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Weighted multi-criteria decision-making with coalition strategies: a framework for citizens public participation

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Abstract

Public planning decisions affect the living conditions of diverse categories of people differently. Therefore, voters should express their support and/or rejection to each alternative. Since their opinions tend to be subjective, intensities of support and rejection should be collected and processed (rather than binary *yes*, *no* voting) to reveal whether an agreement is in favour or against each alternative. However, inconsistent responses (simultaneous high levels of support and rejection) for the same alternative represent a challenge. The next challenge is the different influence of alternatives to diverse citizens subgroups. To address these issues, this work proposes strengthening the consistent answers and weakening the contradictory responses by the convex combination of t-norm and t-conorm function. Next, the impact of coalitions (agreement) among subgroups is formalised by fuzzy measures and Choquet integral, because the impact is different when two of the most affected subgroups or two lightly affected subgroups agree on a specific alternative. In real-life problems through the Traffic Strategy Case Study in the Street of Unterdorf in Geuensee, Lucerne (Switzerland), 13 alternatives were evaluated by four subgroups of voters. Fuzzy measures or weights are assigned to each subgroup and their possible coalitions considering their features. In addition, the sensitivity analysis is performed by Monte Carlo simulation. Finally, topics for future work are outlined.

Keywords Citizens flexible voting · Choquet integral · Coalition strategies · Fuzzy voting · Multi-criteria decision-making · Weighted hierarchical aggregation

1 Introduction

Citizens' voting is influenced by feelings and intuition. Some authors explain intuition as *pattern recognition*, (i.e., trying to find similar patterns in brain memory (or experiences) of past decisions and their consequences to similar problems (Ariely 2009)). Voting is not only a clear rational evaluation, but also a feeling. Like a

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data mining task in uncertain areas (e.g., classification using fuzzy logic approaches into *yes* and *no* classes while assessing inclination by class *Maybe* (Bartoszuk et al. 2021; Hudec et al. 2021)), these voting decisions are also more or less heuristic (Stanovich 1999).

The next problem is participation in the voting. Voters are either non-motivated to vote, or are not able to provide their binary opinion. Instead of clear *yes* or *no*, they are somewhere between. A flexible voting referendum research applied in Basel (Switzerland) concluded that it was a welcomed alternative, particularly accepted by undecided voters who rarely participated in voting (Schwarz et al. 2024).

In public matters, important strategic decisions should consider the diverse groups of stakeholders (Zapletal et al. 2023a). Especially in local spatial planning settings, subgroups are affected at varying levels depending on their geographical location and other factors (Emmenegger et al. 2024a) and therefore this observation should be reflected. Traditionally, such issues were addressed through surveys and related methodologies. These approaches frequently encounter limitations as voters may struggle to provide precise answers (Altig et al. 2022; Ellerby et al. 2022; Kouatli 2020) even when categorical answers are proposed on a Likert scale, where neighbouring answers due to uncertainty were also noticed (and carelessness) (Rakovská and Hudec 2019). Due to the uncertainty regarding the level of support or rejection in relation to specific issues, the need for more nuanced analytical methods on the one hand and less demanding on the voter side is highlighted and this is where the present study fills a gap in the literature. The recognised uncertainty and feelings should not be neglected in the hesitance of the responses: *support* or *reject*.

When examining subgroup votes, weights should be assigned to subgroups (Zapletal et al. 2023a). But weights exhibit uncertainty, which might cause that smaller changes in weights influence the order or raised alternatives. Thus, a sensitivity analysis should be applied. This work significantly extends the initial proposal introduced in (Emmenegger et al. 2024b).

After this introduction, the review or related literature is presented in Sect. 2, followed by the theoretical preliminaries in Sect. 3. Section 4 is devoted to the proposed model, while Sect. 5 applied this model to the real-life voting problem. Next, Sect. 6 discusses the results, answers research questions, and raises future research topics. Finally, Sect. 7 concludes this work.

2 Related works

The literature has already taught voting problems from several perspectives. Participation is not only a clear rational evaluation of possible alternatives, but also a feeling (Ariely 2009), for example, trying to find similar patterns in brain memory (or experiences) of past decisions and their consequences. Next, several citizens are more informed and, therefore, more rational than others about the voting questions raised (Stanovich 1999). Recent work (Schwarz et al. 2024) advocated that flexibility in voting is particularly accepted by undecided voters who rarely participated in voting but considered to participate by providing their vote as an inclination towards clear *Yes* or clear *No*.

Uncertainty is a inherent property in various fields. For example, in financial decisions, where financial knowledge could be defined not only as a proxy for financial expertise, but also as a basis for making decisions that extend in an uncertain way beyond the specific context of the questions raised (Brent and Ward 2018). The same holds for greater citizen participation, where the motivation to explore alternatives, reduce perceived barriers, and improve usage levels is welcomed (Bach et al. 2025).

Hierarchical evaluation of alternatives has been explored in DEX model (Decision Expert, cf. (Trdin and Bohanec 2018)), where the evaluation is realized in a button up way by functions adapted to each attribute. The model allows coping with uncertainty by probabilities, fuzzy data and functions. On the other hand, we focus on the hierarchy of respondents and the logic aggregation functions at each level to evaluate raised alternatives. Naturally, functions are different. However, it might raise possibilities for future work in the search for common functions for both approaches in hierarchical aggregation.

