## Katarina Sakalova ${ }^{1}$, Ingrid Ondrejkova Krcova ${ }^{2}$ PROFIT PARTICIPATION OF POLICYHOLDERS BY THREE FACTOR CONTRIBUTION METHOD

The aim of this article is to describe and analyse three-factor contribution method used to determine and allocate profit to policyholders. This method is based on the idea that each class of policyholders has its share of divisible surplus which is proportionate to contribution of this class to surplus.
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## Катерина Сакалова, Інгрід Ондрейкова Крцова <br> РОЗПОДІЛ ПРИБУТКІВ МІЖ ПОЛІСОТРИМАЧАМИ ЗГІДНО З МЕТОДОМ ТРИФАКТОРНОЇ СПЛАТИ ВНЕСКІВ

У статті описано та проаналізовано метод трифакторного внеску, що використовується для визначення та розподілу прибутків між полісотримачами. Цей метод базується на ідеї про те, що кожсна категорія полісотримачів має власну частку розподіленого прибутку, пропорційну внескам відповідної категорії полісотримачів. Ключові слова: метод сплати внесків; дивіденди; полісотримач.
Форм. 2. Рис. 2. Табл. 13. Літ. 14.

## Катерина Сакалова, Інгрид Ондрейкова Крцова РАСПРЕДЕЛЕНИЕ ПРИБЫЛЕЙ МЕЖДУ ПОЛИСОДЕРЖАТЕЛЯМИ СОГЛАСНО МЕТОДУ ТРИФАКТОРНОЙ ОПЛАТЫ ВЗНОСОВ

В статье описан и проанализирован метод трифакторных взносов, который используется для определения и распределения прибыли между полисодержателями. Данный метод основан на идее о том, что каждая категория полисодержателей имеет собственную долю распределённой прибыли, пропорциональную взносам соответствующей категории полисодержателей.
Ключевые слова: метод оплаты взсносов; дивиденды; полисодержатель.
Introduction. The basic idea of with a profit policy (UK) or participating policy (USA) is that policyholder accepts lower guarantee (sum assured) for the same premium, than he/she would accept under an equivalent not-profit policy. But in return for lower guarantee, he/she has the right to share profits of the life office. In order to qualify for it the policyholder will pay a higher premium than in the case when benefit is an original amount stated in a contract. In the latter case the contract is called without profit contract. A number of methods are used in different parts of the world to determine and allocate profit to policyholders. The most obvious are: additions to benefits or the so called bonus system (in the Commonwealth), contribution method or dividends (USA, Canada) and the revalorisation method (Europe).

Literature review. Special attention in literature is given to analysis of participation of policyhoders in profits of a life insurance company. There exist a lot of approaches to participation in profit but this study will concern only three factor con-

[^0]tribution method. In this paper we present some relevant sources, for example: N.L. Bowers et al. (1986), G. Luffrum (1992), K. Black and H.D. Skipper (1994). Another author described this method more from the stochatic point of view (Norberg, 2002). Following hime are the authors (Cipra, 2006; Promislow, 2011). All these authors applied first sight different variants of the fomula but lessentially one formula. This study is based on one form of this formula.

Basic notions and problem statement. According to contribution method profit is given by a contract in the form of a dividend. This method is based on the analysis of sources of insurer surplus, so it is retrospective. There exist many reasons, why a life office wants to analyse surplus or profit arising from the contract. One reason is distribution of profit between policyholders by identifying the sources of this profit. Profit arises because of divergences between assumptions and what actually happened. Actual sources in any particular case depend on the type of a contract.

Contribution principle is based on the idea that each class of policyholders has a share of divisible surplus proportionate to contribution of this class to surplus. This does not require the return of all surplus. It requires the amount of allocated surplus to be in the same proportion as contributed surplus. Dividends are obviously smaller and sources of surplus over for example interest rate are reduced in extent. In the interests of simplicity we usually work with three major sources of surplus: mortality savings, excess interest and loading savings. So the most used approach is thus called the three-factor contribution method.

