



Application of the analytic hierarchy process method in a comparison of financial leasing and loans

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Abstract

Decision making is an integral part of all business processes. Some decisions are spontaneous and others time consuming, requiring detailed information about their implications. Decision making can be performed according to perspectives that may also be conflicting in nature. Models that are helpful in dealing with such problems are multi-criteria decision-making methods. The most important method in this group is the analytic hierarchy process. The principle of the analytic hierarchy process is the distribution of the main parts into smaller and more detailed elements and thus the creation of a structured problem. The aim of this paper is to select the optimal form of asset acquisition (loan or leasing) according to clients' selected criteria using the analytic hierarchy process (AHP) and a sensitivity analysis to assess the resulting rank of alternatives. Based on the criteria, financial leasing with a high down payment was selected as the best alternative. By means of the sensitivity analysis, it was found that the best alternative is not sensitive to a change in the weights estimated by the AHP.

Keywords

AHP, alternatives, analytic hierarchy process, criteria, leasing, loan, MADM.

JEL Classification: C44, G11, M21, M41

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1. Introduction

Decision making belongs to the everyday activities of people and businesses. It is a process whereby an individual or a group (the decision maker) selects the best alternative from a pool of possible alternatives. It represents an alternative that at best meets most of the decision maker's preferences. Among the most important business activities belongs investment decision making, which is frequently focused on the renewal and expansion of tangible and intangible assets (Fotr and Souček, 2011). The impacts of investment decision making have a long-lasting characteristic and are financially challenging, thus it is advisable during the selection or decision-making process to consider more than one criterion. This is a decision to which multi-criteria decision making can be successfully applied.

The aim of this paper is to select the optimal form of asset acquisition (loan or leasing) according to clients' selected criteria using the analytic hierarchy process (AHP) and a sensitivity analysis to assess the resulting rank of alternatives.

This paper is comprised of a theoretical part, which constitutes a detailed methodology of the AHP and sensitivity analysis, an application part, in which the described method is put into practical use for ranking financing alternatives, the selection of the optimal form of asset acquisition financing and a sensitivity analysis of the estimated ranking of alternatives. The results of the application part are summarized in the conclusion.

2. Multi-criteria decision making

The problem of multi-criteria evaluation of alternatives is foremost a task involving finding the optimal (best) alternative and ranking the alternatives from the best to the worst conceivable. In short, it is the optimization problem. Decomposing multiple attribute methods are among the most convenient when it comes to the evaluation of a finite number of alternatives. One of the most widely used methods is the analytic hierarchy process (Saaty, 1980). The ranking of alternatives and the selection of the optimal one are

based on weighted sum criteria (total weighted utility) of the alternatives, which can be calculated according to the following formula:

$$U(a_i) = \frac{\sum_{j=1}^m x_{i,j} \cdot v_j}{\sum_{j=1}^m v_j}, \quad (1)$$

where v_j represents the non-normalized weight of the j^{th} criteria, $\sum_j v_j$ stands for the sum of all (non-

normalized) criteria weights and x_{ij} represents the evaluation of the i^{th} alternative according to the j^{th} criterion. The normalized weight can be calculated using the following formula:

$$w_j = \frac{v_j}{\sum_j v_j}, \quad (2)$$

where w_j represents the normalized weight of the j^{th} criterion. Then, for the weighted sum criteria of normalized weights, the following formula can be applied (Zmeškal et al., 2013):

$$U(a_i) = \sum_{j=1}^m w_j \cdot x_{i,j}. \quad (3)$$

The fundamental advantages of multi-criteria decision-making methods can be found in the decision maker's ability to evaluate each alternative using a large number of criteria. These methods compel the decision maker to express explicitly (not intuitively) his or her understanding of the importance of each criterion. Thus, the whole process of the evaluation of alternatives becomes more transparent, easy to follow and clear, for other parties that are more or less engaged in the decision-making process as well (Fotr et al., 2010). Based on the information about the criteria or alternatives, the preference methods can be accordingly classified into:

- methods with nominal information, i.e. only the criteria names are known and the preferences are not available;
- methods with ordinal information about criteria preferences (when the rank of the criteria is known);

- methods with cardinal information (when it is possible to know the rank together with the intensity of significance of particular criteria, e.g. by using a points scale, the scoring method of which can display the weights' linear increase and which transforms the scoring evaluation of criteria importance into the vector of weights, and the Saaty method of quantitative pair-wise comparison, which determines the vector of weights from information about the estimation of weight distribution assigned by the decision maker) (Ramík, 1999).

