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Managing and Modelling of Financial Risks 2016
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Managing and Modelling of Financial Risks 2016

8th International Scientific Conference

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Prediction of the risk of bankruptcy of Slovak companies

Peter Adamko, Lucia Svabova [1]

Abstract

The purpose of this paper is to present the theoretical and practical bases of our research and also compare the results we got using new Altman's (2014) logit models on a samples of Slovak companies. Performance of the models was measured using the area under the receiver operating characteristic curve (AUC).

Key words

default, estimation of probabilities of default, logit model

JEL Classification: G17, C52, C53

1. Úvod

Keď Altman, približne pred polstoročím, vytvoril prvý viacrozmerný bankrotný model, dal tak impulz k celosvetovému rozšíreniu tohto prístupu k modelovaniu defaultu. Stal sa z neho dôležitý nástroj bankárov, investorov, ratingových agentúr ale aj samotných firiem, ktoré zaujímalo riziko bankrotu. Jeho Z model bol prototypom pre mnohé interné modely v bankách pre stanovenie kapitálovej primeranosti, obzvlášť po BASEL II (Misankova, Kral, 2015).

Altmanov model bol založený na MDA, ktorý vyžaduje splnenie predpokladov multinormality, homoskedasticity, a linearity, ktoré pri finančných ukazovateľoch často nie sú splnené. V prípade, že niektorá z premenných je kategoriálna, táto metóda dáva skreslené výsledky. (Cisko, S. and Kliestik, T., 2013).

To bol dôvod, prečo sa začala uprednostňovať logistická regresia, ktorá tieto podmienky nemusí spĺňať. Logistická regresia, tiež nazývaná logit model, je používaná hlavne pri modeloch, ktoré majú dichotomickú výstupnú premennú (Kliestik, T. et al, 2015). V logit modeloch je logaritmus pomeru šancí (odds ratio) predikovanej premennej počítaný ako lineárna kombinácia prediktorov (Kliestik, Majerova, Lyakin, 2015). Logit model tak poskytuje možnosť modelovania zložitých vzťahov medzi premennými, ale predpokladá log-lineárny vzťah medzi vysvetľovanou a vysvetľujúcimi premennými. Logistická regresia je obzvlášť užitočná, keď zo spojitých nezávislých premenných predikujeme binárny výstup. Je vhodné spomenúť, že pre malé súbory dát môže byť MDA užitočnejšia, avšak pre väčšie vzorky je logistická regresia vhodnejšia. (Misankova, M. et al 2015).

Pri tvorbe alebo testovaní ľubovoľného predikčného modelu je jednou z najdôležitejších a najnáročnejších náležitostí získanie vhodnej vzorky dát na vytvorenie, resp. testovanie modelu. V prípade predikčného bankrotného modelu sú to účtovné závierky. Donedávna nebolo na Slovensku jednoduché získať vhodné dáta vo väčšom množstve. Situácia sa zmenila od 1.

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januára 2014, odkedy Register účtovných závierok (www.registeruz.sk) povinne zverejňuje dokumenty podľa § 23 ods. 2 zákona zostavené k 31. decembru 2013 a neskôr. Na stránke registra je možné bez poplatku prezerať, ukladať a tlačiť dostupné účtovné závierky, účtovné výkazy a ďalšie dokumenty zverejňované v registri. Z hľadiska hromadného zberu dát je veľkou výhodou existencia rozhrania umožňujúca automatizované sťahovanie dát z verejnej časti. Istou nevýhodou alebo obmedzením je nutnosť vytvorenia programu komunikujúceho s daným rozhraním. V čase vzniku článku bolo celkovo k dispozícii viac ako milión účtovných závierok.

Veľké množstvo testovaných firiem je výhodou. Táto výhoda hromadného spracovania je ale kompenzovaná nedostatkom jednoznačného určenia či je firma v bankrote. Preto sme na určenie stavu firmy použili zákon č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov. Ten definuje predlžený podnik: „ ... hodnota jeho záväzkov presahuje hodnotu jeho majetku.“ Rozdiel medzi majetkom a záväzkami, bude pre potreby nášho testovania vyjadrovať stav firmy, pričom záporná hodnota klasifikuje podnik ako zadlžený (bankrot).

$$Zadlženosť = majetok_spolu - záväzky \quad (1)$$

Ak ukazovateľ nadobúda záporné hodnoty podnik zlyhal (zbankrotoval, tzn. je predlžený), v opačnom prípade podnik nezlyhal (je prosperujúci) (Kocisova, K. and Misankova, M., 2014).

2. Testované modely

Altman vo svojej práci (Altman et al, 2014) predkladá nový MDA model a 7 variantov logit modelu. Použil dáta z 31 krajín Európy, Kolumbie, Číny a USA. Do vzorky sú zaradené firmy, ktorých celkový majetok dosiahol v skúmanom období aspoň raz minimálne 100 000€. Celkovo použil na tréning modelu 2 602 563 zdravých firiem, 38 215 bankrotujúcich, na testovanie to bolo 3 148 079 resp. 43 664 firiem. Slovenské firmy boli zastúpené v trénovacej skupine 7 856 resp. 120 firmami, v testovacej 7 788 resp. 124 firmami. Dáta boli z rokov 2008, 2009 a 2010. AUC sa pre jednotlivé modely pre Slovensko pohybovala od 0,769 po 0,811.

V článku testujeme Altmanov logistický predikčný bankrotný model z roku 2014 a jeho dva varianty, ktoré dosiahli pri jeho testovaní najlepší resp. najhorší výsledok. Testovanie je vykonané na vzorke slovenských firiem z rokov 2013 a 2014, t. j. na dátach o 3 až 8 rokov novších ako použil Altman. K dispozícii sme mali pre každý rok približne 80 000 účtovných závierok.

Table 1: Počty podnikov za jednotlivé roky

	Zadlženosť >= 0	Zadlženosť < 0
2013	60 489	20 035
2014	59 767	17 164

Základný model (AUC = 0,776), ktorý predstavil Altman v roku 2014 má nasledujúcu podobu:

$$Z = 0{,}035 - 0{,}495 \cdot X_1 - 0{,}862 \cdot X_2 - 1{,}721 \cdot X_3 - 0{,}017 \cdot X_4 \quad (2)$$

kde
X_1 = Pracovný kapitál / Celkové aktíva,
X_2 = Nerozdelený zisk / Celkové aktíva,
X_3 = EBIT / Celkové aktíva,
X_4 = Vlastné imanie / Celkové záväzky.

Model, ktorý dosiahol najlepší výsledok (AUC = 0,811) zahŕňal okrem pôvodných štyroch prediktorov dva ďalšie. Prirodzený logaritmus celkových aktív a jeho druhú mocninu.

$$Z = -13{,}466 - 0{,}441 \cdot X_1 - 1.146 \cdot X_2 - 1.619 \cdot X_3 - 0.012 \cdot X_4 + 1.830 \cdot X_5 - 0.061 \cdot X_6 \quad (3)$$

kde
X_1, X_2, X_3, X_4 sú zhodné s modelom 2,
X_5 = ln(Celkové aktíva),
X_6 = $\ln^2$(Celkové aktíva).

Najhorší model (AUC = 0,769), podľa Altmanovho výskumu obsahuje okrem pôvodných prediktorov pomocnú premennú, ktorá nadobúda v závislosti od veku firmy jednu z troch hodnôt.

$$Z = 0{,}007 - 0{,}487 \cdot X_1 - 0{,}846 \cdot X_2 - 1{,}757 \cdot X_3 - 0{,}017 \cdot X_4 + AD \quad (4)$$

kde
X_1, X_2, X_3, X_4 sú zhodné s modelom 2,
AD = 0,135, ak je podnik mladší ako 6 rokov,
AD = -0.058, ak je podnik starší ako 12 rokov,
AD = 0, inak.

3. Meranie efektívnosti modelu

Altman použil na meranie kvality modelu metódu AUC (Area Under Curve), čo je veľkosť plochy pod krivkou ROC (receiver operating characteristic). Túto metódu sme použili aj my.

Skúmané (binárne) logistické modely neurčujú jednoznačne, do ktorej skupiny spadá daný podnik. Určujú pravdepodobnosť s akou podnik patrí do skupiny predlžených. V prípade, že potrebujeme jednoznačne zaradiť podnik do jednej zo skupín (v našom prípade: zdravé alebo predlžené), musíme určiť prah (threshold, cut-off) (Kicova. E, Kramarova K., 2013). Hodnota prahu ovplyvňuje počet správne (a tým aj nesprávne) identifikovaných podnikov v každej skupine.

Najčastejšie sa prah určuje tak, aby sa dosiahlo: Maximum(Špecifickosť + Citlivosť), maximum(Kappa), maximum(PCC), minimum(ROCdist), PredPrev=Obs a Špecifickosť = Citlivosť. PCC (percent correctly classified) je percento správnych klasifikácii. ROCdist je vzdialenosť krivky ROC od ľavého horného bodu (0,1). PredPrev=Obs (Predicted Prevalence = Observed Prevalence)je rovnosť percentuálneho výskytu skutočných a predikovaných predlžených podnikov. Štandardný prah je 0,5.

Kappa je miera zhody skutočnosti a predikcie vyjadrená nasledujúcim vzťahom:

$$\kappa = \frac{p_0 - p_e}{1 - p_e} \quad (5)$$

kde
p_0 = tp + tn,
p_e = (tp+fn)(tp+fp) + (fn+tn)(fp+tn), tp, tn, fp, fn sú z tabuľky 2

Table 2: Confusion matrix

	predikcia	
skutočnosť	tp	fp
	fn	tn

Hodnota Kappa menej ako 0,2 sa považuje za slabú zhodu, hodnoty nad 0,8 sú znakom vysokej zhody.

4. Výsledky testovania

Pre dáta z roku 2014 dosiahli skúmané modely nasledujúce výsledky:

Základný model AUC = 0,8817 a 95% interval spoľahlivosti: 0,8789-0,8845.

Model uvažujúci vek firmy má AUC = 0,8835 a 95% interval spoľahlivosti: 0,8807-0,8863.

Model s logaritmami majetku má AUC = 0,8116 a 95% interval spoľahlivosti: 0,8079-0,8154.

Pre dáta z roku 2013 dosiahli skúmané modely nasledujúce výsledky:

Základný model AUC = 0,8747 a 95% interval spoľahlivosti: 0,8719-0,8775.

Model uvažujúci vek firmy má AUC = 0,8753 a 95% interval spoľahlivosti: 0,8725-0,878.

Model s logaritmami majetku má AUC = 0,8127 a 95% interval spoľahlivosti: 0,8092-0,8162.V tabuľke 2 sú zaznamenané miery popísané v kapitole 3.

Table 2: Porovnanie niektorých mier pre jednotlivé modely, roky a kritéria

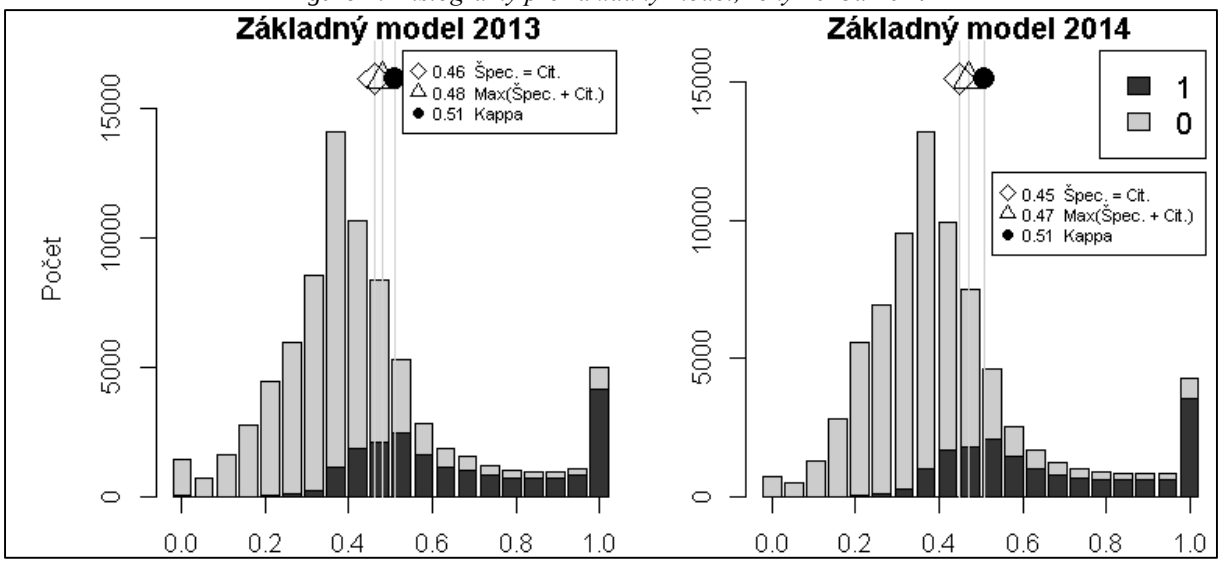
	2013					2014				
	threshold	PCC	Cit.	Špec.	Kappa	threshold	PCC	Cit.	Špec.	Kappa
Cit. = Špec.	0,46	0,80	0,80	0,79	0,52	0,45	0,80	0,81	0,79	0,50
Max(Cit.+Špec.)	0,48	0,82	0,76	0,84	0,55	0,47	0,82	0,77	0,83	0,54
Max(Kappa)	0,51	0,84	0,69	0,89	0,57	0,51	0,85	0,68	0,90	0,57
Max(PCC)	0,52	0,84	0,66	0,90	0,57	0,53	0,85	0,63	0,92	0,56
Min(ROCdist)	0,47	0,81	0,78	0,82	0,54	0,46	0,81	0,79	0,81	0,52
Cit. = Špec.	0,46	0,80	0,80	0,79	0,52	0,46	0,81	0,80	0,81	0,52
Max(Cit.+Špec.)	0,47	0,81	0,78	0,82	0,54	0,46	0,81	0,80	0,81	0,52
Max(Kappa)	0,51	0,83	0,69	0,88	0,56	0,51	0,85	0,69	0,89	0,57
Max(PCC)	0,54	0,84	0,62	0,91	0,56	0,54	0,85	0,61	0,92	0,56
Min(ROCdist)	0,46	0,80	0,80	0,79	0,52	0,46	0,81	0,80	0,81	0,52
Cit. = Špec.	0,39	0,73	0,72	0,73	0,39	0,38	0,72	0,73	0,72	0,36
Max(Cit.+Špec.)	0,43	0,77	0,66	0,80	0,42	0,43	0,77	0,65	0,80	0,41
Max(Kappa)	0,53	0,82	0,48	0,93	0,46	0,52	0,83	0,50	0,92	0,46
Max(PCC)	0,55	0,82	0,45	0,94	0,45	0,56	0,84	0,43	0,95	0,45
Min(ROCdist)	0,40	0,74	0,70	0,75	0,40	0,39	0,73	0,72	0,74	0,37

Table 3:Počty podnikov základných modeloch v závislosti od použitého thresholdu

			predikcia					
			2013				2014	
			1	0	threshold		1	0
Skutočnosť	Cit. = Špec.	1	16 002	12 442	0,46	0,45	13 518	11 142
		0	4 033	48 047			3 646	48 625
	Max(Cit.+Špec.)	1	15 203	9 871	0,48	0,47	12 803	8 797
		0	4 832	50 618			4 361	50 970
	Max(Kappa)	1	13 749	6 628	0,51	0,51	11 623	5 919
		0	6 286	53 861			5 541	53 848
	Max(PCC)	1	13 192	5 997	0,52	0,53	11 152	5 354
		0	6 843	54 492			6 012	54 413
	Min(ROCdist)	1	15 604	11 110	0,47	0,46	13 184	9 936
		0	4 431	49 379			3 980	49 831

Na obrázku 1 sú znázornené histogramy pre základné modely pre rok 2013 a 2014 s prahom: Špecifickosť = Citlivosť, Maximum(Špecifickosť + Citlivosť) a Kappa.

Figure 1: Histogramy pre základný model, roky 2013a 2014



5. Zhrnutie

Modely na predikciu finančného zdravia podniku poskytujú možnosť ako rýchlo a pomerne efektívne klasifikovať skúmané firmy. V článku sme testovali tri varianty Altmanov modelu z roku 2014. Výsledky sú zobrazené v tabuľke 4.

Table 4: Porovnanie AUC pre jednotlivé modely a roky

	Základný model	Model s vekom firmy	Model s log. majetku
AUC -Altman 2008-2010	0,776	0,769	0,811
2013	0,8747	0,8753	0,8127
2014	0,8817	0,8835	0,8116

Z tabuľky vidieť, že nové dáta dávajú približne rovnaké výsledky ako pri pôvodných dátach iba pre tretí model pracujúci s logaritmami majetku. Základný model a model s vekom firmy dávajú s novými dátami lepšie výsledky. Takéto zlepšenie je pomerne neobvyklé (väčšinou možno očakávať zhoršenie) a môže byť spôsobené tým, že v pôvodných dátach boli iba firmy s majetkom aspoň 100 000€ a v dátach z rokov 2013, 2014 sú firmy bez obmedzenia. Hlavným dôvodom môže byť ale spôsob zaradenia podniku do skupín, pretože oproti Altmanovej vzorke, v ktorej bola prevalencia 1,5%, v nami testovaných súboroch bola 25%.

Acknowledgement

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References

[1] Altman, E. I. et al. (2014). *Distressed Firm and Bankruptcy Prediction in an International Context: A Review and Empirical Analysis of Altman's Z-Score Model*. (August 10, 2014). Available at SSRN: http://ssrn.com/abstract=2536340

[2] Cisko, S. a Kliestik, T. (2013). *Financny manazment podniku II*, EDIS Publishers, Zilina, Slovakia.

[3] Kliestik, T. et al. (2015). Metamorphoses and Semantics of Corporate Failures as a Basal Assumption of a Well-founded Prediction of a Corporate Financial Health. *In 3rd International Conference on Economics and Social Science (ICESS 2015), Advances in Education Research,* Vol. 86, IERI 2015, pp. 150 – 154.

[4] Kocisova, K. and Misankova, M. (2014). Prediction of Default by the Use of Merton's Model and Black and Cox Model, *In 4th international conference on applied social science (ICASS 2014), Advances in Education Research*, Singapore, Singapore, Vol. 51, pp. 563-568.

[5] Misankova, M. et al. (2015). Determination of Default Probability by Loss Given Default, *Procedia Economics and finance,* Vol 26, pp. 411- 417.

[6] Misankova, M. and Kral, P. (2015). Application of CreditRisk+ for the calculation of credit risk, *In 10th International Scientific Conference Financial management of Firms and Financial Institutions*, Ostrava, Czech Republic, pp. 807-814.

[7] Kliestik, T., Majerová, J., Lyakin, A. N. (2015). *Metamorphoses and Semantics of Corporate Failures as a Basal Assumption of a Well-founded Prediction of a Corporate Financial Health*, In: 3rd International Conference on Economics and Social Science (ICESS 2015), Advances in Education Research, Vol. 86, pp. 150 – 154, IERI 2015, ISSN 2160-1070

[8] Kicova, E., Kramarova, K. (2013). Possibilities of using financial analysis in the bus transport companies. *In: Financial management of firms and financial institutions.* Ostrava, Czech Republic.

Weak market in the US Efficiency Period 2000-2009

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Abstract

The Hypothesis theory of Market Efficiency (HEM) notes that a stock market is efficient when analysts and investors believe that the market security price is a good estimate of their theoretical or intrinsic price because it is properly valued. This document aims to contrast the series of monthly returns of the US market in the period 2000-2009, to observe their behavior and random validation HEM in its weak form. The data used for the monthly closing price of the stock indices S & P500 and S & P100 has 120 data. With a significance level of 1%, 5% and 10%, the Dickey-Fuller Test results allowed to check that the probability of error assumed by rejecting the null hypothesis is greater than 5% for both series, so we conclude that the series study are stationary and they follow a random walk during the period 2000-2009.

Key words

Hypothesis of efficient markets, random walk, skewness, kurtosis, autocorrelation, unit root.

JEL Classification: C12, G14

1. Introduction

The Hypothesis Theory of Market Efficiency (HEM) remarks that a stock market is efficient when analysts and investors believe that the market price of a security is a good estimate of their theoretical or intrinsic price because it is properly valued. In the market price title publicly available information is reflected and this price also adjusts quickly to the new information appeared.

Therefore, it is impossible to outperform the market permanently, except through privileged or lucky information, because the future information or news that affect the price of securities is random and unknown. Simply, some investors and analysts overreact to the news while others react randomly. Some market events like bubbles, crash and depressions are compatible with the HEM, provided they are not predictable [1]

For Samuelson [2], in an efficient market prices fluctuate randomly and the market must meet the following conditions: there are no transaction costs, that is, they should be zero (0); information is available to all investors without cost to them; all market participants have equal time horizon and homogeneous expectations about prices.

Thus, an efficient market implies the existence of conditions of a perfectly competitive market, the rational behavior of those operating in such market and of free information conditions, although these utopian conditions are difficult to find because in reality it is often costly to collect and process information. However, in a market where this costs money,

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analysts probably could identify poorly evaluated values and its profit, in doing so, would be displaced latter for the costs incurred. [3]

If it is too expensive to obtain good information, investors who have to make the same efforts to get spending, must earn rates of abnormal performance, large enough to cover these expenses. However, when we get the corresponding net figure costs, its net income is the same as the net return of investors who select their portfolios at random. [4]

Another definition of efficient market suggested by Fama, means that prices reflect all available information and guides its participants about how it should be their investment decisions [5]

The ideal market is one in which prices provide accurate indications for resource allocation. This is a market where companies can carry out decisions of production and investment, and in which investors can choose among the securities that represent the ownership of the company, under the assumption that prices thereof, fully reflect all available information at all times.

Subsequently, Fama [6] also noted that the necessary conditions for an efficient market are not very inflexible. The market will be efficient if the information is obtained at the same time by a sufficient number of investors, if transaction costs are reasonable, and if there is no significant difference in the performance of different participants.

Meanwhile, Jacquillat and Solnik [7] defined an efficient market as one market "that instantly includes the consequences of past events and accurately reflects the expectations expressed about future events. Thus, the price of a financial asset is at all times a good estimate of its intrinsic value".

The other concept related to the HEM is the Random Walk Theory, which has been much discussed and achieved empirical validity both in the academic field and in the professional one.

Also it is known as the Random Walk Hypothesis (RWH) and Jacquillat and Solnik [7] call it the theory of erratic behavior or random theory of evolution. This theory assumes that market prices follow a random behavior and collect all relevant information available, i.e., future prices depend on new information and are therefore unpredictable.

The Theory of Random Walk requires compliance with two main assumptions: firstly, successive changes in stock prices are independent of each other, and on the other, price variations come from a certain function of probability distribution [8].

This approach has been the subject of constant discussion since Fama [5] suggested the efficient markets hypothesis. Since then, many studies have shown that fluctuations of stock prices follow a random walk.

2. Weak market in the US Efficiency

The efficiency term weak form indicates that prices of securities on the market have a stochastic behavior. It assumes that an investor or analyst cannot predict new information about historical prices.

In this regard, Gómez-Bezares, Madariaga and Ugarte [9] explain this efficiency as follows: ... If value prices remain stable and systematically go up every year on the same date, for example when the memory of the company is published, speculators will buy before the rise and sell after obtaining extraordinary returns...

The so-called technical analysis tries to take advantage of historical information by using systems such as graphics, filters, etc. These systems lack support if the weak efficiency is given.

In the case of technical analysis or *chartismo* as portfolio management strategy, and if the market were efficient this strategy would be useless; which means that the market is inefficient in its weak form for the followers of this strategy. Supporters of technical analysis sustain the hypothesis that prices keep a trend and the market has a historical memory concerning that past

behavior affects present prices. These price behaviors are reflected in a series of figures and avail themselves of a number of techniques such as filters, mobile averages, autocorrelation, and cross-correlation between companies, overreaction or opposite strategy, and seasonal effects: January effect and weekend, among others.

In finance, it is important to note that the degree of efficiency of a market depends on how quickly prices or quotations are adjusted to new information that appear in it.

According to Watts and Zimmerman 1986 mentioned by said Sampaio Franco de Lima GA, Sarto Marzal JL, Barbosa A, Siqueira Lima, I. and Corrar LJ [10], weak efficiency includes the information available to different people at low cost and, therefore, there is no expectation of abnormal returns.

The HEM in its weak form assumes that the price of the securities reflects past information obtained from historical series so it is not possible to obtain returns higher than the market; the news occur randomly and accordingly the change in prices is also random and technical analysis is useless.

This document shows that the series of monthly returns of the US market in the period 2000-2009, behave randomly and thus the HEM is validated in its weak form. The data used for the monthly closing price of the stock indices $S\&P_{500}$ and $S\&P_{100}$, has 120 data.

Prior to conducting the weak efficiency tests, an analysis of descriptive statistics of the time series is employed in the study, in order to describe the set of data used and verify the existence or non-normality of the same for purposes of statistical inference. The non-normality of the series allows to perform some changes familiar with the ultimate aim of achieving normalcy of it. Weak efficiency tests used were: Serial Autocorrelation Function and Unit Root Test of Dickey-Fuller.

3. Descriptive statistics of the series of monthly market returns

The monthly returns are used to prevent not normally features and Leptokurtoses in the series of daily data [6]. This descriptive statistics analysis was conducted to determine the characteristics of the series. The statistical measures used were: arithmetic means, median, maximum value, minimum value, standard deviation, skewness, kurtosis and Jarque Bera statistics.

In Table 35 the results of descriptive statistics and normality test JB series of monthly returns of stock indices under study for the period 2000-2009 are summarized.

Both the $[\![S\ \&\ P]\!]_{100}$ and $[\![S\ \&\ P]\!]_{500}$ present negative average returns, -0.24 and -0.11, respectively; that is, the indices during the study period generated lower profits or even losses for shareholders, on average. The dispersion of the two American series is very similar, 4.69 of $[\![S\ \&\ P]\!]_{100}$ and 4.65 of $[\![S\ \&\ P]\!]_{500}$. This emphasizes the fact that it is the series that showed the lowest variability of the group under study, associated with profitability negative average showing they were affected by major events which influence the stock market. These series show smaller asymmetric coefficients less than zero, showing that its distribution is slight asymmetrical negative and sharp curves are little leptokurtic.

Table 1. Descriptive statistics of the monthly return

	$S\&P_{500}$	**$S\&P_{100}$**
Maximum value	9.671990	10.791900
Minimum value	-16.942450	-14.590880
Average	-0.119814	-0.249311
Median	0.601957	0.279393
Standard deviation	4.652143	4.690514
Skewness	-0.573841	-0.386807

Kurtosis	3.844951	3.431659
Jarque – Bera	10.155590	3.924041
p-valor	0.006234	0.140574

Source: Own calculations. Data obtained from the statistical program.

In order to avoid doing specific results on skewness and kurtosis analysis, the confidence intervals to verify the rejection or not of the following assumptions are estimated: i) for asymmetry, the system hypothesis H 0: $\gamma_1 = 0$ (symmetry of the series) vs. H_1: $\gamma_1 \neq 0$ (asymmetry of the series); ii) in the case of kurtosis, the system was used hypothesis H 0: γ_2: 3 (normal series) vs. H_1: $\gamma_2 \neq 3$ (non-normality of the series). The results can be seen in the following table.

Table 2. Hypothesis testing using confidence intervals for the coefficients of skewness and kurtosis of the series of monthly returns

	Asymmetry			**Kurtosis**		
	Lower limit	**Upper limit**	**Conclusion**	**Lower limit**	**Upper limit**	**Conclusion**
$S\&P_{500}$	-1	-0.13	Simétrica	2.96	4.71	Non normal
$S\&P_{100}$	-0.81	0.05	Simétrica	2.55	4.30	Normal

Source: Own calculations.

Regarding the regularity of the series, it is used a significance level of 5% ($\alpha = 5\%$) as the basis of the analysis, observing the probability values associated with the Jarque-Bera statistics of each series, being evident that the series the $[\![S \& P]\!]_{500}$ do not present normal probability distribution. It is important to highlight that non-normality of the series does not affect the quality of the estimates made using ordinary least squares (OLS), as normality is not a basic assumption associated with the method.

Table 3. Descriptive statistics of the transformed series to achieve normality

	D $S\&P_{500}$
Average	0.057709
Median	0.164949
Maximum value	19.64732
Minimum value	-12.75157
Standard deviation	5.952773
Skewness	0.401815
Kurtosis	3.928875
Jarque – Bera	7.480293
p-valor	0.023751
Observations	119

Source: Own calculations. Statistical program data. D: first difference of the series
No logarithmic transformation was applied because the returns were negative and not possible mathematically.

However, series $[\![S \& P]\!]_{500}$ transformation was performed in order to achieve normality; however this was not possible because the only viable solution was to increase the sample size based on the Central Limit Theorem, and it cannot be applied because it had different samples sizes and could not be comparable to the series of $S\&P_{100}$ that shows normal recording.

The null hypothesis of normality of the series with a level of significance of 5% is rejected, because the same by its transformation is not distributed normally. It is important to express that when the differentiation of a time series is made an observation is lost, thus recalling the series happens to have bad long-term information, to have good short-term information.

4. Autocorrelation function of the series of monthly return market

The autocorrelation of the series aims to measure the correlation or linear association among its values apart for a period of time t and provides the coefficients of linear correlation between the values of the series under study by k time unit's series (delays). In other words, they indicate the degree of correlation between each value of the series and displaced values 1,2 ... n periods.

In practice behaviors graphics lack precision by which, through hypothesis testing can determine when an estimated coefficient (partial autocorrelation or autocorrelation) is considered null. So, a contrast statistical significance of the coefficients is made in this paper and the Ljung-Box, based on the analysis of significance of a series of delays, is used. The null hypothesis ⟦M⟧ _0 is formulated considering the absence of autocorrelation, which means considering the invalidity of a set of coefficients auto covariance or autocorrelation (ρ). ρ_1 = ρ_2 = ⋯ = ρ_m = 0 and the alternative hypothesis states: H_1: ⟦at least one ρ⟧ _i ≠ 0 so as test HEM in weak form is to be detected or not, the presence of autocorrelation and therefore correlograms are performed, considering up to 36 delays from of the monthly returns of the indices $⟦S\&P⟧_{500}$ y $⟦ S\&P⟧_{100}$.

The autocorrelation coefficients and partial autocorrelation and statistical Ljung-Box (Q-statistic) for a significance level of 5% are obtained to support the analysis of significance in hypothesis testing raised earlier. The following graphs show the correlograms and statistical Q series profitability of markets under study. Both series have no autocorrelation. If the value of the probability of Q_ LB contrast (Q statistics) is more than 5% at a significance level of 5% the null hypothesis of no autocorrelation is not rejected. Failure to reject H_0 in most delays means that series is a stochastic process following steady during the period 2000-2009. Therefore, it is inferred that historical returns cannot be used to predict future performance and get extraordinary returns because memory is not transmitted through mistakes, that is, in an event (of large or small extent) nothing guarantees that it stays in the time; and is an element indicating that the HEM is held in its weak form.

Series $⟦S\&P⟧_{100}$ suggests that the HEM is held in weak forms for all delays, because the H_0 is not rejected. In the case of $⟦S\&P⟧_{500}$, from the 2nd delay absence of autocorrelation was achieved. This situation observed in the last series could be justified by the events outlined in the economy during the period under study.

5. Unit root test of Dickey-Fuller price series of monthly closing of markets

In a time series like the price of monthly market closing, it is common that a random type behavior path is followed with no tendency; it may even be random and non-stationary and may not predict future prices from historical prices, given its characteristics. If on the other hand, a number of market prices do not follow a random walk, but manifest stationary behavior, it indicates that future prices can be predicted from historical prices. Thus, it is possible to design and leverage investment proposals based on historical data. The statistical test commonly used to try the hypothesis of random walk in a series of financial market is the Unit Root Test developed by Dickey-Fuller DF, also known as contrast non stationary. In a random walk market prices respond to returns uncorrelated, that is, prices do independent successive changes.

This test allows detecting non stationary in a series under the null hypothesis of the presence of a unit root in the data of the analyzed series generating process (DGP).

The model based initially this test is an autoregressive first order process AR (1) y_t = α_1 y_ (t-1) + ε_t, which allows determine whether the series or not, a trend component and the coefficients meet stationary conditions, compared to the simple model of random walk

stochastic trend or simply y_t = y_ (t-1) + ε_t. Therefore, it would be contrasted if: α_1 = 1 or ≠ 1 α_1.

In this case the probability distribution estimator asymptotic ordinary least squares (OLS) model AR (1) has a discontinuity when α_1 = 1 and derivative distributions should be used empirically by a Monte Carlo process by Dickey in 1976. The aforementioned Montecarlo process allows to test whether it is the α_1 = 1 or instead parameter α_1 ≠ 1.

The initial null hypothesis H 0: α_1 = 1 becomes H 0: δ = 0 against the alternative hypothesis H_1: δ <0. In this case, by accepting the null hypothesis δ = 0, it is equal to say α_1 = 1, so there is a unit root, the process is not stationary; while if δ <0, it is equal to say that α_1 <1 and the process turns out to be stationary.

Before estimating the model parameters is necessary to decide which will be the data generating process because the statistical reference for the contrast will be different depending on the model chosen. In the present investigation the simple model is chosen, because interest verifies that the study series have a unit root and thus remain a non-stationary random process, which would validate the HEM in its weak form.

Also, the test increased DF (DFA) which allows testing for the presence of the unit root in a series that follows an AR (ρ) process was used, as many time series are better suited to autoregressive processes of higher order AR (2), AR (3), and it would be wrong to contrast the presence of a unit root using the structure of an AR (1) model also test DFA is a possible correction autocorrelation problems that may exist in the term DF basic model error.

Except that it is only necessary to test the DFA model with constant, only if they reject the null hypothesis H_O is also made: δ = 0 and ensure that corresponds to an autoregressive process with non-zero average.

Igualmente, se justificaría el uso del modelo DFA con constante y tendencia determinista, también en caso de no rechazar la hipótesis nula y se desee garantizar la consistencia del contraste. Similarly, the use of DFA model with constant and deterministic trend, also should not reject the null hypothesis and ensure consistency desired contrast is justified.

The results of the test DFA (simple model) is derived from the prices of monthly closing stock indices: $[\![S \& P]\!]_{500}$ and $[\![S \& P]\!]_{100}$ for the period 2000 to 2009. Table 4 presents the probability tables obtained in DFA, together with the respective conclusion, both level and first difference to verify when the series is stationary or not. For this purpose a significance level of 5% to reject or not the hypothesis and the conclusion set was used.

Table 4. Hypothesis testing for Dickey-Fuller unit root

	Level		**First difference**	
	p value (probability)	**Conclusion**	**p value (probability)**	**Conclusion**
$S\&P_{500}$	0.4339	Non stationary	0.0000	Stationary
$S\&P_{100}$	0.2284	Non stationary	0.0000	Stationary

Source: Own calculations. Results of the statistical program.

The above results allow to check that with a significance level of 1%, 5% and 10%, the probability of error assumed by rejecting the null hypothesis H 0: δ = 0 is greater than 5% for both series, so it is not rejected the null hypothesis in the US market and concludes that the series under study are stationary, follow a random walk over the period 2000-2009, that is, is stochastic series and HEM is validated in its weak form.

6. Conclusion

To analysis the weak form efficiency they are considered two time series of the US market during the period 2000-2009, using monthly closing prices of the most representative stock market indices. Summary statistics are presented In Table 5.

Table 5: Summary statistics

Index	Test Jarque– Bera	Test Ljung-Box	Test Dickey – Fuller (First difference)
$S\&P_{500}$	Non normal	Non autocorrelation	Stationary
$S\&P_{100}$	Normal	Non autocorrelation	Stationary

Source: Own calculations

Is worth mentioning that non-normality of the series does not affect the quality of the estimates made by MCO, as normality is not a basic assumption associated with the method. Both series have no autocorrelation.

The Dickey-Fuller test results allowed to check that with a significance level of 1%, 5% and 10%, the probability of error assumed by rejecting the null H 0 hypothesis: $\delta = 0$ is greater than 5% for both series, therefore the null hypothesis is not rejected and concludes that the series under study are stationary, follow a random walk over the period 2000-2009, ie series is stochastic which validates the HEM in its weak form.

References

[1] Cootner, P.H., (1962). "Stock Prices: Random Versus Systematic Changes", *Industrial Management Review*, Vol.3, Num. 2. Primavera, pp. 24-45.

[2] Samuelson P. (1965). "Proof that Properly Anticipated Prices Fluctuate Randomly". *Industrial Management Review*, Vol. 6, pp. 41-49.

[3] Alexander G.J., Sharpe W.F., Bailey J.V. (2003). "*Fundamentos de Inversiones, Teoría y Práctica*". Editorial Pearson Prentice Hall. Tercera Edición, Méjico, pp. 72-76.

[4] Weston J. y Copeland T. (1995). "*Finanzas en Administración*". Novena edición. Volumen I. Editorial Mc Graw Hill, p.111-116.

[5] Fama. E. (1970) "Efficient Capital Markets. A Review of Theory and Empirical Work", *Journal of Finance*. Vol. 25, Nº 2, Mayo, pp. 383-417.

[6] Fama, E. (1976). "*Foundations of Finance*". New York: Basic Books.

[7] Jacquillat, B. y Solnik B. (1975). "*Mercados Financieros y Gestión de Carteras de Valores*", Editorial Tecniban S.A., Madrid, p. 22.

[8] Swoboda C. (2001). "La hipótesis del Random Walk en el Mercado de Valores de la Argentina. Período 1992-2000". http://cdi.mecon.gov.ar/biblio/docelec/uncor/DT7.pdf Fecha de la consulta: 19 de abril de 2004.

[9] Gómez-Bezares F., Madariaga J.A. y Ugarte J.V. (1988). "La eficiencia en el mercado bursátil español." Actualidad Financiera, Nº 42, noviembre pp. 2238-2250.

[10] Sampaio Franco De Lima, G.A., Sarto Marzal J.L, Barbosa A, Siqueira Lima, I. Y Corrar L.J (2006). "El estudio del anuncio de la adhesión a los niveles diferenciados de gobierno corporativo con la utilización de estudio del evento". *VI Conferencia Internacional de Finanzas,* Santiago mayo de 2006, p.95.

Interdependence among Capital Markets of Germany, Poland and Baltic States. Application of DCC-GARCH Model

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Abstract

The growing interdependencies among capital markets are becoming significant factor affecting process of risk management both at macro and microeconomic level. Thus, the aim of the article is the analysis of interdependencies among capital markets of Germany, Poland and Baltic States. In order to measure the interdependencies DCC-GARCH model was applied. The research was conducted for the years 2004-2015. The capital market of Germany was taken as the one that has the biggest influence on the analysed Central and Eastern European markets. The conducted research enabled to determine the different nature of the interdependencies among the capital markets of Baltic States and Poland on one side and German capital market on the other side.

Key words

capital market, conditional variance, conditional correlation, DCC-GARCH model, integration

JEL Classification: G15, C58

1. Introduction

Last global financial crisis has confirmed the importance of interdependencies among economies. The interdependencies in real sphere can lead to transition of shocks in the case of all aspects of socio-economic life such as labour markets and human capital (Müller-Frączek & Pietrzak 2011; Wilk *et al*., 2013; Biczkowski *et al*., 2014; Pietrzak & Balcerzak 2016, Balcerzak, 2016a; Balcerzak & Pietrzak, 2016a; 2016b), sustainability of economies (Balcerzak & Pietrzak, 2016c; Pietrzak & Balcerzak, 2016b), international trading positions of countries and their technological potential (Balcerzak & Pietrzak, 2016c, Balcerzak, 2016b, Pietrzak & Łapińska, 2015; Vitunskiene & Serva, 2015; Łapińska, 2016), institutional order (Balcerzak, 2009; 2015; Balcerzak & Pietrzak 2016d), and last but not least macroeconomic financial stability of countries (Mackiewicz-Łyziak, 2015; Balcerzak & Rogalska, 2016; Balcerzak *et al*., 2016). However, in recent decades the empirical research have confirmed that the most significant interdependencies among economies can be found in the case of capital markets (Forbes & Rigobon, 2002, Pericoli & Sbracia, 2003; Billio & Caporin, 2010, Fałdziński *et al.*, 2012; Heryán & Ziegelbauer, 2016; Pietrzak et al., 2017). As a result, the research devoted to the factors that can destabilise groups of capital markets and finding the transmission channels of potential disturbances is crucial for risk management (Baur, 2003; Corsetti *et al*., 2005; Engle 2009; Zinecker *et al*., 2016).

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The main aim of the article is the analysis of interdependencies among capital markets of Poland, Baltic States and Germany. The research was conducted for the years 2004-2015. The first year of the study was chosen as it was the year of the biggest enlargement of the European Union, which significantly increased the interrelations among analysed economies. The last year of the research was restricted by the availability of data for the whole group of market indices. In order to verify the interdependencies DCC-GARCH model was applied.

In the research two hypotheses were proposed. According to the first one, there is a low level of interdependencies among capital markets of Baltic States and Germany. In the second hypothesis the existence of different mechanisms of reactions of capital markets of Baltic States and Poland to shocks coming from German market is assumed.

2. Methodology

An observation of periods of significantly increased and reduced volatility of capital markets prompted the researchers to introduce GARCH class models (Engle & Bollerslev, 1986). GARCH models allowed estimating the conditional variance for individual assets or indices. The methodology of GARCH models has been extended to the multi-dimensional case, which allowed inclusion of interdependencies among indices (Faldzinski & Pietrzak, 2015; Fałdziński *et al.,* 2016). DCC-GARCH model is an example of multi-dimensional model, which allows describing interdependencies between markets by estimating the conditional correlation (Engle 2002). In DCC-GARCH model the conditional variance depends on the lagged conditional variances and the square returns with the consideration of conditional means. In the case of a conditional correlation equation the standardized residuals from the variance equation and lagged conditional correlations make the describing variables. The estimation of the parameters of DCC-GARCH model can be done with application of the maximum likelihood method (Engle in 2002, 2009). In that case the two-step estimation method is the most commonly used. It assumes a separate estimation of the parameters of conditional variance equation and means. Then, on the basis of the obtained estimates of parameters, the parameters of conditional correlation equation are obtained (Fałdziński & Pietrzak, 2015). DCC-GARCH model can be written as follows:

$$\mathbf{Y_t} = \boldsymbol{\mu_t} + \boldsymbol{\eta_t}, \quad \boldsymbol{\eta_t} \mid \mathrm{F}_{t-1} \sim t(0, \mathbf{D_t R_t D_t}, v), \tag{1}$$

$$\mathbf{D_t^2} = diag\{\mathbf{H_t}\}, \ \mathbf{H_t} = \mathbf{V_{t\text{-}1}}(\boldsymbol{\eta_t}), \ \mathbf{H_t} = \boldsymbol{\omega_i} + \boldsymbol{\alpha_i}\boldsymbol{\eta}_{\mathbf{i,t\text{-}1}}^{\mathbf{2}} + \boldsymbol{\beta_i}\mathbf{H_{i,t\text{-}1}}, \ \boldsymbol{\varepsilon_t} = \mathbf{D_t^{-1}}\boldsymbol{\eta_t} \tag{2}$$

$$\mathbf{R_t} = diag\{\mathbf{Q_t}\}^{-1/2}\mathbf{Q_t}diag\{\mathbf{Q_t}\}^{-1/2}, \ \mathbf{Q_t} = \boldsymbol{\Omega} + \alpha\boldsymbol{\varepsilon}_{\mathbf{t\text{-}1}}\boldsymbol{\varepsilon}'_{\mathbf{t\text{-}1}} + \beta\mathbf{Q_{t\text{-}1}}, \ \boldsymbol{\Omega} = \bar{\mathbf{R}}(1-\alpha-\beta), \tag{3}$$

where: $\mathbf{Y_t}$ - the multivariate process of returns, $\boldsymbol{\mu_t}$ - the vector of conditional means of returns, $\mathbf{H_t}$ - the conditional variance matrix for *i*-th returns, $\mathbf{R_t}$ - the time-varying conditional correlation matrix, $\mathbf{V_{t\text{-}1}}(\boldsymbol{\eta_t})$ - the covariance matrix of the residuals $\boldsymbol{\eta_t}$, $\boldsymbol{\omega_i}, \boldsymbol{\alpha_i}, \boldsymbol{\beta_i}$ - the parameters of the conditional variance equation, α, β - the parameters of the conditional correlation equation, $\bar{\mathbf{R}} = \frac{1}{T}\sum_{i=1}^{T}\boldsymbol{\varepsilon_t}\boldsymbol{\varepsilon_t}'$ - the unconditional covariance matrix.

After estimating conditional correlation the next step in the research can be an analysis of order of integration for conditional correlation for pairs of selected indices. To measure the order of integration of stochastic processes in 1979 Dickey and Fuller proposed the unit root test DF. Its statistics is correct only if the stationary process is a AR(1) process (see Dickey & Fuller, 1979). Then, Phillips and Perron (1988) provided a method of solving the problem of autocorrelation of higher orders in the case of testing non-stationarity (see Phillips & Perron, 1988). They modified the DF statistics in such way that the autocorrelation of higher order does not affect the asymptotic distribution of the test statistic. In the first step of the procedure

for the Phillips and Perron test it is assumed in the null hypothesis that the process is non-stationary of order 1 at least. Rejection of the null hypothesis results in the statement that the process is stationary, ie. an integrated of order 0. In the case of the absence of evidence allowing rejecting the null hypothesis, the test is repeated for the first differences of the process. In the null hypothesis it is assumed that the process is non-stationary of order 2 at least, and in the alternative hypothesis that the process is non-stationary of order 1. Appling the test is continued until the rejection of the null hypothesis.

3. Analysis of interdependencies among markets of Lithuania, Latvia, Estonia, Poland and German capital market

In the conducted empirical research time series for five indices were uses: (OMX Riga, OMX Tallinn, OMX Vilnius, WIG and DAX). For all the indices a daily rates of return in the period 02.01.2004–02.0.2015 were assessed. The year 2004 was chosen because of the accession of the Baltic States and Poland to the European Union. This factor has influenced significantly the development of capital markets in these countries (Meluzin & Zineker, 2016) and could also affect the interdependences among the markets. Therefore, the research for this period seems to be justified by the existence of similar institutional conditions for the functioning of the capital markets of selected countries.

The study began by estimating parameters of DCC-GARCH model. For this purpose the maximum likelihood method with a conditional normal distribution was applied. In the first step separate estimation of the parameters of the conditional variance equation and means was performed, where AR(p)-IGARCH(p,q) model was assumed for all the indices. The analysis of the indices indicated a maximum degree of autoregression equal to one. In addition, preliminary estimation of the volatility models indicated IGARCH model the best fit for all the indices. Therefore, for all the indices the following specification of AR(1)-IGARCH (1,1) model was made:

$$\mathbf{y}_t = \gamma_0 + \gamma_1 y_{t-1} + \eta_t, H_t = \omega_0 + \alpha_1 \eta_{t-1}^2 + \beta_1 H_{t-1},$$

where the sum of parameters α_1 and β_1 equals to 1.

According to the two-step maximum likelihood method in the second step the parameters of the conditional correlation equation were estimated. The results are presented in Table 1. In the case of the conditional mean equations parametr γ_1 was statistically insignificant only for the German capital market. In the case of the conditional variance equations and conditional correlation in DCC-GARCH model for all the indices all the parameters α1 and β1 were statistically significant at the 5% significance level.

Table 1: The results of the estimation of the DCC-GARCH model

The conditional variance equation					
Parameter (stock index)	**Estimate**	**p-value**	**Parameter (stock index)**	**Estimate**	**p-value**
γ_0 (OMX RIGA)	0.058	~0.00	γ_0 (WIG)	0.085	0.005
γ_1 (OMX RIGA)	-0.094	~0.00	γ_1 (WIG)	0.071	~0.00
ω_1 (OMX RIGA)	0.015	0.061	ω_1 (WIG)	0.008	0.007
α_1 (OMX RIGA)	0.089	~0.00	α_1 (WIG)	0.074	~0.00
β_1 (OMX RIGA)	0,911	-	β_1 (WIG)	0.926	-

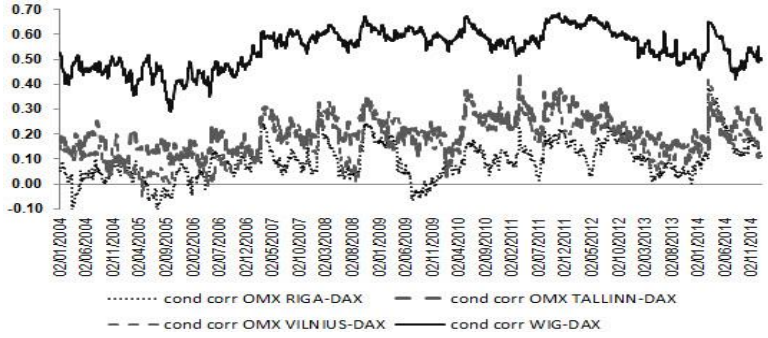
γ_0 (OMX TALLINN)	0.022	0.182	γ_0 (DAX)	0.081	~0.00
γ_1 (OMX TALLINN)	0.143	~0.00	γ_1 (DAX)	-0.007	0.707
ω_1 (OMX TALLINN)	0.013	~0.00	ω_1 (DAX)	0.018	0.006
α_1 (OMX TALLINN)	0.138	~0.00	α_1 (DAX)	0.109	~0.00
β_1 (OMX TALLINN)	0.862	-	β_1 (DAX)	0.891	-
γ_0 (OMX VILNIUS)	0.066	~0.00			
γ_1 (OMX VILNIUS)	0.138	~0.00	**The conditional correlation equation**		
ω_1 (OMX VILNIUS)	0.015	0.212	**Parameter**	**Estimate**	**p-value**
α_1 (OMX VILNIUS)	0.116	0.049	α	0.011	~0.00
β_1 (OMX VILNIUS)	0.884	-	β	0.977	~0.00

Source: own estimation.

DCC-GARCH model allows the measurement of the strength of interdependencies among analysed capital markets by means of the conditional correlations. The estimation of the parameters of DCC-GARCH model allowed to determine the value of the conditional correlation for the next pairs of indices. An analysis of the variability of conditional correlation over time allows to verify tendencies in the interdependences.

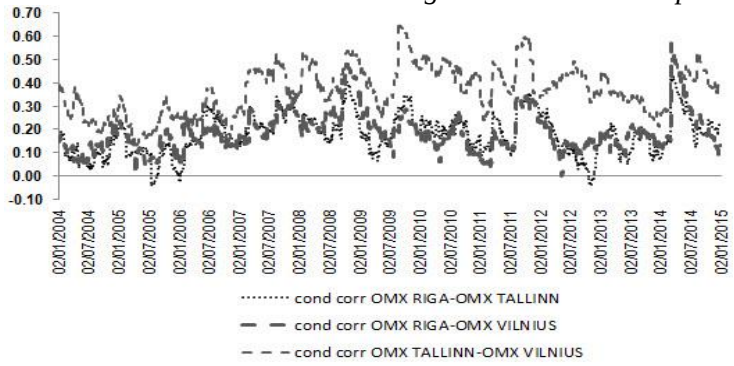
Figure 1 presents the conditional correlation among the DAX index and OMX Riga, OMX Tallinn, OMX Vilnius and WIG indices. The analysis of the correlations indicates that the interdependencies among the German capital market and the capital markets of the Baltic States are at a very low level. Thus, it can be said that since 2004 the shocks occurring on the German capital markets have been transferred to a low extant to the markets of Estonia, Latvia and Lithuania. However, in the case of Poland the interdependence with German capital market is at a much higher level. This means that the valuation of Polish index is largely dependent on the situation on the capital market in Germany.

Figure 1: The conditional correlation among chosen capital markets and market of Germany



On the other hand, Figure 2 presents the conditional correlations among indices of Baltic countries: OMX Riga, OMX Tallinn and OMX Vilnius. One can find weak interdependencies among the capital market of Latvia and the Estonian and Latvian markets. In the case of Estonian and Lithuanian capital markets the interdependencies are at a much higher level. The obtained results for the conditional correlations allowed to verify the hypothesis about the low level of interdependence among the markets of the Baltic Countries and the German capital market.

Figure 2: The conditional correlation among Baltic Countries capital markest



The values of the conditional correlations for the pairs of markets have been used to analyse integration. This allowed to verify the degree of integration of the interdependencies among pairs of selected markets. With application of Phillips-Perron test (Phillips & Perron, 1988) both the stationarity of the processes of conditional correlation I(0), and the integration of order one I(1) were established. The results of the Phillips-Perron test are shown in Table 2. In the case of the processes of conditional correlation for a pair of indices: OMX Riga and DAX, the test allowed to determine the integration of order zero. This means that the correlation process for a pair of indices OMX Riga and DAX is stationary. This stationarity may be the result of weak reaction of the Latvian market to shocks coming from other capital markets, including the capital market of Germany. Additionally, it is probable that if there are any reactions of the Latvian market, they do not case long term changes in the character of the interdependence. This can be confirmed by the values of conditional correlations, which for the pair of OMX Riga index with other indices are at much lower level in comparison to the values for other pairs of indices.

Table 2: The results of the integration analysis

Pair of indices	**Time series**	**Test statistics**	**p-value**	**Order of integration**
OMX Riga, DAX	Levels	-2.632	0.008	I(0)
	Differences	-	-	
OMX Tallinn, DAX	Levels	-1.298	0.1796	I(1)
	Differences	-52.947	~0.00	
OMX Vilnius, DAX	Levels	-1.691	0.086	I(0) for α=0.1 I(1) for α=0.05
	Differences	-52.746	~0.00	
WIG, DAX	Levels	-0.491	0.503	I(1)
	Differences	-52.321	~0.00	

Source: own estimation.

In the case of the processes of conditional correlations for pair of indices OMX Tallinn, DAX and WIG, DAX the test results indicate the integration of order one. For the process of the conditional correlation for a pair of indices OMX Vilnius and DAX one can find the integration of order one with 5% significance level. The increasing of the significance level to 10% results in rejecting the null hypothesis and the conclusion of integration of order zero. This means that other processes of conditional correlations are non-stationary in variance. The non-stationarity may be the result of strong reactions of the selected markets to shocks coming from the German capital market, which contribute to the long term changes in the levels of indices.

The results on the integration of processes of the conditional correlation enable to verify second hypothesis, which pointed to different reaction mechanism of Baltic States capital markets and Polish market to shocks coming from market of Germany.

4. Conclusions

The article concentrates on the problem of interdependencies among capital markets of Baltic States, Poland and Germany. The identification of interrelations among markets is a research problem, which is important form the perspective of risk management. Verifying the transmission channels of shocks between the markets is necessary for mitigating the costs of crisis situations.

In order to measure the interdependencies among selected markets DCC-GARCH model has been used. After estimation of the parameters of the model the conditional correlations showing the interrelationships among capital markets were obtained. The analysis allowed to conclude that there are very weak interrelationships among German capital market and the markets of the Baltic States. The study also allowed the identification of different mechanisms of formation of interdependences among German market and the capital markets in selected countries.

References

[1] Balcerzak, A.P. (2009). Effectiveness of the Institutional System Related to the Potential of the Knowledge Based Economy. *Ekonomista*, 6, pp. 711-739.

[2] Balcerzak, A.P. (2015). Europe 2020 Strategy and Structural Diversity Between Old and New Member States. Application of Zero-unitarizatin Method for Dynamic Analysis in the Years 2004-2013. *Economics & Sociology*, 8(2), pp. 190-210.

[3] Balcerzak, A.P. (2016a). Multiple-criteria Evaluation of Quality of Human Capital in the European Union Countries. *Economics & Sociology*, 9(2), pp. 11-26.

[4] Balcerzak, A.P. (2016b). Technological Potential of European Economy. Proposition of Measurement with Application of Multiple Criteria Decision Analysis. *Montenegrin Journal of Economics*, 12(3). DOI: 10.14254/1800-5845.2016/12-3/4.

[5] Balcerzak, A.P. and Pietrzak, M.B. (2016a). Human Development and Quality of Institutions in Highly Developed Countries. In: M. H. Bilgin, H. Danis, E. Demir, and U. Can (Eds.). *Financial Environment and Business Development. Proceedings of the 16th Eurasia Business and Economics Society*. Springer International Publishing.

[6] Balcerzak, A.P. and Pietrzak, M.B. (2016b). Quality of Human Capital in European Union in the Years 2004-2013. Application of Structural Equation Modeling. In: *Proceedings of the International Scientific Conference Quantitative Methods in Economics Multiple Criteria Decision Making XVIII*. Vratna: Letra Interactive, pp. 7-12.

[7] Balcerzak, A. P. and Pietrzak, M. P. (2016c). Application of TOPSIS Method for Analysis of Sustainable Development in European Union Countries. In: T. Loster & T. Pavelka (Eds.). *The 10th International Days of Statistics and Economics. Conference Proceedings*. September 8-10, 2016. Prague.

[8] Balcerzak, A.P. and Pietrzak, M.B. (2016c). Structural Equation Modeling in Evaluation of Technological Potential of European Union Countries in the Years 2008-2012. In: M. Papież and S. Śmiech (Eds.). *The 10th Professor Aleksander Zelias International Conference on Modelling and Forecasting of Socio-Economic Phenomena. Conference Proceedings*. Cracow: Foundation of the Cracow University of Economics, pp. 9-18.

[9] Balcerzak, A.P. and Pietrzak, M.B. (2016d). Quality of Institutions for Knowledge-based Economy within New Institutional Economics Framework. Multiple Criteria Decision Analysis for European Countries in the Years 2000–2013. *Economics & Sociology*, 9(4).

[10] Balcerzak, A.P., Pietrzak, M.B. and Rogalska, E. (2016). Fiscal Contractions in Eurozone in the years 1995-2012: Can non-Keynesian effects be helpful in future deleverage process?. In: M. H. Bilgin, H. Danis, E. Demir, U. Can (Eds.). *Business Challenges in the Changing Economic Landscape - Vol. 1. Proceedings of the 14th Eurasia Business and Economics Society*. Springer International Publishing. 2016, pp. 483-496.

[11] Balcerzak, A.P. and Rogalska, E. (2016). Non-Keynesian Effects of Fiscal Consolidations in Central Europe in the Years 2000-2013. In: M. H. Bilgin, H. Danis, (Eds.). *Entrepreneurship, Business and Economics - Vol. 2. Proceedings of the 15th Eurasia Business and Economics Society*. Springer International Publishing, pp. 271-282.

[12] Baur, D. (2003). Testing for Contagion – Mean and Volatility Contagion. *Journal of Multinational Financial Management*, 13, pp.. 405-422.

[13] Biczkowski, M., Muszyńska, M., Müller-Frączek, I., Pietrzak, M.B. and Wilk, J., 2014. The Delimitation of Bipolar Metropolitan Area Within the kujawsko-pomorskie Region, *Oeconomia Copernicana*, 5(2), pp. 101-122.

[14] Billio, M. and Caporin, M. (2010). Market Linkages, Variance Spillover and Correlation Stability: Empirical Evidences of Financial Contagion. *Computational Statistics & Data Analysis*, 54(11), pp. 2443-2458.

[15] Corsetti, G., Pericoli, M. and Sbracia, M. (2005). Some Contagion, Some Interdependence: More Pitfalls in Testing for Contagion. *Journal of International Money and Finance*, 24, pp. 1177-1199.

[16] Dickey, D.A. and Fuller W.A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association*, 74, pp. 427–431.

[17] Engle, R.F. and Bollerslev, T. (1986). Modeling the Persistence of Conditional Variances. *Econometric Reviews*, 5, pp. 1-50.

[18] Engle, R.F. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. *Journal of Busines & Economic Statistics*, 20, pp. 339-350.

[19] Engle, R.F. (2009), *Anticipating Correlations A New Paradigm for Risk Management*. Princeton and Oxford: Princeton University Press.

[20] Fałdziński, M., Balcerzak, A.P., Meluzín, T., Pietrzak, M. B., and Zinecker, M. (2016). Cointegration of Interdependencies Among Capital Markets of Chosen Visegrad Countries and Germany. In: *34rd International Conference Mathematical Methods in Economics MME 2016 Conference Proceedings*, September 6-9, 2016 Liberec.

[21] Fałdziński, M., and Pietrzak, M.B. (2015). The Multivariate DCC-GARCH Model With Interdependence Among Markets in Conditional Variances' Equations. *Przeglad Statystyczny*, 62(1), pp. 397-413.

[22] Fałdziński, M., Osińska, M., and Zdanowicz, T. (2012). Detecting Risk Transfer in Financial Markets using Different Risk Measures, *Central European Journal of Economic Modelling and Econometrics*, 4(1), pp. 45-64.

[23] Forbes, K. and Rigobon, R. (2002). No Contagion, Only Interdependence: Measuring Stock Market Comovements. *Journal of Finance*, 57(5), pp. 2223-2261.

[24] Heryán, T. and Ziegelbauer, J. (2016). Volatility of Yields of Government Bonds Among GIIPS Countries During the Sovereign Debt Crisis in the Euro Area. *Equilibrium. Quarterly Journal of Economics and Economic Policy,* 11(1), pp. 61-74

[25] Łapińska, J. (2016). Determinant Factors of Intra-Industry Trade: the Case of Poland and Its European Union Trading Partners. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, 11(2), pp. 251-264.

[26] Mackiewicz-Łyziak, J. (2015). Fiscal Sustainability in CEE Countries – the Case of the Czech Republic, Hungary and Poland. *Equilibrium. Quarterly Journal of Economics and Economic Policy,* 10(2), pp. 53-7.

[27] Meluzín, T., & Zinecker, M. (2016). Trends in IPOs: The Evidence from CEE Capital Markets. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, 11(2), 327-341.

[28] Müller-Frączek, I. and Pietrzak, M. B. (2011). Spatial Analysis of the Unemployment Rate in Poland in 2004-2008. In P. Jedlicka (Ed.). *Hradec Economic Days 2011*. Hradec Králové: Gaudeamus, pp. 205-209.

[29] Pericoli, M. and Sbracia, M. (2003). A Primer on Financial Contagion. *Journal of Economic Surveys*, 17(4), pp. 571-608.

[30] Phillips, P. and Perron, P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrica*, 75(2), pp. 335-346.

[31] Pietrzak, M.B. and Balcerzak, A.P. (2016a). A Spatial SAR Model in Evaluating Influence of Entrepreneurship and Investments on Unemployment in Poland. In: *Proceedings of the International Scientific Conference Quantitative Methods in Economics Multiple Criteria Decision Making XVIII*. Vratna: Letra Interactive, pp. 303-308.

[32] Pietrzak, M.B., and Balcerzak, A.P. (2016b). Assessment of Socio-Economic Sustainability in New European Union Members States in the years 2004-2012. In: M. Papież & S. Śmiech (Eds.). *The 10th Professor Aleksander Zelias International Conference on Modelling and Forecasting of Socio-Economic Phenomena*. *Conference Proceedings*. Cracow: Foundation of the Cracow University of Economics, pp. 120-129.

[33] Pietrzak, M. B., Fałdziński, M., Balcerzak, A. P., Meluzín, T. & Zinecker, M. (2017). (2017). Short-term Shocks and Long-term Relationships of Interdependencies Among Central European Capital Markets. *Economics & Sociology*, 10(1). DOI: 10.14254/2071-789X.2016/10-1/5.

[34] Pietrzak, M. B. and Łapińska, J. (2015). Determinants European Union's trade – Evidence From a Panel Estimation of the Gravity Model. *E & M Ekonomie a Management*, 18(1), pp. 18-27.

[35] Wilk, J., Pietrzak, M. B. and Siekaniec, M. (2013). The Impact of Metropolitan Areas on Internal Migrations in Poland. The Case of Southern Regions. In: M. Papież, and S. Śmiech (Eds.). *Proceedings of the 7th Professor Aleksander Zelias International Conference on Modelling and Forecasting of Socio-Economic Phenomena.* Cracow: Foundation of the Cracow University of Economics, pp. 124-132.

[36] Vitunskiene, V., & Serva, E. (2015). Shifts in Lithuania's Agri-food Industry Export Competitiveness: a Comparative Analysis Versus High- and Medium-high Technology Manufacturing Industries. *Oeconomia Copernicana*, 6(1), pp. 7-31,

[37] Zinecker, M., Balcerzak, A.P., Fałdziński, M., Meluzín, T. and Pietrzak, M.B., (2016). Application of DCC-GARCH Model for Analysis of Interrelations Among Capital Markets of Poland, Czech Republic and Germany. In: *Proceedings of the International Scientific Conference Quantitative Methods in Economics Multiple Criteria Decision Making XVIII.* Vratna: Letra Interactive, pp. 416-421.

Slovak Accounting Legislation: a Basic Framework for Predictive Financial Models

Viera Bartošová[1], Viera Berzáková [2]

Abstract

Every business has to track the accounting laws and their changes. The last amendment to the accounting legislation for businesses in the Slovak Republic brought some important changes in accounting practice, which will affect not only the content of reported information in the framework of regulated financial accounting for its users, but it will also affect most of the indicators of the financial analysis of the Slovak enterprises in its wider context. This kind of amendment influences all the accounting units in the Slovak Republic. This article discuses and deals with the key problems related to the latest amendments of the Act on Accounting and accounting methodology.

Key words

financial accounting, accounting legislation, enterprise, predictive financial models, accounting data, accounting/financial statements

JEL Classification: G3, M4

1. Úvod

Finančné účtovníctvo je údajovou základňou, z ktorej vychádza každá štandardne koncipovaná finančná analýza podniku, čo znamená, že výsledky ukazovateľov finančnej analýzy – finančných indikátorov – sú do veľkej miery ovplyvnené účtovnými zásadami a konvenciami, ktoré slovenské účtovné jednotky pri vedení účtovníctva dodržiavajú. (Kicova and Kramarova, 2013) Regulovaná podoba finančného účtovníctva v Slovenskej republike si aj po akceptovaní právnych aktov EÚ, resp. medzinárodných štandardov IAS/IFRS zachováva svoje špecifiká, ktoré ju odlišujú od iných, príbuzných národných účtovných systémov, napr. Českej republiky, čo je dané konkrétnou právnou úpravou. Právny rámec účtovníctva podnikateľov v SR reprezentuje v prvom rade zákon č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov a celý rad opatrení Ministerstva financií Slovenskej republiky (ďalej MF SR), spomeňme najmä opatrenie MF SR č. 23054/2002-92 zo 16. decembra 2002, ktorým sa ustanovujú podrobnosti o postupoch účtovania a rámcovej účtovej osnove pre podnikateľov účtujúcich v sústave podvojného účtovníctva v znení neskorších predpisov, a tiež opatrenie MF SR č. 4455/2003-92 z 31. marca 2003, ktorým sa ustanovujú podrobnosti o usporiadaní, označovaní a obsahovom vymedzení položiek individuálnej účtovnej závierky a rozsahu údajov určených z individuálnej účtovnej závierky na zverejnenie pre podnikateľov účtujúcich v sústave podvojného účtovníctva v znení neskorších predpisov. (Bartosova and Paliderova, 2014)

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Bez ohľadu na niektoré rozdiely v právnej úprave možno konštatovať, že slovenské účtovníctvo je harmonizované v rámci EÚ a aj v širšom medzinárodnom meradle a tento proces pokračuje. Svedčí o tom neustále prispôsobovanie sa jeho podoby medzinárodným účtovným štandardom a všeobecne uznávaným účtovným zásadám, ktoré zastupujú požiadavku transparentnosti, zrozumiteľnosti a lepšej čitateľnosti účtovníctva a sú vo všeobecnosti akceptované vo svete. (Gregova, 2007)

Posledné novely účtovných predpisov v SR priniesli niektoré významné zmeny, ktoré sa pochopiteľne premietnu aj do záverov finančných analýz a celkového hodnotenia finančnej situácie podnikov v SR. (Cisko and Kliestik, 2013) V tejto súvislosti zaujmeme v príspevku stanovisko najmä k novele postupov účtovania pre podnikateľov.

