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MODELOVANIE GLOBÁLNEHO ROZVOJA A DYNAMIKY OBYVATEĽSTVA SO VZDELÁVANÍM

MODELLING GLOBAL DEVELOPMENT AND POPULATION DYNAMICS WITH EDUCATION

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Článok predstavuje vybudovanie modelu rastu pre viac krajín zapojených do voľného obchodu. Dynamickými silami globálneho rozvoja sú akumulácia národného bohatstva, zmena populácie a dynamika ľudského kapitálu diferencovaná podľa pohlavia. Globálna ekonomika sa skladá z viacerých národných ekonomík a každá národná ekonomika pozostáva z priemyselného a vzdelávacieho sektoru. Model integruje Solowov model rastu, Uzawa-Lucasov model, Becker-Barrov výber fertility, Haavelmov populačný model a Oniki-Uzawov obchodný model. Integrácia sa uskutočňuje funkciou Zhanga a koncepciami súčasných a disponibilných príjmov. Jednotlivé štáty sa líšia v úrovni technológií, ľudskom kapitále, preferenciách rodiny a emocionálnych a finančných investíciách do detí. V článku je opísaná globálna ekonomická dynamika krajín J pomocou 4J nelineárnych diferenciálnych rovníc. Model je simulovaný. Zobrazuje sa pohyb globálneho systému a je vykonaná komparatívna dynamická analýza s ohľadom na efektivitu akumulácie ľudského kapitálu. Skúmajú sa napríklad finančné náklady rodín pri starostlivosti o deti, otázka počtu deti, využívanie voľného času a pod.
Kľúčové slová: ľudský kapitál, deti, rodovo diferencované rozloženie v čase, medzinárodný obchod; endogénna miera pôrodnosti, endogénna miera úmrtnosti

This paper constructs a multi-country growth model with free trade. The dynamic machines of global development are national wealth accumulation, national population change, and gender-differentiated human capital dynamics. The global economy consists of multiple national economies and each national economy is composed of industrial sector and education sector. The model integrates Solow's growth, Uzawa-Lucas', Becker-Barro's fertility choice, Haavelmo's population, and Oniki-Uzawa's trade models. The integration is conducted with Zhang's utility function and concepts of current and disposable incomes. Nations are differenciated in technology, gender-related human capital, family preference, and emotional and pecuniary investment in children. We describe J-country

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global economic dynamics with 4J nonlinear differential equations. The model is simulated. We plot the movement of the global system and carry out comparative dynamic analysis with regards to woman's human capital accumulation efficiency, the propensity to have children, the propensity to save, the total factor productivities, men's propensities to use leisure, and families' pecuniary costs in children caring.
Keywords: gender-based human capital; propensity to have children; gender-differentiated time distribution; international trade; endogenous birth rate; endogenous mortality rate
JEL: O4, I25

1 INTRODUCTION

Modern economies are characterized by complicated dynamic interdependence between many variables such as population growth, human capital accumulation, wealth accumulation, gender division of labor and children caring. But modern dynamic economic theories deal simplified interdependence between a few variables. It is obvious that any genuine modelling of economic dynamics will result in high dimensional nonlinear dynamics. Nevertheless, it is only in recent years that we can examine behavior of highly dimensional dynamics. This partly explains why economic theory has been dominated by modelling dynamic economic systems with a few variables. The purpose of this study is to develop a dynamic general equilibrium model with endogenous wealth, population, and gender-differentiated human capital for any number of countries with free trade.

In our approach the economic mechanisms of growth are countries' wealth accumulation, population change, and gender-differentiated human capital dynamics with free trade. The global economy consists of any number of national economy and each national economy is composed of industrial sector and education sector. Our model is to integrate some dynamic models in economic theory. The dynamic mechanisms of growth are capital accumulation, human capital growth, and population change. The capital accumulation follows the neoclassical growth model (Solow 1956, Burmeister and Dobell 1970, Azariadis 1993, Barro and Sala-i-Martin 1995, and Zhang 2020). We base human capital dynamics on the Uzawa-Lucas model (Uzawa 1965, Lucas 1988). There are many growth models with endogenous human capital and physical capital (Jones et al. 1993; Stokey and Rebelo 1995, de la Croix and Licandro 1999, Mino 1996, Lagerlof 2003, Alonso-Carrera and Freire-Sere 2004, Galor 2005, De Hek 2005, Sano and Tomoda 2010, and Osiobe 2019). Our population dynamics is influenced by the models by Haavelmo (1954, see also Stutzer 1980) and by Becker and Barro (1988). The population change consists of dynamics of birth and death. Many factors may interact with changes in fertility (Barro and Becker 1989, Galor and Weil 1996, Doepke 2004, Adsera 2005, Bosi and Seegmuller 2012, Hock and Weil 2012, and Chu et al. 2013). There are close interdependence between mortality rate and economic growth and (Schultz 1993, 1998, Robinson and Srinivasan 1997, Boucekkine et al. 2002, Blackburn and Cipriani 2002, Chakraborty 2004, Hazan and Zoabi 2006, Fanti and Gori 2011, Balestra and Dottori 2012, Lancia and Prarolo 2012). Our trade pattern is based on the Oniki-Uzawa model (Oniki and Uzawa, 1965). There are other trade models which treat capital accumulation as endogenous variables in the context of international economics (e.g., Ethier and Svensson 1986, Bhagwati 1991, Ikeda and Ono 1992, Wong 1995, Jensen and Wong 1998, Obstfeld and Rogoff 1998, Sorger 2002, Vellutini 2003, and Naito and Ohdoi 2011).

Household behavior is described with Zhang's utility function and concepts of current and disposable incomes (Zhang 1993, 2020). The model of this study is a synthesis of Zhang's two models. Zhang (2014) proposes a growth model with gender-differentiated human capital accumulation. Zhang (2016) develops a growth model with endogenous population and international trade. The rest of the paper is arranged as follows. Section 2 develops the multi-country growth model with endogenous wealth accumulation, gender-differentiated human capital accumulation, and population dynamics. Section 3 simulates a 3-country growth model. Section 4 makes comparative dynamic analysis in some parameters. Section 5 is the conclusion of the study.

2 THE GROWTH MODEL OF A MULTI-COUNTRY WORLD

This section synthesizes the two models by Zhang (2014, 2016). The global economy is composed of J national countries, indexed by j, $j = 1, \ldots J$. The production technology and price determination are based on the Solow growth model. In global economy there is only one homogenous capital good. Each national economy has one education sector and one industrial sector. The industrial sector produces the commodity for consumption and investment. The education sector educates the population. The two sectors' input factors are capital and labor, respectively. Input factors are fully utilized. All the assets belong to households. The disposable incomes of the households are distributed between saving, education, and consumption. International trade is free, and all the markets are perfectly competitive. All the prices are in terms of the capital good. The price of the capital good be unit. We introduce:

- T_0 - fixed available time for work, study, and children caring;
- i, and e − subscript index for production and education sectors, respectively;
- q − subscript index standing for gender, $q = 1$ for man; $q = 2$ for woman;
- $\bar{N}_j(t)$ and $N_j(t)$ − population and labor force of country j at time t;
- $n_j(t)$ and $d_j(t)$ − birth rate and mortality rate of country j;
- $r(t)$ and $w_j(t)$ – the international interest rate and country $j's$ wage rate;
- $w_{jq}(t)$ – the wage rate per unit work hour of worker (j, q);
- $p_j(t)$ – the per unit price of education service in country j;
- $T_{jq}(t)$, $\hat{T}_{jq}(t)$, $\bar{T}_{jq}(t)$ and $\tilde{T}_{jq}(t)$ − times spent on work, education, leisure, and child care by person (j, q);
- $s_j(t)$ and $\bar{k}_j(t)$ − household $j's$ saving and wealth;

- $c_j(t)$ – level of consumption of good by the household in country j;
- $H_{jq}(t)$ − level of human capital of person (j, q);
- $K(t)$ and $\bar{K}_j(t)$ − capital stocks of the world economy and capital owned by country j;
- $K_j(t)$ − total capital stock employed by country j;
- $F_{jm}(t)$ and $Y_j(t)$ − output level of country $j's$ sector m, and national output of country j;
- $K_{jm}(t)$ and $N_{jm}(t)$ – sector $(j, m)'s$ capital input and labor input, $m = i, e$;
- δ_{kj} − country $j's$ physical capital depreciation rate; and
- δ_{jq} − person $(j, q)'s$ depreciation rate of human capital.

National labor force. The total qualified labor force is:

$$N_j(t) = \left(T_{j1}(t)\, H_{j1}^{m_{j1}}(t) + T_{j2}(t)\, H_{j2}^{m_{j2}}(t)\right) \bar{N}_j(t), \tag{1}$$

in which m_{jq} are the efficiencies that worker (j, q) applies human capital.

Neoclassical technologies and marginal conditions of the two sectors. The two sectors' technologies are described with the Cobb-Douglas production functions as follows:

$$F_{jm}(t) = A_{jm}\, K_{jm}^{\alpha_{jm}}(t)\, N_{jm}^{\beta_{jm}}(t), A_{jm}, \alpha_{jm}, \beta_{jm} > 0, \alpha_{jm} + \beta_{jm} = 1, m = i, e, \tag{2}$$

where $A_{jm}, \alpha_{jm},$ and β_{jm} are parameters. The marginal conditions for the two sectors imply:

$$r(t) + \delta_{kj} = \frac{\alpha_{ji}\, F_{ji}(t)}{K_{ji}(t)} = \frac{\alpha_{je}\, p_j(t)\, F_{je}(t)}{K_{je}(t)},$$
$$w_j(t) = \frac{\beta_{ji}\, F_{ji}(t)}{N_{ji}(t)} = \frac{\beta_{je}\, p_j(t)\, F_{je}(t)}{N_{je}(t)}. \tag{3}$$

Worker $(j, q)'s$ wage rate $w_{jq}(t)$ and wage income $W_{jq}(t)$ are given as:

$$w_{jq}(t) \equiv h_{jq}(t)\, w_j(t), h_{jq}(t) \equiv H_q^{m_q}(t), W_{jq}(t) = w_{jq}(t)\, T_{jq}(t).$$

Consumer behaviors. Zhang proposed an alternative approach to household behavior (Zhang, 1993, 2020). This study applies Zhang's concept of disposable income and utility function. Households decide saving, time distribution between work, leisure, children caring and education, number of children, and consumption level of commodity.

The household receives the wage incomes and the interest payment. The household current income is the sum of the wage incomes and interest payment as follows:

$$y_j(t) = r(t)\, \bar{k}_j(t) + w_{j1}(t)\, T_{j1}(t) + w_{j2}(t)\, T_{j2}(t).$$

The household disposable income is the sum of the current income and the value of wealth as follows:

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t) = \bar{R}(t)\, \bar{k}_j(t) + w_{j1}(t)\, T_{j1}(t) + w_{j2}(t)\, T_{j2}(t), \qquad (4)$$

where $\bar{R}(t) \equiv 1 + r(t)$.

We will take account of time of children caring. The parent spend time on children and the following pecuniary cost:

$$p_{bj}(t) = \theta_j\, n_j(t)\, \bar{k}_j(t). \qquad (5)$$

The parent's time on raising children is related to fertility rate as follows:

$$\tilde{T}_{jq}(t) = \theta_{jq}\, n_j(t), \theta_{jq} \geq 0. \qquad (6)$$

The more the children they have, the more time the parent spend on children. We use a linear formation as (6). It implies no increasing or decreasing returns to scale. It is a strict assumption. It is a common sense that the time spent on per child falls as the parents have more children. We consider constant return to scale as this makes the analysis mathematically tractable.