Criteria that represent perspectives of the evaluation of alternatives will be denoted as f_j . It is possible to classify them according to their nature (maximizing or minimizing) and quantification (quantitative or qualitative). Quantified criteria can be set into a criteria matrix. The problem solving of multi-criteria decision making requires a normalized criterion matrix of alternative evaluations \mathbf{X} (where x_{ij} represents the evaluation of the i^{th} alternative by the j^{th} criterion) and weight vector \mathbf{w} (where w_j represents the normalized weight of the j^{th} criterion). In the following matrix \mathbf{X} , the columns relate to the criteria (f_1 to f_m) and the rows relate to the evaluated alternatives (a_1 to a_n). The criteria matrix and weight vector can look like this:

$$X = \begin{matrix} & f_1 & & f_j & & f_m \\ \begin{matrix} a_1 \\ a_i \\ a_n \end{matrix} & \begin{bmatrix} x_{11} \\ \\ x_{n1} \end{bmatrix} & & \begin{bmatrix} \\ x_{ij} \\ \end{bmatrix} & & \begin{bmatrix} x_{1m} \\ \\ x_{nm} \end{bmatrix} \end{matrix}, \quad w = \begin{bmatrix} w_1 \\ w_j \\ w_n \end{bmatrix}. \quad (4)$$

For calculations and comparison, it is necessary to normalize all the criteria values into the unit interval, i.e. $x_{ij} \in [0; 1]$. In general, these values can be acquired from utility functions in which $x_{ij} = u(y_{ij})$ (Zmeškal et al., 2013). A number of methods exist for criteria normalization (e.g. the weighted sum method, TOPSIS, AHP). In problem solving, it is very important to assess the criteria preferences (the ratio of importance, weight). The more important is the criterion, the higher is its weight. Among the methods applied to weight estimation can be named the ranking method, scoring method, Metfessels allocation, pair-wise comparison method, method of quantitative comparison and analytic hierarchy process.

2.1 Analytic hierarchy process

The AHP solves multi-criteria decision-making problems based on a hierarchy (Zmeškal, 2011, 2012). Generally, the hierarchy has three levels: the goal,

criteria and alternatives. The criteria can be broken down into sub-criteria to make a lower level. AHP offers a complex and logical concept for problem structuring, the quantification of problem elements that are linked to goals and the evaluation of alternative solutions. It is widespread in several decision-making situations and areas, e.g. commerce, industry and government authorities. It is also applicable to firm assessment techniques (Ishizaka and Labib, 2011). Another advantage comes from its variability of data evaluation, such as price, supply chain performance, quality, etc. The method mathematically determines the weight of each criterion as opposed to relying on subjective decision making (Deng et al., 2014).

The theoretical procedure of the AHP method consists of four steps: hierarchy design (goal definition, identification of alternatives, identification of evaluation factors, assignment of criteria and factor relationships and finishing of the hierarchy), identification of priorities (application of pair-wise comparison, point evaluation of significance, repetition of the procedure for all the hierarchy levels), combination and evaluation (weighted values of alternative solutions) (Gironimo et al., 2013). According to Saaty and Peniwati (2008), the decision making can be structured into three levels: hierarchy, priority and consistency.

Simultaneously with the creation of a structured hierarchy, an optimized system is developed from a group of criteria (sub-criteria) and alternatives. The most widely employed illustration of the hierarchy is a diagram. Saaty's method of pair-wise comparison has to be applied on each level of the hierarchical structure. The first level of the hierarchy is the goal of the evaluation (the selection of the best alternative, rank of alternatives, etc.). The second level of the hierarchy represents the evaluation criteria (the goal of the evaluation depends on which evaluation criteria will be used). The third level of the hierarchy consists of the evaluation sub-criteria. Finally, the fourth level of the hierarchy includes alternatives of which the utility depends on their relationship with the evaluation criteria and sub-criteria (Jablonský, 2002).

The classification of the model through the hierarchy is important for simple evaluation of the results with regard to criteria, easier verification (when the evaluations are not convincing), more precise content of the criteria and better compliance with expert assessment during evaluations (Roháčková and Marková, 2009).

The identification of priorities (evaluation) is based on expert estimation, in which the factor influences are compared. The scale of evaluation has five basic levels, which are mentioned in Table 1 below.