This method is extensively used in the USA and Canada and is little used in the United Kingdom. By this method premium scales do not include margin for addition to benefit like the bonus system in the UK.

Dividends are typically paid out on the annual basis. Most policies also include a final or terminal payment paid out to a holder when a contract matures. Sometimes dividend would be converted into an addition to benefit, instead of being paid out in cash each year. Dividends can be also paid in a form of extra dividends or terminal dividends in addition to regular annual dividends. Extra dividend may consist of a single payment made after a policy has been in force a specified number of years. Single payment extra dividend is generally used when no first year dividend is paid, thus extra dividend serves as a substitute. Some insurers also pay terminal dividends.

Individual share of a divisible surplus is computed from the so-called dividend formula. Traditionally this formula expresses a mathematical form of contribution made by a particular policy to surplus from these three sources. Therefore, this formula does not concern directly other sources of surplus. The amount of dividend $D_{t}$ at the end of a policy year $t$ to be given under a particular contract is calculated using a formula from (Luffrum, 1992), for $t=1,2, \ldots, n$ :

$$
\begin{gather*}
\text { Dividend = Interest factor + Mortality factor + Loading factor } \\
D_{t}=I_{t}+M_{t}+L_{t}=\left(V_{t-1}+P\right)\left(i^{\prime \prime}-i\right)+\left(q-q^{\prime \prime}\right)\left(S-V_{t}\right)+E(1+i)-E^{\prime \prime \prime}\left(1+i^{\prime \prime}\right) \tag{1}
\end{gather*}
$$

where $V_{t-1}$ - value of a contract at the beginning of year $t$ (on the basis of valuation); $V_{t}$ - value of a contract at the end of year $t$ (on the basis of valuation); $P$ - gross annual premium; $i^{\prime \prime}$ - actual rate of interest earned or the so-called dividend rate in year $t$; $i$ - valuation basis (reserve) rate of interest in year $t ; q^{\prime \prime}$ - actual rate of mor-
tality; $q$ - valuation basis rate of mortality; $S$ - sum assured; $E^{\prime \prime}$ - actual experienced under the contract in year $t ; E$ - expenses under the contract according to the valuation basis in year $t$.

Double accents denote actual experience, unaccented symbols denote valuation bases.

Another formula (Black and Skipper, 1994), with only one small difference in calculating one factor - loading factor - is as follows:

$$
\begin{equation*}
D_{t}=I_{t}+M_{t}+L_{t}=\left(V_{t-1}+P\right)\left(i^{\prime}-i\right)+\left(q-q^{\prime}\right)\left(S-V_{t}\right)+(B-P-E)\left(1+i^{\prime}\right), \tag{2}
\end{equation*}
$$

where $V_{t-1}$ - reserve of a contract at the beginning of year $t$ (on the valuation basis); $V_{t}$ - terminal reserve of contract at the end of year $t$ (on the valuation basis); $p-$ valuation net annual premium; $B$ - gross annual premium; $i^{\prime}$ - actual rate of interest earned or the so-called dividend rate in year $t ; i$ - reserve interest rate in year $t ; q^{\prime}-$ actual rate of mortality; $q$ - reserve mortality rate; $S$ - sum assured; $E$ - expenses charged under the contract in year $t$.

Simple accents denote actual experience, unaccented symbols denote valuation bases.

The interest factor $I_{t}$ is the simplest element but has a strong influence on the dividend, particularly in a long term where the initial reserve is large. From several complications related to the application of dividends the most important refers to evaluation of a dividend rate. It is obvious that the final dividend rate is less than the interest rate used in asset rate calculations. We will describe briefly three methods used by insurers while setting an appropriate dividend rate.

The investment generation method is used by insurers varying the dividend rate by the length of the time the contract has been in force. These insurers segregate contracts into different groups (generations) with respect to the age of the contract. Then the dividend rate for each group depends on duration.

By the portfolio average method insurer uses the same dividend rate for each with profit policy. Of course the advantage of this method is that dividend scale is more stable.

