

VERSIONING GOODS AND JOINT PURCHASES WITH NETWORK EXTERNALITY

Jiangli Dou, Bing Ye*

Abstract

This paper analyses the monopolist's production and pricing decisions on two vertically differentiated versions of a product in the presence of network externality. We show that offering only the higher-quality version of the product is the optimal strategy when negative externality exists and the utility from joint purchase is not large. If both versions are provided, the monopolist will charge a monopoly price for each version to induce separate purchases if these two versions are too close substitutes. Moreover, in the equilibrium with joint purchases, with an increase in externality or the utility from a joint purchase, the prices of both versions increase. In addition, with an increase in network externality, the equilibrium region for separate purchases first increases and then decreases.

Keywords: versioning goods, vertical differentiation, joint purchase, network externality

JEL Classification: D21, D42, L12, L25, M11

1. Introduction

It is very common for a firm to commercialize various versions of the same product (also known as versioning products) to price-discriminate and increase profits. For example, an automobile producer releases different series of cars; a mobile phone manufacturer provides both smartphones and conventional mobile handsets; a monopolist may develop two vertically differentiated versions of a software product; computers with different configurations could be provided by the same firm; and for the same film, watching it in the cinema and on a DVD at home are two different experiences with different qualities.

A number of papers in the literature investigate the monopolist's price discrimination, including Acharyya (1998), Gabszewicz *et al.* (1986), Maskin and Riley (1984), Moorthy (1984), Mussa and Rosen (1978), Salant (1989), and Stokey (1979). In particular, Mussa and Rosen (1978) and Moorthy (1984) show that the introduction of different versions of a product on separate markets is optimal due to consumer self-selection. Salant (1989)

* Jiangli Dou, Zhejiang Gongshang University, School of Economics, Hangzhou, China (jianglidou@zjgsu.edu.cn);

Bing Ye, Zhejiang University, Centre for Research of Private Economy and School of Economics, China (colinyebing@zju.edu.cn).

We are grateful to Simon P. Anderson, Emmanuelle Auriol, Jacques Crémer, Renato Gomes, Doh-Shin Jeon, Yassine Lefouili, Markus Reisinger, Michael Riordan, Ruqu Wang, and participants at various conferences and seminars for their helpful comments and suggestions. Dou thanks the financial support of the research project (No. Y201839457) from the Education Department of Zhejiang Province, China. Ye thanks the financial support by the Fundamental Research Funds for the Central Universities. All remaining errors are our own.

and Stokey (1979) identify conditions under which price discrimination is not optimal for the monopolist. Furthermore, Bhargava and Choudhary (2008) show that versioning is optimal when the optimal market share of the lower-quality version, when offered alone, is greater than the optimal market share of the higher-quality version when offered alone. Belleflamme (2005) considers the versioning strategy in the information economy and proves that versioning information goods is not profitable.

Recently, classic versioning research has been extended in two meaningful directions. First, researchers dropped the traditional *zero or one* demand assumption, where each consumer buys a single unit among various products offered on the market. It is common for consumers to buy several variants of the same product, called a *joint purchase* in the literature. For example, many people have several computers or mobile phones with different configurations; someone may watch a film in the cinema and also buy the same film on DVD to watch again; one person may own two different versions of the same software; and one woman can buy handbags in different price levels from the same brand. Gabszewicz and Wauthy (2003) first analyse versioning products when consumers have the possibility to purchase both versions. They assume that there are two vertically differentiated variants of a product, and that these two variants are partially substitutes; hence, the utility from buying both variants is smaller than the sum of the utilities resulting from consuming each variant. They analyse the price competition between two firms when each owns one variant of the product, and they characterize the price competition equilibria when a joint purchase exists. Depending on the utility attached to the joint purchase relative to separate purchases, there can be no equilibrium, a unique equilibrium or multiple equilibria. Calzada and Valletti (2012) analyse the introduction of two versions of the same film. They obtain the result that it is optimal to introduce both versions if products are not too close substitutes. Martínez-Sánchez (2016) uses the same setup as Gabszewicz and Wauthy (2003) to analyse the monopolist's decision on how to design different versions of a product, *i.e.*, whether to make them substitutes or complements. He shows that the decision depends on the degree of concavity and convexity of the cost function.

In addition, economists want to demonstrate that many products (*e.g.*, computers, software, mobile phones, etc.) possess network externalities and that consumers also care about the number of users as well as the intrinsic value of the products. For example, a game player cares about how many players can join him or her when playing an online game. When someone uses Skype or WeChat to contact his or her friends, the larger the number of those utilizing the software, the larger the probability that he or she can contact friends. The larger the number of users with a mobile phone, the more convenient it is for those who want to make a phone call. On the other hand, the effect of network congestion exists if the number of users becomes too large for online software products. The larger the number of users, the more difficult it is for each one to get connected. Another example of negative network externality is conspicuous consumption or snobbish consumption. The more users with a handbag from one luxury brand, the lower the status it represents for the women who use the brand. Winkelmann (2012) shows that the larger the number of users for luxury cars, the lower the satisfaction for others. Jing (2007) considers a single-period product

market, with different qualities, operated by a monopolist. Besides the intrinsic value, the product also has a network value, which depends on the network size. Each consumer purchases one unit of the product or none. Jing shows that the presence of network externality induces price discrimination. Without network externality, the monopolist sells only the higher-quality product; however, the monopolist's optimal strategy is to offer also a second low-end product with the existence of network externality. When the network externalities become stronger, the monopolist will raise the price of the high-quality product and decrease the price of the low-quality product. When the network externalities are sufficiently large, the monopolist will offer the lower-quality product at a loss and obtain profit from the higher-quality product.

In this paper, we combine these two strands of the literature and take both joint purchasing and network externality into consideration to analyse the monopolist's optimal production strategy. The presence of positive network externality induces the monopolist to expand the product's market coverage more aggressively by introducing different versions to make it more valuable to consumers. With the existence of negative network externality, the larger the number of users, the less valuable the product is to the users. Therefore, the monopolist has less incentive to expand its market size. However, the presence of joint purchasing may change the monopolist's production strategy, depending on the degree of substitution between different versions.