Participation in democratic processes is often expressed as a democratic role model, but citizens in Switzerland in most cases participate less than 50% at the municipal, cantonal, and federal level (Sciarini and Tresch 2023). Uncertainty in voting often stems from the complexity of issues and the inability of traditional binary choices (yes-no) to capture nuanced opinions. Many voters find it challenging to express their preferences in a rigid system where only absolute choices are allowed. Instead, they evaluate options using a spectrum of opinions, often relying on additional arguments, contextual information, and flexibility in decision-making processes (Emmenegger et al. 2024a; Schwarz et al. 2024). Sliders, weighted preferences, and textual justifications are examples of tools that reflect this need for comprehensive and nuanced evaluation (Altig et al. 2022; Ellerby et al. 2022; Kouatli 2020; Emmenegger et al. 2023).

Mathematical and IT solutions, such as fuzzy voting systems, fuzzy logic, and clustering algorithms, address this challenge by allowing voters to express degrees of agreement or preferences (Terán 2014). Fuzzy voting allows for partial preferences, where voters can indicate varying levels of support for different options (Romero et al. 2019; Liu and Mendel 2008; Portmann 2019). Fuzzy clustering, on the other hand, can group voters or issues based on similar preferences, providing insights into collective tendencies (Zapletal et al. 2023a). These approaches offer a more accurate and inclusive way to capture the intent of voters, accommodating the inherent uncertainty and diversity of opinions. However, implementing such systems requires careful design to ensure accessibility, transparency, and trust among voters (e.g., (Colombo 2024)).

The three-level decision and aggregation model, introduced in (Rakovská and Hudec 2019) is suitable for selecting the most relevant software tool in a company. The first level is a usual questionnaire, the second level a quantified aggregation to emphasise each department view on relevant software tools, and the third level is an uninorm aggregation to emphasise tolls preferable for a majority of department. Three levels of aggregation in decision making have been proposed in (Zapletal et al. 2023a), where on the first level, hesitant fuzzy sets are adopted to evaluate the hesitance in answers to a set of survey questions for a wider use. On the second level, an aggregation by quantifier *most of* was adopted, while on the third level Choquet integral without reflection on sensitivity analysis. However, our work requires different functions on the first level (not a survey, but voting) and therefore, on the second level averaging aggregation should be preferred as well as sensitivity analysis, because the practical implication is the key factor in our work.

Hence, these models should be advanced to create an enhanced and robust analytical model capable to address the aforementioned issues in citizens voting among alternatives.

Next, the focus is on ways to collect answers by recording uncertainties or indecisiveness. Various sources of uncertainty and therefore methods for catching them are modelled in group decision-making problems. The study by (Zapletal et al. 2023b) proposed recording the answers using the Likert scale enriched by hesitance expressed like: *I'm absolutely sure*, *I feel a weak hesitance*, and *I feel strong hesitance, (i.e., the answer is based mainly on my feelings*). Although this approach collects relevant information, it is not suitable for voting when we need to collect intensities of support and rejection.

This work focusses on formally enveloping and formalising rational and feelings in citizens voting. Based on the problem explored and literature review, we raised the following research questions:

- Could we formalize a model for voters to express their intensities of supporting and rejecting an alternative, and to recognize voters with more and with less clear view on each particular alternative?
- Could we create a robust way of assigning weights to citizen subgroups and all their possible coalitions?
- Could the proposed method solve the needs for an acceptable decision to solve problems through the Traffic Strategy Case in the Street of Unterdorf in Geuensee, Lucerne (Switzerland)?

3 Preliminaries

This section studies relevant theoretical backgrounds, which are used throughout the paper.

3.1 Recording opinions

Voters have by nature a level of hesitance in expressing their answers which is caused by reasons like lack of knowledge, not clear information received, and responding rather by feelings (e.g., when new development influences their living conditions, either in the positive or negative direction, see, e.g., (Rakovská and Hudec 2019; Zapletal et al. 2023a)). In surveys or opinions collection, it is expected that user experience is observable and measurable (Albert and Tullis 2013). However, surveys often face issues as voters may struggle to provide precise answers (Altig et al. 2022).

Opinions can be recorded by free verbal descriptions or open-ended survey responses, which allow voters to express their thoughts in detail. Structured approaches, such as linguistic scales or Likert scales, provide measurable data on voter sentiments. In addition, visual methods, such as ranking or mapping preferences, can be employed for more intuitive feedback collection (Mendel et al. 2010; Martinez et al. 2009; Emmenegger et al. 2023).

The use of sliders allows individuals to express varying intensities of support or rejection towards different alternatives. They capture a spectrum of preferences, enabling a more detailed understanding of where consensus or divergence lies. By focussing on the degrees of intensity, voters can better evaluate both strong advocates and those with moderate or opposing views, facilitating more informed and balanced outcomes (Schwarz et al. 2024; Emmenegger et al. 2023).

