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Research article

Green investments: Portfolio selection based on risk measure and ESG indicators. Impact of environmental indicators on portfolio selection

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Abstract: Green investments are currently gaining investor attention. The question investors are addressing is to what extent investing in environmental companies or funds increases the risk/reward ratio. The novelty of the theoretical approach lies in the construction of a newly developed portfolio selection model with embedded environmental, social and governance (ESG) indicators that also represent the values of the environmental criteria. This innovative model, based on a portfolio selection model using conditional value at risk (CVaR) measures and analyzed data for the Standard and Poor's 500 (S&P 500) stock index, offers a promising potential for investors. The paper's core is the section where the authors define possible approaches to solve the portfolio selection problem based on the selected environmental criterion E. The analysis itself is contained in section three, where the three variants—the absence of environmental concerns, the maximum preference for environmental requirements, and the combined approach of constructing a portfolio on a defined set of stocks—are analyzed, which is the result of the authors' specific approach. Through a comprehensive analysis, this paper focuses on the aspect of determining the amount of risk difference in the approach of selecting assets with the required value of the relevant indicator. This thorough analysis ensures a narrowing of the set of potential assets in the portfolio, regardless of the amount of their return, and assets for which the group condition on the value of the relevant indicator can be applied, which based on the group condition, allows one to include assets with a higher return in the portfolio. The research results confirm that it is preferable for an investor to consider the environmental criteria on the portfolio as a whole rather than on individual assets.

Keywords: green investments; ESG; portfolio optimization; risk measure; CVaR

JEL Codes: G11, C60

1. Introduction

Environmental investing is also linked to an increasingly accentuated understanding of the sustainable development of society and a modern environmental strategy for society as a whole. This is also linked to an investors' interest in environmentally and socially responsible companies. However, implementing green investments is also associated with increased costs (Drobetz et al., 2024).

Investing in environmental (green) companies has been one of the tools for the sustainable development of society since the 1990s. Environmental investing (Rahko, 2023) (Falcone, 2020) (Falcone, 2018) (Pekovic et al., 2018); generally represents the search for investment opportunities that benefit the environment. It aims to promote those activities that positively impact the natural environment.

Thus, environmental investments are investments where most or all of the revenues and profits are derived from green business activities (Makridou et al., 2024) (Caferra & Falcone, 2023; Du et al., 2019; Eyraud et al., 2013; Hsiao et al., 2019). The main objectives of such green activities include renewable energy technologies such as wind, solar, and hydroelectric power, green transportation that reduces the consumption of fossil fuels primarily through electric vehicles, air and water pollution controls, waste reduction, and sustainable agriculture, which are alternative pathways to environmental protection. Environmental investments can be in securities, electronically traded funds, mutual funds and bonds, and environmental investment products, which are investment instruments that positively impact the environment. For example, Apple's $1.5 billion green bond was the first ever green bond issued by a technology company. The bond even won an award from Environmental Finance in 2016.

Investing in environmental stocks means placing funds in the stock market by buying shares of companies that focus on environmental sustainability (Becchetti et al., 2022). Several green mutual funds also focus on a sustainable or green investment strategy, which can be found, for example, through Morningstar Inc. ratings or State Street's R-Factor.

Investing in green firms may generally evoke perceptions that such investing is riskier than conventional investing under other equity strategies ((Taghizadeh-Hesary & Yoshino, 2020) and (Mokhov et al., 2018)). This fact is determined by many companies which accept that the green accent is in the development stage, with low revenues and a high earnings valuation. However, if investors need to support within such firms, then this type of investing can be an attractive way to invest the available investment funds. This may lead to investor interest in creating an environmental investment portfolio. Portfolio optimization is crucial for investing the available funds, as investing is one of the most effective ways to capitalize on the available funds. From the nature of investing, it is clear that every investor seeks to maximize their return while minimizing the risk. However, it is necessary to assume that higher profits also bring a higher risk.

The risk represents the possibility that the actual return differs from the expected return, thus representing the uncertainty about future income. Therefore, the investment risk is represented because the investment may not achieve the expected return or may even mean a loss. Although this risk cannot be entirely avoided, it can be monitored, optimized, and significantly reduced with appropriate tools.

Diversification can minimize this risk (i.e., the risk of individual assets is spread over the portfolio) (Alexander, 2009). Various factors influence investment risk, depending on the financial instruments themselves.

When creating one's environmental investment portfolio, Environmental, Social, and Governance (ESG, which is shorthand for an investing principle that prioritizes environmental issues, social issues, and corporate governance (Gelles, 2023), can be used. For the investor, this raises the fundamental question of whether an environmental investment is profitable, at least at the desired profit level for a certain, probably higher, investment risk. Essentially, the investor is faced with three options:

- disregard environmental concerns and construct a portfolio from a defined set of stocks while maximizing returns and minimizing the risk, for which a risk-based conditional value at risk (CVaR) portfolio selection model can be used;
- give maximum preference to the environmental requirements and only select from a subset of those actions that meet the specific environmental requirements (e.g., based on the values of selected ESG indicators); and
- a combined approach of portfolio construction on a defined set of stocks while maximizing returns and minimizing the risk based on, e.g., a CVaR model in relation to certain requirements on the values of selected ESG indicators.

The article aims to show how compliance with the environmental requirements influences the amount of investor risk. The analysis is carried out on data from the Standard and Poor's 500 (S&P 500) stock index, which includes the stocks of the 500 largest companies listed on the US Stock Exchange and is considered by many investors to be the best indicator of the US stock market. Other stock indices considered for the analysis were the Dow Jones Industrial Average (DJIA) - 30 stocks, the EURO STOXX 50 - 50 stocks, and the CAC 40 - 40 stocks. However, regarding the analysis of selected groups of stocks (1st decile, 1st and 2nd decile, and 1st quartile), the S&P 500 stock index appears to be the most suitable among the above stock indices. The environmental aspect of the analysis is based on ESG indicators, where the environmental criteria reflect the firm's relationship to environmental protection, the social criteria of the firm's relationship to its employees, and the governance criteria of the firm's relationship to its management.

From an environmental perspective, the most interesting values of the firm's environmental assessment (E) are those that represent important boundaries in the analysis that influence the choice of actions.

Thus, the article develops the CVaR risk-based portfolio selection model with additional bounds that can be formulated based on the values of the E indicator of the ESG concept. In general, such bounds can be applied to the partial values of S and G and the comprehensive ESG indicator. The result compares the values of the risk taken by an investor with the environmental requirements for the construction of their portfolio with investing without the environmental requirements.

The paper is structured into the following sections, thus providing a clear roadmap for the readers. It begins with an introduction to the analyzed problem (Section 1), followed by a section where the authors present the theoretical background and analyze the input data. In this part (Section 2.1), the authors introduce the concept of the ESG indicator development. Then, they (Section 2.2) define the selected measures of risk and return that are standardly used in financial modeling. The main contribution (Section 2.3) in the theoretical area lies in constructing a newly developed portfolio selection model with embedded ESG indicators. The concept of the approach is based on a portfolio selection model based on CVaR risk measures derived from the analyzed data. The visualization and basic characteristics of the input data (S&P 500 stock index) are presented in Section 2.4, where the assets to be analyzed and their ESG indicator values are defined. The paper's core is the third section, in which the authors define possible approaches to solve the portfolio selection problem based on the selected environmental criterion E. The analysis itself is contained in Section 3, where the three mentioned variants—the absence of environmental concerns, the maximum preference for the environmental requirements, and the combined approach of constructing a portfolio on a defined set of stocks—are analyzed, which is the result of the authors' specific approach. The authors' approach is generalized in the final diagram, which represents a general approach to address the relevant issue. A discussion (Section 4) of the obtained results is provided in the last section. This section can be a good basis for further scientific research in addressing new financial modeling challenges concerning the input data structure, the suitability of the individual approaches, and a consideration of the environmental requirements.

2. Materials and methods

2.1. ESG indicators

Including the ESG risks aims to redirect the financial flows toward sustainable investments (Escrig-Olmedo et al., 2017; García et al., 2020; Giese et al., 2019). The environmental criteria consider how a firm protects the environment, including the individual corporate policies. The social criteria examine how a firm manages its relationships with its employees, suppliers, customers, and the communities in which it operates. The governance criteria focus on the firm's leadership, executive compensation, audits, internal controls, and shareholder rights.

Under the ESG approach, institutions are expected to have an overview of the impact of the environmental risks on the business environment in which they operate in the short, medium, and long term to make well-informed strategic and investment decisions. Institutions should identify and quantify these risks as part of their overall capital adequacy process.

The ESG concept generally focuses on criteria and standards for assessing a firm's environmental and social impact performance, governance, and leadership (Boffo & Patalano, 2020). The aim is to encourage a sustainable and responsible behavior by companies and ensure that they prioritize long-term value creation for all stakeholders. These legislative frameworks allow firms to disclose and report on their ESG performance, thus emphasizing transparency and accountability.

Different approaches in calculating the ESG can lead to significantly different results. Several rating agencies are involved in the ESG assessment; the most well-known ones include LSEG Data & Analytics (previously known as Refinitiv), Morningstar Sustainalytics, MSCI, Bloomberg or S&P Global, MSCI Bloomberg, LSEG, and RobecoSAM.