The household expenditure is on saving $s_j(t)$, consumption of goods $c_j(t)$, and bearing children $p_{bj}(t)$. We have the budget constraint as follows:

$$p_j(t)\, \hat{T}_{j1}(t) + p_j(t)\, \hat{T}_{j2}(t) + c_j(t) + s_j(t) + \theta_j\, \bar{k}_j(t)\, n_j(t) = \hat{y}_j(t). \qquad (7)$$

Any person has the following time constraint:

$$T_{jq}(t) + \hat{T}_{jq}(t) + \bar{T}_{jq}(t) + \tilde{T}_{jq}(t) = T_0. \qquad (8)$$

Insert (8) in (7)

$$\bar{p}_{j1}(t)\, \hat{T}_{j1}(t) + \bar{p}_{j2}(t)\, \hat{T}_{j2}(t) + w_{j1}(t)\, \bar{T}_{j1}(t) + w_{j2}(t)\, \bar{T}_{j2}(t) + \widetilde{w}_j(t)\, n_j(t) + c_j(t) + s_j(t) = \bar{y}_j(t), \qquad (9)$$

where we apply (6) and

$$\bar{p}_{jq}(t) = p_j(t) + w_{jq}(t), \tilde{w}_j(t) \equiv w_{0j}(t) + \theta_j\, \bar{k}_j(t),$$
$$w_{0j}(t) \equiv w_{j1}(t)\, \theta_{j1} + w_{j2}(t)\, \theta_{j2}, \bar{y}_j(t) \equiv \bar{R}(t)\, \bar{k}_j(t) + w_{j1}(t)\, T_0 + w_{j2}(t)\, T_0.$$

Utility functions and optimal conditions. We specify the family's utility function of $c_j(t)$, $s_j(t)$, $\overline{T}_{jq}(t)$, $\tilde{T}_{jq}(t)$, and $n_j(t)$ as follows:

$$U_j(t) = c_j^{\xi_{j0}}(t) s_j^{\lambda_{j0}}(t) \hat{T}_{j1}^{\eta_{j01}}(t) \hat{T}_{j2}^{\eta_{j02}}(t) \bar{T}_{j1}^{\sigma_{j01}}(t) \bar{T}_{j2}^{\sigma_{j02}}(t) n_j^{\upsilon_{j0}}(t),$$

where ξ_{j0} is called the propensity to consume, λ_{j0} the propensity to own wealth, η_{j0q} gender $q's$ propensity to receive education, σ_{j0q} gender $q's$ propensity to use leisure time, and υ_{j0} the propensity to have children. The marginal conditions of maximizing $U_j(t)$ subject to (9) imply:

$$c_j(t) = \xi_j\, \bar{y}_j(t), s_j(t) = \lambda_j\, \bar{y}_j(t), \hat{T}_{jq}(t) = \frac{\eta_{jq}\, \bar{y}_j(t)}{\bar{p}_{jq}(t)}, \bar{T}_{jq}(t) = \frac{\sigma_{jq}\, \bar{y}_j(t)}{w_{jq}(t)},$$
$$n_j(t) = \frac{\upsilon_j\, \bar{y}_j(t)}{\tilde{w}_j(t)}, \qquad (10)$$

where

$$\xi_j \equiv \rho_j\, \xi_{j0}, \lambda_j \equiv \rho_j\, \lambda_{j0}, \eta_{jq} \equiv \rho_j\, \eta_{jq0}, \sigma_{jq} \equiv \rho_j\, \sigma_{jq0}, \upsilon_j \equiv \rho_j\, \upsilon_{j0},$$
$$\rho_j \equiv \frac{1}{\xi_{j0} + \lambda_{j0} + \eta_{j10} + \eta_{j20} + \sigma_{j10} + \sigma_{j20} + \upsilon_{j0}}.$$

The birth and mortality rates and population dynamics. The population change rate is the birth rate minus death rate

$$\dot{\bar{N}}_j(t) = \left(n_j(t) - d_j(t)\right) \bar{N}_j(t). \qquad (11)$$

There are different ideas about determinants of birth and mortality rates (e.g., Razin and Ben-Zion 1975, Yip and Zhang 1997, and Chu et al. 2013). Our model is influenced by the literature. As in Zhang (2014), the mortality rate is assumed negatively dependent on the disposable income:

$$d_j(t) = \frac{\bar{\upsilon}_j\, \bar{N}_j^{b_j}(t)}{\bar{y}_j^{a_j}(t)}. \qquad (12)$$

where $\bar{\upsilon}_j \geq 0$, $a_j \geq 0$. Parameter $\bar{\upsilon}_j$ is called mortality rate parameter. The term $N_j^{b_j}(t)$ takes account of possible influences of the population on mortality. Insert (10) and (12) in (11).

$$\dot{\bar{N}}_j(t) = \left(\frac{v_j\, \bar{y}_j(t)}{\widetilde{w}_j(t)} - \frac{\bar{v}_j\, \bar{N}_j^{b_j}(t)}{\bar{y}_j^{a_j}(t)}\right) \bar{N}_j(t). \quad (13)$$

Wealth change. The change in the household's wealth is saving minus dissaving

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t) = \lambda_j\, \bar{y}_j(t) - \bar{k}_j(t). \quad (14)$$

Human capital dynamics. We apply a generalized Uzawa's human capital accumulation as follows (Uzawa 1965):

$$\dot{H}_{jq}(t) = \frac{v_{jq}\left(F_{jq}(t)/2\bar{N}_j(t)\right)^{a_{jq}}\left(H^{m_{jq}}(t)\, \hat{T}_{jq}(t)\right)^{b_{jq}}}{H^{\pi_{jq}}(t)} - \delta_{jq}\, H_{jq}(t), \quad (15)$$

where v_{jq}, m_{jq}, a_{jq}, and b_{jq} are non-negative parameters. We use π_{jq} to measure returns to scale in education. Learning through education shows increasing returns to scale when $\pi_{jq} < 0$ or decreasing returns to scale when $\pi_{jq} > 0$. We also have that human capital accumulation rises in education per capita $F_{jq}(t)/2\bar{N}_{jq}(t)$, and in the (qualified) total study time, $\left(H^{m_{jq}}(t)T_{jq}(t)\right)^{b_{jq}}$.

Balances in education market. The demand and supply for education balances as follows

$$\hat{T}_{j1}(t)\, \bar{N}_j(t) + \hat{T}_{j2}(t)\, \bar{N}_j(t) = F_{je}(t). \quad (16)$$

Equilibrium of good markets. The world output is used up for the global consumption, the global net saving, and the capital depreciations:

$$C(t) + S(t) - K(t) + \sum_{j=1}^{J} \delta_{kj}\, K_j(t) = \sum_{j=1}^{J} F_{ji}(t), \quad (17)$$

where

$$S(t) = \sum_{j=1}^{J} s_j(t)\, N_j(t)\, , C(t) = \sum_{j=1}^{J} c_j(t)\, N_j(t)\, .$$

The value of global capital goods equals the value of global wealth.

$$\sum_{j=1}^{J} \bar{k}_j(t)\, \bar{N}_j(t) = K(t). \quad (18)$$

Full employment of global capital. The global capital is fully employed

$$\sum_{j=1}^{J} K_j(t) = K(t). \tag{19}$$

The national labor force is fully employed.

$$N_{ji}(t) + N_{je}(t) = N_j(t). \tag{20}$$

We define the trade balances as:

$$N_j(t) = \left(\bar{N}_j(t) - K_j(t)\right) r(t). \tag{21}$$

When $B_j(t)$ is positive (negative), country j is in trade surplus (deficit). When $B_j(t)$ is zero, country j trade is in balance.

We built dynamic general equilibrium model. It is an integration of some well-known economic models. We now study its dynamic properties.

3 The model's properties

The previous section developed a gender-based multi-country model with endogenous wealth, population, human capital, and time-distribution. The model is nonlinearly dynamic. It is almost impossible to provide a general solution of the model. We apply computational simulation to show some properties of the system. We define the following variables:

$$z_1(t) \equiv \frac{r(t) + \delta_{k1}}{w_1(t)}, \{\bar{k}_j(t)\} \equiv \left(\bar{k}_2(t), \ldots, \bar{k}_J(t)\right).$$

We show that the dynamics can be given by differential equations with $z_1(t), \{\bar{k}_j(t)\}$, $\left(H_{jq}(t)\right)$, and $\left(\bar{N}_j(t)\right)$ as the variables.

Lemma. We determine the dynamics with the following differential equations:

$$\dot{z}_1(t) = \Omega_{k1}\left(z_1(t), \{\bar{k}_j(t)\}, \left(\bar{N}_j(t)\right), \left(H_{jq}(t)\right)\right),$$
$$\dot{\bar{k}}_j(t) = \Omega_{kj}\left(z_1(t), \{\bar{k}_j(t)\}, \left(\bar{N}_j(t)\right), \left(H_{jq}(t)\right)\right),$$
$$j = 2, \ldots, J, \dot{\bar{N}}_j(t)\Omega_j\left(z_1(t), \{\bar{k}_j(t)\}, \left(\bar{N}_j(t)\right), \left(H_{jq}(t)\right)\right),$$
$$\dot{H}_{jq}(t)\ \Omega_{Hj}\left(z_1(t), \{\bar{k}_j(t)\}, \left(\bar{N}_j(t)\right), \left(H_{jq}(t)\right)\right), j = 1, \ldots, J, q = 1,2, \tag{22}$$

where $\Omega_{kj}(t)$, $\Omega_j(t)$ and $\Omega_{jq}(t)$ are functions of $z_1(t)$, $\{\bar{k}_j(t)\}$, $\left(H_{jq}(t)\right)$, and $\left(\bar{N}_j(t)\right)$ which are given in the Appendix 1. We have the other variables as functions of $z_1(t)$, $\{\bar{k}_j(t)\}$, $\left(H_{jq}(t)\right)$, and $\left(\bar{N}_j(t)\right)$ as follows: $r(t)$, $w_j(t)$, and $w_{jq}(t)$ with (A2) → $z_j(t)$ with (A3) → $p_j(t)$ by (A3) → $\bar{k}_1(t)$ by (A19) → $\bar{y}_j(t)$ by (A5) → $c_j(t)$, $s_j(t)$, $\bar{T}_{jq}(t)$, $\hat{T}_{jq}(t)$, and $n_j(t)$ from (11) → $d_j(t)$ by (12) → $\tilde{T}_{jq}(t)$ by (6) → $T_{jq}(t)$ with (8) → $N_j(t)$ from (1) → $N_{je}(t)$ by (A3) → $K_{jm}(t)$ from (A1) → $F_{jm}(t)$ by (2) → $p_{bj}(t)$ by (5) → $K(t)$ by (8) → $K_j(t) = K_{ji}(t) + K_{je}(t)$ → $\bar{K}_j(t) = k_j(t)\bar{N}_j(t)$ by (2) → $B_j(t)$ from (21).

