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Maximal Covering Location Problem and Bimatrix Games

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***Abstract* – The problem of optimally locating service centers to maximize customer coverage within a specified distance remains both theoretically significant and practically relevant. This article presents a game-theoretic approach to solving such location-allocation problems, offering a comprehensive framework that identifies all viable facility configurations while evaluating their strategic stability. The methodology enables the discovery of alternative solutions that may be preferable under specific conditions, considering both maximum coverage objectives and system-wide stability requirements. The study examines the case of two service centers planned for organizational merger, with the approach being extensible to multiple facilities. The proposed technique provides new theoretical insights into location problems while offering practical applications across various domains. Potential implementations include retail network optimization during corporate consolidations, strategic placement of specialized healthcare facilities, and deployment of critical public infrastructure services. A regional case study illustrates how equilibrium-based solutions outperform traditional approaches by simultaneously maintaining service accessibility and ensuring long-term network stability.**

***Keywords* – Game theory, location models, bimatrix game, stability.**

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1. Introduction

While opening service centers remains a critical problem for individuals, as well as for businesses and governments at both local and national levels, it presents a set of challenges rather than profits for all.

Such facilities could be numerous, taking large housing spaces such as central warehouses, hospitals, or recycling hubs in a particular location [19]. The location-allocation issue, which is about the best placement of the service centers, has been the subject of many controversial discussions among scientists for almost a century [10], [9]. On the other hand, while traditional coverage gives you only one plane of solutions (the problem of enumerating all optimal solutions may be more difficult than optimizing the original problem), game theory posits a more profound insight.

The theoretical framework for evaluating consumers' responses to prices and transport costs for different types of products is Fetter's early law of market areas, established in 1924, which determines the proportionate market share based on the distance between consumers' locations. Hotelling further developed this field with his 1929 article, "Stability in Competition," in which he proposed a linear market model selling identical commodities with consumers evenly distributed along the scale [12], [18].

Martin C. Byford explores the Hotelling model under the realistic assumption of a finite number of consumers [4]. This setup does not allow for a pure strategy Nash equilibrium [12].

The location-allocation problem is a complex optimization challenge that requires balancing various factors, such as transportation costs, customer preferences, and competitive dynamics [1]. One application of the location-allocation problem is in the context of printed circuit board assembly, where the optimal placement of component bins can significantly impact the efficiency of the assembly process [5]. Similarly, in the context of warehouse management, the allocation of inventory across multiple warehouses can be modeled as a location-allocation problem, with the goal of maximizing regional utilization [18].

Another aspect of the location-allocation problem is the consideration of stochastic factors, such as demand uncertainty or supply chain disruptions. By incorporating these elements into the problem formulation, researchers have developed robust optimization techniques to ensure the resilience of location-allocation decisions.

The location-allocation problem is a strategic decision-making process that aims to minimize the total cost or distance between the demand points and the facilities [15], [14]. This problem is often formulated as a mathematical optimization problem, where the objective function aims to minimize total transportation costs, maximize demand coverage, or a combination of these factors [8], [9]. The solution to the location-allocation problem can be found using techniques such as heuristic algorithms, metaheuristics, or exact optimization methods, depending on the complexity of the problem [17], [7].

In a service center framework, it is convenient to consider all the involved entities as a unit entity that can be easily covered through a facilitated process. While the introduction of centers as competitors offers a dynamic aspect to appraisal, it may ultimately lead to more long-lasting solutions in the future. Game theory provides a tool for considering situations in which the centers strive for domination without the possibility of unilateralism over the status quo. One application of game theory focuses on spatial competition, where companies strategically locate their facilities to maximize profits and minimize costs [18].

The location-allocation problem and the bimatrix game are both important concepts in the realm of operations research and game theory [1]. While they have distinct applications, the interplay between these two concepts can offer valuable insights into strategic decision-making across various industries and contexts [2], [3].

