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Systemic Risk in Financial Risk Regulation*

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Abstract

The paper deals with the systemic risk concept which is important in the framework of modern risk regulatory systems in finance and insurance (the most actual examples are Basel III in finance and Solvency II in insurance). Two numerical applications of possible approaches are presented. The first one shows that marginal expected shortfall MES can be a useful risk measure when the systemic risk is examined using the Czech data represented by the composing index PX of Prague Stock Exchange. The second approach based on the common shock can be suitable for risk regulation in insurance.

1. Introduction

The previous financial crises have demonstrated various weaknesses in the global regulatory framework and banks' risk management practices. The paper deals with a special quantitative approach to the systemic risk when portfolio scheme is applied (e.g. particular firms participating in a stock index may present a systemic risk when aggregate capital drops below a given threshold).

The systemic risk seems to be a significant risk in today's financial world (particularly in banks) and is subject of regulation. The Financial Stability Board, i.e., an international body that monitors and makes recommendations about the global financial system, defines so called Systemically Important Financial Institutions (SIFI) as "financial institutions whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity". As Basel III is concerned, which represents an actual regulatory framework for banks (see e.g. BIS (2010)), it introduces for SIFIs special regulations (one uses also acronyms G-SIB for Global Systemically Important Banks or G-SII for Global Systemically Important Institutions, see BIS (2011)) to estimate their need for additional regulation capital and control them for moral hazard due to "Too Big to Fail" (TBTF).

In general practice, the methodologies concerning the systemic risk in the financial sector are frequently based on an indicator measurement approach (see e.g. Cerutti et al (2012), Huang et al (2011), Zheng et al (2012)) and identify factors causing international contagion such as the size of banks, their interconnectedness, the lack of substitutes for their services, their global cross-jurisdictional activity,

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their complexity and others (the group of G-SIB includes e.g. Bank of American, Bank of China, Barclays, BNP Paribas, Citigroup, Credit Suisse, Deutsche Bank, HSBC, ING Bank, JPMorgan Chase, Morgan Stanley, Société Générale, UBS, Unicredit Group, Wells Fargo and others). Moreover, national regulators try to identify national important financial institutions (so called O-SII for Other Systemically Important Institutions).

One can examine various quantitative aspects of systemic risk with many references in research and applied literature and calculation outputs in financial practice. Acharya et al (2010, 2012) develop a simple model in which a group of banks set leverage levels and choose asset positions in a broader economic environment with systemic risk emerging when aggregate bank capital drops below a given threshold (more details are given below). In this model the systemic risk of a firm is the product of three components: (*i*) real social costs of a crisis per dollar of capital shortage, (*ii*) probability of a crisis (i.e., an aggregate capital shortfall) and (*iii*) expected capital shortfall of the firm in a crisis. The model was applied also to European banking system (see Acharya and Stefen (2012)). Billio et al (2012) propose several econometric measures of systemic risk to capture interconnectedness among the monthly returns of hedge funds, banks, brokers and insurance companies; these measures are based on principal components and Granger causality tests, and they show that all four sectors have become highly interrelated over the past decade, increasing the level of systemic risk in the finance and insurance industry. A component expected shortfall approach to systemic risk is suggested in Banulescu and Dumitrescu (2015). Another popular approach to systemic risk is based on the concept of CoVaR which measures what happens to the system's value-at-risk (VaR) when one particular institution is under financial stress, as measured by its own individual VaR (see Adrian and Brunnermeier (2008) with more details given below). The systemic risk measures are frequently backtested (see e.g. Brownlees et al (2015) for backtests during historical bank runs or Guntay and Kupiec (2014)). Moreover, stress tests seem to be further issues of interest in the context of systemic risk (see e.g. Acharya et al (2013), BIS (2015), Boss et al (2006), Brechmann (2013), Canedo and Jaramillo (2009), Henry and Kok (2013)). The analytics of the corresponding systemic risk measures is described in Bisias et al (2012).

This paper concentrates just on the quantitative aspects of systemic risk, in particular, on measuring the systemic risk in a portfolio context. In practice, there are two ways of measuring the contribution of a given firm to the overall risk of the system (see Benoit et al (2013)). The supervisory approach relies on firm-specific information (size, leverage, liquidity, interconnectedness, substitutability and others) and uses data provided by the financial institution to the regulator. The second approach relies on publicly available market data (stock returns, CDS spreads and others) since such data are believed to reflect all information about publicly traded firms. In both cases one must apply a suitable measure of the corresponding systemic risk.

The second part of the paper concerns the insurance sector. Analogously to Basel III, Solvency II (see e.g. CEA (2006), Solvency II (2009)) is a regulatory framework for insurance and reinsurance companies where the capital requirements are quantified according to underlying risks. In the insurance context, the paper addresses the problem of so called common shock since one can look upon it as being a systemic risk which threatens the whole insurance sector. More generally, the common shock uses to be a frequent topic in the whole financial sector, and also in the financial context it can be considered as a special case of situations exposed to the systemic risk (see e.g. Atanasov and Black (2014) or Moreno and Trehan (2000)).

In the insurance context, the paper deals only with a very specific form of systemic risk where the common shock affect all reinsurers of a given insurance company (such a reinsurance portfolio often represents a substantial part of the whole reinsurance market). The typical examples of such common shocks are e.g. the arrival of the financial and economic crisis or recession, a legislative change or reform, a catastrophic event and others. The paper suggests for the described situation a quantitative approach how to calculate solvency capital requirements (SCR) which play the key role in the Pillar 1 of Solvency II.

The paper is organized as follows. Section 2 introduces the concept of MES and discusses the typical model situation when one applies this instrument for systemic risk analysis. Section 3 describes a new method how to deal with systemic risk triggered by common shocks affecting insurance and reinsurance markets. Sections 4 and 5 show applications: in Section 4 one analyses systemic risk by means of stocks of key companies composing the index PX of Prague Stock Exchange while in Section 5 one studies the impact of a common shock affecting reinsurance portfolio. Finally, Section 6 contains conclusions.