3.2 Aggregation functions

An aggregation function $A : [0, 1]^n \rightarrow [0, 1]$ is a function aggregating n opinions together and satisfying:

(i) Axioms of boundary conditions: $A(0, 0, .., 0) = 0$ - if all atomic opinions are clearly non-satisfied, then the alternative is fully rejected, and $A(1, 1, .., 1) = 1$ - if all atomic opinions are ideally satisfied, then the alternative is fully accepted.
(ii) Monotonicity axiom - If the values of the atomic conditions are increasing, then the satisfaction of the alternative is increasing or remains the same (Grabisch et al. 2009).

This is a broad definition to cover a large set of real-world requirements for aggregation.

The standard classification of aggregation functions is the following (Dubois and Prade 2004):

- Conjunctive characterized by $A(\mathbf{x}) \leq \min(\mathbf{x})$.
- Disjunctive given as $A(\mathbf{x}) \geq \max(\mathbf{x})$.
- Averaging characterized by $\min(\mathbf{x}) \leq A(\mathbf{x}) \leq \max(\mathbf{x})$
- Mixed aggregation functions (combination of the functions above)

where $\mathbf{x}$ is a vector of individual opinions.

In Sects. 4.1 to 4.3, we examine the requirements for evaluating votes to recognize the most suitable classes and particular functions from the selected respective classes.

3.3 Monte Carlo simulation

Since weights are often assigned subjectively by expert opinions, it is reasonable to check the robustness of the results with respect to the changes in weights. Monte Carlo simulation (Rubinstein and Kroese 2016) is one of the convenient options to do this check. It replaces the explored parameters with random variables with an assigned probability distribution. The model is executed repeatedly with randomly generated values, and the stability is revealed (Shonkwiler and Mendivil 2024).

4 Methodology and model

In the main part of this section, the three-level aggregation model is proposed. We first formalized the relevance (reliability) of each voter's answer, followed by the aggregation at the subgroup level and the aggregation among subgroups considering their so-called coalitions or impact on decision making (see Fig. 1).

4.1 Relevance of the voter answer

In our approach, voters are asked to express both their intensities of support and rejection of each proposed alternative. When a voter provides both intensities, the reliability of answer is estimated.

We should support this reliability by adopting a suitable aggregation function covering intensities of support and rejection. Let us look at the possible cases:

1. When support (S) and rejection (R) are both high, the consistency of answer is low.
2. When support (S) is high and rejection (R) is low, the consistency of answer is high in supporting the alternative.
3. When support (S) is low and rejection (R) is high, the consistency of answer is high against the alternative.
4. When support (S) is low and rejection (R) is low, the consistency of answer is low.

This observation leads to the adoption of mixed aggregation functions (Grabisch et al. 2009), which are able to emphasise high support and low rejection alongside with attenuating low support and high rejection and averaging when the voter is somewhere between in response. First, we transform rejection into its negation (i.e., $\bar{R} = 1 - R$). This transformation is merely used to simplify the calculations.

Among the mixed aggregation functions, an option is the convex combination of Łukasiewicz t-norm and its dual t-conorm. For example, more about these functions can be found in (Boixader and Recasens 2022; Zong et al. 2025). For simplicity in notation, we denote support (S) by x and the inverse of rejection ($\bar{R}$) by y resulting in the following:

$$A_{\lambda}(x, y) = \lambda \cdot \max(0, x + y - 1) + (1 - \lambda) \cdot \min(1, x + y), \tag{1}$$

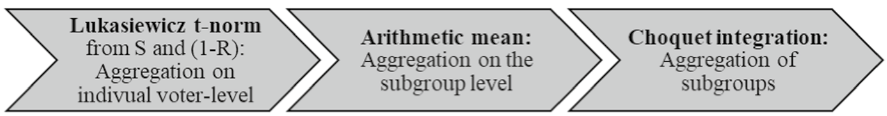

Fig. 1 Overview of the proposed three level aggregation of citizens voting

where $T_L = \max(0, x + y - 1)$, $S_L = \min(1, x + y)$ and $\lambda \in [0, 1]$. When $\lambda = 1$, we get Łukasiewicz t-norm T_L; when $\lambda = 0$, we get Łukasiewicz t-conorm S_L; while when $\lambda = 0.5$, we get the neutral averaging behaviour.

The values of (λ) are determined based on the conditions of support (x) and inverse of rejection (y) as follows:

- When $(x, y) \in [0, 0.5)^2$, then $\lambda = 0.75$ emphasizes inclination to rejection by conjunctive function.
- When $(x, y) \in (0.5, 1]^2$, then $\lambda = 0.25$ emphasizes the inclination to support by a disjunctive function.
- In the other two cases, $(x, y) \in [0, 0.5] \times [0.5, 1] \cup [0.5, 1] \times [0, 0.5]$, $\lambda = 0.5$ emphasizes indecisiveness by means of the averaging function.

This methodology suggests that values near 0.5 aggregate very indecisive or conflicting opinions, while values 0.75 and 0.25 indicate a high inclination toward rejection or acceptance, respectively.

An illustrative example for an alternative is shown in Table 1. The borderline axioms are satisfied (voters V_1 and V_3, that is, full support and without any rejection means clear support, while the opposite means clear rejection). Next, voter V_4 has slightly better support than voter V_1, but a little stronger rejection. This nuance in the answers is reflected in the closeness to rejection. Voters V_5, V_6 and V_7 are indecisive or confused with the alternative, as their answers are quite close to value 0.5.

