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Short-Run Elasticity of Substitution

Karol Szomolányi, Martin Lukáčik, Adriana Lukáčiková

Abstract: *We provide a strategy for estimating a short-run elasticity of factor substitution. The method is based on a co-integration estimation relationship between factor prices and average factor products. From the literature, it is known that this form is useless for estimating a long-run elasticity of substitution coefficient, because it is not consistent with a theory. However, restricting the long-run relationship according to the theory and estimating the short-run coefficients with the co-integration coefficient jointly in one step allows estimating the short-run elasticity of substitution. The co-integration term of the form captures possible underlying long-run processes and so it is useful in obtaining the unbiased estimate.*

To verify the method we use Jorgenson's sector data of the United States of America. In the results U.S. short-run elasticity of substitution is relatively small and it differs in different sectors. These values are between 0.05 and 0.64. In the conclusions we argue that the small and in different sectors different values of the coefficient is supported by the both empirical and theoretical research.

Key words: Elasticity of factor substitution · Long-run and short-run · Co-integration analysis · Neoclassical growth theory

JEL Classification: C13 · E23 · O40

1 Introduction

Chirinko (2008) and Klump, McAdam and Willman (2012) provide rich literature survey of elasticity of input substitution estimation problem. There are many ways to estimate the elasticity of substitution. We focus to the co-integration analysis of the factor prices. Caballero (1994) measures long-run values by exploiting the co-integration relations between the capital/output ratio and the user cost of capital. As argued in Chirinko and Mallick (2011), this estimation strategy faces some econometric difficulties in recovering production function parameters. In this paper we use similar analysis of labour/output and capital/output ratio. We use Chirinko's and Mallick's suggestion to form and estimate a co-integration econometric specification suitable to quantify short-run values of the elasticity of substitution.

Consider the co-integration estimation form

$$\Delta(y_t - x_t) = \alpha_0 + \beta_1 \Delta\left(p_t^x - p_t^y\right) + \lambda\left[\left(y_{t-1} - x_{t-1}\right) - \gamma_0 - \gamma_1\left(p_{t-1}^x - p_{t-1}^y\right)\right] + u_t \qquad (1)$$

where y_t, x_t, p_t^y and p_t^x are the logarithms of output y, factor x, the price of output and the price of factor x, u_t is a white-noise stochastic term. Coefficients β_1 and γ_1 are estimates (suggested by Caballero, 1994) of long-run and short-run elasticity of substitution and $-1 \leq \lambda \leq 0$ is a co-integration coefficient. Chirinko and Mallick (2011) argue that neoclassical growth theory assumes the constancy of the factor share $p_t^x + x_t - p_t^y - y_t$. However after substituting the factor share to the co-integration form (1), "the constancy holds if and only if the influence of relative prices is eliminated. In this case coefficient γ_1 must equal 1" (2011, p. 206) and the coefficient is not a measure of the long-run elasticity of substitution.

In fact, Szomolányi, Lukáčik and Lukáčiková (2013, 2015) estimated values of the coefficient close to 1 using U.S. and V4 (Visegrad Four country club, i.e. Czech Republic, Hungary, Poland and Slovakia) aggregated data of average product of labour and labour price. However, we argue that the estimation form (1) is suitable for estimating the short-run elasticity of substitution β_1.

doc. Ing. Karol Szomolányi, Ph.D., University of Economics in Bratislava, Faculty of Economic Informatics, Department of Operations Research and Econometrics, Dolnozemská cesta 1, 852 35 Bratislava, Slovak Republic, e-mail: karol.szomolanyi@euba.sk

doc. Ing. Martin Lukáčik, Ph.D., University of Economics in Bratislava, Faculty of Economic Informatics, Department of Operations Research and Econometrics, Dolnozemská cesta 1, 852 35 Bratislava, Slovak Republic, e-mail: martin.lukacik@euba.sk