This approach allows us to extensively investigate the problem and form feasible recommendations that achieve the best balance of conditions that go beyond the traditional ones, among which there may be options that deviate enough from the model. How companies interact with each other and where they're located creates a competition for space. In studies of this competition, businesses primarily compete for customers by offering the lowest prices and by locating in the most convenient places [5]. These two factors form the foundation of a company's success in the market [10], [21].

This article proposes an alternative perspective for addressing location-allocation problems through game theory, unconstrained by traditional limitations.

The analysis examines a competitive model involving two service centers, representing a duopolistic framework where each center operates as a strategic player.

This analysis provides new insights to address the service center location issue, contributing to a deeper understanding of the location-allocation problem.

For the aforementioned reasons, this results in instability, which can be triggered when either party, including public or private healthcare service providers, decides to migrate (when given the opportunity) [21], [5].

The location-allocation problem and the bimatrix game are two distinct but interrelated concepts in operations research and game theory. The integration of these two concepts can lead to more realistic and practical decision-making models in various applications, such as supply chain management, logistics, and urban planning [12], [13].

2. Location-Allocation Problem and Bimatrix Game

The section presents the methodological base that will be used in the case study. Assume a complete and valued graph $\bar{G} = (V, \bar{H})$, where $V = \{1,2, \dots n\}, n \in Z^+$ is the set of nodes and $\bar{H}$ is the set of graph edges between all pairs of nodes v_i a v_j, $i, j \in V$, whose value is equal to minimal distance (length, time) between nodes v_i and v_j. This edge evaluation can be written into a matrix $\mathbf{D}_{nxn} = \{d_{ij}\}, i, j \in V$. The following mathematical models address selected location-allocation problems [11].

Consider a scenario where the number of service centers is predetermined, and these centers can be placed at any node of the graph $\bar{G}$. The objective is to determine the minimum distance required for these centers to serve all nodes. The model employs the following notation:

Sets and parameters:

$n \in Z^+$– number of nodes of the graph

$V = \{1,2, \dots n\}$ – set of all nodes

$d_{ij}, i, j \in V$ – minimal distances between *i*-th and *j*-th node

p – maximal (predetermined) number of service centers

Variables:

$z, z \geq 0$ – non-negative variable representing the smallest distance needed to cover all nodes

$y_{ij}, y_{ij} \in \{0,1\}, i, j \in V$– binary variable representing whether the *i*-th node is within reach of the *j*-th center ($y_{ij} = 1$), or not ($y_{ij} = 0$)

$x_i, x_i \in \{0,1\}, i \in V$– binary variable representing whether the serving center opens at the *i*-th node ($x_i = 1$), or not ($x_i = 0$)

The mathematical model is then as follows:

$$f(\mathbf{x},\mathbf{y},z) = z \rightarrow min \quad (1)$$

$$\sum_{i\in V} y_{ij} = 1 \quad j \in V \quad (2)$$

$$y_{ij} - x_i \leq 0 \quad i,j \in V \quad (3)$$

$$\sum_{i\in V} x_i \leq p \quad (4)$$

$$\sum_{i\in V} d_{ij}y_{ij} \leq z, j \in V \quad (5)$$

The objective function (1) minimizes the maximum distance required to achieve complete graph coverage. Constraints (2) enforce the condition that each node must be assigned to exactly one service center. Constraints (3) guarantee that a node can only be served by an open facility, while constraint (4) limits the total number of service centers to the predetermined value p. The coverage requirements are implemented through constraints (5), which ensure all nodes are accessible within the determined maximum distance.

