Ai_AI
F1	EAT/EBT	0,758535	0,833326	0,006173	0,132422	0,040428	0,050461
F2	EBT/EBIT	0,943404	0,946329	0,006173	0,132422	0,032505	0,044436
F3	EBIT/R	0,065357	0,058933	0,000360	0,000502	0,469202	0,713533
F4	ITFA/R	1,190747	0,954255	0,013776	0,056372	-0,010357	-0,017593
F5	LTFA/R	0,246777	0,160167	0,001135	0,007746	-0,010357	-0,017593
F6	Reseivables/R	0,785269	0,672346	0,002888	0,018878	-0,010357	-0,017593
F7	STFA/R	0,286898	0,345567	0,004749	0,024137	-0,010357	-0,017593
F8	Materials/R	0,190397	0,118375	0,001434	0,000619	-0,010357	-0,017593
F9	WIP/R	0,119164	0,052016	0,000108	0,000135	-0,010357	-0,017593
F10	FG/R	0,079870	0,034918	0,000691	0,000071	-0,010357	-0,017593
F11	M /RV	0,035087	0,025657	0,000018	0,000042	-0,010357	-0,017593

F12	Aother/R	0,026725	0,026915	0,000011	0,000107	-0,010357	-0,017593
F13	RC/E	0,408146	0,335317	0,002660	0,003108	-0,015796	-0,019444
F14	OL/E	0,016715	0,023798	0,000008	0,000038	-0,015796	-0,019444
F15	OE/E	0,591854	0,664683	0,002660	0,003108	-0,015796	-0,019444
F16	Reserves/E	0,064014	0,142483	0,000066	0,001460	-0,015796	-0,019444
F17	Payables/E	0,638314	0,836094	0,002256	0,017537	-0,015796	-0,019444
F18	Debt/E	0,222356	0,160314	0,002098	0,003650	-0,015796	-0,019444
F19	RE	0,028097	0,026086	0,000037	0,000039	-1,000000	-1,000000

Source: self-elaboration

After applying method of decomposition of variance was found that economic value added of selected industries was distributed to 19 proposed component financial ratios, while only 5 ratios had negative influence and 14 ratios had positive influence. Positive influence means that if variance of component indicators is growing, then the variance of EVA is increasing. On the other hand, negative influence works in reverse. Figure 3 shows the share of variance of component ratios to variance of EVA/E of manufacturing industry (MI) and automotive industry (AI).

Figure 3: The share of variance of component indicators to variance of relative economic value added of MI and AI



Source: self-elaboration

It is obvious that if the financial performance of manufacturing industry is measured by the economic value added indicator, then the most influential factor is indicator of share of EBIT to revenues (F3), which makes up 46.34 % of variance of economic value added. Less important indicator is costs of equity (F19), which consider of 27.47 % of variance of economic value added. The last important indicator is EAT/EBT (F1), which makes up 13.93

% of variance of economic value added. These indicators have highest positive influence on economic value added of the manufacturing industry. Indicator share of other equity to equity (F15) has negative influence, which makes up -1.71 % of variance of economic value added. Indicator share of payables to equity (F17) is the second indicator with negative influence and makes up -0.97 %. The last important indicator with negative influence is STFA to revenues, which consider of -0.59 % of variance of economic value added.

The financial performance of automotive industry is measured by the economic value added indicator, then the most influential factor is indicator of share of EBIT to revenues (F3), which makes up 42.354 % of variance of economic value added. Less important indicator is EBT/EBIT (F2), which consider of 21.46 % of variance of economic value added. The last important indicator is EAT/EBT (F1), which makes up 13.88 % of variance of economic value added of an automotive industry. These indicators have positive influence on economic value added. Indicator share of STFA to revenues (F7) has negative influence, which makes up -1.26 % of variance of economic value added. Indicator share of reserves to equity (F16) is the second indicator with negative influence and makes up -1.12 %. The last important indicator with negative influence is indicator of share other equity to equity (F15), which consider of -0.47 % of variance of economic value added.

4. Conclusion

The contribution was focused on performance evaluation of selected industries in the Czech Republic, manufacturing and automotive industries. The aim of this paper was to identify factors, which has the most important influence on the relative EVA of selected industries for the period 2007 – 2017 by using variance decomposition analysis.

Firstly, relative EVA of manufacturing and automotive industry was clarified. It was distributed to 19 proposed component financial ratios and by analysis of variance was found that the main influencing ratio is share of EBIT to revenues (F3). The most of the ratios had positive influences while only five ratios had negative influence to variance of relative EVA.

Economic value added is usually influenced by the number of selected indicators. Variance analysis is appropriate to use when time series of financial ratios is known. Meanwhile, analysis of deviation is used just only between two periods. For deeper analysis is better to use variance analysis because in these case changes appeared during analyzed period are taken into account.

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Assessment of The Solvency and Financial Condition Report of the insurance companies in Slovakia for 2017

Katarína Sakálová, Mária Kamenárová [1]

Abstract

The Solvency and Financial Condition Report provides the information about the financial health, risk profile and solvency position of the insurance companies since 2016. This report is required to be published by each insurance company after the end of the financial year as part of Solvency II regulation within the 3rd pillar as public disclosure document. The report format is prescribed by the regulation and thus provide the useful and comparable information to the public audience. This paper shows the analyses of the disclosed information for Slovak market, compares it with information disclosed in 2016 and shows the independent observations and conclusions. The focus is on life insurance companies and their standard capital requirement and fair value of the liabilities under Solvency II requirements. In general analyses show that the solvency position in Slovak market is decreasing either due to capital optimization for some market players or by the changes in the methodologies or calculation improvements. It might be concluded that some companies implemented solvency capital calculation differently than others.

Key words

Solvency and Financial Condition Report, Solvency II Ratio, Fair Value Balance Sheet, Standard Capital Requirement

JEL classification: G22

1. Introduction

The Solvency and Financial Condition Report is the public document that has to be disclosed every year within prescribed date, usually on the web site of the insurance company. The report is prepared by the representatives of the insurance company with the aim to disclose the information required by the Insurance Act No 39/2015 and also by the implementation of the Delegated Act of the EU Commission No 2015/35 (further "Delegated Act").

The Solvency and Financial Condition Report (further on " SFCR report") provides to the public audience the clear and transparent information about the financial status under the new regulatory regime Solvency II. It contains useful information about the performance of the insurance company in area of underwriting and investment income, system of governance, evolution of the risk profile, and valuation differences, financial cover for the solvency capital requirements and solvency adequacy. The structure and the content of the SFCR report are defined in the articles 290 - 303 of this Delegated Act [1] and in its the Appendix XX[1].

The aim of this paper to evaluate the financial position in the Slovak market and to find out if there are some possible risk visible based on provided information.

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2. Collection of basic data from published SFCR reports in year 2017

For the purpose of the evaluation of the financial strength of the insurance companies is important to work with the most recent information, and to compare them with the evolution over the recent period. It the SFCR report the data about the previous year has to be provided. The similar analysis was done based on information from year 2016 [18].

The SFCR report has to be prepared by the stand alone insurance companies established in Slovakia, the branches are reporting together with mother insurance company in their home state. In Slovakia there are 16 stand alone insurance companies, this paper includes 14 of them. Other 2 companies were excluded, one company is in the liquidation[2] and one was merged[3] shortly before time of this paper preparation.

The analyzed information represents 99.7% of the total insurance market in Slovakia as reported in [2] based on Total Premium Written indicator. The information used to evaluate the size of the individual insurance companies by Premium Written Total, Premium Written Life Insurance, Total Value of the Assets valued in line with Solvency II, Market Value of the Technical Provisions valued in line with Solvency II, Profit after Tax and Own Funds IFSR. These information are as shown in Table 1 and used later.

Table 1: Basic information about the selected insurance companies in 2017

in Mio Eur	Premium Written Total	Premium Written Life Insurance	Total Market Value of Assets (Solvency II)	Market Value of Technical Liabilities (Solvency II)	Profit After Tax	Own Funds (IFRS)
Allianz SP	504.7	208.0	2,168.2	1,530.0	71.7	302.7
Kooperativa	468.7	202.3	*1,201.8*	708.3	29.8	304.7
Generali	225.0	94.7	531.5	281.3	9.2	132.6
Komunálna poisťovňa	195.0	108.2	348.6	258.9	8.2	47.8
Uniqa	126.9	33.0	*284.5*	209.2	3.4	50.1
ČSOB poisťovňa	85.1	45.5	355.5	260.0	10.1	59.5
NN poisťovňa	74.6	74.6	530.0	395.3	5.1	62.7
Wuestenrot poisťovňa	57.2	32.0	181.9	117.8	4.2	35.4
Union poisťovňa	53.1	9.9	129.5	91.4	2.0	22.9
Aegon	48.7	48.7	119.4	51.7	4.1	28.3
Novis	25.1	25.1	31.3	(35.3)	2.8	21.3
Cardif poisťovňa Sovakia	24.2	6.4	30.2	(4.1)	2.2	13.4
Poštová poisťovňa	14.3	12.5	34.7	10.1	2.3	19.6
ERGO poisťovňa	6.9	6.0	103.0	90.4	(1.5)	8.2
Total	**1,909.5**	**907.0**	**6,050.0**	**3,965.0**	**153.6**	**1,109.2**

The source of the information are disclosed SFCR reports [3], [5], [6], [7], [8], [9], [10], [11], [14], [16], [16], [15], [16], [17]. Some of the information was not disclosed there and were taken from yearly Statutory Financial Statements (IFRS) for example [4].

3. Assessment of the capital adequacy in Slovak market

The analyzed information represents 99.7% of the total insurance market in Slovakia as reported in [2] based on Total Premium Written indicator. The information used to evaluate the size of the individual insurance companies by Premium Written Total, Premium Written Life Insurance, Total Value of the Assets valued in line with Solvency II, Market Value of the

[2] Rapid Life životná poisťovňa a.s.
[3] Poisťovňa Slovenskej sporiteľne a.s. merge as of 1.4.2018

Technical Provisions valued in line with Solvency II, Profit after Tax and Own Funds IFSR. These information are shown in Table 2 and analyzed later.

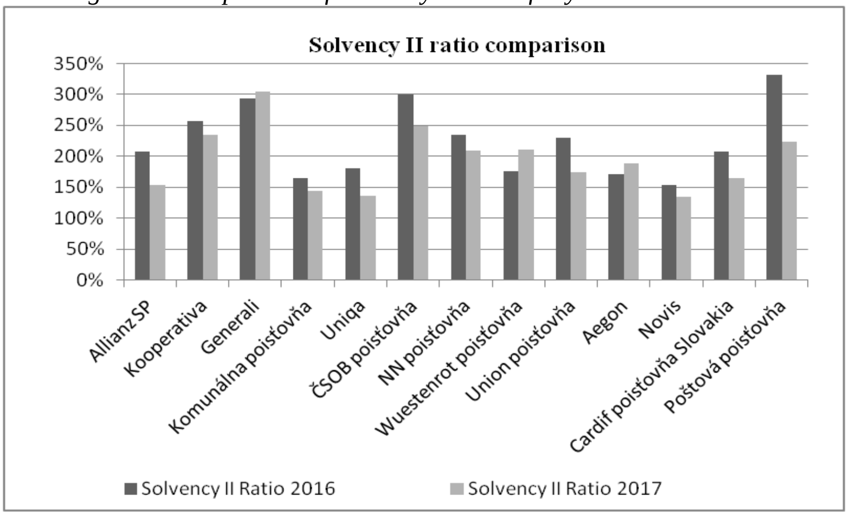
Solvency II ratio = Eligible Own Funds/ Required Capital

Table 2: Basic information on Eligible Own Funds and Required Capital for2016 and 2017

in Mio Eur	Required Capital for Solvency 2016	Eligible Own Funds for Solvency 2016	Required Capital for Solvency 2017	Eligible Own Funds for Solvency 2017	Solvency II Ratio 2016	Solvency II Ratio 2017
Allianz SP	194.2	401.9	191.6	295.5	207%	154%
Kooperativa	128.8	331.0	135.0	316.7	257%	235%
Generali	49.2	144.2	55.7	170.0	293%	305%
Komunálna poisťovňa	30.5	50.2	35.3	50.9	164%	144%
Uniqa	30.6	55.4	37.8	51.5	181%	136%
ČSOB poisťovňa	23.5	70.5	26.2	65.4	300%	250%
NN poisťovňa	26.1	61.3	30.8	64.4	235%	209%
Wuestenrot poisťovňa	20.6	36.1	25.4	53.7	175%	211%
Union poisťovňa	12.8	29.4	16.3	28.3	230%	174%
Aegon	31.5	53.7	35.8	67.7	170%	189%
Novis	16.0	24.5	24.7	33.3	153%	135%
Cardif poisťovňa Slovakia	9.0	18.7	10.1	16.7	208%	165%
Poštová poisťovňa	5.9	19.6	8.8	19.6	332%	223%
ERGO poisťovňa	4.2	11.3	3.8	9.1	269%	239%
Average	**41.6**	**93.4**	**45.5**	**88.8**	**227%**	**198%**

In year 2017 the average Solvency II ratio in the Slovak market was 198%. This shows that the market is sufficiently capitalized, 3 companies has Solvency II ratio below 150%, and 2 companies above 250%. There was mild worsening of the solvency position, as Solvency II ratio decreased by 29% from average value 227% reported in year 2016, as the insurance companies except two reported decrease of the Solvency II ratio in year 2017.

Figure 1: Comparison of Solvency II Ratio for years 2016 and 2017.



The further analysis of reasons of the decrease of the solvency position in the Slovak market was performed. Seven insurance companies 50% showed significant - more than 30%, difference of the Solvency II ratio over one year, which is above the average market. It should be noted that in year 2017, there were no major negative factors reported in the financial market. But in 2017 was applied 8% tax on the premium in Non-Life insurance. This is not assumed to have negative impact on capital adequacy position is short time. This assumption is stated from analyses of NBS[4] [2]. In 2017 the increase in average value of required capital was higher than the growth in Premium Written Total that was 6% [2]. The average Required Capital for Solvency II was 45.5 Mio Eur and increased by 9% is visible for all insurance companies with 2 exceptions.

It is assumed, that significant difference is caused by the other factors than natural evolution of the insurance portfolio. The change in the Solvency II ratio over 1st year of the Solvency II regime adoption might by caused by:

- the capital optimization of the shareholders;
- the possible improvements and corrections in the valuation approach;
- the changes in risk exposure and risk profile.

The capital optimization was made by company Allianz SP a.s., as show in Table 2 from the comparison of the Eligible Own Funds in year 2017, that caused the significant decrease of the Eligible Own Funds in year 2017. This confirmed in [3].

The possible improvements and correction in valuation approach were reported by 43% of companies, that is 5 out of 7 selected, in particular for [9], [11], [12], [13], [14], [16] and one company confirmed change in risk profile [15]. Only one company did not showed the change in the Solvency II ratio above 50% [14] is company Novis.

In Table 3 are data from SFCR reports and it shows the parts of the capital per individual risk as required by standard formula: Market Risk, Life Underwriting (further "UW") Risk, Non-Life UW risk, Health UW risk, Counterparty Default Risk and Operation Risk as defined in the Delegated Act [1] per each insurance company. These risks are diversified and the final Total Required capital is not the simple sum.

Table 3: Modules of the Required Capital for Solvency for year 2017

in Mio Eur	**Capital for Market Risk**	**Capital for Life UW Risk**	**Capital for Non-Life UW Risk**	**Capital for Health UW Risk**	**Capital for Counterparty default risk**	**Capital for Operation Risk**	**Total Required Capital for Solvency II**
Allianz SP	152.7	86.8	89.2	3.3	29.0	18.8	191.6
Kooperativa	84.2	37.1	***69.8***	1.5	11.3	15.8	135.0
Generali	22.3	37.6	17.3	14.2	8.8	6.9	55.7
Komunálna poisťovňa	18.8	5.9	17.6	0.2	3.4	6.3	35.3
Uniqa	16.0	6.1	***18.8***	0.4	8.6	3.9	37.8
ČSOB poisťovňa	15.3	12.0	8.0	4.7	3.7	2.4	26.2
NN poisťovňa	11.2	11.8	-	24.1	1.0	4.1	30.8
Wuestenrot poisťovňa	9.2	13.6	9.2	5.3	1.7	1.9	25.4
Union poisťovňa	2.4	4.5	11.3	1.9	1.8	1.6	16.3
Aegon	7.1	9.3	-	30.6	0.8	1.5	35.8
Novis	12.4	16.5	-	6.4	0.6	1.4	24.7
Cardif poisťovňa Slovakia	0.2	1.3	7.1	3.5	2.4	0.9	10.1
Poštová poisťovňa	7.5	3.9	1.4	2.0	0.5	0.6	8.8
ERGO poisťovňa	0.9	1.3	1.3	0.8	1.2	0.4	3.8
Average	**25.7**	**17.7**	**17.9**	**7.1**	**5.3**	**4.7**	**45.5**

The difference capital for particular risks to market average amongst companies is large. There are defined relative risk indicators for further analyses:

1. ***Life Premium Ratio = Premium written Life Insurance / Premium Written Total*** that expresses the risk profile of insurance company, 100% means pure life insurance company;

[4] NBS: National Bank of Slovakia

2. ***Life Risk Premium Ratio = Capital for Life UW Risk/ Life Premium Written,*** that express the Life Risk Capital as percentage cost of yearly life premium;

3. ***Health Risk Premium Ratio = Capital for Health UW Risk/ Life Premium Written*** that expresses the Health Risk Capital as percentage cost of yearly life premium;

4. ***Total Capital Life Premium Ratio = Total Required Capital/ Premium Written Life*** that expresses Total Required Capital as percentage cost of yearly life premium;

5. ***Total Capital Total Premium Ratio = Total Required Capital/ Total Premium Written*** that expresses Total Required Capital as percentage cost of yearly Total premium;

6. ***Total Capital Asset Ratio = Total Required Capital/ Total Market Value of Assets*** that expresses Total Required Capital as percentage cost of total asset valued based on Solvency II ;

For the calculation of these indicators were taken data from Table 1, Table 2 and Table 3. The values of the defined indicators were calculated and are shown in Table 4.

Table 4: Risk indicators for year 2017

	Life Risk Life Premium Ratio	**Health Risk Life Premium Ratio**	**Total Capital Life Premium Ratio**	**Life Premium Ratio**	**Total Capital Total Premium Ratio**	**Total Capital Asset Ratio**
Allianz SP	41.7%	1.6%	92.1%	41.2%	38.0%	8.8%
Kooperativa	18.3%	0.7%	66.7%	43.2%	28.8%	11.2%
Generali	39.7%	15.0%	58.8%	42.1%	24.8%	10.5%
Komunálna poisťovňa	5.5%	0.2%	32.6%	55.5%	18.1%	10.1%
Uniqa	18.5%	1.2%	114.3%	26.0%	29.8%	13.3%
ČSOB poisťovňa	26.4%	10.3%	57.6%	53.5%	30.8%	7.4%
NN poisťovňa	15.8%	32.3%	41.3%	100.0%	41.3%	5.8%
Wuestenrot poisťovňa	42.5%	16.6%	79.4%	55.9%	44.4%	14.0%
Union poisťovňa	45.5%	19.2%	164.6%	18.6%	30.7%	12.6%
Aegon	19.1%	62.8%	73.5%	100.0%	73.5%	30.0%
Novis	65.7%	25.5%	98.4%	100.0%	98.4%	78.9%
Cardif poisťovňa Slovakia	20.3%	54.7%	157.8%	26.4%	41.7%	33.4%
Poštová poisťovňa	31.2%	16.0%	70.4%	87.4%	61.5%	25.4%
ERGO poisťovňa	21.7%	13.3%	63.3%	87.0%	55.1%	3.7%
Average	**29.42%**	**19.25%**	**83.64%**	**59.78%**	**44.06%**	**18.93%**
Standard deviation	**15.86%**	**19.33%**	**39.20%**	**29.49%**	**21.73%**	**19.45%**

The indicators show high volatility, for example for companies with similar risk profile with Life Premium Ratio of 40% has the Total Capital Life Premium Ratio range of 59% - 92%, or Health Risk Life Premium Ratio range of 0.7% – 15%. Similarly it is for pure life insurance companies where Health Risk Life Premium Ratio has range of 32% – 63%.

The average market value and the standard deviation were calculated for each risk indicator so the benchmark for each company can be made. The best indicators that capture all risks are Total Capital Total Premium Rate and Total Capital Asset Ratio as market risk is also there.

The further analysis was made by using the standardized values from the calculated indicators, their average and standard deviation. The aim was to identify possible outliers that would indicate higher risk based on Normal distribution, so when the standardized value is higher than 1 and higher than 2. The standardized values were chosen as the representation of the Normal Distribution rules to find out the outliers.

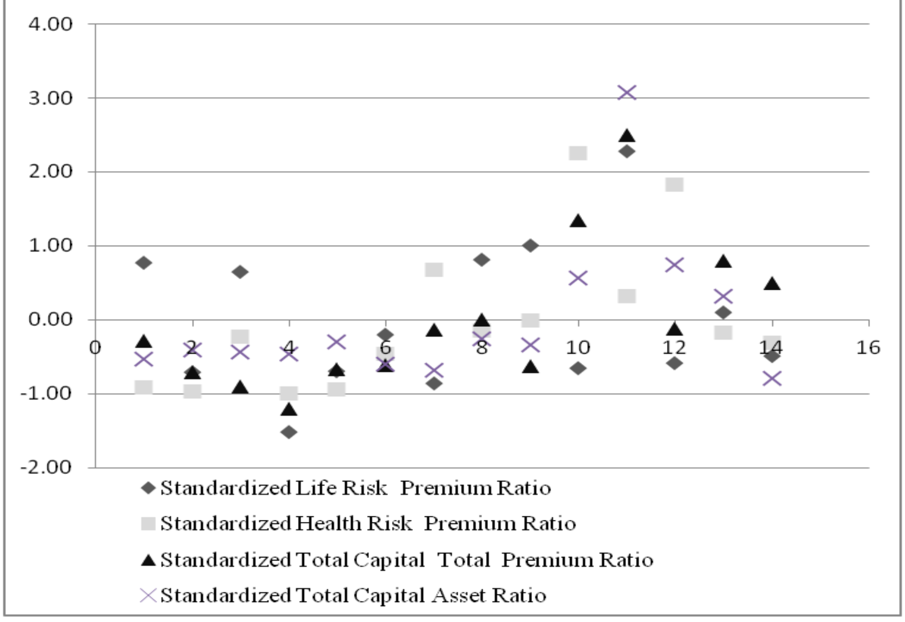
Table 5: Standardized values of the selected indicators

	Standardized Life Risk Premium Ratio	Standardized Health Risk Premium Ratio	Standardized Total Capital Life Premium Ratio	Standardized Total Capital Total Premium Ratio	Standardized Total Capital Asset Ratio	No of hits > 1	No of hits > 2
Allianz SP	0.78	-0.91	0.22	-0.28	-0.52	-	-
Kooperativa	-0.70	-0.96	-0.43	-0.70	-0.40	-	-
Generali	0.65	-0.22	-0.63	-0.89	-0.43	-	-
Komunálna poisťovňa	-1.51	-0.99	-1.30	-1.19	-0.45	3	-
Uniqa	-0.69	-0.93	0.78	-0.66	-0.29	-	-
ČSOB poisťovňa	-0.19	-0.46	-0.66	-0.61	-0.59	-	-
NN poisťovňa	-0.86	0.68	-1.08	-0.13	-0.67	1	-
Wuestenrot poisťovňa	0.83	-0.14	-0.11	0.02	-0.26	-	-
Union poisťovňa	1.01	0.00	2.07	-0.62	-0.33	-	-
Aegon	-0.65	2.25	-0.26	1.36	0.57	2	1
Novis	2.29	0.32	0.38	2.50	3.08	3	3
Cardif poisťovňa Slovakia	-0.57	1.83	1.89	-0.11	0.75	2	-
Poštová poisťovňa	0.11	-0.17	-0.34	0.80	0.33	-	-
ERGO poisťovňa	-0.49	-0.31	-0.52	0.51	-0.78	-	-

The standardized indicators from Table 3 are shown in Figure 2, where on *x* axis is the order in row from Table 5, e.g. the last company in 14th row is shown as value 14, and on *y* axis is the standardized value of the indicators.

Figure 2: Comparison of Solvency II Ratio for years 2016 and 2017.

Standardized values for risk indicators



There are indentified the outlier positions for company:
- in the 11th row, as it shows 3 out of 5 standardized risk indicators in range more than 2,
- in the 4th row show 3 out of 5 indicators in range more than 1.

This might be the indication of higher risk profile of the company, higher risk for the solvency position of the company or the possible inconsistency in current valuation method used.

4. Conclusion

In general analyses showed that Slovak Insurance market is sufficiently capitalized with the average solvency position in Slovak market decreased in year 2018 either due to capital optimization or by improvements and corrections in the valuation approach 43% of companies. The analyses by the selected risk indicators and their standardized values showed that some insurance companies are the outliers. That might indicate the existence of the financial risk for these companies, but it might be also concluded that some companies implemented solvency capital calculation differently than others and some further correction might be expected in future.

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Advantages and disadvantages of particular types of bonus from the point of view of life insurance company

Katarína Sakálová, Ingrid Ondrejková Krčová [1]

Abstract

A basic idea of a with profit policy is that the policyholder accepts a lower guarantee (sum assured) for the same premium, than he would accept under an equivalent without profit policy. In return for a lower guarantee, the policy has the right to share in the profits of the life insurance company. In order to qualify for such entitlement the policyholder will pay a higher premium that if the benefit was an amount stated in the contract. This paper describes one method that is used in in the United Kingdom and many of the countries that are of the British Commonwealth. The profit is distributed by making additions to the basic benefit, i.e. sum assured, under the contract. These additions are known as bonuses and take a number of different forms. We will describe differences between existing bonus systems and demonstrate on the practical example advantages and disadvantages of particular types of reversionary bonus from the point of view of life office.

Key words

With profit contracts, reversionary bonuses, cost of bonus, terminal bonus.

JEL Classification: G22

1. Introduction

This paper considers a particular type of the life insurance product which is known as a with profit contract. With profit policy is a special type of life insurance policy for distribution of the surplus. A number of different methods are used in different parts of the world to allocate surplus or profit to policyholders. These methods have of course their advantages and disadvantages. In United Kingdom and many other countries insurers pay bonuses on with profit policies. In USA and some other countries insurers provide another method - dividends - on so called participating policies, see e. g. [1], [3], [4].

The basic idea of the bonus method is no immediate distribution of the surplus during the term of the policy but when the policy becomes a claim, amount of it is greater than the basic sum assured. So with profit policyholder pays a larger premium for the same basic sum assured as the without profit policyholder, but expects to have his benefit increased by declarations of various kinds of bonus. In this article we will analyse most important part of bonus system – reversionary bonus.

2. Addition to benefits – bonus system

This method is used in the United Kingdom and some of the countries that are of the British Commonwealth, for example Australia and India. The profit is distributed by making

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additions to the basic sum assure of the contract. These additions, which are called bonuses, can take a number of different forms.

Reversionary bonuses

A reversionary bonus is an addition made to the benefit each year. If a reversionary bonus has been added to the sum assured, it cannot be removed. The amount of this bonus is not payable immediately, it is added to the contract, but is paid at the same time (on death or maturity) as the basic benefit is paid. The amount of the bonus can be calculated in one of three ways, see e. g. [5], [6], [7]

- **simple** – a simple reversionary bonus at a percentage rate b of the sum assured S adds $b \cdot S$ to the basic sum assured S each year and after t declarations the total guaranteed benefit is

$$S \cdot (1 + t \cdot b)$$

- **compound** – a compound reversionary bonus at a percentage rate b each year adds b times sum assured and existing reversionary bonuses each year and after t declarations the total guaranteed benefit is

$$S \cdot (1 + b)^t$$

- **super-compound** – (also called a two-tier reversionary bonus) at percentages rates b_1, b_2 per year: one applied to the initial sum assured S and a second applied to any bonuses that have already been given. After t bonus declarations the total guaranteed benefit is

$$S \cdot (1 + b_1 \cdot s_{\overline{t|}}^{b_2\,\%})$$

The second percentage will usually be higher than the first.

Special reversionary bonuses

Special reversionary method is the same as the previous reversionary bonus method except one thing: by declaration of this bonus the company will at the same time declare that policyholder cannot expect this bonus to be repeated.

Terminal bonuses

A terminal bonus is an addition to the benefit that is made when the benefit is actually paid and the amount of the addition will be determined at that point. This could in theory imply a constantly changing bonus, but in practice this does not happen, although a life insurance company will not usually guarantee to maintain the bonus at any particular level. The bonus to give to a particular contract may be specified in a number of different ways, for example

- a percentage, that may vary by duration in force, of the total reversionary bonuses that have already been added to the benefit;
- a percentage of the total claim amount, before addition of terminal bonus, with the percentage varying according to duration in force.

Current practice is the following, see e. g. [8], [9]: the company keeps the sum assured low to start with. It will keep the reversionary bonuses low, in order to reduce the rate at which the guarantees build up. Reducing the reversionary bonuses will reduce the cost of the bonus; so more surplus can be put into the investment reserve. This reserve can cushion the fluctuations of investing in riskier assets; investment freedom of the life insurance company is increased. At maturity, the sum assured plus bonuses will be lower than the policy's asset share. In order to pay the maturity value, company makes a final transfer out of the investment reserve on the maturity date, and use this to pay a terminal bonus.

The key features of a terminal bonus, and the main differences between it and reversionary bonuses, are:

- Terminal bonuses are not guaranteed in advance.
- Terminal bonuses do not increase the guaranteed liabilities, and hence the need for valuation reserves, in the way that reversionary bonuses do.
- Terminal bonus is a retrospective payment, whereas a reversionary bonus is a prospective payment.

3. Modelling various kinds of reversionary bonuses on the example

A 50 years aged person is insured for endowment for a period of 10 years and a sum of €10,000. We use the 2017 mortality tables from the Infostat[2], with the unisex tables created as an average of the tables for men and women. Insurance technical rate (i) will be 0.7%. We want to have the reversionary bonuses to amount closely €2,500 at maturity.

Clearly under the simple bonus system the company may declare a bonus of 2.5% of the sum assured. So for example in the second year is the amount of a simple bonus $0.025 \cdot 10,000 = 250$.

Under the compound system life office must declare a bonus of 2.26% of the basic sum assured and attaching bonuses. So for example in the second year life office calculates the compound reversionary bonus (if the bonus in the first year is €226) as follows $0.0226 \cdot (10,000 + 226)$, i.e. €311.11.

Under the super-compound system company may for example declare a bonus of 2% of the basic sum assured and 4.85% of all attaching bonuses. The new reversionary bonus in second year added to the basic sum assured is then calculated $0.02 \cdot 10,000 + 0.0485 \cdot 200$, i.e. €409.70.

To illustrate the different effect of these three bonus methods, we shall look how the three types of bonuses build up over the lifetime of a policy. So the following table shows the resulting figures for the bonus declared each year (Table 1). The figures in the tables are rounded to two decimal places.

Table 1: Bonus declared in year (in €)

Year	Simple	Compound	Super-compound
1	250.00	226.00	200.00
2	250.00	231.11	209.70
3	250.00	236.33	219.87
4	250.00	241.67	230.53
5	250.00	247.13	241.72
6	250.00	252.72	253.44
7	250.00	258.43	265.73
8	250.00	264.27	278.62
9	250.00	270.24	292.13
10	250.00	276.35	306.30

[2] http://www.infostat.sk/vdc/sk/index.php?option=com_content&view=article&id=17&Itemid=18

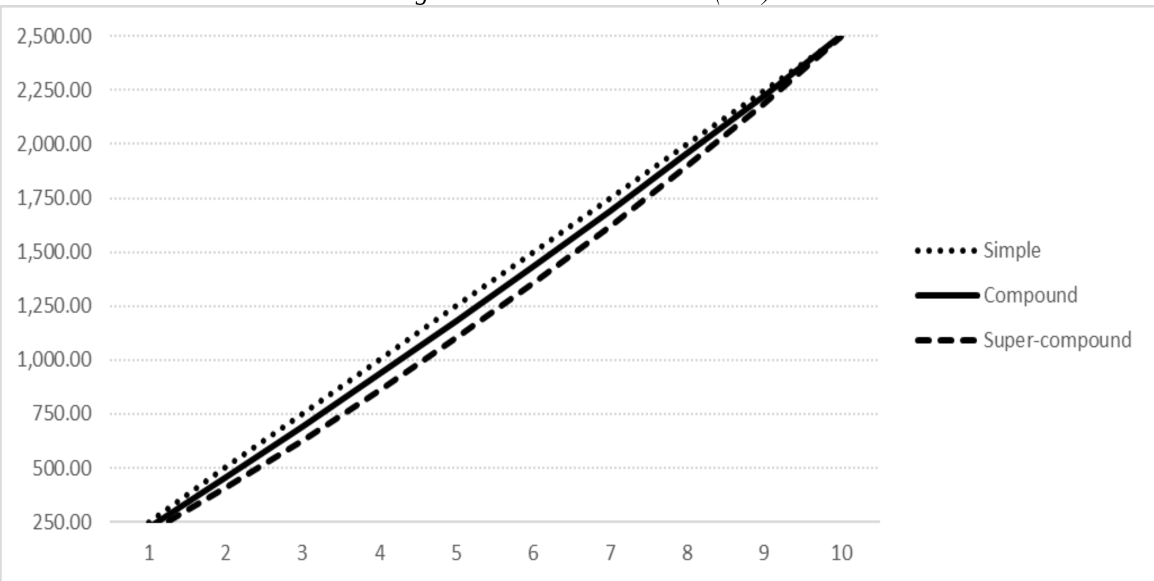
The next table shows the resulting figures for the total attaching bonus in each year of the lifetime of a contract (Table 2).

Table 2: Total bonus to date (in €)

Year	Simple	Compound	Super-compound
1	250.00	226.00	200.00
2	500.00	457.11	409.70
3	750.00	693.44	629.57
4	1,000.00	935.11	860.10
5	1,250.00	1,182.24	1,101.82
6	1,500.00	1,434.96	1,355.26
7	1,750.00	1,693.39	1,620.99
8	2,000.00	1,957.66	1,899.61
9	2,250.00	2,227.91	2,191.74
10	2,500.00	2,504.26	2,498.04

As we wanted the total bonuses at the end of the term are virtually identical under all there bonus systems. The following graph shows how the total bonuses build up under the three bonus systems.

