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The Czech Government Yield Curve Decomposition at the Lower Bound*

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Abstract

The term structure of yields is an important source of information on market expectations about future macroeconomic developments and investors' risk perceptions and preferences. This article presents the decomposition of the Czech government bond yield curve into its components using a shadow–rate affine term–structure model and interest rate and credit default swap quotations. The evolution of the components is interpreted in relation to the macro–financial environment embodied by selected variables. The practical use of the decomposition in estimating and interpreting responses of the Czech government bond yield curve to macroeconomic and financial shock is presented using a vector autoregression model. Finally, the results are evaluated in terms of the lower bound proximity.

1. Introduction

In his speech in February 2005, former Federal Reserve Board chairman Alan Greenspan expressed a "conundrum" over the falling 10-year Treasury yield despite increasing federal funds rate (Greenspan, 2005). The prevailing view risk premium being roughly constant over time, making of long yields an average of expected short yields, proved insufficient. In order for the Federal Reserve Board to understand and steer the long yields, understanding of the factors affecting the yields – beyond the monetary policy conduct – needed to be developed. Indeed, after a wide discussion among researchers, a drop in the risk premium was identified as the source of the decline of 10–year yields (Backus and Wright, 2007). Consequently, the ability of central banks to influence the longer part of the yield curve was admitted to be weaker then originally thought. A proper risk premium modelling and forecasting can help overcome this gap

A similar need for understanding yield factors emerged also in the case of Czech government bonds (GBs). Czech GB yields have been, like global yields, on a

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downward trend on average since the global financial crisis started. They have been negative at maturities of up to six years from the beginning of 2016 until ca. mid–2017. This can hardly be explained solely by market expectations of continued low rates or by the lower Czech sovereign risk premium. Therefore, this paper presents an approach to disentangling the Czech GB yields into multiple components corresponding to the factors crucial for explaining the Czech GB yield curve dynamics. To add to this, we go beyond the usual separation of the yields to risk-neutral expectation and risk premium: we propose a way to obtain four components which we consider necessary to capture the core dynamics of the Czech GB yield curve. As we show, this allows us to improve understanding of the yield curve movements, both in terms of historic dynamics and estimating responses of yields to macroeconomic and financial shocks. As with Greenspan's conundrum, we document that some factors may affect yields in a contrary direction than the monetary policy intended, which may weaken the ability of monetary policy to affect the yield of long-term Czech GBs. As additional contributions, we evaluate the effect of the lower bound on the response of yield curve to shocks. We also comment on the perception of Czech GBs in times of stress – whether they represent a safe haven or a risky instrument in terms of international capital flows.

In yield curve modeling, the first step is to identify the factors affecting the yield curve and corresponding yield components. The factor-based modeling of yields is frequently based on Nelson-Siegel (Nelson and Siegel, 1987) parametrization of the yield curve using three parameters: the level, the slope and the curvature. This approach was further developed by Diebold and Li (2006), who developed the Dynamic Nelson-Siegel model (DNS). They see parameters as time-varying factors, which allows representation of yield curve movements by the dynamics of three factors. The usage of three factors seems sufficient, since they explain the dominant share of the yield curve variation (Litterman and Scheinkman, 1991) as well as of macroeconomic information (Moench, 2008).

Despite its popularity, the drawback of the relative parsimony of the basic DNS model is that it is not able to separate risk premia from the risk-neutral expectation of future rates.[1] Therefore, in term-structure literature, the Gaussian affine model framework is frequently used instead. This approach builds on the short rate process in the sense of Vasicek (1977). Duffie and Kan (1996) developed a framework for affine modeling which allows solution of wide scale of models relatively easily if certain conditions are satisfied (including the affine relations of factors and other processes). Duffee (2002) further demonstrated how the affine framework may be used to extract the term premia from the model. Further improvement in the specification and estimation of the models was presented by Cochrane and Piazzesi (2008) and Adrian et al. (2013); a summary of the most important approaches is outlined in Piazzesi (2010) and Krippner (2015).

One caveat of both the DNS approach and the affine framework is an implicit assumption about the symmetry of yield movements. This means that the probability of yields going up is the same as the probability of yields going down. However, at

[1] The DNS framework was later extended by no-arbitrage conditions (Christensen et al., 2011) and other elements (Diebold and Rudebusch, 2013). That in practice means integration of the DNS and affine framework. We consider the no-arbitrage DNS model as part of the family of Gaussian affine models.

the lower bound, this assumption is not fulfilled. Shadow-rate affine models were shown to efficiently reflect asymmetry close to the lower bound while offering a tractable framework and relative parsimony (Krippner, 2013).

Affine models are able to decompose yields into risk-neutral expectations about yields and term (risk) premia. Term (risk) premia are however still affected by multiple factors, including fiscal and geopolitical uncertainty (Bauer, 2017) or inflation uncertainty (Wright, 2011). In this paper, we build on an affine approach adjusted by the lower bound; however we further decompose the risk premia so that we obtain separatelly (i) "pure" term premium, (ii) credit risk premium and (iii) portfolio effect. Together with the risk-neutral expectations, we obtain four components that correspond to the most important factors affecting the yields of Czech GBs.

The article is structured as follows. Section 2 presents the method used to decompose yield curves. In section 3, the results of the decomposition are shown. In section 4, the analysis focuses on the use of the components to estimate and interpret the response of the Czech GB yield curve to macroeconomic and financial shocks in a vector autoregression framework. Also, the effect of the lower bound and additional sensitivity testing is presented. The final section concludes.

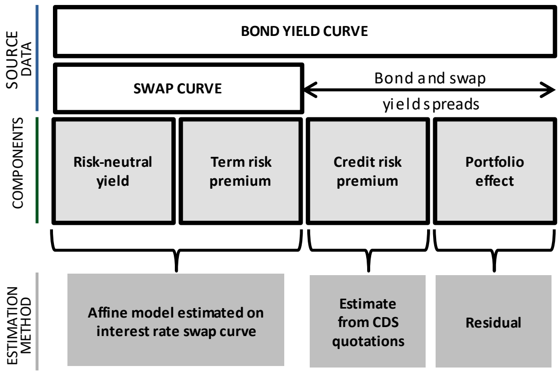
2. Yield Curve Decomposition Methodology

2.1 Decomposition Rationale and Approach

The yield curve is made up of yields on bonds issued by a single entity with various residual maturities at a specific point in time. The shape of the curve is determined by its level (the yields on the long end of the curve), its slope (the difference between yields on short– and long–maturity bonds) and its curvature (allowing for concave or convex maturity–yield relationship). The relative level of short–term and long–term yields should depend on market expectations about the future path of short–term rates. According to the pure expectations hypothesis, a risk–neutral investor should attain the same yield from investing in a long–term bond as from a series of investments in a short–term bond over a period equal to the residual maturity of the long–term bond. The pure expectations hypothesis offers a simple and attractive interpretation of the yield curve. However, it does not hold in reality, as it does not take risk–averse investors into consideration. In other words, investors perceive long–term investment as uncertain and demand a risk premium.

In the literature (for example Wright, 2011), a frequent approach is to consider the government bonds as default free so that the risk premium is only related to the uncertainty of future evolution of yields (i.e. the term premium as described below). However, in the case of Czech GB yields, the premium for the credit risk was historically also an important source of yield variation. At the same time, Czech GBs have an exclusive position among Czech financial assets, representing the only safe and liquid (marketable) asset available in Czech koruna and a vehicle within speculative schemes. Consequently, we go beyond the traditional approach and instead of decomposing the yield into two components (a risk–neutral yield and a risk premium), we propose a novel extension of the decomposition. More specifically, we decompose the Czech GB yield curve into four additive components (see Figure 1): a risk–neutral yield, a term premium, a credit risk premium and a portfolio effect. The latter three components together form the full risk premium of Czech GBs. To keep the model parsimonious, we prefer additive components to a multiplicative approach.

Figure 1 Components of The Swap and Bond Yield Curves



Notes: CDS = credit default swap.
Source: Authors.

The risk–neutral yield reflects expectations about future monetary policy and economic developments (i.e. the expectations hypothesis). If investors expect the monetary policy rate to rise in the future, they also expect the rate of return on holding and regularly reinvesting short–term bonds to go up gradually. The term premium relates to the maturity of the bond and is compensation for interest rate risk. It takes into account investors' uncertainty about the future path of the short–term rate. Committing to long–term bonds will turn out to be relatively less (more) advantageous if future short–term rates are higher (lower) than originally expected. As long-term bonds have higher duration than short-term ones, impact of interest rate changes on long-term bond investors' portfolios is magnified.

Regarding our approach to the decomposition, as an underlying assumption we consider the risk–neutral yield and the term premium to be common for both Czech GB yields and Czech koruna interest rate swap (CZK IRS) rates. Such an assumption follows the intuition that expectations about the future short interest rates as well as the uncertainty related to them is not dependent on financial instruments. A corollary of this assumption is that the spread between the Czech GB yield and the CZK IRS is formed exclusively by the other two components: the credit risk premium and the portfolio effect.

To simplify the method, we assume both credit risk premium and the portfolio effect of IRSs equal to zero. This simplification is in line with the nature of the IRS: IRSs contain only a very limited credit risk premium, as no principal is paid, coupon payments are netted and the way IRSs are traded mitigates counterparty risk. An IRS is meanwhile not an investment asset, because it cannot be used to deposit liquidity. The portfolio effect of an IRS is therefore negligible.

The absence of any other components allows the use of the affine class of models (Duffie and Kan, 1996; the affine model described in 2.2) for the IRS data to be separated into two components. The application of the affine model on the IRS data can be seen as its application on a set of yields of composite bonds, which are risk free in terms of credit risk and are traded on a perfect market (i.e. the portfolio effect is not present). In contrast, the portfolio component in the Czech GB yields can be affected in certain circumstances by specific market effects such as flight to quality, flight to liquidity, search for yield and various types of speculation caused, for example, by unconventional monetary policies and exchange rate regimes. However, these specific effects could disrupt the affine model's assumption of market efficiency and the impossibility of arbitrage. The risk–neutral yield and the risk premium estimated using the affine model from GB yield curves could thus be distorted.

The credit risk premium is a compensation for the risk that bond coupons and principal will not be paid on time and/or in full. This premium tends to increase with increasing maturity. The issuer's position can worsen significantly over time, so, for example, the one–year probability of default in five years' time (i.e. the probability of default between the fifth and sixth years) is usually higher than the current one–year probability of default, i.e. the probability of default between now and 12 months from now (Moody's, 2016). The credit risk premium is estimated from credit default swap (CDS) quotations for Czech GBs.[2] The volatility of the CDS quotations was reduced by smoothing them using the three–month moving average. Furthermore, to obtain quotations for each maturity, the Nelson–Siegel function was fitted to these averaged quotations.