Subsequently, a mathematical formulation is introduced to maximize graph coverage (without requiring complete coverage) given a fixed number of service centers and a specified maximum service distance. The model employs the following sets and parameters:

$n \in Z^+$– number of nodes of the graph
$V = \{1,2, \dots n\}$ – a set of all nodes of a graph
p – maximum (pre-determined) number of serving centers
K – maximum required availability (may not be sufficient to cover the entire graph)
$a_{ij}, i,j \in V$ – parameters representing if the *i*-th and *j*-th nodes are to distance K ($a_{ij} = 1$), or not ($a_{ij} = 0$)
Variables:
$y_j, y_j \in \{0,1\}, j \in V$- a binary variable representing whether the *j*-th node is in athe vailability of at least one center ($y_{ij} = 1$), or not ($y_{ij} = 0$)
$x_i, x_i \in \{0,1\}, i,j \in V$- a binary variable representing whether the sorting center opens in the *i*-th node ($x_i = 1$), or not ($x_i = 0$)

The mathematical model can be formulated as follows:

$$f(\mathbf{x},\mathbf{y},\mathrm{z}) = \sum_{i\in V} y_j \rightarrow min \quad (6)$$

$$\sum_{i\in V} x_i \leq p \quad (7)$$

$$\sum_{j\in V} a_{ij}x_j - y_i \leq 0 \quad j \in V \quad (8)$$

Objective (6) ensures the greatest possible coverage of the graph. Condition (7) represents that the maximum number of operating service centers must not be greater than the predetermined number.

Equations (8) ensures that if the *j*-th service center is within reach of the *i*-th node, it is served from that center.

The location-allocation problem can alternatively be analyzed through a game-theoretic lens. This study examines network coverage through a duopolistic framework, where two competing service providers (players P_1 and P_2) strategically select nodes for facility placement, following the approach established in [10]. The model incorporates two key assumptions: (1) unit demand exists at each graph node, and (2) service coverage is constrained by a maximum distance threshold K between facilities and demand points. This situation can be formulated as a bimatrix game (non-constant sum game), where the payment matrix **A** of the player P_1 and the payment matrix **B** of the player P_2 are of dimension *nxn*. Elements of matrix **A** ($a_{ij}\ i,j \in V$) represent the number of nodes served if player P_1 serves in the *i*-th location and player P_2 serves in the *j*-th location and elements of matrix **B** ($b_{ij}\ i,j \in V$) represent the number of nodes served, if player P_2 serves in the *i*-th location and player P_1 serves in the *j*-th location. It is further assumed that the demand of each node is served at its closer service point, but only if the distance to the operating position is less than or equal to *K*; otherwise, the node remains unserved. In the case that the distances to both players are equal (and less than or equal to *K*), both players share the service equally.

Thus, the payment matrix elements $\left(\mathbf{A} = \{a_{ij}\}, i,j \in V\right)$ can be calculated as follows:

LET $V = \{1,2, \dots n\}$, $\mathbf{D}_{nxn} = \{d_{ij}\}$, K
LOOP $(i,j \in V)$ **DO**
$a_{ij} = 0;$
LOOP $(k,i,j \in V)$ **DO**
andIF $d_{ki} < d_{kj}$**and**$d_{ki} \leq K$ **DO** $a_{ij} = a_{ij} + 1;$
ELSEIF $d_{ki} = d_{kj}$ **and** $d_{ki} \leq K$ **DO** $a_{ij} = a_{ij} + 0{,}5;$
ENDIF
END

In this formulation of the location-allocation problem, pricing strategies between duopolists are not taken into account. Consequently, the payoff matrix for player P_2 is identical to that of player P_1, so **B** = **A.** resulting in a symmetric game. The bimatrix game solution involves identifying optimal strategies where neither player can unilaterally improve their outcome by deviating from equilibrium. Specifically, when a player adopts a Nash equilibrium strategy, any unilateral strategy change by the opponent cannot disadvantage the equilibrium-playing player. This strategic stability, formalized as the Nash equilibrium (named for John Nash, recipient of the 1994 Nobel Memorial Prize in Economic Sciences), ensures that no player benefits from deviating from their equilibrium strategy.

The solution of bimatrix games is based on the following assumptions: Both players have complete information about the conflict situation, the players are intelligent, and each player wants to maximize the payment, and knows that the opponent is also watching this.