Environmental investment opportunities through the CVaR-based portfolio selection model will be presented on data obtained from finance.yahoo, where Sustainalytics, Inc provides the ESG data. The rating agency Sustainalytics provides ESG ratings for over 20,000 companies. The rating for each of the 20,000 companies consists of a quantitative score and a risk severity category. The quantitative score represents units of unmanaged ESG risk. The lower the unmanaged risk, the lower the ESG score. Then, the ESG risk scores can be divided into five different severity categories (Sustainalytics, 2023):

- Negligible -ESG rating 1 - 9.9;
- Low - ESG rating 10 - 19.9;

- Medium - ESG rating 20 - 29.9;
- High - ESG rating 30 - 39.9; and
- Severe - ESG rating above 40.

The Sustainalytics ESG score reflects a company's exposure to material industry-specific ESG risks and its management of those risks. ESG scores are built on corporate governance, material ESG issues, and unforeseen ESG issues. Sustainalytics uses a multidimensional concept, namely exposure, and management, to calculate the ESG scores (Sustainalytics, 2023).

In addition to ESG assessments, the Agency provides partial assessments of the environmental, social, and governance components. The score for the Environmental, Social, and Governance (E/S/G) cluster in the ESG risk assessment is calculated by combining the scores from the material ESG issues. Each cluster score is a blend of the ESG material issues scores with weights assigned based on the importance of management and the event indicators for each issue.

The scores of the environmental (E), social (S), and governance (G) clusters are derived by consolidating the indicators for each cluster—E for environmental, S for social, and G for governance. A firm's holistic ESG Risk Ratings score is determined by assessing the risk, which includes the unmanaged risk across all material ESG issues facing the firm (Sustainalytics, 2023).

2.2. Risk measure and expected return

In this section, we present the method of calculating the model input data as the yield vector, the cumulative return vector, and the method of obtaining the expected return value of the asset. Additionally, we define the selected CvaR measure, which is used in our mathematical programming model formulation.

Let us consider having an asset and let P_t represent the price of an asset in time t =1, 2,...T, where T is the length of the observed time period.

The rate of the return of an asset between time t and t -1 can be expressed as a relative rate of return r_t and is expressed by the following equation:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad . \tag{1}$$

Subsequently, the cumulative return rate of an asset in time t can be expressed by the following equation:

$$y_t = \frac{P_t - P_1}{P_1} \quad . \tag{2}$$

The expected return of an asset based on the value of the cumulative return is as follows:

$$E = \left(1 + y_T\right)^{\frac{1}{T}} - 1 \tag{3}$$

The value at risk (VaR) indicator is a standard for measuring and managing market risk, which quantifies the maximum loss that will not be exceeded with a chosen probability α over a specified time horizon (Yamai & Yoshiba, 2002). The VaR is defined as the one-sided confidence interval of possible losses in value due to changes in the commodity prices, securities prices, interest rates, and exchange rates; thus, the VaR answers the question of how much value an investor can lose with a certain probability over a certain period (Alexander, 2009). However, the VaR says nothing about risks that go beyond the chosen probability α and, as a consequence, can either lead to choosing very risky investment strategies or to underestimating the diversification of investment instruments.

The VaR is most commonly measured, with a probability of 95% ($\alpha = 0.05$). Several different methods are used to measure the VaR. Among the most popular are the historical method, the parametric method, and the Monte Carlo simulation.

If X is a random variable representing returns and $L(X) = -X$ is a loss function, then the VaR can be mathematically defined as follows:

$$P(L(X) < VaR_\alpha) = 1 - \alpha \quad , \tag{4}$$

$$P(-X < VaR_\alpha) = 1 - \alpha \quad , \tag{5}$$

where X is a random variable that represents the value of gains and losses, and α is the confidence level.

Thus, the VaR_α is a threshold that splits the values into two parts, with one part having losses less than the VaR_α at the α significance level and the other part having values greater than or equal to the VaR_α.

The VaR has become a standard risk management tool in the financial sector, primarily because of its conceptual and computational simplicity. Many authors point to several conceptual problems with the VaR (Basak & Shapiro, 200 and Artzner et al., 1999). Because of the shortcomings of the VaR, an alternative measure of risk, namely the CVaR, has been proposed. The CVaR can be defined as the expected loss in excess of the VaR; thus, the CVaR only considers loss values that are higher than the VaR (Arici et al., 2021; Bodnar et al., 2022; Hamdi et al., 2022; Wang & Zhu, 2021; Kang et al., 2020). The CVaR value is defined at a given confidence level α as follows:

$$\mathrm{CVaR}_\alpha\,(\mathrm{X}) = E(L(X)|L(X) \geq \mathrm{VaR}_\alpha)\ , \tag{6}$$

$$\mathrm{CVaR}_\alpha\,(\mathrm{X}) = E(-X|-X \geq \mathrm{VaR}_\alpha), \tag{7}$$

where X denotes the random variable which represents the return, $L(X) = -X$ denotes the loss function of the random variable X, and VaR_α is the value-at-risk described above.

The formulation of the equation for the calculation of the CVaR during the period under review (T) for an asset can be defined in the following manner ((Pekár et al., 2024) and (Rockafellar & Uryasev, 2000)):

$$\begin{aligned} CVaR_\alpha\left(X\right) &= VaR_\alpha - \frac{1}{\alpha}E\left[\left|\mathbf{r} + VaR_\alpha\right|_-\right] \\ CVaR_\alpha\left(X\right) &= VaR_\alpha + \frac{1}{\alpha T}\sum_{t=1}^{T}\max(-\left(r_t + VaR_\alpha\right), 0) \end{aligned} \tag{8}$$

where r represents the vector of returns asset, and α represents the significance level.

2.3. Portfolio optimization with ESG

Optimizing your investment portfolio is the key idea behind any investment. Every investor seeks to maximize his or her return while minimizing the risk. The risk is the possibility that the actual return will differ from the expected return and, therefore, represents the uncertainty of the future income. This risk can be minimized by diversification (i.e., spreading the risk of individual assets across a portfolio, thereby reducing it). Portfolio optimization that accepts the environmental objectives includes the incorporation of the ESG criteria.

The incorporation of the ESG concept can be seen in the classical Markowitz portfolio model, which aims to find an appropriate selection of assets that form an optimal investment portfolio regarding the profitability and risk. This model incorporates additional conditions on the ESG values or partial values of E, S, and G (Xidonas & Essner, 2024; De Spiegeleer et al., 2023; Cesarone et al., 2022; Torricelli & Bertelli, 2022).

This section presents the construction of the mathematical programming model based on the expected return, the CVaR risk measure, and the ESG indicators. In the first step, we define the model variables and the parameters used in its construction. The solution results to this model will be efficient portfolios that take the level of short-term risk (CVaR) and the ESG indicators into account, in contrast to traditional portfolio models (Arcuri et al., 2023; Pekár et al., 2024).

The objective of the optimization is to minimize the risk in the CVaR space. Based on the equation (8), an objective function can take the following the form:

$$\min CVaR\left(w_1, w_2, \ldots w_n, VaR_\alpha\right) = VaR_\alpha + \frac{1}{\alpha T}\sum_{t=1}^{T}\max(-VaR_\alpha - \sum_{j=1}^{n} r_{jt}w_j, 0). \tag{9}$$

Since the objective function (9) is not in standard form, it must be transformed into a linear function by replacing the formulation $\max(-VaR_\alpha - \sum_{j=1}^{n} r_{jt}w_j, 0)$ using the variables z_t, where $z_t \geq 0$ (t=1, 2,...T). Then, the linear programming problem can be formulated using the variables z_t and the constraint $z_t \geq \max(-VaR_\alpha - \sum_{j=1}^{n} r_{jt}w_j, 0)$, ($t$=1, 2,...$T$).

When constructing the model, we will use the variables $z_t \geq 0$ (t = 1, 2,...T), which acquire the value of the difference between the VaR and the portfolio return in state t, if the return is lower than VaR, or will be equal to zero based on the significance level α. Let VaR_α be a variable that represents the threshold for the portfolio return based on the significance level α. In addition to the mentioned free variables, the variables w_1, w_2 ,...w_n, also appear in the model, which acquires optimal values of the weights of the individual assets (j = 1, 2, ... n).

The parameter E_p indicates the minimum expected return of the portfolio (i.e., the value determined by the investor that the calculated portfolio must reach). Additionally, the model also uses criteria that consider the ESG indicators (4 criteria - ESG, E, S, G). The following parameters have been used in the construction of the model: ESG required value (E_{ESG}), environmental (E_E), social (E_S), governance (E_G). As an input parameter, the following values from finance.yahoo of Sustainalytics were used: ESG (ESG_j, j-th asset), environmental (EN_j, j-th asset), social (S_j, j-th asset), and governance (G_j, j-th asset).