The pair-wise comparison is conducted between two criteria and the value of preference is noted in a matrix of pair-wise comparisons $\mathbf{S} = (s_{ij})$, which has a square shape ($m \cdot m$). For the elements on the main diagonal of the matrix, the relationship is $s_{ij} = 1$ (each criterion is equal to itself). This matrix is reciprocal, i.e. inverse elements are determined by the following formula according to Saaty and Peniwati (2008):

$$s_{ij} = \frac{1}{s_{ji}}. \quad (5)$$

Table 1 Point scale with descriptors

No. of points	Descriptor
1	Criteria i and j are equal
3	Low preference for criterion i before j
5	Strong preference for criterion i before j
7	Very strong preference for criterion i before j
9	Absolute preference for criterion i before j
2, 4, 6, 8	Medium values between two neighbouring criteria for more precise preference determination.

The information about the significance (s_{ij}) consists of values that determine the ratio of the evaluation criterion's significance in relation to the other criteria. The values of s_{ij} are then set into a matrix of relative significances \mathbf{S} . The elements in the matrix s_{ij} are an estimate of the weight ratios of criteria v_i and v_j , so the following applies:

$$s_{ij} \cong \frac{v_i}{v_j}, \quad i, j = 1, 2, \dots, m. \quad (6)$$

The matrix elements are generally not absolutely consistent. However, the evaluation requires a certain level of matrix consistency, i.e. that the elements are linearly independent. That can be assessed by employing the consistency ratio (C.R.) as follows:

$$C.R. = \frac{C.I.}{R.I.} = \frac{\lambda_{\max} - m}{m - 1} \cdot \frac{1}{R.I.}, \quad (7)$$

where $C.I.$ is the consistency index, λ_{\max} is the highest eigenvalue of the matrix and m represents the number of independent rows of the matrix. The λ_{\max} can be calculated as follows:

$$\lambda_{\max} = \sum_{j=1}^m \frac{(\mathbf{S} \cdot \mathbf{v})_j}{m \cdot v_j}, \quad (8)$$

where \mathbf{S} represents the pair-wise comparison matrix and \mathbf{v} means the matrix eigenvector. R.I. means the random index and represents an average number that is selected according to a particular number of matrix rows, as shown in Table 2 (Alonso and Lamata, 2006).

Table 2 Values of the random index for different numbers of criteria

m	1	2	3	4	5	6	7	8	9	10
R.I.	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Weights v_j can be estimated according to a condition that the matrix \mathbf{S} should be close to the matrix \mathbf{V} , i.e. to minimize the sum of squares of deviations according to following formula

$$\min F = \sum_{i=1}^m \sum_{j=1}^m \left(s_{ij} - \frac{v_i}{v_j} \right)^2, \quad (9)$$

under the necessary condition of $\sum_{j=1}^m v_j = 1$.

The weights v_j can be obtained through an algorithm based on the geometric mean method (the method of least logarithmic squares) under the same necessary condition; then, the solution is a normalized geometrical mean of the matrix as follows:

$$w_j = \frac{\left[\prod_{j=1}^m s_{ij} \right]^{\frac{1}{m}}}{\sum_{j=1}^m \left[\prod_{j=1}^m s_{ij} \right]^{\frac{1}{m}}}. \quad (10)$$

The geometrical mean can be calculated using the MS Excel function *GEOMEAN*. This function will be employed for calculations in the application part.

To apply the AHP method correctly, it is necessary to follow several major principles. Afterwards, the complete matrix of pair-wise comparison can be obtained: when the alternative a_1 is preferred m times to a_2 , then the alternative a_2 is $1/m$ times preferred to a_1 , the elements have to be comparable and the comparison on the lower level depends on the element on the higher level, i.e. transitivity; if the criteria are changed, it might be necessary to undertake a new evaluation for the new hierarchy (Roháčková and Marková, 2009).

The practical AHP approach consists of: the creation of the hierarchy, weight quantification for each criterion (sub-criterion), comparison of the alternatives according to the identified criteria, analysis of consistency (C.R.) and determination of the optimal alternative (with the highest aggregate weight) (Zmeškal et al., 2013).

2.2 Sensitivity analysis of investment project alternatives

Based on the acquired values of the weighted sum criterion $U(a_i)$ of particular alternatives, bearing in mind the rule of the highest value, the best alternative can be found. According to the results, all the alternatives can be organized by different measures (e.g. best

to worst alternative, etc.). Then, it is possible to assess the created ranking with various exclusion methods and select a number of alternatives to be considered further. After that, it is important to determine whether this selection is stable, i.e. prone to changes in weights. To assess the stability of the rank of alternatives, a sensitivity analysis of weight changes of alternatives can be performed.