2. Zmeny v metodike účtovania u podnikateľov

Novela opatrenia Ministerstva financií Slovenskej republiky zo 16. decembra 2002 č. 23054/2002-92, ktorým sa ustanovujú podrobnosti o postupoch účtovania a rámcovej účtovej osnove pre podnikateľov účtujúcich v sústave podvojného účtovníctva v znení neskorších predpisov, priniesla do slovenského účtovníctva a účtovného vykazovania podnikateľských subjektov – právnických osôb niektoré významné zmeny. Ustanovenia tejto novely nadobudli účinnosť od 31. decembra 2014, najmä ustanovenia týkajúce sa zrušenia mimoriadnej činnosti, a od 1. januára 2015 nadobudli účinnosť ostatné ustanovenia, ktoré sa týkajú najmä účtovania podielových účastí v iných účtovných jednotkách.

Jednou z najvýznamnejších zmien je *zrušenie mimoriadnej činnosti účtovnej jednotky,* ktorá mala svoje miesto v štruktúre rámcovej účtovej osnovy pre podnikateľov: účtové triedy 5 – Náklady a 6 – Výnosy, a následne v štruktúre výsledku hospodárenia za účtovné obdobie. Výsledok hospodárenia sa po zmene postupov člení iba na *výsledok hospodárenia z hospodárskej činnosti a výsledok hospodárenia z finančnej činnosti*. V súvislosti s tým úplne zanikli účtové skupiny 58 – Mimoriadne náklady a 68 – Mimoriadne výnosy, ako aj prevodové účty v účtových skupinách 59 a 69.

Výsledok hospodárenia z hospodárskej činnosti a výsledok hospodárenia z finančnej činnosti po novom spolu tvoria *výsledok hospodárenia za účtovné obdobie*, bez členenia na bežnú a mimoriadnu činnosť, ktoré takisto stratilo svoje opodstatnenie. Je dôležité doplniť, že *všetky skutočnosti, ktoré doposiaľ predstavovali mimoriadnu činnosť účtovnej jednotky, sa aktuálne musia účtovať na príslušných účtoch nákladov a výnosov v rámci hospodárskej činnosti podniku.* Tabuľka 1 uvádza prehľad zmien v platnej rámcovej účtovej osnove pre podnikateľov.

Tabuľka 1: Zmeny v RÚO pre podnikateľov

Účet	**Zmena**
591 – Splatná daň z príjmov z bežnej činnosti	premenovaný na Splatná daň z príjmov
592 – Odložená daň z príjmov z bežnej činnosti	premenovaný na Odložená daň z príjmov
593 – Splatná daň z príjmov z mimoriadnej činnosti	zrušený
594 – Odložená daň z príjmov z mimoriadnej činnosti	zrušený
597 – Prevod nákladov na hospodársku činnosť	zrušený
598 – Prevod finančných nákladov	zrušený

Pokračovanie tabuľky 1: Zmeny v RÚO pre podnikateľov

Účet	**Zmena**

582 – Škody	zrušený
588 – Ostatné mimoriadne náklady	zrušený
682 – Náhrady škôd	zrušený
688 – Ostatné mimoriadne výnosy	zrušený
697 – Prevod výnosov z hospodárskej činnosti	zrušený
698 – Prevod finančných výnosov	zrušený

Zdroj: vlastné spracovanie podľa: Novela opatrenia o postupoch účtovania. Dostupné na: <http://www.podnikajte.sk/dane-a-uctovnictvo/c/1837/category/uctovnictvo/article/novela-o-postupoch-uctovania.xhtml>

Predmetom novely opatrenia o postupoch účtovania sú aj zmeny v účtovaní cenných papierov, podielov, pôžičiek, dividend a pod., o ktorých sa účtuje v rámci podielovej účasti. Na účely účtovania a vykazovania cenných papierov a podielov sa do opatrenia dostávajú nasledujúce definície, ktoré je potrebné na účely správneho účtovania a vykazovania považovať za veľmi dôležité:

- *materskou účtovnou jednotkou* je účtovná jednotka, ktorá má rozhodujúci vplyv v jednej alebo vo viacerých dcérskych účtovných jednotkách, pričom rozhodujúci vplyv sa posudzuje podľa § 22 ods. 3 zákona č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov,
- *dcérskou účtovnou jednotkou* je účtovná jednotka, v ktorej má iná účtovná jednotka rozhodujúci vplyv, a to vrátane každej dcérskej účtovnej jednotky konečnej materskej účtovnej jednotky,
- *skupinu účtovných jednotiek* tvorí materská účtovná jednotka a všetky jej dcérske účtovné jednotky,
- *prepojenými účtovnými jednotkami* sú dve alebo viaceré účtovné jednotky v rámci skupiny účtovných jednotiek,
- *spoločným rozhodujúcim vplyvom* je zmluvne dohodnuté zdieľanie rozhodujúceho vplyvu,
- *podielovou účasťou* sa rozumie existencia aspoň 20 % podielu na základnom imaní v inej účtovnej jednotke.

Na účte *061 – Podielové cenné papiere a podiely v dcérskej účtovnej jednotke* sa účtujú cenné papiere a podiely priamo v dcérskej účtovnej jednotke vymedzenej v § 22 ods. 4 zákona o účtovníctve. Na účte *062 – Podielové cenné papiere a podiely s podstatným vplyvom* sa účtujú cenné papiere a podiely na základnom imaní v obchodnej spoločnosti, v ktorej má účtovná jednotka podstatný vplyv, t. j. najmenej 20 % podiel na jej hlasovacích právach, cenné papiere a podiely v rámci podielovej účasti a tiež cenné papiere a podiely v účtovnej jednotke so spoločným rozhodujúcim vplyvom.

Nový názov a obsahové vymedzenie má účet *066 – Pôžičky v rámci podielovej účasti*. Na tomto účte sa účtujú dlhodobé pôžičky poskytnuté voči prepojenej účtovnej jednotke, účtovnej jednotke so spoločným rozhodujúcim vplyvom a účtovnej jednotke s podielovou účasťou. Povinnosťou veľkých účtovných jednotiek bude na osobitnom analytickom účte evidovať dlhodobé pôžičky poskytnuté voči prepojenej účtovnej jednotke.

Ďalšou zmenou je spresnenie obsahového vymedzenia účtov, na ktorých sa účtujú *pohľadávky a záväzky v rámci podielovej účasti*. Účet 351 – Pohľadávky v rámci konsolidovaného celku sa premenoval na účet *351 – Pohľadávky v rámci podielovej účasti* a rozšírilo sa aj jeho obsahové vymedzenie. Na účte 351 – Pohľadávky v rámci podielovej účasti sa s výnimkou pohľadávok z obchodného styku budú účtovať pohľadávky voči prepojenej účtovnej jednotke, účtovnej jednotke so spoločným rozhodujúcim vplyvom a účtovnej jednotke s podielovou účasťou. Predmetom účtovania na tomto účte teda budú

napríklad krátkodobé pôžičky, dlhodobé dividendy a krátkodobé dividendy, úroky z omeškania pri oneskorenom splácaní peňažného vkladu spoločníka obchodnej spoločnosti. Veľká účtovná jednotka má povinnosť na osobitnom analytickom účte evidovať tieto pohľadávky voči prepojenej účtovnej jednotke. V účtovníctve mikro účtovnej jednotky sa tieto účtovné prípady budú účtovať na účte *355 – Ostatné pohľadávky voči spoločníkom a členom.* Účet 361 – Záväzky v rámci konsolidovaného celku sa premenoval na účet *361 – Záväzky v rámci podielovej účasti* a jeho obsahové vymedzenie sa tiež rozšírilo. Na účte 361 – Záväzky v rámci podielovej účasti sa s výnimkou záväzkov z obchodného styku budú účtovať záväzky voči prepojenej účtovnej jednotke, účtovnej jednotke so spoločným rozhodujúcim vplyvom a účtovnej jednotke s podielovou účasťou. Veľká účtovná jednotka má povinnosť na osobitnom analytickom účte sledovať tieto záväzky voči prepojenej účtovnej jednotke. V účtovníctve mikro účtovnej jednotky sa tieto účtovné prípady budú účtovať na účte 365 – *Ostatné záväzky voči spoločníkom a členom*. Účet 471 – Dlhodobé záväzky v rámci konsolidovaného celku zmenil svoj názov na účet *471 – Dlhodobé záväzky v rámci podielovej účasti* a jeho ekonomický obsah sa takisto rozšíril. Na účte 471 sa s výnimkou záväzkov z obchodného styku budú účtovať záväzky voči prepojenej účtovnej jednotke, účtovnej jednotke so spoločným rozhodujúcim vplyvom a účtovnej jednotke s podielovou účasťou. Veľká účtovná jednotka musí na osobitnom analytickom účte účtovať tieto záväzky voči prepojenej účtovnej jednotke. V účtovníctve mikro účtovnej jednotky sa tieto dlhodobé záväzky v rámci podielovej účasti budú účtovať na účte *479 – Ostatné dlhodobé záväzky.*

Spresnilo sa obsahové vymedzenie účtov, na ktorých sa účtujú ostatné pohľadávky a záväzky voči spoločníkom a členom. Na účte *355 – Ostatné pohľadávky voči spoločníkom a členom* sa s výnimkou pohľadávok z obchodného styku účtujú pohľadávky voči účtovnej jednotke, v ktorej nemá účtovná jednotka podielovú účasť (menej ako 20 % podiel na základnom imaní v inej účtovnej jednotke). Môže ísť napríklad o krátkodobé pôžičky, dlhodobé dividendy a krátkodobé dividendy, úroky z omeškania pri oneskorenom splácaní peňažného vkladu spoločníka obchodnej spoločnosti a podobne. V účtovníctve mikro účtovnej jednotky sa na tomto účte budú účtovať aj účtovné prípady, ktoré sa účtujú na účte 351 – Pohľadávky v rámci podielovej účasti. Na účte *365 – Ostatné záväzky voči spoločníkom a členom* sa s výnimkou záväzkov z obchodného styku účtujú záväzky voči účtovnej jednotke, v ktorej nemá účtovná jednotka podielovú účasť; môže ísť napríklad o krátkodobé pôžičky, úroky z omeškania pri oneskorenom splácaní peňažného vkladu a podobne. V účtovníctve mikro účtovnej jednotky sa na tomto účte budú účtovať aj účtovné prípady, ktoré sa účtujú na účte 361 – Záväzky v rámci podielovej účasti. Mikro účtovná jednotka má zakázané účtovať o odloženej dani z príjmov Povinnosť účtovania o odloženej dani z príjmov sa vzťahuje len na tie účtovné jednotky, ktoré majú povinnosť overenia účtovnej závierky audítorom podľa § 19 zákona o účtovníctve. Ostatné účtovné jednotky mohli o odloženej dani z príjmov účtovať tiež, ak sa tak dobrovoľne rozhodli. *Novelou opatrenia o postupoch účtovania sa do § 10 ods. 4 tohto predpisu dostáva ustanovenie, ktoré vyslovene hovorí, že v účtovníctve mikro účtovnej jednotky sa o odloženej dani z príjmov neúčtuje.*

Novela opatrenia priniesla tiež výnimky z účtovania časového rozlíšenia podľa kategorizácie účtovných jednotiek do troch veľkostných skupín. *Mikro účtovná jednotka* nemusí, ale dobrovoľne môže, na účtoch časového rozlíšenia účtovať nevýznamné a stále sa opakujúce účtovné prípady, ktoré súvisia s účtovaním nákladov a výnosov medzi dvoma účtovnými obdobiami. Výnimka sa nesmie uplatňovať na účtovanie časového rozlíšenia, ktoré sa vzťahuje na účtovné prípady týkajúce sa účtovania dotácií a emisných kvót. *Malé účtovné jednotky a veľké účtovné jednotky* o časovom rozlíšení musia účtovať povinne. Výnimka z účtovania časového rozlíšenia sa môže v malých účtovných jednotkách a veľkých účtovných jednotkách uplatňovať len vtedy, ak ide o nevýznamný a stále sa opakujúci účtovný prípad

týkajúci sa časového rozlíšenia nákladov alebo výnosov posledného a prvého mesiaca účtovného obdobia.

Zmeny sa dotkli aj účtovania v účtovej skupine 25-Krátkodobý finančný majetok. Na účte *252 – Vlastné akcie a vlastné obchodné podiely* sa naďalej budú účtovať vlastné akcie a vlastné obchodné podiely v obstarávacej cene; na vlastné akcie a vlastné obchodné podiely sa však v účtovníctve bude vytvárať zákonný rezervný fond s použitím účtu *417 – Zákonný rezervný fond z kapitálových vkladov* alebo účtu *421 – Zákonný rezervný fond*. Doteraz sa účet 252 – Vlastné akcie a vlastné obchodné podiely vykazoval v účtovnej závierke na strane pasív vo vlastnom imaní so znamienkom mínus. *Počnúc účtovnou závierkou zostavenou za rok 2014 sa tento účet vykazuje na strane aktív v krátkodobom finančnom majetku v kladnej hodnote*, čo mení bilančnú sumu aktív a podskupín aktív.

Podľa § 42 ods. 3 opatrenia sa *analytické účty k účtom účtovej triedy 3 – Zúčtovacie vzťahy* vytvárajú podľa jednotlivých dlžníkov a veriteľov, ich rovnorodých skupín a podľa jednotlivých druhov mien. Veľká účtovná jednotka je v zmysle novely navyše povinná na osobitných analytických účtoch sledovať aj pohľadávky z obchodného styku a záväzky z obchodného styku voči prepojenej účtovnej jednotke a voči účtovnej jednotke s podielovou účasťou.

Význam má aj zmenená kategorizácia účtovných jednotiek do troch skupín s odlišnou mierou vykazovania účtovných skutočností: mikro účtovná jednotka, malá účtovná jednotka a veľká účtovná jednotka. Zákon o účtovníctve uvádza podmienky, za ktorých sa účtovná jednotka zaradí do príslušnej kategórie, pričom výrazná zmena prišla s vytvorením spoločnej veľkostnej kategórie pre stredné a veľké účtovné jednotky, ktorá znamenala, že požiadavky dovtedy kladené na veľké podniky sa s novelizáciou preniesli aj na stredné podniky.

Nový legislatívny rámec kategorizácie účtovných jednotiek priniesol zmeny vo vykazovaní účtovných informácií, konkrétne sa toto rozdelenie odráža v rozsahu a obsahu účtovnej závierky. Novovzniknutá kategória je mikro účtovná jednotka, ktorá sa do veľkostnej skupiny radí na základe splnenia aspoň 2 z 3 zákonom stanovených kritérií (týkajúcich sa celkovej sumy majetku, čistého obratu a priemerného počtu stálych zamestnancov v sledovanom období), zostavuje účtovnú závierku v skrátenej podobe, obsahujúcu všeobecné náležitosti, súvahu, výkaz ziskov a strát a poznámky. Súvaha mikro účtovnej jednotky sa odlišuje agregáciou viacerých súvahových účtov do jednej položky (čo sa prejavuje napríklad vykazovaním položiek časového rozlíšenia súčasne s pohľadávkami alebo záväzkami), ako aj vykazovaním súvahových položiek výlučne v hodnote netto. Výkaz ziskov a strát mikro účtovnej jednotky má tiež špecifickú podobu, obdobne ako pri súvahe ide o skrátenú podobu prostredníctvom agregácie položiek výkazu, v niektorých prípadoch až na úroveň účtovných skupín. Obsahuje aj jeden ukazovateľ – Pridanú hodnotu. Ďalším špecifikom výkazu je formálne usporiadanie nákladov a výnosov do dvoch skupín – hospodárska a finančná činnosť. Obsahová náplň poznámok mikro účtovnej jednotky sa skrátila na 3 strany, avšak umožňuje poskytnutie informácií aj nad rámec požadovaného obsahu.

Na účely zostavenia účtovnej závierky je malá účtovná jednotka povinná sa riadiť iným opatrením ako veľká účtovná jednotka alebo subjekt verejnej správy, a to aj napriek tomu, že majú jednotný vzor účtovnej závierky pre malé aj pre veľké účtovné jednotky. Novovznikajúce účtovné jednotky sa do veľkostnej kategórie začlenia na základe vlastného rozhodnutia.

3. Diskusia

Pretože účtovníctvo je základným informačným zdrojom finančných analýz, bez ohľadu na metodickú rôznorodosť a náročnosť použitých postupov, je mimoriadne dôležité, aby

finančný analytik nepretržite sledoval zmeny v účtovníctve, dokázal posúdiť ich vplyv na výsledky výpočtov finančných ukazovateľov a ich zmysluplnú ekonomickú interpretáciu, ako aj správne a fundovane formulovať závery pre manažment podniku. (Buc and Kliestik, 2013)

Pre slovenské účtovné jednotky účtujúce v sústave podvojného účtovníctva a zostavujúce individuálnu účtovnú závierku podľa slovenských právnych predpisov je z novelizovaných postupov účtovania **najvýznamnejšia zmena týkajúca sa vykazovania výsledku hospodárenia** za účtovné obdobie s následnou väzbou na daň z príjmov. Novelizovaná účtovná legislatíva v SR zrušila vykazovanie *výsledku hospodárenia z mimoriadnej činnosti*, čomu je prispôsobená aj štruktúra účtových tried RÚO (rámcovej účtovej osnovy) – ÚT 5-Náklady a ÚT 6-Výnosy. Predchádzajúca úprava účtovného zobrazenia výsledkových položiek a následne výsledku hospodárenia rešpektovala skutočnosť, že v nepretržitej činnosti podniku môže dochádzať sporadicky k vzniku nákladov alebo výnosov, ktoré nemajú opakovaný charakter, sú zriedkavé, ojedinelé alebo jednoducho nepredvídateľné, a teda nemajú byť v účtovníctve evidované rovnako ako spotreba a jej ekvivalent vo výnosoch, ktoré majú zámerný, plánovaný charakter a sú neodmysliteľnou súčasťou a zmyslom podnikania. *Bežnú činnosť* teda finančné účtovníctvo chápalo v jadre ako *hospodársku (prevádzkovú) činnosť* (z iného uhla pohľadu hlavnú činnosť – core business plus nevyhnutné podporné činnosti) a *finančné operácie podniku*, potrebné pre fungovanie prevádzky, prípadne finančné investície. (Majercak, Cisko and Majercakova, 2013) Na báze takéhoto členenia nákladov a výnosov boli koncipované aj ukazovatele štandardnej, tradične koncipovanej finančnej analýzy – už len v pomerovej analýze sú to všetky ukazovatele, v ktorých vystupujú v čitateli alebo menovateli náklady, výnosy (tržby) alebo výsledok hospodárenia v jeho rôznych variantoch (ukazovatele rentability, aktivity, ...), ale aj ďalšie matematicky i myšlienkovo náročnejšie ukazovatele (metódy, modely), v ktorých predchádzajúce ekonomické veličiny figurujú iba ako parciálny ukazovateľ. Ovplyvňovanie výsledku hospodárenia náhodnými nákladmi či výnosmi v ukazovateľoch rentability neumožňuje poskytnúť dostatočne zmysluplnú informáciu o výnosnosti (vlastného imania, aktív atď.), pretože mimoriadne vplyvy nie sú v réžii podniku a takto chápaná výnosnosť napr. vloženého kapitálu nie je výsledkom podnikateľského snaženia (samozrejme, akceptujeme existenciu systematického rizika), ale kumuláciou rozmanitých vplyvov, vrátane pôsobenia prírodných síl.

Na ilustráciu môžeme ako príklad uviesť aj ukazovatele nákladovosti: ak sme napr. mohli doposiaľ na základe finančného účtovníctva kvantifikovať *ukazovatele nákladovosti* samostatne pre *hospodársku, finančnú a mimoriadnu činnosť* a z nich odvodiť závery o hospodárnosti týchto činností (s tým, že ťažiskom hodnotenia bola logicky hospodárska činnosť, pričom vo vnútroorganizačnom a manažérskom účtovníctve by sa mohla ďalej analyzovať kapacitná a spotrebná odchýlka a uplatňovať ďalšie postupy), v súčasnosti bude *ukazovateľ nákladovosti hospodárskej činnosti* vďaka zakomponovaným „mimoriadnym“ nákladom a výnosom skreslený a hodnotiť na jeho základe hospodárnosť prevádzkovej / operatívnej činnosti by nemalo hlbší zmysel. Pripomeňme si matematické vzťahy týchto ukazovateľov:

- *ukazovateľ celkovej nákladovosti* vyjadruje úroveň hospodárnosti podniku v kontexte s vynaloženými nákladmi a ich kompenzáciou výnosmi:

$$celková\ nákladovosť = \frac{N}{V} \qquad (1)$$

- pre *nákladovosť jednotlivých činností* platili donedávna vzťahy:

$$nákladovosť_{HČ} = \frac{NzHČ}{VzHČ} \qquad (2)$$

$$nákladovosť_{FČ} = \frac{FN}{FV} \quad (3)$$

$$nákladovosť_{MČ} = \frac{MN}{MV} \quad (4)$$

a súčasne rovnica:

$$\frac{N}{V} = \frac{NzHČ}{V} + \frac{FN}{V} + \frac{MN}{V} \quad (5)$$

kde: N.....náklady celkom,
V.....výnosy celkom,
NzHČ.....náklady z hospodárskej činnosti,
VzHČ.....výnosy z hospodárskej činnosti,
FN.....finančné náklady,
FV.....finančné výnosy,
MN.....mimoriadne náklady,
MV.....mimoriadne výnosy,
ukazovatele v peňaž. j.

Podľa aktuálnej právnej úpravy sa vzťah (4) nezisťuje. Treba povedať, že aj v minulosti bola pozícia ukazovateľa nákladovosti mimoriadnej činnosti v celkovom hodnotení viac-menej formálna – vzhľadom na charakter mimoriadnych nákladov a výnosov stanovený v postupoch účtovania pre podnikateľov.

Celková nákladovosť podniku má teda v súčasnosti tvar:

$$\frac{N}{V} = \frac{NzHČ}{V} + \frac{FN}{V} \quad (6),$$

pre nákladovosť hospodárskej a finančnej činnosti platia vzťahy (2) a (3), pričom pre nákladovosť hospodárskej činnosti má zmysel robiť ďalšiu úroveň rozkladu podľa jednotlivých skupín nákladov – nákladových druhov. (Ponisciakova, Gogolova and Ivankova, 2015)

4. Záver

Ďalšie z prezentovaných zmien sa týkajú najmä *účtovania v konsolidovanom celku*; novela (2015) priniesla niektoré spresnenia definícií relevantných pojmov a postupov, ktoré treba zohľadniť *v konsolidovanej účtovnej závierke*. Pri posúdení závažnosti týchto zmien však môžeme konštatovať, že ide skôr o „kozmetické“ úpravy doposiaľ jestvujúceho stavu.

Hoci sme priniesli prehľad všetkých zmien v postupoch účtovania pre podnikateľov v SR, z nášho pohľadu bude v hodnotení finančnej spôsobilosti a finančnej výkonnosti podniku najväčšia úloha patriť najmä **inej filozofii výpočtu výsledku hospodárenia**, ako tomu bolo doteraz. Táto zmena sa netýka podnikateľov účtujúcich v sústave jednoduchého účtovníctva, ktoré nekladie dôraz na spotrebu a jej kompenzáciu vo výnosoch, ale pri výpočte výsledku hospodárenia vychádza z pohybu peňažných prostriedkov – príjmov a výdavkov – u ktorých navyše uplatňuje striktne daňové hľadisko, pričom náklady a výnosy ako účtovné kategórie definované v predmete účtovníctva (zákon č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov) majú v tejto účtovnej sústave iba „sekundárny“, latentný charakter.

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References

[1] Zákon č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov.

[2] Novela opatrenia o postupoch účtovania. 2015. Available at: <https://www.podnikajte.sk/dane-a-uctovnictvo/c/1837/category/uctovnictvo/article/novela-o-postupoch-uctovania.xhtml> [Accessed 1 August 2016]

[3] Bartosova, V., Paliderova, M. 2014. *Účtovníctvo podnikateľov I. Podvojné účtovníctvo.* Žilina: Žilinská univerzita v Žiline vo vydavateľstve EDIS. p. 432.

[4] Buc, D., Kliestik, T. 2013. *Aspects of Statistics in Terms of Financial Modelling and Risk.* In: 7th International Days of Statistics and Economics. Prague. pp. 215-224.

[5] Cisko, S., Kliestik, T. 2013. Finančný manažment podniku II. Žilina: Žilinská univerzita v Žiline vo vydavateľstve EDIS. p. 775.

[6] Gregova, E. 2007. *Malé a stredné podniky v podmienkach transformujúcich sa ekonomík SR a RF*. Žilina: Žilinská univerzita v Žiline vo vydavateľstve EDIS. p.160.

[7] Kicova, E., Kramarova, K. 2013. *Possibilities of Using Financial Analysis in the Bus Transport Companies.* In: 9th International Scientific Conference on Financial Management of Firms and Financial Institutions. Ostrava. pp. 332-341.

[8] Majercak, P., Cisko, S., Majercakova, E. 2013. *The Impact of Theory of Constraints on the Management Accounting.* In: 7th International Days of Statistics and Economics. Prague. pp. 894-904.

[9] Ponisciakova, O., Gogolova, M., Ivankova, K. 2015. Calculations in Managerial Accounting, *Procedia Economics and finance*, 26, pp. 431-437.

Predictive models from the point of view of current business accounting legislation in the Slovak Republic

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Abstract

The usefulness and objectivity of an enterprise financial situation by applying predictive models is limited by many factors. One of the most important factors seems to be information/ data in the legal framework of the financial accounting regulation. In this article we discuss in brief some problems of predictive financial models vs. current state in businesses accounting of the Slovak Republic.

Key words

Enterprise, predictive models, financial analysis, indicators of financial health, financial accounting, accounting legislation

JEL Classification: G3, M4

1. Úvod

V praxi slovenských podnikov sa veľmi často v rámci analýzy účtovných výkazov používajú rôzne predikčné – bonitné a bankrotné modely – ako rozšírenie tradičnej, v prvom rade pomerovej finančnej analýzy. Interné analýzy, zabezpečované pracovníkmi podniku "vo vlastnej réžii" však majú často mnohé nedostatky: od použitia nevhodných modelov, cez nesprávne vybrané účtovné údaje na dosadenie do vzťahov až po skreslené výsledky a nesprávnu, a teda nepoužiteľnú interpretáciu výsledkov v riadiacej praxi. Je pravdou, že teória podnikových financií, resp. finančná ekonómia, ponúka nespočetné množstvo metód a modelov na hodnotenie a predikciu finančnej situácie podniku, nie všetky sú však rovnako vhodné na účely analýzy v našich podmienkach, a to nielen preto, že ich výpovede sa vzťahujú k odlišným východiskovým makro- a mikroekonomickým podmienkam v ekonomike, kde vznikli, ale aj z hľadiska času – niektoré prestávajú bez výrazných skreslení plniť svoju funkciu, pretože zastarali.

Každý z načrtnutých problémov si zaslúži osobitnú pozornosť; v príspevku sa sústredíme na účtovné údaje, tvoriace vstupy predikčných modelov, a ich aktuálnu podobu v slovenskej právnej úprave účtovníctva podnikateľov.

2. Stručný exkurz do predikcie finančnej situácie podniku

Vo finančnej analýze sa okrem metód, ktoré sú zamerané na hodnotenie dosiahnutej finančnej situácie podniku, čiže na hodnotenie jeho minulosti (analýza ex post), presadili

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viaceré metódy zamerané na prognózovanie finančného vývoja, v odbornej literatúre známe ako *predikčné modely, metódy finančnej analýzy ex ante,* príp. *systémy včasného varovania.* Sú založené na poznaní, že vybrané podnikové ukazovatele (indikátory úpadku alebo naopak indikátory bonity) sa už v časovom predstihu začnú v porovnávaných podnikoch odlišovať. Ich najväčší význam však spočíva v upozornení na riziko finančného úpadku podniku, ktoré môže vyústiť až do bankrotu. K najčastejším symptómom patria problémy s bežnou likviditou, výškou čistého pracovného kapitálu, dosahovanie záporného výsledku hospodárenia, problémy s rentabilitou vloženého kapitálu a i.

Základom predikcie, a finančnej analýzy vo všeobecnosti, je vytypovať také ukazovatele dokumentujúce ekonomickú činnosť podniku, ktorých výpovede budú v spojitosti s hodnotením finančnej situácie a spôsobilosti podniku relevantné a signifikantné. Správne zvolené indikátory finančného zdravia odrážajú pôsobenie najdôležitejších činiteľov externého a interného charakteru, v lepšom prípade zahŕňajú do hodnotenia aj často takmer nepostihnuteľné synergické efekty, ktoré môžu nastať. Okrem zmysluplného ekonomického obsahu závisí konkrétna podoba modelu od použitých matematických, matematicko-štatistických alebo neštatistických metód, ktorým je v odbornej literatúre venovaný dostatočný priestor.

Známe predikčné modely často diskutované v teórii a pomerne frekventovane používané v praxi sa vyznačujú viac alebo menej zložitou konštrukciou a myšlienkovým základom. Problémom použiteľnosti a úspešnej aplikácie predikčných modelov môže byť prílišná jednoduchosť alebo naopak príliš zložitý, sofistikovaný charakter riešení, ktorý neprispieva k zrozumiteľným a ani spoľahlivým výsledkom. Vzájomná konfrontácia výstupov viacerých modelov je pritom užitočná, ale treba tiež vziať do úvahy, o aké modely ide, kedy vznikli atď.

Finančná situácia podniku je ovplyvňovaná veľkým množstvom činiteľov. Úlohou finančnej analýzy je určiť, ktoré činitele a s akou intenzitou sa na formovaní finančnej situácie podieľajú. Tieto činitele sa vzhľadom na charakter pôsobenia delia na externé a interné.

Externé (objektívne) činitele pôsobia na podnik zvonka, majú charakter štátnych zásahov alebo opatrení, napr. daňová, menová, colná politika štátnej správy, životná úroveň, sociálny vývoj krajiny, ekonomické vplyvy krajiny a pod. Pôsobenie týchto činiteľov podnik nemôže ovplyvniť, v inom kontexte o nich hovoríme, že vytvárajú systematické riziko.

Interné (subjektívne) činitele vznikajú v interakcii iných subjektov na mikroúrovni (obchodní partneri, zákazníci, zamestnanci podniku, konkurenti, banky a iní veritelia) s analyzovaným podnikom, alebo vo vnútri samotného podniku, a podnikový manažment ich môže ovplyvniť. Zastupujú tzv. špecifické riziko. Tieto činitele sa z iného aspektu členia na kvantitatívne (napr. objem výroby, tržby, obrat), ktoré by mali mať primeranú úroveň a dynamiku, a kvalitatívne, vyjadrujúce celkovú úroveň, s akou sa v podniku transformujú vstupy na výstupy (výsledok hospodárenia, ukazovatele rentability a iné).

Za počiatky predikcie sa považujú štúdie zakladateľov vedeckého prognózovania finančnej situácie: Fitzpatricka, Merwina, Ansoffa, Tamariho, Beavera a Altmana, ktorých modely sa datujú cca od tridsiatych rokov minulého storočia. Ďalšie významné práce a modely sa spájajú s menami/názvami: Weibel, Beerman, Deakin, Edmister model, Argentiho model (A score), Tafflerov & Tishawov model (základná a modifikovaná verzia), metóda RSW R. Schmidta (Fundamental score), Altmanov et al ZETA model, modely autorov Springate, Marais, Mayr, Weinrich, Gebhart, Fulmerov H factor, Ohlsonov, Horriganov, Mertonov model, H score spoločnosti Company Watch, faktorová analýza Ezzamela, Brodieho a Mar-Molinera, model Richardsona et al, model LVQ (Learning Vector Quantiser), systém hodnotenia BP-14, model Zmijewski, Dimitras et al, Lin & McClean, Beynon & Peel, Shin & Lin, Park & Han, Shirata model, Kim & Han, McKee, Thun, Kralickov Quick Test a mnoho ďalších teórií a modelov, ktorých vznik sa zaraďuje do minulého aj tohto storočia. Okrem ideového základu sa odlišujú

tým, že na analýzu a hodnotenie podniku používajú rôzne výrazové prostriedky, nástroje a čiastkové ukazovatele, sú to napr. metódy bodového hodnotenia, jednorozmerná a viacrozmerná diskriminačná analýza, metódy LOGIT, PROBIT, faktorová analýza, gnostická teória dát, neurónové siete a iné. Veľký význam majú aj priamo v praxi vytvárané modely, uveďme napr. Index bonity creditreform (spol. Creditreform, s.r.o.), skóringové modely (bank credit scoring), Index Českej národnej banky, Model Banque de France, Model US Bank, ratingové modely (CreditMetrics, CreditRisk+, KMV), Model spol. ASPEKT Central Europe Group a.s., ICV – index celkovej výkonnosti podniku v expertnom systéme REVALEX atď. Treba však doplniť, že ide o systémy hodnotenia v širšom kontexte ako je predikcia finančnej situácie podniku.

V podmienkach Slovenskej republiky, a takisto aj Českej republiky, sa používajú niektoré modely, ktoré vznikli v týchto národných ekonomikách, pričom sa vychádza z predpokladu podobnosti podmienok, v ktorých vznikli. Medzi najčastejšie používané modely českých autorov patria: Grűnwaldov index bonity, Model INFA (ideový základ indexov IN Neumaierovcov), samotné IN indexy, Bilančná analýza R. Douchu; zo slovenských sú to: Jakubík – Teplý model, Binkertov model, ktorý vznikol v spolupráci s prof. K. Zalaiom, významným predstaviteľom finančnej analýzy na Slovensku, Chrastinovej CH-index a Gurčíkov G-index pre oblasť agrorezortu.

3. Účtovníctvo podniku

Hlavným informačným zdrojom a údajovou základňou predikcie, rovnako ako finančnej analýzy vo všeobecnosti, je účtovníctvo podniku, t. j. bežné účtovníctvo a účtovná dokumentácia každého druhu, a účtovná závierka zahrňovaná do analýz v časových radoch hodnôt ukazovateľov umožňujúcich posúdiť hospodárenie a vývoj finančnej situácie podniku. Touto schopnosťou výpovede disponujú čiastočne už samotné účtovné položky (absolútne a rozdielové ukazovatele súvahy a výsledovky), a samozrejme, na základe nich vytvorené ukazovatele finančnej analýzy, predovšetkým pomerové ukazovatele, bonitné a bankrotné modely, ako aj indikátory finančnej výkonnosti a hodnoty podniku. Vstupné účtovné dáta sú v podstatnej miere determinované právnou úpravou účtovníctva, ako aj určitou prípustnou mierou „tolerancie“, v ktorej sa účtovná jednotka v systéme regulovaného, unifikovaného finančného účtovníctva pohybuje.

3.1 Aktuálna právna úprava účtovníctva podnikateľov v SR

Právnou úpravou účtovníctva podnikateľov v SR je v prvom rade zákon č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov (ďalej zákon o účtovníctve), opatrenie Ministerstva financií Slovenskej republiky č. 23054/2002-92 zo 16. decembra 2002, ktorým sa ustanovujú podrobnosti o postupoch účtovania a rámcovej účtovej osnove pre podnikateľov účtujúcich v sústave podvojného účtovníctva v znení neskorších predpisov (ďalej postupy účtovania), opatrenie MF SR č. 4455/2003-92 z 31. marca 2003, ktorým sa ustanovujú podrobnosti o usporiadaní, označovaní a obsahovom vymedzení položiek individuálnej účtovnej závierky a rozsahu údajov určených z individuálnej účtovnej závierky na zverejnenie pre podnikateľov účtujúcich v sústave podvojného účtovníctva v znení neskorších predpisov, a ďalšie opatrenia a metodické usmernenia MF SR vydávané v rámci regulácie finančného účtovníctva.

Účtovníctvo podnikateľov v SR prešlo v posledných rokoch niektorými významnými zmenami, ktoré síce nezmenili razantne jeho podobu, ale zmenil sa obsah a rozsah vykazovania v závislosti od typu účtovnej jednotky, došlo k zmenám v štruktúre účtovnej závierky – v súčasnosti ju netvoria samostatné účtovné výkazy, ale jediný komplexný výkaz

zjednocujúci súvahu a výkaz ziskov a strát, zmeny nastali aj vo vymedzení a usporiadaní položiek účtovnej závierky, napr. výsledku hospodárenia. Najvýznamnejšie zmeny v posledných rokoch majú účinnosť od 1. 1. 2015, 1. 1. 2016 a 1. 1. 2017 (iné termíny sa viažu k prechodným ustanoveniam a drobným korekciám).

Účtovná závierka podnikateľov zostavená za účtovné obdobie (riadne: kalendárny rok, hospodársky rok; mimoriadne) rešpektuje *typ účtovnej jednotky.* Platný zákon o účtovníctve rozlišuje 3 typy (veľkostné skupiny) účtovných jednotiek: *mikro účtovná jednotka, malá účtovná jednotka* a *veľká účtovná jednotka.* Táto kategorizácia sa vzťahuje na obchodné spoločnosti, družstvá, pozemkové spoločenstvá a vybrané fyzické osoby, nevzťahuje sa na účtovné jednotky, ktoré musia (povinnosť) alebo môžu (voliteľnosť) zostavovať individuálnu účtovnú závierku podľa IFRS v znení prijatom EÚ, Národnú banku Slovenska a subjekty verejného záujmu. Rozhodovacími kritériami pre zaradenie sú: celková suma majetku v EUR, čistý obrat v EUR a priemerný prepočítaný počet zamestnancov. Čo je podstatné aj z hľadiska analýzy, pre každú veľkostnú kategóriu účtovných jednotiek sú definované osobitné požiadavky pre účtovnú závierku. Pre zostavenie riadnej individuálnej účtovnej závierky účtovnej jednotky, ktorej účtovným obdobím je kalendárny rok, sú relevantné tieto Opatrenia Ministerstva financií SR, ktoré ustanovujú vzory účtovných závierok:

Tabuľka 1: Vzory účtovnej závierky podľa typu účtovnej jednotky

Účtovná jednotka	**Vzor účtovnej závierky**
mikro	Opatrenie MF SR č. MF/15464/2013-74
malá	Opatrenie MF SR č. MF/23378/2014-74
veľká	Opatrenie MF SR č. MF/23377/2014-74

Zdroj: www.finance.gov.sk

Stručne zhrňme najdôležitejšie zmeny, ktoré majú dopad nielen na výstupy účtovníctva, ale aj napĺňanie predikčných modelov údajmi z účtovníctva. Ich nerešpektovanie by mohlo viesť k viac alebo menej významným skresleniam výsledkov o budúcej finančnej spôsobilosti podniku a jeho celkovom ekonomickom statuse, príp. výsledky výpočtov z hodnotenia úplne diskvalifikovať:

- Boli zrušené účty mimoriadnych nákladov a mimoriadnych výnosov, s čím súvisí aj zrušenie vykazovania mimoriadnej činnosti, a teda aj bežnej činnosti v časti výsledovka v účtovnej závierke; výsledok hospodárenia sa sleduje osobitne za hospodársku činnosť, finančnú činnosť a za účtovné obdobie celkom (v poznámkach k ÚZ sa vyžaduje uviesť opis a sumu nákladov a výnosov, ktoré majú výnimočný rozsah alebo výskyt).
- V spojitosti s predchádzajúcim boli zrušené prevodové účty 597-Prevod nákladov na hospodársku činnosť, 598-Prevod finančných nákladov, 697-Prevod výnosov z hospodárskej činnosti a 698-Prevod finančných výnosov.
- Mikro účtovná jednotka nepoužíva účty súvisiace s pohľadávkami a záväzkami v rámci podielovej účasti (351, 361, 471) a účty súvisiace s preceňovaním na reálnu hodnotu a používaním metódy vlastného imania (414, 373, 376, 377, 564, 567, 664, 667).
- Mikro účtovná jednotka nemá povinnosť účtovať o odloženej dani.
- Spresnilo sa účtovanie na účtoch časového rozlíšenia nákladov a výnosov (účt. skupina 38); mikro účtovná jednotka nemusí účtovať na účtoch časového rozlíšenia, ak ide o nevýznamný a stále sa opakujúci účtovný prípad medzi dvoma účtovnými obdobiami, pričom nejde o dotácie a emisné kvóty; malé a veľké účtovné jednotky nemusia časovo rozlišovať náklady a výnosy, ak ide o nevýznamný a stále sa opakujúci účtovný prípad týkajúci sa posledného a prvého mesiaca účtovného obdobia.

- Zmeny vo finančnom majetku: bola zavedená nová klasifikácia pri vykazovaní súvahových položiek v rámci konsolidovaného celku, ktoré sa rozdeľujú na podiely a pôžičky v prepojených účtovných jednotkách, podiely s podielovou účasťou a ostatné, podobne sa rozdeľujú aj pohľadávky, záväzky a finančné výnosy vo výsledovke.
- Finančný majetok sa rozdeľuje na dlhodobý a krátkodobý. V krátkodobom sa vykazujú aj „Vlastné akcie a vlastné obchodné podiely“, ktoré boli predtým vykazované na strane pasív (so znamienkom mínus). Súčasťou dlhodobého finančného majetku sú aj bankové účty s dobou viazanosti dlhšou ako jeden rok.
- Zmeny v dlhodobých a krátkodobých pohľadávkach: zaviedla sa podrobnejšia štruktúra obchodných a ostatných pohľadávok v členení na prepojené účtovné jednotky, podielové účasti a ostatné.
- Podobne ako u pohľadávok zmeny nastali aj vo vykazovaní krátkodobých a dlhodobých záväzkov.
- Pohľadávky a záväzky z derivátových operácií sa vykazujú na osobitných riadkoch ako dlhodobé a krátkodobé.
- Časť „Finančné účty“ sa zredukovala na „Peniaze“ a „Účty v bankách“, keďže vznikla časť „Krátkodobý finančný majetok“ a bankové účty s dobou viazanosti dlhšou ako jeden rok sú evidované ako súčasť položky „Dlhodobý finančný majetok“ (ale na účte 221, a nie v účt. skupine 06-DFM).
- Ako sme už uviedli, zo základného imania sa riadok „Vlastné akcie a vlastné obchodné podiely“ presunul do aktív.
- Zmenila sa štruktúra vlastného imania, vznikli nové položky „Emisné ážio“, „Ostatné kapitálové fondy“, „Zákonné rezervné fondy“, „Ostatné fondy zo zisku“ a „Oceňovacie rozdiely z precenenia“, ktoré sa doteraz prezentovali ako „Kapitálové fondy“ a „Fondy zo zisku“.
- V časti krátkodobé záväzky sa nevyfakturované dodávky pred úpravou vykazovali na osobitnom riadku, podľa nového opatrenia sa vykazujú na príslušných riadkoch v závislosti od povahy záväzku, s ktorým súvisia.
- Bankové úvery sa prezentujú v dvoch osobitných častiach súvahy „Dlhodobé bankové úvery“ a „Krátkodobé bankové úvery“, rezervy sa rozdelili na dlhodobé a krátkodobé a tiež sú vykazované v rôznych častiach súvahy.
- V časti ÚZ – výsledovka pribudol nový riadok „Čistý obrat“. Podľa novelizovaného zákona o účtovníctve sa na účely tohto zákona do čistého obratu zahŕňajú výnosy z predaja výrobkov, tovarov a služieb po odpočítaní zliav, ako aj iné výnosy po odpočítaní zliav tej účtovnej jednotky, ktorej predmetom činnosti je dosahovanie iných výnosov ako výnosov z predaja výrobkov, tovarov a služieb, ale do čistého obratu nepatria výnosy, ktoré nemajú opakovaný charakter v spojitosti s podnikaním účtovnej jednotky, napríklad výnosy z predaja nepotrebného majetku a pod.
- Prevádzková časť výsledovky sa rozdeľuje na samostatné časti: „Výnosy z hospodárskej činnosti spolu“ a „Náklady na hospodársku činnosť spolu“.
- Zanikli riadky „Obchodná marža“, „Výroba“ a „Výrobná spotreba“, riadok „Pridaná hodnota“ sa presunul pod výsledok hospodárenia z hospodárskej činnosti.
- Finančná časť výsledovky sa rozdeľuje na samostatné časti: „Výnosy z finančnej činnosti spolu“ a „Náklady na finančnú činnosť spolu“. V časti výnosov sa zavádza detailnejšia prezentácia výnosov v členení na prepojené osoby, podielové účasti a ostatné. Výnosové a nákladové úroky sa tiež rozdeľujú na položky týkajúce sa prepojených osôb a ostatné. V nákladovej časti sa vykazujú aj „Opravné položky k finančnému majetku“.

Prevažná časť týchto zmien bola uskutočnená v dôsledku povinnosti SR implementovať smernicu č. 2013/34/EÚ o individuálnej a konsolidovanej účtovnej závierke do svojej právnej úpravy. V súvahe zostal bez zmeny dlhodobý nehmotný a hmotný majetok, zásoby, výsledok hospodárenia minulých rokov a časové rozlíšenie nákladov a výnosov v aktívach a v pasívach. Vo výsledovke sa väčšina pôvodných riadkov iba premiestnila na iné miesto bez toho, aby sa zmenila ich obsahová náplň. Z úprav účinných od 1. 1. 2016 uveďme najmä zmeny v oceňovaní: oceňovanie reálnou hodnotou má teraz širšie použitie. Dôsledkom rozšírenia použitia reálnej hodnoty pri oceňovaní majetku a záväzkov je aj zrušenie reprodukčnej obstarávacej ceny, tzn., že majetok, ktorý sa do 31. 12. 2015 oceňoval reprodukčnou obstarávacou cenou, sa od 1. januára 2016 oceňuje reálnou hodnotou, ide o: majetok bezodplatne nadobudnutý, okrem peňažných prostriedkov, cenín a pohľadávok, ktoré sa oceňujú menovitou hodnotou; majetok preradený z osobného vlastníctva do podnikania, okrem peňažných prostriedkov, cenín a pohľadávok, ktoré sa oceňujú menovitou hodnotou; nehmotný majetok a hmotný majetok novozistený pri inventarizácii a v účtovníctve doteraz nezachytený; majetok obstaraný verejným obstarávateľom bezodplatne od koncesionára za plnenie vo forme koncesie na stavebné práce podľa osobitného predpisu V prípade cenných papierov a podielov určených na predaj (účtovaných na účte 063 – Realizovateľné cenné papiere a podiely) má účtovná jednotka pri prvotnom ocenení na výber: ocenenie v obstarávacej cene alebo ocenenie v reálnej hodnote.

3.2 Účtovné položky v ilustratívnych predikčných modeloch

Na základe empirických poznatkov získaných z praxe slovenských podnikov môžeme konštatovať, že k najčastejšie používaným predikčným modelom v SR, v rámci uskutočňovaných finančných rozborov a analýz hospodárenia, stále patrí Altmanovo Z-skóre, Index bonity B (zjednodušený Beerman), Kralickov Quick Test, IN indexy Neumaierovcov, ale aj Tamariho, Beaverov, Beermanov a Tafflerov model. Tieto modely používajú rôzne ekonomické premenné na analýzu a hodnotenie podniku a zaraďujú sa k rôznym typom modelov z hľadiska ich konštrukcie. Na ilustráciu sme do tabuľky 2 zaradili modely, ktoré nevznikli u nás, ani v susednej ČR, ale v našich podmienkach sa používajú. Pripomíname, že samotné hodnotenie nás v tomto prípade nezaujíma, zaujímajú nás výhradne účtovné položky.

Tabuľka 2: Ilustratívne modely a ich premenné

Model/metóda	**Typ**	**Premenné (ukazovatele)**
Beaverov test	UDA	CF/Cudzí kapitál .....Čistý zisk/Celkový majetok Cudzí kapitál/Celkový kapitál ...Čistý pracovný kapitál/Celkový majetok Obežný majetok/Krátkodobé záväzky
Diskusia: Celkový majetok - celkový majetok = aktíva celkom = pasíva celkom alebo aktíva brutto, aktíva netto? Cudzí kapitál - všetky cudzie zdroje krytia? T.j. v IÚZ všetky záväzky vrátane rezerv plus pasívne položky časového rozlíšenia? Aký význam bude mať nezahrnutie niektorých položiek do cudzieho kapitálu? Čistý zisk - VH v novej štruktúre po zdanení.		
Altmanovo Z score$_{1983}$ (pre spoločnosti, ktoré nemajú verejne obchodovateľné akcie)	MDA	Čistý pracovný kapitál/Majetok.....Nerozdelené zisky/Majetok EBIT/Majetok.....Účtovná (trhová) hodnota vlastného kapitálu/Cudzí kapitál ..... Tržby/Majetok
Diskusia: Nerozdelené zisky - totožné s našou účtovnou položkou – účet 428-Nerozdelený zisk minulých rokov? Alebo s riadkom 97 IÚZ Výsledok hospodárenia minulých rokov? Účtovná hodnota vlastného kapitálu vs. trhová hodnota VK ... ako sa dá zistiť u spoločností s neobchodovanými akciami na trhu?		
Kralickov Quick Test	MBH	Miera samostatnosti = Vlastný kapitál/Celkový kapitál Doba splácania dlhu = (Cudzí kapitál – Finančné účty)/CF Podiel CF na celkových výnosoch = CF/Výnosy Rentabilita celkového kapitálu na báze EBIAT = EBIAT/Celkový kapitál

Pokrač. tabuľky 2 Ilustratívne modely a ich premenné

Diskusia: Celkový kapitál - pasíva celkom, t. j. vrátane položiek časového rozlíšenia? Alebo iba v prípade, ak ide o významné položky z hľadiska ich výšky? Cudzí kapitál - vrátane rezerv? Finančné účty - podľa novej úpravy Peniaze a bankové účty. Rovná sa/nerovná sa stav finančných účtov CF? EBIAT? VH po zdanení znížený o nákladové úroky? Časovo rozlíšené/nerozlíšené?		
Beermanov test	MDA	Odpisy DHM/(začiatočný stav DHM + prírastok za obdobie) Prírastok DHM za obdobie/Odpisy DHM EBT/Tržby ..... Bankové úvery a výpomoci/Cudzí kapitál Zásoby/Tržby .....CF/Cudzí kapitál Cudzí kapitál/Majetok ..... EBT/Majetok Tržby/Majetok ..... EBT/Cudzí kapitál
Diskusia: Odpisy DHM - v sústave podvojného účtovníctva brať do úvahy účtovné odpisy v súlade s požiadavkami zákona o účtovníctve; stav DHM brutto alebo netto? Pri výpočte EBT rešpektovať novú štruktúru VH (EBT = VH za účt. obdobie pred zdanením); Tržby - tržby celkom bez ohľadu na to, či majú priamy súvis s podnikaním, pravidelnou činnosťou podniku? Tržby po odpočítaní zliav? Tržby zodpovedajúce v novej úprave „čistému obratu"? Bankové úvery a výpomoci – bankové úvery krátkodobé, dlhodobé, ale KFV iba krátkodobé; Majetok - aktíva spolu - brutto, netto? Cudzí kapitál - v našom účtovníctve iba úvery a záväzky alebo aj rezervy a pasívne položky časového rozlíšenia?		
Tafflerov model	MDA	EBT/Krátkodobé záväzky (v širšom zmysle) Obežný majetok/Cudzí kapitál Krátkodobé záväzky (v širšom zmysle)/Majetok Tržby/Majetok
Diskusia: Krátkodobé záväzky v širšom zmysle - čo to v našom účtovníctve znamená? Striktne v kontexte IÚZ (indiv. účt. závierky). Majetok = aktíva celkom = pasíva celkom alebo aktíva netto?		

Zdroj: vlastné spracovanie;
Vysvetl.: UDA – jednorozmerná diskriminačná analýza; MDA – viacrozmerná diskriminačná analýza; MBH – metóda bodového hodnotenia

4. Záver

Správne údaje sú ťažiskom každého objektívneho a spoľahlivého hodnotenia. Vo finančnej analýze, kde je hlavným informačným zdrojom účtovníctvo podniku a jeho evidencia, to platí dvojnásobne, pretože ani ten najdokonalejší systém hodnotenia nebude pre podnik užitočný, ak bude používať nesprávne a nepresné údaje, a súčasne, ak informácie nebudú relevantné, objektívne, zrozumiteľné, porovnateľné a včas k dispozícii. V súvislosti s údajmi je niekedy problémom nielen dosadzovanie účtovných položiek do modelov, ktoré vznikli v inej krajine, s odlišnou podobou účtovníctva, v inom čase, ale k chybám, bohužiaľ, dochádza aj v modeloch a pri ukazovateľoch domáceho pôvodu, kedy sú jednotlivé položky (parciálne ukazovatele) popísané nejednoznačne alebo sú nesprávne interpretované používateľom, čo môže viesť k skresleniam, chybným záverom a v konečnom dôsledku nesprávnym rozhodnutiam v praxi.

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References

[1] Bartošová, V. (2016). Rešerš literatúry k APVV-14-0841: Comprehensive Prediction Model of the Financial Health of Slovak Companies.

[2] Bartošová, V. (2008). *Financial Analysis and Planning.* Žilina: University of Žilina in EDIS.

[3] Bartošová, V., Paliderová, M. (2015). *Účtovníctvo podnikateľov 1. Podvojné účtovníctvo.* Žilina: Žilinská univerzita v Žiline vo vydavateľstve EDIS.

[4] Bartošová, V., Paliderová, M., Hrašková, D. (2014). *Basic Rules and Methodological Framework of the Financial Analysis of the Group of Companies.* Newark: Information Engineering Research Institute, p. 86-91.

[5] Buzgová, Ľ. (2014). Zmeny v postupoch účtovania pre podnikateľov účtujúcich v sústave podvojného účtovníctva. Dostupné na: <http://www.cfo.sk/articles/zmeny-v-postupoch-uctovania-pre-podnikatelov-uctujucich-v-sustave-podvojneho-uctovnictva#.V7bfbcbr1jr>.

[6] Farkaš, R. (2014). Zmeny v účtovných predpisoch k 31. decembru 2014 a 1. januáru 2015. Dostupné na: <http://www.cfo.sk/articles/zmeny-v-uctovnych-predpisoch-k-31-decemu-2014-a-1-januaru-2015#.V7bCWsbr1jr>.

[7] Mišanková, M., Kráľ, P. (2015). *Application of CreditRisk+ for the calculation of credit risk.* Ostrava: VŠB - Technická univerzita, s. 807-814.

[8] Nedeliaková, E., Sekulová, J., Nedeliak, I. (2014). Manažment rizika v podmienkach manažéra železničnej infraštruktúry. *Ekonomicko-manažérske spektrum,* 1(8), s. 22-28.

[9] Opatrenie MF SR č. 23054/2002-92 zo 16. decembra 2002, ktorým sa ustanovujú podrobnosti o postupoch účtovania a rámcovej účtovej osnove pre podnikateľov účtujúcich v sústave podvojného účtovníctva v znení neskorších predpisov

[10] Opatrenie MF SR č. 4455/2003-92 z 31. marca 2003, ktorým sa ustanovujú podrobnosti o usporiadaní, označovaní a obsahovom vymedzení položiek individuálnej účtovnej závierky a rozsahu údajov určených z individuálnej účtovnej závierky na zverejnenie pre podnikateľov účtujúcich v sústave podvojného účtovníctva v znení neskorších predpisov

[11] Opatrenia MF SR č. MF/15464/2013-74, č. MF/23378/2014-74, č. MF/23377/2014-74

[12] Poniščiaková, O., Litvaj, I. (2010). Riadenie podniku a ekonomický informačný systém. *Ekonomicko-manažérske spektrum,* 2(4), s. 80-85.

[13] Poniščiaková, O., Tokarčíková, E. (2015). The use of accounting information system for the management of business costs. *Manažment podnikov,* 3(5), s. 125-128.

[14] Švábová, L., Ďurica, M. (2016). Korelačná analýza prediktorov použitých v bankrotných predikčných modeloch na Slovensku. *Ekonomicko-manažérske spektrum,* 1(10), s. 2-11.

[15] Tokarčíková, E., Poniščiaková, O., Litvaj, I. (2014). *Key performace indicators and their exploitation in decision-making process.* Kaunas: University of Technology, pp. 372-375.

[16] http://www.finance.gov.sk/Default.aspx?CatID=4485

[17] zákon č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov

Tax Risk in Financial Management of Enterprises in the Slovak Republic

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Abstract

The paper is focused on the risks which are a part of the enterprise decision-making processes and on the factors influencing the tax risk in corporate financial management. Tax risk is the risk of loss resulting from changes in tax laws or unpredictable tax. Tax risk belongs to systematic risks since it is related to permanent changes in tax legislation in the Slovak Republic. In this paper, we analyze the most significant current changes in tax legislation, evaluate not only the negative impact on the business environment, but also the increase of tax burden, worsening of the financial situation and decreasing of the legal security of businesses. At the same time we present several suggestions to eliminate tax risk.

Key words

financial management, costs, revennes, tax risk, tax legislation.

JEL Classification: M40, H26

1. Úvod

Podnikanie je objektívne zaťažené celým radom rizík. Finančný manažment by mal poznať nielen možné zdroje a druhy rizika, ale aj riziko zhodnotiť a prijať opatrenia na odstránenie príčin a jeho dôsledkov. K najzložitejším rizikám z hľadiska ich ovplyvnenia a predikcie patria riziká spojené s externými faktormi, medzi ktoré patrí aj daňové riziko. Daňové riziko súvisí s permanentnou zmenou daňovej legislatívy a nepredvídateľným zdanením, ktoré má negatívne dôsledky na majetkovú a finančnú situáciu podniku.

2. Riziko ako súčasť podnikateľských rozhodovacích procesov

Medzi dlhodobými zámermi podniku má najvyššiu prioritu rast trvalo udržateľnej hodnoty zabezpečenej prostredníctvom rastu cash flow a zisková orientácia podniku. Hlavným, trvalým a bezpečným zdrojom financovania rastu hodnoty podniku je zisk. Zisk ako strategický cieľ je primárnym motívom podnikania so stimulačným efektom pri všetkých závažných rozhodnutiach a uskutočňovaných aktivitách. Všetky rozhodnutia a procesy prebiehajúce v podniku sa v konečnom dôsledku vždy priamym alebo nepriamym spôsobom prejavia v nákladoch alebo vo výnosoch, t. j. vo výsledku hospodárenia. Preto by mal manažment hľadať spôsoby a prijímať ekonomické rozhodnutia, realizáciou ktorých dosiahne nielen zvýšenie zisku, ale efektivitu celého podnikového systému.

Každý podnikový výkon by mal prinášať pozitívnu súčasnú hodnotu, tzv. NPV (anglicky Net Present Value), ktorá berie do úvahy časovú hodnotu peňažných tokov. Každá nákladová položka by mala prispieť k rastu hodnoty pre podnik a jeho vlastníkov (Bahnub, 2010).

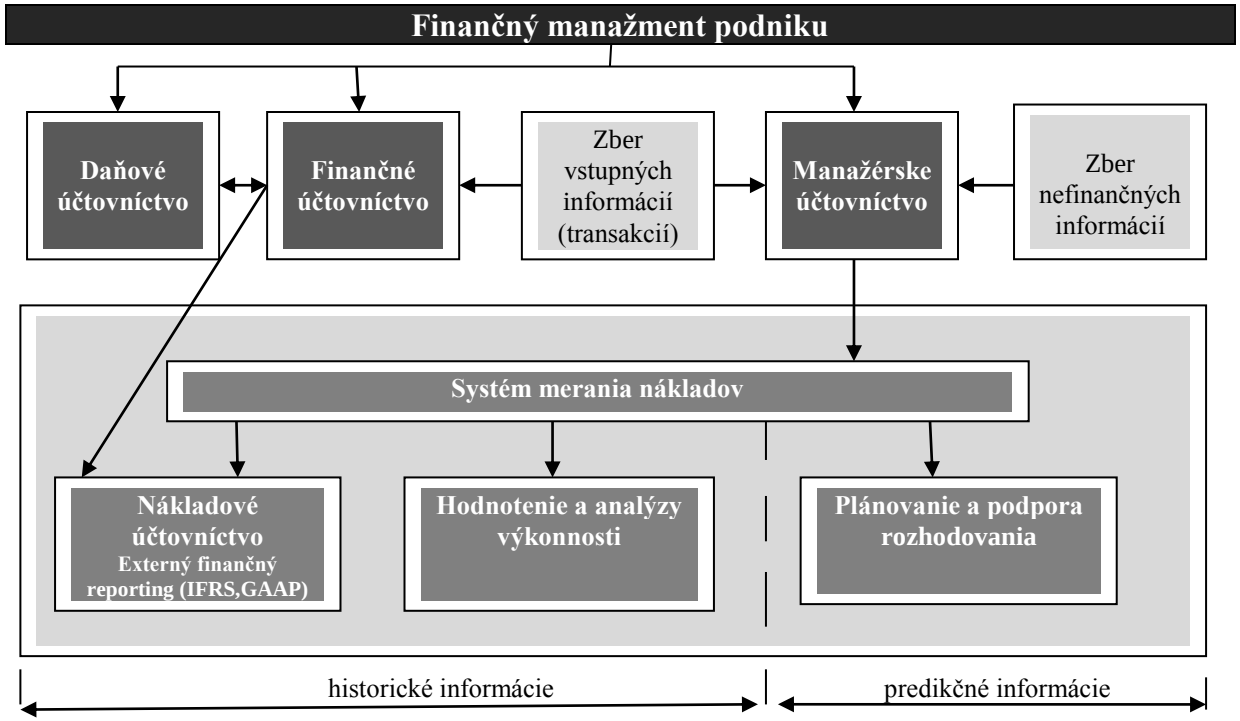
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Východiskovým ukazovateľom pri hodnotení kvality fungovania podnikových procesov je výsledok hospodárenia podniku. Všeobecne ho možno vypočíať ako rozdiel výnosov a nákladov. Pri súčasnom saturovanom trhu sa kladie väčší akcent na sledovanie a ovplyvňovanie nákladov. Samotné meranie, vykazovanie, plánovanie a riadenie nákladov je v nestabilnom trhovom prostredí veľmi náročné a vyžaduje si sofistikované nástroje. Finančné, nákladové a manažérske účtovné systémy tieto nástroje obsahujú a napomáhajú manažmentu porozumieť štruktúre podnikových nákladov a ich vzájomným väzbám.

IFAC (International Federation of Accountans) prezentuje postavenie systémov merania nákladov v rámci finančného riadenia podniku. Uvedené vzťahy ilustruje obrázok 1.

Obrázok 1: Postavenie nákladových systémov v rámci finančného manažmentu podniku



Zdroj: prepracované podľa IFAC - Professional Accountants in Business Committee, 2009. International Good Practice Guidance: Evaluation and Improving Casting in Organizations [online]. New York: International Federation of Accountants, s. 7 [cit. 2014.11.10]. Dostupné na internete: <http://www.ifac.org/sites/default/files/publications/IGPG-Evaluating-and-Improving-Costing-July-2009.pdf>

Nie všetky zložky systému prinášajú rovnaký typ informácií. Pre ekonomicky správne rozhodnutie, pre maximalizáciu využitia zdrojov, ktoré podnik má, je kľúčové použitie tých informácií, ktoré boli pre tento účel pripravené. Použitie informácií pripravených na iný účel môže viesť k chybe (Staněk, 2003).

Úlohou finančného účtovníctva je poskytnúť verný a pravdivý obraz o majetkovej a finančnej situácii podniku predovšetkým pre rozhodovanie externých subjektov (investorov, banky, burzy, dodávateľov, odberateľov). Pre zabezpečenie spoľahlivosti a porovnateľnosti sú informácie z finančného účtovníctva regulované právnou normou, národnými alebo medzinárodnými štandardami a sú prezentované formou účtovnej závierky.

Úlohou daňového účtovníctva je zobraziť aktivity podniku s akcentom na náklady, výnosy, daňové záväzky a daňové pohľadávky. Poskytuje primárne informácie manažmentu na

stanovenie a optimalizáciu jednotlivých daní, t. j. na daňové plánovanie. Je zamerané na testovanie a analyzovanie jednotlivých transakcií z hľadiska ich vplyvu na základ dane a daňovú povinnosť, na výpočty smerujúce k minimalizácii daňových povinností pri prijímaní dôležitých ekonomických rozhodnutí (Baštincová, 2016). Tieto informácie sledujú a posudzujú aj daňové úrady ako správcovia daní s cieľom maximalizovať výber daní podľa platnej daňovej legislatívy.

Manažérske účtovníctvo poskytuje manažmentu relevantné finančné a nefinančné informácie pre rozhodovanie.

Finančné riadenie síce využíva informácie o ukončených resp. prebiehajúcich aktivitách a procesoch podniku, ale je orientované na budúcnosť. Všetky rozhodnutia vychádzajú z hodnotenia možných variant budúceho vývoja podniku a jeho okolia, ktoré ho ovplyvňujú. Odhad budúceho vývoja podniku je spojený s určitou mierou neurčitosti, ktorá sťažuje rozhodovanie. Túto skutočnosť je nevyhnutné rešpektovať pri rozhodovaní a mieru rizika zohľadňovať v kritériách rozhodovania.

Riziko, ktoré je súčasťou podnikateľských rozhodovacích procesov, nie je v odbornej literatúre definované jednoznačne. Existujú rôznorodé pohľady na chápanie rizika a na jeho vyjadrenie. Riziko a jeho vplyv na výsledok podnikania je dané interakciou vnútorných a vonkajších podmienok, pričom miera a spôsob minimalizácie rizika je determinované rozhodnutím podniku a objektívnou povahou rizika.

K najzložitejším rizikám z hľadiska ich posudzovania a predikcie patria riziká spojené s externými faktormi, ktoré ovplyvňujú vývoj trhov, realizáciu podnikových výkonov, chovanie zákazníkov, konkurencie. Medzi tieto riziká patria riziká zvýšenia úrokových sadzieb, zdraženia úverov, kurzové riziká, daňové riziká (Korába, 2007). Daňové riziko ako súčasť obchodného rizika je rizikom straty zo zmeny daňových zákonov alebo nepredvídateľného zdanenia (Jílek, 2000).

3. Daňové riziko pri finančnom riadení podnikov v SR

Daňové riziko patrí medzi systematické riziká, pretože súvisí s permanentnou zmenou daňovej legislatívy. Zákony a predpisy upravujúce oblasť podnikania v SR prechádzajú už niekoľko rokov intenzívnymi zmenami. Tieto legislatívne zmeny väčšinou neslúžia podpore a rozvoju podnikania, pretože sú v nich implementované nielen rôzne administratívne prekážky, ale aj permanentne sa sprísňujúce kritéria a limity pre uplatňovanie nákladov súvisiacich s výpočtom základu dane a dane z príjmov.