The six-criteria problem (all six formulated objectives are respected) can be transformed into a mathematical programming problem that minimizes the CVaR risk objective function concerning the constraints that will model the minimum required value of the expected return, E_p, and the maximum required values of the ESG indicators E_{ESG}, E_E, E_S, E_G:

$$\min CVaR\left(w_1, w_2, \ldots w_n, z_1, z_2, \ldots z_T, VaR_\alpha\right) = VaR_\alpha + \frac{1}{\alpha T}\sum_{t=1}^{T} z_t \tag{10}$$

$$z_t + \sum_{j=1}^{n} r_{jt} w_j + VaR_\alpha \geq 0, t = 1, 2, \ldots T \tag{11}$$

$$\sum_{j=1}^{n} w_j = 1 \tag{12}$$

$$\sum_{j=1}^{n} E_j w_j \geq E_P \tag{13}$$

$$\sum_{j=1}^{n} ESG_j w_j \leq E_{esg} \tag{14}$$

$$\sum_{j=1}^{n} EN_j w_j \leq E_E \tag{15}$$

$$\sum_{j=1}^{n} S_j w_j \leq E_S \tag{16}$$

$$\sum_{j=1}^{n} G_j w_j \leq E_G \tag{17}$$

$$w_1, w_2, \ldots w_n \geq 0, z_1, z_2, \ldots z_T \geq 0 \tag{18}$$

The objective function of the model represents the Equation (10). Constraint (11) is used to obtain the minimum CVaR value. The condition to invest all capital is included in Constraint (12). Constraint (13) ensures that the minimum required value of the yield criteria and Constraints (14) to (17) of the four criteria based on the ESG indicators (not exceeding the required weighted value) are achieved. Of course, not every Constraint (14)–(17) has to be respected in the individual calculations, but only those constraints which are emphasized by the investor. For example, when prioritizing the environmental requirements, only Constraint (15) is incorporated into the model; when emphasizing the total ESG values, it is either Constraint (14) or a combination of individual constraints. The solution of the formulated mathematical programming problem (10)–(18) results in the values of the variables w_1, $w_2, \ldots w_n$, which represent the weights of the individual assets in the computed portfolio.

2.4. Database

For a correct application of the presented model, it is important to correctly determine the time period for analyzing historical stock prices. In the presented analysis, the evolution of the weekly stock prices of the S&P 500 stock market during a specific period, namely from 29 August 2016 to 26 August 2024, was accepted. The evolution of the stock index during the period under study is shown in Figure 1. The analysis used the values of 422 stocks for which the ESG ratio information was provided by finance.yahoo.com as of September 1, 2023. The presented data were obtained using the yesg module in Python, which can be used to retrieve data on ESG indicators from Sustainalytics, Inc.

This time period is key to obtaining the relevant data, which is then used to calculate the weekly return according to relation (1). Figure 2 shows a histogram of the calculated weekly return values for the 417-week observation period for the S&P 500 index, and the corresponding statistical indicators are in Table 1.

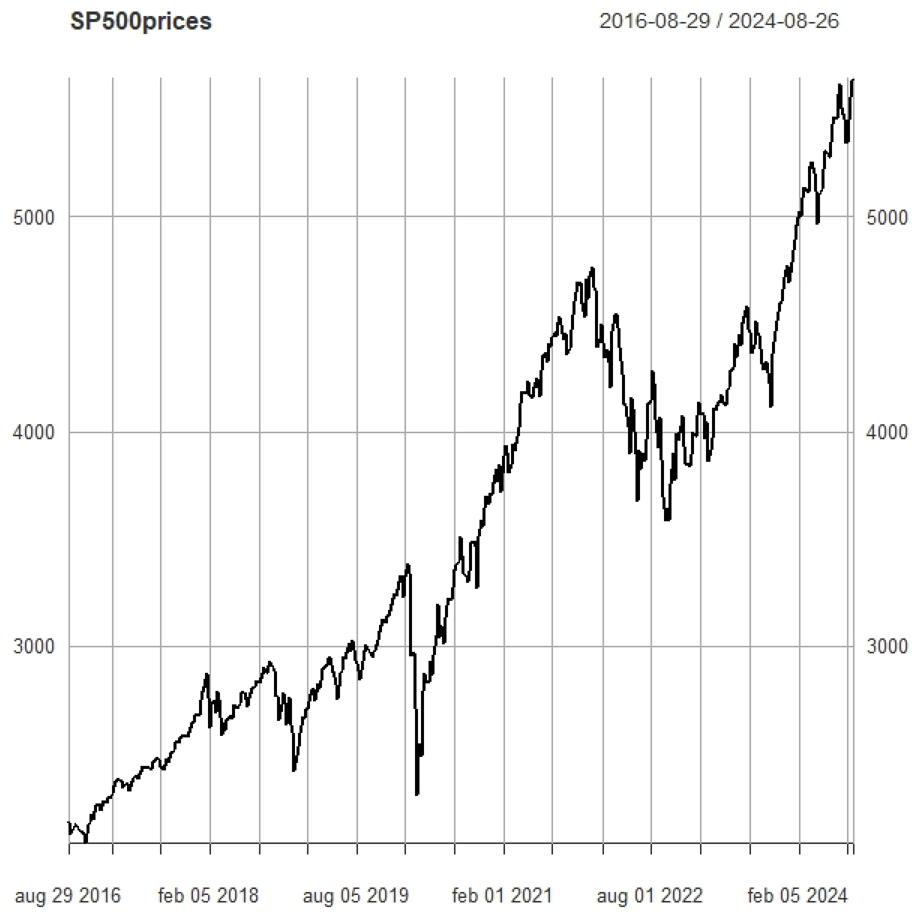

Figure 1. Evolution of the S&P 500 stock index during the period under review.

Table 1. Statistical indicators for the S&P 500 stock index.

	mean	sd	median	mad	min	max	range	skew	kurtosis
SP500	0	0.02	0	0.02	−0.15	0.12	0.27	−0.63	6.71

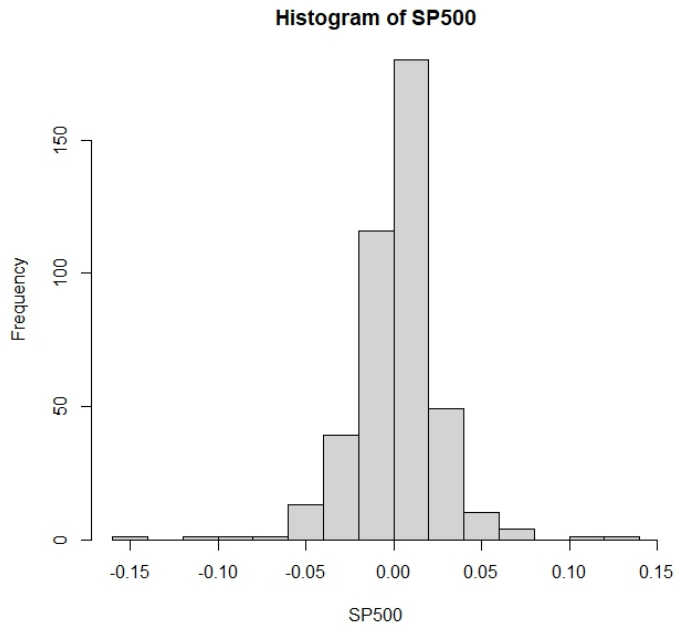

Figure 2. Histogram of calculated values of weekly returns for the period under review.

The skewness coefficient for the normal distribution takes the value of 3, and the skewness coefficient takes the value of 0, which is not the case for the calculated values of the weekly returns. Therefore, it is not appropriate to use approaches based on the normal data distribution for the financial analysis (Pekár & Pčolár, 2022). For this reason, in the presented analysis of the impact of the environmental ESG indicators on portfolio selection, a CVaR-based portfolio selection model was used, which does not require the assumption of a normal distribution of the data (Arcuri et al., 2023).

The ESG values for S&P 500 stocks were obtained from the Sustainalytics rating agency data as of September 1, 2023, which provides a solid database for the model. Figure 3 shows a boxplot of the individual ESG indicator values.

Sustainalytics' scoring methodology underwent significant changes in November 2019. The scoring logic used until then was set up so that the higher a company's ESG score, the higher the risk it indicated. Currently, the lower the score, the lower the sustainability risk.

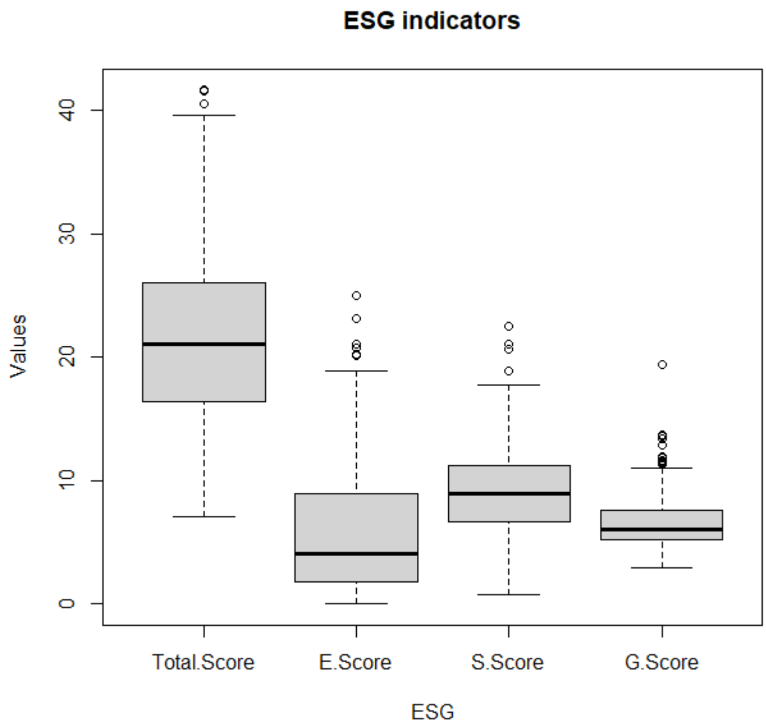

Figure 3. SP500 stocks ESG indicator values Box plot visualization.