System (20) consists of $4J$ nonlinear differential equations and the same number of endogenous variables. We cannot generally analyze dynamic properties of the system as it is too complicated. We simulate the model to illustrate behavior of the system. We fix the available time and the parameters in mortality rate functions as follows:

$$T_0 = 24, a_j = 0.4, b_j = 0.5, j = 1, \dots, J.$$

Mortality is positively related to the population and negatively related to the disposable income. The parameters in the production functions and δ_{km} are specified as follows:

$$\begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1.2 \\ 1 \end{pmatrix}, \begin{pmatrix} A_{1e} \\ A_{2e} \\ A_{3e} \end{pmatrix} = \begin{pmatrix} 1.3 \\ 1.1 \\ 0.8 \end{pmatrix}, \begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.32 \\ 0.34 \end{pmatrix}, \begin{pmatrix} \alpha_{1e} \\ \alpha_{2e} \\ \alpha_{3e} \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.27 \\ 0.3 \end{pmatrix}, \begin{pmatrix} \delta_{k1} \\ \delta_{k2} \\ \delta_{k3} \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.27 \\ 0.3 \end{pmatrix}.$$

The total productivity factors of the countries vary. In the literature of empirical studies of economic growth, the value in the Cobb-Douglas production function is approximately $1/3$ (Miles and Scott 2005, Abel et al. 2007). We specify the utilization efficiencies of human capital and depreciation rates as follows:

$$\begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0.65 \\ 0.6 \end{pmatrix}, \begin{pmatrix} m_{12} \\ m_{22} \\ m_{32} \end{pmatrix} = \begin{pmatrix} 0.65 \\ 0.5 \\ 0.45 \end{pmatrix}, \begin{pmatrix} \delta_{11} \\ \delta_{21} \\ \delta_{31} \end{pmatrix} = \begin{pmatrix} 0.035 \\ 0.045 \\ 0.05 \end{pmatrix}, \begin{pmatrix} \delta_{12} \\ \delta_{22} \\ \delta_{32} \end{pmatrix} = \begin{pmatrix} 0.045 \\ 0.045 \\ 0.05 \end{pmatrix}.$$

We assume that the woman applies human capital less effectively than the country man. The man from country 1 applies human capital more effectively than from country 2, while the man from country 2 more than the man from country 3. Similarly, we assume women's human capital application efficiency differences between countries. The preferences are specified as follows:

$$\begin{pmatrix}\xi_{10}\\ \xi_{20}\\ \xi_{30}\end{pmatrix}=\begin{pmatrix}0.2\\ 0.2\\ 0.2\end{pmatrix},\begin{pmatrix}\lambda_{10}\\ \lambda_{20}\\ \lambda_{30}\end{pmatrix}=\begin{pmatrix}0.5\\ 0.5\\ 0.45\end{pmatrix},\begin{pmatrix}v_{10}\\ v_{20}\\ v_{30}\end{pmatrix}=\begin{pmatrix}0.5\\ 0.6\\ 0.7\end{pmatrix},\begin{pmatrix}\sigma_{110}\\ \sigma_{210}\\ \sigma_{310}\end{pmatrix}=\begin{pmatrix}0.15\\ 0.22\\ 0.15\end{pmatrix},$$
$$\begin{pmatrix}\sigma_{120}\\ \sigma_{220}\\ \sigma_{320}\end{pmatrix}=\begin{pmatrix}0.2\\ 0.25\\ 0.25\end{pmatrix},\begin{pmatrix}\eta_{110}\\ \eta_{210}\\ \eta_{310}\end{pmatrix}=\begin{pmatrix}0.15\\ 0.009\\ 0.008\end{pmatrix},\begin{pmatrix}\eta_{120}\\ \eta_{220}\\ \eta_{320}\end{pmatrix}=\begin{pmatrix}0.011\\ 0.007\\ 0.007\end{pmatrix},$$
$$\begin{pmatrix}\bar{v}_1\\ \bar{v}_2\\ \bar{v}_3\end{pmatrix}=\begin{pmatrix}0.02\\ 0.03\\ 0.035\end{pmatrix},$$
$$\begin{pmatrix}\theta_{11}\\ \theta_{21}\\ \theta_{31}\end{pmatrix}=\begin{pmatrix}0.1\\ 0.2\\ 0.2\end{pmatrix},\begin{pmatrix}\theta_{12}\\ \theta_{22}\\ \theta_{32}\end{pmatrix}=\begin{pmatrix}0.6\\ 0.6\\ 0.6\end{pmatrix},\begin{pmatrix}\theta_{1}\\ \theta_{2}\\ \theta_{3}\end{pmatrix}=\begin{pmatrix}0.7\\ 0.8\\ 0.9\end{pmatrix}.$$

Countries differ in preferences. We will show how changes in the preferences affect global economies. We specify the human capital accumulation parameters as follows:

$$\begin{pmatrix}v_{11}\\ v_{21}\\ v_{31}\end{pmatrix}=\begin{pmatrix}0.6\\ 0.5\\ 0.4\end{pmatrix},\begin{pmatrix}v_{12}\\ v_{22}\\ v_{32}\end{pmatrix}=\begin{pmatrix}0.55\\ 0.4\\ 0.35\end{pmatrix},\begin{pmatrix}a_{11}\\ a_{21}\\ a_{31}\end{pmatrix}=\begin{pmatrix}0.4\\ 0.37\\ 0.35\end{pmatrix},\begin{pmatrix}a_{12}\\ a_{22}\\ a_{32}\end{pmatrix}=\begin{pmatrix}0.37\\ 0.35\\ 0.33\end{pmatrix},$$
$$\begin{pmatrix}b_{11}\\ b_{21}\\ b_{31}\end{pmatrix}=\begin{pmatrix}0.5\\ 0.46\\ 0.43\end{pmatrix},\begin{pmatrix}b_{12}\\ b_{22}\\ b_{32}\end{pmatrix}=\begin{pmatrix}0.46\\ 0.43\\ 0.45\end{pmatrix},\begin{pmatrix}\pi_{11}\\ \pi_{21}\\ \pi_{31}\end{pmatrix}=\begin{pmatrix}0.2\\ 0.3\\ 0.4\end{pmatrix},\begin{pmatrix}\pi_{12}\\ \pi_{22}\\ \pi_{32}\end{pmatrix}=\begin{pmatrix}0.25\\ 0.35\\ 0.45\end{pmatrix}.$$

The initial conditions are chosen:

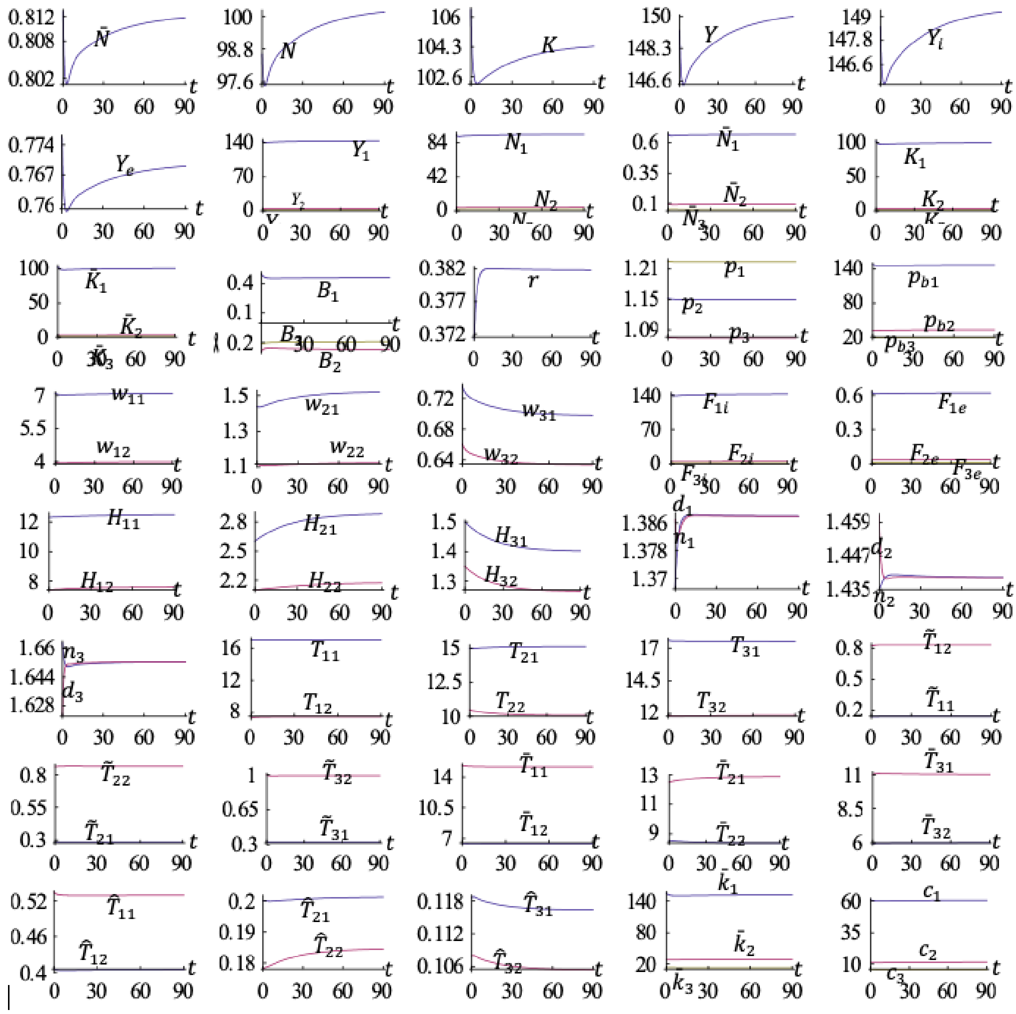
$$\begin{pmatrix}z_1(0)\\ \bar{k}_2(0)\\ \bar{k}_3(0)\end{pmatrix}=\begin{pmatrix}0.39\\ 28\\ 13\end{pmatrix},\begin{pmatrix}\bar{N}_1(0)\\ \bar{N}_2(0)\\ \bar{N}_3(0)\end{pmatrix}=\begin{pmatrix}668300\\ 94300\\ 50200\end{pmatrix},\begin{pmatrix}H_{11}(0)\\ H_{21}(0)\\ H_{31}(0)\end{pmatrix}=\begin{pmatrix}12.3\\ 2.6\\ 1.5\end{pmatrix},\begin{pmatrix}H_{12}(0)\\ H_{22}(0)\\ H_{32}(0)\end{pmatrix}=\begin{pmatrix}7.8\\ 2.1\\ 1.35\end{pmatrix}.$$

The simulation result is plotted in Figure 1, in which we introduce a few variables to describe the global economy

$$\begin{gathered}\bar{N}(t)\equiv\bar{N}_1(t)+\bar{N}_2(t)+\bar{N}_3(t), N(t)\equiv N_1(t)+N_2(t)+N_3(t),\\ K(t)\equiv K_1(t)+K_2(t)+K_3(t), Y(t)\equiv Y_1(t)+Y_2(t)+Y_3(t),\\ Y_i(t)\equiv Y_{1i}(t)+Y_{2i}(t)+Y_{3i}(t), Y_e(t)\equiv Y_{1e}(t)+Y_{2e}(t)+Y_{3e}(t).\end{gathered}\quad(23)$$

In Figure 1, $N_j(t)$, $\bar{N}_j(t)$, $K_j(t)$, $\overline{K}_j(t)$, $B_j(t)$, $F_{ji}(t)$, $F_{je}(t)$, and the variables in (23) are scaled by 10^{-6}. The global population and global labor force grow from their low initial conditions. For country 1 the mortality rate is higher than the corresponding rate before they converge. For country 2 the mortality rate is higher than the birth rate initially and then the mortality rate is lower than the birth rate before they converge. For country 3 the mortality rate is lower than the birth rate initially and then the mortality rate is higher than the birth rate before they converge. It should be noted that some studies identify declines of fertility rates in the process of economic growth (Kirk 1996, Ehrlich and Lui 1997, Galor 2012). The relations between economic growth and birth rates vary between different countries, as shown in Figure 1. According to the wealth level and consumption levels' rankings, we call countries 1, 2, and 3 respectively the advanced, middle-income, and developing economies.

