

## Slovak Economy in a Period of Recession: Nonlinear DSGE Model with Time-varying Parameters<sup>1</sup>

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### Abstract

*In this paper, we study the dramatic changes in the structure and behaviour of the Slovak economy in a period of the accession to the Euro area and the Great Recession and subsequent return to the long-run growth equilibrium. This small and very open economy is represented by nonlinear dynamic stochastic model of a general equilibrium with financial accelerator. The development of time-varying structural parameters is identified using the second order approximation of a nonlinear DSGE model. The model is estimated with the use of nonlinear particle filter. Analogous model was estimated for the economy of the Euro area. It is our goal to identify the most important changes in behaviour and underlying structure of the Slovak economy. In order to distinguish the country specific changes from broader Europe-wide trends we also compare the time-varying estimates of the Slovak economy and the Euro area.*

**Keywords:** *nonlinear DSGE model, Great Recession, monetary union, time-varying parameters, nonlinear particle filter, second order approximation*

**JEL Classification:** E32, E44, E58

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## Introduction

Slovakia became a member of the Euro area on January 1, 2009 in a period of global economic difficulties and general downturn of economic activity that is today called the Great Recession. This extraordinary confluence of events that are critical to the development of this small open economy calls out for closer investigation of possible short-term and long-term changes in the structure of the Slovak economy. The aim of this paper is to identify the most important changes of the structural parameters during and after the Great Recession and to interpret these changes in terms of behaviour of representative economic agents. We also seek to find out to what extent did the structural changes of the Slovak economy follow the common trends in the Euro area and what were its unparalleled specifics.

In this paper, we estimate a DSGE model of the Slovak small open economy with time-varying parameters and an analogous model of the economy of the Euro area. We perform a filtration of a nonlinear model with the use of nonlinear particle filter (NPF) to identify the unobserved trajectories of the time-varying structural parameters.

The results presented in this paper are an outcome of a line of research whose earlier results were published in Tvrz, Tonner and Vašíček (2012), Tvrz and Vašíček (2013; 2014). In this paper, we further elaborate on our previous research and present a comprehensive set of empirical results with focus on the Slovak economy and the economy of the Euro area.

## 1. Model

Since we focus on the period of financial and subsequent economic crisis, a DSGE model with financial frictions<sup>2</sup> is used for the analysis. In our paper we use the model framework developed by Shaari (2008) that incorporates financial accelerator mechanism proposed by Bernanke, Gertler and Gilchrist (1999) into the small open economy setting of Justiniano and Preston (2010) and Galí and Monacelli (2005). This tractable medium-sized<sup>3</sup> model of a small open economy incorporates important real as well as nominal rigidities and allows us to describe the Slovak economy in a reasonable detail. Structure of the model is quite standard, therefore, we will describe only the most important features of the model.

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<sup>2</sup> The implications of the financial frictions for the monetary policy of the Czech small open economy are investigated in detail in Ryšánek et al. (2012).

<sup>3</sup> The model contains 25 endogenous variables (including 8 observables), 8 exogenous variables, 20 structural parameters (of which 5 are calibrated and 15 are estimated and considered time-varying), exogenous processes are described using 20 estimated parameters (8 standard deviations, 4 AR(1) parameters, 12 parameters describe foreign SVAR(1) block).

The model contains households, entrepreneurs, retailers, central bank and foreign sector. The households receive wages for supplied labour, government transfers, profits made by retailers and domestic and foreign bonds returns. Domestic bonds pay fixed nominal return in domestic currency while foreign non-contingent bonds give a risk adjusted nominal return denominated in foreign currency. The debt-elastic risk premium contains exogenous AR(1) component of risk premium or uncovered interest parity (UIP) shock. The households then spend their earnings on consumption and domestic and foreign bonds acquisition.

### 1.1. Entrepreneurs

The entrepreneurs play two important roles in the model. They run wholesale goods producing firms and they produce and own the capital. Market of intermediate goods as well as capital goods market is assumed to be competitive. The wholesale goods production is affected by domestic productivity AR(1) shock and the capital goods production is subject to capital adjustment costs. Entrepreneurs finance the production and ownership of capital  $K_t$  by their net worth  $N_t$  and borrowed funds. Cost of borrowed funds is influenced by borrower's leverage ratio via external finance premium

$$EFP_t = \left( \frac{N_t}{Q_{t-1} K_t} \right)^{-\chi} \quad (1)$$

where

- $Q_t$  – real price of capital or Tobin's  $Q$ ,
- $\chi$  – financial accelerator parameter.

To maximize profit, the entrepreneurs choose the optimal level of capital and borrowed funds. Each period a proportion  $(1 - A_t^{NW})\zeta$  of entrepreneurs leaves the market and their equity  $(1 - A_t^{NW})\zeta V_t$  is transferred to households in a form of transfers.  $A_t^{NW}$  is a shock in entrepreneurial net worth. It influences the development of net worth by changing the bankruptcy rate of entrepreneurs and its positive innovations increase the survival rate of entrepreneurs. Its logarithmic deviation from steady state is assumed to evolve according to AR(1) process.  $\zeta$  is the steady-state bankruptcy rate.