To the best of our knowledge, in term structure modeling, such explicit calculation of the credit risk component from CDS quotations has not been used so far. The literature mostly focuses on yields of U.S. government bonds, where the credit risk premium is considered as negligible and rather constant over time, and hence interpreted as a part of the total risk premium. By contrast, in the Czech GB yield curve, the increase of the credit risk premium significantly affected the yield of longer maturities between 2009–2012 (see Figure 2). As a contrasting approach, it would be also possible to estimate the credit risk premium from the Czech GB yields by using intensity–based modeling of credit risk as in Lando (2009). This approach extends the risk–neutral pricing used in the affine model to include the credit risk premium as well. However, since this approach would further increase the technical requirements for building and estimating the model, we leave this possibility for further research and stick to the estimation of the credit risk premium directly from the CDS quotations.

The portfolio effect of the yield reflects demand for GBs as an investment asset or a tool usable for speculative trades (Kladívko, 2010). Many investors prefer GBs to other assets, mainly because of their low credit risk, their relatively high market liquidity, their low haircuts when used as financial collateral and their preferential regulatory treatment. Additionally, GBs can serve as a tool to perform speculative or arbitrage trades. The portfolio effect can take positive (negative)

[2] The advantages of this approach are the objective existence of quotations (which should represent the direct cost of hedging credit risk), its forward–lookingness and the availability of a range of maturities in daily periodicity. On the other hand, we also need to take into account certain sovereign CDS market anomalies that may limit the use of CDS quotations as a sovereign solvency indicator (see Komárek et al., 2013, Box 4 of CNB (2010) and Box 4 of CNB (2012)). Short time series are another potential limitation for some maturities.

values if the yield demanded by the investor for holding the bond is higher (lower) than the expected short–term rate plus the term premium and the credit risk premium. The portfolio effect is calculated as a difference between the observed GB yield and the observed rate of an IRS rate of identical maturity minus the fitted credit risk premium. The average portfolio effect in the model therefore depends on the estimate of the credit risk premium and also includes possible measurement fitting error from the calculation of the credit risk premium. Therefore, in the interpretation of the results, we reflect the fact that certain dependencies between the portfolio effect and credit risk premium may still persist. The portfolio effect may also reflect the supply side of the GB market – a prospective scarcity of the supply of bonds with certain maturities may be reflected by a lower required yield for those maturities, i.e. lower portfolio effect.

The presented approach does not consider the components to be orthogonal. In practice, the correlation of the term premium and the credit risk premium can be expected to be significantly positive, since both reflect the risk-attitude of investors. In our approach, the correlation is data-driven, i.e. results from the correlation of the term premium priced in the CZK IRS rates and the CDS quotations. The separate estimation of the components does not allow modeling of these variables jointly and utilizing the correlation in the framework. An intensity–based modeling of credit risk (Lando, 2009) could bridge their relations in a joint framework, which however goes beyond the extent of this paper and is left for future research.

2.2 Interest Rate Swap Curve Decomposition

To decompose the IRS curve we use an affine model (Duffie and Kan, 1996, or Málek, 2005). The basic building block of this model is its assumption that there are several underlying factors (X_t) that determine the entire term structure of rates. The affine model presented here uses three factors, in line with the standard approach employed in the literature (Litterman and Scheinkman, 1991). These factors are unobservable and we estimate them within the model. In the model, their dynamics under a risk-neutral $\mathbb{Q}$ measure are set in the form of a mean–reverting (Ornstein–Uhlenbeck) process (Krippner, 2015):

$$\mathrm{d}X_t = \kappa(\theta - X_t)\mathrm{d}t + \sigma \mathrm{d}W_t, \tag{1}$$

where W_t is a 3–dimensional independent Wiener process, θ is a 3×1 vector representing the level of the mean–reversion, κ is a 3×3 matrix determining the speed of the mean–reversion and σ is a 3×3 matrix allowing the innovations to $X_t$to be correlated.

Since the model falls into the family of affine models, affine representation is used to obtain both the market price of risk λ_t:

$$\lambda_t = \sigma^{-1}(\gamma + \Gamma X_t) \tag{2}$$

and the short (infinitesimal) rate r_t:

$$r_t = \alpha_0 + \alpha_1 X_t \tag{3}$$

From the short rate, the longer rates $R_t(\tau)$ of any maturity τ are derived by applying the expectation hypothesis under the $\mathbb{Q}$ measure. The expected path of the factors is obtained from the equation (1). The expected short rate path under the $\mathbb{Q}$ measure is then again calculated by the transformation in the equation (3). The longer rates $\hat{R}_t(\tau)$ are derived by integrating the short rate over the expected path:

$$\hat{R}_t(\tau) = \frac{1}{\tau}\int_0^\tau \left[E_t^{\mathbb{Q}}(r_{t+u}|X_t) - VE(u)\right]\mathrm{d}u, \tag{4}$$

where $VE(u)$ is a volatility effect, which corrects the expectations with respect to Jensen's inequality (Heath et al., 1992). The Jensen's inequality results from the nature of the expectations hypothesis in terms of bond prices and the convexity of the exponential function:

$$e^{-R_t(\tau)\tau} = E_t^{\mathbb{Q}}\left[e^{-\int_0^\tau r_{t+u}\mathrm{d}u}|X_t\right] \leq e^{-E_t^{\mathbb{Q}}\left[\int_0^\tau r_{t+u}\mathrm{d}u|X_t\right]}, \tag{5}$$

$VE(u)$ value requires calculation of a double integral (Heath et al., 1992) and depends only on the maturity and the parameters from the equation (3). $\hat{R}_t(\tau)$ differs from $R_t(\tau)$ by the measurement error, which appears since the time–invariant parameters in the equations (1)–(3) do not allow fitting the observed yields exactly.

Under the $\mathbb{Q}$ measure, the investors are risk-neutral, i.e. the term premium does not exist (it is incorporated into the risk-neutral path of the factors). Therefore, to obtain the term premium, the factor process in equation (1) is transformed into a data-generating $\mathbb{P}$ measure. This transformation is performed using the market price of risk λ_t: $\mathrm{d}\widetilde{W}_t = \mathrm{d}W_t - \lambda_t \mathrm{d}t$, where the Wiener process $\mathrm{d}\widetilde{W}_t$ is under the $\mathbb{P}$ measure. Given the affine representation in the equation (2), the formulas under the $\mathbb{P}$ measure are identical; only the parameters of the equation (1) are adjusted (see Duffee, 2002, for details). Through the equations (3)–(5) under the $\mathbb{P}$ measure, the $\hat{R}_t(\tau)$ is obtained – in this case, it represents only the risk-neutral yield, i.e. these do not fit the observed yields. The term premium is finally obtained as the difference between $R_t(\tau)$ and $\hat{R}_t(\tau)$ fitted under the $\mathbb{P}$ measure.

A plausible consequence of the affine form of equations (2) and (3) is that the affine model under the $\mathbb{Q}$ measure can be solved analytically. This means that the equations (1)–(5) result in a representation of the fitted yields as a function of the factors and the maturity: $\hat{R}_t(\tau) = F(X_t, \tau)$. The difference between $\hat{R}_t(\tau)$ and $R_t(\tau)$ under the $\mathbb{Q}$ measure is given only by the measurement error. Moreover, the factors X_t are unobservable, therefore it is necessary to estimate them. As a solution the affine model is specified in a state space representation. The measurement equation linking the observed yields and the factors is given by a no-arbitrage $\mathbb{Q}$ measure pricing equation $R_t(\tau) = \hat{R}_t(\tau) + e_t(\tau) = F(X_t, \tau) + e_t(\tau)$, whereas the $\mathbb{P}$ measure factor process represents the state equation. It can be shown (Meucci, 2010) that the solution to the continuous time process in equation (1) forms a first–order vector autoregression process for a discrete time step Δt. The final state–space representation is thus:

$$R_t(\tau) = F(X_t, \tau) + e_t(\tau), \tag{6}$$

$$X_t = \tilde{\theta} + exp(-\tilde{\kappa}\,\Delta\, t)\big(X_{t-1} - \tilde{\theta}\big) + \upsilon_t, \tag{7}$$

where υ_t is a 3×1 vector of random innovations to state variables. The equation (7) represents the discrete time process for the factors under the $\mathbb{P}$ measure. Given the state-space representation, the affine model can be estimated using the maximum likelihood, with a Kalman filter (Durbin and Koopman, 2012) utilized to obtain the factors.

If interest rates are already close to their lower bound, the probability of them falling further is lower than the probability of them rising. This leads to a violation of the assumptions underlying the affine model – the process W_t in the equation (1) is no longer a Wiener process. To take this asymmetry into account, the model uses the concept of shadow rates (Krippner, 2013). In this concept, which builds on the model of Black (1995), the yield on investing in a bond equals the sum of the yield on investing in a shadow bond whose yield is not bounded below by zero and the yield from the sale of a bond option. The option bears the right to purchase the bond at such price so that its yield is equivalent to the lower bound value. For details, see Krippner (2013).

In practice, in the shadow–rate affine model, the equation (3) holds for the shadow short rate (rs_t). The expected value of the observed short rate (r_{t+u}) entering into the equation (4) is then obtained as a sum of the expected shadow short rate and the option effect:

$$E_t^{\mathbb{Q}}(r_{t+u}|X_t) = E_t^{\mathbb{Q}}[rs_{t+u}|X_t] + E_t^{\mathbb{Q}}[\max(r_L - rs_t, 0)|X_t], \tag{8}$$

where r_L is the value of the lower bound. Krippner (2013) derives a closed–form solution for $E_t^{\mathbb{Q}}(r_{t+u}|X_t)$ in terms of parameters from the equations (1–3). After plugging into the equation (4), the whole term structure is derived, i.e. the function $F(X_t, \tau)$is obtained. The final formulas are complex and therefore omitted here for parsimony; a complete description can be found for example in Krippner (2015). Due to the option effect, the function $F(X_t, \tau)$ is not linear in case of the shadow–rate affine model., Therefore, an extended iterative Kalman filter (Durbin and Koopman, 2012) is employed in the estimation. However, the shadow rates can still be expressed as an affine transformation of the factors. For this reason, the model still falls into the category of affine models.

To estimate the shadow–rate affine model in its state space representation defined by the equations (6) and (7), it is necessary to set identifying restrictions. We used the shadow–rate stationary Gaussian affine (SR–SGA) model described in

Krippner (2015, section 4.4.4)[3]. The restrictions imposed by the SR–SGA model on the parameters from the equations (3) and (7) are as follows:

$$\tilde{\kappa} = \begin{bmatrix} \tilde{\kappa}_1 & 0 & 0 \\ 0 & \tilde{\kappa}_2 & 0 \\ 0 & 0 & \tilde{\kappa}_3 \end{bmatrix}, \tilde{\theta} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \alpha_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{9}$$

Furthermore, σ is lower–triangular. The measurement errors $e_t(\tau)$ from the equation (6) form a random normally distributed vector with a zero mean and a time–invariant diagonal covariance matrix. The state innovations v_t from equation (7) are normally distributed with zero mean and a variance given by the σ matrix (derivation can be found in Krippner, 2015). The two error vectors are assumed to have a zero covariance matrix.

