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USING MARKOV CHAINS IN THE SOCIAL SECURITY SYSTEM

Abstract. Submitted article is primarily focused on Markov chains and their possible application in practice. They represent a calculation of the likely progress of a certain event. In practical terms these are insurance events such as job loss, childbirth, or illness. All these events have a common characteristic, they have an equal probability at any point in time. It is necessary for insurance companies to know what the probabilities of transition are. Suitable mathematical tools are therefore homogeneous Markov chains. This article is investigating the application of absorption chains and interprets results using a particular situation. We pay attention to the Markov processes, because software ability to work with continuous models, therefore we consider the changes of system state in arbitrary time. Microsoft Office Excel was used to provide the calculations.

Key Words: social security, multi-state models, Markov chains, transition probability, matrix

JEL Classification: J1, K3

1. Introduction

The Slovak Republic is a country respecting the fundamental rights and freedoms of citizens and accepting their integrity. For every citizen without distinction of age there is necessary right for retirement security, security in case of working disability, or loss of breadwinner, and the right to assistance in material deprivation. Under the special protection of the law there are family and marriage rights, which integral part is the right to care and protection during pregnancy, childcare, and last but not least, the right to education. Social Security represents a form of assistance provided by the state to its citizens if they fall into a difficult, or for them unmanageable life situation. The largest part of social security is the social insurance, a public insurance based on paying contributions and receiving social security benefits.

The role of the Social Security is to protect the majority of the population against the life risks, but also to ensure the redistribution of income in the national economy. State social support focuses primarily on adverse life events as the part of individual as well as family life of every citizen. Support of the families in poor social situation the country solve through providing allowances for children, housing, etc., from the state budget. One-time, as well as regular allowances are provided within the social assistance system. For example, the amount of childbirth allowance is $\notin 151.37$ in the Slovak Republic ($\notin 227.06$ in case of multiple birth at once, respectively) [17]. Other kinds of support are the parental allowance, family allowances, child allowances, allowances to the contributions, tax credits, and others (more in details in [8]).

2. Sickness insurance

Within the social security system the sickness insurance provides the insurance against earned income reduction or loss of, and to generate the income in case of temporary disability to work, or in case of pregnancy and maternity. From the sickness insurance system the sickness benefits are granted, including sickness, maternity, nursing, and balancing allowances. These allowances are designed to help bear the financial burden, which is today due to the living standards in most of the cases much higher. In such cases, it is highly appropriate to ensure living standard by the additional insurance in one of life insurance companies, which offer their customers adequate financial protection.

Each citizen can only get allowances from the insurance, to which he pay the contributions. In case of exemption from payment of certain contributions there is no possibility to get allowances from this type of insurance. Contributions to sickness insurance are paid by all people working on a work performance agreement, with an exception of pensioners, disability pensioners and students. They are also paid work performance agreement by all employees working on a working contract, and all compulsory insured sole traders (self-employed). They can obtain the sickness, maternity, and nursing allowance from the sickness insurance.

Sickness allowance is provided to the insured person in case he/she was recognized as temporarily disable to perform employed activities, or imposed under quarantine due to illness or injury [11]. Temporary inability to work must arise during sickness insurance, or in the withdrawal period (7 days after the end of sickness insurance). During the period of inability to work the insured person cannot have an income, considered as the assessment base. The amount of sickness allowance from 1st to 3rd day of inability to work represents 25% of the daily assessment base [16]. Since the 4th day the sickness allowance reaches 55% of the daily assessment base. Sickness allowance is paid for calendar days. First 10 days of inability to work is the sickness allowance to employees and those who work on work performance agreement paid by the employer, afterwards, since the 11th day it is paid by the Social Insurance Office. Sickness allowances to self-employed (sole traders) are paid by the Social Insurance Office since the 1st day. Entitlement

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to sickness allowance ends by termination of temporary inability to work, the longest duration of receiving it takes 52 weeks.

Entitlement to nursing allowance arises in case of need for personal and all day nursing a sick child, a sick husband, sick wife, or sick parent, which health status according to medical confirmation necessarily requires care of another person, or personal and all day nursing a healthy child under 10 years, if the child was imposed under quarantine. Amount of nursing allowance reaches 55% of the daily assessment base. This allowance is provided for calendar days, but for maximum 10 calendar days.