With the above information in mind (the rank of alternatives), which was produced by employing the weighted sum criterion $U(a_i)$ with formulas (1) and (2), the following sensitivity analysis will deal with the evaluation of alternatives according to the weights estimated for this criterion (Zmeškal, 2009).

Consequently, the approach of the weight sensitivity assessment will be deduced for the weighted sum criterion $U(a_i)$. The weighted sum criterion $U(a_i)$ has to be calculated for particular evaluated alternatives. The aim is to find a limited value that would cause a change in the ranking of alternatives m and n :

$$U(a_m) = \frac{\sum_j x_{m,j} \cdot v_j}{\sum_j v_j}, \quad U(a_n) = \frac{\sum_j x_{n,j} \cdot v_j}{\sum_j v_j}, \quad (11)$$

where $U(a_m) > U(a_n)$. Following the increase in the k^{th} weight for alternatives m and n by the value of $\alpha_k^{m,n}$ to $v'_k = v_k + \alpha_k^{m,n}$, it makes the relation uneven and the following will be obtained $U'(a_m) < U'(a_n)$. The new value of the total weighted sum criterion can be written as follows:

$$U'(a_m) = \frac{\sum_j x_{m,j} \cdot v_j + x_{m,k} \cdot \alpha_k^{m,n}}{\sum_j v_j + \alpha_k^{m,n}},$$

$$U'(a_n) = \frac{\sum_j x_{n,j} \cdot v_j + x_{n,k} \cdot \alpha_k^{m,n}}{\sum_j v_j + \alpha_k^{m,n}}. \quad (12)$$

Following the change in disparity and substitution to $U'(a_m) < U'(a_n)$, it leads to

$$\frac{\sum_j x_{m,j} \cdot v_j + x_{m,k} \cdot \alpha_k^{m,n}}{\sum_j v_j + \alpha_k^{m,n}} < \frac{\sum_j x_{n,j} \cdot v_j + x_{n,k} \cdot \alpha_k^{m,n}}{\sum_j v_j + \alpha_k^{m,n}}. \quad (13)$$

Hence, in formula (13), the denominators are the same, leading to

$$\sum_j x_{m,j} \cdot v_j + x_{m,k} \cdot \alpha_k^{m,n} < \sum_j x_{n,j} \cdot v_j + x_{n,k} \cdot \alpha_k^{m,n}. \quad (14)$$

To determine the limits of sensitivity (boundaries) of the weights, it is beneficial to follow a simplified expression from the above-mentioned formulas and then determine an inequality:

$$A_m = \sum_j x_{m,j} \cdot v_j, \quad A_n = \sum_j x_{n,j} \cdot v_j, \quad (15)$$

$$A_m - A_n < (x_{n,k} - x_{m,k}) \cdot \alpha_k^{m,n}. \quad (16)$$

According to the above-mentioned formula, this relationship can be further simplified to make the following general rules for the determination of the sensitivity limits of the weights:

$$\alpha_k^{m,n} > \frac{A_m - A_n}{x_{n,k} - x_{m,k}}, \quad \text{for } x_{n,k} - x_{m,k} > 0, \quad (17)$$

$$\alpha_k^{m,n} < \frac{A_m - A_n}{x_{n,k} - x_{m,k}}, \quad \text{for } x_{n,k} - x_{m,k} < 0, \quad (18)$$

$$\alpha_k^{m,n} = \infty, \quad \text{for } x_{n,k} - x_{m,k} = 0. \quad (19)$$

In formulas (17) and (18), when the coefficients $\alpha_k^{m,n}$ are smaller, the more sensitive the alternatives m and n become towards the weight k . The last formula (19) represents a situation in which the alternatives m and n are equal based on their weights, i.e. these alternatives are insensitive to changes in weights. It is possible to create a list of weight changes (limits, boundaries) that lead to a change in the rank of alternatives. This enables a comparison not only concerning the best alternative but also across all the alternatives. Based on the identified limits $\alpha_k^{m,n}$, the new weights for particular alternatives can be determined (Zmeškal, 2009; Zmeškal et al., 2013).

3. Application to the model problem

The following chapter will illustrate the theoretical approach that was described in the last chapter.