After the SR–SGA model is estimated, it can be used to decompose the observed rates into the two components. This decomposition is done independently for each period *t* in the sample. The vector of factors X_t, which was extracted during the SR–SGA model estimation, is the starting point. Using the equations (3) and (7), the expected path of the factors $\{E^{\mathbb{P}}[X_{t+j\,\Delta\,t}|X_t]\}_{j=1,2,\ldots,\mathrm{T}/\Delta\mathrm{t}}$ and the short rate $\{E^{\mathbb{P}}[r_{t+j\,\Delta\,t}|X_t]\}_{j=1,2,\ldots,\mathrm{T}/\Delta\mathrm{t}}$ is calculated, where T represents the longest maturity in the sample (in years). For any maturity $\tau < T$, the average expected short rate $\Delta\,t/\tau * \sum_{k=1}^{\tau/\Delta\,t} E^{\mathbb{P}}[r_{t+k\,\Delta\,t}|X_t]$ represents the risk–neutral component of the yield $\hat{R}_t(\tau)$. As with the equation (4), the volatility effect (VE) adjustment applies. The expectations are under the $\mathbb{P}$ measure: the risk–neutrality of the component means that it is calculated as an average of future short rates (i.e. without any risk premia). The second estimated component – the risk premium (which is equal to the term premium in case of the swap rates) – is then obtained as the difference between the fitted swap rate $\hat{R}_t(\tau)$ and the risk–neutral component.

3. Decomposition of the Czech Government Bond Yield Curve

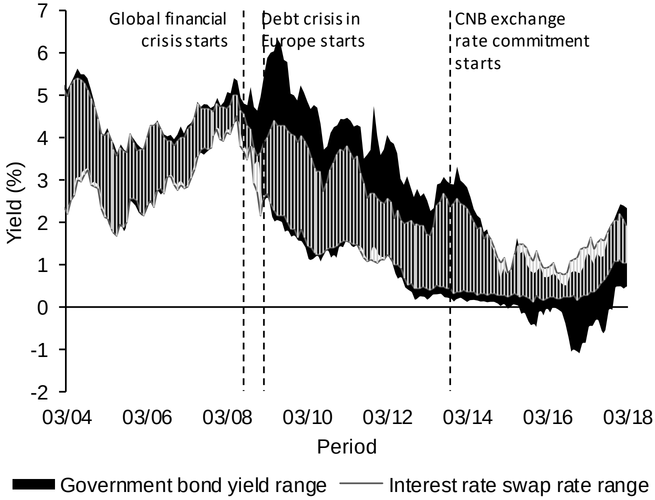
The yield curve is decomposed using the yields on zero–coupon bonds of relevant maturities, since those yields are not affected by the size and distribution of the coupons over the life of the bond and hence are an exact indicator of the rate of return demanded for investing for the relevant time period.[4] For this reason, a zero–coupon curve was constructed using Czech government bonds in Czech koruna. As the risk–neutral yield and the term premium are estimated using swap rates, it was also necessary to construct a zero–coupon koruna swap curve. The two zero–coupon

[3] We also evaluated the results using an arbitrage–free Dynamic Nelson–Siegel (ADNS) model specification established by Christensen et al. (2011) and extended with the shadow rates by Christensen and Rudebusch (2013). However, due to a relatively short maximum maturity included in the sample, the shadow–rate ADNS model does not converge well and is very sensitive to initial conditions. As described in the next section, we used maturities up to 15 years. Contrary, in case of the US or EA yield curves, maturities 20–30 years are usually available. Apart from the sample, the shape of the Czech yield curve is also slightly different than in case of other countries – the Czech yield curve was never inverted in the sample period.

[4] The use of yield to maturity of coupon bonds could potentially lead to underestimation of the yields demanded for a given maturity (Livingston and Jain, 1982).

curves were constructed for maturities of 1 to 15 years[5] as of the end of each month over the period of 7/2003–03/2018. The Fama–Bliss bootstrap method (Fama and Bliss, 1987), which assumes constant forward rates among the closest maturities, was used for the construction. The advantage of this method over the alternatives (such as Nelson and Siegel, 1987, or Svensson, 1994)[6] consists in its ability to replicate any yield curve shape exactly, which eliminates problems with imperfect fit on some segments of the curve.

Figure 2 Ranges for Zero–Coupon Czech Government Bond Yields and Koruna Interest Rate Swap Rates (in %; ranges between 1Y and 15Y maturities)



Notes: Vertical lines mark the last monthly observation before the event described. The start of the global financial crisis is related to the collapse of Lehman Brothers in September 2008. The start of the debt crisis is related to the negative assessment of Greek public finances by the International Monetary Fund and the European Commission in February 2009.

Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations.

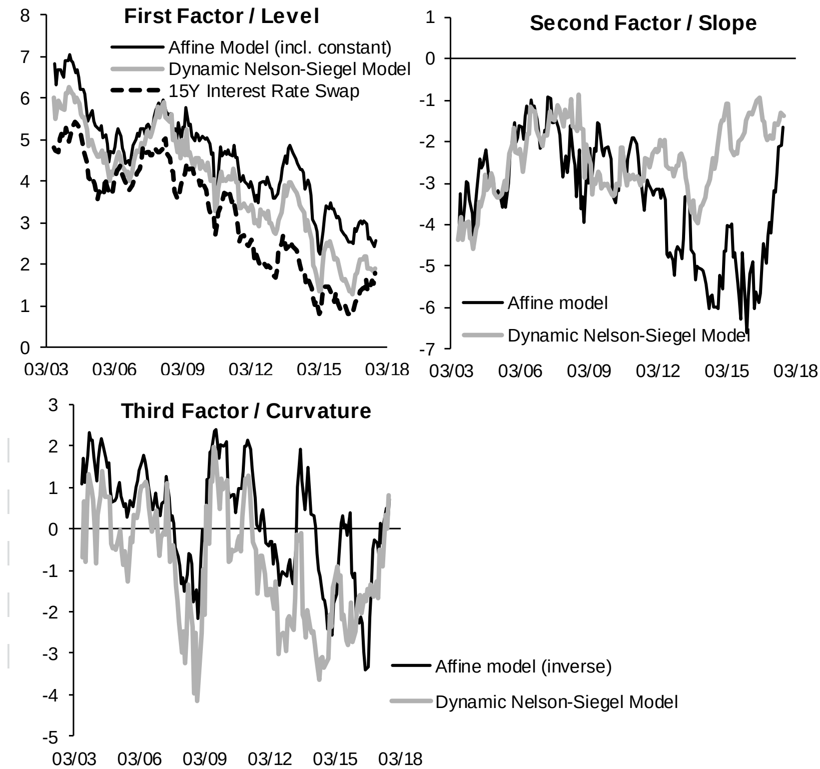
The maturity spreads for zero–coupon GB yields and swap rates in 2003–2018 show mixed developments (see Figure 2). Until the outbreak of the global financial

[5] The range of maturities considered was chosen with regard to data availability and quality. Bonds with maturities of less than one year are not used in such studies because their prices can be distorted by specific effects due to lower liquidity (BIS, 2005). In addition, Czech koruna interest rate swaps are not available for maturities of less than one year. The time series for bonds and swaps with maturities of over 15 years are shorter and their prices may be less reliable due to their lower trading volumes. The empirical strategy assumes the quotes stated for both bonds and swaps are reasonable. Surveys conducted by CNB and market intelligence suggest good liquidity of Czech koruna swaps up to 10 years. There is also certain turnover above 10 years which allows market participants to deal with market makers at their stated quotes. The liquidity of swaps is comparable with that of Czech government bonds, which are also less liquid at longer maturities. The usability of swaps for yield curve construction is further confirmed by EIOPA's current use of swaps (up to 15 years) rather than bonds for discounting insurers' liabilities for regulatory purposes.

[6] Previous works on yield curve construction from Czech government bonds that use Nelson and Siegel, and Svensson models include Hladíková and Radová (2012), Kladívko (2010) and Slavík (2001).

crisis in September 2008, yields and rates followed similar patterns. This is consistent with the findings in Kladívko (2010). From then until the second half of 2009, yields were affected by the fear of the emerging debt crisis in Europe. Owing to the responses of the various relevant authorities to the crisis, yields began to trend downwards in mid–2009 and a positive gap opened up between yields and rates at longer maturities. At the end of 2013, yields started falling faster than rates – until 2015 for long maturities and then exclusively for short maturities. It is clear from this simple historical excursion that yields and rates were affected by different factors with different intensity, including for individual maturities

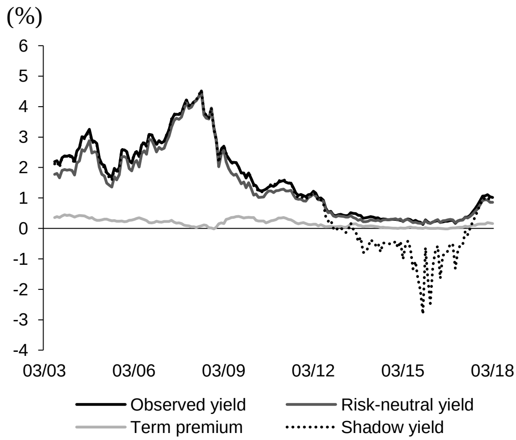
Figure 3 Affine Model Factors (values of factors, multiplied by 100)



Source: Authors' calculations.

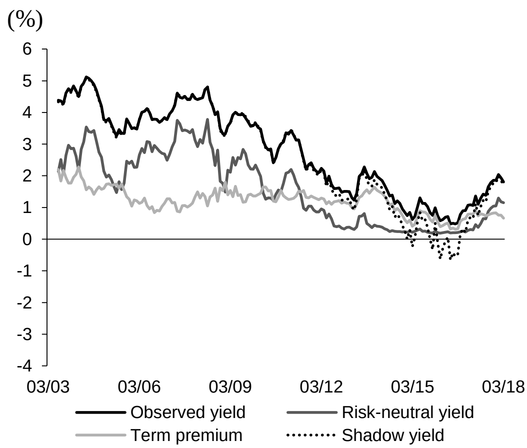
The factors X_t resulting from the affine model are shown in the Figure 3. To understand their dynamics, we also estimate DNS model (Diebold and Li, 2006). DNS results in three yield factors with straightforward interpretation, following the Nelson-Siegel (1987) function: the level of the yield curve, its slope and its curvature. As is obvious from Figure 3, the factors obtained from the affine model are not far from the DNS factors, which therefore can be similarly interpreted level, slope and curvature. Such a finding is important for understanding the extent to which the non-linearity embedded in both affine and DNS framework could affect the results of the autoregression analysis presented in section 4. Both the long end of the yield curve (i.e. the level) and its short end (instantaneous rate as a combination of the level and the slope) preserve linearity; non-linearity is related only to the yields between. Therefore, although non-linearity might be an issue in the autoregression analysis below, the results will be to a certain extent always anchored by the linearly-dependent ends of the yield curve. A certain non-linearity issue is also related to the option effect in the shadow-rate model. We show in section 4 to what extent the option effect, i.e. the binding lower bound, affects the results.

Figure 4 One–Year Swap Rate Decomposition



Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations.