Maternity allowance is a sickness benefit paid by the Social Insurance Office due to pregnancy or taking care of a new born child. It is provided since the beginning of the 6th week before the expected birth date determined by a physician. The condition for obtaining maternity allowance is at least 270 days of sickness insurance in the last 2 years before the childbirth. Entitlement to maternity allowance must arise either during the sickness insurance, or within the withdrawal period, which in this case takes 8 months. The maternity allowance reaches 65% of the daily assessment base. Maternity allowance is calculated from a maximum of 1.5 times the average wage 2 years ago. Since the next year maternity allowance should increase up to 70% of the assessment base.

3. Homogeneous Markov Chains

Markov chains are the simplest type of Markov processes. They are discrete-state processes with discrete timing. Markov feature is expressed by the conditional probabilities between considered states at a certain time point as follows:

 $P\{s(n) = j / s'(n-1) = i, s'(n-2) = k, \dots, s'(0) = m\} = P\{s(n) = j / s'(n-1) = i\}, \quad (1)$

where s(n) means the system status at the time point n.

Markov feature system is defined by the vector of absolute probabilities

 $p(n) = (p_1(n), p_2(n), \dots, p_m(n))$ and by transition probability

matrix $P(n) = [p_{ii}(n)]$. The probabilities $p_i(n)$ express the probability, that the

system is at the moment *n* in condition *i* for i = 1, 2, ..., m, and the probabilities

 $p_{ii}(n)$ are conditioned probabilities of transition from state *i* to state *j*, which may

occur between the time periods n-1 and n. The time period between n-1 and n is called step (in practice, a day, a month or a year...). If $p_{ii}(n) = p_{ii}$ is true, i.e.

independence from the transition moment, we call it homogeneous Markov chain. Transition matrix does not change its shape during the process, and therefore P(n) = P (more in [2]).

4. Absorption Markov chains

Recurrent, transient, and absorbing states could be recognized within the homogeneous Markov chains. Recurrent states allow return to a starting position at

any time, i.e. after any number of steps. In transition states is this return not possible. If remaining at some state is the certain event with no return, it is called the absorption state [13].

Absorption chains contain transient and absorption states only. Transition probability matrix is then adjusted so the compact blocks are created. By transposing rows and columns we will get the matrix in the following form:

$$\mathbf{P} = \begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix}$$

In the total number of states n, and the number of transient state r the size of the unit matrix E is (n-s).(n-s). Matrix R containing transition probabilities between transient and absorption states has a dimension s(n-s), and matrix Q containing the transition probability between the transient states has a dimension (n-s).s.

Fundamental matrix $N = (E - Q)^{-1}$ is a feature describing the median of transitions number of transient states. Elements of the matrix indicate the average number of system transitions into transition states. Moreover, if we assume that the transitions will happen in the unit time intervals, we can investigate the average residence time in individual transition states.

Transition matrix between transient states is expressed by matrix $H = (N - E)(\hat{N})^{-1}$, where $(\hat{N})^{-1}$ is matrix inverse to the matrix N, containing only the diagonal elements. Matrix H elements describe the probabilities of residence in the system of transient states.

The transition from transient to absorption states can occur directly, or after transition through a series of transient states. Probability of transition is marked as b_{ii} and expressed by the formula:

$$b_{ij} = p_{ij} + \sum_{s_k \in T} p_{ik} b_{kj},$$
(2)

where p_{ij} is a direct transition to the absorption state, the second term in sum expresses all possible indirect transitions. In matrix form, we can express given probabilities as B = R + QB, and after adjustment is the matrix B expressed as:

 $\mathbf{B} = (\mathbf{E} - \mathbf{Q})^{-1} \mathbf{R}$ (3)

Rows of the matrix B contain the transition probabilities to the absorption state and the sum is equal to one, since the system must end in one of them, according to [2].

5. Application of absorption Markov chains

Application of absorption chains in practice was performed within the frame of social security, namely in securing mothers in case of pregnancy and maternity. This life event brings a lot of changes, especially in financial terms by increasing expenditures. This paper is focused on women aged 30 to 40 years, when the probability of pregnancy in women is the highest. Using the absorption chains the frequency of the occurrence of the insurance events (the childbirths) was recorded.