3.1 Problem description

An entrepreneur (decision maker) wants to acquire a long-term asset (personal vehicle) within a budget of 15 000 EUR. The entrepreneur intends to use outside financing: a loan or financial leasing in particular. The decision maker has four different alternatives (two variants of bank loans and two variants of financial leasing). The actual firms that provide the information about these financial services do not want to be mentioned in this paper, so all the alternatives are assigned as follows: consumer loan 1, consumer loan 2, capital lease 1 and capital lease 2. The aim of this chapter is to assess these alternatives and select the alternative that is the best option for acquiring the asset (personal vehicle) based on the identified parameters employing the AHP method. Finally, the sensitivity analysis of changes in alternatives' weights is performed. A similar scheme with financial and non-financial criteria was used by Lee and Kwak (1995).

3.2 Initial data set and analysis of alternatives

Based on information gathered from the AHP method, the most convenient financial product will be selected. The following criteria are considered: the total cost of the purchase (f_1), fixture period (f_2), monthly payment (f_3), related fees (f_4) and annual percentage rate (APR) (f_5). The details of these financing alternatives are shown in Tables 3 and 4.

Table 3 Numerical summary of the alternatives

Criteria Alternative	Total cost (EUR)	Fixture period (months)	Monthly payment (EUR)	Other fees (EUR)	APR (%)
Consumer loan 1 (a_1)	24 413.75	120	190.217	597.80	11.34
Consumer loan 2 (a_2)	17 930.74	36	286.28	9 009.60	31.93
Capital lease 1 (a_3)	21 177.70	60	284.52	4 106.50	16.70
Capital lease 2 (a_4)	18 420.20	48	245.21	6 710.12	8.97
Criteria assignment	f_1	f_2	f_3	f_4	f_5

Source: own elaboration based on data from Finance.cz (2013)

3.3 Hierarchical structure of investment decision making and performance of the AHP method

The initial part of the AHP method approach is the creation of the hierarchy for the selection of the best asset acquisition alternative; see Figure 1. The goal appears at the top of the scheme and then two levels of criteria (criteria and sub-criteria) follow. The bottom of the scheme is represented by alternatives that have to be connected to all the criteria. This structure represents a four-level AHP structure.

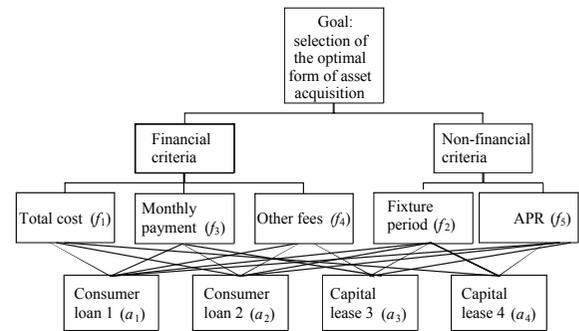


Figure 1 Hierarchical structure of the AHP

After the hierarchical structure, the initial design follows the next step in the form of quantification of the criteria weights. According to the suggested scale (Table 1), the criteria are due for pair-wise comparison (criteria and sub-criteria). The result is the matrix of pair-wise comparisons. Then, a normalized geometrical mean of rows is calculated using formulas (5), (6) and (10), to estimate the actual weights of the criteria and sub-criteria. The actual problem is structured into four levels (see Figure 1). The first level is the goal representing the selection of the best alternative. The second level consists of the major criteria: financial and non-financial. Both criteria are comprised of further sub-criteria. The following sub-criteria are presented in the diagram: three financial sub-criteria (f_1 total cost, f_3 – monthly payment, f_4 – sum of related fees) and two non-financial sub-criteria (f_2 – fixture period and f_5 – APR). The bottom level represents the set of alternatives for which the utility is linked to relationships with the sub-criteria. Detailed information about these alternatives can be found in Table 3. The weight estimation process according to the scale from Table 1 is shown in the following tables. The first-level Saaty pair-wise comparison matrix is presented in Table 5.

Table 4 Alternatives and detailed description of the criteria

Alternative	Description
Consumer loan 1 (a_1)	The fixture period is 120 months; the monthly payments are in two versions (first 12 months 27.74 EUR/month; then 208.27 EUR/month); the average monthly payment is 190.217 EUR. The processing fee is 239 EUR and the monthly fee is 2.99 EUR (120 months = 358.80 EUR). The interest rate is 9.9% p.a. and the APR is 11.34%. The total cost is 24 413.75 EUR.
Consumer loan 2 (a_2)	The fixture period is 36 months; the monthly payment is 286.28 EUR. The processing fee is 153.60 EUR, the interest rate is 26.07% p.a. and the APR is 31.93%. The client agreed to pay the initial part of the purchase price of the vehicle of 8 856 EUR (the actual loan is 6 144). The total cost is 17 930.74 EUR (= 8 856 + 9 074.74 including the accessories).
Capital lease 1 (a_3)	The fixture period is 60 months; the monthly payment is 284.52 EUR. There are no processing fees or monthly fees. The interest rate is 8.87% p.a. The APR is 16.70%. The purchase price at the end of the fixture period is 96.24 EUR. The down payment is 4 010.26 EUR. The total cost is 21 177.70 EUR.
Capital lease 2 (a_4)	The fixture period is 48 months; the monthly payment is 245.21 EUR. There are no processing fees or monthly fees. The interest rate is 7.1% p.a. The APR is 8.97%. The purchase price at the end of the fixture period is 60 EUR. The down payment is 6 650.12 EUR. The total cost is 18 420.20 EUR.