Figure 1: The movement of the economy



Source: processed by author.

The variables are stationary in the long term. The simulation shows the following equilibrium point:

$$\bar{N} = 812417, N = 1.003 \times 10^8, K = 1.045 \times 10^8, Y = 1.503 \times 10^8, r = 0.382,$$
$$Y_i = 1.495 \times 10^8, Y_e = 769864, Y_1 = 1.44 \times 10^8, Y_2 = 4.73 \times 10^6, Y_3 = 1.51 \times 10^6,$$
$$\bar{N}_1 = 667664, \bar{N}_2 = 94328, \bar{N}_3 = 50424.7, N_1 = 94.4 \times 10^6, N_2 = 42.4 \times 10^6,$$
$$N_3 = 1.8 \times 10^6, K_1 = 10^8, K_2 = 3.45 \times 10^6, K_3 = 1.134 \times 10^6, \bar{K}_1 = 1.02 \times 10^8,$$
$$\bar{K}_2 = 2.74 \times 10^6, \bar{K}_3 = 634186, B_1 = 461264, B_2 = -270264, B_3 = -191000,$$
$$F_{1i} = 143 \times 10^6, F_{2i} = 4.73 \times 10^6, F_{3i} = 1.5 \times 10^6, F_{1e} = 623493, F_{2e} = 36414.7,$$
$$F_{3e} = 11176.7, K_{1i} = 9.95 \times 10^7, K_{2i} = 3.42 \times 10^6, K_{3i} = 1.13 \times 10^6, K_{1e} = 415141,$$
$$K_{2e} = 23884, K_{3e} = 9084, N_{1i} = 93.85 \times 10^6, N_{2i} = 4.21 \times 10^6, N_{3i} = 1.73 \times 10^6,$$
$$N_{1e} = 503253, N_{2e} = 37352, N_{3e} = 16801, H_{11} = 12.5, H_{21} = 2.89, H_{31} = 1.4,$$
$$H_{12} = 7.6, H_{22} = 2.18, H_{32} = 1.27, w_{11} = 7.1, w_{21} = 1.52, w_{31} = 0.4, w_{12} = 4,$$
$$w_{22} = 1.13, w_{32} = 0.63, p_1 = 1.15, p_2 = 1.07, p_3 = 1.22, p_{b1} = 147.2, p_{b2} = 33.43,$$
$$p_{b3} = 18.71, n_1 = d_1 = 1.388, n_2 = d_2 = 1.439, n_3 = d_3 = 1.653, \bar{k}_1 = 151.51,$$
$$\bar{k}_2 = 29.03, \bar{k}_3 = 12.58, c_1 = 60.61, c_2 = 11.61, c_3 = 5.59, T_{11} = 17.06,$$
$$T_{21} = 15.12, T_{31} = 17.54, T_{12} = 7.47, T_{22} = 10.07, T_{32} = 11.88, \tilde{T}_{11} = 0.139,$$
$$\tilde{T}_{21} = 0.288, \tilde{T}_{31} = 0.331, \tilde{T}_{12} = 0.833, \tilde{T}_{22} = 0.864, \tilde{T}_{32} = 0.992, \bar{T}_{11} = 6.4,$$
$$\bar{T}_{21} = 8.39, \bar{T}_{31} = 6.01, \bar{T}_{12} = 15.17, \bar{T}_{22} = 12.88, \bar{T}_{32} = 11.02, \hat{T}_{11} = 0.404,$$
$$\hat{T}_{21} = 0.201, \hat{T}_{31} = 0.116, \hat{T}_{12} = 0.53, \hat{T}_{22} = 0.186, \hat{T}_{32} = 0.105.$$

The eigenvalues at equilibrium point are as follows:

$$-0.84, -0.74, -0.7, -0.65, -0.62, -0.47, -0.065, -0.055, -0.051, -0.046, -0.037, -0.027.$$

We have real negative eigenvalues. The equilibrium point is locally stable. This guarantees the validity of dynamic comparative analysis.

4 DYNAMIC COMPARATIVE ANALYSIS

The previous section plotted the movement of the global economy by computer simulation. It is important to see how the global economy changes its path of development when some countries' preferences or technologies are changed. As the Lemma gives the computational procedure to calibrate the movement of the economic system, we can study effects of changes in any parameter on transitory processes and equilibrium values of all the variables. A variable $\bar{\Delta} x_j(t)$ is introduced to stand by the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value.

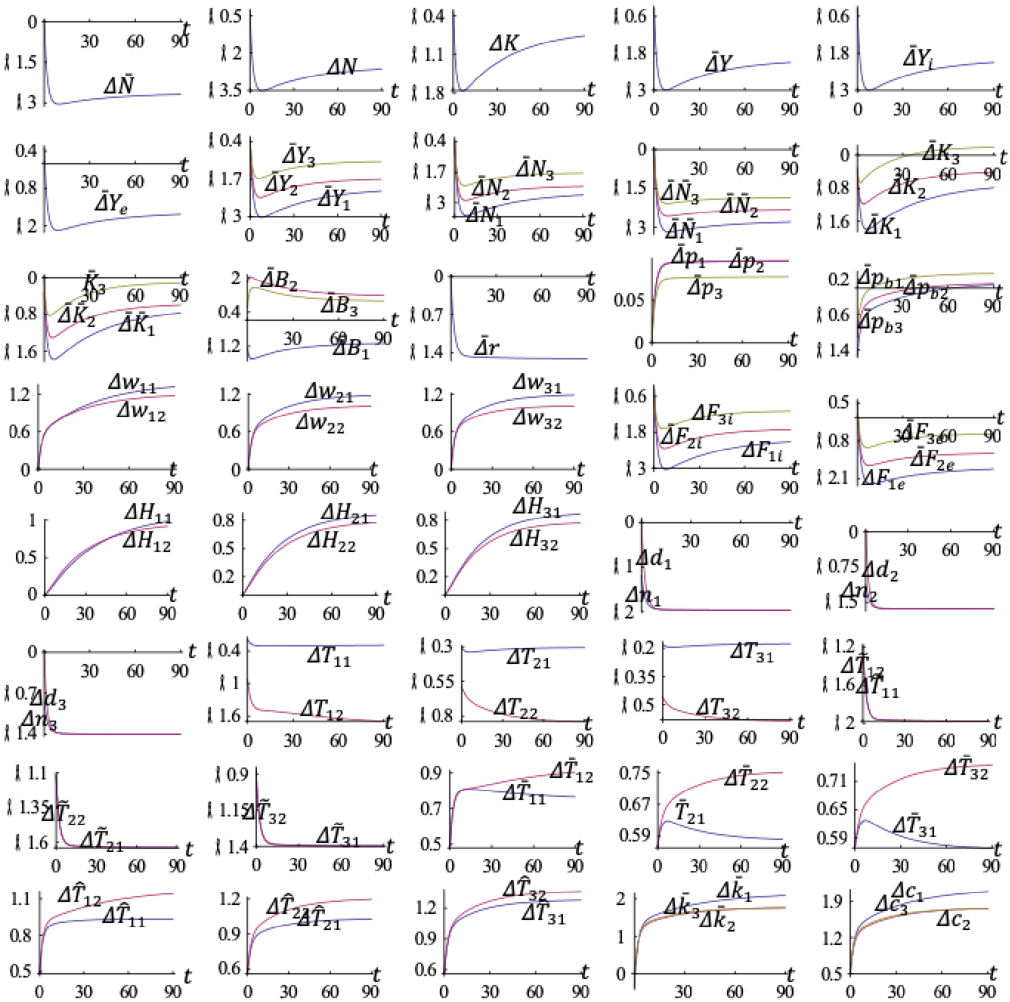
4.1. Propensities to have children are reduced

Tournemaine and Luangaram (2012, p. 925) show ambiguous effects of population change on national economic growth in the following way: “depending on the country, population growth may contribute, deter, or even have no impact on economic development. This ambiguous result is explained by the fact that the effects of population growth change over time. For example, a higher fertility rate can have a short-term negative effect caused by the cost of expenditures on children whereas it has a long-run positive effect through the larger labor force it generates.” We now examine study the effects that the propensities to have children are enhanced in all the three economies as follows:

$$\upsilon_{10}: 0.5 \Rightarrow 0.49, \upsilon_{20}: 0.6 \Rightarrow 0.59, \upsilon_{30}: 0.7 \Rightarrow 0.69.$$

The result is given in Figure 2. The reduced propensities to have children reduce the birth rates and mortality rates. The global population, each country’s population, the global labor force, and each country’s labor force fall. Similarly, the other macro variables such as the global capital stock, national capital stocks, and capital stocks employed by each country, global income, and each country’s income, are reduced. The two sectors in the three countries shrink. We conclude that in macro levels national and global economies suffer from reduced preferences for having children. The rate of interest falls. The wage rates are enhanced. The prices of education change slightly. The costs of children caring fall initially and rise in the long term. Both men’s and women’s human capital levels are increased. The households’ wealth and consumption levels are increased. The time distributions are changed. The parents increase leisure and education hours, and reduce work and children caring hours.

Figure 2: The propensities to have children are reduced



Source: processed by author.

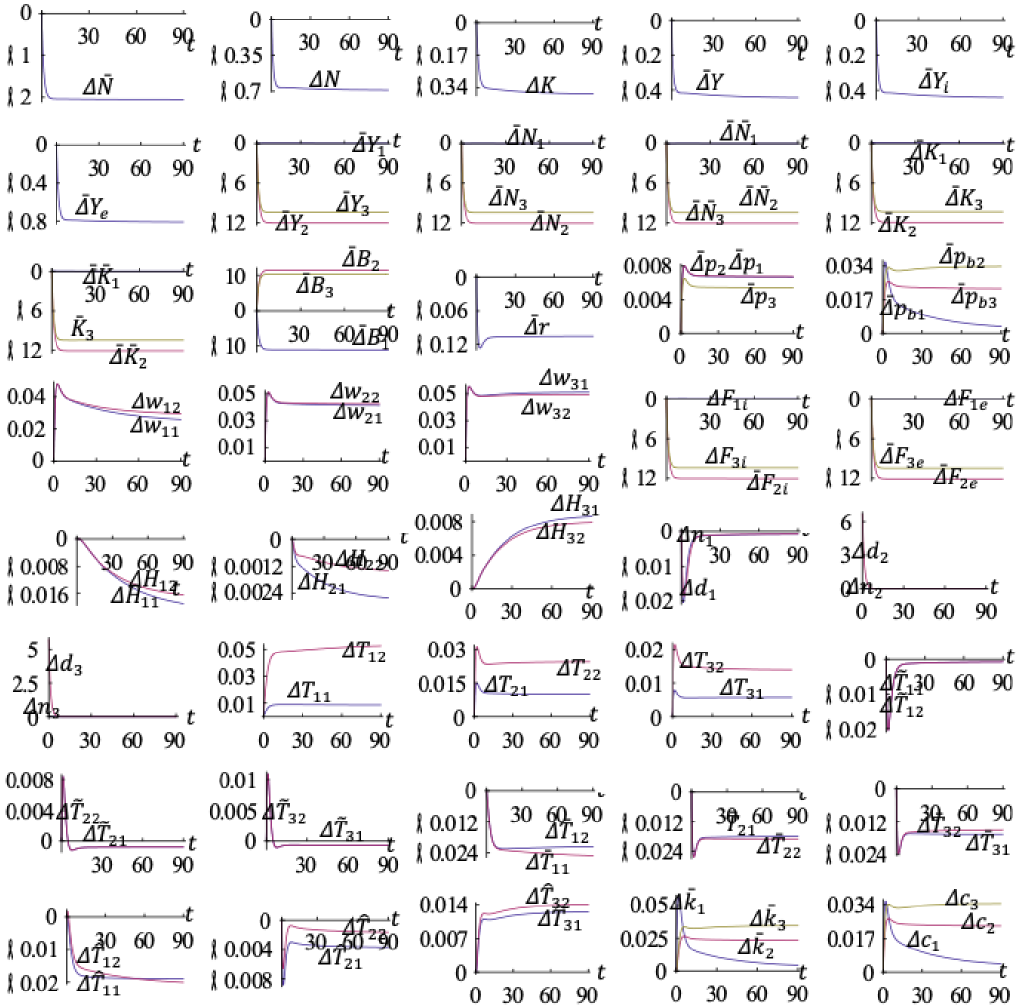
4.2. The middle-income and developing economies have higher mortality