### 1.2. Retailers

There are two types of retailers in the model – home goods retailers and foreign goods retailers. Both types of retailers are assumed to operate in conditions of monopolistic competition. Home good retailers buy domestic intermediate

goods at wholesale price and sell the final home goods to the consumers. Foreign good retailers buy goods from foreign producers at the wholesale price and resell the foreign goods to the domestic consumers. The difference between foreign wholesale price expressed in domestic currency and final foreign goods price, i.e. deviation from law of one price is determined by exogenous AR(1) law of one price (LOP) shock. By Calvo-type price setting and inflation indexation of the retailers the nominal rigidities are introduced into the model.

### 1.3. Central Bank

The central bank determines the nominal interest rate in accordance with following forward/backward-looking Henderson McKibbin Taylor interest rate rule (small letter variables denote deviations from steady state, i.e. gap)

$$r_t = \rho \cdot r_{t-1} + (1 - \rho) \cdot [\beta_\pi \cdot E(\pi_{t+1}) + \Theta_y \cdot E(y_{t+1})] + \varepsilon_t^{MP} \quad (2)$$

where

- $r_t$  – nominal policy interest rate,
- $\rho$  – a smoothing parameter,
- $\beta_\pi$  – weight parameter of expected inflation  $E(\pi_{t+1})$ ,
- $\Theta_y$  – weight parameter of expected output gap  $E(y_{t+1})$ .

Deviations of interest rate from the interest rate rule are explained as monetary policy i.i.d. shocks  $\varepsilon_t^{MP}$ .

### 1.4. Foreign Sector

The foreign economy variables – real output, consumer price index (CPI) inflation and nominal interest rate, are modelled using a structural VAR(1) model as described in equation (3).

$$\begin{pmatrix} y_t^* \\ \pi_t^* \\ r_t^* \end{pmatrix} = \begin{pmatrix} \rho_{y^*y^*} & \rho_{y^*\pi^*} & \rho_{y^*r^*} \\ \rho_{\pi^*y^*} & \rho_{\pi^*\pi^*} & \rho_{\pi^*r^*} \\ \rho_{r^*y^*} & \rho_{r^*\pi^*} & \rho_{r^*r^*} \end{pmatrix} \begin{pmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ r_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \sigma_{\pi^*y^*} & 1 & 0 \\ \sigma_{r^*y^*} & \sigma_{r^*\pi^*} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^{y^*} \\ \varepsilon_t^{\pi^*} \\ \varepsilon_t^{r^*} \end{pmatrix} \quad (3)$$

### 1.5. Time-varying Parameters

All the estimated model parameters are considered time-varying with the exception of shock autoregression parameters and standard deviations. Time-varying parameters are defined as unobserved endogenous variables with following law of motion

$$\theta_t = (1 - \alpha_t^\theta) \cdot \theta_{t-1} + \alpha_t^\theta \cdot \bar{\theta} + v_t^\theta \quad (4)$$

where

- $\theta_t$  – a general time-varying parameter,
- $\bar{\theta}$  – initial value of this parameter,
- $\alpha_t^\theta$  – a time-varying adhesion parameter common for all the remaining time-varying parameters,
- $v_t^\theta \sim N(0, \sigma_v^\theta)$  – exogenous innovation in the value of parameter  $\theta_t$ .

Setting of the adhesion parameter  $\alpha_t^\theta$  influences the tendency of the time-varying parameter  $\theta_t$  to return to its initial value  $\bar{\theta}$ . With  $\alpha_t^\theta = 0$ , the time-varying parameter would be defined as random walk, while with  $\alpha_t^\theta = 1$ , the parameter would be white noise centred around the initial value  $\bar{\theta}$ . For the purposes of this paper, we set the initial value of the adhesion parameter to a value of  $\alpha_0^\theta = 0.25$ .<sup>4</sup> Since the adhesion parameter is also considered time-varying we calibrated its adhesion to a fixed value of 0.05.

## 2. Estimation Technique

Nonlinear particle filter (NPF) is used to identify the unobserved states of the DSGE model, including the time-varying parameters, in this paper. In this section we briefly describe the main principles of this nonlinear particle filter.

### 2.1. Nonlinear State-space Model

In this subsection we introduce the notation used to describe the nonlinear state-space system. The state transition is described by the transition equation

$$x_t = g(x_{t-1}, w_{t-1}) \quad (5)$$

where

- $x_t \in R^{n_x}$  – the vector of unobserved states,
- $w_t \in R^{n_w}$  – the process noise with covariance matrix  $Q$ .