Figure 1: Total bonus to date (in €)



As we see compound bonus builds up more slowly than the simple bonus and super-compound bonus builds up more slowly than either two. Although the amount of bonus matters, from the policyholders point of view, he or she will receive in this example the same amount at maturity. So the policyholder who continuous to maturity will be indifferent as to how the bonuses are created or calculated. But from the life offices point of view situation is rather different.

As was said the final value of the reversionary bonus is payable only with the sum assured by the death or maturity of the contract. The cost to the life insurance company of the reversionary bonus is therefore present value of bonus allowing of course for interest rate and mortality.

So in the next we can calculate the present value (the cost) of all three types of the reversionary bonus declared each year of the lifetime of contract. We assume that the bonus attaches at the end of each year.

For the following calculations, we use the formula for the calculating endowment using probabilities (p, q) and interest rates, see e. g. [2], [10]

$$A_{x+t,\overline{n-t}|} = \sum_{t=1}^{10} {}_{t-1}p_x \cdot q_{x+t-1} \cdot v^t + {}_n p_x \cdot v^n$$

where x is the policyholders age, n is the duration of the insurance and v is calculated from the formula $v = \dfrac{1}{1+i}$.

So for example for $t = 2$ we have

$$A_{52,\overline{8}|} = \sum_{t=1}^{8} {}_{t-1}p_{52} \cdot q_{52+t-1} \cdot v^t + {}_8 p_{52} \cdot v^8 = 0.946936292$$

In the second year of bonus declarations is then cost of bonus in this year for all three types of bonuses:

For simple bonus: $250 \cdot A_{52,\overline{8}|}$, i.e. €236.73,

for compound bonus: $231.11 \cdot A_{52,\overline{8}|}$, i.e. €218.84,

for super-compound bonus: $209.70 \cdot A_{52,\overline{8}|}$, i.e. €198.57.

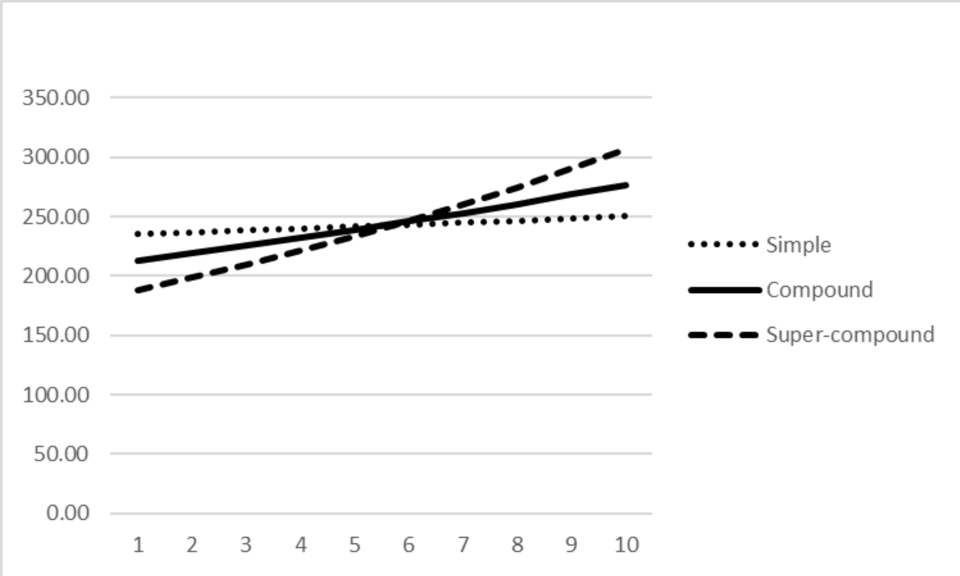
Table 3: Cost of bonus declared in year (in €)

Year	Simple	Compound	Super-compound
1	235.16	212.58	188.13
2	236.73	218.84	198.57
3	238.33	225.29	209.60
4	239.93	231.94	221.25
5	241.56	238.79	233.55
6	243.20	245.85	246.55
7	244.86	253.12	260.27
8	246.55	260.62	274.77
9	248.26	268.36	290.10
10	250.00	276.35	306.30

In the preceding table were calculated cost of bonus declared in each year. As you can see, by using the compound and especially super compound bonuses the life office saves its money in the early years. The surplus which was not distributed in the early years of time of insurance will augment the free assets of the life insurance company. This can be reason for more adventurous investment policy of life office and of course this can be reason for higher return from life office assets.

Figures which are shown in Table 3 are illustrated also in the following graph, where we can see that the best possibility for life insurance office is super compound bonus or better to say it is the less bad method.

Figure 2: Cost of bonus declared in year (in €)



We illustrate effect of preceding technic (using compound or super-compound bonus instead of simple bonus) in Table 4, where we will assume, that the not distributed surplus earns 4 %, i.e. the return that life insurance company obtain from using this technic is higher that was assumed in costing the bonuses. Than accumulated savings for two types of bonuses are calculated as follows.

For compound bonus: 1. year: $235.16 - 212.58 = 22.58$

2. year: $22.58 \cdot (1 + 0,04) + 236.73 - 218.84 = 41.37$

3. year: $41.37 \cdot (1 + 0,04) + 238.33 - 225.29 = 56.05$

For super-compound bonus: 1. year: $235.16 - 188.13 = 47.03$

2. year: $47.03 \cdot (1 + 0,04) + 236.73 - 198.57 = 87.07$

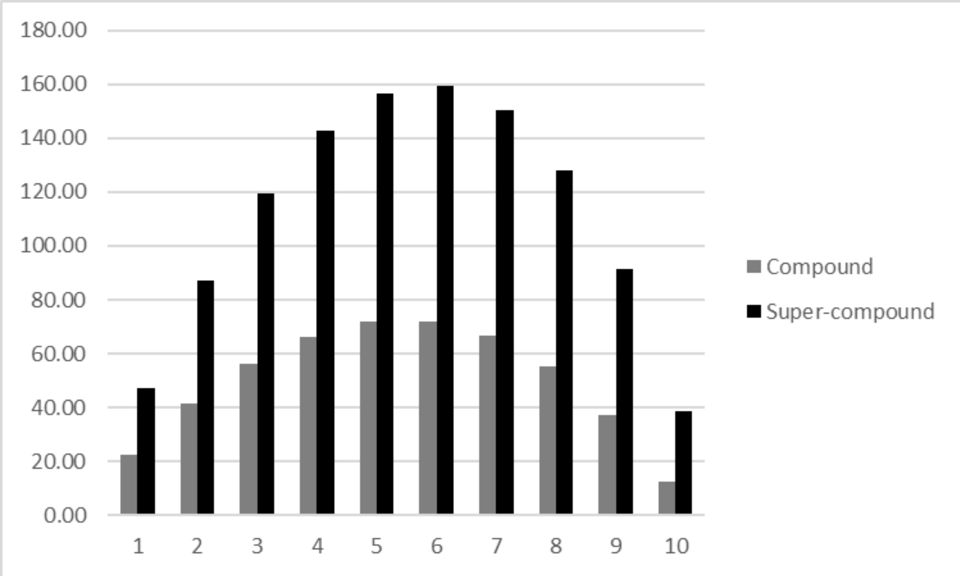
3. year: $87.07 \cdot (1 + 0,04) + 238.33 - 209.60 = 119.28$

Table 4: Accumulated saving (in €)

Year	Compound	Super-compound
1	22.58	47.03
2	41.37	87.07
3	56.05	119.28
4	66.29	142.73
5	71.71	156.45
6	71.93	159.36
7	66.55	150.33
8	55.14	128.12
9	37.25	91.41
10	12.38	38.76

Again the figures which are shown in the preceding table (Table 4) are illustrated also in the following graph.

Figure 3: Accumulated saving (in €)



All these calculations can give a good reason why life offices often use the following scheme:

1. step: Keep sum assured (guaranteed benefit) low at the start of with profit policy.
2. step: Then in order to reduce guarantees, reduce the reversionary bonus (using instead of simple bonus compound or super-compound bonus) and reduce the cost of the bonus. Then more surplus can be used for more intensive and free investment.
3. step: Then at maturity, the company makes a final transfer out of investment reserve and pays policyholder not only sum assured with reversionary bonus but from investment reserve also terminal bonus.

4. Conclusion

The main points to notice are these:

- The compound bonus builds up more slowly than the simple bonus, and the super-compound bonus builds up more slowly than other two.
- The cost of the simple bonus starts at a relatively high level, but is lowest by the end of the term.
- The cost of the compound bonus is lower than the cost of the simple bonus to start with, but higher at the end of the term.
- The super-compound bonus is cheapest to start with, but very much the most expensive at the end of the term.

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Modelling Non-life Insurance Price using Generalized Linear Models

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Abstract

Regression analysis plays an important role in statistics, which uses tools for analysing relationships among variables in datasets. Linear regression or widely General Linear Model, is not fully suitable for setting the price in Non-life Insurance, though, it assumes normally distributed random errors, respectively residuals, while the number of claims costs are non-negative and often right-skewed. Moreover, in linear model, the mean is a linear function of the covariates. It follows that Generalized Linear Models (GLMs) are standard methods more reasonable for pricing. This paper focuses on GLM technique from theoretical point but first and foremost demonstrates this knowledge on practical example from Non-life Insurance.

Key words

Tariff analysis, Generalized Linear Models, Claim Frequency, Claim Severity, Pure Premium

JEL Classification: C10, C31, C53, G22

Introduction

This paper deals with the problem of non-life insurance pricing in establishing the pure premium. That is a complex endeavour, which can be best conceptualised as a process or even as a project: that is to say, an undertaking with a beginning and an end and a number of different tasks to be executed in a certain order. A non-life insurance policy is an agreement between a policyholder and an insurance company. The fundamental role is to undertake to compensate the customer, provide financial protection for certain unpredictable losses during a time period, usually one year, against a fee, the premium. Due to the fact that not all risks are equal, then each individual will be charged with the premium or tariff corresponding with gravity of this risk. Policyholders are divided into several tariff groups, where for each group different expected values of claims are assumed.

The statistical study an actuary performs to obtain a tariff is called a tariff analysis. The actuary tries to find a statistical model which describes, for example, how the claim severity or frequency depends on the number of covariates respectively explanatory variables (Ohlsson and Johansson, 2010). The premium is determined by the given values of a number of variables, the rating factors. In practise, however, the rating factors are divided into classes, e.g. age into age intervals, etc. Basic indicators or key ratios include the claim frequency, claim severity, pure premium and loss ratio. Here, the ratio means a ratio between the outcome of a random variable and a volume measure, which is called exposure. The exposure w results in a response X, for example the claim amount or the number of claims. Table 1 summarizes some important key ratios. Many things in real world are normally distributed due to the central limit theorem and linear models are therefore often sufficient. In non-life insurance pricing, however the dependent variable Y is often not normally distributed with non-trivial connection between the predictors and the response (Frees, 2010). In this context, linear regression, used to evaluate the impact of explanatory variables of studied risk, has been replaced by the Generalized Linear models (GLMs) which provide a solution.

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Table 1: Key ratios

Exposure w	Response X	Key ratio $Y = X/w$
Duration	Number of Claims	Claim Frequency
Duration	Claim costs	Pure Premium
Number of Claims	Claim costs	Claim Severity
Earned Premium	Claim costs	Loss Ratio
Number of Claims	Number of Large Claims	Proportion of Large Claims

Research Methodology

The aim of this paper is to present a rich class of statistical methods related to the GLMs which generalizes the ordinary linear models. Standard linear regression model has very rigorous assumption of normally distributed observations, which doesn't allow appropriate modelling of premium (Haberman and Renshaw, 1996). This section also obtains some specific issues related to building, evaluating and selecting suitable model.

1.1 Generalized Linear Models (GLMs)

The key concept of all GLMs is that are often used within the framework of the Exponential Dispersion Family (EDF), which generalizes the normal distribution used in linear models (McCullagh and Nelder, 1989). In this class of statistical methods certain forms of non-linear models can be fitted. The purpose of these statistical models is to estimate response variable Y which depends on a certain number of explanatory variables X_i. The variables $Y_1, \dots, Y_n$ are independent, but not identically distributed (Kaas, 2009). The generalized linear models are based on the following three keystones (Jong and Zeller, 2008):

i. ***An Exponential Family of Distributions***: the probability distribution of an EDM is given by the following frequency function

$$f_{Y_i}(y_i; \theta_i, \phi) = exp\left\{\frac{y_i\theta_i - b(\theta_i)}{\phi/w_i} + c(y_i, \phi, w_i)\right\}, \tag{1.1}$$

where θ_i is an unknown parameter that is allowed to depend on i. Another unknown parameter is called dispersion parameter $\phi > 0$ is the same for all i. A cumulant function $b(\theta_i)$ is assumed twice continuously differentiable. For every choice of such a function, we get a family of probability distributions, e.g. the normal, Poisson and gamma distributions. Given the choice of $b(\cdot)$, the distribution is completely specified by the parameters θ_i and ϕ. The function $c(\cdot,\cdot,\cdot)$ does not depend on θ_i. Elementary rules for mean and variance of the dependent variable Y_i according to the (1.1) can be yield

$$E(Y_i) = \mu_i = b'(\theta_i), \tag{1.2}$$
$$Var(Y_i) = \phi b''(\theta_i)/w_i. \tag{1.3}$$

ii. ***A Linear Predictor $\boldsymbol{\eta_i}$*** is defined as linear function of predictors

$$\eta_i = \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} = \sum_{j=1}^{r} x_{ij}\beta_j\,; \quad \forall\, i \in \{1, \dots, n\}, \tag{1.4}$$

where $\boldsymbol{\beta} = (\beta_1, \dots, \beta_r)^{\mathrm{T}}$ is a r-dimensional vector of unknown parameters and x_{ij} is a given value of covariate x_j for observation i. The structure of linear predictor reminds the structure of the standard linear model (SLM). And as in SLM we can use transformations of predictors to create extended versions of GLM. Expression (1.4) is called a systematic component of the model.

iii. ***The Link Function g*** is strictly monotonic and twice differentiable. It is also the fundamental object in the GLMs and transforms the mean of the response variable to the linear predictor. Then the generalized linear model is given by

$$g(\mu_i) = \eta_i = \mathbf{x}_i^{\mathrm{T}}\boldsymbol{\beta} = \sum_{j=1}^{r} x_{ij}\beta_j \,; \quad \forall\, i \in \{1, \dots, n\}, \tag{1.5}$$

The choice of $g(\cdot)$ depends on character of the data and is somehow arbitrary. Common link functions $g(\cdot)$ are given in the next Table 2, in relation with specific probability distributions of the data.

Table 2: Common link functions

Distribution	**Link Function**	$\boldsymbol{g(\mu)}$
normal	identity	μ
Poisson	log	$\ln(\mu)$
binomial	logit	$\ln\left(\frac{\mu}{1-\mu}\right)$
	complementary log-log	$\log(-\log(1-\mu))$
gamma	inverse	μ_i^{-1}

The main objective of this paper is to determine the pure premium, which is given by using statistical models of the claim frequency and severity.

1.2 Probability Distribution of the Claim Frequency

In the research (Denuit, et.al, 2007) has been described, that the most popular model used to estimate the frequency of claims in non-life insurance has a Poisson structure. Assume that $N(t)$ is the number of claims for an individual policy in the time period $[0, t]$, where $N(0) = 0$. The stochastic process $\{N(t); t \geq 0\}$ is called the claims process. This process may be referred to as Poisson process, when the claims are independent, homogeneous and do not cluster. The next part refers to Ohlsson and Johansson (2010), into the section (2.1.1).

Let's denote X_i as number of claims in the tariff cell with duration w_i and expectation μ_i. Then we have $E(X_i) = w_i\mu_i$ where X_i has Poisson distribution with the following frequency function

$$f_{X_i}(x_i; \mu_i) = e^{-w_i\mu_i}\frac{(w_i\mu_i)^{x_i}}{x_i!}, \quad x_i = 0,1,2, \dots \tag{1.6}$$

Our interest is the distribution of the claim frequency $Y_i = X_i/w_i$. Now we provide a transformation of that distribution, we will denote it as relative Poisson distribution. The frequency function is, for y_i, such that $w_i y_i$ is a non-negative integer

$$f_{Y_i}(y_i; \mu_i) = P(Y_i = y_i) = P(X_i = w_i y_i) = e^{-w_i\mu_i}\frac{(w_i\mu_i)^{w_i y_i}}{(w_i y_i)!}$$
$$= \exp\{w_i[y_i \log(\mu_i) - \mu_i] + c(y_i, w_i)\}, \tag{1.7}$$

where $c(y_i, w_i) = w_i y_i \log(w_i) - \log(w_i y_i!)$. If we transform (1.7) to (1.1) by re-parametrizing it through $\theta_i = \log(\mu_i)$, then the Poisson frequency function in EDM is given by following expression

$$f_{Y_i}(y_i; \theta_i) = \exp\{w_i(y_i\theta_i - e^{\theta_i}) + c(y_i, w_i)\}, \tag{1.8}$$

with $\phi = 1$ and the cumulant function $b(\theta_i) = e^{\theta_i}$. The parameter space is $\mu_i > 0$.

1.3 A Model for Claim Severity

In actuarial practice, it is more difficult to estimate claim amount than claim frequency. The model of claim severity in each tariff cell will be the quantity of our interest in this part. Indexation will be temporarily omitted for simple demonstration. The exposure information, the number of claims is denoted by w. Total claim cost in tariff cell is X and claim severity $Y = X/w$ (Ohlsson and Johansson, 2010). In this case, the analysis it is not at all obvious which distribution has X. There are no distributions, for positive real values. The distribution is typically skewed to the right, so the normal distribution is not suitable here (Frees, 2010).

For modelling of claim costs, the gamma distribution has become the standard approach in GLM analysis of claim severity regression problem.

We assume that the cost of individual claim is independently gamma distributed $X \sim G(\alpha, \beta)$, where the frequency function for X is given by

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}; \quad x > 0 \tag{1.9}$$

index parameter $\alpha > 0$ and scale parameter $\beta > 0$. If $X \sim G(w\alpha, \beta)$ then we conclude it as the sum of w independent gamma distributed in random variables. The frequency function for $Y = X/w$ is

$$f_Y(y) = w f_X(wy) = \frac{(w\beta)^\alpha}{\Gamma(w\alpha)} y^{w\alpha-1} e^{-w\beta y}; \quad y > 0 \tag{1.10}$$

and $Y \sim G(w\alpha, w\beta)$.
To transform (1.10) into the form (1.1), we will reparametrize it through $\mu = \alpha/\beta$ and $\phi = 1/\alpha$. Then the frequency function can be re-written

$$f_Y(y) = f_Y(y; \mu, \phi) = \frac{1}{\Gamma\left(\frac{w}{\phi}\right)} \left(\frac{w}{\mu\phi}\right)^{\frac{w}{\phi}} y^{\left(\frac{w}{\phi}\right)-1} e^{-\frac{wy}{(\mu\phi)}} =$$

$$= \exp\left\{\frac{-y/\mu - \log(\mu)}{\phi/w} + c(y, \phi, w)\right\}; \quad y > 0, \tag{1.11}$$

Now we will transform (1.11) to EDM, where the first parameter is changed to $\theta = -1/\mu$ and the new parameter takes value from the open set $\theta < 0$. Then the frequency function of the claim severity Y is expressed as

$$f_Y(y) = f_Y(y; \theta, \phi) = \exp\left\{\frac{-y/\theta + \log(-\theta)}{\phi/w} + c(y, \phi, w)\right\}. \tag{1.12}$$

It follows that the gamma distribution in form of (1.12) belongs to EDM with $b(\theta) = -log(-\theta)$.

1.4 Model Selection

It is very important to investigate whether the chosen model fits the reality well, whether we can add additional rating factor or change the levels for existing ones. In this evaluation, the following statistical tests can be used and give some guidance.

1.5 Measuring the Goodness of Fit

There are several ways to consider the goodness of fit. In this paper we present the deviance and the Pearson chi-square statistics.

A model that has the maximum number of parameters, i.e. the number of parameters is equal of response variables $(r = n)$, is called *saturated model*. It is also referred as a *maximum* or a *full model*. While this model is trivial and of no practical interest, it is often used as a benchmark in measuring the goodness-of-fit of other models, since it has a perfect fit (Kaas, 2009). The maximized log-likelihood function of the full model with maximum achievable value is denoted by

$$l^*(\mathbf{y};\widehat{\boldsymbol{\theta}}^*,\phi)=\sum_{i=1}^{n}\frac{y_i\hat{\theta}_i^*-b(\hat{\theta}_i^*)}{\phi/w_i}+\sum_{i=1}^{n}c(y_i,\phi,w_i)\,, \tag{1.13}$$

where we assume the dispersion parameter ϕ to be fixed. The scaled deviance D^* of the investigated fitted model is then given by

$$D^*=2\left[l(\mathbf{y};\widehat{\boldsymbol{\theta}}^*,\phi)-l(\mathbf{y};\widehat{\boldsymbol{\theta}},\phi)\right]=\frac{2}{\phi}\sum_{i=1}^{n}w_i\left[y_i(\hat{\theta}_i^*-\hat{\theta}_i)-\left(b(\hat{\theta}_i^*)-b(\hat{\theta}_i)\right)\right], \tag{1.14}$$

and is defined as the likelihood-ratio-test (LRT) statistic of the model under consideration, against the saturated model. The LRT is two times the logarithm of the likelihood-ratio, where $\hat{\theta}_i=(b')^{-1}\left(g^{-1}(\mathbf{x}_i^{\mathrm{T}}\widehat{\boldsymbol{\beta}})\right)$.

The other important statistic used for measuring the discrepancy of fit is Pearson´s chi-square χ^2. In a generalized form used in GLMs is defined by

$$\chi^2=\sum_{i=1}^{n}\frac{(y_i-\hat{\mu}_i)^2}{\mathrm{Var}(Y_i)}=\frac{1}{\phi}\sum_{i=1}^{n}w_i\frac{(y_i-\hat{\mu}_i)^2}{\mathrm{v}(\hat{\mu}_i)}, \tag{1.15}$$

where $\mathrm{v}(\hat{\mu}_i)$ is the estimated variance function for the distribution in concern. The Pearson χ^2 statistic is approximately χ^2 distributed with $n-r$ degrees of freedom, where r is the number of estimated parameters β.

1.6 Submodel testing the Significance of Explanatory variable

In the model fitting it is very important to test for the significance of rating factors, since models give more accurate estimates when not overparametrized (Ohlsson and Johansson, 2010). This holds especially in case of small data samples, which are common in non-life insurance.

We would like to test whether we need all rating factors or whether a lower dimensional model can equally well explain the observations. As an aid in deciding whether or not to include the components or a rating factor in a GLM model we can use a hypothesis test. We may perform a likelihood-ratio-test (LRT) of the two models against each other, with and without the particular rating factor. We assume model with p, $1\le p<r$, parameters β omitted as a submodel. This test can be used only for two nested models with a null hypothesis

$H_0\!: submodel\ holds$ vs.alternative $H_1\!: model\ holds.$

The LR statistic is defined as

$$LR=2\log\left[\frac{L(model)}{L(submodel)}\right]=2[L(model)-L(submodel)], \tag{1.16}$$

where LR statistic is always non-negative and under H_0 is approximately χ^2 distributed with p degrees of freedom. The null hypothesis is rejected when the large values of LR are observed. In the distribution of the LR statistic also the parameter ϕ is included and therefore has to be estimated. The LR test is related to the deviance considering the possibility to rewrite the LR statistic into a difference of the deviances (1.14), when in both log-likelihoods the same estimator of ϕ is used. Analogously, this test can be applied recursively to a sequence of nested models. This may lead to a step-wise reduction of complexity. It is preferred cross-validation for model selection which applies in mode generality (Ohlsson and Johansson, 2010).

To compare models, the test statistic LR can also be related to Akaike´s information criterion (AIC)

$$AIC=-2\cdot(\mathbf{y};\widehat{\boldsymbol{\theta}},\hat{\phi})+2\cdot(number\ of\ parameters). \tag{1.17}$$

The preferred model is considered to be that with the lowest AIC.

Empirical Study for a Partial Casco Insurance

In this section we demonstrate the practical usage of GLM that has been introduced above. The data for this empirical study comes from the Swedish insurance company Wasa, and concerns partial Casco insurance. Our portfolio is based on 38 508 policies and 860 associated claims during the years 1994 - 1998. Originally, the data contained some nil claims, which are excluded from the analysis here. We use standard R software environment for fitting GLMs, which can easily be used for our analysis. For each of these insurance policies we have feature information and contains the following variables (with Swedish acronyms):

- ***agarald:*** the owners age, between 0 and 99,
- ***kon:*** gender of a policyholder, M (male) or K (female),
- ***zon:*** geographic zone numbered from 1 to 7, in a standard classification of all Swedish parishes:
    1. central and semi-central parts,
    2. suburbs and middle-sized towns,
    3. lesser towns, except those in 5 or 7,
    4. small towns and country side, except 5 – 7,
    5. northern towns,
    6. northern countryside,
    7. Gotland (Sweden's largest island),
- ***mcklass:*** MC class, a classification by so called EV ratio, defined as (Engine power in kW × 100) / (Vehicle weight in kg + 75), rounded to the nearest lower integer. The 75 kg represent the average driver weight. The EV ratios are divided into seven classes:
    1. 0 – 5,
    2. 6 – 8,
    3. 9 – 12,
    4. 13 – 15,
    5. 16 – 19,
    6. 20 – 24,
    7. > 24,
- ***fordald:*** vehicle age, between 0 and 99,
- ***bonuskl:*** Bonus class, contains values from 1 to 7. A new driver starts with bonus class 1, for each claim-free year the bonus class is increased by 1. After the first claim the bonus is decreased by 2, the driver cannot return to class 7 with less than 6 consecutive claim free years,
- ***duration:*** the number of policies,
- ***antskad:*** the number of claims,
- ***skadkost:*** the claim cost.

The first task that is needed to do is to aggregate the data to the cells of the current tariff and then compute the empirical claim frequency and severity at this level.

4. Claim Frequency

After these data changes have been made, we will start with the average annual number of claims per policy, respectively claim frequency. We will try to find the best fitting model for the claim frequency in terms of given rating factors.

As we mentioned in section (2.1.1), Poisson distribution with a logarithmic link function will be used, where the weights is duration. Table 3 shows the estimates Poisson regression coefficients with their standard errors, Wald z-statistics and p-values as well. We tested null hypothesis $H_0: \beta_j = 0$ vs. $H_1: \beta_j \neq 0$ where for $j = 1,2,\ldots,24$.

Table 3: Summary of Model Frequency

```
              Estimate   Std.Error  z value   Pr(>|z|)
(Intercept)  -4.0923951  0.2159510 -18.951  < 2e-16 ***
konM          0.3352715  0.1348995   2.485  0.01294 *
zon2         -0.5258523  0.1076579  -4.884 1.04e-06 ***
zon3         -1.0058287  0.1176580  -8.549  < 2e-16 ***
zon4         -1.4516881  0.1043295 -13.914  < 2e-16 ***
zon5         -1.6669593  0.3420466  -4.873 1.10e-06 ***
zon6         -1.3549160  0.2482220  -5.458 4.80e-08 ***
zon7         -1.8423136  1.0027848  -1.837  0.06618 .
mcklass2      0.2377589  0.1989874   1.195  0.23215
mcklass3     -0.3779353  0.1692510  -2.233  0.02555 *
mcklass4     -0.2608418  0.1806614  -1.444  0.14879
mcklass5      0.1062366  0.1715856   0.619  0.53582
mcklass6      0.5653134  0.1724443   3.278  0.00104 **
mcklass7      0.2201059  0.4369518   0.504  0.61445
agarald2.L   -1.1248976  0.0811274 -13.866  < 2e-16 ***
agarald2.Q    0.4536069  0.0705986   6.425 1.32e-10 ***
fordald2.L   -0.9359030  0.0909772 -10.287  < 2e-16 ***
fordald2.Q    0.0004939  0.0721518   0.007  0.99454
bonuskl2     -0.0277772  0.1461203  -0.190  0.84923
bonuskl3      0.0127685  0.1587649   0.080  0.93590
bonuskl4      0.2318189  0.1534803   1.510  0.13094
bonuskl5      0.0052189  0.1742598   0.030  0.97611
bonuskl6     -0.0398944  0.1775054  -0.225  0.82217
bonuskl7      0.1593872  0.1133496   1.406  0.15968
```

The base class is made by the young females whose has the new car where is taken the first mc class and the first bonus class. The average number of claims, for example, for a young male who owns an old car and has fourth bonus class and lives in suburbs is equal to exp{-4.092 + 0.335 – 0.526 + 0.565 – 1.125 + 0.001 + 0.232}= 0.010, that is, in average one claim occurs each 100 years.

In addition, we can see, that all p-values of bonus classes are higher than 0,05. We will analyse the overall effect rating factor *bonuskl* where it will be compared the deviance from the full model with the deviance of the model, where the bonus class was excluded.

Table 4: Analysis of Deviance table

Model	Residual Df	Residual Deviance	Df	Deviance	Pr(>Chi)
adjusted model	3838	1439.7			
full model	3832	1434.2	6	5.4785	0.4841

From the summary output of Table 4, chi-squared test indicates that the rating factor bonus class is statistically insignificant predictor of outcome. For this reason, we will reject this rating factor when building our model. For a goodness of fit, we can use residual deviance. From the reference of (1.14) results that if residual difference is small enough, this model is suitable and fits the data.

Table 5: The Goodness of fit Chi-squared test

Residual deviance	Df	p-value
1434.237	3832	1

As it can be seen from the summary output in Table 5, the model fits well because the goodness of fit chi-squared test is not statistically significant, that is, p-value equals to one.

Claim Severity

The next stage in establishing premium is to estimate the claim cost divided by number of claims, i.e., the average cost per claim. Relying on the part (2.1.2) we will model using a Gamma distribution for the errors. Note the point that this is only one of several plausible candidate distributions. Further, we fit gamma distribution with different link functions in full models. The following Table 6 compares deviances together with p-values for presented models, where we can find out how well these models fit the data.

Table 6: Residual Deviance table

Model	Null Deviance	Residual Deviance	p-value
Gamma with inverse link	1520.6	1025.3	$6.015 \cdot 10^{-51}$
Gamma with log link	1520.6	1030.9	$1.162 \cdot 10^{-51}$

From the obtained results, we have chosen the gamma with inverse link model, because it has the smallest residual deviance comparing it with the gamma with log link model. P-values of the both presented models are lower than 0.05, that is, all goodness of fit tests are significally significant.

The summary output in Table 7 shows the estimates inverse gamma regression coefficients with their standard errors, Wald z-statistics and p-values. Just like with claim frequency, here we test null hypothesis too, where $H_0: \beta_j = 0$ vs. $H_1: \beta_j \neq 0$ where for $j = 1,2, \ldots ,24$.

From summary output Table 7, we can see that all p-values are higher than 0.05 in case of bonus class. For this reason we will test the overall effect of this factor by comparing deviance of the full model with adjusted model where bonuskl is excluded.

Table 7: Analysis of Parameter estimates for Severity

```
               Estimate  Std. Error  t value Pr(>|t|)
 (Intercept)  7.572e-05  1.146e-05    6.608 1.16e-10 ***
 konM        -2.203e-05  7.150e-06   -3.081  0.00220 **
 zon2         3.290e-09  1.551e-06    0.002  0.99831
 zon3         9.241e-06  3.468e-06    2.665  0.00799 **
 zon4         2.158e-06  1.747e-06    1.236  0.21723
 zon5         5.217e-05  3.722e-05    1.402  0.16174
 zon6         3.781e-05  2.022e-05    1.870  0.06216 .
 zon7         1.535e-03  2.240e-03    0.686  0.49335
 mcklass2     4.867e-07  1.179e-05    0.041  0.96709
 mcklass3    -2.351e-05  8.251e-06   -2.849  0.00459 **
 mcklass4    -1.871e-05  8.491e-06   -2.204  0.02807 *
 mcklass5    -1.755e-05  8.528e-06   -2.058  0.04017 *
 mcklass6    -2.339e-05  8.277e-06   -2.825  0.00494 **
 mcklass7    -1.034e-05  2.367e-05   -0.437  0.66231
 agarald2.L   5.836e-06  2.328e-06    2.507  0.01256 *
 agarald2.Q   3.680e-06  1.552e-06    2.371  0.01816 *
 fordald2.L   3.705e-05  8.152e-06    4.545 7.15e-06 ***
 fordald2.Q   1.159e-05  5.006e-06    2.316  0.02104 *
 bonuskl2    -8.268e-07  2.962e-06   -0.279  0.78030
 bonuskl3     2.265e-06  4.137e-06    0.547  0.58435
 bonuskl4     1.220e-06  3.151e-06    0.387  0.69881
 bonuskl5     2.572e-06  4.285e-06    0.600  0.54877
 bonuskl6     4.675e-06  5.535e-06    0.845  0.39874
 bonuskl7    -2.619e-06  2.125e-06   -1.233  0.21837
```

P-value from R summary output in Table 8 is equals to 0.3083, which indicates that bonus class is a statistically insignificant predictor. It follows that the rating factor bonus class is for our analysis meaningful and we will reject it from the full initial model.

Table 8: Model Comparison by Deviance

Model	Residual Df	Residual Deviance	Df	Deviance	Pr(>Chi)
Adjusted model	434	1040.4			
Full model	428	1025.3	6	15.124	0.3083

Modeling the Pure Premium

The standard approach in GLM tariff analysis is to do separate econometric modelling for claim frequency and claim severity. There is a question, why we have built our analysis into two GLM models? Ohlsson and Johanesson (2010) have both shown in section (2.3.4) that:

i. Claim frequency is usually much more stable than claim severity and often much of the power of rating factors is related to claim frequency: these factors can be estimated with greater accuracy;
ii. A separate analysis gives more insight into how a rating factor affects the pure premium.

The Pure premium is the expectation of the annual cost of claims of policyholders and is obtained by multiplying given GLMs model for estimated claim frequency and claim severity.

As a basic tariff cell (2, 4, 2, 3, 2) we have chosen the class with the highest duration. In the next step, it is necessary to calculate the duration and number of claims for each level of each rating factor. After the changes have been applied, it can be determined the relativities for claims frequency and claim severity separately by using GLMs. Then the relativities for the pure premium are found by multiplying the results. Consequently, the pure premium is the product of the claim frequency and the claim severity, what can be expressed as

$$Pure\ Premium = Claim\ frequency\ \times Claim\ severity$$

General results are shown in Table 9, where all relativities with duration and number of claims for each rating factor.