The main idea behind the third method is that excess interest rate depends directly on any policy loan activity and rate. For example, if a policyholder borrows as part of a policy contract and at interest rate relative to market rate, it implies lower dividends. With the constant dividend rate of interest, the interest factor increases with duration, if reserves increase.

The mortality factor $M_{t}$ (and also mortality contribution to surplus) is normally smaller than the interest factor and makes a decreasing contribution with greater durations. Of course, it is a more significant factor for term assurance policies. In practice many insurers express the mortality factor as a percentage of assumed cost of insurance (the net amount at risk at the end of the year times probability of death during this year). This percentage can vary from higher values (50\%) at younger ages to lower values ( $5 \%$ ) at lower ages.

The loading factor $L_{t}$ of the dividend depends on the difference between actual expenses and valuation expenses. But there exist wide differences among life offices in charging gross premiums and expenses. So, the importance of this factor varies with an insurer.

Any of the three preceding terms from dividend formula may be negative in any year. Also in practice again in the interests of simplicity the experienced rates are often based on the average of recent trends, especially for the factors which tend to fluctuate.

Given now the ready availability of significant computer resources, a more complicated approach would usually be used as in (Cipra, 2006; Sekerova and Bilikova, 2007; Sakalova, 2001, 2006). For example, by taking into account more factors than does the simple formula above. Whichever method is used to calculate the dividend, a terminal dividend may also be given to reflect any profit, which has not yet been given to a policyholder.

In the following part we will consider briefly operations from the preceding formula from Luffrum. We will work with two typical contracts. Term assurance (Krcova and Sakalova, 2006) and a best-selling life insurance product - endowment.

We will deal with the cases of regular premium payments and assume these premiums are level, meaning each of the same amount and payable annually in advance through the whole time of assurance. After calculation of each factor for these two life insurance contracts, we will analyse and discuss them.

## Key results.

Term assurance. A person aged 40 purchases a term assurance with the term of 10 years and the sum assured 5,000 EUR payable at the end of the year of death.

Elements of the pricing bases are as follows. Mortality follows Statistical office (archiv.statistics.sk) unisex mortality tables for the year 2012 (more recent data are not available). Life office calculates with the following costs: initial expenses $=$ 100 EUR, marketing expenses = 12 EUR, first year commission is $60 \%$ from the level premium paid in the first year. Per policy renewal expenses for the second year are 7 EUR annually. Particular costs as absolute values are the average costs for all contracts of the same type.

Financial assumptions are: technical interest rate is $1.9 \%$ annually (www.nbs.sk), the rate of inflation $1.4 \%$ (appropriate value for years around 2012) annually applied to per policy renewal expenses. Regular level yearly netto premium is calculated from our assumptions using classical methods of actuarial mathematics.

Table 1 presents the values for the probability of death $q_{x+t-1}$ for various ages. With respect to the fact that we will be modelling the real mortality by rejuvenation, here are the values since the age of 37 . Table 2 shows the reserves for our 40 y.o. person at the beginning of each year of the term of a contract calculated from the preceding assumptions. It should be noted that the reserve at the beginning of the first year and at the end of the tenth year is zero. Note also that in case of term assurance reserves as compared to sum assured of this type of product are small. Of course, term assurance is a typical risk insurance product.

Influence of the actual values of interest rate and mortality on the interest rate factor $I_{t}$ and the mortality factor $M_{t}$ of dividend we list in the following tables. Before starting the analysis of the achieved results it should be noted that in all the tables for term assurance the death factor (component) of the whole dividend is in the last year relatively high compared to other years. It is due to a zero reserve at the end of the tenth year.