3.1 Daňová legislatíva SR

Dane reprezentujú významné ekonomické nástroje riadenia, ktoré pôsobia na výšku zisku a cash flow podniku. Najdôležitejšou právnou normou, ktorá ovplyvňuje ekonomické rozhodnutia finančného manažmentu podniku v SR, je zákon o dani z príjmov (zákon č. 595/2003 Z. z.), účinný od 1. januára 2004. Zákon o dani z príjmov upravuje daň z príjmov fyzickej osoby, daň z príjmov právnickej osoby, spôsob platenia a vyberania dane, pričom najpodstatnejšiu časť zákona tvorí identifikácia daňovo uznaných a daňovo neuznaných nákladov, súvisiacich s výpočtom základu dane a dane z príjmov.

Pri daňových zákonoch všeobecne je potrebná čo najväčšia stabilita s minimálnym množstvom zmien, pretože ustálenosť a jednoznačnosť daňových predpisov je kľúčovým predpokladom ich bezproblémovej aplikácie v praxi. Zákon o dani z príjmov bol počas svojej účinnosti 64-krát novelizovaný, pričom najpočetnejšie a najzávažnejšie novelizácie zákona sa uskutočnili v rokoch 2014 - 2015. Deklarovaným zámerom daňových úprav je predovšetkým

systematický boj proti daňovým špekuláciám a podvodom vrátane boja proti únikom a stratám daňových príjmov. Neustále zmeny zákona o dani z príjmov narúšajú právnu istotu podnikov, neprispievajú k vytvoreniu motivačného podnikateľského prostredia, neumožňujú daňové plánovanie a prognózovanie finančnej situácie. Zmeny daňového zákona by bolo možné tolerovať, keby prispievali k zjednodušeniu výpočtu základu dane a dane z príjmov a administratívnej záťaže podnikov. V súčasnosti však smerujú k tomu, aby permanentne a často latentnými formami zvyšovali daňové zaťaženie podnikov.

Ďalším problémom je daňová nejednoznačnosť zákona. I keď mnohí finanční manažéri, daňoví analytici a účtovníci poznajú jednotlivé ustanovenia zákona, v praxi vzniká problém s ich správnou interpretáciou. Preto vznikajú paradoxné situácie, kedy správcovia daní pri rovnakom probléme odporučia rozdielny postup. Ministerstvo financií SR a Finančné riaditeľstvo vydávajú metodické pokyny a usmernenia, z ktorých je možné čerpať výklad niektorých ustanovení zákona; problémom je ich veľký počet a nesystematičnosť. Naviac, sú v nich často závery, ktoré prekračujú rámec zákona tak, že pri rôznych možných interpretáciách znejú v neprospech podniku ako daňovníka.

Novelizácie zákona o dani z príjmov od roku 2014 nemožno hodnotiť z podnikového hľadiska pozitívne, pretože ich hlavným cieľom bolo obmedziť daňovú uznateľnosť nákladov pri výpočte základu dane a dane z príjmov. Medzi významné negatívne opatarenia patria jednoznačne zmeny v odpisovaní dlhodobého majetku, pričom predĺženie doby odpisovania sa týkalo aj majetku nadobudnutého a zaradeného do používania pred účinnosťou tejto novelizácie. Rovnaké negatívne dôsledky majú aj zmeny týkajúce sa nákladov súvisiacich s finančným a operatívnym prenájmom dlhodobého majetku, zmeny v možnosti umorovania daňovej straty a zavedenie daňových licencií stratových podnikov.

3.2 Dôsledky zmien daňovej legislatívy na finančnú situáciu podnikov SR

Zmeny novelizovaného zákona o dani z príjmov sa týkajú predovšetkým oblasti odpisovania dlhodobého majetku. Ich rozsah bol natoľko významný, že sa týkal každého podniku bez ohľadu na veľkosť, právnu formu či oblasť podnikania. Na účely daňového odpisovania bol dlhodobý majetok do konca roka 2014 zaradený do štyroch odpisových skupín s dobou odpisovania od 4 do 20 rokov. Od roku 2015 sa rozšíril počet odpisových skupín na šesť s dobou odpisovania od 4 do 40 rokov. Najdlhšia doba odpisovania sa týka najmä budov na administratívu, kultúru, vzdelávanie, hotelov a pod. Zmeny v zaradení majetku do odpisových skupín sa týkali nielen novoobstaraného majetku, ale aj majetku odpisovaného podľa zákona o dani z príjmov pred jeho novelizáciou.

Každý podnik bol povinný vykonať zmeny v zaradení dlhodobého majetku do odpisových skupín a od roku 2015 pokračovať v odpisovaní tohto majetku v nových odpisových skupinách, pričom už uplatnené odpisy sa spätne neupravovali. Súčasne zákon obmedzil možnosť využívať zrýchlený spôsob odpisovania dlhodobého majetku. Do konca roka 2014 platilo, že podnik, bež ohľadu na odpisovú skupinu, v ktorej bol dlhodobý majetok zaradený, sa mohol rozhodnúť pre uplatňovanie rovnomerného alebo zrýchleného spôsobu odpisovania. Od roku 2015 je rovnomerný spôsob odpisovania stanovený ako všeobecný spôsob, pričom možnosť voľby spôsobu odpisovania umožňuje len pri dlhodobom majetku zaradenom v druhej alebo tretej odpisovej skupine. Zároveň stanovil nové koeficienty pre zrýchlené odpisovanie tohto majetku. V tomto prípade ide o precedens, pretože podľa samotného zákona o dani z príjmov nemožno zmeniť spôsob odpisovania po celú dobu jeho odpisovania.

Od roku 2015 zákon o dani z príjmov obmedzil uplatňovanie daňových odpisov z osobných automobilov so vstupnou cenou 48 000 eur a viac. Toto obmedzenie vychádza z testovania vykázaného základu dane voči úhrnu odpisov vypočítaných z limitovanej ceny z

týchto osobných automobilov v danom zdaňovacom období (ročný odpis z limitovanej ceny jedného osobného automobilu je 12 000 eur). Ak vykázaný základ dane je nižší ako úhrn limitovaných odpisov z týchto automobilov, základ dane sa zvýši o rozdiel medzi úhrnom skutočne uplatnených daňových odpisov a úhrnom ročných odpisov vypočítaných z limitovanej vstupnej ceny. Ak vykázaný základ dane je vyšší ako úhrn limitovaných odpisov z týchto automobilov, úprava základu dane sa nevykoná. Zmeny v odpisovaní dlhodobého majetku znamenajú retroaktívne opatrenie, keďže podniky pri obstaraní tohto majetku a jeho zaradení do príslušnej odpisovej skupiny uvažujú s určitou dobou odpisovania a presnou sumou daňových odpisov, ktoré budú ovplyvňovať základ dane a daň z príjmov. Predĺžením doby odpisovania a znížením sumy daňových odpisov sa podnikom zvýšil základ dane a daň.

Zákon o dani z príjmov súčasne zrušil zvýhodnené lízingové odpisovanie majetku obstaraného formou finančného prenájmu. Majetok obstaraný formou finančného prenájmu do konca roka 2014 podnik odpisoval rovnomerne počas doby prenájmu dohodnutej v nájomnej zmluve. Od roku 2015 tento majetok odpisuje podnik ako nájomca počas doby odpisovania v príslušnej odpisovej skupine a to rovnomerným alebo zrýchleným spôsobom odpisovania.

Rovnako obmedzil aj zahrnovanie nájomného pri operatívnom prenájme z osobných automobilov so vstupnou cenou 48 000 eur a viac do daňových výdavkov. Podnik ako nájomca je povinný testovať dosiahnutý základ dane s limitovaným nájomným vo výške 14 400 eur za jedno auto (uvedená suma predstavuje ročný odpis z limitovanej vstupnej ceny 12 000 eur zvýšený o 20 % ziskovú prirážku u prenajímateľa). Ak je základ dane nižší ako limitované nájomné, nájomca je povinný základ dane zvýšiť o rozdiel medzi úhrnom skutočne uplatneného nájomného a úhrnom limitovaného nájomného.

Pri využívaní majetku, ktorý môže mať charakter osobnej spotreby a s ním súvisiacich nákladov (napr. spotrebnej elektroniky, nábytku a pod.) je náklad uznaný len v pomernej časti, v akej sa majetok používa na dosiahnutie, zabezpečenie a udržanie zdaniteľných príjmov. Uvedená striktnejšia úprava núti podniky prehodnotiť prístup k uplatňovaniu nákladov na taký majetok, ktorý súčasne využívajú aj na iný účel ako na podnikanie, t. j. na súkromné účely. Zákon o dani z príjmov umožňuje využiť dva spôsoby uplatňovania nákladov a to:

- vo forme paušálnych výdavkov vo výške 80 %, ak je tento majetok využívaný aj na súkromné účely,
- v preukázateľnej výške v závislosti od pomeru používania tohto majetku na zabezpečenie zdaniteľných príjmov.

Tým sa do zákona zaviedol princíp prezumpcie viny, podľa ktorej sa každý podnik snaží ukrátiť štát na daniach. Týka sa to 80 % nákladov z majetku, ktorý sa môže týkať aj osobnej spotreby. Ak si chce podnik uplatniť náklady v plnej výške, musí jednoznačne preukázať, že tento majetok používa výlučne na podnikateľské účely. Daňový úrad by mal dokazovať nevyužívanie majetku na podnikanie; týmto spôsobom sa dôkazné bremeno presunulo na podnik ako daňovníka.

V daňovom zákone bolo zrušené aj spravodlivé pravidlo, podľa ktorého výnosy, vzťahujúce sa k daňovo neuznaným nákladom, boli z daňového hľadiska neuznané, teda nezdaniteľné. Ide napr. o zmluvné pokuty a penále, ktoré sú u dlžníka aj po ich zaplatení veriteľovi neuznaným daňovým nákladom, ale u veriteľa sú zdaniteľným príjmom už pri ich zaúčtovaní do výnosov.

Zákon o dani z príjmov rozšíril okruh nákladov, ktoré môže podnik zahrnúť do základu dane až po ich zaplatení. Ide predovšetkým o náklady na poradenské a právne služby, náklady na marketingové štúdie a prieskum trhu, náklady na nájomné, na sprostredkovanie obchodu.

Pre objektívnosť treba pripomenúť, že výnosy, ktoré sa vzťahujú k uvedeným nákladom sa u veriteľa zahrnujú do základu dane už pri ich zaúčtovaní v účtovníctve. Toto opatrenie výrazným spôsobom zvyšuje sumu daňovo neuznaných nákladov na konci zdaňovacieho

obdobia a znižuje transparentnosť sledovania a preukazovania podkladov pre výpočet základu dane a dane z príjmov.

Novelizovaný zákon o dani z príjmov obmedzil aj niektoré legálne možnosti zníženia daňového zaťaženia, konkrétne možnosť odpočítať daňovú stratu od základu dane. Do roku 2014 mali podniky možnosť odpočítať od základu dane daňovú stratu počas siedmich bezprostredne po sebe nasledujúcich zdaňovacích období a to podľa vlastného rozhodnutia. Od roku 2015 majú podniky možnosť odpočítať daňovú stratu rovnomerne počas štyroch bezprostredne po sebe nasledujúcich zdaňovacích období. Ak podnik počas štyroch zdaňovacích období nevykáže každoročne dostatočný základ dane, od ktorého si môže odpočítať jednu štvrtinu z celkovej daňovej straty, nárok na jej odpočítanie mu definitívne zaniká. Toto opatrenie malo rovnako retroaktívny účinok, pretože sa týkalo aj strát vykázaných pred účinnosťou novely zákona o dani z príjmov.

Zákon o dani z príjmov zaviedol od roku 2014 minimálnu daň formou daňovej licencie. Ak podnik vykáže v daňovom priznaní daňovú stratu alebo daňovú povinnosť nižšiu ako výška daňovej licencie, je povinný platiť daňovú licenciu. Zákon zaviedol tri pásma na uplatnenie daňovej licencie v závislosti od výšky ročného obratu a to diferencovane pre platiteľov a pre neplatiteľov dane z pridanej hodnoty. Najnižšiu daňovú licenciu v sume 480 eur ročne platí podnik, ktorý nie je platiteľom dane z pridanej hodnoty s ročným obratom neprevyšujúcim 500 tis. eur (ak je platiteľom dane platí daňovú licenciu 960 eur). Ostatné podniky platia daňovú licenciu 2 880 eur. Kladný rozdiel medzi daňovou licenciou a daňou vypočítanou v daňovom priznaní je možné započítať na daňovú povinnosť počas nasledujúcich troch bezprostredne po sebe nasledujúcich zdaňovacích obdobiach.

4. Možnosti eliminácie daňového rizika pri finančnom riadení podnikov v SR

Podnik nemôže byť dlhodobo úspešný, ak nie je ochotný akceptovať a znášať určité podnikateľské riziká. Riziko pri podnikaní je spojené s očakávaním úspechu. Daňové riziko súvisí predovšetkým s preferovanou daňovou politikou vlády, ktorá sa premieta do aktuálnej daňovej legislatívy, s jej prístupom pri aplikácii všeobecne uznávaných daňových zásad, s mierou akceptácie požiadaviek podnikateľskej sféry a pod.

Štát svojimi zásahmi v oblasti daní výrazným spôsobom ovplyvňuje správanie a ekonomické rozhodovanie podnikov ako daňových subjektov, pretože priamo vplýva na výšku zisku a cash flow. Pri preferovaní reštriktívnej daňovej politiky dochádza k vyššiemu daňovému zaťaženiu podnikov, k obmedzovaniu daňových úľav, k úpravám daňových odpisov, ktoré vedú k predlžovaniu doby odpisovania, k obmedzovaniu uznávania nákladov z daňového hľadiska a pod. Dôsledkom takejto politiky môže byť napr. útlm podnikateľskej aktivity, snaha o nelegálnu daňovú optimalizáciu, zmena miesta podnikania. Výsledkom uplatňovania expanzívnej daňovej politiky je znižovanie daňového zaťaženia podnikov, rozširovanie daňových úľav, opatrenia smerujúce k znižovaniu základu dane, znižovanie sadzieb dane a pod. Cieľom tejto politiky je zvýšiť podnikateľskú aktivitu, zamestnanosť, prilákanie zahraničných investorov.

Z uvedeného vyplýva, že vláda SR uplatňuje rôzne kombinácie uvedených daňových politík. Jej zámerom v súčasnosti by malo byť preferovanie expanzívnej daňovej politiky prostredníctvom daňových zásad a opatrení tak, aby slúžili na presadzovanie ekonomických, sociálnych a politických cieľov a prestíže medzi krajinami EÚ. Pri realizácii daňovej politiky by sa mala uplatňovať predovšetkým zásada spravodlivosti, jednoduchosti, efektívnosti, daňovej istoty (t. j. minimalizácie zmien v oblasti daní), transparentnosti.

Daňovú spravodlivosť teoretici z oblasti finančnej vedy zaraďujú medzi hlavné daňové zásady. Z teoretického hľadiska predstavuje stav, ktorý by sa dosiahol, ak bude na úhradu spoločných výdavkov formou daní prispievať každý subjekt práve adekvátnym, spravodlivým dielom (Schultzová a kol., 2011). Princíp daňovej spravodlivosti nerieši problémy spravodlivosti výberu daní, ale spôsob vyrubovania daní, ktorý by mal byť čo najspravodlivejší. To znamená, že subjekt s vyššími príjmami alebo vyššou hodnotou majetku resp. vyššou spotrebou by mal platiť vyššiu daň.

Zásada daňovej istoty a transparentnosti znamená stabilitu daňovoprávnych zákonov (i keď určitá pružnosť je pomienkou jeho účinnosti), zosúladenie nižších právnych noriem, teda vládnych nariadení a vyhlášok ústredných orgánov so zákonmi, zosúladenie rozhodovacích kompetencií príslušných daňových orgánov.

Zásada efektívnosti spočíva v tom, že uplatňovaná daňová politika musí obsahovať dostatok prvkov stimulujúcich oživenie podnikateľských činností a pracovných aktivít. Jej aplikácia súvisí aj s úpravou vykázaného účtovného zisku podniku na základ dane, so spôsobom umorovania daňovej straty, s možnosťou uznávania nákladov preukázateľne vynaložených na dosiahnutie, zabezpečenie a udržanie zdaniteľných príjmov, s určením výšky povinných odvodov do fondov a pod.

Podľa navrhovaných opatrení vlády SR v nasledujúcom volebnom období by malo ísť o zmeny viacerých daňových zákonov so zámerom prehĺbenia zásady spravodlivosti medzi jednotlivými daňovými subjektmi. Pre riešenie problémov príjmovej nerovnosti by mali byť zavedené tieto dane:

- daň z dividend vo výške 15 % pre vlastníkov veľkých akciových spoločností, pretože dividendy v SR ako v jedinej krajine EÚ nepodliehajú dani z príjmov; súčasne by mali byť zrušené zdravotné odvody z dividend vo výške 14 %, pretože predstavujú európsky unikát,
- zrážková daň z platieb do daňových rajov vo výške 35 %,
- vyššie dane pre banky a vybrané odvetvia ekonomiky pri zachovaní súčasného osobitného odvodu pre regulované odvetvia,
- vyššia registračná daň na motorové vozidlá s vyšším výkonom pri prvej registrácii automobilu,
- zvýšenie dane z hazardných hier a spotrebnej dane z tabaku.

Sadzba dane z príjmov právnických osôb by sa mala znížiť z 22 % na 21 %.

Z navrhovaných opatrení vlády SR vyplýva, že ide o celý rad neprehľadných a nesystematických legislatívnych zmien. Medzi uvedenými plánovanými opatreniami a riešením príjmovej nerovnosti neexistuje relevantný vzťah. Vláda neplánuje reciprocne znižovať napr. dane z príjmov fyzickým osobám. Absentuje záujem o objektívne zhodnotenie pozitívnych a negatívnych opatrení súčasnej daňovej politiky.

Pre zníženie daňového rizika podnikateľskej sféry je nevyhnutné, aby pri prijímaní súčasných a budúcich opatrení v oblasti daní, vláda SR vychádzala z dôkladnej komplexnej analýzy predpokladaných dopadov jednotlivých druhov daní na príslušné skupiny daňovníkov. Ak má daňový systém pôsobiť na aktivitu podnikateľských subjektov pozitívne, mal by rešpektovať aj ich požiadavky. Bolo by vhodné zaviesť určité daňové moratórium, napr. v rozsahu dvoch alebo troch rokov, počas ktorého by sa testovali a hodnotili dôsledky prijatých legislatívnych zmien na základe exaktných údajov a nie na základe záujmov určitých lobistických skupín. Zmeny v daňovej legislatíve by sa mali uskutočňovať systematicky na základe jasne definovaných dôvodov bez retroaktívnej účinnosti.

5. Záver

Dane predstavujú významné ekonomické nástroje riadenia, ktoré pôsobia na výšku zisku a cash flow podniku. Najdôležitejšou právnou normou, ktorá ovplyvňuje ekonomické rozhodnutia finančného manažmentu v SR je zákon o dani z príjmov. Neustále zmeny tohto zákona narúšajú právnu istotu podnikov, nevytvárajú motivačné podnikateľské prostredie, pretože permanentne sprísňujú kritériá a limity pre uplatňovanie nákladov súvisiacich s výpočtom základu dane a dane z príjmov, neumožňujú daňové plánovanie. Medzi významné negatívne opatrenia s retroaktívnym účinkom patria najmä zmeny v odpisovaní dlhodobého majetku, zmeny v daňovej uznateľnosti nákladov súvisiacich s finančným a operatívnym prenájmom dlhodobého majetku, zmeny v možnosti umorovania daňovej straty, zavedenie daňových licencií pre stratové podniky. Aj z navrhovaných opatrení vlády SR v nasledujúcom období možno očakávať celý rad nesystematických legislatívnych zmien. Pre zníženie daňového rizika pri finančnom riadení podnikov je nevyhnutné, aby východiskom pri prijímaní opatrení v oblasti daní bola dôkladná analýza predpokladaných dôsledkov na príslušné skupiny daňovníkov a rešpektovanie relevantných požiadaviek podnikateľských subjektov. Zmeny v daňovej legislatíve by sa mali uskutočňovať systematicky na základe exaktných údajov a jasne definovaných dôvodov bez retroaktívnej účinnosti.

6. Potvrdenie

Príspevok bol spracovaný ako jeden z výstupov riešenia projektu grantovej agentúry Vedecká grantová agentúra MŠ SR a Slovenskej akadémie vied (VEGA) č. 1/0512/16 (2016-2018) "Zachovanie a rast majetkovej podstaty ako relevantný nástroj trvalo udržateľného rozvoja podniku".

References

[1] Bahnub, B. (2010). *Activity Based Management for Financial Institutions*. Driving Bottom - Line Results. 1. vydanie. USA: John Wiley & Sons, s. 199.

[2] Baštincová, A. (2016). *Účtovníctvo a dane právnických osôb.* 1. vydanie. Bratislava: Wolters Kluwer, s. 23-24.

[3] IFAC - Professional Accountants in Business Committee (2009). International Good Practice Guidance: *Evaluation and Improving Casting in Organizations* [online]. New York: International Federation of Accountants, s. 7 [cit. 2014.11.10]. Dostupné na internete:<http://www.ifac.org/sites/default/files/publications/IGPG-Evaluating-and-Improving-Costing-July-2009.pdf>.

[4] Jílek, J. (2000). *Finanční riziká*. 1. vydanie. Praha: Grada, s. 95

[5] Koráb, V., Peterka, J., Režňáková, M. (2007). *Podnikatelský plán.* 1. vydanie. Brno: Computer Press, s. 88-92.

[6] Schultzová, A. a kol. (2015). *Daňovníctvo - daňová teória a politika I.* 2. vydanie. Bratislava: Wolters Kluwer, s. 19-23.

[7] Staněk, V. (2003). *Zvyšování výkonnosti procesním řízením nákladu*. 1. vydanie. Praha: Grada Publishing, s. 29.

[8] Zákon č. 595/2003 Z. z. *o dani z príjmov* v znení neskorších predpisov.

Adequacy test for longevity risk reserves for a portfolio of model annuities from Pillar II of the Slovak pension system

Mária Bilíková, Jana Mihalechová, Katarína Sakálová [1]

Abstract

The paper gives a concise description of Pillar II of the Slovak pension system and describes a model whole-life annuity with a guarantee of payment if death occurs in the first seven years of the contract. For the given model portfolio the values of the annuity payment and the reserve in the fourth and tenth years of insurance are calculated deterministically. The paper focuses on analysing the financial risk relating to longevity which the life insurer takes on by selling such products. Longevity risk assumes that the mortality of those insured will be lower than that expected for the population in the mortality tables used. For the given portfolio a mortality risk margin is applied in the form of a percentage reduction in the mortality rates and a reduction of age in the mortality tables. In respect of this risk reserve adequacy testing is carried out for the given portfolio and the results produced are compared using graphs.

Key words

Pillar II of Slovak pension system, adequacy test, longevity risk, annuity

JEL Classification: G 22, C 65

1. Úvod

Systém dôchodkového zabezpečenia na Slovensku je v súčasnosti zložený z troch pilierov. I. pilier je povinný a priebežne financovaný, II. pilier príspevkovo definovaný a III. pilier predstavuje dobrovoľné doplnkové dôchodkové sporenie[2]. V príspevku sa zameriavame na dôchodky z II. piliera, ktorých výplatu zabezpečujú, podľa Zákona č. 183/2014 Z. z. o starobnom dôchodkovom sporení [9], komerčné poisťovne, ktoré na to získali licenciu od Národnej banky Slovenska (NBS). Výplatnú fázu budeme analyzovať z pohľadu poisťovne, ktorá pri určovaní výšky dôchodku musí zohľadniť na jednej strane svoju konkurencie schopnosť (aby ňou určené dôchodky boli dostatočne vysoké a bol o ne záujem) a na druhej strane svoju solventnosť (aby bola stále schopná plniť svoje záväzky voči poistencom). Na to, aby poisťovňa mala dostatok finančných prostriedkov na krytie záväzkov, je povinná tvoriť technické rezervy. Výška dôchodkov a následne aj technických rezerv závisí od toho, aké predpoklady o úmrtnosti, úrokovej miere, nákladoch a ostatných podmienkach poisťovňa zvolí. Vystavuje sa tak rizikám toho, že niektorý predpoklad nastaví nesprávne. V prípade dôchodkov z II. piliera je významným riziko dlhovekosti, ktoré vplýva na výšku technických rezerv, a teda aj na zisk poisťovne. To, že riziko dlhovekosti nie je zanedbateľné, ukáže test

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[2] Viac informácií možno nájsť napr. v [5]

primeranosti rezerv. Pri testovaní budeme skúmať modelové portfólio jedného z typov doživotných dôchodkov z II. piliera.

2. Modelový dôchodok z II. piliera dôchodkového zabezpečenia

Sporenie na dôchodok má najprv sporiacu fázu, kedy si sporiteľ ukladá svoje prostriedky na účet vo zvolenej dôchodkovej správcovskej spoločnosti, ktorá ich zhodnocuje. Ak klient požiada o výplatu dôchodku, vystaví sa certifikát o výške nasporených prostriedkov a poistitelia[3] mu dajú ponuku na výplatu dôchodku. Zákon č. 183/2014 Z. z. charakterizuje jednotlivé typy doživotných a dočasných dôchodkov, resp. programový výber, ktorý si klient môže zvoliť. Spomedzi viacerých možností sa zameriame na doživotný dôchodok bez zvyšovania a pozostalostných dôchodkov, ktorý bude vyplácaný mesačne pozadu s garanciou výplaty sumy pri úmrtí poberateľa dôchodku počas siedmych rokov. Podľa § 32 uvedeného zákona je totiž umožnené dedenie vo fáze výplaty doživotného dôchodku, ak poberateľ zomrie skôr, ako mu bolo vyplatených 84 mesačných dôchodkov. V tomto prípade sa vyplatí suma, určená rozdielom medzi výškou 84 mesačných dôchodkov a už vyplatenými dávkami.

V tejto časti príspevku odvodíme vzťah na výpočet mesačnej výšky takéhoto dôchodku, ktorú označíme Q. Túto dávku bude poistiteľ vyplácať vždy na konci mesiaca za predpokladu, že je poberateľ nažive. Súčasnú hodnotu dávok (označenú $SH(D)$) takto vyplácaného dôchodku určíme vzťahom:

$$SH(D) = 12Q \cdot a_x^{(12)} + Q \cdot SH(VP) \tag{1}$$

kde $a_x^{(12)}$ je aktuársky symbol pre súčasnú hodnotu jednotkového polehotného doživotného dôchodku x-ročnej osoby vyplácaného mesačne. Jeho hodnotu možno určiť s využitím lineárnej aproximácie (podľa [7], str. 102) nasledovne:

$$a_x^{(12)} \cong \frac{N_{x+1}}{D_x} + \frac{11}{24} \tag{2}$$

kde jednotlivé symboly znamenajú:

$D_x = l_x \cdot v^x$ – komutačné číslo počtu žijúcich osôb vo veku x,

$N_x = \sum_{k=0}^{\omega - x} D_{x+k}$ – komutačné číslo druhého rádu,

l_x – počet osôb žijúcich vo veku x zo zvolenej úmrtnostnej tabuľky,

$v = (1+i)^{-1}$ – odúročiteľ, vyjadrený pomocou technickej úrokovej miery i,

ω – najvyšší dosahovaný vek v úmrtnostnej tabuľke.

Symbol $SH(VP)$[4] označuje dočasné poistenie na úmrtie s každomesačne sa znižujúcou vyplatenou dávkou pozostalým v prípade úmrtia poistenej osoby počas prvých 7 rokov. Vyplatená dávka je vo výške rozdielu sumy 84 mesačných dávok a už vyplatených dôchodkov. Súčasnú hodnotu takéhoto poistenia vyjadríme vzťahom:

$$SH(VP) = \sum_{s=0}^{83} {}_{\frac{s}{12}}p_x \cdot {}_{\frac{1}{12}}q_{x+\frac{s}{12}} \cdot v^{\frac{s+1}{12}} \cdot (84 - s) \tag{3}$$

kde:

${}_n p_x = \frac{l_{x+n}}{l_x}$ – je pravdepodobnosť, že osoba vo veku x rokov sa dožije veku $x+n$,

[3] Komerčné poisťovne s licenciou od NBS, v súčasnosti sú to: Allianz – Slovenská poisťovňa, a. s., Generali Poisťovňa, a. s. a Union poisťovňa, a. s.

[4] Odvodené od „Súčasná hodnota výplaty pozostalým“.

${}_nq_x = 1 - {}_np_x$ – je pravdepodobnosť, že osoba vo veku *x* rokov sa nedožije veku $x+n$, pričom pravdepodobnosti vo vzorci (3) určíme z úmrtnostných tabuliek lineárnou interpoláciou.

Uvažujeme štyri typy nákladov spojených so spravovaním dôchodkového poistenia. Jednorazové začiatočné náklady α určíme percentom z nasporenej sumy (π). Bežné správne náklady β stanovíme percentom z ročnej výplaty dôchodku. Ďalšími sú náklady γ pri výplate pozostalým a δ náklady potrebné na výplatu mesačného doživotného dôchodku (určené pevnou sumou). Súčasnú hodnotu nákladov $SH(N)$ určíme nasledovne:

$$SH(N) = \alpha \cdot \pi + (\beta \cdot Q + \delta) \cdot 12 \cdot a_x^{(12)} + \gamma \cdot {}^{(12)}A^1_{x,\overline{k}|} \tag{4}$$

pričom ${}^{(12)}A^1_{x,\overline{k}|}$ reprezentuje faktor výplaty garancie a určíme ho na základe vzťahu:

$${}^{(12)}A^1_{x,\overline{k}|} = \sum_{s=0}^{12k-1} {}_{\frac{s}{12}}p_x \cdot {}_{\frac{1}{12}}q_{x+\frac{s}{12}} \cdot v^{\frac{s+1}{12}} \tag{5}$$

kde *k* počet rokov garancie, teda *k* = 7.

Výšku nasporenej sumy alebo jednorazového poistného π vyjadríme na základe princípu ekvivalencie a princípu fiktívneho súboru, teda $\pi = SH(D) + SH(N)$ a po úprave môžeme určiť výšku mesačného dôchodku *Q* zo vzťahu:

$$Q = \frac{\pi \cdot (1-\alpha) - \delta \cdot 12 a_x^{(12)} - \gamma \cdot {}^{(12)}A^1_{x,\overline{k}|}}{(1+\beta) \cdot 12 a_x^{(12)} + SH(VP)} \tag{6}$$

3. Rezervy a test ich primeranosti

Poisťovňa vytvára technické rezervy v takej výške, v akej predpokladá v budúcnosti záväzky voči klientom, ktoré vzniknú z uzatvorených poistných zmlúv. Tieto rezervy predstavujú prostriedok krytia, ktorý je vytvorený účelovo a zaručuje schopnosť poisťovne uhrádzať v každom okamihu svoje záväzky.

Podľa toho, akú poisťovaciu činnosť poisťovňa vykonáva, vytvára v súlade so zákonom č. 39 /2015 Z. z. o poisťovníctve a o zmene a doplnení niektorých zákonov [10] rôzne technické rezervy, v životnom poistení najmä technickú rezervu na poistné plnenie a technickú rezervu na životné poistenie.

Keďže takmer vždy je medzi vytvorením technickej rezervy a jej použitím na poistné plnenie pomerne dlhý časový úsek, poisťovňa môže prostriedky tvoriace technické rezervy investovať tak, aby sa zhodnotili. Avšak kvôli zachovaniu bezpečnosti rezerv je poisťovňa povinná podľa zákona o poisťovníctve dodržiavať nasledujúce zásady pre výpočet technických rezerv v životnom poistení (uvádzame výber zo zásad podľa § 172):

- výška technických rezerv v životnom poistení sa vypočítava dostatočne obozretnou prospektívnou metódou, pričom pri výpočte sa vychádza z budúceho splatného poistného a budúcich záväzkov, ktoré sú určené v poistných podmienkach pre každú poistnú zmluvu,
- dostatočne obozretná metóda ocenenia technických rezerv nie je určenie najlepšieho odhadu, ale mala by zahŕňať primeranú maržu pre negatívne odchýlky predpokladov ocenenia,
- technické rezervy v životnom poistení sa vypočítavajú poistno-matematickými metódami pre každú poistnú zmluvu samostatne.

Pri prospektívnom výpočte poistnej rezervy poistenia *x*-ročnej osoby v *r*-tom roku vychádzame zo vzťahu:

$${}^{B}{}_rV_x^{prosp} = {}_rH_b^D + {}_rH_b^N - {}_rH_b^P \tag{7}$$

kde:

${}_rH_b^{D/N/P}$ – vyjadruje hodnotu budúcich dávok/nákladov/poistných určených k dátumu výpočtu rezervy v *r*-tom roku poistenia.

V prípade modelového dôchodkového poistenia, ktorého poistné je zaplatené jednorazovo nasporenou sumou π, hodnota budúcich poistných do výpočtu nevstupuje, teda ${}_rH_b^P = 0$. Ďalšie dve hodnoty (dávky a náklady) určujeme využitím analogických vzťahov ako boli (1), (3) a (4) vyjadrených v *r*-tom roku poistenia. Keďže v tomto poistení dochádza k 7-ročnej garancii výplaty dávky pre pozostalých, zvolili sme dva dátumy výpočtu rezervy, a to vo štvrtom a v desiatom roku poistenia ($r = 4$ a $r = 10$).

Technické rezervy sú stanovované na základe predpokladov, ktorých hodnota je určená v čase oceňovania produktu (výpočtu dávky). Avšak poistné zmluvy v životnom poistení majú dlhodobý charakter, čo spôsobuje, že sa tieto predpoklady môžu zmeniť, čo môže byť príčinou, že technická rezerva bude nedostatočná na pokrytie záväzkov. Poisťovňa nemôže meniť metódu na výpočet výšky technickej rezervy, ale ich musí mať v každom momente vytvorené v dostatočnej výške na krytie svojich záväzkov. Preto poisťovne vykonávajú test primeranosti rezerv[5], ktorý predstavuje dôležité kritérium zisťovania solventnosti a kapitálovej vybavenosti poisťovne.

Od 2. 2. 2016 platí na slovenskom poistnom trhu aktualizovaná Odborná smernica SSA[6] č.1 v3 [3], ktorej cieľom je stanoviť základné zásady testovania primeranosti v životnom poistení požadované platnou legislatívou. Zásady ich výpočtu v životnom poistení, ktoré by mali obozretne vyjadrovať finančnú situáciu poisťovní, zohľadňujúc aktuálne poistné uvedené v poistných zmluvách a očakávané hodnoty parametrov, ktoré ovplyvňujú výšku záväzkov vyplývajúcich z poistných zmlúv, predovšetkým úrokové miery, náklady a úmrtnosť. Smernica je navrhnutá tak, že zohľadňuje zásady medzinárodného účtovného štandardu IFRS 4 pre poistné zmluvy. Smernica reaguje na vývoj v aktuárskych predpokladoch a modeloch, ktorý spôsobuje, že účtovné technické rezervy v životnom poistení počítané napríklad podľa aktuárskych predpokladov prvého rádu (stanovených pri uzatvorení poistenia) alebo podľa inej metodiky, môžu byť nedostatočné. V takom prípade je potrebné rozhodnúť o prípadnom navýšení týchto technických rezerv tak, aby poisťovňa bola schopná plniť svoje záväzky plynúce z poistných zmlúv.

4. Modelové portfólio a test primeranosti rezerv

Uvažujeme modelové portfólio[7] skladajúce sa zo 100 poistných zmlúv doživotného modelového dôchodku vyplácaného z II. piliera charakterizovaného v časti 2. Použitím generátora pseudonáhodných čísel sme v programe R[8] vygenerovali 100 náhodných hodnôt výšky jednorazového poistného[9] v rozsahu od 7000 € do 9000 €. Zvyšné údaje vstupujúce do výpočtov sme uviedli v Tab. 1.

[5] Problematike testovania primeranosti rezerv sa venuje príspevok [4].

[6] Slovenská spoločnosť aktuárov (v skratke SSA) je profesijné združenie slovenských aktuárov. Dokument [3] i ďalšie podrobnosti o spoločnosti možno nájsť na stránke www.aktuar.sk.

[7] Hodnoteniu portfólia poistných zmlúv sa venujú Šoltés, Šoltésová (2013) v [6].

[8] Program R je open source program určený na štatistickú analýzu a grafiku.

[9] Ide o hodnotu π a teda nasporenú sumu na osobnom dôchodkom účte sporiteľa, ktorá prechádza do životnej poisťovne po uzavretí zmluvy ako jednorazové poistné.

Tabuľka 1: Predpoklady modelového dôchodku

Symbol:	*Charakteristika:*	*Hodnota:*
x	vek osoby pri odchode do dôchodku	62 rokov
i	technická úroková miera	1,5 %
α	začiatočné náklady (z π)	5 %
β	správne náklady (z ročnej výplaty dôchodku)	3 %
δ	náklady na výplatu mesačnej výšky doživotného dôchodku	1 €
γ	náklady na výplatu pri úmrtí	5 €

Pri výpočtoch sme použili zmiešané (unisex) úmrtnostné tabuľky za rok 2014 z databázy Human Mortality Database[10] pre Slovensko a pre veky 0 až 100 rokov.

4.1 Výpočet rezerv

Vzhľadom na to, že výpočet rezerv sme zvolili v 4. a v 10. roku, je potrebné zistiť pravdepodobnosť, že osoba vo veku 62 rokov sa nedožije 66 rokov (v prípade rezervy počítanej v 4. roku) a pravdepodobnosť toho, že 62 ročná osoba sa nedožije 72 rokov (pri rezerve počítanej v 10. roku). Dosadením príslušných hodnôt počtu dožitia osôb l_x z úmrtnostných tabuliek (do vzťahu uvedeného vo vysvetlivke vzorca (3)), dostávame pravdepodobnosti úmrtia rovné ${}_4q_{62} = 0{,}05960$ a ${}_{10}q_{62} = 0{,}17796$. Budeme teda predpokladať, že za štyri roky zomrie približne 6 ľudí, čím v danom portfóliu zostane 94 penzistov a za desať rokov zomrie približne 18 ľudí, a teda dôchodok bude naďalej poberať už iba 82 dôchodcov.

Keďže výšku rezerv budeme počítať pre celé portfólio spolu, zostavili sme tri scenáre:

- V prvom prípade zomrie 6 (18) ľudí, ktorí poberali najnižšie dôchodky, a teda aj výška ich jednorazového poistného bola najnižšia z spomedzi 100 penzistov.
- Druhý scenár zahŕňa smrť 6 (18) ľudí s priemernými mesačnými výškami dôchodkov a s priemernými sumami jednorazového poistného.
- Pri treťom scenári uvažujeme smrť 6 (18) penzistov s najvyššími hodnotami mesačných dôchodkov, a teda aj s najvyššími sumami jednorazového poistného.

Výšku rezerv sme pre jednotlivé zmluvy vypočítali na základe nasledujúcich vzťahov. Pre rezervu v 4. roku to je:

$$ {}^{B}{}_{4}V_{62}^{prosp}{}_{i} = 12Q_i \cdot a_{66}^{(12)} + SH(VP) + \beta \cdot 12Q_i \cdot a_{66}^{(12)} + \delta \cdot 12a_{66}^{(12)} + \gamma \cdot {}^{(12)}A^{1}_{66,\bar{3}|} \tag{8} $$

kde Q_i je výška mesačného modelového dôchodku určeného podľa vzťahu (6) pre $i = 1,..,100$ jednotlivých poistných zmlúv a $SH(VP)$ vyjadríme analógiou vzorca (3) v tvare:

$$ SH(VP) = \sum_{s=0}^{35} {}_{\frac{s}{12}}p_{66} \cdot {}_{\frac{1}{12}}q_{66+\frac{s}{12}} \cdot v^{\frac{s+1}{12}} \cdot (36 - s) \tag{9} $$

a ${}^{(12)}A^{1}_{66,\bar{3}|}$ analogicky ako v (5) na základe vzťahu:

$$ {}^{(12)}A^{1}_{66,\bar{3}|} = \sum_{s=0}^{35} {}_{\frac{s}{12}}p_{66} \cdot {}_{\frac{1}{12}}q_{66+\frac{s}{12}} \cdot v^{\frac{s+1}{12}} \tag{10} $$

Pri rezerve v 10. roku poistenia už nezarátavame garanciu a s ňou súvisiace náklady, preto vzorec pre rezervu v tomto roku má jednoduchší tvar:

$$ {}^{B}{}_{10}V_{62}^{prosp}{}_{i} = 12Q_i \cdot a_{72}^{(12)} + \beta \cdot 12Q_i \cdot a_{72}^{(12)} + \delta \cdot 12a_{72}^{(12)} \tag{11} $$

[10] Ide o rozsiahlu databázu, ktorá vznikla v roku 2000 a obsahuje detailné informácie o úmrtnosti a vývoji populácie pre 37 krajín sveta vrátane Slovenska. Je určená najmä pre študentov, výskumníkov, novinárov a analytikov a dáta poskytuje zadarmo.

4.2 Vplyv rizika dlhovekosti na výšku rezerv

Keďže je príspevok zameraný práve na riziko dlhovekosti, jeho vplyv na výšku rezerv zohľadníme testom primeranosti rezerv, a to tak, že pri výpočte rezerv aplikujeme rizikovú prirážku. Tú pri výpočtoch vyjadríme dvoma spôsobmi:

- znížime úmrtnosť v použitých úmrtnostných tabuľkách o 15 %[11],
- posunieme vek osoby o 2 roky (smerom nadol) v použitých úmrtnostných tabuľkách.

Výška rezerv celého portfólia počítaná v 4. roku poistenia podľa vzťahu (8) je po aplikácii týchto prirážok k úmrtnosti uvedená v Tab. 2.

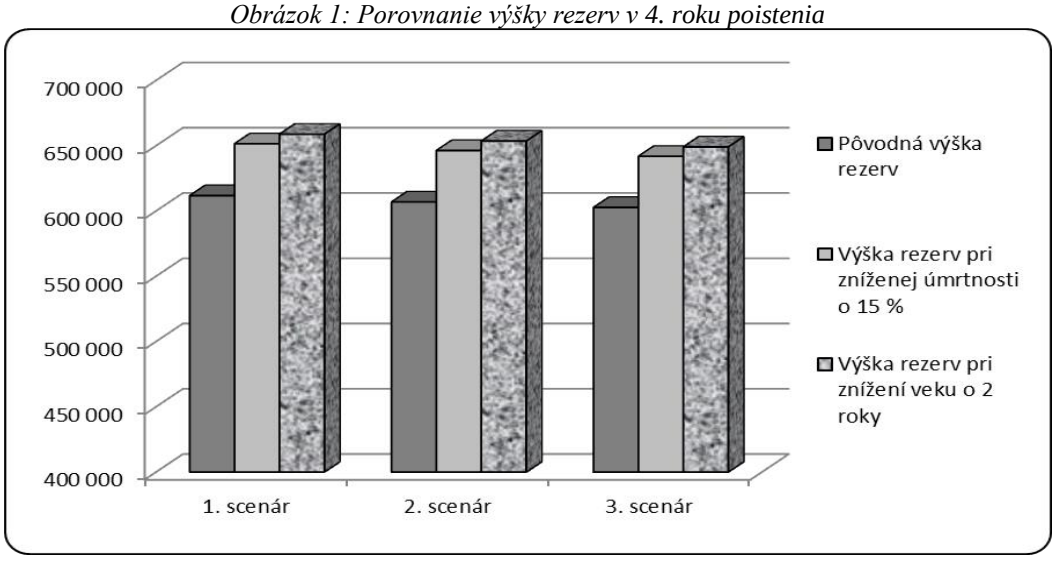
Tabuľka 2: Porovnanie výšky technických rezerv v 4. roku pred a po zohľadnení rizika dlhovekosti

Portfólio poisťovne	Pôvodná výška rezerv v 4. roku v €	Výška rezerv v 4. roku po aplikovaní rizikovej prirážky v €	
		Znížením úmrtnosti o 15 %	Znížením veku o 2 roky
1. scenár (bez najnižších hodnôt)	**611 715,14**	**651 475,02**	**658 782,20**
	rozdiel	-39 759,88	-47 067,06
2. scenár (bez priemerných hodnôt)	**606 895,53**	**646 342,02**	**653 591,59**
	rozdiel	-39 446,49	-46 696,06
3. scenár (bez najvyšších hodnôt)	**602 668,56**	**641 840,21**	**649 039,25**
	rozdiel	-39 171,65	-46 370,69

Rozdiel v rezervách v 4. roku pri zníženej úmrtnosti o 15 % oproti pôvodnej sume rezerv je 6,50 % a pri posune veku osoby o 2 roky ide o percentuálny rozdiel vo výške 7,69 % oproti pôvodnej sume. Pri oboch prirážkach je výsledná hodnota rezerv vyššia ako pôvodná, ktorá sa na základe toho javí ako nedostatočná.

Výšku rezerv celého portfólia v 4. roku poistenia z Tab. 2 znázorníme na Obr. 1. Zníženie úmrtnosti vyplývajúce z rizika dlhovekosti spôsobilo vyššie hodnoty požadovaných rezerv.

Obrázok 1: Porovnanie výšky rezerv v 4. roku poistenia



Pri rezerve portfólia poisťovne v 10. roku je test primeranosti rezerv zachytený v Tab. 3. Rezervy boli počítané na základe vzťahu (11).

[11] Minimálna hodnota prirážky v prípade úmrtnosti, ktorú odporúčala Smernica SSA č. 1 v2 je 10 %.

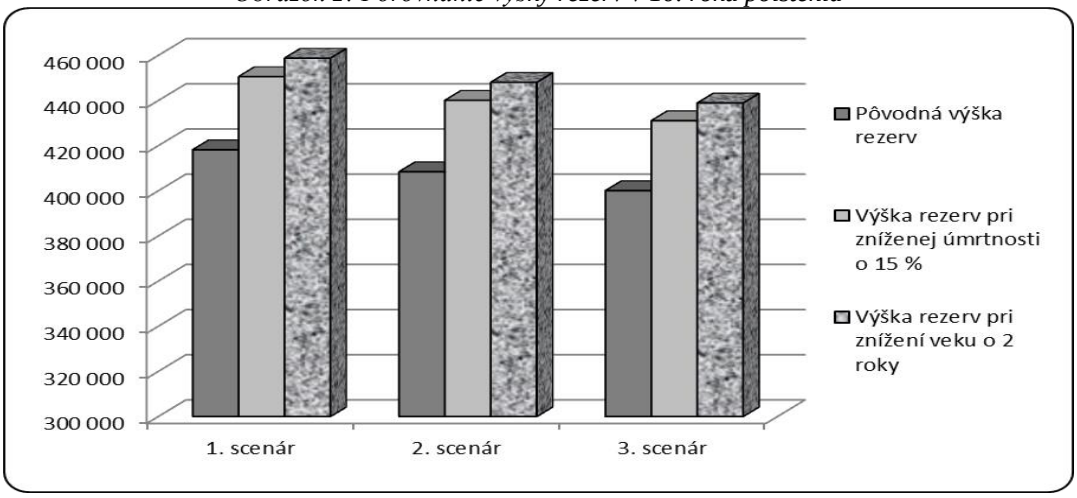
Tabuľka 3: Porovnanie výšky technických rezerv v 10. roku pred a po zohľadnení rizika dlhovekosti

Portfólio poisťovne	Pôvodná výška rezerv v 10. roku v €	Výška rezerv v 10. roku po aplikovaní rizikovej prirážky v €	
		Znížením úmrtnosti o 15 %	Znížením veku o 2 roky
1. scenár (bez najnižších hodnôt)	**418 176,19**	**450 566,43**	**458 733,41**
	rozdiel	-32 390,24	-40 557,22
2. scenár (bez priemerných hodnôt)	**408 430,91**	**440 066,28**	**448 042,95**
	rozdiel	-31 635,37	-39 612,04
3. scenár (bez najvyšších hodnôt)	**400 057,20**	**431 043,99**	**438 857,14**
	rozdiel	-30 986,79	-38 799,94

Rozdiel v rezervách v 10. roku pri zníženej úmrtnosti o 15 % oproti pôvodnej rezerve je 7,75 % a pri posunu veku osoby o 2 roky ide o percentuálny rozdiel vo výške 9,70 % oproti pôvodnej sume rezerv, a teda aj pri rezervách v 10. roku pri vplyve rizika dlhovekosti (v prípade, že mu je poisťovňa vystavená) sú pôvodné výšky rezerv nedostatočné.

Aj v prípade rezerv počítaných v 10. roku poistenia graficky znázorníme výsledky z Tab. 3, a to na Obr. 2. Rovnako aj pri takto počítanej rezerve spôsobuje zníženie úmrtnosti vyššiu hodnotu požadovaných rezerv.

Obrázok 2: Porovnanie výšky rezerv v 10. roku poistenia



Percentuálne rozdiely v rezervách vo 4. roku (6,50 % a 7,69 %) a v 10. roku poistenia (7,75 % a 9,70 %) sú spôsobené tým, že riziko dlhovekosti má opačný vplyv na výplatu garantovanej dávky pri úmrtí, ktoré boli v rezerve len vo 4. roku poistenia, ako pri dôchodkových dávkach.

Je samozrejmé, že ani príliš vysoká hodnota rezerv nie je pre poisťovňu výhodná. Pripravuje ju o možnosť tieto finančné prostriedky volne investovať. Preto je pre poisťovňu dôležité správne odhadnúť budúcu úmrtnosť a vývoj dlhovekosti a čo najpresnejšie odhadnúť výšku potrebných rezerv.

5. Záver

Prístup k testu primeranosti rezerv závisí aj od veľkosti poisťovne. Na Slovensku poisťovne, ktoré patria k najväčším na trhu, využívajú na test primeranosti rezerv rôzne aktuárske softvéry. V menších poisťovniach sa na test využíva najmä Excel. Tento prístup je

však časovo náročný, ak sa aplikuje na veľké portfóliá poistných zmlúv. Vtedy poisťovne rozsiahle portfólio rozčlenia do skupín na základe podobných parametrov. Pri menej rozsiahlych portfóliách je možné testovanie každej zmluvy, čo prináša najpresnejšie výsledky.

Dôchodky z II. piliera, ktoré sa začali vyplácať v roku 2015, sú aj v súčasnosti na Slovensku veľmi aktuálnou témou. Prioritou zákona [9] je, aby dôchodok z II. piliera bol vyplácaný vo forme doživotného dôchodku, s ktorým sa viaže riziko dlhovekosti. Ide o riziko, ktoré predpokladá, že úmrtnosť poistených osôb bude nižšia, ako je očakávaná pre populáciu z použitých úmrtnostných tabuliek pri stanovení dôchodkových dávok. Rozdiel medzi očakávanou výškou záväzkov pri uzatváraní zmluvy a výškou záväzkov po aktualizovaní predpokladov je možné vypočítať testom primeranosti rezerv. Pre účely príspevku sme v teste zmenili výšku úmrtnosti, ktorú sme upravili dvoma spôsobmi, znížením pravdepodobnosti úmrtia o 15%, resp. znížením veku o dva roky. Analyzovali sme tak vplyv rizika dlhovekosti na výšku rezerv za predpokladu, že ostatné vstupné údaje ostali nezmenené. Z výsledkov vyplýva, že po zohľadnení tohto rizika sú rezervy v obidvoch prístupoch nedostatočné vo 4. aj v 10. roku poistenia. Preto poisťovňa, aby bola pripravená kryť svoje záväzky vyplývajúce z poistných zmlúv, musí rezervy navýšiť, čím ovplyvní výšku svojho zisku.

Literatúra

[1] Bilíková, M. (2003). *Spojité metódy v poistnej matematike.* Bratislava: Vydavateľstvo EKONÓM, 2003. 96 s. ISBN 80-225-1698-8.

[2] Mihalechová, J. (2016). *Modelovanie rizika dlhovekosti v súvislosti s II. pilierom dôchodkového zabezpečenia*. Bratislava: FHI EU, 2016. Dizertačná práca.

[3] Odborná smernica SSA č.1 v3. (2016). *Test primeranosti technických rezerv v životnom poistení*. Dostupná na: http://www.aktuar.sk/sk/dokumenty/smernice-ssa/.

[4] Ondrejková Krčová, I. (2013). *Použitie metódy testovania zisku pri oceňovaní produktov a ohodnocovaní rezerv v životnom poistení*. In Aktuárska veda v teórii a v praxi: proceedings from the 9th international scientific conference. 7th November 2013, Bratislava, Slovakia. Bratislava: Vydavateľstvo EKONÓM, 2013. ISBN 978-80-225-3639-4. S. 87-97.

[5] Simonka, Zs. – Škrovánková, L. (2015). *Systém dôchodkového poistenia v SR*. 16. medzinárodná vedecká konferencia „AIESA – Budovanie spoločnosti založenej na vedomostiach“. FHI EU v Bratislave, Ekonóm, 5.-6.11.2015 Bratislava. ISBN 978-80-225-4151-0 S. 508-517.

[6] Šoltésová, T. – Šoltés, E. (2013). *Embedded value as the value reporting tool of the life insurance companies*. In The 7th professor Aleksander Zelias international conference on modeling and forecasting of socio-economic phenomena: proceedings, May 7-10, 2013, Zakopane, Poland. Cracow: Foundation of the Cracow University of Economics, 2013. ISSN 978-83-62511-57-0. S. 165-172.

[7] Špirková, J. – Urbaníková, M. (2012). *Aktuárska matematika – životné poistenie*. Bratislava: Iura Edition, 2012. 241 s. ISBN 978-80-8078-514-7.

[8] Zákon č. 43/2004 Z. z. o starobnom dôchodkovom sporení.

[9] Zákon č. 183/2014 Z. z., ktorým sa mení a dopĺňa zákon č. 43/2004 Z. z. o starobnom dôchodkovom sporení.

[10] Zákon č. 39/2015 Z. z. o poisťovníctve a o zmene a doplnení niektorých zákonov.

Conflicts of interest and their consequences for financial management and risk in a company

Joanna Błach[1]

Abstract

The stakeholder theory explains the importance of various groups of stakeholders in a corporate financial management. Different needs and objectives of stakeholders may lead to the conflicts of interest, which may have negative consequences for the company, so they should be identified, analyzed and reduced or eliminated. This can be achieved by the effective process of conflicts of interest management being part of the stakeholder management.

The main objective of this paper is to identify the most important types of conflicts of interest in a company and to analyze their consequences for the company's financial management and financial risk. This conceptual paper based on the literature review presents the theoretical model illustrating the relationship between conflicts of interest and financial management with related financial risk. A proposal of conflicts of interest management process is also described.

Key words

stakeholder theory, agency theory, asymmetric information, moral hazard, corporate governance

JEL Classification: G31, G34, G14,

1. Introduction

Activity of a modern company – in terms of financial management - is usually explained based on one of two basic theories – the shareholder theory and the stakeholder theory. The assumptions of the shareholder theory place owners (equity holders) and their interest in the centre of the decision making process of the company with the main objective defined as the shareholder wealth (value) maximisation.

As the Corporate Social Responsibility and the sustainable development concepts become more and more popular, the second theory – based on the stakeholder approach is often applied to explain the conditions and objectives of the corporate financial management. Nowadays the efficient financial management requires taking into consideration the objectives and needs of all primary stakeholders of the company, such as: shareholders, creditors, managers, employees, customers, local community, etc. However, as these stakeholders represent various groups of interest their conflicting objectives and needs may hamper the smooth development of the company.

The main objective of this paper is to identify the most important types of conflicts of interest in the company and to analyze their consequences for the company's financial situation, its performance with regard to the realized strategy and the level of risk. This conceptual paper based on the literature review presents the theoretical model illustrating the relationship between conflicts of interest and financial management with related financial risk.

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This paper is organized as follows. Section 2 presents the basic concepts of the stakeholder theory and analyzes the objectives of various groups of interest. Section 3 identifies the main types of conflicts of interest in the company while section 3 analyzes the most important consequences of these conflicts for the company's financial management and financial risk. It also presents the model illustrating relationships between conflicts of interest and financial management with related financial risk. A proposal of a conflicts of interest management process is also described.

2. Stakeholder theory of a company

The stakeholder theory was firstly formalised by Freeman in 1984 [1] and developed by Cornell and Shapiro in 1987[2]. According to this theory, a company is described as a nexus of contracts (formal and informal ones) or a network of relationships with different groups of stakeholders. These networks are becoming easier to use, richer in content, more interactive and instantaneous in operations due to the rapid development of technology that reduces costs of building and operating complex stakeholder networks [3].

Stakeholder is defined as any group or individual who is affected by or can affect the achievement of the company's objectives [2]. These stakeholders bear certain risk as a result of their involvement in the company – in a form of financial capital, human capital or other resources. Without their participation the company can not survive. Primary stakeholders include: capital suppliers (shareholders and creditors), employees, customers, other resource suppliers, local community, the natural environment, the government and its agencies.

Nowadays the success of the company depends not only on its owners and customers but also on achieving permanent balance the satisfies all the stakeholders as a result of the sound integrated stakeholder management process. As it was already stressed, stakeholders consist of various group of interest with different objectives and expectations (see table 1). Despite these differences, all stakeholders should be considered by the company. Shareholders will continue investing in a company only if it satisfies their expectations in respect of dividends or capital gains. Also creditors will provide debt financing only if a company is able to repay it on time with the required interest payments. Employees will develop their activities and knowledge if the company provides them satisfaction in work. Customers will buy products and services that respond to their needs, price, quality and service demands. Suppliers will continue providing their knowledge, abilities and resources to the company only if it facilitates them the opportunity to obtain reasonable profit. Communities will tolerate company only if it satisfies its legal duties and guarantees a sustainable development.

Table 1: Main stakeholders - their objectives and expectations

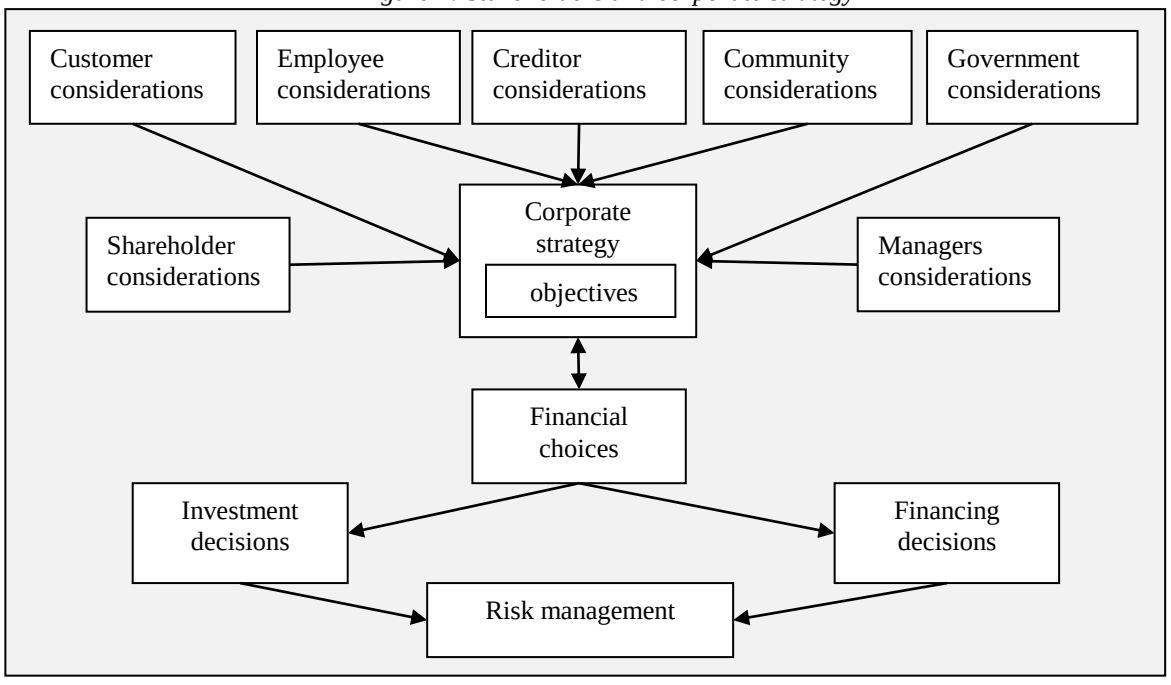
Group of stakeholders	Objective	Expectations
Shareholders (equity holders)	Attractive investment	Increase of share prices, high dividend
Management	Individual professional success	High salary, professional and social prestige, extra benefits
Creditors (debt holders)	Attractive investment	Creditworthiness, debt repayment with interest
Employees	Individual professional success	High salary, personal satisfaction, good working condition, job security and career opportunities
Customers	Good quality and safety	High quality, reasonable prices, professional service and advice

Government	Survival and development of a company	Tax revenues, employment opportunities, competitiveness and economic growth
Local community and society	Survival and development of a company	Employment opportunities, local tax revenues, environment protection, social welfare

Source: own elaboration based on [4].

Also, the inter-stakeholders relationship management become more and more important. It is necessary to maintain a balance between the contributions and benefits in respect of each group of stakeholders. In case of voluntary stakeholders (such as: shareholders, creditors, employees, customers, suppliers) the basic principle of stakeholder management is mutual benefit. These stakeholders contribute directly to the company's operations and expect to receive benefits as a result. In case of involuntary stakeholders (such as: local community, society) the main principle has to be the reduction or avoidance of harm and/or the creation of offsetting benefits. Their expectation is to protect their lives from the negative consequences of the company's activity. Within stakeholder management process a company must identify its stakeholders, determine their benefits-contributions profile and find its own balance in respect of each group. The creation of value and the development of a company is possible when a satisfactory relationship of benefits-contributions is achieved and maintained. As a consequence, choices and decisions within integrated financial management of a company are defined as a function of balancing stakeholders' influences and expectations (see figure 1).

Figure 1: Stakeholders and corporate strategy



Empirical studies proved that efficient and effective stakeholder relationship management leads to improved shareholder value [5], generates long-term competitive advantage for the company and for society as well [6].

3. Conflicts of interest in a company

The effective stakeholder relationship management may be hindered by the conflicting expectations and objectives of various groups of stakeholders. The main reasons for conflicts of interest in the company may be listed as follows [7]:

- different access to information and asymmetric information among stakeholders (as one of the financial market imperfections);
- separation of ownership and control function (as result of hiring professional managers to run the business on behalf of the owners);
- different risk attitude (risk aversion) of stakeholders;
- behavioural mechanisms impeding rational choices of stakeholders;
- decisions and activity of primary and secondary stakeholders (e.g. block-holders/strategic investors control not only their own capital but also the investment of minority shareholders);

There are distinguished four main areas of conflicts in a modern company and as a result four types of conflicts between:

1) managers and shareholders,
2) company and capital providers,
3) controlling shareholders and minority shareholders,
4) shareholders and creditors.

The first type of conflict of interest – between **managers (as agents) and shareholders (as principals)** lies in the centre of the agency theory formulated by Jensen and Mecklin in 1976, which explains the relationships between managers and shareholders under the separation theorem [8]. The managers should act in the best interest of the shareholders, however they may use their information advantage (problem of asymmetric information between managers as insiders and shareholders as outsiders) to make decisions that help them to achieve their own particular interest, which may have negative consequences for the company and its shareholders. This leads to the agency conflict hindering the achievement of value-maximization objective due to the increase of agency costs, including monitoring costs and costs of incentive programmes.

Possible conflicts of interest between managers and shareholders may include [9]:

- payment of excessive salaries or benefits for managers and the problem of "empire building",
- reluctance to undertake high-risk, high NPV projects,
- retaining profits rather than paying them out in dividends,
- working less efficiently than the shareholders might desire.

As a consequence, the conflicts of interest between managers and shareholders may be analyzed in four key areas:

- moral hazard,
- earnings retention,
- time horizon,
- risk aversion.

The first area is related to the moral hazard problem. Jensen and Meckling in 1976 first proposed a moral hazard explanation for agency conflicts [8]. They developed a model whereby the incentive of the manager to undertake private perquisite consumption (e.g. benefits, privilege, extra payment, exclusive business trips) rather than invest in positive net present value projects increases as the manager's ownership stake in the company declines. Empirical research proved that managers may choose to invest in assets best suited to their personal skills or preferences (investing in visible or fun industry for extra personal non-

financial benefits e.g. meeting with celebrities). Such investment increase their individual value to the company and make manager replacement more difficult. Moral hazard problems are also related to a lack of managerial effort and to the size of the company. Decreasing managerial effort may damage corporate value. Larger companies are difficult to be effectively monitored which increases the monitoring costs.

The second area of conflict of interest between managers and shareholders is related to the earnings retention decisions. Jensen in 1986 found out that managers prefer to retain earnings whereas shareholders prefer higher levels of cash distributions especially when the company has few positive NPV investment projects [10]. Managers benefit from retained earnings because company's size growth gives them a larger power base, greater prestige, ability to dominate the board of directors and allows them to extract higher levels of remuneration. This reduces also the amount of firm-specific risk within the company and strengthens their job security. It should be stressed that earnings retention reduces the need for external financing for new investment projects. At the same time, it reduces the monitoring function of external financing that should encourage management to undertake value-maximizing decisions.

Conflicts of interest may also arise between shareholders and managers with regard to the timing of cash flows. Shareholders are concerned with all future cash flows of the company as these are reflected in the current share price. While, managers may only be concerned with company cash flows for their term of employment. Thus, they may prefer short-term projects with high accounting rates of return, ignoring their long-term potentially negative consequences for the company's value.

The next area of conflicts is related to the managerial risk aversion which arises because of portfolio diversification constraints. Shareholders are concerned only with systematic risk while company managers are concerned with both systematic and firm-specific risk. As a consequence managers may decide to apply investment and financing policies that minimize the risk of a company. If risky investment projects are successful, the shareholders make the gain, if unsuccessful – the shareholders will lose only a small part of their diversified portfolios but the managers may lose their jobs. Similarly, although higher debt is expected to reduce agency conflicts and gives the potential of tax shield and positive financial leverage effect, risk-adverse managers prefer equity financing due to the potential bankruptcy risk related to debt financing.

The second type of conflict of interest is observable between a **company and capital providers** (potential and existing ones), both equity holders and debt holders. This type of conflict is related to the asymmetric information problem described by Leland and Pyle in 1977 [11]. Asymmetric information may occur in one of two following forms [4]:

- hidden information and hidden knowledge possessed by a company, that can not be accessed by the capital providers and may result in a adverse selection problem (it occurs before the transaction is made and capital is invested in the company);
- hidden action of a company, that can not be assessed and monitored by the capital providers and may result in a moral hazard problem (it occurs after the transaction is made and capital is invested in the company).

Under such circumstances the company may decide to increase equity capital by issuing shares to transfer the potential loss to the new shareholders (when the prospects are bad) and to increase debt by taking a bank loan or issuing bonds when the prospects are good. Companies characterize by higher level of risk are more willing to accept higher required rate of return expected by the capital providers. Thus, the asymmetric information between company and capital providers may result in the limited access to capital (capital gap problem), higher cost of capital, lower market prices of shares and limited development of a company.