Table 9: Relativities from a Poisson GLM for claim frequency and an inverse gamma GLM for claim severity

Rating factor	Class	Duration	No. of Claims	Relative Frequency	Relative Severity	Relative Pure Premium
Gender	*F*	7 126	61	1.40	1.42	1.99
Gender	***M***	**58 111**	**636**	**1.00**	**1.00**	**1.00**
Zone	1	6 205	183	0.23	0.98	0.23
Zone	2	10 103	167	0.36	0.89	0.32
Zone	3	11 677	123	0.59	1.00	0.59
Zone	**4**	**32 628**	**196**	**1.00**	**1.00**	**1.00**
Zone	5	1 582	9	0.25	0.67	0.17
Zone	6	2 800	18	0.19	0.60	0.11
Zone	7	241	1	0.16	0.05	0.08
Age	1	11 225	353	1.55	0.94	1.47
Age	**2**	**33 676**	**235**	**1.00**	**1.00**	**1.00**
Age	3	20 336	109	0.33	0.94	0.31
MC class	1	5 190	46	1.16	1.30	1.50
MC class	2	3 990	57	1.86	1.47	2.73
MC class	**3**	**21 666**	**166**	**1.00**	**1.00**	**1.00**
MC class	4	11 740	98	0.69	1.46	1.02
MC class	5	13 440	149	1.27	1.00	1.27
MC class	6	8 880	175	0.79	1.33	1.05
MC class	7	331	6	1.31	1.16	1.52
Vehicle age	1	17 228	314	0.99	0.87	0.88
Vehicle age	**2**	**27 160**	**304**	**1.00**	**1.00**	**1.00**
Vehicle age	3	20 849	79	0.39	0.69	0.26

5. Conclusions

In this study, we calculated the pure premium for insurance data from the former Swedish insurance company Wasa in the years 1994 – 1998. This paper considers an analysis by using the GLMs. At a first stage, the frequency model of claims was built through Poisson regression. Then, by using the inverse gamma model were estimated average claim costs in term of given rating factors.

Eventually, the research results have shown that younger people, obviously females have more claims that men. The rating factor zone affects the claim frequency and the claim severity in the same direction, which means that vehicles in age among 5 – 15 from small towns are more risky – they are more often stolen. Based on the summary output from Table 7, for zone 7 and for MC class 7, it is rather obvious that no conclusions can be drawn due to very small number of claims, and hence very uncertain estimate of claim severity.

The conclusions of this study can be useful for many insurance companies. But it is also necessary to mentioned that they can't be applied in generalized character, e.g. in all insurance companies or in their portfolios. Every company uses different strategy, benefits and information and it just requires a unique and professional approach to such a GLM model that will be reliably capture the results.

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Hedging from Accounting Perspective: Theory versus Reality

Jiří Strouhal [1]

Abstract

Paper provides a critical evaluation of the hedge accounting principles and real practices. There is discussed the shift in hedge effectiveness measurement from old international financial reporting standard IAS 39 (Financial Instruments: Reporting and Measurement) towards a new IFRS 9 (Financial Instruments) being its full replacement. There are also discussed the effect on the Czech accounting legislature. Within the analytical part is provided the critical evaluation of these changes from the practical point of view based on the real behaviour of Czech companies having hedge relations. Practical position proves that the legislature (and theory) as well is now even farther from the corporate reality and companies have to rely on professional preparations of hedge documentations and hedge effectiveness testing probably even more than before.

Key words

hedge accounting, fair value hedge, cash flow hedge, hedge effectiveness, hedge documentation, IFRS 9, IAS 39, Czech accounting legislature

JEL Classification: M41, G30

1. Introduction

There shall be stated, that hedge accounting is just a possible legal alternative for a company. It may voluntary decide whether to treat all derivatives as held for trading or will adopt the hedge accounting rules. In such case there shall be fully met following conditions of hedge accounting:

- at the inception of the hedge there is formal designation and documentation of the hedging relationship and the entity's risk management objective and strategy for undertaking the hedge. That documentation shall include identification of the hedging instrument, the hedged item or transaction, the nature of the risk being hedged and how the entity will assess the hedging instrument's effectiveness in offsetting the exposure to changes in the hedged item's fair value or cash flow attributable to the hedged risk;
- the hedge is expected to be highly effective in achieving offsetting changes in fair value or cash flows attributable to the hedged risk, consistently with the originally documented risk management strategy for that particular hedging relationship
- the effectiveness of the hedge can be reliably measured, i.e. the fair value or cash flows of the hedged item that are attributable to the hedged risk and the fair value of the hedging instrument can be reliably measured;

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- for cash flow hedges, a forecast transaction that is the subject of the hedge must be highly probable and must present an exposure to variations in cash flows that could ultimately affect profit or loss;
- the hedge is assessed on an ongoing basis and determined actually to have been highly effective throughout the financial reporting periods for which the hedge was designated (IASB, 2015).

The reason why companies mostly aim to apply hedge accounting is linked to tax deductibility, despite in real practice significant percentage of companies do not test the hedge effectiveness, neither prepare the hedge documentations. However this is mostly done by non-audited SMEs.

Pirchegger (2006) is concerned with the fact that accounting entities tend to note primarily the high level of disclosure obligations in relation to hedge accounting and the costs related thereto. On the other hand, the primary goal of the standard-issuing authority is the incontestable effort to provide investors with highly relevant information. The fact that the information on hedge accounting should form an indivisible part of the financial statements is motivated by the effort to assure investors that the criteria applicable to the field of hedge accounting were applied correctly rather than by the fact that the information on hedge accounting causes considerable additional costs to accounting units.

Numerous studies in our professional practice have dealt with the bond between the economic and the accounting concept of hedging. Melumad et al. (1999) for instance, indicates that the application of hedge accounting in compliance with the US standard SFAS 133 leads to deviations from optimum hedging in the economic sense. However Barnes (2001) draws attention to the fact that these deviations from economic hedging are the very consequence of the set hedge accounting model, pointing out that hedge accounting may motivate poorly performing companies to speculate and influence their economic results on a short-term basis.

Several studies have also dealt with the information and control effects of hedge accounting (Hughes et al., 2002; Jorgensen, 1997). The most interesting finding lies in the fact that the voluntary application of hedge accounting leads to a deviation from the optimum hedging strategy (as opposed to the exclusive application of economic hedging without the application of the principles of hedge accounting).

The reminder of this paper is as follows. Firstly there would be discussed general rules of hedge accounting based on IAS 39, IFRS 9 and Czech accounting legislature. There would be provided the information what shall be covered by hedge documentation and which hedge relationships we may differ. Analytical part would be focused on most common cash flow hedges in current practice discussing the major problems companies shall deal with.

2. Hedge Accounting: Basic Principles

2.1 A Hedging Instrument

Very important term linked with hedge accounting is a hedging instrument. This one is mostly understood as a designated derivative or, for a hedge of the risk of changes in foreign currency exchange rates only, a designated non-derivative financial asset or non-derivative financial liability whose fair value or cash flows are expected to offset changes in the fair value or cash flows of a designated hedged item. As possible hedging instruments might be mentioned derivatives; non-derivatives for exchange rate risks and lastly intrinsic values of derivatives (not the decreasing time values – for options).

2.2 Hedge Effectiveness: Shift from IAS 39 Approach towards IFRS 9 Approach

The biggest practical problem is measuring the hedge effectiveness, i.e. the degree to which changes in the fair value or cash flows of the hedged item that are attributable to a hedged risk are offset by changes in the fair value or cash flows of the hedging instrument.

The main aim of prospective testing of the hedge effectiveness is to provide an evidence of highly effective relationship between hedging instrument and hedged item when compensating the defined hedged risk. Prospective tests are realized upon the dates of interim financial statements as well as annual financial statements.

The main aim of retrospective testing is to provide evidence that the hedge relationship was effective in compensation of the defined hedge risk during tested accounting period.

Among most popular methods how to calculate the effectiveness of hedge relationship can be stated "dollar-offset" method and regression analysis (Strouhal and Bonaci, 2011).

"Dollar-offset" method is considered as a more practically oriented one for the computation of the effectiveness, however it may cause, that a substantial volume of hedge relationships could be considered as ineffective. This method could be used upon following conditions: (i) the basic characteristics of hedging instrument and hedged item are covered (and are compensated), or (ii) these characteristics are not covered, however the expected extent of change is very slight and will not generate significant changes in fair values between hedging instrument and hedged item what otherwise might cause an inefficiency of hedge relationship.

Regression analysis tests the statistical relation between the hedged item and hedging instrument. It provides the best tool for determination of the level of dependence. Entities typically use regression analysis to measure the effectiveness (prospective as well as retrospective) in areas where they did implemented various strategies to hedge the portfolio risks.

Based on "old" IAS 39 rules, a hedge is regarded as highly effective only if both of the following conditions are met:

- at the inception of the hedge and in subsequent periods, the hedge is expected to be highly effective in achieving offsetting changes in fair value or cash flows attributable to the hedged risk during the period for which the hedge is designated. Such an expectation can be demonstrated in various was, including a comparison of past changes in the fair value or cash flows of the hedged item that are attributable to the hedged risk with past changes in the fair value or cash flows of the hedging instrument, or by demonstrating a high statistical correlation between the fair value or cash flows of the hedged item and those of the hedging instrument;
- the actual results of the hedge are within a range of 80 – 125 %. (IASB, 2015)

A "new" IFRS 9 treatment of hedge effectiveness is quite fuzzy. Within this standard is said that the hedging relationship shall meet all of the following requirements:

- there is an economic relationship between the hedged item and the hedging instrument (i.e. same requirement as within IAS 39);
- the effect of credit risk does not dominate the value changes that result from that economic relationship (not mentioned previously); and
- the hedge ratio of the hedging relationship is the same as that resulting from the quantity of the hedged item that the entity actually hedges and the quantity of the hedging instrument that the entity actually uses to hedge that quantity of hedged item. However, that designation shall not reflect an imbalance between the weightings of the hedged item and the hedging instrument that would create hedge ineffectiveness (irrespective of whether recognised or not) that could result in an accounting outcome that would be inconsistent with the purpose of hedge accounting.

Especially the last requirement sounds from the practical point of view quite discussable, so from this perspective we may employ and explanatory guidance in IFRS 9 (B6.4.9-11). Within B6.4.9 is mentioned if an entity hedges less than 100 % of the exposure on an item, such as 85 %, it shall designate the hedging relationship using a hedge ratio that is the same as that resulting from 85 % of the exposure and the quantity of the hedging instrument that the entity actually uses to hedge those 85 %. Similarly, if, e.g. an entity hedges an exposure using a nominal amount of 40 units of a financial instrument, it shall designate the hedging relationship using a hedge ration that is the same as that resulting from that quantity of 40 units (i.e. the entity must not use a hedge ratio based on a higher quantity of units that it might hold in total or a lower quantity of units) and the quantity of the hedged item that it actually hedges with those 40 units (IASB, 2018).

2.3 Czech Legal Approach

Till the end of 2017 there was fully applied the approach of IAS 39 in terms of hedge accounting in the Czech accounting legislature. The year 2018 brought a change for the reporting of financial institutions where shall be applied the new IFRS 9 treatment, however for business entities is applied the following paragraph from the Decree 500/2002 Col.: "For the reporting, measurement and presentation of the information about derivatives shall be applied the decree 501/2002 Col. for financial institutions in the form valid as at 31st December 2017". What does this really fuzzy information mean? Basically there shall be applied very same rules like in previous years, but Ministry of Finance decided to change understandable paragraphs with logical explanations to this sentence (Strouhal, 2018).

To sum it up – business entities have to have the results of hedge effectiveness on retrospective bases between 80 – 125 % (hedging instrument against hedged item), however banks and other financial institutions according the last IFRS 9 requirement on the hedge ratio.

Basically the IFRS 9 requirement works well for "Big4" companies who always pushed the business entities to account for hedge ratio higher than 100 % as ineffective one.

3. Critical Evaluation of the Hedge Accounting from Practical Insight

In practice of business entities might be applied these three hedge relationships:

- fair value hedge
- cash flow hedge
- hedge of a net investment in a foreign transaction

Fair value hedge is understood as a hedge of the exposure to changes in fair value of a recognised asset or liability or an unrecognised firm commitment, or an identified portion of such an asset, liability or firm commitment, that is attributable to a particular risk and could affect profit or loss.

If a fair value hedge meets the condition during the period, it shall be accounted as follows:

- the gain or loss from remeasuring the hedging instrument at fair value shall be recognised in profit or loss; and
- the gain or loss on the hedged item attributable to the hedged risk shall adjust the carrying amount of the hedged item and be recognised in profit or loss. This applies if the hedged item is otherwise measured at cost. Recognition of the gain or loss attributable to the hedged risk in profit or loss applies if the hedged item is an AFS financial asset.

Cash flow hedge is understood as a hedge of the exposure to variability in cash flows that is attributable to particular risk associated with a recognised asset or liability or a highly probable forecast transaction, and could affect profit or loss. Gains or losses from the hedging instrument revaluation are accounted against the revaluation fund (part of other comprehensive income)

If a cash flow hedge meets the condition during the period, it shall be accounted as follows:

- the portion of the gain or loss on the hedging instrument that is determined to be an effective hedge shall be recognised in other comprehensive income; and
- the ineffective portion of the gain or loss on the hedging instrument shall be recognised in profit or loss.

Hedges of a net investment in a foreign operation shall be accounted for similarly to cash flow hedges.

As mentioned before, one of the required conditions for the hedge accounting application is the preparation of hedge documentation. The hedge documentation has to cover following areas:

- purpose and strategy of risk management
    - the entity has to clearly explain the reason for the negotiation of the hedge relationship. There has to be provided the statement of compliance of this relationship with company's risk management policy, eventually also some cross-reference which is valid to a specific type of hedge relationship
- type of hedge relationship
- the entity has to identify the type of hedging relationship (see below), i.e. fair value hedge, cash flow hedge or hedge of net foreign investment
- character of hedged risks
    - the entity has to clearly identify all risks being hedged, e.g. currency risks, interest rate risks, etc.
- identification of hedging instrument
    - the entity has to provide a detailed description of the hedging instrument to be clearly be identified
    - hedging instrument is a designated derivative or (for a hedge of the risk of changes in foreign currency exchange rates only) a designated non-derivative financial asset or non-derivative financial liability whose fair value or cash flows are expected to offset changes in the fair value or cash flows of a designated hedged item
- identification of hedged item
    - the entity has to provide a detailed description of the hedged item to be clearly be identified
    - hedged item is an asset, liability, firm commitment, highly probable forecast transaction or net investment in a foreign operation that (i) exposes the entity to risk of changes in fair value or future cash flows and (ii) is designated as being hedged.
- estimated transaction
    - the entity has to declare the high probability of the estimated transaction realization:
        - the transaction is highly probable in case, that there is a certainty of almost 100 %,
        - information, whether the transaction represents a threat to the volatility of cash flow due to the hedged risk which may affect company's profit,
    - documentation has to contain the information about the time determination, nature and volume of estimated transaction, i.e. hedged quantity or amount,

- identification of the mechanism for the effectiveness testing
    - the entity has to clearly describe at the very beginning of the hedge relationship the method, throughout which will test the prospective as well as retrospective effectiveness of this relationship and have to disclose following information:
        - whether the entity use the whole amount of change in fair value of hedging instrument or eliminates its concrete part (e.g. time value of option which could not be used as a hedging instrument),
        - methodology for the calculation of the changes in fair value of the hedging instrument for the effectiveness tests,
        - methodology for the calculation of the changes in fair value of hedged item for the effectiveness tests,
    - the entity is required to expect the high level of effectiveness throughout the duration of the hedge relationship.

An entity shall discontinue prospectively the hedge accounting if:

- the hedging instrument expires or is sold, terminated or exercised;
- the hedge no longer meets the criteria for hedge accounting; or
- the entity revokes the designation.

In any of the following circumstances an entity shall discontinue prospectively the hedge accounting:

- the hedging instrument expires or is sold, terminated or exercised. In this case, the cumulative gain or loss on the hedging instrument that has been recognised in other comprehensive income from the period when the hedge was effective shall remain separately in equity until the forecast transaction occurs;
- the hedge no longer meets the criteria for hedge accounting. In this case, the cumulative gain or loss on the hedging instrument that has been recognised in other comprehensive income from the period when the hedge was effective shall remain separately in equity until the forecast transaction occurs;
- the forecast transaction is no longer expected to occur, in which case any related cumulative gain or loss on the hedging instrument that has been recognised in other comprehensive income from the period when the hedge was effective shall be reclassified from equity to profit or loss as a reclassification adjustment. A forecast transaction that is no longer highly probable may still be expected to occur;
- the entity revokes the designation. For hedges of a forecast transaction, the cumulative gain or loss on the hedging instrument that has been recognised in other comprehensive income from the period when the hedge was effective shall remain separately in equity until the forecast transaction occurs or is no longer expected to occur. If the transaction is no longer expected to occur, the cumulative gain or loss that had been recognised in other comprehensive income shall be reclassified from equity to profit or loss as a reclassification adjustment.

Now we may focus on the most common cash flow hedge relationships in current Czech corporate practice:

- hedging of rent against loan payments
- loan secured by swap
- forward sales of foreign currency hedging domestic payments

3.1 Hedging of Rent against Loan Payments

For the hedge of exchange rate risks might be also use non-derivative contracts. Among developers became popular the hedge of loans against received rentals in foreign currency.

The procedure of this hedge relation will be following. As a hedging instrument are understood loans being paid in foreign currency. As a hedged item are understood rental payments to be received in foreign currency. From this perspective it is understandable that we are hedging the exchange rate risk and because of the forecasting of hedged item we are dealing with cash flow hedge.

At the very beginning there is estimated similar evolution of hedging instrument and hedged item. There are valid following details:

- all transactions are negotiated in very same currency,
- value of loans is less than present value of future cash flows from rental contracts,
- there are not estimated significant changes in hedge items (e.g. dramatic increase/decrease of rental payments).

From this point of view there it is estimated the hedge effectiveness throughout its whole duration. Changes in rental payments in foreign currency will be compensated by changes in values of loan funding in foreign currency.

For the retrospective testing the entity will compare the cumulative change in fair value of received rentals in foreign currency and cumulative change in fair value from loan funding in foreign currency. In case that present value of the rental payments received will be higher than present value of the loan funding, for the test of effectiveness will be used only present value up to the value of loan funding.

This type of hedging is very popular among developers, but also within the relationships between parent company and subsidiary (e.g. renting of warehouses from each other). Problem of large development projects (e.g. large shopping malls) is the duration of the hedge relationship. Mostly the loans shall be repaid in 30+ years and there is an issue how to estimate the rental payments what the owner will receive from lessees in 2040 or even later. These estimates are really fuzzy and the reliability strongly varies. That’s why there is understood as an effective hedge the case when the present value of rental payments is higher than present value of loan payments. The reason why companies are using this hedge relationship is the fact that loans is revalued at fair value via revaluation surplus and from this surplus are compensated interest payments (based on the effectiveness) – from this perspective there is minimised the interest rate risk and its effect on corporate profit/loss.

3.2 Loan Secured by Interest Rate Swap

A few years ago it became extremely popular by banks to provide clients loans secured by the interest rate swaps. The case might be the situation that your company will get a loan with the interest rate of 3 % p.a. + 1M PRIBOR (EURIBOR or another float interest rate). But to get this loan, you have to sign a swap contract that you will receive from bank 1M PRIBOR (EURIBOR, etc.) and will pay to bank 2.5 % p.a. on monthly basis.

From the hedge accounting point of view this is an optimum 100 % hedge as you have to pay 1M PRIBOR and this one you will get from bank. So there are just back-tested present values of those PRIBOR payments on annual basis (estimates done based on yield curves and bank predictions).

Question is whether Czech banks will continue in this offer these days when interest rates are rising and contracts negotiated by 2016 does not sound such profitable for them in long term run.

If the previous hedge relationship effectiveness is sort of crystal ball challenge, here the only problem for the back-testing is the future interest rates expectations (despite if the expectation would be wrong, by same amount are affected both swap as well as loan, so the hedge ratio will not be affected).

3.3 Forward Sales of Foreign Currency when Hedging Liabilities in Domestic Currency

This type of hedge relationship is common for exporters gaining sales in foreign currency while majority of their liabilities is in domestic currency.

As an optimal hedge instrument might be used a series of FX forwards (rather than one FX swap) which are copying the prospective sales of the company. There shall be estimated the future liabilities in domestic currency linked to these sales. Mostly we are dealing with liabilities from material and energy consumptions, salaries and VAT aspects. On the prospective basis the estimates of liabilities shall be nearly close to those translated FX forward sales on domestic currency. For the retrospective testing there are computed the real liabilities settled within the period.

The computation of a hedge ratio is not a big issue in this hedge relationship. However there might be a problem when the sales are delayed whether this hedge relationship is highly probable or not (e.g. sales delayed for one month). From the practical point of view the decision is linked to materiality and the proportion of the delay towards the contract duration (or intervals of payments).

3.4 Corporate Hedge Accounting: Urban Legends

The most proper grand finale of this chapter might be to discuss the so called urban legends in terms of hedge accounting in Czech corporate practice.

Legend No. 1. Auditors sometimes have a no idea that company is using derivatives or apply hedge accounting. Companies in this case are aiming to play in a market casino and e.g. FX forwards are just accounted on monthly basis as exchange rate gains or losses. In case that these results are under the materiality level there might be possible that auditor will not recognise that company is using derivatives. The biggest problem are open positions at the year-end which shall be revaluated at fair value (mostly with profit/loss impact).

Legend No. 2. Company is using derivatives and because the accountant understood from the legislature there is a possibility to revaluate certain derivatives through revaluation surplus in equity simply does that. This possibility is however applicable only for the case of cash flow hedge or a hedge of a net foreign investment. Sometimes when a new auditor is overtaking a client he is surprised that company did this case for many years (of course with a heavy impact on income tax) and now the company is surprised that these items shall be re-taxed (with significant penalty from the fiscal authority). Moreover there is no evidence about hedge accounting and sometimes it is applied for a cases where is applicable fair value hedge only.

Legend No. 3. This case may happen in multinational companies with asset liability management at parent. The ALM department at parent decide to hedge some relationships of subsidiaries without informing them. These subsidiaries are after that surprised at the year-end when receiving the bank confirmations that there were negotiated (on behalf of themselves) some hedge relationships. Then is mostly used hotline to parent what really happened and what to do.

4. Conclusive Remarks

The change from IAS 39 towards IFRS 9 requirements is challenging, however currently it doesn’t affect anyhow companies reporting under Czech accounting rules. For listed companies of course the change shall be applied for the financial statements of 2018 at the latest. The most significant difference is the new computation of a hedge ratio and just the future will show whether this change was logical or not and to what extent it might be really applicable in common practice.

From the reporting point of view many companies still don't have a no idea how to properly report derivatives and hedge accounting relationship what is illustrated on two legends in subchapter 3.4 which are still very common.

As a potential limitation of this paper might be mentioned it's a critical evaluation only, so it is based on authors' professional experience with the approached area and the reality among Czech accounting entities. For the future research there might be employed qualitative analysis (interviews with stakeholders about the hedge accounting) rather than quantitative research where the results may vary between entities in difference sectors.

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Interest Rate Risk in the Banking Book (IRRBB) – Forecast quality of the EBA scenarios comparing to the Historical Simulation

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Abstract

Interest rate risk arising from non-trading activities, so called Interest Rate Risk in the Banking Book (IRRBB), becomes more and more important in times of low interest rates. Especially a historical low level of yields within the strategic expansion of maturity transformation stresses the profitability of institutes. Setting a standard for an adequate risk measurement the EBA defined six interest rate risk scenarios. Hence, measuring IRRBB with these scenarios leads to the question whether there is an empirical proof of the forecast quality of these scenarios comparing to the Historical Simulation in a monthly backtesting. In addition, the empirical analysis has the aim to show if the six EBA scenarios generate an adequate forecast quality in case of increasing yields.

Key words

Value at Risk, EBA, IRRBB, forecast quality

JEL Classification: G21, G24, G28, G32

1. Introduction

Interest rate risk is, especially in the German banking sector, one of the main risks [Reuse (2012), pp. 7 ff.]. Strengthened by the low level of yields, German banks offers long maturities to their customers and deal with a high level of long term orientated maturity transformation [Reuse and Svoboda (2014a), pp. 37 ff.; Reuse and Svoboda (2014b), pp. 376 ff.].

Setting a standard on measuring Interest Rate Risk in the banking book (IRRBB) the EBA published a Guideline in 2017, which was finalized on July 19th, 2018 [EBA (2017); EBA (2018)]. The final Guidelines offer nearly no material changes compared to the consultation. The Guideline defined six scenarios which states an adequate measure of risk [EBA (2018), p. 34]. While several characteristics of interest rate risk management and measures were already analysed, the empirical validity of the six EBA scenarios as an adequate risk measurement method for the banking book was not analysed yet in total. First comparisons were done by the authors in 2018 [Svoboda, Reuse, Rüder and Boka (2018), pp. 712 ff.]. This article extends these results and has the aim to analyse the empirical validity of the EBA scenarios comparing to the Historical Simulation in a monthly backtesting.

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Supposing that the forecast quality of the Historical Simulation is caused by the length of the historical dataset and also by different cashflow structures, the empirical analysis considers different cashflow structures and a forward-looking scenario of increasing yields [Boka and Rüder (2018), pp. 28 ff.; Wegner and Sievi (2017)]. Extending the results of Svoboda, Reuse, Rüder and Boka (2018), pp. 712 ff., the following academic aspects should be answered:

a) *Are the six EBA interest risk scenarios an adequate measurement approach during time?*
   This paper analyses the forecast quality of the six EBA scenarios during time through the historical dataset.
b) *Are the six EBA scenarios a better measurement than the Historical Simulation during time?*
   Within this monthly backtesting the results were compared to the Historical Simulation which is widely used and accepted in theory and practice.
c) *Are the six EBA scenarios and the Historical Simulation in case of an increasing yield curve structure an adequate risk measurement method during time?*
   In addition, the paper answers the question, whether the forecast quality of both risk measurement methods in a forthcoming scenario of increasing yields lead to the consumption of an adequate risk measurement.

2. Measuring IRRBB under the actual EBA Guideline

2.1 General requirements of the EBA Guideline

As a pillar 2 Guideline the EBA "… highlight that institutions should develop and use their own internal arrangements to identify, measure, monitor and control IRRBB, while respecting supervisory expectations set out in these guidelines" [EBA (2018), p. 3]. Considering the supervisory expectations on the management of IRRBB it affects the specific supervisory requirements of to the SREP (Supervisory Review and Evaluation Process) as a pillar 1 instrument. It states that IRRBB is an important risk and should considered in the internal risk management process [EBA (2018), p. 3].

The Guideline distinguish in gap risk, basis risk and option risk. Furthermore, the guideline provides an earning and also an economic value measure. The earnings measure focuses the future profitability including individual planning's of the banks, the economic value measure focuses changes of the static economic value due to yield changes [EBA (2018), p. 13 f.]. Interest bearing positions of IRRBB includes off-balance sheet positions as well as interest rate derivatives and other interest rate sensitive instruments which are not held in the trading book. Depending on the riskiness, complexity and the level of sophistication and the total investment of the instruments, the Guidelines provides adequate investment, risk und management strategies including a business environment to operate into [EBA (2018), pp. 20 ff.]. The internal measurement should meet both perspectives of IRRBB (economic value and earnings).

Regarding to the internal model approach the EBA requires six interest rate risk scenarios which should be implement in the risk management framework. These scenarios are described in the following section. Quarterly, in times of high volatility e.g. more frequent, the institutions should measure their IRRBB-exposure. Within, two shock scenarios, known as a +/-200bp shift, should calculate quarterly in an EVE-perspective. Institutes with 20% economic value changes are assumed to have high IRRBB-risk [EBA (2018), pp. 34 ff.].

In general, the Guideline reflects the Internal Capital Adequacy Assessment Process (ICAAP) and the SREP and regulate different perspectives of IRRBB. In order to provide a comparable risk measurement six scenarios to measure interest rate risk and benchmark institutes are introduced by the Guideline.

2.2 EBA-interest-rate-risk scenarios

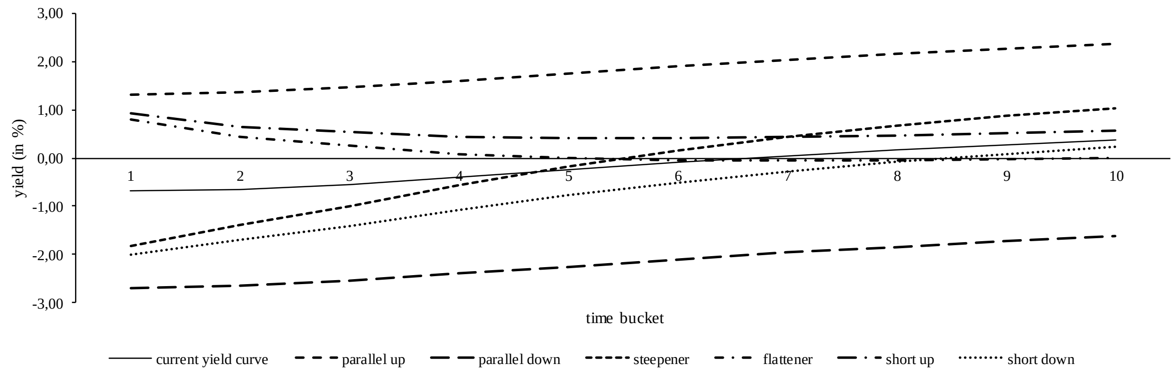
Focussed on the six EBA scenarios they cannot be defined as a professional risk measurement, although they are used for a supervisory review and outlier test [Reuse (2018), pp. 148 ff.; EBA (2018), p. 4 f]. In addition to this, an early warning indicator is implemented. The present value losses of the economic value should be less than 15% of Tier 1 capital [EBA (2018), pp. 8 ff. and pp. 62 ff.].

Nevertheless, these scenarios seem to be highly important for IRRBB-measurement and managing additional capital buffers. Beginning with the estimation of the scenarios there are three shift parameters for Euro yield curves [EBA (2018), pp. 49 ff.]:

- parallel: 200 bp,
- short: 250 bp,
- long: 100 bp.

In Annex III [EBA (2018), pp. 49 ff.] the EBA displays a way of estimation the shock parameters for each currency. All parameters are based on historical interest rates from 2000 to 2015. According to this shift parameters, the EBA provides functions to estimate shock parameters and of course a yield curve which considers each maturity. Even though the Basel II interest rate shocks only require a parallel shift, the additional scenarios also require a rotation of the yield curve. Therefore, the EBA Guideline implement a steepener shock (short rate ↓ long rate ↑), a flattener shock (short rate ↑ long rate ↓), and a short rate shock up and down. For these scenarios, the Guidelines provide a bank-individual risk-free curve. Negative interest rates are possible. The following figure 1 shows the six different scenarios, estimate from the actual yield curve [Deutsche Bundesbank (2018); EBA (2018), pp. 49 ff.].

Figure 1: yield curves of the EBA interest rate risk scenarios



	1 Yr.	2 Yr.	3 Yr.	4 Yr.	5 Yr.	6 Yr.	7 Yr.	8 Yr.	9 Yr.	10 Yr.
current yield curve	-69,0 bps.	-65,0 bps.	-54,0 bps.	-40,0 bps.	-25,0 bps.	-10,0 bps.	4,0 bps.	16,0 bps.	27,0 bps.	37,0 bps.
parallel up	200,0 bps.	200,0 bps.	200,0 bps.	200,0 bps.	200,0 bps.	200,0 bps.	200,0 bps.	200,0 bps.	200,0 bps.	200,0 bps.
parallel down	-200,0 bps.	-200,0 bps.	-200,0 bps.	-200,0 bps.	-200,0 bps.	-200,0 bps.	-200,0 bps.	-200,0 bps.	-200,0 bps.	-200,0 bps.
steepener	-112,9 bps.	-73,0 bps.	-45,2 bps.	-15,3 bps.	8,0 bps.	26,2 bps.	40,3 bps.	51,3 bps.	59,8 bps.	66,5 bps.
flattener	148,9 bps.	107,9 bps.	79,2 bps.	48,4 bps.	24,4 bps.	5,7 bps.	-8,8 bps.	-20,1 bps.	-28,9 bps.	-35,8 bps.
short up	160,7 bps.	129,1 bps.	107,1 bps.	83,4 bps.	64,9 bps.	50,6 bps.	39,4 bps.	30,7 bps.	23,9 bps.	18,6 bps.
short down	-130,6 bps.	-104,9 bps.	-87,0 bps.	-67,7 bps.	-52,8 bps.	-41,1 bps.	-32,0 bps.	-24,9 bps.	-19,4 bps.	-15,1 bps.

Source: taken from Svoboda, Reuse, Rüder and Boka (2018), p.715 based on data of Deutsche Bundesbank (2018).

2.3 Measuring IRRBB with the Historical Simulation

As mentioned, the EBA scenarios provided a simple measure of economic value changes but cannot characterized as a professional risk measurement method [Reuse (2018), pp. 148 ff.]. Therefore, the following empirical analysis implements the Historical Simulation as well. Of course, there are some other risk measurement methods like variance/covariance and monte-carlo as well as a copula model as a modern approach for IRRBB [Svoboda, Reuse, Rüder and Boka (2018), pp. 712 ff.; Reuse, Boka and Rüder (2018-E); Daníelsson (2012), pp. 73 ff.]. Furthermore, the Historical Simulation is widely spread in theory and practice of risk management. This could justify by its very simple approach of calculating historical changes of the interest rate and expecting that they are representative for the future [Choudhry, Moskovic and Wong (2014), pp. 79]. Hereby, the observed changes are transmitted to the actual yield curve in order to measure the varying economic values. In difference to the economic value there are historic observable changes of the economic value which represent the future economic value changes [Schierenbeck, Lister and Kirmße (2008) pp. 96 ff.]. The quartile, respectively the Value at Risk, is the maximum unexpected change of the actual present value in a specific confidence interval and holding period [Eddari (2007), pp. 26 ff.].