4.2 Aggregation on the subgroup level

Voters belong to subgroups according to their geographical proximity to the planned development area, or the membership to particular sets such as landlords, living in affected area, and commuting.

In each subgroup, we have a certain number of voters responding to voting, where each answer is in the unit interval (Sect. 4.1). Hence, a suitable aggregation for the opinion of a subgroup is by arithmetic mean, due to its averaging and neutral logical behaviour (Dujmović 2018), (i.e., neither an inclination to the full rejection, like by

Table 1 An illustrative example of aggregating support and rejection by (1)

Voter	Support S	Rejection R	$1 - R$	λ	S
V_1	0.20	0.80	0.20	0.75	0.10
V_2	1.00	0.00	1.00	0.75	1.00
V_3	0.00	1.00	0.00	0.25	0.00
V_4	0.21	0.88	0.12	0.75	0.08
V_5	0.50	0.50	0.50	0.50	0.50
V_6	0.88	0.90	0.10	0.50	0.49
V_7	0.30	0.40	0.60	0.50	0.45
V_8	0.90	0.30	0.70	0.25	0.90
V_9	0.90	0.10	0.90	0.25	0.95

geometric mean, nor an inclination to full support, like by quadratic mean). Hence, the solution for each subgroup $g = 1...G$ is obtained as:

$$A_g(x,y) = \frac{1}{m_g}\sum_{j=1}^{m_g} A_\lambda(x_j, y_j), \tag{2}$$

where m_g is the number of voters in a subgroup g and $A_\lambda(x_j, y_j)$ is the aggregated support x and rejection y of the voter j in subgroup g by (1).

4.3 Aggregation of subgroups

The aggregation of subgroups could be realised by some simple averaging function. However, this could result in biased conclusion, because there can be some mutual interactions between subgroups, leading to strengthening or weakening when joining them together. Thus, we should create a robust aggregation of subgroups.

The aggregation of coalitions is managed by the discrete Choquet integral (Choquet 1954) where fuzzy measures (or capacities) indicate the importance or weight of each possible coalition. Choquet integration is based on not necessarily additive, but monotone fuzzy measures $v : 2^{\mathcal{N}} \to [0,1]$. A discrete fuzzy measure (Horanská and Šipošová 2018; Wang and Klir 1992) is a set function on $\mathcal{N} = \{1, 2, \dots, n\}$ that is monotonic ($v(\mathcal{A}) \le v(\mathcal{B})$ whenever $\mathcal{A} \subseteq \mathcal{B}$) and satisfies the boundary conditions $v(\emptyset) = 0$ and $v(\mathcal{N}) = 1$.

A subset $\mathcal{A} \subseteq \mathcal{N}$ is considered as a coalition, where $v(\mathcal{A})$ explains the importance of coalition (Takáč et al. 2022). The discrete Choquet integral with respect to a fuzzy measure v is given by (Grabisch et al. 2009) as:

$$C_v(\boldsymbol{x}) = \sum_{i=1}^{n} x_{(i)}\left[v\left(\{j | x_j \ge x_{(i)}\}\right) - v\left(\{j | x_j \ge x_{(i+1)}\}\right)\right], \tag{3}$$

where $\left(x_{(1),}x_{(2),} \dots x_{(n)}\right)$ is a permutation of non-decreasing values, n is the number of coalitions, and $x_{(n+1)} = \infty$. An alternative expression, more suitable for computing is (Beliakov et al. 2020):

$$C_v(\boldsymbol{x}) = \sum_{i=1}^{n}\left[x_{(i)} - x_{(i-1)}\right] v(H_i), \tag{4}$$

where $x_{(0)} = 0$ and $H_i = \{(i), \dots, (n)\}$ is the subset of indices of the $(n - i + 1)$ largest components of vector $\boldsymbol{x}$.

When the collation of two citizen subgroups A and B should be emphasised, we model it as $v(\mathcal{A} + \mathcal{B}) > v(\mathcal{A}) + v(\mathcal{B})$ (the property of super additivity). When it should be attenuated, we model it as $v(\mathcal{A} + \mathcal{B}) < v(\mathcal{A}) + v(\mathcal{B})$ (the property of subadditivity) and finally, when coalition does not bring nothing new, we model it as $v(\mathcal{A} + \mathcal{B}) = v(\mathcal{A}) + v(\mathcal{B})$ (the property of additivity). In all three cases, we should keep the monotonic property (i.e., $\max(v(\mathcal{A}), v(\mathcal{B})) \le v(\mathcal{A} + \mathcal{B}) \le 1$).

The main challenge is how to assign weights that reflect importance for each subgroup individually and for all their possible coalitions. Since weights are often assigned by expert opinions subjectively, a thorough sensitivity analysis is recommended. The overview of the proposed model is highlighted in Fig. 1.

5 Real-world application in traffic planning

This section presents the Traffic Strategy Case in the Street of Unterdorf in Geuensee, Lucerne (Switzerland). The case study was designed to address the challenges in traffic planning for a specific street within the community. It was conducted and accompanied by the authors of this paper. First, the collected data, including the votes from stakeholders, are explained, followed by a detailed description of the three levels of aggregation used in the analysis. Finally, the case study concludes with a sensitivity analysis, highlighting the robustness and reliability of the proposed methodology.

