Analysis of the Relationship between Money Growth and Inflation Using Wavelet Coherency*

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Different theories of money lead to different conclusions about whether there is a stable long-run relationship between money growth and inflation. In this analysis, the author examines the wavelet coherency between the growth in the M2 monetary aggregate and inflation in Hungary. Based on the findings, there is no robust long-run relationship between the two variables. When filtering out movements in the HUF exchange rate, the coherency between money growth and inflation is eliminated even over the period from the mid-2000s to the early 2010s. This also empirically supports the assumption that monetary policy does not affect inflation by shaping the money supply.

Journal of Economic Literature (JEL) codes: C10, E31, E40

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1. Introduction

Regarding the impact of monetary policy, one recurring question is whether there is a clear, stable relationship between the evolution of money supply and inflation. This is not an easy issue to assess as different theories of money yield different conclusions about whether there is a relationship, and if so, how robust and how volatile it is over time.

The quantity theory of money suggests a linear relationship between money growth and inflation, with an increase in the former, *ceteris paribus*, resulting in an increase in the latter. However, in other theoretical models of money, the link between money growth and inflation is far from clear: for example, proponents of Modern Monetary Theory and Post-Keynesian Economics argue that there is no simple, linear relationship between money growth and inflation (*Mitchell et al. 2019*). Non-linearity is particularly conspicuous where there is a high level of dollarisation in an economy. In such cases, money growth may lead to a high rate of inflation (*Levy*

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Yeyati 2006), posing significant challenges for monetary policy (*Alvarez-Plata – Garcia-Herrero 2008*).

Moreover, empirical studies have not clearly confirmed the relationship between inflation and money growth. Based on an analysis by *McCallum – Nelson* (*2010*), macroeconomic data from the G7 nations support the conclusions based on the model of the quantity theory of money. The relationship between the two variables has deteriorated considerably since the 1980s and shows significant heterogeneity among countries depending on the level of development (*Gertler – Hofmann 2018*). In economies with more liberalised financial systems and lower rates of inflation, the link is weaker (*Estrella – Mishkin 1997; De Grauwe – Polan 2005; Teles et al. 2016; Benati 2009*). In the current global economic environment characterised by supply shocks, the relationship between the two variables may also be distorted by the evolution of supply factors due to inflation being at a high level (*Fornaro – Wolf 2023*).

At the same time, the question has practical significance for central banks as many of them also conduct asset purchase or sale programmes, in addition to shaping interest rates. If money growth results in inflation, a central bank may generate inflation via asset purchases or contribute to curbing inflation via asset sales. However, analyses show that the asset purchase programmes implemented by major global central banks over the last decade had no clear inflationary effect, despite considerable money growth (*Borio – Zabai 2018; Csiki 2022*). In New Keynesian monetary policy models – such as the basic monetary policy model applied by the Magyar Nemzeti Bank (the central bank of Hungary, MNB) (*Békési et al. 2016*) – money growth is either not reflected in or does not affect the equilibrium of real variables.

This empirical analysis assesses the robustness of the relationship between domestic money growth and inflation. To the best of my knowledge, it is the first in Hungary to examine the relationship between these two variables by using wavelet coherency, which allows the evolution of the relationship to be tracked over time, while also being able to describe the changes in different frequency components of the time series. Many studies underline the variability of the relationship between the two components over time (*Hofmann 2006; Gertler – Hofmann 2018; Berger et al. 2023*). Tracking both the time and frequency dimensions allows for studying general, non-stationary time series, while also exploring how close the relationship between the two variables is in different frequency domains (*Hajnal et al. 2023*). In addition, the wavelet decomposition procedure also makes it possible to study the lead-lag relationship.

Wavelet decomposition involves using the sum of functions of limited lengths (so-called "wavelets") to approximate a time series at each point in time. Wavelets are translated and scaled to varying degrees and thus are capable of appropriately describing the time series components at different frequencies. Wavelet coherency is a key indicator in this study and, by analogy with correlation, it shows how robust the relationship between the two variables is at a given point in time and a given frequency.

The findings show that, overall, no robust relationship can be established between the increase in the domestic money supply and inflation. In the context of Hungary, *Komáromi* (*2007*) argues that the monetary base is a given for the central bank and thereby does not directly provide any input to expected inflation. The present analysis expands on that argument by calling into question not only the inflationary effect of the change in the monetary base, but also whether there is a stable relationship between total money supply in the economy and inflation. The findings were verified using a number of robustness tests. Significant coherency between money growth and inflation was identified during the period from the mid-2000s to the early 2010s, which, however, is eliminated when one filters out the movements in the HUF exchange rate. Consequently, it can be concluded that the evolution of money supply did not have any effect on inflation in the period under review.

Section 2 of the study provides an overview of the main arguments for and against the relationship between inflation and money growth with both an international and a domestic dimension. In *Section 3*, the wavelet decomposition process that was the basis for this analysis is discussed, along with the data used in the analysis. *Section 4* contains the key findings of the study. A summary of the conclusions is presented in *Section 5*.

2. Review of literature

A central theme in monetary economics is the theory of money and, as a related issue, the long-run relationship between money growth and inflation. In the period before the 2008 global financial crisis, many economists found empirical evidence for the existence of a one-to-one relationship between an increase in broad money supply in the economy and in the price level (*e.g. Vogel 1974; Lucas 1996; Gertler – Hofmann 2018*). These observations are consistent with the quantity theory of money, where the key idea is that each unit of exogenous money growth results, *ceteris paribus*, in a unit increase in nominal variables in the long term, without affecting real variables in the economy (*McCallum – Nelson 2010*). This is equivalent to the long-run neutrality of money.