Especially, the assumed stationarity of the process is an important disadvantage of the Historical Simulation, but it is nearly the only one [Schierenbeck, Lister and Kirmße (2008) pp. 96 ff.; Boka (2018), pp. 50 ff.]. In contrast, the freedom of model assumptions is a great advantage. In addition, the goodness of fit depends in the stationarity of the process and also on the length of the data set. Due to the length of the data set, the historic observable extreme events affect the forecast quality of extreme events. Implied volatilities or fat fails, which are not observed in the data set, are still not considered in the Historical Simulation [Wiedemann (2013), pp. 65 ff.; Boka (2018), pp. 50 ff.].

So the conclusion could be drawn that the Historical Simulation is widely spread in theory and practice because of the low effort to implement and the very simple mathematical approach. Therefore, we use the Historical Simulation to compute an alternative risk measurement method to the EBA scenarios.

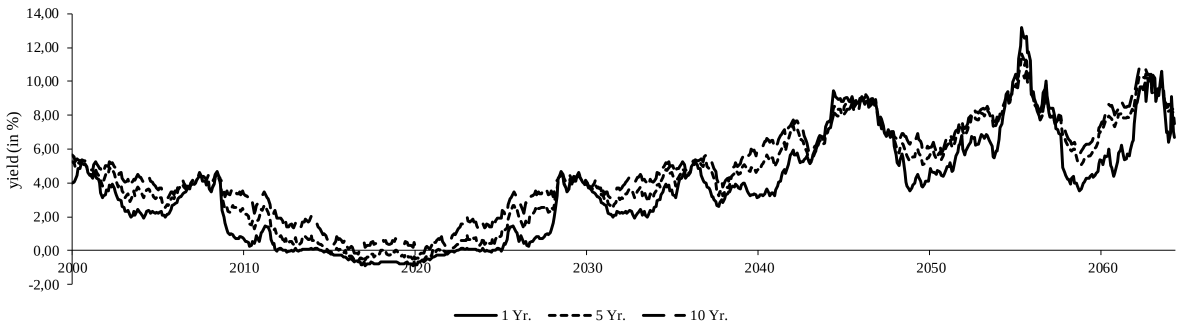
3. Data and Methodology

3.1 Used Data

The following analysis compares the EBA scenarios with the Historical Simulation. Therefore, German sovereign bond yields are used as a risk-free yield curve [Deutsche Bundesbank (2018)]. The long history is an advantage of the chosen yield curve although the EBA provides a historic data set since 2000. The dataset is available for the past 46 years. Similar to the EBA scenarios we compute our risk estimation from 2000 up to 2018 on historic data.

Next to a historic backtesting of the EBA scenarios from 2000 up to 2018 it is necessary to assess the future forecast quality. Therefore, we extrapolate 46 years historic yield curve changes into 46 upcoming years. Nevertheless, these yield curve changes do not reflect the future yield curve changes in time and level exactly. Even though this form of reflecting the historic yield curves meet the assumption of stationary changes for the Historical Simulation.

Figure 2: Development of the 1, 5 and 10-year German sovereign bond yields



Source: own calculations

In order to compare different IRRBB-strategies we analysed different cashflow positions. All positions are modelled up to ten years maximum cashflow period. The gliding 10-year position is a typical strategy for German banks and is analysed too [Reuse and Svoboda (2014b), pp. 376 ff.].

Table 1: Analysed cashflow positions

in TEUR	1 Yr.	2 Yr.	3 Yr.	4 Yr.	5 Yr.	6 Yr.	7 Yr.	8 Yr.	9 Yr.	10 Yr.
gliding 10 Yr.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
long-term										10.000
short-term	10.000									

Source: own table

3.2 Description of Methodology

The paper uses idealized portfolios. Effects of reinvestments or reduction of the residual maturity are isolated. We use a holding period of one year and regarding to the newest requirements to the ICAAP risk-bearing capacity a confidence interval of 99.9% [BaFin (2018), Note 61]. The Historical Simulation uses a rolling data set of 5 years as well as an increasing history for the VaR-estimation. The yield curve changes are computed as absolute changes.

Regarding to the six EBA scenarios, we compute on each observation shifted scenarios. In difference to the actual economic value, we compute economic value changes. The forecast quality is assessed by comparing the risk forecast with the ex-post performance. If the ex-post performance is worse than the risk forecast we ablate a VaR excess.

4. Discussing the results

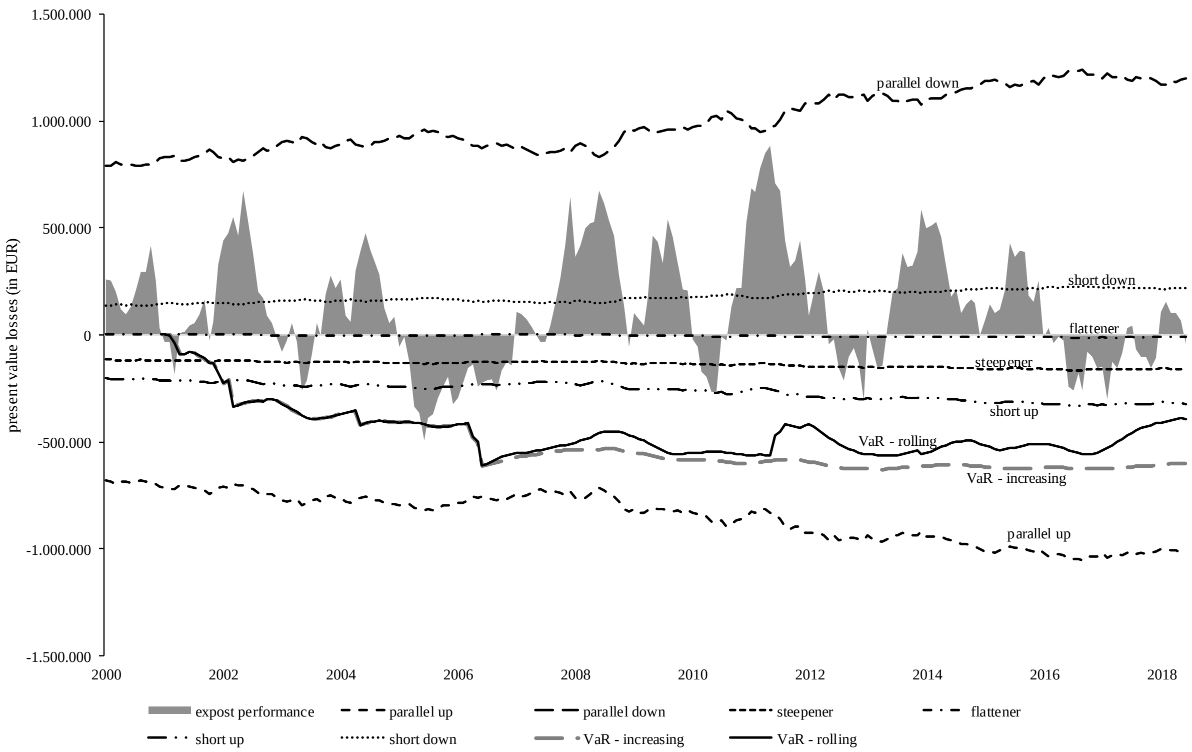
4.1 Backtesting the Historical Simulation and EBA risk scenarios

Starting with the backtesting between 01/2000 and 06/2018, figure 3 shows the ex post performance and the risk forecast for a 1-year-holding-period and a 99.9% confidence interval. From this point of view, we observe a very good forecast quality of the parallel up (+200bp) EBA scenario, as shown in [Svoboda, Reuse, Rüder and Boka (2018), pp. 712 ff.]. Of course, this scenario also determinates the highest positive change of the yield curve which meant the highest loss of economic value. This observation is relevant for all portfolios.

Both VaR-measurement methods of the Historical Simulation expect for the first 5 years an identical risk forecast. After the first period of 5 years the rolling VaR is more sensitive for the volatility of the yield curve changes. Within, the rolling VaR is more volatile than the increasing VaR. Nevertheless, both VaR measurement methods offer a very good risk forecast especially after an increasing dataset of ca. 5 years. The forecast quality depends on the length of the dataset which is used to compute a stable risk forecast. While this period of an increasing dataset the forecast quality leads to the expectation that it offers a worse forecast quality, which is not suitable for a good risk forecast during this period. After this period of 5 years we observe a very good forecast quality without any outlier during time in both VaR perspectives. All outliers of both VaR measures could be observed before completing the input-dataset of 5 years. It is necessary to mention that there is a very good performance of the economic value during time including a predominant increasing economic value.

At least, all other EBA scenarios do not meet the standard of an acceptable forecast quality. Especially the parallel down scenario forces a really bad forecast quality and could not held as a valid risk measurement method of the economic value for all portfolios. Probably, the effectiveness of this scenario could have constituted for an earnings perspective. In the parallel down scenario, we observe increasing economic value and decreasing earnings [Reuse (2016), pp. 138 ff.]. Furthermore, the other scenarios short up, short down, steepener und flattener do not offer adequate risk values for linear portfolios – maybe in a nonlinear perspective these scenarios constitute a good forecast quality.

Figure 3: Ex post performance of gliding 10-year-portfolio and risk forecast, 1-year-holding-period, 99.9% confidence interval between 2000 and 2018



Source: own calculations

Regarding to the different portfolios the evaluation of the forecast quality is sometimes different. We can observe that the flattener and the short up scenario has a very good forecast quality for the short-term portfolio. Compared with this both VaR forecasts are not able to predict a sufficient forecast quality for the short-term portfolio. In addition, one could assess that the length of the portfolio strategy correlate to the forecast quality of the VaR measurements.

Table 2: Relative VaR excesses between 2000 and 2018

	parallel up	**parallel down**	**steepener**	**flattener**	**short up**	**short down**	**VaR increasing**	**VaR rolling**
gliding 10 Yr.	0,00% „green“	38,46% „red“	17,31% „red“	37,50% „red“	5,77% „red“	38,46% „red“	0,96% „yellow“	0,96% „yellow“
short-term	0,00% „green“	34,62% „red“	34,62% „red“	0,00% „green“	0,00% „green“	34,62% „red“	5,29% „red“	5,29% „red“
long-term	0,00% „green“	38,46% „red“	0,96% „yellow“	38,46% „red“	28,85% „red“	38,46% „red“	0,96% „yellow“	0,96% „yellow“

Source: own calculations

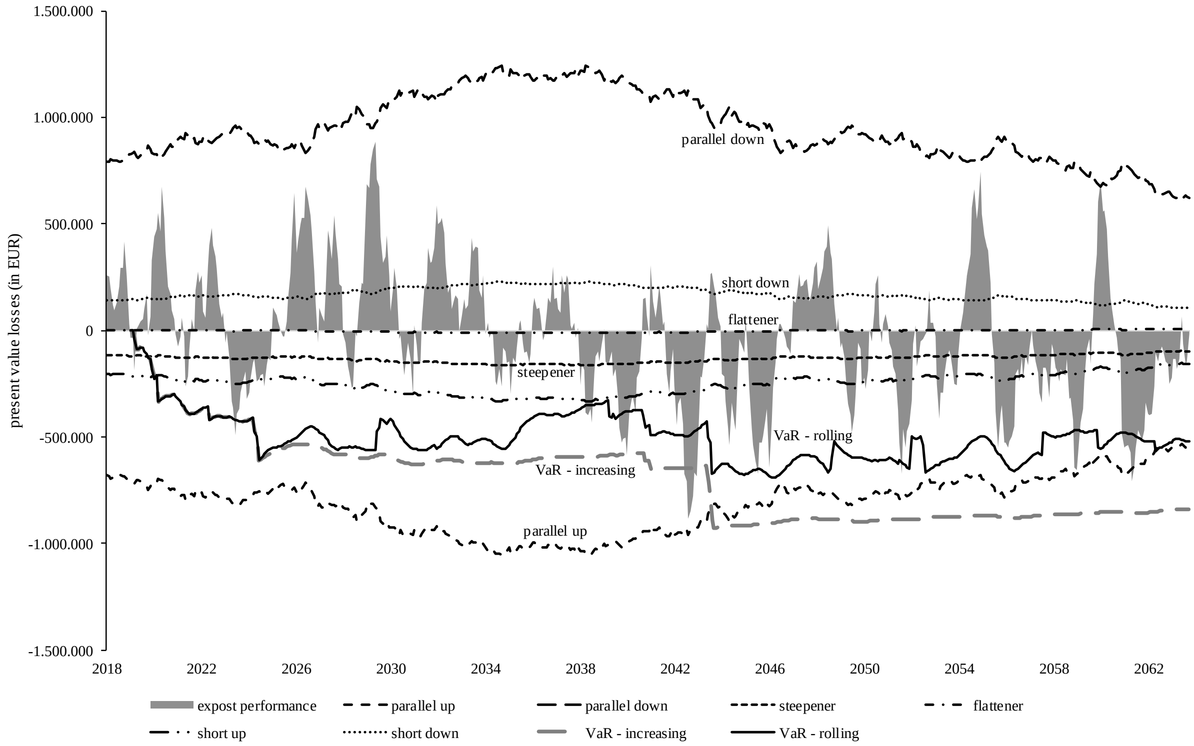
According to the Basel model risk traffic light approach relative VaR excesses above 0.1% are not “green”. Hence, for these excesses there is a detailed analysis necessary. In addition to the mentioned forecast quality, only the parallel up scenario fit to the regulatory backtesting. Although there is also a yellow traffic light, all other EBA scenarios do not meet the standard of an adequate risk measurement method of economic value because of too many outliers. Maybe, both VaR methods could held for an adequate risk measurement. We observe all outliers in the first 5 years which are used to compute an adequate data set. Regarding to the complete data set of minimum 5 years we observe a very good forecast quality.

In distinction between the portfolios the forecast quality of the short-term portfolio is worse than the gliding 10-years portfolio and the long-term portfolio. For the relevant risk scenarios, the forecast quality regarding the two portfolios gliding 10-year and long-term is nearly the same. In case of a short-term portfolio we can observe a good forecast quality for the flattener and the short up scenario. Furthermore, these scenarios are not able to predict a good forecast quality for any other portfolio.

4.2 Forecast quality of Historical Simulation and the EBA scenarios in the future

Although the actual forecast quality of the parallel shift up and both VaR scenarios is very good, the future forecast quality is probably affected by the future yield scenario. In addition to that a forthcoming scenario of increasing yields is probable. In this case, the future forecast quality shows a different perspective on each risk forecast.

Figure 4: Ex post performance of gliding 10-year-portfolio and risk forecast, 1-year-holding-period, 99.9% confidence interval between 2018 and 2062 mirrored from 1972 to 2018



Source: own calculations

Table 3: Relative VaR excesses between 2000 and 2062

	parallel up	**parallel down**	**steepener**	**flattener**	**short up**	**short down**	**VaR increasing**	**VaR rolling**
gliding 10 Yr.	2,28% „yellow“	54,09% „red“	39,06% „red“	53,83% „red“	27,65% „red“	54,09% „red“	1,34% „yellow“	6,71% „red“
short-term	7,25% „red“	52,75% „red“	52,75% „red“	10,74% „red“	9,93% „red“	52,75% „red“	3,49% „yellow“	11,28% „red“
long-term	0,81% „yellow“	54,50% „red“	19,33% „red“	54,50% „red“	50,07% „red“	54,50% „red“	1,21% „yellow“	4,43% „red“

Source: own calculations

Regarding to the EBA scenarios the steepener, flattener, short up, short down and parallel down shock scenarios leads to a bad forecast quality in times of increasing yields. Furthermore, only the parallel up scenario is an adequate risk measurement method. Although the relative VaR excesses leads to a refusal of the scenario, it shows the best risk forecast. Extreme events like the German Reunification (1991, reflected in 2042) with afterwards extremely decreasing yields get covered by this scenario. Although we can observe outliers for the parallel up scenario in 2058-2061, which reflects the historical yield development during oil crisis in 1973. It is quite surprising that the extreme shift scenario (+ 200 bp) is not able to cover such a risk event. Regarding to the 99.9% risk quartile of the EBA scenarios such extreme events should be covered. Though, the relative excesses lead to the assumption that such events could nearly classified as a black swan [Taleb (2008), pp. 261 ff.]. In addition, this event leads to the refusal of this scenario. The complete risk forecast before the reflected oil crisis is without any VaR outlier for the parallel up scenario. For such black swans the risk measurements are not able to predict a good forecast quality. In times of increasing yields, the EBA scenarios, especially the parallel up scenario, have a good forecast quality [Svoboda, Reuse, Rüder and Boka (2018), p. 719].

In addition, the parallel up scenario has the best forecast quality while increasing yields and supposes a better goodness of fit than the “internal” VaR models. Both VaR models lead to the assumption that both models do not offer good risk measurement results in times of increasing yields. Of course, these assumptions are determined by the used dataset and the absolute yield changes. Relative yield changes could lead to a better forecast quality in times of increasing yields [Boka and Rüder (2018), pp. 34 ff.]. Although the used dataset and confidence interval have the identical length and the identical confidence interval like the EBA dataset, the forecast quality is insufficient.

5. Conclusions

Summing up the main results leads the following aspects:

- Based on the used length of dataset only the EBA parallel up scenario and the internal VaR models are able to predict a good risk forecast in the actual interest rate environment.
- In times of increasing yields, neither the EBA scenario nor the internal VaR scenarios have a good risk forecast.
- The forecast quality of the scenarios depends on the used portfolio. Especially the short-term shocks compute a good risk forecast for short-term portfolios in the actual interest rate environment.
- Overall, only the parallel up shift, often discussed as a bad scenario, leads to adequate risk measurement method during time and increasing yields.
- Black swans like the oil crisis cannot get covered by any risk measurement method.
- We got indications that the length of the dataset, used for the EBA scenarios and transmitted to the internal VaR methods, is not sufficient to predict a good risk forecast during time.

Last, the the academic questions should be answered:

a) *Are the six EBA interest risk scenarios an adequate measurement approach during time?*
   No, only the +200 bp shock offers acceptable results.
b) *Are the six EBA scenarios a better measurement than the Historical Simulation during time?*
   This question cannot be answered clearly. Regarding the EBA scenarios, only the +200 bp shock leads to adequate results. The two VaR results offer good results as well, but it cannot be stated, which of these three remaining measurement methods is the best.
c) *Are the six EBA scenarios and the Historical Simulation in case of an increasing yield curve structure an adequate risk measurement method during time?*
   In sum, this could be stated – even though black swans cannot be quantified and could be defined as the “area after the quartile”.

Regarding forthcoming empirical analysis, it could be interesting to analyse the forecast quality with a longer input dataset or relative yield curve changes. Regarding to the used linear portfolios, using nonlinear portfolios could result in a different assessment of the forecast quality. In general, dealing with black swans like the oil crisis could lead to the Expected Shortfall as an adequate risk measurement method for such extreme events.

Acknowledgement

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Quality of reporting under IAS 14 Revised in comparison with the new reporting requirements under IFRS 8 of selected companies with a statutory seat in the Czech Republic

Jiří Šimůnek [1], Jiřina Bokšová [2]

Abstract

This article is focusing on the research of reporting quality according to the standard IAS 14 Revised (Segment Reporting) of issuers of the quoted securities when fulfilling their information liability. The primary source of the data is The Business Register, the official online server of the Czech judiciary; the years analyzed are 2006 to 2008. Compared to IAS 14 Original (Segment Reporting) valid till December 31, 1998, IAS 14 R effective for accounting periods beginning on or after July 1, 1998, provides more guidance for identifying business and geographical segments, based on internal structure and internal reporting system. It also provides segregation that one basis of segmentation is primary with additional information required and the other is secondary with considerably less information required. The main purpose of the article is to analyze whether IFRS 8 provides more quality information about company segments than IAS 14 R. To ensure comparability, the accounting entities will be divided into a group with prevailing core activity of „Financial and insurance activities" and other entities according to the Lists of NACE codes published by the EU.

Key words

International Accounting Standards, IAS, IAS 14 R, IFRS 8, Information Asymmetry, Business Segment, Geographical Segment, Segment Reporting, Management Approach, Czech Republic

JEL Classification: M41, G32

1. Introduction

This article’s theoretical part is focusing on a description of the IAS 14 Revised Standard (Segment Reporting), effective for accounting periods beginning on or after July 1, 1998, which replaced the standard IAS 14 (Reporting Financial Information by Segment) and was superseded by IFRS 8 Operating Segments with effect from annual periods beginning on or after 1 January 2009. Expert users of financial statements will almost always say that the information about the segments is by far the most interesting part of the package. On the other hand, the competitive edge might be lost if business rivals and customers will learn too much about the enterprise. The key issues in segment reporting are concerned with deciding how much detail is desirable and how to define segments so the information provided is comparable across a range of enterprises and from one period to the next (Roberts, Weetman, Gordon, 2005). The standard reduces the scope to which entities it should be applied by excluding other economically significant entities. It also provides more detailed guidance for identifying business and geographical segments, mostly based on the internal structure. It also increases

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the number of quantitative criteria to be disclosed for primary segments, while reducing the reporting requirements for secondary segments. The aim of the practical part is the research of the causality existence between selected financial indicators and the number of reported segments of the subjects under regulation of the Czech National Bank and the comparison of the segment requirements based on IFRS 8 [3]. The conclusion sums up the research findings and also provides ideas for further research in this area.

2. Background

IAS 14 revised (referred to hereinafter as IAS 14 R) was issued in August 1997 as a revision of a reformatted 1995 version of the previous IAS 14 standard issued in 1981. IAS 14 R substantially changed segment reporting requirements in response to numerous criticisms of the original standard. It requires disclosing the financial information by business or geographical area and distinguishing for primary and secondary segment reporting formats, with the primary format based on whether the entity's risks and returns are affected predominantly by the products and services it produces or by the fact that it operates in different geographical areas. The purpose of the standard is to secure for the users of financial reports the information about performance and financial position of the individual operating segments. The scope requires the application of this standard for individual or consolidated financial statements of the accounting entity when at least one of the condition is met:

- its debt or equity securities are traded in a public market; or
- the entity is in the process of issuing equity or debt securities in public securities markets.

In case that a single financial report contains both consolidated statements of an entity which securities are publicly traded and the separate financial statements of the parent, segment information need to be presented only in the consolidated statements.

The standard provides more detailed guidance for identifying business and geographical segments. Business segment is defined as “a distinguishable component of an enterprise that is engaged in providing an individual product or service or a group of related products or services and that is subject to risks and returns that are different from those of other business segments”. A business segment can also therefore be made up of several business subsystems relating to various products or services, on condition that these are homogenous in terms of their nature, the technology used, the class of customer served, the channel of distribution used, the risks to which they are subject and returns.

Geographical segments are identified according to the location in which production takes place (the location of its assets), if risks and returns are principally influenced by factors linked to the geographical areas in which the entity has its production facilities; alternatively, on the basis of the location of its markets, if risks and returns are more dependent on factors linked to the geographical areas in which sales are made.

The choice of segmentation criterion to be used in the primary segment reporting format must fall upon the one considered to have the greater capacity of information for highlighting the main source of risks and returns of the segment (Nicoló, 2006). In the case in which risks and returns are principally affected by differences among products/services, then this criterion

[3] Šimůnek, J. (2017). Quality of the reporting under IFRS 8 of issuers of the quoted securities in the Czech Republic. *In Financial Management of Firms and Financial Institutions - 11th International Scientific Conference.* Ostrava: VŠB - Technická univerzita Ostrava, p. 814-820.

has to be selected for drawing up primary segment reporting, while the secondary format will be used for geographical segments. If, on the other hand, the risk/return profile is more heavily influenced by variables characterizing the economic environment of the countries in which the entity makes or distributes its products, the primary reporting format will be geographical segments, while the secondary one will be for business sectors.

IAS 14 sets out the financial information to be reported for each segment (Lenormand & Touchais, 2014). The segment information should be prepared using the accounting policies adopted for preparing the consolidated financial statements. One of the key changes vs. the original standard is that greater disclosure is required for primary segments, and should include external and intersegment sales revenue, result, the carrying value of assets, liabilities, depreciation and amortization, and tangible and intangible capital additions for the financial year. The required disclosure items for secondary segments are limited to external sales revenue, assets, and tangible and intangible capital additions. In the table below, you can find the overview of financial information to be disclosed for individual segments based on the revised and original standard.

Table 1: Overview of disclosure requirements under IAS 14 and IAS 14 R

	IAS 14 requirements		IAS 14R requirements	
	Business	Geographic	Primary	Secondary
Revenue	Yes	Yes	Yes	Yes
Results	Yes	Yes	Yes	No
Assets	Yes	Yes	Yes	Yes
Liabilities	No	No	Yes	No
Capital expenditures	No	No	Yes	Yes
Depreciation and amortization	No	No	Yes	No
Other non-cash expenses	No	No	Yes	No
Equity method and JV income	No	No	Yes	No
Reconciliations to consolidated amounts	No	No	Yes	No
Basis for intersegment pricing	Yes	Yes	Yes	No

Source: Prather-Kinsey, Meek, 2004

3. Methodology and hypotheses for the research

This research is focused on the quality of the reporting under IAS 14 R and assessment of the benefit of IFRS 8, as a standard replacing IAS 14 R. For comparability purposes the relevant sample of subjects reporting in years 2006 – 2008 under IAS 14 R contains only subjects, which were reporting under IFRS 8 also in years 2013 – 2015 (for outcomes of this analysis see Simunek, 2017). Collection of Documents, which is part of Czech Business register as of February 12, 2018 was used as a data source. The table below summarizes the selection of relevant sample for the research (relevant sample used in the article Simunek, 2017, is marked as “2017”).

Table 2: Assessment of relevant research sample

44	• The issuers which reported information under IFRS 8 at least in one financial statements in the years 2013 - 2015 (hereinafter "Relevant sample 2017").
14	• No. of entities in the Relevant sample with core business "K - Financial and insurance activities" according to the List of Nace codes issued by EU (hereinafter "Relevant sample K 2017").
30	• No. of entities in the the Relevant sample with core business different than "K - Financial and insurance activities" (hereinafter "Relevant sample different than K 2017").
24	• No. of entities in the Relevant sample 2017, which were reporting under IAS 14 R also in years 2006 - 2008 (hereinafter "Relevant sample IAS 14 R").
11	• No. of entities in the Relevant sample IAS 14 R with core business "K - Financial and insurance activities" according to the List of Nace codes issued by EU (hereinafter "Relevant sample K IAS 14 R").
13	• No. of entities in the Relevant sample IAS 14 R with core business different than "K - Financial and insurance activities" according to the List of Nace codes issued by EU (hereinafter "Relevant sample different than K IAS 14 R").

Author: Own creation

From the original sample of 44 subjects (Relevant sample 2017) only 24 subjects were reporting in years 2006-2008 also under IAS 14 R. Investment funds were again excluded from the research due to the limited legal personality and shared sources of financing. Out of those 24 subjects (Relevant sample IAS 14 R) there was 11 of them represented in the category “K – Financial and insurance activities” (Relevant sample K IAS 14 R) and remaining 13 subjects were represented in a category other than “K – Financial and insurance activities” (Relevant sample different than K IAS 14 R). Segregation to category “K – Financial and insurance activities” and subjects with core business different than “K – Financial and insurance activities” were chosen due to significantly different requirements e.g. on capital investments, required profit and a structure and size of the assets. This segregation will help to better classify the research results.

Zero hypothesis of this research (h0) is expressing the assumption that there is a higher positive correlation between profit before taxes or total assets (independent variables) and the number of reported segments (dependent variable) among the subjects from the Relevant sample 2017 according to IRFS 8 compared to Relevant samples IAS 14 R and also positive correlation measured according to IFRS 8 is higher than 10% at least for one independent variable. Alternative hypothesis (h1) is determined compared to zero hypothesis. If zero hypothesis will be met, it can be said that reporting under IFRS 8 provides better transparency for users of financial statements than reporting under IAS 14 R. Segment reporting might be beneficial to the entities in a sense that it might help them to obtain more capital at regulated markets and/or will make higher profit before taxes.

The following is a summary of hypotheses for the research:

H0: There is a higher positive correlation between profit before taxes or total assets (independent variable) and the total reported segments (dependent variables) among the subjects from the Relevant sample 2017 under IFRS 8 compared to Relevant sample IAS 14 R and also positive correlation measured according to IFRS 8 is higher than 10% for at least one independent variable.

H1: There is a lower positive correlation between profit before taxes or total assets (independent variables) and the total reported segments (dependent variable) among the subjects from the Relevant sample 2017 under IFRS 8 compared to Relevant sample IAS 14 R or positive correlation measured according to IFRS 8 is lower than 10% for at least one independent variable.

For the hypotheses testing the correlation matrix method was selected.

4. Hypotheses testing, research conclusions

Table 3: Correlation matrix – all subjects in the Relevant sample 2017

Total assets	***Profit before taxes***	***No. of segments***	
1,0000	0,9225	0,2181	***Total assets***
	1,0000	0,1380	***Profit before taxes***
		1,0000	***No. of segments***

Author: Jiri Simunek, 2017

No. of subjects: 44
No. of observations: 114
Critical test value for level alpha= 0,1840 pro n = 114

From Table 3 we can see a weak positive correlation of the number of segments on both total assets (+ 0,2181) and profit before taxes (+ 0,1380). The critical value (5%) is at least higher at the independent variable profit before taxes therefore we will test whether the positive correlation profit before taxes (+ 0,1380) is greater than the same independent variable from sample “Relevant sample IAS 14 R".

Table 4: Correlation matrix – all subjects in the Relevant sample IAS 14 R

Total assets	***Profit before taxes***	***No. of segments***	
1,0000	0,5959	-0,0362	***Total assets***
	1,0000	-0,0682	***Profit before taxes***
		1,0000	***No. of segments***

Author: Own creation based on the collected data

No. of subjects: 24
No. of observations: 71
Critical test value for level alpha= 0,2335 pro n = 71

From Table 4 we can see a weak negative correlation of the number of segments on both total assets (-0,0362) and profit before taxes (- 0,0682). The critical value (5%) is at least higher for the independent variable profit before taxes and total assets.

Considering that the correlation between number of segments and the independent variable total assets from sample Relevant sample 2017 (+ 01380) is greater than for same values from Relevant sample IAS 14 R (- 0,0362), **we decline alternative hypothesis H1 and conclude on zero hypothesis H0.**

Table 5: Correlation matrix – subjects in the Relevant sample without financial and insurance institutions 2017

Total assets	***Profit before taxes***	***No. of segments***	
1,0000	*0,9589*	*0,0794*	***Total assets***
	1,0000	*0,1091*	***Profit before taxes***
		1,0000	***No. of segments***

Author: Jiri Simunek, 2017

No. of subjects: 30
No. of observations: 78
Critical test value for level alpha = 0,2227 pro n = 78

From Table 5 we can see a weak positive correlation of the number of segments on both total assets (+ 0,0794) and profit before taxes (+ 0,1091). The critical value (5%) is higher at the independent variable profit before taxes and total assets, therefore we will test whether the positive correlation total assets (+ 0,0794) or profit before taxes (+ 0,1091) is greater than the same independent variable (total assets or profit before taxes) from Relevant sample IAS 14 R.

Table 6: Correlation matrix – subjects in the Relevant sample without financial and insurance institutions

Total assets	***Profit before taxes***	***No. of segments***	
1,0000	*0,9838*	*0,0047*	***Total assets***
	1,0000	*-0,0062*	***Profit before taxes***
		1,0000	***No. of segments***

Author: Own creation based on the collected data

No. of subjects: 14
No. of observations: 38
Critical test value for level alpha = 0,3202 pro n = 38

From Table 6 we can see a weak positive correlation of the number of segments on total assets (+ 0,0047) and negative correlation of the profit before taxes (- 0,0062) on total assets. The critical value (5%) is at least higher for the independent variable profit before taxes and total assets, therefore we will test whether positive correlation total assets (+ 0,0047) or negative correlation profit before taxes (- 0,0062) is greater than the same independent variables (total assets or profit before taxes) from the sample Relevant sample different than K IAS 14 R.

Considering that correlation between number of segments and independent variable total assets from sample Relevant sample 2017 (+ 0,0794) and also and independent variable profit before taxes (+0,1091) is greater than for same values from sample Relevant sample IAS 14 R (+ 0,0047 – total assets; - 0,0062 – profit before taxes), **we decline alternative hypothesis H1**

and conclude on zero hypothesis H0 (proven for independent variable profit before taxes only).

Due to not reliable correlation matrix model of Relevant sample K 2017, hypotheses testing by comparison of conclusions from data of Relevant sample K 2017 and Relevant sample K IAS 14 R will not be done. Hypotheses analyses would not be reliable at this case.

6. Conclusion

The aim of this article has been a theoretical specification of requirements of IAS 14 R standard and empiric comparison of their quality to the reported information under IFRS 8. It found out that reporting under IFRS 8 shows a weak positive correlation between number of segments and total assets or profit before taxes for all entities or for entities with prevailing activity other than finance or insurance, which is higher than for the same indicators reported under IAS 14 R. Weak positive correlation occurred especially with subjects other than financial and insurance institutions, which differentiate from other subjects (together with excluded investment funds) by some specific characteristics – high value of total assets, less stable values of indicators dependent on macroeconomics of the country etc. (Simunek, 2017). It has been good progress of the amendment of accounting standards contributing to (quantified) higher quality of information in the financial statements. An additional research topic could be the accounting standard IFRS 5 Non-current Assets Held for Sale and Discontinued Operations which partially relates to the reporting of a relatively separated part of an accounting entity.

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Insurance Premium Calculation Using Generalized Linear models

Adéla Špačková[1]

Abstract

The non-life insurance pricing consist estimation of claim frequency and claim severity. Insurance premium is paid by the policyholders to the insurance company exchange for the transfer of risk. In the insurance practise, the question arises, how these risks price correctly. The possible way to obtain the insurance premium is a combination of the conditional expectation of claim frequency and claim severity, where each of them is described a differenent distribution of probability. The aim of this paper is pure premium calculation given by the individual characteristic of the policyholders using Generalized Linear Models. All calculations are computed in statistical software STATA.

Key words

Pure Premium, Generalized Linear Models, Vehicle Insurance, Gamma distribution, Negative-binomial Distribution, Multiplicative model

JEL Classification: C13, G22

1. Introduction

Historically, actuarial science and econometrics modelling have been limited by standard Gaussian linear regression includes many restrictive conditions. The non-life insurance pricing depends on many individual risk factors given by the policyholders. The most suitable method for estimation and subsequently pricing in insurance practise are generalized linear models. The final pricing consist of estabilishing a premium or a tariff paid by the policyholders exchange for a risk transfer. Premium calculation is given by multiplicative model of estimation claim frequency and claim severity.