We adopt a setup similar to Gabszewicz and Wauthy (2003), adding network externality. The larger the market size for the product, the more (less) valuable the product is to the consumers with positive (negative) network externality. Similarly, as in duopoly competition, as presented by Gabszewicz and Wauthy (2003), we show that there exist two types of equilibria when both versions are produced in the monopoly setting. Separate purchase under which the monopolist charges a monopoly price for each version is better for the monopolist when the two versions are too close substitutes; otherwise, the monopolist will decrease the price of both versions to attract some consumers to purchase both versions. Differently from the equilibrium result in Jing (2007) without a joint purchase, we find that, in the equilibrium with the joint purchase, as a result of an increase in network externality or the utility from a joint purchase, the monopolist charges a higher price for each version. Furthermore, the existence of joint purchase options increases the monopolist's incentive to release another low-end quality product. Compared with Calzada and Valletti (2012) and Martínez-Sánchez (2016) without network externality, the existence of network externality also increases the monopolist's incentive to version the product. Offering only the higher-quality version is the optimal strategy if the network congestion effect exists and the utility from joint purchase is not large. With an increase in network externality, the equilibrium region of separate purchases first increases and then decreases.

The paper is organized as follows. In Section 2, we present the model and derive the possible equilibrium price segmentation when both versions are provided. Next, we characterize the price equilibria and versioning strategy in Section 3. Finally, we conclude in Section 4.

2. The Model

2.1 Setup

We consider a monopolist who can produce two vertically differentiated versions of a product with quality u_H and u_L ($u_L < u_H$), respectively. For the sake of simplicity, we assume that the monopolist produces at zero cost.¹ Consumers are indexed by type $\theta \in [0,1]$ which indicates their tastes for the product. No additional utility can be obtained from consuming more than one unit of the same version; hence, the classic zero or one assumption is satisfied for each version. We assume that θ follows a uniform distribution on the support set. The consumer with type θ obtains utility:

$$u = \begin{cases} \theta u_H - P_H + \gamma N & \text{if he or she consumes only the H version,} \\ \theta u_L - P_L + \gamma N & \text{if he or she consumes only the L version,} \\ \theta u_2 - P_H - P_L + \gamma N & \text{if he or she consumes both versions.} \end{cases} \quad (1)$$

We denote $u_2 \in (u_H, u_H + u_L)$ as the inherent value that can be obtained by consuming both versions, where $u_2 < u_H + u_L$ indicates that these two versions are partially substitutes. N is the total network size of consumers who purchase at least one version of the product; γ , which can be positive or negative, denotes the strength of the network effect on the valuation of a consumer in purchasing the product, indicating the marginal value derived from the network effect. The linear setup of the network effect, with respect to the network size and the additive formulation, is commonly used and consistent with previous studies (Choi, 1994; Jing, 2007; Prasad *et al.*, 2010; Shy, 2011; and Zhao, 2014). It is possible that higher-quality products generate higher positive network externality to the users, such as in Liu *et al.* (2015). Here, we use the setup that consumers only care about the network size or the installed base of this product, but do not care about the version of each user. For example, a mobile phone user cares about the number of phone owners but not each owner's mobile phone type. An online game player only cares about the number of players online when he or she is playing a game but not the version of software that each player uses. Hence, the utility derived from network externality is linear with respect to the user base of the product.² P_H and P_L denote the price charged by the monopoly firm for versions H and L , respectively.³ If a consumer purchases neither version of the product, he or she obtains

1 Our analysis in the following still holds for a unit cost function that is non-decreasing and linear with quality, under the setup of the quasi-linear consumer utility function. The constant marginal cost assumption is widely used for analytical simplification in the model of vertical differentiation. See, for example, Shaked and Sutton (1982), Gabszewicz *et al.* (1986) and Gabszewicz and Wauthy (2003), among others.

2 However, we should remember that the definition of the "quality" of a product is multi-dimensional. Take mobile phones or game software as examples, these dimensions can be functionalities, performance speed, after-sale support, compatibility with other products, *etc.* Our main results still hold for the quality-dependent network effect.

3 Here, we analyse the monopolist's production strategy based on consumer self-selection. There is no bundling price when consumers purchase both versions together.

the status-quo utility of zero. We assume that consumers have perfect foresight (rational expectations) about equilibrium demand.

We use θ_i ($i = H, L$) to denote users who are indifferent about buying one unit of the variant i or buying no version of the product. Therefore, $\theta_H = \frac{P_H - \gamma N}{u_H}$, and $\theta_L = \frac{P_L - \gamma N}{u_L}$. The user who is indifferent about buying both versions or buying nothing is denoted by $\theta_2 = \frac{P_H + P_L - \gamma N}{u_2}$. We use $\theta_{HL} = \frac{P_H - P_L}{u_H - u_L}$ to denote the type of the user who is indifferent about buying either the H version product or the L version product; $\theta_{2H} = \frac{P_L}{u_2 - u_H}$ denotes the typical user who is indifferent about buying both versions or buying only the H version product; and $\theta_{2L} = \frac{P_H}{u_2 - u_L}$ denotes the user who is indifferent about buying both versions or buying only the L version.

In the following, we only consider the case when $\gamma < u_L$. That is, when each consumer purchases one or the other version of the product, the largest network benefit γ is smaller than the value of the L product; hence, the consumers purchase the product mainly because of the intrinsic utility it provides, not because of the network value. The larger the number of users, the lower the satisfaction the product brings to consumers with the existence of negative network effect.