Ing. Adriana Lukáčiková, Ph.D., University of Economics in Bratislava, Faculty of Economic Informatics, Department of Operations Research and Econometrics, Dolnozemská cesta 1, 852 35 Bratislava, Slovak Republic, e-mail: adriana.lukacikova@euba.sk

According to Chirinko and Mallick (2011), three cases consistent with a general economic knowledge may exhibit the co-integration form (1). Firstly, co-integration relation holds. This may be reasonable according to the neoclassical growth theory, if labour is the factor. Then $\gamma_1 = 1$. Secondly, co-integration relation does not hold. This may be reasonable according to the theory, if capital is the factor. Finally, co-integration relation does not hold, but variables are driven by different underlying co-integration processes. By the first case, we can estimate co-integration form with $\gamma_1 = 1$. To estimate all coefficients in one step we rewrite the co-integration relation into the form suggested by Stock and Watson (1993).

$$\Delta\left(y_t - x_t\right) = \beta_0 + \beta_1 \Delta\left(p_t^x - p_t^y\right) + \lambda\left[\left(y_{t-1} - x_{t-1}\right) - \left(p_{t-1}^x - p_{t-1}^y\right)\right] + u_t \tag{2}$$

Szomolányi, Lukáčik and Lukáčiková (2015) showed that the co-integration form (2) is consistent with the normalised constant elasticity of substitution production function suggested by De La Grandville (1989) and Klump, McAdam and Willman (2012).

2 Data and Methods

To estimate the coefficients of the form (2) we use yearly data of logarithms of average labour and capital product quantities, $y_t - l_t$ and $y_t - k_t$, and labour and capital prices, $p_t^l - p_t^y$ and $p_t^k - p_t^y$, in the period 1960 – 2005 and in the 35 U.S. economy sectors obtained from Jorgenson (2008). In the first look on data we focus to the stationarity tests. Using augmented Dickey-Fuller and Phillips-Perron tests of unit roots (see Lukáčik and Lukáčiková, 2008) we state that the most of average factor product data series are non-stationary. Therefore we need to use their first differences in the estimation forms. Both (1) and (2) forms use the first differences of average factor products.

According to the first Chirinko's and Mallick's case in the introduction, we expect that the labour share ($p_t^l + l_t - p_t^y - y_t$), data series are stationary with no exogenous variable. In fact, using augmented Dickey-Fuller and Phillips-Perron tests of unit roots we state that the labour share data series are statistically significantly stationary at the 5% level in the most industries. Labour share data series of "Metal mining", "Coal mining", "Machinery non-electrical" and "Electrical machinery" industries are not stationary. Labour share of "Communications" industry is statistically significantly stationary at the 10% level. Therefore we state that co-integration form (2) is suitable for estimating short-run elasticity of substitution, β_1.

Considering the third Chirinko's and Mallick's case in the introduction, we expect that the capital share ($p_t^k + k_t - p_t^y - y_t$) data series are not stationary. However, using Dickey-Fuller and Phillips-Perron tests of unit roots we state that the capital share data series are statistically significantly stationary at the 5% level in the most industries. Capital share data series of "Metal mining", "Coal mining" and "Motor vehicles" industries are not stationary. Note that tests using exogenous variables as constant and trend suggest unit root in the capital share data series. We explain the capital share stationarity by Chirinko's and Mallick's assumption. Average capital product and price ratio are co-integrated because of different underlying co-integration processes. Then the (1) form is not suitable for long-run elasticity of substitution estimate. But for the unbiased short-run elasticity of substitution β_1 estimate, we need to include variables capturing the processes into the estimation form. Considering that capital share is stationary, it is good candidate. Estimation form (2) uses this variable.