Figure 5 Ten–Year Swap Rate Decomposition



Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations.

The crucial ability of the affine model is to decompose the swap rate into the risk–neutral yield and the term premium (see Figures 4 and 5). The results show that the model generates components in line with general intuition. The term premium of the short rate is rather insignificant, whereas for the longer maturity, it explains a significant share of the rate movements. The term premium for the longer rates behaved countercyclically in the period 2005–2007, which led to a smaller increase of the long rates, compared to the short end of the term structure. Such behavior was widely discussed in terms of U.S. yields (Greenspan, 2005). From these observations, we conclude that the model–generated components have plausible properties.

Figures 4 and 5 also display the estimated shadow 1–year and 10–year swap rates. Although estimation of the shadow rates themselves was not the main aim of the paper, their evolution offers interesting insight into the monetary policy stance over the lower bound period. More specifically, until 2012, the shadow rates were almost identical to the observed rates. However, after approaching the lower bound in 2012, the shadow short rate gradually decreased to -1%, where they remained for some time. However, during 2015–6, with the growth of the CNB balance due to the forced intervention to keep the exchange rate floor, the shadow short rate further decreased up to -3%. Longer shadow rates followed a similar evolution, although the drop of the shadow rates was much smaller, since the longer observed rates are more distant from the lower bound. During 2017, as the exchange rate floor was exited and the monetary policy rates were increased, the shadow rates returned to positive values, again very close to the observed values.

The estimated values of the affine model parameters and the statistical properties are presented in the Appendix. The overall fit of the model is good (see Figures A1 and A2). Measurement error contains residual autocorrelation and heteroscedasticity (see Appendix), which is a consequence of the changing ability of the model to fit various shapes of the yield curve. However, the measurement error is low, its absolute value on average does not exceed 2 b.p. Therefore, the consequences of the residual patterns are expected to be negligible.[7]

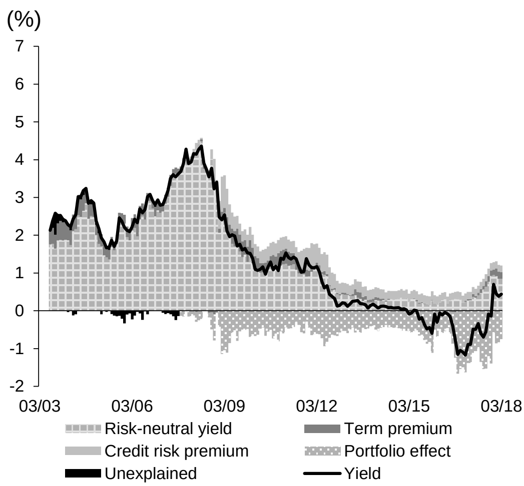
To obtain the credit risk premium, month–end CDS quotations for maturities of 1–5, 10, 20 and 30 years were used in the estimation. We included CDS quotations with 20–year and 30–year maturities (which were not included in the yield sample) in the estimation because of the absence of quotations for 15–year CDS. After smoothing them by a three–month moving average, the Nelson–Siegel function was used to obtain the credit risk premium values for all required maturities. Due to the limited liquidity on the Czech GBs CDS market[8], CDS quotations for Czech public debt of shorter maturities are close to those of longer maturities. However, this was not reflected in the yields on Czech GBs of short maturities. The Nelson–Siegel function was therefore specified so that the credit risk premium converged to zero with decreasing maturity, which is in line with the economic intuition.

[7] Our approach to the affine model estimation and decomposition is in line with the best practices (see Krippner, 2015, for discussion). Therefore, we are not much concerned with the properties of the measurement error. Instead, we consider the plausible properties of the components and reasonable implied shadow yields as a sufficient check of the model quality.

[8] Data from trade repositories available to the CNB indicate the Czech sovereign CDS turnover is rather limited. The most frequent maturity is 5y (about 35% of total turnover by notional principal), followed by maturities of 1–4 years. Maturities longer than 5 years are rare (jointly about 15% of turnover) and maturities longer than 10 years virtually nonexistent. The turnover substantially declined when the market tempered after the debt crisis in Europe subsided. Nevertheless, the low turnover does not necessarily imply illiquity as long as the market makers are willing and ready to transact at the quotes they contribute.

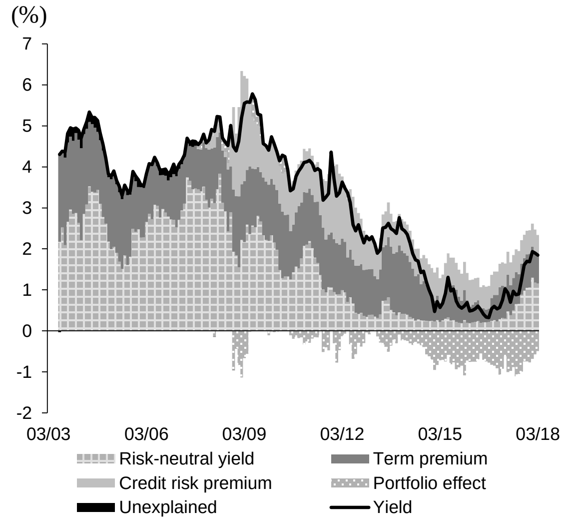
The zero–coupon Czech GB yield curve was decomposed into four introduced components for one–year and ten–year maturities (see Figures 6 and 7). In the case of the one–year bond, yield was made up predominantly of risk–neutral yield until the global financial crisis broke out in 2008 (see Figure 6). From the end of 2008 onwards, the one–year bond yield declined due to falling risk–neutral yield. The decline in this component was linked with market expectations that short–term rates would stay very low. In addition, starting in the second half of 2008, key central banks gradually released large amounts of liquidity as part of their monetary and lender–of–last–resort policies. For reasons of flight to quality and search for yield, Czech GBs represented an attractive opportunity for foreign investors. Owing to the negligible risk of sovereign default over such a short time scale, the credit risk premium was relatively low in the period under review. The negative portfolio component was linked with investors' preference for holding shorter–maturity bonds at a time of market stress. In 2015, the portfolio component exceeded all the other components combined for the first time and the one–year bond yield thus turned negative. Since then, the yield on short–maturity Czech GBs has reflected strong interest among foreign investors speculating on appreciation of the Czech koruna against the euro upon the exit from the CNB's exchange rate commitment combined with a relatively limited supply of the bonds of certain maturities (see CNB, 2017, section 2.1). Since 2017, monetary policy normalization has led to an increase of the yields through both higher risk-neutral yield and in absolute terms decreasing portfolio effect.

Figure 6 Decomposition of the One–Year Zero–Coupon Bond Yield



Notes: Reliable data on CDS quotations are not available until 2008. As a result, the difference between the bond yield and the swap rate could not be decomposed and is reported as unexplained.
Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations.

Figure 7 Decomposition of the Ten–Year Zero–Coupon Bond Yield



Notes: Reliable data on CDS quotations are not available until 2008. As a result, the difference between the bond yield and the swap rate could not be decomposed and is reported as unexplained.
Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations

The significance of the different components on the level of the ten–year Czech GB yield changed substantially over the 13 years under review (see Figure 7). Until the global financial crisis broke out, ten–year bond yields were almost equal to swap rates of the same maturity. The risk–neutral yield and the term premium each made up around half of the yield. When the US investment bank Lehman Brothers collapsed in mid–September 2008, the global financial market situation worsened sharply. Uncertainty and risk aversion increased, giving rise to higher market price volatility. Owing to the high level of global market integration, the market stress passed to the Czech GB market, as evidenced by growth in the credit risk premium. In mid–October 2008, market liquidity on the Czech GB market dropped sharply as a result of excess supply of Czech GBs from foreign institutional investors. The CNB responded by introducing extraordinary liquidity–providing repo operations in which Czech GBs were accepted as eligible collateral for the first time. This fostered a slight reduction in the credit risk premium. For the same reasons as for the one–year bond, the risk–neutral yield and the term premium began to fall in mid–2008. The term premium increased in late May/early June 2013 in response to a change in market expectations about the timing of the tapering of bond purchases by the US Federal Reserve in the QE3 programme. This change in expectations triggered an unusually sharp price adjustment in a whole range of asset categories across global markets, accompanied by market turbulence. In November 2013, however, the ECB reduced its base rate and in June 2014 it announced the use of other unconventional instruments, including a plan to purchase euro area GBs. In November 2013, the CNB started to use the koruna exchange rate as an additional monetary policy instrument. This combination of measures and a relative scarcity of supply led not only to a fall in the term premium of the ten–year bond, but also to a negative portfolio component. From 2011 onwards, the credit premium and the portfolio component were also affected by the debate about, and subsequent phasing in of, new financial market regulatory measures (Basel III, CRD IV/CRR). A signal of preferential treatment of GBs in the capital and liquidity requirements was sent out to the market.

In order to confirm the theoretical interpretation of the estimated components, their profiles were compared with those of selected macroeconomic and financial variables with which they should theoretically be closely linked. Besides that, we ran correlation analysis between the components and the macroeconomic and financial variables. The correlation analysis was run on monthly changes of components and variables (reported in Table 1) and also on their levels (Kučera et al., 2017).

The risk–neutral yield should correspond to market expectations about future short–term rates. A comparison with analysts' expectations about the CNB's two–week repo rate one year ahead confirmed this theoretical assumption (Kučera et al., 2017). The dynamics of risk–neutral yields turned out to be more closely correlated with the dynamics of expected monetary policy rates than with that of actual policy rates (see Table 1). When considering levels correlation, the correlation between the risk–neutral yields and expected policy rates was the highest among all investigated variables (0.96).

The term premium should theoretically be closely correlated with the level of difficulty in forecasting future short–term rates at a given maturity horizon. A relatively strong correlation between the term premium and the present and expected interest rate levels lent some support to the theoretical assumption. Generally speaking, when interest rates are low, their volatility is also low. This enables investors to make better forecasts and demand a lower term premium. Although the presented long–term correlation patterns of the term premium and the risk–neutral yield are similar, it should be noted that their short–term movements differ significantly. For instance, in case of the 10–year GB, the volatility of the term premium was lower than the volatility of the risk–neutral yields. Additionally, whereas the largest decrease of the risk–neutral yield was observed over the years 2011–2012, the term premium decreased only slightly in this period. Instead, the largest drop of the term premium appeared in between end of 2013 and half of 2015, when the uncertainty about the future monetary policy rate movements was suppressed by the introduction of an unconventional monetary policy tool.

The credit risk premium should be correlated with investor perceptions about Czech GB credit risk. Given the method for estimating the credit risk premium, the correlation between it and CDS spreads was very high (even with CDS spreads of other maturities; see Table 1). Another market indicator of credit risk – the spread between Czech and German five–year GB yields – was also highly correlated with the credit risk premium. The credit risk premium turned out to be closely correlated with GDP growth and exchange rate and market uncertainty indicators (see Table 1). Worsening economic performance and a weakening Czech koruna could signal potential macroeconomic instability and require foreign investors to monitor more closely the fiscal position of the government. Global uncertainty as measured by the VIX index was also significantly correlated with the credit risk premium. When uncertainty rises, investors become more cautious and require higher compensation for bearing credit risk; sometimes this surge in credit premium is not fully justified by the development of Czech fundamentals.