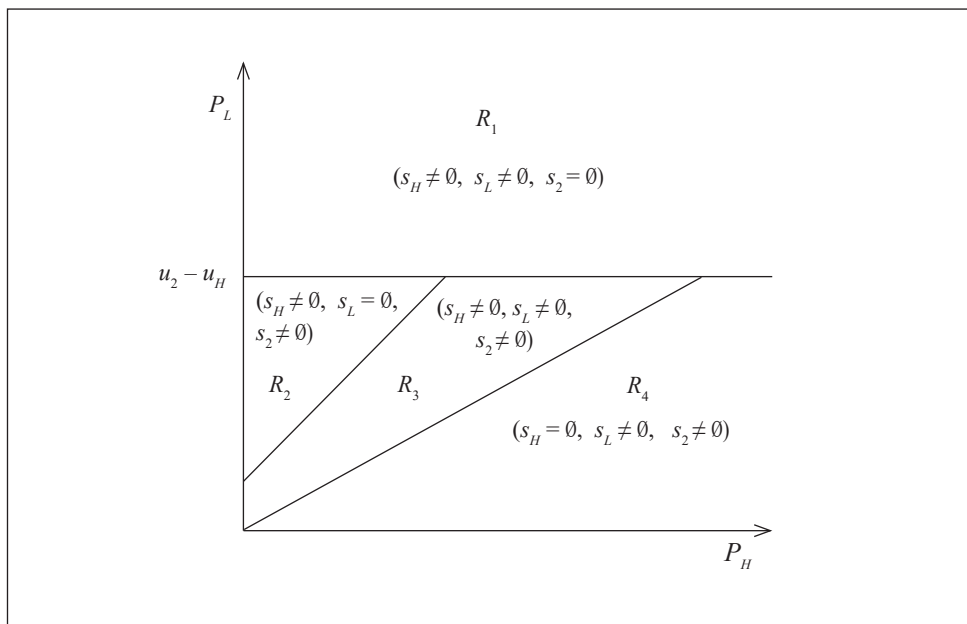
2.2 Price segmentation

With the definitions in Section 2.1, we can derive the demand formula for each variant of the monopolist, which can be best depicted by using Figure 1 to partition the (P_H, P_L) space into four regions.

In Figure 1, no consumer purchases the L version product alone in region R_2 , and no one purchases the H version product alone in region R_4 . Therefore, the demand formulas for the H version product in R_2 , and for the L version product in R_4 , are the standard monopoly demands. The demand for the H version product coincides with the demand in the vertical differentiation model in R_3 ; whereas the demand for the L version product is different from that in the classical vertical differentiation model because of the existence of joint purchase. Compared with the standard vertical differentiation model, the existence of the joint purchase essentially amounts to the circumstance when the monopolist charges a price for the L version product below the value $u_2 - u_H$. We summarize the partition and demand formulas for each price region in Table 1, where, in the demand formula in region R_3 ,

$$K = \frac{1}{u_2 - u_H} + \frac{1}{u_H - u_L} + \frac{1}{u_L - \gamma}.$$

Figure 1 | The Partition of Price Regions



Note: We use s_H , s_L and s_2 to denote the set of consumers who purchase only the H version, L version and both versions, respectively. In region R_1 , the price for the L product P_L is high; hence, there is no joint purchase. There exist some consumers that purchase both versions in regions R_2 , R_3 and R_4 .

Source: This figure is depicted to illustrate the partition of price regions based on formulas in Table 1.

Table 1 | The Partition of Price Regions and the Demand Formula

	Region partitions	Demand formulas
R_1	$R_1 = \{(P_H, P_L) : P_L \geq u_2 - u_H\}$	$D_H = 1 - \frac{P_H - P_L}{u_H - u_L}; \quad D_L = \frac{P_H - P_L}{u_H - u_L} - \frac{P_L - \gamma}{u_L - \gamma}$
R_2	$R_2 = \{(P_H, P_L) : \frac{P_H(u_L - \gamma)}{u_H - \gamma} + \frac{\gamma(u_H - u_L)}{u_H - \gamma} \leq P_L < u_2 - u_H\}$	$D_H = 1 - \frac{P_H - \gamma}{u_H - \gamma}; \quad D_L = 1 - \frac{P_L}{u_2 - u_H}$
R_3	$R_3 = \{(P_H, P_L) : P_L < u_2 - u_H, \frac{P_H(u_2 - u_H)}{u_2 - u_L} \leq P_L \leq \frac{P_H(u_L - \gamma)}{u_H - \gamma} + \frac{\gamma(u_H - u_L)}{u_H - \gamma}\}$	$D_H = 1 - \frac{P_H - P_L}{u_H - u_L};$ $D_L = 1 + \frac{\gamma}{u_L - \gamma} - P_L K + \frac{P_H}{u_H - u_L}$
R_4	$R_4 = \{(P_H, P_L) : P_L < u_2 - u_H, P_L \leq \frac{P_H(u_2 - u_H)}{u_2 - u_L}\}$	$D_H = 1 - \frac{P_H}{u_2 - u_L}; \quad D_L = 1 - \frac{P_L - \gamma}{u_L - \gamma}$

3. Results and Discussion

3.1 Optimal pricing strategies

In this subsection, we analyse the monopolist's equilibrium pricing strategy for the given rational expectations of the equilibrium demand for each version in each region. Then we obtain the optimal versioning strategy.

First, we try to obtain the optimal price strategy for different regions as described in Section 2.2. In region R_1^4 , the firm's maximization problem is:

$$\max_{P_H, P_L} \pi = P_H D_H + P_L D_L = P_H \left[1 - \frac{P_H - P_L}{u_H - u_L} \right] + P_L \left[\frac{P_H - P_L}{u_H - u_L} + \frac{\gamma - P_L}{u_L - \gamma} \right],$$

which yields $P_H = \frac{u_H}{2}$ and $P_L = \frac{u_L}{2}$. Therefore, we obtain $D_H = \frac{1}{2}$, $D_L = \frac{\gamma}{2(u_L - \gamma)}$.

If $0 \leq \gamma \leq u_L / 2$, we have $D_L \leq \frac{1}{2}$. The firm obtains a profit of $\frac{u_H}{4} + \frac{\gamma u_L}{4(u_L - \gamma)}$.

If $\gamma > u_L / 2$, the market is fully covered, we have $N = 1$, and the market share for each product is $1 - \frac{P_H - P_L}{u_H - u_L}$ and $\frac{P_H - P_L}{u_H - u_L}$ respectively. To ensure that the consumer with type

$\theta = 0$ purchases the L version, we must have $P_L \leq \gamma$. By solving the firm's maximization problem, we can easily find that $P_L = \gamma$ and $P_H = (u_H - u_L) / 2 + \gamma$. This pricing strategy

belongs to R_1 if $u_2 \leq u_H + \frac{u_L}{2}$.⁵

If $\gamma < 0$, there exists negative network externality, and no one consumes the L product. Hence, the firm's profit is the monopoly profit from selling only the H product, which is equal to $u_H^2 / 4(u_H - \gamma)$.

