

IPO PRICE, HETEROGENEOUS PRIORS, AND GRADUAL INFORMATION FLOW

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Abstract

This paper attempts to develop a theoretical framework that builds on heterogeneous beliefs to explain the financial anomalies related to IPO stocks. In particular, we develop a dynamic analysis framework to study the valuation of IPO price and the short-term probability of falling below IPO price based on perspectives of investors' heterogeneous priors and gradual information flow. Our study shows that the valuation of IPO price increases as the degree of heterogeneity due to investors' heterogeneous priors increases. Moreover, the short-term probability of falling below IPO price increases as the degree of investors' cognitive biases caused by gradual information flow increases.

Keywords: heterogeneous priors, gradual information flow, IPO price, probability of falling below IPO price

JEL Classification: G02, G14

1. Introduction

Traditional asset pricing theory states that a stock's return is determined by measures of its riskiness, such as its beta, in an efficient market. However, a lot of evidence has been found that there exist financial anomalies related to stock returns and these anomalies are not able to be explained by traditional asset pricing theory. Many scholars have tried to make technical modifications to the traditional asset pricing theory, but these effects are not satisfactory for explanations of such anomalies (Shiller, 2003). More and more academic researchers start to challenge the basic assumption of homogeneous beliefs in the traditional asset pricing theory and the idea of heterogeneous beliefs has been introduced to study financial anomalies.

An early work by Miller (1977) points out that in the presence of heterogeneous beliefs and short-sales constraint, the valuations of optimistic investors will be reflected in a stock price because they can buy and hold stocks, but the valuations of pessimistic investors will not realize due to the short-sales constraint. Thus, the stock will be overpriced in short term, while the stock price will tend to the true value in long run as beliefs of two types of investors gradually converge with the flow of market information. Miller's model is based on qualitative analysis and Jarrow (1980) develops a theoretical framework for Miller's model.

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The innovative work of Miller (1977) applies investors' heterogeneous beliefs to explain abnormal stock returns, providing a new idea for interpreting financial anomalies. The model of Miller (1977) is static, that is, the initial level of investors' heterogeneous beliefs is given and does not change before the stock liquidates. Hong and Stein (2007) point out that heterogeneous beliefs are originated from investors' heterogeneous priors due to differences in personal experience, educational background, age, occupation, gender, *etc.*, however, investors can continually update their valuations based on their personal interpretations of incoming news, that is, heterogeneous beliefs can be also derived from gradual information flow. Regarding this, dynamic models that allow investors' heterogeneous beliefs to be updated are better suited to study financial anomalies than static models. The dynamic model was initiated by Harrison and Kreps (1978) and other contributions include Morris (1996), Scheinkman and Xiong (2003), Hong *et al.* (2006), David (2008), Gallmeyer and Hollifield (2008), Tsyrennikov (2012), *etc.* Hong and Stein (2007) and Xiong (2013) provide surveys of the research work of heterogeneous belief. Hong and Stein (2007) summarize that mechanisms that can generate heterogeneous beliefs are heterogeneous priors, gradual information flow and limited attention.

In this paper, we attempt to develop a theoretical framework that builds on heterogeneous beliefs to explain the financial anomalies related to IPO stocks, the overvaluation of a stock IPO price and the high probability of falling below IPO price in short term. Empirical studies have already found the evidence for the relationship between the overvaluation of IPOs and heterogeneous beliefs (*e.g.* Douge *et al.*, 2001; Cornelli *et al.*, 2006; Gao *et al.*, 2006; Dorn, 2009), however, few theoretical studies focus on this issue. Xiong (2013) points out that it is reasonable and natural for investors to have heterogeneous priors about IPOs because they do not have any useful information to form any beliefs, and investors can gradually update their beliefs about the stocks according to the steam of market information over time. Regarding this, our theoretical framework is based on the perspectives of heterogeneous priors and gradual information flow.

The contributions of this paper are threefold to the best of our knowledge. First, from the perspectives of heterogeneous priors and gradual information flow, we derive a theoretical analysis framework for IPO price and short-term probability of falling below IPO price. Second, it is the first approach to theoretically show that the expectation of IPO price increases as the degree of heterogeneity due to investors' heterogeneous priors increases. Third, it is the first work to theoretically prove that IPO stock's short-term probability of falling below IPO price increases as investors' cognitive biases caused by gradual information flow increases.