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We consider the women aged 30 years, which are planning maternal leave within the next 10 years. The insurance company in terms of fulfilment of obligations is interested in, when the childbirth can occur. As a transition period one year was chosen (pregnancy lasts 9 months, i.e. childbirth will take place within 1 year). The system will describe the event through the transient states $s_1, s_2...s_{10}$, that express, in which year a woman gives a birth (the insured event occurs), and two absorption s_{11} (state of mother), and s_{12} (woman during the 10 years did not give a birth, i.e. insurance event not occurred), in which the insurance expires. For simplification, abortion or death of a woman is not considered. As the childbirth term does not depend on time, we can describe the system using absorption chain. Data on newborn children according to women age in 2013 according to the Statistical Office of the SR are presented in the Table 1.

Age	Number of live-born children according to the age of woman	Number of women	Number of live-born children to number of women ratio
30	3850,4	45 022	0,085523
31	3850,4	47 367	0,081289
32	3850,4	46 738	0,082383
33	3850,4	46 028	0,083653
34	3850,4	46 081	0,083557
35	3682,2	45 307	0,081272
36	3682,2	44 865	0,082073
37	3682,2	42 893	0,085846
38	3682,2	40 510	0,090896
39	3682,2	37 944	0,097043

Table 1: The number of women and live-born children in 2013

Construction of the transition probability matrix P is based on the following assumptions:

1. woman becomes a mother at most once during the insurance period, i.e. 10 years;

2. if woman does not give a birth, she goes to the state s_{12} ;

3. if she does not give a birth until 10 years, the insurance expires.

Because in the chain there is only a birth tracking that in each year occurs or does not, every line of matrix P will have two non-zero elements for the transit states. Transition probabilities for absorbing states are $p_{1111} = p_{1212} = 1$, and therefore other elements in that row equals zero. Based on data from the Table 1 transition probabilities p_{i11} i = 1, 2...10, were calculated for describing an event (women giving birth) in a certain year with an assumption she did not give a birth in the

Source: Statistical Office of the SR, [18].

previous years. Then $p_{i11} = (1 - p)^i p$ for $i = 0, 1, \dots 9$, where *p* is the probability that a woman will give a birth at the age 30–39 years. Since this probability is at any age of the women with small deviations almost the same (fourth column of the Table 1), we choose p = 0,085 for each age category as the average value of those inputs. Other elements of the matrix **P** were calculated from the relationship $\sum_{j=1}^{12} p_{ij} = 1$ for $i = 1, 2, \dots 10$.

Transition probability matrix dimensions are 12x12 and following form:

		S_1	<i>S</i> ₂	<i>S</i> ₃	S_4	<i>S</i> ₅ <i>S</i>	₆ S ₇	<i>S</i> ₈	S ₉	S_{10}	S ₁₁	s ₁₂	
	s ₁	0	0,915	0	0	0	0	0	0	0	0	0,085	0]
	<i>s</i> ₂	0	0	0,9222	0	0	0	0	0	0	0	0,0778	0
	<i>s</i> ₃	0	0	0	0,9288	0	0	0	0	0	0	0,0712	0
	S ₄	0	0	0	0	0,9349	0	0	0	0	0	0,651	0
	<i>s</i> ₅	0	0	0	0	0	0,9404	0	0	0	0	0,0596	0
P =	<i>s</i> ₆	0	0	0	0	0	0	0,9455	0	0	0	0,0545	0
r =	<i>s</i> ₇	0	0	0	0	0	0	0	0,9501	0	0	0,0,499	0
	<i>s</i> ₈	0	0	0	0	0	0	0	0	0,9544	0	0,0456	0
	<i>s</i> ₉	0	0	0	0	0	0	0	0	0	0,9582	0,0418	0
	<i>s</i> ₁₀	0	0	0	0	0	0	0	0	0	0	0,0382	0,9618
	<i>s</i> ₁₁	0	0	0	0	0	0	0	0	0	0	1	0
	<i>s</i> ₁₂	0	0	0	0	0	0	0	0	0	0	0	1

By interchanging the rows and columns compact blocks were created in matrix P, or sub-matrices E_2 , R, and Q respectively.