Therefore, the monopolist's price and profit in R_1 are respectively:

$$P_1^* = (P_{H1}^*, P_{L1}^*) = \begin{cases} (u_H / 2, u_L / 2) & \text{if } \gamma < 0, \\ (u_H / 2, u_L / 2) & \text{if } 0 \leq \gamma \leq u_L / 2, \\ ((u_H - u_L) / 2 + \gamma, \gamma) & \text{otherwise} \end{cases} \quad (2)$$

$$\pi_1^* = \begin{cases} u_H^2 / 4(u_H - \gamma) & \text{if } \gamma < 0, \\ u_H / 4 + \gamma u_L / 4(u_L - \gamma) & \text{if } 0 \leq \gamma \leq u_L / 2, \\ (u_H - u_L) / 4 + \gamma & \text{otherwise} \end{cases} \quad (3)$$

4 Region R_1 is the same as the standard vertical differentiation model. When the firm charges an infinitely high price for one version, we obtain the same result as if providing only the other version.

5 It is obvious that if $0 \leq \gamma \leq u_L / 2$, the pricing strategy belongs to R_1 if and only if $u_2 \leq u_H + u_L / 2$. If $\gamma > u_L / 2$, the pricing strategy belongs to R_1 if and only if $\gamma > u_2 - u_H$, which is satisfied if $u_2 \leq u_H + u_L / 2$ and $\gamma > u_L / 2$.

In region R_1 with separate purchases, if the externality γ is negative, there is no demand for the L version of the product; hence, the firm acts as a monopoly for only the H version of the product. If γ is positive but no larger than $u_L / 2$, the firm charges the monopoly price for each variant. The demand for the L version of the product increases with γ and equals 0 when $\gamma = 0$. The existence of positive network externality expands its market size by attracting previous non-consumers to consume the L version because each version is more valuable to the users. The demand for the H version is lower than the monopoly demand because the existence of the L version cannibalizes the market coverage of the H version. If the externality γ is larger than $u_L / 2$, one half of the consumers purchase the H version and the remaining half purchase the L version; hence, the market is fully covered, and the price charged for each version is higher than the monopoly price. Indeed, the externality is so large that the very low-type users purchase the product because of the network effect, and the existence of these low-type consumers makes the product more valuable to the high-type consumers. Therefore, the monopolist can charge a higher price for each version.

Proposition 1. *To induce consumers to buy at most one version of the product, the optimal pricing strategies are characterized by equations in (2). Under those pricing schemes, the monopolist sells two versions of the product on the market when the network effect is positive; otherwise, the monopolist just provides the H version on the market. The market is fully covered when the network effect is large ($\gamma > u_L / 2$).*

In region R_2 , the maximization problem is

$$\max_{P_H, P_L} \pi = P_H D_H + P_L D_L = P_H \left[1 - \frac{P_H - \gamma}{u_H - \gamma} \right] + P_L \left[1 - \frac{P_L}{u_2 - u_H} \right],$$

which yields $P_H = \frac{u_H}{2}$ and $P_L = \frac{u_2 - u_H}{2}$. This pricing strategy does not belong to R_2 ,⁶

Hence, the optimal price strategy should lie at the boundary of R_3 , which, by continuity, is dominated by the optimal strategy in the interior of R_3 . From the symmetry of the price partition of regions R_2 and R_4 , we conclude that the optimal strategy is not in region R_4 either.⁷

In region R_3 , from the maximization problem

$$\max_{P_H, P_L} \pi = P_H D_H + P_L D_L = P_H \left[1 - \frac{P_H - P_L}{u_H - u_L} \right] + P_L \left[1 + \frac{\gamma}{u_L - \gamma} + \frac{P_H}{u_H - u_L} - P_L K \right],$$

we have

$$P_H = \frac{(u_L - \gamma)(u_2 - u_L) + u_H(u_2 - u_H)}{2(u_2 + u_L - u_H - \gamma)}, \text{ and } P_L = \frac{(2u_L - \gamma)(u_2 - u_H)}{2(u_2 + u_L - u_H - \gamma)}.$$

6 We can easily prove that the condition for this price belonging to R_2 is a contradiction with $u_H < u_2 < u_H + u_L$, and $u_H > u_L > \gamma$.

7 From the maximization problem in R_4 , we obtain $P_H = (u_2 - u_L) / 2$, $P_L = u_L / 2$. In order for this price strategy to belong to R_4 , we need $P_L \leq P_H(u_2 - u_H) / (u_2 - u_L)$, which is equivalent to $u_L / 2 \leq (u_2 - u_H) / 2$ which is a contradiction with $u_2 < u_H + u_L$. Hence, the optimal strategy is not in R_4 .

Both P_H and P_L increase with γ and u_2 . The intuition is that larger γ increases the network value to the consumers; therefore, the monopolist can extract a higher surplus from each variant. A larger u_2 benefits consumers who purchase the L version in addition to the H version; therefore, the monopolist can charge a higher price for the L version. Yet, since consumers have a higher willingness to pay for the H version than for the L version, it is optimal for the firm to charge higher prices for both versions.

The demand for each version is $D_H = 1/2$, $D_L = u_L / 2(u_L - \gamma)$, which increases with γ and is equal to $1/2$ when $\gamma = 0$. When $\gamma \leq u_L / 2$ and $u_2 > u_H + u_L / 2$, this pricing strategy belongs to R_3 , and the market is not fully covered.⁸

With this equilibrium price, we have $\theta_{2H} = \frac{2u_L - \gamma}{2(u_2 + u_L - u_H - \gamma)} > \theta_{HL} = \frac{1}{2}$. Therefore, there always exist some consumers who purchase the H version product alone.

If $\gamma \geq 0$, we have $D_L \geq \frac{1}{2}$; hence, some consumers purchase the L version alone for a positive value of γ . If $u_2 - u_H - \sqrt{(u_2 - u_H)^2 + u_L(u_H + u_L - u_2)} \leq \gamma < 0$, we have $\theta_L < \theta_{HL} = 1/2$; hence, there still exist a group of consumers who purchase only the L version.