Table 1. Probabilities of dying for different ages, own calculations

| Age | $q_{x+t-1}$ |
| :---: | :---: |
| 37 | 0.001282 |
| 38 | 0.001399 |
| 39 | 0.001500 |
| 40 | 0.001610 |$\quad$| Age | $q_{x+t-1}$ |
| :---: | :---: | :---: |
| 41 | 0.001812 |
| 42 | 0.002151 |
| 43 | 0.002425 |$\quad$| Age | $q_{x+t-1}$ |
| :---: | :---: |
| 44 | 0.002686 |
| 45 | 0,002920 |
| 46 | 0.003201 |$\quad$| Age | $q_{x+t-1}$ |
| :---: | :---: | :---: |
| 47 | 0.003547 |
| 48 | 0.003995 |
| 49 | 0.004440 |

Table 2. Reserves for term assurance, own calculations

| Year $t$ | Reserve $V_{t-1}$ at the <br> beginning of the year |
| :---: | :---: |
| 1 | 0.00 |
| 2 | 6.08 |
| 3 | 11.29 |
| 4 | 14.90 |
| 5 | 17.22 |$\quad$|  | Reserve $V_{t-1}$ at the <br> beginning of the year |  |
| :---: | :---: | :---: | :---: |
|  | 7 | 18.29 |
|  | 8 | 18.22 |
|  | 9 | 16.73 |

Let us assume that the actual interest rate is increasing to $2 \%$ and mortality is decreasing. Decrease of assumed mortality is modelled by standard mortality tables assuming that the life is shorter a certain number of years (3) than the original age (we will sometimes use the notion: 3 years rejuvenation). The actual administrative costs decreased by 3 EUR. Suppose that the assumed inflation of costs remains unchanged. From the formulas for the dividend factors follows that the interest rate affects the interest factor as well as the loading factor of the dividend. Given the changes we have achieved the following results: Table 3.

Table 3. 3 years younger, interest increase of 0.001, decrease of costs = 3 EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.013860365 | 1.63849 | -0.12032 | 1.532034 |
| 2 | 0.019944133 | 2.057139 | 3.095742 | 5.172825 |
| 3 | 0.025145497 | 3.242792 | 3,139082 | 6.40702 |
| 4 | 0.028761289 | 4.061373 | 3.18303 | 7.273163 |
| 5 | 0.031084737 | 4.356269 | 3.227592 | 7.614946 |
| 6 | 0.032154201 | 3.832453 | 3.272778 | 7.137385 |
| 7 | 0.032077839 | 3.8685 | 3.318597 | 7.219175 |
| 8 | 0.030594613 | 4.294311 | 3.365057 | 7.689963 |
| 9 | 0.027347542 | 5.365448 | 3.412168 | 8.804964 |
| 10 | 0.021784347 | 6.191607 | 3.459939 | 9.67333 |

As we see a loading factor $L_{t}$ is in the first year negative due, on the one side, to the unchanged initial, marketing expenses and commission, but on the other side to the increasing actual interest rate (see the role of interest rate in the formula for loading factor). Because term assurance is a typical risk assurance, a change of the assumed mortality has a significant impact on the mortality factor $M_{t}$ and also on the mortality contribution to the whole dividend.

To confirm our statements about mortality and its impact on the dividend we introduce Table 4, in which a person is two years younger and Table 5, in which a person is one year younger.

Table 4. 2 years younger, interest increase of $\mathbf{0 . 0 0 1 ,}$ decrease of costs $=3$ EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.013860365 | 1.051865 | -0.12032 | 0.94541 |
| 2 | 0.019944133 | 1.552969 | 3.095742 | 4.668655 |
| 3 | 0.025145497 | 2.696588 | 3.139082 | 5.860816 |
| 4 | 0.028761289 | 3.056202 | 3.18303 | 6.267993 |
| 5 | 0.031084737 | 2,666471 | 3.227592 | 5.925148 |
| 6 | 0.032154201 | 2.466684 | 3.272778 | 5.771616 |
| 7 | 0.032077839 | 2.56737 | 3.318597 | 5.918045 |
| 8 | 0.030594613 | 3.127264 | 3.365057 | 6.522916 |
| 9 | 0.027347542 | 3.961889 | 3.412168 | 7.401405 |
| 10 | 0.021784347 | 4.461672 | 3.459939 | 7.943395 |