We now study the effects that the middle-income and developing economies have higher mortality as follows:

$$\bar{v}_2 : 0.03 \Rightarrow 0.32, \bar{v}_3 : 0.035 \Rightarrow 0.037.$$

The result is given in Figure 3. The middle-income and developing economies have the higher mortality rates initially and almost same level of the mortality rates in the long term. The developed economy's mortality rate is slightly affected. The world has less population, less labor force, less capital stock, lower income, lower industrial output, and lower education output. The middle-income and developing economies trade balances are improved, while the developed economy's trade balance is deteriorated. The developed economy's macroeconomic variables are slightly changed, while the macro real variables of the middle-income and developing economies are reduced. The prices of education are slightly affected, while the pecuniary costs of children caring are reduced. The wage rates and human capital levels are slightly affected. The time distributions, per family's consumption and wealth are slightly affected.

Figure 3: The middle-income and developing economies have higher mortality



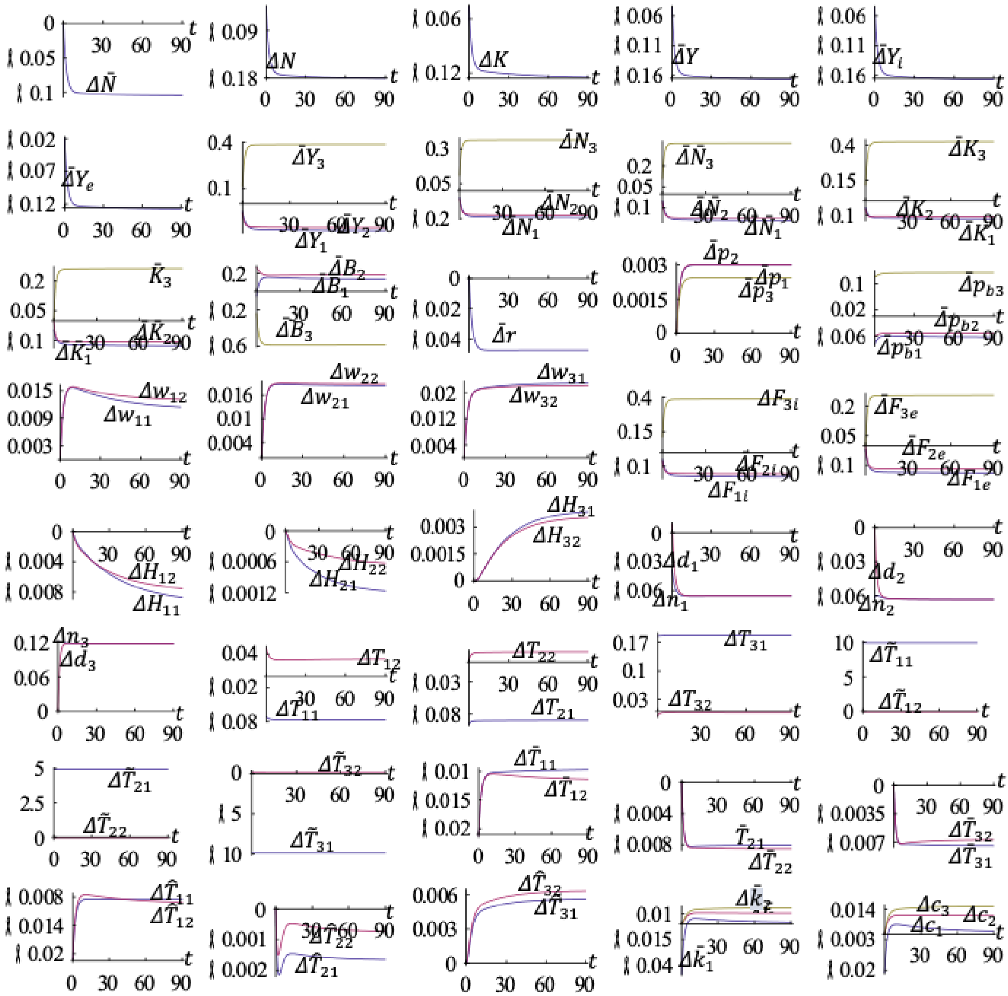
Source: processed by author.

4.3. Men change their hours on taking care of each child

We now examine study the effects that men in the developed and middle-income economies spend more hours on per children caring and men in the developing economy reduce hours on taking care of each child as follows:

$$\theta_{11}: 0.1 \Rightarrow 0.11, \theta_{21}: 0.2 \Rightarrow 0.21, \theta_{31}: 0.2 \Rightarrow 0.18.$$

Figure 4: Men change their hours on taking care of each child



Source: processed by author.

The result is given in Figure 4. The developed and middle-income have the higher mortality rates initially and almost same level of the mortality rates in the long term. The developed economy's mortality rate is slightly affected. The world has less population, less labor force, less capital stock, lower income, lower industrial output, and lower education output. The trade balances of developed and middle-income economies are improved, while the developing economy's trade balance is deteriorated. The national incomes of developed and middle-income economies fall, while the developing economy's national income rises. The pecuniary cost of children caring rises in the developing economy, while the pecuniary cost of children caring change slightly in the other two economies. The developing economy's population labor force, national wealth, capital stocks employed, production scales of the two sectors are all increased, while the corresponding variables in the other two economies are reduced. The time distributions vary. The wealth and consumption per family are slightly affected.

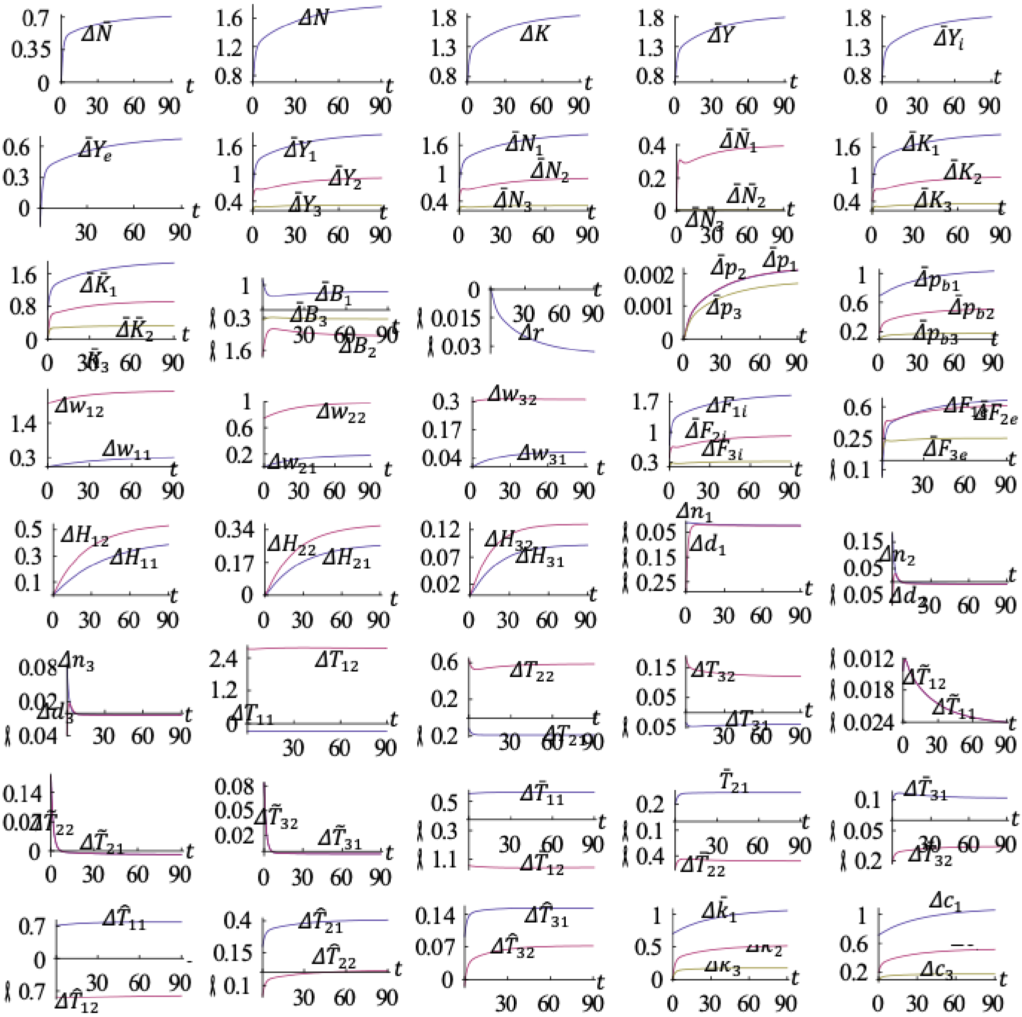
4.4. Women in the world apply human capital more effectively

We examine how the world economy is affected when women in all the countries increase their human capital utilization efficiencies as follows:

$$m_{12}: 0.65 \Rightarrow 0.66, m_{22}: 0.5 \Rightarrow 0.51, m_{32}: 0.45 \Rightarrow 0.46.$$

Figure 5 plots the result. The birth rates and mortality rates are slightly affected in the long term. The macroeconomic real variables in international and national levels are all increased. The developed economy's trade balance is improved, while the other two economies' trade balances are deteriorated. The rate of interest falls. The wage rates are enhanced. The prices of education change slightly. The pecuniary costs of children caring rise. Both men's and women's human capital levels are enhanced. The households' wealth and consumption levels are enhanced. The time distributions are changed. Women work more hours, while men work less hours.

Figure 5: Women in the world apply human capital more effectively



Source: processed by author.