If $\gamma < u_2 - u_H - \sqrt{(u_2 - u_H)^2 + u_L(u_H + u_L - u_2)}$, there is no consumer who purchases the L version alone, and we have the demand of $D_H = \frac{1}{2}$; and $D_L = 1 - \frac{2u_L - \gamma}{2(u_2 + u_L - u_H - \gamma)}$.

Now, let us calculate the equilibrium price strategy when the market is fully covered. If $\gamma \geq u_L / 2$, each consumer purchases one unit of the L version. Hence, we have

$1 + \frac{\gamma}{u_L - \gamma} + \frac{P_H}{u_H - u_L} - P_L K = 1$. Combined with the monopolist's maximization problem, it is easy to obtain the pricing strategy:

$$P_H = \frac{(u_L - \gamma)(u_H - u_L) + (u_H + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}, \text{ and } P_L = \frac{(u_L + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}.$$

The price for each version increases with both γ and u_2 .

Therefore, when $\gamma \geq u_2 - u_H - \sqrt{(u_2 - u_H)^2 + u_L(u_H + u_L - u_2)}$, we have the equilibrium price and profit in region R_3 is:

$$P_3^* = \begin{cases} \left(\frac{(u_L - \gamma)(u_2 - u_L) + u_H(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}, \frac{(2u_L - \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)} \right) & \text{if } \gamma \leq \frac{u_L}{2}, \\ \left(\frac{(u_L - \gamma)(u_H - u_L) + (u_H + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}, \frac{(u_L + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)} \right) & \text{otherwise.} \end{cases} \quad (4)$$

⁸ We can check that this pricing strategy belongs to R_3 if $\gamma < 2(u_2 - u_H)$, which is clearly satisfied if $\gamma \leq u_L / 2$ and $u_2 \geq u_H + u_L / 2$.

$$\pi_3^* = \begin{cases} \frac{(u_L - \gamma)(u_2 - u_L) + u_H(u_2 - u_H)}{4(u_2 + u_L - \gamma - u_H)} + \frac{u_L(u_L - \gamma)(u_2 - u_H)}{4(u_L - \gamma)(u_2 + u_L - \gamma - u_H)} & \text{if } \gamma \leq \frac{u_L}{2}, \\ \frac{(u_L - \gamma)(u_H - u_L) + (u_H + \gamma)(u_2 - u_H)}{4(u_2 + u_L - \gamma - u_H)} + \frac{(u_L + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)} & \text{otherwise.} \end{cases} \quad (5)$$

Proposition 2. *To induce some consumers to make a joint purchase of both versions, the optimal pricing strategies are characterized by equations in (4).*

When the network effect is not too negative, under those pricing schemes, the high-type consumers buy both versions, the medium-type consumers buy only the H version, and the low-type consumers buy the L version; when the network effect is too negative, there is no consumer who buys only the L version; the market is fully covered when the network effect is large ($\gamma > u_L / 2$).

With negative network externality, the larger the number of users, the less valuable the product for each consumer. Therefore, each consumer is less willing to purchase the product. Proposition 2 indicates the consumers' purchase decision in region R_3 . If the two versions are not too close substitutes, there exist domains for joint purchasers and purchasers for each version.

Proposition 3. *In the equilibrium with joint purchases, when the externality (γ) or the utility from the joint purchase (u_2) increases, both P_H and P_L increase.*

Proof: If $\gamma \leq u_L / 2$,

$$\frac{\partial P_H}{\partial \gamma} = \frac{\partial P_L}{\partial \gamma} = \frac{(u_2 - u_H)(u_H + u_L - u_2)}{2(u_2 + u_L - \gamma - u_H)^2} > 0,$$

$$\frac{\partial P_H}{\partial u_2} = \frac{\partial P_L}{\partial u_2} = \frac{(u_L - \gamma)(2u_L - \gamma)}{2(u_2 + u_L - \gamma - u_H)^2} > 0.$$

If $u_L / 2 \leq \gamma < u_L$,

$$\frac{\partial P_H}{\partial \gamma} = \frac{\partial P_L}{\partial \gamma} = \frac{(u_2 - u_H)(u_2 - u_H + 2u_L)}{2(u_2 + u_L - \gamma - u_H)^2} > 0,$$

$$\frac{\partial P_H}{\partial u_2} = \frac{\partial P_L}{\partial u_2} = \frac{u_L^2 - \gamma^2}{2(u_2 + u_L - \gamma - u_H)^2} > 0.$$

It is easy to prove that $\frac{(u_L - \gamma)(u_H - u_L) + (u_H + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)} = \frac{(u_L - \gamma)(u_2 - u_L) + u_H(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}$

and $\frac{(u_L + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)} = \frac{(2u_L - \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}$ when $\gamma = u_L / 2$.

Proposition 3 is different from Jing (2007) without joint purchase, in which an increase in network externality will increase the price for the H version and decrease the price for the L version. The reason is that, with the option of joint purchase, the cannibalization effect is insignificant as the previous H version consumers have the option to purchase both versions, instead of purchasing the L version only.

From the above analysis for different regions, the optimal pricing strategy is either P_1^* or P_3^* with an equilibrium profit given by (3) or (5). By comparing π_1^* and π_3^* , we obtain the optimal pricing strategy for the monopolist, which is P_1^* for negative γ if $u_2 \leq u(u_H, u_L, \gamma)$ where $u(u_H, u_L, \gamma)$ is defined as $[u_L^2 u_H (u_L + 3u_H) - 2\gamma u_H (2u_L - \gamma) (u_L + u_H) - \gamma u_L (u_L + \gamma)^2] / [u_H u_L (3u_L - 4\gamma) - 3\gamma u_L (u_L - \gamma) + \gamma^2 (2u_H - \gamma)]$; otherwise it is P_3^* . If $0 \leq \gamma \leq u_L / 2$, the optimal price is P_1^* if $u_2 \leq u_H + u_L^2 / (3u_L - \gamma)$; otherwise, it is P_3^* . If $u_L / 2 \leq \gamma < u_L$, the optimal price is P_1^* if $u_2 < u_H + 4\gamma(u_L - \gamma) / (3u_L - \gamma)$; otherwise, it is P_3^* . This means that the monopolist charges the price inducing separate purchase if the utility from the joint purchase option is low. We summarize the results with the small network congestion effect; therefore, the market segmentation in region R_3 is described as in Proposition 2, in the following proposition.