Observations are related to the unobserved states by the measurement equation

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<sup>4</sup> The choice of the calibration of adhesion parameter  $\alpha_0^\theta$  does not qualitatively change the results. Sensitivity analysis was performed with value of this parameter set to 0.01, 0.05, 0.10, 0.25 and 0.50. The persistence of the identified trajectories of the time-varying parameters is larger for the lower values of adhesion. However, main characteristics of the identified development (such as direction and timing of major shifts in the value of given parameter) stay more or less the same. From the point of view of interpretation in terms of the changes in the behaviour of the representative agents, the main conclusions remain valid.

$$y_t = h(x_t, v_t) \quad (6)$$

where

$y_t \in R^{n_y}$  – the vector of observations,

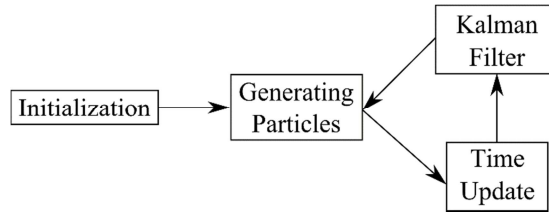
$v_t \in R^{n_y}$  – the measurement noise with covariance matrix  $R$ .

## 2.2. Nonlinear Particle Filter

Unlike basic Kalman filter that is optimal only for linear systems with Gaussian noise, the nonlinear particle filter is a more sophisticated tool that can be used even for nonlinear state-space systems with non-Gaussian noise. In this section, we provide only the basic principles of the algorithm. A detailed description can be found for example in van der Merwe et al. (2000) or Haug (2005).

Figure 1

Nonlinear Particle Filter



Source: Authors.

Figure 1 contains a diagram of the NPF algorithm. In a condensed form, the NPF algorithm can be described as follows:

- I. Initialization:  $t = 0$ , set the prior mean  $\bar{x}_0$  (steady state) and covariance matrix  $P_0$  for the state vector  $x_t$ .
- II. Generating particles: Draw a total of  $N$  particles  $x_t^{(i)}$ ,  $i = 1, \dots, N$  from distribution  $p(x_t)$  with mean  $\bar{x}_t$  and covariance matrix  $P_t$ .
- III. Time Update:  $t = t + 1$ , for each particle ( $i = 1, \dots, N$ ) propagate the particle into future with the use of (nonlinear) transition and measurement equation and calculate means  $\bar{x}_{(t|t-1)}$ ,  $\bar{y}_{(t|t-1)}$  and covariance matrices  $P_{(t|t-1)}$ ,  $P_{(y|y)}$ ,  $P_{(x|y)}$ .
- IV. Kalman filter: Calculate Kalman gain  $K_t = P_{(x|y)} P_{(y|y)}^{-1}$ ,  
 $\bar{x}_t = \bar{x}_{(t|t-1)} + K_t (y_t - \bar{y}_{(t|t-1)})$  and  $P_t = P_{(t|t-1)} - K_t P_{(y|y)} (K_t)^T P_{(y|y)} (K_t)^T$ .
- V. Continue by step II until  $t = t_{max}$ .

In our application we performed 20 runs of the NPF with 30 000 particles each<sup>5</sup> for the second order approximation of the nonlinear DSGE model.

### 2.3. Initial Values

Before the application of the NPF algorithm we estimated the model with constant parameters to obtain estimates of autoregression parameters and standard deviations of structural shocks that are considered constant even in the NPF. Also, the posterior means of the structural parameters were used as initial values of the time-varying parameters ( $\bar{\theta}$ ) in the NPF estimation. Standard deviations of time-varying parameter innovations ( $\sigma_v^\theta$ ) were set proportional to the standard deviations of posterior estimates of the model with constant parameters. Constant model parameters were estimated using Random Walk Metropolis-Hastings algorithm as implemented in Dynare toolbox for Matlab. Two parallel chains of 1 000 000 draws each were generated during the estimation. First 50% of draws were discarded as burn-in sample. The scale parameter was set to achieve acceptance rate around 30%.

## 3. Data

Quarterly time series of eight observables were used for the purposes of estimation. These time series cover the period between the second quarter of 1999 and the third quarter of 2013 and contain 58 observations. Input data expressed in per cent deviations from steady state are depicted in Figure 2.