Within the location-allocation problem framework, the analysis focuses exclusively on pure strategy solutions, defined as strategy pairs (i_0,j_0) where for all: $i,j \in V$: $a_{i_0 j_0} \geq a_{ij_0}$ and $b_{j_0 i_0} \geq b_{ji_0}$ i.e. These conditions correspond to identifying column maxima in payoff matrices A and B. Four distinct cases may emerge:

1. A unique equilibrium point exists, clearly identifying optimal pure strategies for both players.
2. Multiple equilibrium points exist, with one dominant solution Pareto-superior for both players.
3. Multiple non-dominated equilibrium points exist, necessitating mixed strategy solutions.
4. No equilibrium point exists in pure strategies, requiring mixed strategy analysis.

However, the interpretation of mixed strategies in the case of a location-allocation problem is problematic, although not impossible [21]. This study, therefore, restricts its consideration to pure strategies due to their more practical interpretation in facility location contexts. This approach aligns with the cooperative game solution concept where the optimal location pair maximizes the joint coverage function $max\{a_{ij} + b_{ji}\}$. The principle is illustrated in the next section.

3. Case Study

This study applies the proposed methodology to analyze service coverage in the Banská Bystrica region of Slovakia. Region border splitting of Slovakia is of great importance to provide services and resources to the citizen. It is organized as an autonomous entity comprising eight self-managing districts, each with its own capital. Slovakia regional structure offers a platform for service delivery and organization. However, the most effective service centers within regions are placed in different factors, such as population, facilities like roads and housing, and, of course, economic activities. The game-theoretic approach to the location-allocation problem provides a rigorous framework for determining optimal service point distribution within Slovakia's regional organizational structure.

The district towns of the Banská Bystrica region consists of thirteen cities ($V = \{1,2,\ldots 13\}$), which will be indexed as follows: 1 – Banská Bystrica, 2-Banská Štiavnica, 3-Brezno, 4-Detva, 5-Krupina, 6-Lučenec, 7-Poltár, 8-Revúca, 9-Rimavská Sobota, 10-Veľký Krtíš, 11-Žarnovica, 12-Žiar nad Hronom, 13-Zvolen.

The shortest distances (in km) between district towns of the Banská Bystrica region are represented in Table 1.

The application of Model (1)-(5) demonstrates that complete network coverage can be achieved with two service centers operating within a maximum service distance of 59.7 km. Subsequent optimization using Model (6)-(8) with parameter p=2 yields an optimal solution locating facilities at nodes 1 (Banská Bystrica) and 7 (Poltár). All numerical optimizations were performed using GAMS programming language with CPLEX 12.10.0.0 as the solver [6].

Table 1. District towns of Banská Bystrica – shortest distances

Order	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	46,8	42	50	52	83	89,8	91,1	109	79,1	57,2	41,5	21,1
2	46,8	0	89	54	21	87	94,2	138	114	60,5	33,9	34,2	31
3	42,1	88,8	0	52	94,2	77	59	49	67,6	95,8	99.3	83,6	63,2
4	49,9	54,2	52	0	51,7	35	42,3	89,3	61,1	54,1	64,8	49,2	26,7
5	52	21	94	52	0	84	91,3	142	111	39,8	67	51,4	28,9
6	82,8	87,1	77	35	84,2	0	18,6	81,1	31	35	98	82,4	59,9
7	89,8	94,2	59	42	91,3	19	0	59,7	25,6	49,2	105	89,5	67
8	91,1	138	49	89	142	81	59,7	0	57,4	116	148	133	112
9	109	114	68	61	111	31	25,6	57,4	0	64,5	123	108	85
10	79,1	60,5	96	54	39,8	35	49,2	116	64,5	0	94	78,4	55,9
11	57,2	33,9	99.3	65	67	98	105	148	123	94	0	19,7	41,5
12	41,5	34,2	84	49	51,4	82	89,5	133	108	78,4	19,7	0	25,7
13	21,1	31	63	27	28,9	60	67	112	85	55,9	41,5	25,7	0

Now formulate the problem as a bimatrix game, where K=59,7. The game is characterized by a payoff bimatrix with elements $c_{ij} = \{a_{ij}; b_{ji};\}, i, j \in V$, where the first element a_{ij} represents the number of served nodes of player P_1, the second element b_{ji} represents the number of served nodes of player P_2 in the case the player P_1 serves in node i and the player serves in node j. The payoffs related to the nature of nodes at each decision point imply the extent to which the service center will cover the area and thus, will be accessible.