Table 2. Descriptive statistics for individual indicators.

	mean	sd	median	Mad	min	max	range	skew	kurtosis
ESG	21.56	6.82	21.07	7.03	7.08	41.66	34.58	0.43	−0.3
E	5.77	5.05	4.08	4.42	0	24.98	24.98	1.04	0.48
S	9.1	3.65	8.93	3.34	0.76	22.48	21.72	0.36	0.22
G	6.7	2.17	6.1	1.62	2.96	19.43	16.47	1.3	2.76

ESG values can be included in the model in the portfolio construction process in the form of a bounding condition or they can directly constrain the list of assets on which the portfolio selection model will be applied in the space of expected return and the CVaR risk measure presented in the fourth part of the article. In doing so, the analysis presented here will emphasize the values of pillar E. The descriptive statistics (García García et al., 2022) of the individual indicators are presented in Table 2.

If the investor's objective is to construct an investment portfolio with a risk-minimizing strategy at a given value of return, they will use the presented model (10)–(18) without bounds (14), (15), (16), and (17). Data on the price values of 422 stocks are required for the calculation.

Since it can be assumed that an environmentally focused investor is only interested in stocks of firms that emphasize the environmental goals and Pillar E, they can narrow their stock selection to stocks of firms with a certain Pillar E value. The analyses considered firms from the first decile D1 (i.e., 43 firms with the lowest E values), from the first and second decile D2 (i.e., 86 firms with the lowest E values), and from the first quartile (i.e., 113 firms with the lowest E values). Based on these requirements, the E indicator values from the Sustainalytics rating agency data can be used.

If an investor wants to create a portfolio in which, in addition to minimizing risk at a given level of return, then emphasis is also placed on the environmental aspect; only the maximum values of the respective pillar E for the first decile D1, for the first and second deciles D2, and for the first quartile Q1 were considered in the calculation of the threshold values of E in the structural boundaries from all values for the 422 shares of the firms.

The individual values of the right-hand sides of bounds (14)–(17) are shown in Table 3. In the analyses presented here, the values for the E variable for the first quartile (1.7825), the first decile (0.645), and the first and second deciles (1.46) were used for bound (15).

Table 3. Values at the upper limit of quartiles and first two deciles.

	ESG	E	S	G
Q1	16.4425	1.7825	6.6725	5.25
Q2	21.07	4.085	8.925	6.095
Q3	26.0025	8.9925	11.2125	7.6375
D1	13.09	0.645	4.502	4.64
D2	15.318	1.46	6.004	5.11

3. Results

This section will analyze the portfolio selection results on the stocks of the S&P 500 Index. The inputs were the S&P 500 stock prices from August 29, 2016, to August 26, 2024. The environmental aspect was represented by the E values of the ESG indicators as of September 1, 2023. In order to implement the calculations based on the mathematical programming model (10)–(18), the MIP module in Python was used for optimization.

Three options were considered when constructing a custom environmental investment portfolio for the investor:

1. Disregard environmental concerns and construct a portfolio of 422 S&P 500 stocks using a CVaR-based portfolio selection model without ESG constraints (Tables 4 and 5);
2. Give maximum preference to the environmental requirements and select only from stocks for which the E value is in the 1st decile (Tables 6 and 7); and
3. A combined portfolio construction approach on the entire set of stocks, with the requirement of a weighted value of E for the portfolio less than the value at the upper bound of the 1st decile (i.e., 0.645) (Tables 8 and 9).

Table 4 shows the CVaR risk and expected return values E_p for the efficient portfolios calculated (E1 - E8) based on the formulated model using relations (10)–(13) and (18).

The weights for each effective portfolio are shown in Table 5, and 25 assets occurred in at least one portfolio.

The results of the second variant analyzed are shown in Tables 6 and 7, with the first step being the selection of stocks based on the values of E, which had to be lower than the value at the upper limit of the 1st decile (i.e., 0.645). Subsequently, the model was applied using relations (10)–(13) and (18) for a subset of 43 stocks with an E ratio less than 0.645. The weights for each efficient portfolio are shown in Table 7, and 10 assets occurred in at least one portfolio.

Table 4. Risk and return of efficient portfolios in the absence of E.

	CVaR	E_p
E1	3.410%	0.348%
E2	3.739%	0.461%
E3	4.216%	0.575%
E4	4.835%	0.688%
E5	5.710%	0.801%
E6	6.853%	0.914%
E7	8.130%	1.027%
E8	9.706%	1.141%

Table 5. Composition of efficient portfolios and stock ratios when E is ignored.

Action	ESG	E	Yield shares	CVaR	E1	E2	E3	E4	E5	E6	E7	E8
ANET	16.67	0.40	0.86%	9.1%	0.00%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AVGO	19.98	7.34	0.71%	7.7%	0.00%	0.00%	5.67%	8.07%	1.45%	0.00%	0.00%	0.00%
AZO	11.02	0.11	0.43%	11.8%	0.00%	2.67%	3.23%	3.74%	3.58%	0.00%	0.00%	0.00%
CDNS	12.02	2.33	0.65%	7.9%	2.57%	2.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CINF	25.61	1.76	0.24%	12.0%	0.08%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CME	17.8	2.13	0.15%	7.5%	3.42%	11.54%	6.06%	0.00%	0.00%	0.00%	0.00%	0.00%
CPT	16.09	4.00	0.50%	6.6%	0.00%	0.39%	1.55%	5.75%	0.00%	0.00%	0.00%	0.00%
DPZ	29.19	10.65	0.24%	6.7%	0.32%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
FAQ	27.01	11.52	0.34%	5.5%	3.96%	4.63%	2.57%	0.00%	0.00%	0.00%	0.00%	0.00%
ED	23.81	6.86	0.21%	9.0%	0.15%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
GLW	16.48	5.53	0.14%	7.2%	14.25%	5.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
HSY	26.05	10.52	0.05%	8.0%	16.44%	8.91%	3.23%	0.00%	0.00%	0.00%	0.00%	0.00%
KR	21.27	6.55	0.25%	8.1%	3.13%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
LLY	24.28	2.54	0.72%	7.0%	7.49%	17.90%	21.04%	30.46%	33.95%	39.86%	32.74%	17.24%
MSCI	16.27	1.57	0.55%	6.4%	3.39%	0.39%	4.29%	3.89%	0.00%	0.00%	0.00%	0.00%
NEM	20.52	7.05	0.23%	11.5%	7.75%	1.73%	2.37%	0.00%	0.00%	0.00%	0.00%	0.00%
NOW	16.59	3.99	0.70%	9.6%	3.47%	8.80%	9.55%	8.74%	7.43%	2.26%	0.00%	0.00%
NVDA	13.59	2.3	1.25%	10.9%	0.00%	0.00%	2.78%	7.43%	20.65%	34.87%	52.41%	74.40%
PGR	20.02	1.45	0.59%	7.2%	17.17%	27.29%	26.62%	22.37%	26.58%	13.47%	3.32%	0.00%
PSA	13.05	4.74	0.23%	8.9%	1.88%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
QRVO	28.52	12.32	0.29%	9.2%	0.45%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
REGN	18.04	0.34	0.36%	6.2%	0.24%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
TSLA	25.23	3.31	1.01%	13.9%	0.00%	0.00%	2.33%	3.94%	6.36%	9.54%	11.54%	8.36%
VRTX	22.72	0.75	0.48%	5.1%	3.92%	1.27%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
WMT	25.26	5.86	0.36%	9.4%	9.91%	7.34%	8.71%	5.62%	0.00%	0.00%	0.00%	0.00%

Table 6. Risk and return of efficient portfolios at E less than selected thresholds.

selection 1st decile			selection of 1st and 2nd decile			selection 1st quartile		
	CVaR	E_p		CVaR	E_p		CVaR	E_p
E1	4.815%	0.434%	E1	4.132%	0.394%	E1	4.108%	0.397%
E2	4.966%	0.487%	E2	4.235%	0.452%	E2	4.201%	0.454%
E3	5.218%	0.540%	E3	4.451%	0.510%	E3	4.408%	0.512%
E4	5.658%	0.593%	E4	4.766%	0.567%	E4	4.775%	0.569%
E5	6.325%	0.645%	E5	5.335%	0.625%	E5	5.349%	0.627%
E6	7.145%	0.698%	E6	6.246%	0.683%	E6	6.266%	0.684%
E7	8.159%	0.751%	E7	7.599%	0.740%	E7	7.619%	0.741%
E8	9.665%	0.804%	E8	9.343%	0.798%	E8	9.353%	0.799%

Table 7. Composition of efficient portfolios and stock ratios for E less than 0.645.