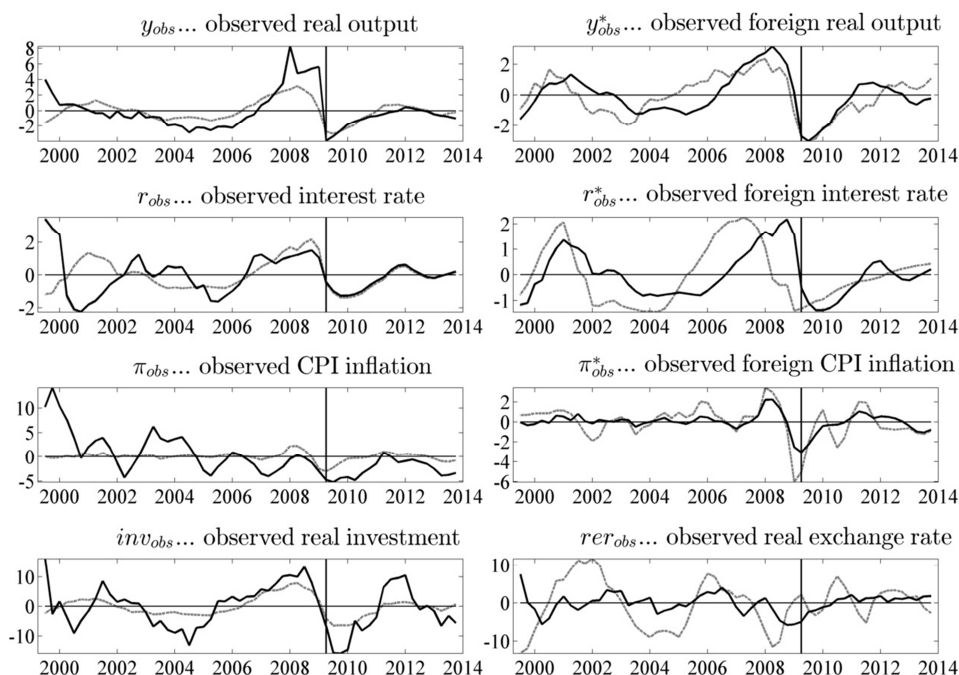
Time series of real gross domestic product (GDP), harmonised consumer price index (CPI), 3-month policy interest rate and real investment are used for the domestic economy. The foreign economy is represented by the 17 Euro area countries in the case of the Slovak economy while the rest of the world is represented by the economy of the United States in the case of Euro area. Seasonally adjusted time series of real GDP, CPI and 3-month policy interest rate are used. Time series of SKK/EUR and EUR/USD real exchange rate is also used. These seasonally adjusted time series were obtained from the Eurostat, National Bank of Slovakia, European Central Bank and Federal Reserve Bank of St. Louis databases.

The original time series were transformed prior to estimation so as to express the logarithmic deviations from their respective steady states. Logarithmic deviations of most observables from their trends were calculated with the use of Hodrick-Prescott (HP) filter.<sup>6</sup> Time series of the domestic and foreign CPI inflation were demeaned.

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<sup>5</sup> Setting of the particle simulation is chosen as a compromise between accuracy and time demands of the calculation. By experimenting with the setting of the particle filter algorithm we found out that the results do not change significantly when the number of runs or the number of particles is further increased.

Figure 2  
Observed Variables



Note: Black solid line – SK, grey dashed line – EA, vertical line – 2009Q1.

Source: Authors' calculations.

#### 4. Calibration

We decided to calibrate several deep structural parameters because they are difficult to estimate. These parameters were assigned values commonly reported in the literature. The value of discount factor  $\beta$  of 0.995 implies real interest rate of approximately 2% p.a., similar values are reported by Christensen and Dib (2008) or Christiano, Trabandt and Walentin (2011). Capital share in production  $\alpha$  corresponds to the national income share of capital of 0.35. Values around one third are usually used in the literature, see Adolfson, Laséen and Villani (2007) or Christiano, Trabandt and Walentin (2011). Capital depreciation rate  $\delta$  of 2.5% per quarter is also standard and follows Christensen and Dib (2008). Households' share of the labour supply  $\Omega$  is calibrated according to Shaari (2008) to 99% which leaves the remaining 1% of the labour supply to be provided by the entrepreneurs. Calibration of parameter  $\mu$  follows Shaari (2008) as well.

<sup>6</sup> Parameter of the HP filter  $\lambda$  was set to 1 600, a value commonly used for quarterly data.



## 5. Empirical Results

### 5.1. Posterior Estimates

Prior and posterior densities of estimated parameters are presented in Table 1. Table 2 shows the posterior estimates of the autocorrelation coefficients of the AR(1) shock processes and the standard deviations of shock innovations. Prior densities are the same for both economies in order to identify the structural differences in the data. Our posterior estimates are similar to the values obtained by Senaj, Výškrabka and Zeman (2010), who estimated two-country DSGE model for the Slovakia and Euro area, and are comparable to the calibration used in Zeman and Senaj (2009).

Table 1

**Priors and Posteriors, Structural Parameters**

Parameter	Distribution	Prior		SK Posterior		EA Posterior		
		Mean	Std	Mean	Std	Mean	Std	
<i>Structural parameters</i>								
$\gamma$	Habit persistence	B	0.60	0.05	0.59	0.05	0.67	0.06
$\Psi$	Inv. elast. of lab. supply	G	2.00	0.50	1.48	0.36	1.00	0.23
$\psi^B$	Debt-elastic risk premium	G	0.05	0.02	0.03	0.01	0.02	0.01
$\eta$	Home/foreign elast. subst.	G	0.65	0.10	0.62	0.09	0.43	0.02
$\kappa$	Price indexation	B	0.50	0.10	0.48	0.09	0.41	0.09
$\gamma$	Pref. bias to foreign goods	B	0.40	0.15	0.44	0.06	0.27	0.04
$\theta_H$	Home goods Calvo	B	0.70	0.10	0.80	0.02	0.81	0.03
$\theta_F$	Foreign goods Calvo	B	0.70	0.10	0.77	0.03	0.82	0.04
$\psi^I$	Capital adjustment costs	G	8.00	3.00	13.68	2.77	16.24	2.89
<i>Financial frictions</i>								
$\Gamma$	Capital/net worth ss ratio	G	2.00	0.50	1.34	0.15	1.31	0.11
$\zeta$	Bankruptcy rate	B	0.025	0.015	0.06	0.02	0.03	0.01
$\chi$	Financial accelerator	G	0.05	0.015	0.05	0.01	0.04	0.01
<i>Taylor rule</i>								
$\rho$	Interest rate smoothing	B	0.70	0.10	0.81	0.03	0.73	0.04
$\beta_\pi$	Inflation weight	G	1.50	0.20	1.83	0.21	1.83	0.19
$\theta_y$	Output gap weight	G	0.50	0.20	0.25	0.06	0.22	0.05