Table 2. Bimatrix game

Order	1	2	3	4	5	6	7	8	9	10	11	12	13
1	4;4	4;4	7;3	(6);5	6;3	(7);5	(7);6	8;3	(8);4	(7);4	5;3	3;5	2;(8)
2	4;4	3,5;3,5	5;5	5;6	4;4	6;5	6;(7)	7;4	7;4	6;4	5;2	2;5	3;6
3	3;7	5;5	2,5;2,5	3;8	4;6	3;6	3;5	4;2	4;3	4;5	5;4	3;7	3;(9)
4	5;(6)	6;5	8;3	5;5	(7);3	(7);4	(7);5	(9);3	(8);4	(7);3	(8);3	5;(6)	5;(6)
5	3;6	4;4	6;4	3;(7)	3,5;3,5	5;5	6;6	7;4	7;4	6;3	6;2	3;5	3;6
6	5;(7)	5;6	6;3	4;(7)	5;5	3;3	4;4	6;2	5;2	4;3	5;5	5;6	4;(7)
7	6;(7)	(7);6	5;3	5;(7)	6;6	4;4	3,5;3,5	5;2	5;2	6;3	7;5	(7);6	(6);(7)
8	3;8	4;7	2;4	3;(9)	4;7	2;6	2;5	2;2	2;3	3;6	4;5	4;7	4;9
9	4;(8)	4;7	3;4	4;(8)	4;7	2;5	2;5	3;2	2;2	4;4	4;5	4;7	4;(8)
10	4;(7)	4;6	5;4	3;(7)	3;6	3;4	3;6	6;3	4;4	3;3	5;5	4;6	3;(7)
11	3;5	2;5	4;5	3;(8)	2;6	5;5	5;7	5;4	5;4	5;5	2,5;2,5	2;5	2;7
12	5;3	5;2	7;3	(6);5	5;3	6;5	6;(7)	7;4	7;4	6;4	5;2	3,5;3,5	2;(7)
13	(8);2	6;3	(9);3	(6);5	6;3	(7);4	(7);(6)	(9);4	(8);4	(7);3	7;2	(7);2	4,5;4,5

For both players, the best answers are marked (13,7). Therefore, it is obvious that the game has two Nash equilibria; to build service centers in nodes 13 and 7.

The cooperative solution space is examined through joint payoff analysis. Table 2 presents the coalition values: $\{a_{ij} + b_{ji}\}$:

Table 3. Cooperative solution

Order	1	2	3	4	5	6	7	8	9	10	11	12	13
1	8	8	10	11	9	12	13	11	12	11	8	8	10
2	8	7	10	11	8	11	13	11	11	10	7	7	9
3	10	10	5	11	10	9	8	6	7	9	9	10	12
4	11	11	11	10	10	11	12	12	12	10	11	11	11
5	9	8	10	10	7	10	12	11	11	9	8	8	9
6	12	11	9	11	10	6	8	8	7	7	10	11	11
7	13	13	8	12	12	8	7	7	7	9	12	13	13
8	11	11	6	12	11	8	7	4	5	9	9	11	13
9	12	11	7	12	11	7	7	5	4	8	9	11	12
10	11	10	9	10	9	7	9	9	8	6	10	10	10
11	8	7	9	11	8	10	12	9	9	10	5	7	9
12	8	7	10	11	8	11	13	11	11	10	7	7	9
13	10	9	12	11	9	11	13	13	12	10	9	9	9

The table shows that all network coverage can be achieved by building centers in nodes 1 and 7; 2 and 7; 7 and 12; 7 and 13; 8 and 13.