The remainder of this paper is organized as follows. Section 2 establishes the time setting of our study. Section 3 and 4 develop the theoretical framework for the valuation of IPO price and probability of falling below IPO price, respectively. Section 5 concludes.

2. Time Setting of the Study

Figure 1 indicates the time setting of our theoretical framework. Assume at time $T - 2$, a company's IPO is approved by the regulatory commission and investors have an initial valuation of the company's intrinsic value based on the existing information. During the period of $(T - 2, T - 1)$, the company gradually discloses detailed information related to the company through road show. Then at time $T - 1$, investors revise their valuations. During the period of $(T - 1, T)$, orders from investors are received and then the issuer decides

on the IPO price at time T . After time T , the stock is traded publicly on the stock market. Because investors' heterogeneous priors will affect their valuations of the company's intrinsic value and thus affect the IPO price of the stock, our study on the valuation of IPO price covers the period from $T - 2$ through T .

Figure 1 | Time Setting of the Study



Source: Authors.

It is noted that investors will form their private signals according to external and internal information shocks when the IPO price is determined at time T . During the period of $(T, T+1)$, private signals will be disclosed gradually. Investors will adjust their valuations based on received signals, which thus affects the trading price of the stock. We study the probability of falling below the IPO price at time $T+1$ due to investors' heterogeneous priors and gradual information flow.

3. IPO Price

The IPO price of a stock should not only reflect the company's net asset value and future growth (*i.e.* the intrinsic value of the company) but also be accepted by investors. Thus, the IPO price (S_{IPO}) of the stock is considered to be composed of company's intrinsic value (V) and investors' valuation (S_I) of company's intrinsic value as follows

$$S_{IPO} = \alpha V + (1 - \alpha) S_I, \quad (1)$$

where $0 \leq \alpha \leq 1$. In an IPO process, the IPO price is determined by the issuer. Because the issuer is a rationally economic man, the IPO pricing follows the principle of expected utility maximization. Assume the utility function of the issuer is constant absolute risk aversion (CARA):

$$U(W) = -\exp(-W/\eta), \quad (2)$$

Where W is the amount of capital raised from the IPO and η is the risk-bearing capacity of the issuer. The corresponding expected utility function is

$$E[U(W)] = -\exp\left\{-\frac{1}{\eta}\left[E(W) - \frac{1}{2\eta}Var(W)\right]\right\}. \quad (3)$$

To maximize the expected utility function, it is equivalent to maximize $E(W) - \frac{1}{2\eta}Var(W)$. If the number of shares issued is Q , then

$$W = S_{IPO} \cdot Q = [\alpha V + (1 - \alpha) S_I] Q \quad (4)$$

and

$$E(W) - \frac{1}{2\eta}Var(W) = \alpha QV + (1 - \alpha)QE(S_I) - \frac{1}{2\eta}(1 - \alpha)^2 Q^2 Var(S_I). \quad (5)$$

The first order condition of (5) is

$$\frac{\partial \left[E(W) - \frac{1}{2\eta} \text{Var}(W) \right]}{\partial \alpha} = QV - QE(S_I) + \frac{1}{\eta} (1 - \alpha) Q^2 \text{Var}(S_I) = 0, \quad (6)$$

which yields

$$\alpha = 1 + \frac{\eta}{Q\text{Var}(S_I)} [V - E(S_I)]. \quad (7)$$

Combining (1) and (7), we have

$$S_{IPO} = V + \frac{\eta}{Q\text{Var}(S_I)} [E(S_I) - V] (S_I - V), \quad (8)$$