The key idea of the quantity theory of money is far removed from real-world applications. The monetary base directly controlled by central banks does not fundamentally provide any direct information on the expected evolution of inflation, as even this includes many factors that are external and should be thought of as a given for the central bank (*Komáromi 2007*). The endogenous theory of money is more suitable to describe the functioning of modern monetary policy (*Ábel et al. 2016*). According to this theory, banks make decisions on their activities in response to the relevant economic circumstances, and thus, overall, the banking system, which provides a significant share of the money supply, shapes its money supply endogenously. *McCallum – Nelson* (*2010*) and *Ábel* (*2019*) point out that, fundamentally, money creation by central banks is also an endogenous response to economic processes, and accordingly money growth cannot be considered exogenous. The endogenous theory of money can be supplemented to establish a consistent framework by assuming that inflation is essentially determined by labour market conditions in the economy, including, in particular, the state of the labour market in relation to full employment (*Mitchell et al. 2019*). Consequently, the theory suggests no linear relationship between the two variables. Non-linearity is particularly conspicuous where there is a high level of dollarisation in an economy, because in such cases money growth may lead to a high rate of inflation (*Levy Yeyati 2006*). As a result, the margin for monetary policy is significantly limited (*Alvarez-Plata – Garcia-Herrero 2008*).

Empirical results support the conclusions of the quantity theory of money in certain cases and those of the endogenous theory in others. In general, a robust empirical relationship was observed between money growth and inflation in the pre-2000s (*Lucas 1996; Haug – Dewald 2004*). However, a number of findings suggest that, from the 2000s, the strength of the relationship deteriorated (*Hofmann 2006; Carstensen 2007; Gertler – Hofmann 2018*). This observation is also supported by the fact that the asset purchase programmes implemented by central banks in the wake of the crisis have had unclear direct effects on inflation in developed economies (*Yu 2016; Borio – Zabai 2018; Csiki 2022*). The deterioration of this link can also be explained by the liberalisation of financial systems and an environment characterised by persistently lower inflation (*Estrella – Mishkin 1997; De Grauwe – Polan 2005; Teles et al. 2016; Benati 2009*). However, *McCallum – Nelson* (*2010*) note that using cross-country averages and taking long moving averages of time series in empirical analyses may also distort the relationship and, by excluding them, a strong link is identified between the two variables based on G7 data.

There are certain cases where supply shocks may have a lasting impact on inflation, also distorting the link. Studying the period of the COVID pandemic and the energy price shock, *Fornaro – Wolf* (*2023*) concluded that negative supply shocks may generate permanent output losses. This, however, is accompanied by a permanent rise in inflation that cannot be addressed by tight monetary policy in itself, but rather only in coordination with fiscal policy (*Fornaro – Wolf 2023*). Against this background, in the current period of inflation, other factors, such as energy prices or the evolution of the tightness of the measures taken during the pandemic, may have significantly distorted the link between money growth and inflation.

As the quantity theory of money assumes a long-run relationship between money growth and inflation, empirical analysis also looks back at a long-run period in order to work with low-frequency time series components. In these types of analyses, one of the processes that is gaining increasing attention in literature is wavelet decomposition. Using wavelets, *Mandler – Scharnagl* (*2014*) found that comovements between the 8- to 16-year components of money growth and inflation in the Eurozone deteriorated after the 1990s.[1] Based on US data, the link between the two variables also weakened after the turn of the millennium (*Scharnagl – Mandler 2015*). At the same time, *Jiang et al.* (*2015*) also used wavelets to show that, from the mid-2000s, the relationship between money growth and inflation strengthened in China as opposed to developed countries.

The wavelet decomposition process is not widespread in the domestic economic literature. Applications in Hungary include *Uliha* (*2016*), who identified links between oil prices and economic variables in Sweden and Norway. *Uliha – Vincze* (*2018*) studied the relationship between the evolution of exchanges rates and consumer prices, while *Hosszú – Lakos* (*2021*) used the wavelet transform to determine the credit-to-GDP gap with the best early warning performance. In its Financial Stability Report, the MNB uses the wavelet transform to analyse the time required for monetary transmission in different countries in the region (*MNB 2023: pp. 65–66*). The Hungarian application closest to the present study in its theme is *Hajnal et al.* (*2023*). Its authors used wavelet decomposition and error correction models to analyse household and corporate deposit transmission in the Central and Eastern European region. Their findings show that in recent years the region's higher-inflation environment was characterised by weakening deposit transmission and a slowdown in deposit-rate repricing, particularly in the household segment (*Hajnal et al. 2023*). To my knowledge, my study is the first to deploy wavelet decomposition to examine the relationship between money growth and inflation in Hungary.

[1] However, for components with periods longer than 16 years, the authors found a stable, almost one-to-one relationship.

3. Methodology

3.1. Wavelet transform

In my analysis, I apply the wavelet decomposition process, which is a part of the spectral analysis toolkit. Spectral analysis is aimed at decomposing the time series into the sum of periodic functions at different frequencies, while maintaining variance. The best-known example for this is the Fourier transform that uses sinusoidal functions for decomposition. The disadvantage of the Fourier transform is that it requires the stationarity of the time series as a prerequisite. Moreover, as the decomposition is constant over time, it does not provide any information about the temporal distribution of each frequency component. Thus, the Fourier transform does not allow for identifying any structural breaks that may change the dominant frequency.

Wavelet decomposition addresses these shortcomings of the Fourier transform. The process and the associated mathematical/theoretical background is a result of multidisciplinary efforts (*Daubechies 1992; Torrence – Compo 1998; Grinsted et al. 2004*). The decomposition itself is about locally approximating a time series with the sum of so-called "wavelets" of limited lengths, instead of breaking it up into sinusoidal functions at different frequencies. Accordingly, with wavelet decomposition, the time series is not bound by stationarity restrictions.