Action	**ESG**	**E**	**Yield shares**	**CVaR**	**E1**	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**	**E8**
AAPL	17.22	0.46	0.61%	8.6%	6.54%	10.97%	14.83%	19.23%	32.51%	38.53%	40.14%	17.72%
AJG	21.16	0.09	0.50%	9.2%	8.02%	32.86%	35.44%	42.85%	24.65%	14.50%	0.00%	0.00%
ANET	16.67	0.4	0.86%	9.1%	6.38%	6.84%	17.03%	24.09%	31.31%	43.41%	57.38%	78.26%
AZO	11.02	0.11	0.43%	11.8%	23.25%	14.23%	3.72%	0.00%	3.43%	0.63%	0.00%	0.00%
EA	13.29	0.13	0.21%	8.3%	13.93%	2.92%	0.50%	0.00%	0.00%	0.00%	0.00%	0.00%
NFLX	16.41	0.09	0.65%	13.6%	3.98%	2.52%	0.08%	1.14%	2.11%	1.78%	2.48%	4.02%
ORCL	13.89	0.5	0.39%	6.5%	13.42%	0.00%	4.54%	0.00%	0.00%	0.00%	0.00%	0.00%
REGN	18.04	0.34	0.36%	6.2%	16.19%	18.99%	19.32%	12.69%	0.00%	0.00%	0.00%	0.00%
SPGI	13.04	0	0.41%	6.2%	8.31%	10.67%	1.43%	0.00%	0.00%	0.00%	0.00%	0.00%
UNH	15.3	0.03	0.45%	10.3%	0.00%	0.00%	3.10%	0.00%	5.99%	1.15%	0.00%	0.00%

In the last part, the calculation was carried out based on the model using relations (10)–(13), (18), and (15). Thus, the condition of the weighted sum of E values with a value lower than 0.645 (1st decile) was added, while the input was 422 S&P 500 stocks, since, in this case, stocks with a higher value of the E indicator can also enter the portfolio, though the weighted sum cannot exceed the required value. The results are shown in Tables 8 and 9. Additionally, the weights for the individual efficient portfolios are shown in Table 9, with 26 assets that occurred in at least one portfolio.

The analysis aims to compare the results based on the three options. Figure 4 shows the efficient portfolios in the return-risk space represented by the CVaR values. Based on Figure 4, an increase in the risk-return ratio can be observed with additional environmental conditions. However, it is interesting to compare the ratios shown (i.e., to what extent an investor considering environmental criteria has to increase the riskiness of the investment).

The above analysis was also carried out for the values of the environmental indicator E in decile 1 and 2 and quartile 1 (Tables 6 and 8). Figure 5 shows the effective portfolios for the three options in deciles 1 and 2, and Figure 6 shows the effective portfolios for the three options in quartile 1.

Table 8. Risk and return of efficient portfolios.

condition sum 1st decile				condition sum of 1st and 2nd decile				condition sum 1st quartile			
	CVaR	E_p	CVaR difference without ESG		CVaR	E_p	CVaR difference without ESG		CVaR	E_p	CVaR difference without ESG
E1	4.167%	0.348%	22.2%	E1	3.760%	0.348%	10.3%	E1	3.689%	0.348%	8.2%
E2	4.357%	0.461%	16.5%	E2	3.947%	0.461%	5.5%	E2	3.897%	0.461%	4.2%
E3	4.896%	0.575%	16.1%	E3	4.398%	0.575%	4.3%	E3	4.300%	0.575%	2.0%
E4	5.820%	0.688%	20.4%	E4	5.045%	0.688%	4.4%	E4	4.923%	0.688%	1.8%
E5	7.410%	0.801%	29.8%	E5	6.053%	0.801%	6.0%	E5	5.837%	0.801%	2.2%
				E6	7.504%	0.914%	9.5%	E6	7.160%	0.914%	4.5%
				E7	9.120%	1.027%	12.2%	E7	8.647%	1.027%	6.4%
								E8	10.372%	1.141%	6.9%

Table 9. Composition of efficient portfolios and stock indicators under the condition of a weighted sum of E values for the 1st decile.

Action	**ESG**	**E**	**Yield shares**	**CVaR**	**E1**	**E2**	**E3**	**E4**	**E5**
AAPL	17.22	0.46	0.61%	8.6%	2.94%	9.83%	11.39%	0.00%	0.00%
AJG	21.16	0.09	0.50%	9.2%	0.00%	11.09%	21.46%	22.53%	11.55%
ANET	16.67	0.4	0.86%	9.1%	1.17%	4.00%	15.44%	21.07%	37.86%
AZO	11.02	0.11	0.43%	11.8%	13.09%	16.29%	7.51%	16.65%	0.00%
CME	17.8	2.13	0.15%	7.5%	1.15%	0.00%	0.00%	0.00%	0.00%
CNC	19.89	0.09	0.59%	11.6%	0.00%	0.17%	0.00%	6.26%	19.99%
COF	22.5	0.13	0.22%	7.0%	10.96%	1.67%	0.00%	0.00%	0.00%
CRM	14.88	2.01	0.08%	6.2%	10.81%	10.07%	0.00%	0.00%	0.00%
CSCO	13.87	0.45	0.58%	8.3%	0.00%	0.00%	0.00%	2.43%	0.00%
CVS	21.97	0.03	0.55%	9.9%	0.00%	0.00%	0.00%	0.00%	1.43%
FITB	16.84	1.19	0.19%	6.7%	1.49%	0.00%	0.00%	0.00%	0.00%
JNJ	23.98	0.92	0.16%	6.0%	9.91%	0.00%	0.00%	0.00%	0.00%
LLY	24.28	2.54	0.72%	7.0%	0.00%	1.18%	4.80%	0.00%	0.00%
MMC	21.07	0.09	0.36%	4.9%	1.21%	0.00%	0.00%	0.00%	0.00%
NDAQ	13.09	0.03	0.35%	8.5%	2.88%	0.00%	0.00%	0.00%	0.00%
NFLX	16.41	0.09	0.65%	13.6%	0.00%	0.40%	0.00%	0.00%	0.00%
NVDA	13.59	2.3	1.25%	10.9%	0.00%	0.00%	0.29%	14.89%	20.09%
ORCL	13.89	0.5	0.39%	6.5%	2.82%	0.00%	0.00%	0.00%	0.00%
PANW	13.92	0.7	0.63%	13.8%	0.00%	0.35%	0.22%	0.00%	0.00%
PGR	20.02	1.45	0.59%	7.2%	5.73%	13.86%	19.77%	5.97%	0.00%
REGN	18.04	0.34	0.36%	6.2%	3.24%	5.93%	12.81%	0.00%	0.00%
ROL	19.31	1.76	0.41%	7.7%	1.86%	0.00%	0.00%	0.00%	0.00%
SPGI	13.04	0	0.41%	6.2%	11.69%	11.40%	0.59%	0.00%	0.00%
UNH	15.3	0.03	0.45%	10.3%	0.00%	0.00%	0.00%	0.00%	9.09%
VRSK	17.53	0.05	0.34%	8.4%	5.50%	0.84%	0.00%	0.00%	0.00%
VRTX	22.72	0.75	0.48%	5.1%	13.57%	12.92%	5.71%	10.20%	0.00%

The increase in risk for the exact value of the required return for Options 1 and 3 (Table 8) averages 21.0%, which represents a significant increase in the risk.

In Option 2, the investor cannot achieve the risk values of Option 1 and Option 3, with the lowest risk level being 4.815% at an expected return value of 0.434%. For Variants 1 and 3, the investor has a higher expected return and a lower risk for an efficient E2 portfolio (Variant 1 risk of 3.739%, expected return of 0.461%, and Variant 3 risk of 4.357%, expected return of 0.461%). The increase in risk with a lower expected return are 28.79% compared to Option 1 and 10.53% compared to Option 3.

In the first and second variants, only three stocks—ANET[1] , AZO[2] , REGN[3]—are used in each portfolio, which results in a high return-risk ratio, a return path to other assets (asset correlation), and a low value of the E indicator for these assets. The above phenomenon indicates that environmental companies do not achieve a high risk-return ratio. In contrast to the first and third variants, where there are 7 common stocks—ANET, AZO, CME[4] , LLY[5] , NVDA[6] , PGR[7] , and REGN—there is a higher correlation since all assets are considered in the portfolio construction and are also assets with a higher value of the E indicator, which is compensated by the selection of assets with a lower value of this indicator. The highest match, namely 9 stocks—AAPL[8] , AJG[9] , ANET, AZO, NFLX[10] , ORCL[11] , REGN, SPGI[12] , and UNH[13]—is in the second and third variants, as almost all stocks from the second variant are selected, and in the third variant, they are complemented by assets with higher E ratio, which will cause an improvement in the return-risk ratio in efficient portfolios.

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[1] https://finance.yahoo.com/quote/ANET/. Arista Networks, Inc. is a leading supplier of cloud networking solutions for internet companies, cloud service providers, and next-generation data centers.