Proposition 4. *If the network congestion effect is not significant, the monopolist's optimal price strategy is as follows:*

If $u_2 - u_H - \sqrt{(u_2 - u_H)^2 + u_L(u_L + u_H - u_2)} \leq \gamma \leq 0$,

$$P^* = \begin{cases} \left(\frac{u_H}{2}, \frac{u_L}{2} \right) & \text{if } u_2 \leq u(u_H, u_L, \gamma), \\ \left(\frac{(u_L - \gamma)(u_2 - u_L) + u_H(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}, \frac{(2u_L - \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)} \right) & \text{otherwise.} \end{cases}$$

If $0 \leq \gamma \leq u_L / 2$,

$$P^* = \begin{cases} \left(\frac{u_H}{2}, \frac{u_L}{2} \right) & \text{if } u_2 \leq u_H + \frac{u_L^2}{3u_L - \gamma}, \\ \left(\frac{(u_L - \gamma)(u_2 - u_L) + u_H(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}, \frac{(2u_L - \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)} \right) & \text{otherwise.} \end{cases}$$

If $u_L / 2 \leq \gamma < u_L$,

$$P^* = \begin{cases} \left(\frac{u_H - u_L}{2} + \gamma, \gamma \right) & \text{if } u_2 \leq u_H + \frac{4\gamma(u_L - \gamma)}{3u_L - \gamma}, \\ \left(\frac{(u_L - \gamma)(u_H - u_L) + (u_H + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)}, \frac{(u_L + \gamma)(u_2 - u_H)}{2(u_2 + u_L - \gamma - u_H)} \right) & \text{otherwise.} \end{cases}$$

The demand formula in region R_1 is the same as the separate demand in the vertical differentiation model. If the externality (γ) is not large so that the market is not fully covered, the price charged under separate purchases is the monopoly price for each version, which is larger than the price charged under the joint purchase. The intuition is that, to attract consumers to purchase both versions, the monopolist should charge a lower price. If γ is large and the market is fully covered, the price charged under a separate purchase is larger than the monopoly price because the larger γ , the higher the price the monopolist charges.

Now, we consider the equilibrium regions of separate purchase and joint purchase when the monopolist releases both versions onto the market. From comparison, the monopolist charges a higher price for each version; hence, there is no joint purchase if, and only if:

- i) $0 \leq \gamma \leq u_L / 2$ and $u_2 < u_H + \frac{u_L^2}{3u_L - \gamma}$;
- ii) $u_L / 2 \leq \gamma < u_L$ and $u_2 < u_H + \frac{4\gamma(u_L - \gamma)}{3u_L - \gamma}$. Therefore, we have the following:

Proposition 5. *If both versions are offered, the monopoly firm prefers to charge prices to induce a separate purchase rather than a joint purchase if, and only if, the utility from the joint purchase is not large.*

We depict the distribution of a separate purchase and a joint purchase in Figure 2.

If $0 \leq \gamma \leq u_L / 2$, the demand for the L version increases with γ , as larger γ provides a higher utility to each consumer. Some previous non-consumers obtain positive utility from buying the L version; therefore, the monopolist's preference for separate demand increases with γ .

If $u_L / 2 \leq \gamma \leq \gamma^*$, the market is fully covered. Therefore, the number of consumers under separate purchases can no longer increase. With an increase in γ , the profit under separate purchases increases as the price charged for each version increases with γ ; at the same time, the profit under joint purchases increases as the monopolist charges higher prices for both versions. The profit increment under separate purchases is larger than the profit increment under joint purchases for these values of γ . Therefore, the firm's preference for separate purchases increases with γ .

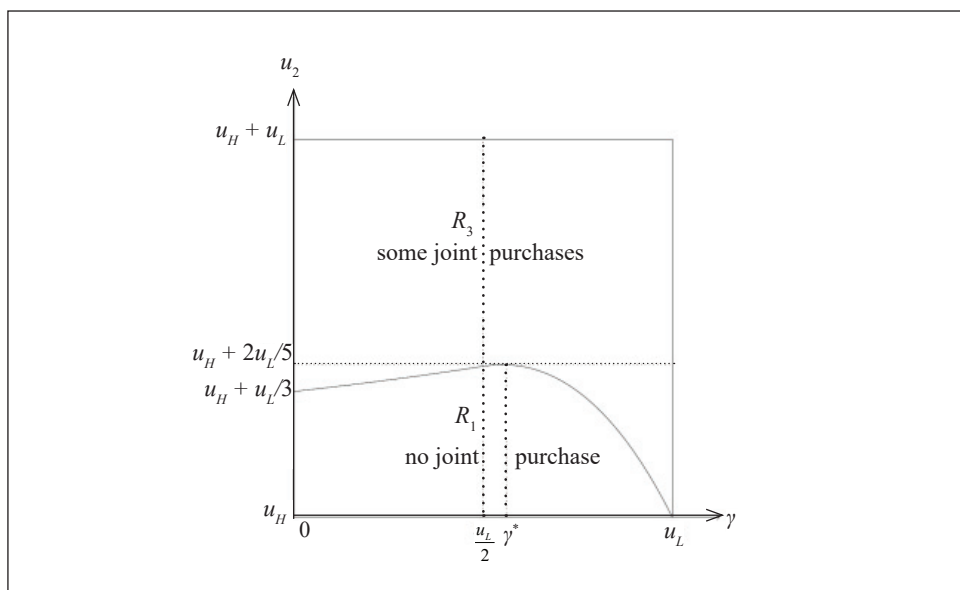
If $\gamma^* \leq \gamma < u_L$, with an increase in γ , the profit increment under a joint purchase is larger than that under a separate purchase; as a result, the preference for separate purchases decreases with the increase in γ .