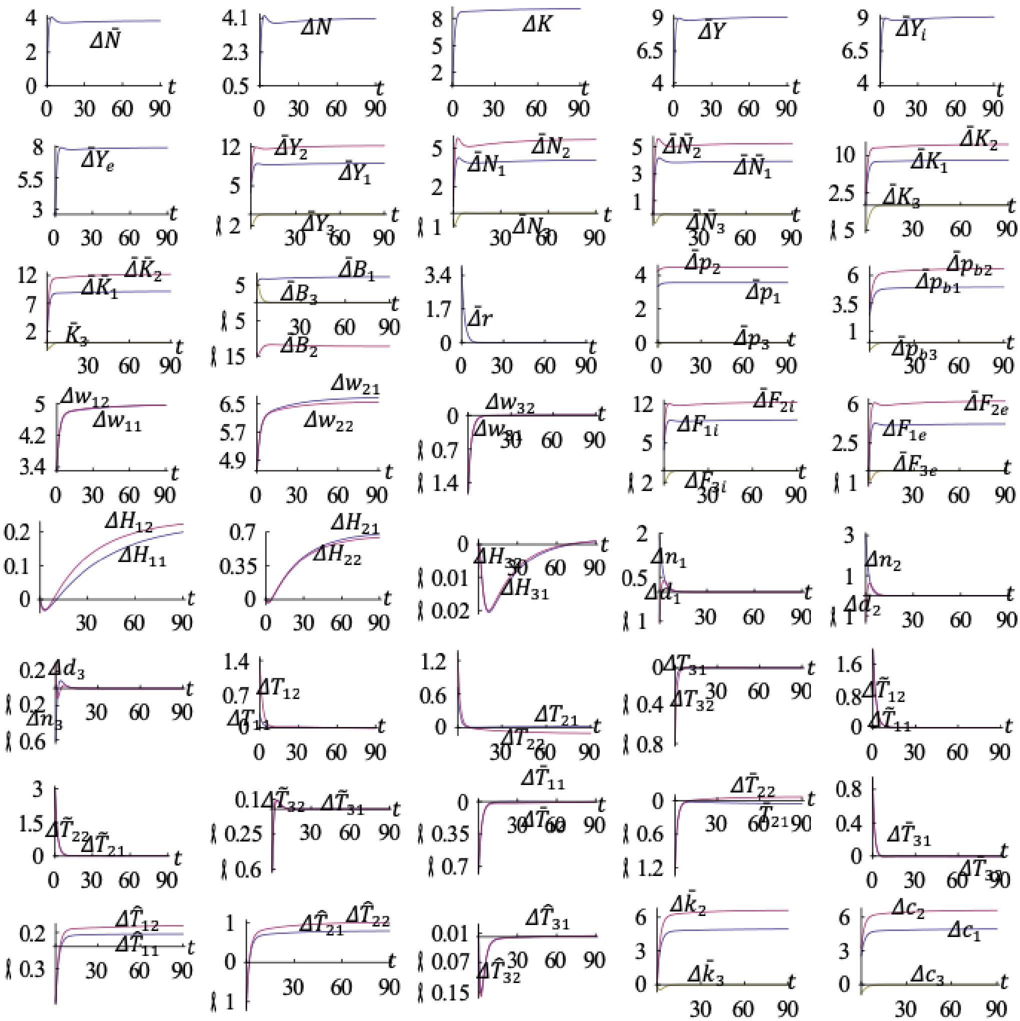
4.5. The industrial sectors' total factor productivities are enhanced

We analyze how the world economy is affected when the industrial sector's total factor productivities in the developed and middle-income economies are enhanced as follows:

$$A_{1i}: 1.5 \Rightarrow 1.65, A_{2i}: 1.2 \Rightarrow 1.25.$$

The result is given in Figure 6. Changes in the middle-income and developed economy's technologies have almost no effects on the developing economy in the long term. In the long term the birth rates and mortality rates are slightly affected. The global capital stock, total industrial output, total education output, population, labor force, and total income are all increased. The macroeconomic real variables of the developed and middle-income economies are augmented. The developed economy's trade balance is improved, while the middle-income economy's trade balance is deteriorated. The education prices and pecuniary costs of children caring are enhanced. The wage rates and human capital of the developed and middle-income economies are enhanced. In the long term the time distributions are slightly affected.

Figure 6: The industrial sectors' total factor productivities are enhanced



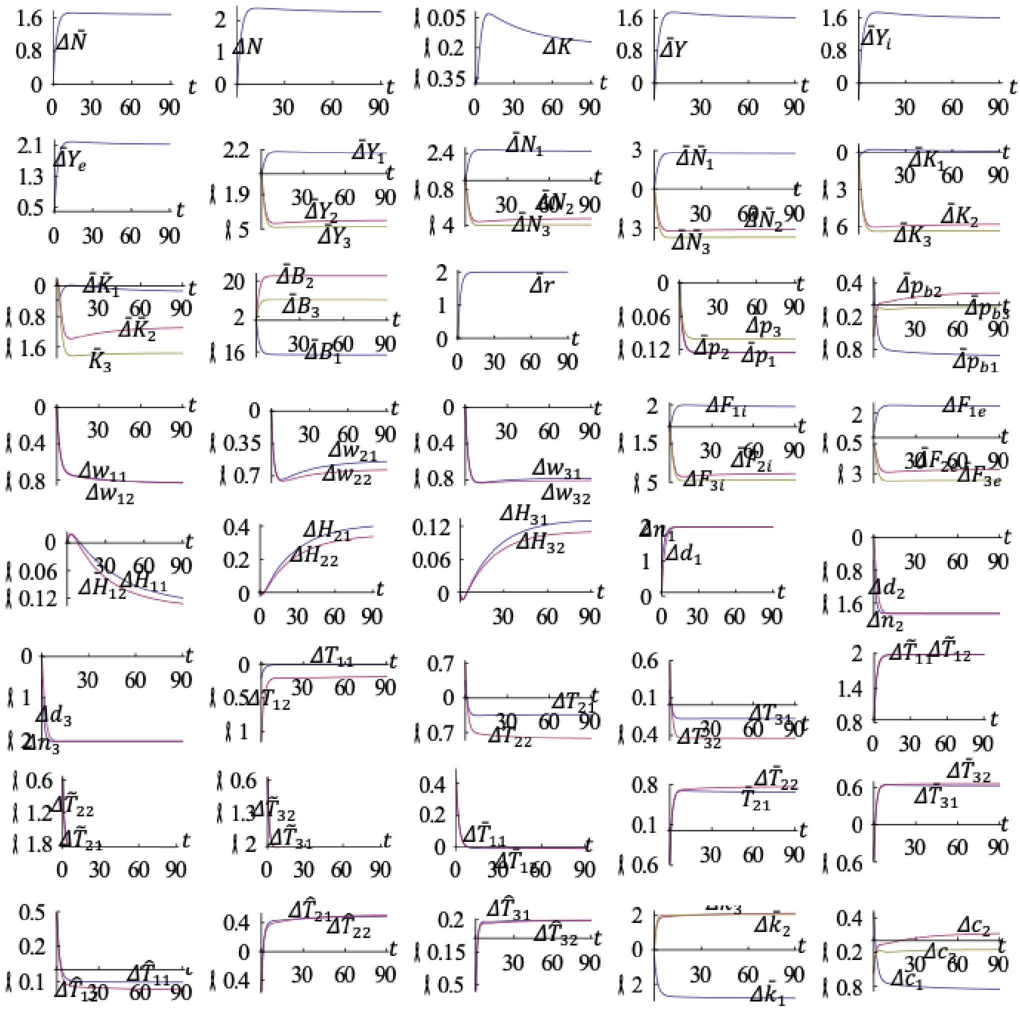
Source: processed by author.

4.6. The propensities to save are changed

We analyze how the world economy is affected when the developed economy reduces its propensity to save and the other countries increase the propensities to save as follows:

$$\lambda_{12}: 0.5 \Rightarrow 0.49, \lambda_{20}: 0.5 \Rightarrow 0.51, \lambda_{30}: 0.45 \Rightarrow 0.46.$$

Figure 7: The propensities to save are changed



Source: processed by author.

The result is given in Figure 7. The birth and mortality rates of the developed economy fall, while the birth and mortality rates of the middle-income and developing economies rise. The global population and labor force and global income are augmented, while the global capital stock falls. The global industrial and education sectors' output levels are enhanced. The developed economy increases the national output, population, and labor force, while the other two countries reduce the national output, populations, and labor forces. The capital stock employed by the developed economy changes slightly, while the capital stocks employed by the other economies are reduced. The industrial and education sectors in the developed economy supply more, while the industrial and education sectors in the middle-income and developing economies supply less. The developed economy's wealth changes slightly, while the other two economies' wealth are reduced. The wage rates fall. The developed economy's per family wealth on fall, while The middle-income and developing economies' per family wealth rise.

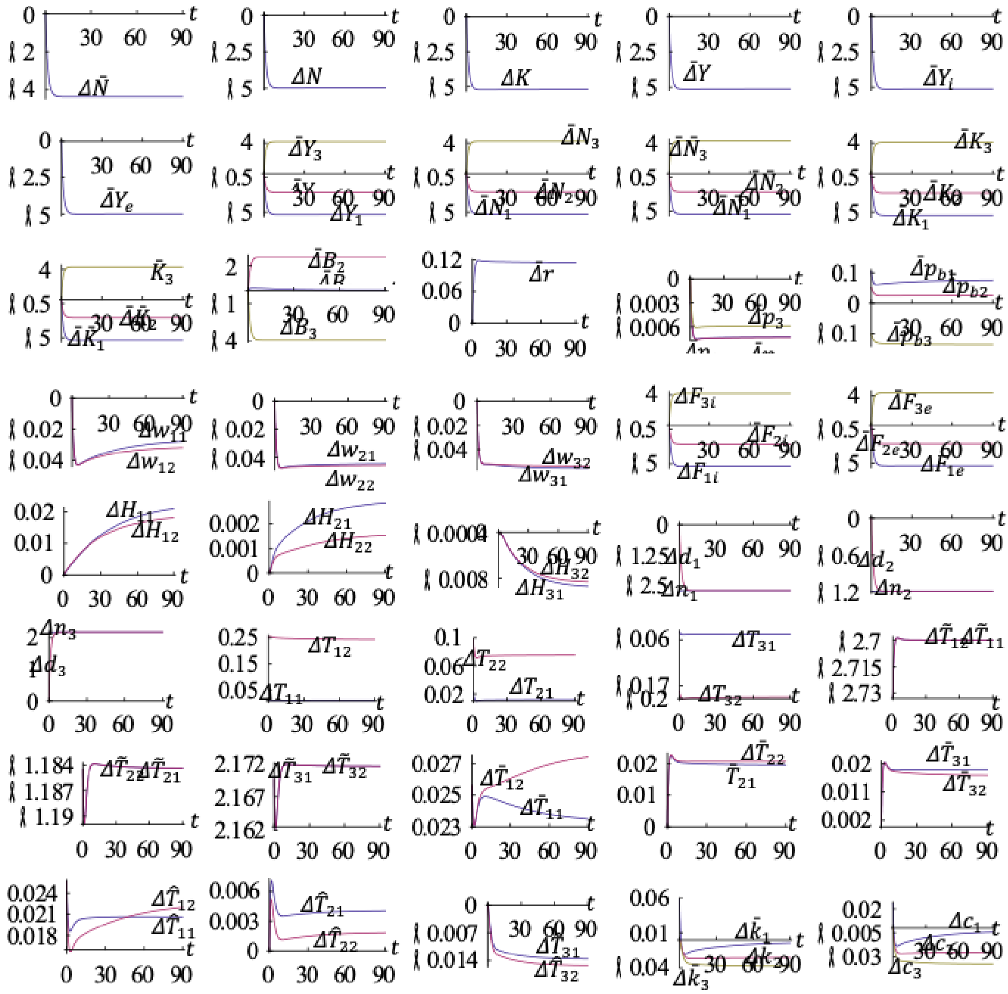
4.7. The pecuniary costs per child are changed

We now study the effects that the developed and middle-income economies have higher pecuniary costs and the developing economy has lower pecuniary cost as follows:

$$\theta_1: 0.7 \Rightarrow 0.72, \theta_2: 0.0.8 \Rightarrow 0.82, \theta_3: 0.9 \Rightarrow 0.88.$$

The result is given in Figure 8. The developed and middle-income economies have higher birth and mortality rates. The developing economy's birth and mortality rates are enhanced. The developed and middle-income economies' population and labor force are decreased, while the developing economy's population and labor force are increased. The global population, labor force, wealth, and income are all reduced. The birth and mortality rates are enhanced. The rest variables are shown in Figure 8.