and the expectation of IPO price is

$$E(S_{IPO}) = V + \frac{\eta}{Q\text{Var}(S_I)} [E(S_I) - V]^2. \quad (9)$$

Now, we consider the heterogeneous beliefs in investors' valuation (S_I) of company's intrinsic value. Referring to Hong *et al.* (2006), we categorize investors into two types according to investors' sentiment. The two types are pessimistic investors (type A investor) and optimistic investors (type B investor). Due to investors' heterogeneous priors, investors have different prior valuations of company's intrinsic value. Denote by $\tilde{V}_{T-2}^A$ the type A investor's valuation and $\tilde{V}_{T-2}^B$ the type B investor's valuation at time $T-2$. We assume that $\tilde{V}_{T-2}^A \sim N(V - \delta', 1/\rho)$ and $\tilde{V}_{T-2}^B \sim N(V + \delta'', 1/\rho)$, where $V - \delta'$ ($\delta' > 0$) is the expectation of pessimistic investors' valuation $V + \delta$ ($\delta > 0$), is the expectation of optimistic investors' valuation, and $1/\rho$ represents the degree of dispersion in investors' valuations¹. Since information related to the company is disclosed through road show, investors modify their valuations at time $T-1$. Without loss of generality, let's assume investors receive positive information about the company², valuations of pessimistic investors and optimistic investors then change into $\tilde{V}_{T-1}^A \sim N(V, 1/\rho)$ and $\tilde{V}_{T-1}^B \sim N(V + \delta'', 1/\rho)$ ³, respectively, where $\delta'' > 0$ is the difference between the expectations of investors' valuations, reflecting the degree of heterogeneity due to investors' heterogeneous priors. Then, investors' valuation of company's intrinsic value S_I at time $T-1$ is determined by

$$S_I = \beta \tilde{V}_{T-1}^A + (1 - \beta) \tilde{V}_{T-1}^B, \quad (10)$$

where β ($0 \leq \beta \leq 1$) and $1 - \beta$ are the weights. Since there is no information exchange between two types of investors $\tilde{V}_{T-1}^A$, and $\tilde{V}_{T-1}^B$ are considered to be mutually independent. Substituting (10) into (9), we have

1 Similar to Hong *et al.* (2006), we assume that valuations of both types of investors have the same degree of dispersion.

2 For the case that negative information is received, the following analysis could be applied similarly.

3 For the simplicity of derivations, we assume that the variance of investor's valuations does not change and this assumption will not affect the conclusion of the following analysis.

$$\begin{aligned}
 E(S_{IPO}) &= V + \frac{\eta}{QVar(\beta\tilde{V}_{T-1}^A + (1-\beta)\tilde{V}_{T-1}^B)} \left[\beta E(\tilde{V}_{T-1}^A) + (1-\beta)E(\tilde{V}_{T-1}^B) - V \right]^2 \\
 &= V + \frac{\rho\eta}{Q[\beta^2 + (1-\beta)^2]} \beta^2 (\delta'')^2.
 \end{aligned} \tag{11}$$

Differentiating $E(S_{IPO})$ with respect to δ'' yields

$$\frac{\partial E(S_{IPO})}{\partial \delta''} = \frac{2\beta^2 \rho\eta}{Q[\beta^2 + (1-\beta)^2]} \delta'' > 0. \tag{12}$$

Therefore, (12) indicates that the expectation of IPO price increases as the degree of investors' heterogeneous priors increases.

4. Probability of Falling below IPO Price

According to the disclosure of company's information during the IPO process and the impact of market information, investors modify their valuations of company's intrinsic value, which forms private signals for both types of investors accordingly. As the private signals are gradually disclosed after the stock is publicly traded, one type of investor can further modify their valuations according to the signals received from the other type of investor. However, as information flows gradually, it is possible that one type of investor may or may not observe the modification made by the other type of investor, which then leads to investors' cognitive biases and eventually results in fluctuations of stock trading price. In this section, we focus on this situation and study the IPO stock short-term probability of falling below IPO price due to investors' heterogeneous beliefs.

Following the analysis framework in Section 3, the valuation of pessimistic investors (type A investor) changes from $\tilde{V}_{T-2}^A \sim N(V - \delta', 1/\rho)$ into $\tilde{V}_{T-1}^A \sim N(V, 1/\rho)$ and the valuation of optimistic investors (type B investor) changes from $\tilde{V}_{T-2}^B \sim N(V + \delta, 1/\rho)$ into $\tilde{V}_{T-1}^B \sim N(V + \delta'', 1/\rho)$ at time $T - 1$ as investors receive positive information about the company. Changes in investors' valuations are usually reflected by their behaviours, such as cash flows, price inquires, responses to road show, *etc.* However, due to the information asymmetry and low efficiency of information transfer, behaviours of one type of investor may or may not be noticed by the other type of investor, which then results in the situation that one type of investor notices the change in the other type of investor's valuation but the other type of investor does not. Without loss of generality, we assume that type A investor notices type B investor's change in valuation, while type B investor does not notices type A investor's change⁴, that is to say, type B investor still believes that type A investor's valuation at time $T - 1$ is $\tilde{V}_{T-1}^A \sim N(V - \delta', 1/\rho)$.