[2] https://finance.yahoo.com/quote/AZO/. AutoZone, Inc. is an American retailer of spare parts and accessories for the automotive industry.

[3] https://finance.yahoo.com/quote/REGN/. Regeneron Pharmaceuticals, Inc. is a leading biotechnology company using the power of science to bring new medicines to patients in need.

[4] https://finance.yahoo.com/quote/CME/. CME Group Inc. is a financial services company.

[5] https://finance.yahoo.com/quote/LLY/. Regeneron Pharmaceuticals, Inc. Lilly is a medicine company turning science into healing to make life better for people around the world.

[6] https://finance.yahoo.com/quote/NVDA/. NVIDIA Corporation is an American company that designs and develops processors for graphics cards and chipsets for motherboards.

[7] https://finance.yahoo.com/quote/PGR/. The Progressive Corporation is an American insurance company.

[8] https://finance.yahoo.com/quote/AAPL/. Apple, Inc. engages in the design, manufacture, and sale of smartphones, personal computers, tablets, wearables and accessories, and other varieties of related services.

[9] https://finance.yahoo.com/quote/AJG/. Arthur J. Gallagher & Co. is an American global insurance brokerage company providing risk management services.

[10] https://finance.yahoo.com/quote/NFLX/. Netflix, Inc. is an American on-demand Internet streaming media provider and film studio.

[11] https://finance.yahoo.com/quote/ORCL/. Oracle Corporation is one of the largest companies that develop relational databases, tools for database development and management, or customer relationship management systems.

[12] https://finance.yahoo.com/quote/SPGI/. S&P Global provides data and benchmarks to capital and commodity market participants.

[13] https://finance.yahoo.com/quote/UNH/. UnitedHealth Group Incorporated is an American multinational health insurance and services company.

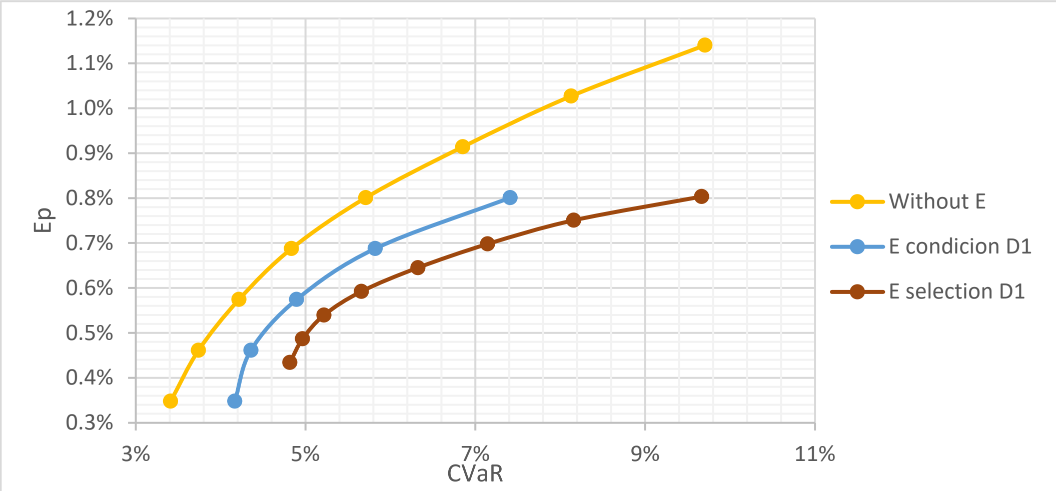

Figure 4. Effective frontier of portfolios for analyzing E values from the 1st decile.

The above analysis was also carried out for the environmental indicator values in deciles 1 and 2 and quartile 1. Figure 5 shows the effective portfolios for the three variants in deciles 1 and 2.

The increase in risk for the exact value of the required return for Options 1 and 3 (Table 8) averages 7.45%, which represents a significant increase in risk.

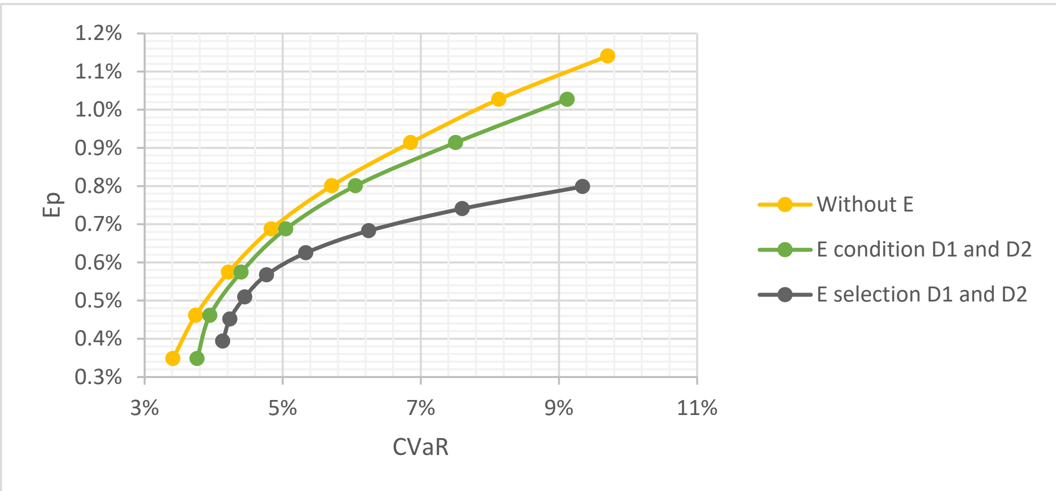

Figure 5. Effective frontier of the portfolios for analyzing E values from decile 1 and decile 2.

Figure 6 shows the analysis of the values calculated for the 1st quartile. As can be observed from the curves in Figures 5 and 6, the risk-return ratio for the higher-return portfolios significantly deteriorates for the second quartile, thus causing the selection of assets with lower environmental indicators that achieve significantly lower expected return values.

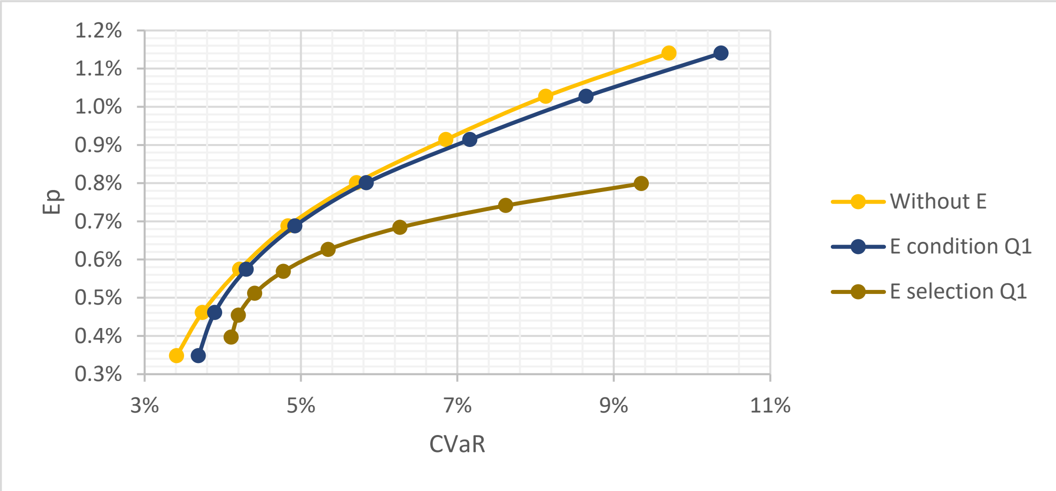

Figure 6. Effective frontier of portfolios for analyzing E values from the 1st quartile.

The increase in risk for the same value of required return for Options 1 and 3 (Table 8) averages 4.51%, thus representing a significant increase in the risk. Therefore, it is clear from the analyses performed that as the environmental aspects increase (calculations for the 1st decile), the risk/expected return ratio increases for portfolios with higher returns, thus causing the selection of assets with lower environmental indicators. In the calculations for the 1st quartile, this ratio decreases, and in subsequent calculations with a lower emphasis on E (e.g., second quartile), the difference between the different options considered decreases for the calculated efficient portfolios and the same value of the required return.

The above procedure can be generalized in a flowchart:

1. Acquisition and analysis of input data
2. Does the input yield data have a normal distribution?

a. Yes - select a measure of risk requiring a normal distribution (e.g., Markowitz portfolio selection model)

b. No - select a risk measure not requiring a normal distribution (e.g., CVaR-based portfolio selection model)

3. Should the environmental aspect be implemented?

a. No - Portfolio selection model without environmental criteria

b. Yes

i. Apply the portfolio selection model to a set of stocks selected based on the calculated value of E (or ESG, S, G)

ii. Extend the portfolio selection model to include the conditions of meeting the calculated value of E (resp. ESG, S, G)

4. Analysis of results.

In the presented analysis, the portfolio selection model based on CVaR was used after a thorough examination of the input data, which did not confirm their normal distribution.