2. Marginal expected shortfall

There are three prominent systemic risk measures preferred in practice. To describe them in a simple way we shall use the following model situation (see Benoit et al (2013)). Let's consider N firms and denote r_{it} the return of firm i at time t. The corresponding market return is

$$r_{mt} = \sum_{i=1}^{N} w_{it} r_{it} \ , \tag{2.1}$$

where w_{it} is the relative market capitalization of firm i at time t:

(1) *Marginal Expected Shortfall* (MES), see e.g. Acharya et al (2010), Brownlees and Engle (2012): The MES is based on the well-known concept of the expected shortfall ES (the ES at level α is the expected return in the worst α% of the cases). The expected shortfall is usually preferred among risk measures in today's financial practice (due to its coherence and other properties giving to it preferences e.g. in comparison with the classical value-at-risk VaR, see Artzner et al (1999) or Yamai and Yoshiba (2005)). The modification of ES to the MES means a conditional version of ES, in which the global returns exceed a given market drop C ($C < 0$, i.e.,

MES similarly as ES or VaR or CoVaR are typically negative). Such a modification seems to be productive just when dealing with systemic risk:

$$MES_{it}(C) = \mathrm{E}_{t-1}(r_{it} | r_{mt} < C) \, . \tag{2.2}$$

If the conditional ES of the system is formally defined as

$$ES_{mt}(C) = \mathrm{E}_{t-1}(r_{mt} | r_{mt} < C) = \sum_{i=1}^{N} w_{it} \mathrm{E}_{t-1}(r_{it} | r_{mt} < C) \, , \tag{2.3}$$

then it holds

$$MES_{it}(C) = \frac{\partial ES_{mt}(C)}{\partial w_{it}} \tag{2.4}$$

(one must understand the symbols $MES_{it}(C)$ and $ES_{mt}(C)$ conditionally at time as $MES_{i,t|t-1}(C)$ and $ES_{m,t|t-1}(C)$, i.e., computed at time t given the information available at time $t-1$). Hence the marginal expected shortfall measures the increase in the risk of the system (measured by the ES) induced by a marginal increase in the weight of firm i in the system (the higher the firm MES, the higher the individual contribution of the firm to the risk of the financial system, see Scaillet (2005) or Idier et al (2013)). Due to this natural meaning we make use of the MES also in the case study in Section 4.

(2) *Systemic Risk Measure* (SRISK), see e.g. Acharya et al (2010, 2012): The SRISK extends the MES in order to take into account both the liabilities and the size of the given financial institution. One of possible definitions of SRISK can be formally written as

$$\begin{aligned} SRISK_{it} &= \max\{\, 0; k[D_{it} + (1 - LRMES_{it})W_{it}] - (1 - LRMES_{it})W_{it}\} = \\ &= \max\{\, 0; kD_{it} - (1-k)W_{it}(1 - LRMES_{it})\} \end{aligned} \, , \tag{2.5}$$

where k is the prudential capital ratio, D_{it} is the book value of total liabilities and W_{it} is the market capitalization of market value of equity (the SRISK is obviously an increasing function of the liabilities and a decreasing function of the market capitalization; in any case, it is positive by convention). Finally, the SRISK also considers the interconnection of a firm with the rest of the system through the long-run marginal expected shortfall LRMES. The LRMES corresponds to the expected drop in equity value, the firm would lose, should the market falls more than a given threshold within next six months. Acharya et al (2012) propose to approximate it using the daily MES defined for the threshold C equal to 2%:

$$LRMES_{it} \approx 1 - \exp(18 \cdot MES_{it}) \tag{2.6}$$

(this approximation represents the firm expected loss over a six-month horizon, obtained conditionally on the market falling by more than 40% within the next six months).

(3) *Delta Conditional Value-at-Risk* (ΔCoVaR), see Adrian and Brunnermeier (2008): This risk measure is based on the concept of value-at-risk VaR(α) (the VaR at level α is the maximum loss within the α%-confidence interval). The CoVaR corresponds to the VaR of the market return obtained conditionally on some event $C(r_{it})$ observed for firm i:

$$\mathrm{P}(r_{mt} \leq CoVaR_t^{m|C(r_{it})} | C(r_{it})) = \alpha \ . \tag{2.7}$$

The ΔCoVaR of firm i is then defined as the difference between VaR of the financial system conditional on this particular firm i being in financial distress and the VaR of the financial system conditional on firm i being in its median state. In practice, the usual choice of the event $C(r_{it})$ is the situation in which the loss of firm i is precisely equal to the corresponding VaR:

$$CoVaR_{it}(\alpha) = CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)} - CoVaR_t^{m|r_{it}=median(r_{it})} . \tag{2.8}$$

In this paper we prefer the modification of the MES approach (see above in this Section) since it seems suitable for the PX index application presented in Section 4. Even if various numerical procedures for calculation of MES in practice have been proposed (see e.g. Cai et al (2015), Caporin et al (2010), Chen (2008) and others, see also below), we shall deal with the MES in the framework of the simple model described in Section 1, which is supplemented in such a way that one can model time-varying dependencies using the multivariate GARCH-DCC modeling class (see Brownlees and Engle (2012) and Engle (2009)). Moreover, such an econometric method enables to estimate the capital shortfall over a potentially long time period (e.g. a quarter or six months), which is useful in financial practice.

The final model (e.g. for a whole portfolio of stocks composing an exchange index) can be explained by means of a bivariate conditionally heteroskedastic model that characterizes the dynamics of the daily firm and market returns. Let r_{it} and r_{mt} denote the i-th firm and market log returns on day t similarly as in Section 1. The bivariate process of the daily firm and market returns is modeled as

$$\begin{aligned} r_{mt} &= \sigma_{mt}\varepsilon_{mt}, \\ r_{it} &= \sigma_{it}\varepsilon_{it} = \sigma_{it}\rho_{it}\varepsilon_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\,\zeta_{it}, \end{aligned} \tag{2.9}$$

where the shocks (ε_{mt}, ζ_{it}) are independent and identically distributed over time with zero mean, unit variance and zero covariance. A mutual independence of these shocks is not assumed: on the contrary, there are reasons to believe that extreme values of disturbances ε_{mt} and ζ_{it} interact (when the market is in its tail, the firm disturbances may be even further in the tail if there is serious risk of default). The stochastic specification is completed by a description of the two conditional standard deviations and the conditional correlation. These quantities can be formulated as follows:

Applying the principles of conditional heteroskedasticity, the volatility is modeled by means of the simplest threshold GARCH model (usually denoted by the acronym GJR-GARCH(1,1)) as

$$\begin{aligned} \sigma_{it}^2 &= \omega_i + \alpha_i r_{i,t-1}^2 + \gamma_i r_{i,t-1}^2 I_{i,t-1}^- + \beta_i \sigma_{i,t-1}^2, \\ \sigma_{mt}^2 &= \omega_m + \alpha_m r_{m,t-1}^2 + \gamma_m r_{m,t-1}^2 I_{m,t-1}^- + \beta_m \sigma_{m,t-1}^2 \end{aligned} \tag{2.10}$$

with $I_{i,t}^- = 1$ for $r_{it} < 0$ and 0 otherwise, $I_{m,t}^- = 1$ for $r_{mt} < 0$ and 0 otherwise. According to the applied threshold modification, the model can cover the leverage effect, i.e. the tendency of volatility to increase more with negative news rather than with positive ones (see e.g. Cipra (2013)).