Table 5. 1 year younger, interest increase of 0.001 , decrease of costs = 3 EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.013860365 | 0.54717 | -0.12032 | 0.440714 |
| 2 | 0.019944133 | 1.006369 | 3.095742 | 4.122055 |
| 3 | 0.025145497 | 1.690948 | 3.139082 | 4.855176 |
| 4 | 0.028761289 | 1.366041 | 3.18303 | 4.577832 |
| 5 | 0.031084737 | 1.300723 | 3.227592 | 4.5594 |
| 6 | 0.032154201 | 1.165941 | 3.272778 | 4.470873 |
| 7 | 0.032077839 | 1.401082 | 3.318597 | 4.751757 |
| 8 | 0.030594613 | 1.725268 | 3.365057 | 5.120921 |
| 9 | 0.027347542 | 2.234696 | 3.412168 | 5.674212 |
| 10 | 0.021784347 | 2.223429 | 3.459939 | 5.705152 |

The following two tables illustrate how changes in interest rates influence the interest rate factor $I_{t}$ of the whole dividend. However, we can state that the impact of interest rates on the overall size of dividends for this type of insurance is generally small. Also as we see although the interest rate impact the loading factor, this impact is negligible. Whatever the interest rate increases from $2.4 \%$ (Table 6) to $2.9 \%$ (Table 7), other assumptions from Table 3 remain unchanged.

Table 6. 3 years younger, interest increase of 0.005 , decrease of costs $=3$ EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.069301827 | 1.63849 | -0.60158 | 1.10621 |
| 2 | 0.099720664 | 2.057139 | 3.079518 | 5.236377 |
| 3 | 0.125727484 | 3.242792 | 3.122631 | 6.49115 |
| 4 | 0.143806444 | 4.061373 | 3.166348 | 7.371527 |
| 5 | 0.155423685 | 4.356269 | 3.210677 | 7.72237 |
| 6 | 0.160771006 | 3.832453 | 3.255626 | 7.24885 |
| 7 | 0.160389196 | 3.8685 | 3.301205 | 7.330095 |
| 8 | 0.152973065 | 4.294311 | 3.347422 | 7.794707 |
| 9 | 0.13673771 | 5.365448 | 3.394286 | 8.896472 |
| 10 | 0.108921733 | 6.191607 | 3.441806 | 9.742334 |

Table 7. 3 years younger, interest increase of 0.01 , decrease of costs = 3 EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.138603654 | 1.63849 | -1.20316 | 0.573931 |
| 2 | 0.199441327 | 2.057139 | 3.059238 | 5.315818 |
| 3 | 0.251454967 | 3.242792 | 3.102067 | 6.596314 |
| 4 | 0.287612889 | 4.061373 | 3.145496 | 7.494482 |
| 5 | 0.31084737 | 4.356269 | 3.189533 | 7.85665 |
| 6 | 0.321542011 | 3.832453 | 3.234187 | 7.388182 |
| 7 | 0.320778392 | 3.8685 | 3.279465 | 7.468744 |
| 8 | 0.305946129 | 4.294311 | 3.325378 | 7.925635 |
| 9 | 0.27347542 | 5.365448 | 3.371933 | 9.010857 |
| 10 | 0.217843465 | 6.191607 | 3.41914 | 9.82859 |

By comparing Tables 4-7 we can confirm the dominant influence of the mortality factor $M_{t}$ on the total dividend. This fact is also illustrated in Figure 1, which graphically shows the proportion of the mortality factor on the total dividend as a percentage. In the first column are the values of $M_{t}$ and $D_{t}$ from Table 3, in the second column are the same values from Table 4 and in the third column are these values from Table 5 . We see that with decreasing mortality (from one year rejuvenation to three years rejuvenation) the mortality factor (in the last tenth year of the term) is increasing in absolute values from 2.223429 to 6.191607 .


Figure 1. Term of insurance product, authors'
Endowment. Now we will analyse the impact of three assumptions (interest rate, mortality and loading) on the dividend factors for an endowment. Endowment provides a payment not only in the event of death during the term of a contract, but also at the end of the term in case of survival. All assumption are as in the preceding example of term assurance. Regular annual net premium is in the amount of 455.64 EUR.