5.1 Collected data

As part of the data collection process, voters assessed each of the 13 alternatives. Data were collected using the Comparative Linguistic Expressions (CLEs), used to assess support and rejection (Emmenegger et al. 2023). Each slider displayed four statuses (e.g., ranging from 'I am against it and resist active' to 'I am not resisting at all', as shown in Fig. 2). The answers are converted into the unit interval [0, 1], where 1 means complete rejection, while 0 means no rejection. Analogously, the slider was constructed to express the support of the voters. CLEs were used to allow voters to assess the intensity of options more intuitively, in a manner that is more similar to human reasoning.

This resulted in the collection of 26 numerical values per voter. In addition, demographic data were gathered, including voters' place of residence, age, gender, and their relationship or involvement with the traffic topic (Emmenegger et al. 2023). These supplementary variables enabled the definition of specific subgroups to facilitate more nuanced analyses and interpretations of the data.

5.2 Groups and weights

In the case, four distinct stakeholder groups were formed based on the geographic and demographic characteristics of the voters. These groups reflect variations in

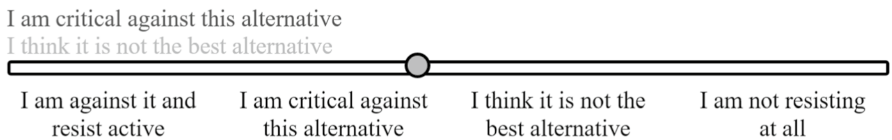

Fig. 2 Example of a slider showing CLEs (Emmenegger et al. 2023)

their proximity to the affected area and their connection to the traffic issue, ensuring that diverse perspectives were included in the evaluation. The expert team, who guided the entire participation process, evaluated the groups using four key criteria: demographic characteristics, geographic proximity, their direct or indirect relationship to the traffic problem, and their overall engagement in the issue. This structured evaluation ensured that the groups were defined logically and fairly, maintaining a robust foundation for the case study in Geuensee:

- **Direct Involvement**: This criterion measures whether a participant is actively engaged as a crucial part of the solution team. It involves their participation in creating, discussing, and executing solutions, indicating their role is not just advisory but integral to the implementation process. This reflects the degree to which a participant is embedded in the solution's development and outcome.
- **Perception of Urgency**: Assesses how participants perceive the relevance of the alternatives presented in the voting process. It evaluates whether they desire change and their awareness of the need for solutions, measuring how pressing they find the issues at hand. This criterion helps gauge the motivational drivers behind participants' decisions.
- **Influence and Property**: Evaluates participants' stakes in the outcomes based on their ownership, such as land or other significant assets, that directly affects their involvement. Participants who own property that is impacted by the voting issues are not only more likely to be influenced by the outcomes but also have substantial influence themselves if they oppose or support alternatives.
- **Democratic Power**: Measures the degree of participation in the decision-making process, specifically through voting on the final costs and project alternatives. This criterion assesses how many people are involved in the decision-making process, with the principle that each vote represents an equal share of power, thereby highlighting the democratic engagement across the participant base.

These criteria were assigned equal weights of 25% each for analysis purposes, since there was no objective and unbiased way to set the weights differently. Equal weighting of the criteria ensures that all aspects of the evaluation are considered equally important in the analysis.

The four groups were evaluated in terms of the four criteria above by the expert team, using the scale from 1 to 10. The weighted sum of these expertly assigned values and the weights of criteria for each group determine the normalized weight of each group. These weights will be used further for aggregation using the Choquet integral, when calculating the final evaluation of the alternatives (see Sect. 4.3).

The initial scores of four subgroups (Group 1, Group 2, Group 3, and Group 4) based on these four criteria are shown in Table 2. For the criterion of Direct Involvement, which evaluates the level of engagement or participation of each subgroup, Group 4 scores the highest with a rating of 10, indicating the highest level of involvement, while Group 2 scores the lowest with a 3. In terms of Perception of Urgency, which measures how urgent or important the subgroups perceive the situation, Group 4 again scores the highest with a rating of 10, while Groups 1 and 3 receive a lower score of 3.

Table 2 Table with updated first column and groupings Criteria: Direct Involvement, Perception of Urgency, Influence and Property, Democratic Power

Group/ Coalition	Crit. 1	Crit. 2	Crit. 3	Crit. 4	Weighted Sum	Normalized Weight
Group 1	7	3	6	5	5.25	22.34%
Group 2	3	3	2	9	4.25	18.09%
Group 3	7	3	6	5	5.25	22.34%
Group 4	10	10	8	7	8.75	37.23%
Gr1-2	12.0	6.0	10.0	16.0	11.00	13.79%
Gr1-3	14.0	6.0	12.0	10.0	10.50	13.17%
Gr1-4	18.5	16.5	15.0	13.0	15.75	19.75%
Gr2-3	12.0	6.0	10.0	16.0	11.00	13.79%
Gr2-4	16.5	16.5	13.0	17.0	15.75	19.75%
Gr3-4	18.5	16.5	15.0	13.0	15.75	19.75%
Gr1-2-3	12.7	6.0	10.7	15.3	11.17	19.59%
Gr1-2-4	16.7	15.3	13.3	16.0	15.33	26.90%
Gr1-3-4	18.0	15.3	14.7	12.7	15.17	26.61%
Gr2-3-4	16.7	15.3	13.3	16.0	15.33	26.90%

In the Influence and Property criterion, which assesses the level of influence or ownership each subgroup holds, Group 4 is rated the highest with an 8, while Group 2 has the lowest score of 2. Finally, the Democratic Power criterion reflects the democratic values and power represented by each subgroup. Group 2 leads in this criterion with a rating of 9, while Groups 1 and 3 are rated at 5.