The time-varying correlations are captured by using the *asymmetric dynamic conditional correlation* (*ADCC*) modeling scheme (see e.g. Engle (2009)). Let R_t denote the time-varying correlation matrix of the market and firm return, we shall assume that

$$\mathrm{var}_{t-1}\begin{pmatrix} r_{it} \\ r_{mt} \end{pmatrix} = D_t R_t D_t = \begin{pmatrix} \sigma_{it} & 0 \\ 0 & \sigma_{mt} \end{pmatrix}\begin{pmatrix} 1 & \rho_{it} \\ \rho_{it} & 1 \end{pmatrix}\begin{pmatrix} \sigma_{it} & 0 \\ 0 & \sigma_{mt} \end{pmatrix}, \tag{2.11}$$

where

$$R_t = diag(Q_t)^{-1/2} Q_t \; diag(Q_t)^{-1/2} \tag{2.12}$$

with

$$Q_t = \Omega_Q + A_Q^{\mathrm{T}} \varepsilon_{t-1}^{*} \varepsilon_{t-1}^{*\,\mathrm{T}} A_Q + B_Q^{\mathrm{T}} Q_{t-1} B_Q + C_Q^{\mathrm{T}} \varepsilon_{t-1}^{**} \varepsilon_{t-1}^{**\,\mathrm{T}} C_Q, \tag{2.13}$$

where the matrix operator $diag(\bullet)$ creates a diagonal matrix by extracting diagonal elements of the input matrix, Ω_Q is a (2×2) symmetric positive definite intercept matrix, A_Q, B_Q and C_Q denotes the (2×2) matrices of parameters, $\varepsilon_t^* = diag(D_t)^{-1/2}(r_{mt}, r_{it})^{\mathrm{T}}$ are "degarched" financial returns, and finally $\varepsilon_t^{**} = \min(0, \varepsilon_t^*)$. All cross products of ε_t^{**} elements will be nonzero only if both multiplied components are negative. Therefore, the model allows that dynamic correlations may be different for negative financial returns than they are for positive ones. Note that the matrix Q_t is symmetric and positive definite by construction. For simplicity, one may assume that the matrices A_Q, B_Q and C_Q are diagonal. Particularly, we shall put $A_Q = \alpha_Q I$ with some $\alpha_Q \geq 0$, $B_Q = \beta_Q I$ with some $\beta_Q \geq 0$ and $C_Q = \gamma_Q I$ with some $\gamma_Q \geq 0$, where I denotes the (2×2) identity matrix. Model estimation and other related issues are studied in detail by Engle (2009) and the works cited therein.

From the practical viewpoint, the model should enable to construct predictions in order to find the future capital shortfall. The one-period ahead MES can be expressed as a function of volatility, correlation and tail expectation of the standardized innovations distribution

$$\begin{aligned} MES^1_{i,t-1}(C) &= \mathrm{E}_{t-1}(r_{it}|r_{mt} < C) = \sigma_{it}\mathrm{E}_{t-1}(\varepsilon_{it}|\varepsilon_{mt} < C/\sigma_{mt}) = \\ &= \sigma_{it}\rho_{it}\mathrm{E}_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} < C/\sigma_{mt}) + \sigma_{it}\sqrt{1-\rho^2_{it}}\,\mathrm{E}_{t-1}(\zeta_{it}|\varepsilon_{mt} < C/\sigma_{mt}), \end{aligned} \tag{2.14}$$

where the conditional probability of a systemic event is defined as

$$\mathrm{P}^1_{t-1}(C) = \mathrm{P}_{t-1}(r_{mt} < C) = \mathrm{P}(\varepsilon_{mt} < C/\sigma_{mt}) . \tag{2.15}$$

The conditional tail expectation $\mathrm{E}_{t-1}(\zeta_{it}|\varepsilon_{mt} < C/\sigma_{mt})$ in the expression (2.14) captures the tail spillover effects from the financial system to the financial institution (firm) that are not captured by the conditional correlation. Assuming that innovations ε_{mt} and ζ_{it} are *i.i.d.*, the nonparametric estimates of the tail expectations introduced in (2.14) can be obtained as (see Chen (2008))

$$\begin{aligned} \hat{\mathrm{E}}_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} < C/\sigma_{mt}) &= \frac{\sum_{\tau=1}^{T}\varepsilon_{m\tau}\cdot I[\varepsilon_{m\tau} < C/\sigma_{m\tau}]}{\sum_{\tau=1}^{T} I[\varepsilon_{m\tau} < C/\sigma_{m\tau}]}, \\ \hat{\mathrm{E}}_{t-1}(\zeta_{it}|\varepsilon_{mt} < C/\sigma_{mt}) &= \frac{\sum_{\tau=1}^{T}\zeta_{i\tau}\cdot I[\varepsilon_{m\tau} < C/\sigma_{m\tau}]}{\sum_{\tau=1}^{T} I[\varepsilon_{m\tau} < C/\sigma_{m\tau}]}, \end{aligned} \tag{2.16}$$

where T denotes the sample size and $I[\bullet]$ is the binary indicator of an event •. Alternatively, nonparametric kernel methods might be considered to estimate the tail expectations. Finally, the threshold C characterizing the systemic event is given, e.g., as the unconditional or conditional value-at-risk of r_{mt}.

The multi-period ahead MES is usually realized by simulations (Brownlees and Engle (2012)). In order to construct the h-period ahead MES given information until and including time $t-1$, one generates S paths of financial returns r_m and r_i of length h by simulating corresponding paths of the innovations ε_m and ζ_i (e.g. by assuming an a priori distribution or using residual bootstrap techniques). To be more specific, having generated $(\varepsilon^s_{m,t-1+\tau}, \xi^s_{i,t-1+\tau})^h_{\tau=1}$ for $s = 1, \ldots, S$ the model (2.9)-(2.13) is used to evaluate $(r^s_{m,t-1+\tau}, r^s_{i,t-1+\tau})^h_{\tau=1}$ (the current levels of volatility and conditional correlation serve as starting conditions). Afterwards, one calculates the cumulative returns for $s = 1, \ldots, S$, i.e.

$$\bar{R}^s_{i,t-1+h} = \exp\left\{\sum_{\tau=1}^{h} r^s_{i,t-1+h}\right\} - 1 \quad \text{and} \quad \bar{R}^s_{m,t-1+h} = \exp\left\{\sum_{\tau=1}^{h} r^s_{m,t-1+h}\right\} - 1\,. \tag{2.17}$$

Then, the multi-period ahead MES is the Monte Carlo average of the S simulated paths, i.e.

$$MES^h_{i,t-1}(C) = \frac{\sum_{s=1}^{S} \bar{R}^s_{i,t-1+h} \cdot I[\bar{R}^s_{m,t-1+h} < C]}{\sum_{s=1}^{S} I[\bar{R}^s_{m,t-1+h} < C]}, \quad h = 1,2,\ldots \tag{2.18}$$

Analogously, the multi-period ahead conditional probability of a systemic event is computed as

$$POS^h_{t-1}(C) = \frac{1}{S}\sum_{s=1}^{S} I[\bar{R}^s_{m,t-1+h} < C]\,. \tag{2.19}$$

Alternatively, the simple historical one-period ahead MES can be used as a simple benchmark (Brownlees and Engle (2012)). It is given straightforwardly as

$$MES^{HIST}_{i,t-1}(C) = \frac{\sum_{\tau=t-M}^{t-1} r_{i\tau} \cdot I[r_{m\tau} < C]}{\sum_{\tau=t-M}^{t-1} I[r_{m\tau} < C]}, \tag{2.20}$$

where M is the width of moving window. This estimator is inspired by the financial practice, where various rolling window averages are commonly used.