At time T , type A and B investors form their private signals S^A and S^B , respectively, and the private signals are structured as $S^i = \tilde{V}_{T-1}^i + \varepsilon^i$ and $\varepsilon^i \sim N(0, 1/\gamma)$ ($i = A, B$), where ε^i and $\tilde{V}_{T-1}^i$ are mutually independent. According to its own private signal, type A and B investors modify their valuations to $\hat{V}_T^i \sim N(\hat{f}_T^i, 1/\tau_0)$ ($i = A, B$). Following Hong (2006)'s work, the means of valuations are

4 For the case of opposite situation, the following analysis could be applied similarly.

$$\begin{aligned}\hat{f}_T^A &= \frac{\rho}{\tau_0} V + \frac{\gamma}{\tau_0} S^A \\ \hat{f}_T^B &= \frac{\rho}{\tau_0} (V + \delta'') + \frac{\gamma}{\tau_0} S^B\end{aligned}\quad (13)$$

and $\tau_0 = \rho + \gamma$. During the period $(T, T+1)$, private signals are gradually disclosed. According to the private signal of the other type, type A and B investors further modify their valuations to $\tilde{V}_{T+1}^i \sim N(\hat{f}_{T+1}^i, 1/\tau_1)$ ($i = A, B$) at time $T+1$, where

$$\begin{aligned}\hat{f}_{T+1}^A &= \frac{\rho}{\tau_1} V + \frac{\gamma}{\tau_1} (S^A + S^B) \\ \hat{f}_{T+1}^B &= \frac{\rho}{\tau_1} (V + \delta'') + \frac{\gamma}{\tau_1} (S^A + S^B)\end{aligned}\quad (14)$$

and $\tau_1 = \rho + 2\gamma$.

Let h_t denote the degree of heterogeneous beliefs in the valuations of type A and B investors at time $T+t$. From (13) and (14), we have $h_0 = \hat{f}_T^B - \hat{f}_T^A = \frac{\rho}{\tau_0} \delta''$ and $h_1 = \hat{f}_{T+1}^B - \hat{f}_{T+1}^A = \frac{\rho}{\tau_1} \delta''$.

This is also type A investor's understanding of heterogeneous beliefs since type A investor noticed type B investor's modification of valuation at time $T-1$. However, since type B investor did not notice type A investor's modification at time $T-1$ and still believes that $E(\tilde{V}_{T-1}^A) = V - \delta'$ instead of V , thus type B investor's understanding of heterogeneous

beliefs is $h_0^B = \frac{\rho}{\tau_0} (\delta' + \delta'')$ and $h_1^B = \frac{\rho}{\tau_1} (\delta' + \delta'')$. This means that there exists cognitive bias between two types of investors where δ' reflects the degree of cognitive bias between

two types of investors, and such cognitive bias eventually affects the stock price.

Assume that the amount of shares that type A and B investors obtain in the primary market is $(1-k)Q$ and kQ ($0 < k < 1$), respectively. According to the market clearing conditions as well as the maximization of investors' expected utility function⁵, the valuations of the equilibrium stock price at time T for type A and B investors are

$$P_{T+1}^A = \begin{cases} \hat{f}_{T+1}^A - \frac{Q}{2\eta\tau_1} + \frac{1}{2}h_1, & 0 \leq h_1 \leq \frac{Q}{\eta\tau_1} \\ \hat{f}_{T+1}^A - \frac{Q}{\eta\tau_1} + h_1, & h_1 > \frac{Q}{\eta\tau_1} \end{cases}\quad (15)$$

and

$$P_{T+1}^B = \begin{cases} \hat{f}_{T+1}^A - \frac{Q}{2\eta\tau_1} + \frac{1}{2}h_1^B, & 0 \leq h_1^B \leq \frac{Q}{\eta\tau_1} \\ \hat{f}_{T+1}^A - \frac{Q}{\eta\tau_1} + h_1^B, & h_1^B > \frac{Q}{\eta\tau_1} \end{cases}.\quad (16)$$

Since type A investor has noticed type B investor's change in valuation at time $T-1$, type A investor's valuation of the equilibrium stock price (P_{T+1}^A) reflects the real stock price (P_{T+1}) at time $T+1$, i.e. $P_{T+1} = P_{T+1}^A$. Since type B investor did not notice type A investor's

5 The utility function of investors is CARA: $U(W) = -\exp(-W/\eta)$.

change in valuation at time $T - 1$, type B investor's valuation of the equilibrium stock price (P_{T+1}^B) is biased from P_{T+1} .