Source: own elaboration based on data from Finance.cz (2013)

Table 5 Matrix of financial and non-financial pair-wise comparisons

Criteria	Financial	Non-financial	Geometrical mean of the row $[v_i]$	Normalized weight $[w_j]$
Financial	1	5	2.236	0.833
Non-financial	$1/5$	1	0.447	0.167
Sum			2.683	1

The consistency index (7) is $0.0 \leq 0.1$; the matrix in Table 5 is consistent.

The following quantitative pair-wise comparisons of sub-criteria are shown in Table 6 and Table 7.

Table 6 Financial sub-criteria weights

Criteria	f_1	f_3	f_4	Geometrical mean of the row $[v_i]$	Normalized weight $[w_j]$
f_1	1	3	9	3.000	0.655
f_3	$1/3$	1	7	1.326	0.289
f_4	$1/9$	$1/7$	1	0.251	0.054
Sum				4.577	1.000

The consistency index (7) is $0.0692 \leq 0.1$; the matrix is consistent.

Table 7 Non-financial sub-criteria weights

Criteria	f_2	f_5	Geometrical mean of the row $[v_i]$	Normalized weight $[w_j]$
f_2	1	$1/3$	0.577	0.250
f_5	3	1	1.732	0.750
Sum			2.309	1

The consistency index (7) is $0.0 \leq 0.1$; the matrix is consistent.

Table 8 Final weight estimation by the AHP method

Item	Weight	Local criteria weights	Global criteria weights
Financial criteria*		0.833	0.833
Non financial criteria		0.167	0.167
f_1 – Total cost		0.655	0.546
f_2 – Fixture period		0.250	0.042
f_3 – Monthly payment		0.290	0.241
f_4 – Sum of related fees		0.055	0.046
f_5 – Annual percentage rate		0.750	0.125

* Financial criteria in italics.

The global weight is calculated as the multiplication of the criteria weight and the local weight of the sub-criteria. The financial sub-criteria have the local weight of 0.833 (applied to sub-criteria f_1, f_3, f_4) and the non-financial criteria have the local weight of 0.167 (for f_2, f_5). The estimation of criteria weights is followed by the third level of the hierarchy, i.e. the comparison of alternatives. This has to proceed with

the employment of further matrixes in which these alternatives can be compared with regard to particular criteria, as shown in the example in Table 9. The other comparisons are performed similarly. The results are shown in Table 10.

Table 9 Matrix of pair-wise comparisons of alternatives for criterion f_1

f_1 – Total cost	a_1	a_2	a_3	a_4	v_j	w_j
a_1	1	$1/5$	$1/3$	$1/4$	0.359	0.074
a_2	5	1	2	$1/3$	1.351	0.279
a_3	3	$1/2$	1	$1/2$	0.931	0.192
a_4	4	3	2	1	2.213	0.456
Σ					4.854	1

The consistency index (7) is $0.09 \leq 0.1$; the matrix is consistent.

To estimate the weights of the sub-criteria, the alternatives are compared according to the comparison criteria, as shown in Table 10. These evaluations represent a complex assessment of particular financing alternatives according to each criterion. Each alternative evaluation is summed up and multiplied by a particular weight to calculate the total evaluation of alternatives $U(a_i)$. This value is used for the final ranking of alternatives.

Table 10 Results of the AHP for asset financing alternatives

Criterion \ Alternative	f_1	f_2	f_3	f_4	f_5	$U(a_i)$	Rank
Weight $[w_j]$	0.546	0.042	0.241	0.046	0.125	–	–
a_1	0.074	0.067	0.635	0.661	0.199	0.252	2
a_2	0.278	0.352	0.061	0.052	0.046	0.190	3
a_3	0.192	0.171	0.096	0.170	0.141	0.160	4
a_4	0.456	0.407	0.207	0.119	0.612	0.398	1

According to the results of the AHP method, the best alternative can be selected: (a_4) – Capital lease 2.