Note that different modeling strategies would be introduced in order to predict MES by (2.14) and (2.18). In particular, distinct modeling specifications for calculating volatilities and correlations given by the formulas in (2.10)-(2.13) could be considered. See Hendrych and Cipra (2016) and many others.

3. Common shock

The common shock approach is another possible method in the framework of regulation of the risk of systemic character. We present its variant which can be recommended e.g. for the reinsurance risk regulation in the framework of Solvency II (2009), see e.g. Berg (2008), Cipra (2010), Ferreiro (2011), Lindskog and McNeil (2003), Sandström (2011)). Moreover, we shall show that the MES concept can be applied also in this context.

More generally, the presented method can be suitable also for financial risk regulation (i.e., for the capital requirements to cover the risk). For instance, the asymptotic formulas recommended when applying the advanced internal rating based approach IRB in Basel III (see BIS (2010)) have been suggested for large portfolios of homogenous risks. Therefore, they should not be used for the adjustment of regulatory capital if the credit counterparties form a small group (units or tens) of very heterogeneous subjects (see McNeil et al (2005)). In such a situation the suggested common shock method is more appropriate.

The common shock method can be described in several steps:

(*i*) Let us denote the common shock affecting all reinsurers of a given insurance company (e.g. the arrival of the financial and economic crisis or recession, a legislative change or reform, a catastrophic event) as a random variable R ranging in the interval between zero and one. For the values of R near to zero or one, the common influence of the given phenomenon is low or high, respectively. The behavior of R can be described by the probability density function of the form

$$f(r) = \beta r^{\beta-1}, \quad 0 < r < 1, \tag{3.1}$$

where β is a parameter ($0 < \beta < 1$), see Figure 1. Such a probability density function can be looked upon as a special case of the beta distribution (see e.g. Cipra (2015)), and it is acceptable from the practical point of view since according to it (1) the small shocks are the most probable ones, while (2) the probability of more intensive shocks declines to small positive values (however, the zero value is never achieved). Another interpretation is also possible: the maximal shock among n mutually independent annual shocks (i.e., during n years) has the probability density function of the form (3.1) but with the parameter $n\beta$ instead of β. Therefore e.g. for $\beta = 0.05$ and $n = 20$, the probability density function (3.1) corresponds to the uniform distribution so that the maximal shock during the period of 20 years attains each of its values with the same probability (one can make use of this property when calibrating the parameter β).

(*ii*) The probabilities of default will depend on the common shock R. A suitable functional relation seems to be

$$PD_i(r) = p_i + (1 - p_i) r^{\gamma / p_i}, \quad 0 < r < 1, \tag{3.2}$$

where p_i is a basic level of the probability of default of the i-th counterparty ($i = 1, \ldots, k$) in the credit portfolio (it is a benchmark if excluding the influence of the common shock R). Moreover, in (3.2) one adds to this basic level a component that depends on R by means of a positive parameter γ. The power exponent in (3.2) is a decreasing function of the basic level of the probability of default p_i since counterparties with a low p_i are not very sensitive to the random shocks while the higher probability of default increases the default sensitivity even if the attained values of R are not high. In any case, the function (3.2) increases from the basic level p_i to the value 1. Moreover, this function is concave for $\gamma < p_i$ (and the probability of default PD_i is considered more likely as large), it is convex for $\gamma > p_i$ (and the probability PD_i is considered more likely as small); for $\gamma = p_i$ one obtains an increasing line (see Figure 2 for the fixed $\gamma = 0.1$).

Figure 1 The probability density (3.1) of the common shock *R* (with $\beta = 0.05$)

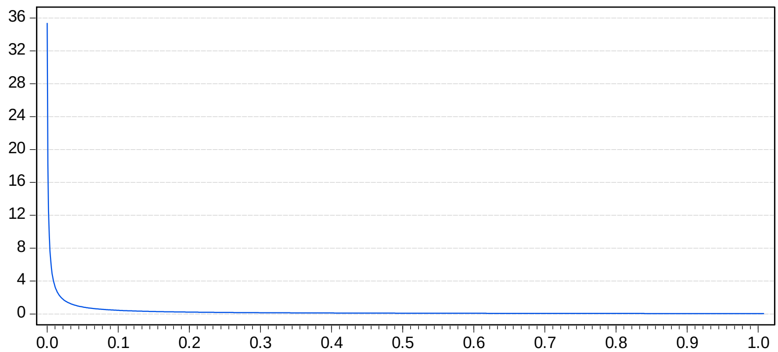
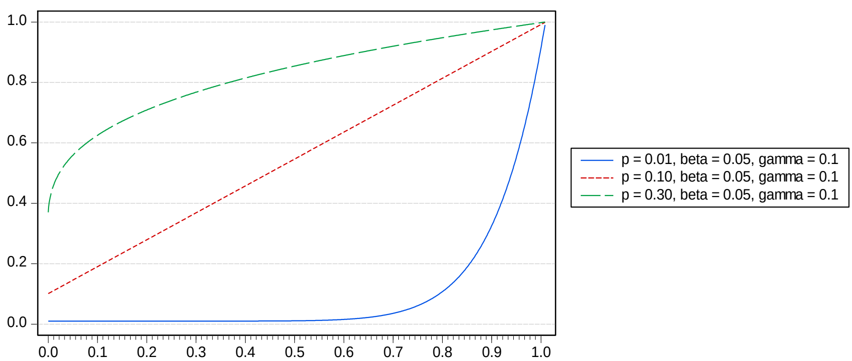


Figure 2 The probability of default as the function (3.2) of the common shock *R*



(*iii*) By integrating the function (3.2) over r using the probability density function (3.1) one obtains the formula for the probability of default of the i-th counterparty as

$$PD_i = \mathrm{E}(PD_i(R)) = \int_0^1 PD_i(r)\, f(r) dr = \frac{(\gamma+\beta)p_i}{\gamma+\beta p_i}\,. \tag{3.3}$$

Conversely, if one takes the numerical values of default probabilities PD_i of particular counterparties as an external rating provided by specialized agencies (see e.g. *S&P* in *Table 4*) then one can find the basic level of the probability of default p_i of the *i*-th counterparty evidently as

$$p_i = \frac{\gamma\, PD_i}{\beta(1-PD_i)+\gamma}\,. \tag{3.4}$$

One can summarize that the behavior of particular counterparties in the credit portfolio may be modeled in a suitable parametric way using two parameters β and γ.

