# USING PORTFOLIO THEORY IN THE SECOND PENSION PILLAR IN SLOVAKIA - SEARCHING FOR AN EFFICIENT SET OF PORTFOLIOS 

MARTIN DLUHOŠ ${ }^{1}$

## Využitie teórie portfólia v druhom dôchodkovom pilieri na Slovensku - hl’adanie efektívnej množiny portfólií


#### Abstract

Slovak savers who are investing in the second pension pillar save part of financial means for a pension in pension management companies offering more funds - notably bond fund, mixed fund, stock fund, and index fund. At present, savers can spread their savings and other future pension payments over two funds with any ratio. This has allowed them to create an optimal investment portfolio due to their tendency for risk. The goal of this paper is to examine the possibilities of creating a portfolio of various funds at Slovak pension management companies and compare the risk and profitability of individual portfolios. The presented results indicate positive values in the covariance matrix of logarithmic returns of individual assets, and in most of the examined pension management companies, the efficient frontier consists of a combination of investment in an index fund and bond fund. Passively managed index funds, index-replicating with a high level of spatial diversification with time diversification which is associated with regular payments, represent a real possibility of saving for one's pension.


Keywords: second pension pillar, diversification, efficient frontier
JEL Classification: G 11, G 23

## 1 Introduction

Slovak savers investing in the second pension pillar have the opportunity to choose from six pension management companies, offering the most common four funds, which are characterized by different levels of return and risk. Funds of the pension management companies consist of different types of financial assets. At present, savers have two choices: the first option is the possibility of investing all finances in only one fund; the second option is the possibility of splitting finances into two funds in any percentage ratio, while

[^0]the necessary condition is that one of the two funds must be a bond fund. The possibility of allocating the financial resources into two funds enables the use of the Markowitz modern portfolio theory (Markowitz, [7]), which seeks to find an efficient set of portfolios (called efficient frontier) with respect to the maximum return at a given level of risk.

The goal of the present paper is to examine and assess the possibility of creating a portfolio of several funds in individual pension management companies and assess the effective frontiers of pension management companies for the period April 2012 - December 2016. Part of the research is also assessing whether actively managed funds achieve and beat the profitability of passively managed funds with respect to the comparable level of risk.

## 2 Literature review

Modern portfolio theory (MPT) or mean-variance analysis was first introduced by Markowitz [7]. The aim of MPT is to create a portfolio of assets and to find the maximum level of expected return for a given level of expected risk, which is measured most often by the variance or standard deviation. The problem in the Markowitz approach is expected return and expected risk, which we do not currently know, but it is often estimated by historical return and historical risk, which are different from future real return and risk. Mandelbrot [6], Rom and Ferguson [9] confront these problems of expected return and expected risk with the fact that expected return and expected risk have problems with normal distribution and follow up on the MPT with the new post-modern portfolio theory (PMPT). Following Markowitz's MPT, Black and Litterman [2] used a different approach, taking into account the current risk and return, and the expected risk and return, based on the idea that the differences between these current and expected values can be used in deals speculating on the decline in assets.

The original Markowitz mean-variance analysis showing historical return and historical risk provides information about the effective frontier for the period under examination, and what has been optimal for investors during this period due to their tendency to risk. Based on this information, interesting results for recommendations for future strategy and results for the comparison of actively and passively managed funds can be presented. A lot of research and studies indicate that an actively managed portfolio of financial assets in the long-term period do not beat the return of a passively managed portfolio copying various stock indexes, which use spatial diversification (Ferguson, [4]; Malkiel, [5]; Cremers and Petajisto, [3]; Bhootra et al., [1]; Tudor, [10]. Martí [8] realized research for the period 2006-2010 and examined what the impact of the active managed portfolio in the Spanish pension system is and
came to the conclusion that there is no added value of the active management of the portfolios. It states that there are only higher costs for savers in the pension system when the funds are actively managed in contrast to passively managed funds. Similarly, this article examines the comparison of actively managed funds in several pension companies in Slovakia through the search for historical effective frontiers.

## 3 Data

There are six pension management companies in Slovakia - AXA, VÚB Generali, NN, Post bank (Poštová banka), Aegon, Allianz. AXA, VÚB Generali, NN and Post bank publish historical data about the pension unit of each fund on a daily basis on website, while Aegon and Allianz do not offer these data (it is only offered in the form of graphs). For that reason, searching for an efficient frontier was examined with only four pension management companies.