The aim of this paper was pure premium calculation given by the individual characteristic of the policyholders using Generalized Linear Models.

At first part it is introduced generalized linear model, including test of verification and validation. Next subchapter includes description of models especially claim severity and claim frequency including probability and fitting into exponenential family framework.

Subsequently, it was described the relationship between these models and the calculation of pure premium.

Generalized linear models were applied into a real data of insurance contracts, collected during the years 2008-2010 in the area of Czech Republic. All the calculations were computed in the statistical software Stata.

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2. Generalized Linear Models

Generalized linear models have been formulated by John Nelder and Robert Wedderburn as the way of others regression statistical models, including linear that permit for independent variable utilize other than normal distribution.

The basic of these models are defined as an extentions of the Gaussian linear predictor derived from the exponential family. The main purpose of theese models is to estimate random explanation variable (denoted *y*), depending on certain number of explanatory variables (X_i).

Generally, GLM includes three main assumptions:

- A probability distribution have to be from an exponential family
- A linear predictor is transform by link function, such as: $n = x'\beta$
- A link function $g\left(\frac{\mu}{n}\right) = x'\beta$,

where g is a link function, μ is mean, n is called the exposure.

Link function can be diverse, but for the purposes of this paper the logarithm link function is selected (more in JONG, Piet de a Gillian Z. HELLER and GRAY, Roger J. a Susan M. PITTS) .

Thus, link function g is log, that becomes:

$$\ln\left(\frac{\mu}{n}\right) = x'\beta = \ln\mu = \ln n + x'\beta \ , \tag{2.1}$$

where $\ln n$ is called an „offset". This offset is another variable x, where the coeficient β is equal to one. Offsets are usualy used to correct differing time period of observation.

All probability distribution can be decribed of the general form:

$$f(y) = c(y,\phi)\exp\left\{\frac{y\theta - a(\theta)}{\phi}\right\}, \tag{2.2}$$

where θ is the canonical parameter and ϕ is called the dispersion parameter. $a(\theta)$ and $c(y,\phi)$ are functions determining the actual probability function such as normal, gamma, binomial etc.

For the purposes of this paper the negative binomial and gamma distribution are choosen. Description of exponencial family parameters is shown in Table 2.1.

Table 2.1 Parameters distributions of exponential family

Distribution	θ	$a(\theta)$	ϕ	$E(y)$	$V(\mu) = \frac{Var(y)}{\phi}$
Negative-binomial (μ,κ)	$\ln\frac{\kappa\mu}{1+\kappa\mu}$	$-\frac{1}{\kappa}\ln(1-\kappa e^{\theta})$	1	μ	$\mu(1+\kappa\mu)$
Gamma (μ,v)	$-\frac{1}{\mu}$	$-\ln(-\theta)$	$\frac{1}{v}$	μ	μ^2

source: JONG, Piet de a Gillian Z. HELLER

2.1 Estimation of parameters

The standard method of estimation parameters β is maximum likelihood estimation. This method is based on selecting parameter estimates and maximize the likelihood of the observed sample:

$$\ell(\theta,\phi) \equiv \sum_{i=1}^{n} \ln f(y_i;\theta,\phi) \ , \tag{2.3}$$

where $f(y_i)$ is a probability function depends on the canonical parameter θ and the dispersion parameter ϕ.

When the maximum likelihood estimation is the exponential family probability function:

$$\ell(\beta,\phi) = \sum_{i=1}^{n} \ln f(y_i;\beta,\phi) = \sum_{i=1}^{n} \left\{ \ln c(y_i,\phi) + \frac{y_i\theta_i - a(\theta_i)}{\phi} \right\} \tag{2.4}$$

Maximalization of likelihood called the log-likelihood is a logarithm of the likelihood with respect to β_j:

$$\frac{\partial \ell}{\partial \beta_j} = \sum_{i=1}^{n} \frac{\partial \ell}{\partial \theta_i} \frac{\partial \theta_i}{\partial \beta_j}, \tag{2.5}$$

where the parameters are folowing:

$$\frac{\partial \ell}{\partial \theta} = \frac{y_i - \dot{a}(\theta_i)}{\phi} = \frac{y_i - \mu_i}{\phi}, \tag{2.6}$$

$$\frac{\partial \theta_i}{\partial \beta_j} = \frac{\partial \theta_i}{\partial \eta_i} \frac{\partial \eta_i}{\partial \beta_j} = \frac{\partial \theta_i}{\partial \eta_i} x_{ij} \ . \tag{2.7}$$

x_{ij} is a component i of x_j.

When the equation 2.5 is equal to zero, than estimation equations for β are:

$$\sum_{i=1}^{n} \frac{\partial \theta_i}{\partial \eta_i} x_{ij} (y_i - \mu_i) = 0 \Leftrightarrow X'D(y-\mu) = 0 \, , \tag{2.8}$$

According to the equation (2.8) it is clear, that parameter β is implicit and working throught μ and D.

Generalized linear models are estimated using Newton-Raphson method, or the method of IRLS (method of iteratively weighted least squares). Using the algorithm Newthon-Rapson can obtain the observed information matrix (OIM), on the contrary, the method of IRLS we obtain the expected information matrix (EIM) see GRAY, Roger J. a Susan M. PITTS (2012).

2.2 Testing the significance of explanatory variables

The estimated parameters of the model have to be statiscically significant, thus it is necesary to test them. The most appropriate method for parameter testing is a Wald test.

2.1.1 Wald test

The basic of Wald test consist in testing hypotheses which have been estimated by maximum likelihood.

The null hypothesis:

$$H_0 = \hat{\beta} = 0 \tag{2.9}$$

is compared to alternative:

$$H_A = \hat{\beta} \neq 0 \tag{2.10}$$

If the null hypothesis is rejected, it suggest that parameters should be involved to the model (LONG, J. Scott a Jeremy FREESE).

2.1.2 Likelihood test

LR test is a statistical method used for comparing a godness of fit of two models. It is a situation, when the one model is nested to the other. The formula of LR test is following:

$$2\ln\lambda = 2(\hat{\ell} - \tilde{\ell}), \tag{2.11}$$

where the likelihood ratio is defined:

$$\lambda = \hat{L} / \tilde{L} \tag{2.12}$$

$\hat{L}$ is likelihood of full (unrestricted model), $\tilde{L}$ is likelihood of nested (restricted) model.

2.2 Claim Frequency Model

Claim frequency model (or observed number of claims) is a situation, where the random dependent variable is discrete and conditioned by vector of explanatory variables (characteristic of risk based on individual characteristic of shareholders) is negative-binomial distributed. The probability of random variable Y fitting into the exponential family framework (2.2) is given:

$$\ln\{f(y)\} = y\ln\frac{\mu}{1+\kappa\mu} - \frac{1}{\kappa}\ln(1+\kappa\mu) = \frac{y\theta - a(\theta)}{\phi}, \tag{2.13}$$

where the dispersion parameter is $\phi = 1$ and canonical parameter $\theta = \ln\frac{\kappa\mu}{1+\kappa\mu}$ (see table of parameters 2.1), (JONG, Piet de a Gillian Z. HELLER).

Mean and variance function is denoted:

$$E(y) = \dot{a}(\theta) = \frac{e\theta}{1-\kappa e^{\theta}} = \mu \tag{2.14}$$

$$Var(y) = \phi\ddot{a}(\theta) = \frac{e^{\theta}}{(1-\kappa e^{\theta})^2} = \mu(1+\kappa\mu) \tag{2.15}$$

where $\dot{a}(\theta)$ and $\ddot{a}(\theta)$ are first and second derivates of $a(\theta)$ with respect to θ.

Next part is going to be focused on claim severity model, the second part of calculation which is necessary for premium.

2.3 Claim Severity Model

Claim severity model (claim amount) is the second essential part of pure premium calculation. In this case dependent variable is continous and also conditioned by vector of explanatory variaboles given by individual characteristic of shareholders. A several literature explains that suitable econometric model permiting classic econometric modeling of claim severity is the Gamma model (JONG, Piet de a Gillian Z. HELLER and HARDIN, James W. a Joseph HILBE). The density function, which is fit into the exponential family framework (see formula 2.2) is folowing:

$$\ln\{f(y)\} = \left\{\frac{y\theta - a\theta}{\phi}\right\} + (v-1)\ln y - \ln\Gamma(v) + v\ln v, \tag{2.16}$$

where the canonical parameter $\theta = -\frac{1}{\mu}$ (table 2.1) etc.

And then mean and variance function is denoted:

$$E(y) = \dot{a}(\theta) = -\frac{1}{\theta} = \mu \tag{2.17}$$

$$Var(y) = \phi\ddot{a}(\theta) = v^{-1}\frac{1}{\theta^2} = \frac{\mu^2}{v} \tag{2.18}$$

where $\dot{a}(\theta)$ and $\ddot{a}(\theta)$ are first and second derivates of $a(\theta)$ respecting to θ.

2.4 Calculation of pure premium

In the case of non-life insurance, the pure premium is given by expected cost of all claims caused by the policyholders during the time insured period. Mostly, calculation of pure premium includes estimation of the frequency and severity model. Basicly, the mathematical formula can be obtained by multiplying the two estimated components, claim frequency and claim severity:

$$E\left[\sum_{i=1}^{N} sev_i\right] = E(freq_i)\cdot E(sev_i) \tag{2.19}$$

3 Model estimation

In this part are giong to be estimated two type of models. First, claim frequency by using negative-binomial distribution and the second claim severity which is going to be estimated with gamma distribution. Both model are going to be tested, compared and then the pure premium is going to be obtained.

3.1 Data

The aim of this paper is pure premium calculation given by the individual characteristic of the policyholders using GLMs.

For the purposes of this paper a random selection of real data in vehicle insurance is used and collected during the years 2005-2010 in the Czech republic teritory. The file contains 5 383 contracts.

All calculations are computed in statistical software STATA 14.0. Each vehicle insurance contract includes this following individual characteristic of the policyholders:

Table 3.1 Parameters description

variable	**description**	**category**
frequency	Claim frequency	0,1,2,3
severity	Claim severity	/
fuel	Type of fuel	1,2,3,4
gender	Gender of driver	0 – man, 1
catagecar	Vehicle age	1-new car, 2, 3, 4,
catageman	Age of driver	1 - the youngest, 2, 3, 4
catprice	Vehicle price	1 – the cheapest, 2, 3, 4
catwolumkw	Engine power	1 – the weakest, 2, 3, 4

3.2 Claim frequency

In this part it is estimated claim frequency model, with negative-binomial distribution and log-link function. The parameters are estimated by maximum likelihood method and results of insignificant variables are shown in table 4.2.

Table 3.2 Parameters β and p-value

variable	β parameters	p-value
fuel		
diesel	0,2399	0,017
p-butane	-9,3512	0,986
other	0,5924	0,575
district		

Královehradecký	-0,4100	0.200
Pardubický	-0,4291	0.180
Ústecký	0,1933	0.411
Jihomoravský	-0,3163	0.188
Liberecký	0,3712	0.133
Plzeňský	-0,3968	0.215
Zlínský	-0,3648	0.219
Karlovarský	-0,2831	0.389
Moravskoslezský	-0,1754	0.430
Praha	0,1139	0.556
Vysočina	-1,0669	0.005
Olomoucký	0,0127	0.961
Středočeský	-0,1282	0.532

P-value of variables fuel and the district was higher than 0.05, which of course signifies the irrelevance to the variable count. On the basic of Wald test were those variables found to be significant and were thus included to the model. In Table 4.3 the results of the Wald test are presented.

Table 3.3 Wald test

$\chi^2(3)$	prob. > χ^2
fuel	
61,93	0,0000
$\chi^2(13)$	prob. > χ^2
district	
54,84	0,0000

It was found that the fuel category and district are statistically significant, will not therefore from the model omitted.

According to LR test procedure, it is necessary to estimate a second model. Thus, next step contains second model, which is estimated without variables fuel and district. This model, called model 2 is nested to the model 1 and subsequently it is possible to test it by LR test. The results of likelihood ratio test are in table 4.4.

Table 3.4 Likelihood ratio test

LR test $\chi^2(16)$	prob. > χ^2
46,1	0,0001

According to the results it is clear that the more accurate model is model 1 including all variables.

Following histogram shows empirical (left) and expected frequency estimated by model 1.

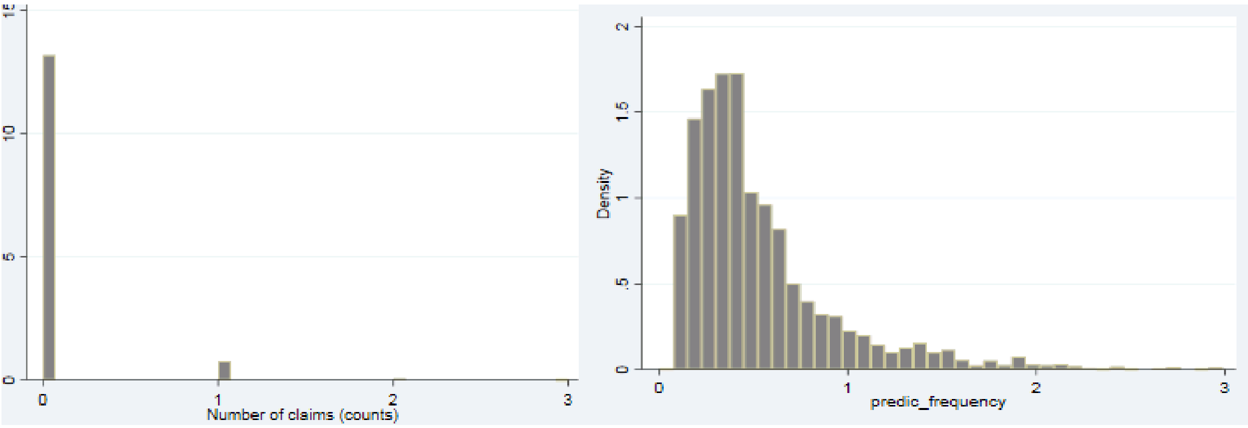

Figure 3.1Empirical counts and expected counts

According to the histogram it is clear that expected counts are slightly higher than empirical counts.

3.3 Claim severity

This part is focused on claim severity model, with gamma distribution and log-link function. The parameters are also estimated by maximum likelihood method. It should be noted, that p-value of variables fuel and the district was again higher than 0.05. Thus, it is necessary test it by Wald test. In Table 4.5 are presented the results of the Wald test.

Table 3.5 Wald test

$\chi^2(2)$	prob. > χ^2
fuel	
33,34	0,0000
$\chi^2(13)$	prob. > χ^2
district	
19,43	0,0000

On the basis of the results it is clear that the omission of these two variables would lead to a distortion of the model. Therefore, they will be left in the model. In the next step is going to be estimated restricted model and then compared with first model. The results of LR test are described in following table 4.6.

Table 3.6 Likelihood ratio test

LR test $\chi^2(14)$	prob. > χ^2
18,43	0,0000

According to table 4.6, the statistical significance of the variables was confirmed. The omission the above-mentioned variables from the model could lead to the distortion of final estimation. In the following figure are shown the histograms of an empirical and the expected severity estimated by full model 1.

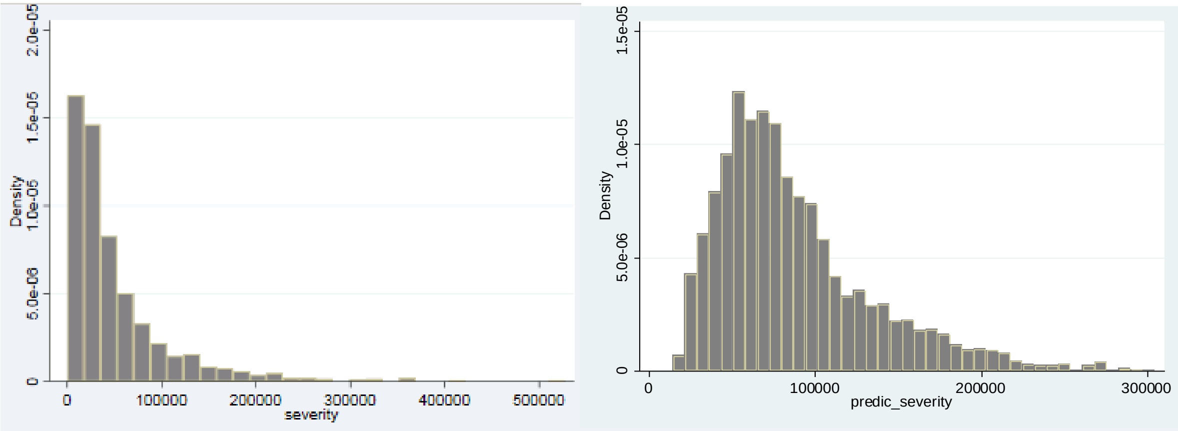

Figure 3.2 Empirical and expected severity

According to the histogram of severities it is clear that expected severity are slightly higher than the empirical.

3.4 Pure premium calculation

This part is focused on pure premium calculation in non-life insurance. The process of estimation the pure premium is given by formula (2.19) and obtained by multiplication estimated means of claim severity and the claim frequency. The amount of the pure premium have to be equal to the amount of insurance claims. It is clear, that the main problem of GLM, as we can see on fig 4.3, is overestimation of insurance risk.

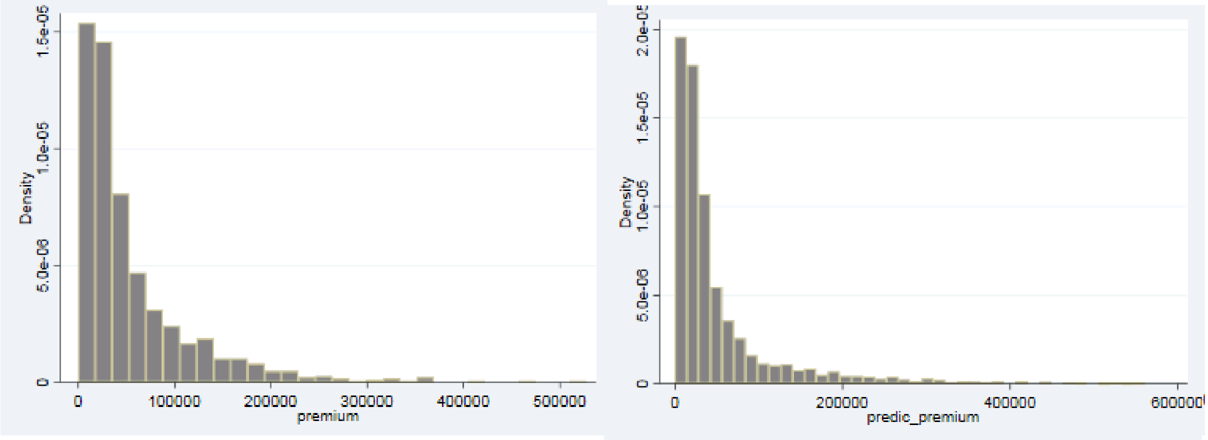

Figure 3.3 Empirical and expected premium

It is clear, that prediction of pure premium is slightly overestimated. This situation can be given by selection of a probability distribution or link function, but still, the combination of negative-binomial distribution and gamma distribution is the most suitable for this type of estimation.

4 Conclusion

The non-life insurance pricing depends on many risk factors. It consist of estabilishing a premium or a tariff paid by the policyholders exchange for a risk transfer. Premium calculation is given by multiplicative model of estimation claim frequency and claim severity. The most suitable method for estimation is and subsequently pricing is generalized linear models.

The aim of this paper was pure premium calculation given by the individual characteristic of the policyholders using Generalized Linear Models.

At first it was introduced generalized linear regression model, including test of verification and validation of model. Next subchapter includes description of models especially claim severity and claim frequency including probability and fitting into exponenential family framework.

Subsequently, it was described the relationship between these models and the calculation of pure premium.

Generalized linear models were applied into a real data of insurance contracts, collected during the years 2008-2010 in the area of Czech Republic. All the calculations were computed in the statistical software Stata.

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Pricing of CDO under Copula Framework

Yuan Tian [1]

Abstract

The topic of the paper is pricing of CDO under copula framework. CDO is widely applied to the risk management of banks and other financial institutions. The goal of the paper is to compare the CDO tranche spreads based on different factor copula models and figure out how the maturity and the recovery rate can influence the CDO tranche spreads. Factor copula models enable to derive the portfolio cumulative loss distribution and then the expected value of the cumulative tranche loss, which is the key issue for pricing CDO. Consider a portfolio of 100 reference with 5-year maturity traded on iTraxx Europe, Gaussian, *t*, and Clayton copula can get close tranche spreads after the appropriate calibrations. Furthermore, longer maturity and lower recovery rate can provide a higher CDO tranche spread, which is coordinating with the theoretical analysis.

Key words

CDO, copula model, cumulative loss distribution, credit risk, LHP

JEL Classification: G15, G17, G21, H63

1. Introduction

CDO is widely applied to the risk management of banks and other financial institutions as one of most popular credit derivatives in the financial market. Pricing of CDO is a research priority because of not only the rapid development of CDO but also the US subprime crisis. In this paper, we focus on factor copula model for CDO pricing. The goal of the paper is to compare the CDO tranche spreads based on different factor copula models and figure out how the maturity and the recovery rate can influence the CDO tranche spreads. Copula models are widely extended to CDO pricing and risk measurement because of the good extensibility. The one-factor Gaussian copula model has already been the benchmark of pricing models. The key issue for pricing CDO is to determine the cumulative loss distribution function, which can be derived from factor copula models.

2. Collateralized debt obligations

A collateralized debt obligation (CDO) is a structural financial product that pools together credit risky assets, which are called collaterals, and then repackages the asset pool into varying tranches. When the collaterals are bank loans, the CDO is called a collateralized loan obligation (CLO); when they are high-yield bonds, the CDO is called a collateralized bond obligation (CBO). A CDO may also include fixed income securities, subordinated debt, emerging market corporate debt, etc. The CDO is managed by a sponsoring organization, namely a special purpose vehicle (SPV). The tranches in a CDO are categorized as senior, mezzanine, and junior/subordinated tranches according to the level of credit quality, which is usually a credit rating received from a rating agency (Moody’s, Fitch, S&P, etc.). The standard prioritization scheme is a simple subordination: senior CDO notes are paid before mezzanine and lower

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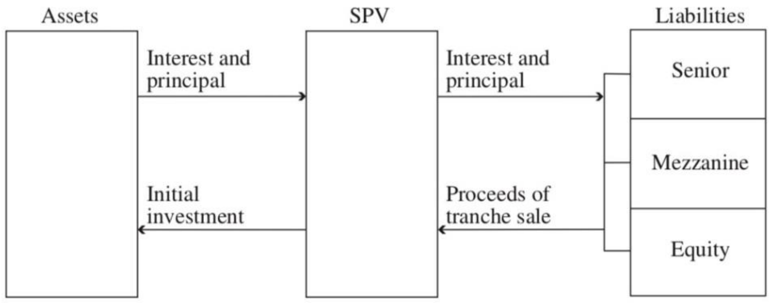
subordinated notes are paid, with any residual cash flows paid to junior notes. This mechanism is known as the waterfall. Figure 1 shows a simplified CDO structure. The junior tranche is usually called the equity tranche because the holders of the junior tranche are similar with the equity holders in a corporation in receiving the residual cash flows.

Figure 1: Simplified CDO structure



Source: McNeil, A. J., Frey, R., Embrechts, P. (2015). P477.

3. CDO pricing models

The study of CDO pricing models usually focuses on the probability of defaults of the obligors and the default correlations between the obligors. Among the early pricing models, binomial expansion technique (BET) is applied to pricing CBOs and CLOs. However, BET is not good enough to estimate the credit loss because of its over-simplified assumptions. Copula models are widely extended to CDO pricing and risk measurement. The one-factor Gaussian copula model, the simplest copula model, has already been the benchmark of pricing models in the industry. The good extensibility of copula models enables them to improve the imitative effects of CDO pricing.

3.1 BET model

Binomial expansion technique (BET), first proposed by Cifuentes and O'Connor (1996) and widely applied by Moody's, uses the diversity score concept to build a hypothetical asset pool of uncorrelated and homogeneous bonds or loans to mimic the default behavior of the original asset pool. Assume that *DS* is the diversity score of the fictitious asset pool and *p* is the weighted average probability of default, the probability of default of *i*-th asset in the pool is:

$$P_i = \frac{DS!}{i!\,(DS-i)!} p^i (1-p)^{DS-i}. \tag{1}$$

The expected loss of the fictitious asset pool is:

$$EL = \sum_{i=1}^{DS} P_i L_i\,, \tag{2}$$

where L_i is the loss of the *i*-th asset when the default occurs. Then according to Moody's idealized historic data, the expected loss can be transformed into a rating.

Since the underlying asset pool is highly heterogeneous and centralized, Moody's improved BET model to multi-binomial expansion technique (MBET) and correlated binomial method (CBM). These models are easy and convenient to use, but it is unreasonable to assume the probability of default is the same for every asset in the portfolio and follows binomial distribution. Besides, Garcia et al. (2005) proved that BET model underestimates the credit loss of the portfolio with high default correlation compared with copula model.

3.2 Copula model

Copula model is based on the copula functions and the marginal probability distribution functions of the underlying assets to get the joint distribution function. Consider a CDO of *n* underlying assets with default times τ_i, loss given default δ_i, and notional amount A_i. The marginal probability distribution functions of the underlying assets at the time of default are $F_1(t_1), F_2(t_2), \cdots, F_n(t_n)$. The joint distribution function is F. There exists a copula C: $[0,1]^n \to [0,1]$ such that:

$$F(t_1, t_2, \cdots, t_n) = C\big(F_1(t_1), F_2(t_2), \cdots, F_n(t_n)\big). \tag{3}$$

3.2.1 General framework

Generate n uniformly distributed random variables U_i based on the selected copula first and then transform U_i into default times τ_i in ascending order. Compute the default intensity $\lambda(t)$ and default times τ_i is given by $\tau_i = -\frac{\ln U_i}{\lambda}$. The cumulative loss of the portfolio up to time t is given by:

$$L(t) = \sum_{i=1}^{n} A_i(1 - R_i)\mathbf{1}_{\{\tau_i \le t\}}, \forall i = 1,2, \cdots, n, \tag{4}$$

where A_i is the notional amount and R_i is the recovery rate.

Assume that there is a tranche γ and the attachment/detachment points are denoted by $\left(K_L^\gamma, K_U^\gamma\right)$, then the cumulative loss of the tranche is:

$$L\left(K_L^\gamma, K_U^\gamma, t\right) = \begin{cases} 0, & if\ L(t) < K_L^\gamma \\ L(t) - K_L^\gamma, & if\ K_L^\gamma \le L(t) \le K_U^\gamma \\ K_U^\gamma - K_L^\gamma, & if\ L(t) > K_U^\gamma \end{cases}. \tag{5}$$

It can be summarized as the payoff a call spread:

$$L\left(K_L^\gamma, K_U^\gamma, t\right) = \max\left\{\min\left[L(t), K_U^\gamma\right] - K_L^\gamma, 0\right\} = \left\{\min\left[L(t), K_U^\gamma\right] - K_L^\gamma\right\}^+. \tag{6}$$

The expected value of the cumulative tranche loss with the continuous portfolio loss distribution function $F_L(x)$ is:

$$\begin{aligned} \mathrm{E}\left[L\left(K_L^\gamma, K_U^\gamma, t\right)\right] &= \frac{1}{K_U^\gamma - K_L^\gamma}\int_{K_L^\gamma}^{1}\left[\min\left(x, K_U^\gamma\right) - K_L^\gamma\right]\mathrm{d}F_L(x) \\ &= \frac{1}{K_U^\gamma - K_L^\gamma}\left[\int_{K_L^\gamma}^{1}\left(x - K_L^\gamma\right)\mathrm{d}F_L(x) - \int_{K_U^\gamma}^{1}\left(x - K_U^\gamma\right)\mathrm{d}F_L(x)\right] \end{aligned}. \tag{7}$$

The expected value of the default leg and the premium leg can be computed respectively by:

$$\mathrm{E}(DL) = \mathrm{E}\left[\int_0^T \mathrm{B}(0,t)\mathrm{d}L\left(K_L^\gamma, K_U^\gamma, t\right)\right], \tag{8}$$

$$\mathrm{E}(PL) = \mathrm{E}\left[\sum_{i=1}^{n} s\Delta t_i\, \mathrm{B}(0, t_i)\min\left\{\max\left[K_U^\gamma - L(t_i), 0\right], K_U^\gamma - K_L^\gamma\right\}\right], \tag{9}$$

where $T = t_n$ is the maturity, $\mathrm{B}(0,t)$ is the discount factor until time t, s is the par spread of the tranche, $\Delta t_i = t_i - t_{i-1}$, and $L(t_i) = \delta_i A_i \mathbf{1}_{\{\tau_i \le t\}}$.

Based on the general semi-analytic approach, each tranche conducts a premium so that the premium leg equals the default leg, namely $\mathrm{E}(PL) = \mathrm{E}(DL)$. The par spread s^* is therefore:

$$s^* = \frac{\mathrm{E}\left[\int_0^T \mathrm{B}(0,t)\mathrm{d}L\left(K_L^\gamma, K_U^\gamma, t\right)\right]}{\mathrm{E}\left[\sum_{i=1}^{n} \Delta t_i\, \mathrm{B}(0, t_i)\min\left\{\max\left[K_U^\gamma - L(t_i), 0\right], K_U^\gamma - K_L^\gamma\right\}\right]}. \tag{10}$$

It is clear that the key issue for pricing CDO is to determine the cumulative loss distribution function $F_L(x)$, which is the important element of the expected value of the cumulative tranche loss $\mathrm{E}\left[L\left(K_L^\gamma, K_U^\gamma, t\right)\right]$. However, it is not easy to derive $F_L(x)$ because of influences of the default correlation between the reference entities. Thus, the remaining introduce factor copula model to derive the portfolio cumulative loss distribution $F_L(x)$.

3.2.2 Factor copula model

One factor copula model is a copula model with the latent variables decomposed into one systematic or common factor and n idiosyncratic factors. It assumes that those factors are distributed based on a certain copula function. The first factor copula model for portfolio credit risk was given by Li (2000), which is based on the Gaussian copula. General copula models were introduced in Schönbucher and Schubert (2001).

The value of the *i*-th asset is:

$$V_i = \sqrt{\rho_i}Y + \sqrt{1-\rho_i}Z_i, \forall i = 1,2,\cdots,n, \tag{11}$$

where *Y* is the systematic factor, Z_i is the idiosyncratic factors, and $\rho_i \in [0,1]$ is the correlation coefficient between the i-th asset and the systematic factor. Y and Z_i are mutually independent random variables. Denote the probability distribution function of Y, Z_i, and V_i by F_Y, F_Z, andF_V.

Let K_i be the default barrier of the *i*-th asset, the default time is defined as:

$$\tau_i = \inf\{t \geq 0 : V_i \leq K_i\}, \forall i = 1,2,\cdots,n. \tag{12}$$

The probability of default of the *i*-th asset is p_i, then the default barrier $K_i = F_V^{-1}(p_i)$.

The conditional default probability of the *i*-th asset is:

$$p_i(y) = P(V_i \leq K_i|Y = y) = P\left(z_i \leq \frac{K_i - \sqrt{\rho_i}Y}{\sqrt{1-\rho_i}}\middle|Y = y\right) = F_Z\left[\frac{K_i - \sqrt{\rho_i}y}{\sqrt{1-\rho_i}}\right]. \tag{13}$$

The most frequently used one is one-factor Gaussian copula model, first proposed by Vasicek (2002). The systematic factor Y and the idiosyncratic factors Z_i are independent and identically distributed standard normal random variables, then the random vector $\boldsymbol{V} = (V_1, V_2, \cdots, V_n)'$ is also normally distributed.

3.2.3 Loss distribution of the large homogeneous portfolio

There exists a so-called large homogeneous portfolio approximation (LHP) that allows to derive an analytical situation for the portfolio loss distribution and the then the expected value of the cumulative tranche loss. The approximation, proposed by Vasicek (1987), assumes that the number of the obligors *n* in the portfolio is extremely large. All obligors are homogeneous, which means they are identical in notional amounts, recovery rates, and unconditional default probabilities. Thus, the approximating distribution of the portfolio loss is:

$$\begin{aligned} F_L(x) &= P[(1-R)p_i(y) \leq x] = P\left\{F_Z\left[\frac{K_i - \sqrt{\rho}y}{\sqrt{1-\rho}}\right] \leq \frac{x}{1-R}\right\} \\ &= P\left[y \geq \frac{K_i - \sqrt{1-\rho}F_Z^{-1}\left(\frac{x}{1-R}\right)}{\sqrt{\rho}}\right] \\ &= 1 - F_Y\left[\frac{K_i - \sqrt{1-\rho}F_Z^{-1}\left(\frac{x}{1-R}\right)}{\sqrt{\rho}}\right] \end{aligned} . \tag{14}$$

According to formula (7), the expected value of the cumulative tranche loss is:

$$\begin{aligned} \mathrm{E}\left[L\left(K_L^\gamma, K_U^\gamma, t\right)\right] &= \frac{1}{K_U^\gamma - K_L^\gamma}\left[\int_{K_L^\gamma}^{1}\left(x - K_L^\gamma\right)\mathrm{d}F_L(x) - \int_{K_U^\gamma}^{1}\left(x - K_U^\gamma\right)\mathrm{d}F_L(x)\right] \\ &= \frac{1}{K_U^\gamma - K_L^\gamma}\left\{\begin{aligned} &\int_{F_Z^{-1}\left(\frac{K_L^\gamma}{1-R}\right)}^{F_Z^{-1}(1)}\left[(1-R)F_Z^{-1}(y) - K_L^\gamma\right]f_Y\left(\frac{K_i - \sqrt{\rho}y}{\sqrt{1-\rho}}\right)\mathrm{d}y \\ &-\int_{F_Z^{-1}\left(\frac{K_U^\gamma}{1-R}\right)}^{F_Z^{-1}(1)}\left[(1-R)F_Z^{-1}(y) - K_U^\gamma\right]f_Y\left(\frac{K_i - \sqrt{\rho}y}{\sqrt{1-\rho}}\right)\mathrm{d}y \end{aligned}\right\} \end{aligned} . \tag{15}$$

4. Numerical results

Assume that there is a portfolio of 100 reference with 5-year maturity. The recovery rate is 0.4 and the risk-free interest rate is 0.05. The attachment and detachment points are 3% and 10% respectively. The dimension for all copulas we would like to compare equals the number of reference entities, namely 100. We consider Gaussian copula model first and compute the margins with respect to the correlation parameter ρ as shown in Table 1. There is a negative dependence of the equity tranche and a positive dependence of the senior tranche with respect to the correlation parameter. As for the mezzanine tranche, there is a bumped curve, which means it is not sensitive to the correlation parameter.