The portfolio effect should theoretically be linked with investors' preference for Czech GBs over other assets – denominated in korunas or other currencies. The inflow of short–term foreign assets into the Czech economy and the proportion of Czech GBs held by non–residents were strongly correlated with the portfolio effect when we correlated their levels. A strong level correlation was also found between the portfolio effect and the currency-hedged profit on investing in Czech assets. Rising yields on this type of investment were associated with a lower portfolio effect. These correlations can be interpreted as meaning that an inflow of foreign portfolio investment motivated by hedged profits boosted demand for Czech GBs as an attractive instrument, causing their yields to turn negative. Czech GB trading volume itself was not significantly correlated with the portfolio effect.

4. Czech GB Yield Curve Response Analysis

In this section, we estimate a Bayesian vector autoregression model with an exogenous variable (BVARX model) of the Czech economy to obtain some basic responses of Czech GB yield curve to macroeconomic shocks. In particular, we focus on shocks to expectations, which allows to evaluate the extent to which the yield curve can be considered as a forward-looking indicator of the business cycle. Using the presented decomposition methodology, we split the yield curve responses into the responses of individual components. This allows us to interpret the yield curve movements in a greater detail and obtain insight into monetary policy transmission. The aim of the presented analysis is to illustrate the usefulness of the decomposition in relation to the interpretation of macroeconomic dynamics. The development of more sophisticated macroeconomic VAR models is left for future research. The model can be written in the following form:

$$V_t = A_0 + A_1 V_{t-1} + A_2 W_t + \epsilon_t \tag{10}$$

V_t is a vector of endogenous variables, W_t is a vector of exogenous variables, A_0, A_1 and A_2 are parameter matrices and ϵ_t is a i.i.d. vector of random disturbances. In the presented model, we use seven endogenous variables and a single exogenous variable, i.e. the matrices A_0, A_1 and A_2 have dimensions 7×1, 7×7 and 7×1, respectively.

The seven endogenous variables include two macroeconomic variables, three IRS latent factors and the level and slope of the asset swap spread (ASWS). ASWS represents the difference between the Czech GB yield and the IRS rates, which means that it equals the sum of the credit risk premium and the portfolio effect. The reason why we use the ASWS instead of its two components is to keep the model parsimonious. Additionally, the separation of these two components was not possible until the end of 2008 due to the unavailability of reasonable CDS quotations. Modeling them jointly therefore allows use of a longer sample period. Nevertheless, we also evaluate the results from the shorter sample period with these two components kept separate to obtain additional insight into the dynamics of ASWS responses.

The two macroeconomic variables in the model are forward-looking Consensus Forecasts of growth of GDP and Consumer prices (Consensus Economics, 2018). Since the Czech yield curve comprises maturities up to 15 years, we prefer using long-term forecasts (forecasts of average annual growth rates 5–10 years ahead). However, the long-term forecasts are published only with a 6-month periodicity. Therefore, to introduce sufficient monthly variability of the forecasts, we obtain the series as a weighted average of the interpolated long-term forecasts and 1Y forecasts. In a baseline estimation, the weights were arbitrarily set to 80% for long-term forecasts and 20% for 1Y forecasts, which puts most emphasis on the long-term trends. However, below, we also show results for alternative weights.

Unlike a canonical monetary VAR (Bernanke and Blind, 1992, for instance), we do not use the monetary policy rate, since the monetary policy rate hit its lower bound during the sample period. As a result, the monetary authority used the exchange rate intervention as an alternative tool, which can be considered a structural change. Including monetary policy or money market rates would consequently lead to biased results. Instead, we evaluate the effect of the shadow rate, which we calculate from the three IRS latent factors X_t extracted from the estimated affine model (SR–SGA model specification, see section 2). This means that by including the three IRS latent factors into the BVARX model, we allow both an unbiased monetary policy proxy (the shadow short rate) and the interest rates to enter the model. The shadow–rate specification of the affine model avoids the bias caused by the lower bound. The shadow short rate can be easily obtained in the SR–SGA model as a sum of the latent factors X_t (plus the α_0 parameter from equation (3)). This shadow short rate can be seen as a proxy for overall monetary policy conduct, reflecting both interest rate tools and any unconventional tools (Krippner, 2015). Further, the estimated affine model also allows us to translate responses of the latent factors X_t to responses of the rates via equation (6) and to decompose these responses to the two components: the risk–neutral yield and the term premium.

To be able to infer the responses of the Czech GB yield curve to macroeconomic shocks, we need to introduce another two components: the credit–risk premium and the portfolio effect. These are jointly expressed by the ASWS. Using the first two elements of the Nelson–Siegel function fitted over ASWS, we represent the ASWS term structure by two variables: its level and its slope. These enter the BVARX model as the sixth and the seventh variables. As noted before, to obtain additional insight, a model with shorter sample period and separated credit risk premium and portfolio effect is also estimated and discussed below. The results of this alternative estimation might however be biased by the shortened period covering only the low-yield environment.

Apart from endogenous variables, we also consider the effect of general market uncertainty on the Czech GB yield curve. We assume these effects come from global financial markets; hence we use the VIX index as a proxy. Since the Czech economic situation has a negligible effect on the global markets, we do not assume any feedback loops, which supports the exogeneity of this variable.

To identify the model, we utilize recursive identification with Choleski decomposition. In this case, ordering of the endogenous variables is crucial. We order them as they are described above. Ordering macroeconomic variables first is reasonable because of the lag in their publication and an overall sluggish response of the real economy to financial shocks. The IRS factors are ordered afterwards, since we assume that they react immediately to shocks to the macroeconomic variables but not to shocks to ASWS. Finally, the ASWS factors are ordered at the end.

We use three lags for the BVARX model. The Akaike information criterion for the VARX model hints that at least two lags should be used (see Figure A4 in the Appendix). Although the criteria reach their minimum for two lags, we use three lags which is more common in the macroeconomic literature (the difference between the Akaike information criterion for two and three lags is small). To support robustness, we provide comparison of results for various numbers of lags in the Appendix.

For estimation, we use the sample period identical to the rest of the paper, i.e. 177 months from July 2003 until March 2018. We use flat priors and obtain the final distributions of parameters and impulse-response functions using Gibbs sampling.[9] The use of flat priors has its negative effects, especially for forecasting performance, which we do not evaluate (Giannone et al., 2012). However, the aim of the presented BVARX analysis is to highlight and further support the interpretation of the obtained components. A complex analysis of the optimal BVARX specification, prior selection and evaluation of the forecasting performance is beyond the scope of this paper and we leave it for future research.

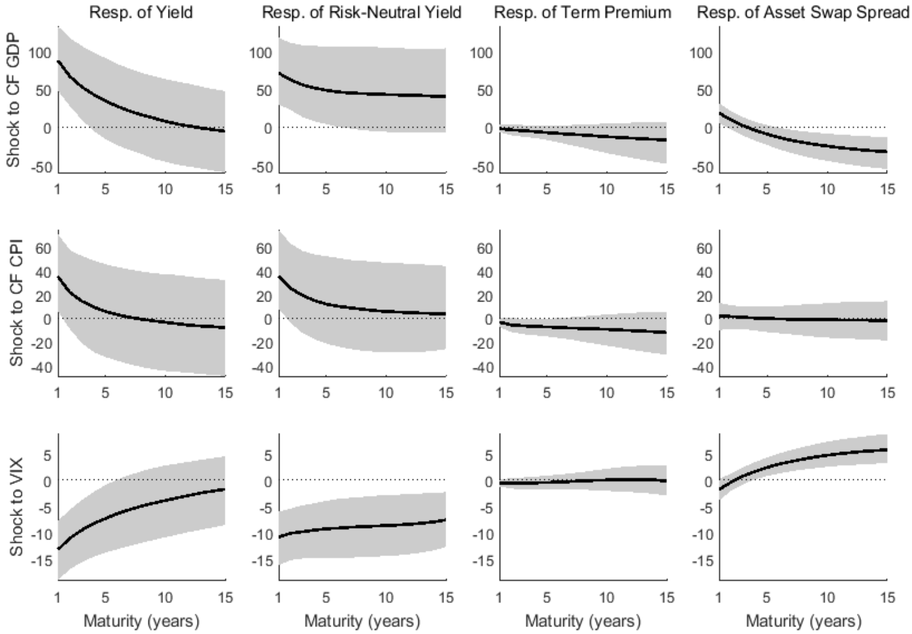
To discuss the results of the model, we mainly focus on the impulse–response functions. In our calculation we proceed in three steps. First, from the model parameters from each Gibbs iteration, we calculate responses of the yield factors (i.e. three IRS factors and the level and the slope of ASWS) to shocks in the macro–financial variables (Consensus Forecasts of growth of GDP and CPI and the exogenous VIX index) over a 4–year horizon. Afterwards, we translate the responses of the yield factors to responses of the yield components. That means that (i) from IRS factors' responses, the responses of the risk–neutral yield and the term premium are calculated; and (ii) from the responses of ASWS level and slope, ASWS responses are calculated. ASWS responses aggregate the response of the other two components – the credit risk premium and the portfolio effect. Finally, we sum up the response of the risk–neutral yield, the term premium and ASWS to obtain the response of the Czech GB yield curve in the particular iteration.

Figure 8 shows the responses of the whole yield curve (in basis points) after 12 months from the initial shocks. The left column of the figure shows the response of the whole yield curve, which equals to the sum of responses of the components in the other columns. Each row presents a response to a shock in the variable denoted at the left of the figure. Figure 9 shows the dynamics of the response for the 5–year Czech GB yield over the 4–year horizon. In the figures, the solid line shows the median response, whereas the grey area displays 80% credible intervals of the responses.

We put the shocks to forecasted growth in GDP and CPI equal to the average absolute annual change of the series, which means 0.32 pp in case of GDP and 0.22 pp in case of CPI. In the case of VIX, it enters the model in a logarithmic form; we set the size of the shock equivalent to a growth of the VIX index from 12 points (the historical level related to low risk) to 30 points (the average value achieved in periods with an increased uncertainty).

[9] In the Gibbs recursion, we use 5,000 iterations, of which the first 2,000 iterations are discarded.

Figure 8 Response of the Whole Yield Curve After 12 Months (in bps)



Notes: The figure displays the response of the whole Czech GB yield curve (1–15 years) to the shocks after 12 months. The shocks are defined as a 0.32pp increase of forecasted GDP growth, 0.22pp increase of forecasted inflation and as an increase of VIX index from 12 to 30 points. The line displays the estimated response; the grey area displays 80% credible intervals.

As Figure 8 shows, the response of the Czech GB yield curve has various shapes, including both parallel shift and rotation. In response to a positive shock to the expected GDP growth rate, the yields increase in line with the growth of the risk-neutral yields. Simultaneously, the positive economic growth prospects slightly decrease the premium of uncertainty (reflected by the term premium) and lead to a decrease of slope of the Asset Swap Spread. This can be explained by a decline in credit risk and portfolio reallocation towards longer maturities in case of positive economic news. As a result, the increase of yields of long maturities caused by the risk-neutral expectations is compensated by the effect of the other components. The yield curve therefore flattens.