The Post bank, VÚB Generali and NN have four types of funds at their disposal - bond, mixed, stock and index, AXA offers only three funds bond, stock and index. The funds in these pension companies differ mainly in the name and content of the individual assets. The bond funds in all four companies include various bonds and money-market assets, while mixed funds in all three companies have a different mix of bonds, money-market assets and stocks. Stock funds in all four companies differ in the types of selected shares. Index funds in three companies (AXA, VÚB Generali and Post Bank) copy a passively selected stock index, while the index fund in pension company NN copy a share index with active management through special algorithm, which seeks to change the percentage ratio of shares for this index in order to find above-average return.

Pension management companies included index funds in the offer from April 2012, therefore the examination was carried out during the period April 2012 - December 2016. Historical return represents the logarithmic returns of individual assets (logarithmic returns of the pension unit of individual funds) and historical risk is represented by historical standard deviations of the pension unit of individual funds.

## 4 Methodology

Modern portfolio theory examines return and risk, while in this research, return represents the logarithmic returns of the pension unit in individual funds, calculated as:

$$
\begin{equation*}
r_{i}=\ln \left(\frac{x_{i}}{x_{i-1}}\right) \tag{1}
\end{equation*}
$$

where $x_{i}$ represents the value of the pension unit in the specific fund at the time $i, x_{i-1}$ represents value of the pension unit in the specific fund at the time $i-l$ and $r_{i}$ is the logarithmic return of the pension unit at the time $i$.

In this paper, risk represents standard deviation of individual funds $-s$, calculated as:

$$
\begin{equation*}
s=\sqrt{\frac{1}{n} \sum_{i=1}^{n}\left(r_{i}-\bar{r}\right)^{2}} \tag{2}
\end{equation*}
$$

where $n$ is the total number of observations (the number of logarithmic returns of the pension unit) and $\bar{r}$ is the average of logarithmic returns of pension units with $n$ observations.

When a portfolio consist of two assets (or funds), then the average portfolio return $\left(R_{\mathrm{p}}\right)$ is calculated as the weighted average rate of return, where the weights are the percentage ratio of individual assets or funds in portfolio:

$$
\begin{equation*}
R_{p}=w_{1} r_{1}+w_{2} r_{2} \tag{3}
\end{equation*}
$$

where $w_{1}$ and $w_{2}$ represents weights as the ratio of individual funds in portfolio, $w_{1}+w_{2}=1$ and $r_{1}$ and $r_{2}$ are returns of funds calculated as (1).

When a portfolio consists of two assets (funds), the standard deviation of portfolio is calculated as $s_{p}$ :

$$
\begin{equation*}
s_{p}=\sqrt{w_{1}^{2} s_{1}^{2}+w_{2}^{2} s_{2}^{2}+2 w_{1} w_{2} \operatorname{cov}(X, Y)} \tag{4}
\end{equation*}
$$

where $w_{1}$ and $w_{2}$ represents weights as the ratio of individual funds in the portfolio, $s_{1}$ is the standard deviation of first fund and $s_{2}$ is standard deviation of the second fund and $\operatorname{cov}(X, Y)$ is the covariance of $X$ and $Y$, which is calculated as:

$$
\begin{equation*}
\operatorname{cov}(X, Y)=\frac{1}{n} \sum_{i=1}^{n}\left(r_{1 i}-\bar{r}_{1}\right)\left(r_{2 i}-\bar{r}_{2}\right) \tag{5}
\end{equation*}
$$

where $r_{1 i}$ represents the specific values of logarithmic returns at time $i$ in the first fund, $r_{2 i}$ represents specific values of logarithmic returns at time $i$ in the second fund, $\bar{r}_{1}$ is an average return of the first fund and $\bar{r}_{2}$ is an average return of the second fund.

## 5 Results

At first, the rate of return and risk was only calculated for the specific funds in four pension management companies. The calculated results are shown in Table 1 Historical return and risk in different pension companies and different funds:

Table 1

## Historical return and risk in different pension companies and different funds

| Company | Return - r and <br> Risk - s | Bond fund | Mixed fund | Share fund | Index fund |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ AXA | r | 0,00005 | - | 0,00018 | 0,00045 |
|  | s | 0,00062 | - | 0,00268 | 0,00931 |
| VÚB | r | 0,00010 | 0,00020 | 0,00022 | 0,00040 |
|  | s | 0,00048 | 0,00318 | 0,00442 | 0,00928 |
| $\mathbf{N N}$ | r | 0,00004 | 0,00015 | 0,00020 | 0,00035 |
|  | s | 0,00032 | 0,00236 | 0,00346 | 0,01204 |
| Post bank | r | 0,00012 | 0,00012 | 0,00011 | 0,00035 |
|  | s | 0,00097 | 0,00287 | 0,00529 | 0,00994 |

Source: own processing.
In the second step, the calculations for each set of portfolios were performed between just two specific funds with a different percentage ratio for the first and second fund. These results along with the results for investment only in specific funds were plotted on a return-risk graph. Each of the following figures (showing return and risk) contain the grey point O , which represents the bond fund, the grey point $\Delta$ - mixed fund, the black point $\boldsymbol{\Delta}$ - share fund and the black point - index fund in the pension management company under examination, the black dashed curve represents the set of portfolios consisting of the bond fund and index fund in the pension management company under examination.