4. Discussion

ESG-enabled green investing is increasing, with estimates that global sustainability-related assets could reach $50 trillion by 2025. Many decision-support tools have been developed to help make green investment decisions, such as the S&P 500 ESG Index, which is comprise of S&P 500 companies that meet the sustainability criteria. Thus, the S&P 500 ESG provides information for investors who reflect legislative rules, such as Article 8 of the EU Sustainable Finance Disclosure Regulation, in their decision-making.

However, many investors also require additional information, particularly on the return and riskiness of such investments. Markowitz model-based analyses are preferred in the literature (De Spiegeleer et al., 2023; Torricelli & Bertelli, 2022; Xidonas & Essner, 2024). Moreover, statistical analyses revealed that the input data often do not adhere to the assumption of a normal distribution. This raises questions about the suitability of such an approach for all analyses designed for portfolio construction (Arcuri et al., 2023; Pekár & Pčolár, 2022). These findings open up avenues for future research to discuss the suitability of different approaches and models.

At present, many investors are grappling with the question of how environmental requirements influence the asset returns. The literature presents two primary approaches to integrate ESG criteria into the portfolio selection models. Some authors concentrated on selecting assets with the desired value of selected indicators (ESG, E, S, G), as seen in (Jia, 2024; Varmaz et al., 2024). Another group of authors examined the group condition on the value of the respective criterion on the constructed portfolio (Cesarone et al., 2022). However, their correlation has yet to be analyzed, thus making it challenging to make a clear recommendation on which approach is more suitable for implementation. This paper delved into the aspect of determining the amount of risk difference in the approach of selecting assets with the required value of the relevant indicator, which led to a narrowing of the set of potential assets in the portfolio regardless of their return. Additionally, it explored assets for which the group condition of the value of the relevant indicator could be applied, thus allowing the inclusion of assets with a higher return in the portfolio based on the group condition.

The article deals with the modification of the portfolio selection model based on the criteria of maximization of the expected return and minimization of the risk based on the CVaR rates and ESG indicators, in which, six criteria are considered, unlike the models known from the literature. The values of the indicator E, which represents the prioritization of environmental goals, were used in the analyses. The solution for the presented model was performed in the Python programming language. The result of the process obtained effective solutions for the modified model in the Python programming environment, namely the MIP module, which documents the possibility of calculating the solution and serves as a basis for the investor's decision.

The article aimed to analyze the impact of introducing environmental criteria into the portfolio construction decisions for the modern investor. As there is now an increased emphasis on green investments, the investor must know the impact of incorporating green investments into the portfolio, as the investor must be willing to accept a certain level of increased risk in their investments based on past experience. The article presents a tool that can be used to analyze the impact of an increase in the investment risk for a selected group of assets using the S&P 500 index stocks. Up-to-date ESG indicators available on finance.yahoo.com were used to determine the environmental criteria. To automate the calculations, in addition to the MIP module for optimization, the yesg module was used to retrieve data on the ESG indicators from Sustainalytics, Inc.

When analyzing the impact of introducing the environmental criteria into the portfolio construction decisions, three options were considered that can be applied to the investment strategy. In the first variant, the investor does not accept the risk associated with the impact of the inclusion of the environmental indicators and uses a classical, conservative approach of maximizing returns while minimizing risk, to which the CVaR risk measure can be applied when constructing the portfolio. This approach, which was implemented for models (10) - (18) without constraints (14)–(17), provides a baseline for the investor to compare the obtained values with environmental constraints (Table 4). Based on the calculations, the portfolio represented 25 assets that occurred in at least one portfolio.

Respecting the environmental criteria, two cases were analyzed (second and third variants of the analysis). For the first case of emphasizing the environmental requirements, the assumption was considered that the investor only invests in the environmental firms that met a certain value of indicator E. This selection was only made for firms in the 1st decile (first 43 firms), the 1st and 2nd decile (first 86 firms), and the1st quartile (first 113 firms) with the lowest E values out of the 422 firms analyzed in the S&P 500 stock index. For these selected sets of firms, calculations were performed based on the portfolio selection model (10)–(18) without bounds (14) to (17).

In the second case of emphasizing the environmental requirements, the investor does not restrict the selection of assets from each group based on any requirement, but is interested in the overall value of the environmental indicator of the constructed portfolio E (i.e., the value of the weighted environmental indicator of the portfolio E was located in the 1st decile, 1st and 2nd decile and the 1st quartile (Table 3), thus applying the bound (15) of the presented model (10)–(18) without bounds (14), (16), and (17)).

From the results of the analyses of the environmental criterion represented by the indicator E presented in Tables 6 to 13 and Figures 4 to 6, it is clear that the more appropriate way of portfolio selection is the variant with the calculation of the total value of the weighted environmental indicator E (bound (15) of the presented model); this is because there are also stocks with a higher environmental burden in the formed portfolio, but with a high return, which compensate for the selection of assets with a lower environmental burden, but with a lower return. This allows the construction of portfolios that have a higher expected return-to-risk ratio. These observations are also evident in Figures 4 to 6, where the expected return/risk ratios were close to those calculated in the model without considering the environmental criteria. Thus, the investor achieves similar ratios even after considering the additional criteria.

Therefore, based on the analyses carried out, it can be concluded that considering the environmental criteria can lead to an increase in the investment risk for the same expected return. Strictly investing in the environmental firms with the required E values leads to a substantial increase in this risk and the selection of assets with lower returns. Applying the environmental constraint in the form of constraint (15) of the presented model with the computation of the total value of the weighted environmental portfolio indicator E seems to be a more appropriate way of environmental investing because, in this case, stocks with a higher environmental burden but high return are also included in the portfolio. This innovative approach, represented by model (10) - (18), allows the ESG criteria, either overall or partial, to be incorporated into portfolio construction to suit the investor's requirements.

Tools for green investor decision-making are continuously being refined to reflect an investors' preference for integrating environmental sustainability into investment decision-making. Investors aim to seek out investment opportunities that benefit the environment. Accordingly, investors are currently rethinking their strategies for reallocating investment capital and fundamentally realigning their finances.

Companies often offer various ESG metrics as part of their financial reporting. However, there are no universal standards for ESG evaluation. ESG-focused assets are currently assessed based on the same approaches as the rest of the equity market.

Therefore, investors should not only carefully examine a company's actual sustainability impacts, but also look beyond traditional securities analyses to find other ways to make meaningful investments in a sustainable future.

Therefore, the approach to an investment portfolio construction based on classical models, as well as the formulation of new approaches and models, should be modified in the future. The authors' research focused on modifying the CVaR model through ESG indicators, and especially focused on the environmental portfolio indicator E. For a comprehensive assessment of the firm, it is also interesting to consider other indicators of the ESG concept and to analyze the possible impacts of the calculated values of the complete ESG indicator and its components. From the calculations carried out in the author's research, it is clear that when considering the individual boundary values from the higher deciles of the set of firms under study, the portfolio formed minimally differed compared to the portfolio formed without ESG boundaries. However, the analyses were interesting when combining different thresholds for individual indicators.

In the present paper, a set of recommended portfolios were computed for the analyzed data to determine the value of returns. In the final decision-making of the investor, it is necessary to select the one that most satisfies the investor's requirements from the given set of efficient portfolios, thereby taking the different risk aversions of the investors based on the determined risk aversion curve in the literature into account (Chhatwani, 2024; Su & Li, 2024)). Then, the investor's strategy can be incorporated into a scheme for selecting a portfolio from a set of efficient alternatives as follows:

1. Determine the degree of risk aversion
2. If risk aversion is high, then implement portfolio selection with minimal risk
3. If risk aversion is low, then implement portfolio selection with maximum return
4. As the risk aversion decreases, portfolio selection with a higher return and a higher risk is implemented.

From the results obtained in Tables 4, 6, and 8, it is possible to interpret the differences in the two extreme strategies for an investor with high (point 2 of the scheme) and low (point 3 of the scheme) risk aversion (Table 10 represents the values for the first decile).

Table 10. Comparison of extreme strategies for the investor for the 1st decile.

	absence of E		selection 1st decile		condition sum 1st decile	
	CVaR	E_p	CVaR	E_p	CVaR	E_p
Min E_p	3.410%	0.348%	4.815%	0.434%	4.167%	0.348%
Max E_p	9.706%	1.141%	9.665%	0.804%	7.410%	0.801%

From the values in Table 10 for the calculated efficient strategies, it is evident that the highest rate of return (with the lowest risk aversion) can be achieved for the strategy without considering the environmental aspect (1.141%) with a corresponding risk rate of 9.706%. When investing in assets with a specified maximum value of E (1st decile), a maximum rate of return of 0.804% can be achieved at a risk rate of 9.665%. When investing in assets by accepting the group condition on the value of criterion E, a maximum return rate of 0.801% can be achieved at a risk rate of 7.410%. Thus, it is clear that the ratio of the rates of return to the risk is worst when investing in stocks with a specified maximum value of E. Adequately, the values for the investor with the highest risk aversion (min Ep), for whom the risk rate is the highest among the alternatives considered (4.815%), can also be interpreted, as shown in Figure 4.

Thus, the presented modeling approach provides the investor with a basis for determining their portfolio strategy. In addition to classical approaches and models, environmental aspects can be used in such portfolio construction. Determining the relevant cutoff values for complex analyses of combinations of the ESG, E, S, and G indicators through the general model (10)-(18) may be an interesting area for future research.