Let P_T be the IPO price of the stock. Now, we evaluate P_T in three cases according to the values of h_1 and h_1^B and then investigate $\Pr(P_{T+1} < P_T)$, the probability of falling below IPO price at time $T + 1$.

Case 1: $0 \leq h_1 \leq h_1^B \leq \frac{Q}{\eta\tau_1}$, i.e. $0 \leq \delta' \leq \delta'' \leq \frac{Q}{\eta\rho}$
In this case,

$$P_{T+1} = P_{T+1}^A = \hat{f}_{T+1}^A - \frac{Q}{2\eta\tau_1} + \frac{\rho}{2\tau_1}\delta'' \quad (17)$$

and $P_{T+1}^B = \hat{f}_{T+1}^A - \frac{Q}{2\eta\tau_1} + \frac{\rho}{2\tau_1}(\delta' + \delta'')$. Since private signals are not disclosed at time T ,

type A and B investors have to estimate the other type investor's private signal based on their own beliefs. Hence, type A investor's estimation of type B investor's private signal at time T is $\tilde{S}_B = \tilde{V}_T^A + \varepsilon^A$ and the corresponding expectation and variance are

$$E(\tilde{S}_B) = \hat{f}_T^A = \frac{\rho}{\tau_0}V + \frac{\gamma}{\tau_0}S^A \text{ and } Var(\tilde{S}_B) = \frac{1}{\tau_0} + \frac{1}{\gamma}.$$

Similarly, type B investor's estimation of type A investor's private signal is $\tilde{S}_A = \tilde{V}_T^B + \varepsilon^B$ and $E(\tilde{S}_A) = \hat{f}_T^B = \frac{\rho}{\tau_0}(V + \delta'') + \frac{\gamma}{\tau_0}S^B$

and $Var(\tilde{S}_A) = \frac{1}{\tau_0} + \frac{1}{\gamma}$. Accordingly, the expectation and variance of type A and B investors' estimations on P_{T+1} at time T are

$$\begin{aligned} E_T^A[P_{T+1}] &= V + \frac{\gamma}{\tau_0}\varepsilon^A - \frac{Q}{2\eta\tau_1} + \frac{\rho}{2\tau_1}\delta'' \\ E_T^B[P_{T+1}] &= V + \frac{\gamma}{\tau_0}\varepsilon^B - \frac{Q}{2\eta\tau_1} + \frac{\rho}{2\tau_1}(\delta' + \delta'') + \frac{\gamma(\rho + \tau_1)}{\tau_0\tau_1}\delta'' \end{aligned} \quad (18)$$

and

$$Var_T^A[P_{T+1}] = Var_T^B[P_{T+1}] = \frac{\gamma}{\tau_0\tau_1}. \quad (19)$$

The market clearing conditions and the maximization of investors' expected utility yield

$$\begin{cases} \frac{\eta(E_T^B[P_{T+1}] - P_T)}{Var_T^B[P_{T+1}]} = kQ \\ \frac{\eta(E_T^A[P_{T+1}] - P_T)}{Var_T^A[P_{T+1}]} = (1-k)Q \end{cases} \quad (20)$$

Substituting (18) and (19) into (20), we get

$$P_T = E_T^B[P_{T+1}] - \frac{k\gamma Q}{\eta\tau_0\tau_1} = V + \frac{\rho}{2\tau_1}(\delta' + \delta'') - \left(\frac{Q}{2\eta\tau_1} + \frac{k\gamma Q}{\eta\tau_0\tau_1}\right) + \frac{\gamma}{\tau_0}\varepsilon^B + \frac{\gamma(\rho + \tau_1)}{\tau_0\tau_1}\delta''. \quad (21)$$