3.4 Sensitivity analysis of the alternative evaluation using the estimated weights

After the final selection or ranking of alternatives, it is important to determine whether this selection is stable and robust. This means the application of sensitivity analysis to weight changes. The comparison will proceed with the best alternative a_4 towards the rest (a_1, a_2, a_3) with regard to the rank of evaluation values acquired by AHP. The purpose of this analysis is to assess the sensitivity of weight changes in the ranking of alternatives. Each alternative is pair-wise compared with another to find the limit value for a change in the rank of alternatives.

It is based on the rules for the determination of the sensitivity of weight limits described in the second chapter, the ratio of the value difference of the sum criteria in both compared alternatives to the difference between normalized values of both compared alternatives. In this way, the measure of weight change can be found and used to determine the rank change. The results of the sensitivity analysis for alternatives a_4 and a_1 regarding a weight change in the first criteria are shown in Table 11. The sensitivity analysis is characterized by formulas (17–19). It is apparent that the smaller these coefficients are, the more the alternatives are sensitive to a change in the weights. An example is shown in Table 11.

With regard to formula (17), it stands that $\alpha_k^{m,n}$ has to be lowered by 0.383 or more to cause a change in the rank of both alternative (a_4) and alternative (a_1) and that (a_1) could be the better alternative.

Table 11 Sensitivity analysis of the evaluation of alternatives a_4 and a_1 for criteria f_1

Item	Sum criteria	Normalized value	Difference	Value difference	Change of weight
Symbol	$A_{m,n}$	$X_{m,n,k1}$	$A_m - A_n$	$x_{m,k1} - x_{n,k1}$	$\alpha_k^{m,n}$
Alternative (a_4)	0.399	0.456	–	–	–
Alternative (a_1)	0.252	0.074	0.146	–0.382	–0.383

A similar approach is applied to the other analysed criteria and alternatives. A summary of the weight and rank changes of alternatives towards the best alternative selected according to the result from the AHP is provided by the following Table 12.

Table 12 The list of changes in weights towards a_4

Criteria \ Alternative	Change of particular weight				
	f_1	f_2	f_3	f_4	f_5
a_1	–0.383	–0.434	0.342	0.270	–0.353
a_2	–1.175	–3.830	–1.423	–3.138	–0.367
a_3	–0.900	–1.010	–2.136	4.664	–0.503

The smaller is coefficient $\alpha_k^{m,n}$, the more sensitive the alternatives are towards changes in the weights. The most sensitive is alternative a_1 (with regard to the best alternative). Alternatives a_2 and a_3 can be considered insensitive because the weight change required to cause a rank shuffle is not possible (the normalized weight cannot be over 1). This is the reason why the further analysis works only with a rank of the second-best alternative a_1 . The changes that are necessary to cause a change in the order of the original rank are shown in the following Table 13.

Table 13 The summary of rank changes for alternative a_1

Criteria	f_1	f_2	f_3	f_4	f_5
Weight [w_j]	0.546	0.042	0.241	0.046	0.125
Change of weight	–0.383	–0.434	0.343	0.270	–0.353
New weight [w'_j]	0.163	–0.393	0.584	0.315	–0.230

For a better understanding, a graphical result is provided in Figure 2.

A decrease in the weight of the first criterion or a decrease in the weight of the third or even the fourth criterion that could materialize would have to be quite high. This leads to the conclusion that the selected alternatives can be perceived as stable and robust. A decrease in the second and fifth criteria is not realistic (the weights would have to be negative).

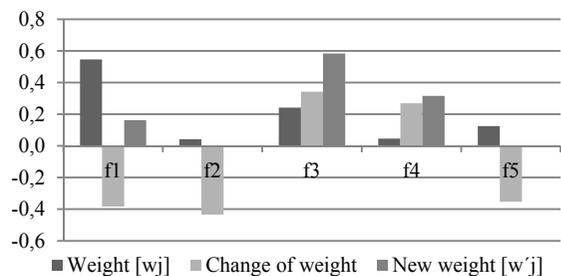


Figure 2 Changes of weights for alternative a_1

3.5 Results and discussion

In the present economic depression, decision making plays a crucial role in entrepreneurs' activities. Thus, in the long term, monetary decision making is important to assess a wide range of factors and multiple criteria relevant to the investment case at hand. The result of such a decision is also biased by the correct selection of the method employed. In the example that was presented in this paper, the analytic hierarchy process (AHP) was selected. The reason was its fundamental versatility and hierarchical structure, which can accommodate multiple levels of criteria. The method also monitors the consistency of decision makers' choices.