Figure 8: The pecuniary costs per child are changed



Source: processed by author.

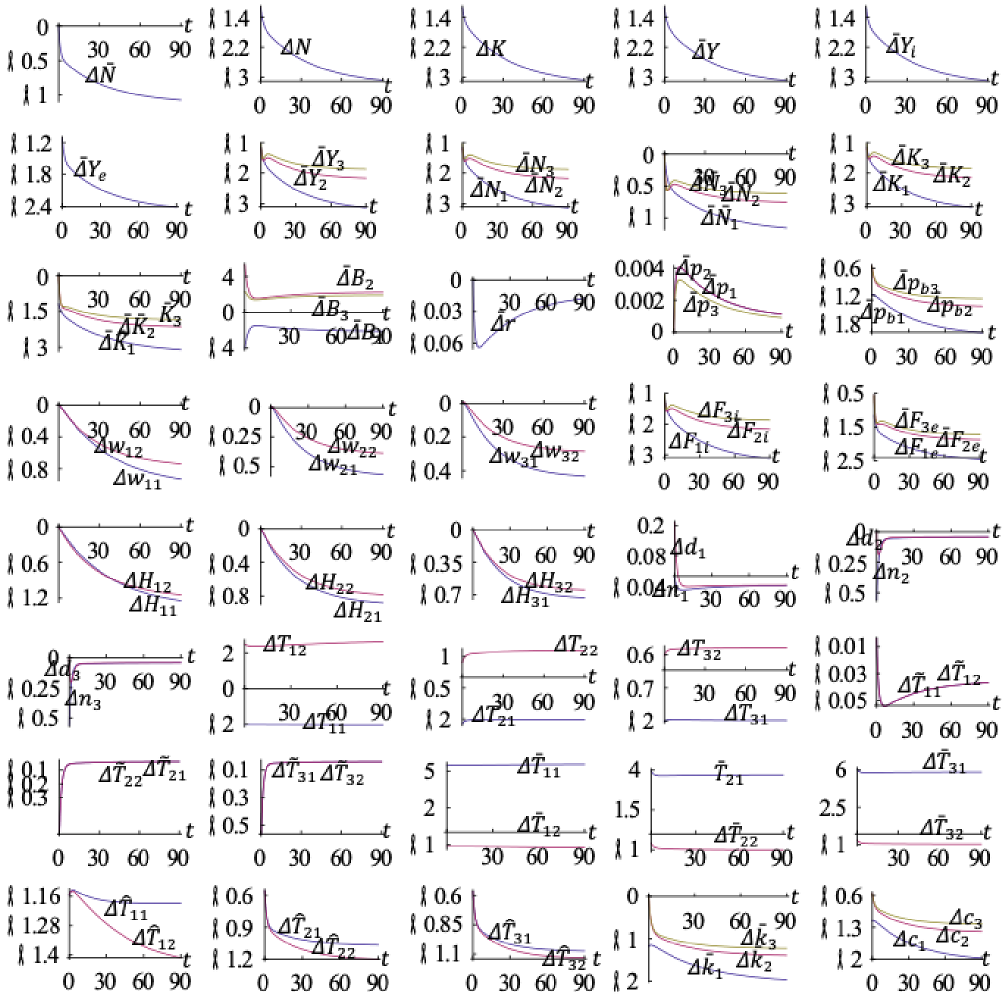
4.8. Men increase their propensities to enjoy leisure

We now analyze the effects that men in all the countries increase their propensities to enjoy leisure as follows:

$$\sigma_{11}: 0.15 \Rightarrow 0.16, \sigma_{21}: 0.22 \Rightarrow 0.23, \theta_{31}: 0.15 \Rightarrow 0.16.$$

The result is given in Figure 9. The global and national macroeconomic real variables are reduced as men spend more hours at home and less hours on working. Families' consumption and wealth are reduced. Women work more hours and spend less hours at home.

Figure 9: Men increase their propensities to enjoy leisure



Source: processed by author.

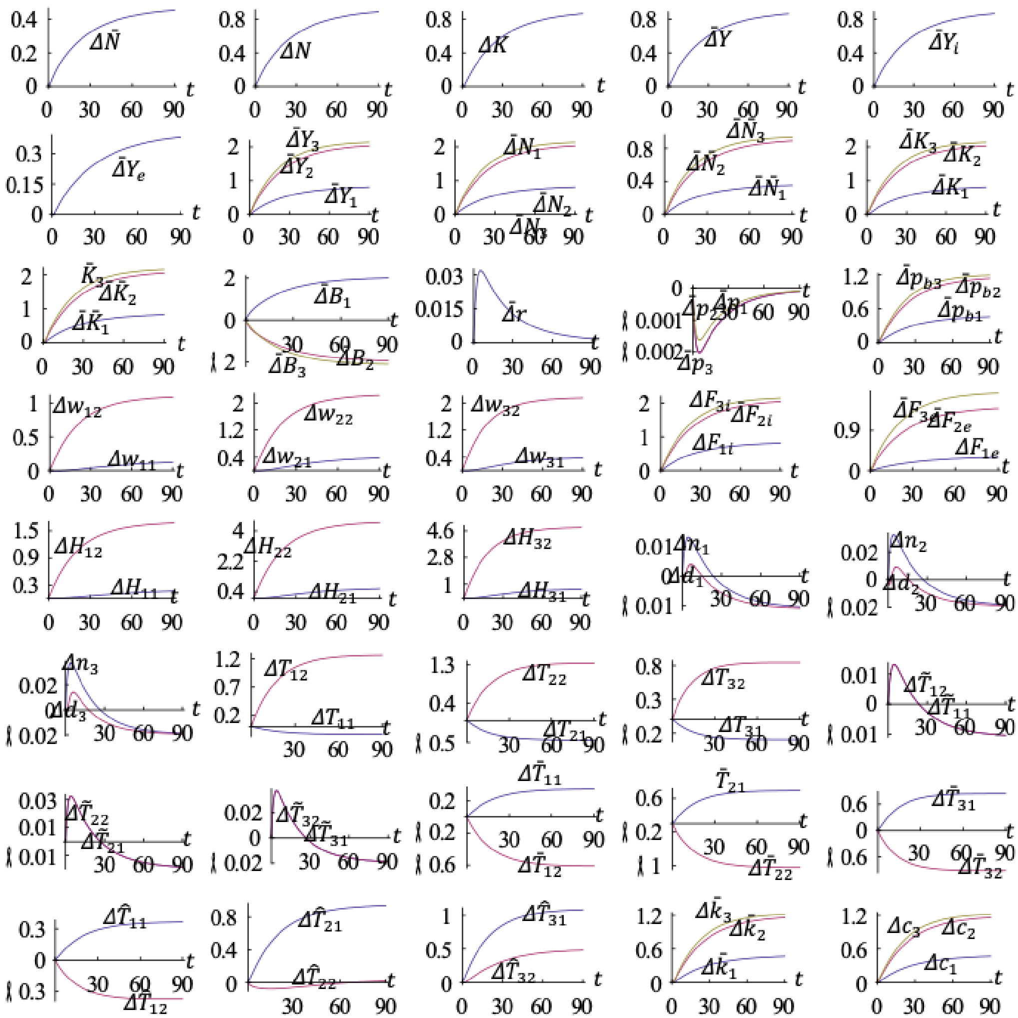
4.9. Women accumulate human capital more effectively

We analyze how the world economy is affected when women in all the countries accumulate human capital more effectively:

$$v_{12}: 0.55 \Rightarrow 0.56, v_{22}: 0.4 \Rightarrow 0.42, v_{32}: 0.35 \Rightarrow 0.37.$$

Figure 10 plots the result. Women's human capital and wage rates are increased. The global and national macroeconomic real variables are enhanced. The developed economy's trade balance is improved, while the middle-income and developing economies' trade balances are deteriorated. Women increase working hours and reduce leisure hours. Men reduce working hours and increase leisure hours.

Figure 10: Women accumulate human capital more effectively



Source: processed by author.

5 CONCLUDING REMARKS

The purpose of this study is to examine global economic growth with endogenous wealth and population. A few theoretical models deal with the important issues within a compact framework.

This paper constructed a multi-country growth model with free trade. The dynamic machines of the world economic growth are the wealth accumulation, the population change, and gender-differentiated human capital dynamics with free trade. The global economy consists of any number of national economies. Each national economy has an industrial sector and an education sector. The population is female and male. The model is built by integrating some economic models in the literature economic theory. The capital accumulation follows the neoclassical growth model by Solow; the human capital dynamics follows the two-sector model by Uzawa and Lucas; the population dynamics follows the population models by Haavelmo and by Barro and Becker; and the trade pattern follows the model by Oniki and Uzawa. Gender division and gender-based time distribution is according to Zhang. Household behavior is described with Zhang's utility function and concept of current and disposable incomes. Nations are differences in technology, gender-related human capital, family preference, and emotional and pecuniary investment in children. We describe J-country global economic dynamics with 4J nonlinear differential equations – one country with one equation for endogenous wealth, two for human capital, and one for population. The model was simulated. We identified one stable equilibrium point and plotted the movement of the global system. We carried out comparative dynamic analysis in woman's human capital accumulation efficiency, the propensity to have children, the propensity to save, the total factor productivities, men's propensities to use leisure, and families' pecuniary costs in children caring. Since our model is built on the well-developed models in the literature of theoretical economics and each of these models has been extended in different directions, we can easily make our model more realistic or more general on basis of these extensions. We can also further simulate our model by other combinations of parameter changes or take on more general production functions or/and utility functions.