Summing up, the final econometric model is in the form:

$$\begin{aligned}\Delta\left(y_{ti} - l_t\right) &= \beta_{l0i} + \beta_{1i}\Delta\left(p_{ti}^l - p_{ti}^y\right) + \lambda_{li}\left[\left(y_{ti-1} - l_{ti-1}\right) - \left(p_{ti-1}^l - p_{ti-1}^y\right)\right] + u_{til} \\ \Delta\left(y_{ti} - k_t\right) &= \beta_{k0i} + \beta_{1i}\Delta\left(p_{ti}^k - p_{ti}^y\right) + \lambda_{ki}\left[\left(y_{ti-1} - l_{ti-1}\right) - \left(p_{ti-1}^l - p_{ti-1}^y\right)\right] + u_{til}\end{aligned} \qquad \forall i = 1, 2, \ldots, 35 \tag{3}$$

where an estimate of the short-run elasticity substitution in the sector i is denoted by β_{1i}. To capture cross-sectional dependence in the stochastic terms we use Seemingly Unrelated Regression method to estimate coefficients of (3). The set of instrument consists of the levels of the average factor products, and prices and first differences of the prices in all 35 sectors.

3 Research results

The estimates of the coefficients are in the Table 1. The most of coefficients are statistically significant at the 5% level (white rows). For testing the co-integration coefficients, λ_{li} and λ_{ki}, we use tables suggested by Banerjee, Dolado and Mestre (1998). Moreover, the co-integration coefficients are statistically significant for "Machinery, non-electical", industry at the 10% level. We cannot confirm the capital co-integration relation, but we confirm labour co-integration relation for "Oil and gas extraction" and "Chemicals" industries. We do not consider estimates with statistically insignificant labour co-integration relation, because it is not consistent with the theory (under-painted rows). From the consistent estimates, we state that the short-run elasticity of substitution differs from industry to industry and it is between 0.05 ("Food and kindred products") and 0.64 ("Transportation equipment & ordnance").

Table 1 Estimate of the short-run elasticity of substitution in 35 U.S. sectors

Industry	β_1	$s_{\beta 1}$	λ_l	$t_{\lambda l}$	λ_k	$t_{\lambda k}$
Agriculture	0.149	0.007	-0.067	-8.195	-0.060	-12.483
Metal mining	0.372	0.014	-0.085	-6.509	-0.164	-9.509
Coal mining	0.205	0.009	0.046	5.863	0.006	0.952
Oil and gas extraction	0.570	0.008	-0.176	-12.210	-0.022	-2.394
Non-metallic mining	0.608	0.007	-0.024	-1.454	-0.097	-10.773
Construction	0.449	0.010	-0.050	-3.974	-0.026	-3.546
Food and kindred products	0.048	0.005	-0.147	-6.074	-0.009	-5.805
Tobacco	0.240	0.006	-0.030	-4.148	-0.056	-11.668
Textile mill products	0.227	0.011	-0.107	-4.483	-0.024	-3.390
Apparel	0.183	0.008	-0.110	-9.141	-0.023	-3.649
Lumber and wood	0.405	0.008	-0.204	-16.813	-0.120	-16.149
Furniture and fixtures	0.217	0.010	-0.133	-5.548	-0.157	-19.412
Paper and allied	0.256	0.007	-0.273	-25.274	-0.039	-6.151
Printing, publishing and allied	0.200	0.008	-0.282	-10.570	-0.039	-5.512
Chemicals	0.553	0.008	-0.080	-6.637	-0.010	-2.073
Petroleum and coal products	0.156	0.007	-0.074	-8.838	-0.025	-4.937
Rubber and misc plastics	0.383	0.009	-0.219	-9.115	-0.106	-12.859
Leather	0.174	0.010	-0.044	-4.521	-0.052	-9.609
Stone, clay, glass	0.410	0.008	-0.084	-6.172	-0.084	-13.725
Primary metal	0.416	0.011	-0.278	-13.576	-0.168	-15.381
Fabricated metal	0.276	0.008	-0.072	-9.122	-0.068	-13.919
Machinery, non-electical	0.477	0.011	-0.060	-3.206	-0.028	-3.005
Electrical machinery	0.269	0.009	0.074	9.217	-0.119	-21.365
Motor vehicles	0.445	0.009	-0.500	-20.917	-0.136	-16.539
Transportation equipment & ordnance	0.635	0.012	-0.719	-18.525	-0.325	-24.009
Instruments	0.605	0.011	-0.075	-6.469	-0.156	-17.152
Misc, manufacturing	0.218	0.008	0.033	3.925	-0.070	-14.009
Transportation	0.305	0.010	-0.033	-3.551	-0.070	-9.523
Communications	0.575	0.012	-0.041	-9.724	-0.039	-6.419
Electric utilities	0.430	0.011	-0.342	-18.226	-0.046	-7.995
Gas utilities	0.272	0.010	-0.014	-1.757	-0.116	-24.160
Trade	0.394	0.007	-0.048	-7.007	-0.056	-10.877
Finance Insurance and Real Estate	0.517	0.011	-0.112	-6.641	-0.130	-14.928
Services	0.298	0.008	0.001	0.098	-0.118	-22.443
Government enterprises	0.272	0.002	-0.215	-14.887	-0.031	-15.505