These ratings provide a clear picture of how the subgroups differ in their engagement, urgency, influence, and democratic power, offering valuable insights for further analysis. The four evaluated criteria are summarized and then normalized, resulting in a value between [0, 1] as the weight for each group (across four groups).

In this scenario, 16 possible coalitions can be formed, ranging from the empty set to a coalition involving all four groups. The individual ratings of the groups are already presented above. To evaluate the coalitions, the sum of maximum and mean values is created:

$$g^j(\boldsymbol{x}^j) = \max(\boldsymbol{x}^j) + \frac{1}{k} \sum_{x_i^j \in \boldsymbol{x}^j} x_i^j \tag{5}$$

where $\boldsymbol{x}^j = \{x_1^j, x_2^j, \ldots, x_G^j\}$ represents the rating of the coalition in terms of the criterion j, consisting of G participating groups.

If the maximum was used without the mean, then it would not matter what are the ratings of other groups in the coalition accept of the one with the maximum value (e.g., (10,10,10) and (10,1,1) would lead to the same evaluation). On the other hand, the maximum part ensures that coalitions, where each criterion is evaluated with high rating, are preferred over coalitions with medium rating across the criteria.

The final values of coalition weights were calculated using the weighted sum of $g^j(\boldsymbol{x}^j)$ values and weights of 4 considered criteria (i.e. 0.25 for all of them), see the column 'Weighted Sum' in Table 2.

It is important to note that all ratings are always normalized to [0, 1], ensuring that the sum of all individual coalition ratings ultimately equals 1 (see the column 'Normalized Weight'). This guarantees that the ratings of the individual coalitions are aggregated in a consistent and comparable manner, without any group or coalition receiving disproportionate weight.

5.3 Three-level aggregation

In this section, the results of aggregation at all three levels are provided one by one.

5.3.1 First-level aggregation

The results of the first level of aggregation can be seen in Table 3. The first column indicates which of the 13 alternatives each individual voting row corresponds to. The second and third columns (*S* and *1-R*) display the effective slider settings. These are followed by user-related data, such as the user ID and their subgroup affiliation. The λ column represents the selection used for the first aggregation, while the last column shows the result. The table presents selected values corresponding to alternatives 1, 3, and 9. It is intended as an excerpt to provide illustrative insights into

Table 3 Result of the First-level aggregation for selected voters

Alt.	Support (x)	Rejection (y)	Voter	Group	λ	$A_\lambda(x, y)$ (1)
1	0.643	0.958	1	Group 4	0.25	0.900
3	0.403	0.657	1	Group 4	0.50	0.530
9	0.125	0.109	1	Group 4	0.75	0.059
1	0.627	1.000	2	Group 3	0.25	0.907
3	0.220	0.435	2	Group 3	0.75	0.164
9	0.280	0.629	2	Group 3	0.50	0.455
1	0.234	0.329	3	Group 2	0.75	0.141
3	0.559	0.538	3	Group 2	0.25	0.774
9	0.864	0.769	3	Group 2	0.25	0.908
1	0.199	0.213	4	Group 4	0.75	0.103
3	0.317	0.412	4	Group 4	0.75	0.182
9	0.236	0.191	4	Group 4	0.75	0.107
1	0.491	0.542	5	Group 3	0.50	0.517
3	0.491	0.517	5	Group 3	0.50	0.504
9	0.450	0.520	5	Group 3	0.50	0.485
1	0.998	1.000	6	Group 4	0.25	1.000
..	..	..	..	..	..	..

Table 4 Results of the Second-Level Aggregation by arithmetic mean (2)

Alt	Group 1	Group 2	Group 3	Group 4
1	0.045	0.571	0.665	0.738
2	0.552	0.318	0.479	0.248
3	0.614	0.812	0.711	0.766
4	0.439	0.178	0.333	0.205
5	0.685	0.454	0.212	0.416
6	0.424	0.224	0.052	0.162
7	0.718	0.473	0.456	0.449
8	0.794	0.532	0.668	0.691
9	0.895	0.454	0.403	0.190
10	0.747	0.425	0.646	0.460
11	0.546	0.288	0.351	0.415
12	0.699	0.428	0.546	0.515
13	0.711	0.415	0.634	0.571

Group sizes: $n_1 = 16$, $n_2 = 27$, $n_3 = 14$, $n_4 = 25$

the data, rather than a complete representation. The complete data set is available online.[1]

5.3.2 Results of the second-level aggregation

The results of the second-level aggregation can be found in Table 4. The values represent the arithmetic mean of the first-level aggregation results for each subgroup and alternative (2) in Table 3. The data shown in the table reflect the mean performance per subgroup (Group 1, Group 2, Group 3, and Group 4) across various alternatives (Alt). Each cell in the table contains the arithmetic mean value for the respective group and alternative.