(*iv*) Finally, one should extend the results for particular counterparties to the whole credit portfolio. Let I_i is a random default indicator of the *i*-th counterparty, i.e., $I_i = 1$ or $I_i = 0$ depending whether the default has occurred or not, respectively. The loss L generated by the whole credit portfolio can be expressed in the form

$$L = \sum_{i=1}^{k} LGD_i \cdot I_i\;, \tag{3.5}$$

where LGD_i is the particular loss following from the default of the *i*-th counterparty. It holds

$$\mathrm{E}(L) = \sum_{i=1}^{k} LGD_i \cdot PD_i\,, \quad \mathrm{var}(L) = \sum_{i=1}^{k}\sum_{j=1}^{k} LGD_i \cdot LGD_j \cdot \sigma_{ij}\;, \tag{3.6}$$

where

$$\sigma_{ii} = PD_i(1-PD_i), \quad \sigma_{ij} = \frac{\beta(1-p_i)\cdot(1-p_j)}{\beta+\gamma p_i^{-1}+\gamma p_j^{-1}} - (PD_i - p_i)\cdot(PD_j - p_j), \quad i \neq j\,. \tag{3.7}$$

For instance, to derive σ_{ij} for $i \neq j$ one can write using (3.3)

$$\sigma_{ij} = \mathrm{cov}(I_i, I_j) = E(I_i, I_j) - E(I_i)\cdot E(I_j) = E(I_i, I_j) - PD_i \cdot PD_j =$$

$$= \int_0^1 [p_i + (1-p_i) r^{\gamma/p_i}][p_j + (1-p_j) r^{\gamma/p_j}]\, \beta r^{\beta-1} \mathrm{d}s - \frac{(\gamma+\beta)p_i}{\gamma+\beta p_i}\cdot\frac{(\gamma+\beta)p_j}{\gamma+\beta p_j} =$$

$$= \frac{\beta(1-p_i)\cdot(1-p_j)}{\beta+\gamma p_i^{-1}+\gamma p_j^{-1}} - (PD_i - p_i)\cdot(PD_j - p_j) .$$

In the special case of a single counterparty (i.e., $k = 1$) the relations (3.6) and (3.7) are simplified to

$$\mathrm{E}(L) = LGD \cdot PD, \quad \mathrm{var}(L) = LGD^2 \cdot PD \cdot (1-PD) . \tag{3.8}$$

In any case, the capital requirements calculated by the methodology of value-at-risk $VaR_{0.995}$ (with the confidence level 99.5 % under the assumption of normality, see Cipra (2015)) can be obtained substituting (3.6) and (3.7) to the formula

$$VaR_{0.995} = \mathrm{E}(L) + 2.576\sqrt{\mathrm{var}(L)} \ . \tag{3.9}$$

One can also calculate the marginal expected shortfall for *i*-th reinsurer as

$$MES_i = LGD_i \cdot \mathrm{E}(I_i | R > VaR_\alpha) , \tag{3.10}$$

where the value-at-risk VaR_α of R is obviously

$$VaR_\alpha = \alpha^{1/\beta} . \tag{3.11}$$

A simple technical arrangement gives

$$\begin{aligned} &\mathrm{E}(I_i | R > VaR_\alpha) = p_i(1-\alpha) + \frac{\beta(1-p_i)}{\gamma p_i^{-1}+\beta}(1 - VaR_\alpha^{\gamma p_i^{-1}+\beta}) = \\ &\frac{(\gamma+\beta)p_i}{\gamma+\beta p_i}(1 - VaR_\alpha^{\gamma p_i^{-1}+\beta}) + p_i(VaR_\alpha^{\gamma p_i^{-1}+\beta} - \alpha) \end{aligned} \tag{3.12}$$

4. Applications of MES approach in finance

This section presents a case study of the Prague Stock Exchange index (PX index) constituents. In order to calculate the marginal expected shortfall (MES) for each involved firm, we shall implement a (slightly) modified estimation method originally considered by Brownlees and Engle (2012). In particular, we shall follow the modeling framework introduced in Section 2. The conditional volatilities σ_{mt} and σ_{it} are modeled by the GJR-GARCH(1,1) scheme given by (2.10). The time-varying conditional correlations ρ_{it} are described by means of the asymmetric DCC model fully specified by (2.11)-(2.13). We estimate the model in two consecutive steps by applying quasi maximum likelihood similarly as Engle (2009). Given estimated the above-mentioned quantities, the marginal expected shortfall MES and the conditional probability of a systemic event POS are then computed by (2.14) or (2.20) and (2.15), respectively.

We analyze the panel of companies constituting the PX index basis. The panel contains fourteen firms, which have been incorporated into the PX index basis according to their market capitalization as of the end of June 2008. This panel is unbalanced in that sense that not all companies have been continuously traded during the sample period. We extracted the daily logarithmic returns from January 6, 2000 to May 9, 2016. The full list of institutions involved in our study is reported in Table 1. Table 2 delivers selected sample characteristics of the studied logarithmic returns.

Figure 3 displays conditional volatilities of all investigated firms jointly with the PX index (market) conditional volatility. Apparently, all graphs are significantly influenced by the explosion in variability during the financial crisis. Furthermore, one identifies a similar trend over many charts that is in line with the market volatility trend. On the contrary, several return time series are dominated by other effects, which are not common for the whole market or for other returns. For instance, one can emphasize the volatility of *O2* financial returns, which was increased due to the split of the company in 2015.

Figure 4 shows the estimated time-varying correlations ρ_{it} between returns of the i-th company and the PX index. It is evident that the financial returns of involved firms are significantly positively correlated with the market financial returns. However, one identifies different behavior over correlations displayed in those figures. Some correlations are relatively stable when comparing with others, see e.g. the results for *ČEZ*, *Komerční banka*, *Philip Morris* or *Vienna Insurance Group*. However, some conditional correlations demonstrate a trend, which varies in time, e.g. *O2* or *Unipetrol*.

Figure 5 compares the estimated one-step ahead MES evaluated according to (2.14) or (2.20) with the rolling window width $M = 250$. Figure 6 presents the estimated conditional probability of a systemic event POS as it was given in (2.15). Point out that the threshold C was set as the 1% unconditional value-at-risk of the PX index log returns. At the first sight, the estimated POS is evidently very high during the financial crisis. Accordingly, all one-step ahead MES values calculated by (2.14) are also excessive during the financial crisis period. Nevertheless, high one-step ahead MES values occur also in other time instants, see e.g. *O2* (the company split observed in 2015) or *Unipetrol* (in 2005 and 2006). It should be highlighted that the *Philip Morris* returns indicate the lowest average one-step ahead MES overall, i.e. it shows the firm stability. Finally, the MES computed by (2.20) evidently does not represent a suitable measure of systemic risk. For instance, as can be seen from Figure 5, during the years 2012-2015 the historical one-period ahead MES predictions are mostly zero due to absence of any systemic event declared by the threshold C. It is in sharp contrast to the MES forecasts obtained by (2.14).