Note: B – beta distribution, G – gamma distribution.

Source: Authors' calculations.

The differences in the posterior estimates of the Slovak economy and the economy of the Euro area are intuitive and seem to capture the specifics of the two economies. Habit persistence  $\gamma$  is higher in the Euro area, which means that the households in the Euro area are more sensitive to the variations in the level of consumption. Lower inverse elasticity of labour supply  $\Psi$  in the Euro area implies more elastic labour supply and, therefore, stronger response of labour supply to the variations in wages. Higher debt-elastic risk premium  $\psi^B$  in Slovakia

corresponds to the fact that the Slovak bonds are perceived as riskier than the bonds of Euro area and their yield is more sensitive to the development of net foreign assets. Elasticity of substitution between domestic and foreign intermediate goods  $\eta$  is lower than one in both economies, which suggests that the domestic and foreign goods work as complements in the production of final goods. Indexation of prices to the past inflation  $\kappa$  is lower in the Euro area because of less volatile development of the CPI inflation. Preference bias to foreign goods  $\gamma$  also measures the openness of the economy, and therefore, it is not a surprise that the estimate for the Slovak economy is much higher than the estimate of this parameter for the much bigger economy of the Euro area. Calvo parameters of domestic intermediate goods  $\theta_H$  and imported intermediate goods  $\theta_F$  are estimated to be slightly higher in the Euro area, which suggests higher price stickiness in the Euro area. Capital adjustment costs  $\psi'$  are lower in the Slovak economy, hence it is less costly to invest and make new capital. This result corresponds to the fact that the Slovak economy is an emerging market economy with available investment opportunities.

Table 2

**Priors and Posteriors, Shock Processes**

Parameter	Distribution	Prior		Posterior	Mean	
		Mean	Std	SK	EA	
<i>Autocorrelation coefficients</i>						
$\rho_Y$	Domestic productivity	B	0.50	0.20	0.56	0.45
$\rho_{UIP}$	Uncovered interest parity	B	0.50	0.20	0.70	0.77
$\rho_{LOP}$	Law of one price	B	0.50	0.20	0.74	0.83
$\rho_{NW}$	Entrepreneurial net worth	B	0.50	0.20	0.63	0.40
<i>Standard deviations</i>						
$\sigma_Y$	Domestic productivity	IG	1.00	$\infty$	1.46	0.49
$\sigma_{UIP}$	Uncovered interest parity	IG	0.50	$\infty$	0.46	0.28
$\sigma_{LOP}$	Law of one price	IG	0.50	$\infty$	3.80	5.33
$\sigma_{NW}$	Entrepreneurial net worth	IG	1.00	$\infty$	2.61	1.79
$\sigma_{MP}$	Monetary policy	IG	0.50	$\infty$	0.16	0.10
$\sigma_{y^*}$	Foreign output	IG	1.00	$\infty$	0.55	0.56
$\sigma_{\pi^*}$	Foreign inflation	IG	0.50	$\infty$	0.14	0.31
$\sigma_{r^*}$	Foreign interest rate	IG	0.50	$\infty$	0.08	0.11

Note: B – beta distribution, IG – inverse gamma distribution.

Source: Authors' calculations.

Capital/net worth steady state ratio  $F$  is estimated to be very similar in both economies. The posterior estimates suggest that on average the entrepreneurs finance approximately 25% of their capital stock by external funds. Bankruptcy rate of entrepreneurs  $\zeta$  is lower in the more stable economic environment of the Euro area as compared to the emerging Slovak economy. Financial accelerator  $\chi$  is slightly higher in the Slovak economy, which suggests that the commercial

banks are more sensitive to the changes in the capital/net worth ratio (leverage ratio) of the firms. The parameters of the Taylor rule are estimated very similarly in both economies. This is given by the fact that Slovakia was already a member of the Euro area at the end of our data sample and its monetary policy was determined by the ECB, i.e. it was the same as in the rest of the Euro area. Also, as Slovakia prepared for the accession to the monetary union, its monetary policy converged to that of the ECB even before the entry.