The third type of conflict of interest occurs between **controlling shareholders and minority shareholders**. This conflict is also related to the moral hazard problem. By controlling the company through majority voting rights, large shareholders (strategic investors, institutional investors, family members) can undertake action that expropriate wealth from non-controlling (minority) shareholders. This may also lead to the suboptimal investment decisions to lower the cash flow risk. In case of family controlled companies there may be also a problem of:

- suboptimal capital structure - applying lower level of debt and tax shield effect due to a fear of bankruptcy;
- related-party transactions - making contracts with related-parties (e.g. companies controlled by family members) at prices below market value;
- costs of succession - founders may continue to be involved in running the family business even when it is not in the best interests of the company and its shareholders.

The negative consequences of such problems create agency costs for non controlling (non-family) shareholders because of the loss of corporate value. Minority shareholders dissatisfied with company's performance may decide to sell the shares, which may decrease share prices and increase the threat of a hostile takeover.

The last type of conflict of interest may be observed between **shareholders and creditors** (bondholders) and is also related to the moral hazard problem. Shareholders prefer value-maximizing decisions, while creditors are concern about the safety of their capital involved in a company. As their income on invested capital is fixed, they will not participate in the increasing value of the company. Thus, the decisions that may be assessed as negative from the creditors point of view include [12]:

- undertaking risky investment projects increasing the overall level of company's risk,
- increasing level of debt financing (related to the claim dilution problem),
- increasing level of dividend payments (related to the claim dilution problem),
- disposal and substitution of assets (replacing old assets with the new ones of different risk-return profile).

These actions are usually taken after the transaction with creditors was made, which means that creditors can not increase interest rate on debt to cover the increased risk. However, they may decided to cancel the agreement and demand from the company to repay the debt before maturity. They may be reluctant to provide company with additional debt financing in future (adverse selection problem) which may lead to suboptimal capital structure and underinvestment problems. If they decide to provide capital, they may insist on implementing debt covenants which increase creditors safety but at the same time they decrease the independence of a company. As a result, this may have negative consequences for the company and its shareholders in a long-term horizon.

4. Consequences of conflicts of interest

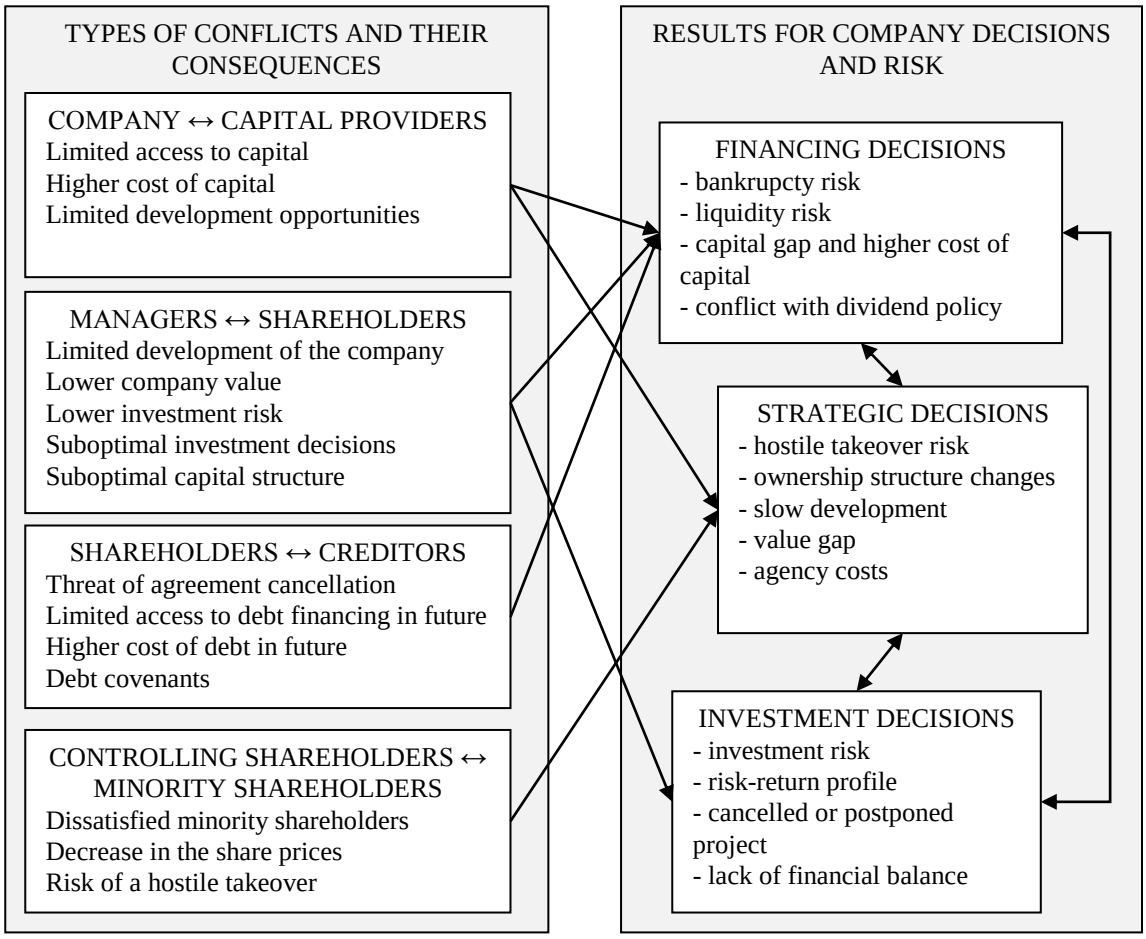
Consequences of conflicts of interest are observable in different areas of financial management and as result in the level of financial risk. The most important consequences include [13]:

- ineffective risk-sharing between capital providers and other stakeholders that influences the expected rate of return and cost of capital for a company;
- suboptimal capital structure together with the application of financial instruments inadequate to the actual market conditions, company's financial situation and its ability to meet the expectations of capital providers (existing and potential ones),

- suboptimal investment decisions impeding the development of a company and the realization of a company's strategy and objectives,
- conflicting free cash flow management decisions with regard to earnings retention and dividend policy,
- udervaluation or overvalution of shares (market value of a company) in comparison to the economic value of a company (value gap).

Figure 2 presents in synthetic way the relationships between four aforementioned types of conflicts of interest and the main areas of financial management with related types of financial risk. As financing and investment decision influence the realization of the company's strategy, the third area of consequences was added, uderlining the strategic dimension of the conflicts of interest.

Figure 2: Conflicts of interest and their consequences – a model approach



Thus, it is important to identify types of conflicts of interest in the company together with their potential consequences for the company's performance, its financial management and the level of financial risk.

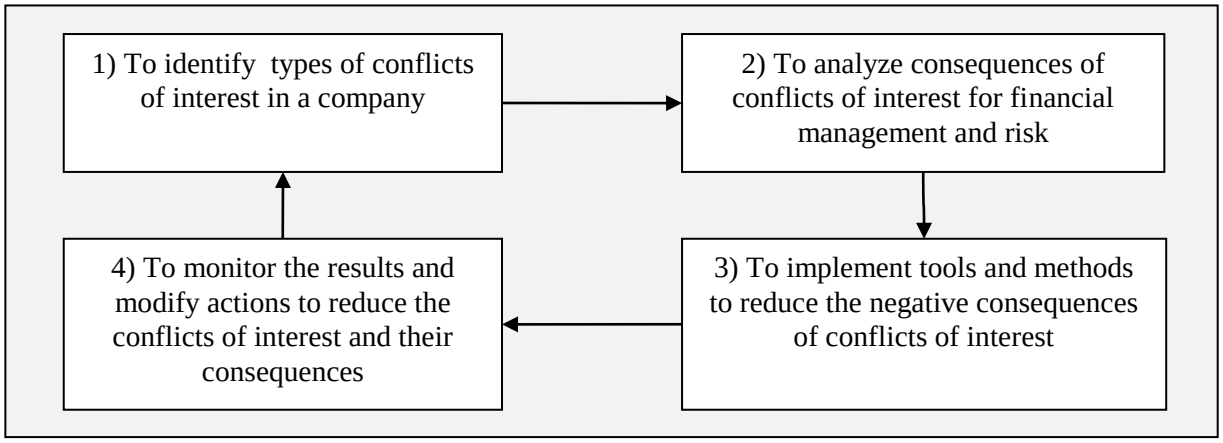
Based on the identification of these types of conflicts of interest, appropriate tools and methods aiming at reducing their negative consequences can be selected and applied. The most important ways of reducing conflicts of interest and their consequences consist of [14], [15]:

- sound corporate governance mechanisms based on internal solutions including independent supervisory board with audit, risk management and remuneration committees;
- transparent financial reporting together with effective investor relations programme;

- external control of independent auditors guarantying quality of disclosures and financial statements of a company;
- equity-based earnings systems with stock option incentive programmes for managers;
- institutional and strategic investors controlling and monitoring the activity of management;
- law regulations protecting rights of different stakeholders e.g. minority shareholders, creditors, employees;
- capital market regulations and actions taken by market supervisory agencies promoting codes of best practices;
- education programmes increasing the awarness of corporate governance problems within society;

These methods and tools (voluntary and obligatory ones) may be applied both at the micro-level (by a company for its own purposes) and the macro-level (for entire financial market or economy by international ogranisations or government agencies and market regulators). The process of conflicts of interest management is continuous as it should be verified and modified on a regular basis. Four stages of this process – discussed above – are presented in figure 3.

Figure 3: Conflicts of interest management as 4-stages process



Conclusion

In a modern company efficient financial management integrated with financial risk management requires taking into consideration the conflicting interest of various groups of stakeholders. To reduce the negative consequences of these conflicts a company should implement a stakeholder management programme with regard to the conflicts of interest. The first step of this programme is to identify the types of conflicts of interest in a company. The most important types are conflicts between: managers and shareholders, company and capital providers, controlling shareholders and minority shareholders, shareholders and creditors. These conflicts are connected with the asymmteric information (together with moral hazard and adverse selection problems) and the agency problem. The second step is to analyze the potential consequences of these conflicts for the financial situation of the company with regard to the financing and investment decisions and related financial risk. The strategic consequences of such conflicts should be also analyzed. The next step is to implement the suitable tools and methods aiming at reduction of conflicts of interest and their consequences. A company may select tools and methods adequate to its needs and current market situation, using: internal and external corporate governance mechanisms, financial law and accounting

regulations, investor relations programmes, disclosures and financial statments, etc. The process of conflicts of interest management is continuous – it should be regulary monitored, verified and modified.

References

[1] Freeman R.E. (1984). *Strategic Management: A Stakeholder Approach*. Boston: Pitman Books Limited.

[2] Cornell, B. and Shapiro, A.C. (1987). Corporate Stakeholders and Corporate Finance. *Financial Management*, Spring, p. 5-14.

[3] McVea, J.F. and Freeman, R.E. (2005). A Names-and-Faces Approach to Stakeholder Management. *Journal of Management Inquiry*, March, p. 57-69.

[4] Bogacz-Miętka, O. (2011). *Kompendium wiedzy o nadzorze i kontroli nad przedsiębiorstwem.* Warszawa: CeDeWu.

[5] Hillman, A.J. and Keim, G.D. (2001). Shareholder Value, Stakeholder Management, and Social Issues: What's the Bottom Line? *Strategic Management Journal*, 22, p. 125-139.

[6] Lorca, P. and Garcia-Diez, J. (2004). The Relation between Firm Survival and the Achievement of Balance Among its Stakeholders: an Analysis. *International Journal of Management*, 21(1), March, p. 93-99.

[7] Łukasik, G. (2006). Konflikty interesów uczestników rynku kapitałowego i ich wpływ na wybory przedsiębiorstwa. [in:] Mamcarz, H. (Ed.) *Rynki finansowe*. Lublin: UMCS.

[8] Jensen, M. and Meckling, W. (1976). The Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure. *Journal of Financial Economics*, 3, p. 305-360.

[9] Bender, R. and Ward, K. (2009). *Corporate Financial Strategy*. London: Routledge.

[10] Jensen, M.C. (1986). Agency cost of free cash flow, corporate finance, and takeovers. *American Economic Review*, 76(2), p. 323-329.

[11] Leland, H. and Pyle, D. (1977). Informational asymmetries, financial structure and financial intermediation. *Journal of Finance*, 32(2), p. 371-387.

[12] Stradomski, M. (2004). *Zarządzanie strukturą zadłużenia przedsiębiorstwa*. Warszawa: PWE.

[13] Łukasik, G. (ed.), (2007). *Przedsiębiorstwo na rynku kapitałowym*. Katowice: Prace Naukowe Akademii Ekonomicznej im. K. Adamieckiego w Katowicach.

[14] Megginson, W.L., and Smart, S.B. (2011). *Introduction to Corporate Finance*. Chichester: Wiley.

[15] Lumby, S. and Jones, Ch. (2011). *Corporate Finance, Theory and Practice*. Andover: Cengage Learning, South-Western.

Agency theory and the audit engagement[1]

Michal Bobek [2]

Abstract

Audit has been important and integral part of market economy because it enables to mitigate costs of capital, transactions etc. The article presents agency theory applied to the area of audit which has not been much analyzed yet. Audit is explained as the result of agency relationships within the company. Audit works as a tool to mitigate the agency costs within a company. The article brings short description of the agency relationships within the company and with a person of auditor. There is presented the concept of Internal-agency-relationship-not-affected auditor, a theoretical model of auditor who does not succumb to the internal relationship within the auditee. Following to this concept, the internal-agency-relationship-affected auditor is described as well. The results of both concepts are described mathematically as well.

Key words

Agency theory, audit, By-internal-agency-relationship-not-affected auditor

JEL Classification: M42

1. Úvod

Důležitost ověřovacích zakázek je neoddiskutovatelná, jelikož svým ujištěním snižují riziko, zvyšují kvalitu informací, snižují náklady na kapitál a mají mnoho dalších výhod. Ověřovací zakázky však, stejně jako úplně všechny lidské činnosti, podléhají ovlivnění ze strany vztahů zastoupení, což způsobuje, že ani ověřovací zakázka není schopná podat zcela nezaujatý pohled na věc. Tento problém je z pohledu teorie auditu, resp. z pohledu teorie ověřování chápán především jako problém nezávislosti ověřovatele. Z finančního pohledu však vidíme problém pohledem vztahů zastoupení. Vztahy zastoupení v ověřovacích službách, resp. v jejich užším pojetí při provádění auditu, jsou v tomto příspěvku analyzovány.

1.1 Motivace pro výzkum

Motivací pro výzkum v oblasti vztahů zastoupení v ověřovacích zakázkách vychází z toho, že audit[3], je jedním ze způsobů, kterým se snižují náklady zastoupení, které vznikají při vztazích zastoupení (Jensen, Meckling, 1976, s. 308; Alchian, Demsetz, 1972, s. 782). Tzn. auditor je klíčovou osobou, která pomáhá k tomu, aby nedocházelo k nárůstu nákladů zastoupení, které mezi jednotlivými subjekty existují z důvodu vztahů zastoupení. Ovšem jak poznamenávají Jensen a Meckling (1976, s. 309), Ross (1973, s. 134) a mnoho dalších, kteří

[1] The paper is processed as an output of the research project „Theoretical Approaches for the Optimalization of Capital Structure“ registered by the University of Economics, Prague under the evidence registration number F1/48/2016 and under support of a research project of the Faculty of Finance and Accounting of University of Economics in Prague, No. IP100040.
[2] Ing. Michal Bobek, MBA, Ph.D. candidate, University of Economics, Prague, Faculty of Finance nad Accounting, Department of Corporate Finance and Valuation. michael.bobek@gmail.com
[3] V rámci doprovodného výzkumu (Bobek, 2015) bylo rozlišeno mezi auditem, atestovými službami a ověřovacími službami. Zde se zabýváme pouze auditem, proto se nadále bude hovořit jen o auditu.

se oblastí vztahů zastoupení zabývají, žádná strana není od vztahů zastoupení osvobozena a vztahy zastoupení vznikají mezi všemi smluvními subjekty. Z těchto výroků jasně plyne, že vztahy zastoupení vznikají taktéž mezi auditorem, vlastníkem, manažerem a ostatními stakeholdery (přesná definice jednotlivých osob je uvedena v Bobek, 2014). Otázkou je, jakým způsobem se ovšem mohou tyto vztahy dotknout výsledku, tedy auditu, který má zajistit, aby mezi vlastníkem a manažerem nedocházelo k vysokým nákladům zastoupení. Na výsledek ověření se spoléhá nejen vlastník, který je takto ujišťován, že ve společnosti nedochází k žádným problémům, tak i ostatní stakeholdeři, kteří získávají ujištění, že společnost je v pořádku a mohou s ní obchodovat, proto velikost nedostatků v ověření jim může způsobit ztráty. Motivací našeho výzkumu je definovat vztahy zastoupení, které mezi jednotlivými subjekty vznikají a určit, jak na ně reaguje auditor. Z tohoto pohledu je definován teoretický koncept – tzv. názorově neovlivněný auditor, jehož koncept vylučuje vliv interních vztahů zastoupení na tuto osobu. Následně je definován názorově ovlivněný auditor, v praxi běžný stav, který je ovlivněn vztahy zastoupení mezi jednotlivými subjekty uvnitř společnosti.

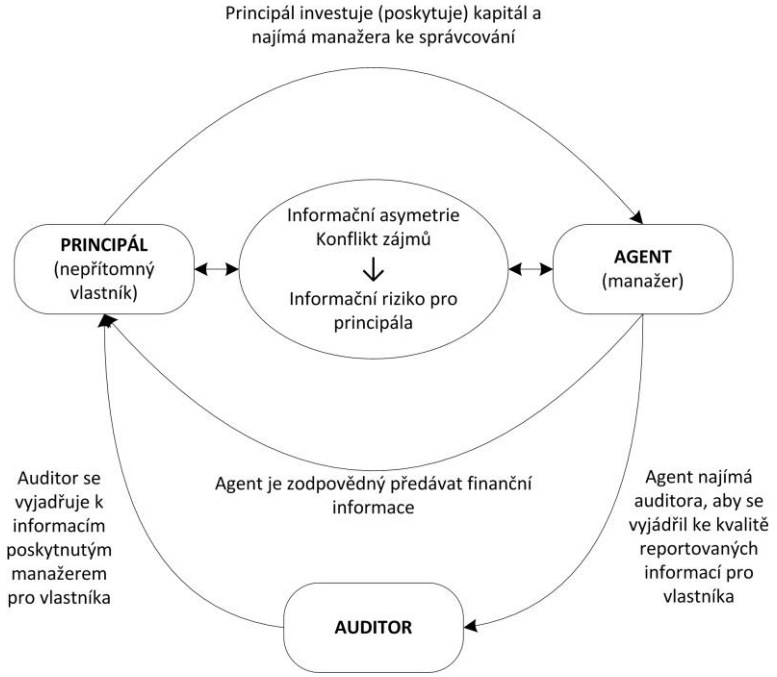
2. Podstata auditu

Audit je možné podle American Accounting Association (AAA, 1973) definovat jako „systematický proces nezaujatého získání důkazů a jejich ohodnocení, které se týkají tvrzení o ekonomických činnostech a událostech za účelem zjištění úrovně shody mezi těmito tvrzeními a stanovenými kritérii a sdělení těchto výsledků zainteresovaným uživatelům.“[4] Audit je také důležitým monitorovacím nástrojem v rámci vztahů manažera a vlastníka a je tedy významnou součástí vědeckých prací týkajících se vztahů zastoupení. Audit, resp. veškeré ověřování ve své podstatě vzniklo a existuje právě díky existenci vztahů zastoupení. Jak uvádí Messier et al. (2008) vztah mezi investory, kteří byli daleko od místa své investice, a správci majetku přímo v místě provozování činnosti dal vzniknout kontrole účetnictví a tím i auditu. A je potřeba si uvědomit, že i samotné finanční účetnictví, stejně jako audit, je ve vztahu manažera a vlastníka důležité a tvoří monitorovací kontrolu. Právě pojem principál a agent i sám Messier et al. (2008, s.6) používají při definování důvodu, proč audit existuje. Messier et al. (2008, s.7) vyjadřují důvod pro existenci auditu a vztahy mezi jednotlivými subjekty na obrázku č. 1.

Mezi manažerem a vlastníkem existuje informační asymetrie, která je zmenšená vlivem auditu. Manažer připravuje účetní výkazy, či případně jiné reporty, které shrnují aktuální stav a předkládá je vlastníkům. Jelikož však vlastník nemá možnost situaci sám kontrolovat, ať již kvůli geografické vzdálenosti, nebo kvůli nedostatku vlastních znalostí, najímá si auditora, aby provedl ověření informací, které mu byly představeny manažerem. Auditor provede postupy, které potřebuje k tomu, aby mohl vyjádřit své tvrzení k informacím, které jsou obsaženy v reportech, jež mu byly předloženy managementem. Díky tomuto ověření se kvalita obsažených informací zvýší, jelikož vlastník se kromě tvrzení manažera může spolehnout také na ujištění, která mu byla předložena auditorem. Vzhledem k předchozímu výzkumu (Bobek, 2014) došlo k úpravě obrázku, původně prezentovaného jako obrázek č. 1 do podoby uvedené na obrázku č. 2. Na tomto obrázku také došlo k znázornění vztahů zastoupení formou čárkované spojnice s popiskem konflikt zájmu.

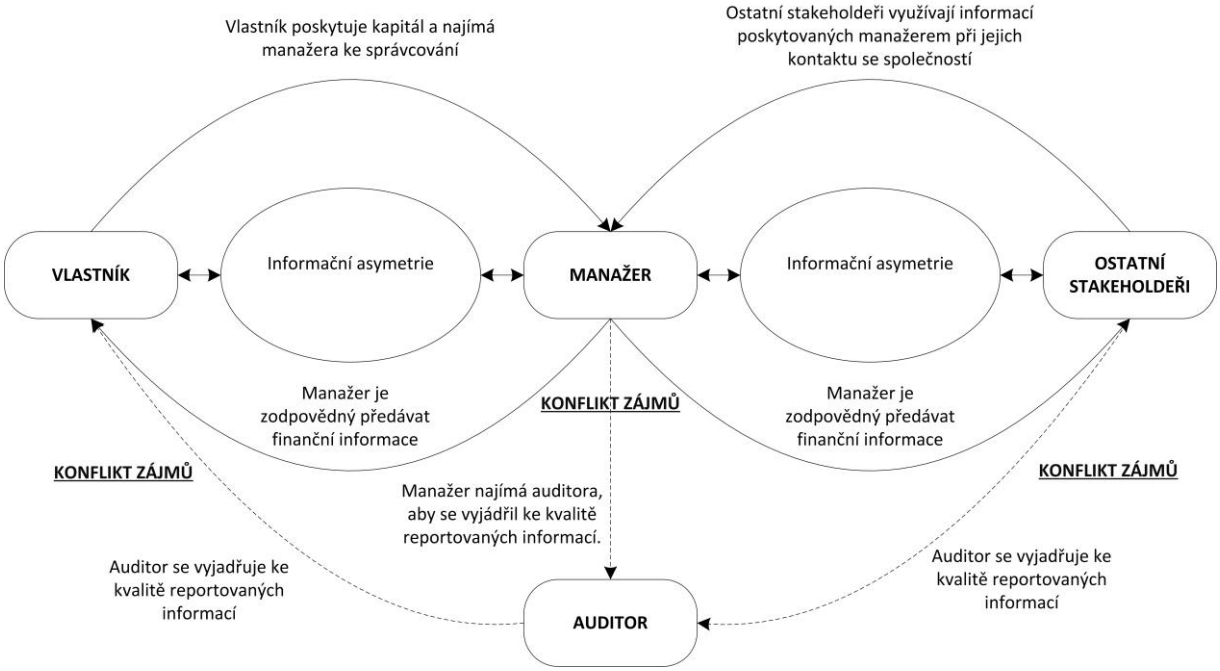
[4] V anglickém originále jako: „Auditing is a systematic process of objectively obtaining and evaluating evidence regarding assertions about economic actions and events to ascertain the degree of correspondence between those assertions and establish criteria and communicating the results to interested users.“ (Messier at al., 2008, s. 11).

Figure 1: Důvod pro vznik a existenci auditu



Vyjdeme-li z teorie zastoupení, tak důvodem, proč dochází k rozdílným preferencím agenta a principála je v tom, že obě strany vztahu, tj. jak principál, tak agent, mají (1) jiné cíle, (2) jiný přístup k informacím a (3) jiný vztah k riziku. (Podrug a kol., 2010). Je zřejmé, že tyto případy u vztahu u provádění auditu nastávají a částečně již byly popsány v předchozím výzkumu (Bobek, 2014).

Figure 2: Koncept auditu s vyjádřením vztahů zastoupení



3. Základní vztahy mezi subjekty

V rámci schématu byly identifikovány vztahy zastoupení, které mezi jednotlivými subjekty vznikají. Mezi vlastníkem a manažerem vzniká konflikt zájmu, tímto vztahem se zabývá

většina literatury na oblast vztahů zastoupení (Ross, 1973, Jensen, Meckling, 1976, Fama 1980.). Vztah mezi manažerem a ostatními stakeholdery je spojen se snahou ostatních stakeholderů získat objektivní informace o společnosti. Manažer ovšem má tendenci ostatním stakeholderům předkládat ty informace, které se mu nejvíce líbí a které se mu nejvíce hodí pro jeho strategii. Vztah mezi ostatními stakeholdery a vlastníkem je závislý na tom, do jaké míry je vlastník zaangažován na řízení společnosti. Pokud vlastník na řízení společnosti zaangažován není, pak je z hlediska přístupu k informacím o společnosti v podobné pozici jako ostatní stakeholdeři. Pokud na řízení zaangažován je, pak je ve vůči externím stakeholderům ve stejné pozici, jako manažer. Z pohledu tohoto příspěvku jsou důležitější vztahy, které vznikají v důsledku auditu.

Od auditora se očekává, že odvede ve všech aspektech bezchybnou práci, která povede k identifikaci nedostatků v účetní závěrce a následně k provedení nápravy ze strany jednotlivých subjektů. Tato očekávání se nemusí naplnit, jednak proto, že auditor není schopen odhalit veškeré nedostatky (což odpovídá způsobu práce s hranicí významnosti) a také proto, že na auditora může být přenesen konflikt zájmu, který existuje mezi jednotlivými osobami (vlastník – manažer – ostatní stakeholdeři). Jelikož auditor přichází z venku, předpokládáme, že je při vstupu do zakázky názorově neutrální, tedy není ovlivněn žádným vztahem zastoupení, který existuje mezi výše zmíněnými subjekty.

Auditor je ovšem nucen pro splnění zakázky spolupracovat s jednotlivými subjekty, které jsou na činnosti společnosti zaangažovány. Z toho důvodů se názorově neutrální auditor musí vyrovnat se vztahem mezi manažerem a vlastníkem, ve kterém existuje informační asymetrie a ve vztahu mezi společností (tj. manažerem a vlastníkem operujícími ve shodě[5]) a ostatními stakeholdery. Auditor, ačkoliv je na začátku názorově neutrální může být ovlivněn spoluprací a interními tlaky na to, aby svou neutralitu opustil a tím pádem byl ovlivněn interními vztahy mezi jednotlivými subjekty.

Pojem názorově neutrální auditor je možné definovat následujícím způsobem:

> *Názorově neutrální auditor je auditor, který není ovlivněn vztahy zastoupeními, které vznikají v rámci subjektu, jež audituje, ale je ovlivněn pouze vlastními preferencemi a vlastními zájmy, které také generují vztahy zastoupení.*

Názorově neutrální auditor nemá žádné jiné preference, nežli maximalizovat vlastní užitek, který plyne z bezproblémově provedené zakázky, která auditora nevyčerpá na zdrojích, tj. nezpůsobí mu zbytečně vysoké náklady, ale přinese mu co nejvyšší užitek. Pod pojmem užitek se chápe nejen krátkodobý finanční přínos, ale současná hodnota všech budoucích příjmů včetně hodnoty nefinančních výhod, jako je prestiž, dlouhodobý vztah s auditovanou společností, ocenění v oboru. Auditor se v takovém případě snaží maximalizovat svůj vlastní čistý užitek, který je možné definovat jako součet výnosu auditora ze zakázky v podobě honoráře a případných dalších fakturací a současné hodnoty všech budoucích výnosů (ať již ve finanční nebo nefinanční oblasti), které budou plynout ze současného jednání auditora, snížené o náklady auditora na konkrétní zakázce v aktuálním roce.

Názorově neovlivněný auditor je teoretický koncept, dá se říci, že jde o podobnou situaci, jako dokonalá konkurence. Názorově neovlivněný auditor může při vstupu do zakázky existovat, ale v okamžiku, kdy je vystaven vlivu prostředí u auditované společnosti, je ovlivněn vztahy zastoupení, které u dané společnosti vznikají. Nebrat tyto vztahy v potaz by byla chyba, ale stejně jako v případě dokonalé konkurence i zde je možné tento koncept použít

[5] Zde vycházíme z Jensena a Mecklinga (1976), kteří upozorňují, že neexistuje něco jako společnost, ale vždy jde o rozhodování a o motivy konkrétních lidí. Předpokládáme, v tomto zjednodušujícím případě, že manažer a vlastník jednají ve shodě vůči externím subjektům.

pro teoretické prozkoumání vztahů, ze kterých se následně bude vycházet při zkoumání vztahů ovlivněných prostředím.

3.1 Vztahy zastoupení názorově neovlivněného auditora

Názorově neutrální auditor se snaží především vykonat svou práci s co nejnižšími náklady, aby mohl dosáhnout co největšího čistého užitku (viz výpočet výše). Vůči vlastníkovi se názorově neutrální auditor staví jako vůči osobě, které předává výsledky své práce (tj. zprávu auditora a prezentaci hlavních zjištění) a se kterým si sjednává smlouvu. V případě názorově neutrálního auditora je vlastník bodem, ve kterém může maximalizovat své příjmy, proto se auditor snaží přesvědčit vlastníka společnosti, že vykonal co největší díl práce s co nejvyšší kvalitou. Tímto způsobem auditor zajišťuje, že dostane v budoucnu vyšší honorář, nebo že nedojde k jeho snížení a zároveň si vytváří základ pro svou prestiž a doporučení do budoucna. Výsledek auditu, neboli jestli dojde k modifikaci výroku auditora je pro auditora položkou, kterou si nenechá ovlivnit. Vlastník může mít zájem tlačit na auditora, aby došlo ke korekci výroku, ale auditor tomuto tlaku odolává, jelikož se případná korekce výroku neslučuje s dodržením všech aspektů auditorských standardů a vystavuje auditora vysokému riziku, které nehodlá akceptovat (toto riziko je pro něj vysokým nákladem). Auditor se domnívá, že vyššího výnosu dosáhne tím, že striktně dodržuje své hlavní poslání, čímž je shoda nemodifikovaného výroku s finančními výsledky bez materiálních nedostatků. Při prezentaci výsledků auditu má auditor tendenci prezentovat velké množství zjištění, aby vlastníkovi dokázal náročnost auditu a přesvědčil jej ve svůj prospěch.

Vztah s manažerem je pro auditora zdrojem k minimalizaci nákladů, které jsou v rámci zakázky vynakládány. Zároveň manažer neovlivní velikost honoráře[6], který může stanovit jen vlastník. K minimalizaci dochází tím, že auditor se snaží co nejvíce pochopit manažera, zjistit, kde má slabá místa a zaměřit se pouze na kontrolu těchto slabých míst. Druhým způsobem, jak může auditor ovlivnit velikost nákladů je vybudování dobrého vztahu s manažerem. Dobrého vztahu s manažerem auditor dosáhne tak, že je ochoten méně zasahovat do věcí, které se manažerovi nelíbí, neboť se dotýkají jeho problémových záležitostí (jde například o jeho chyby, nadspotřeba a další – viz Marek, 2007).

Hlavním rysem názorově neutrálního auditora je, že vždy dodržuje pravidlo, že cokoliv, co by mělo vlastníka zajímat (tedy jde o signifikantní[7] záležitost) a zároveň jde položku materiální, bude reportovat. Názorově neutrální auditor je tedy ochoten pro dobrý vztah s manažerem nereportovat nemateriální položky, které pro svou signifikanci by měly být vlastníkovi reportovány. Velikost hranice, kterou si auditor stanoví je na úrovni neevidovatelné chyby, jelikož cokoliv, co je nad neevidovatelnou chybou, musí být reportováno.

Pro maximalizaci vlastního užitku jedná názorově neutrální auditor tak, aby předal všechny zásadní (tj. signifikantní) informace vlastníkovi a pouze v případě, že mu to pomůže k minimalizaci nákladů, tak je ochoten některé záležitosti, které jsou pod neevidovatelnou chybou, vlastníkovi neprezentovat. Další specifickou vlastností názorově neutrálního auditora je, že není náchylný k přehnané důvěře k manažerovi, neboli, pokud auditor vyhodnotí riziko jako významné, tak se nenechá přesvědčit manažerem o opaku a je ochoten danou oblast testovat i přes zvýšené náklady.

[6] Pokud by dokázal auditor ovlivnit ve vztahu k manažerovi velikost honoráře, snažil by se primárně ovlivnit velikost této proměnné a mohlo by docházet k záměrnému ovlivnění výsledků auditu. Situace by byla podobná s tím, co bylo popsáno (Bobek, 2014, s. 65) v případě vztahů auditora a ostatních stakeholderů. Neboli, že by auditor vlastníka „ponížil“ na úroveň ostatních stakeholderů.

[7] Tento pojem použit kvůli sjednocení názvosloví s překladem českých standardů ISA.

Názorově neutrální auditor nemá žádný specifický vztah vůči ostatním stakeholderům. Řídí se ovšem svým posláním, tedy, že účetní závěrka musí být správná ve všech materiálních aspektech. Tedy jeho hlavní snahou je, aby účetní závěrka neobsahovala chybu.

Pokud by názorově neutrální auditor postupoval jako steward (viz stewardship theory – Davis a kol, 1997), tak by takový auditor primárně následoval vždy vlastníka společnosti, který si jej najímá. Jelikož auditor by v takové chvíli měl jednat v prospěch vlastníka, nesnažil by se maximalizovat velikost své odměny, ale najít optimální velikost odměny podle skutečně vykonané práce, kterou by vázal na pracnost podle potřeby skutečně vykonat veškeré testy. Ze své podstaty nepředpokládáme, že by auditor měl jednat jako steward, jelikož, jak uvádí Davis a kol. (1997), stewardship theory se projevuje jen v určitých případech a my zde předpokládáme, že audit je doménou vztahu zastoupení (zejména kvůli potřebě hledat stále nové zakázky, zvyšovat honoráře apod.).

Podle Bobek (2014) tedy platí pro informace prezentované názorově neovlivněným auditorem následující vztahy:

$$D_{\mathrm{Prez}} < D_{Odh} \leq D_{Exis} \leq M \wedge D_{\mathrm{Prez}} = \sum_{i=1}^{n-x} D_{i(Odh)}$$

$$D_{i(Odh)} \geq M_{NeCh} \Rightarrow D_{i(Odh)} \in D_{\mathrm{Prez}}$$

kde D_{Prez} – Diference prezentované vlastníkovi,
D_{Odh} – diference odhalené v rámci auditu,
D_{Exis} – diference skutečně existující v účetní závěrce,
M – materialita stanovená pro danou zakázku,
$D_{i(Odh)}$ – konkrétní odhalená chyba,
n – celkový počet odhalených signifikantních chyb,
x – počet odhalených signifikantních chyb, které jsou nemateriální,
M_{NeCh} – neevidovatelná chyba zakázky.

3.2 Vztahy zastoupení názorově ovlivněného auditora

Názorově ovlivněný auditor[8] se v mnoha aspektech chová velmi podobně jako názorově neovlivněný auditor – stejně jako on se primárně snaží maximalizovat svůj čistý užitek (viz výpočet výše), ale v důsledku toho, že přejímá vztahy zastoupení panující v auditované jednotce, mění své chování.

Vztah vůči vlastníkovi je taktéž jako v případě názorově neovlivněného auditora spojen se snahou auditora o maximalizaci svých výnosů skrze jeho přesvědčování o náročnosti zakázky, provedené práci apod. Vlastník považuje zprávu auditora nejen za způsob kontroly managementu společnosti, ale také jako způsob prezentování se firmy vůči ostatním stakeholderům. Názorově ovlivněný auditor ovšem nepovažuje výrok za neutrální je pro něj činitelem, který přímo ovlivňuje jeho výnosy v budoucnu, zejména přes dlouhodobost kontraktu, ale také skrze jeho prestiž a postavení na trhu. Auditor je vždy omezen materialitou (Bobek, 2014) – tzn., auditor nemůže vydat výrok bez výhrad, pokud suma chyb je větší než jím stanovená materialita[9]. Pokud je auditor tlačen k vydání nemodifikovaného výroku v případě, že by měl vydat modifikovaný výrok, pak auditor posuzuje své náklady a výnosy z jednotlivých variant a volí tu, která je pro něj nejvýhodnější. Auditor zejména posuzuje

[8] Názorově ovlivněný auditor NENÍ závislým auditorem, tak jak jej chápou ISA. Názorově ovlivněný auditor mění své chování, ale nedochází u něj k porušování auditorských standardů (například ISA).
[9] Vzhledem k tomu, že performance materialita vyjadřuje jen způsob promítnutí rizika do testování, budeme používat v tomto příspěvku pracovat jen s pojmem materialita.

budoucí výnosy od auditovaného subjektu (tendence výrok nemodifikovat) a svou prestiž a postavení na trhu (tendence výrok modifikovat).

Vlastník společnosti může názorově ovlivněného auditora využít k činnostem, které on sám považuje za důležité, tj. chtít po auditorovi, aby se zaměřil na konkrétní oblasti zkoumání (chyby managementu, oblast, ve které chce vlastník řešení apod.). V tuto chvíli auditor tedy nepostupuje podle vlastního úsudku a vyhodnocení rizik, ale podle doporučení někoho jiného, případně dělá procedury, které by měly být spíše součástí forenzního zkoumání. Pokud auditor není dostatečně kompenzován za tyto vícepráce, tak zvažuje, zdali snížit náročnost jiných testů (a tím jejich nákladovost), nebo jestli snížit velikost čistého užitku ze zakázky. V případě, že by auditor neprováděl všechny potřebné testy tak se vystavuje riziku, že neodhalí materiální chybu, která by následně ovlivnila kvalitu účetní závěrky a tím způsobem i jeho auditorovu pověst.

Stejně jako v případě vztahu názorově neovlivněného auditora je v případě názorově ovlivněného auditora vztah s manažerem zdrojem k minimalizaci nákladů. Auditor opět nezasahuje do věcí, které jsou signifikantní, ale nejsou materiální. Rozdílem je, že auditor, ve snaze maximalizovat svůj užitek může, skrze snahu o zlepšení vztahů s manažerem porušit zásadu, že se reportují veškeré položky, které jsou nad neevidovatelnou chybou. Auditor záměrně některé položky nemusí vlastníkovi prezentovat, aby zlepšil vztah s manažerem. Samozřejmě při tomto rozhodování auditor posuzuje, zdali pro něj má větší užitek danou informaci prezentovat (zlepšení vztahů s vlastníkem) nebo neprezentovat (zlepšení vztahu s manažerem a minimalizace nákladů).

V případě názorově ovlivněného auditora dochází také k rezignaci na předpoklad, že auditor není přehnaně důvěřivý vůči managementu. Auditor je v tomto případě, pokud je to v mantinelech materiality, ochoten věřit vysvětlení manažera, které mu umožní snížit počet testovaných položek a tak sníží své náklady.

Vztah vůči ostatním stakeholderům se nemění – stejně jako v případě názorově neovlivněného auditora jej neovlivňují napřímo, pouze přes zásadu, že prezentované finanční informace mají být správné. V důsledku vztahu s vlastníkem je auditor ochoten porušit tuto zásadu nepřímo (tj. přes zvýšení materility, nikoliv přes otevřené vydání pozitivního výroku, který neměl být vydán). V souladu s výsledky v Bobek (2014) se nepředpokládá, že by auditor byl ochoten výrok měnit kvůli managementu, ale pouze kvůli vlastníkovi, resp. kvůli osobě, přes kterou je auditor schopen maximalizovat své výnosy. Management není schopen ovlivnit výrok auditora[10].

Matematicky lze vztahy názorově ovlivněného auditora popsat následovně:

$$D_{\mathrm{Prez}} \le D_{Odh} \le D_{Exis} \le M_f \wedge M \le M_f$$

$$D_{\mathrm{Prez}} = \sum_{i=1}^{n-y} D_{i(Odh)} \wedge x \le y; \forall x, y \in \mathrm{N}^0$$

$$D_{i(Odh)} \ge M_{NeCh} \wedge i \in \langle x, y \rangle \Rightarrow D_{i(Odh)} \notin D_{\mathrm{Prez}}$$

kde D_{Prez} – Diference prezentované vlastníkovi,
D_{Odh} – diference odhalené v rámci auditu,
D_{Exis} – diference skutečně existující v účetní závěrce,
M – materialita stanovená pro danou zakázku v případě názorově neovlivněného auditora,

[10] Tento předpoklad platí pouze za situace, že management není schopen ovlivnit velikost výnosů auditora.

M_f	–	materialita stanovená pro danou zakázku v případě názorově ovlivněného auditora,
$D_{i(Odh)}$	–	konkrétní odhalená chyba,
n	–	celkový počet odhalených signifikantních chyb,
x	–	počet odhalených signifikantních chyb, které jsou nemateriální,
y	–	počet odhalených signifikantních chyb, které jsou jak materiální, tak nemateriální,
M_{NeCh}	–	neevidovatelná chyba zakázky,
i	–	konkrétní odhalená chyba, která nabývá hodnot od 0 do n ($i \in N^0$), přičemž tyto chyby jsou seřazeny podle materiality a signifikantnosti od nejmenší po největší v pevném pořadí.

Ostatní stakeholdeři si mohou být jistí, že pokud nedošlo k modifikaci výroku, tak účetní závěrka neobsahuje materiální chyby. Jelikož však nemají informaci o tom, jak velká byla materialita, tak mohou pouze odhadovat, jak přiměřená materialita byla použita. Externí stakeholdeři jsou odkázáni na kontrolní prostředí v rámci auditorské profese a na obecnou pravděpodobnost, že účetní závěrka obsahuje chyby. Tuto pravděpodobnost je externí stakeholder schopen pouze odhadovat z minulých případů.

Externí stakeholder v takovém případě postupuje podle Akerlofova (1970) principu pro výběr v nejistotě – posuzuje kvalitu závěrky podle auditora, který výrok vydal. Ostatní stakeholder v takovém případě ví, že u některých auditorů (skupina I) má vyšší pravděpodobnost π, že výrok byl vydán bez vlivu interních nákladů zastoupení na auditora, než u jiných auditorů (skupina II s pravděpodobností μ), přičemž platí, že $\pi > \mu$. Stakeholder tedy při posuzování závěrky bere v potaz typ auditora a podle toho, do které skupiny auditor patří, upravuje i své subjektivní vnímání kvality auditu.

4. Závěr

Příspěvek prezentuje nový pojem názorově ovlivněného, resp. názorově neovlivněného auditora. Názorově neovlivněný auditor představuje koncept, který je ideální, jelikož auditor, ačkoliv je vystaven tlaku prostředí, nepodléhá mu a chová se pouze dle svých vlastních zájmů. Jeho zájmy jsou sobecké, auditor se primárně snaží maximalizovat vlastní užitek, ale názorově neovlivněný auditor není ochoten slevit z obecných zásad, které jsou nutné k tomu, aby provedl odpovídající procedury k odhalení všech materiálních nedostatků účetní závěrky.

Vzhledem k tomu, že koncept názorově neovlivněného auditora je nový, je prostor pro další výzkum především v oblasti postavení názorově neovlivněného auditora, jeho základů chování a toho, jak sám zachází s vlastními vztahy zastoupení. Zaměření dalšího výzkumu by z našeho pohledu bylo možné i na auditora – stewarda, který by nebyl ovlivněn vztahy zastoupení a nechoval by se sobecky.

References

[1] Akerlof, G.A. (1970). The Market for „Lemons“: Quality Uncertainty and the Market Mechanism. *The Quarterly Journal of Economics*, 84(3), p. 488 – 500.

[2] American Accounting Association - AAA (1973). *Committee on Basic Auditing Concepts. A Statement of Basic Auditing Concepts*. Sarasota: American Accounting Association.

[3] Alchian, A.A. and Demsetz, H. (1972). Production, Information Costs, and Economic Organization. *The American Economic Review*, 62(5), p. 777 - 795

[4] Bobek, M. (2014). Vliv vztahů zastoupení na kvalitu informací poskytovaných ověřovacími službami. *Český finanční a účetní časopis*, 9(2), p. 52–68.

[5] Bobek, M. (2015). Ověřovací služby – koncept a standardizace. *Český finanční a účetní časopis*, 10(2), p. 23–57.

[6] Davis, J.H., Schoorman, F.D., Donaldson, L. (1997). Toward a Stewardship Theory of Management. *Academy of Management Review*, 22(1), p. 20 – 47.

[7] Fama, E.F. (1980). Agency Problems and the Theory of the Firm. *The Journal of Political Economy*, 88(2), p. 288-307.

[8] Jensen, M. C. and Meckling, W. H. (1976). Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure. Journal of Financial Economics, 3(4), p. 305-360.

[9] Marek, P. (2007). Vliv teorie zastoupení na teorii podnikových financí. *Český finanční a účetní časopis*, 2(2), p. 6-16.

[10] Messier, W. F., Jr., Glover, S. M. and Prawitt, D. F. (2008). *Auditing & Assurance Services: A Systematic Approach*. New York: McGraw-Hill/Irwin.

[11] Podrug, N., Filipovic, D., Milic, S. (2010). Critical Overview of Agency Theory. p. 1227 – 1228. In Katalinic, B. (eds.). *Annals of DAAAM for 2010 & Proceedings of the 21st international DAAAM Symposium*, 20-23rd October 2010, Zadar, Croatia, 21(1)

[12] Ross, S. A. (1973). The Economic Theory of Agency: The Principal's Problem. *The American Economic Review*, 63(2), p. 134-139.

The S & P dimensions in the SCP paradigm: the case of Slovak insurance companies

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Abstract

Building on the premise of the SCP (Structure – Conduct – Performance) paradigm, the paper investigates the relationship between market structure and performance that exists in the Slovak insurance sector. Whereas the influence of market structure on insurance companies is captured by their market shares, their performance is represented in a traditional way by their profitability ratios. Using annual data for three years from 2013 to 2015, the ambition is to investigate the nexus between the S and P dimensions while controlling for net loss ratio reported by individual insurance companies that implicitly measures the risk arising form a particular composition of their portfolio of insurance policies. It is nevertheless found that higher market shares were not contributive to higher performance, which challenges the validity of this behavioural hypothesis for the Slovak insurance sector.

Key words

The SCP paradigm, risk, insurance company, market share, profitability, net loss ratio.

JEL Classification: G22, L11.

1. Introduction

A fair deal of attention is dedicated in both theoretical and empirical literature to investigations into the sources from which performance of financial companies sprouts and into the conditions under which their profitability is attained. Toward this end, theoretical constructs of the theory of industrial organization are applied in the belief that the ideas that fit to manufacturing or service industries are transferable to the description of financial enterprises and of their behaviour. An illustrious example is the SCP paradigm that finds the roots of performance of individual (here financial) enterprises in the factors that decide the character and form of competition in a given market. This prestigious hypothesis posits a unidirectional cause-and-effect relationship between market structure (S), conduct (C) and performance (P), in which it attributes extraordinary profits to an advantageous (anti)competitive position that some financial enterprises may occupy in the market. The SCP paradigm stipulates that the market structure and its concentration (reflected by the number of market participants and their participation in providing the market supply) influences the way the market participants behave and determines their pricing behaviour, which has then an immediate eventual impact upon their performance. Too intense a concentration (possibly marked with signs of oligopoly) incites market participants to collude, to build entry barriers and in their profit-maximization effort to charge higher prices without a need to effectively minimize costs. In consequence, their operations are both technically and allocatively inefficient and result in the loss of social welfare. If the maintained hypothesis is valid, then it

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also has profound implications for regulatory action because it suggests for the regulator that (not only financial) markets should be monitored or screened for anti-competitive effects, and simultaneously it vindicates all elements of competition or antitrust policy.

These considerations give rise to examinations of the structural characteristics of financial markets and are the reason why the SCP paradigm is frequently subjected to verification. Admittedly, the centre of these verifications are especially banking institutions and only then insurance companies. Other financial companies (such as stock brokerages, investment funds, leasing companies, consumer finance companies) are as a rule not investigated as to whether there is a linkage between market structure and performance and whether anti-competitive behaviour underlies the conduct of market competitors in their struggle for higher performance. The reason being twofold: first, they are not as important on a comparative basis as banks or insurance companies, and second, there is a multitude of theoretical issues that need be addressed and they are not sufficiently resolved even for banks and insurance companies (regarding e.g. their model and essence of production). On the one hand, the Slovak banking and insurance sectors have already been investigated for the validity of the SCP paradigm (see Kočišová, 2014; Brokešová et al., 2014, 2016); yet, the scarcity of such studies suggests that the research intensity has been sort of marginal. On the other hand, this is not true for studies into efficiency, an area that is closely linked to the SCP paradigm, which were quite numerous (for the Slovak insurance sector see e.g. Grmanová and Jablonský, 2009; Grmanová, 2014; Zimková, 2015, 2016). The paper arises in an attempt to rectify this neglect and investigates whether there exists a link between the first (S) and the last (P) dimensions and whether higher concentration and higher market shares of Slovak insurance companies actually help their higher performance. Using data for three fiscal years, 2013, 2014 and 2015, the paper investigate the relationship between the market shares of Slovak insurance companies and their profitability ratios whereas controlling for net loss ratio reported. This variable is intended to implicitly capture a specific level of risk arising from a particular composition of a portfolio of insurance policies that every insurance company keeps, and fills in for the jointly shared exposure to macro and micro environmental factors. Two primary means of statistical analysis are employed in the investigation: scatter analysis (based on scatter graphs) and correlation analysis (based on correlation and partial correlation measures). Whereas the former intermediates a direct visual assessment of the relationship between two variables that are plotted on perpendicular axes and is ancillary in reading any non-linear patterns that appear between them, the latter is a traditional approach to measuring the strength of linear relationship. The ascertainment that follows from the investigation is somewhat unrewarding: higher market shares do not indicate higher performance as opposed to the stand defended by the SCP paradigm. Although this statement encompasses only a small (though fairly representative) sample of Slovak insurance companies for three recent consecutive years, one is free to doubt whether incorporation of other years and enlargement of the sample would yield a different result.

The remainder of the paper is organized into three other sections. Section 2 exposits the fundamentals and comments on empirical translation of the SCP paradigm. Section 3 then conducts the planned investigation of the structure-performance relationship for the Slovak insurance sector. Finally, Section 4 concludes and discusses.

2. Review of the SCP paradigm for insurance companies

Albeit the fundamentals of the SCP paradigm were laid by Mason (1939) and developed into a solid theory by Bain (1951), some claim that the relationship it describes is largely empirical and relies on empirical data (e.g. Heffernan, 2005, p. 495). This rebuke is then

supported by the fact that the empirical findings regarding its validity are to a great extent controversial and vary across research studies depending upon their setting and methodological framework (see e.g. Choi and Weiss, 2005, pp. 637-638).

The structure-performance hypothesis introduces a link between structure, conduct and performance whose working can be schematically represented by the following causal chain:

Structure (higher concentration) → Conduct (higher prices) → Performance (higher profits).

In a concentrated market whose considerable part is serviced only by a handful of enterprises these enterprises tend naturally to collude or fall into lax monopoly-like behaviour, in consequence of which they either press for joint profit maximization by cooperatively charging higher prices (here insurance premiums) or by effectively reducing their compulsory expenditure (here payments on claims). In either way, the effect is seen in boosted performance and increased profitability. An appendant feature of higher concentration is building up entry barriers that should discourage potential participants from entering the market. The inherent flaw of the SCP paradigm is that it views market structure as exogenous and pre-determined for market participants in the sense that it is not an outcome of their managerial excellence or extraordinary conduct, but possibly the product of their colluding behaviour. This is then revisited by a competing approach, the ES (efficient structure) hypothesis, that argues that higher market shares (and the resultant market concentration) follow from higher (technical and/or scale) efficiency that allowed firms to grab larger portions of the market. Under the ES hypothesis, (higher) efficiency is a determinant of conduct (reflected in higher output and/or lower prices) that works through (higher) market shares up to (satisfactory) performance. In short, efficient firms can simply afford to charge lower prices and then seize larger market shares and enjoy higher economic rents with an effect of increased concentration.

Returning back to the SCP paradigm, although the logic of the causal chain "Structure → Conduct → Performance" is clear, it must be operationalized – especially in empirical applications and partly also in theoretical considerations – what is meant by the individual elements of this chain and how these elements are represented by measurable quantities. The elements that are recognized under each of these dimensions are summarized briefly (and not exhaustively) in Table 1.

Table 1: Factors of the SCP dimensions

Market structure	Firm Conduct	Firm performance
• the degree of seller concentration • the degree of buyer concentration • the degree of product differentiation • the condition of entry to the market	• the pricing policies of sellers (determination of selling prices and calculation of selling costs) • the production and product policies of sellers (determination of output levels, product designs and qualities) • the sales promotion policies of sellers (determination of distribution channels and advertising expenditure) • the mechanism of coordination and cross-adaptation of price, product and sales-promotion policies of sellers (predatory and exclusionary tactics)	• the profit margin and the height of selling price relative to the long-run marginal cost of production and to the long-run average cost of production • the relative technical efficiency of production • the character of the products (including design, level of quality and variety)

Source: the author (based on Bain, 1968, pp. 7-12, 303-304).

Table 1 lists the factors that represent collectively each of the SCP dimensions. It is praiseworthy noting that market structure is in general understood as a combined result of the factors appearing both on the part of buyers and on the part of sellers. For some markets, the sellers' part of the market may be concentrated but buyers may be just few, which is an

asymmetry that puts buyers to an advantage and affects the conduct of buyers. Nonetheless, insurance providers usually have well diversified product lines oriented both on retail and corporate customers, and their market focus is somewhat universal with low buyers' concentration in effect. The last category of factors under the C dimension, viz. the mechanism of coordination and cross-adaptation, in fact represents whether the sellers tacitly or openly cooperate (i.e. collude) or the decision-making is independent in a purely competitive fashion. Lastly, the criteria in the P dimension encompass also non-financial criteria of performance that come to be relevant from a societal point of view, and are scarcely a motivation to firms as such. Inasmuch as firms are driven by their profit maximization goals, what is of relevance to them is the profit they make relative to the sources needed in production and the capital invested into the business. This also holds for insurance providers.

An empirical extension of Table 1 is Table 2 that shows how most of the factors in Table 1 are operationalized and translated into measurable quantities.

Table 2: Operationalization of SCP variables in insurance

Market structure	Firm Conduct	Firm performance
• individual market shares • a measure of market concentration (e.g. the k-firm concentration ratio, the Herfindahl index) • a measure of barriers to entry (e.g. economies of scale or absolute cost advantages)	• a measure of technical, scale and/ or allocative efficiency • advertising expenditure • R&D expenditure • a measure of reinsurance utilization • a degree of diversification (mix of business) • the type of distribution system • the Learner index of competition • a measure of collusion	• a gross profitability ratio (e.g. return on assets, return on equity, return on capital) • an excess profitability ratio (i.e. gross profitability less opportunity costs such as the required rate of return) • a profit margin (offsetting premiums earned with losses incurred and/or underwriting expenses with premiums written)

Source: the author (based on Heffernan, 2005, pp. 496-506; Jedlicka and Jumah, 2006, pp. 10-16; Choi and Weiss, 2005, p. 645).

Market structure is captured by individual market shares as the market share is an indicator of the degree of control that a firm exerts in a market, and for a highly concentrated market, it is an generic instrument for assessment the tendency of this firm to behave uncompetitively in an attempt at higher profits. Concentration itself is frequently measured by the portion of the market operated by the k largest firms (the k-firm concentration ratio, where often k is set to 3 or 4) or by the sum of squared market shares of all the firms in the market (the Herfindahl index). Sometimes, also barriers of entry are measured by means of an indicator that captures economies of scale (although defined as a ratio of minimum efficient scale to market output, it is proxied for reasons clear, see Jedlicka and Jumah, 2006, p. 11). Suitable measures of firm conduct are technical, scale and allocative efficiencies that quantify the degree of uncompetitive conduct of firms in terms of their production and pricing policies. The loss of efficiency may be either owing to the environmental conditions or on account of managerial mismanagement in conditions when efficiency is not the ultimate goal because larger market power is an augury of higher profits. Paradoxically, from a societal point of view, firm efficiency is a factor of performance, but to firms (in)efficiency is an indicator of economic (mis)conduct, whether objective given the circumstances or desired. In addition, there are some other indicators that measure advertising or R&D intensity (a proportion of advertising R&D on total expenditure), the inclination to make use of reinsurance (a ratio of reinsurance ceded to premiums written) or the diversification of business (e.g. shares of individual lines of business and their concentration). Some insurers utilize independent agency systems and others have their own points of sales and carry out direct underwriting. The Lerner index is defined as the relative deviation of price from marginal costs and captures the degree to which a firm can exercise monopoly power. Although appealing, there are troubles with estimating

it. As is reasoned by Jedlicka and Jumah (2006, p. 12) it is difficult to explicitly measure collusion (e.g. because it is a legal term), yet this need usually tackled by employing variables describing market structure, viz. firm sizes, concentration and barriers to entry. They conclude that "an SCP analysis could be broken down to only the structure-profitability relationship" (ibid.). Eventually, performance is reduced to profitability, in which there are complications as to how profitability should be measured, especially when there are multiple products and services. Therefore, comprehensive synthetic indicators of profitability are preferred such as return on assets, return on equity or return on capital, sometimes netted of the opportunity rate of return associated with achieving a given level of profitability. Another possibility is to set up a profit margin that would measure the deviation of earned/written premiums from associated incurred losses / underwriting expenses.

3. Application to the Slovak conditions

The last paragraph of the preceding section arrived at the observation that in an empirical verification of the SCP paradigm, one is free to confine oneself to examining the relationship between the S and P dimensions solely, and this is what empirical tests devised to this end actually do. They are usually rendered in a regression manner, in which a profitability indicator is regressed on a set of structural indicators and variables that control for specific environmental conditions (see e.g. Heffernan, 2005, pp. 497, 500) unless it is the aim to arbitrate between the SCP paradigm and the ES hypothesis.

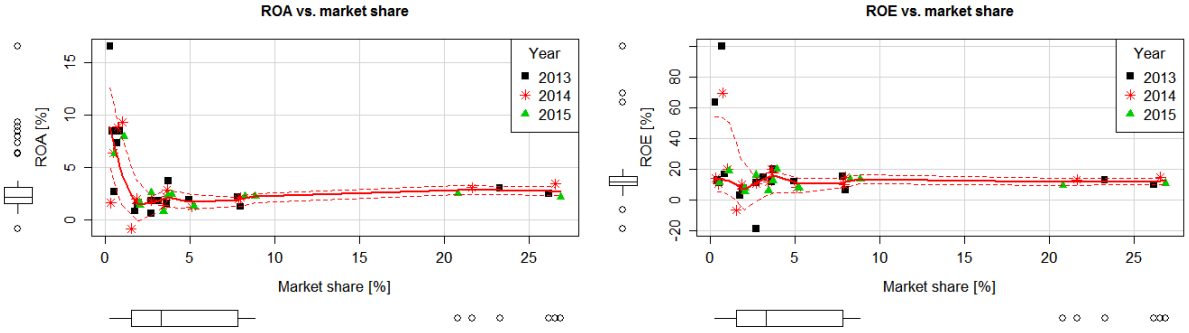
Hence, with a confinement to the S and P dimensions, the content of this section is to gain an insight into the validity of the SCP paradigm for the Slovak insurance sector. Truthfully, the study provides a limited insight using the data on several Slovak insurance companies for the consecutive years 2013, 2014 and 2015. The data set is not exhaustive of the entire Slovak insurance sector in these three years and for some insurance companies observations on some variables were not available.[2] The data come from separate financial statements of individual insurance companies as prepared under IAS/IFRS and compiled by TREND Analyses of News and Media Holding, a. s., Bratislava. The S dimension is represented by the market shares of individual insurers derived from their premium written in a given year (including deposits from investment policies) and the P dimension is represented by two profitability ratios, return on equity ("ROE") and return on assets ("ROA"). These are defined as net income to equity or net income to assets, respectively. The relationship between market shares and profitability may be affected by a number of environmental conditions, out of which it is controlled here for net loss ratio ("Loss ratio") defined as net claims incurred to net premium earned. On the one hand, the indicator is connected directly to profitability; still, on the other

[2] The list of insurance companies comprises a total of 22 subjects established in providing life assurance and/or non-life insurance, namely Allianz - Slovenská poisťovňa, a. s.; Kooperatíva poisťovňa, a. s., Vienna Insurance Group; Generali Slovensko poisťovňa, a. s.; Komunálna poisťovňa, a. s., Vienna Insurance Group; MetLife Europe Ltd (the earlier MetLife Amslico, Ltd), branch office; Uniqa poisťovňa, a. s.; Poisťovňa Slovenskej sporiteľne, a. s., Vienna Insurance Group; ČSOB Poisťovňa, a. s.; NN Životná poisťovňa, a. s. (the earlier ING Životná poisťovňa, a. s.); Wüstenrot poisťovňa, a. s.; AXA životní pojišťovna, a. s., branch office; Aegon Životná poisťovňa, a. s.; Union poisťovňa, a. s.; Poisťovňa Cardif Slovakia, a. s.; Colonnade Insurance S.A. (the earlier QBE Europe, Ltd), branch office; Poštová poisťovňa, a. s. (the earlier Poisťovňa Poštovej banky, a. s.); AXA pojišťovna, a. s., branch office; AIG Europe Ltd, branch office; HDI Versicherung AG, branch office; Mitsui Sumitomo Insurance (Europe) Ltd, branch office; D.A.S. Rechtsschutz AG, branch office; Ergo životná poisťovňa, a. s.

hand, it captures the overall riskiness of an insurer and all the conditions that made its business unfavourable in a given period.

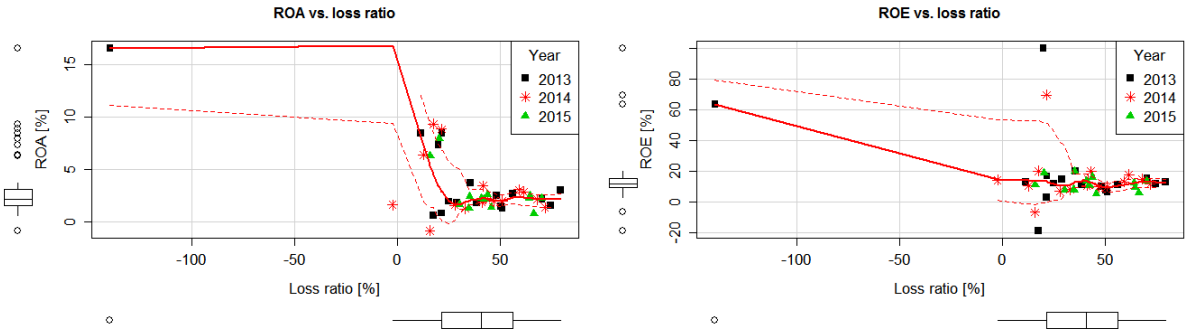
The analysis is carried out in the statistical program R (R Core Team, 2013) using functionalities of the packages `car` (Fox and Weisberg, 2015) and `ppcor` (Kim, 2012). The statistical output is organized in Figures 1 and 2 and in Table 3. Whereas the two figures present the results of the scatter analysis, the table does this for the correlation analysis. Figure 1 displays two scatter graphs showing the dependence of ROA and ROE upon market shares and Figure 2 displays two analogical plots for dependence of ROA and ROE on Loss ratio. Each scatter plot is equipped with side box plots that instruct on the location and variability of the market shares, loss ratio and two profitability ratios in the Slovak insurance sector for the period of three years analyzed. The observations associated with different years are marked with different colours and plotted in different symbols. Additionally, in each scatter graph an estimated LOESS curve with an estimate of (the square root of) the variance function is tended through the bulk of observations. This estimate indicated by a dashed curve serves in judging the uncertainty around the non-parametric estimate of the regression relationship captured by a full solid curve.

Figure 1: Influence of market shares upon profitability ratios



Source: the author.

Figure 2: Influence of loss incidence upon profitability ratios



Source: the author.

It is apparent from Figure 1 that the profitability of the Slovak insurance companies as represented by either ROA and ROE is not related (directly) to their market shares and that larger market shares are not associated with higher profitability. In a fairly frequent number of cases, comparatively high values of profitability ratios are attained at small market shares. The relationships revealed in Figure 2 for the dependence of the profitability of the Slovak insurance companies upon their net loss ratio somewhat differ, though they do not point to a particularly strong (linear or non-linear) relationship. There is no informational value in distinguishing between the three different years. Whereas the profitability ratios appear unrelated to market shares, one must have an impression of a weak negative relationship for

the net loss ratio. This is confirmed by the correlation analysis that ensues in Table 3. The left-hand part of Table 3 displays traditional pairwise correlation coefficients for three distinct methods of computation, i.e. Pearson's, Spearman's and Kendall's. The values of the correlation coefficients indicate – though possibly affected by the presence of outlying values – that both profitability ratios are almost uncorrelated or very weakly negatively correlated with market shares and negatively correlated with net loss ratio. Additional information displayed therewith is that ROA and ROE are, naturally, positively (and moreover strongly) correlated, and that there is, surprisingly, a positive correlation between market shares and net loss ratio.

Table 3: Results of the correlation analysis

Traditional Pearson bivariate correlations				**Pearson correlations net of LossRatio**		
	ROE	**ROA**	**Loss ratio**		**Share vs. ROE**	**Share vs. ROA**
Share	-0.143	-0.198	0.358	**Partial**	-0.012	0.058
ROE		0.653	-0.389	**Semi-partial**	-0.011	0.054
ROA			-0.693			
Traditional Spearman bivariate correlations				**Spearman correlations net of LossRatio**		
	ROE	**ROA**	**Loss ratio**		**Share vs. ROE**	**Share vs. ROA**
Share	-0.084	-0.183	0.650	**Partial**	-0.025	-0.062
ROE		0.747	-0.100	**Semi-partial**	-0.019	-0.047
ROA			-0.209			
Traditional Kendall bivariate correlations				**Kendall correlations net of LossRatio**		
	ROE	**ROA**	**Loss ratio**		**Share vs. ROE**	**Share vs. ROA**
Share	-0.050	-0.062	0.464	**Partial**	-0.027	-0.003
ROE		0.568	-0.056	**Semi-partial**	-0.023	-0.002
ROA			-0.124			

Source: the author.