Table 3 reports the examined stocks listed in ascending ordered regarding the one-step ahead MES predicted for October 20, 2008, for which the highest POS was anticipated. Moreover, Table 4 contains the estimated multi-period ahead MES and

POS ($h = 125$, i.e. the half-year MES and POS predictions starting from May 9, 2016) for actually traded stocks by respecting (2.18) and (2.19). The bivariate distribution of innovations was replicated by using the residual bootstrap method. This approach reflects the structure of standardized residuals computed during estimation of the one-step ahead MES. The threshold C was set as minus 5% and minus 20%, respectively. To be more precise, an investor can identify and anticipate potential capital shortfall under condition that a systemic event occurs half a year after the investment (i.e. that the market cumulative return (2.17) is less than the threshold C half-year after the investment). We have estimated that a systemic event occurs with probability 31.2% for $C = -0.05$ and 7.2% for $C = -0.20$. Consequently, *New World Resources PLC* and *Erste Group Bank* have been identified as the riskiest assets assuming $C = -0.05$ (multi-period ahead MES of –14.7%) and $C = -0.20$ (multi-period ahead MES of –25.5%), respectively. One should remind that some drawbacks related to the Czech stocks market as low liquidity might bring some distortions into the previous results.

As the methodology used is concerned, Brownlees et al (2011/12) explore the performance of volatility forecasting by exercising it on a wide range of domestic and international equity indices and exchange rates. They find that across asset classes and volatility regimes, the simplest GARCH specification very similar to our approach is the most often the best forecaster of future risk. Prague Stock Exchange (PSE) in the risk context has been examined e.g. in Seidler and Jakubík (2009a, b) using traditional Merton's approach with the aim to obtain estimates of *LGD* for companies listed in PSE. Hanousek and Novotný (2014) employ the high-frequency data from PSE (and from NYSE) to analyze the variation in extreme price movements and market volatility around the period of fall of Lehman Brothers but they employ the price jump indicators for this purpose.

Table 1 List of companies involved in the MES analysis introduced in Section 4

Stock name (abrev.)	*Stock name*	*Obs. from*	*Obs. to*
PX	Prague Stock Exchange Index	Jan 6, 2000	May 9, 2016
AAA	AAA Auto	Sep 25, 2007	Jul 3, 2013
VIG	Vienna Insurance Group	Feb 6, 2008	May 9, 2016
CEZ	ČEZ	Jan 6, 2000	May 9, 2016
CETV	Central European Media Enterprises	Jun 28, 2005	May 9, 2016
ECM	ECM Real Estate Investments	Dec 8, 2006	Jul 20, 2011
ERSTE	Erste Group Bank	Oct 2, 2002	May 9, 2016
KB	Komerční banka	Jan 6, 2000	May 9, 2016
NWR	New World Resources PLC	May 7, 2008	May 9, 2016
O2	O2 Česká republika	Jan 6, 2000	May 9, 2016
ORCO	Orco Property Group SA	Feb 2, 2005	Sep 19, 2014
PEGAS	Pegas Nonwovens SA	Dec 19, 2006	May 9, 2016
PHILMOR	Philip Morris ČR	Oct 9, 2000	May 9, 2016
UNIPETROL	Unipetrol	Jan 6, 2000	May 9, 2016
ZENTIVA	Zentiva	Jun 29, 2004	Apr 27, 2009

Table 2 Sample characteristics of the log returns used for the MES analysis

Stock name	*# obs*	*Mean*	*Std. dev*	*Median*	*Min*	*Max*	*Skew*	*Kurt*
PX	4101	0.00014	0.01429	0.00051	–0.16185	0.12364	–0.45054	12.21528
AAA	1450	–0.00057	0.02885	0.00000	–0.23107	0.34179	1.14937	23.07040
VIG	2071	–0.00049	0.02215	0.00000	–0.17920	0.13539	–0.51478	8.42960
CEZ	4099	0.00039	0.01932	0.00090	–0.19834	0.19517	–0.38745	9.95323
CETV	2705	–0.00114	0.03921	0.00000	–0.70628	0.47994	–1.98131	56.46028
ECM	1155	–0.00356	0.03748	–0.00253	–0.41313	0.24481	–1.02567	20.24787
ERSTE	3127	–0.00005	0.02668	0.00000	–0.26834	0.19382	–0.37664	11.05523
KB	4099	0.00048	0.02138	0.00000	–0.19392	0.10410	–0.43406	6.76452
NWR	2008	–0.00436	0.05186	0.00000	–0.49590	0.51083	–0.86226	16.88669
O2	4101	–0.00022	0.02509	0.00000	–0.94253	0.13056	–13.02806	487.11037
ORCO	2385	–0.00181	0.03614	–0.00168	–0.26397	0.29480	0.14712	11.66799
PEGAS	2353	0.00000	0.01784	0.00000	–0.23370	0.13509	–0.93750	21.07422
PHILMOR	3882	0.00020	0.01756	0.00000	–0.13709	0.14842	–0.63086	8.43172
UNIPETROL	4097	0.00028	0.02233	0.00000	–0.21770	0.26472	–0.12378	14.04297
ZENTIVA	1214	0.00061	0.01929	0.00000	–0.17839	0.09733	–1.31104	13.24937

Table 3 One-step ahead MES predicted for October 20, 2008

Stock name (abr.)	*MES*
ERSTE	–0.25211
ORCO	–0.23861
NWR	–0.21770
CETV	–0.20791
KB	–0.20197
ECM	–0.18318
UNIPETROL	–0.17690
PEGAS	–0.15227
CEZ	–0.14778
AAA	–0.12997
VIG	–0.11844
O2	–0.11019
PHILMOR	–0.05774

Table 4 Multi-period ahead MES and POS ($h = 125$) for the actually traded stocks and the PX index itself

Stock name (abr.)	$MES_{5/9/2016}^{125}(C = -5\%)$	$MES_{5/9/2016}^{125}(C = -20\%)$	$POS_{5/9/2016}^{125}(C = -5\%)$	$POS_{5/9/2016}^{125}(C = -20\%)$
AAA	NA	NA	0.31156	0.07204
VIG	–0.12630	–0.21913		
CEZ	–0.09328	–0.19380		
CETV	–0.09321	–0.16910		
ECM	NA	NA		
ERSTE	–0.13347	–0.25510		
KB	–0.09235	–0.19925		
NWR	–0.14654	–0.19410		
O2	–0.09833	–0.16540		
ORCO	NA	NA		
PEGAS	–0.02161	–0.08087		
PHILMOR	–0.03092	–0.07247		
UNIPETROL	–0.02253	–0.09114		
ZENTIVA	NA	NA		