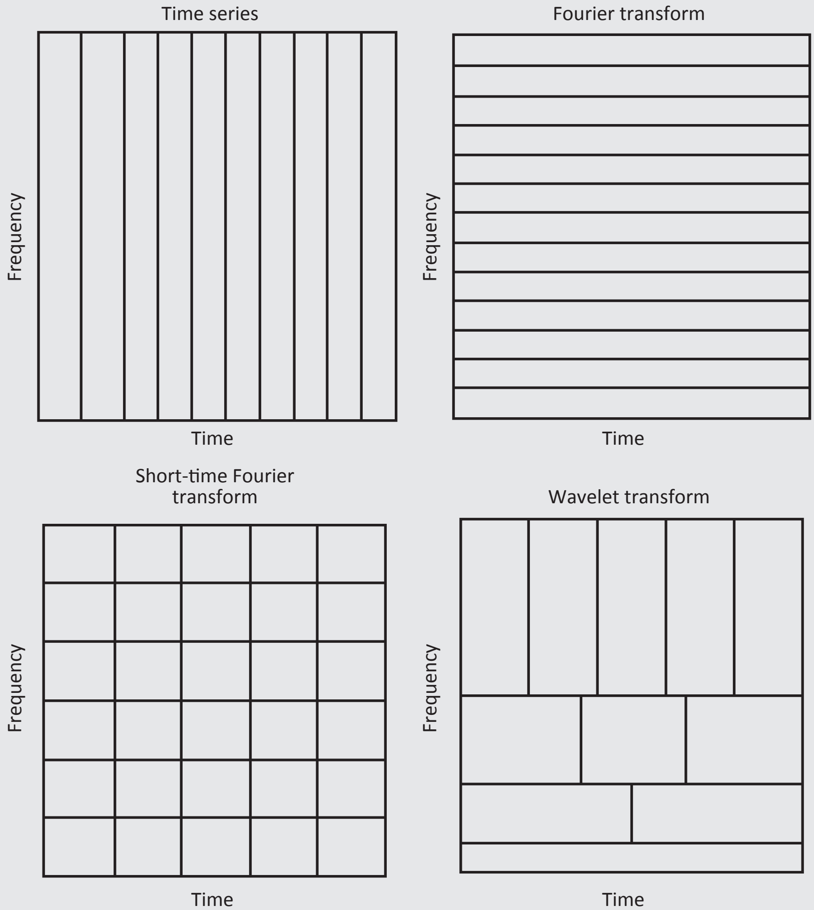
Wavelets have a window of interpretation that can be translated and also stretched or compressed (scaled), making them suitable for describing periodical movements at different frequencies. More time-stretched wavelets capture the information content of the lower-frequency components of a time series, while more compressed wavelets capture the higher-frequency components. By choosing an appropriate form for the function (the so-called "mother wavelet"), all square-integrable functions can be generated as a sum of translated, stretched and compressed wavelets, meaning that this process can be applied to any economic time series. Wavelet decomposition can be carried out through discrete or continuous wavelet transform. I used the continuous version in this analysis.[2]

As a result of compressing, the resolution of the wavelet decomposition is scale-dependent. This means that it assigns shorter time windows to higher-frequency components of a time series and longer time windows to lower-frequency components (*Figure 1*). This is important because it shows higher-frequency components in the best detail possible, while also being capable of representing the evolution of lower-frequency components. However, due to methodological specificities, the resolution will be distorted at certain points in time and on certain

[2] I used the R package "biwavelet" by *Gouhier et al.* (*2021*) for the analysis.

scales. The distortion-free area is called the cone of influence (COI) in wavelet literature (*Torrence – Compo 1998*). Results outside the COI are generally ignored.

Figure 1
Comparison of different time-frequency resolutions



Source: Uliha (2016: 512)

A correlation-like indicator, called wavelet coherency, can be calculated between two time series at a given point in time and on a given scale. This indicator is between 0 and 1 and essentially represents how strong the relationship between the two time series is at a given time-frequency point. If the evolution of wavelet coherency between the two time series is to be studied while also eliminating the effect of an additional variable, partial wavelet coherency can be used, as defined and interpreted analogously to partial correlation.

In a wavelet decomposition analysis framework, an indicator that can be thought of as similar to the sign of correlation is wavelet phase difference. It shows the lead-lag relationship between the wavelets of the two time series at the relevant time-frequency point. It is important to note that a lead-lag relationship does not mean causality: if time series *x* is ahead of *y*, it is still possible that, for example, it is actually *y* that causes *x*, but its effect is spread over more than the entire period of the wavelet.

One central task of the present analysis is to identify the level of wavelet coherency between inflation and money growth at different points in time and frequency. I ran Monte Carlo simulations to determine the points where coherency is significantly different from noise-generated coherency. A detailed description of the methodology applied is included in the *Appendix*.

3.2. Data

In the empirical analysis, I explore the robustness of the relationship between M2 money growth and inflation in Hungary. M2 is a monetary aggregate that includes assets that can be liquidated quickly and at little cost; its growth is considered by many as an indication of inflationary pressures (*Hallman et al. 1991; De Grauwe – Polan 2005*).[3]

I used monthly observations for the period from January 1999 to September 2022. CPI inflation data for Hungary were obtained from the Hungarian Central Statistical Office (HCSO). The year-on-year growth of the unadjusted M2 monetary aggregate was calculated based on MNB data. Besides the two main variables, additional time series were also used in order to assess the robustness of the findings in several aspects. These included the core inflation indicator (HCSO), the year-on-year growth of unadjusted M1 and M3 monetary aggregates (calculated based on MNB data), annualised GDP growth observed on a quarterly basis and adjusted for seasonality and calendar effects (HCSO) and the 3-month interbank (BUBOR) rate and HUF/EUR exchange rate observed on a daily basis (Bloomberg). Monthly averages of daily-frequency data were calculated. In preparation for the wavelet decomposition process, the time series were standardised and differentiated.[4]

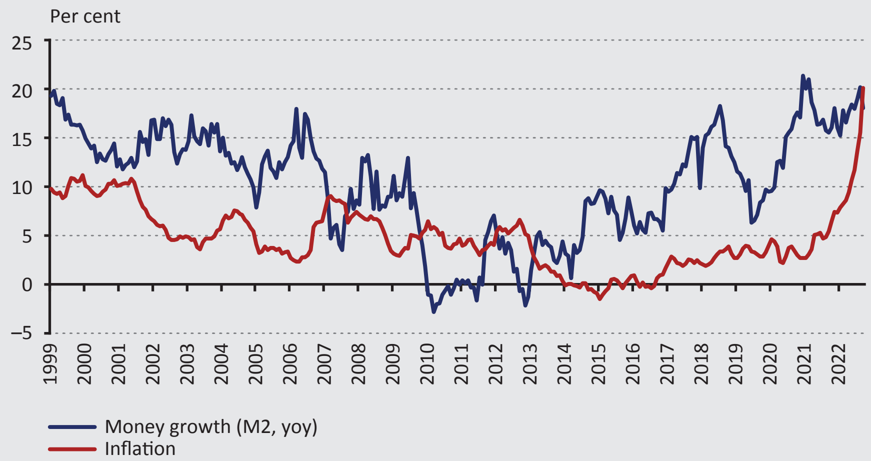
The two main time series studied in the analysis are presented in *Figure 2*. Based on this figure, there is no stable long-run relationship between the two variables. In certain periods, the change in the rate of money growth is followed by a change in inflation, with a few months' delay. Of these, the period from July 2019 to December

[3] M2 is comprised of cash outside monetary financial institutions, demand and current-account deposits and time deposits with an original maturity of up to 2 years.