A positive shock to expected inflation similarly leads to an increase in risk-neutral yields, although of a lesser magnitude. Unlike a shock to GDP, in this case, the shock causes yield curve upward shift together with rotation. Such a response is triggered by the risk–neutral yield, i.e. the expectation of a reaction from the monetary authority, which translates into the adjusted expected future short rate path. The response reaches its maximum around 60bps, 6 months after the shock for short maturities, whereas for the longer maturities, the response is smaller but quite persistent (see Figure 9). The term premium slightly rises for long maturities, which may reflect uncertainty about the new short rate path.

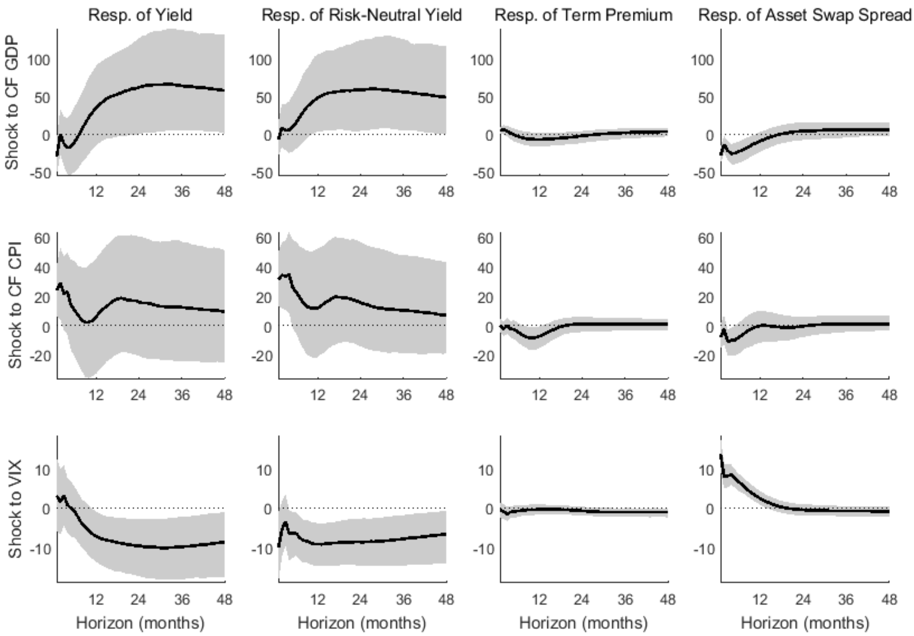
In the case of both economic shocks, the increase in the long yields due to a change in the risk-neutral yield is compensated by other effects. The results here presented need to be cautiously interpreted in the context of the sample period: most of the sample covers the period following the global financial crisis, which was specific in terms of the non-standard monetary policy conduct and persistent low-yield environment. We interpret the results as showing that positive expectation shocks regarding the real economy, and price inflation, lead to an increase of yields, although compensated by specific factors for long maturities. Such an interpretation is in line with the latest empirical evidence from the years 2016–2017: the expected monetary policy tightening, which was triggered by the positive inflation data since the end of 2016, was not fully reflected by the long end of the yield curve. The whole yield curve was at that time pushed down by an increase in the demand for Czech GBs following foreign speculation on Czech koruna appreciation. A similar compensation of responses was also present after the negative GDP shock during the crisis. The shock led to a drop in short yields, but the long yields were kept high due to increased risk pricing and capital outflows.

The persistence of the shocks is high (see Figure 9). This could imply a predictability of yields in the Czech GB market and limited efficiency of the market and therefore cast doubt regarding the validity of the affine model. The persistence however needs to be considered in terms of uncertainty: the response of monetary policy to economic shocks is uncertain in terms of both timing and extent. Therefore, the gradual response of yields reflects the gradual monetary policy response. Furthermore, the sample period includes the global financial crisis which represents a highly persistent real shock – yields of long maturities decreased for several periods after the initial shock (see Figure 2). The new low-yield environment following the crisis could be seen as a structural shock, which the model does not incorporate and therefore interprets the transition towards new normal as a sluggish response. Future possible extensions of the presented framework could include regime shifting, which would partially correct this issue.

Finally, the exogenous shock to VIX, representing a shock towards increased market uncertainty, rotates the yield curve by pushing the low end of the yield curve downwards. In absolute terms, the response is lower than in case of the shocks to the real economy. The response is a result of two opposing effects. The shock has a negative impact on the expected monetary policy rate path, i.e. the risk–neutral yields of all maturities decrease (Figure 8). On the other hand, it increases the risk premium of Czech GBs and reduces their attractiveness to investors, which pushes the ASWS up for long maturities. As a result, the yield curve rotates. However, the response changes over time: for the 5–year yield, the effect of the ASWS prevails for the first 6 months; afterwards, the effect of decreased monetary policy rate expectations becomes dominant (see Figure 9).

The outflow of investors (presumably foreign), which causes the ASWS to increase after increased market uncertainty, signals that their perception of the Czech GB bonds as a safe asset is limited. The role of foreign investors in determining the ASWS is dominant due to the high turnover these investors have in the Czech GB bond market, relative to domestic investors (CNB, 2014). Also, in this case the results reflect the beginning of the Global financial crisis, when global financial market uncertainty and its effect on the real economy was one of the reasons for the global yields decreasing towards record-low levels. Similarly, during the subsequent gradual global process of return towards non-zero yields, events triggering market uncertainty (the August 2015 Asian market crash or the Brexit vote, for example) further postponed the expected monetary policy tightening.

Figure 9 Response of 5Y Yield, Response Horizon up to 4 Years (in bps)

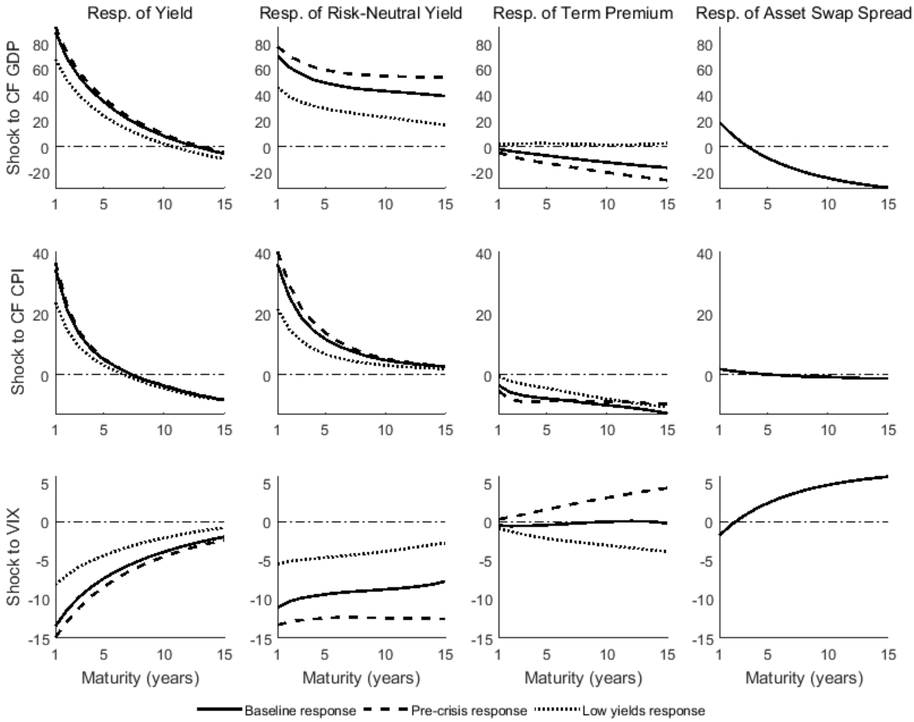


Notes: The figure displays the dynamics of the response of the 5 year Czech GB yield to shocks over a 48 month response horizon. See note to Figure 8 for the description of the plot elements.

The results demonstrate that the response of the yield curve is truly complex and that movements of the components may mutually offset. The drop of the term and credit premia and the response of the portfolio component after a positive economic shock may have important implications for both the monetary and financial stability policies of the CNB. Similarly, the combination of macroeconomic and financial responses to the market uncertainty shock (VIX) needs to be accounted for when considering possible policy measures.

To measure the effect of the lower bound on responses of the yield curve, the steady state of the VAR analysis is shifted towards the lower bound. In the alternative setup, we set the steady state of yields equal to average factor values before/after the recent global financial crisis. Due to the non–linearity of the $F(X_t, \tau)$ function in case of the shadow–rate model, the steady state level matters for the yield responses within the VAR analysis (except for the ASWS, which does not enter the affine model). After doing so, the responses to the impulses are altered (see Figure 10).

Figure 10 Responses at The Lower Bound (in bps)

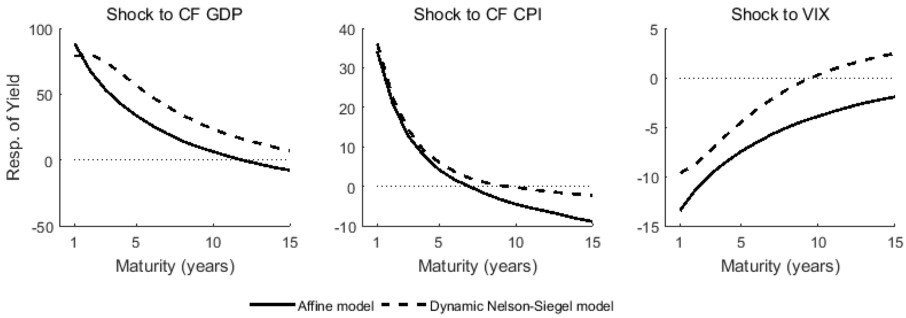


Notes: The figure displays the dynamics of the response of the yield curve under various steady states at the 12–month horizon. See note to Figure 8 for the description of the plot elements.

In general, the lower bound suppresses the responses. This is caused by the lesser reaction of the monetary authority at the lower bound in terms of interest rate adjustment. Technically, following the concept of the shadow rates, the shadow rate response translates to only limited observed rate response where rates are close to their lower bound (see Figures 4 and 5). Economically, this can be explained by the presence of unconventional monetary policy tools. In the case of the monetary policy response, it can be expected that unconventional policies will be altered first, creating a buffer against the monetary policy rate change. In practice, such a situation was present in the Czech monetary policy during the exchange rate commitment regime: the commitment needed to be abandoned before changing the policy rates. One interesting finding was that the pre-crisis and low-yields response of the term premium are exactly opposite after a shock to VIX. This is however not surprising: before the crisis, a shift in financial market uncertainty raised questions about the possible monetary policy response, which was in line with growth in the term premia. In contrast, since the crisis, VIX shock meant extending the low yield regime, i.e. decreasing the uncertainty about monetary policy steps in the near term.