Sizes (n) for each group are as follows: Group 1: $n_1 = 16$, Group 2: $n_2 = 27$, Group 3: $n_3 = 14$ and Group 4: $n_4 = 25$ (i.e. 82 in total). These values represent the number of voters in each subgroup. It is important to note that the sample size is relatively small and should be considered when interpreting the results.

5.3.3 Results of the third-level aggregation

The ranking of all 13 alternatives, based on the aggregation results using the Choquet integral (Eq. 4), is shown in Table 5. In this case, the best-ranked alternative is Alt. no. 3, followed by Alt. No. 8. The corresponding Choquet integral values are displayed in the last rows (3LA).

[1] www.unterdorfstr.ch.

Table 5 Results of aggregation at third level by (4) for different alternatives

#Rank	1	2	3	4	5	6	7
Alt. No.	3	8	7	12	13	10	11
3LA	0.658	0.595	0.507	0.489	0.482	0.481	0.346

#Rank	8	9	10	11	12	13
Alt. No.	9	5	2	1	4	6
3LA	0.336	0.323	0.299	0.233	0.226	0.134

5.4 Sensitivity analysis by Monte Carlo simulation

Two subjective numerical inputs have been added within the methodology into the model. Namely, the ratings of groups in terms of defined criteria (using 1 to 10 point scale), and weights of these criteria. Both have been used to calculate the weights of groups for aggregation. As written in the previous sections, it is reasonable to explore to what extent these expertly set inputs impact the results. Moreover, when looking at the resulting values of the alternatives, there are very small differences between some positions. This signalizes that the obtained ranking is potentially not so robust. For all these reasons, the sensitivity analysis is performed under three different scenarios:

- **Scenario 1:** The model has been run for changed ratings of groups x_i^j. In particular, the original values have been changed by up to ±2 points (i.e., still bounded on the original ten-point scale, thus still considering only integers). All feasible values could occur with the same probability. This scenario treats the fact that it can be uneasy to choose the most suitable value on a qualitative scale.
- **Scenario 2:** Here, as well as in Scenario 1, values x_i^j are changed. All ratings are uniformly distributed while keeping the same order of the values across groups for the given criterion j. For example, if the original values were (2, 8, 5, 1), this scenario can work with (4, 10, 8, 3).
- **Scenario 3:** This scenario, unlike the previous two, considers different weights of criteria for evaluation the weights of groups while keeping the ratings x_i^j untouched.

The results of the sensitivity analyses for all three scenarios can be found in Fig. 3. In all three cases, the ranking is quite robust. For Scenario 1, only four pairs of alternatives can possibly switch their position: 1 with 4 (34% of cases), 5 with 11 (24% of cases), and 10 with 13 (34% of cases), and 5 with 9 (<1% of cases). For Scenario 2, the ranking is the most volatile. Only the first three positions (Alternatives 3, 8 and 7) are absolutely stable. Alternative 5 can be ranked at the highest number of positions – 4 different positions (from the 7th to the 10th position). On the other hand, the original ranking is preserved in at least 90% for 7 out of 13 alternatives. The last scenario, Scenario 3, provided the most robust results. Only alternatives 9 and 11 switched their positions in less than 1% of cases. To conclude, despite quite large number of subjectively set inputs, the results turned out to be very stable, and thus reliable.

6 Discussion

This section discusses the results, limitations, answering research questions and speculating future research topics.

Rank/	Scenario 1												
Alternative	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th
1											66.0%	34.0%	
2										100%			
3	100%												
4											64.0%	66.0%	
5							24.0%	75.0%	1.0%				
6													100%
7			100%										
8		100%											
9								1.0%	99.0%				
10					66.0%	34.0%							
11							76.0%	24.0%					
12				100%									
13					34.0%	66.0%							
Rank/	Scenario 2												
Alternative	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th
1											37.4%	62.5%	0.1%
2								1.0%	4.4%	94.6%			
3	100%												
4											62.6%	37.4%	
5							53.0%	43.1%	2.9%	1.0%			
6												0.1%	100%
7			100%										
8		100%											
9								2.9%	92.7%	4.4%			
10				7.4%	77.8%	14.8%							
11							47.0%	53.0%					
12				91.7%	6.4%	1.9%							
13				0.9%	15.8%	83.3%							
Rank/	Scenario 3												
Alternative	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th
1											100%		
2										100%			
3	100%												
4												100%	
5								99.1%	0.9%				
6													100%
7			100%										
8		100%											
9								0.9%	99.1%				
10						99.4%	0.6%						
11						0.6%	99.4%						
12				100%									
13					100%								

Fig. 3 Sensitivity Analysis Results from three scenarios (grey background indicates the original ranking without simulation)

The proposed mixed logic aggregation function (1) emphasizes a stronger inclination to support or reject less reluctant voters, while attenuating the inclination of more hesitant voters. Generally, the referendum voting system (i.e., expressing opinions on one or several alternatives) relies on a higher number of active voters, which might prefer as simple as possible voting, but allowing them to express their inclinations or uncertainty. Thus, we proposed sliders that allow voters to express their support and rejection. This answers the first research question and raises topics for future research, such as whether other mixed logic aggregation functions might be applied. A different perspective is supported by the intuitionistic fuzzy sets, which cover intensity of belonging, intensity of non-belonging, and intensity of indecisiveness. Intuitionistic fuzzy sets should satisfy the property (Atanassov 2012):

$$0 \leq \mu_A(x) + \upsilon_A(x) + \pi_A(x) \leq 1 \tag{6}$$

where $\mu_A(x)$ is the intensity of belonging to the concept A (supporting alternative), $\upsilon_A(x)$ is the intensity of non/belonging to the concept A (rejecting alternative) and $\pi_A(x)$ is the intensity of indecisiveness of belonging to the concept A (neither supporting nor rejecting).