Table 1: CDO margins based on Gaussian copula (bps)

ρ	Equity	Mezzanine	Senior
0	3647	150	0
0.1	2389	279	1
0.3	1463	364	9
0.5	997	342	19
0.7	626	268	30
1	94	107	68

Similarly, we can compute CDO margins based on *t* copula and Clayton copula with respect to the parameter ρ and θ as summarized in Table 2 and Table 3. After the calibration of the parameters, it is possible to get the same equity tranche premium. For example, when the parameter of Gaussian copula is 0.3, the parameters of *t*(6), *t*(12), and Clayton copula must be 0.1, 0.2, and 0.18.

Table 2: CDO margins based on t copula (bps)

ρ	***t*(6)**			ρ	***t*(12)**		
	Equity	**Mezzanine**	**Senior**		**Equity**	**Mezzanine**	**Senior**
0	1794	330	3	**0**	2296	277	0
0.1	1463	351	7	**0.2**	1463	361	8
0.3	1075	370	15	**0.4**	1128	331	16
0.5	728	302	21	**0.6**	697	271	23
0.7	564	261	27	**0.8**	447	233	31
1	113	96	61	**1**	98	84	48

Table 3: CDO margins based on Clayton copula (bps)

θ	Equity	Mezzanine	Senior
0	3568	151	0
0.05	2268	286	1
0.18	1463	364	9
0.36	935	368	20
0.66	626	282	23
10	138	105	47

The attachment and detachment points of tranches are different between the US and the European market as illustrated in Table 4. We assume that the portfolio is traded on iTraxx Europe, which comprises 125 equally-weighted European names. The attachment and detachment points are 3%, 6%, 9%, 12%, and 22%.

Table 4: The attachment and detachment points

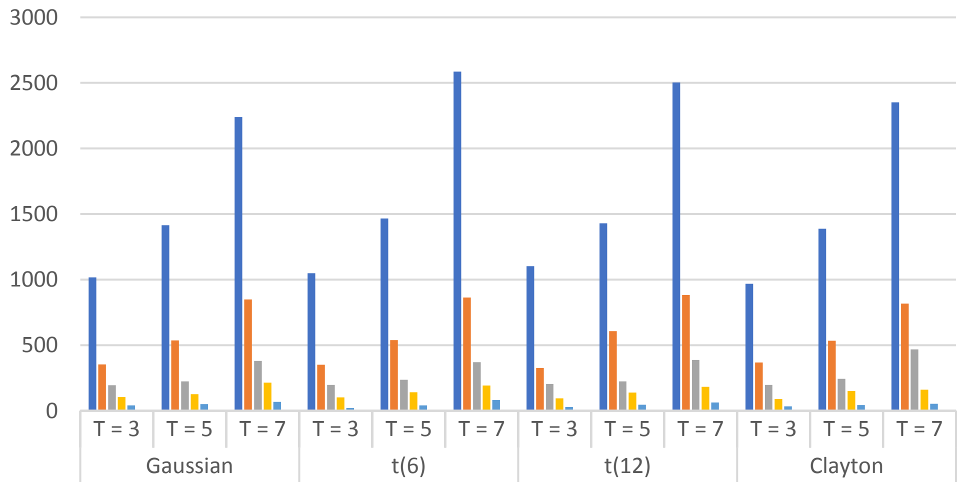
Tranche	CDX US		iTraxx Europe	
	K_L^γ (%)	K_U^γ (%)	K_L^γ (%)	K_U^γ (%)
Equity	0	3	0	3
Mezzanine junior	3	7	3	6
Mezzanine senior	7	10	6	9
Senior junior	10	15	9	12
Senior	15	30	12	22
Super senior	30	100	22	100

Consider the correlation ρ for Gaussian, *t*(6), and *t*(12) copula is 0.3, 0.1, and 0.2; while θ for Clayton copula is 0.18 as we discussed. The CDO maturity is 5 years. The CDO tranche spreads based on different copulas are summarized in Table 5. The results of each tranche are similar, which further proves Gaussian, *t*, and Clayton copula can get close tranche spreads after the appropriate calibrations. The tranche spreads under every single copula are decreasing from the equity tranche to the super senior tranche, because the higher the risk, the higher the par spread. Moreover, most spreads based on *t* copula are slightly higher because *t* copula is better at revealing the extreme values.

Table 5: Tranche spreads based on copulas (bps)

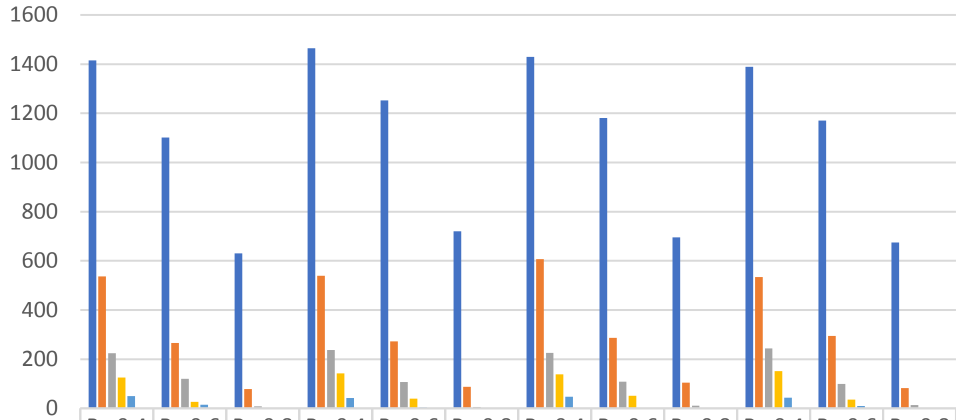
Copula	Equity	Mezzanine junior	Mezzanine senior	Senior junior	Senior	Super senior
Gaussian	1415	537	224	126	50	0.93
***t*(6)**	1465	539	237	142	42	0.72
***t*(12)**	1429	607	225	139	47	0.72
Clayton	1389	534	244	152	43	1.18

Figure 2: Tranche spreads with different maturities (bps)



	Gaussian T = 3	Gaussian T = 5	Gaussian T = 7	t(6) T = 3	t(6) T = 5	t(6) T = 7	t(12) T = 3	t(12) T = 5	t(12) T = 7	Clayton T = 3	Clayton T = 5	Clayton T = 7
Equity	1016	1415	2239	1050	1465	2585	1102	1429	2504	969	1389	2351
Mezzanine junior	354	537	848	352	539	864	327	607	883	367	534	816
Mezzanine senior	196	224	381	197	237	370	204	225	387	198	244	469
Senior junior	104	126	214	103	142	192	95	139	182	91	152	160
Senior	40	50	69	22	42	82	28	47	62	34	43	53
Super senior	0,58	0,93	1,97	0,11	0,72	0,75	0,52	0,72	1,01	0,61	1,18	1,58

Figure 3: Tranche spreads with different recovery rates (bps)



	Gaussian			t(6)			t(12)			Clayton		
	R= 0.4	R= 0.6	R= 0.8	R= 0.4	R= 0.6	R= 0.8	R= 0.4	R= 0.6	R= 0.8	R= 0.4	R= 0.6	R= 0.8
Equity	1415	1101	630	1465	1252	720	1429	1181	696	1389	1170	675
Mezzanine junior	537	266	79	539	272	88	607	287	104	534	294	83
Mezzanine senior	224	120	8	237	107	6	225	109	11	244	99	14
Senior junior	126	26	0	142	39	1	139	51	3	152	35	2
Senior	50	15	0	42	4	0	47	2	0	43	10	0
Super senior	0,93	0	0	0,72	0	0	0,72	0	0	1,18	0,03	0

Equity Mezzanine junior Mezzanine senior Senior junior Senior Super senior

Then we can test how different maturities and discovery rates influence the tranche spreads. The results are summarized in Figure 2 and Figure 3. Note that we fix the recovery rate at 0.4 when we change the maturities and fix the maturity at 5 years when we change the recovery rates. When the maturity is longer, no matter which copula is chosen, the CDO tranche spread is higher. It is reasonable because the risk is higher when the maturity is longer. Moreover, when the recovery rate is higher, the CDO tranche spread is lower, which also agrees with the theoretical analysis.

5. Conclusion

CDOs promote the development of the financial markets as well as a good investment choice for investors. However, CDOs are usually associated with great hidden peril. Pricing of CDO therefore becomes a research priority. In this paper, we focused on copula models for CDO pricing, especially so-called factor copula models. Factor copula model assumes the latent variables decomposed into one systematic or common factor and *n* idiosyncratic factors. It enables us to compute the conditional default probability and the approximating distribution of the portfolio loss.

We considered a portfolio of 100 reference with 5-year maturity traded on iTraxx Europe, the results illustrate that once calibrated correctly, *t* copula and Clayton copula can provide close results as Gaussian copula provides. *t* copula usually provides slightly higher CDO tranche spreads because *t* distribution is fat tailed and better at revealing the extreme values. Moreover, the CDO tranche spread is higher with longer maturity and lower recovery rate.

Acknowledgments

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Copula function estimation using FT technique

Tomáš Tichý[1], Aleš Kresta[2]

Abstract

Modelling of multidimensional distribution function can be in finance divided into two steps: modelling of univariate marginal distributions and modelling of a dependency. The dependency is usually handled by means of copula function, which links together the marginal distributions and thus create a multidimensional distribution function. However, the estimation of the copula function parameters is problematic as these are not directly observable. In this contribution, we apply fuzzy transform (FT) filter for estimation and smoothing of copula functions. Based on illustrative study of two stock market indices we show some interesting properties of this novel approach.

Key words

Copula function, non-parametric estimation, fuzzy transform

JEL Classification: C58

1. Introduction

Copula functions are, in opposed to marginal or joint distribution functions, not directly observable and thus, their estimation is challenging, see, e.g. Cherubini et al. (2004) or Nelsen (2006). On the other hand, such kind of dependency structure can be very useful in many financial applications.

Obviously, assuming some kind of marginal and joint probability distribution, the copula function can easily be estimated parametrically. In other cases, semiparametric or fully non-parametric approaches should be preferred. In the past, several researchers suggested various ways for efficient estimation of copula functions, see, e.g., Genest et al. (1995) and Chen and Fan (2006) or Fine and Jiang (2000) for semiparametric approach, and Deheuvels (1979), Fermanian and Scaillet (2003) or Fermanian (2005) for non-parametric approach.

In this contribution, we focus on rather novel approach of Fuzzy Transform (FT) technique proposed by Perfilieva (2006) and suggest its usage in non-parametric estimation of empirical copula function. We proceed as follows, in Section 2 copula functions are defined; in Section 3 brief definition of FT approach is provided; finally, in Section 4 an illustrative study is presented.

2. Copula functions

Copula function is in fact a real function, which maps the dependency among particular distribution functions into [0,1],

$$C : [0,1]^n \rightarrow [0,1] \text{ on } R^n. \tag{1}$$

Every copula function has to satisfy the following three conditions: (*i*) $C(u) = 0$, whenever u=$[0,1]^n$ has at least one component being equal to 0, (*ii*) $C(u) = u_i$ whenever $u = [0,1]^n$ has all

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the components being equal to 1 except the i-th, which is equal to u_i and finally (iii) $C(u)$ is n-increasing.

Actually, any copula function can be regarded as a multidimensional distribution function with marginals in the form of standardized uniform distribution. Assume potentially dependent random variables with marginal distribution functions $F_1, \ldots, F_n$ and joint distribution function $F_{1,\ldots,n}$. Then following the Sklar's theorem (Sklar, 1959):

$$F_{1,\ldots,n}(x_1, \ldots, x_n) = C\,(F_1(x_1), \ldots, F_n(x_n)). \tag{2}$$

If all distribution functions are continuous, a copula function C is unique. Sklar's theorem implies also an inverse relation,

$$C(u_1, \ldots, u_n) = F_{1,\ldots,n}(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)). \tag{3}$$

When we have m empirical observations $\left(x_1^m,\ldots,x_n^m\right)$, the empirical copula function can be defined as follows,

$$C\left(u_1,\ldots,u_n\right) = \frac{1}{m}\sum_{i=1}^{m} 1\left(\tilde{U}_1 \le u_1,\ldots,\tilde{U}_n \le u_n\right). \tag{4}$$

However, the above function is not smooth and less data we observe (m) or higher is the dimension (n) of the problem the more uneven the function is. In this problem the fuzzy smoothing filter can be applied.

3. Discrete fuzzy transforms

The technique of fuzzy transform was originally proposed by Perfilieva (2004, 2006) as a tool for image processing. Recently, in Holčapek and Tichý (2010, 2011) it was suggested to use this method for financial time series smoothing.

Following Perfilieva (2006), when we apply fuzzy transform approach, as we can guess from the term fuzzy, the independent variables are fuzzyfied according to the proximity to a given point, which can be compared to the concept of weights obtained via probability distribution function in kernel regression approach. Next, the observations of the dependent variable are averaged, which forms a functional relation for a given point. This step of fuzzy transform is called direct fuzzy transform. Obviously, the second step is inverse fuzzy transform, within which we return back to the original crisp space and obtain a smoothed function describing the relation of both variables.

Assuming that R is a real interval, g is a finite real function given at the nodes $x_1 < \cdot \cdot \cdot < x_n$ with $\mathrm{Dom}(g) \subseteq \mathrm{R}$ and $A = \{A_i | i \in I\}$ is a fuzzy r-partition of R determined by (T, S) such that $\mathrm{Dom}(g)$ is sufficiently dense with respect to A, one can say that a collection of real numbers $\{F_i | i \in I\}$ is discrete (direct) fuzzy transform of g with respect to A, if

$$\mathrm{F_i} = \frac{\sum_{j=1}^{n} \mathrm{g(x_j)A_i(x_j)}}{\sum_{j=1}^{n} \mathrm{A_i(x_j)h}}. \tag{5}$$

The numbers F_i are called components of the discrete F-transform. The F-transform of an original function serves as its discrete representation, which can be e.g. successfully used in numerical computations. To bring the F-transform back we use the inverse F-transform,

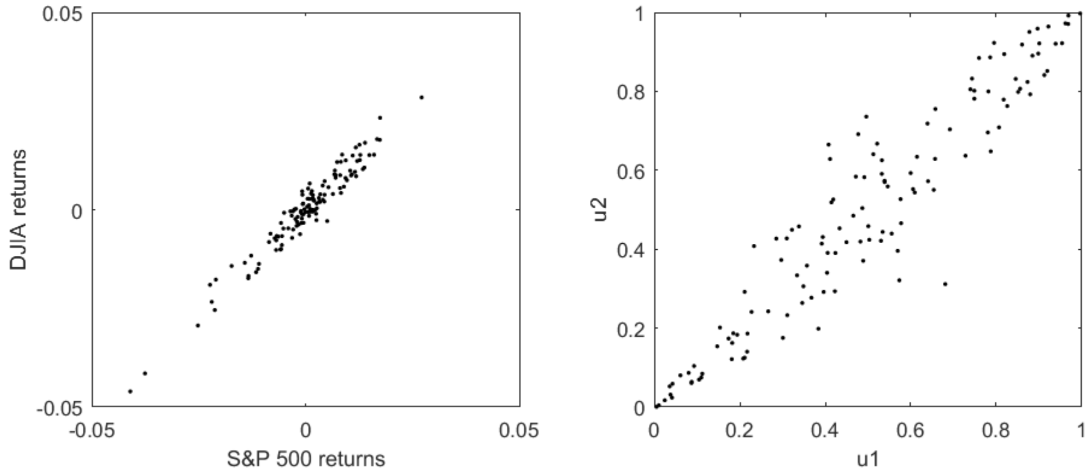
$$g(x_j) = \sum_{i=1}^{n} F_i A_i(x_j). \tag{6}$$

The above mentioned functions and the whole concept can be easily generalized to higher dimensions, see e.g. Martino et al. (2011). This can be helpful when smoothing the empirical cumulative density function (cdf) or empirical copula function (4), which represents only the dependence structure in cdf.

4. Illustrative example using stock market indices

In the real world application, we assume daily returns of two important American stock market indices DJIA and S&P 500 over the last year (19.6.2017 – 18.6.2018), see Figure 1.

Figure 1: Daily returns (left) transformed to uniform marginal distributions (right)



In figure 2 we show the comparison of empirical copulas (left graphs) smoothed by FT technique (right graphs). The copulas are calculated from last 10 (upper graphs) and 21 (lower graphs) daily returns. In figure 3 we show contour plot of empirical copula (left graph) smoothed by FT technique (right graph). As can be seen from the figures, the effect of the smoothing is higher for lower numbers of daily returns utilized in the estimation.

Figure 2: Empirical copula (left) smoothed by FT (right) based on 10 (upper) and 21 (lower) data points

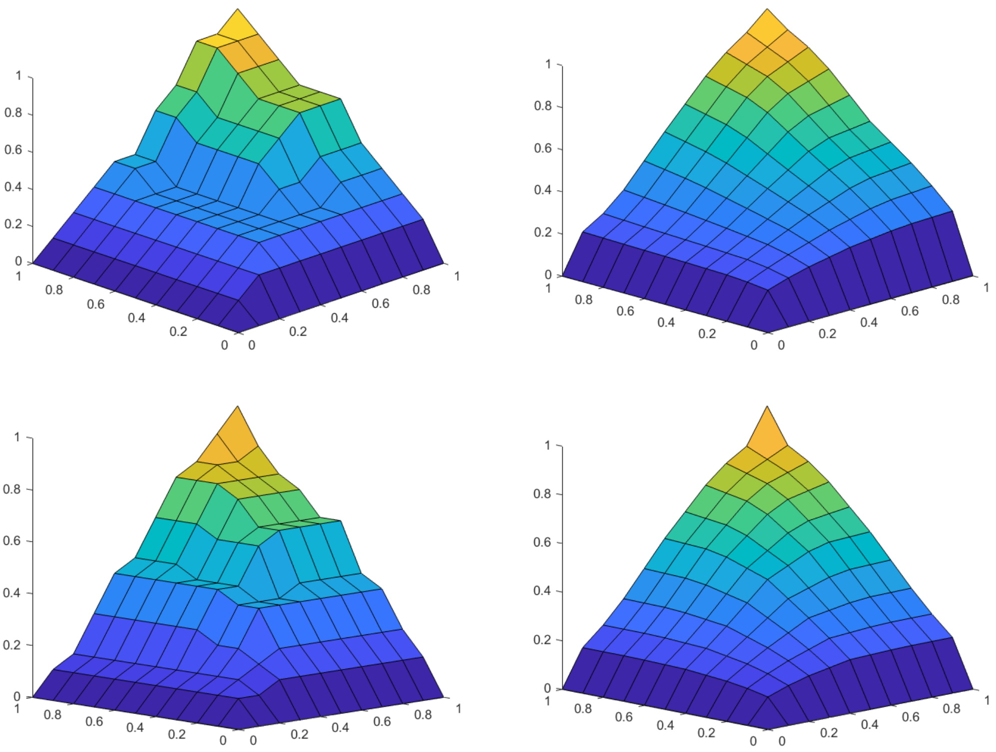
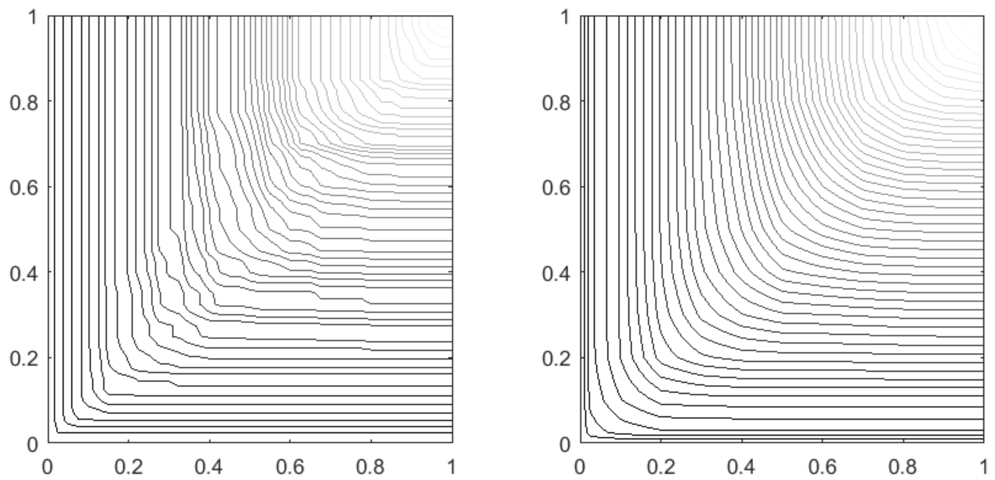


Figure 3: Contour plot of empirical copula function (left) smoothed by FS filter (right) from 252 data points



The advantage of the smoothed copula is also that it is specified by less parameters. In our application, we assumed 25 nodes (5 for each dimension). From these data (i.e. 25 values) we can obtain function values $C(u_1, u_2)$ for every $u \in [0,1]^n$. On the other hand, the empirical copula functions are specified by 10/21/252 pairs of daily returns. As we can see, by smoothing the empirical copula function there is generally less parameters compared to the empirical copula.

Moreover, we compare the fitness of smoothed copula function to the fitness of well-known parametrical copulas. In Table 1, we depict the mean absolute difference between empirical copula and smoothed copula (FT). The benchmark is the mean absolute difference between empirical copula and estimated parametrical copulas.

As it is apparent from the table, the smoothed copula has the lowest mean absolute error for small number of observations (10 days), however, as the number of observations increases (21 or 63 days) the parametrical copulas fit the empirical data better. For 21 days Gaussian copula provides best fit, for 63 days Gumbel copula best fits the data. It is important to note that all parametrical copulas except Student copula have only one parameter to be fitted, while by applying FT we obtain 25 parameters.

Table 1: The mean absolute difference between empirical copula and smoothed copula

	10 days	21 days	63 days
FT	**0.045157**	0.030987	0.027566
Gaussian	0.04841	**0.030765**	0.018143
Student	0.055401	0.063889	0.069763
Clayton	0.053177	0.039112	0.019791
Frank	0.052689	0.031261	0.018394
Gumbel	0.068189	0.032741	**0.017905**

5. Conclusion

In this contribution we have extended the FT filter for estimation and smoothing of copula functions, i.e., a function to specify the joint distribution of random variables. Illustrative study based on stock market index data shows interesting properties of this novel approach. We therefore suggest further analysis of the method for various kind of data and parameter settings, in order to provide complex evaluation of the strengths and weakness of this specific approach of smoothing and time series forecasting.

Acknowledgements

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Network conditional tail risk estimation in the European banking system

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Abstract

CoVaR is one of the most popular measures of systemic risk. It is the VaR (Value at Risk) of the system (represented as a broad market index) conditional to the fact that a certain institution is in distress (i.e. at its VaR). One of the limits of CoVaR is that it does not consider the relations among institutions in the system, failing to represent interconnectedness, that is a relevant component of systemic risk. Instead, it reflects more the *systematic* component of risk, that is, the one related to a common component. A popular approach to analyze interconnectedness is to consider an economic system as a network. In this work we deal with network-ΔCoVaR, a multivariate extension of ΔCoVaR, that measures the marginal tail dependence among institutions while controlling for the effects of the others. We discuss the properties of the model and we propose an estimation methodology based on quantile regression with Smoothly Clipped Absolute Deviation (SCAD) penalty. Finally, we use these tail risk networks to develop systemic risk indicators and to study the characteristic of the European banking system and its evolution over time.

Key words: ΔCoVaR; Systemic risk; Quantile regression; Financial networks

JEL Classification: C58, G01, G15

1. Introduction

Since the global financial crisis in 2008, systemic risk became a relevant topic for scholars, investors and regulators. Many approaches have been defined to measure and handle it. One of the main challenges is the lack of a shared definition. This leads to very different modelization approaches, each one focused on different aspects of systemic risk.

Some works take systemic risk in terms of potential for the spreading of financial distress, measuring this increase in tail comovement. These works focus on the analysis of tail risk under stress scenarios, measuring either the effect of a systemic shock to the value of an institution, or the effect of the distress of an institution to the entire system. The most known approaches in this group are probably the CoVaR and the closely related ΔCoVaR [1]. In particular, CoVaR measures the Value at Risk (VaR) of the system conditional to a particular asset being distressed, and ΔCoVaR compares the CoVaR to its VaR in a non distressed situation.

Other works use a network approach to study the interdependence of assets. They embrace a vision of systemic risk more focused on the presence of risk spillover and contagion (see e.g. the definition of systemic risk given by [2]. These approaches model institutions as nodes in a network, and their relations as edges. They can be used to uncover structural features of the system that may not emerge from aggregated data. A main challenge is the estimation of the network: it can be modelled in several ways, either considering physical measures of interconnectedness or with statistical measures based on time series.

In this work we join the network approach to the ΔCoVaR approach, by studying systemic risk from a network perspective, extending some of the results developed in the CoVaR

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framework to a multivariate setting, studying the theoretical properties of these networks and comparing them in an empirical application focused on the European market.

The extension of ΔCoVaR is based on a concept recently introduced in the statistical literature. That is, quantile graphical models [3,4]. This approach models the quantiles of a variable in a system conditional to the value of the other variables, and provides a rich and flexible modelization of a multivariate system. The estimation is typically done using quantile regression. We define network-ΔCoVaR (also denoted in matrix form as **ΔCoVaR**) and we show how such model can also be considered an extension of partial correlation networks, in which we consider conditional quantiles instead of the conditional means.

After discussing some of the properties of the model under parametric specifications, we propose an estimation procedure based on SCAD penalized quantile regression and an empirical application focused on the European banking system.

2. Market-based systemic risk measures

2.1 CoVaR and -ΔCoVaR

Value at Risk (VaR) is a popular risk measure that indicates the potential loss of a position in a given time period with a certain level of confidence $1-\tau$. We can define it implicitly as the τ - quantile:

$$\Pr\{X_i \leq VaR_\tau^{X_i}\} = \tau.$$

Such measure is based on the univariate distribution of the variable X_i, and it does not allow to measure the risk related to interaction among assets. CoVaR is an alternative measure proposed by [1] that allows to consider conditional tail risk. In particular, CoVaR measures what happens to the system's VaR when one institution is under stress.
we denote CoVaR as a value such that:

$$\Pr\left\{X_i \leq CoVaR_\tau^{X_{sys}|X_i} | X_i = VaR_\tau^{X_i}\right\} = \tau,$$

where X_{sys} is the return of the entire system, and X_i the returns of the *i*th institution.
A high CoVaR could be caused by an overall high VaR of the system. Therefore, we compare CoVaR to a reference value, typically the VaR of the system when institution i is in a normal state (i.e. its median value). We consider therefore ΔCoVaR:

$$\Delta CoVaR_\tau^{X_{sys}|X_i} = CoVaR_\tau^{X_{sys}|X_i} - CoVaR_{50\%}^{X_{sys}|X_i}.$$

ΔCoVaR is a popular measure of systemic risk. The estimation is commonly performed using quantile regression. When estimated using quantile regression, ΔCoVaR can be expressed as:

$$\Delta CoVaR_\tau^{X_{sys}|X_i} = \beta_{tau}^{X_{sys}|X_i}\left(VaR_\tau^{X_i} - VaR_{50\%}^{X_i}\right),$$

where $\beta_{tau}^{X_{sys}|X_i}$ is the τ-quantile regression parameter of the quantile regression of X_{sys} over X_i.

2.1.1 Limits of ΔCoVaR

Although very popular in the literature, CoVaR and ΔCoVaR have some issues as systemic risk indicators. First, as highlighted by [5], ΔCoVaR can be interpreted as a measure of *systematic* risk, rather than *systemic*. Indeed, [5] show that this measure under certain assumptions is simply a proxy for systemic risk beta, that is easier to estimate.

Two more arguments regard the shape of the quantile functions: first, the slope of the quantile function can be influenced by the characteristics of the bivariate distribution, and it is not always related to the level of tail dependence. Then, the estimation performed using quantile regression impose the linearity of the conditional quantile function, leading to potential underestimation of the conditional tail risk.
Finally, the ability of conditional tail measures to predict actual distress in financial institions has been questioned by empirical analyses, see e.g. [6].

2.2 Network models for systemic risk – partial correlation networks

Network models allow to consider the component of contagion across companies or financial institutions. In particular, several works use network centrality measures to assess the relevance of a bank in a system. One of the most relevant challeges is the estimation of the network structure, and the literature describes several approaches. A relevant strand of literature focuses on measures of co-movement of time series, e.g. [7,8]. We follow this strand, and we focus in particular on partial correlation networks.
Partial correlation networks allow to model the correlation between any couple of variables while controlling for all the others in the system. The edge *ij* of the network is defined as the partial correlation between nodes *i* and *j*, conditional to all the other variables in the system. An important form of partial correlation networks are Gaussian graphical models that, under the assumption that data have a jointly normal distribution, imply that two nodes that are not connected by an edge are conditionally independent. The reader can refer to [8,9] for more details, where partial correlation models are presented, together with estimation procedures under Gaussian and t-Student distribution.

3. Network-ΔCoVaR

Here we extend ΔCoVaR to the network case, and we highlight some of the properties. The concept is strictly related to the quantile graphical models [3]. In general, we can express the quantiles of the distribution of an item conditional to the others as

$$Q_{X_{i,\tau}} = f\left(X_{\setminus i}, \tau\right),$$

where f is a generic function. The estimation of f is particularly challenging in high dimension, and several simplifying assumptions can help the estimation. A common choice is to use an additive form: $f\left(X_{\setminus i}, \tau\right) = \sum_{j \neq i} f_j\left(X_j, \tau\right)$, where $f_j\left(X_j, \tau\right)$ are smooth functions [4]. A further simplification is to assume that $f_i\left(X_j, \tau\right)$ are linear. In this case we have $f_i\left(X_j, \tau\right) = a_i + \beta^{i|\cdot} X_{\setminus i}$. The coefficients $\beta^{i|\cdot}$ can be conveniently represented by the matrix B, consisting in the squared matrix consisting in the stacked vectors $\beta^{i|\cdot}$, with zero on the main diagonal.
Given the quantile graphical model, we can extend the concept of CoVaR to the multivariate setting: we measure the quantile function of asset i conditional to the fact that asset j is in distress, and the other assets in the system are in their normal (i.e. median) state. More formally:

$$\Pr\left\{X_i \leq CoVaR_\tau^{X_i|X_j} | X_j = VaR_\tau^{X_j}, X_{\setminus\{ij\}} = VaR_{50\%}^{\setminus\{ij\}}\right\} = \tau.$$

We explicitly we have:

$$CoVaR_\tau^{X_i|X_j} = VaR_\tau\left(X_i \middle| X_j = VaR_\tau^{X_j}, X_{\setminus\{ij\}} = VaR_{50\%}^{\setminus\{ij\}}\right).$$

Then, we can compute ΔCoVaR as:

$$\Delta CoVaR_\tau^{X_i|X_j} = CoVaR_\tau^{X_i|X_i} - CoVaR_{50\%}^{X_i|X_j}.$$

We can construct a weighted and directed network using the set of all bilateral $\Delta CoVaR_\tau^{X_i|X_j}$. For convenience we define the weighted adjacency matrix **ΔCoVaR**, where the element *ij* is equal to $\Delta CoVaR_\tau^{X_i|X_j}$.

3.1 Parametric examples

We consider first a parametric specification characterized by a *p*-variate Gaussian distribution, and consider the following partition:

$$X = \begin{bmatrix} X_{\setminus i} \\ X_i \end{bmatrix} \sim \mathcal{N}_p(\mu, \boldsymbol{\Sigma}) = \mathcal{N}_p\left(\begin{bmatrix} \mu_{\setminus i} \\ \mu_i \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{\setminus i \setminus i} & \boldsymbol{\Sigma}_{\setminus ii} \\ \boldsymbol{\Sigma}_{i \setminus i} & \boldsymbol{\Sigma}_{ii} \end{bmatrix}\right).$$

Focusing for simplicity on the case $\mu = \mathbf{0}$, we represent the conditional quantile of $X_i|X_{\setminus i}$ as:

$$Q_\tau\big(X_i \big| X_{\setminus i}, \tau\big) = \phi^{-1}(\tau)\boldsymbol{\Sigma}_{ii|\setminus i}^{0.5} + \mu_{i|\setminus i}$$

where $\mu_{i|\setminus i} = \boldsymbol{\Sigma}_{i\setminus i}\boldsymbol{\Sigma}_{\setminus i\setminus i}^{-1}X_{\setminus i}$ and $\boldsymbol{\Sigma}_{ii|\setminus i} = \boldsymbol{\Sigma}_{ii} - \boldsymbol{\Sigma}_{i\setminus i}\boldsymbol{\Sigma}_{\setminus i\setminus i}^{-1}\boldsymbol{\Sigma}_{\setminus ii}$.
It follows that all the conditional quantile functions are affine, and parallel to the OLS regression coefficients, as we notice that $\beta_{OLS}^{i|\setminus i} = \boldsymbol{\Sigma}_{i\setminus i}\boldsymbol{\Sigma}_{\setminus i\setminus i}^{-1}$.
We can then express network-CoVaR and network-*Δ*CoVaR in terms of $\beta_{OLS}^{i|\setminus i}$, as:

$$CoVaR_\tau^{X_i|X_j} = \phi^{-1}(\tau)\boldsymbol{\Sigma}_{ii|\setminus i}^{0.5} + \beta_{OLS}^{i|\setminus i}X_{\setminus i},$$

$$\Delta CoVaR_\tau^{X_i|X_j} = \beta_{OLS}^{i|\setminus i}VaR_\tau^{X_j}.$$

In matrix form we can express network-$\Delta CoVaR$ as $\boldsymbol{\Delta CoVaR_\tau}$:

$$\boldsymbol{\Delta CoVaR_\tau} = \phi^{-1}(\tau)\boldsymbol{B}\,\boldsymbol{D}_{\boldsymbol{\Sigma}}^{0.5},$$

where $\boldsymbol{B}$ is the matrix of OLS coefficients, and $\boldsymbol{D}_{\boldsymbol{\Sigma}}$ is a diagonal matrix with the diagonal elements of the covariance matrix $\boldsymbol{\Sigma}$. Following [8,9,10], we know that the matrix $\boldsymbol{B}$ is a rescaling of the partial correlation matrix. It follows that in the Gaussian case network-*Δ*CoVaR carry the same information of partial correlation networks.