Following equations capture the estimated foreign SVAR(1) block in the model of the Slovak economy (7) and the economy of the Euro area (8).

$$\begin{pmatrix} y_t^* \\ \pi_t^* \\ r_t^* \end{pmatrix} = \begin{pmatrix} 0.90 & 0.39 & -0.59 \\ 0.11 & 0.26 & -0.54 \\ 0.06 & 0.06 & 0.59 \end{pmatrix} \begin{pmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ r_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0.16 & 1 & 0 \\ 0.09 & -0.06 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^{y^*} \\ \varepsilon_t^{\pi^*} \\ \varepsilon_t^{r^*} \end{pmatrix} \quad (7)$$

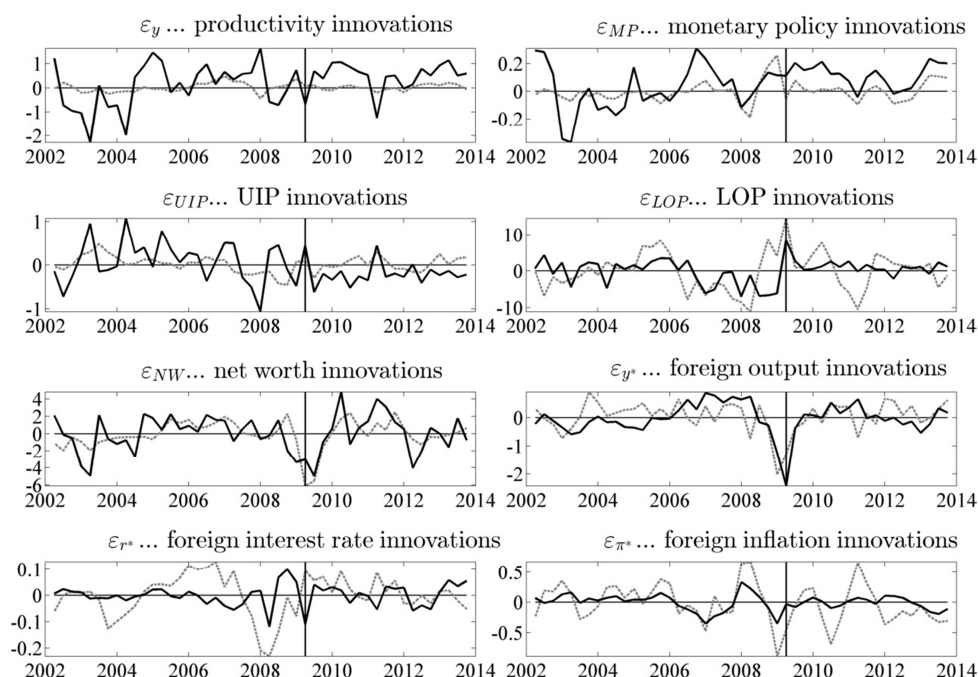
$$\begin{pmatrix} y_t^* \\ \pi_t^* \\ r_t^* \end{pmatrix} = \begin{pmatrix} 0.81 & 0.23 & 0.46 \\ -0.03 & 0.27 & 0.35 \\ 0.03 & -0.02 & 0.83 \end{pmatrix} \begin{pmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ r_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0.26 & 1 & 0 \\ 0.10 & 0.02 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^{y^*} \\ \varepsilon_t^{\pi^*} \\ \varepsilon_t^{r^*} \end{pmatrix} \quad (8)$$

## 5.2. Shock Innovations

Shock innovations obtained by the NPF filtration are depicted in Figure 3. Innovations of the foreign output capture the downturn of foreign demand that began in 2008 and culminated in 2008Q4 in Euro area and in 2009Q1 in the Slovakia. Negative innovations in the net worth shock show problems in the financial sector during 2008 – 2009 as well. Also, the positive innovations in the law of one price shock are distinct in the period of 2008 – 2009 crisis. This development can be linked to closing law of one price gap and real exchange rate depreciation. These results suggest, that the immediate impact of the crisis in the Slovak economy and in the Euro area came in a form of foreign demand slump accompanied by a worsening conditions in the credit market leading to increased interest rate spreads.

Torój and Konopczak (2012) identified the foreign demand shock to be the main driving force of the Slovak economy in the late 2008 and early 2009, however, their DSGE model did not include the financial frictions. Giri (2014) estimated a closed economy DSGE model of the Euro area with financial frictions proposed by Gerali et al. (2010) and found that a large part of the development of the endogenous variables after 2007 can be explained by financial shocks, including the rise of interest rates on the credit market in 2008.

Figure 3  
Filtered Shock Innovations



Note: Black solid line – SK, grey dashed line – EA, vertical line – 2009Q1.

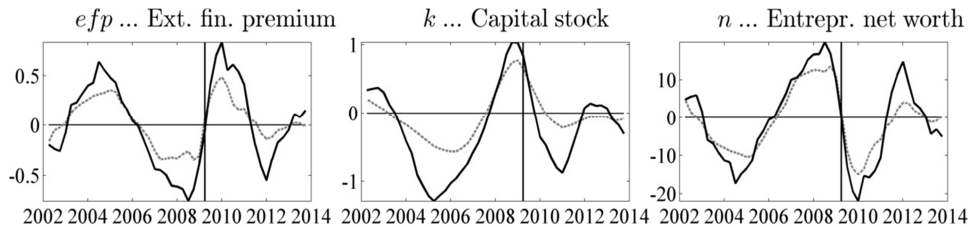
Source: Authors' calculations.