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APPENDIX 1: CHECKING THE LEMMA

We now prove the Lemma. From (3), we get:

$$z_j \equiv \frac{r+\delta_{kj}}{w_j} = \frac{\tilde{\alpha}_{jm} N_{jm}}{K_{jm}}, j = i, e, \tag{A1}$$

where $\tilde{\alpha}_{jm} \equiv \alpha_{jm}/\beta_{jm}$. From (2) and (3), we have

$$r = \alpha_{ji} A_{ji} \left(\frac{z_j}{\tilde{\alpha}_{ji}}\right)^{\beta_{ji}} - \delta_{kj}, w_j = \beta_{ji} A_{ji} \left(\frac{\tilde{\alpha}_{ji}}{z_j}\right)^{\alpha_{ji}}, w_{jq} = h_{jq}\, w_j. \tag{A2}$$

With (A2) we solve

$$r(z_1) = \alpha_1 A_1 \left(\frac{z_1}{\tilde{\alpha}_{1i}}\right)^{\beta_{ji}} - \delta_{kj}, z_j(z_1) = \tilde{\alpha}_{ji} \left(\frac{r+\delta_{kj}}{\alpha_{ji} A_{ji}}\right)^{\frac{1}{\beta_j}}. \tag{A3}$$

From (3) we solve

$$p_j = \frac{w_j}{\beta_{je}\, A_{je}} \left(\frac{z_j}{\tilde{\alpha}_{je}}\right)^{\alpha_{je}}. \tag{A4}$$

The definition of $\bar{y}$ and (3) imply

$$\bar{y}_j = \bar{R}\, \bar{k}_j + h_{j0}, \tag{A5}$$

where

$$h_{j0} \equiv \left(h_{j1} + h_{j2}\right) w_j T_0.$$

By (8) and (11), we have

$$T_{jq} = T_0 - \frac{\eta_{jq}\, \bar{y}_j}{\bar{p}_{jq}} - \frac{\sigma_{jq}\, \bar{y}_j}{w_{jq}} - \tilde{T}_{jq}. \tag{A4}$$

Insert (A6) and (10) in (A6)

$$T_{jq} = T_0 - \widehat{w}_{jq}\, \bar{y}_j, \tag{A7}$$

where

$$\widehat{w}_{jq} \equiv \varphi_{jq} + \frac{\upsilon_j\, \theta_{jq}}{\widetilde{w}_j}, \varphi_{jq} \equiv \frac{\eta_{jq}}{\bar{p}_{jq}} + \frac{\sigma_{jq}}{w_{jq}}.$$

From (A7) and (1) we have

$$N_j = \tilde{n}_j - \left(h_{j1}\, \widehat{w}_{j1} + h_{j2}\, \widehat{w}_{j2}\right)\bar{N}_j\, \bar{y}_j, \tag{A8}$$

where

$$\tilde{n}_j \equiv \left(h_{j1} + h_{j2}\right)\bar{N}_j\, T_0.$$

From (2) and (A1) we have

$$f_{jm} \equiv \frac{F_{jm}}{N_{jm}} = A_{jm}\left(\frac{\tilde{\alpha}_{jm}}{z_j}\right)^{\alpha_{jm}}. \tag{A9}$$

With (16), (10) and (A9) we have

$$N_{je} = \bar{f}_{je}\, \bar{y}_j, \tag{A10}$$

where

$$\bar{f}_{je} = \left(\frac{\eta_{j1}}{\bar{p}_{j1}} + \frac{\eta_{j2}}{\bar{p}_{j2}}\right)\frac{\bar{N}_j}{f_{je}}.$$

From (A8), (A10) and (20) we have

$$N_{ji} = \tilde{n}_j - \left(h_{j1}\, \widehat{w}_{j1}\, \bar{N}_j + h_{j2}\, \widehat{w}_{j2}\, \bar{N}_j + \bar{f}_{je}\right)\bar{y}_j. \tag{A11}$$

The definitions of z_j and K_j imply:

$$K_j = \frac{\tilde{\alpha}_{ji}\, N_{ji}}{z_j} + \frac{\tilde{\alpha}_{je}\, N_{je}}{z_j}. \tag{A12}$$

Insert (A10) and (A11) in (A12)

$$z_j\, K_j = \tilde{\alpha}_{ji}\, \tilde{n}_j - \left(h_{j1}\, \widehat{w}_{j1}\, \bar{N}_j + h_{j2}\, \widehat{w}_{j2}\, \bar{N}_j + \bar{f}_{je}\right)\tilde{\alpha}_{ji}\, \bar{y}_j + \tilde{\alpha}_{je}\, \bar{f}_{je}\, \bar{y}_j. \tag{A13}$$

From the definitions of $\widehat{w}_{j1}$ and (A13), we have

$$K_j = \frac{\tilde{\alpha}_{ji}\tilde{n}_j}{z_j} - \left(\omega_{1j} + \frac{\omega_{2j}}{\tilde{w}_j}\right)\bar{y}_j, \qquad \text{(A14)}$$

where

$$\omega_{1j} \equiv \frac{\left(h_{j1}\,\varphi_{jq} + h_{j2}\,\varphi_{j2}\right)\tilde{\alpha}_{ji}\,\bar{N}_j + \left(\tilde{\alpha}_{ji} - \tilde{\alpha}_{je}\right)\bar{f}_{je}}{z_j}, \omega_{2j} \equiv \left(h_{j1}\,\theta_{jq} + h_{j2}\,\theta_{j2}\right)\frac{\upsilon_j\,\tilde{\alpha}_{ji}\,\bar{N}_j}{z_j}.$$

From the definitions of (A5) and (A14), we have:

$$K_j = \omega_{0j} - \omega_{1j}\,\bar{R}\,\bar{k}_j - \frac{(\bar{R}\,\bar{k}_j + h_{j0})\omega_{2j}}{\tilde{w}_j}, \qquad \text{(A15)}$$

where

$$\omega_{0j} \equiv \frac{\tilde{\alpha}_{ji}\,\tilde{n}_j}{z_j} - h_{j0}\,\omega_{1j}.$$

From (18) and (19) we have

$$\sum_{j=1}^{J} \bar{k}_j\,\bar{N}_j = \sum_{j=1}^{J} K_j. \qquad \text{(A16)}$$

Insert (A15) in (A16)

$$\frac{(\bar{R}\,\bar{k}_1 + h_{10})\omega_{21}}{\tilde{w}_1} + (\bar{N}_1 + \omega_{11}\,\bar{R})\bar{k}_1 + \omega_0 = 0, \qquad \text{(A17)}$$

where

$$\omega_0 \equiv -\sum_{j=1}^{J}\omega_{0j} + \sum_{j=2}^{J}\left(\bar{N}_j + \omega_{1j}\,\bar{R}\right)\bar{k}_j + \sum_{j=2}^{J}\frac{\left(\bar{R}\,\bar{k}_j + h_{j0}\right)\omega_{2j}}{\tilde{w}_j}.$$

Substitute $\tilde{w}_1 = w_{01} + \theta_1\,\bar{k}_1$ into (A18)

$$\bar{k}_1^2 + m_1\,\bar{k}_1 + m_2 = 0, \qquad \text{(A18)}$$

where

$$m_1\left(z_1, \{\bar{k}_j\}, (H_{jq}), (\bar{N}_j)\right) \equiv \frac{\bar{R}\omega_{21} + (\bar{N}_1 + \omega_{11}\,\bar{R})w_{01} + \theta_1\,\omega_0}{m_0},$$

$$m_2\left(z_1,\{\bar{k}_j\},(H_{jq}),(\bar{N}_j)\right)\equiv\frac{\omega_{21}\,h_{10}+\omega_0\,w_{01}}{m_0},$$
$$m_0\left(z_1,\{\bar{k}_j\},(H_{jq}),(\bar{N}_j)\right)\equiv(\bar{N}_1+\omega_{11}\,\bar{R})\theta_1.$$

Solve (A19)

$$\bar{k}_1=\phi\left(z_1,\{\bar{k}_j\},(H_j),(\bar{N}_j)\right)\equiv\frac{-m_1\pm\sqrt{m_1^2-4\,m_2}}{2}. \tag{A19}$$

The simulation confirms the following solution of (A19) meaningful:

$$\bar{k}_1=\frac{-m_1+\sqrt{m_1^2-4\,m_2}}{2}.$$

We thus can represent the variables as functions z_1, $\{\bar{k}_j\}$, (H_j), and $(\bar{N}_j)$: r, w_j, and w_{jq} by (A2) → z_j by (A3) → p_j by (A3) → $\bar{k}_1$ by (A19) → $\bar{y}_j$ by (A5) → c_j, s_j, $\bar{T}_{jq}$, $\hat{T}_{jq}$, and n_j by (11) → d_j by (12) → $\tilde{T}_{jq}$ by (6) → T_{jq} by (8) → N_j by (1) → N_{je} by (A3) → K_{jm} by (A1) → F_{jm} by (2) → p_{bj} by (5) → K by (8) → $K_j=K_{ji}+K_{je}$ → $\bar{K}_j=k_j\bar{N}_j$ by (2) → B_j by (21). From this procedure, and (13)-(15), we have

$$\dot{\bar{k}}_1=\Omega_0\left(z_1,\{\bar{k}_j\},(\bar{N}_j),(H_{jq})\right)\equiv s_1-\bar{k}_1, \tag{A20}$$
$$\dot{\bar{k}}_j=\Omega_{kj}\left(z_1,\{\bar{k}_j\},(\bar{N}_j),(H_{jq})\right)\equiv s_j-\bar{k}_j, j=2,\dots,J,$$
$$\dot{\bar{N}}_j=\Omega_j\left(z_1,\{\bar{k}_j\},(\bar{N}_j),(H_j)\right)\equiv\left(\frac{\upsilon_j\,\bar{y}_j}{\widetilde{w}_j}-\frac{\bar{\upsilon}_j\,\bar{N}_j^{b_j}}{\bar{y}_j^{a_j}}\right)\bar{N}_j, j=1,\dots,J,$$
$$\dot{H}_{jq}=\Omega_{Hj}\left(z_1,\{\bar{k}_j\},(\bar{N}_j),(H_{jq})\right)\equiv\frac{\upsilon_{jq}\left(\frac{F_{jq}}{2\bar{N}_j}\right)^{a_{jq}}\left(H^{m_{jq}}\,\hat{T}_{jq}\right)^{b_{jq}}}{H^{\pi_{jq}}}-\delta_{jq}\,H_{jq}. \tag{A21}$$

Take derivatives of (A19) in t

$$\dot{\bar{k}}_1=\frac{\partial\phi}{\partial z_1}\dot{z}_1+\sum_{j=2}^{J}\frac{\partial\phi}{\partial\bar{k}_j}\dot{\bar{k}}_j+\sum_{j=1}^{J}\frac{\partial\phi}{\partial H_j}\dot{H}_j+\sum_{j=1}^{J}\frac{\partial\phi}{\partial\bar{N}_j}\dot{\bar{N}}_j. \tag{A22}$$

Insert (A20) in (A22)

$$\dot{\bar{k}}_1=\frac{\partial\phi}{\partial z_1}\dot{z}_1+\sum_{j=2}^{J}\Omega_{kj}\frac{\partial\phi}{\partial\bar{k}_j}+\sum_{q=1}^{2}\sum_{j=1}^{J}\Omega_{jq}\frac{\partial\phi}{\partial H_{jq}}\dot{H}_{jq}+\sum_{j=1}^{J}\Omega_j\frac{\partial\phi}{\partial\bar{N}_j}. \tag{A23}$$

From the right-hand sides of (A22) and (A23), we have:

$$\dot{z}_1=\Omega_{k1}\left(z_1,\{\bar{k}_j\},(\bar{N}_j),(H_{jq})\right)\equiv$$

$$\left(\Omega_0 - \sum_{j=2}^{J} \Omega_{kj} \frac{\partial \phi}{\partial \bar{k}_j} - \sum_{q=1}^{2} \sum_{j=1}^{J} \Omega_{jq} \frac{\partial \phi}{\partial H_{jq}} \dot{H}_{jq} - \sum_{j=1}^{J} \Omega_j \frac{\partial \phi}{\partial \bar{N}_j}\right) \left(\frac{\partial \phi}{\partial z_1}\right)^{-1}. \quad \text{(A24)}$$