The following Figure 1 Return-risk analysis of pension funds - AXA show the historical return and historical risk of pension funds in AXA.

## Return-risk analysis of pension funds - AXA



Source: own processing in R.
We can notice that the AXA provides only three funds - bond, share and index funds. The share fund, which is actively managed provides only a slightly higher rate of return than a portfolio with a similar risk (standard deviation) consisting of a bond fund and index fund. The effective frontier in AXA is composed of two curves - the first black curve represents the set of portfolios constituted by investments in bond and share fund and the second black curve represents the set of portfolios constituted by investments in the share fund and index fund.

The following Figure 2 Return-risk analysis of pension funds - Post bank shows the historical return and historical risk of pension funds in Post bank. The following Figure 2 shows the investment portfolios generated between just two funds, which are represented by black curves and dashed curve. It is seen that the effective frontier in Post bank consists of a combination of a bond and index fund. Actively managed funds - the mixed and share fund did not achieve a return at a comparable risk as a combination of index and bond portfolio. From January 2017, the mixed fund in Post bank was canceled and connected with bond fund.

Figure 2

## Return-risk analysis of pension funds - Post bank



Source: own processing in R.

Figure 3

## Return-risk analysis of pension funds - VÚB



Source: own processing in R.
In Figure 3 Return-risk analysis of pension funds - VÚB we can see that the efficient frontier is very close to the curve, which combines the bond fund with the index fund. The set of portfolios consists of the share fund and the
index fund (black curve between the grey point $\Delta$ and black point $\bullet$ ) reached only a slightly higher historical return than set of portfolios between the bond fund and index fund respecting the same level of risk.

Figure 4 shows an analysis of historical return and historical risk of the NN pension management company. At first sight, it is clear that the combination of the bond fund and index fund was the worst possible way for pension saving in the past (April 2012 - December 2016), because the mixed fund and share fund achieved a higher return than the combination of the share and index fund. Important information is that the index fund in the NN company achieved a much higher level of risk and a much lower return than index funds in companies AXA, Post bank and VÚB, which may be due to the active managing and modification of this index fund by NN company managers.

Figure 4
Return-risk analysis of pension funds - NN


Source: own processing in R.
The effective frontiers for each company at the period under examination are viewed in an aggregated manner in a single figure, Figure 4. It should be noted that the possibility of allocating finances to two funds in any pension managed company is conditional upon allocating a part of savings into the bond fund. For this reason, some effective frontiers are not continuous (this is the case of AXA and NN).

Figure 5 shows the historical efficient frontiers, which represents an optimal investment strategy across the different levels of risk (standard deviation). For each level of risk, the maximum of historical return in each company was
chosen. It is evident that the NN achieved the worst results - especially the index fund with active management, which reduce the possibility of spatial diversification (this index fund achieved much worse results in return and risk than index funds in the other three companies). For investors with risk aversion savings in AXA and NN was advantageous invest in bond found, respectively spread finance into the bond fund and stock fund. In the case of AXA, it was due to only marginally higher performance of the stock fund, which beats the portfolio composed of the bond fund and index fund with a similar risk as just a stock fund. For NN investors accepting higher level of risk, the combination of the bond and index fund with a higher percentage ratio for the index fund was appropriate. It should also be noted that the index fund in NN, which is actively managed and adjusted by managers of the company, achieves a lower return and higher risk than index funds in the other three companies.

Figure 5

## Historical efficient frontiers of the four pension management companies



Source: own processing in R.
Notes: grey dashed curve - efficient frontier of VÚB, black curve - efficient frontier of AXA, black dashed curve - efficient frontier of Post bank, grey curve - efficient frontier of NN The most interesting