The fact that there is no relationship between the market shares and the profitability ratios is also attested by the partial and semi-partial correlation coefficients displayed in the right-hand side of Table 3 for each of three methods. Here, the partial correlation coefficients measure the correlation between market shares and ROE/ROA while controlling for net loss ratio (after adjusting both market shares and ROE/ROA by the effect of net loss ratio). Similarly, the semi-partial correlation coefficients measure the correlation between the market shares and ROE/ROA, but the variation of net loss ratio is now removed only from ROE/ROA. The reason is that the conceivable causal effect of net loss ratio should be on profitability rather than on market shares. If it were present also on market shares, it should only happen by chance and be random. The almost zero values (although mostly negative) of the partial and semi-partial correlation coefficients provide compelling evidence that the profitability of Slovak insurance companies did not seem to be determined (primarily) by their market shares in the analyzed period of three years.

4. Concluding remarks

The paper looks into the relationship between market structure and performance that existed for the Slovak insurance sector, using a limited sample of 22 insurance companies for three consecutive fiscal years from 2013 to 2015. Although the sample did not originate as a random sample and does not permit statistical testing, it is representative in the sense that it covers the major Slovak insurance companies and those insurance companies that operate a non-negligible portion of the Slovak insurance sector. The study may be not extensive and may have approached the question of the validity of the SCP paradigm with dispatch as it reduced the scope of investigations to the first and last dimensions built up about this

behavioural hypothesis. In this, market structure constitutes the S dimension and performance represents the P dimension of the SCP paradigm. The omission of the intermediary C dimension capturing conduct was excused by noticing that in many respects firm conduct may be represented by proxy factors that appear actually in the S dimension, which is the fact that is accommodated also in empirical tests of the SCP paradigm if they are carried out on an isolated basis. This paper has no ambition to prove or disapprove with authority whether the SCP paradigm is in operation for the Slovak insurance sector, it just provides a first look into the relationship maintained by the SCP paradigm with a somewhat disappointing outcome – that higher market shares are no warranty for better performance (if understood in terms of return on equity or return on assets). Of course, the absence of the relationship pre-ordained by the SCP paradigm was found for three years only whereas this ascertainment was arrived at by using the exploratory approach of scatter analysis and by the approach of correlation analysis. Despite the limitations of both methods and despite the dominance of regression analysis in verifications of the SCP paradigm, the findings remain valid and credible. That being said, there are two main vectors toward which further research might head. One of them is the inclusion of other control variables that represent micro and macro environmental conditions of Slovak insurance companies, and another is a possible distinction between life and non-life insurance. In theory, different behavioural models might hold for companies operating life assurance than for those established in non-life insurance. If desirable, such a course of research is not easily implementable as the majority of the Slovak insurance companies has lines of business diversified both in life and non-life insurance and it is not straightforward to tell apart what fragment of their reported business is related to life or non-life insurance.

As far as the compatibility of the present findings with the previous research in this area is concerned, only three studies are known troubled with the task of investigating the relationship between the market structure and performance of Slovak insurance companies, and these were listed in the introduction of the paper. Brokešová et al. (2014, 2016) focused only on the part of the Slovak insurance sector providing life insurance for a different period. Their period of 20 years from 1993 until 2012 span the transformation phase of the Slovak economy through various turbulent moments and ends in the times when the insurance sector in Slovakia may be described as consolidated and stable (regardless of natural entries and exits). During this period, the insurance sector in Slovakia developed from a close-to-monopoly market with one dominant insurance company into an unconcentrated market as it remains now. It is therefore not surprising that they find that decreasing concentration increases profit and performance of the insurance sector. It is a conclusion natural given the trend of lowering concentration and the stabilization of economic outcomes between 1993 and 2012. The present paper considers the stable situation of the next three years, from 2013 to 2015, and in these stabilized conditions, no relationship is found between market shares and profitability of the Slovak insurance sector. This does not contradict, it only answers to the changed conditions. Furthermore, the present paper operates directly with market shares than with concentration measured in Brokešová et al. (2014, 2016) by the Herfindahl index. The concentration in the Slovak insurance sector remained virtually unchanged between 2013 and 2015 as is proven by almost constant values of the Herfindahl index: 1428 (for 2013), 1423 (for 2014), and 1473 (for 2015). Of course, these numbers follow from the data on premium written used in the present analysis.

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References

[1] Bain, J.S. (1951). Relation of profit-rate to industry concentration: American manufacturing, 1936 – 1940. *Quarterly Journal of Economics* 65(3), pp. 293-324.

[2] Bain, J.S. (1968). *Industrial organization*, 2nd ed. New York: Wiley.

[3] Brokešová, Z., Ondruška, T. and Pastoráková, E. (2014). Market structure and performance of the life insurance industry in the Slovak Republic. In Deev, O., Kajurová, V. and Krajíček, J. (eds.) *11th International scientific conference on European financial systems 2014*. Lednice, Czechia, 12-13 June 2016. Brno: Masaryk University in Brno, Czechia.

[4] Brokešová, Z., Ondruška, T. and Pastoráková, E. (2016). Industry concentration and performance: case of life insurance industry. In Dudycz, T., Osbert Pociecha, G. and Brycz, B. (eds.) *6th international conference on efficiency as a source of the wealth of nations. Essence and measurement of organizational efficiency*. Wroclaw, Poland, 9-10 March 2015. Springer proceedings in Business and Economics, pp. 29-37.

[5] Choi, B.P. and Weiss, M.A. (2005). An empirical investigation of market structure, efficiency, and performance in property-liability insurance. *The Journal of Risk and Insurance* 72(4), pp. 653-673.

[6] Fox, J. and Weisberg, S. (2015). *car: Companion to applied regression.* R package version 2.0-25, http://cran.r-project.org/package=car.

[7] Grmanová, E. (2014). New business and efficiency of insurance companies in the Slovak Republic. In *International multidisciplinary scientific conferences on social sciences and arts (SGEM 2014)*. Albena, Bulgaria, 1-10 September 2014. Political sciences, law, finance, economics and tourism, vol. II, pp. 511-518.

[8] Grmanová, E. and Jablonský J. (2009). Analysis of efficiency of Slovak and Czech insurance companies by data envelopment analysis models. *Ekonomický časopis* 57(9), pp. 857-869.

[9] Heffernan, S. (2005). *Modern banking.* Chichester (England): Wiley.

[10] Jedlicka, L. and Jumah, A. (2006). *The Austrian insurance industry: A structure, conduct and performance analysis*. Wien: Institut für Höhere Studien.

[11] Kim, S. (2012). *ppcor: Partial and semi-partial (part) correlation*. R package version 1.0, http://cran.r-project.org/package=ppcor.

[12] Kočišová, K. (2014). Koncentrácia slovenského bankového sektora a výkonnosť bánk. *Finančné trhy* 11(3), pp. 1-12.

[13] Mason, E.S. (1939). Price and production policies of large-scale enterprise. *American Economic Review* 29(1), pp. 61-74.

[14] R Core Team (2013). *R: A language and environment for statistical computing.* Vienna: R Foundation for Statistical Computing, http://www.R-project.org/.

[15] Zimková, E. (2015). Technical efficiency and super-efficiency of the insurance sector in Slovakia. *Acta Universitatis Agriculturae et Silviculturae Mendeleniae Brunensis* 63, pp. 2205-2211.

[16] Zimková, E. (2016) *The retail production process in commercial banking and its evaluation*. Prague: Wolters Kluwer.

Financial risk and financial condition of Slovak forestry and logging enterprises

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Abstract

The paper investigates the financial condition of Slovak forestry and logging enterprises during the time span from 2007 to 2014 as captured by conventional financial indicators derived from financial statements and aggregated on an industry level. This investigation is undertaken with the intention to identify principal factors that contributed to the financial condition of the forestry and logging sector of the Slovak economy in the period of 8 years under scrutiny and to pinpoint the types and sources of financial risks that challenged the business of Slovak forestry and logging enterprises. Albeit the industry during this period flourished in terms of profitability, a considerable proportion of enterprises operated at a large level of liquidity or with a weaker capacity to service debt through generated earnings.

Key words

Forestry enterprise, logging enterprise, financial condition, financial ratio, quartile value.

JEL Classification: Q23, G32.

1. Introduction

The paper studies the financial condition of Slovak forestry and logging enterprises with the ambition to describe the financial risks to which they are exposed during their entrepreneurial activities. The inquiry comes with the recognition that forestry and logging business is characteristic of specific features that make the forestry and logging industry different from "traditional" manufacturing or service-providing industries. Be that as it may, the financial condition and risks of Slovak forestry and logging enterprises are examined by financial ratio analysis covering all the four main aspects of financial operations: liquidity, activity, leverage and profitability. The analysis is conducted using a set of financial ratios aggregated on an industry level for the period of 8 years from 2007 to 2014, and – in point of fact – represents an analysis of the Slovak forestry and logging industry rather than of enterprises that with their activities fall statistically under it. Such an industry-aggregated analysis is helpful in identifying the aspects of operations that are jointly troublesome to the entire economic segment. It is found that a considerable proportion of Slovak forestry and logging enterprises operated between 2007 and 2014 with excessive liquidity or with a weaker capacity to service debt through generated earnings, though they would not suffer from crippled profitability.

The remainder of the paper consists of 4 other sections. Sect. 2 is an overview of literature with an emphasis on specifics of forestry and logging, Sect. 3 is methodological and expands into Sect. 4 that presents the results and summarizes the findings. Finally, Sect. 5 concludes.

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2. Literature review

As it happens, forestry and logging as an economic segment are characteristic of several singularities in comparison to other segments of a national economy. Not only are forests an important source of wood as a renewable production factor in which they provide timber, fuel wood and other tradable wood products and services, but they are also essential with respect to a number of other functions that depend on the creation and preservation of natural ecosystems. From an economic viewpoint, the main aim of forestry is creation, management, planned and sustainable development of forests and associated resources. Sustainable development of forestry is presently closely linked with factors of an economic and non-economic trait, amongst which a special standing goes to natural factors. Forestry and logging are simultaneously an economic segment, an industry of the economy, for which it is decisive to find and preserve a reasonable compromise between environmental, ecological or purely economic and other factors that contribute significantly to activities of individual entrepreneurial subjects (i.e. to those established in forestry and logging activities). In relation to financial aspects of forestry and logging operations, both Mura and Buleca (2012) and Hajdúchová et al. (2011) accentuate that it is necessary to pay paramount attention to issues of providing financial resources to secure them and that a successful management of forestry and logging enterprises must be based on thorough production analysis and on efficiency and profitability analysis. Financial analysis of forestry and logging enterprises requires a special approach as is proposed e.g. by Lenoch (2009, 2010) who published an alternative methodology of special financial analysis of forestry enterprises. His methodology distinguishes several types of special financial analysis in forestry according to the principles that guide the analysis: the principle of capital returns, the principle of labour effects and the principle that combines both these criterial viewpoints. The third variation applies in enterprises in which both capital and human labour are equally important. Lenoch (2010) stresses that utilization of this new approach is possible only in forest estates that meet the criterion of logging balance and sustainability as regards the size of areas covered by trees and forest age classes. The said criterion represents then a possibility of approximately balanced all-year-round volumes of logged wood. Instead of assessing expenses and revenue generated by every vegetation component of a forest, Lenoch (2010, p. 60) recommends evaluating the overall economics of forestry management in terms of balanced and continuous forestry activities carried out in the course of a whole year. There is a variety of authors who in their research addressed the issues of financial and economic management of forestry and logging enterprises, and many of them either explicitly or marginally occupied themselves with a topic of modelling financial risks. The ideas can be traced to a fairly long history. A good example is the model suggested by Faustmann in 1849 and its augmentation by Ohlin in 1921 into what is known as the Faustmann-Ohlin theorem that introduces a formalism into investigations of the optimal rotation period in forest management based on the expected discontinued income (see Löfgren, 1983). Some other useful sources and overviews are the studies by Sedliačiková et al. (2011, 2016), Hajdúchová and Hlaváčková (2014), Hajdúchová et al. (2016), Klieštik and Cúg (2015), Mura et al. (2015), Levá et al. (2016), Potkány et al. (2016). These authors also therewith state specifics that should be taken into account when performing a financial analysis of forestry and logging enterprises, and simultaneously formulate main theoretical points underlying measurement and modelling of financial risks in forestry and logging.

3. Methodology and data

Under the Statistical classification of economic activities in the European Community (SK) NACE Rev. 2 the economic segment in question is cast into division "02 Forestry and logging" and is further broken down into four groups: "021 Silviculture and other forestry activities", "022 Logging", "023 Gathering of wild growing non-wood products", "024 Support services to forestry". In what follows, the industry and its subindustries are sometimes tagged by the respective two-digit or three-digit codes, i.e. "02", "021", "022", "023" and "024", respectively. The number of enterprises engaged in forestry and logging activities between 2007 and 2014 inclined to rise at a considerable pace (in 2012 there was a 6.6 % decline; otherwise in each year there was a rise, and the annual rates of growth varied from 7.1 % in 2009 to 152.3 % in 2010). These trends are readable from the yearly numbers of enterprises in the forestry and logging industry declared in Table 1 compiled from the statistical reporting produced by one leading financial analyst company in Slovakia (SCB - Slovak Credit Bureau, 2010; CRIF - Slovak Credit Bureau, 2014). The percentage figures in parentheses correspond to the absolute values one row above and they all jointly suggest significant structural changes between 2007 and 2014. First of all, subindustry 021 was steadily growing in importance at the expense of subindustry 024 that was becoming less populated in terms of enterprises registered for this support subindustry. Whereas in 2007 there were about 23 % of enterprises occupied primarily with silvicultural and other forestry activities (i.e. falling into 021) and about 58 % of enterprises carrying out chiefly support forestry services (i.e. falling into 024), these proportion were after 8 years somewhat reversed to about 64 % (for 021) and about 23 % (for 024). Only one subject was registered for 023, and it was only between 2010 and 2014. Finally, the relative proportion of logging enterprises fluctuated steadily roughly around 20 % between 2007 and 2012, but in 2013 it revealed a small decrease until 13.2 % in 2014.[3]

Table 1: Structure of the data set for the Slovak forestry and logging industry between 2007 and 2014

Industry and single activities	2007	2008	2009	2010	2011	2012	2013	2014
021 Silviculture and other forestry...	**68**	**72**	**84**	**471**	**431**	**822**	**899**	**1374**
	(22.8 %)	(20.5 %)	(22.3 %)	(49.5 %)	(48.5 %)	(56.6 %)	(54.6 %)	(63.8 %)
022 Logging	**56**	**66**	**77**	**195**	**190**	**290**	**280**	**285**
	(18.8 %)	(18.8 %)	(20.4 %)	(20.5 %)	(21.4 %)	(20.0 %)	(17.0 %)	(13.2 %)
023 Gathering of wild growing...	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**
	(0.0 %)	(0.0 %)	(0.0 %)	(0.0 %)	(0.1 %)	(0.1 %)	(0.1 %)	(0.0 %)
024 Support services to forestry	**174**	**214**	**216**	**285**	**266**	**340**	**466**	**493**
	(58.4 %)	(60.7 %)	(57.3 %)	(30.0 %)	(30.0 %)	(23.4 %)	(28.3 %)	(22.9 %)
02 Forestry and logging	**298**	**352**	**377**	**951**	**888**	**1453**	**1646**	**2153**
	(100 %)	**(100 %)**	**(100 %)**	**(100 %)**	**(100 %)**	**(100 %)**	**(100 %)**	**(100 %)**

Source: SCB - Slovak Credit Bureau (2010), CRIF - Slovak Credit Bureau (2014).

The data obtained from SCB - Slovak Credit Bureau (2010) and CRIF - Slovak Credit Bureau (2014) were summarized values of financial ratios aggregated on the level of industrial activities pre-processed in the form of quartile values available for each year. The financial condition and then financial risk in the Slovak forestry and logging industry was captured by means of two liquidity financial ratios (quick ratio, current ratio), four activity ratios (total asset turnover, inventory turnover in days, receivable turnover in days, accounts payable turnover in days), four leveraging ratios (debt ratio, long-term debt ratio, equity-to-debt ratio

[3] Although the database from which these data are gathered is fairly extensive, it does not have to include all the enterprises that were actually registered for a division or group of (SK) NACE Rev. 2. Despite this deficiency such occurrences are not frequent and it is maintained that their additional incorporation would not change the results. Their structural or statistical influence is anticipated marginal. Moreover, the majority of medium-sized and large enterprises and enterprises with a reliable economic record and a history are part of the database.

and debt leverage ratio) and, finally, by six profitability ratios (return on equity, return on assets, operating return on assets, value-added-to-sales ratio, value-new-created-to-sales ratio, EBITDA-to-sales ratio). The definition of these financial indicators is declared in Table 2. The summary quartile values that are provided by SCB - Slovak Credit Bureau (2010, 2014) are derived from year-end balances and yearly profit and loss items disclosed by individual Slovak forestry and logging enterprises. Seasonality of their production may induce an inevitable bias that must be accounted for.

Table 2: Definition of the financial indicators employed in the analysis

Financial indicator	Definition
Liquidity ratios	
Quick ratio	(current assets – stock inventory) / current liabilities
Current ratio	current assets / current liabilities
Activity ratios	
Total asset turnover	net sales / total assets
Inventory turnover in days	stock inventory / net sales × 360
Receivable turnover in days	receivables / net sales × 360
Accounts payable turnover in days	accounts payable / net sales × 360
Leveraging ratios	
Debt ratio	liabilities / total assets
Long-term debt ratio	long-term liabilities / total assets
Equity-to-debt ratio	equity / liabilities
Debt leverage ratio	liabilities / (net income + amortization and depreciation charges)
Profitability ratios	
Return on equity	net income / equity
Return on assets	(net income + taxes) / assets
Operating return on assets	earnings before interest and taxes / assets
Value-added-to-sales ratio	(net sales – costs of trade – costs of production – costs of services) / net sales
Value-new-created-to-sales ratio	value-added-to-sales ratio – (personal costs + amortization and depreciation charges) / net sales
EBITDA-to-sales ratio	(net income + taxes + interest expense + amortization and depreciation charges) / net sales

Source: the methodology of SCB - Slovak Credit Bureau (2010), CRIF - Slovak Credit Bureau (2014).

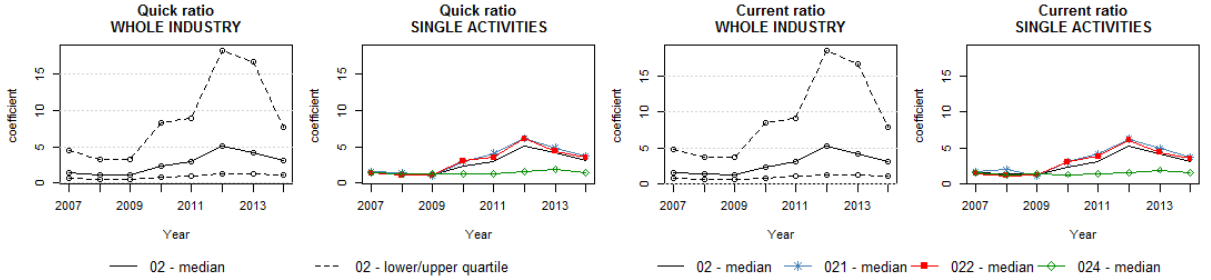
4. Results and findings

As was observed in Table 1, none or only one enterprise was operational in subindustry 023 for the period of interest, which is the reason that this subindustry is dropped out of considerations and its financial condition or financial development is further unreported. For the entire forestry and logging industry and its three relevant subindustries 021, 022 and 024, the development of each financial ratio summarized in Table 2 is visualized in the following four figures (Figure 1 for liquidity, Figure 2 for activity, Figure 3 for leveraging and Figure 4 for profitability). The development of each financial ratio between 2007 and 2014 is displayed in two adjoining plots. Whereas the left-hand plot exhibits the median values (solid line) and lower or upper quartiles (dashed lines) for the entire industry of a respective indicator, the right-hand plot shows the median values of this indicator also for the three subindustries (distinguished by different colouring and plotting symbols).

Figure 1 reports almost an identical pattern of development for quick and current ratio. Although both liquidity ratios measure the ability to pay short-term liabilities with liquid assets, their difference lies in the fact that current ratio also considers stock inventory and counts it among liquid assets. The fact that these ratios are for the entire period virtually equal merely implies that the forestry and logging industry does not tend to hold any inventory. This is in compliance with the seasonality of forestry and logging activities – production of standing timber (subindustry 021) as well as production of roundwood and wood for energy (subindustry 022) is concentrated between March and September and at the end of a fiscal year (as of 31 December) all or most of timber or wood stock is depleted. Forestry service

activities such as forest management consulting services, timber evaluation, forest pest control (subindustry 023) does not require holding inventories. This observation implies that the production of wood in Slovakia in the period in question was healthy without any disposition to pile up stock unplaced on the market. Figure 1 displays that liquidity somewhat increased between 2007 to 2012 and afterwards decreased between 2012 to 2014 for the entire industry and subindustries 021 and 022. No matter whether measured by quick or current ratio, one half of such enterprises in 2007 recorded liquidity higher than about 1.4, in 2012 higher than about 5.1 and in 2014 higher than about 3.2. Whereas between 2007 and 2009 the liquidity levels in the industry appeared quite low, in 2012 and 2013 they were noticeably high. In 2014 liquidity seems still high. One may speculate that the reason rests in high amounts of accounts receivable. Understandably, the exception to this pattern is subindustry 024 that showed a relatively steady pattern of median liquidity over the whole period that oscillated about 1.2 to 1.5.

Figure 1: Liquidity in the forestry and logging industry and its subindustries between 2007 and 2014

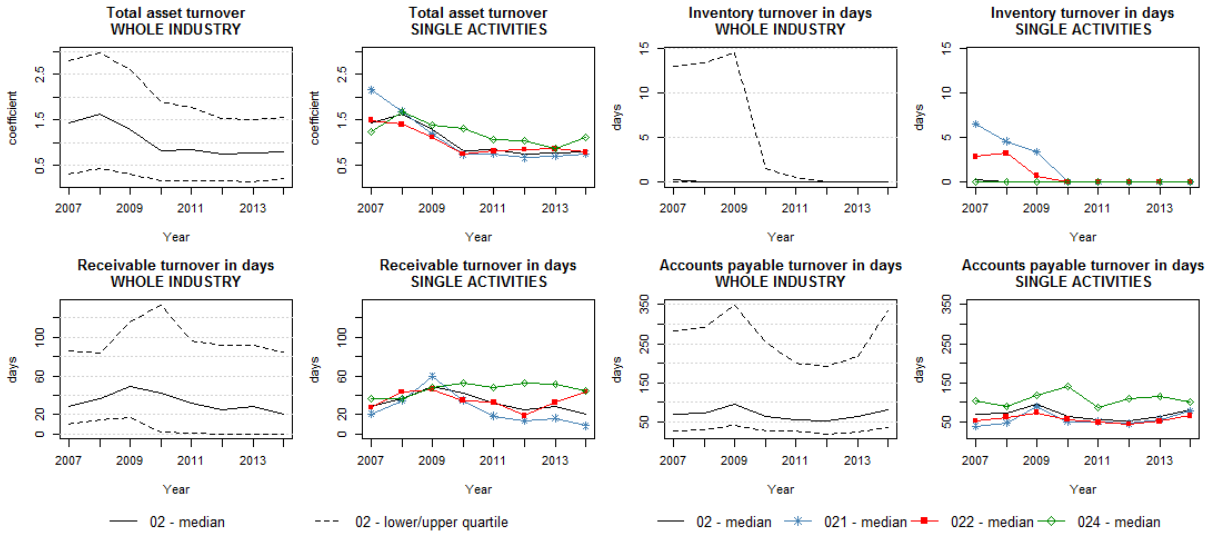


Source: the authors.

As regards activity, the quartile values of four activity ratios displayed in Figure 2 signal possible troublesome areas that are detectable from the trends of these ratios and their relationship. The overall activity of the industry and its three subindustries exhibits a unanimous decline and may seem relatively low. The median total asset turnover stabilized between 2010 and 2014 at about 0.8 and three fourths of forestry and logging enterprises would not turn over their assets in net sales more than about 1.5 times, which is not a sign of eminent sales-generation efficiency. Nevertheless, the total asset turnover values only map the situation at then end of fiscal years as the underlying ratios shown in Figure 1 were derived using end-year balances of total assets. In the middle of the year total assets might be expected to be much larger owing to the maintained stock of production that is most likely realized by the end of the year. In consequence, actual values are even lower. The observation made at inspecting the financial condition in liquidity translates to the pattern manifested by the inventory turnover of the Slovak forestry and logging industry. Negligible or non-existent stocks of inventory in the majority of enterprises are discernible in the zero median values of the inventory turnover ratio of the entire industry (between 2008 and 2014) and its subindustries 021 and 024 (between 2007 and 2014). In addition, the descending upper quartile levels falling to zero in 2012 and remaining so until 2014 testify to successful placement of forestry and logging production. More or less, the receivables turnover and payables turnover reveal roughly comparable trends: a slight increase between 2007 and 2009 and then a return to the starting values of 2007 between 2010 and 2014. Focusing on the entire industry, the median receivable collection period measured by receivable turnover in days varied between about 20 to 30 days in 2007 and between 2011 and 2014 whereas the median payable payment period measured by account payable turnover in days was about 70 days in 2007 and 2008 and about 80 days in 2014. The latter activity ratio was systematically higher by about 20 to 60 days. The observed systematic excess of average payment period over average collection period signalizes a satisfactory degree of operational efficiency as far as

payment and collection policies are concerned. Still, the industry and its subindustries have a tendency to have payables high relative to receivables and the disproportion does not appear trifling when comparing the upper quartile values. Such a grave mismatch between payment obligations and receivables to collect may lead to insolvency difficulties and certainly is not favourable in terms of business risk.

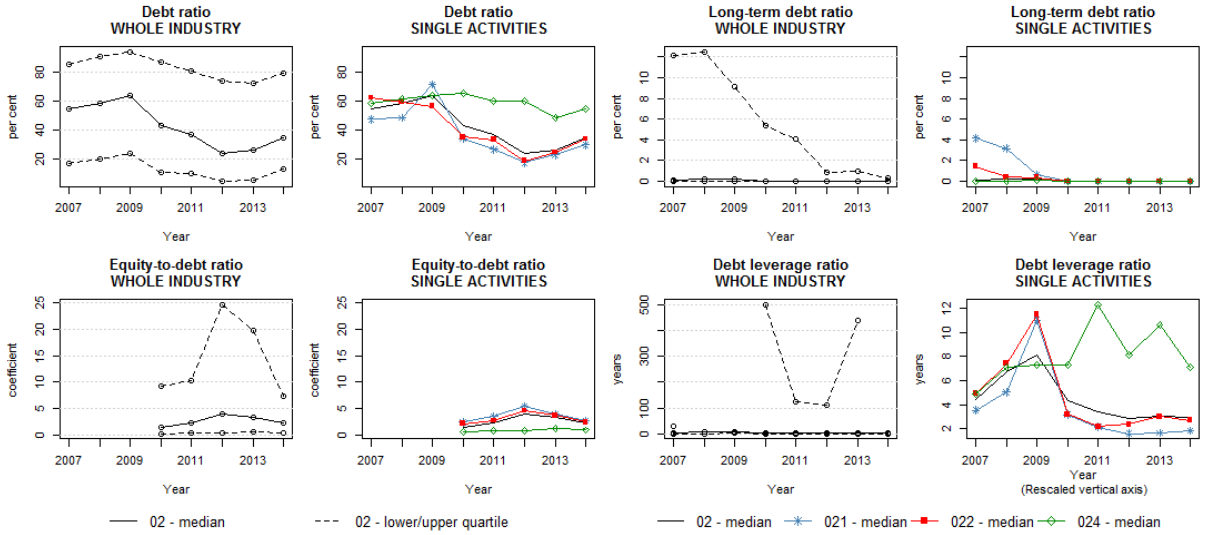
Figure 2: Activity in the forestry and logging industry and its subindustries between 2007 and 2014



Source: the authors.

Figure 3 reflects the fact that the time series of data for equity-to-debt ratio began only in 2010 and that some quartile values of debt leverage ratio were $+\infty$, which are not then displayed. The quartile bands of the debt ratio shows that during the period from 2007 to 2009 Slovak forestry and logging enterprises preferred debt financing over equity financing whereas for the other 5 years most of them were financed more by equity. The general trends of the industry to suppress debt financing were also copied by subindustries 021 and 022. Only subindustry 024 defied them insomuch as the level of debt financing was preserved in the entire period, one half of the enterprises in this subindustry financed more than 60 % of their activities using debt.

Figure 3: Leverage in the forestry and logging industry and its subindustries between 2007 and 2014

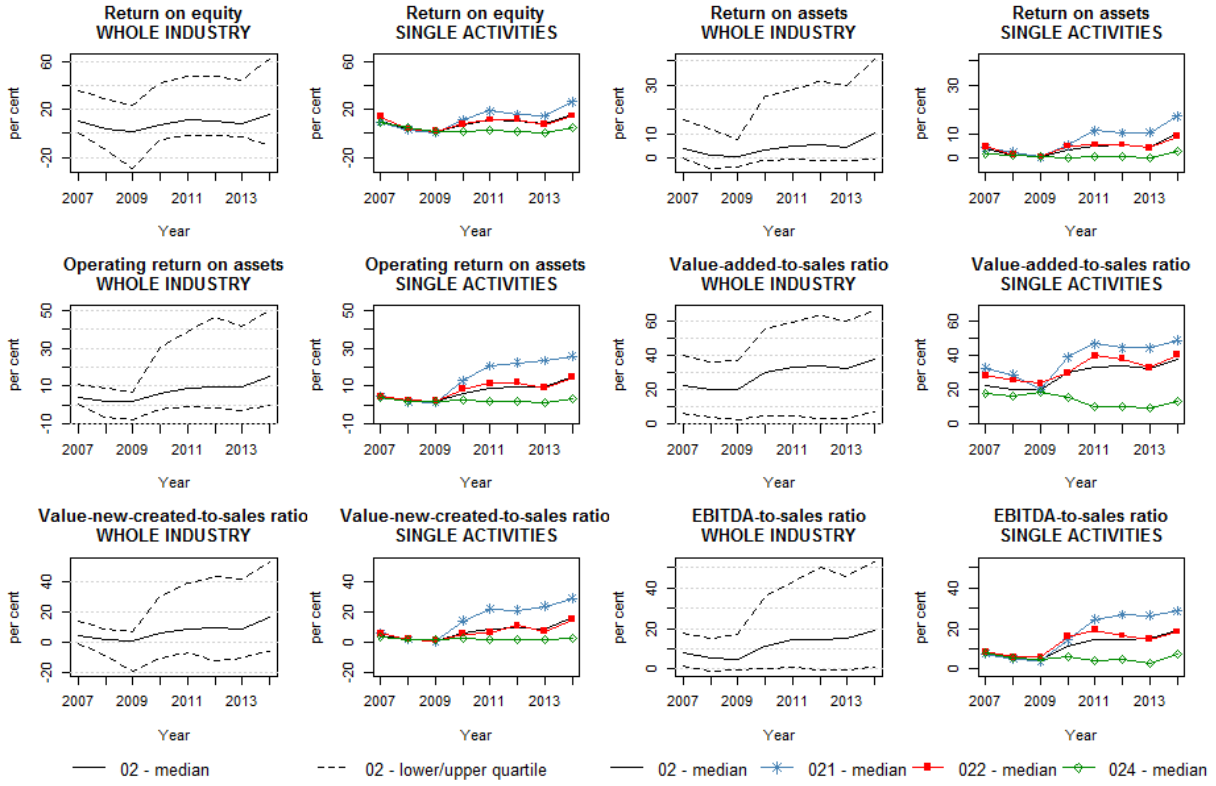


Source: the authors.

An interesting tendency is also unveiled by the long-term debt ratio whose tendency showed growing disfavour toward long-term debt financing. The majority of Slovak enterprises in the forestry and logging industry did not use long-term debt or almost no long-term debt, and if, it was particularly the enterprises affiliated with subindustries 021 and 022 in the first four years of the period of interest. In respect of debt servicing between 2007 and 2009 the trend was unfavourable as the capacity of the industry and equally of its subindustries to pay debts by generated earnings worsened, but as of 2010 it began to recover except for the enterprises in subindustry 024. Between 2012 and 2014 for the majority of enterprises it took less than 1.9 to 3.4 years to produce cash-flow sufficient *ceteris paribus* to completely pay off their debt. Only in subindustry 024 between 2010 and 2014 the pattern was fluctuating and the period necessary to generate cash-flow seems on the whole much longer. All in all, this subindustry tended in the period from 2007 to 2014 to utilize a larger level of debt financing and its cash-flow generating capability appeared lower in comparison to the other subindustries of the Slovak forestry and logging industry.

Figure 4 gives a consonant picture on the profitability in the industry in the 8-year period and on its sources. It is apparent that the profitability of Slovak forestry and logging enterprises either bettered, or at least did not deteriorate during the period in question following the general ordering: subindustry 024 ≤ the entire industry ≤ subindustry 022 ≤ subindustry 021. The symbol "≤" signalizes that the profitability of the enterprises on the left-hand side was not worse than the profitability of the enterprises on the right-hand side. Whilst generally the trends in returns on equity, assets or sales were promising and after a slight retrogression between 2007 and 2009 they were upward, it is again subindustry 024 that escapes the general tendency and the median levels of all the indicators of profitability in fact stagnated with a small revival in 2014. That said, the median levels of return indicate that most enterprises in the Slovak forestry and logging industry performed well and no less than one fourth, as a matter of fact, had performance *par excellence*.

Figure 4: Profitability in the forestry and logging industry and its subindustries between 2007 and 2014



Source: the authors.

5. Conclusion

The paper turned its attention to the identification of the trends that manifested themselves and prevailed in the forestry and logging industry in the time span of 8 years from 2007 to 2014. Toward this end, this industry as well as its subindustries were subjected to a financial analysis based on a selection of traditional financial ratios. It must be accentuated thereupon that the definition of the selected financial ratios obeyed the methodology of the company CRIF - Slovak Credit Bureau, s. r. o. (SCB - Slovak Credit Bureau, 2010; CRIF - Slovak Credit Bureau, 2010) that precludes a possibility to take into full account the specific features with which undertaking in this industry is associated and abounds. Most of them were in the paper highlighted and summarized. For a more complex assessment of potential determinants affecting or triggering financial risks of forestry and logging enterprises it would be desirable in the future to examine the situation and trends in the area of cash-flow generation as well as the effect of many other external factors. Such factors include possible impacts of European Union legislation with a focus upon ecology and sustainability of timber harvesting and production, and especially in consequence of the changing climatic conditions it would be desirable to approach the issue from the standpoint of so-called ecological economics. The reason is that the correct interpretation of the obtained results faces in some respects the question whether the observed condition is on account of the global economic crisis, or an effect of the euro adoption in Slovakia in 2009, or a consequence of the enforcement of the *acquis communautaire* associated with the membership of Slovakia in the European Union.

Another specific of the forestry and logging industry is paradoxically that many enterprises with good economic results and exemplary performance are prone to artificially worsen their actually achieved economic results, by which they would belong to "mediocre" enterprises and by which the value created would pass into other ancillary entrepreneurial subjects or would be further "recycled" during forest restoration. On the other hand, those forestry and logging enterprises whose economic results are unsatisfactory prefer improving them artificially by so-called long-term investment into forests and forestry activities with the intention of distorting financial risks they face.

In their future research the authors of the paper find it also suitable to apply procedures of the so-called American approach of special financial analysis developed for forestry enterprises, which is advocated by Lenoch (2010) as was brought into attention in Sect. 2. This approach brings along additional and explicative information or comments with which they enrich so-called standard financial analysis. Moreover, it stresses the importance of factors that determine the quality of forestry management from an economic point of view and in relation to production and economic potential of forest properties. Special attention must be devoted to profitability analysis, which in this case should be oriented methodologically e.g. on forestry-adjusted earnings proposed by Lenoch (2010).

Acknowledgment

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References

[1] CRIF - Slovak Credit Bureau (2014). *Stredné hodnoty finančných ukazovateľov ekonomických činností v Slovenskej republike za rok 2013*. Bratislava: CRIF - Slovak Credit Bureau.

[2] Hajdúchová, I. (2011). *Finančná stabilita podniku.* Zvolen: Technická univerzita vo Zvolene.

[3] Hajdúchová, I. and Hlaváčková, P. (2014). Impact of global economy of forestry and forest-based industry in the Czech and Slovak Republic. Acta Facultatis Xylologiae Zvolen, 56(2), pp. 135–146.

[4] Hajdúchová, I. et al. (2016). The Slovakian forest-based sector in the context of globalization. *BioResources* 11(2), pp. 4808-4820.

[5] Klieštik, T. and Cúg, J. (2015). Comparison of selected models of credit risk. *Procedia Economics and Finance* 23, pp. 356 – 361.

[6] Lenoch, J. (2009). *Návrh metodiky speciální finanční analýzy lesních majetků*. Brno: Mendelova zemědělská a lesnická univerzita v Brně.

[7] Lenoch, J. (2010). Ekonomika obhospodařování lesních majetků. *Zprávy lesnického výzkumu* 55(1), pp. 59-67.

[8] Levá, M. et al. (2016). Hodnocení rentability u vybraných podniků poskytujících služby lesnímu hospodářství v České republice. *Zprávy lesnického výzkumu* 61(2), pp. 145-150.

[9] Löfgren, K.G. (1983). The Faustmann-Ohlin theorem: A historical note. *History of Political Economy* 15(2), pp. 261-264.

[10] Mura, L. and Buleca, J. (2012). Evaluation of financing possibilities of small and medium industrial enterprises. *Procedia Economics and Finance* 3, pp. 217-222.

[11] Mura L. et al. (2015). Quantitative financial analysis of small and medium food enterprises in a developing country. *Transformation in Business & Economics* 14(1), pp. 212-224.

[12] Potkány, M. et al. (2016). Outsourcing in conditions of SMEs – the potential for cost savings. *Polish Journal of Management Studies* 13(1), pp. 145-156.

[13] SCB - Slovak Credit Bureau (2011). Stredné hodnoty finančných ukazovateľov ekonomických činností v Slovenskej republike za rok 2010. Bratislava: SCB - Slovak Credit Bureau.

[14] Sedliačiková, M. (2011). Prognózovanie finančnej situácie v podniku drevárskeho priemyslu. *Acta Facultatis Xylologiae Zvolen,* 53(2), pp. 93-101.

[15] Sedliačiková, M. et al. (2016). Improving the performance of small and medium wood-processing enterprises. *BioResources* 11(1), pp. 439-450.

Reporting of investments in commercial insurance companies after the amendment of the Act on Accounting in 2016 in the Czech Republic

Jiřina Bokšová[1], Josef Horák [2]

Abstract

The amendment to Act no. 563/1991 Coll., on Accounting, as amended by Act no. 221/2015 Coll., effective from 1st January 2016 brings changes in the valuation of investments at the reporting date in commercial insurance companies and reinsurance companies. Amendment to Decree no. 502/2002 Coll. as amended, which are effective from 1st January 2016 in the decree should reflect changes in the Act on Accounting. However, it is in some respects inconsistent with the amended Act. As a result, commercial insurance companies are not allowed to follow the amended ordinance because it is in conflict with the Act on Accounting. They can not strictly apply the method laid down by decree because new reporting will not provide enough true and fair view of their financial situation. At the same time, they have no support on how to deal with changes to the Act on Accounting in the decree. The paper deals with changes in the presentation of financial placements before and after the amendment to the Act on Accounting, including the impact on the financial position of commercial insurance or reinsurance companies from 1st January 2016.

Key words

commercial insurance companies, land and real estate, legislative changes 2016, reporting of investments

JEL Classification: M41, M42

1. Úvod

Oceňování představuje jeden z nejdůležitějších metodických prvků účetnictví, který má vliv na vypovídací schopnost účetních výkazů. Zvolený způsob oceňování ovlivňuje nejen výši aktiv a pasiv, ale i velikost nákladů a výnosů účetní jednotky, má tedy zásadní vliv na výsledek hospodaření běžného účetního období. Každá regulace účetnictví – ať na národní či nadnárodní úrovni – obsahuje poměrně důkladná ustanovení týkající se oceňování.

2. Prvotní ocenění v účetnictví komerčních pojišťoven

Ocenění při prvotním rozpoznání vychází v České republice z pořizovacích nákladů (cen), které je třeba vynaložit k pořízení aktiva, nebo z odhadu jejich ocenění. Konkrétní způsob ocenění závisí na způsobu pořízení aktiva (vzniku závazku).

K prvotnímu okamžiku ocenění – k okamžiku uskutečnění účetního případu – používají české účetní předpisy následující oceňovací báze:

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[2] Ing. Josef Horák, Ph.D., ŠKODA AUTO University, josef.horak3@savs.cz.

- historické náklady – představují částku peněz či peněžních ekvivalentů, kterou je třeba vynaložit v době pořízení aktiva. V případě závazků vyjadřuje tato hodnota částku přijatého prospěchu získaného za povinnost závazek uhradit;
- běžná vstupní cena (reprodukční cena) – vyjadřuje částku peněz, která by byla v současnosti zaplacena za nabytí ekvivalentního nebo téhož aktiva, či kterou by bylo třeba ke splacení závazku v současné době.

Bázi historických nákladů je možné rozdělit v České republice na pořizovací cenu, vlastní náklady a ocenění nominální hodnotou.

Pořizovací cena představuje ocenění, za které byl majetek pořízen, a to včetně nákladů spojených s pořízením daného majetku. Takto je oceňován v České republice hmotný majetek s výjimkou hmotného majetku vytvořeného vlastní činností, zásob s výjimkou zásob vytvořených vlastní činností, nehmotného majetku kromě pohledávek s výjimkou nehmotného majetku vytvořeného vlastní činností, majetkové podíly, ostatní cenné papíry a deriváty, pohledávky při nabytí za úplatu nebo vkladem a pořízení závazků. Ocenění vlastními náklady se použije v případě, kdy dojde k vytvoření majetku vlastní činností. Nominální hodnota je použita u peněžních prostředků, cenin, pohledávek a závazků při jejich vzniku. [1]

Báze běžné vstupní ceny představuje v ČR reprodukční pořizovací cena. Toto ocenění odpovídá veličině, za kterou by byl majetek pořízen v době, kdy je majetek zařazen do účetnictví účetní jednotky. Ocenění je využíváno především u majetku pořízeného bezúplatně (s výjimkou peněžních prostředků, cenin a pohledávek). Tabulka 1 ukazuje vývoj oceňování při prvotním rozpoznání v komerčních pojišťovnách.

Tabulka 1: Vývoj oceňování finančního umístění k datu uskutečnění účetního případu

Oceňování k datu uskutečnění účetního případu	**Do roku 2002**	**Rok 2002 - 2009**	**Rok 2010 - 2015**	**Rok 2016**
	Historické náklady/ případně běžná vstupní cena			

Zdroj: vlastní zpracování

3. Ocenění k datu účetní závěrky v účetnictví komerčních pojišťoven

Oceňování aktiv a závazků k datu účetní závěrky vychází v souladu s českými účetními předpisy ze dvou koncepcí, a to koncepce historických nákladů a koncepce reálné hodnoty.

Koncepce historických nákladů vychází z původních opatřovacích cen (pořizovací náklady či vlastní náklady, případně z nominální hodnoty) a je možné toto ocenění v závislosti na znehodnocení majetku snižovat v důsledku trvalého snížení hodnoty aktiva (odpisy dlouhodobých hmotných a nehmotných aktiv, odpisy nepoužitelných zásob, odpisy pohledávek) či v důsledku přechodného snížení hodnoty aktiva (pomocí tvorby opravných položek). Pominou-li důvody, v důsledku kterých došlo ke snížení hodnoty aktiva, je možné v České republice opětovně zvýšit hodnotu aktiva pouze však na úroveň pořizovacích nákladů či na úroveň zůstatkové ceny (u odpisovatelných aktiv). [1], [2]

Koncepce reálné hodnoty naopak umožňuje k bilančnímu dni hodnotu majetku snižovat, ale i zvyšovat nad úroveň původních opatřovacích cen. Označení reálná hodnota, dle české účetní legislativy, není však totožné s oceněním fair value podle IFRS. Přestože oba přístupy vychází při stanovení hodnoty ze stejných základů, jsou v některých případech aplikovány v české legislativě odlišným způsobem. [3], [4]

Koncepce ocenění reálnou hodnotou přináší řadu základních otázek. Tento příspěvek si neklade za cíl polemizovat o vhodnosti tohoto ocenění, proto jsou čtenářům pouze naznačena dilemata této koncepce. Z jakého základu mají ekonomické entity vycházet při aplikaci koncepce oceňování v reálné hodnotě k datu účetní závěrky? Mají vycházet při ocenění z podmínek očekávaného specifického užitku z aktiva (či odtoku užitku u závazku) u

konkrétní účetní jednotky (tj. dvě účetní jednotky budou mít stejné aktivum oceněné v reálné hodnotě v jiné výši, protože každá společnost očekává od daného aktiva jiný užitek), nebo by mělo ocenění vycházet z ocenění na tržní bázi, které je nezávislé na konkrétních podmínkách ekonomické entity (dvě společnosti budou mít totožné aktivum oceněné v reálné hodnotě ve stejné výši, přestože každé společnosti bude přinášet jiný užitek? Zde je nezbytné zdůraznit, že očekávání odrážející konkrétní podmínky ekonomické entity je založeno na očekáváních vykazující účetní jednotky, která se mohou významně odlišovat od očekávání, které formují reálnou hodnotu na významném trhu. [5], [6]

Nemohou ekonomické entity pomocí ocenění v reálných hodnotách ovlivňovat výsledek hospodaření běžného účetního období (je-li aktivum přeceňováno na vyšší reálnou hodnotu výsledkově) a ovlivňovat výši vlastního kapitálu (je-li přecenění na vyšší reálnou hodnotu s rozvahovým dopadem). Nepreferují tuto koncepci oceňování vlastníci (akcionáři), vrcholní manažeři účetních jednotek z důvodu získání svých podílů na zisku a odměn za dosažené výsledky hospodaření? Nemohou, tímto způsobem manažeři účetních jednotek, zkreslovat finanční situaci účetní jednotky k rozvahovému dni? Nebylo by uživatelsky přínosnější oceňovat k datu účetní závěrky aktiva a závazky účetní jednotky v účetních výkazech v historických cenách a případný „zisk z držby“, tj. jejich vyšší reálné ocenění zjištěné tomuto datu uvádět pouze v příloze k účetní závěrce?[3] [5], [6]

Neexistují „toxická aktiva“, o kterých hovoří v médiích i čeští představitelé politických stran, to jen zákonodárci umožnili ekonomickým entitám přeceňovat aktiva na vyšší reálné hodnoty. Podnikům se tak v aktivech mohou nacházet položky sice vysoce oceněné (v reálných hodnotách), avšak bez užitku pro danou účetní jednotku (např. bez možnosti prodeje takového aktiva za vykazovanou „reálnou“ cenu). Nyní se podívejme, jak se v historickém vývoji oceňovaly investice k datu účetní závěrky v komerčních pojišťovnách.

3.1 Ocenění reálnou hodnotou k datu účetní závěrky ve vazbě na zákon o účetnictví

Ocenění reálnou hodnotou v účetnictví komerčních pojišťoven upravoval do 31. 12. 2015 zákon o účetnictví v § 27, písm. c: *„finanční umístění, s výjimkou dluhopisů držených do splatnosti, jejichž emitentem je členský stát Organizace pro ekonomickou spolupráci a rozvoj a jejichž hodnocení bylo alespoň dvěma mezinárodně uznávanými ratingovými agenturami stanoveno na úrovni České republiky nebo vyšší, nejedná-li se o finanční umístění vztahující se k závazkům ze životního pojištění, je-li nositelem investičního rizika pojistník, a technické rezervy u účetních jednotek, které provozují činnost pojištění nebo zajištění podle zvláštních předpisů; kromě veřejného zdravotního pojištění;“* [7]

Tento paragraf zákona o účetnictví komerčním pojišťovnám vymezovat povinnost ocenit k datu účetní závěrky téměř všechny složky finančního umístění reálnou hodnotou.

Finančním umístěním (od 1. 1. 2016 nazývané jako „Investice“) se rozumí způsoby a formy umístění prostředků, jejichž zdroji jsou buď pojistně technické rezervy či vlastní zdroje pojišťovny. V České republice je skladba finančního umístění (investic) upravena legislativou pro komerční pojišťovny (zákonem o pojišťovnictví, prováděcí vyhláškou k zákonu o pojišťovnictví). Od roku 2002 je v České republice finanční umístění oceňováno k datu účetní závěrky reálnou hodnotou. Zákon o účetnictví však nikdy nenaznačil cestu, jakým způsobem je možné reálnou hodnotu aktiv a pojistně technických rezerv zjistit či stanovit.

Podle zákona o pojišťovnictví patří do skladby finančního umístění v komerčních pojišťovnách tyto druhy investic:

- pozemky a stavby – jedná se jak o provozní nemovitosti, které pojišťovna používá výhradně k pojišťovací a zajišťovací činnosti, tak o nemovitosti neprovozní,

[3] „Ztráta z držby“ bude vykázána v modelu oceňování v historických cenách k datu účetní závěrky formou tvorby opravné položky (pro aktiva) či tvorbou rezerv (pro závazky).

- podíly, majetkové účasti dlouhodobé/ krátkodobé;
- dluhopisy vydané státy OECD dlouhodobé/ krátkodobé,
- ostatní dluhopisy (držené do splatnosti dlouhodobé/krátkodobé)
- jiná finanční umístění – majetkové cenné papíry k obchodování, ostatní dluhopisy realizovatelné či k obchodování, deriváty, bankovní depozita apod.,
- finanční umístění životních pojištění, je-li nositelem investičního rizika pojistník;
- depozita při aktivním zajištění. [8], [9]

3.2 Oceňování finančního umístění k datu účetní závěrky

Do roku 2002 bylo finanční umístění oceňováno k datu účetní závěrky na bázi historických cen. U odpisovatelného majetku byl vytvořen odpisový plán a každoročně docházelo v účetnictví k trvalému snížení hodnoty tohoto majetku. Při přechodném snížení hodnoty majetku bylo možné tvořit v souladu se zásadou opatrnosti k položkám finančního umístění opravnou položku.

V roce 2002 prošlo oceňování finančního umístění k datu účetní závěrky v komerčních pojišťovnách bouřlivými změnami, v důsledku kterých byla po několik let potlačena meziroční srovnatelnost informací v účetních výkazech. V tomto roce poprvé došlo k přeceňování položek finančního umístění na reálnou hodnotu. Některé položky finančního umístění (např. dlouhodobé cenné papíry) byly přeceňovány přímo do vlastního kapitálu (do fondu z přecenění), jiné položky finančního umístění např. pozemky a nemovitosti byly přeceněny na reálnou hodnotu do výsledku hospodaření běžného účetního období. Protože do roku 2002 se majetek oceňoval k datu účetní závěrky vždy jen v historických nákladech, byla hodnota tohoto majetku v účetních výkazech podhodnocena. Rozdíly z přecenění mezi historickými náklady (pořizovací cenou, vlastními náklady, zůstatkovou cenou, nominální hodnotou) a novou reálnou hodnotou, které byly zachyceny do výnosů, vedly k nadhodnocení výsledku hospodaření běžného období. V tomto období mohly nerealizované zisky z přecenění finančního umístění na vyšší reálnou hodnotu opustit prostřednictvím vykázaného zisku pojišťovnu (mohly být vyplaceny jako dividendy, podíly na zisku). Účetní legislativa tak v roce 2002 umožnila, v souladu se zákonem, tunelování pojišťoven. [10]

Zcela nesmyslně se od roku 2002 začaly přeceňovat na reálnou hodnotu i dluhopisy držené do splatnosti (na rozdíl od IAS 39).

V roce 2003 se česká legislativa pokusila řešit zcela zásadní dopady povolené legislativou v roce 2002. Např. u pozemků a staveb docházelo v tomto roce k přeceňování na reálnou hodnotu ve prospěch/ na vrub fondu z přecenění, které jsou součástí vlastního kapitálu pojišťoven. Při opětovném přecenění na reálnou hodnotu, se původní přeceňovací rozdíl převedl do nákladů/výnosů běžného účetního období a nově vzniklý přeceňovací rozdíl (mezi starší reálnou hodnotou a reálnou hodnotou ke dni účetní závěrky) se vykázal nově ve fondu z přecenění. Došlo jen k posunutí vykázání dopadu do výsledku hospodaření v čase. [11]

V roce 2004 došlo opět ke změnám v oceňování k datu účetní závěrky. Např. v oblasti pozemků a staveb došlo ke změně účetní metody. Pozemky a stavby se od roku 2004 přeceňovaly do kapitálových fondů a nemovitosti se od 1. 1. 2004 v účetnictví komerčních pojišťoven neodepisovaly. Opotřebení nemovitostí se promítalo do reálné hodnoty tohoto majetku, která podléhá výkyvům a meziročně se mohla zvyšovat i snižovat. Snížení reálné hodnoty v důsledku zohlednění opotřebení vedlo ke snížení vlastního kapitálu.

V oblasti finančních aktiv jsou v roce 2004 nadále přeceňovány na reálnou hodnotu všechny cenné papíry, tedy i dluhopisy držené do splatnosti (na rozdíl od IAS a IFRS, kde byly dluhopisy držené do splatnosti vykazovány v amortizované hodnotě).

Tento způsob oceňování finančního umístnění k datu účetní závěrky platil až do roku 2010. V roce 2010 dochází k úpravě zákona o účetnictví a z oceňování reálnou hodnotou k datu

účetní závěrky jsou vyjmuty dluhopisy vydané státy OECD v případě, že jsou k dispozici 2 ratingová hodnocení na úrovni ČR nebo vyšší. Celá česká odborná veřejnost chápala tuto změnu v oceňování dluhopisů vydaných státy OECD jako posun správným směrem. Stále více bylo v této době slyšet hlasy odborné veřejnosti k rozdílnosti oceňování dluhopisů držených do splatnosti. Přeceňováním dluhopisů držených do splatnosti k datu účetní závěrky na reálnou hodnotu docházelo u pojišťoven k umělému ovlivňování velikosti vlastního kapitálu po dobu, než byly dluhopisy za předem dohodnutou cenu vypořádány. [12] Položky finančního umístnění, jako jsou depozita z aktivního zajištění a finanční umístnění jménem pojištěných, byly od roku 2002 až do roku 2015 přeceňovány k datu účetní závěrky na reálnou hodnotu výsledkovým způsobem, tj. s dopadem do výsledku hospodaření běžného účetního období.

4. Oceňování investic k datu účetní závěrky v roce 2016

Zásadní změny přinesla pro oceňování finančního umístění v komerčních pojišťovnách novela zákona o účetnictví, novela prováděcí vyhlášky pro komerční pojišťovny k tomuto zákonu, a to s účinností od 1. 1. 2016[4]. V zákoně o účetnictví došlo ke zrušení odstavce, který nařizoval komerčním pojišťovnám oceňovat k datu závěrky položky investic (finančního umístění) reálnou hodnotou. Z původního znění odstavce zbylo pouze oceňování reálnou hodnotou u pojistně technických rezerv. [13] Pro širokou laickou veřejnost je možné učinit závěr, že změna zákona o účetnictví počínaje rokem 2016 dává na roveň oceňování investic (finančního umístění) jak u podnikatelů, tak u komerčních pojišťoven.

Bohužel, však k této změně zákona nebyla přizpůsobena prováděcí vyhláška k zákonu o účetnictví pro komerční pojišťovny účinná rovněž od 1. 1. 2016. Tím se tyto dvě legislativní normy dostaly do příkrého rozporu. V čísle 1/2016 odborného časopisu Účetnictví – zástupce Ministerstva financí ČR – podal v příspěvku „2016: Novelizovaný zákon o účetnictví a prováděcí vyhláška pro pojišťovny“ přehled oceňování k datu účetní závěrky v roce 2016 u komerčních pojišťoven. V tomto článku uvádí, že: *...v § 27 zákona o účetnictví bylo „vymazáno“ dosavadní oceňování investic reálnou hodnotou a to tak, že nakonec odstavec 1 písm. c) ve znění zákona o účetnictví č. 221/2015 Sb., upravuje oceňování technických rezerv“*. Příspěvek toto hodnotí jako omyl a dále nabádá k tomu, aby pojišťovny zachovaly dosavadní rozsah oceňování finančního umístění (investic) reálnou hodnotou. [7], [14]

Tyto vzájemně rozporuplné informace ohledně oceňování finančního umístění počínaje rokem 2016 připravily komerčním pojišťovnám složitou situaci. Platí novela zákona o účetnictví a k němu nově přijatá prováděcí vyhláška má věcné chyby nebo vypadl paragraf o finančním umístění z novely zákona o účetnictví a nově přijatá prováděcí vyhláška k zákonu o účetnictví je ve správném znění. Kde se stala chyba. Čím se mají komerční pojišťovny řídit? Dle příspěvku pracovníka ministerstva financí v časopisu Účetnictví 1/2016 to vypadalo, že chyba byla na straně zákona. [14]

V březnu 2016 se k tomuto problému sešli zástupci Ministerstva financí ČR a ekonomická sekce České asociace pojišťoven. Závěr tohoto jednání byl, že novelizovaný zákon o účetnictví je v pořádku a chyby vykazuje novelizovaná vyhláška. Dne 19. května 2016 vyšlo ve Finančním zpravodaji sdělení, které by mělo komerčním pojišťovnám pomoci s vyřešením zásadního problému, jak se mají v roce 2016 oceňovat k datu účetní závěrky složky finančního umístění. Jak se tedy budou v roce 2016 oceňovat jednotlivé druhy investic? [15]

[4] Novela zákona o účetnictví přinesla řadu změn, jako např. rozdělení účetních jednotek na mikro, malé, střední a velké, změnila rozsah zveřejňovaných součástí účetní závěrky.

Z důvodu omezeného rozsahu příspěvku bude legislativní změna názorně ukázána u pozemků a staveb vykazovaných v účetních závěrkách komerčních pojišťoven.

5. Pozemky a stavby, popřípadě jiné věci nemovité

Z Tabulky 2 vyplývá, že pozemky a stavby byly v roce 2002 přeceňovány na reálnou hodnotu výsledkově a ovlivnily výsledek hospodaření běžného účetního období (podhodnocené rozvahy v historických cenách vedly k nárůstu hodnoty aktiv v řádek 10 – 100 %) a mohly být vyplaceny akcionářům ve formě dividend a podílů na zisku.

V roce 2002 a 2003 se stavby a nemovitosti odepisovaly z původní historické ceny (korekce v rozvaze), ale oceňovaly se reálnou hodnotou. Hodnota sloupce brutto představovala součet reálné hodnoty a oprávek, sloupec korekce vyjadřoval dosavadní oprávky a netto hodnota představovala reálnou hodnotu majetku k datu účetní závěrky. Je možné konstatovat, že informace zveřejněné v účetních závěrkách postrádaly potřebnou vypovídací schopnost o finanční situaci pojišťovny. [16], [17]

Ministerstvo financí vyřešilo tuto situaci v roce 2004 tak, že se nemovitosti v komerčních pojišťovnách přestaly účetně odepisovat. Daňové odepisování majetku samozřejmě pokračovalo. Tím, že se účetní odpisy nepromítly na vrub nákladů, realizovaly pojišťovny vyšší výsledek hospodaření běžného účetního období, který se opět mohl vyvést z účetní jednotky ve formě podílů na zisku směrem k akcionářům.

Tato situace trvala až do roku 2016, kdy se v oceňování této položky aktiv legislativa vrací před rok 2002, tj. oceňování k datu závěrky historickou cenou.

Jak však mají pojišťovny postupovat při změně oceňování této složky finančního umístění. Výchozí ocenění pozemků a staveb pro rok 2016 bude reálná hodnota těchto aktiv k 31. 12. 2015. Je nezbytné si uvědomit, že proti „přeceněným“ aktivům stojí na straně vlastního kapitálu položka – ostatní kapitálové fondy neboli fond z přecenění. Jak se tento fond rozpustí?

Tabulka 2: Vývoj oceňování pozemků a staveb k datu účetní závěrky u komerčních pojišťoven v ČR

<table>
<tr><th colspan="7">Oceňování k datu účetní závěrky</th></tr>
<tr><th>ROK</th><th>do 2002</th><th>2002</th><th>2003</th><th>2004</th><th>2010 - 2015</th><th>2016</th></tr>
<tr><td rowspan="2"></td><td>Historická nebo tržní cena (nižší z nich)</td><td colspan="4" rowspan="2"></td><td>Historická nebo tržní cena (nižší z nich)</td></tr>
<tr><td>Pozemky, stavby</td><td>Pozemky, stavby</td></tr>
<tr><td colspan="2">Rozdíl z přecenění na RH:</td><td colspan="4">Reálná hodnota (RH)</td><td></td></tr>
<tr><td colspan="2">do vlastního kapitálu (fond z přecenění)</td><td></td><td>Pozemky, stavby</td><td>Pozemky, stavby</td><td>Pozemky, stavby</td><td></td></tr>
<tr><td colspan="2">do výsledku hospodaření běžného roku</td><td>Pozemky, stavby</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Odpisy</td><td colspan="3">Ano, z historické ceny</td><td>Ne</td><td>Ne</td><td>Není zatím vyřešeno</td></tr>
<tr><td>Opravné položky</td><td>Ano</td><td colspan="3">Ne</td><td>Ne</td><td>Ano</td></tr>
</table>

V případě, že dojde v průběhu roku 2016 k prodeji pozemků či nemovitostí, je fond z přecenění rozpuštěn do výkazu zisku a ztráty a zpětně ovlivní výši vlastního kapitálu pojišťovny. V případě, že se bude jednat o pozemky či nemovitosti, které bude pojišťovna vykazovat ve svém portfoliu k datu závěrky, pak by mělo dojít ke zrušení této položky se souvztažným zápisem na konkrétní majetkový účet (pozemky, nemovitosti). Tímto způsobem dojde zcela jednoznačně ke změně výše vlastního kapitálu pojišťovny. V případě, že fond z přecenění vykazuje zůstatek na straně DAL, pak výsledný dopad bude snížení výše vlastního kapitálu komerční pojišťovny. [15] Jestliže se mají pojišťovny vrátit k oceňování pozemků a staveb na historické ceny, pak bude v praxi bezesporu problém s nemovitostmi, které se v historickém modelu odepisují. Mají pojišťovny nasimulovat odpisové plány od roku 2004, který byl prvním rokem, kdy se přestalo s účetním odpisováním těchto aktiv? Takto nasimulované odpisy by se proti oprávkám vykázaly v ostatním výsledku hospodaření a snížily by výši vlastního kapitálu, a tak solventnosti pojišťovny.

A jak to bude v případě, že pojišťovny znají historické ceny pozemků a staveb, ale v roce 2002 je přecenily na nové vyšší (v řádek 10 – 100 %) ceny do výnosů roku 2002, a dosáhly tak vysokých výsledků hospodaření za rok 2002? Mělo by skutečně snížit ocenění až na historické ceny a přecenění z roku 2002 nyní z majetkových účtů a zachytit na vrub vlastního kapitálu? Je toto reálně možné? Na jaké ocenění se u položky pozemků a nemovitostí vlastně mohou komerční pojišťovny vrátit? Toto všechno jsou v současné době otevřené otázky, jejichž řešením se budou zabývat pojišťovny při sestavení účetní závěrky za rok 2016.

6. Závěr

Cílem příspěvku bylo poukázat na vývoj v oceňování investic v komerčních pojišťovnách od roku 2002 do současnosti a upozornit na úskalí, které s sebou přináší koncepce oceňování reálnou hodnotou k datu účetní závěrky. Položky investic byly v komerčních pojišťovnách počínaje rokem 2002 přeceňovány k datu účetní závěrky na reálnou hodnotu. Od roku 2010 se vykazovaly k datu účetní závěrky v historických cenách, při splnění určitých podmínek, dluhopisy držené do splatnosti emitované státy OECD. Počínaje rokem 2016 se v komerčních pojišťovnách výrazně změnila koncepce oceňování rozvahových položek k datu účetní závěrky.

Novela zákona o účetnictví č. 563/1991 Sb., v platném znění a prováděcí vyhlášky č. 502/2002 Sb., v platném znění vyvolala v lednu 2016 v odvětví komerčních pojišťoven velké otazníky. Obě právní normy byly ve vzájemném rozporu a několik měsíců (leden – duben 2016) nebylo vůbec jasné, která z obou norem vykazuje chybná ustanovení. Zaměstnanci ministerstva financí se domnívali, že pochybení je v paragrafu zákona o účetnictví, a doporučovali pojišťovnám oceňovat investice v reálných hodnotách, s tím, že toto pochybení bude napraveno. Finanční ředitelé komerčních pojišťoven, kteří se řadu let potýkali s nesmyslným přeceňováním některých položek investic na reálnou hodnotu, vycházeli z toho, že chybné paragrafy obsahuje prováděcí vyhláška k zákonu o účetnictví. Vše vyřešila až společná schůzka zástupců Ministerstva financí ČR a ekonomické sekce ČAP. Výsledky tohoto jednání byly zveřejněny ve finančním zpravodaji v květnu 2016 s tím, že platí ustanovení zákona o účetnictví a chybné paragrafy obsahuje vyhláška.

Toto rozhodnutí, které se vrací u některých položek investic (finančního umístění) k oceňování k datu účetní závěrky na historickou cenu, s sebou v tomto roce přinese ještě řadu otázek, které budou muset být spolu s auditory na konci právě probíhajícího účetního období v jednotlivých komerčních pojišťovnách vyřešeny.

Autoři tohoto článku jednoznačně podporují oceňování investic k datu účetní závěrky v historických cenách, neboť přeceňování na vyšší reálné hodnoty především s dopadem do

výsledku hospodaření vede k možnosti „vyvádění aktiv“ z účetní jednotky. Stát by se měl snažit ochraňovat zájmy velké většiny pojištěnců minimálně tím, že legislativně neumožní výplaty nerealizovaných zisků. K tomu, aby pojišťovna informovala uživatele účetní závěrky o reálné hodnotě vykazovaných investic, by měla jednoznačně sloužit pouze příloha k účetní závěrce.

References

[1] Bokšová, J. (2010). *Účetnictví komerčních pojišťoven*. Praha: Wolters Kluwer.

[2] Kovanicová, D. (2005). *Finanční účetnictví světový koncept*. Praha: Polygon.

[3] Kovanicová, D. (2009). Fair Value in a New Overcoat? About IFRS ED “Fair Value Measurement”. *Účetnictví.*

[4] Mezinárodní standardy účetního výkaznictví (IFRS) 2005: včetně Mezinárodních účetních standardů (IAS) a Interpretací k 1. lednu 2005 : úplné znění všech Mezinárodních standardů účetního výkaznictví k 1. lednu 2005. Praha: Svaz účetních.

[5] Catty, J. P. (2010). *Wiley Guide to Fair Value Under IFRS: International Financial Reporting Standards*. New York: Wiley.

[6] Chorafas, D. N. (2006). *IFRS, Fair Value and Corporate Governance: The Impact on Budgets, Balance Sheets and Management Accounts*. Oxford: CIMA Publishing.

[7] Zákon č. 563/1991 Sb., o účetnictví, ve znění zákona č. 221/2015 Sb.