However, the intuitionistic fuzzy sets approach requires the transformation of answers to meet this requirement (Marasini et al. 2016) and further complex computations, but a benefit could balance them. Further research in this field is welcome.

New road construction, or any other construction, is known to influence different subgroups of citizens differently (Emmenegger et al. 2023). Hence, a robust voting model supported by assigning weights to subgroups is desirable. However, it is a hard task to assign precise weights to each subgroup. The geographical distance and structure of the subgroup support this assignment. Then, coalitions of subgroups should also be included. For this reason, fuzzy measures and their computation using Choquet integral were adopted. Even though it is able to cover increased, decreased, and neutral influences of coalitions, particular weights for each subgroup might change within these intervals and therefore influence final ranking of alternatives. To explore this problem, we applied a Monte Carlo simulation. The results have shown that even the expert evaluation of profiles of voters groups together can provide highly robust and reliable answers. We emphasize that a sensitivity analysis should be performed within each application of the proposed model.

The analysis of results shows that the proposed method, utilizing coalitions and a three-level aggregation model, provides a clear and robust solution for addressing complex decision-making processes in groups with differing preferences. For the specific Traffic Strategy Case Study in Street of Unterdorf in Geuensee, Lucerne (Switzerland), this method demonstrates its ability to achieve an acceptable decision, which was implemented as the first-ranked alternative was pursued in reality. The results indicate that even when the weights and the evaluation of the criteria differ, the outcomes remain highly robust. This robustness can be attributed to the structure of the aggregation model: The first level enforces consistent voting patterns, while the second and third levels maintain logical and comprehensible aggregation, ensuring consistency throughout the process. This answers the second research question.

This model has shown its benefits for searching the most suitable alternative to road construction in the municipality of Geuensee, together with the complete ranking prioritizing the alternatives (Emmenegger et al. 2023). It can be considered beneficial that the application has been made in Switzerland, – a country with a long tradition of direct voting to variety of questions and the recent results in (Schwarz et al. 2024), where voters welcomed the initial basic principle of flexible or fuzzy voting. Thus, it is advisable to examine the application in other countries to evaluate the universality of the proposed model.

For the future work it is valuable to explain the interactions and effects between the data of interest (in our case votes and decisions) with the environmental observations (Weber et al. 2010). These relations also exhibit data uncertainty and vagueness.

A recognized limitation of the study is a small sample of voters per group, which hindered statistical significance and generalizability, though representativeness was not its focus. The total number of respondents is mentioned in Section 5.3.2. The findings demonstrated the applicability of the proposed model on a small-scale problem (citizens in a city), and the results can be seen as preliminary basis for future large-scale applications.

7 Conclusion

When collecting votes from various voter subgroups, we face the problems of uncertainty in supporting/rejecting the alternatives, diverse citizens subgroups, and the relevance of the subsets of various citizens subgroups. The answer, proposed in this article, is the three-level model of voting aggregation. At the first level, we proposed the convex combination of Łukasiewicz t-norm and t-conorm to formalize relevance of each answer towards supporting or rejecting alternatives. The second level calculates the aggregated intensity of support with each subgroup by arithmetic mean. Next, we implemented the third level of aggregation using the discrete Choquet integral considering the weights of each subgroup independently and the weights of each possible coalition. Finally, we realized a Monte Carlo simulation to evaluate sensitivity.

The resulting ranking turned out to be sufficiently stable and robust against the changes in the evaluation of the voters' groups (their weights and profiles). Therefore, the winning alternative can be considered reliable enough and easy to justify for local authorities.

The future research direction will also focus on the aggregation of uncertainty in votes when voters might provide intensities of support, rejection, and uncertainty to each alternative. Hence, a research focused on adjusting intuitionistic fuzzy sets is advisable. Next, it is welcome to apply the proposed model in other countries and for other projects to evaluate its robustness and universality.

The proposed model could be useful in financial decision contexts where individuals should weigh factors such as risk aversion or acceptance. For example, when households decide whether to buy investment or invest in financial assets, they are not simply making a binary evaluation, but include subjective perceptions of financial risk (between *full acceptance* and *full rejection*) and the influence of peer groups (Tversky and Kahneman 1991). Thus, the proposed first level of aggregation is suitable for handling these inconsistencies in answering the questionnaire, while the third level handles behavior of peer groups.

Generally, the proposed model is suitable in any field where we expect hesitance in providing answers and face with non-compact sizes of groups or groups affected differently, where simple averaging might oversimplification (Vaníček et al. 2009). For example, when a company or public institution is evaluating the benefits of educational curses or software tools among departments.

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Declarations

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