Of these points, however, only 7 and 13 or 13 and 7 respectively are Nash equilibriums, so this solution should be preferred regarding the stability of the whole system.

The matrix exhibits the payoffs of both parties outlined by their strategies. As will be illustrated by the example, if player P_1 chooses the node 1 and player P_2 choose the node 7, player P_1 serves 7 nodes and player P_2 serves 6 nodes. The same coverage of the net is achieved if player P1 serves from node 13 and player P_2 serves from node 7 (or vice versa).

However, assuming the primary concern is the "common good" and the secondary concern is system stability, management should prefer a Nash equilibrium solution rather than building centers in nodes 1 and 7 (marked in red).

A critical distinction emerges when examining the stability of alternative solutions. The configuration with facilities at nodes 1 and 7, while providing complete coverage, represents a strategically unstable solution. This instability arises because P_2 has a rational incentive to unilaterally deviate to node 8, as such relocation would increase its served nodes by two. This potential deviation demonstrates the non-equilibrium nature of the (1,7) solution, as it fails to satisfy the Nash condition where neither player can benefit from unilateral strategy changes.

However, in that case, player P_1 would serve only two nodes, and the total number of nodes served would decrease to 10. Player P_1 would then want to move the service to node 7, where they would serve six nodes (compared to two), and the game would end in a Nash equilibrium.

The established Nash equilibria denote the strategic patterns where neither of the players has a motivation to break it, it is offered without prompting a similar change by his opponent. As a result, at those points of equilibrium, both will be happy with their choice, as their counterparts also made the same decision.

In both cases, all nodes are served; however, the configuration with facilities at nodes 13 and 7 yields a stable system.

The presence of the game's equilibriums provides information about decision-making processes' strategic. These equilibriums occasionally offer stable outcomes, but they also are not the dominant ones, which means that the "best" solution strongly depends on many factors including the preferences of decision makers, and the broader service provision context [10], [21].

For this reason, management should give close attention to the consequences of each Nash equilibrium, so that the intentions towards ensuring that there are maximum number of people covered is not being complied, the same time accessibility is not compromised while stability is maintained. One way to achieve this is by choosing to employ the Nash equilibrium as a basis for decision-making to attain a stable solution while also making good progress toward meeting the service delivery objectives in the Banská Bystrica region.

The analysis proceeds by examining solutions across varying service distance thresholds $K = \{50,40,30,20\}$. Table 3 presents the optimal coverage configurations for each threshold value, along with the corresponding minimal inter-nodal distances: $\min_{i,j}\{d_{ij}\}, i,j \in V$.

Table 4. Solution for different distances

K	50	40	30	20
# of covered nodes	11	10	8	4
Location of service centers	2,6;2,7; 6,13;7,13	6,2; 6,13	7,1	6,11;6,12; 7,11;7;12
Nash equilibrium	Not	6,1	7,1	6,11;6,12; 7,11;7;12

Table 4 shows that at the required maximum distance of 50 highest distance, the 11 nodes of the graph (possible 4 solutions) will be covered. However, all four plans regarding the service center's location fall into non-Nash equilibrium classes.

At $K = 40$ (a lower distance), ten nodes are covered (two solutions). A Nash equilibrium solution only has one service center localized, in either node 6 or node 13 (the nodes for the service centers), for stability.

At $K = 30$, the point of service provides coverage for the eight locations. The only placements are nodes 7 and 13, which repeat after a certain time.

With $K = 20$ distance, coverage is extremely low (four at most). Surprisingly, for all four of these the possible moves, every move becomes a Nash equilibrium. Thus, this means that such a network has a high stability degree where the providers' migration to other regions is not a priority because no migration is at the advantage of them.

The analysis demonstrates an inherent trade-off between service coverage and system stability mediated by the distance threshold parameter K. Two distinct operational regimes emerge:

At elevated K values, expanded service coverage is achieved through greater nodal accessibility, though this potentially compromises system stability due to the possible absence of Nash equilibrium solutions. Under these conditions, service providers face intensified incentives for unilateral relocation, which can potentially undermine coordinated spatial arrangements.