[4] The wavelet methodology is based on a local approximation, and so dispensing with standardisation would not have had a qualitative impact on findings. Differentiation was necessary due to outliers at the end of the inflation time series and the coefficient estimation process applied during wavelet decomposition; this, however, also did not have any effect on the findings.

2020 stands out as one when the M2 growth rate rose from 6.5 per cent to over 20 per cent, followed by inflation moving on an upward trajectory from January 2021. Overall, however, there was a substantial part of the period under review when inflation did not follow the fluctuations in the growth rate of money, nor is there an inverse correlation to be established based on the figure.

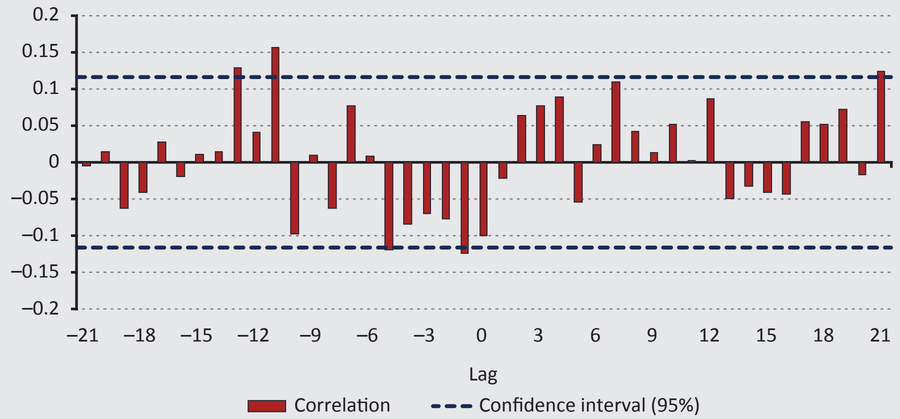
Figure 2
Annual M2 money growth and the evolution of inflation in Hungary



Source: HCSO, MNB

Prior to wavelet decomposition, I explored the cross-correlation structure between money growth and inflation, as presented in *Figure 3* with different lags. With most lags, the correlation between the two variables does not significantly differ from 0. However, with lag configurations where inflation was 13, 11, 5 months or 1 month ahead of money growth, the correlation differed significantly from 0, as it did when money growth was 21 months ahead of inflation (*Figure 3*). This may suggest that changes in inflation might lead to the transformation of lending processes and thereby a change in money growth in the short run, while, in the long run, the evolution of money supply may determine inflation. However, these are the only handful of cases where correlation indicators differ significantly from zero, meaning that the relationship can also be attributed to coincidence, especially considering correlations calculated with nearby lags.

Figure 3
Correlation between annual M2 money growth and inflation with different lags



Note: For negative lag values, money growth follows inflation, while positive values indicate the opposite.

Source: Calculation based on HCSO and MNB data

4. Findings

4.1. Coherency between money growth and inflation

The correlation structure provides no clear evidence on whether there is a relationship between money growth and inflation. Accordingly, it is necessary to continue the analysis using wavelet decomposition. In this section, I present the resulting findings.

The colour scale in *Figure 4* shows how robust wavelet coherency is between the two time series at a given point in time and on a given scale. Coherency should be thought of as a correlation coefficient in this context. The areas with black outlines indicate wavelet coherency significantly different from the coherency structure generated by red noise,[5] where significance was determined using Monte Carlo simulations.

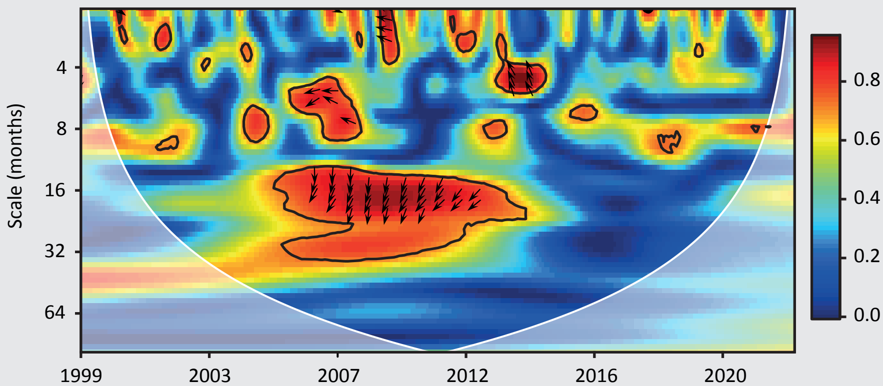
Using wavelet decomposition, it is established that there is no stable long-run relationship between the two variables. *Figure 4* shows that changes in money growth and inflation in the short run (scales of up to a few months) exhibit significant wavelet coherency over certain periods, but this phenomenon is essentially not stable. However, the roughly 16- to 32-month trends of the two

[5] The coherency structure of red noise is one where coherency increases as frequency decreases (scale increases). On the other hand, the coherency structure of white noise is constant across frequencies (see *Torrence – Compo 1998*).

variables were significantly correlated between 2005 and 2013, and the value of the wavelet coherency indicator was typically over 0.8 in this time frame and over those scales. Coherency here may well be related to an event in economic history. In 2001, Hungary's central bank implemented inflation targeting, which may have contributed to a considerable shift in the relationship between inflation and money growth.

Within the significant ranges, the direction of the arrows shows the phase difference between coherent wavelets: for arrows pointing down, money growth is ahead of inflation, for arrows pointing up, inflation is ahead of money growth, for arrows pointing right, the two wavelets are in phase and for arrows pointing left, the two wavelets are in an antiphase relation. Based on the phase difference between coherent wavelets, the direction of the relationship evolved in line with the quantity theory of money: the change in money growth rate was ahead of the change in inflation. However, apart from this range that spans 16 scales and 8 years, wavelet coherency was quite low and did not differ significantly from 0 in the time frame and over the scales under review (*Figure 4*).