### 5.3. Financial Sector Variables

Figure 4 depicts selected unobserved endogenous variables describing the development on the financial market in the Slovak economy and in the economy of the Euro area. Decline of the entrepreneurial net worth in 2008 and 2009 caused a sharp increase of the external finance premium demanded by the commercial banks and lead to an increase of the interest rate spreads. As the conditions at the financial markets tightened, the growth of the capital stock slowed down and in 2009 turned to a decline. According to the filtered trajectories, the development of the financial market variables was very similar in both economies.

Giri (2014) identified a significant interbank market riskiness shock in the Euro area that pushed the interest rate spread between the policy interest rate and interbank interest rate up especially in late 2008 and 2009. The propagation of this interbank shock in the economy causes a decline of total output, investment and capital stock while at the same time lowering the value of the collateral the entrepreneurs could use to acquire credit. According to Giri (2014), this development was strengthened by a credit market riskiness shock in 2009 that further exacerbated the situation.

Figure 4  
**Filtered Financial Sector Variables**



Note: Black solid line – SK, grey dashed line – EA, vertical line – 2009Q1.

Source: Authors' calculations.

#### 5.4. Time-varying Parameter Estimates

Filtered trajectories of selected time-varying structural parameters are presented in Figure 5 and Figure 6, which depicts the parameters of the Taylor interest rate rule. The development of the time-varying parameters is expressed in per cent deviations from initial values.

Some parameters that were estimated as time-varying showed only negligible deviations from their initial values,<sup>7</sup> and therefore, their trajectories are not presented. These parameters, that can be considered deep, are namely the Calvo parameters  $\theta_H$  and  $\theta_F$  the habit in consumption  $\gamma$ , inflation indexation  $\kappa$  and elasticity of substitution between domestic and foreign goods  $\eta$ .

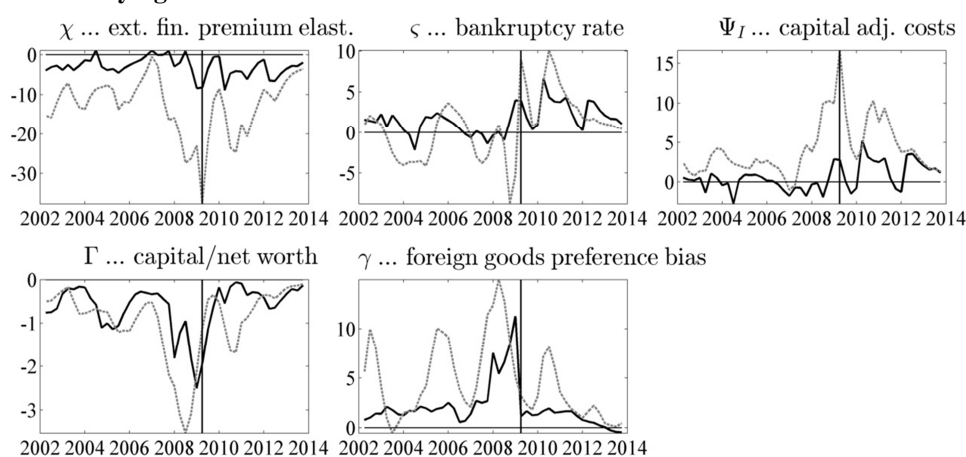
Most of the parameters of the financial sector showed significant deviations from the initial values, especially the external finance premium elasticity  $\chi$ , bankruptcy rate  $\varsigma$  and capital adjustment costs  $\psi^I$ . As the entrepreneurial net worth increased in 2006 – 2007, the interest rate spreads lowered, approached the zero lower bound and became less sensitive to the variations in the leverage ratio, i.e.  $\chi$  decreased. Positive development of the entrepreneurs is also reflected in the decreasing estimate of the steady state leverage ratio  $I^*$  in this period, which would suggest that the firms were becoming slightly more independent of the external financing. At the same time, as the capital stock deviated above the steady state it was increasingly difficult to find suitable investment opportunities and the capital adjustment costs  $\psi^I$  increased. The onset of the crisis in 2009 meant a correction and the values of the financial sector parameters returned to the vicinity of their initial values. As the overall economic conditions worsened, the bankruptcy rate  $\varsigma$  slightly increased in 2009 but the biggest increase of the bankruptcy rate did not occur until the second quarter of 2010. This delay in the reaction of the number of bankruptcies is not unexpected as the firms that were

<sup>7</sup> I.e. less than one 1% of the initial value.

getting into difficulties during the crisis would continue to operate for as long as possible. According to the filtered trajectories of the financial parameters, the situation in the financial sector seems to be returning to normal as the parameter values returned to the initial values during 2013.