	<i>s</i> ₁₁ <i>s</i>	12 S ₁		S ₂	s ₃ s ₄	<i>S</i> ₅	<i>s</i> ₆	<i>S</i> ₇	s ₈ s	₉ <i>S</i> ₁₀			
5	n 1	0	0	0	0	0	0	0	0	0	0	0]	
\$	0	1	0	0	0	0	0	0	0	0	0	0	
	s ₁ 0,085	0	0	0,915	0	0	0	0	0	0	0	0	
1	s ₂ 0,0778	8 0	0	0	0,9222	0	0	0	0	0	0	0	
	s ₃ 0,0712	2 0	0	0	0	0,9288	0	0	0	0	0	0	,
$\mathbf{P} = \mathbf{A}$	s ₄ 0,0651	0	0	0	0	0	0,9249	0	0	0	0	0	
	s ₅ 0,0596	5 0	0	0	0	0	0	0,9404	0	0	0	0	
	s ₆ 0,0545	i 0	0	0	0	0	0	0	0,9455	0	0	0	
1	s, 0,0499	0	0	0	0	0	0	0	0	0,9501	0	0	
	s ₈ 0,0456	i 0	0	0	0	0	0	0	0	0	0,9544	0	
	s, 0,0418	8 0	0	0	0	0	0	0	0	0	0	0,9582	
\$	₁₀ 0,5038	2 0,9618	0	0	0	0	0	0	0	0	0	0	

re: 0 0	0,915 0	0	0						
0		0	0						
	0		0	0	0	0	0	0	0]
Δ		0,9222	0	0	0	0	0	0	0
U	0	0	0,9288	0	0	0	0	0	0
0	0	0	0	0,9349	0	0	0	0	0
0	0	0	0	0	0,9404	0	0	0	0
0	0	0	0	0	0	0,9455	0	0	0
0	0	0	0	0	0	0	0,9501	0	0
0	0	0	0	0	0	0	0	0,9544	0
0	0	0	0	0	0	0	0	0	0,9582
0	0	0	0	0	0	0	0	0	0
0,0	85	0]							
0,1	63	0							
0,2	.34	0							
0,2	.99	0							
0,3	59	0							
0,4	13	0							
0,4	-63	0							
0,5	09	0							
	0 0 0 0 0,0 0,1 0,2 0,2 0,3 0,4 0,4	$\begin{array}{cccc} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \\$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$

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0

0,55

0,589 0,411

In the next step, the fundamental matrix N, matrix H, and matrix B were calculated, using which the characteristics of absorption chain were investigated, and the results were interpreted.

	[1	0,915	0,8438	0,7837	0,7327	0,689	0,6515	0,619	0,5908	0,5661	
	0	1	0,9222	0,8565	0,8008	0,7531	0,712	0,6765	0,6456	0,6186	
	0	0	1	0,9288	0,8683	0,8166	0,7721	0,7336	0,7001	0,6708	
	0	0	0	1	0,9349	0,8792	0,8313	0,7898	0,7538	0,7223	
N _	0	0	0	0	1	0,9404	0,8891	0,8448	0,8063	0,7726	
IN =	0	0	0	0	0	1	0,9455	0,8983	0,8574	0,8215	
	0	0	0	0	0	0	1	0,9501	0,9068	0,8689	
	0	0	0	0	0	0	0	1	0,9544	0,9145	
	0	0	0	0	0	0	0	0	1	0,9582	
	0	0	0	0	0	0	0	0	0	1	

If the matrix N will be multiplied by the number of days in a year, individual rows of the matrix N can be interpreted as an average number of days, during which woman remains childless at a certain age, regarding to her entry age.