Figure 4
Wavelet coherency and phase difference between annual M2 money growth and inflation



Note: The horizontal axis shows time and the vertical axis shows wavelet scale s (in months) which corresponds to the component of the time series with the same period. The colour scale shows how robust wavelet coherency is between the two time series at a given point in time and on a given scale, to be understood as a correlation coefficient in the time-frequency domain. In the figure, the white outlines indicate the COI, outside of which the wavelet decomposition is distorted.

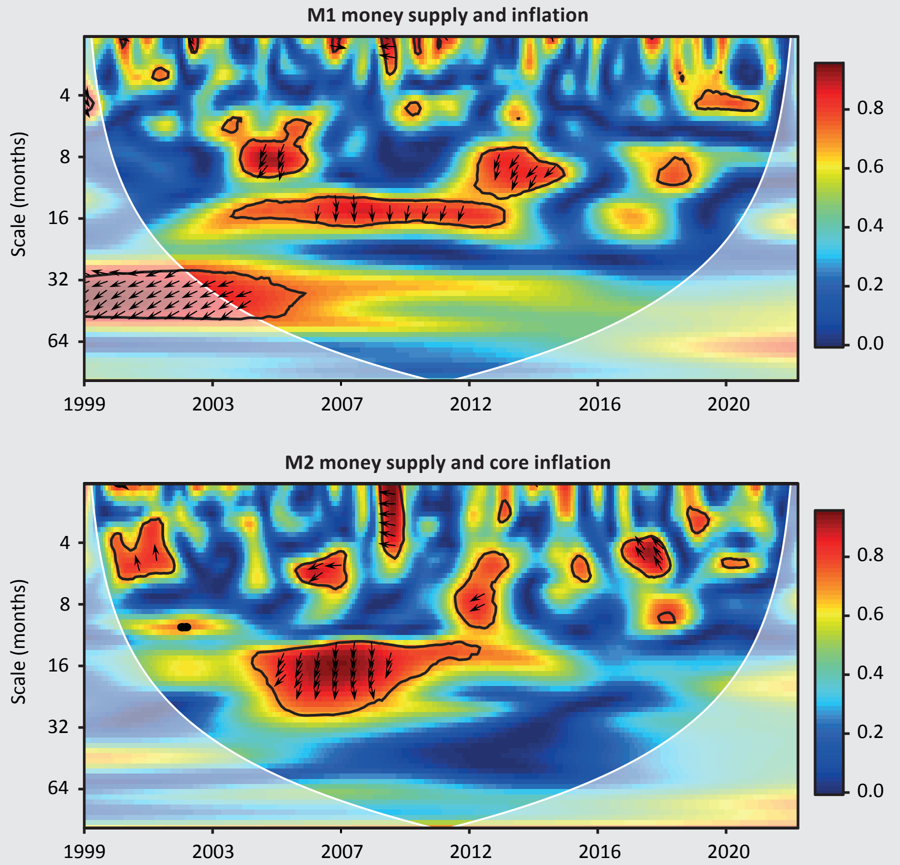
Source: Compiled from MNB and HCSO data

4.2. Robustness testing

The findings are robust with respect to the selection of both the monetary aggregate and the inflation indicator. As seen in *Figure 5*, there is no qualitative change in the key findings compared to the previous subsection if the growth rate of the

M1 aggregate is studied instead of that of the M2 aggregate or if core inflation is considered instead of the headline inflation indicator.[6] As before, these cases also show periodic coherency over the short scales as well as coherency at 16- to 32-month scales from the mid-2000s to the early 2010s. Overall, however, wavelet coherency did not differ significantly from 0 in most time-frequency ranges.

Figure 5
Wavelet coherency and phase difference between the annual growth of different money supplies and inflation/core inflation



Note: The horizontal axis shows time and the vertical axis shows wavelet scale s (in months) which corresponds to the component of the time series with the same period. The colour scale shows how robust wavelet coherency is between the two time series at a given point in time and on a given scale, to be understood as a correlation coefficient in the time-frequency domain. In the figure, the white outlines indicate the COI, outside of which the wavelet decomposition is distorted.

Source: Compiled from MNB and HCSO data

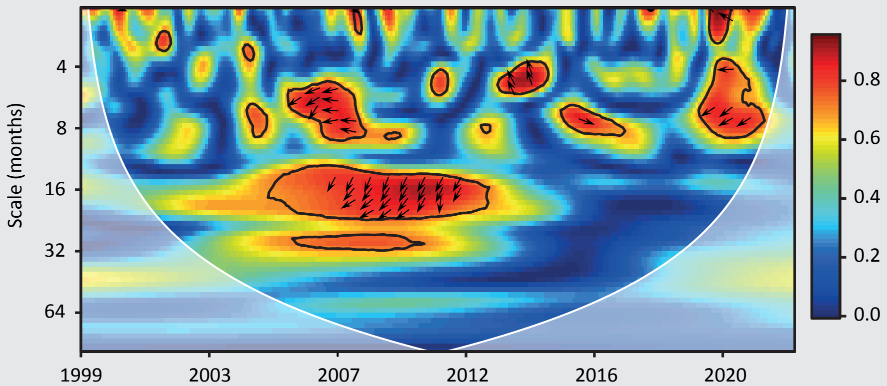
[6] In addition to M1, robustness testing was also carried out using the M3 monetary aggregate, with no changes in the key findings.

4.3. A look at omitted explanatory variables

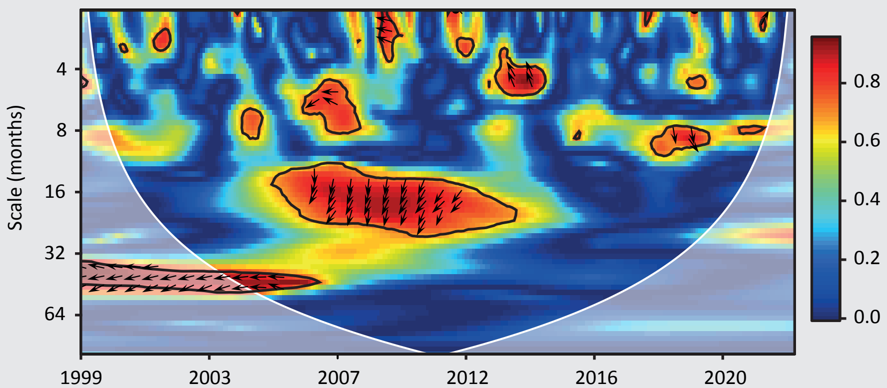
I also studied the impact of additional variables on the findings. High economic growth may generate considerable money demand and thereby significant money growth. As the omitted variable might distort findings, I also performed the analysis with money growth adjusted for GDP growth.[7] I made the adjustment using rolling GDP growth. The key findings remained unchanged in this case as well.