Conversely, diminished K values ensure guaranteed stability through Nash equilibrium solutions but impose stricter distance limitations that constrain overall service coverage.

This constrained regime naturally reduces providers' incentives to deviate from optimal locations, fostering system-wide stability at the expense of reduced accessibility.

The parametric analysis demonstrates that while extensive coverage distances (large K) improve service accessibility, they may undermine strategic stability. Conversely, constrained coverage distances (small K) ensure equilibrium conditions but require accepting more limited-service availability. This trade-off presents policymakers with crucial design considerations when planning facility networks.

The value of K that maximizes the overall objective depends on the priorities. When coverage is the primary objective, it may be justifiable to prefer larger values of K. However, if stability is prioritized, then a lower value of K can be recommended, with fewer accessible places but converging to the Nash equilibrium.

The findings from the location-allocation analysis in Slovakia's Banská Bystrica region highlight two critical and interdependent considerations: The trade-off between stability and coverage. By continuously varying the distance threshold *K, the study reveals how adjustments influence both the extent of service coverage and the conditions for a* Nash equilibrium.

Higher values of K enhance regional inclusivity by expanding service coverage, but this comes at the cost of potential instability, as equilibrium conditions may no longer hold. Conversely, lower values of *K reinforce stability through Nash equilibria; however,* this restricts coverage, leaving significant portions of the region underserved.

While the two-player model captures essential strategic dynamics, real-world scenarios often involve more complex interactions among multiple stakeholders. The theory of *n-player games extends this framework, providing tools to analyze decision-making in systems with numerous participants and complex* relationships. The generalized solution to the location-allocation problem aligns with cooperative game theory, where optimal placements maximize coverage while satisfying equilibrium conditions in a non-cooperative setting.

Although efficiently identifying all equilibrium points in n-player non-cooperative games remains a computational challenge, the present problem can be addressed by verifying pure-strategy equilibria, which admit tractable solutions under certain constraints [10]. This approach ensures the practical applicability of facility placement strategies while maintaining theoretical rigor.

4. Conclusion

This study investigates the location-allocation problem in Slovakia's Banská Bystrica region through a game-theoretic optimization framework. The research employs a bimatrix game formulation to model strategic interactions between two competing service providers, with the dual objectives of maximizing coverage while maintaining system stability.

The analysis identifies Nash equilibrium solutions that represent mutually optimal facility placements, where neither provider has an incentive to deviate from their chosen location unilaterally.

This equilibrium provides a stable solution, ensuring that the system remains robust and predictable over time. The results show that by focusing on these equilibrium points, decision-makers can avoid disruptions in service delivery that would otherwise occur if players deviated from their optimal strategies.

In the specific case of the Banská Bystrica region, the model demonstrated that placing service centers in nodes 7 and 13 achieved both full coverage and system stability. This configuration was identified as a Nash equilibrium, ensuring that both service centers can operate without the risk of relocation or instability induced by competition. Moreover, alternative locations, such as nodes 1 and 7, were found to be less stable, as one of the centers would be incentivized to change its location, resulting in a reduction in overall coverage and system efficiency.

The present approach provides policymakers with practical insights, demonstrating that prioritizing equilibrium-based solutions yields stable and efficient service center placement. By solving the problem using both location-allocation model and game theory, the study highlights the importance of strategic decision-making in minimizing costs and disruptions. The stability of the Nash equilibrium ensures that service centers remain in their optimal positions, providing continuous and reliable service to the entire region.

This study also offers a framework for further application in regions with similar service center needs. The game-theoretical approach enables decision-makers to consider competition between service centers while ensuring maximum coverage without compromising long-term stability. The findings presented in this case study can serve as a reference for addressing the challenge of balancing competitive dynamics with operational efficiency, demonstrating the practical benefits of integrating game theory into location-allocation problems.

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