Finally, we also present several sensitivity analyses which provide additional insight into the presented results. First, we re-estimate the BVARX model using the DNS approach of Diebold and Li (2006). That means that we estimate the DNS model and gather the factors – the level, the slope and the curvature. Afterwards, we estimate the BVARX model using these three factors. From the estimated responses of factors, the yields are calculated using the Nelson and Siegel (1987) function. As the results show (see Figure 11), the responses in both DNS and affine models are roughly similar. This supports the robustness of our results. However, the DNS framework does not allow us to decompose the yields. Therefore, sticking to DNS framework would lead to a conclusion that the response of yields of long maturities are insignificant without the knowledge of mutually offsetting components, which was presented above.

Figure 11 Response in Comparison with Dynamic Nelson-Siegel Framework Results (in bps)



Notes: The figure displays the response of the whole Czech GB yield curve (1–15 years) to shocks after 12 months. See note to Figure 8 for the description of the plot elements.

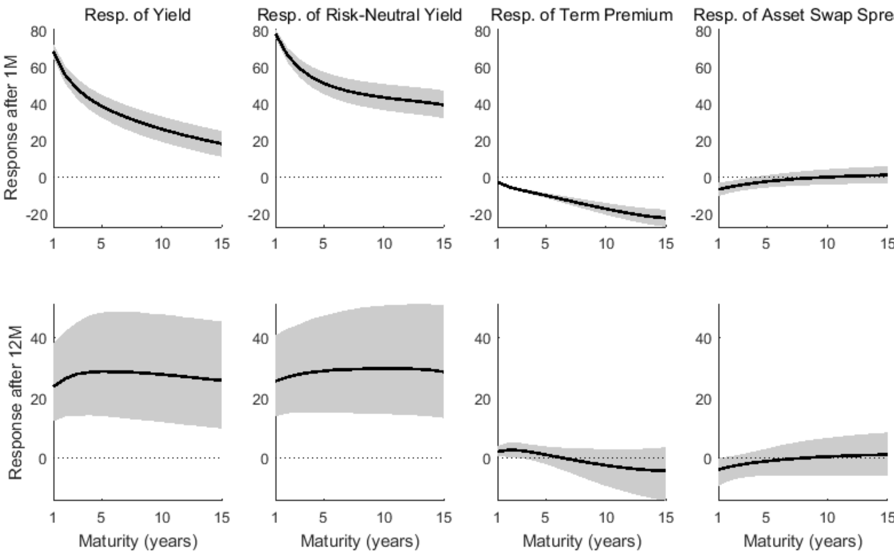
Second, we also show the effect of monetary policy. Since we use a shadow-rate model, we utilize the shadow rate as a monetary policy proxy. It can be obtained as a sum of the three affine yield factors. The response of yields after a 1pp increase in the shadow rate is first almost proportional for the shortest maturities and gradually decreases over time (see Figure 12). For the longer maturities, the response of yields is only partial but still significant. The less than proportional response may be explained by a mix of (i) the opposite effect of decreasing term premium, (ii) the one-off definition of the shock (vs. construction of the yields of long maturities as average value over the full horizon) and (iii) the option effect embedded in the shadow rate, which matters close to the lower bound.

Finally, we provide additional sensitivity checks. We evaluate various shares of long-term forecasts when constructing the CF GDP and CF CPI series (see description above). The results show (see Figure A5) that the shape of the responses is preserved. However, by increasing the share of long-term forecasts, the response increases in its magnitude.[10] This reflects the empiric ability of the yield curve to reflect changes in expectations rather than realized shocks. In an additional sensitivity check, we show that the results are stable across a varying number of lags in the BVARX model (see Figure A6). The last sensitivity check shows that the

[10] The different response of the series consisting of 100% long-term forecasts is given by the interpolation. The absence of any short-term forecasts in the calculation results in biased behaviour – therefore, we utilize an 80% share of long-term forecasts in our baseline estimation as an optimal combination of high share of long-term forecasts while still allowing for short-term variability.

results for the macroeconomic variables are preserved also when the ASWS factors are excluded from the model. This means that the BVARX is estimated for the CZK IRS rates rather than the Czech GB yields. The results show (see Figure A7) that the basic direction of response to macroeconomic shocks is preserved, although the extent of the response differs for some maturities. Exclusion of ASWS might lead to an omitted variable bias; we therefore argue for keeping it in the baseline estimation.

Figure 12 Responses to the Monetary Policy Shock (in bps)



Notes: The figure displays the response of the whole Czech GB yield curve (1–15 years) to the 1pp increase in the shadow rate after 12 months. The line displays the estimated response; the grey area displays 80% credible intervals.

5. Conclusion

The yield curve is an important indicator of the economic cycle, as it aggregates the expectations of market participants. The factors that affect the shape of the yield curve do so to different extents in different circumstances. To interpret the evolution of the yield curve correctly, it is therefore useful to decompose it. This article presented a method to decompose the Czech government bond yield curve.

We decomposed the Czech GB yield curve into four components: a risk–neutral yield, a term premium, a credit risk premium and a portfolio effect. The first two were obtained by decomposing the zero–coupon koruna swap curve using an shadow-rate stationary Gaussian affine model. The credit risk premium was estimated from credit default swap quotations for Czech GBs. The portfolio effect formed the residual. Inclusion of a credit risk premium and a portfolio effect improve the understanding of the dynamics of interest rate. These two additional components are necessary for government bonds in which investors see a non-zero default risk.

A comparison of the four estimated components with selected macroeconomic and financial variables confirmed the strong theoretical interpretation of these components. As the theory had anticipated, for example, the risk–neutral yield matched analysts' expectations about future short–term policy rates, and the portfolio effect became highly negative as the removal of the CNB's exchange rate floor neared.

The above decomposition allowed for a more detailed interpretation of the responses of the Czech GB yield curve to macroeconomic and financial shocks. Using a vector autoregression analysis, we show that the yield curve responds both by changing its level and by rotation. Such responses result from a combination of various responses of the yield components to the shocks. Most importantly, the upward response of yields following positive shocks to expectations about GDP and CPI is partially compensated by shifts in portfolio allocation and risk pricing. Such a finding is in line with international experience (Greenspan's conundrum in the U.S. in 2005–7) and has important implications for both monetary policy conduct and the usage of the yield curve as an indicator of the business cycle. Also, a rise in global risk awareness proxied by VIX leads to a rise in yields via the portfolio effect, which suggests that Czech government bonds do not possess a status of a safe haven asset. Finally, at the lower bound, we show that the yield response is generally suppressed, which is in line with economic reality.

Table 1 Correlation of Monthly Changes Between the Components of the 10–Year Zero–Coupon Bond and Economic and Financial Variables

Type of variable	*Name of variable*	*Risk-neutral yield*	*Term premium*	*Credit risk premium*	*Portfolio effect*
Macroeconomic	Inflation (CPI)	0.08	0.05	-0.21 (**)	0.15
	GDP growth	-0.04	0.06	-0.20 (**)	0.01
	CZK/EUR exchange rate	-0.07	0.05	0.21 (**)	-0.13
Short interest rates and market expectations	CZEONIA index	0.01	0.10	-0.06	-0.05
	CNB 2W repo rate (current)	0.18 (**)	-0.08	-0.28 (***)	0.22 (**)
	3M PRIBOR	0.40 (***)	-0.31 (***)	-0.13	0.14
	3M OIS in CZK	0.24 (***)	-0.22 (***)	-0.20 (**)	0.20 (**)
	CNB 2W repo rate (1-year expectations)	0.37 (***)	-0.17 (**)	-0.56 (***)	0.29 (***)
	Inflation (1-year expectations)	0.05	-0.04	-0.14	-0.02
Fluctuations in short interest rates and market uncertainty	Variability# of inflation	-0.20 (**)	0.13 (*)	0.17 (*)	-0.06
	Variability# of 1-year inflation expectations	0.06	-0.05	-0.07	0.19 (**)
	Variability# of 1-year expectations about CNB 2-week repo rate	-0.16 (**)	0.13	0.15	-0.14
	VIX volatility index	-0.07	-0.01	0.24 (***)	0.01
Credit risk of Czech state and Czech interbank market	Czech GBs issued/GDP	-0.23 (**)	-0.08	0.02	0.17
	5-year CDS spread for Czech GB	0.06	-0.16 (*)	0.54 (***)	-0.11
	Spread between 3-month PRIBOR and 3-month OIS	0.09	-0.02	0.15	-0.13
	Spread between Czech and German 5-year GB yields	0.21 (***)	-0.22 (***)	0.26 (***)	0.28 (***)
Investment flows	Czech GB trading volume	0.08	-0.04	-0.06	0.13
	Proportion of foreign holders of Czech GBs	0.09	0.06	0.05	0.08
	Profit on hedged investment in Czech GBs##	-0.14 (*)	0.21 (***)	-0.21 (**)	0.15
	Net portfolio and other investment in balance of payments	0.02	-0.15 (*)	0.07	-0.06

Notes: The asterisks refer to statistical significance at 1% (***), 5% (**) and 10% (*) levels. The explanatory power of the correlations may be limited by the short length for some of the time series. #Variability is measured by the standard deviation of the last four monthly observations. ##The average profit on (1) an investment consisting in converting euros into korunas, depositing them at the CNB deposit rate and then converting them back into euros at the 3–month forward rate, and (2) an investment consisting in converting euros into korunas, buying a 2–year Czech GB and then converting it back into euros at the 2–year forward rate. The return that could have been achieved by depositing the funds for three months at the ECB deposit rate was deducted from the first investment and the 2–year German GB yield was deducted from the second.

Source: Bloomberg, CNB, MTS Czech Republic, Prague Stock Exchange, Thomson Reuters, authors' calculations.

APPENDIX

Estimated Values Of Parameters

$$\tilde{\kappa} = \begin{bmatrix} -0.01 & 0 & 0 \\ 0 & 0.13 & 0 \\ 0 & 0 & 1.03 \end{bmatrix} \quad \tilde{\theta} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \alpha_0 = 0.02 \quad \alpha_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\kappa = \begin{bmatrix} 0.002 & -11.62 & -0.15 \\ -0.001 & 10.94 & -0.002 \\ -0.051 & 6.93 & 65.43 \end{bmatrix} /10^2 \quad \theta = \begin{bmatrix} 0.012 \\ -0.000 \\ -0.320 \end{bmatrix} /10^2$$

$$\begin{aligned} std(e_t) = diag(&2.92, 2.98, 3.11, 3.18, 3.21, \\ &3.24, 3.26, 3.28, 3.29, 3.30, \\ &3.30, 3.30, 3.29, 3.28, 3.28)/10^4 \end{aligned}$$

$$\sigma = \begin{bmatrix} 79 & 0 & 0 \\ -101 & 93 & 0 \\ 0.003 & 0.005 & 136 \end{bmatrix} /10^4$$

Lower bound value = 0.0019

Notes: Small values of parameters and the lower bound reflect the fact that the model uses yields in decimal representation.