[8] Ducháčková, E. (2009). *Principy pojištění a pojišťovnictví*. Praha: Ekopress, s. r. o.

[9] Meluchová, J. (2009). *Accounting and Reporting of Insurance Companies under IFRS*. Bratislava: Wolters Kluwer.

[10] *Discussion Paper: Fair value measurements* (2007). London: IASB.

[11] Bokšová, J. (2008). Placement and Technical Reserves in Insurance Accounting. *European Financial and Accounting Journal*, 3(2), pp. 87-100.

[12] Bokšová, J. (2011). Measurement Issues in Financial Placement and Technical Provisions of Czech Insurance Companies. *European Financial and Accounting Journal*, 3.

[13] Bokšová, J. (2008). Oceňování pojistných závazků. *Český finanční a účetní časopis*, pp. 54-61.

[14] Huleš, J. (2016). Novelizovaný zákon o účetnictví a prováděcí vyhláška pro pojišťovny. *Účetnictví*, 1, pp. 22-26.

[15] Finanční zpravodaj MF 06/2016, vydaný 19. května 2016.

[16] Vyhláška č. 502/2002 Sb., ve znění vyhlášky č. 252/2015 Sb.

[17] Vyhláška 434/2009 Sb., kterou se provádí zákon o pojišťovnictví.

Insurance Market Assessment in the Czech Republic

Martina Borovcová [1]

Abstract

Insurance is one of the most important sectors of the economy and has an irreplaceable position in the national economy. The insurance market is very much intertwined in the financial markets, therefore assessment its level is important. The assessment and analysis of the instance market performs assess the level of insurance market using indicators (claims frequency ratio, written premium ratio, insurance payment ratio, Herfindahl-Hirschman index, et al.). Paper is focused on the insurance market assessment in the Czech Republic on 1996-2015years.

Key words

Insurance market, indicators of insurance market, insurance market assessment, written premium ratio, insurance payment ratio.

JEL Classification: G22, G19

1. Úvod

V rozvinutých ekonomikách patří pojišťovnictví mezi jejich důležité oblasti a podobně jako např. bankovnictví je, s ohledem na realizované činnosti a nabízené produkty, oblastí regulovanou. Pojistný trh tedy podléhá sledování, analýzám a hodnocením a hodnocena a analyzována je jak úroveň pojistného trhu, tak jaho vývoj. Mimo jiných přístupů je možné k posouzení a hodnocení úrovně pojistného trhu použít ukazatele hodnocení úrovně pojistného trhu. Ty je možné obecně rozdělit na ukazatele kvalitativní (ukazatel škodovosti, ukazatel pojištěnosti) a ukazatele kvantitativní (ukazatel hrubého předepsaného pojistného, ukazatel pojistného plnění, ukazatel počtu komerčních pojišťoven, ukazatel počtu uzavřených pojistných smluv, ukazatel počtu vyřízených pojistných událostí, ukazatel počtu zaměstnanců v pojišťovnictví či ukazatel koncentrace pojistného trhu). V příspěvku je hodnocen trh jak pomocí výše uvedených ukazatelů kvalitativních a kvantitativních, tak pomocí ukazatelů modifikovaných (ukazatel průměrného předepsaného pojistného, ukazatel průměrného pojistného plnění) a to v letech 1996-2015. Vývoj ukazatelů je sledován jak za pojistný trh celkem, tak za jeho jednotlivá odvětví, životní pojištění a neživotní pojištění.

2. Popis vybraných ukazatelů hodnocení pojistného trhu

Většina kvantitativních ukazatelů je vyjádřena hodnotou, jejíž interpretace není, sama o sobě, složitá či problematická. V případě kvalitativních ukazatelů je nutné tyto nejprve popsat a vysvětlit jejich konstrukci pro snazší interpretaci výsledku. V níže uvedeném textu je proto popsán ukazatel škodovosti, pojištěnosti, ale rovněž Herfindahlův-Hirschmanův index, vyjadřující úroveň koncentrace pojistného trhu.

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2.1 Ukazatel škodovosti

Škodovost vyjadřuje poměr mezi objemem poskytnutých pojistných plnění a výší hrubého předepsaného pojistného za sledované období, jak je uvedeno ve vzorci (1). Ukazatel je zpravidla sledován jak za jednotlivá odvětví pojištění, tak za trh jako celek.

$$Škodovost = \frac{PPL}{HPP} \cdot 100, \tag{1}$$

kde *PPL* je označením pro pojistná plnění a *HPP* je vyjádřením hrubého předepsaného pojistného. Snahou by měl být klesající vývoj ukazatele. Škodovost na úrovni 100% či vyšší signalizuje ekonomicky nevyrovnané odvětví pojištění či trh, kdy výdaje na pojistná plnění z daného pojištění jsou stejné či podstatně vyšší než strana příjmová, zastoupená přijatým pojistným. Důvodem k tolerovatelným krátkodobým výkyvům ve výši ukazatele škodovosti mohou být živelní události, přírodní katastrofy apod.

2.2 Ukazatel pojištěnosti

Ukazatel pojištěnosti umožňuje zjistit, v jaké míře, či s jakou intenzitou, jsou na sledovaném území využívány pojistné produkty. Ukazatel je vyjádřen poměrem velikosti hrubého předepsaného pojistného k hrubému domácímu produktu v běžných cenách.

$$Pojištěnost = \frac{HPP}{HDP} \cdot 100, \tag{2}$$

kde *HDP* je označení hrubého domácího produktu v běžných cenách. Čím vyšších hodnot ukazatel pojištěnosti dosahuje, tím silnější je postavení pojištění v ekonomice a za žádoucí vývojovou tendenci je tak považován růst hodnoty ukazatele. Tento ukazatel je označován za jeden z nejvýznamnějších ukazatelů úrovně a rozsahu pojistného trhu.

2.3 Herfindahlův-Hirschmanův index

Stupeň či intenzita koncentrace pojistného trhu v závislosti na tržním podílu jednotlivých pojistitelů je zjistitelná pomocí ukazatele koncentrace pojistného trhu. Ten má podobu Herfindahlova-Hirschmanova indexu (HHI), vypočteného dle vztahu (3).

$$HHI = \sum_{i=1}^{n} x_i^2, \tag{3}$$

kde *n* je počet všech pojistitelů v odvětví a x_i^2 je podíl *i*-tého pojistitele na celkovém obratu odvětví. Herfindahlův-Hisrchmanův index je založený na předpokladu, že konkurenční síla roste s druhou mocninou tržního podílu. Maximální výše ukazatele může nabývat hodnoty 10 000, a v takovém případě existuje ve sledovaném odvětví pouze jeden pojišťovací subjekt se stoprocentním tržním podílem. Ostatní výsledné hodnoty indexů jsou zařazeny do jednoho ze tří pásem. V tabulce 1 jsou uvedeny původní hodnoty intervalů HHI, ale i hodnoty aktuální, tak jak je doporučuje a používá např. Antimonopolní divize ministerstva spravedlnosti USA a stejně tak úřady ostatních států OECD.

Table 1: Intervaly pro hodnoty Herfindahlova-Hischmanova indexu

Stupeň koncentrace trhu	Původní intervaly hodnot HHI	Aktuální intervaly hodnot HHI
Monopol	HHI = 10 000	HHI = 10 000
Vysoce koncentrované odvětví	HHI > 1 800	HHI > 2 500
Středně koncentrované odvětví	1 000 < HHI < 1 800	1 500 < HHI < 2 500
Nízko koncentrované odvětví	HHI < 1 000	HHI < 1 500
Dokonalá konkurence	HHI = 0	HHI = 0

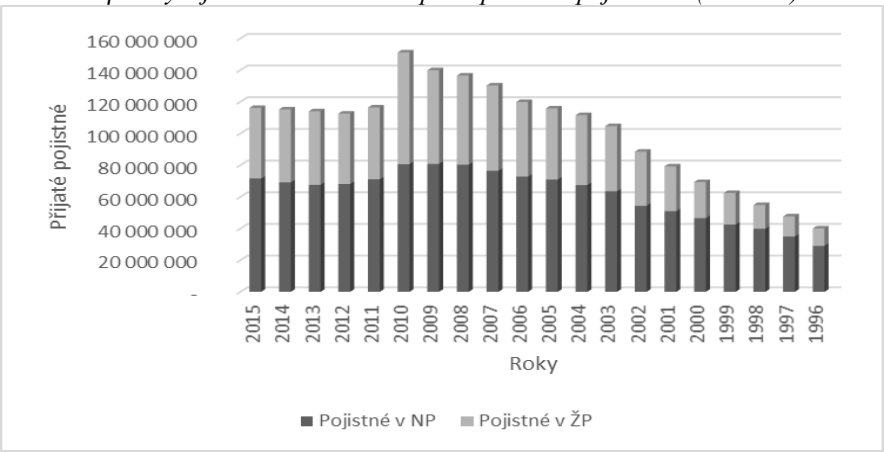
3. Hodnocení pojistného trhu v České republice

K hodnocení úrovně pojistného trhu v České republice jsou použity jak ukazatele kvantitativní, tak ukazatele kvalitativní. Konkrétně je trh hodnocen na bázi vývoje ukazatelů hrubého předepsaného pojistného, pojistného plnění, počtu komerčních pojišťoven, koncentrace pojistného trhu, počtu uzavřených pojistných smluv, počtu vyřízených pojistných událostí, počtu zaměstnanců, škodovosti a pojištěnosti,. K hodnocení úrovně trhu jsou použity rovněž modifikované ukazatele průměrného předepsaného pojistného a průměrného pojistného plnění. Vývoj je analyzován v období let 1996-2015 na základě dat České asociace pojišťoven.

3.1 Hodnocení pojistného trhu dle kvantitativních ukazatelů

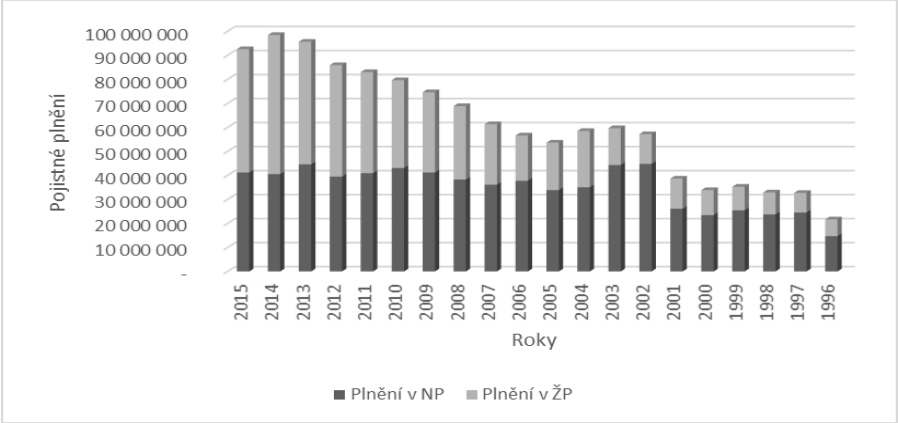
K hodnocení úrovně jsou zprvu použity ukazatele kvantitativní a to ukazatel hrubého předepsaného pojistného, ukazatel pojistného plnění, ukazatel počtu komerčních pojišťoven, ukazatel koncentrace pojistného trhu, ukazatel počtu uzavřených pojistných smluv, ukazatel počtu vyřízených pojistných událostí a ukazatel počtu zaměstnanců.

Graf 1: Vývoj ukazatele hrubého předepsaného pojistného (v tis. Kč)



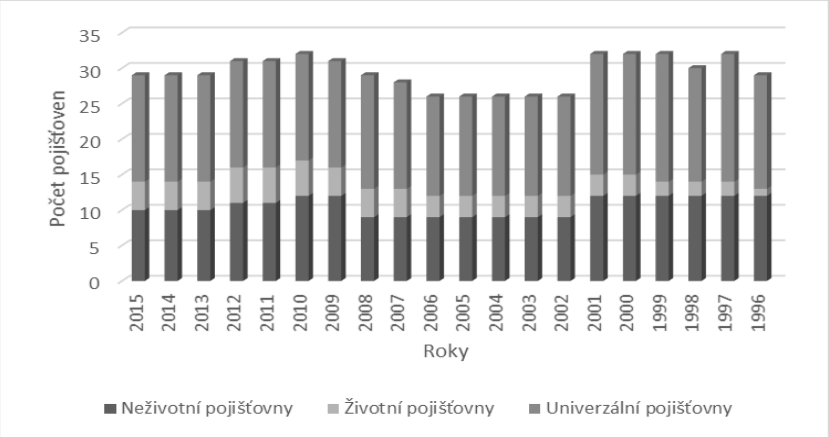
Vývoj ukazatele vykazuje, vyjma roku 2011 a 2012, růst, i když se zcela odlišnou dynamikou v letech 1996-2010 a v obdobích následujících. A to jak globálně, tak v jednotlivých odvětvích pojištění. V celém sledovaném období se na celkovém hrubém předepsaném pojistném zásadním způsobem podílí pojistné vybrané v NP. Propad v pojistném, který nastal v roce 2011, je možné připsat především vývoji ve výběru pojistného v životním pojištění.

Graf 2: Vývoj ukazatele pojistného plnění (v tis. Kč)



Vývoj ukazatele za celý pojistný trh je téměř po celé sledované období rostoucí, což pro hodnocení úrovně pojistného trhu není pozitivním zjištěním, protože pojišťovny musejí vyplácet stále větší objemy prostředků ve formě pojistných plnění a to jak úhrad reálných škod v neživotním pojištění, tak výplat pojistných částek v pojištění životním. Zatímco v neživotním pojištění je vývoj hodnoty ukazatele rozkolísaný, v životním pojištění je zřejmý stálý růst. Po roce 2010 se pak pojistné plnění vyplacené v životním pojištění stává převažující hodnotou. To však může mít opodstatnění v samotném principu životního pojištění, a pokud by pak rostl počet uzavřených pojištění a počet vyřízených pojistných událostí, je logické předpokládat růst i v hodnotě vypláceného pojistného plnění.

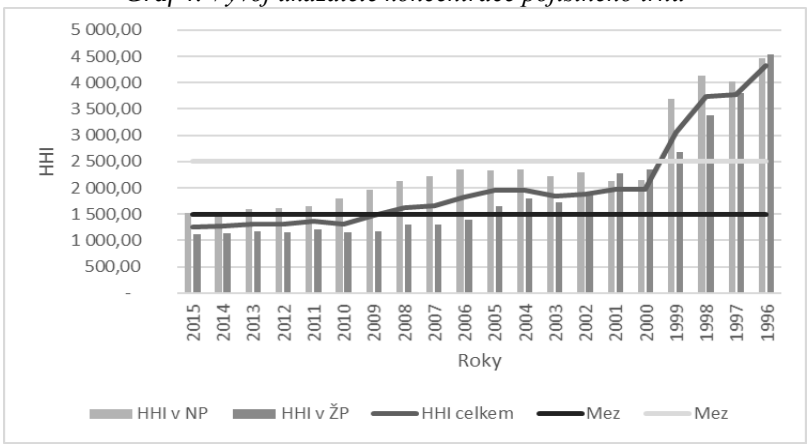
Graf 3: Vývoj ukazatele počtu komerčních pojišťoven



Mezi pojišťovnami, které jsou členy České asociace pojišťoven, převažují, a to ve všech sledovaných obdobích, pojišťovny univerzální. Ve všech sledovaných obdobích je také na trhu více pojišťoven, které nabízení produkty neživotního pojištění. A i když je vývoj počtu pojišťoven ukazatelem s ne zcela jednoznačnou vývojovou tendencí, ve všech obdobích je na pojistném trhu dostatečný počet subjektů zabezpečujících nabídku produktů jak životního, tak neživotního pojištění. Pokud by byl sledován ukazatel počtu pojišťoven dle kapitálu, pak převažují tuzemské pojišťovny, ovšem s převahou zahraničního kapitálu.

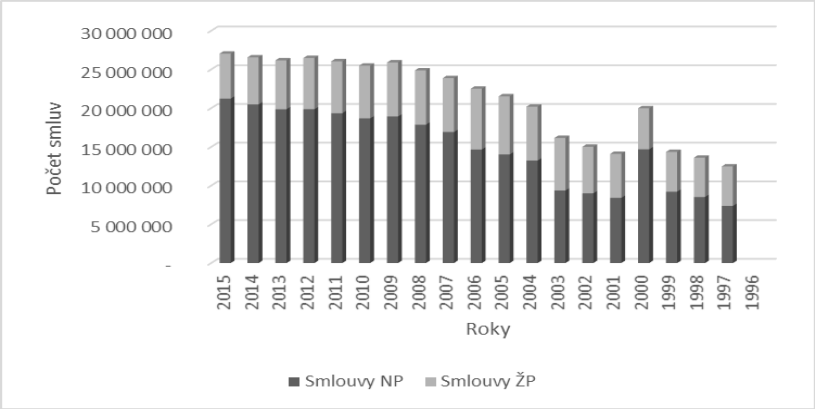
Zajišťovala-li by pojištění na trhu jediná pojišťovna, šlo by o monopol. Období monopolu však v českém pojišťovnictví skončilo rokem 1991. Jak je zřejmé z vývoje hodnot Herfindahlova-Hirschmanova indexu, je možné od roku 2000 považovat český pojistný trh za středně koncentrované odvětví a od roku 2009 dokonce za odvětví s nízkou koncentrací.

Graf 4: Vývoj ukazatele koncentrace pojistného trhu



Téměř v celém analyzovaném horizontu je více koncentrován trh s neživotním pojištěním. Důvodem byl a stále je produkt pojištění odpovědnosti zaměstnavatele za pracovní úrazy a nemoci z povolání, realizovaný, z rozhodnutí státu, dvěma pojišťovnami, které tak mají podstatný tržní podíl, promítající se právě do hodnoceného ukazatele.

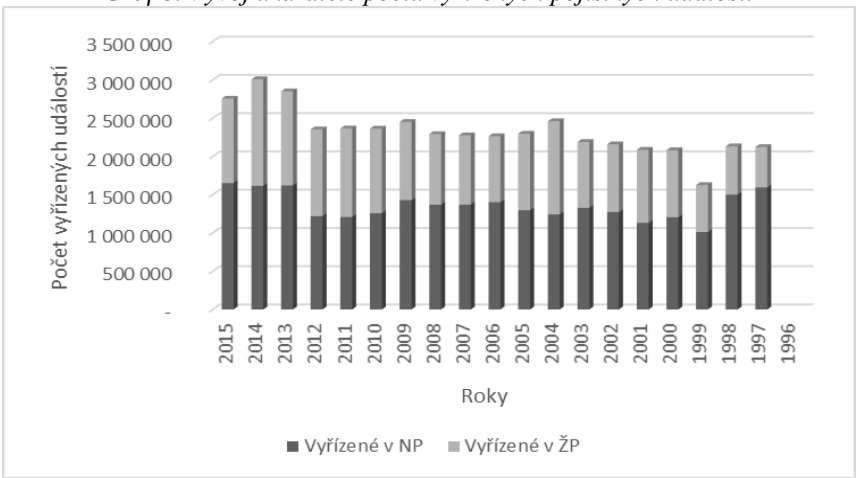
Graf 5: Vývoj ukazatele počtu uzavřených pojistných smluv



Rostoucí počet uzavřených smluv vypovídá o zvýšeném zájmu o pojištění a vývoj hodnot ukazatele tak lze považovat za pozitivní. Podstatnou měrou se přitom na celkovém počtu uzavřených smluv podílí uzavřené smlouvy na neživotní pojištění. Pokles podílu smluv na pojištění životní je markantní, ze zhruba 45% podílu na 20%.

S růstem počtu uzavřených smluv by mělo růst předepsané pojistné a následně i pojištěnost. Zjištěný vývoj hrubého předepsaného pojistného tomu však neodpovídá, dá se tedy předpokládat, že jsou uzavírány smlouvy s nižšími pojistnými částkami či na méně rizik a proto jsou stanovovány nižší hodnoty pojistného, případně může tato skutečnost vypovídat o pojistných podvodech. To však musí následně logicky vést k problému ve vývoji ukazatele pojištěnosti.

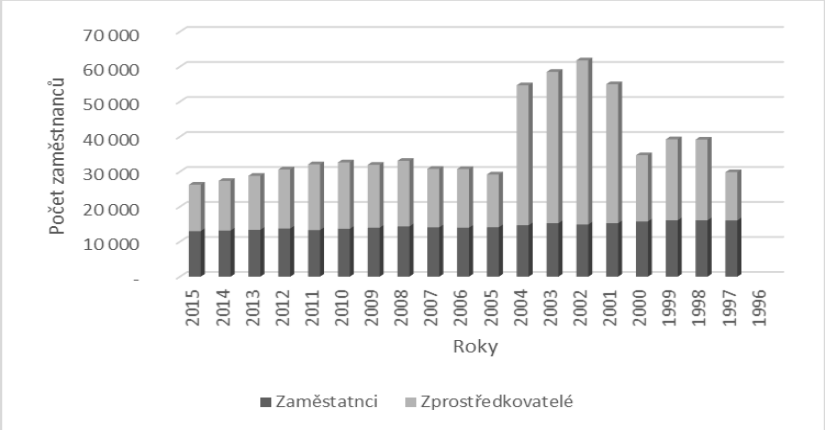
Graf 6: Vývoj ukazatele počtu vyřízených pojistných událostí



Vyřízení pojistné události je spojeno s vyplacením pojistného plnění. Než k tomuto dojde, musí být pojistná událost nahlášena a zahájeno a ukončeno šetření nutné ke stanovení povinnosti pojišťovny plnit. Hodnoty, zachycené v grafu 6 jsou pak výstupem takto probíhajícího procesu likvidace pojistných událostí v pojišťovnách. V tomto případě je však při hodnocení vývoje ukazatele nutno brát v potaz odlišný charakter odvětví pojištění.

V případě neživotního pojištění je vyřízenou pojistnou událostí např. rozhodnutí o výplatě škody vzniklé havárií, krádeží apod., které se navíc mohou opakovat. V případě životního pojištění, však jde o vyplacení úspor, pokud se pojištěná osoba dožila ve smlouvě stanoveného dne, nebo o výplatu pojistné částky, pakliže pojištěná osoba zemřela. Vyšší hodnoty ukazatele v odvětví neživotního pojištění mohou souviset s opakujícími se povodněmi (1997, 2002, 2009, 2013). Hodnoty vztahující se k životnímu pojištění takové růstové výkyvy nezaznamenaly.

Graf 7: Vývoj ukazatele počtu zaměstnanců

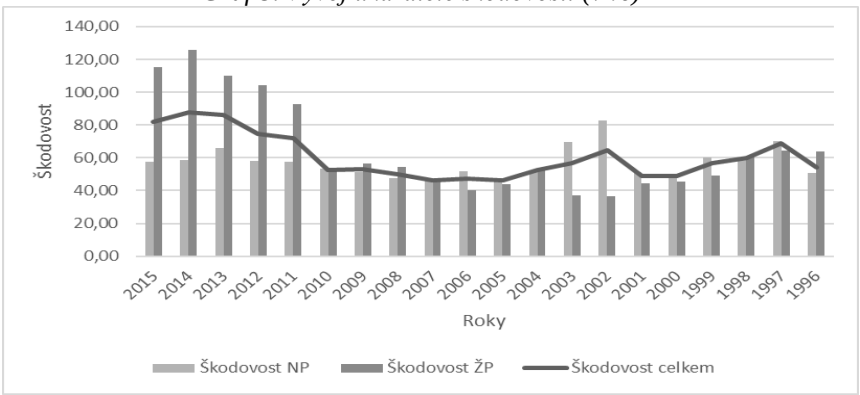


Snaha o snižování počtu zaměstnanců je zřetelná i v oblasti pojišťovnictví. Rozvoj technologií umožňuje uzavírání pojištění řady produktů on-line, navíc úspory v nákladech pojišťoven, stejně jako v jiných odvětvích ekonomiky jsou, v první řadě, spojeny se zefektivňováním práce a snižováním počtu zaměstnanců, snižováním počtu poboček, apod. Jak z údajů v grafu 7 vyplývá, počet zaměstnanců se za sledované období téměř plynulým vývojem snížil zhruba o čtvrtinu. Zajímavým je však vývoj počtu zprostředkovatelů pojištění. Jak uvádí Majtánová, Vachálková (2011) jsou zprostředkovatelé nedílnou součástí systému pojištění a jejich služeb využívá okolo 70% subjektů při uzavírání pojistných smluv. Zprostředkování by tak mělo být činností vysoce odbornou a kvalifikovanou. Ze strany regulátora jsou nyní zřejmé tendence o zkvalitnění zprostředkovatelské činnosti.

3.2 Hodnocení vývoje pojistného trhu dle kvalitativních ukazatelů

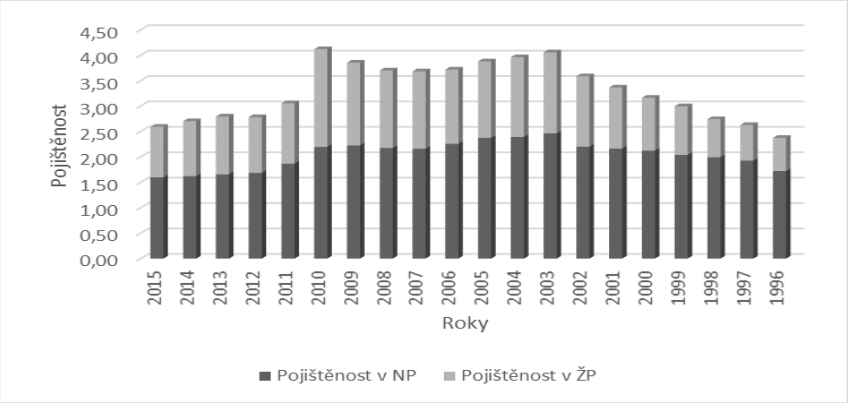
K hodnocení úrovně pojistného trhu v České republice jsou použity dva základní kvalitativní ukazatele hodnocení úrovně pojistného trhu a to ukazatel škodovosti a ukazatel pojištěnosti. Vývoj hodnot ukazatelů je zachycen v grafu 8 a 9.

Graf 8: Vývoj ukazatele škodovosti (v %)



Interpretace vývoje ukazatele, s ohledem na jeho konstrukci, musí zohledňovat jak vývoj ukazatele hrubého předepsaného pojistného (viz graf 1), tak vývoj ukazatele vyplaceného pojistného plnění (viz graf 2). Negativní vývoj ukazatele celkové škodovosti, zvláště pak po roce 2010, je způsoben škodovostí v ŽP, kdy výše vybraného pojistného není dostatečná s ohledem na objem vyplácených pojistných plnění. A i když v posledním sledovaném roce nastává mírné zlepšení, stále je hodnota ukazatele škodovosti v ŽP vysoko nad sto procenty. Dá se předpokládat, že vývoj může stimulovat pojistitele ke zdražování nabízených komerčních produktů, převážně pak v ŽP.

Graf 9: Vývoj ukazatele pojištěnosti (v %)

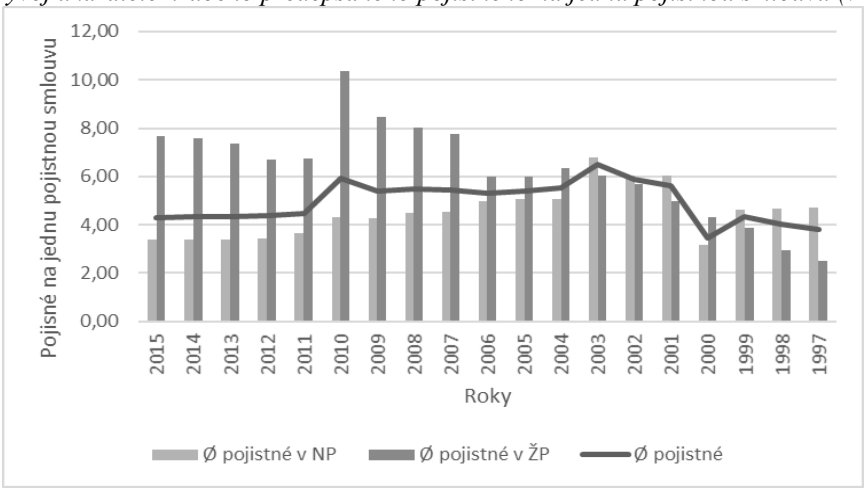


Dle vývoje ukazatele pojištěnosti je možné konstatovat, že nejvíce bylo pojištění využíváno, dle analyzovaných dat České asociace pojišťoven, v letech 2003-2010, poté hodnota ukazatele klesá, což odpovídá vývoji ukazatele hrubého předepsaného pojistného. Ani hodnocení trhu tímto ukazatelem tedy nevypovídá o pozitivním směru vývoje v několika posledních letech, přičemž podstatně horší je opět situace v ŽP, což souvisí s již výše uvedeným vývojem ukazatele hrubého předepsaného pojistného.

3.3 Hodnocení pojistného trhu dle modifikovaných ukazatelů

V subkapitole jsou analyzovány ukazatele průměrného pojistného a ukazatele průměrného vyplaceného pojistného plnění, které doplňují provedenou analýzu pojistného trhu, založenou na kvantitativních a kvalitativních ukazatelích.

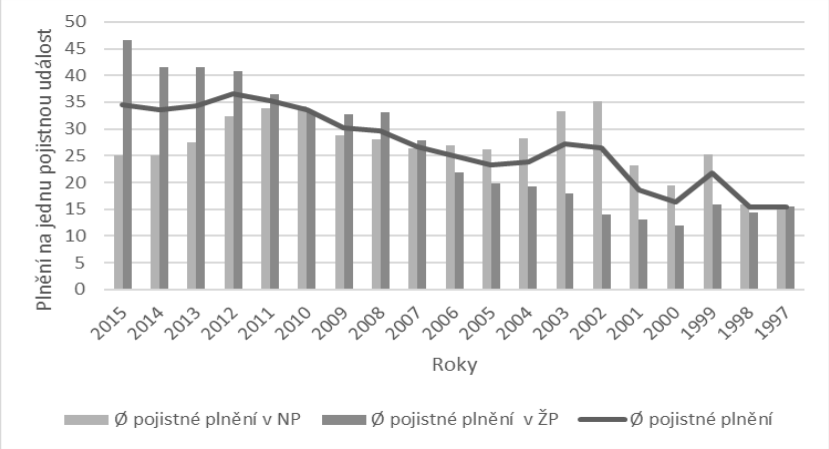
Graf 8: Vývoj ukazatele hrubého předepsaného pojistného na jednu pojistnou smlouvu (v tis. Kč)



Pomocí ukazatele průměrného pojistného je vyjádřena hodnota hrubého předepsaného pojistného připadající na každou jednu uzavřenou smlouvu. Zatímco počet uzavřených smluv

v analyzovaném období roste (viz graf 3), hrubé předepsané pojistné nikoli (viz graf 1) a tato skutečnost pak ovlivňuje vývoj sledovaného ukazatele. I když je předepsané pojistné vždy vyšší v NP, počet uzavřených smluv průměrnou hodnotu sráží pod průměr v ŽP.

Graf 10: Vývoj ukazatele pojistného plnění na jednu pojistnou událost (v tis. Kč)



Vývoj ukazatele vypovídá o zvyšující se finanční náročnosti každé jedné pojistné události, která je pojišťovnami vyřešena. Každá pojistná událost v neživotním pojištění byla, až do roku 2006 spojena každoročně s vyšší hodnotou vyplaceného pojistného plnění, od roku 2007 se naopak vyšší vyplacená hodnota pojistného plnění stává typickou pro pojistnou událost v pojištění neživotním. Z pozice hodnocení pojistného trhu není tento vývoj ideální. V životním pojištění může být důvodem zřetelná snaha pojištěných o lepší zabezpečení ve stáří či zabezpečení pozůstalých, což se promítá ve vyšších uzavřených pojistných částkách, které jsou předmětem výplat pojistných plnění. V neživotním pojištění je pak možné sledovat vysoké hodnoty průměrného pojistného plnění téměř výhradně v obdobích povodňových či popovodňových.

4. Závěr

Pojistný trh, tvořený členy České asociace pojišťoven, je možné v hodnoceném období označit za vyvíjející se. Jednoznačná stagnace, dokladovaná dlouhodobě neměnnými hodnotami ukazatelů, není vysledovatelná ve vývoji žádného ukazatele, který je použit k hodnocení úrovně pojistného trhu. Pozitivně se na vývoji pojistného trhu, který je možné označit za středně až nízko koncentrovaný, projevuje množství pojišťoven zajišťujících dostatečnou nabídku produktů, klesající počet zaměstnanců a růst počtu uzavřených smluv. Ten ovšem není doprovázen odpovídajícím očekávaným růstem předepsaného pojistného, což se negativně dále projevuje ve zjištěné škodovosti i pojištěnosti či hodnotě průměrného pojistného na jednu smlouvu. V důsledku rostoucího počtu vyřízených pojistných událostí pak dochází k růstu objemu vyplacených pojistných plnění. Vývojový trend několika posledních let by měl být předmětem hlubší analýzy a realizace opatření ke zlepšení.

References

[1] Böhm, A. and Mužáková, K. (2010). *Pojišťovnictví a regulace finančních trhů*. 1st ed. Praha: Professional Publishing.

[2] Borovcová, M. (2013). Insurance Market Assessment in the Czech Republic. In: *Proceedings form 9th International Scientific Conference Financial Management of Firms and Financial Institutions*. Ostrava: VŠB-TU Ostrava.

[3] ČAP. (2015 - 1997). *Výroční zpráva 2015-1997*. Česká asociace pojišťoven.

[4] Čejková, V. (2002). *Pojistný trh.* 1st ed. Praha: Grada.

[5] ČNB. (2015). *Zpráva o finanční stabilitě 2014/2015*. Dostupné na: http://www.cnb.cz/miranda2/export/sites/www.cnb.cz/cs/financni_stabilita/zpravy_fs/fs_2014-2015/fs_2014-2015.pdf.

[6] Daňhel, J a kol. (2005). *Pojistná teorie*. 1st ed. Praha: Professional Publishing.

[7] Department of Justice, Antitrust Division. (2015) *Herfindahl-Hirschman Index*. In: https://www.justice.gov/atr/herfindahl-hirschman-index.

[8] Ducháčková, E. (2009). *Principy pojištění a pojišťovnictví*. 3rd ed. Praha: Ekopress.

[9] Ducháčková, E. and Daňhel, J. (2010). *Teorie pojistných trhů*. 1st ed. Praha: Professional Publishing.

[10] Majtánová, A. and Vachálková, I. (2011). Actual problems of financial intermediation in the Slovak Republic. In: *Conference proceedings from 8th international scientific conference Financial management of firms and financial institutions*. Ostrava: VŠB-TU Ostrava.

[11] Majtánová, A., Daňhel, J., Ducháčková, E. and Kafková, E. (2006). *Pojišťovnictví: teorie a praxe.* 1st ed. Praha: Ekopress.

[12] OECD. (2015). *Hearing on Oligopoly Markets*. In: http://www.oecd.org/officialdocuments/publicdisplaydocumentpdf/?cote=DAF/COMP/WD(2015)45&docLanguage=En.

Relocation as the answer to weakening of social structures in the globalisation

Michal Butek[1]

Abstract

The aim of this article is to describe the relocation of economic and social relations in the globalized society. The issue of globalization is widely discussed in the scientific sociological, economic, political and anthropological literature. On the economic level, there is the removal of barriers between national economies creating one global market. Thaks to globalisation the growth of welth production has been indisputable. However the progressive automation of production, declining power of nation states and recently the weakening of global economic growth are creating tensions at the social level. The global society is at the beginning of a major transformation process, which will result in more self-sustaining regions. This article will analyze in more detail the causes and symptoms of relocation.

Key words

Globalisation, relocation, sustainability,

JEL Classification: O10, O30, O18

1. Úvod

Ekonomické, sociálne a a politické procesy sú úzko prepojené, závislé jeden od druhého a vzájomne sa ovplyvňujúce. Spoločnosť sa v každej dobe potýka s novými problémami a nachádza riešenia, ktoré sú vždy odpoveďou na aktuálne problémy a nikdy nie sú konečným ideálnym riešením. Snaha o nájdenie toho správneho spoločenského zriadenia, zavedenie toho správneho modelu emisie peňazí, právneho poriadku, daňovo-odvodového zaťaženia sú z dlhodobého hľadiska dopredu odsúdené na neúspech práve preto, že sú vždy odpoveďou na špecifické problémy svojej doby. Spoločnosť je ako živý neustále sa vyvýjajúci organizmus. Cieľom článku je analyzovať a popísať procesy a trendy súčasných socioekonomických zmien**.** Kríza súčasnej doby nie je krízou riešiteľnou v rámci pravidiel na základe ktorých spoločnosť funguje. Jedná sa o rozsiahly transformačný proces na inú formu spoločnosti s inými pravidlami a iným základným nastavením. Relokalizácia spoločnosti sa javí byť prirodzeným východiskom z krízovej situácie vyspelých demokratických spoločností. Podnik ako základná stavebná ekonomická jednotka spoločnosti bude musieť na rozsiahle transformačné procesy prirodzene reagovať a nachádzať si miesto v novovytvorenom usporiadaní spoločnosti. Bude prispôsobovať ciele a metódy fungovania. Z pohľadu strategického plánovania v podniku je z existenčního hľadiska nevyhnutné pochopenie transformačních procesov. Sledovanie rôznych druhov ukazovateľov finančných rizík nebude v meniacej sa dobe dostatočne spoľahlivým ukazovateľom perspektívneho budúceho fungovania podniku.

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2. Faktory podmieňujúce relokalizáciu

2.1 Integrácia suverénnych národných štátov do vyšších štruktúr

Pre národné štáty v procese globalizácie je charakteristická liberalizácia svojho trhu spojená s postupovaním moci vyšším inštitúciám. Strata štátnej moci sa týka takmer všetkých oblastí: hospodárstvo, zákonodárstvo, politika. [6] Postupným integrovaním štátnych ekonomík do globálneho trhu dochádza k čiastočnej strate významu národného štátu. Zrušením protekcionistických štátnych bariér, ktoré boli jeden z dôvodov jeho existencie, predovšetkým ako ochrana krajiny pred zahraničnou konkurenciou, stráca národný štát jednu zo svojich kľúčových funkcií.

2.2 Neudržateľnosť deficitného štátneho hospodárenia

Vo vyspelých demokratických krajinách vzniká model, kedy sa národné štáty v rámci zapájania do globalizovaného sveta zbavujú moci nad svojim hospodárstvom, no na svojich pleciach naďalej nesú ťarchu spočívajúcu v zabezpečovaní sociálnej oblasti. V snahe vytvárať pracovné miesta vytvárajú štáty rôzne druhy subvencií, čím sa len viac zaťažuje štátny rozpočet. Daňová záťaž sa tak týka predovšetkým domácich podnikov, zatiaľ čo zisk nadnárodných korporácií sú v rôznej miere od daňovej povinnosti oslobodené, prípadne vďaka rôznym účtovným metódam sa dokážu daniam vyhýbať. [7] Niektoré odhady udávajú, že polovina svetových peňazí sídli v daňových rajoch alebo nimi prechádza. Japonsko jedna z najvyspelejších ekonomík sveta dosahuje deficit štátneho rozpočtu na hranici 7 percent HDP, ktorý je možný len vďaka centrálnej banke BoJ, ktorá skupuje aktíva vo výške 15 percent HDP ročne.

2.3 Rast chudoby

Rast svetového hospodárstva neprospieva všetkým vrstvám spoločnosti rovnako a niektoré vrstvy z neho neťažia vôbec. V niektorých vyspelých krajinách dochádza k stagnácii prípadne aj zníženiu reálnych miezd. Podľa Fedu došlo napríklad v USA v rokoch 2010-2013 k zníženiu mediánovej mzdy o 5 percent. Aj samotný FED tak potvrdzuje závery ku ktorým došiel Thomas Piketty vo svojej knihe Kapitál v 21. storočí, že z rastu HDP neťažia všetky spoločenské vrstvy rovnako a niektoré si dokonca pohoršili. [2] Dochádza ku koncetrácii rastu príjmov mezi úzkou skupinou najbohatších. Rodiny, ktoré majú najnižšie príjmy zaznamenali pokračujúci významné poklesy svojich príjmov. [3]

2.4 Technologický rozvoj

Technologický rozvoj spôsobil v priebehu vývoja spoločnosti presun pracovnej sily z poľnohospodárstva do priemyslu a neskôr aj z priemyslu do služieb. Široké masy obyvateľstva vždy našli pracovné uplatnenie v inom sektore hospodárstva. V USA zamestnáva poľnohospodárstvo, ako najproduktívnejší sektor menej ako jedno percento pracovnej sily. [11] Počas priemyselnej revolúcie došlo k rastu zamestnanosti v priemysle, následnou automatizáciou bola pracovná sila postupne vytlačená do sektoru služieb. V súčasnej dobe dochádza vo vyspelých krajinách postupne k automatizácii aj v oblasti služieb. Automatizácia a zvyšovanie produktivity má však aj negatívny dopad na spoločnosť vo forme nezamestnanosti. Na mieste je tak otázka kam sa presunie pracovná sila z oblasti služieb. Sociálne dopady súčasného trendu sú zjavné. [8] Niektorí autori pomenovávajú proces znižovania významu ľudskej práce ako výrobného faktoru v ekonomike a sociálne implikácie tohoto procesu ako „závod ke dnu“.

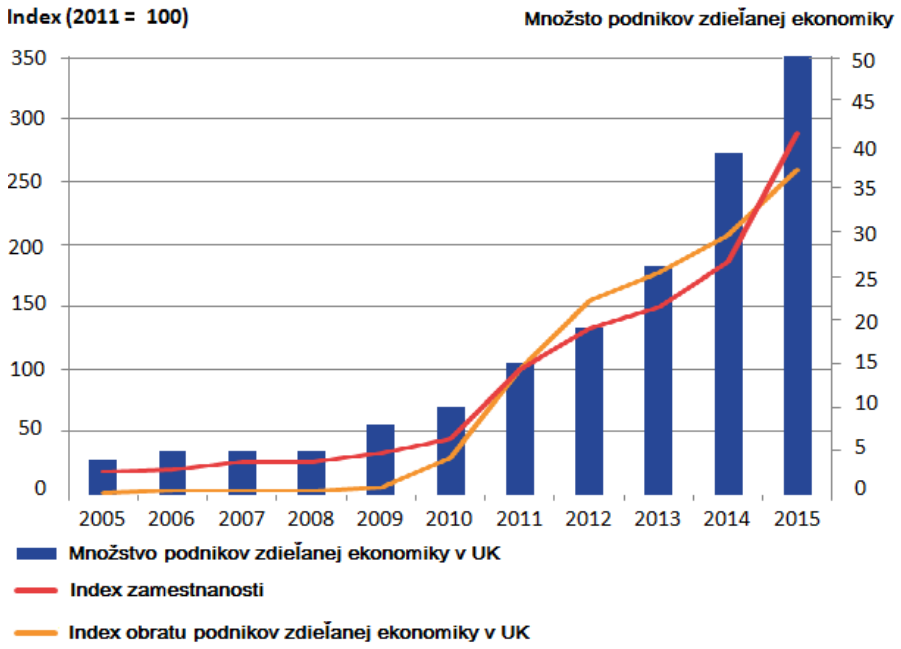
3. Prejavy relokalizačných procesov

Problémy a nestabilita spoločnosti pod vplyvom globalizácie vytvára v západných spoločnostiach prirodzené odpoveďe vo forme relokalizujúcej sa ekonomiky s heslom „mysli globálne jednaj lokálne“. Nejedná sa teda o proces spočívajúci v návrate uzatvoreným ekonomikám, ale naopak ide o proces, ktorý naplno využíva poznatky a technológie globálu za účelom posilnenia sebestačnosti a samoudržateľnosti lokálnu. Posilnenie významu regiónu dochádza na úrovňach hospodárskej sebestačnosti, politickej samosprávy, kultúrnej autonómie. [13]

3.1 Zdieľaná ekonomika

Zhoršujúce sa ekonomické podmienky širokých vrstiev obyvateľstva budú viesť k efektivnejšiemu využívaniu lokálnych zdrojov. Výrazným prejavom tohto trendu je zdielaná ekonomika. Jedná sa o fenomén, kde poskytovanie služby už nie je výsostným prejavom správania firmy. Dochádza k stieraniu striktne zadefinovanej role podnik/spotrebiteľ, kde podnik je subjekt ktorý službu alebo produkt vytvára a zákazník je ten, kto službu spotrebováva. V zdieľanej ekonomike sa každý čiastočne stáva podnikateľom. Základom pre rozvoj zdieľanej ekonomiky sú informačné technológie a platformy existujúce vo sfére internetu. [5] Internet extrémne znižuje čas a náklady na komunikáciu. Mení sa základný koncept poskytovania služby. Vznikajúce platformy už tak nie sú štandardným poskytovateľom služieb, ale skôr priestorom umožňujúcim obyvateľstvu pomôcť si navzájom. Príkladom takýchto služieb sú zdieľanie dopravy pomocou platformy Uber, zdieľanie ubytovanie cez Airbnb, požičanie náradia cez 1000tools, donáška obedu niekomu domov cez IstaCart. V procese relokalizovanej ekonomike si tak ľudia pomáhajú navzájom lokálne no za pomoci platforiem pôsobiacich na globálnej úrovni.

Graf 1: vývoj významu zdieľanej ekonomiky v UK



Zdroj: Vlastné spracovanie na základe Office for National Statistics [15]

3.2 Zánik medzičlánkov

Dochádza k eliminácii či oslabovaniu medzičlánkov, sprostredkovateľov, dílerov a podobne. Samoobslužné činnosti sú predpokladom k zániku medzičlánkov. Výrobci budu sami hľadať kratšiu a priamejšiu cestu k zákazníkovi. [12] Tím znižujú zbytočné vysoké náklady na medzičlánky, zvyšujú konkurencieschopnosť a lojalitu svojich priamych zákazníkov. Spotrebitelia takisto budú preferovať možnosti priameho nákupu od výrobcov pri nižších cenách.

3.3 Prispôsobenie výrobkov podľa špecifických požiadaviek zákazníka.

Vyspelé technológie ako 3D tlačiarne, kompaktné CNC stroje umožňujú prispôsobenie parametrov produktu zákazníka a jeho následnú výrobu bez nákladov na skladovanie a dopravu. [9] Tradičným príkladom výroby na mieru je spoločnosť Dell Computer. Pomocou priamej komunikácie so zákazníkom vyrába len to, čo bolo objednané. Nevyrába dopredu, na regály, do zásob, či překupnických medzičlánkov. Vďaka 3D tlačiarňam je napríklad možné vyrobiť bicykel priamo na mieru podľa fyzických dispozícií zákazníka. Dochádza tak k posunu od typizovanej výroby a následnému hľadaniu vhodného zákazníka k výrobe produktu priamo na mieru. [4]

3.4 Vyspelé technológie

Rozvoj výpočtovej techniky zaznamenáva v našej spoločnosti extrémne rýchly vzostup. Integrácia mikroprocesorov do bežných zariadení každodenného života sa stáva samozrejmosťou. Príkladom je vývoj v automobilovom priemysle. V minulosti bolo fungovanie a riadenie automobilu čisto mechanickou záležitosťou, V súčasnosti je auto systém vzájomne komunikujúcich riadiacich jednotiek, ktoré vyhodnocujú nielen vstupy pochádzajúce od samotného vodiča, ale aj signály z takmer všetkých častí automobilu. Novinkou v tejto oblasti sú autonómne vozidlá, ktoré po zadaní cieľovej destinácie dokážu samostatne absolvovať trasu bez nutnosti zásahov vodiča. Posun od človekom riadeného mechanického stroja k autonómnemu počítačom riadenému stroju prebieha vo všetkých oblastiach spoločnosti. [1] Produkcia sa tak stane doménou automatizovaných liniek s malým podielom ľudskej práce ako výrobnému faktoru. Technológia 3D tlače bola označená výskumným týmom Crédit Suisse jako technológia, ktorá rozvráti tradičné výrobné procesy, podobne bola označená aj Goldman Sachs jednou z 8 prevratných technológií, ktorým sa budú musieť podniky prizpôsobiť. [16]

3.5 Znalosti namiesto informácií

Rozvojom internetu došlo k masívnemu pokroku v oblasti šírenia informácií. Informácia sa tak stáva voľne dostupnou pre každého. Informácií bude stále viac a ich hodnota bude stále menšia. Naproti tomu znalosti a schopnosti vyhodnotiť informácie, porovnať ich, posúdiť, aplikovať atd. budú tým, čo bude čím dalej viac cenené a bude predstavovať skutočnú hodnotu. [14]

3.6 Rozvoj regionálnyh štruktúr

S postupujúcim procesom globalizácie ekonomiky silnejú trendy vytvárania regionálnej identity. V Európe dochádza k nárastu snáh o vyššiu autonómnosť regiónov jako je napríklad Katalánsko alebo Bavorsko. Prejavom týchto procesov je napríklad vytváranie lokálnych menových systémov, ktorých cieľom nie je nahradiť globálnu menu, ale paralelne pôsobiť a rozvíjať lokálnu ekonomiku. Príkladom je aj systém Sardex so svojím pôvodom na Sardínii. Jedná sa o model doplnkovej meny, ktorý sa od svojho založenia v roku 2008 rozvinul v zúčtovací lokálny systém s obratom 51 miliónov eur ročne. Systém bol replikovaný

v ďalších 10 regiónoch Talianska. Prvkom podtrhujúcim relokalizačný proces je práve vzájomná uzavretosť jednotlivých regionálnych systémov.

Regióny sa tak budú snažiť o vybudovanie systémov a štruktúr, ktoré budú pôsobiť na ekonomickej aj sociálnej úrovni ako ochrana pred otrasmi na globálnych trhoch.

4. Záver

Globálna ekonomika stojí na prahu veľkej transformácie. Pre podnik bude kľúčové pochopenie týchto transformačných procesov a následné prispôsobenie sa. Správna identifikácia rôznych druhov rizík a ich kvantifikácia bude možná len s prihliadaním na tieto procesy. V prostredí znižovania objemu umelo nafúknutých ekonomík sa každý stane podnikateľom. Vlastník firmy, živnostík aj ich zamestnanec. Bude sa meniť forma podniku, niektorí budú pracovať doma, iní budú na voľnej nohe poskytovať prácu viacerým zamestnávateľom, ďalší budú pracovať sezónne. Budú sa však musieť naučiť podnikať so svojim talentom, vzdelaním a ponúkať na trhu viac, ako druhí. Pre úspech bude nevyhnutná schopnosť adaptácie na nové prostredie, nové podmienky a pravidlá. Podnikanie bude mať stále viac rozmer sociálny a ekologický.

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References

[1] BRETTEL M., FRIEDERICHSEN N., KELLER M. and ROSENBERG M., (2014) How Virtualization, Decentralization and Network Building Change the Manufacturing Landscape: An Industry 4.0 Perspective, *International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering*,

[2] BRICKER, J. et al, (2014) Changes in U.S. Family Finances from 2010 to 2013: Evidence from the Survey of Consumer Finances, *Federal Reserve Bulletin*

[3] JANOSKOVA, K., KRAL, P. (2012). Energy poverty - the new socio-economic problems. *Ekonomicko-manazerske spectrum*

[4] MAJERNIK M., DANESHJO N., STOFKOVA K.R. (2016), Modeling the process of business, *Communications: Scientific letters of the University of Žilina,* 18(2)

[5] MARKOVA, V., LESNIKOVA, P. (2015). Strategic level of sustainable development of enterprises. *Ekonomicko-manazerske spektrum*,

[6] NAGDY M. and ROSER M. (2016), International Trade, *Published online at OurWorldInData.org*

[7] O´ BRIEN, R AND WILIAMS, M. R. (2010), *Global political economy*,

[8] SOROS G. (1998), The Crisis of Global Capitalism, *PublicAffairs*

[9] STOFKOVA J., MADLENAK R., STOFKOVA K.R. (2015), *Business management*, Bratislava: DOLIS

[10] STOFKOVA J., STRICEK I. STOFKOVÁ K. (2016) Data analysis in quality management of the network enterprise, *Production management and engineering sciences*. London: CRC Press/Balkema

[11] STOFKOVA K., STOFKOVA J. (2010), Implementation of the knowledge management as a competitive advantage, *International conference on education and new learning technologies*, Barcelona

[12] STOFKO, S. SOLTES V., STOFKOVA Z. (2016) Options of using the integrated management system, *Production management and engineering sciences.* . London: CRC Press/Balkema

[13] ZELENY, M. (2009), The economy is an organisme not a machine, *Czech Business Weekly*

[14] ZELENY M. (2006), From Knowledge to Wisdom: On Being Informed and Knowledgeable, Becoming Wise and Ethical, *Xiangshang Forum*

[15] HARDY M. (2016), The feasibility of measuring the sharing economy, dostupné online https://www.ons.gov.uk/economy/economicoutputandproductivity/output/articles/thefeasibilityofmeasuringthesharingeconomy/2016-04-05

[16] WILLE, R. (2013), GOLDMAN: The 8 Extraordinary Technologies Forcing Businesses To Adapt Or Die, *Businessinsider*

Wavelet methods for pricing European options

Dana Černá [1]

Abstract

The paper is concerned with numerical methods for pricing European options. We focus on the classic Black-Scholes model and solve it by two methods: the wavelet-Galerkin method and an adaptive method that uses wavelets as basis functions. The advantages of the adaptive method are a small number of degrees of freedom, high-order accuracy and a relatively small number of iterations needed to resolve the problem with desired accuracy. We present numerical examples of a European vanilla put option and a European put option on the maximum of two assets that are based on the real market data.

Key words

Black-Scholes model, European option, wavelets, adaptive wavelet method, spline wavelet, vanilla option, rainbow option.

JEL Classification: C63, G13.

1. Introduction

Several models and methodologies for option pricing are known, namely the Black-Scholes model, the Heston model and the Lévy model. In this paper, we focus on the Black-Scholes model for pricing of European put and call options.

Classical methods such as the finite difference method, the finite element method or Monte Carlo method (Achdou and Pironneau, 2005) and also some modern methods such as discontinuous Galerkin method (Hozman and Tichý, 2015) or wavelet methods (Černá, 2016; Finěk, 2016) can be used for solving the equation. We use the Crank-Nicolson scheme for time discretization and the wavelet-Galerkin method and an adaptive wavelet method for discretization on the given time level. To compare the results with other methods such as the discontinuous Galerkin method we solve the numerical examples of European options with the same parameters as in (Hozman and Tichý, 2014, 2015, 2016).

2. Wavelet basis

We briefly review a concept of a wavelet basis, for more details one can see e.g. (Urban, 2009). Let us consider a separable Hilbert space H with the norm $\|.\|_H$. Let $\|.\|$ and (.,.) denote the L^2-norm and the L^2-inner product, respectively. Furthermore, let J be an index set such that each index $\lambda \in J$ takes the form $\lambda = (j, k)$ and $|\lambda|$ denotes a level and k represents a spatial location. For $v = \{v_\lambda\}_{\lambda \in J}$, $v_\lambda \in \mathbb{R}$, we define

$$\|v\|_2 = \left(\sum_{\lambda \in J} |v_\lambda|^2 \right)^{1/2}, l^2(J) = \{v : \|v\|_2 < \infty\}.$$

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Definition: A family $\Psi = \{\psi_\lambda,\ \lambda \in J\}$ is called a wavelet basis of the space H if it satisfies the following conditions:

- Ψ is a Riesz basis for H, i.e. the closure of the span of Ψ is H and there exist constants $c, C \in (0, \infty)$ such that for all $b = \{b_\lambda\}_{\lambda \in J} \in l^2(J)$ we have
$$c\,\|b\|_2 \leq \left\| \sum_{\lambda \in J} b_\lambda \psi_\lambda \right\|_H \leq C\|b\|_2.$$
- For all $\lambda \in J$ a diameter of the support of ψ_λ is bounded by $C\,2^{-|\lambda|}$ with C independent on $|\lambda|$ and at a given level the supports of only finitely many wavelets overlap at any point x.

Wavelet basis Ψ on a bounded domain has typically a hierarchical structure
$$\Psi = \Phi_{j_0} \cup \bigcup_{j=j_0}^{\infty} \Psi_j$$
Let I_j and J_j be finite index sets and
$$\Phi_j = \{\varphi_{j,k}, k \in I_j\}, \qquad \Psi_j = \{\psi_{j,k}, k \in J_j\}.$$
Functions $\varphi_{j,k}$ are called scaling functions and functions $\psi_{j,k}$ are called wavelets on the level j. The set
$$\Psi_s = \Phi_{j_0} \cup \bigcup_{j=j_0}^{s} \Psi_j$$
is a finite dimensional subset of Ψ and is called a multicale wavelet basis. We further assume that there exists $L \geq 1$ such that wavelets $\psi_{j,l}$ have L vanishing moments, i.e. for any polynomial p of degree $m \leq L-1$ we have
$$\int_{supp\ \psi_{j,l}} p(x)\,\psi_{j,l}(x)\,dx = 0, l \in J_j, j \geq j_{j_0}.$$

3. Black-Scholes model

Let $S_1, S_2 > 0$ represent prices of the assets, t represents time to maturity, r is a risk-free rate, $\sigma_i = \sigma_i(S_1, S_2, t)$, $i = 1, 2$, are the corresponding volatilities of the assets and the parameter ρ is the correlation factor. Then for several types of options the market price $V(S_1, S_2, t)$ of the option at time t can be computed as the solution of the Black-Scholes equation (Black and Scholes, 1973):
$$\frac{\partial V}{\partial t} - \mathcal{L}_{BS}(V) = 0, t \in (0, T),$$
where the Black-Scholes operator $\mathcal{L}_{BS}$ is given by
$$\mathcal{L}_{BS}(V) = \frac{\sigma_1^2 S_1^2}{2}\frac{\partial^2 V}{\partial S_1^2} + \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} + \frac{\sigma_2^2 S_2^2}{2}\frac{\partial^2 V}{\partial S_2^2} + rS_1\frac{\partial V}{\partial S_1} + rS_2\frac{\partial V}{\partial S_2} - rV.$$

We approximate the unbounded domain $\mathbb{R}_+^2$ by a domain $\Omega = (0, S_1^{max}) \times (0, S_2^{max})$, where S_1^{max} and S_2^{max} are large enough.

We assume that g and h are given functions and that initial and boundary conditions are of the form $V(S_1, S_2, 0) = g(S_1, S_2), (S_1, S_2) \in \Omega$, and $V(S_1, S_2, t) = h(S_1, S_2, t)$, $(S_1, S_2, t) \in \partial\Omega \times (0, T)$. The functions g and h depend on the type of options.

Let $W \in H^1\left(\Omega \times (0,T)\right)$ be such that $W = h$ on $\partial\Omega \times (0,T)$ and set $\tilde{V} = V - W$. Then we can restrict ourselves to a problem

$$\frac{\partial V}{\partial t} - \mathcal{L}_{BS}\left(\tilde{V}\right) = f, t \in (0,T), f = -\frac{\partial W}{\partial t} + \mathcal{L}_{BS}(W),$$

with initial conditions

$$\tilde{V}(S_1, S_2, 0) = g(S_1, S_2) - W(S_1, S_2, 0), (S_1, S_2) \in \Omega,$$

and homogeneous Dirichlet boundary conditions.

We use the Crank-Nicolson scheme for time discretization. Let $M \in \mathbb{N}$, $\tau = M^{-1}$, $t_l = l\tau$, $l = 0, \cdots, M$, and denote $\tilde{V}_l\ (S_1, S_2) = \tilde{V}(S_1, S_2, t_l)$, and $f_l\ (S_1, S_2) = f(S_1, S_2, t_l)$. The scheme has the form:

$$\frac{\tilde{V}_{l+1} - \tilde{V}_l}{\tau} - \frac{\mathcal{L}_{BS}\left(\tilde{V}_{l+1}\right)}{2} - \frac{\mathcal{L}_{BS}\left(\tilde{V}_l\right)}{2} = \frac{f_{l+1} + f_l}{2}.$$

4. Wavelet-Galerkin method

The standard weak formulation of the scheme has the form:

$$\frac{\left(\tilde{V}_{l+1}, v\right)}{\tau} - \frac{a\left(\tilde{V}_{l+1}, v\right)}{2} - \frac{a\left(\tilde{V}_l, v\right)}{2} = \frac{\left(\tilde{V}_l, v\right)}{\tau} + \frac{(f_{l+1} + f_l, v)}{2},$$

where $v \in H_0^1(\Omega)$ and $a(u, v) = (\mathcal{L}_{BS}(u), v)$.

Let $\Psi = \{\psi_\lambda,\ \lambda \in J\}$ be a wavelet basis for $L^2(\Omega)$ such that Ψ rescaled with respect to the H^1-seminorm is a wavelet basis for $H_0^1(\Omega)$ and let $\Psi_s \subset \Psi$ be a multiscale wavelet basis. The Galerkin method insists in approximating the solution $\tilde{V}_{l+1}$ by a function

$$\tilde{V}_{l+1}^s = \sum_{\psi_\lambda \in \Psi_s} u_\lambda^s \psi_\lambda,$$

where the vector of coefficients $u^s = \{u_\lambda^s\}$ is given by

$$A^s u^s = f^s,$$

with

$$A_{\mu,\lambda}^s = \frac{\left(\psi_\lambda, \psi_\mu\right)}{\tau} - \frac{a\left(\psi_\lambda, \psi_\mu\right)}{2}, \psi_\lambda, \psi_\mu \in \Psi_s,$$

and

$$f_\mu^s = \frac{a\left(\tilde{V}_l, \psi_\mu\right)}{2} + \frac{\left(\tilde{V}_l, \psi_\mu\right)}{\tau} + \frac{\left(f_{l+1} + f_l, \psi_\mu\right)}{2}, \psi_\mu \in \Psi_s.$$

It is clear that A^s, f^s and u^s can depend on the time level t_l, but for simplicity we omit the index l. For more details one can see e.g. (Urban, 2009).

5. Adaptive wavelet method

We use an adaptive wavelet method for discretization with respect to the variables S_1 and S_2. The method is a modification of the method from (Cohen, Dahmen and DeVore, 2002). While the classical adaptive methods use refining a given mesh according to a-posteriori local error estimates, the wavelet approach is different.

We start with a variational formulation but instead of turning to a finite-dimensional

approximation, we expand the solution $\tilde{V}_{l+1}$ in a basis Ψ, i.e.

$$\tilde{V}_{l+1} = \sum_{\lambda \in J} u_\lambda \psi_\lambda,$$

and transform the continuous problem into an infinite-dimensional l^2-problem

$$Au = f$$

with $u = \{u_\lambda\}_{\lambda \in J}$,

$$A_{\mu,\lambda} = \frac{(\psi_\lambda, \psi_\mu)}{\tau} - \frac{a(\psi_\lambda, \psi_\mu)}{2}, \qquad \lambda, \mu \in J,$$

and

$$f_\mu = \frac{a(\tilde{V}_l, \psi_\mu)}{2} + \frac{(\tilde{V}_l, \psi_\mu)}{\tau} + \frac{(f_{l+1} + f_l, \psi_\mu)}{2}, \qquad \mu \in J.$$

As above for simplicity we omit the index l in notations of A, f and u.

Then we use an iteration scheme for solving this infinite-dimensional problem. We have to replace all infinite-dimensional vectors by finitely supported ones and we have to design a routine for an approximate computation of a multiplication of a bi-infinite matrix with a finitely-supported vector. The matrix A has so-called finger structure, see e.g. (Černá, 2016). It is generally not sparse but quasi sparse. However, for wavelet basis from (Cvejnová, Černá and Finěk, 2015) that is used in Section 6 the matrix is sparse.

We use the standard Jacobi diagonal preconditioner and solve the problem by the method of generalized residuals (GMRES). For the given time level t_l we choose some initial vector v_0 and then for $j = 0,1, ...,$ we perform the following steps:

1. Compute sparse representation f_j of the right-hand side f such that the error is smaller than a given tolerance ε_j^1. The computation of a sparse representation insists in thresholding the smallest coefficients and working only with the largest ones.
2. Compute K steps of GMRES for solving the system $Av = f_j$ with the initial vector v_j. Each iteration of GMRES requires multiplication of the infinite-dimensional matrix with a finitely supported vector. It is computed approximately with the given tolerance ε_j^2 by the method from (Černá and Finěk, 2013) or exactly in the case that the stiffness matrix is sparse. We denote the resulting vector by z.
3. Compute sparse representation v_{j+1} of z with the error smaller than ε_j^3.

We repeat the steps 1., 2., and 3. until the residual is not smaller than the required tolerance. Since we work with the sparse representation of the right-hand side and the sparse representation of the vector representing the solution, the method is adaptive. For more details about adaptive wavelet methods see (Cohen, Dahmen and DeVore, 2002; Urban, 2009).

6. Numerical examples

For comparison with other methods we present numerical examples for European put options with the same parameters as in (Hozman and Tichý, 2014, 2015, 2016). We denote the exact solution at time to maturity T by V_T, an approximate solution computed at time to maturity T by V_T^{app} and we compute the relative errors

$$\rho_0 = \frac{\left\|V_T - V_T^{app}\right\|}{\|V_T\|}, \quad \rho_1 = \frac{\left\|V_T - V_T^{app}\right\|_{H^1(\Omega)}}{\|V_T\|_{H^1(\Omega)}}.$$

In the case of the Galerkin method the errors ρ_0 and ρ_1 typically depend on the step size h: $\rho_i \approx C_1 h^{r_i}$ with the constant C_1 independent on h. Since $h \approx C_2 N^{-1/d}$, where d denotes the dimension and N is the number of basis functions, we can rewrite the error estimate as

$$\rho_i \approx C_3 N^{-\frac{r_i}{d}}.$$

The optimal value $r_i = D - i$, where D is the polynomial exactness of the basis Ψ, is achieved under assumptions on the smoothness of the solution and the functions g and h.

In the case of an adaptive wavelet method the error estimate for ρ_1 also has the form

$$\rho_1 \approx C_4 N^{-\frac{r_1}{d}}.$$

and the optimal value is $r_1 = D - 1$.

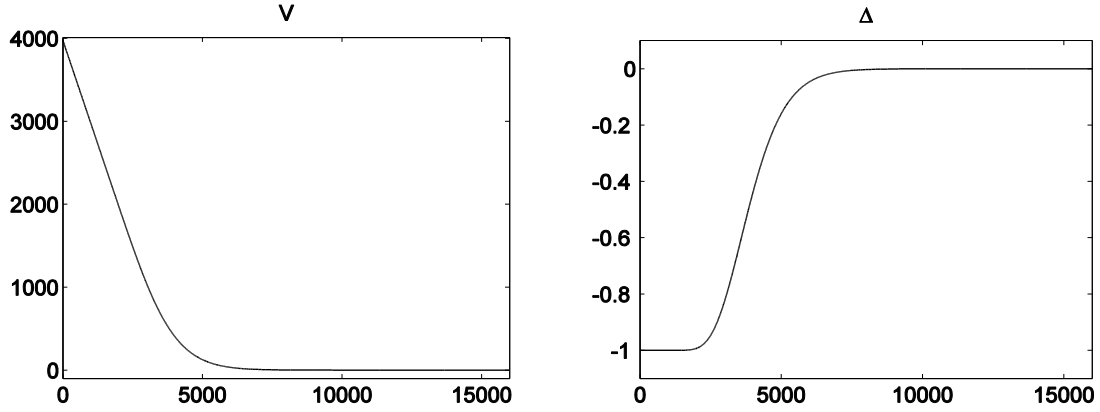
We compute the estimates of r_i numerically. If $\rho_i(N_j)$ is a computed error for N_j functions, $j = 1,2$, then the approximate order of convergence is

$$r_i \approx \frac{d\ log\left(\frac{\rho_i(N_1)}{\rho_i(N_2)}\right)}{log\left(\frac{N_2}{N_1}\right)}.$$

Example 1. We consider a vanilla put option with the real data of the German DAX on 15 September 2011 as in (Hozman and Tichý, 2014, 2016). The strike price is $K = 4000$ Euro and the option is maturing in 4 months, i.e. $T = 1/3$. The corresponding volatility is $\sigma = 0.4594$ and the risk-free interest rate is $r = 0.0176$.