Figure 6
Impact of GDP growth and interest rates on findings

Wavelet coherency and phase difference between GDP-growth-adjusted M2 money growth and inflation



Wavelet coherency and phase difference between annual M2 money growth and inflation, with partial effects of 3-month BUBOR rate eliminated



Note: The horizontal axis shows time and the vertical axis shows wavelet scale s (in months) which corresponds to the component of the time series with the same period. The colour scale shows how robust wavelet coherency is between the two time series at a given point in time and on a given scale, to be understood as a correlation coefficient in the time-frequency domain. In the figure, the white outlines indicate the COI, outside of which the wavelet decomposition is distorted.

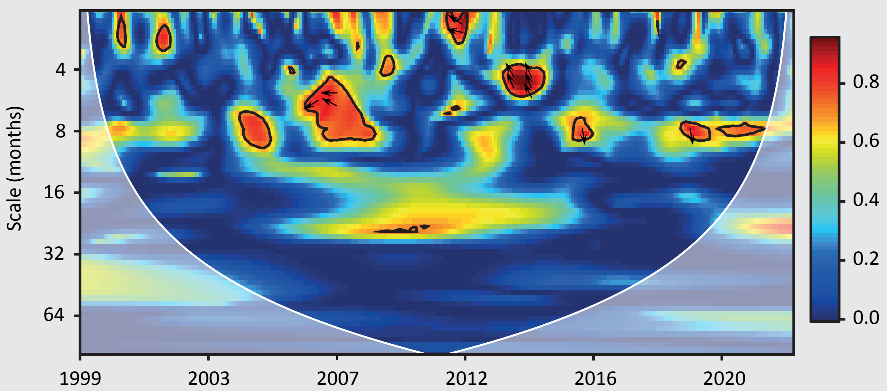
Source: Compiled from MNB, HCSO and Bloomberg data

[7] I.e. by subtracting the GDP growth rate from the monetary aggregate growth rate.

In addition, the evolution of interest rates may affect both inflation and money growth at the same time, thus producing endogeneity issues. Accordingly, as an additional robustness test, I repeated the wavelet analysis of the relationship between the two variables by eliminating the partial effects of the 3-month BUBOR rate. To eliminate rate effects, I calculated partial wavelet coherency analogously with partial correlation.[8] This also resulted in a coherency heat map similar to the original findings (*Figure 6*).

However, additional conclusions could be drawn from filtering out the effects generated by the fluctuations in the exchange rate.[9] When also considering the evolution of the exchange rate, coherency between the 16- to 32-month scales of the variables is found to be almost completely eliminated over the period from the mid-2000s to the early 2010s. As shown in *Figure 7* presenting the evolution of partial wavelet coherency, the range where wavelets were coherent in previous figures essentially never differs significantly from 0 when the partial effects of movements in the HUF/EUR exchange rate are eliminated (*Figure 7*). Therefore, it can be concluded that the coherency between the two variables in the early 2010s is partly attributable to exchange rate movements. Consequently, this provides stronger support for the key finding of the analysis that there is no permanent, long-run relationship between money growth and inflation.

Figure 7
Wavelet coherency and phase difference between annual M2 money growth and inflation, with partial effects of HUF/EUR exchange rate eliminated



Note: The horizontal axis shows time and the vertical axis shows wavelet scale s (in months) which corresponds to the component of the time series with the same period. The colour scale shows how robust wavelet coherency is between the two time series at a given point in time and on a given scale, to be understood as a correlation coefficient in the time-frequency domain. In the figure, the white outlines indicate the COI, outside of which the wavelet decomposition is distorted.

Source: Compiled from MNB, HCSO and Bloomberg data

[8] Partial wavelet coherency is presented in the *Appendix*.
[9] The partial effects of exchange rate movements were also eliminated by calculating partial wavelet coherency.

The above suggest that foreign currency lending reaching significant proportions in the mid-2000 may have been the underlying factor driving the previously seen relationship. Foreign currency lending is sensitive to the evolution of the HUF exchange rate as a stronger forint has the effect of reducing repayments of foreign currency loans. As a result, the exchange rate maintained through tight monetary policy increased money supply through lending, while interest rate spreads and relatively stable exchange rates made foreign currency lending more attractive than HUF-denominated loans, providing the population access to more credit in this structure (*Bethlendi et al. 2005; Kolozsi et al. 2015*). By boosting consumption *ceteris paribus*, this generated inflationary pressures on the demand side. Despite tight monetary conditions, inflation overall rose in the Hungarian economy, also driven by imported inflation and fiscal imbalances. A strong forint in this period contributed to the overheating of the economy, resulting in an apparent relationship between the two variables in question.

5. Conclusions

The analysis revealed that there is no stable relationship between money growth and inflation in Hungary which is consistent with the branch of literature that argues that no clear relationship can be found between the two variables since the 2000s (*Gertler – Hofmann 2018; Csiki 2022*). These findings are supported by a number of robustness tests. Of these, the analysis of partial exchange rate effects is highlighted. In the few years starting from the mid-2000s, comovements between the medium-run trends of the two variables disappeared with exchange rate effects eliminated, suggesting that the movements of both variables in question during part of the period under review were determined by the evolution of the exchange rate.

In the context of Hungary, *Komáromi* (*2007*) argues that the monetary base does not directly convey any information concerning expected inflation. The present analysis calls into question not only the monetary base, but also the relationship between total money supply in the economy and inflation. However, an important limitation to the study is that, although it looks at coherency between time series across multiple frequencies, there is only limited opportunity to interpret the relationship between lower frequencies due to the narrow COI.