Source: Own processing

We test the stationarity of residuals by augmented Dickey-Fuller and Phillips-Perron tests. Using the Phillips-Perron test we reject the unit root hypothesis at 1% level of the statistical significance in all cases. Using the augmented Dickey-Fuller test we reject the unit root hypothesis at 1% level of statistical significance in all cases, but the residuals for capital estimates in the "Communication" sector are statistically significantly stationary at the 5% level.

4 Conclusions

Authors of the latest estimates prefer the long-run elasticity of substitution value 0.40 (Chirinko and Mallick. 2016), 0.60 (Klump. McAdam and Willman. 2007) and 0.70 (Klump. McAdam and Willman. 2008). These values are larger than our short-run estimates which are between 0.04 and 0.64. Moreover, all three papers consider the long-run elasticity substitution to be universal for each sector as well as for whole aggregated economy. However, in the conclusions, we support a statement that for a short term, elasticity of substitution is smaller than for a long term, and it can be different in a different industry. Our support is based on both empirical and theoretical research.

A summary by Chirinko (1993) found the short-run elasticity estimates varied widely, but they tended to be small and less than 0.30. The small value of the short-run elasticity is consistent with the theory of Jones (2003. 2005) and Jürgen (2009. 2010). Jürgen (2009) suggests that transition state economies have smaller substitution elasticity than the steady state economies. Jürgen (2010) predicts that competitive equilibrium can be realized only under Cobb-Douglas technology with the unit elasticity of substitution. Jones (2003) and (2005) consider Cobb-Douglas production function form in the long-run view and CES production function with an elasticity of substitution less than one from the short-run view, because input innovations are Pareto distributed from the long-run. This theory suggests that in the long-run, the elasticity of substitution is larger than in the short-run. The later Jones' paper suggests that elasticity of substitution differs similarly in the "local" and "global" perspective. From the local perspective input innovations are distributed similarly as from the short term perspective, while from the global perspective input innovations are distributed similarly as rom the long term perspective. Using the theory we should expect that the elasticity values depend on the aggregation rate in each industry. Moreover, Chirinko and Mallick (2016) estimated elasticity of substitution with properly aggregated data to be larger (0.657) than their "benchmark value of 0.406 based on homogeneity assumption" (2016, p. 24). This result supports Jones' idea that the global elasticity of substitution is larger than local.

The estimation strategy presented in this paper could be useful for estimating an elasticity of substitution in the countries that suffer from small datasets and missing capital and capital income data-series as the post-communist countries. The results of the paper emphasize problems about analysis based on the dynamic stochastic general equilibrium (DSGE) model that maintain that elasticity of substitution equals to 1 (using Cobb-Douglass production function) highlighted by Chirinko and Mallick (2016). Such models surely overvalue the effect of price movements. Considering that DSGE models focus on short term economic relations, their bias is even larger, if a true short-run elasticity of substitution is very small.

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