Figure 5

**Time-varying Parameter Estimates**



Note: Black solid line – SK, grey dashed line – EA, vertical line – 2009Q1.

Source: Authors' calculations.

Apart from the financial parameters, the foreign goods preference bias  $\gamma$  (openness parameter) also showed interesting behaviour. During 2007 – 2008, its value increased and together with the volume of international trade the interdependency of world economies strengthened. In the beginning of 2009 came a sharp decline, the volume of international trade dropped and globalization trends weakened.<sup>8</sup>

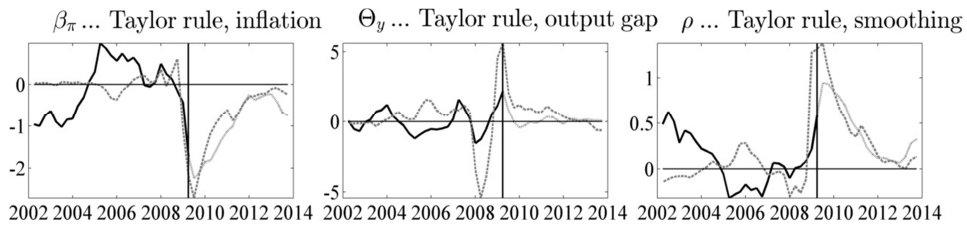
The estimated trajectories of the Taylor interest rate rule show a decline in the weight of inflation during 2008 that can be explained by the zero lower bound that limited the expansionary response of the monetary policy in this disinflationary period. Filtered trajectory of the weight of the output gap shows a decline in 2007 when the economies were beginning to overheat. This result would suggest that the monetary policy was slightly looser in the period of economic boom. In the end of 2008 the weight of the output gap increased as the central banks lowered the interest rates in response to a fall in GDP and fading inflation pressures.

<sup>8</sup> This result is in line with Vašíček, Tonner and Polanský (2011) who examined the stability of structural parameters of a DSGE model of the Czech small open economy. It can be related to the estimates of the trade openness technology.

Remaining Taylor rule parameter of the interest rate smoothing increased very slightly in 2008 when the interest rates were dropped and kept at near-zero levels for a prolonged period of time.

Figure 6

**Time-varying Parameter Estimates, Taylor Rule**



Note: Black solid line – SK, grey dashed line – EA, vertical line – 2009Q1.

Source: Authors' calculations.

Overall, the main characteristics and often even the scale and timing of the changes of the time-varying parameters are the same in both the Slovak economy and the economy of the Euro area. There are, however, distinct differences in the variation of some financial sector parameters. Especially the external finance premium  $\chi$  and capital adjustment costs  $\psi^I$  fluctuated much more strongly in the Euro area than in the Slovak economy. Also, the development of some parameters, such as capital/net worth steady state ratio  $I$  or foreign goods preference bias  $\gamma$ , followed a similar path as in the Euro area with a lag of 2 – 4 quarters. These results correspond to considerably more dramatic course of the financial crisis in the Euro area as compared to relatively sheltered Slovak economy.

## Conclusion

In this paper, we estimated two DSGE models of a small open economy with financial accelerator for the Slovak economy and for the Euro area. We applied a two-step approach to the estimation of the time-varying parameters. First, we estimated the models with time-invariant parameters using the Random Walk Metropolis-Hastings algorithm and then employed the obtained results for the initial setting of the second estimation technique of Nonlinear Particle Filter that was used for the estimation of the models with time-varying parameters.

Obtained results suggest that there is a subset of structural parameters that can be considered deep. The estimated trajectories of these parameters showed only negligible deviations of less than 1% of the initial value. This set of parameters includes the Calvo parameters, the habit in consumption, inflation

indexation and elasticity of substitution between domestic and foreign goods. The development of the financial sector parameters together with the parameter of foreign goods preference bias (openness parameter) and the parameters of the Taylor interest rate rule showed substantially more dynamic behaviour, especially during the financial crisis of 2007 and the Great Recession. The estimated changes of the time-varying parameters were similar in the Slovak economy and the economy of the Euro area with some differences in the magnitude of the deviations and timing. Most notably, the financial sector parameters of external finance premium elasticity and capital adjustment costs fluctuated much more strongly in the Euro area than in the Slovak economy, which corresponds to considerably more dramatic course of the financial crisis in the Euro area as compared to relatively unscathed financial sector of the Slovak economy. Also, the development of some parameters, such as capital/net worth steady state ratio or foreign goods preference bias, followed a similar path as in the Euro area with a distinct delay of 2 – 4 quarters.

We found that the behaviour of the representative economic agents probably changed to a certain extent during the Great Recession. Therefore, the economic development in that period was not determined solely by exogenous (especially foreign demand) shocks, but the adaptation of the representative economic agents to the unstable economic conditions also played a role. Nevertheless, the deviation was only temporary and it mostly faded away by 2013. According to the filtered trajectories of the structural parameters, the situation in the Slovak economy and in the economy of the Euro area seems to be largely stabilized as the parameter values returned to the vicinity of their respective initial values.

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