The reasons for the absence of a relationship between inflation and money growth were partly revealed; other reasons, however, may be explained by different theories of money. Accordingly, future analyses may address the impact of other factors on the link between money supply and inflation in Hungary, such as financial innovations, financial regulation and the state of the labour market in relation to full employment. A deeper understanding of this relationship may also be facilitated by applying additional approaches besides wavelet decomposition, such as those based on error correction models.

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Appendix: The Mathematics of Wavelet Transform

As a starting point, the mother wavelet is defined. A square-integrable function $\psi(t) \in L^2(\mathbb{R})$ may be a mother wavelet if it meets the following admissibility condition (*Daubechies 1992*):

$$0 < C_\psi := \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|}{|\omega|} d\omega < \infty, \tag{1}$$

where $\Psi(\omega)$ represents the Fourier transform of $\psi(t)$, that is $\Psi(\omega) = \int_{-\infty}^{\infty} \psi(t)\, e^{-i\omega t} dt$. Here, ω represents the angular frequency from which the Fourier frequency can be derived using the formula $f = \frac{\omega}{2\pi}$. However, when identifying the mother wavelet, other criteria are often applied to the decay of the function in addition to condition (1). For a mother wavelet function decaying at the appropriate rate, a condition equivalent to the admissibility condition is that $\Psi(0) = \int_{-\infty}^{\infty} \psi(t)\, dt = 0$ holds. This means that the function oscillates a few times, and then vanishes.

The $\psi_{\tau,s}$ family of wavelets is generated by translating in time and scaling the mother wavelet ψ.[10] Temporal translation by τ means that the wavelet at time t takes the mother wavelet's value at time $t - \tau$, while scaling by *s* means that the wavelet takes *s* times the value of the (translated) mother wavelet. Thus, if $|s| > 1$, it is stretched, otherwise it is compressed.

Now, let $x(t) \in L^2(\mathbb{R})$. The continuous wavelet transform of *x* can be defined with respect to the mother wavelet ψ as follows:

$$W_{x,\psi}(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t) \psi^* \left(\frac{t-\tau}{s} \right) dt, \tag{2}$$

where * denotes complex conjugation. The wavelet's temporal position is determined by τ and the degree of scaling by *s*. Thus, the wavelet transform localises the time series in both time and scale dimensions. This, however, results in significant redundancy as it decomposes a time series consisting of *n* observations into *n* time series of *n* elements.

Clearly, the time series includes no observations for many points in time where the application of formula (2) requires the mother wavelet be translated. In most applications, the start and end of the time series is padded with zeros in this case; I did the same in this analysis.[11] However, zero padding causes distortion at the edges of the interpretable time series. The level of distortion depends greatly on the scale as the greater *s* is, the longer time series is needed to generate the wavelet transform $W_{x,\psi}(\tau, s)$. The line within which there is no distortion is referred to as

[10] In the analysis, "mother wavelet" always means a function that meets the admissibility condition in equation (1), while "wavelet" means its translated and scaled form.
[11] It is also possible to loop the time series so that it "restarts from the beginning" at the end.

the cone of influence (COI) in the wavelet literature (*Torrence – Compo 1998*).[12] Results outside the COI are usually disregarded.

Generally, due to redundancy, the inversion of the wavelet transform is not well-defined. However, the transform has an energy-preserving feature and thus, if the mother wavelet is real-valued, the original time series can be reconstructed from the transform using the formula $x(t)=\frac{2}{C_\psi}\int_0^\infty\left[\int_{-\infty}^\infty W_{x,\psi}(\tau,s)\,\psi_{t,s}(t)d\tau\right]\frac{ds}{s^2}$. If the wavelet is complex-valued, but analytical, that is $\Psi(\omega)=0$ for each $\omega<0$ (meaning that its Fourier transform vanishes at negative frequencies), and $x(t)$ is real, the inversion can also be performed with the so-called Morlet formula (*Aguiar-Conraria – Soares 2014*):

$$x(t) = 2\Re\left[\frac{1}{K_\psi}\int_0^\infty W_{x,\psi}(\tau,s)\,\frac{1}{s^{3/2}}ds\right], \tag{3}$$

where $\Re$ denotes the real part of the complex number and $K_\psi := \int_0^\infty \frac{\Psi^*(\omega)}{\omega}d\omega$. Based on equation (3), it is possible to perform band-pass filtering of the time series x, which can be accomplished by performing the integration in the equation on a selected subset of scales.

So far, the functional form of the mother wavelet ψ has not been addressed. Following the majority of the wavelet literature, I used the Morlet mother wavelet for my analysis, which takes the following functional form:

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}}\,e^{i\omega_0 t}\,e^{-\frac{t^2}{2}}, \tag{4}$$

where ω_0 is a parameter.[13] The most frequently used value in the literature is $\omega_0 = 6$ (e.g. *Torrence – Compo 1998; Aguiar-Conraria – Soares 2014; Scharnagl – Mandler 2015*). This parameter value makes the conversion between scale and angular frequency simple as in this case $\omega\approx\frac{1}{s}$. Furthermore, this is when the wavelet reaches optimal joint time-frequency resolution (*Aguiar-Conraria – Soares 2014*). Generally, the Morlet mother wavelet is complex-valued and is analytical, thus allowing the original time series to be clearly reconstructed from the wavelet transform using equation (3). Additionally, the time series can also be band-pass filtered. As ψ is the same in almost all applications found in literature, hereinafter the simpler $W_{x,\psi}$ will be used instead of W_x.

The wavelet power spectrum can be calculated from the continuous wavelet transform as follows:

$$WPS_x(\tau,s) = \frac{1}{s}|W_x(\tau,s)|^2, \tag{5}$$

[12] On the functional form of the COI, see also *Torrence – Compo* (*1998*).
[13] The functional form defined in equation (4) does not meet the admissibility condition set in equation (1); however, when $5 \le \omega_0$, the difference becomes negligible.

where $|\cdot|$ denotes the absolute value operator. The wavelet power spectrum captures the local variance of the time series x. The correction of the power spectrum with $\frac{1}{s}$ is necessary so that the local variance is not distorted at higher frequencies, and therefore it does not underestimate variance in the physical sense (*Liu et al. 2007*).