The most interesting example is the VU'B, where effective frontiers consist of a combination of bond fund and index fund. An optimal strategy for riskaverse savers was saving the major part of finance in a bond fund, while for savers searching for risk, saving the major part of finance in an index fund was optimal. Interesting funds are mixed and stock funds in the VÚB, which achieved similar results in terms of risk and return than a portfolio consisting of a bond and index fund at a given level of risk. A more detailed examination indicates that the VU'B in mixed fund and stock fund does not
select stocks with potential growth (active management). This company applies the passive strategy when the mixed fund is formed by a combination of a bond and index fund (around $35 \%$ of index fund), and the stock fund is formed by a combination of the bond and index fund with a larger share in the index fund (around $60 \%$ of index fund). Since 2013 the VÚB has increased percentage of the index fund in the mixed and stock fund, which moved these two funds on the efficient frontier to a higher risk and higher returns, thereby more evenly placing all four funds on the efficient frontier. The bond fund in this pension company achieved the highest return with respect to the low level of risk. It should be noted that this bond fund in the VUB contains the largest percentage of money-market assets compared to other bond funds of other pension companies. The major proportion of the bond funds in other companies (AXA, Post bank and NN) are investments in bonds.

## 6 Conclusion

The covariance matrix of logarithmic returns contains positive values, therefore the created portfolios consisting of the two funds could not be achieved at a lower level of risk (lower level of standard deviation) than the risk of a single fund. The presented results indicate that in most of examined pension management companies, the efficient frontier consisting of a combination of investment in indexed and bond funds, where different levels of risk may be determined by the percentage change between these two funds. For AXA and NN savers, it was most profitable in the previous period to invest at a given level of risk only in a stock fund rather than a combination of a bond and an index fund (in the case of AXA, this performance was only slightly higher). This fact (a higher performance of the stock fund in AXA) was caused by a stock fund containing bonds and more stock indexes, which provide a greater diversification than the combination of the same bonds and only one stock index (combination of a bond fund and index fund).

I can summarize that the passively managed index fund consisting of a specific stock index or of several stock indexes is the optimal solution (in the long period) in terms of return-risk for those who like risk. For very riskaverse savers, bond funds are optimal. If the level of risk a specific investor is somewhere in between risk-averse and looking for risk, the optimal strategy is to allocate part of all finances to a bond fund and part of all finances to an index fund rather than allocating all finances to a specific mixed or stock fund (which are very often actively managed). A positive exception is the VUB, where the mixed fund and stock fund are a combination of the bond fund and index fund. A negative exception is NN implementing the active management in the index fund, which reduces the diversification of this fund compared with other indexes funds.

## Acknowledgement

This research was supported by project No. I-17-101-00 "Valuation of real estate in Slovak republic in the context of the new system of real estate taxation based on their market prices".

## References

[1] BHOOTRA, A. et al. (2015). Mutual Fund Performance: Luck or Skill? In: International Journal of Business. Vol. 20, Issue 1, pp. 52. ISSN 1083-4346.
[2] BLACK, F. - LITTERMAN, R. (1992). Global portfolio optimization. In: Financial analysts journal. Vol. 48, Issue 5, pp. 28-43. ISSN 0015-198X.
[3] CREMERS, K. J. - PETAJISTO, A. (2009). How active is your fund manager? A new measure that predicts performance. In: Review of Financial Studies. Vol. 22, Issue 9, pp. 3329-3365. ISSN 1465-7368.
[4] FERGUSON, R. (1975). Active Portfolio Management-How to Beat the Index Funds. In: Financial Analysts Journal. Vol. 31, Issue 3, pp. 1-11. ISSN 0015198X.
[5] MALKIEL, B. G. (2003). Passive investment strategies and efficient markets. In: European Financial Management. Vol. 9, Issue 1, pp. 1-10. ISSN 1468036X.
[6] MANDELBROT, B. (1960). The Pareto-Levy law and the distribution of income. In: International Economic Review. Vol. 1, Issue 2. pp. 79-106. ISSN 1468-2354.
[7] MARKOWITZ, H. (1952). Portfolio selection. In: The journal of finance. Vol. 7, Issue 1, pp. 77-91. ISSN 1540-6261.
[8] MARTÍ, C. (2011). The performance of Spanish pension plans in the period 2006 to 2010. In: African Journal of Business Management. Vol. 5, Issue 29, pp. 11686-11695. ISSN 1993-8233.
[9] ROM, B. - FERGUSON, M. - KATHLEEN, W. (1993). Post-modern portfolio theory comes of age. In: The Journal of Investing. Vol. 2, Issue 4, pp. 27-33. ISSN 1068-0896.
[10] TUDOR, C. (2012). Active portfolio management on the Romanian Stock Market. In: Procedia-Social and Behavioral Sciences. Vol. 58, pp. 543-551. ISSN 1877-0428.


[^0]:    1 Martin Dluhoš, PhD. Student, University of Economics in Bratislava, Faculty of Business Economics with a seat in Košice, Department of Quantitative Methods, Košice, Slovak Republic, e-mail: martin.dluhos8@gmail.com