Figure A1 Affine Model Fit and Shadow Rate (One–Year Swap Rate) (%)

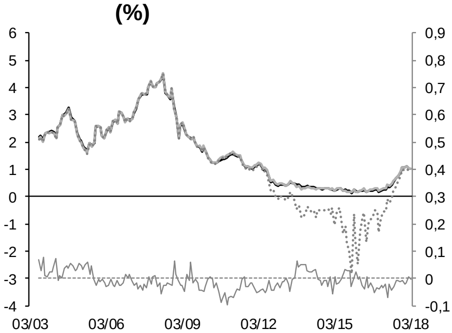
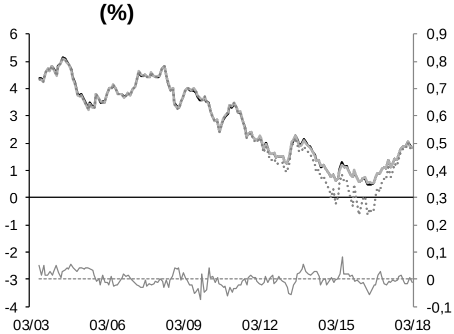
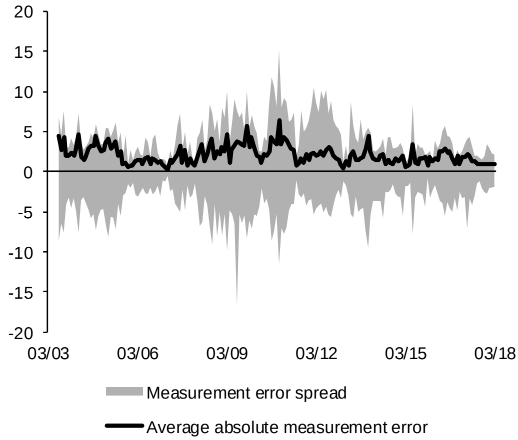


Figure A2 Affine Model Fit Rate and Shadow (Ten–Year Swap Rate) (%)



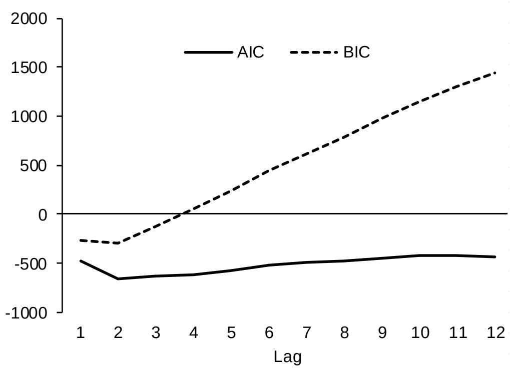
Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations.

Figure A3 Measurement Error Series Diagnostics (b.p.)



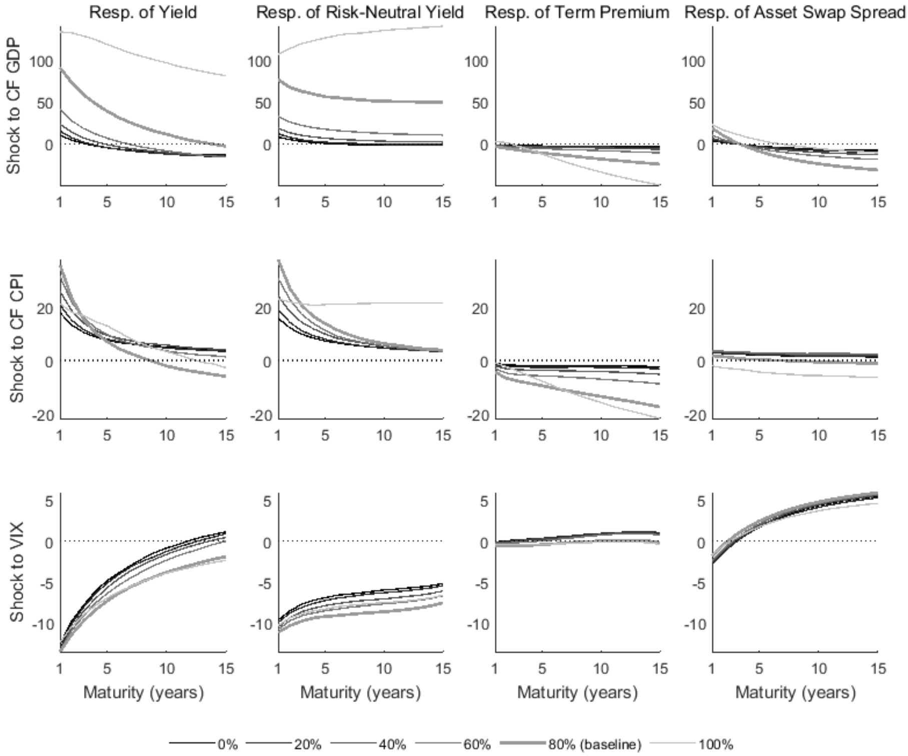
Source: Authors' calculations.

Figure A4 Information Criteria (Information criteria values, x-axis: lag)



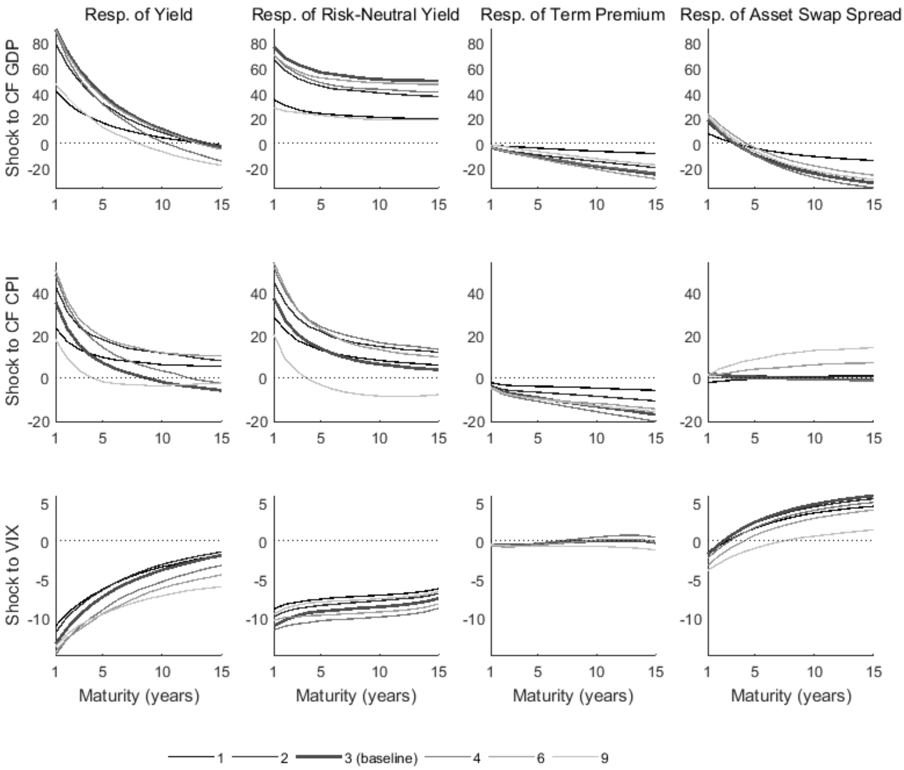
Source: Authors' calculations.

Figure A5 Response for Various Shares of Long-Term Forecasts (in bps)



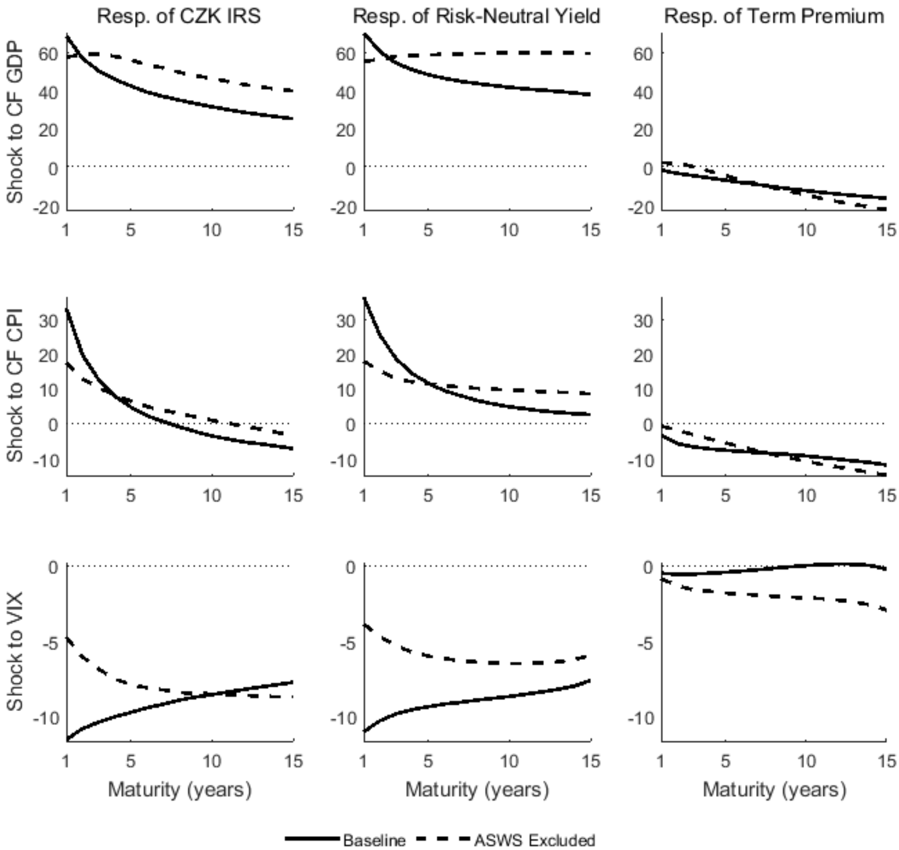
Notes: The figure displays the response of the whole yield curve (1–15 years) to shocks after 12 months. See note to Figure 8 for a description of the plot elements. The shares 0–100% refer to the share of the long-term forecasts in the calculation of the CF GDP and CF CPI series.

Figure A6 Response for Various Lags (in bps)



Notes: The figure displays the response of the whole yield curve (1–15 years) to shocks after 12 months. See note to Figure 8 for the description of the plot elements.

Figure A7 Response of CZK IRS After Excluding ASWS (in bps)



Notes: The figure displays the response of the whole CZK IRS curve (1–15 years) to shocks after 12 months. See note to Figure 8 for the description of the plot elements

Table A1 Measurement Error

Maturity	***1***	***5***	***10***	***15***
Normality (Shapiro-Wilk test)	YES	NO	YES	NO
Autocovariance (Ljung-Box)	YES	YES	YES	YES
Homoscedasticity (Ljung-Box on squares)	NO	NO	NO	YES
Cross- correlations (%):				
1	100	-10	68	-71
5	-10	100	-38	22
10	68	-38	100	-86
15	-71	22	-86	100

Notes: The tests were evaluated for 5% significance level.
Source: Authors' calculations.

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