Unlike the term assurance in the case of an endowment reserves are higher and are approaching the sum assured as the policy year tends to the term of the contract (endowment is investment contract with high reserves).

Table 8. Reserves for endowment, own calculations

| Year $t$ | Reserve at the <br> beginning $V_{t-1}$ |
| :---: | :---: |
| 1 | 0.00 |
| 2 | 456.98 |
| 3 | 922.57 |
| 4 | 1396.64 |
| 5 | 1879.90 |


| Year $t$ | Reserve at the <br> beginning $V_{t-1}$ |
| :---: | :---: |
| 6 | 2372.85 |
| 7 | 2876.03 |
| 8 | 3389.81 |
| 9 | 3914.66 |
| 10 | 4451.14 |

The real interest rate has increased as compared to the estimated value of $2 \%$ and mortality fell by three years, actual administrative expenses decreased by 3 EUR. Interest rate factor as compared to the previous values of this factor is high, especially in the last tenth year, which is caused precisely by the fact that the last year reserve is zero. Effect of changes in particular assumptions is provided in Table 9.

Table 9. $\mathbf{3}$ years younger, interest increase of 0.001 , decrease of costs $=3$ EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.013860365 | 1.490553 | -0.13085 | 1.373562 |
| 2 | 0.470838713 | 1.681364 | 3.1008 | 5.253003 |
| 3 | 0.936426684 | 2.343977 | 3.162816 | 6.443219 |
| 4 | 1.410497428 | 2.543139 | 3.226072 | 7.179709 |
| 5 | 1.893759568 | 2.297317 | 3.290594 | 7.481671 |
| 6 | 2.38671317 | 1.633959 | 3.356406 | 7.377077 |
| 7 | 2.889887384 | 1.249988 | 3.423534 | 7.563409 |
| 8 | 3.403669523 | 0.934681 | 3.492004 | 7.830355 |
| 9 | 3.928518222 | 0.589915 | 3.561845 | 8.080278 |
| 10 | 4.464996405 | 6.191607 | 3.633081 | 14.28968 |

The formula for the components of the dividend shows that the interest rate affects the interest factor $I_{t}$ and the loading factor $L_{t}$, although in the case of the loading factor the impact is low. The following tables illustrate the fact that changes in interest rates have the most significance influence on the amount of total dividends for this type of life insurance product.

As we see the loading factor decreases with the increasing interest rate, but not to the extent that would significantly affect the amount of dividend. However, evident is the considerable influence of changes in interest rate. Changing mortality for this type of a contract is not negligible but small (endowment paid sum assured in the case not only of a survival but also death) as it is illustrated in the following two tables (compare Table 9 to Tables 12 and 13).

The dominant influence of the interest factor on the total dividend is illustrated in Figure 2 which graphically shows the proportion of the interest rate factor on the total dividend as a percentage. In the first column are the values of $I_{t}$ and $D_{t}$ from Table 9, in the second column are the same values from Table 10 and in the third col-
umn are the values from Table 11. We see that with increasing interest rate (from 2\% to $2.9 \%$ ) the interest rate factor (in the last tenth year of endowment) is increasing in the absolute values from 4.464996405 to 44.64996405 .

Table 10. 3 years younger, interest increase of 0.005 , decrease of costs = 3 EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.069301827 | 1.490553 | -0.60158 | 0.958273 |
| 2 | 2.354193564 | 1.681364 | 3.079518 | 7.115076 |
| 3 | 4.682133422 | 2.343977 | 3.122631 | 10.14874 |
| 4 | 7.052487138 | 2.543139 | 3.166348 | 12.76197 |
| 5 | 9.468797842 | 2.297317 | 3.210677 | 14.97679 |
| 6 | 11.93356585 | 1.633959 | 3.255626 | 16.82315 |
| 7 | 14.44943692 | 1.249988 | 3.301205 | 19.00063 |
| 8 | 17.01834762 | 0.934681 | 3.347422 | 21.30045 |
| 9 | 19.64259111 | 0.589915 | 3.394286 | 23.62679 |
| 10 | 22.32498202 | 6.191607 | 3.441806 | 31.95839 |