Based on the evaluations performed and the analysis of four potential forms of asset acquisition, which were assessed by five sub-criteria, it can be suggested that the optimal alternative (with the highest utility) of vehicle purchase for the entrepreneur (decision maker) is the fourth alternative, i.e. a_4 (Capital lease 2), with total utility of 39.83%. This alternative is characterized by a total cost of 18 420.2 EUR, a fixture period of 48 months and monthly payments of 245.21 EUR. There are no processing fees or monthly fees. The interest rate is 7.1% p.a. and the APR is 8.97%. The purchase price at the end of the fixture period is 60 EUR. The down payment is 6 650.12 EUR. The

second-best alternative was identified as a_1 (Consumer loan 1), with total utility of 25.19%. The third- and fourth-placed alternatives, a_2 and a_3 , did not reach a utility over 20%.

It was also found that the form of asset acquisition is not as important as particular factors (criteria – total cost, fixture period, monthly payment, other fees and APR) that are considered in the decision-making process. This supports the fact that a capital lease with a fixture period of 48 months (a_4) has been evaluated as the best alternative, when a capital lease (a_3) with a longer fixture period has been evaluated as the worst alternative based on the considered criteria.

A sensitivity analysis was performed to verify the best alternative among all the alternatives. The limits that were found by the sensitivity analysis enabled the determination of new weights and the realization of a new rank of alternatives. The changes in weights that would need to occur to shuffle the original ranking are quite high. The particular changes in weights that would have to occur to elevate the second-best alternative a_1 are illustrated in Figure 2 and Table 12. It can be concluded that the ranking of alternatives based on the results from the AHP is stable and robust enough.

In the case of a larger number of alternatives, the application of a limit of acceptance for the alternatives can be suggested. This would divide the set of acceptable alternatives from the unacceptable alternatives and both their rankings. Further research can be focused on combination with different multiple attribute decision-making methods, such as TOPSIS and VIKOR.

4. Conclusion

It is common knowledge that the least effective and also the most expensive form of investment project financing is to use one's own resources (own capital). Hence, more consumers are looking for other options. One of them is the *purchase on loan*. Among the most frequently offered asset purchasing financing services are the lease and the consumer loan. Leasing companies and banks create a large market with a large number of options and products. Some of them are especially focused on entrepreneurial clients. Then they face the problem of selection and decision making.

Decisions have a profound and long-term character and are constrained by a budget. That is the reason why a decision maker (entrepreneur) should consider more than one criterion. This leads to the difficult problem of the evaluation and comparison of several criteria simultaneously. Multi-criteria decision-making methods provide a solution that can be applied to a

large variety of problems. The selected method of AHP is specifically designed for a decision-making problem focused on the evaluation of alternatives. The purpose of this method is to select the optimal alternative, evaluate more alternatives of the problem and determine the ranking.

The presented method was applied in the paper to an example of four financing alternatives for an asset purchase. The comparison of these alternatives led to the conclusion that the best alternative, according to the selected criteria, is Capital lease 2. The stability and robustness were verified by sensitivity analysis.

The paper represents a real business example of how a decision maker, in this case an entrepreneur, can use a simple yet more sophisticated technique for procurement. In this case, the problem is the selection of the best alternative (the means of financing of the purchase). The entrepreneur can choose relevant factors (criteria) and estimate their importance using the AHP method. This method enables the entrepreneur to evaluate a larger number of criteria using pairwise comparisons and also to calculate the consistency. This raises the credibility of his decision making. The AHP estimates weights and priorities and suggests the best alternative considering all the criteria.

The example that is presented in this paper is focused on an application that is useful for small and medium-sized businesses because it can be conducted using the affordable and familiar MS Excel interface. Many other AHP applications forget to provide a sensitivity analysis, which is an important step in decision making. In the practical sense, the presented approach should be considered for implementation in loan and leasing calculators, which are available to entrepreneurs from various online and offline sources. It is important to mention that the presented comparison is an example of optional agreements for asset purchase. This paper does not consider the legal and tax relations that are specific to consumer loans and capital leasing.

In the post-crisis business environment, decision-making processes are very important. The results of a decision have in most cases a profound influence not only on the entrepreneur him- or herself but also on the whole network of suppliers, customers and other business partners.

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