We set $S^{max} = 4K = 16000$. For this choice the error is not significantly influenced by approximate boundary conditions: $V(0,t) = Ke^{-rt}$, $V(S^{max},t) = 0$. The approximate solution V_T^{app} and its delta function $\left(V_T^{app}\right)'$ are displayed in Figure 1.

Figure 1: Approximate solution (left) and its delta function (right) for the wavelet-Galerkin method with 128 functions and wavelets from (Černá and Finěk, 2015b).



We choose small time step $\tau = 1/3000$ in order to guarantee that the error depends mainly on the parameters in spatial discretization and is not significantly influenced by error caused by semidiscretization in time. We solve the problem by both wavelet-Galerkin method and adaptive wavelet method. Convergence history for several wavelet bases is shown in Table 1, Table 2 and Table 3. We can see that in all cases the optimal convergence rates was achieved

and that smaller number of functions and thus smaller number of degrees of freedom is needed to obtain a solution with a desired accuracy in the case of an adaptive wavelet method.

Table 1: Convergence history for the Galerkin method and adaptive wavelet method with quadratic spline-wavelet basis from (Černá and Finěk, 2015b).

Wavelet-Galerkin method					Adaptive wavelet method				
N	ρ_0	r_0	ρ_1	r_1	N	ρ_0	r_0	ρ_1	r_1
16	9.07e-4		1.08e-2		9	9.05e-4		7.44e-3	
32	7.35e-5	3.62	2.20e-3	2.30	14	1.91e-4	3.53	2.73e-3	2.27
64	8.14e-6	3.17	5.20e-4	2.08	25	3.54e-5	2.90	8.56e-4	2.00
128	9.89e-7	3.04	1.28e-4	2.02	40	6.76e-6	3.52	2.98e-4	2.24
256	1.23e-7	3.01	3.19e-5	2.01	67	1.31e-6	3.19	9.56e-5	2.21
512	1.79e-8	2.78	8.00e-6	2.00	115	2.87e-7	2.81	3.30e-5	1.97

Table 2: Convergence history for the Galerkin method and adaptive wavelet method with cubic spline-wavelet basis from (Černá and Finěk, 2015a).

Wavelet-Galerkin method					Adaptive wavelet method				
N	ρ_0	r_0	ρ_1	r_1	N	ρ_0	r_0	ρ_1	r_1
9	5.80e-3		3.21e-2		7	6.46e-3		3.23e-2	
17	5.77e-4	3.33	5.49e-3	2.55	18	3.44e-5	5.54	3.76e-4	4.71
33	1.14e-5	5.66	2.99e-4	4.20	26	1.01e-5	3.32	1.59e-4	2.34
65	5.05e-7	4.49	3.03e-5	3.30	38	1.53e-6	4.99	3.89e-5	3.71
129	3.00e-8	4.07	3.60e-6	3.07	59	2.61e-7	4.01	1.01e-5	3.07
257	5.32e-9	2.50	4.46e-7	3.01	92	6.70e-8	3.06	2.85e-6	2.84

Table 3: Convergence history for the Galerkin method and adaptive wavelet method with cubic spline-wavelet basis from (Cvejnová, Černá and Finěk, 2015).

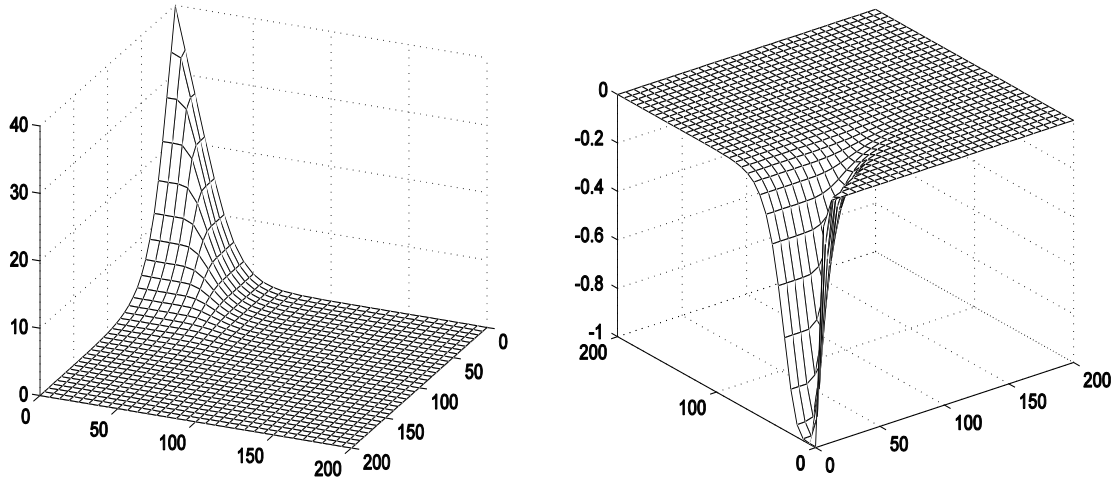
Wavelet-Galerkin method					Adaptive wavelet method				
N	ρ_0	r_0	ρ_1	r_1	N	ρ_0	r_0	ρ_1	r_1
8	1.24e-2		5.78e-2		9	7.55e-3		3.74e-2	
16	8.15e-4	3.93	7.47e-3	2.95	18	2.93e-4	4.69	3.13e-3	3.58
32	6.66e-5	3.61	1.25e-3	2.57	25	5.22e-5	5.25	9.20e-4	3.73
64	5.84e-6	3.51	2.01e-4	2.64	36	1.10e-5	4.28	2.72e-4	3.35
128	4.21e-7	3.80	2.74e-5	2.88	53	2.08e-6	4.30	8.43e-5	3.03
256	2.81e-8	3.90	3.51e-6	2.96	75	3.94e-7	4.79	2.58e-5	3.40
512	6.06e-9	2.21	5.22e-7	2.75	120	6.72e-8	3.76	5.27e-6	3.38

Example 2. We consider a European put option on the maximum of two assets. This example is based on the real data as in (Hozman and Tichý, 2015, 2016), i.e. Allianz stock and Deutsche Bank stock, the striking price is $K = 40$ Euro, and the option is maturing in 94 days. We assume that the current date is September 13, 2011. Then the two stocks trades at $S_1 = 59.79$ and $S_2 = 23.40$ Euro, respectively. The last year estimate of the Pearson linear correlation is $\rho = 0.88$ and market implied risk-free interest rate for a given horizon $r = 0.01557$ p.a. The corresponding volatilities are $\sigma_1 = 0.6392$ and $\sigma_2 = 0.9461$.

We set $S_1^{max} = S_2^{max} = 200$ and the time step $\tau = 1/(2 \cdot 365)$. We use a quadratic spline wavelet basis from (Černá and Finěk, 2015b). The approximate solution 94 days to maturity and the delta function for the first asset, i.e. the partial derivative of the solution with respect to S_1, are displayed in Figure 2. We set boundary conditions as in (Hozman and Tichý, 2015). The initial condition has the form $V(S_1, S_2, 0) = \max(K - \max(S_1, S_2), 0)$. Note that this function has not continuous first derivative. Therefore in our numerical experiments the

optimal rate r_i was not achieved. We observe the rate $r_0 \in (2,3)$ and $r_1 \in (1,2)$. It is better rate than the rate computed for finite difference method and similar type of option in (Jo and Kim, 2013). In the case of adaptive wavelet method the smaller number of functions was needed to obtain a solution with a desired accuracy than in the case of wavelet-Galerkin method. The difference was much more significant than for one dimensional problem in Example 1. For example, using wavelet-Galerkin method with 4096 functions the error ρ_0 was $1.15e-3$, while using adaptive wavelet method only 341 functions was needed to represent V_T with the same error ρ_0. In (Černá, 2016), the numbers of iterations needed to get the solution with desired accuracy were compared for several wavelet bases for a model example using the Black-Scholes equation with the same parameters σ_1, σ_2, ρ and r.

Figure 2: Approximate solution (left) and the delta function (right) for the first asset for the Galerkin method with 1024 functions and wavelets from (Černá and Finěk, 2015b) for Example 2.



Conclusion

In this paper, we presented two methods based on wavelets for option pricing, namely the wavelet-Galerkin method and an adaptive wavelet method. Since the used wavelet bases were quadratic and cubic spline wavelet bases both methods are high-order accurate with respect to the variables representing prices. In the case of the adaptive wavelet method the smaller number of parameters is needed to obtain the solution with desired accuracy than in the case of the Galerkin method. The difference is significant especially for two-dimensional problems. We provided numerical examples of a European vanilla put option and a European put option on the maximum of two assets. In these two special examples explicit formulas are known. Therefore we were able to compute the errors of approximation. However, described methods can be used for pricing any option that is represented by Black-Scholes equation with Dirichlet boundary conditions, e.g. European basket options.

References

[1] Achdou, Y., and Pironneau, O. (2005). *Computational Methods for Option Pricing*. Society for Industrial and Applied Mathematics, Philadelphia.

[2] Black, F., and Scholes, M. (1973). The pricing of options and corporate liabilities. *J. Polit. Econ.*, 81, p. 637-659.

[3] Cohen, A., Dahmen, W., and DeVore, R. (2002). Adaptive wavelet methods II - beyond the elliptic case. *Found. Math*., 2, p. 203-245.

[4] Černá, D., and Finěk, V. (2013). Approximate multiplication in adaptive wavelet methods. *Cent. Eur. J. Math*., 11, p. 972-983.

[5] Černá, D., and Finěk, V. (2015a). Wavelet basis of cubic splines on the hypercube satisfying homogeneous boundary conditions. *Int. J. Wavelets Multiresolut. Inf. Process.,* 3, 1550014 (21 pages).

[6] Černá, D., and Finěk, V. (2015b). Quadratic spline wavelets satisfying homogeneous boundary condition. Submitted for publication.

[7] Cvejnová, D., Černá, D., and Finěk, V. (2015). Hermite Cubic Spline Multi-wavelets on the Cube. In: 41st International Conference Applications of Mathematics in Engineering and Economics, *AIP Conference Proceedings*, 1690, 030006 (6 pages).

[8] Černá, D. (2016). Numerical Solution of the Black-Scholes Equation Using Cubic Spline Wavelets. In: 42st International Conference Applications of Mathematics in Engineering and Economics, *AIP Conference Proceedings*, (8 pages).

[9] Finěk, V. (2016). Fractional Step Method for Wavelet Based Solution of Black-Scholes Equation. In: 42st International Conference Applications of Mathematics in Engineering and Economics, *AIP Conference Proceedings*, (4 pages).

[10] Hozman, J., and Tichý, T. (2014). Black-Scholes option pricing model: comparison of h-convergence of the DG method with respect to boundary condition treatment. *ECON - Journal of Economics Management and Business*, 24, p. 141-152.

[11] Hozman, J., and Tichý, T. (2015). A discontinuous Galerkin method for pricing of two-asset options. In: 33rd International Conference Mathematical Methods in Economics, Cheb, ZČU Plzeň, p. 273-278.

[12] Hozman, J., and Tichý, T. (2016). DG solver for one-factor and two-factor Black-Scholes models. In: Managing and Modeling of Financial Risks, Ostrava.

[13] Jo, J., and Kim, Y. (2013). Comparison of numerical schemes on multi-dimensional Black-Scholes equations. *Bull. Korean Math. Soc.*, 50, p. 2035-2051.

[14] Urban, K. (2009). *Wavelet methods for elliptic partial differential equations*, Oxford University Press, Oxford.

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Risk Management in Venture Capital Investment Process within Business Accelerators Owned by Private Corporations

Petr Čížek[1]

Abstract

The risk management in the investment process is mostly divided into two main parts. First is due diligence which is defined as one-off activity which is used to accepting or rejecting of investment proposal. The other is investment monitoring which is frequently executed activity with purpose of information gathering about the investment to address potential crisis. The aim of the paper is to examine what is the situation and difference in risk management in the investment process in the business accelerators owned by private corporation. To fulfill the aim, the management of the business accelerators were interviewed and the information related to this type of investor was gathered regarding due diligence and investment monitoring. According to the results of the research conclusion and recommendations were made.

Key words

Risk management, Due diligence, Business accelerator, Startup, Investment process

JEL Classification: D92, G31, M13, G24

1. Theory background

The chapter deals with theoretical definitions of financial market where investors and invested companies meet. Moreover, the different types of private equity are presented enriched with current situation in the EU and the Czech Republic. As it is explained the venture capital in the Czech Republic is very underrated next to other (especially western) countries. In the next part, there is shown venture capital investment process as it is presented in the literature and applicable to venture capital funds and banks. The integral part of venture capital is risk management – the whole topic is shown and explained in the following part. In the last part of the theory background, the topic of business accelerators is examined with emphasis on the role of business accelerators in the investment process. The paper examines the benefits and impacts on investment process and risk management in the case where business accelerator acts as primary investor instead typical role as catalyst.

1.1 Definition of private equity and investment

The financial market is place where demand and supply of financial sources meet. The private equity capital which is used on financial market can be divided into these main domains – Venture capital, Growth capital, Rescue turnaround capital, Replacement capital and Buyouts. (Nývltová, 2007)

Venture capital is divided into the three main types which differ by stage of the company which is invested into. The types of venture capital are:

- Seed Capital

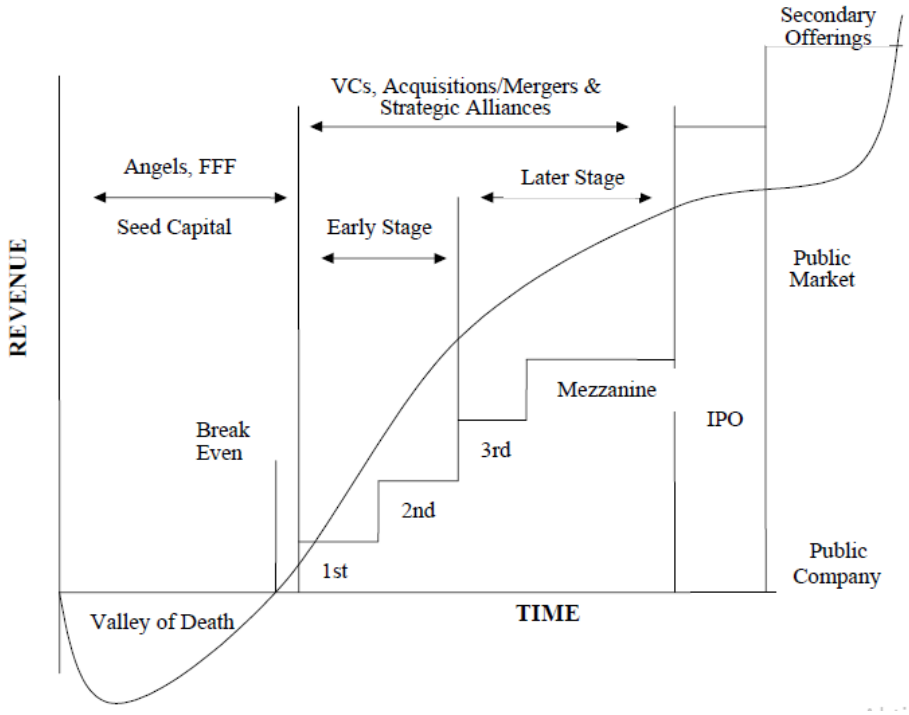
[1] Ing. Petr Čížek, M.A.,University of Pardubice,Petr.cizek@upce.cz

- Startup Capital
- Later Stage Capital

(Czech Private Equity & Venture Capital Association, 2014)

Cardullo shows the connection between stage of the invested company and the typical investor involved in the investment. As it is possible to see in Figure 1 in early stages of the company (seed capital) the typical investors are business angels or FFF (Family, Friends and Fools). In the later stages of the company other investors joins the investment opportunity – venture capital funds, strategic alliances etc. (Cardullo, 199)

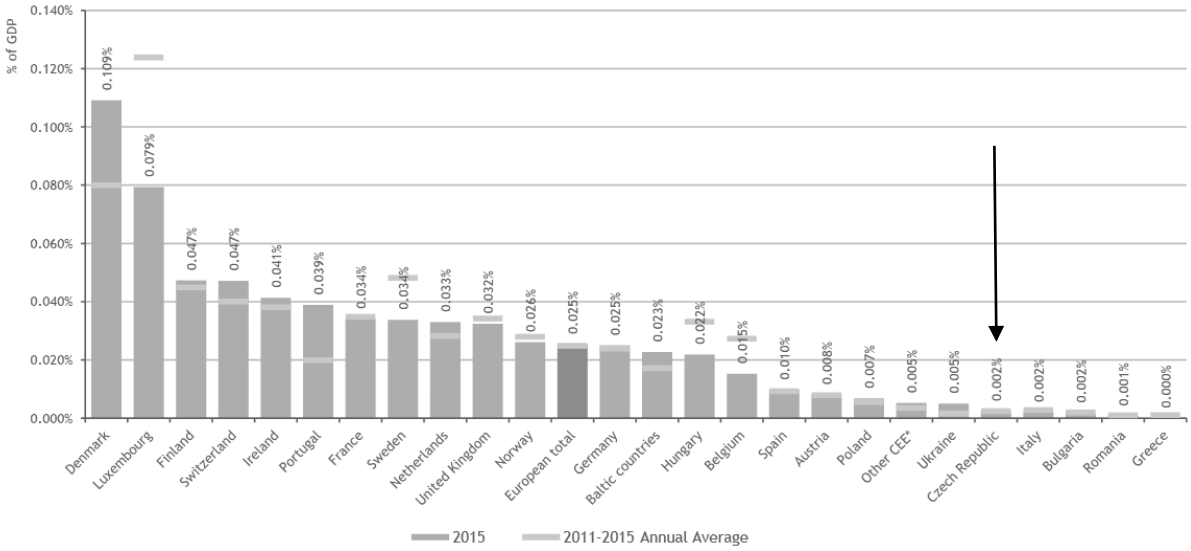
Figure 1: Private equity stages and typical investors



(Cardullo, 1999)

The Czech Private Equity & Venture Capital Association and European Private Equity and Venture Capital Association monitor situation of venture capital in the Czech Republic with comparison with other EU countries. Figure 2 shows the difference in amount of investments as percentage of GDP in EU countries. The highlighted amount - venture capital investments as 0,002% of Czech GDP – is very below EU average. It indicates underrated situation of venture capital in the Czech Republic with huge potential in the future. (EVCA, 2015)

Figure 2: Investments as percentage of GDP in EU countries



(EVCA, 2015)

1.2 Venture capital investment process and investment risk management

The investment process in the venture capital investments is showed in the Figure 3. The process itself is comprised of six steps.

First step is detection of investment opportunities. In this step investor tries to identify as much opportunities as possible to choose from. There are three main ways how investor can gather potential opportunities. Firstly, clients (firms looking for investment) are coming by themselves to the investor. Investor should be well known so clients know where they can go asking for the investment. Secondly investor approaches potential clients – this is according to the low effectivity the least used method. Thirdly other subjects (ie. banks, consultants or business incubators) are searching for investment opportunities. The 3rd party organization is the mediator between potential clients and investor.

Second step in the investment process is separation of investment opportunities. In this step investor separates the potential clients which the best suits the needs of investor.

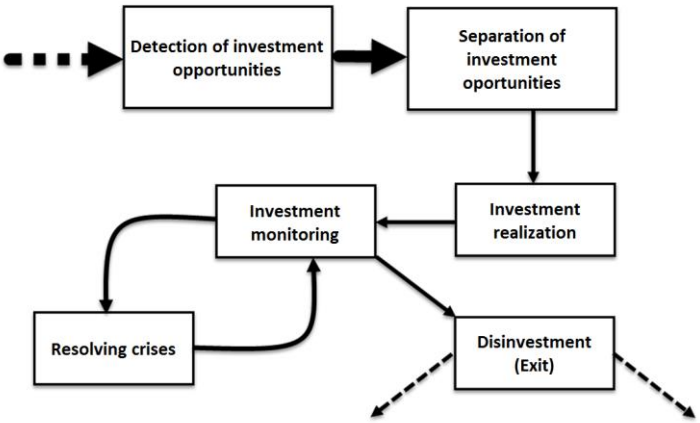
Third step is investment realization where the due diligence, contract preparation and contract signing happen.

Fourth step is investment monitoring which could be (and most time is) followed by resolving crisis which is identified by investment monitoring activity.

Last step is disinvestment where investor after certain time allocated to the investment realize investment exit and creates desired profit.

(Dvorak, 1998)

Figure 3: Investment process from the view of the investor



(Dvořák, 1998)

During the investment process there are two crucial risk management events. Firstly, there are due diligence which happens in the step of separation of investment opportunities and there is investment monitoring.

European Private Equity and Venture Capital Association defines difference in due diligence and monitoring in Table 1. The main difference is where due diligence is one-off activity with purpose of accepting or rejecting of investment proposal, monitoring is frequently executed activity with purpose of information gathering about the investment to address potential crisis. (EVCA, 2013)

Table 1: Differentiation between due diligence and monitoring

DUE DILIGENCE	MONITORING
Focus on individual transaction	Focus on individual Transaction
One-off with detailed analysis of individual fund proposal	Frequently executed
Mainly pre-investment for investment decision making	On-going activity (pre- and post-investment)
Purpose is accepting or rejecting of investment proposal	Purpose is monitoring of the development of an individual fundand addressing potential issues in dialogue
Conservative bias with stringent cut-off criteria	Information gathering
Focus on achieving high performance for individual investment proposal, i.e. capital risk	Focus on protecting investment in fund

(EVCA, 2013)

McClure defines three main areas which is considered in the due diligence of the investment. (McClure, 2010)

- Company's management – which is considered as the most important
- Size of market
- Product with competitive edge

1.3 Business accelerators

Business accelerator is defined by Romans as short term (usually 3 months) innovation clearly defined incubation program where company span from one phase to next. Very often companies which finishes business accelerator program continues to the business incubator program. (Romans, 2013)

Business accelerators are considered in the investment process as catalyst or 3rd party which mediates investment between investor and invested company. Business accelerators can also act as primary investor. However, the situation in the Czech Republic is dismal as, according to BIC (2012) the business accelerators and incubators don't provide their own venture capital to the incubated companies. The exception are business accelerators which are owned (and operated) by private corporates. The examples of these business accelerators are Wayra (owned by Telefónica), Alza Ventures (owned by Alza.cz) or Be Bolt (owned by O2 Czech Republic).

2. Methodology

The aim of the paper is to examine what is the situation and difference in risk management in the investment process in the business accelerators owned by private corporation.

To fulfil the aim of the paper the interviews were set with managers of business accelerator owned by private corporation. The names of corporations are anonymous as it was agreed during the interview. The number of interviewees was limited to two due to narrow specification of examined business accelerators and amount of these types of business accelerators in the Czech Republic.

According to Saunders (2007) the nature of research is explanatory therefore it was chosen the semi-structured type of interview with open questions. The main focus of the questions was to examine the methods and preferences of risk management during the investment process in the business accelerator. Especially the emphasis was made into due diligence and investment monitoring.

Secondary quick questionnaire amongst incubated firms were spread to gather information whether business accelerators are viewed as favorable primary investor. The questionnaire was handed to 255 companies via email and 27 replied back which makes 10% response rate.

3. Research results

The results of the research are divided to the results of the interview with business accelerator managers about risk management during the investment process and the results of the questionnaire handed to managers of incubated firms.

3.1 Risk management in the investment process within the business accelerators

The interview was divided into two main parts. Firstly, the information about the due diligence in the investment process was gathered and secondly the information about the investment monitoring was collected.

3.1.1 Due diligence

According to due diligence, both interviewees stated that during the due diligence the quality of management team is crucial. The reasons mentioned were that very often the management of the company has to pivot with focus and nature of the product which could be troublesome for someone. However strong and compact management team can overcome these difficulties quite easily which can result into further success. Moreover, it was stated

that there are some signs which indicates more perilous investment risk such as situation when the company management is consisted only by one person. The statement that company management is the most important factor in the due diligence is in agreement with theory stated by McClure (2010).

The other – less important factors are stated:

- Technology which company possess
- Market attractiveness
- Product sold by the company – this interesting result where product plays minor role in the due diligence which is de facto investment decision making. As it was stated above the reason is due to frequent pivot of the product and company direction.
- Synergy with corporation which owns the business accelerator – the decision making about the risk is also influenced by the level of synergy with business of maternal corporation. The reason is that invested company can during the acceleration process utilize the resources of the corporation to fasten the growth and proof concept of the product. These factors lower the risk of failure in the mass scale product launch and therefore the success of entire invested company.

3.1.2 Investment monitoring

Both interviewees stated that the monitoring during the investment is far more intense when the business accelerator is in daily contact with invested company. There are many different means how to gather the information about the company whether there are any further crisis rising or if the company struggles in some area of business. This is considered by both interviewees as distinct advantage prior to classical investors such as venture capital funds or banks because mostly these types of investors don't have sufficient resources and time to properly monitor their investment as it is possible to perform in the business accelerator.

The examples how to receive relevant information about the invested company are:

- Regular meetings with program coordinator (usually on 14-day basis)
- Feedbacks from the mentors
- Defined milestones and monitoring whether the invested company fulfills them

All the information is used to overcome the potential failure of the company and therefore the investment.

Milestones are carefully set and they are used as the motivation for the company. The investor use monitoring of the fulfillment of the milestones and thus the expectations of the investor as metric for the second round of investment money as it is bonded to these milestones. This is also the way how to reduce the risk of failure and loss of the investment.

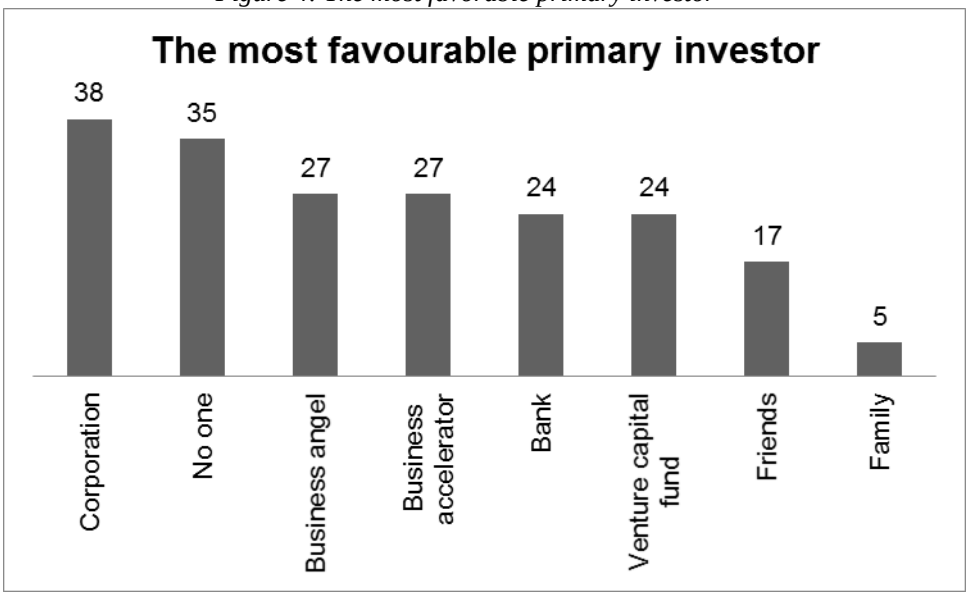
One of the inquired business accelerator uses the investment option for further investment into the company in the situation that the accelerated company will decide to make another round of the investments. This secures and increases the profit for the investment in the case of the success of the company. Other interviewed business accelerator doesn't use investment option due to lack of experience in the later stages of the company and therefore it can't provide sufficient support to the invested company.

3.2 Questionnaire

The other part of the research focuses on the view of the incubated companies at the business accelerator as primary investor. The reason is that investment is two-way contract where both sides have to be comfortable by the condition.

The results of the questionnaire were received by 27 respondents where respondent identified three types of investor with which he would like to make an investment contract arranged from the most to less favorable. If certain type of investor would appear on the first place it would receive 5 points, second would receive 3 point and third one would receive 1 point. As it is possible to see from Figure 4 – the most favorable investor is corporation followed by business angel and business accelerator. Least favorable are friends and family. Which shows that the business accelerators are perceived as quite favorable primary investor especially if they are associated (or owned) by private corporation.

Figure 4: The most favorable primary investor



(Author)

4. Conclusion and recommendation

The risk management in the investment process is mostly divided into two main parts. First is due diligence which is one-off activity which is used to accepting or rejecting of investment proposal. On the other hand, monitoring is frequently executed activity with purpose of information gathering about the investment to address potential crisis.

The aim of the paper is to examine what is the situation and difference in risk management in the investment process in the business accelerators owned by private corporation. To fulfill the aim, the management of the business accelerators were interviewed and the information related to this type of investor was gathered regarding due diligence and investment monitoring.

Results of the interviews show the main fragments of the due diligence and the investment monitoring. Regarding the due diligence, the main factor of the risk management was identified as management team. Followed by less important aspects such as market attractiveness, technology used by invested company, product sold and synergy with corporation.

Regarding the investment monitoring it was concluded that the business accelerators have noticeable advantage in risk management next to more classical investors (i.e. venture capital funds, banks etc.) It is due to more intensive supervision and feedback received from acceleration program. Moreover, the well-adjusted milestones can lower the risk of the failure of the investment.

References

[1] Cardullo, M. (1999) *Technological Entrepreneurism: Enterprise Formation, Financing and Growth*, Research Studies Press Ltd., Baldock, England.

[2] Czech Private Equity & Venture Capital Association. *Central and Eastern Europe Statistics* [cit. 2016-08-17]. Available at: <http://www.cvca.cz/files/EVCA/CEE/EVCA_CEE2014.pdf>

[3] Dvořák, I. (1998) *Rizikový a rozvojový kapitál: venture capital*. 1st ed. Praha: Management Press. ISBN 80-859-4374-3.

[4] European Business and Innovation Centre Network. *BIC Observatory 2012*. [cit. 2013-03-20]. Available at: <http://www.ebn.be/assets/assets/pdf/2012/bic_obs_2012.pdf.>

[5] European Venture Capital Associtation. (2013) *Risk Measurement Guidelines.* [cit. 2016-08-19]. Available at: <http://www.investeurope.eu/media/10083/evca-Risk-Measurement-Guidelines-January-2013.pdf>

[6] European Private Equity and Venture Capital Association. *European Private Equity Activity* [cit. 2016-07-29]. Available at: <http://www.evca.eu/knowledgecenter/statisticsdetail.aspx?id=6392>

[7] McClure, B. (2010) *How Venture Capitalists Make Investment Choices.* [cit. 2016-08-19]. Available at: <http://www.investopedia.com/articles/financial-theory/11/how-venture-capitalists-make-investment-choices.asp>

[8] Nývltová, R. *Mezinárodní kapitálové trhy: zdroj financování*. 1st ed. Praha: Grada, ISBN 978-80-247-1922-1.

[9] Romans, A. (2013) *The entrepreneurial bible to venture capital: inside secrets from the leaders in the startup game.* 1st ed. New York: McGraw-Hill Education. ISBN 0071830359.

[10] Saunders, M., Lewis, P. and Thornhill, A., (2007) *Research methods for business students*. Harlow: Pearson Educated

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Valuation of Advanced Real Options[1]

Miroslav Čulík, Petr Gurný [2]

Abstract

This paper focuses on the valuation of selected advanced (exotic) real options. Particularly, sequential compounded option, option with changing volatility and rainbow option are described and illustrative examples are stated to provide the way how they are valued. For valuation binomial lattice and replication strategy is applied. Results are commented and presented graphically.

Key words

Option, real options, replication strategy, binomial lattice, rainbow option, sequential compound option,

JEL Classification: G12, G32

1. Introduction

Real options methodology is a relatively new approach for solution of a wide range of valuation and decision-making issues. This approach works with traditional methods and models used for financial options valuation and enables to determine assets value. Compared to the traditional passive valuation approaches (NPV, IRR etc.), real options approach takes into consideration two important aspects: (a) riskiness of cash flow generated by the assets and (b) flexibility, i.e. capability of management to change past decisions or to make new ones in already undertaken projects. These future possible decisions (depends on the future state of the world) are modelled as formal call and put options, which have their values and can be exercised by company´s management. Real asset value determined by the real options methodology is then given as a sum of two components: the present value of directly measurable cash flows and the flexibility value, which captures future managerial possibilities (real options).

There are many applications in corporate finance where the financial option valuation models are applied on the real assets valuation (real options approach). Black and Scholes (1973) were the first authors to state that it is possible to take the equity in a levered company as a call option on the company value. Since then, new pioneering work appeared developing real option analysis, see for example Merton (1973) Cox, Ross and Rubinstein (1979) or Brennan and Schwartz (1985). During the last two decades, significant increase in publishing activities on this topic is obvious. This area has been studied and developed by many authors, and new possible applications appear for solutions for a wide array of financial-decision and valuation problems. The key papers and books that focus on the real options methodology application are those of Dixit and Pindyck (1994), Smith and Nau (1995), Trigeorgis (1999),

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Brennan and Trigeorgis (2000), Copeland and Antikarov (2000), Grenadier (2000), Brach (2002), Trigeorgis and Schwartz (2001). Trigeorgis and Smit (2004) or Damodaran (2006).

This paper aims to determine the project (asset) value with embedded advanced (exotic) real options. Particularly sequential compounded real option, rainbow option and option with changing volatility will be evaluated. Due to the characteristics of the most real options (options of American type), binomial lattice and replication strategy will be applied for valuation. Valuation process will be demonstrated via illustrative examples accompanied by the valuation process description and graphical presentation of the results.

2. Real options – managerial flexibility, applicability and valuation

Real options methodology represents an approach where a financial options pricing theory and models are applied on real assets valuation.

Real options methodology application on real projects is justifiable only if:

- There is risk.
- Risk drives project value.
- Management has flexibility.
- Flexibility strategies (real options) are creditable and executable.
- Management is rational in executing real options.

Future managerial investment opportunities captured in real options and quantified by financial option valuation models represent the flexibility component (active part) of the project value. The total project's NPV then consists of two components: the traditional static (passive) NPV of directly measurable expected cash flows, and the flexibility value capturing the value of real options under active management, i.e.,

Expanded NPV = standard (static, passive) NPV of directly measurable cash flows + flexibility value (value of real options from active management).

For flexibility quantification, traditional models for financial options pricing are employed. These models can be classified as follows: (a) analytical (Black-Scholes model), discrete (binomial, trinomial, multinomial), and simulation (Monte Carlo).

Because the real options are mostly the American options (decisions can be made at any time until the investment opportunity disappears), more possible decisions can exist at a given point of time, so the discrete valuation model on the basis of replication strategy is frequently applied.

2.1 Types of managerial flexibility

Generally, risk is understood as an uncertainty in the future evolution of financial variables (expected cash flows). Compared to traditional approaches, here we do not work with one, but more possible scenarios (states) of evolution cash flows. Flexibility (adaptability) reflects the management's opportunities (options) to make new future decisions or to correct the past ones in once undertaken projects to adapt the projects to the new market conditions. These future possible opportunities are modelled as call and put options, which can be in some situations and states exercised either only at some point in time (European option) or at any time during given period (American option).

The future opportunities (options) can be classified according to the flexibility they provide. Options related to the project size are option to expand (management can expand the scale of the production if the market conditions turn out to be more favourable than initially expected), *option to contract* (scale of production can be contracted if market conditions are worse than initially expected), *option to shut down and restart* (production can be stopped and restarted again when market temporarily drop down), etc. Options relating to project life and

timing include *option to abandon* (project can be completely abandoned if market conditions turn overall unfavourable), *option to defer* (management has the right to postpone the project initiation by next year until the uncertainty is not resolved), *sequential options* (project can be abandoned during construction if investment outlay is not a single expenditure at the outset but sequence of outlays extending throughout the project's life) etc. Real options relating to project operations include *output mix options* (product flexibility, input mix options) *process flexibility* etc. Detailed description or other classification possibilities can be found in Trigeorgis (2000), Trigeorgis and Schwartz (2001) etc.

Above mentioned options are plain-vanilla (non-exotic) options and their valuation is easy and straightforward. Presence of the real options can have one of the following forms: only one individual real option is present (only one possible decision at given time) or combinations (portfolio) of real options (more possible decisions at given time). Valuation procedure both individual real options and portfolio of real options is identical; the main difference is that in the case of portfolio of real options, at each decision node we choose from more possible decisions (with respect to prevailing conditions) maximizing intrinsic value.[3]

Advanced (exotic) options are option with more complicated payoff functions. The advanced options could have one or more of the following features:

- payoff doesn´t depend only at the underlying asset value on maturity day, but the average value over given time (*Asian options*), minimum or maximum value (*lookback options*), if some level is reached (*barrier options*), the difference in price between two underlying assets (*spread options*), etc.
- payoff depends on the performance of the asset or assets that are in portfolio underlying: for example, as the weighted sum or average of different (*basket options*), performance of the best asset in the portfolio (*Himalaian options*), worst-performing asset in the portfolio (*Everest options*), etc.
- payoff depends on the moneyness of the option on the maturity day (*cash or share options*)
- options including callability and putability rights, etc.

Applications based on exotic option theory are widely employed, as well. Geske and Johnson (1984a) use exotic multi-fold compound options for the American put option, while Carr (1988) presents the pricing formula for sequential exchange options. Corporate debt (Chen, 2003 and Geske and Johnson, 1984b) and chooser options (Rubinstein, 1992), as well as capletions and floortions (options on interest rate options) (Musiela and Rutkowski, 1998) are also priced by compound options, etc.

In this paper, the attention will be focused on the valuation of the following exotic options: compound options, rainbow options, and option with variable parameters. These will be described in the following part.

2.1.1 Compound real options

Compound real options are option, where the payoff involves the value of another option. Compound option then has two or more maturity days and two or more exercise (strike) prices. Such options can be typical for multistage projects where project undertaking can be divided into more phases, where next phase depends on the success of the previous one. Thus, the success of the first stage creates right but not the obligation to start the following stage and so on. Compound options can be either sequential or parallel. Sequential options are options where first option must be exercised to create the next one (for example the R&D of a new product must be successful to allow its commercial production). On the other hand, parallel compound options are available at the same time.

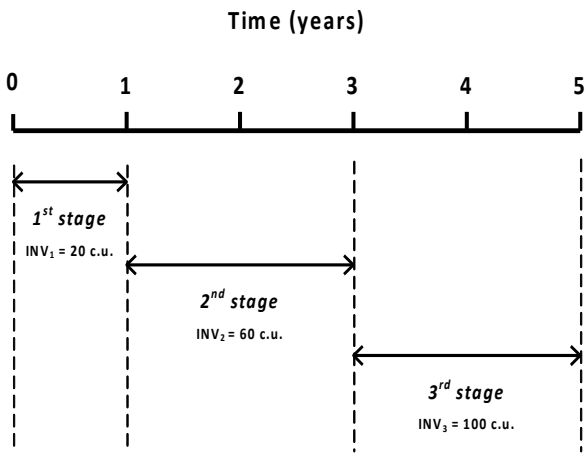
[3] For more details se efor example Čulík (2013) pp. 123-129.

A: Valuation of the sequential compound real option

A1) Problem specification

A company is considering undertaking a project. The investment stage is divided into three intermediate stages. Each stage must be completed and successful before the next stage is started. Following Figure 1 shows the length and investment outlays of each stage.

Figure 1: Stages of the investment



The present value of the cash flow generated by the project (gross project value) if undertaken now is 150 c.u. with volatility of log return of 25%. The future project value follows GBM and binomial lattice is applied. Risk free rate (annual) is 5%. The goal is to determine the net present value of such project.

A2) Steps of the problem solution

Process of the solution can be decomposed into the following steps:

I. Calculation of the upward and downward coefficients, risk-neutral probabilities of up and down ward movements in the underlying asset (gross project value) according to the formulas,

$$u = \exp\left(\sigma \cdot \sqrt{dt}\right), \tag{1}$$

$$d = \exp\left(-\sigma \cdot \sqrt{dt}\right), \tag{2}$$

$$p^u = \frac{\exp(r_f \cdot dt) - d}{u - d}, \tag{3}$$

and construction of the binomial lattice of the underlying asset (gross project value, *V*) for the whole period, see Figure 2.

Figure 2: Binomial lattice of the gross project value (V)

0	1	2	3	4	5
					524
				408	
			318		318
		247		247	
	193		193		193
150		150		150	
	117		117		117
		91		91	
			71		71
				55	
					43

II. Calculation of value for each sequential options.

II-a) Valuation of the longest option (option value ${}_3C_t$)

Valuation is performed by backward induction, i.e. from the final nodes of the binomial lattice to the initial one. For the last nodes of the binomial lattice (period $T = 5$) is the option value given as,

$${}_3C_T = \max(V_T - INV_3; 0), \tag{4}$$

for the previous ones (period t from 0 to 4) according to the formula,

$${}_3C_t = \max\left[V_t - INV_3; \left({}_3C^u_{t+1} \cdot p^u + {}_3C^d_{t+1} \cdot p^d\right) \cdot \exp(-r_f)\right]. \tag{5}$$

Calculation is repeated until the value in initial node of the binomial lattice is reached.

II-b) Valuation of the intermediate option (option value ${}_2C_t$)

Valuation is again performed by backward induction. Values for each final nodes (period $T = 3$) are given by the formula,

$${}_2C_T = \max\left({}_3C_T - INV_2; 0\right), \tag{6}$$

for the previous ones (period t prom 0 to 2) according to the formula,

$${}_2C_t = \max\left[{}_3C_t - INV_2; \left({}_2C^u_{t+1} \cdot p^u + {}_2C^d_{t+1} \cdot p^d\right) \cdot \exp(-r_f)\right], \tag{7}$$

Valuation is again repeated until the value in initial node is reached.

II-c) Valuation of the shortest option (option value ${}_1C_t$)

Similarly, the valuation is performed by the same procedure as the valuation of the option ${}_2C_t$, i.e. for the terminal nodes (here $T = 1$),

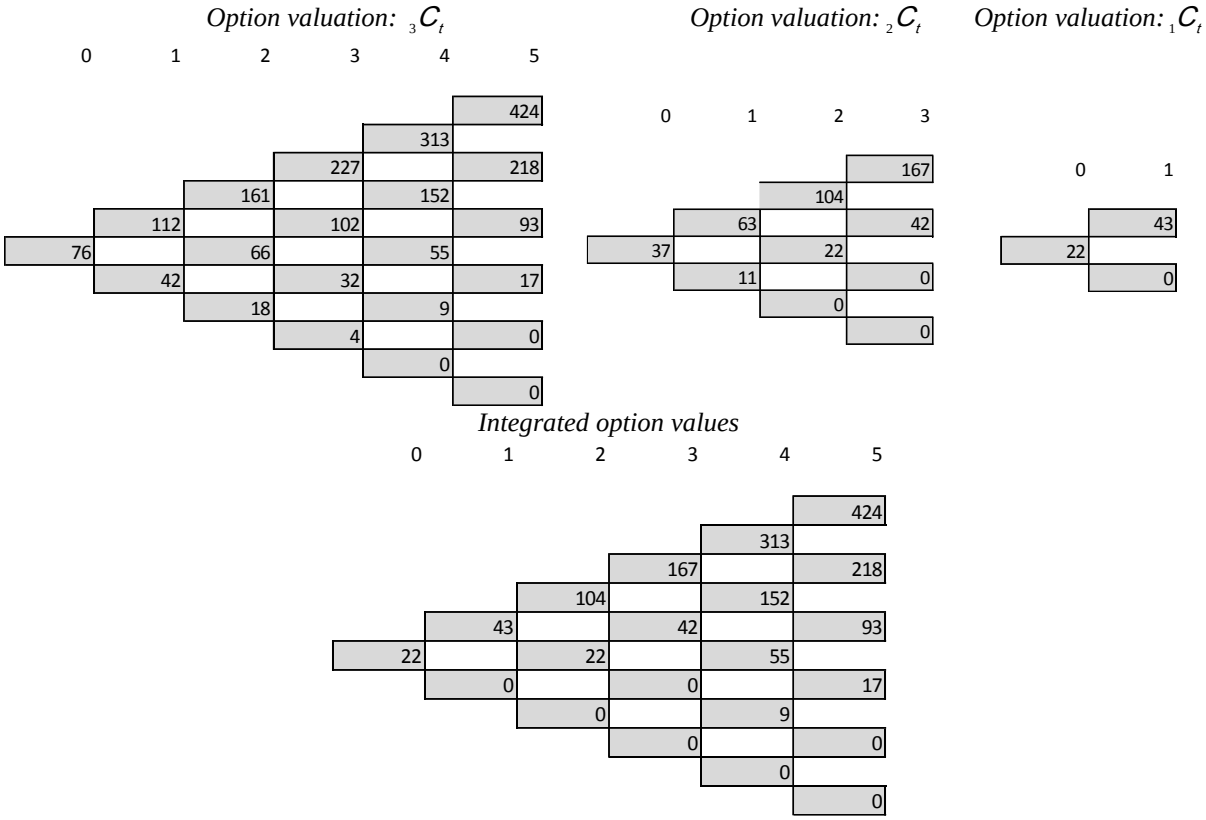
$${}_1C_T = \max\left({}_2C_T - INV_1; 0\right), \tag{8}$$

and for the initial node at $t = 0$,

$${}_1C_t = \max\left[{}_2C_t - INV_1; \left({}_1C^u_{t+1} \cdot p^u + {}_{13}C^d_{t+1} \cdot p^d\right) \cdot \exp(-r_f)\right]. \tag{9}$$

Option values for each sequential option and integrated option values are depicted in the following Figure 3.

Figure 3: Binomial lattice of the sequential options and integrated option values



B: Project valuation with embedded option and changing volatility

B1) Problem specification

The goal is to determine the project value by applying binomial lattice (with the option to expand a project). The option is an American-type option, i.e. the project can be expanded at any time during the expected life span. The underlying asset (incremental project value if expanded) follows GBM with the initial value V_0 = 100 c.u. Company has the option to expand the project at any time during the life span with the costs on expansion I_E = 70 c.u. The annual risk-free rate is r_f = 5%. The volatility of the log returns in project´s cash flows is σ_1=30% (period 0-2) and σ_2=50% (period 3-4).

B2) Steps of the problem solution

Valuation with changing volatility includes following steps:

I) Calculation of upward and downward coefficients for each period and volatility according to (1) and (2) and corresponding risk-neutral probabilities. Recall, that the volatility σ_1 applies for the first two periods, σ_2 for period three and σ_3 for period four. Results are summarized in the following Table 1.

Table 1: Up(down) ward coefficients and transition probabilities for given volatility level

	volatility (%)	period	upward factor (u)	downward factor (d)	p^u (%)
σ_1	40	0–1 and 1–2	1,349859	0,740818	51,0
σ_2	50	2–3 and 3–4	1,648721	0,606531	42,7

II) Simulation of the underlying asset *V* via non-recombining binomial lattice.

III) The intrinsic values calculation, IV , as,
$IV_t = \max(V_t - INV; 0)$

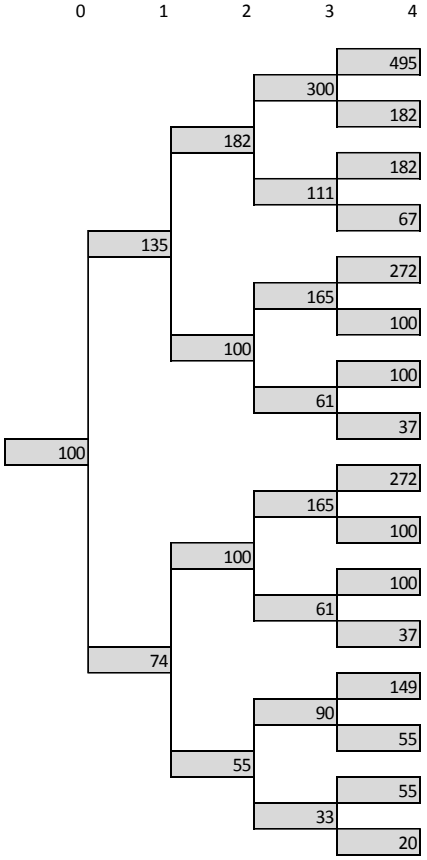
IV) Option value calculation according to the formula
$C_t = \max\left[\left(C^u_{t+1} \cdot p^u + C^d_{t+1} \cdot p^d\right) \cdot \exp(-r_f); IV_t \right].$ (10)

V) Construction of the decisions lattice.

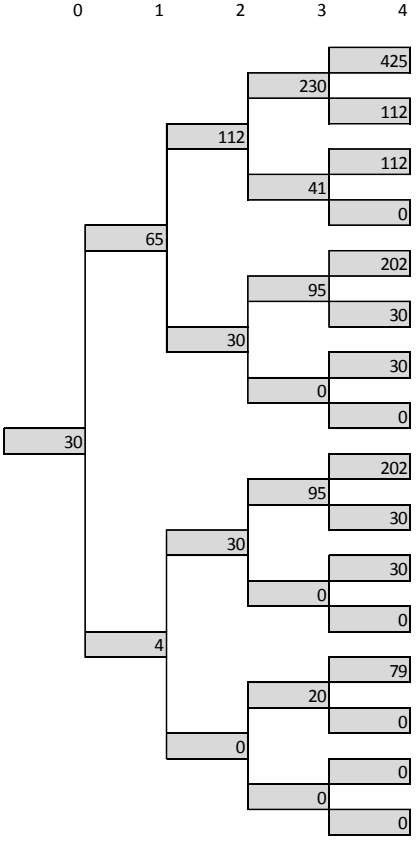
Following Figure 4 shows underlying asset lattices; intrinsic vales lattice, option price lattice and decisions lattice.

Figure 4: Non-recombining binomial lattices of option value determination

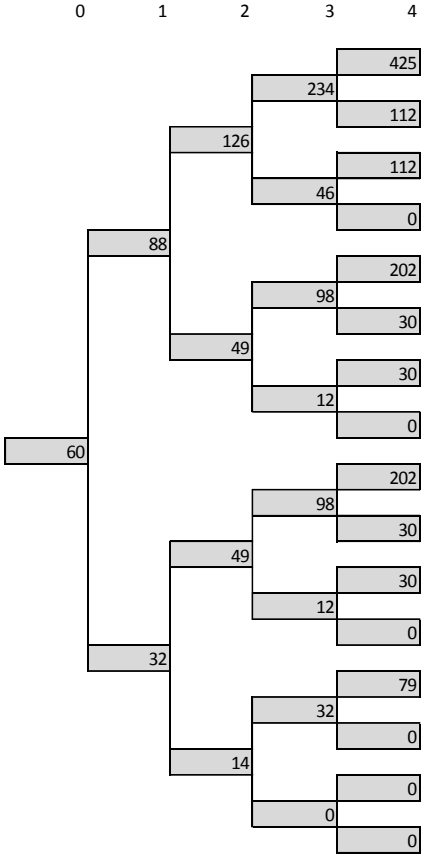
a) Non-recombining underlying asset lattice



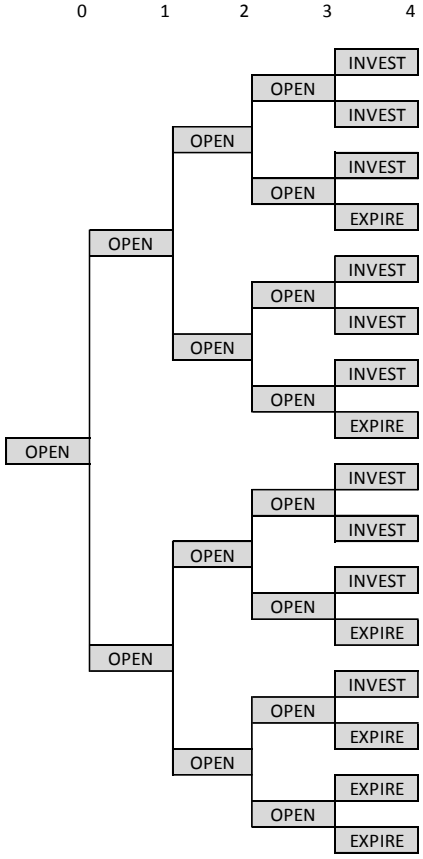
b) Non-recombining intrinsic values lattice



c) Non-recombining valuation lattice



d) Non-recombining decisions lattice



C: Rainbow option valuation

C1) Problem specification

The company is considering to undertake an investment project with the expected life span of two years. The cash flows generated by the project is influenced by two random variables: quantity (*Q*) and price (*P*); the fixed costs of the project are constant and equals 50 c.u. For simplicity, no correlation between both random variables exists. The volatility of the log returns of quantity is 10% and 25% for the price. Both these random variables are assumed to follow GBM via binomial lattice. Project cash flow in each node are given according to the formula $FCF_t = Q_t \cdot P_t - FC$. The goal is to (*a*) calculate the project value if no real option is present and (*b*) calculate the project value and the flexibility (option) value if the company has the opportunity to abandon the project at the beginning of the second year and sell it for salvage price A = 1000 c.u. The annual risk-free rate is 1.25%.

C2) Steps of the problem solution

Solution of the problem can be again decomposed into the following steps:

I) Calculation of the upward and downward coefficients according to (1) and (2), risk-neutral probabilities of up and downward movement according to (3) for both random variables and construction of the binomial lattices, see Figure 5.

Figure 5: Binomial lattices of random variables: quantity (left) and price (right)



II) Construction of the cash flow binomial lattices for project with no flexibility according to the formula,

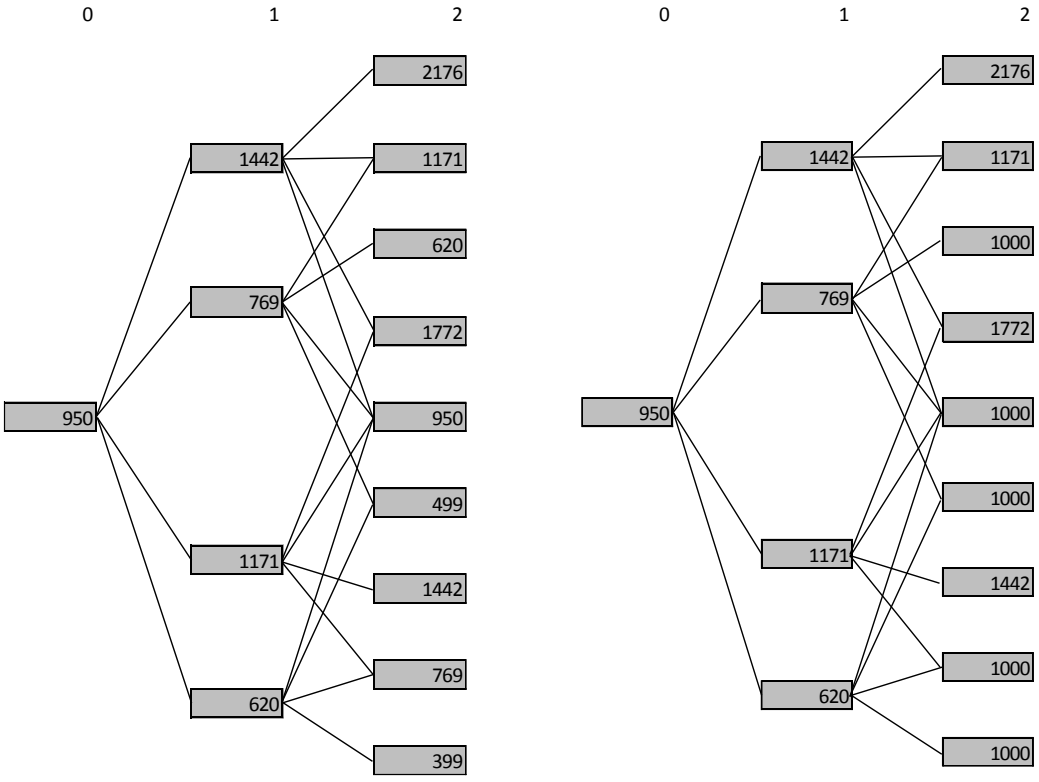
$$FCF_t = Q_t \cdot P_t - FC, \tag{11}$$

if option to abandon a project is considered, the cash flow for the last nodes of the binomial lattice is given as,

$$FCF_T = \max[Q_T \cdot P_T - FC; A_T], \tag{12}$$

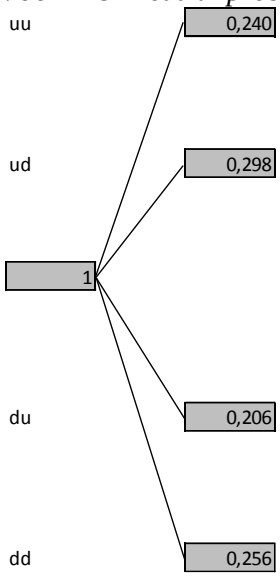
for the previous once according to (11). Results are shown in Figure 6.

Figure 5: Binomial lattices of cash flow: with no option (left) and with option to abandon (right)



III) Calculation of join risk-neutral probabilities, see Figure 6.

Figure 6: Join risk-neutral probabilities

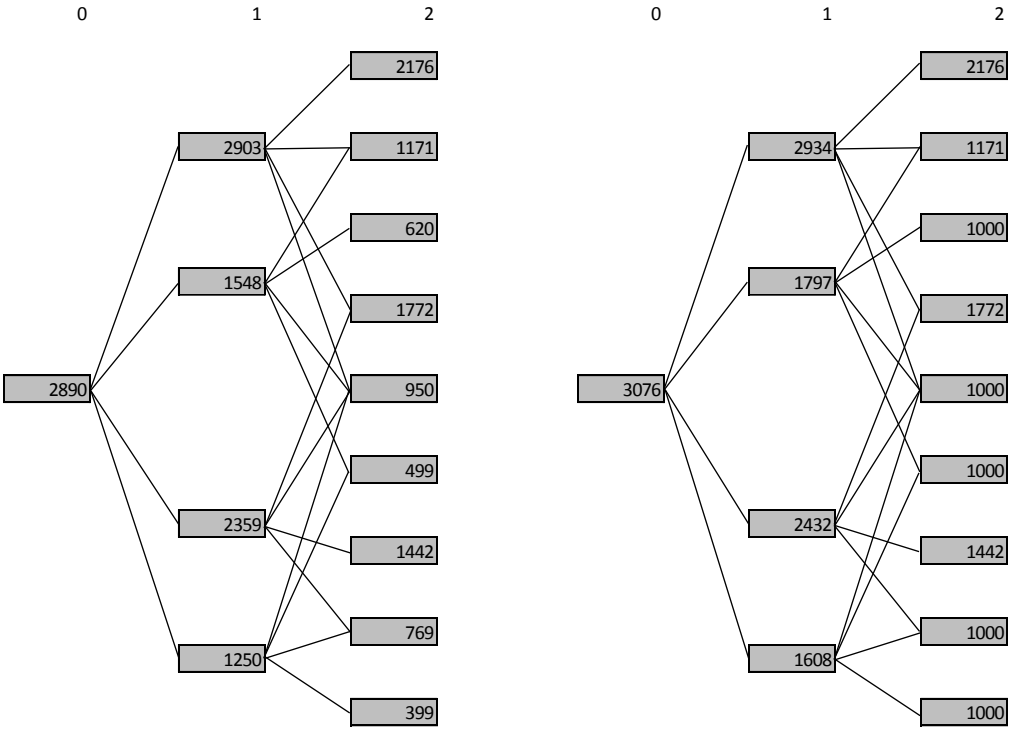


IV) Project valuation by backward induction from the end nodes to the initial one according to the formula,

$$V_t = FCF_t + \left[FCF_{t+1}^{uu} \cdot p^{uu} + FCF_{t+1}^{ud} \cdot p^{ud} + FCF_{t+1}^{du} \cdot p^{du} + FCF_{t+1}^{dd} \cdot p^{dd}\right] \cdot \exp(-r_f). \quad (13)$$

Results of the project valuation with no flexibility and option to abandon is shown in Figure 7. It is obvious that the option to abandon increases the project value by 186 c.u.

Figure 7: Project valuation: no flexibility (left) and with option to abandon (right)



3. Conclusion

This paper aims at the valuation of the advanced (exotic) real options. Specifically, sequential compound real option, option with changing variables (volatility) and rainbow

options are evaluated. For the valuation purposes, binomial lattice and replication strategy is used. Compared to the traditional valuation models, real option methodology assumes active management that can revise the past decisions or make new ones in response to the market conditions development. Flexibility and adaptability of the projects represent significant part of the total project value, which is not measurable by traditional methods (they assume passive approach of the management). This enables to undertake more projects even those that would be by applying traditional methods rejected.

References

[1] BLACK, F., SCHOLES, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy* 81(3): pp. 637–654.

[2] BRACH, M. A. (2002). *Real Options in Practice.* New Jersey: Wiley.

[3] BRENNAN, M. J., SCHWARTZ, E. S. (1985). Evaluating natural resource investments. Journal of Business 58(2): pp. 135–157.

[4] BRENNAN, M.J., TRIGEORIS, L. (2000). Project Flexibility, Agency and Competition: New Developments in the Theory and Application of Real Options. 1st Ed. London: Oxford University Press.

[5] COPELAND, T. E., ANTIKAROV, V. (2003). Real Options, Revised Edition: A Practitioner's Guide. 2st Ed. New York: Texere.

[6] COX, J. C., ROSS, S. A., RUBINSTEIN, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics* 7(3): pp. 229–263.

[7] DAMODARAN, A. (2006). *Damodaran on Valuation: Security Analysis for Investment and Corporate Finance*. 2nd Ed. New Jersey: Wiley.

[8] DIXIT, A.K., PINDYCK, R.S. (1994). *Investment under Uncertainty*. 1st Ed. New York: Princeton University Press.

[9] GRENADIER, S. (2000). *Game Choices: The Intersection of Real Options and Game Theory*. London: Riskbooks.

[10] GUTHRIE, G. (2011). Learning Options and Binomial Trees. *Wilmott Journal*, Vol. 3(1), pp. 1–23.

[11] HAAHTELA, T. (2010). Recombining Trinomial Tree for Real OptionValuation with Changing Volatility. In *Annual Real Option Conference*.

[12] HOEK, J., ELLIOT, R.J. (2006). *Binomial Models in Finance*. New York: Springer.

[13] HULL, J.C. (2014). *Options, Futures, and Other Derivatives*. 9th Ed. Prentice Hall.

[14] LEE, M.Y., *et all* (2008). The generalized sequential compound options pricing and sensitivity analysis. *Mathematical Social Sciences*, Vol. 55, Issue 1, pp. 38–54.

[15] MERTON, R. (1973). The theory of rational option pricing. *Bell Journal of Economics and Management Science* 4(1): pp. 141–183.

[16] MUN, J. (2005). *Real Option Analysis. Tool and Techniques for Valuing Strategic Investments and Decisions*. 2nd Ed. New Jersey: Wiley.

[17] MUN, J. (2003). *Real Options Analysis Course: Business Cases and Software Applications*. New Jersey: Wiley.

[18] SMITH, J.E., NAU, R.F. (1995). Valuing risky projects: Option pricing theory and decision analysis. *Management Science* 14: pp. 795–816.

[19] TICHÝ, T. (2008). *Lattice models. Pricing and hedging at (In)omplete Markets*. VŠB-TU Ostrava.

[20] TRIGEORGIS, L. (1999). *Real Options and Business Strategy: Applications to Decision Making.* London: Riskbooks.

[21] TRIGEORGIS, L. (2000). *Real Options in Capital Investments: Models, Strategies and Applications*. London: Praeger Westport.

[22] TRIGEORGIS, L., SCHWARTZ, E. S. (2001). *Real Options and Investments under Uncertainty*. Cambridge: MIT Press.

[23] TRIGEORGIS, L., SMITH, H. T. J. (2004). *Strategic Investment: Real Options and Games*. New York: Princeton University Press.

The Impact of Market Factor Changes on The Capital Requirements in an Insurance Company

Petra Daníšek Matušková [1]

Abstract

The new insurance regulation, known as Solvency II, which has been fully implemented into national legislations of the member states of the European Union during this year, introduces major changes in the existing risk management in an insurance company. The key idea of Solvency II is to increase the protection of policyholders and insured persons using higher demands for high-quality and function of risk management, systems of internal controls and transparency in the insurance market. The main aim of this paper is to estimate capital requirements for market risk (equity and currency) under Solvency II and evaluate the impact of exchange rate changes to capital requirements in an insurance company.

Key words

Solvency II, Insurance, Capital, Capital Requirement

JEL Classification: G17, G22, C3

1. Introduction

Insurance sector represents one of the most complex sectors of financial services which aim is to provide insurance protection against negative and unpredictable events, which bring material and financial losses. The first attempts to regulate and harmonize the insurance industry at the transnational level date back to the 70s of the last century, but major changes occurred in 2002 with the acceptance of a regulatory directive Solvency I. Although the Solvency I brought considerable changes in existing approach of the regulation of insurance sector, it has been insufficient. The European Union (EU) has initiated a revision of the European regulatory framework for the insurance industry, which is based on the all risks and not just the risks derived from the premium volume or insurance costs. The final form of Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) was accepted in 2009. The regulation, which should have been initially implemented in 2012, was fully implemented after many years of discussion, procrastination and replenishment (Omnibus II) during 2016 into national legislations of countries of EU. The Czech Republic missed deadline for implementation and did not approve amendment to insurance Act authorize, which would implement elements of Solvency II into national law. Therefore, Czech Republic may be sanctioned by the European supervisory authorities.

Solvency II is based on the regulation of risk management, balance approach and it implements more fundamental and more complex evaluation of a total financial situation of the insurer. Solvency II is based on three pillars (requirements for sufficient capital of the insurance companies, rules for risk management and requirement for the insurance's activity and transparency); and is similar as Basel II, which is used in banking sector.

The main objective of Solvency II is to strengthen of financial stability of insurers and reinsurers and achieve greater protection of insured persons against unguarded processes of

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these institutions. At the same time Solvency II unifies the rules of insurance and reinsurance business in the single European market, so that the offered conditions and insurance protection would be equal for all EU citizens. Solvency II also extends the power of the central banks as supervisory institutions, thereby it is strengthening its responsibility for the stability of the insurance market.

Major changes were made in the calculation of capital requirements. That changes contained formula adjustment, extent of risks and techniques how to reduce value of capital requirements.