Combining (17) and (21) yields

$$P_{T+1} - P_T = -\frac{\rho}{2\tau_1}\delta' + \frac{\gamma}{\tau_1}\varepsilon^A + \frac{\gamma^2}{\tau_0\tau_1}\varepsilon^B + \frac{k\gamma Q}{\eta\tau_0\tau_1} - \frac{\gamma(\rho + \tau_1)}{\tau_0\tau_1}\delta'' . \quad (22)$$

Since $\varepsilon^i \sim N(0, 1/\gamma)$ ($i = A, B$), we obtain the probability of falling below IPO price at time $T + 1$

$$\Pr(P_{T+1} < P_T) = \Phi\left(\frac{\eta\rho\tau_0\delta' - 2k\gamma Q + 2\eta\gamma(\rho + \tau_1)\delta''}{2\eta\gamma\sqrt{\gamma^2 + \tau_0^2}}\right) \quad (23)$$

and

$$\frac{\partial \Pr(P_{T+1} < P_T)}{\partial \delta'} = \Phi'\left(\frac{\eta\rho\tau_0\delta' - 2k\gamma Q + 2\eta\gamma(\rho + \tau_1)\delta''}{2\eta\gamma\sqrt{\gamma^2 + \tau_0^2}}\right) \cdot \frac{\rho\tau_0}{2\gamma\sqrt{\gamma^2 + \tau_0^2}} > 0 . \quad (24)$$

Case 2: $0 \leq h_1 \leq \frac{Q}{\eta\tau_1} < h_1^B$, i.e. $0 \leq \delta'' \leq \frac{Q}{\eta\rho} < \delta' + \delta''$

Following similar analysis as described in Case 1, it can be shown that

$$\Pr(P_{T+1} < P_T) = \Phi\left(\frac{2\eta\rho\tau_0\delta' - Q\tau_0 - 2k\gamma Q + 2\eta\gamma(\rho + \tau_1)\delta''}{2\eta\gamma\sqrt{\gamma^2 + \tau_0^2}}\right) \quad (25)$$

and

$$\frac{\partial \Pr(P_{T+1} < P_T)}{\partial \delta'} = \Phi'\left(\frac{2\eta\rho\tau_0\delta' - Q\tau_0 - 2k\gamma Q + 2\eta\gamma(\rho + \tau_1)\delta''}{2\eta\gamma\sqrt{\gamma^2 + \tau_0^2}}\right) \cdot \frac{\rho\tau_0}{\gamma\sqrt{\gamma^2 + \tau_0^2}} > 0 . \quad (26)$$

Case 3: $0 < \frac{Q}{\eta\tau_1} < h_1 < h_1^B$, i.e. $0 < \frac{Q}{\eta\rho} < \delta'' < \delta' + \delta''$

Similarly, we can show that

$$\Pr(P_{T+1} < P_T) = \Phi\left(\frac{\eta\rho\tau_0\delta' - k\gamma Q + \eta\gamma(\rho + \tau_1)\delta''}{\eta\gamma\sqrt{\gamma^2 + \tau_0^2}}\right) \quad (27)$$

and

$$\frac{\partial \Pr(P_{T+1} < P_T)}{\partial \delta'} = \Phi'\left(\frac{\eta\rho\tau_0\delta' - k\gamma Q + \eta\gamma(\rho + \tau_1)\delta''}{\eta\gamma\sqrt{\gamma^2 + \tau_0^2}}\right) \cdot \frac{\rho\tau_0}{\gamma\sqrt{\gamma^2 + \tau_0^2}} > 0 . \quad (28)$$

Recall that the value of δ' indicates the degree of cognitive bias in type A and B investors' understanding of heterogeneous beliefs. Therefore, (24), (26) and (28) together imply that the probability of falling below IPO price increases as the degree of cognitive bias between investors increases.

If investors receive negative information about the company, then the valuation of pessimistic investors (type A investor) changes from $\tilde{V}_{T-2}^A \sim N(V - \delta', 1/\rho)$ into $\tilde{V}_{T-1}^A \sim N(V - \delta'', 1/\rho)$ and the valuation of optimistic investors (type B investor) changes from $\tilde{V}_{T-2}^B \sim N(V + \delta, 1/\rho)$ into $\tilde{V}_{T-1}^B \sim N(V, 1/\rho)$ at time $T - 1$. Following similar analysis described above, it also can be proved that the probability of falling below IPO price increases as the degree of cognitive bias between investors increases.