	365	333,975	307,987	286,0505	267,4355	251,485	237,7975	225,935	215,642	206,6265
	0	365	336,603	312,6225	292,292	274,8815	256,88	246,9225	235,644	225,789
	0	0	365	339,012	316,9295	398,059	281,8165	267,764	255,5365	244,842
	0	0	0	365	341,2385	320,908	303,4245	288,277	275,137	263,6395
365 * N =	0	0	0	0	365	343,246	324,5215	308,352	294,2995	281,999
303 · IN =	0	0	0	0	0	365	345,1075	327,8795	312,951	299,8475
	0	0	0	0	0	0	365	346,7865	330,982	317,1485
	0	0	0	0	0	0	0	365	348,356	333,7925
	0	0	0	0	0	0	0	0	365	349,743
	0	0	0	0	0	0	0	0	0	365

We can expect that 30 year old insured woman remains childless at age 31 for 333 days in average, then this number is reduced and at the age of 39 years it takes approximately half a year. At the female entry age 35 years we can expect that she will remain childless in the following year for 345 days in average. This number is reduced only a little and at the age of 39 years, it remains at 300 days on average.

	0	0,915	0,8438	0,7837	0,7327	0,689	0,6515	0,619	0,5908	0,5661
	0	0	0,9222	0,8565	0,8008	0,7531	0,712	0,6765	0,6456	0,6186
	0	0	0	0,9288	0,8683	0,8166	0,7721	0,7336	0,7001	0,6708
	0	0	0	0	0,9349	0,8792	0,8313	0,7898	0,7538	0,7223
H =	0	0	0	0	0	0,9404	0,8891	0,8448	0,8063	0,7726
11 –	0	0	0	0	0	0	0,9455	0,8983	0,8574	0,8215
	0	0	0	0	0	0	0	0,9501	0,9068	0,8689
	0	0	0	0	0	0	0	0	0,9544	0,9145
	0	0	0	0	0	0	0	0	0	0,9582
	0	0	0	0	0	0	0	0	0	0

Persistence of the process in the system of transient state is characterized by matrix H. In practice it means, we can determine the probability that woman remains childless in a certain year, depending on an entry age. Interpretation of individual rows for each age category was provided separately. Woman aged 30 years remains childless at the age of 31 years with 91.5% probability, and at age of 32 years with 84.38% probability, then the probability gradually declines until it reaches the level of 56.61%. We can observe that with an increasing entry age probability of childbirth increases.

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	0,4556	0,5444
	0,405	0,595
	0,3548	0,6452
	0,3053	0,6947
B =	0,257	0,743
D =	0,2099	0,7901
	0,1643	0,8357
	0,1204	0,8796
	0,0784	0,9216
	0,0382	0,9618

Matrix B contains the probability that a woman becomes the mother or remain childless during insurance period according to the entry age. From a theoretical point of view it represents a transition to absorption state (first column for the state s_{11} , the second column for s_{12}). Sum of row elements in the matrix B equals one, because the birth either occurs or does not during the insurance period.

If the woman entry age is 30 years, the probability of becoming a mother in 10 years is 45.56%. If a woman is 34 years old, we can expect that she will become a mother in six years with a probability of 25.7%. This probability decreases with an increasing age (in average by 4.6%), and for 39 year woman it is very unlikely (3.82%) that she will become a mother in one year.

6. Conclusion

Computed probabilities represent important information for calculation of insurance premium of insurance product. They are useful in valuation of risks and fulfilment of obligations of insurance company, which need to simulate expected number of insurance events.

Social security systems in different countries vary considerably, in particular by using actuarial mathematics to calculate insurance premiums and claims. Through detailed analysis high quality proposals applicable to the Slovak Republic, to improve social security system status can be obtained (detailed in [12]). Therefore this paper is focused on some Markov models and through the examples their utilization in the specific insurance claim was shown. The authors would like to highlight the wide range of application of Markov chains within the frame of social security.

This approach is used in operational analysis in modelling the reconstruction process and for modelling of inventories and mass services. The paper is introducing the concept of transition probability matrix, which is also called the stochastic matrix. For the calculations of insurance company it is important to know the probability that the insured person will be in certain state for a certain period of time, i.e. healthy, sick, or dead. For example, an insurance company

might be interested, what is the probability that the insured person will be in several weeks or months sick when he is sick now. In this paper the use of homogeneous Markov chains is shown, because for present technologies there is no problem to work with continuous model when considering a changes in system status at any point of time. The model becomes more precise reflection of reality. All theoretical findings were presented in the illustrative example. Microsoft Office Excel was used for the calculations.

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