Corollary 1. *For any network externality (γ), there exists a function $u_2^*(\gamma)$, such that there exist joint purchases in equilibrium if and only if $u_2 > u_2^*(\gamma)$. There exists $\gamma^* \in (u_L / 2, u_L)$, such that $u_2^*(\gamma)$ is increasing when $\gamma < \gamma^*$ and decreasing when $\gamma > \gamma^*$.*

In Figure 2, we can see that, when $u_H + u_L / 3 \leq u_2 \leq u_H + 2u_L / 5$, for a small value of γ , in equilibrium region R_3 , some consumers purchase both versions. When γ increases to the middle range, no one makes joint purchases at the given equilibrium prices as charged by the monopolist. The reason is that, from Proposition 3, in equilibrium with joint purchase, the price for both versions increases with γ . Therefore, with an increase in γ , the equilibrium

price with joint purchase increases up to the monopoly price, and the equilibrium result moves from R_3 to R_1 , in which no one makes joint purchases. For a large value of γ , some consumers make joint purchases under the monopolist's equilibrium prices. From Proposition 4, when $u_L/2 \leq \gamma$, the prices increase with γ under both types of equilibria. The price increment for the L version of the product in R_1 is faster than that in R_3 if γ is not large, and vice versa. As a result, for a large value of γ , the monopolist's optimal strategy is to decrease the price for the L version to induce some consumers to purchase both versions. If γ is close to u_L , in equilibrium there exist some joint purchases because of the network effect, even if these two versions are too close substitutes.

Figure 2 | The Equilibrium Region of Separate Purchases and Joint Purchases



Note: When the externality increases, the equilibrium region of separate purchases increases until $\gamma = \gamma^*$ and then decreases.

Source: This is depicted by assuming that $u_H = 10$ and $u_L = 8$.

3.2 Versioning or not

From Proposition 4, we have the monopolist's optimal versioning strategy:

Proposition 6. *Offering only the H version of the product is the optimal strategy if, and only if, $\gamma \leq 0$ and the utility from the joint purchase is small. Otherwise, the monopolist versions the product.*

If the externality γ is positive but not large, the market is not fully covered and there exist some consumers who purchase neither version of the product. If the utility from the joint purchase u_2 is not large, we come to region R_1 if both versions are provided and the monopolist charges a monopoly price for each version. The release of another L

version has two effects: the first one is the market expansion effect by attracting some previous non-consumers to purchase the L version of the product; the second effect is the cannibalization effect where some previous H version consumers switch to the L version of the product. The market expansion effect prevails over the cannibalization effect because of the existence of positive network externality. If the utility of the joint purchase u_2 is large, and if both versions are provided, the equilibrium lies in region R_3 , where some consumers make joint purchases of both versions and the monopolist charges a price lower than the monopoly price for each version. This joint purchase prevails over offering only the H version of the product, as the monopolist faces a higher demand.

If γ is large, the market coverage condition is satisfied, and each consumer purchases at least one version if both versions are provided. If u_2 is small, offering only the H version is dominated by separate purchases under which the monopolist charges a price higher than the monopoly price for the H version because of the presence of L version of the product and the network externality. Offering only the H version is also dominated by joint purchases under which the monopolist faces a larger demand.

If γ is negative, the effect of network congestion exists among consumers for this product. Hence, the monopolist is eager to decrease its market coverage. Therefore, it is optimal for the monopolist to offer only the H version if the two versions are too close substitutes.

Compared to Jing (2007), who excludes the option of joint purchases and shows that offering only the highest version is the optimal strategy if there is no network externality; the presence of joint purchases reduces the region for offering only the highest version.

Compared to Calzada and Valletti (2012) and Martínez-Sánchez (2016), who consider the case without network externality and show that versioning is optimal if the utility from joint purchase is high enough, the presence of network externality enlarges the region for offering both versions.

Proposition 7. *With the presence of the joint purchase or network externality, the monopolist has more incentive to release different versions. Even if there is no network externality ($\gamma = 0$) or a small network congestion effect, providing both versions is beneficial for the monopolist if these two versions are not too close substitutes (u_2 is large).*

4. Concluding Remarks

This paper analyses the monopoly firm's product strategy when it has the option to introduce different versions of the same product, and when consumers care about the number of users as well as the basic value of the product. The larger the number of users, the higher (lower) each consumer's utility with a positive (negative) network externality. For example, network externality exists among users of online software products. If the number of users is not very large so that each user has ample access within the network bandwidth, the larger the number of users of the software, the more valuable the product is to each user as it is more convenient to contact each other. If the number of users is very large, the larger the number of users, the more difficult it is for users to get connected.

We find that offering only the H version of a product is the monopolist's optimal strategy if the network congestion effect exists and the utility of the joint purchase is small. This can explain why many software developers stop producing the lower version of a product when they have the capability to produce a higher version of it, as different versions of the same software are too close substitutes. If both versions are offered, even if consumers obtain more utility from purchasing both versions than purchasing only one version, the monopolist may set prices to induce separate purchase as in the classical vertical differentiation model where no consumer makes joint purchases if the two versions are too close substitutes. The monopolist charges a monopoly price for each version under which there is no joint purchase if the two versions are too close substitutes. With an increase in externality, the equilibrium region of separate purchases first increases and then decreases.

In reality, the monopolist may introduce different versions sequentially. Due to technological constraints, the monopolist can only produce the L version and later upgrade it to the H version. For example, many game developers upgrade the product's quality very frequently. One possible extension for this paper is to analyse the monopolist's optimal production strategy in the sequential case. Given that the L version of a product is durable, the consumers of the L version of the product in the first period will therefore not consume the L version again; thus, there exists an optimal time to upgrade the product. Another objective for future research is to relax the assumption that each consumer earns the same network benefits from consuming a product, and to analyse the case when consumers derive different network benefits from different groups of users. In such a case, each consumer earns a higher network benefit from the number of users consuming the same version of a product than from the number of users consuming a different version.