As the Morlet mother wavelet is generally complex-valued, it can be decomposed into real and imaginary parts. The real part represents the local amplitude of the time series based on the formula $AMP_x(\tau, s) = \sqrt{WPS_x(\tau, s)}$. The imaginary part provides information on what phase each wavelet is in. This can be described with the phase angle: $\phi_x(\tau, s) = Arctan\left(\frac{\Im(W_x(\tau,s))}{\Re(W_x(\tau,s))}\right)$ where $\Im$ represents the imaginary part.

Now, let $x(t), y(t) \in L^2(\mathbb{R})$. In this case, cross-wavelet transform can be defined as follows (*Hudgins et al. 1993*):

$$W_{x,y} = W_x W_y^*, \tag{6}$$

where W_x and W_y are the wavelet transforms of time series x and y. The local cross-spectrum describing the covariance between the two time series can be calculated analogously to the power spectrum WPS. Spectrum correction is needed in this case as well (*Veleda et al. 2012*):

$$WCS_{x,y}(\tau, s) = \left|\frac{1}{s} W_{x,y}(\tau, s)\right|. \tag{7}$$

Following the definition of the correlation indicator, the so-called wavelet coherency indicator can be defined in the following form:

$$R_{x,y}(\tau, s) = \frac{S(WCS_{x,y}(\tau,s))}{\left[S(WPS_x(t,s))\, S\left(WPS_y(t,s)\right)\right]^{1/2}}, \tag{8}$$

where S is a smoothing operator in the time and scale dimensions.[14] Calculated using formula (8), $0 \leq R_{x,y}(\tau, s) \leq 1$, while $R_{x,y}(\tau, s)$ would be equally 1 across all points in time and scales without smoothing.[15]

For more than two variables, similarly to partial correlation, partial wavelet coherency can be calculated. To do so, first, the concept of complex wavelet coherency should be introduced. The latter can be defined in the following form:

$$\varrho_{x,y}(\tau, s) = \frac{S(\frac{1}{s} W_{x,y}(\tau,s))}{\left[S(WPS_x(t,s))\, S\left(WPS_y(t,s)\right)\right]^{1/2}}, \tag{9}$$

[14] On the functional form of the smoothing operator see, for example, *Torrence – Webster* (*1999*) and *Cazelles et al.* (*2007*).
[15] If the denominator in equation (8) is 0 somewhere, $R_{x,y}$ is set to 0.

meaning that the difference between equation (9) and equation (8) is that the former has a complex number in its numerator in a general case. Then, complex partial coherency is calculated using the following formula (*Aguiar-Conraria – Soares 2014*):

$$\varrho_{x,y\,-z}(\tau,s) = \frac{\varrho_{x,y}(\tau,s)-\varrho_{x,z}(\tau,s)\,\varrho^{*}{}_{y,z}(\tau,s)}{\sqrt{\left(1-R^{2}{}_{x,z}(\tau,s)\right)\left(1-R^{2}{}_{y,z}(\tau,s)\right)}}, \tag{10}$$

where $\varrho_{x,y\,-z}$ represents partial coherency between x and y calculated by eliminating the effects of time series $z(t) \in L^2(\mathbb{R})$. Based on this, the (real-valued) partial wavelet coherency is given as:

$$R_{x,y\,-z}(\tau,s) = \left|\varrho_{x,y\,-z}(\tau,s)\right|. \tag{11}$$

The phase angle can be generalised for the two-variable case as follows (*Grinsted et al. 2004*):

$$\phi_{x,y}(\tau,s) = Arctan\left(\frac{\Im(W_{x,y}\,(\tau,s))}{\Re(W_{x,y}\,(\tau,s))}\right), \tag{12}$$

therefore $\phi_{x,y} = \phi_x - \phi_y$. As a result, the indicator defined in equation (12) is also called wavelet phase difference.[16] The indicator can take values between $-\pi$ and π and shows the lead-lag relationship between the wavelets of the two time series at a given scale and at a given point in time. When $\phi_{x,y} = 0$, there is perfect comovement between the two wavelets; when $\phi_{x,y} \in \left(0, \frac{\pi}{2}\right)$, x and y are in phase (i.e. their peaks coincide) and time series x is ahead of y; when $\phi_{x,y} \in \left(0, -\frac{\pi}{2}\right)$, the two variables are also in phase and time series y is ahead of x. When $\phi_{x,y} \in \left(\frac{\pi}{2}, \pi\right)$, the two time series are not in phase (i.e. their peaks do not coincide) and y is ahead and when $\phi_{x,y} \in \left(-\frac{\pi}{2}, -\pi\right)$, x is ahead. If $\phi_{x,y} = \pi$ or $-\pi$, the two variables are in an antiphase relation (i.e. their peaks are in an exact opposite position). Phase difference can be converted to time difference as follows:

$$\Delta T_{x,y}(\tau,s) = \frac{\phi_{x,y}(\tau,s)}{\omega(s)}. \tag{13}$$

With the Morlet mother wavelet, this is approximately $s\phi_{x,y}(\tau,s)$, that is the product of the phase difference and the scale. It is important to note that a lead-lag relationship does not mean causality: if the time series x is ahead of y at a given point in time and on a given scale, it is still possible that, for example, it is actually y that causes x, but its effect is spread over more than one entire period.

[16] Other studies consider the argument of the smoothed cross-wavelet transform to be the phase angle, i.e. perform smoothing in equation (12) using operator S. In this case, generally, the relation $\phi_{x,y} = \phi_x - \phi_y$ does not hold (*Aguiar-Conraria – Soares 2014*).

I ran Monte Carlo simulations to test the significance of power spectrum (and coherency). Although a number of studies address the asymptotic distribution of wavelet coefficients and wavelet coherency (e.g. *Ge 2008; Cohen – Walden 2010*), the analytical density function always depends on the form of the mother wavelet and so it cannot be generalised. Furthermore, asymptotic distributions are also derived from Monte Carlo simulation, so it is evident to apply this procedure directly. I fit an $ARIMA(p, i, q)$ model to the time series and generated new time series using the estimated coefficients and by drawing from the standard normal distribution. Finally, I also calculated wavelet transforms and wavelet power and cross-spectra for them, thereby constructing the approximation of the asymptotic distribution (*Torrence – Compo 1998*).[17]

[17] Following the practice established in the literature, I only calculated phase differences for significantly coherent points in time and scales. I did not perform significance testing on phase differences as they are always distributed evenly between $-\pi$ and π so critical values cannot be established for the test (*Ge 2008*).