Table 11. $\mathbf{3}$ years younger, interest increase of $\mathbf{0 . 0 1}$, decrease of costs = 3 EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.138603654 | 1.490553 | -1.20316 | 0.425994 |
| 2 | 4.708387128 | 1.681364 | 3.059238 | 9.448989 |
| 3 | 9.364266843 | 2.343977 | 3.102067 | 14.81031 |
| 4 | 14.10497428 | 2.543139 | 3.145496 | 19.79361 |
| 5 | 18.93759568 | 2.297317 | 3.189533 | 24.42445 |
| 6 | 23.8671317 | 1.633959 | 3.234187 | 28.73528 |
| 7 | 28.89887384 | 1.249988 | 3.279465 | 33.42833 |
| 8 | 34.03669523 | 0.934681 | 3.325378 | 38.29675 |
| 9 | 39.28518222 | 0.589915 | 3.371933 | 43.24703 |
| 10 | 44.64996405 | 6.191607 | 3.41914 | 54.26071 |

Table 12. $\mathbf{2}$ years younger, interest increase of $\mathbf{0 . 0 0 1}$, decrease of costs $=3$ EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.013860365 | 0.956894 | -0.12032 | 0.850438 |
| 2 | 0.470838713 | 1.26929 | 3.095742 | 4.835871 |
| 3 | 0.936426684 | 1.949166 | 3.139082 | 6.024675 |
| 4 | 1.410497428 | 1.913724 | 3.18303 | 6.507251 |
| 5 | 1.893759568 | 1.406187 | 3,227592 | 6.527539 |
| 6 | 2.38671317 | 1.051666 | 3,272778 | 6.711157 |
| 7 | 2.889887384 | 0.829568 | 3,318597 | 7.038052 |
| 8 | 3.403669523 | 0.680666 | 3,365057 | 7.449393 |
| 9 | 3.928518222 | 0.435598 | 3,412168 | 7.776284 |
| 10 | 4.464996405 | 4.461672 | 3,459939 | 12.38661 |

Conclusion. First and the most important statement is: Influence of all assumptions on calculating each dividend factor depends generally on the formula applied. Given now the availability of more significant computer resources, also a more com-
plicated approach would normally be used. For example, by taking into account more than three factors as the formula above does.

Table 13. One year younger, interest increase of 0.001, decrease of costs $=3$ EUR, own calculations

| year | $I_{t}$ | $M_{t}$ | $L_{t}$ | $D_{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.013860365 | 0.497767 | -0.12032 | 0.391311 |
| 2 | 0.470838713 | 0.822537 | 3.095742 | 4.389118 |
| 3 | 0.936426684 | 1.222263 | 3.139082 | 5.297772 |
| 4 | 1.410497428 | 0.855384 | 3.18303 | 5.448911 |
| 5 | 1.893759568 | 0.685948 | 3.227592 | 5.807299 |
| 6 | 2.38671317 | 0.497096 | 3.272778 | 6.156588 |
| 7 | 2.889887384 | 0.452717 | 3.318597 | 6.661202 |
| 8 | 3.403669523 | 0.375514 | 3.365057 | 7.144241 |
| 9 | 3.928518222 | 0.245698 | 3.412168 | 7.586385 |
| 10 | 4.464996405 | 2.223429 | 3.459939 | 10.14836 |



Figure 2. Endowment, authors'
The interest factor $I_{t}$ is the simplest element but it has a strong influence on the whole dividend, particularly in a long term when the initial reserve is large. So changes in interest rates have the most significance influence on the amount of total dividends, especially for endowment assurances. It is well known also that the mortality factor $M_{t}$ is normally smaller than the interest factor and makes a decreasing contribution with greater durations but it is more a significant factor for term assurance policies.

Comparison of the tables above confirms the dominant influence of mortality on dividend, especially on the mortality factor of it. In contrast to term assurance, the interest rate contribution is highly significant for endowment policies. Obviously, there exist more differences between the impact of particular assumptions for differ-
ent types of insurance products and consequently in practice various dividend scales for different types of products are used.

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