Figure 3 Conditional volatilities of the PX index constituents and the PX index itself

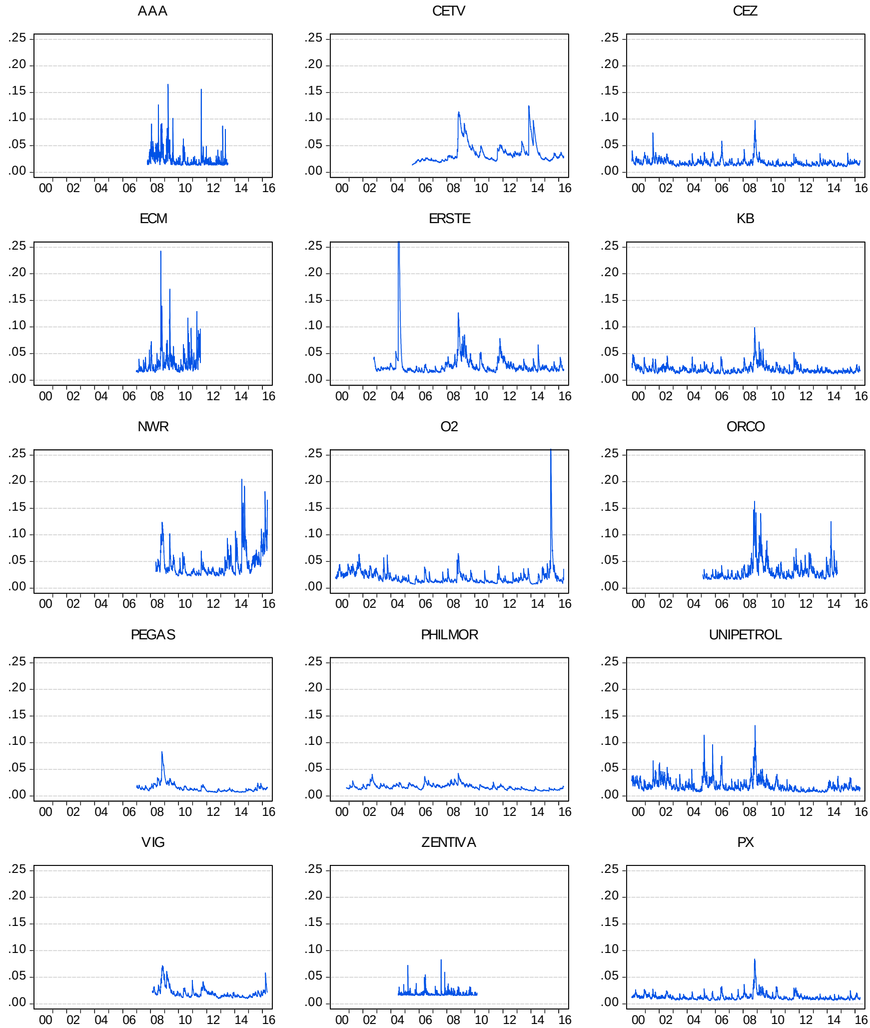


Figure 4 Conditional correlations among the PX index constituents and the PX index

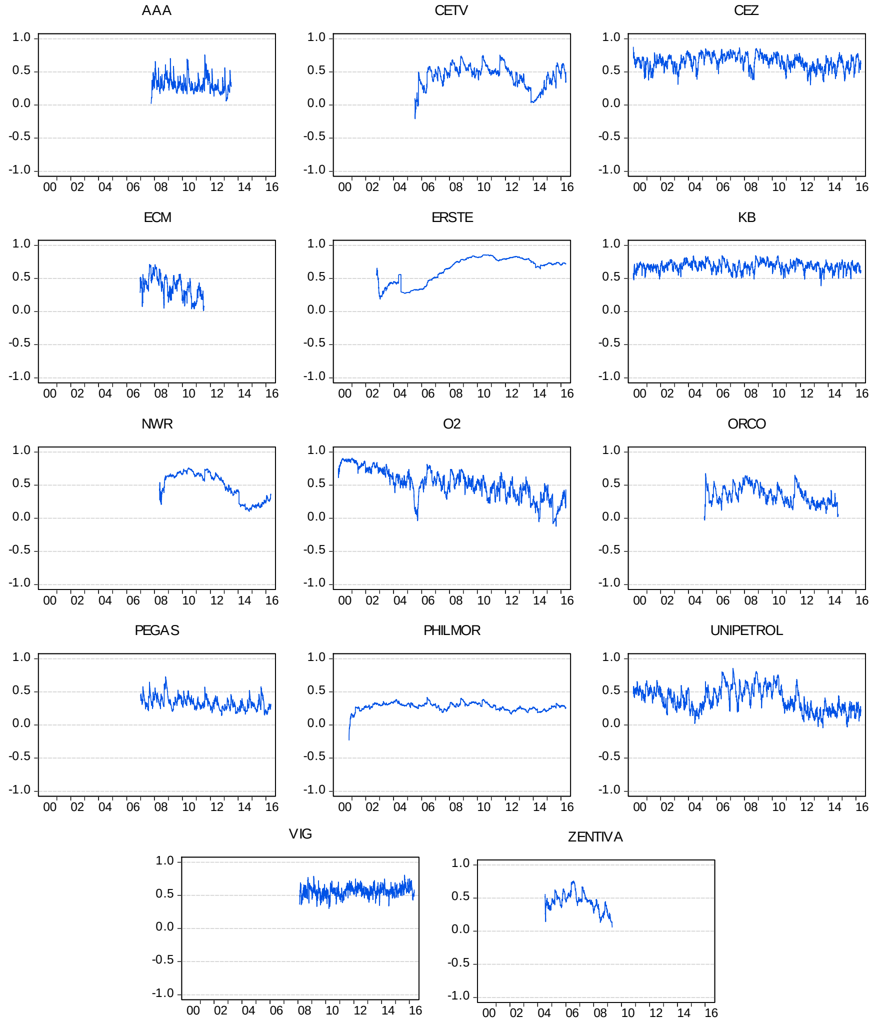


Figure 5 Comparison of MES estimates calculated by (2.14) and (2.20) for the PX index constituents