Author contributions

Conceptualization, JP, IB and MR; data collection and curation, JP, IB and MR; formal analysis, JP, IB and MR; investigation JP, IB and MR; methodology JP, IB and MR; project administration, JP, IB and MR; software, JP, IB and MR; supervision, JP, and IB; visualization, JP, IB and MR; writing—original draft, JP, IB and MR; writing—review and editing, JP, IB and MR. All authors have read and agreed to the published version of the manuscript.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The author declares no conflicts of interest.

References

Alexander C (2009) *Market risk analysis, value at risk models* (Vol. 4): John Wiley & Sons.

Arcuri MC, Gandolfi G, Laurini F (2023) Robust portfolio optimization for banking foundations: a CVaR approach for asset allocation with mandatory constraints. *Cent Eur J Oper Res* 31: 557–581. https://doi.org/10.1007/s10100-022-00821-5

Arici G, Campi MC, Carè A, et al. (2021) A theory of the risk for empirical CVaR with application to portfolio selection. *JSSC* 34: 1879–1894. https://doi.org/10.1007/s11424-021-1229-3

Artzner P, Delbaen F, Eber JM, et al. (1999) Coherent measures of risk. *Math Financ* 9: 203–228. https://doi.org/10.1111/1467-9965.00068

Basak S, Shapiro A (2001) Value-at-risk-based risk management: optimal policies and asset prices. *Rev Financ Stud* 14: 371–405. https://doi.org/10.1093/rfs/14.2.371

Becchetti L, Cordella M, Morone P (2022) Measuring investments progress in ecological transition: The Green Investment Financial Tool (GIFT) approach. *J Clean Prod* 357: 131915. https://doi.org/10.1016/j.jclepro.2022.131915

Bodnar T, Lindholm M, Niklasson V, et al. (2022) Bayesian portfolio selection using VaR and CVaR. *Appl Math Comput* 427: 127120. https://doi.org/10.1016/j.amc.2022.127120

Boffo R, Patalano R (2020) ESG investing: practices, progress and challenges. *Éditions OCDE, Paris*.

Caferra R, Falcone PM (2023) From the "age of instability" to the "age of responsibility": economic uncertainty and sustainable investments. *J Econ Stud* 50: 1297–1316. https://doi.org/10.1108/JES-06-2022-0353

Cesarone F, Martino ML, Carleo A (2022) Does ESG impact really enhance portfolio profitability? *Sustainability* 14: 2050. https://doi.org/10.3390/su14042050

De Spiegeleer J, Höcht S, Jakubowski D, et al. (2023) ESG: a new dimension in portfolio allocation. *JSF&I* 13: 827–867. https://doi.org/10.1080/20430795.2021.1923336

Drobetz W, El Ghoul S, Fu Z, et al. (2024) Institutional investors and corporate environmental costs: The roles of investment horizon and investor origin. *Eur Financ Manag* 30: 727–769. https://doi.org/10.1111/eufm.12444

Du HS, Zhan B, Xu J, et al. (2019) The influencing mechanism of multi-factors on green investments: A hybrid analysis. *J Clean Prod* 239: 117977. https://doi.org/10.1016/j.jclepro.2019.117977

Escrig-Olmedo E, Rivera-Lirio JM, Muñoz-Torres MJ, et al. (2017) Integrating multiple ESG investors' preferences into sustainable investment: A fuzzy multicriteria methodological approach. *J Clean Prod* 162: 1334–1345. https://doi.org/10.1016/j.jclepro.2017.06.143

Eyraud L, Clements B, Wane A (2013) Green investment: Trends and determinants. *Energ Policy* 60: 852–865. https://doi.org/10.1016/j.enpol.2013.04.039

Falcone PM (2018) Green investment strategies and bank-firm relationship: A firm-level analysis. *Econ Bull* 38: 2225–2239.

Falcone PM (2020) Environmental regulation and green investments: The role of green finance. *IJGE* 14: 159–173. https://doi.org/10.1504/IJGE.2020.109735

García F, González-Bueno J, Guijarro F, et al. (2020) Forecasting the environmental, social, and governance rating of firms by using corporate financial performance variables: A rough set approach. *Sustainability* 12: 3324. https://doi.org/10.3390/su12083324

García García F, Gankova-Ivanova T, González-Bueno J, et al. (2022) What is the cost of maximizing ESG performance in the portfolio selection strategy? The case of The Dow Jones Index average stocks. *Entrep sustain issues* 9: 178–192. https://doi.org/10.9770/jesi.2022.9.3(9)

Gelles D (2023) How environmentally conscious investing became a target of conservatives. *The New York Times*. *Retrieved* 2: 408.

Giese G, Lee L-E, Melas D, et al. (2019) Foundations of ESG investing: How ESG affects equity valuation, risk, and performance. *J Portf Manag* 45: 69–83. https://doi.org/10.3905/jpm.2019.45.5.069

Hamdi A, Karimi A, Mehrdoust F, et al. (2022) Portfolio Selection Problem Using CVaR Risk Measures Equipped with DEA, PSO, and ICA Algorithms. *Mathematics* 10. https://doi.org/10.3390/math10152808

Hsiao HF, Zhong T, Dincer H (2019) Analysing Managers' Financial Motivation for Sustainable Investment Strategies. *Sustainability* 11: 3849. https://doi.org/10.3390/su11143849

Chhatwani M (2024) Financial awareness, risk aversion and portfolio choice. *Appl Econ* 1–16. https://doi.org/10.1080/00036846.2024.2356896

Jia S (2024) Impact of ESG Risk on Portfolio Optimization and Returns: An Analysis Using the Markowitz Model. *HBEM* 40: 1184–1192. https://doi.org/10.54097/2dksd066

Kang Z, Li X, Li Z (2020) Mean-CVaR portfolio selection model with ambiguity in distribution and attitude. *JIMO* 16. https://doi.org/10.3934/jimo.2019094

Makridou G, Doumpos M, Lemonakis C (2024) Relationship between ESG and corporate financial performance in the energy sector: empirical evidence from European companies. *Int J Energy Sect Manag* 18: 873–895. https://doi.org/10.1108/IJESM-01-2023-0012

Mokhov VGe, Chebotareva GS, Khomenko PM (2018) Modelling of "green" investments risks. *Bulletin SUSU MMCS* 11: 154–159. https://doi.org/10.14529/mmp180213

Pekár J, Brezina I, Reiff M (2024) Determining the investors' strategy during the COVID-19 crisis based on the S&P 500 stock index. *Strateg Manag J* 29: 028–042. https://doi.org/10.5937/StraMan2200029P

Pekár J, Pčolár M (2022) Empirical distribution of daily stock returns of selected developing and emerging markets with application to financial risk management. *Cent Eur J Oper Res* 30: 699–731. https://doi.org/10.1007/s10100-021-00771-4

Pekovic S, Grolleau G, Mzoughi N (2018) Environmental investments: too much of a good thing? *Int J Prod Econ* 197: 297–302. https://doi.org/10.1016/j.ijpe.2018.01.012

Rahko J (2023) The effects of environmental investments on the economic performance of industrial plants – Evidence from Finland. *J Clean Prod* 394: 136142. https://doi.org/10.1016/j.jclepro.2023.136142

Rockafellar RT, Uryasev S (2000) Optimization of conditional value-at-risk. *J Risk* 2: 21–42. https://doi.org/10.21314/JOR.2000.038

Su X, Li Y (2024) Robust portfolio selection with subjective risk aversion under dependence uncertainty. *Econ Model* 132: 106667. https://doi.org/10.1016/j.econmod.2024.106667

Sustainalytics (2023) *ESG risk ratings: a 360° review*. Available from: https://connect.sustainalytics.com/inv-esg-risk-ratings-a-360-review-ebook?_gl=1*1jaaf8s*_ga*MTY0NzU0MTQ4NS4xNjg1ODk4NTcy*_ga_C8VBPP9KWH*MTY5MDg4NjIzOS4yLjEuMTY5MDg4NzQ5Ny4yOS4wLjA.

Taghizadeh-Hesary F, Yoshino N (2020) Sustainable solutions for green financing and investment in renewable energy projects. *Energies* 13: 788. https://doi.org/10.3390/en13040788

Torricelli C, Bertelli B (2022) ESG compliant optimal portfolios: The impact of ESG constraints on portfolio optimization in a sample of European stocks. *CEFIN WORKING PAPERS*.

Varmaz A, Fieberg C, Poddig T (2024) Portfolio optimization for sustainable investments. *Ann Oper Res* 341: 1151–1176. https://doi.org/10.1007/s10479-024-06189-w

Wang GJ, Zhu CL (2021) BP-CVaR: A novel model of estimating CVaR with back propagation algorithm. *Econ Lett* 209: 110125. https://doi.org/10.1016/j.econlet.2021.110125

Xidonas P, Essner E (2024) On ESG portfolio construction: A multi-objective optimization approach. *Comput Econ* 63: 21–45. https://doi.org/10.1007/s10614-022-10327-6

Yamai Y, Yoshiba T (2002) On the validity of value-at-risk: comparative analyses with expected shortfall. *Monetary Econ Stud* 20: 57–85.

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