We consider then a t-Student specification. Let $X \sim t_p(\mu, \boldsymbol{\Sigma}, \nu)$, where ν is the number of degrees of freedom of the distribution, and μ and $\boldsymbol{\Sigma}$ are the mean and scatter parameters, respectively. Similarly to the Gaussian case, we consider the partition $X = \begin{bmatrix} X_{\setminus i} \\ X_i \end{bmatrix}$. Focusing again on the case with $\mu = \mathbf{0}$, according to [11] we can compute the conditional quantile of X_i given $X_{\setminus i}$:

$$Q_\tau\big(X_i \big| X_{\setminus i}, \tau\big) = Q_{t,\nu+p-1}(\tau)\left(\frac{\nu + d_1\big(X_{\setminus i}\big)}{\nu + p - 1}\boldsymbol{\Sigma}_{ii|\setminus i}\right)^{0.5} + \mu_{i|\setminus i},$$

where $\mu_{i|\setminus i}$ and $\boldsymbol{\Sigma}_{ii|\setminus i}$ are defined as in the Gaussian case, and $d_1\big(X_{\setminus i}\big) = X_{\setminus i}'\boldsymbol{\Sigma}_{\setminus i\setminus i}^{-1}X_{\setminus i}$ is the squared Mahalanobis distance of $X_{\setminus i}$ from the origin with scale matrix $\boldsymbol{\Sigma}_{\setminus i\setminus i}$.

The function is not linear in $X_{\setminus i}$ due to the term $d_1\big(X_{\setminus i}\big)$. It follows that, differently from the Gaussian case, the conditional quantile cannot be computed simply as a translation of the $\beta_{OLS}^{i|\setminus i}$.

We can then compute network-CoVaR and network-ΔCoVaR as:

$$CoVaR_{\tau}^{X_i|X_j} = Q_{t,\nu+p-1}(\tau)\left(\frac{\nu + d_1(X_{\setminus i})}{\nu+p-1}\boldsymbol{\Sigma}_{ii|\setminus i}\right)^{0.5} + \beta_{OLS}^{i|\setminus i}X_{\setminus i}^{d},$$

$$\Delta CoVaR_{\tau}^{X_i|X_j} = Q_{t,\nu+p-1}(\tau)\boldsymbol{\Sigma}_{ii|\setminus i}^{0.5}\left(\left(\frac{\nu + d_1(X_{\setminus i})}{\nu+p-1}\right)^{0.5} - \left(\frac{\nu}{\nu+p-1}\right)^{0.5}\right) + \beta_{OLS}^{i|\setminus i}X_{\setminus i}^{d},$$

Where $X_{\setminus i}^{d}$ denotes a state of the system in which $X_j = VaR_{\tau}^{X_j}, X_{\setminus\{ij\}} = VaR_{0.5}^{X_{\setminus\{ij\}}}$.

In a t-Student setting, a shock to a variable j will therefore have a non-linear impact on the conditional value at risk of an asset i, and the impact will be higher compared to the Gaussian setting.
Moreover, the estimation of a linear quantile function (as the one obtained using a quantile regression procedure) will underestimate the network- ΔCoVaR, as for $\tau < 0.5$ the quantile function will underestimate the value of the network- ΔCoVaR due to the shape of the quantile function, that is concave for $\tau < 0.5$.
Overall, this t-Student setting show us that interconnectedness may have a particularly strong effect in the tails of the distributions, that does not happen in a Gaussian setting.

4. Estimation of network-ΔCoVaR

Quantile regression is a powerful framework for the estimation of non-parametric quantile graphical model. We consider a linear specification of penalized quantile regression, that allows to perform model selection and estimation at the same time by setting some of the parameters to zero, increasing the efficiency of the estimator in setting with a high number of variables in relation to the observations [3]. Contrarily to previous literature, we use SCAD penalized quantile regression [12].
Quantile regression allows to summarize the relationship between a set of regressors and an outcome variable. Differently from the more common mean regression framework, we model the value of the conditional quantile. It is possible to obtain the quantile regression estimator by minimizing an asymmetric loss function. The optimization problem is linear and can be solved efficiently using the simplex method, or interior point methods [13]. We can obtain a sparse estimate of the model (i.e. a model where some of the parameters are exactly equal to zero) by introducing a penalization in the optimization problem [13, 3]:

$$\min_{\beta_\tau} \mathbb{E}\left[\rho\left(X_i - \beta_\tau X_{\setminus i}\right)\right] + n\sqrt{\tau(1-\tau)}p_\lambda(\beta_\tau)$$

Where $\rho(u) = \left(\tau - \mathbb{I}_{\{u\leq 0\}}\right)u$ is an asymmetric loss function, n is the number of observations, β_τ the set of parameters and $p_\lambda(\cdot)$ a penalty function. The most common penalty function is the *lasso*, that penalized the sum of absolute values of the vector of parameters inducing sparsity [3]. We consider instead the SCAD penalty that is shaped as follows:

$$p_{\lambda}^{SCAD}(\beta) = \begin{cases} \lambda|\beta| & if\ |\beta| < \lambda \\ -\dfrac{|\beta|^2 - 2a\lambda|\beta| + \lambda^2}{2(a-1)} & if\ \lambda \leq |\beta| < a\lambda. \\ \dfrac{(a+1)\lambda^2}{2} & if\ |\beta| \geq a\lambda \end{cases}$$

The function is linear and equivalent to a lasso penalization near the origin, then quadratic for a trait and finally flat [14].
[12] proved that the model estimation performed using SCAD penalty has the oracle property (i.e., asymptotically, it identifies the right subset model and it has an optimal estimation rate). This is an advantage over lasso, that does not. The non-convexity of the penalization makes the optimization problem much harder to solve, however several specific algorithms have been developed. For penalized quantile regression problems, we rely on the procedure outlined by [12]. They propose a Difference Convex Algorithm (DCA) that is based on the representation of SCAD penalty as the difference between a linear and a convex function, and solves a sequence of convex problems to approximate the SCAD problem efficiently. The SCAD penalization requires the calibration of two parameters: the penalization factor λ and the parameter a that regulates the shape of the penalty. We use a value of $\alpha = 2.7$ as proposed by [14], and we calibrate λ using Bayesian Information Criterion (BIC).

Quantile regression typically estimates a linear model, where the conditional quantile functions are straight lines (it is possible to model quantile function using different specifications, but this would increase the number of parameters of the model to estimate and would increase the computational complexity). We have seen in Section 3.1 that, except for the Gaussian case, linearity is not respected. We propose therefore to estimate the penalized quantile regressions in a specific sub-set of the data, that is the data points where the market obtained the lowest returns. In this way, we focus on the slope of the quantile function in the observations of distressed markets. In order to guarantee a good balance between size of the estimation dataset and localization of the quantile, we focus on the lowest 50% observations of an equally weighted index of all the data.

5. Network indicators

Network centrality measures are commonly used to assess the relevance of nodes in a network [7, 8, 10]. We consider here strength centrality as an indicator of relevance of nodes, and we develop a novel indicator that combines network tail risk and a measure of idiosyncratic credit risk (i.e. Non-Performing Loans – NPL). Since network- ΔCoVaR is a directed network, we can compute two set of indicators: in-strength c_{in}^{i}, representing systemic fragility, and out-strength c_{out}^{i}, representing systemic relevance:

$$c_{in}^{i} = \sum_{j=1}^{p} \Delta CoVaR_{\tau}^{i|j} , \qquad c_{out}^{i} = \sum_{j=1}^{p} \Delta CoVaR_{\tau}^{j|i} .$$

Another set of indicators includes the NPL in the computation:

$$c_{NPL-in}^{i} = \sum_{j=1}^{p} \Delta CoVaR_{\tau}^{i|j}\ NPL_{j}, \qquad c_{NPL-out}^{i} = \sum_{j=1}^{p} \Delta CoVaR_{\tau}^{j|i}\ NPL_{i},$$

where NPL_i is the NPL ratio of bank i, computed as the ratio between non performing loans over total loans. This measure allows us to provide a richer and nuanced assessment of systemic risk, as interconnection alone may not be enough to cause financial risk. The credit quality of the neighbors may influence the risk of a bank: a strong connection with a solid bank may indeed not represent a threat, while a fragile company may significantly affect the stability of the neighbors.

6. Empirical analysis

We estimate network-ΔCoVaR on the equity returns of a set of 36 large European banks. Due to the presence of heteroskedasticity in the data, we proceed in two steps: first we fit a CCC-GARCH model to the data, and then we estimate the conditional quantile model on the residuals. We report here for brevity the result of the analysis performed in the period Jan2007 - Dec2012. The analysis has been repeated also for the period Jan2013-Jun2018, and a more extensive analysis will be presented in future works.

For comparison, we compute two networks: network-ΔCoVaR and a network based on the quantile graphical model with $\tau = 0.5$, that is, the conditional medians (median QGM). In this way we highlight the different behavior of the tails and the center of the distributions. Note that for the network-ΔCoVaR we consider $\tau = 0.1$; we did not choose a smaller value to avoid estimation instability in finite samples.

Figure 1 shows a graphical representation of the network-ΔCoVaR and the median conditional quantile network. We see that network-ΔCoVaR presents a denser network compared to median QGM, denoting that the interconnectivity is stronger in the tails of the distribution. Moreover, we see that both networks have a community structure (i.e. nodes are grouped in dense clusters), but such structure is stronger in median QGM compared to ΔCoVaR. Together these two facts suggest that conditional tail risk is a relevant channel for the transmission of financial distress, highlighting how shocks to individual institutions can spread to the entire network fast, despite the presence of a community structure.

Figure 2 reports the in- and out-strength centrality for each bank, as well as the NPL-weighted centrality. We see that banks in Northern and Central Europe have particularly low NPL-adjusted in- and out-strength centrality, denoting less systemic relevance. Italian, Irish and Spanish banks on the other hand have high NPL-weighted centrality. The traditional strength centralities instead are more evenly distributed, and banks in central Europe are among the most relevant, suggesting that they are bridges for the transmission of distress.

Figure 1: Representation of ΔCoVaR (τ = 0.1) and median QGM model for the European banking system (2007-12). Colors represent countries.

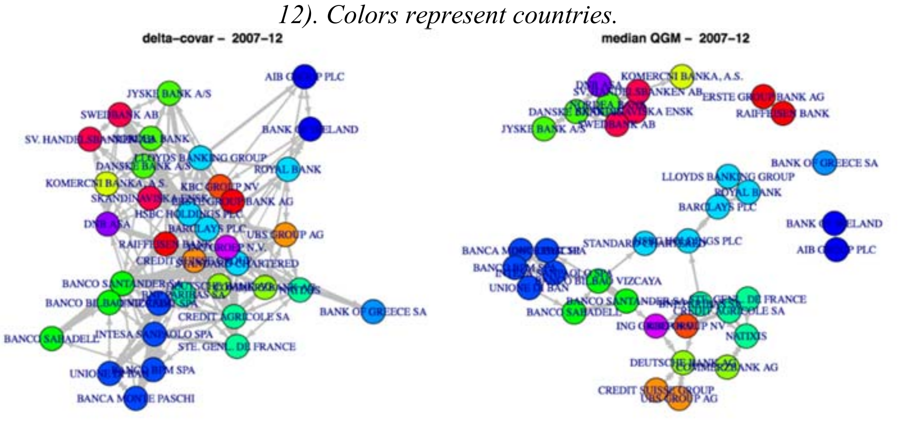
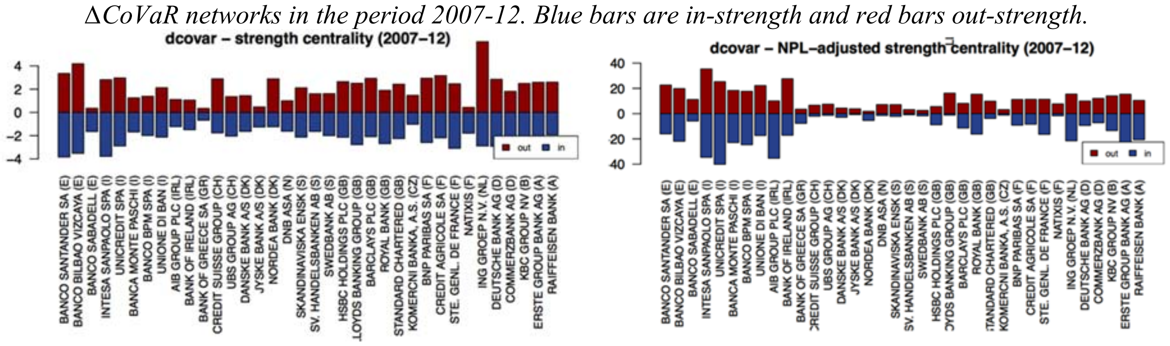


Figure 2: Strength centrality (left) and NPL adjusted strength centrality (right) of the banks in the system for ΔCoVaR networks in the period 2007-12. Blue bars are in-strength and red bars out-strength.



7. Conclusion

We introduced network-ΔCoVaR to characterize the connectivity structure in the tails of banks' equity, extending the concept of ΔCoVaR, and relating it to partial correlation networks.
After showing some properties under Gaussian and t-Student assumptions, we introduce an estimation procedure based on SCAD-penalized quantile regression, and we propose an empirical application to the European banking system, finding that tail risk network is strongly connected and less clustered than median-QGM. This work opens several research lines: first, we may study more in details the theoretical properties of the network-ΔCoVaR, then we can extend the empirical analysis to different time periods and different network indicators, then it would be interesting to assess the ability of the network indicators to forecast realized losses due to systemic risk, and to relate it to regulators' tools. Finally, it would be interesting to use network-ΔCoVaR in a portfolio management framework.

Acknowledgements

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Selection of optimal vehicle insurance by stochastic optimisation

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Abstract

The paper is focused on the finding of the optimal vehicle insurance including policy conditions. We respect that the loss, or severity respectively, is a random variable that follows an exponential probability distribution. Firstly, we set the objective function that includes coinsurance, deductibles as well as limit value that vary across all variants of insurance product and we formulate an optimisation problem. Then, using an illustrative example made up of real variants of vehicle insurance, we identify the optimal choice that minimizes the expected retained part of the loss as well as the premium.

Keywords

Coinsurance, deductibles, limit value, optimal coverage, selection, stochastic optimisation, vehicle insurance

JEL Classification: C44, C61, G22

1. Introduction

Decisions about the optimal insurance product are usually done by involving multi-criteria decision methods where policy conditions, as well as prices, are compared and the "best" product is identified using various methods, for instance (Borovcová, 2014, 2017). However, the final choice represents the best coverage for the given price rather than optimal because this approach does not respect the stochastic nature of insurance claims and even that the premium is supposed to pay if no insured accident occurs. As a result, the potential loss might be totally covered but the policyholder, on the other hand, pays expensive insurance that will be never or rarely used (with very small probability).

In addition, the approach considering several criteria is based on one out of the multi-criteria methods that involve the calculation of weights representing how important each of the criteria is for the policyholder. In the process, the weights need not be determined via AHP process of (Saaty, 1980), see for instance (Borovcová, 2017), but may be derived also in another more understandable manner, see (Tzeng and Huang, 2011), (Zmeškal et al., 2013) or (Rao, 2014) for the overview of methods. However, this process is sensitive to correct evaluation of policyholder's preferences and the final choice may be significantly influenced by these importance weights.

By contrast, an active approach to selection provides the optimal solution when the insurance is not so expensive and the potential loss is not fully covered only with very low probability. However, this alternative approach requires one out of stochastic programming techniques. For an introduction to stochastic optimization, we recommend some general books addressed to this issue, e.g. (Kall and Mayer, 2011) or (King and Wallace, 2012).

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The goal of the paper is to determine an optimal vehicle insurance. We respect the stochastic nature of insurance claims to avoid the purchase of useless and expensive high coverage. The remainder of the paper is organized as follows. The optimisation problem of stochastic programming, as well as the approximation to closed-form objective function, is described in Section 2. Finally, there is an illustrative example in Section 3 where the optimal vehicle insurance is determined for a specific policyholder using sample average approximation. Section 4 provides the discussions and concluding remarks of this brief study.

2. Stochastic optimisation problem

An individual potential loss S is split as follows

$$S = S^{[I]} + S^{[P]}, \tag{1}$$

where $S^{[P]}$ is a retained part by the policyholder and $S^{[I]}$ represents the loss reimbursement claimed by the policyholder and paid by the insurer, thus the insurance benefit.

The method of how the benefit $S^{[I]}$ is determined depends on policy conditions θ, let's say that it is a function of the individual potential loss, thus $S^{[I]} = f(\theta, S)$.

It follows that

$$S^{[P]} = S - S^{[I]} = S - f(\theta, S) = g(\theta, S). \tag{2}$$

The goal of the optimization problem is to find the optimal product to minimize the potential loss that is retained by the policyholder, i.e. $S^{[P]}$, who is also supposed to pay the premium P.

Thus, the total financial costs related to each insurance product can be defined as

$$V_i = g(\theta_i, S) + P_i \text{ for } i = 1, \ldots, N, \tag{3}$$

where N is the number of products (variants) and where θ_i represents policy conditions that vary across the products as well as premium P_i.

Then, the problem of stochastic optimisation can be formulated as follows

$$\sum_{i=1}^{N} x_i V_i \to \min \tag{4}$$

$$\text{s. t. } \sum_{i=1}^{N} x_i = 1,$$

$$x_i \in \{0,1\} \text{ for } i = 1, \ldots, N.$$

The objective function minimizes the potential loss of policyholder (3) and the x_i represents the product that is selected. Therefore, the sum of all x_i must equal to one because the only one out of all products is meaningful to buy.

Clearly, each product yields different financial loss (3) that depends on the random loss and specific policy conditions θ_i. However, the decision about the purchase must be made before the random loss is known. Therefore, it is necessary to find such optimal insurance that minimizes the expected total financial costs of the policyholder. Letting $\sum_{i=1}^{N} x_i V_i = H(x,\theta,S)$, we have the objective function in the form of

$$\min_x E\left[H(x,\theta,S)\right]. \tag{5}$$

However, the closed-form solution to (5) is not always available. In these cases, it is possible to approximate it by Monte Carlo technique referred to as sample average approximation (SAA).

First, the sufficient number of scenarios $j = 1,\ldots,M$ are drawn from the assumed probability distribution. Second, the value of $H(x,\theta,S)$ is evaluated for each of the scenarios and then the expected value of the objective function is approximated by averaging, thus

$$E\left[H(x,\theta,S)\right] \approx \bar{h}_M(x) = \frac{1}{M}\sum_{j=1}^{M} H\left(x,\theta,S^{(j)}\right), \tag{6}$$

where M is the number of scenarios and $S^{(j)}$ is the loss of jth scenario.

3. A practical example for selection of vehicle insurance

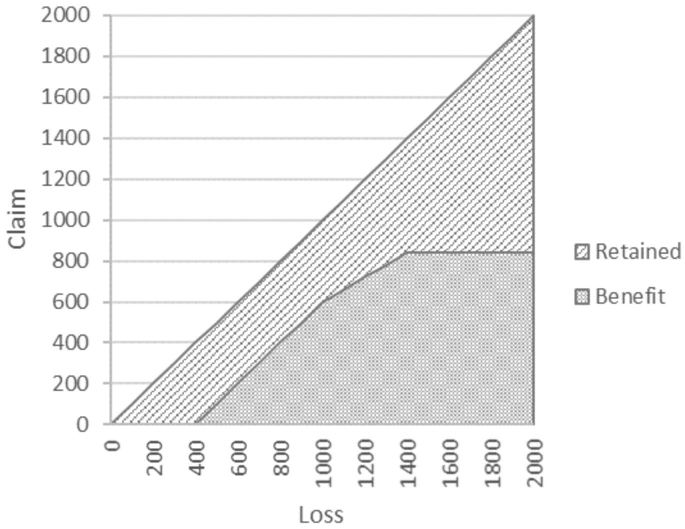
In this section, we identify the optimal product of vehicle insurance by solving the stochastic optimisation problem. Using the website of Česká pojišťovna, we obtained 155 various combinations of premiums P, the coinsurance a, fixed deductibles A and the limit values L. For that purpose, we assumed Honda Civic Tourer with the spark-ignition engine of 1798 ccm, 104 kW and with the weight of 1870 kg. The hypothetical policyholder who lives in Ostrava bought the car as a new for 700 000 CZK and uses it for common purposes.

Further, according to the policy conditions, the policyholder pays a proportional part of the loss, but at least the fixed deductibles. In addition, the insurer firstly applies the limit value and then calculates the coinsurance. The function yielding the benefit can be defined as follows

$$f(\theta,S) = \begin{cases} 0, & L < S < A; \\ S - A, & L > S > A \wedge a \cdot S < A; \\ (1-a)\cdot S, & L > S > A \wedge a \cdot S > A; \\ L - a\cdot L, & S \geq L. \end{cases} \tag{7}$$

For instance, the coinsurance 40 %, fixed deductibles of 400 and the limit value 1400 yields a specific benefit function that is shown in next figure.

Figure 1: Retained part and reimbursement claimed by the policyholder



Clearly, the difference between the benefit and claim represents the fixed deductibles 400. However, when the loss exceeds 1000, the coinsurance exceeds that level of deductibles and the retained part increases. The figure also shows that if the loss exceeds the limit value, the insurer applies the coinsurance and the benefit corresponds to the limit value decreased by this proportion.

Further, the function (7) implies the retained part of the loss, thus

$$g(\theta,S)=\begin{cases} S, & L<S<A; \\ A, & L>S>A \wedge a\cdot S<A; \\ a\cdot S, & L>S>A \wedge a\cdot S>A; \\ S-L+a\cdot L, & S\geq L, \end{cases} \tag{8}$$

or alternatively

$$g(\theta,S)=\max\left(S-L+a\cdot L,\min\left[\max\left(A,a\cdot S\right),S\right]\right). \tag{9}$$

Let us recall that the optimisation problem is formulated as follows

$$E\left[\sum_{i=1}^{N} x_i V_i\right]\rightarrow \min$$

$$\text{s.t. } \sum_{i=1}^{N} x_i = 1,$$

$$x_i \in \{0,1\} \text{ for } i=1,\ldots,N,$$

where $V_i=\max\left(S-L_i+a_i\cdot L_i,\min\left[\max\left(A_i,a_i\cdot S\right),S\right]\right)+P_i$. We solved this problem by SAA where we drew 100,000 scenarios for the random loss that followed the exponential distribution with parameter 0.00001. The optimal vehicle insurance, or policy conditions respectively, was such where coinsurance was at 5 %, but at least 5,000 CZK and the limit value was determined at 320,000 CZK. The insured is supposed to pay 16,857 CZK as the premium and the total expected financial costs (the value of the objective function) equals to 27,261.44 CZK. It follows that the expected retained part corresponds to the 10,404.44 CZK.

Clearly, the optimal insurance does not cover all potential losses. However, the probability that the loss exceeds the limit value at 320,000 CZK is 0.0408, which might be sufficiently small. Otherwise, the probability condition can be added into the optimisation problem and another vehicle insurance, satisfying for instance that the loss is covered with 99% confidence, can be found.

4. Discussion and conclusion

The paper showed an illustrative example how to identify the optimal insurance including policy conditions when the random individual loss is considered. The presented optimization problem is actually general and various relevant probability distribution may be considered, not only exponential distribution. In addition, the problem can be extended by the chance constraint representing that individual loss exceeds the limit value at certain pre-defined probability. Also adding minimal or maximal fixed as well as variable deductibles might be an interesting extension. However, it is necessary to point up some facts that may appear as crucial.

First, the parameters of the loss probability distribution are not known. However, it may be estimated from publicly available statistics or just simply assumed. Clearly, various distributions with various parameters yield different optimal insurance. Second, the exponential distribution, as well as others, are unlimited, i.e. the loss can be infinitely large. By contrast, the maximal loss is actually constrained to the car value rather than unlimited. In addition, the assumed objective function did not respect the probability of the insured accident. However, the goal of the problem was to optimize the coverage that was about the potential loss rather than the expected loss.

Finally, although we identified insurance with the limit value lower than the car value, the preferred insurance might be different due to the policyholder's preferences that were omitted in the optimisation process. However, it would require involving utility theory into the optimisation problem.

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Aluminium Price Management Transaction Costs: Exploratory Case Study

Jan Vašek[1]

Abstract

Commodity aluminium price management is informed by the commodity reference price discovered on the London Metal Exchange. Based on the multiple case study of 14 organizations, this paper explores the key transaction costs related to commodity aluminium price search, contract negotiation, transaction monitoring, compliance enforcement, and adaptation to changed circumstances. Data suggests that the overall transaction costs are very low thanks to the highly transparent commodity reference price.

Key words

Commodity, Procurement, Reference price, Price management, Transaction costs

JEL Classification: H57, Q02, D23

1. Introduction

Commodity reference prices (CRP) are discovered through different mechanisms (Radetzki, 2013). Aluminium CRP is discovered on organized commodity exchange, the London Metal Exchange (LME). Previous research documents that CRP discovered on commodity exchanges are more transparent and have higher information content than alternative price discovery mechanisms such as producer posted prices or price reporting agencies (Figuerola-Ferretti & Gilbert, 2005).

Even though the aluminium CRP is widely used by market participants in day-to-day business, little is known about how it actually shapes the commodity price management practice and in particular how it impacts the different types of Transaction costs (TC). This paper seeks to shed light on this highly relevant topic and enhance our understanding of commodity price management. A multiple case study of 14 aluminium procuring companies served as basis for in-depth exploration of individual phases of aluminium commodity price management as well as qualitative assessment of the related TC.

The data suggests that the aluminium CRP is not only instrumental in all commodity price management phases but actually significantly reduces the accompanying TC: Firstly, the CRP Accuracy and Methodological robustness allows market participants to leverage the CRP as the actual transaction price and triggers only marginal price discovery TC. Secondly, the CRP Completeness contributes to contract standardization as well as the existence of widely accepted safeguards which greatly reduces the contract negotiation TC. Thirdly, the CRP Accessibility and Timeliness practically eliminates the price monitoring related TC. Fourth, the CRP Completeness makes contractual safeguards easy to implement and entailing low TC.

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Finally, the CRP Completeness allows parties to define the suitable adaptation formula ex-ante and thus limit the key commodity price management uncertainty.

The rest of the paper is organised as follows. Firstly, a short review of the aluminium reference price and the transaction costs is provided, followed by the research method and findings. The paper terminates with a short discussion, limitations and suggestions for further research.

2. Literature review

This section shortly reviews existing research on Aluminium reference price and delineates the relevant commodity price management transaction costs.

2.1 Aluminium reference price

The London Metal Exchange publishes a range of aluminium CRP: the spot price, the official price, the official forward curve, the closing price or the average price. The aluminium CRP are discovered through the double auction mechanism (Radetzki, 2013) and reflect true transaction prices valid for all customers (Slade, 1989) with the exception of the Closing price which is determined by the LME Quotations Committee (LME, 2018). Unlike other price discovery mechanisms, commodity exchanges have traders specialized in centralizing price information (Rauch, 1999).

The Commodity reference price transparency index assesses CRP information quality along the continuum ranging from Black hole to Dazzling (Vasek et al., 2018). Aluminium CRP is classified as Dazzling in terms of Accuracy, Completeness, Accessibility and Timeliness, and Methodological robustness. **Table 1** shows rationale for this assessment.

Table 1: Aluminium CRP Transparency assessment following (Vasek et al., 2018)

Feature	Assessment	Classification
Accuracy	True transaction prices (Slade, 1989), Authoritative quotation whose reliability and relevance is rarely questioned (Radetzki, 2013), A common reference price for all transactions (Figuerola-Ferretti & Gilbert, 2005)	Dazzle (4 points)
Completeness	Wide range of prices, selected transaction information shared. Liquid and active futures market (Berg et al., 2013), Facility for hedging providing predictable prices for future transactions (Radetzki, 2013)	Transparent (3 points)
Timeliness and Accessibility	Published daily. Transactors have equal access to exchange prices (Figuerola-Ferretti & Gilbert, 2005)	Transparent (3 points)
Methodological robustness	Existence of trading rulebook (Florin, 2014)	Dazzle (4 points)
Aggregate assessment:	Black hole (0-3.2), Opaque (3.2-6.4), Translucent (6.4-9.6), Transparent (9.6-12.8), Dazzling (12.8-16.0)	Dazzle (14 points)

While aluminium CRP discovered on organized exchanges have many advantages such as assuring equilibrium prices, providing liquidity and inventory holding financing as well as hedging facility (Radetzki, 2013), little is known about its impact on commodity price management defined as a process of identification and evaluation of commodity price risks and opportunities, and selection and implementation of suitable purchase strategies to meet company objectives. This paper seeks to shed light on this topic.

2.2 Transaction costs

Transaction is a transfer of goods or services across a technologically separable interface (Williamson, 1981) between firms or between stages in a vertically integrated company (Hobbs, 1996). Transaction costs have been criticised for being used too widely and loosely and for lacking consensus about what the most important elements are (Wang, 2007). For any meaningful analysis, it is therefore necessary to delineate the construct. **Table 2** outlines TC related to commodity price management.

Table 2: Commodity price management related transaction costs

Price discovery	Information and search costs related to accessing the reference price information, transaction price discovery, price from alternative sources, or price uncertainty (Woldie & Nuppenau, 2011).
Negotiation	Drafting, negotiating and safeguarding an individual contract for each transaction or specify accurately details in a long-term contract (Williamson, 1985) as well as physically carry out the transaction (Hobbs, 1997).
Monitoring	Screening and supervisory actions to ensure supplier performance during the supply agreement execution (Noordewier, John, & Nevin, 1990).
Enforcement	Severity of disciplinary response to supplier´s violation of contractual obligation (Antia & Frazier, 2001).
Adaptation	Contract modification to reflect changes in the external environment (Rindfleisch & Heide, 1997).

3. Research method

This section shortly discusses the rationale for the multiple-case study design, selection of cases, data collection and analysis approach and in particular the qualitative TC measurement approach.

3.1 Multiple case study

A multiple exploratory case study approach was adopted to explore and assess TC related to aluminium commodity price management. A case study approach was selected because this paper seeks insight into a "how" question, explore a contemporary phenomenon with its real life context, and prior knowledge is limited (Yin, 2014). Compared to single case studies, multiple case studies enable broader exploration of the phenomenon of interest (Eisenhardt & Graebner, 2007). Each case study is treated as an experiment (Yin, 2014) where the commodity price management transaction costs are explored and assessed.

3.2 Selection of cases

The case selection follows the replication logic leveraging both the literal and theoretical sampling recommended by Yin (2014). We first followed the theoretical replication logic, where contrasting results are expected but for predictable reasons, and defined three aluminium product groups differing in aluminium share of the total price: aluminium alloys, simple aluminium components and complex aluminium components. We then followed the literal replication logic for each product group and explored between three to six cases where we expected similar results. In total, 14 case studies were analysed.

3.3 Data collection and analysis

Data was collected through semi-structured interviews with key informants lasting between 40-60 minutes. Respondents also produced internal documents such as price lists, dates of purchase, supplier quotes, commodity strategy, third party reports to substantiate their claims but, due to their extremely sensitive nature, they could not be taken away. Each interview was transcribed and any unclear points or missing information discussed with the respondents over the phone. Data was analysed with the Nvivo 10 software.

A particular care was paid to TC measurement. Scholars point out that the Transaction cost economy theoretical development was not accompanied by successful TC measurement (e.g. Hobbs, 1996; Delmas & Tokat, 2005) and that TC remain conceptual and lacking widely accepted operationalized standards (Benham & Benham, 2000). Especially TC valuation in financial terms remains elusive (Hobbs, 1997) and many scholars simply assume the TC existence and mediating effect (Grover & Malhotra, 2003).

Following Hobbs´ (1997) recommendation to focus on measuring the relative importance of individual transaction costs rather than trying to obtain accounting-based data, this paper adopts the qualitative approach and classifies TC as either *Low* or *High*. *Low* TC entail little effort in terms of resources employed (time, money) and are manageable through standard processes which require little expertise to complete the task (Schneider et al., 2013). *High* TC, on the other hand, demand more resources and expertise and cannot be fully standardized. Following Boyatzis (1998), a detailed coding scheme was developed for each TC group and the TC assessment.

4. Findings

This section reports and assesses the TC related to aluminium commodity price management. **Table 3** summarizes the TC assessment and **Table 4** provides some illustrative quotations.

Table 3: Commodity aluminium price management transaction costs assessment

Product group	Company	Search	Negotiation	Monitoring	Enforcement	Adaptation
Aluminium alloys	**Industrial goods**	L	L	L	N/A	L
	Foundry	L	L	L	N/A	L
	Mill	L	L	L	L	L
Simple components	**Windows**	L	L	L	N/A	L
	Construction	N/A	N/A	N/A	N/A	N/A
	Sensors	L	L	L	L	L
	Sensors II	L	L	L	N/A	L
	Automotive I	L	L	L	N/A	L
	Automotive II	L	L	L	N/A	L
Complex components	**Petrochemical**	L	L	L	N/A	L
	Automotive III	L	L	L	N/A	L
	Building materials	L	L	L	N/A	L
	Brewery I	L	L	L	N/A	L
	Brewery II	L	L	L	N/A	L

4.1 Search costs

All but one respondents separate the commodity aluminium from transformation, or added value, cost. LME issued CRP not only inform the price discovery but actually stand for the transaction price including future physical deliveries which are based on the LME official prices curve. As all suppliers offer the same price for commodity aluminium at a given moment, the commodity aluminium price is not a price differentiator. **Sensors** even requires that all suppliers quote the same aluminium level and only later marks it to market.

In summary, thanks to the aluminium CRP Accuracy and Completeness the price discovery is very simple and the TC remain very low.