References

- Acharyya, R. (1998). Monopoly and Product Quality, Separating or Pooling Menu? *Economics Letters*, 61(2), 187–194, [https://doi.org/10.1016/S0165-1765\(98\)00173-6](https://doi.org/10.1016/S0165-1765(98)00173-6)
- Andaluz, J. (2000). On Protection and Vertical Product Differentiation. *Regional Science and Urban Economics*, 30(1), 77–97, [https://doi.org/10.1016/S0166-0462\(99\)00029-0](https://doi.org/10.1016/S0166-0462(99)00029-0)
- Belleflamme, P. (2005). Versioning in the Information Economy: Theory and Applications. *CESifo Economic Studies*, 51(2–3), 329–358, <https://doi.org/10.1093/cesifo/51.2-3.329>
- Bhargava, H. K., Choudhary, V. (2008). Research Note: When is Versioning Optimal for Information Goods? *Management Science*, 54(5), 1029–1053, <https://doi.org/10.1287/mnsc.1070.0773>
- Calzada, J., Valletti, T. M. (2012). Inter-Temporal Movie Distribution: Versioning When Customers Can Buy Both Versions. *Marketing Science*, 31(4), 649–667, <https://doi.org/10.1287/mksc.1120.0716>
- Choi, J. P. (1994). Network Externality, Compatibility Choice, and Planned Obsolescence. *Journal of Industrial Economics*, 42(2), 167–182, <https://doi.org/10.2307/2950488>
- Church, J., Gandal, N. (1992). Network Effects, Software Provision, and Standardization. *Journal of Industrial Economics*, 40(1), 85–103, <https://doi.org/10.2307/2950628>
- Gabszewicz, J., Shaked, A., Sutton, J., Thisse, J. (1986). Segmenting the Market: The Monopolist's Optimal Product Mix. *Journal of Economic Theory*, 39(2), 273–289, [https://doi.org/10.1016/0022-0531\(86\)90046-3](https://doi.org/10.1016/0022-0531(86)90046-3)

- Gabszewicz, J., Sonnac, N., Wauthy, X. Y. (2001). On Price Competition with Complementary Goods. *Economic Letters*, 70(3), 431–437, [https://doi.org/10.1016/S0165-1765\(00\)00383-9](https://doi.org/10.1016/S0165-1765(00)00383-9)
- Gabszewicz, J., Wauthy, X. Y. (2002). Quality Under-Provision by a Monopolist When Quality Is Not Costly. *Economic Letters*, 77(1), 65–72, [https://doi.org/10.1016/S0165-1765\(02\)00092-7](https://doi.org/10.1016/S0165-1765(02)00092-7)
- Gabszewicz, J., Wauthy, X. Y. (2003). The Option of Joint Purchase in Vertically Differentiated Markets. *Economic Theory*, 22(4), 817–829, <https://doi.org/10.1007/s00199-003-0359-2>
- Jing, B. (2007). Network Externalities and Market Segmentation in a Monopoly. *Economics Letters*, 95(1), 7–13, <https://doi.org/10.1016/j.econlet.2006.08.033>
- Krishnan, U., Ulrich, K. (2001). Product Development Decisions: A Review of the Literature. *Management Science*, 47(1), 1–21, <https://doi.org/10.1287/mnsc.47.1.1.10668>
- Liu, Z., Li, M., Kou, J. (2015). Selling Information Products: Sale Channel Selection and Versioning Strategy with Network Externalities. *International Journal of Production Economics*, 166, 1–10, <https://doi.org/10.1016/j.ijpe.2015.04.006>
- Martínez-Sánchez, F. (2016). Versioning Goods and Joint Purchase: Substitution and Complementary Strategies. *Prague Economic Papers*, 25(5), 577–590, <https://doi.org/10.18267/j.pep.575>
- Maskin, E., Riley, J. (1984). Monopoly with Incomplete Information. *Rand Journal of Economics*, 15(2), 171–196, <https://doi.org/10.2307/2555674>
- Moorthy, K. (1984). Marketing Segmentation, Self-Selection, and Product Line Design. *Marketing Science*, 3(4), 288–307, <https://doi.org/10.1287/mksc.3.4.288>
- Moorthy, K., Png, I. (1992). Market Segmentation, Cannibalization, and the Timing of Product Introduction. *Management Science*, 38(3), 345–359, <https://doi.org/10.1287/mnsc.38.3.345>
- Motta, M. (1993). Endogenous Quality Choices: Price vs. Quantity Competition. *The Journal of Industrial Economics*, 41(2), 113–131, <https://doi.org/10.2307/2950431>
- Mussa, M., Rosen, S. (1978). Monopoly and Product Quality. *Journal of Economic Theory*, 18(2), 301–317, [https://doi.org/10.1016/0022-0531\(78\)90085-6](https://doi.org/10.1016/0022-0531(78)90085-6)
- Padmanabhan, V., Rajiv, S., Srinivasan, K. (1997). New Products, Upgrades, and New Releases: A Rationale for Sequential Product Introduction. *Journal of Marketing Research*, 34(4), 456–472, <https://doi.org/10.2307/3151964>
- Prasad, A., Venkatesh, R., Vijay, M. (2010). Optimal Bundling of Technological Products with Network Externalities. *Management Science*, 56(12), 2224–2236, <https://doi.org/10.1287/mnsc.1100.1259>
- Salant, S. (1989). When is Inducing Self-selection Suboptimal for a Monopolist? *Quarterly Journal of Economics*, 104(2), 391–397, <https://doi.org/10.2307/2937854>
- Shaked, A., Sutton, J. (1982). Relaxing Price Competition Through Product Differentiation. *Review of Economic Studies*, 49(1), 3–13, <https://doi.org/10.2307/2297136>
- Shy, O. (2011). A Short Survey of Network Economics. *Review of Industrial Organization*, 38(2), 119–149, <https://doi.org/10.1007/s11151-011-9288-6>
- Stokey, N. (1979). Intertemporal Price Discrimination. *Quarterly Journal of Economics*, 93(3), 355–371, <https://doi.org/10.2307/1883163>
- Winkelmann, R. (2012). Conspicuous Consumption and Satisfaction. *Journal of Economic Psychology*, 33(1), 183–191, <https://doi.org/10.1016/j.joep.2011.08.013>
- Zhao, D., Chen, H., Hong, X., Liu, J. (2014). Technology Licensing Contracts with Network Effects. *International Journal of Production Economics*, 158, 136–144, <https://doi.org/10.1016/j.ijpe.2014.07.023>