5. Conclusion

Many theoretical and empirical studies have shown that financial anomalies in the stock market can be interpreted by investors' heterogeneous beliefs. In this paper, we focus on the anomalies related to IPO stocks. We develop a dynamic analysis framework to study the valuation of IPO price and the probability of falling below IPO price based on perspectives of investors' heterogeneous priors and gradual information flow. We show that (1) in a stock market composed of pessimistic investors and optimistic investors, the expectation of IPO price increases as the heterogeneity of two types of investors' belief on the company's intrinsic value increases; (2) the probability of falling below IPO price increases as the degree of cognitive bias between investors caused by gradual information flow increases.

References

- Cornelli, F., Goldreich, D., Ljungqvist, A. P. (2006). Investor Sentiment and Pre-IPO Markets. *Journal of Finance*, 61(3), 1187–1216, <http://dx.doi.org/10.1111/j.1540-6261.2006.00870.x>
- David, A. (2008). Heterogeneous Beliefs, Speculation, and the Equity Premium. *Journal of Finance*, 63(1), 41–83, <http://dx.doi.org/10.1111/j.1540-6261.2008.01310.x>
- Dorn, D. (2009). Does Sentiment Drive the Retail Demand for IPOs? *Journal of Financial and Quantitative Analysis*, 44(1), 1075–1103, <http://dx.doi.org/10.1017/s0022109009090024>
- Douge, T., Loughram, T., Suchanek, G., Yan, X. (2001). Divergence of Opinion, Uncertainty, and Quality of IPOs. *Financial Management*, 30(4), 5–23, <http://dx.doi.org/10.2307/3666256>
- Gallmeyer, M., Hollifield, B. (2008). An Examination of Heterogeneous Beliefs with a Short Sale Constraint in a Dynamic Economy. *Review of Finance*, 12(2), 323–364, <http://dx.doi.org/10.1093/rof/rfm036>
- Gao, Y., Mao, C., Zhong, R. (2006). Divergence of Opinion and Long-term Performance of Initial Public Offerings. *Journal of Financial Research*, 29(1), 113–129, <http://dx.doi.org/10.1111/j.1475-6803.2006.00169.x>
- Harrison, J. M., Kreps, D. M. (1978). Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations. *Quarterly Journal of Economics*, 92(2), 323–336, <http://dx.doi.org/10.2307/1884166>
- Hong, H., Stein, J. (2007). Disagreement and the Stock Market. *Journal of Economic Perspectives*, 21(2), 109–128, <http://dx.doi.org/10.1257/jep.21.2.109>
- Hong, H., Scheinkman, J., Xiong, W. (2006). Asset Float and Speculative Bubbles. *Journal of Finance*, 61(3), 1073–1117, <http://dx.doi.org/10.1111/j.1540-6261.2006.00867.x>
- Jarrow, R. (1980). Heterogeneous Expectations, Restrictions on Short Sales and Equilibrium Asset Prices. *Journal of Finance*, 35(5), 1105–1113, <http://dx.doi.org/10.2307/2327088>
- Miller, E. M. (1977). Risk, Uncertainty, and Divergence of Opinion. *Journal of Finance*, 32(4), 1151–1168, <http://dx.doi.org/10.1111/j.1540-6261.1977.tb03317.x>
- Morris, S. (1996). Speculative Investor Behavior and Learning. *Quarterly Journal of Economics*, 111(4), 1111–1133, <http://dx.doi.org/10.2307/2946709>
- Scheinkman, J., Xiong, W. (2003). Overconfidence and Speculative Bubbles. *Journal of Political Economy*, 111(6), 1183–1219, <http://dx.doi.org/10.1086/378531>
- Shiller, E. (2003). From Efficient Markets Theory to Behavioral Finance. *Journal of Economic Perspectives*, 17(1), 83–104, <http://dx.doi.org/10.1257/089533003321164967>

- Tsyrennikov, V. (2012). Heterogeneous Beliefs, Wealth Distribution, and Asset Markets with Risk of Default. *American Economic Review*, 102(3), 156–160, <http://dx.doi.org/10.1257/aer.102.3.156>
- Xiong, W. (2013). *Bubbles, Crises, and Heterogeneous Beliefs*. NBER Working Paper No. w18905.