Table 4: Illustrative quotations

Search	"We fix the commodity price at the actual LME level. … Some suppliers may also use the day before official LME price." (**Automotive I**)
	"We take the daily settlement price and calculate the monthly average. That´s all." (**Brewery II**)
Negotiation	"When we agree [the commercial conditions], the supplier sends us the contract … It specifies the price, quantities, chemical composition, payment terms and that´s it. … It is our biggest supplier and the contract is always the same. " (**Foundry**)
	"The supplier insists on signing the contract. … it is simple, a letter of intent, where we declare that we want to work together, … annual volume of 300 tons plus minus 20 % and a six month notice period. … It also defines how we adjust the aluminium prices." (**Sensors**)
Monitoring	"I log into LME once a month to download the historical prices. Even though we have our internal aluminium data, I always use the LME data to avoid any discussion." (**Industrial goods**)
	"Once we agree the deal, there is nothing to monitor. We send an order and they send the invoice." (**Mill**)
Enforcement	"At the end of the quarter [the supplier] would charge us 8 % interest on any outstanding aluminium until we consume it." (**Sensors**)
Adaptation	"You have one format when the [LME] average from December till February is the reference for the 2nd quarter. Then there is the six months [average] format … . And there is another format: we sit down with our customer and discuss the demand for the next quarter, say 500 tons, and then we review the market price of the day. And then you produce the products and when the 500 tons are used up, you sit again together and you consider the next period demand." (**Automotive II**)

4.2 Negotiation costs

Contract negotiations are significantly simplified thanks to the fact that there is a standard commodity aluminium technical specification and that the commodity aluminium share of total price can be easily calculated. As the CRP value is considered accurate and valid across regions, it is used at face value directly in the contracts. Contracts tend to be long-term, highly standardized with well-defined adaptation mechanisms. As commodity aluminium adaptation problems can be addressed ex-ante, there is limited need for relational safeguards. Instead, market participants impose stringent contractual safeguards concerning the price discovery and adaptation mechanisms as well as volume commitment.

In summary, Aluminium CRP Accuracy and Completeness greatly simplify the contracting phase and allow for straightforward and easy to draft long-term contracts. TC can therefore be assessed as low.

4.3 Monitoring costs

Commodity aluminium price monitoring is simple because the CRP is easily Accessible on the LME webpage and Timely. There is no behavioural uncertainty as buyers can check the actual LME price, the price history or the forward curve. Even **Sensors**, who does not have an automatic, LME based adaptation mechanism on specialized, low-volume extrusions, argues that "of course, we monitor the price evolution and it more or less follows the LME."

In summary, thanks to the CRP Accessibility and Timeliness the monitoring TC remain very low.

4.4 Enforcement costs

Automotive I reports a case when the contract breach occurred following customer´s administrative issues and personnel turnover. As soon as the problem was discovered, parties simply applied the right CRP level, calculated the amounts due, and settled it.

Several respondents highlighted the recurring problem with contracted volume discrepancies. As long as the contracted commodity has not been transformed into a specific product, it may be sold to another customer, financially settled, or rolled over into the next contractual period.. If, on the other hand, the buyer has already used up the contracted volume, they have to buy additional material. The CRP always serves as basis for the subsequent financial settlement between partners.

In summary, contract breaches are easily enforced thanks to contractual safeguards and the CRP which informs the financial settlement.

4.5 Adaptation costs

The CRP is instrumental in all observed adaptation mechanisms: Natural hedging, Financial hedging, Forward buy or Price escalator.

Natural hedging covers all instances when the respondent synchronizes the input and output commodity price adaptation mechanism and eliminates the commodity price risk. Natural hedging takes multiple forms and is adapted to final customer preferences: fixed price, trigger point or price escalator. The Accessible and Accurate CRP avoids haggling because partners can easily check the CRP value and evolution.

Financial hedging is enabled by the CRP Completeness. **Brewery I** underscores the separation of commodity physical and financial flows by defining an annual administrative CRP price level with the supplier which is then marked-to-market following the CRP monthly average. The hedging fees are well below one percent.

Forward transaction prices are calculated based on the CRP official prices curve. Forwards cover a fixed quantity and the price normally remains valid until the material is fully used up.

The price escalator is the most popular commodity price adaptation method. **Petrochemical** prefers the price escalator because it alleviates the inherent risk of the “wrong” timing. **Windows**, on the other hand, appreciates that the escalator can be fine-tuned to company needs: it combines the monthly escalator with one supplier with quarterly updates with the other. Finally, the quarterly escalator allows **Sensors** to avoid the necessary forward buy volume commitment.

Three particular instances where the respondents do not use the CRP for adaptation should be mentioned. The first two cases concern low volume, high added value monopoly supplier products: **Sensors** simply accepts supplier price changes as long as they are coherent with the CRP evolution, and **Brewery II** uses the CRP only as a support argument in negotiation. Finally, **Construction** does not treat the commodity aluminium separately, tenders the full price only and passes the commodity price risk through to the supplier.

In summary, the CRP is key for smooth operation and low TC of all the reviewed adaptation mechanisms.

5. Discussion and concluding remarks

This paper explored the commodity aluminium price management related TC. Previous research already highlighted the superiority of commodity exchange discovered CRP in terms of information content (Figuerola-Ferretti & Gilbert, 2005) and options available to market participants (Radetzki, 2013).

This paper contributes to knowledge by documenting how Dazzling aluminum CRP inform and shape all commodity price management phases and minimize the TC by: (1) leveraging CRP Accuracy in the price Search phase, (2) streamlining and standardizing the contract Negotiation phase thanks to the CRP Completeness, (3) practically eliminating the Monitoring phase thanks to the CRP Accessibility and Timeliness, (4) making the Enforcement a simple administrative task, (5) encouraging long-term contracts as adaptation becomes simple and transparent.

The aluminium CRP highlighted practical utility and TC reduction potential gives strong support for the unstoppable progress of commodity exchanges in commodity price discovery hypothesis formulated by Radetzki (2013).

This research has some important limitations. Firstly, only the aluminium CRP price was explored which significantly limits the findings generalizability. Secondly, even though a particular care was paid to the qualitative TC assessment methodology and the multiple case study approach with both literal and theoretical replication was chosen, some subjectivity cannot be ruled out. Finally, the results are still interim and were not reviewed with experienced practitioners.

Further research may replicate this study on another dazzling CRP to gain confidence in the interim conclusions. Scholars may also compare the impact of less transparent CRP on commodity price management and the related TC. Finally, a different TC assessment methodology might bring important additional insight.

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Continuous innovation strategy: Czech economy between 2008 and 2014

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Abstract

This paper analyses strategical behavior of innovators in the Czech Republic by using enterprise-level data from the Community Innovation Survey (CIS) waves of 2008, 2010, 2012, and 2014. A probit model is applied that describes innovation activities based on the frequency of the innovation process. There are three types of enterprises – none to very limited innovators, occasional innovators, and continuous innovators. Current literature lacks similar research and suggests that continuous process contributes to flexibility which allows enterprises to achieve superior performance and to reduce financial risks. Results suggest that there are differences between the Czech continuous and discontinuous (limited, irregular, and radical) innovators. Continuous innovation is more probable in knowledge-intensive services and highly depends on the technological level of the industry. The internal pressures of the group are contributing to a higher continuous innovation probability. Foreign-owned companies are more likely to irregular or no innovators at all. Market orientation is a dominant feature of continuous innovation and it is less important in case of discontinuous innovation. Continuous innovators face an uncertain demand for innovations and compete against dominant enterprises.

Key words

continuousness, discontinuousness, radical, irregular, innovation process, technological intensity, knowledge-intensive services.

JEL Classification: O31, D23, L60, L80

1. Introduction

Continuous innovation (CI) as a business economic term is not well defined. In general, it means to be innovative on a regular basis. In terms of organizational studies, there is a business process called innovation. It is very well established in the practice and has a strategic importance for many companies and acquires resources based on company goals. The extent of the term innovation as a business process is however not well defined in theoretical literature. Does it concern only goods (product and services)? Does it include failed innovation projects? Does it describe only goods introduced onto markets? The term innovation is mainly understood as a business process.

The definition of CI will be based on the business process logic. In this paper, CI is defined as a regular annual process of spending research and development (R&D) resources with uncertain innovation result. It will encompass only the ex-ante motives of the organization to innovate, not the ex-post results. The ex-ante motives of organizations are to introduce something new-to-the-enterprise or new-to-the-market. Ex-post, the results of innovation projects can be introduced to the market using many time-to-market strategies. However, innovation projects can be also abandoned or commercialized without satisfying results.

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It is also problematic to distinguish the (dis)continuousness based on customer segments and time. Organizations with innovation as their business process are most likely incremental innovators. On occasion, they introduce something radical, but it is not really appreciated by customers. Some ex-ante incremental innovation projects can unanticipatedly fill a market gap and attract new segments of customers and have a radical impact. The nature of (dis)continuousness is fluid (Table 1), especially in medium-tech, high-tech and knowledge-intensive industries where time-to-market strategies are usually not important, i.e. the faster innovation pace is a key success factor. It will be problematic to discard radical innovation as not being continuous. Perhaps it is possible only in cases where enterprises self-report to pursue discontinuous innovation strategy and the introduction of new-to-the-world-market radical innovations.

Sometimes, an organization is being shaped after the introduction of new-to-the-market innovation. New employees are hired, processes are adapted and re-engineered. Continual talent management is observed in more competitive, knowledge-intensive, and high-tech industries. The ability of top management to use new technologies and to organize talent is considered to be a new method and an important skill, which is a driver of continuous social technology adoption and organizational innovation (Horváthová and Kashi 2016). Continuous knowledge flow is also increased by supplier and customer feedback.

Table 1: Innovation continuousness – a fluid concept

<table>
<tr><td>Type of innovation</td><td colspan="2">No innovation</td><td colspan="2">New-to-the-enterprise</td><td colspan="2">New-to-the-market</td><td colspan="2">New-to-the-world</td></tr>
<tr><td>Indicator</td><td>No planned R&D, adaptation, use of modern technology</td><td colspan="2">Planned R&D expenditures in the business processes</td><td colspan="2">Innovation as a business process</td><td colspan="3">Innovation as a strategic process, some uncertain (risky) innovation projects</td></tr>
<tr><td>Continuousness probability</td><td>Low</td><td colspan="3">High</td><td colspan="3">Highest</td><td>Medium</td></tr>
<tr><td>Technology and knowledge level based on Eurostat (2016)</td><td colspan="4">Low-Tech to Medium-low Tech manufacturing industries

Less knowledge-intensive services</td><td colspan="4">Medium-Tech to High-Tech manufacturing industries

Knowledge-intensive services</td></tr>
</table>

Source: Author's elaboration

The first research question is aimed at the technological or knowledge level of the industry. The technological level hypothesis will be tested. The manufacturing industry is classified by technological level (Low-, Low-Medium-, Medium-High-, and High-Tech) and services are rated according to their knowledge intensity (Low and High).

H_1: The probability of being a continuous innovator depends on technological and knowledge level of the industry.

Higher probability of continuous innovation process is in high-tech manufacturing industries, knowledge-intensive services, and information and communication technology industry. An alternative explanation is that the probability depends on more factors like management decisions, and learned skills, or other macroeconomic factors which are external to the company, f. e. expectations about the economic cycle and high concentration of the industry where companies are following the incumbent price policy (kinked-demand curve oligopoly model).

Continuous innovations are important success factors. Case studies showed that proclaimed organizational culture plays an important role, f. e. Google's proclaimed organizational culture is viewed as a system for continuous innovation, and i.e. the regular work of certain teams of employees is the continuous innovation process (Steiber and Alänge 2013).

The theoretical field of continuous innovation is mostly based on the intersection of many factors. The definition by Martini et al. (2013) is built on three internal factors: exploration-exploitation, organizational ambidexterity, and paradoxical thinking. Kianto (2011) offers another explanation which is also based on internal features: individual creativity; knowledge implementation and commercialization, and organizational strategic flexibility. According to Taherparvar, Esmaeilpour, and Dostar (2014), the process of imitation and learning, as well as the process of understanding new technologies and their applications, can lead to value added and the process of invention. However, knowledge management without knowledge about customer needs (customer-valued innovation) cannot lead to continuous innovation. There are five factors which can be interpreted as "shifts" towards continuous innovation (Denning 2011). These five transformations are: the enterprise's goal to become engaging customers, the role of managers transforming into enablers instead of controllers, the mode of managerial coordination switching from static to dynamic linking, the values practiced shifts from a single focus on shareholder value to values relevant to all stakeholders, and the communications mode of management changing from command to conversation. Laevy (2015) suggests three key capabilities: creative abrasion, creative agility, and creative resolution. Chen (2016) bases the theory on three aspects: knowledge innovation capability, production innovation capability, and market innovation capability. Two search routines as factors of CI are offered by Karlsson and Björk (2017): cooperation and external acquisition of knowledge. Intra-organizational networks are good at invoking organizational pressures like competition in the market.

The theoretical field of CI shows quite unclear and complex interpretation of continuous innovation. There are also authors of current CI frameworks. According to Martini et al. (2012), CI research seems to be stuck.

The second research question is aimed at the organizational features. There are many factors which are going to be tested.

H2: The probability of being a continuous innovator depends on organizational features. Tested features include: Being part of a group of companies (internal pressures), high importance of innovation factors – lack of information sources, and lack of skilled employees.

In practice, these factors can be seen in all kind of Total Quality Management approaches like Six Sigma, Kaizen and lean/agile concepts (Dluhošová et al. 2014). The Six Sigma framework is the practical representation of continuous innovation (Eng 2011). Manufacturing companies have been implementing these concepts since the 1990s; they also try to integrate their long-term goals with new methods of management. The question is whether this lean thinking usual in automotive, ICT, and manufacturing industries is suitable for the rest of the service industry as well?

2. Materials and Methods

There are four Czech Community Innovation Survey (CIS) datasets of 2008, 2010, 2012, and 2014 (Czech Statistical Office 2017). The dataset contains 22,232 observations about enterprises from different industries. It is very heterogeneous (Table 2) and the variation of financial data is quite substantive. There are 35 % of continuous innovators, the rest is not (radical and not on regular basis). This variable is based on self-reported information. Data are cleared, and some observations were deleted because of empty values in employees, sales, and R&D expenditures. This dataset does not contain micro enterprises with 9 or fewer employees. The results will be interpreted for small (10 or more employees), medium-sized, and large enterprises. An innovator in this paper is defined as a company with non-zero R&D expenditures.

Table 2: Summary statistics of the Czech industry 2008-2014 – Innovation data

Variable	Obs.	Mean	Std. Dev.	Min	Max
Being part of a group	22,232	0.37	0.48	0	1
R&D Expenditure	9169	3.31E+04	2.80E+05	0	1.51E+07
Sales from goods and services	22,232	9.37E+05	6.60E+06	2	3.62E+08
Number of employees	22,232	193.37	799.33	10	36,332
Foreign ownership	22,232	0.31	0.46	0	1
Innovator	22,233	0.41	0.49	0	1
Continuous innovator	9,082	0.35	0.48	0	1

Source: Author's elaboration based on data from the Czech Statistical Office (2017).

The estimation method, the CDM model, is based on Crepon, Duguet, and Mairesse (1998). The first step in this innovation process model is a Heckman procedure, i.e. we selected only innovators in second equation (R&D intensity equation) and we must account for selection bias by using the Mill's ratio (non-selection hazard variable). The ability to capture profits of innovated products and services can be analyzed in the third step. Due to the data availability (lack of instrumental variables), most of the coefficients will be biased. The endogeneity is quite problematic in the full model. The term $X_{1it}\beta_1$ expresses vectors of explanatory variables; ε_{it1} is the random-error term which can be estimated with random or fixed (ρ_i) effects. The error terms are assumed to be independent of the exogenous variables, but there is a bias from omitted variables and the endogeneity.

Table 3: Innovation probability – econometric model

Innovation probability	$r_{it}^{*}\begin{cases} 1 \ if \ r_{it} = \left(X_{1it}\beta_1 + \rho_i + \varepsilon_{it_1}\right) > 0 \\ 0 \ otherwise \ \ (r_{it} \leq 0) \end{cases}$

Complete model can be interpreted. In the Heckmann procedure, the error term is estimated as a system. The first equation (r_{it}^{*}) accounts for selection into R&D activities, and it deals with the probability of an enterprise *i* to engage in all R&D activities in a year *t*. This is specified as a panel Probit random effects model, i.e. $P(r_{it}^{*}>0) = \Phi(X_{1it}\beta_1)$, where r_{it}^{*} equals 1 if enterprise *i* is an innovator. The second linear equation describes innovation input (the log of internal and external R&D expenditures to the number of employees in an enterprise *i*). In all the equations, there are several potential determinants such as an enterprise's size, foreign ownership, being part of a group of enterprises, cooperation, etc. Some of them are used uniquely to identify each equation in the CDM simultaneous estimation (i.e. hampering factors).

3. Results

Continuous innovators are different than discontinuous innovators. First hypothesis (H1) deals with the technological level of industries and knowledge intensity. Continuous innovation is more probable in knowledge-intensive services and proportionally depends on the technological level of industry. Discontinuousness probability is dependent only on the technological level and in a less prominent way than in case of continuous innovators.

Table 4: Innovation probability, continuous vs. discontinuous innovators

Probability to engage in R&D and innovation activities	(1) Innovation	(2) Continuous innovation	(3) Discontinuous innovation
Technological level	0.203***	0.620***	0.128***
Low-Medium Tech	(0.05)	(0.11)	(0.05)
Technological level	0.435***	1.343***	0.211***
Medium Tech	(0.05)	(0.12)	(0.05)
Technological level	0.586***	1.777***	0.242**
High Tech	(0.09)	(0.19)	(0.10)
Knowledge intensive services	0.047	0.607***	-0.064
	(0.04)	(0.09)	(0.04)
Log of number of employees	0.291***	0.620***	0.213***
	(0.01)	(0.03)	(0.01)
Being part of a group	0.570***	0.854***	0.507***
	(0.04)	(0.08)	(0.04)
Foreign owned	-0.260***	-0.635***	-0.162***
	(0.04)	(0.09)	(0.04)
Market orientation	0.543***	1.282***	0.394***
National	(0.04)	(0.09)	(0.04)
Market orientation	0.462***	1.286***	0.273***
Europe	(0.05)	(0.11)	(0.05)
Market orientation	0.613***	1.852***	0.191**
World	(0.07)	(0.14)	(0.08)
HF: Insufficient internal	0.449***	0.532***	0.448***
finances	(0.04)	(0.09)	(0.05)
HF: Innovation was not	-0.581***	-1.090***	-0.427***
required	(0.04)	(0.09)	(0.04)
HF: Not required because of	0.270***	0.139	0.284***
previous innovation	(0.05)	(0.09)	(0.05)
HF: Uncertain demand for	0.044	0.180**	-0.001
innovation	(0.05)	(0.09)	(0.05)
HF: Market dominated by	0.158***	0.343***	0.082
incumbents (large enterprises)	(0.05)	(0.09)	(0.05)
HF: Lack of innovation	-0.412***	-0.508***	-0.388***
partners, difficult cooperation	(0.05)	(0.10)	(0.06)
HF: Lack of information about	1.350***	1.574***	1.246***
markets	(0.05)	(0.09)	(0.05)
HF: Lack of qualified	0.378***	0.574***	0.323***
personnel	(0.04)	(0.08)	(0.05)
Constant	-2.798***	-7.029***	-2.503***
	(0.08)	(0.26)	(0.08)
Year (control variables)	Yes	Yes	Yes
Observations	22184	16249	19051

*Source: Author's estimations based on data from Czech Statistical Office (2017). ** $p < 0.05$, *** $p < 0.01$*

The probability to engage in continuous innovation is higher for larger enterprises. The internal pressures of the group are contributing to a higher innovation probability (H2) of all innovators, in case of continuous innovation, the intensity of the probability is even higher. Foreign-owned companies are more likely to be non-innovators. Foreign-owned companies are more likely to irregular innovators than continuous innovators. Market orientation is a dominant feature of the probability of continuous innovation and it is less important in case of discontinuous innovation. Hampering factors were different for both groups of innovators. Continuous innovators were stressing the fact that they face an uncertain demand for innovations, and that the market is dominated by a large enterprise (incumbent). As expected lower probability of continuous innovation was in case of the factor "Innovation was not required" and higher probability to be discontinuous innovator was in case of the factor "Innovation was not required because of previous innovation".

4. Conclusion

This paper analyzed the Czech industry between 2008 and 2014. It focused on continuous innovation as a business process and tested two research questions in the econometric model. This approach is different than discontinuous innovation process at least in terms of probability to engage in (dis)continuous innovation. Knowledge-intensive services in the Czech economy adopted continuous innovation process. There are substantial differences between these two groups which are dependent on internal factors such as company size, industry, market orientation and foreign ownership. The understanding of some hampering factors is different for the continuous innovators. The continuous innovators face uncertain demand and are aware of the subsequent financial risks, the same applies for the information sources which are necessary for continuous innovation as well as qualified personnel. This paper proposes a full CDM model in the further research. It will be interesting to capture differences in terms of external knowledge acquisition, cooperation partners and networks and ability to translate R&D inputs into financial income in terms of sales of innovated goods and services.

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Proposal and verification of network prediction assets model

Zdeněk Zmeškal, Dana Dluhošová[1]

Abstract

The paper is focused on proposal and verification of network prediction model application for stock exchange indexes. Prediction network is described. Model is verified on stock exchange indexes. It was verified possibility of estimation of model on vector autoregressive method. Better preciseness in comparison with naïve prediction model was verified.

Key words

Stock exchange index, VAR model, network analysis, network prediction model, correlation

JEL Classification: C51, C52, G12, G17

1. Introduction

Assets return prediction is one of a crucial problem in financial analysis and decision making. A relations among other assets create a systematic risk and is characterised by correlation matrix. This novelty approach allows to model assets as system and impulses of financial markets, e. g. contagion. This aspect is important in non-stable situations, especially crisis etc. There is in the paper described a methodology of prediction of financial assets value under network conditions. Vector autoregressive model is applied. Two components are distinguished, autoregressive one and systemic risk one. Methodology is applied for apprising stock exchange index prediction, and network is given by relationship of stock indexes of various stock exchanges. Stock exchange (market portfolio) is crucial parameter for assets price model, e. g. CAPM, Arbitrage, Fama-French models. Precision of network prediction model is compared with a naïve prediction model.

Goal of the paper is to verify possibility and preciseness of prediction by network assets valuation model.

2. Network prediction model description

Autoregressive cross-sectional random model is a generalised model for prediction. It consists of three elements (autoregressive, systemic, random). It belongs to vector autoregressive model. Formulation of model is following

$$R_{i,t} = \sum_{p=1}^{M} \alpha_{i,p} \cdot R_{i,t-p} + \sum \beta_{i,j} \cdot R_{j,t} + \varepsilon_{i,t}, \tag{1}$$

here $R_{i,t}$ is asset return, $\alpha_{i,p}, \beta_{i,j}$ are parameters, $\varepsilon_{i,t}$ is random residuum, M is number of delay, N is number of assets.

Expression of the model in matrix form,

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$$\vec{R}_t = \sum_{p=1}^{M} \hat{A}_p \cdot \vec{R}_{t-p} + B \cdot \vec{R}_t + \vec{\varepsilon}_t \,, \tag{2}$$

$\vec{R}_t$ is vector of dimension (N,1), $\alpha_{i,p} \in \hat{A}_p$ is diagonal matrix of dimension (N,N), $\vec{\varepsilon}_t$ is vector of random residuum of dimension (N,1), $\beta_{i,j} \in B$ is matrix of dimension (N,N).

Estimation of the model is following. Under normal (Gaussian) distribution systemic risk is estimated by parameters beta as covariance or correlation in this way,

$$\beta_{i,j} = \frac{\operatorname{cov}(R_i;R_j)}{\operatorname{var}(R_j)} = \frac{corr(R_i;R_j)\cdot\sigma_i}{\sigma_j}\,. \tag{3}$$

Matrix B consist on $\beta_{i,j}$ coefficients and on diagonal are nulls.

Autoregressive coefficients of diagonal matrices $\hat{A}_p$ are estimated from time-series of $\vec{R}_t$ and $\vec{R}_{t-p}$ vectors and equation (2),

$$(E - B)\vec{R}_t = \sum_{p=1}^{M} \hat{A}_p \cdot \vec{R}_{t-p} + \vec{\varepsilon}_t \,. \tag{4}$$

Prediction network model is as follows

$$\vec{R}_t = \sum_{p=1}^{M} (E - B)^{-1} \cdot \hat{A}_p \cdot \vec{R}_{t-p} + \vec{\varepsilon}_t \,. \tag{5}$$

For comparison a naïve prediction model is based on idea, that the best estimation of the future value is present value, so

$$\vec{R}_t = \vec{R}_{t-1} + \vec{\varepsilon}_t \,. \tag{6}$$

3. Verification of a network prediction model

We suppose a simplified network model with only one delay of auto-regression,

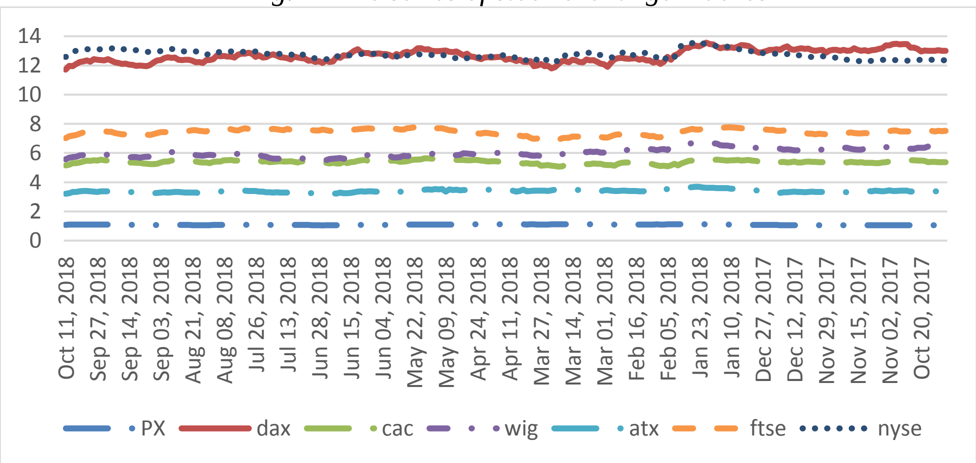
$$\vec{R}_t = (E - B)^{-1} \cdot \hat{A}_1 \cdot \vec{R}_{t-1} + \vec{\varepsilon}_t \,, \tag{7}$$

and naïve prediction model

$$\vec{R}_t = \vec{R}_{t-1} + \vec{\varepsilon}_t \,. \tag{8}$$

Furthermore a group of stock exchange indexes, PX Prague, DAX Frankfurt, CAC Paris, WIG Warsaw, ATX Vienna, FTSE London, NYSE New York are considered. Time series of indexes from October 2017 to October 2018 are on Fig. 1.

Fig. 1 Time series of stock exchange indexes

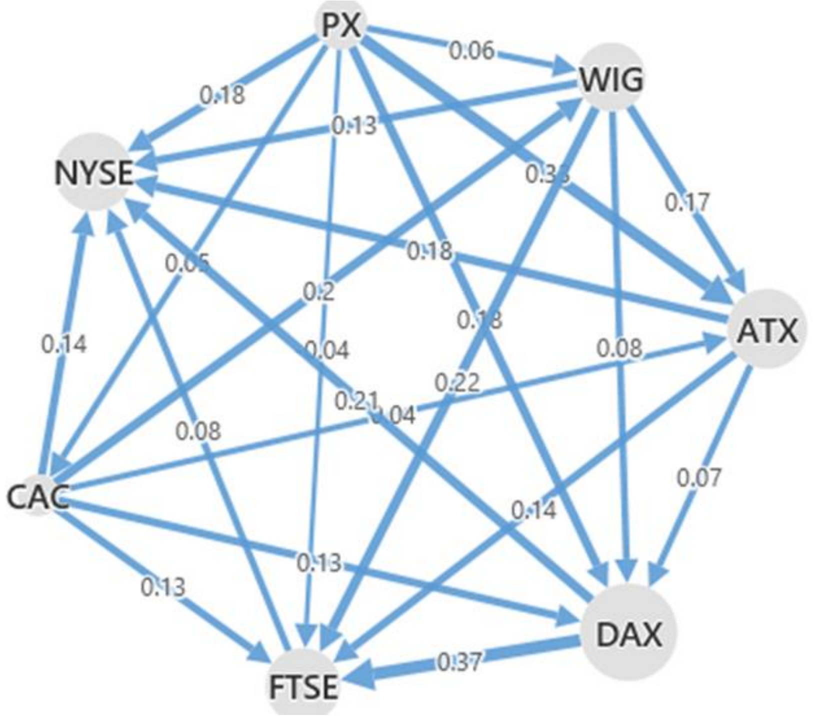


There is in Tab 1 correlation matrix and Fig 2 network of stock indexes.

Tab. 1 Correlation matrix

	PX	CAC	WIG	ATX	DAX	FTSE	NYSE
PX	1	0,046991799	0,056719	0,332719	0,18104681	-0,03918	0,181218
CAC	0,046992	1	0,199556	0,04265	0,13271889	0,12635	0,14138
WIG	0,056719	0,199556408	1	0,172138	0,07987124	0,221569	0,131688
ATX	0,332719	0,042649546	0,172138	1	0,07383035	0,142894	0,180117
DAX	0,181047	0,132718885	0,079871	0,07383	1	0,372571	0,211896
FTSE	-0,03918	0,126349812	0,221569	0,142894	0,37257146	1	0,082552
NYSE	0,181218	0,141379742	0,131688	0,180117	0,21189577	0,082552	1

Fig. 2 Stock indexes network given by correlations



The time series is divided into two samples, in sample from October 2017 to March 2018, out of sample from April 2018 to October 2018.

Calculated B matrix is on Tab 2 and estimated diagonal matrix A is on Tab. 3.

Tab. 2 Matrix B of beta coefficients

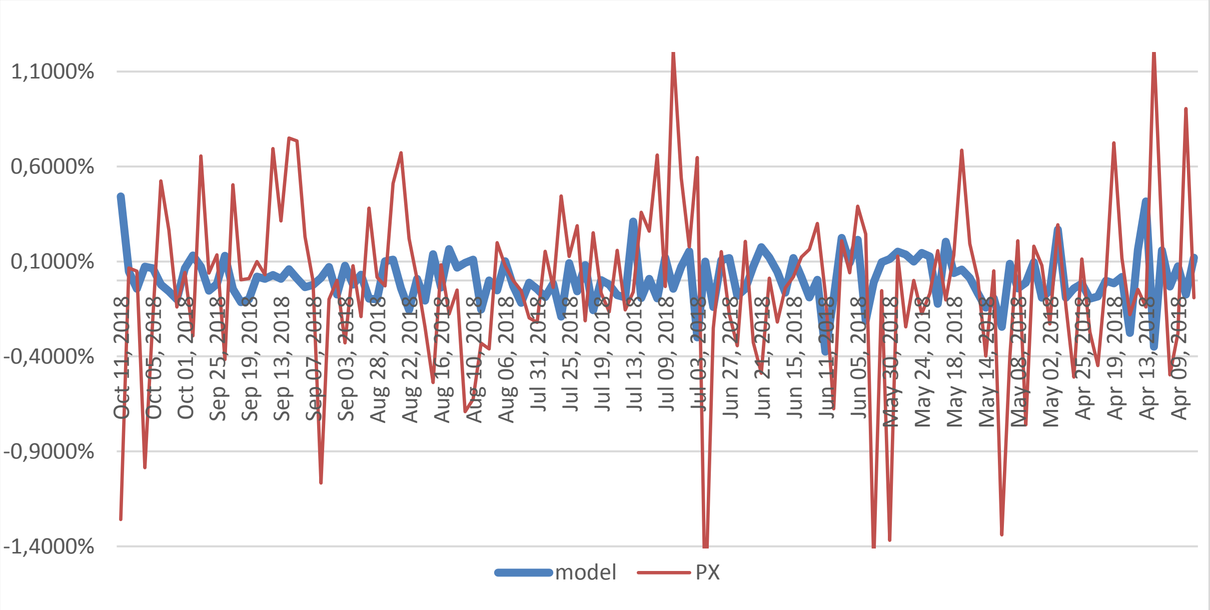
	PX	CAC	WIG	ATX	DAX	FTSE	NYSE
PX	0	-0,10588	-0,0277	0,885811	0,139935	-0,15713	0,434177
CAC	-0,10588	0	0,148122	-0,07342	0,09115	0,106086	0,035535
WIG	-0,0277	0,148122	0	0,060339	-0,00016	0,057195	-0,02289
ATX	0,885811	-0,07342	0,060339	0	0,059784	-0,02883	0,14036
DAX	0,139935	0,09115	-0,00016	0,059784	0	0,379072	0,211858
FTSE	-0,15713	0,106086	0,057195	-0,02883	0,379072	0	0,07714
NYSE	0,434177	0,035535	-0,02289	0,14036	0,211858	0,07714	0

Tab. 3 Estimated diagonal matrix A of alpha coefficients

	PX	*CAC*	*WIG*	*ATX*	*DAX*	*FTSE*	*NYSE*
PX	0,033249	0	0	0	0	0	0
CAC	0	0,061583	0	0	0	0	0
WIG	0	0	-0,20601	0	0	0	0
ATX	0	0	0	0,151487	0	0	0
DAX	0	0	0	0	0,123141	0	0
FTSE	0	0	0	0	0	0,021822	0
NYSE	0	0	0	0	0	0	0,315853

Preciseness of a network prediction model and a naïve prediction model given by a standard deviation from real data. Standard deviation of network model is 0, 3395%, standard deviation of naïve model is 0, 4569%. It is clear that network model is better. On Fig. 3 is network model prediction compared with real data in out of sample period.

Fig. 3 Comparison of network prediction model and real data for PX index



Preciseness of all indexes network prediction models shows Tab. 4. It is certain that network models are better.

Tab. 4 Comparison of standard deviations of predictive models

Stock exchange index	**Prediction model**	
	network	**naive**
PX	0,3395%	0,4569%
CAC	0,5238%	0,7370%
WIG	0,5649%	0,7817%
ATX	0,6830%	0,8512%
DAX	0,6268%	0,8747%
FTSE	0,4428%	0,6232%
NYSE	0,4256%	0,5989%

5. Conclusion

There were in paper described network prediction model and verified on real data of stock exchanges. It was confirmed, that network model is better in comparison of naïve model and applicability and usefulness in prediction process. Importance of methodology described consist in role of stock exchange in assets price modelling.

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