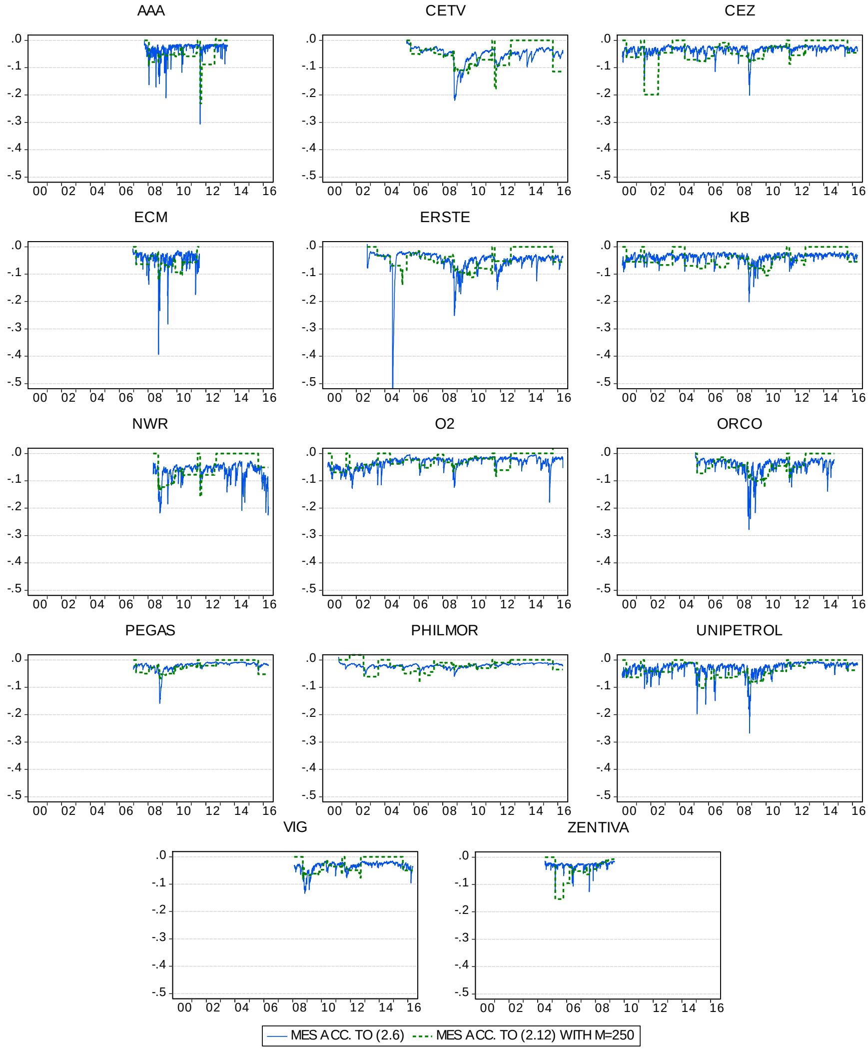
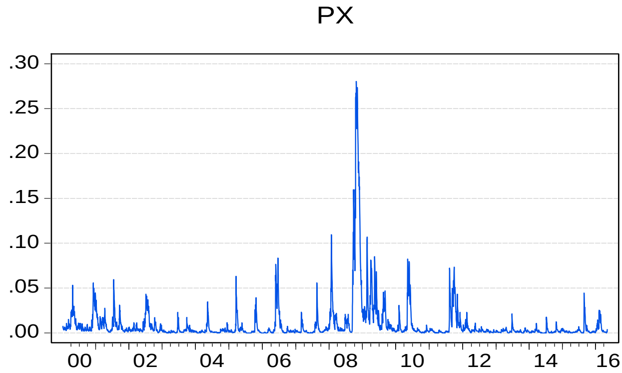


Figure 6 Estimated probability of a systemic event POS of the PX index



5. Application of common shock approach

Table 5 contains numerical values that are used for the calculation of the credit capital requirements as a numerical example of the methodology described in Section 3. In this example an insurer is reinsured by three reinsurers ($i = 1, 2, 3$) with the default probabilities corresponding to the external rating by *S&P* (the mapping *PD*s and *S&P* ratings has been estimated by *S&P* using data from 1981-2004, see Table 10.2.2 in Cipra (2015)). The applied values of parameters β and γ are taken from Sandström (2011) but in practice they should be prescribed by the regulator. *LGD*s are chosen to correspond to values usual in Czech insurance companies according to experience of authors.

Table 5 Numerical values used for the calculation in Section 5

	Symbol	*Numerical value*
Parameters	β	0.05
	γ	0.1
Probability of default from Table 4 (S&P)	PD_1 (~*AA*)	0.005
	PD_2 (~*A*)	0.008
	PD_3 (~*BBB*)	0.083
Loss given default	LGD_1	300 m CZK
	LGD_2	150 m CZK
	LGD_3	80 m CZK

In *Table 6* we have calculated the basic levels of the probabilities of default of particular counterparties (according to (3.4)) and the variances σ_{ii} and covariances σ_{ij} (according to (3.7)) necessary for the calculation of variance of loss in (3.6).

Table 6 Auxiliary values calculated in Section 5

i:	p_i	σ_{ij}		
		j: *1*	*2*	*3*
1	0.003	0.005	0.001	0.001
2	0.005	0.001	0.008	0.002
3	0.057	0.001	0.002	0.076

It enables to calculate according to (3.6) the expected value and the standard deviation of the loss *L* following from the credit default of counterparties

$$\mathrm{E}(L) = 9.3\,\mathrm{m\ CZK}, \quad \sqrt{\mathrm{var}(L)} = 36.4\,\mathrm{m\ CZK}$$

and hence finally the capital requirements as the value-at-risk $VaR_{0.995}$ according to (3.9) as

$$VaR_{0.995} = 9.3 + 2.576 \cdot 36.4 = 103.2\ \mathrm{m\ CZK}\ .$$

Similarly, one obtains $VaR_{0.95}$ = 69.3 m CZK.

Finally, *Table 7* contains the corresponding marginal expected shortfalls for particular reinsurance for α = 0.995 and 0.950.

Table 7 Marginal expected shortfalls of particular reinsurers in Section 5

i	***MES$_i$* (m CZK)**	
	α = 0.995	α = 0.950
1	0.480	0.550
2	0.340	0.440
3	0.370	1.990

6. Conclusion

The present paper was focused on special quantitative approaches to the systemic risk, which seem to be a significant type of risk in today's financial world. In particular, we discussed two different methods in greater detail. Firstly, we have slightly modified the common modeling framework used for estimating the marginal expected shortfall (MES) and the conditional probability of a systemic event occurrence by considering the GJR-GARCH(1,1)-ADCC model, which respects the character of financial returns more conveniently. The introduced method enabled to estimate the capital shortfall over a potentially long time horizon, which could be truly useful in financial practice. The methodology was applied in Section 4 in the case study of the portfolio of PX index constituents with discussion of obtained results.

Secondly, we have proposed the common shock approach to the risk of systemic character. It has been inspired by the insurance and reinsurance practice. This method is based on an idea that the probability of a common shock is driven by beta distribution, and then modeled in a specific way. After some calculations, the MES formula for this particular scheme was expressed explicitly. Section 5 provided a useful numerical demonstration of this approach.

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