The main aim of this paper is to estimate capital requirements for market risk (equity and currency) under Solvency II and evaluate the impact of exchange rate changes to capital requirements in an insurance company.

First part of this paper is aimed at theoretical basis of capital requirements and methods for their calculation. Second part is an application part, where equity portfolio with minimum risk is defined and for that portfolio capital requirements are estimated by Value at Risk and Expected Shortfall. Then there is also analysis of impact of exchange rate changes to value of capital requirements. The last part contains an evaluation of the results.

2 Capital requirements under Solvency II

As mentioned above, Solvency II is based on three basic pillars. The first pillar is focused on the quantitative aspects imposed on insurers. These are the rules for valuing assets and liabilities, the rules for determining the amount of technical provisions, investment rules, rules on formation of equity capital and so on. Main part contains the rules for calculation of solvency capital requirement (SCR) and minimal capital requirement (MCR), which are the essential elements of risk management for all institutions.

SCR reflects a level of capital that insurance must hold, that with 99.5 % probability it will be able to fulfil its obligations to its clients in one-year time horizon. SCR is calculated once a year and applies to all quantifiable risks including diversification effects, which the Solvency I to ignores.

In first pillar of Solvency II there is also MCR defined. The MCR is value at risk of primary capital, which with 85% probability will not be exceeded during one year. At the same time the MCR must be between 25-45 % of the SCR and it does not fall below the lower limit, which is prescribed for the type of insurance. MCR is calculated on a quarterly basis. When the primary capital falls under MCR, it is approached to intervention by the supervisory authority.

Solvency II specifies three approaches of calculation of capital requirements. According to the standard model the final SCR is sum of the basic solvency capital requirement (which minimally consists of the modules: life risk, non-life risk and health underwriting risk, market risk and credit risk), capital requirement for operational risk and adjustments of ability technical provisions and deferred taxes absorb losses. Partial internal model allows replace a part of the calculation of the standard model by internal sub-modules and full internal model which must faithfully capturing the risk profile of an insurance company and thus better reflect the characteristics of the insurance business and its relationships.

The capital requirements are covered by equity capital evaluated by market which is set as the difference between market evaluated assets and reserves.

Calculation of solvency ratio pointer, which is determined as the ratio of SCR and equity capital, was adjusted too. Its value should be above 100% but necessary solvency ratio values are given by differing requirements and levels of capitalization.

The second pillar focuses on qualitative requirements for insurers and captures unquantifiable risks like requirements for the management, control system and supervisors. Last but not least, the second pillar deal with stress testing, which is an integral part of the internal risk management system which identifies potential losses and uncover the vulnerability of the portfolio against hypothetical changes in risk factors.

The third pillar is not essential for this paper, but it focuses on requirements for quality reporting and transparency of an insurance company.

2.1 Value at Risk and Expected Shortfall

Value at Risk (VaR) or Expected Shortfall (ES) are two methods, which are accredited by directive Solvency II.

Value At Risk is probably most used method for measuring and managing risks in practice because it is relatively simple and understandable conception. VaR represents the risk value, which with given probability α will not be exceeded during certain time period N, see Hull (2007). Mathematically VaR can be expressed as one sided quantile of distribution of profits and losses for certain time of holding, and it is determined based on certain historical period. It is a function which consists of two parameters: time horizon (N) and the significance level (α).

$$\Pr\left(\Delta\tilde{X} \leq -VaR\right) = \alpha\,, \tag{1}$$

when $\Delta\tilde{X}$ is joint probability distribution of loss.

No standard calculation method exists for the determination of VaR. Alexander (2008) shows, that there are three basic methods used to calculate VaR in practice: **Variance and covariance method**, which is used for estimation of potential portfolio's losses volatility and correlation, which are acquired from historical data, **Historical simulation**, where the potential future loss is estimated based on losses which happened in the past, **Monte Carlo simulation**, which works with large number of simulations of portfolio's value development.

Expected Shortfall (ES) is second method, which is recommended by directive Solvency II. This method represents a coherent risk measure and captures extreme losses that are occurred with low probability, which is typical for insurance sector. ES can be defined as the average of expected losses that exceeding the Value at Risk on the confidence level α. The Expected Shortfall of X at confidence level $\alpha \in (0, 1)$ is defined as:

$$ES_\alpha = E(X \,|\, X > VAR_\alpha) \tag{2}$$

where ES is expected extreme loss, X is random variable with $E(|X|) < \infty$, VaR represent Value at Risk on confidence level α.

2.2 Monte Carlo Simulation

Monte Carlo simulation (SMC) is a flexible tool modelling stochastic processes. Method is deriving from the law of big numbers; where the large numbers of randomly generated risk factors with selected characteristics come close to theoretical assumption, see Tichý (2010). Large numbers of scenarios generate estimation's error than corresponds to standard deviation of result. In 1997 Boyle et al. (1997), introduced techniques, which are trying to lower estimation error (dispersion of result) and by this increasing simulation's Monte Carlo effectiveness. Among these procedures of MC belong: Antithetic Sampling Monte Carlo, Stratified Sampling Monte Carlo, Control variants Monte Carlo, Moment matching Monte Carlo and others.

Primitive Monte Carlo Simulation (PMC) is based on generate of random elements so that they correspond to characteristics of selected probability distribution. The correlation structure is also integrated in the calculation. Subsequently development of yield (x) of assets

with selected model specifying behaviour of individual portfolio instruments is simulated e.g. Brownian motion, Levy's model etc. This method is comprehensible, easy and relatively quick, but reliable estimate just in case sufficient number of scenarios.

Antithetic Sampling Monte Carlo (ASMC) is for its simplicity and comprehensibility used very often in finance. The method is based on negative correlation among vectors of random elements, meaning, see Tichý (2010). Supposing *X* random elements from normalized normal distribution then by multiplying vector with coefficient -1 we will get vector of random elements. By unification of both vectors we can achieve double the amount of random elements, which better fulfil characteristics of selected distribution. It is a method, which leads to the decrease of the time consumption and achievement of zero mean, meaning the symmetry of probability distribution. The method's limitations lie in the fact that it can be used only when generating random elements from symmetric probability distributions.

Latin Hypercube sampling (LHS) is based on stratified sampling. The method principle lies in dividing domain of the cumulative distribution function *F(x)*, which is corresponded to the probability density function *f(x)*, into *N* disjunctive intervals (stratas). Individual stratas have same probabilities of *1/N*. From each strata one value is selected, which represents the entire interval and is to be used only once in the simulation. There are many methods, how to choose representing value e.g. by generating random numbers *n* from the interval (0,1) of even distribution and assign them to appropriate intervals by linear transformation. By using inverse transformation of the distribution function the values of stratas are obtained:

$$x_{i,k} = F_i^{-1}\left(\frac{n+(k-1)}{N}\right), \tag{3}$$

where F_i^{-1} is the inverse distribution function of random variable X_i, k is the kth strata of the ith random variable X_i, n is a randomly generated number from the interval (0;1) and N is the number of intervals.

When representative stratas for all random variables are generated, they are selected for individual simulations in the form of random permutations of whole numbers *1, 2, … N*.

Very often correlation between individual variables is arising, which affects the accuracy and quality of results. It is possible to supply this method with capturing the correlation structure between variables (LHSD). Again, there are several ways how to include the correlation structure to calculation. In many cases the statistical correlation is implemented in the form of exchanging the sequence of samples at individual variables without changing their values. In this way, the probability distribution of each random quantity is preserved.

The advantage of this method is the way of the selection of realisations, where the entire scope of the random variable input is covered uniformly with respect to the distribution function and then no real value is a priori excluded.

2.3 Stress testing

Stress testing assesses the impact of extreme events and their combinations to the solvency of insurers or other suitable metric e.g. the value of assets/liabilities, economic profit, economic capital, etc. Simultaneously, stress testing is used for verifying the effectiveness of regulatory measures. The objective of stress testing is to identify the size of potential losses, not the frequency of formation, and uncover the vulnerability of the portfolio against hypothetical risk factors changes. Testing uses the pre-defined scenarios of development of risk factors that characterize a possible future situation. Pre-defined scenarios are based on historical information and hypotheses. Scenarios are generally determined on the basis of expert opinion and they are composed of extreme changes in risk factors, such as for example,

the exchange rate changes, decrease of shares prices or other assets. It is possible to generate erroneous data, because the scenarios are subjectively determinated. EIOPA[2], in the context of Solvency II, issues technical specifications, which also include predefined scenarios for stress testing.

3 Estimation of capital requirements and changes of exchange rate

The portfolio of an insurance company is composed of foreign shares with a total value of 6.3 billion CZK (NAV = value of equity capital). The insurance company is then exposed to currency and equity risk, among other things. The daily closing prices of three stock indexes, which are denominated in three different currencies, were used as input data: Down Jones Industrial Average (DJI) denominated in USD, Deutsche Aktien IndeX (DAX) denominated in EUR and FTSE 100 (FTSE) denominated in GBP. Prices of individual indexes were inquired on daily basis in the period of January 1st, 2003 to January 1st, 2015.[3] For the same period exchange rates of foreign currencies to CZK were also inquired. The exchange rates announced by Czech National Bank[4] were considered. In case that some stock markets did not trade, the missing data were substituted by the value from previous business day. So that, there was time line of 3110 daily logarithmic yields of stock markets and exchange rates. Further it is supposed that portfolio with minimum risk (M), which was determined, based on conditions of Markowitz model and its composition is shown in Table 1.

Table 1: Market portfolio´s composition

	DJI (%)	DAX (%)	FTSE (%)	$E(R_p)$* (%)	$\sigma(R_p)$* (%)	S	K
M	0.2827	0.3053	0.4120	4.127	15.728	0.1274	9.9446

**annual values*

From the aforementioned table it is evident that the portfolio does not have normal probability distribution. The mean value is approximately 0.2 % and standard deviation is 16 %. The data are more or less symmetrically distributed around mean value; there is identified low positive skewness (S) and higher kurtosis (K) compared to normal distribution. This phenomenon is also known as Leptokurtic probability distribution, which is typical for financial assets. There is slight dependence between indexes. The strongest correlation is between DJI/DAX and the value is about 0.51.

Next, it is supposed that yields have multivariate student distribution with 4.8157 degrees of freedom, which has been estimated by method of moments. Furthermore, the analysis of sensitivity of degrees of freedom to kurtosis was performed. This analysis confirmed the well-known rule; that with growing value of degrees of freedom, kurtosis degreases and vice versa. If value of degrees of freedom approaches to infinity, the Student distribution approaches to normal probability distribution.

Based on all the above assumptions and assuming that behaviour of individual portfolio's instrument follows Brownian motion multivariate Student distribution portfolio with minimal risk was simulated by PMC, ASMC and LHSD. The descriptive characteristics are shown below in Table 2.

[2] European Insurance and Occupational Pensions Authority
[3] Data available at: <http://finance.yahoo.com/>
[4] Data available at :< http://www.cnb.cz/cs/index.html>

Table 2: Descriptive characteristics for Student distribution

PMC		ASMC		LHSD	
$E(R_p)$ %*	$\sigma(R_p)$ %*	$E(R_p)$ %*	$\sigma(R_p)$ %*	$E(R_p)$ %*	$\sigma(R_p)$ %*
5.065	20.515	4.127	20.715	5.065	20.515
S	**K**	**S**	**K**	**S**	**K**
-0.120	9.2776	-0.000	8.4216	-0.120	9.2776

** annual values*

From the above table it can be seen that individual estimations of standard deviation based on ASMC corresponds with empirical values. However, other descriptive characteristics are significantly different. The remaining used simulations provide the same results. It can be explained by the fact that if a sufficient quantity of random scenarios is used, the error of the PMC estimation is reduced. The results are comparable with much more sophisticated method LHSD. Both simulation methods capture the kurtosis better than other methods and with comparison to ASMC standard deviation of the portfolio is also better. Other descriptive characteristics of the simulated probability distribution differ from the empirical values.

Thereafter the solvency capital requirement was calculated as VaR or ES at 0.5 significant level for a yearlong time horizon and minimal capital requirement as VaR or ES at 15% significant level for a yearlong time horizon. Results are shown in Table 3.

Table 3: Capital requirements in %

VALUE AT RISK						EXPECTED SHORTFALL					
PMC		ASMC		LHSD		PMC		ASMC		LHSD	
SCR	MCR	SCR	MCR	SCR	MCR	SCR	MCR	SCR	MCR	SCR	MCR
59.27	13.22	61.34	14.11	59.27	13.22	79.40	26.13	83.04	27.34	79.40	26.13

The above table shows the capital requirements for market risk (consists of equity and currency risk module) calculated by the standard approach under Solvency II. The insurance company must hold equity capital amounting to approximately 60 % (3.8 bil CZK) to cover its risks, if VaR method was used. This equity capital must not fall below approximately 800 mils. CZK in the following year, that means not below 13.2 % of the equity capital.

Capital requirements estimated by ES lead to higher values. It is necessary to keep approximately 80 % of the value of equity it means approximately 5 bil. CZK. The value of MCR rose too so equity capital must not fall below approximately 1.7 bil. CZK. Assuming that the value of the portfolio is equal to the value of equity capital, it is clear that the insurance company disposes of capital in the value of approximately 165% of the SCR in the case of using the VaR method. In the case of using the EC method, the value of capital is approximately 120 % of the SCR. Based on these results it can be said that the insurance company is adequately capitalized. The results show that the choice of the value at risk (VaR, ES) has a significant impact on the final value of the capital requirements. The choice of simulation method has only little effect on capital requirements in this case.

The value of capital requirements is also influenced by using VaR and ES methods, which are limited because historical data was used and assuming that the market will involve as same as in the past. Other problem can be time horizon. The capital requirements are calculated for one year ahead. It is obvious that market does not develop as same as in the past and extreme and unexpected events that have an impact on the capital of insurance companies may arise. For example, changes of macroeconomic variables (exchange rate, interest rate etc.), changes of new business strategy (new risk factors) or extreme events (financial, economic crisis etc.).

Based on these facts, it is assumed that selected portfolio will be exposed to currency shocks in the following form. The first scenario is a hypothetical release of CZK and its

appreciation to 25.670 CZK/EUR. This value represents an annual average exchange rate for buying euro before the Czech national bank made interventions in November 2013. The other extreme scenario is a historic depreciation of GBP on average value 32.189 CZK/GBP which occurred after the July´s referendum about leaving of Britain from the EU. The both stress scenarios lead to an appreciation of the CZK against foreign currencies. These facts represent decrease of portfolio value. The results of stress testing are summarized in Table 4.

Table 4: Value of NAV after stress testing in CZK

NAV before stress	6 300 006 300.00	
NAV after 1. stress scenario	5 698 837 040.24	
NAV after 2. stress scenario	6 149 620 827.92	
NAV after 1. a 2. stress scenario	5 548 451 568.16	
	SCR	MCR
Capital requirement by VaR (PMC, LHSD)	3 734 013 734.01	832 860 832.86
Capital requirement by ES (PMC, LHSD)	5 002 205 002.20	1 646 191 646.19
Capital requirement by VaR (ASMC)	3 864 423 864.42	888 930 888.93
Capital requirement by ES (ASMC)	5 231 525 231.52	1 722 421 722.42

From the Table 4 it is evident that, the biggest drop in NAV was caused by two shocks at the same time, when the NAV fell by more than 755 mils down. CZK. Although the GBP pound has the largest part of the portfolio, its appreciation has not been significant compared with appreciation of Euro. NAV does not even once fall below the value of SCR and therefore the insurance company would not have a problem with solvency. The final score is affected again by the choice of the value at risk, which is used for determining capital requirements. While the insurance company dispose equity capital of 150 % of SCR in the case of VaR method and first shock, the equity capital decrease to 108 % of SCR in the case of ES with ASMC. Effects of both shocks in the same time cause decrease of the equity capital even to 106 % of SCR.

4 Conclusion

During this year the insurance sector is undergoing significant changes which intervene in functioning of system of insurance companies. Solvency II, which has been fully implemented into national legislations of member states of European Union (except the Czech Republic), provides more fundamental and more complex evaluation of a total financial situation of the insurer. Solvency II also aims to provide adequate protection to all clients and harmonize premium conditions on the level of European market.

Solvency II brought considerable changes in the determination of capital requirements, which represent the necessary equity capital for risk coverage and contribute to solvency of an insurance company. The aim of this paper was to estimate the capital requirements for market risk (equity and currency) under Solvency II and evaluate the impact of exchange rate changes to capital requirements in an insurance company.

The results show that the choice of the value at risk, i.e. VaR and ES, or method of Monte Carlo simulation lead to different results and have a major impact on the capital requirements. Value of SCR calculated by VaR fluctuated around 60 % and MCR was approximately 13.2 %. The SCR calculated by ES method increased to approximately 80 % and MCR to approximately 26 %. This fact can affect actuaries who should intentionally choose procedure setting lower capital requirements and thereby expose the insurance company to excessive risk or vice versa procedure setting higher capital requirements, so the insurance company should hold capital which could be used more effectively.

The procedures of calculation VaR and ES have some limitations. They are based on historical data, calculated for one year ahead etc. Based on these facts, the solvency of

insurance companies and other appropriate metrics are subjected to stress testing, which identifies the size of potential losses from extreme events. In this paper two stress scenarios in the form of CZK appreciation of against foreign currencies were formulated.

In neither case, the equity capital of insurance did not fall below the value of the SCR and the insurance company should not have solvency problems. However, in some cases the equity capital decreased significantly and approached to 100 % of SCR.

Acknowledgements

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References

[1] Alexander, C., 2008. *Market Risk Analysis Volume IV: Value at Risk Models*. Chichester: John Wiley & Sons Inc.

[2] Boyle, P., Broadie, M. and Glasserman, P., 1997. Monte Carlo methods for security pricing. *Journal of Economic Dynamics and Control*, 21, pp. 1267-1321.

[3] Česká národní banka, 2016. *ČNB* [online] Available at: <http://www.cnb.cz/cs/index.html> [Accessed 21 January 2015].

[4] Česká asociace pojišťoven, 2010. ČAP [online] Available at: <http://www.cap.cz/images/tiskove-zpravy/2011-2-2-11-36.pdf> [Accessed 15 June 2016].

[5] European Insurance and Occupational Pensions Authority, 2016. EIOPA. [online] Available at: < https://eiopa.europa.eu/ > [Accessed 30 June 2016].

[6] European Insurance and Occupational Pensions Authority, 2016. QIS5 Technical Specifications. [pdf] EIOPA. Available at: < https://eiopa.europa.eu/Publications/QIS/QIS5-technical_specifications_20100706.pdf> [Accessed 15 May 2016].

[7] Glasserman, P., 2004. *Monte Carlo Methods in Financial Engineering*. Berlin: Springer.

[8] Hull, J. C., 2007. *Risk Management and Financial Institutions*. Upper Saddle River (USA): Pearson Prentice Hall.

[9] Jäckel, P., 2002. *Monte Carlo methods in finance.* Chichester: John Wiley & Sons Inc.

[10] McKay, M. D., Conover, W. J., Beckman, R. J., 1979. *A comparison of three methods for selecting input variables in the analysis of output from a computer code*. Technometrics, 21(2), pp. 239–245.

[11] Sandström, A.: *Handbook of solvency for actuaries and risk managers theory and practice*. CRC Press, FL., 2011.

[12] Parkham, N., Schmidt, W, 2008. *Latin hypercube sampling with dependence and applications in finance.* Frankfurt School of finance & Management: Frankfurt.

[13] Stein M., 1987. *Large Sample Properties of Simulations Using Latin Hypercube Sampling*, Technometrics, (29)2, pp. 143-151.

[14] Svoboda, T., Hilar, M., 2012. Pravděpodobnostní analýza metodou Latin Hypercube Sampling. *Tunel*, (21)4, pp. 80-89.

[15] Tichý, T., 2010. *Simulace Monte Carlo ve financích: Aplikace při ocenění jednoduchých opcí*. Ostrava: VŠB-TU Ostrava.

[16] Vávrová, E., 2012. *Regulatorní opatření Solvency II v kontextu současné finanční krize.* Acta Academica karviniensia (12)4, pp. 173-182.
[17] Zmeškal, Z. et al., 2013. *Finanční modely*. Praha: Ekopress.

Financial Statement Audit and Risk of Material Misstatement Formed by Fraud

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Abstract

Audit of financial statements shall be performed in accordance with International Standards on Auditing (hereinafter the „ISA"), issued by the International Auditing and Assurance Standards Board (hereinafter the "IAASB"). Auditors should obtain a reasonable assurance that financial statements do not contain any material misstatement caused either by a fraud or by an error. In other words, audit procedures are designed to detect only material misstatements, and thus auditors might not be able to detect all fraud even if they apply reasonable procedures indicating that a fraud was committed.
The main objective of this article is to illustrate and summarize the reasons why auditors of financial statements can be expected to detect only a very small percentage of fraud.

Key words

Audit of financial statement, risk, fraud

JEL Classification: M42

1. Introduction

Audit of financial statements is in European terms including Slovakia conducted in accordance with the ISA, issued by the IAASB. Conducting an audit of financial statements in accordance with the ISA is in Slovakia a legal obligation pursuant to Act No. 423/2015 Coll. on Statutory Audit and on Amendments and Supplements to Act. No. 431/2002 Coll. on Accounting, as amended, para. 19 (1).

In literature, many authors divide the objective of an audit into primary and secondary objective.

1.1 Primary and secondary objective of an audit of financial statements

The primary objective of an audit of financial statements is to allow the auditor to express an opinion whether the financial statements are prepared, in all material aspects, in accordance with the applicable financial reporting framework.[2]

It applies for both entities that are obligated to have their financial statements audited in accordance with Act. No. 431/2002 Coll. on Accounting, as amended, para. 19 (1) or para. 22 (10) as well as entities that have decided to have the financial statements audited voluntarily.

The secondary objective refers to detection and prevention of fraud and errors. The auditor´s role is to make ensure users of the financial statements that there are no material misstatements resulting from an either unintentional error, or intentional error or a fraud.

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[2] Kareš. L. 2010. *Audítorstvo*. Bratislava: Iura Edition, 2015. s. 9. ISBN 978-80-8078-334-1.

The final opinion shall provide a reasonable assurance that no material misstatement of financial statements (regardless of the reason) has been identified by auditor.

The term "misstatement" is defined in the ISA 450 - Evaluation of Misstatements Identified During the Audit as a difference between the amount, classification, presentation or disclosure of a reported financial statement item and the amount, classification, presentation or disclosure that is required for the item to be in accordance with the applicable financial reporting framework.

This means that auditors are responsible for provision of a certain degree of assurance (reasonable assurance), accuracy and legality of the operations and accounts of an entity as well as the proper functioning of the system in accordance with specified requirements.

Generally, misstatements might arise from a fraud or an error caused by a difference between the actually reported figures, and figures that are expected to be reported in order to financial statements be truly and fairly presented.

Misstatements are considered "material" if they are significant enough to make a difference to a user of the financial statements. The question of materiality of disclosures in financial statements has been discussed as well as at the international level.[3]

It is important for the users of information stated in financial statements to know whether the financial statements contain any misstatements or not. If the users find out inconsistencies, confidentiality to other reported items fell proportionally.[4]

The level of materiality is determined in the context of a risk assessment and auditor´s professional judgment.

1.2 Audit of Financial Statements and Risk Assessment

With respect to the significant issues relating to detection of a fraud, the IAASB issued the relatively straightforward process for auditors to assess a fraud whilst performing an audit of the financial statements.

Table 1: ISA Standards for assessing fraud

ISA Standard	Title of ISA Standard
240	*THE AUDITOR'S RESPONSIBILITIES RELATING TO FRAUD*
315	*IDENTIFYING AND ASSESSING THE RISKS OF MATERIAL MISSTATEMENT THROUGH UNDERSTANDING THE ENTITY AND ITS ENVIRONMENT*
330	*THE AUDITOR'S RESPONSES TO ASSESSED RISKS*

According to the ISA so called "risk-based audit" we can differ the approach that is in audit of financial statements from the subject as well as time. In terms of time, it is ultimately a comprehensive identification and assessment of individual types of risk before accepting the audit engagement, upon receipt of the audit engagement and during the conducting an audit.

Risk-based audit forms the point of gravity field of the auditor's examination and consideration, which also directly affects the scope, complexity, delay and final auditor´s opinion presented in the auditor´s report.

Components of audit risk are (i) **inherent risk**, (ii) **control risk** and (iii) **detection risk**. **Inherent risk** is a measure of the likelihood that there are material misstatements caused

[3] Mokošová, D. 2013. Materiality of disclosures in financial statements. In: Journal of East European Science and Research. 2013, vol. 4, no. 1. p. 90. ISSN 1338-3302.

[4] Juhászová, Z. – Lepieš, M. – Ješš, M. 2015. Uplatnenie zásady kontinuity v účtovnej závierke poisťovne. In Grant Journal, 2015, vol. 4, no. 7. p. 58. ISSN 11805-0638.

either by an error or fraud in a segment before considering the effectiveness of internal controls. Auditor should assess the factors that make up the inherent risk and take them into consideration.

Control risk is the assessment of the likelihood that a misstatement that could occur and that could be material will not be avoided or detected by the internal control system.

Detection risk shows a measure of the risk that audit evidence will fail to detect misstatements that could be material.

Audit evidence fulfill the role of persuasion about a certain state of facts so that they cannot be considered as absolute. For this reason, the auditor is in a risk that some irregularities will not be detected.

Auditors should pay attention, mainly to audit evidence that contradict other audit evidence already obtained, information that may challenge the reliability of documents used as audit evidence and circumstances that could indicate a fraud.

This risk needs to be reduced on the minimum level (the minimum acceptable level of audit risk) especially by observation, recalculation, testing of internal controls, substantive testing, independent qualified estimation, collecting evidence, analytical procedures and other additional methods.

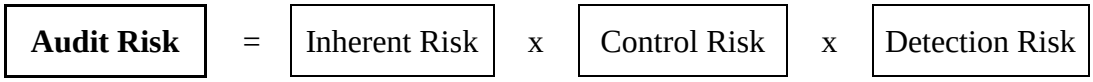
From a range of risks defined by relevant ISA, the auditor is unable to affect the inherent risk and control risk. Part of those risks are usually also the potential risk of a fraud. Auditors are able to affect only their audit risk by adequate identification and recording important findings a standardized audit documentation.

The importance of principle of professional skepticism should not be overlooked. Through the application of this principle, the risk that material misstatements will remain undetected is indirectly reduced.

The auditor´s role in context of detecting the potential fraud cases is limited to findings and recording suspicious circumstances through the facts identified by audit. For this reason, the auditor needs to:

- identify and understand the entity and its environment,
- follow up on the recommendations from previous auditors,
- evaluate the entity´s internal control system and
- identify and evaluate risks including fraud.

Figure 1: Audit Risk Components



1.3 Auditor´s professional judgment

If the auditor considers that there are some indicators of a suspected fraud, he has to decide for proper action to be taken. It is important to evaluate all circumstances of the case.

Particular attention according to auditor´s professional judgment must be paid to the circumstances indicating the intention.

He can decide for additional testing and inspection as well as expansion of the segment.

In case a potential fraud is related to a whole system or its part, the auditor´s responsibility is to inform the competent authorities.

Auditors should be able to determine the limit between audit and investigation. At that point, the fraud case should be transferred to competent authorities in accordance with national rules and practices.

2. Conclusion

This article follows on financial statement audit and risk of a material misstatement formed mostly by a fraud.

As it is stated also in the ISA 240 – The Auditor´s Responsibilities Relating to Fraud in an Audit of Financial Statements, the primary responsibility for implementation of an internal control system and detection of a fraud has the entity´s management.

It is in line with auditor´s objective: to conduct an audit in accordance with the ISA and obtain a reasonable assurance that the financial statements are taken as a whole and are free from material misstatement caused by a fraud or an error.

Because of the inherent limitations of an audit, there is an unavoidable risk that material misstatements in the financial statements will not be detected, even though the audit was planned properly and performed in accordance with the ISA.

References

[1] Act. No. 431/2002 Coll. on Accounting, as amended.

[2] Act No. 423/2015 Coll. on Statutory Audit and on Amendments and Supplements to Act. No. 431/2002 Coll. on Accounting, as amended.

[3] IAASB. (2014). Handbook of International Quality Control, Auditing, Review, Other Assurance and Related Services Pronouncements. Retrieved from: https://www.ifac.org/publications-resources/2014-handbook-international-quality-control-auditing-review-other-assurance-a

[4] Juhászová, Z., Lepieš, M., Ješš, M. (2015). Uplatnenie zásady continuity v účtovnej závierke poisťovne. *Grant Journal*, 4 (7), p. 55 – 58.

[5] Kareš, L. (2010). *Audítorstvo.* 2nd ed. Bratislava: Iura Edition.

[6] Kareš, L. (2015). *Teória auditu.* 1st ed. Bratislava: Wolters Kluwer.

[7] Mokošová, D. (2013). Materiality of disclosures in financial statements. *Journal of East European Science and Research*, 4 (1), p. 89 – 95.

[8] Singelton, T. W., Singelton, A. J. (2010). *Fraud Auditing and Forensic Accounting.* 4th ed. New Jersey: John Wiley & Sons, Inc.

Multi-criteria decision methods for financial management in SMEs

Anna Doś [1]

Abstract

The aim of the paper is to find appropriate multi0criteria decision methods to support SMEs financial management. Through literature analysis the paper demonstrates that SMEs owners make financial decisions within a complex framework, where tangible and intangible goals co-exist, where long-term and short-term perspective must be conciliated and where some spheres of financial managements decisions perform at the expense of other spheres. Basing at the analysis of financial ratios of Polish SMEs for 2014 the paper provides evidence on imbalance between investment decisions and working capital decisions made by Polish SMEs. Than multi-criteria decision methods are compared regarding requirements for their implementation and contribution to the quality of decisions.

Key words

Profitability, working capital, SMEs management, analytical hierarchy process

JEL Classification: G30

1. Financial management in SMEs

Tools and procedures recommended in the literature for corporate financial management are very well developed, sophisticated and goal-oriented. The goal is to create value for stakeholders while shareholder interests predominate [Jensen 2001]. The financial toolkit includes planning, evidence, analysis and controlling within numerous fields: capital structure, asset management, liquidity, profitability, risk management, tax planning etc. The literature provides with sophisticated tools for financial management including those based at advanced mathematics and statistics usually requiring specified hardware and software as well as databases. Consequently due to economies of scale most of advanced tools and procedures can be harnessed by large enterprises that strengthen their competitive advantage over SMEs [Aragón-Sánchez & Sánchez-Marín 2005].

However predominating position in European economies countries (including Poland) is held by SMEs, which is reflected in their number, productivity, number of jobs created, elasticity and openness to innovation [Dosi et al. 2015; Wynarczyk et al. 2016; Brunswicker & Vanhaverbeke 2015]. The success of SME's is thus translated into success of the economy. Problems with financial management are important barriers to SMEs success. Poor financial management is a critical and common cause of business failures in SMEs [Jindrichovska, 2013; Ahmad, Seet 2009]. At the same time good practices related to financial management are among factors having biggest importance for SME's success and competitiveness

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[Piatkowski 2012; Chittithaworn et al. 2011; Ng & Kee 2012]. Thereof the improvement of SMEs financial management can contribute to overall economic growth.

The financial management in SMEs is concentrated in four main areas: investment decisions (capital budgeting decisions), financial decisions (capital structure), working capital management and dividend policy. The literature on financial characteristics of SMEs is focused mainly on financial problems of small and medium sized companies with respect to their funding requirements [eg. Holton et al. 2013]. Several studies concentrate on working capital management in SMEs and SMEs capital structure [eg. Sundat 2011; Degryse et al. 2012; Mateev et al. 2013]. However updated literature on investment decision in SMEs is scarce (some insights are provided by Rosse [2015] or Zarębski [2012]).

In the literature there are studies focused on detection of stimulators and barriers to good financial management among SMEs. They reveal, that the accounting system, long-term planning, financial literacy and the use of external financing and external advisors are important factors [Karadag 2015; Kaya, Alpkan 2012; DeSaulles 2007; Nunoo & Andoh 2012; Ates et al. 2013; McMahon 2001, McChlery et al. 2005, Berry et al. 2006]. Nevertheless financial management described as „sound" can be inappropriate regarding needs of SME's owners and managers. Financial management techniques are designed to maximise firm's value [Michalski 2008]. However the implementation of value-based-management in the day-to-day management of SMEs is very difficult due to the fact, that financial management of SMEs is different than the management of large enterprises and requires different approach.

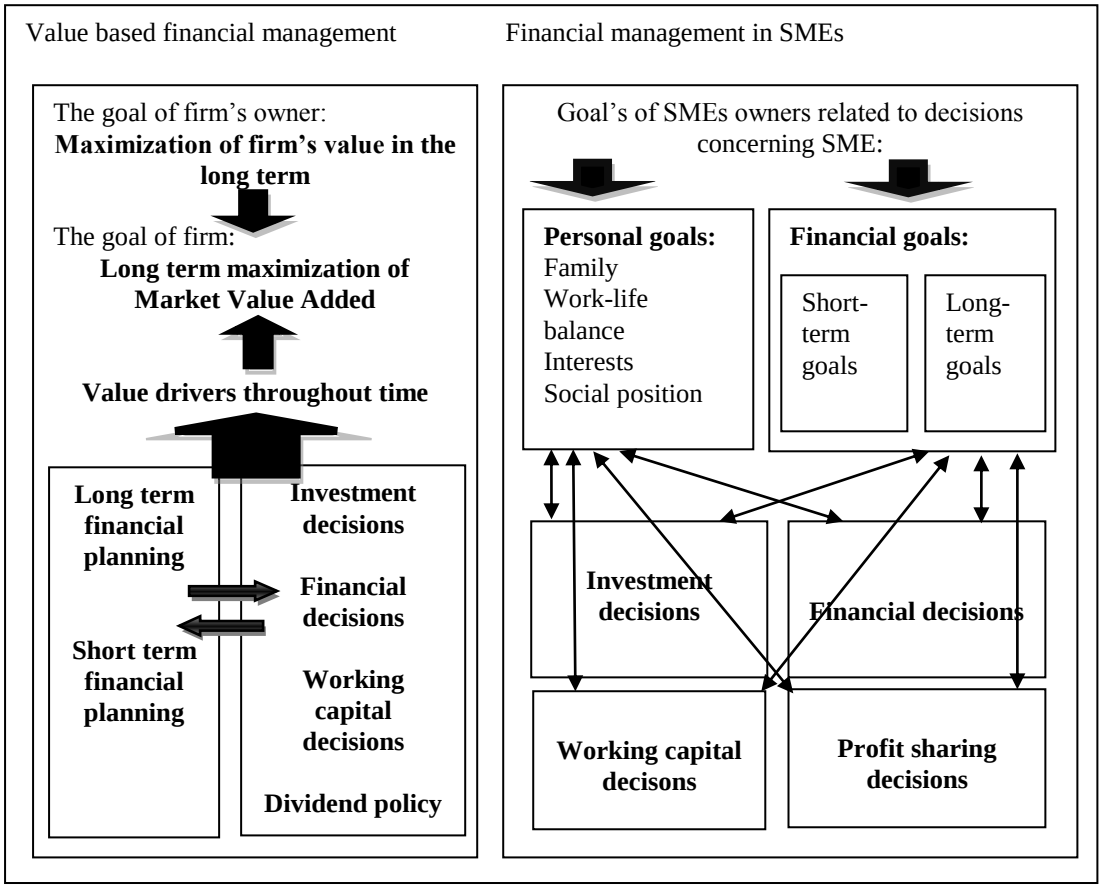
Coordination in small organizations mainly happens through direct instruction and supervision, minimizing the need for formal management (i.e. planning and control) processes. SMEs goals rotate around owners goals. Especially in small businesses and new ventures the influence of the founder in defining the business concept and mode of operation is of paramount importance. There is no simple pattern of motives driving SMEs owners. Entrepreneurs are sometimes classified as "necessity driven" or "opportunity driven" [Tyszka et al. 2011]. This however is still a focus on wealth (earnings) as primary motive driving business decisions whereas in case of individuals "utility" is a more appropriate term describing one's economic goals rather than "wealth". Entrepreneurs personal utility function is a very complex issue. For the majority of SME owners and operators, business performance often ranks far behind intangible goals such as autonomy, personal satisfaction and lifestyle. Quality of life issues, willing to deal with variety of issues, personal problems and family matters as well as occupational stress can be very important for small business owners [Newby et al. 2003; Watson et al. 1998; Fernet et al. 2016 Cinamon & Rich 2002]. These goals may change over time. Mentioned issues are now significantly under-investigated leading to adaptation of narrowed perspective towards SMEs management techniques requirements.

The second problem connected with adaptation of financial management tools recommended in the literature is the problem of integration of decisions within financial management. Value based management prescribes, that the integration of decision concerning different fields of management, levels of hierarchy and different time-scopes comes through examination of how this decisions change value drivers over time and how this changes shape firms value (measured by EVA and MVA). This process is very complex regarding the fact, that investment decisions, financing decisions, working capital decisions and dividend decisions simultaneously affect costs, incomes, level of risk, cost of capital both in short-term and long-term perspective. Thereof the integration of financial management within value based management in practice is time-consuming, costly and requires high-skilled personnel. Consequently in most of SMEs financial management is multi-pillar and remains multi-

objective. Strategic and operational decisions are often disconnected due to diverse sources on information, differences in precision and different purposes (usually SMEs concentrate at short-term planning and suffer from scarcity of long-term planning [Mitchelmore & Rowley 2013; Skokan el at. 2013]).

The comparison of framework for financial decision in SMEs and financial decisions in large enterprises, where value based management can be fully implemented, is presented in Figure 1.

Figure 1. Financial management in SMEs and value based financial management.



Source: authorial preparation.

The Figure 1 demonstrates the framework for financial management in SMEs, whose owners follow multiple tangible and intangible objectives. It additionally shows, that integration of short-term and long-term planning and goals is very difficult for SMEs. Additionally different spheres of financial management remain disconnected. There is no evidence for this provided in the literature, thereof for the purpose of the study the analysis of SMEs performance within two spheres of financial management was performed.

2. Disconnection between investment decisions and working capital decisions among Polish SMEs .

The results of investment decisions are reflected in firm's profitability. The results of working capital decisions are reflected in firm's liquidity. There is a well-known trade-off between profitability (which should be a long-term goal of an enterprise) and liquidity (which should be a short-term goal of financial management). The managers of SMEs must focus on

balancing profitability and liquidity goals to keep the firm solvent in the short-term and profitable in the long-term. For the purpose of the study the financial statements of 761 Polish SMEs from different sectors providing data for 2014 (via EMIS Database) were analyzed. From among the population of analyzed SMEs two groups were distinguished: over-performing and under-performing firms within following spheres of financial management: profitability (measured by return on assets and return on equity) and liquidity (measured by current ratio, quick ratio and cash ratio). All enterprises, whose ratios were higher than the 3rd quartile of the sample were classified as over-performing and all enterprises whose ratios were lower than the 1st quartile were classified as under-performing. Than enterprises over-performing and under-performing within one sphere (profitability) were checked on how they perform within the other sphere (liquidity) to find, if their results are balanced. The thresholds, that is 1st and 3rd quartiles for profitability ratios and liquidity ratios are provided in table 1.

Table 1. Thresholds for over-performing and under-performing Polish SMEs within profitability and liquidity.

Financial management sphere	The ratio	Formula	1st quartile	3rd quartile
Profitability	ROA	Net income/total assets	-0,67%	10,7%
	ROE	Net income/equity	0,29%	27,49%
Liquidity	Current ratio	Current assets/current liabilities	1,0975	3,19
	Quick ratio	(cash&cash equivalents +short-term investment+accounts receivable)/current liabilities	0,9375	3,2325
	Cash ratio	(cash&cash equivalents+invested funds)/current liabilities	0,05	0,8525

Source: authorial computation.

After finding thresholds all enterprises that performed above the 3rd quartile (over-performing) in one of financial management spheres were scanned if they under-perform in the other sphere. The results are provided in table 2.

Table 2. Comparison between performance within profitability and liquidity among Polish SMEs.

Financial management sphere	The ratio for over-performing firms	Number of over-performing firms	Ratio for comparison to over-performing firms	Share of under-performing (basing at ratio for comparison) firms among over-performing firms
Profitability	ROA	190	Current ratio	10%
			Quick ratio	12,63%
			Cash ratio	17,89
	ROE	190	Current ratio	28,94%
			Quick ratio	32,105%
			Cash ratio	28,94%
Liquidity	Current ratio	190	ROA	26,84%
			ROE	35,26%
	Quick ratio	190	ROA	26,31%
			ROE	30,52%
	Cash ratio	190	ROA	26,31%
			ROE	29,47%

Source: authorial computation.

Data provided in Table 2 show that there is a significant imbalance between performance within chosen fields of financial managements. On average 25,5% of firms, which over-

perform in one of the fields of financial management (investment decisions, working capital decisions), underperform in the other field (respectively working capital decision, investment decisions). Thereof the practice of financial management in Polish SMEs calls for managerial tools to improve balancing financial goals.

3. Multi-criteria methods for balancing goals of SMEs financial management.

SMEs owners make financial decisions within a complex framework, where tangible and intangible goals co-exist, where long-term and short-term perspective must be conciliated and where some spheres of financial managements decisions perform at the expense of other spheres. This calls for managerial tools appropriate to support financial management in SMEs. The well-known and well-investigated support of financial management in SMEs is advisory from accountants [Carey 2015; Kamyabi & Devi 2012]. However some studies give the insight into the problem of dependency and resulting frustration [Stone 2015]. Ultimately the owner is the one, who takes the responsibility and must balance his goals (including intangible ones). Decision making is the third most-important competence of an SMEs manager [Dominguez et al 2010]. Thereof there is a need for a tool that would support decision-making of SMEs owners. In order to be utile this tool should be simple and adaptable to a standard software.

Multi-criteria decision making (MCDM) methods deal with the process of making decisions in the presence of multiple objectives. A decision-maker is required to choose among quantifiable or non-quantifiable and multiple criteria. The objectives are usually conflicting and therefore, the solution is highly dependent on the preferences of the decision-maker and must be a compromise. The methods help to improve quality of decisions by making them more explicit, rational and efficient [Pohekar & Ramachandran 2004].

Multi-criteria decision methods (MCDM) are a family of methods that includes several methods based on weighted averages, priority setting, outranking, fuzzy principles and their combinations. The methods can also be classified as deterministic, stochastic and fuzzy methods. The comparison of the most popular MCDMs regarding thei implementation in the financial management of SMEs is provided in table 3.

Table 2. Conditions for implementing MCDM in the financial management of SMEs

Method	Contribution to the improvement of decision's quality	Possibility to use MS Excel	Software - examples
Weighted Sum Method (WSM)	1. The same set of criterions applied to all decisions. 2. Simple ranking of criterions (by introducing weights).	Multiplication, statistical function: average	Not necessary
Analytical Hierarchy Process (AHP)	1. The same set of criterions applied to all alternatives and decisions. 2. Ranking of cirterions established through pairwaise comparison in Saaty's scale. 3. Criterions are put into comparison matrix. Consistency ratio for the matrix is checked.	Multiple cross tabulation, using rank and weighted score or Sanna application to MS Excel	Priority Estimation Tool (AHP), free. AHP online calculator (BPMSG), free More advanced tools with friendly interface - payable
PROMETHEE I and II	1. The same set of criterions applied to all alternatives and decisions 2. The method uses preference function to put a value on an alternative regarding chosen set of criterions.	Not available	PROMETHEE-GAIA - payable

TOPSIS	1. The same set of criterions applied to all alternatives and decisions. 2. Global utilities of alternatives are established according to their distance from ideal and basal alternatives. 3. The geometric distance between each alternative and the ideal alternative (which is the best score in each criterion) and basal alternative, is calculated.	Multiple cross tabulation, using rank and weighted score or Sanna application to MS Excel	Triptych (SDI), payable.
ELECTRE	1. The same set of criterions applied to all alternatives and decisions. 2. Outranking of alternatives and thresholds underscore the method. The threshold can be 0. 3. The concordance and discordance are necessary for outranking. For each criterion it is checked whether, for every pair of alternatives (a,b), there is harmony or disharmony with the assertion aSb; that is, a is at least as good as b.	Sanna application to MS Excel	The ELECTRE TRI Software 2.0.

Source: authorial preparation.

Regarding the controbution to decision rationality and optimal outcomes as well as possibility learn the method and to use standard software two MCDMs can be recommended for financial management in SMEs: AHP and TOPSIS. WSM lacks any developmnets that would address the problem of setting weights. PROMETHEE I and II as well as ELECTRE requires extraordiary software (some SMEs can afford it and benefit from applying the acquired software also in other field of management, eg. human resources management, supply chain management etc.). ELECTRE can be applied with Sanna application to MS Excel, but this tool is not well known among SMEs.

4. Conclusion

Financial management is theoretically single-goal and well integrated at the foundations of value based management. Nevertheless in the realm of SMEs financial decisions have to be made within a complex framework, where tangible and intangible goals co-exist, where long-term and short-term perspective must be conciliated and where some spheres of financial managements decisions perform at the expense of other spheres. The paper provides evidence on imbalance between investment decisions and working capital decisions made by Polish SMEs reflected in their financial rations for 2014. Multi-criteria decision making (MCDM) methods deal with the process of making decisions in the presence of multiple objectives reflected in quantifiable or non-quantifiable and multiple criteria. From among most popular MCDM the AHP method and TOPSIS method seem to be the most valuable and the easiest to implement in financial management ins SMEs.

References

[1] Ahmad, N. H., & Seet, P. S. (2009). Dissecting behaviours associated with business failure: A qualitative study of SME owners in Malaysia and Australia. Asian Social Science, 5(9), 98.

[2] Aragón-Sánchez, A., & Sánchez-Marín, G. (2005). Strategic orientation, management characteristics, and performance: A study of Spanish SMEs. Journal of Small Business Management, 43(3), 287-308.
[3] Ates, A., Garengo, P., Cocca, P., & Bititci, U. (2013). The development of SME managerial practice for effective performance management. Journal of Small Business and Enterprise Development, 20(1), 28-54.
[4] Berry, A. J., Sweeting, R., & Goto, J. (2006). The effect of business advisers on the performance of SMEs. Journal of small business and enterprise development, 13(1), 33-47.
[5] Brunswicker, S., & Vanhaverbeke, W. (2015). Open innovation in small and medium-sized enterprises (SMEs): External knowledge sourcing strategies and internal organizational facilitators. Journal of Small Business Management,53(4), 1241-1263
[6] Carey, P. J. (2015). External accountants' business advice and SME performance. Pacific Accounting Review, 27(2), 166-188.
[7] Chittithaworn, C., Islam, M. A., Keawchana, T., & Yusuf, D. H. M. (2011). Factors affecting business success of small & medium enterprises (SMEs) in Thailand. Asian Social Science, 7(5), 180.
[8] Cinamon, R. G., & Rich, Y. (2002). Gender differences in the importance of work and family roles: Implications for work–family conflict. Sex Roles, 47(11-12), 531-541.
[9] Degryse, H., de Goeij, P., & Kappert, P. (2012). The impact of firm and industry characteristics on small firms' capital structure. Small Business Economics,38(4), 431-447.
[10] De Saulles, M. (2007, January). Information literacy amongst UK SMEs: an information policy gap. In Aslib proceedings (Vol. 59, No. 1, pp. 68-79). Emerald Group Publishing Limited.
[11] Dominguez, C., Varajão, J., Morgado, L., Oliveira, I., & Sousa, F. (2010, October). SME managers' most important entrepreneurship and business competences. In International Conference on ENTERprise Information Systems (pp. 274-282). Springer Berlin Heidelberg.
[12] Dosi, G., Moschella, D., Pugliese, E., & Tamagni, F. (2015). Productivity, market selection, and corporate growth: comparative evidence across US and Europe. Small Business Economics, 45(3), 643-672.
[13] Fernet, C., Torrès, O., Austin, S., & St-Pierre, J. (2016). The psychological costs of owning and managing an SME: Linking job stressors, occupational loneliness, entrepreneurial orientation, and burnout. Burnout Research, 3(2), 45-53.
[14] Holton, S., Lawless, M., & McCann, F. (2013). SME financing conditions in Europe: credit crunch or fundamentals?. National Institute Economic Review,225(1), R52-R67.
[15] Jensen, M. C. (2001). Value maximization, stakeholder theory, and the corporate objective function. Journal of applied corporate finance, 14(3), 8-21.
[16] Jindrichovska, I. (2013). Financial management in SMEs. European Research Studies, 16(4), 79.
[17] Kaya, S., & Alpkan, L. (2012). Problems and solution proposals for SMEs in Turkey. EMAJ: Emerging Markets Journal, 2(2), 30-45.
[18] Kamyabi, Y., & Devi, S. (2012). The impact of advisory services on Iranian SME performance: An empirical investigation of the role of professional accountants. S African Journal of Business management, 43(2), 61-72.
[19] Karadag, H. (2015). Financial management challenges in small and medium-sized enterprises: A strategic management approach. Emerging Markets Journal, 5(1), 26

[20] Mateev, M., Poutziouris, P., & Ivanov, K. (2013). On the determinants of SME capital structure in Central and Eastern Europe: A dynamic panel analysis.Research in International Business and Finance, 27(1), 28-51.

[21] McChlery, S., Godfrey, A. D., & Meechan, L. (2005). Barriers and catalysts to sound financial management systems in small sized enterprises. Journal of Applied Accounting Research, 7(3), 1-26.

[22] McMahon, R. G. (2001). Growth and performance of manufacturing SMEs: The influence of financial management characteristics. International Small Business Journal, 19(3), 10-28.

[23] Michalski, G. (2008). Corporate inventory management with value maximization in view. ZEMEDELSKA EKONOMIKA-PRAHA-, 54(5), 187.

[24] Mitchelmore, S., & Rowley, J. (2013). Growth and planning strategies within women-led SMEs. Management Decision, 51(1), 83-96.

[25] Newby, R., Watson, J., & Woodliff, D. (2003). Using focus groups in SME research: the case of owner-operator objectives. Journal of Developmental Entrepreneurship, 8(3), 237.

[26] Ng, H. S., & Kee, D. M. H. (2012). The issues and development of critical success factors for the SME success in a developing country. International Business Management, 6(6), 680-691.

[27] Nunoo, J., & Andoh, F. K. (2012, August). Sustaining Small and Medium Enterprises through Financial Service Utilization: Does Financial Literacy Matter?. In Unpublished Paper) presented at the Agricultural & Applied Economics Association's 2012 AAEA Annual Meeting, Seattle, Washington.

[28] Piatkowski, M. (2012). Factors Strengthening the Competitive Position of SME Sector Enterprises. An Example for Poland. Procedia-Social and Behavioral Sciences, 58, 269-278

[29] Rossi, M. (2015). The use of capital budgeting techniques: an outlook from Italy. International Journal of Management Practice, 8(1), 43-56.

[30] Skokan, K., Pawliczek, A., & Piszczur, R. (2013). Strategic planning and business performance of micro, small and medium-sized enterprises. Journal of Competitiveness, 5(4).

[31] Stone, G. (2015). Power, dependence and frustration: A study of power in Australian accountants' advisory relationship with small business. Meditari Accountancy Research, 23(3), 250-275.

[32] Sunday, K. J. (2011). Effective working capital management in small and medium scale enterprises (SMEs). International Journal of Business and Management, 6(9), 271.

[33] Tyszka, T., Cieślik, J., Domurat, A., & Macko, A. (2011). Motivation, self-efficacy, and risk attitudes among entrepreneurs during transition to a market economy. The Journal of Socio-Economics, 40(2), 124-131.

[34] Watson, K., Hogarth-Scott, S., & Wilson, N. (1998). Small business start-ups: success factors and support implications. International Journal of Entrepreneurial Behavior & Research, 4(3), 217-238.

[35] Wynarczyk, P., Watson, R., Storey, D. J., Short, H., & Keasey, K. (2016).Managerial labour markets in small and medium-sized enterprises. Routledge.

[36] Zarębski, S. (2012). Plany bezpośrednich reinwestycji dywidend (DRIPs) jako element polityki finansowej małych i średnich przedsiębiorstw. Zeszyty Naukowe Uniwersytetu

Szczecińskiego. Ekonomiczne Problemy Usług, (81 Strategie zarządzania mikro, małymi i średnimi przedsiebiorstwami-mikrofirma 2012), 179-188.

Closet Indexing: the Case of Czechia

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Abstract

Discussion between advocates of active and passive portfolio management have been lasting for a couple of decades. Evidence of enduring popularity of active management among retail investors can be seen in an astounding amount of actively managed funds offered mainly on vast developed markets. An overwhelming majority of these funds require corresponding charges, in exchange for active management and an opportunity to outperform respective benchmark, which lie significantly higher than those of e.g. ETFs. However, declared effort for active management and its subsequent implementation in practice could turn out to be fundamentally different, and simultaneously there arises a phenomenon of so called closet indexing. Evidence show that closet indexing has been widespread among European funds. The aim of this paper is providing an analysis and assessing an occurrence of closet indexing among mutual funds offered in Czechia in context with the occurrence of this phenomenon abroad.

Key words

Closet indexing, active investing, mutual funds, benchmark, Czechia

JEL Classification: G11, G23

1. Introduction

Active investment strategies has been an only option for individual investors for many decades. Passive investment products became offered as late as in the 1970s since when passive investment strategies have been gaining ground among public. Such products include index certificates, index funds or ETFs. The clash between advocates of active and passive management has been vigorous for at least last three decades and a decision to invest either actively or passively or to blend both approaches is one of the greatest dilemmas within investment theory and practice. We shall start with defining actively and passively managed investments (Dráb 2015: 100): actively managed investments are characterized by an effort to outperform market return and reach an excessive return. On the contrary, passively managed investments restrict themselves to achieve market return. Active investing carries a chance for investor to reach an excessive return in exchange for the risk of non-reaching even market return. Passive investing can never lead by definition to lower than market return.

Despite the fact that since the 1970s index products hand in hand with passive investment strategy enjoyed worldwide expansion, active investing including actively managed mutual funds is still holding a strong position among investors' public. This can be documented by enormous number of active funds offered at least over developed countries of Europe. The main challenge lying before each active fund is generally to achieve an excessive return over the market. However, the true value added for investors comes only when this excessive return is achived repeatedly and not by chance - which is not the case of an overwhelming

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majority of active funds. From legal and market point of view this is perfectly alright, because investors paid for a chance to outperform the market while being aware of the risk of underperformance. But how about a situation, when investors bona fide pay for this opportunity but the fund do not really struggle to grasp this opportunity in practice? This leads us to a phenomenon of so called "closet indexing" or "closet index tracking" or "index hugging".

An exact rigorous definition of closet indexing does not exist. Nonetheless, we will try to clarify the term as much as possible. Whole concept of closet indexing is to a significant extent based upon research of A. Petajisto and M. Cremers (2009). They defined a term of an active share. The active share represents the share of portfolio holdings that differ from the benchmark index holdings. Closet index fund is then claimed to be resembling its benchmark from at least 40 %, so the share of active shares is up to 60 %, but no less than 20 % as a frontier of pure index fund. The authors found strong evidence that closet index funds underperform their benchmarks while funds with high active share exhibit excessive return over their benchmarks (Cremers, Petajisto 2009: 3329, 3341 - 3342).

Let us turn to an alternative specification of closet indexing. The European Securities and Markets Authority (ESMA), which has been recently examining the matter, defines closet indexing rather vaguely: "…*according to their fund rules and investor information documentation, to manage their funds in an active manner while the funds are, in fact, staying very close to a benchmark and therefore implementing an investment strategy which requires less input from the investment manager* (ESMA 2016: 1)." J. Mikeš (2013: 16) suggests that closet index fund has "virtually no difference" in asset allocation in comparison to benchmark, yet without sound scientific clarification, though. He later conforms to the view of Petajisto and Cremers and takes over the frontier that closet index fund is resembling its benchmark from at least 40 % and further widens this frontier to "round" 40 % (2015: 18).

This paper shall contribute to research into this newly emerged issue in the realm of Czech financial market and its retail segment, where it has not been fully covered yet. The aim of this paper is providing an analysis and assessing an occurrence of closet indexing among mutual funds offered in Czechia in context with occurrence of this phenomenon abroad.

2. Literature Review

As a relatively newly investigated phenomenon, closet indexing is not much covered across scientific literature. This is even more true for Czechia where this issue is enjoying little interest. In fact the term is not new at all. It appeared at the latest in the 1990s and since then has been occasionally cited in business magazines or newspapers. J. M. Laderman (1997: 162 - 163) used the term in the very same manner as us in a BusinessWeek article. He investigated U.S. equity funds and compared them to S&P 500 and came up with 70 funds out of 508 "suspected" of closet indexing while charging hefty fees. BusinessWeek raised the issue once more in 2004 when L. Braham (2004: 108 - 109) published a list of top index huggers among American funds investing in U.S. equity. J. Taylor (2004: 431 - 440) brought certainly one of the first scientific insights into closet indexing. Based on an analysis of trading opportunities he concluded closet indexing was not widespread phenomenon in the American mutual funds industry during the 1990s. Petajisto (2013: 73 - 92) extended his previous research with Cremers and confirmed their findings fit to financial crisis era too. Cremers et al. (2016: 539 - 541) also extended the research and found that about 20 % of the worldwide mutual fund assets are managed by closet indexers according to the active share and above mentioned 40 % frontier. The only analysis of presence of closet index funds in Czechia known to authors of this paper is that of Mikeš (2015: 18 - 19). He compared 44 active funds investing in

European stocks and found only 4 that fell into the category of closet indexers. However, this sample is rather small and only partially covers the Czech financial market, since it does not actually include any domestic fund.

3. Methodology and Data

Closet indexing can relate in principle to any fund that has a benchmark set and claims itself to be managed in an active manner. However, practical application of the concept has been limited to equity funds, which are mostly affected by this issue. Table 1 shows the amount of mutual funds offered in Czechia according to data provided by Czech National Bank (ČNB).

Table 1: Funds offered in Czechia

Type of fund	**Number of funds as at 01/08/2016**
Domestic mutual funds	197
of that standard and special open-ended	132
of that closed-ended	7
of that funds of funds	11
of that qualified investors funds (QIF)	47
Foreign investment funds	1 399
of that standard open-ended	1 232
of that others	167
Total domestic and foreign funds	1 596

Source: ČNB, own calculations.

According to these data there is 1 596 funds altogether offered in our country. In order to cover retail funds, we divided overall number into several categories. Out of 197 domestic mutual funds, 132 are standard or special open-ended funds (OPF). Standard fund is a fund that has to meet criteria of the European law, namely the UCITS directive, unlike special fund, that does not have to. The rest consists of closed-ended funds, funds of funds, and QIF. These are particular types of special funds that do not count to the previous group. Out of 1 399 foreign investment funds, 1 232 is deemed standard according to ČNB. Concerning foreign funds only standard ones are going to be included in the analysis. The selection of domestic funds will be wider and includes standard and special funds and funds of funds, since they are generally aimed at retail customers and are mutually comparable and standardized.

From this basic starting point we impose several conditions upon our selection and we further deal with minor obstacles concerning availability of information. First, we limit ourselves to equity funds or funds which invest to equity funds. Second, we exclude every fund that is not designated for retail customers or its documentation is not freely available. Third, there is relatively high amount of funds attracting negligible amount of capital and therefore can be omitted without adverse impact on the validity of the analysis. As our focus is targeted mainly on Czech funds, which are most available and common to local customers, we put a dual restriction on fund size. We set a limit of 100 000 000 CZK in assets in case of domestic fund and 3 000 000 000 CZK (in respective currency equivalent) in case of foreign fund as at 01/08/2016 or latest available before that date. Fourth, the exact asset allocation is not always available in full. In case there would be a fund with a significant portion of unclear allocation, it will be omitted. Fifth, many funds do not state their benchmark. We assign a benchmark to such funds artificially where possible. Sixth, there is still a huge amount of foreign standard funds suitable for inclusion to the analysis, but their entire coverage would

require much greater scope and depth than is feasible within this paper. Great portion of these funds has been also decently investigated within the US market or in Mikeš (2015). Based on above stated we are going to analyze only such foreign standard funds that simultaneously appear in the fund list of Czech Capital Market Association (AKAT).

4. Results and Discussion

Based on the mentioned criteria we extracted a total of 62 funds appropriate for the investigation. They are all clearly ordered by fund size in a table below.

Table 2: Closet indexing of mutual funds in Czechia according to the active share in %

Fund Name	Dmcl	Curr	NAV in CZK	Focus	Y/N	Benchmark	AS
Pioneer - Top Eur. Players	LUX	CZK	37 923 000 000	Europe mid&large caps	Y	MSCI Europe	80,66
Pioneer - U.S. Pioneer Fund	LUX	CZK	36 744 100 000	US equity	Y	S&P 500	69,62
KBC Equity Fund Buyback America	BEL	USD	14 057 327 498	US buyback companies	N	N/A	N/A
Raiffeisen - Global Aktien	AUT	EUR	13 007 607 120	global equity	N	MSCI World	82,77
Raiffeisen - Eurasien Aktien	AUT	EUR	11 925 154 264	Asia, Russia, Turkey eq.	N	N/A	N/A
KBC Index Fund Euroland	BEL	EUR	9 792 764 842	index f. EUROSTOXX 50	Y	EURO STOXX 50	0,6[5]
Raiffeisen - Europa Aktien	AUT	EUR	9 511 729 704	Europe equity	N	MSCI Europe	69,98
HSBC Indian Equity	LUX	USD	8 687 351 986	India equity	Y	S&P/IFCI India Gross	N/A
ČS Top Stocks	CZ	CZK	8 438 162 433	global equity	N	MSCI World	96,99
Raiffeisen - US Aktien	AUT	EUR	8 340 619 776	US equity	N	MSCI USA	80,17
Conseq Act. Inv. Dynamický	CZ	CZK	7 272 343 436	equity funds	Y	composite[1]	100,00
KBC Equity Fund Europe	BEL	CZK	7 218 272 327	Europe equity	N	MSCI Europe	46,28
CS EF (Lux) Small and Mid Cap Germany	LUX	EUR	6 358 280 461	German small&mid caps	Y	Midcap Market Index TR	27,63
Raiffeisen - Emerging Markets Aktien	AUT	EUR	5 773 678 797	EM equity	N	MSCI EM	81,15
ESPA Stock Global	AUT	CZK	5 695 720 517	global equity	N	MSCI World	77,35
ESPA Stock Biotec	AUT	CZK	5 473 734 701	biotechnology equity	N	MSCI World Pharmacy BioLife	76,88
KBC Equity Fund High Dividend	BEL	EUR	5 445 362 059	global dividend equity	N	MSCI World High Div. Yield	N/A
HSBC Brazil Equity	LUX	USD	5 020 573 207	Brazil equity	Y	MSCI Brazil 10/40 Net	42,07
ESPA Stock Europe	AUT	EUR	4 972 226 674	Europe equity	N	MSCI Europe	58,76
Pioneer - Em. Europe and Mediterranean Eq.	LUX	CZK	4 735 100 000	EM and Middle East eq.	Y	MSCI EM Europe and Midd. East 10/40	N/A
Amundi Equity Global Luxury and Lifestyle	LUX	CZK	4 708 850 000	sectoral equity	Y	MSCI World Cons. Discretionary	60,57
KBC ECO Fund Water	BEL	EUR	4 603 313 738	water management eq.	N	N/A	N/A
KBC Multi Track Germany	BEL	CZK	4 562 755 956	index fund German equity	Y	MSCI Germany	11,17
Raiffeisen - Pazifik Aktien	AUT	EUR	4 073 910 280	Asia-Pacific equity	N	MSCI Asia Pacific	45,60
CS (Lux) USA Growth Opportunities Equity Fund	LUX	USD	3 928 006 096	US mid & large caps	Y	MSCI USA	79,91
KBC Equity Fund America	BEL	CZK	3 798 921 538	North American equity	N	MSCI North America	77,30
KB privátní spr. aktiv 5D - A	CZ	CZK	3 590 821 952	global&CEE dividend eq.	N	N/A	N/A
ESPA Stock Global - Emerging Markets	AUT	CZK	3 583 410 532	EM equity	N	MSCI EM	48,93

Raiffeisen - TopDividende Aktien	AUT	EUR	3 509 803 798	global dividend equity	N	MSCI World High Div. Yield	N/A
Amundi Eq. Europe Concentrated	LUX	CZK	3 390 500 000	Europe equity	Y	MSCI Europe	73,95
CS SICAV One (Lux) European Equity Dividend Plus	LUX	EUR	3 320 107 021	Europe dividend equity	N	MSCI Europe High Div. Yield	N/A
AXA Selection Global Equity	CZ	CZK	2 209 667 300	global equity funds	N	MSCI World	14,55
Generali Fond glob. značek	CZ	CZK	2 113 289 190	global equity	N	MSCI World	61,95
ČS Sporotrend	CZ	CZK	1 935 039 495	CEE equity	Y	composite[2]	46,00
Conseq Invest Akciový	IRL	CZK	1 907 967 457	central Europe equity	Y	composite[3]	50,05
Generali Fond nemov. akcií	CZ	CZK	1 890 023 710	global property equity	N	MSCI World Real Estate	73,46
KB privátní spr. aktiv 5D - D	CZ	CZK	1 638 296 379	global&CEE dividend eq.	N	N/A	N/A
ČSOB Akciový	CZ	CZK	1 535 569 112	global equity	N	MSCI World	59,59
AXA CEE Akciový fond	CZ	CZK	1 526 942 379	V4 equity	Y	composite[4]	29,82
ČSOB Akc. f. dividend. firem	BEL	CZK	1 242 165 790	global dividend equity	N	MSCI World High Div. Yield	N/A
Pioneer - akciový fond	CZ	CZK	1 239 889 704	OECD equity	N	MSCI World	86,24
Generali Fond farm. a biotech.	CZ	CZK	1 184 824 006	US&EU sectoral equity	N	MSCI World Pharm. BioLife	43,18
Generali Fond ropy a energ.	CZ	CZK	1 073 766 099	EU&US energy equity	N	MSCI World Energy	59,10
AXA Realitní fond	CZ	CZK	999 923 843	global property equity	N	MSCI World Real Estate	77,89
AXA Selection Emerging Eq.	CZ	CZK	979 303 898	global EM equity funds	N	MSCI EM	14,15
ČS Global Stocks FF	CZ	CZK	754 733 261	global equity funds	Y	MSCI World	96,39
KBC Equity Fund Japan	BEL	JPY	700 432 635	Japan equity	N	MSCI Japan	61,63
ČSOB Akc. fond vod. bohatství	BEL	CZK	542 266 895	water management eq.	N	N/A	N/A
ČSOB Akciový realitní	CZ	CZK	537 463 881	Europe&US property eq.	N	MSCI World Real Estate	48,50
Pioneer All Star	CZ	CZK	527 820 000	equity funds	N	N/A	>96,43
Generali Fond nov. ekonomik	CZ	CZK	502 591 551	EM equity ex Russia	N	MSCI EM ex Russia	N/A
IKS Akc. - stř. a vých. Evropa	CZ	CZK	332 115 766	CEE equity	N	MSCI Eastern Europe	44,92
Generali Fond živé planety	CZ	CZK	313 596 507	sectoral equity	N	N/A	N/A
AKRO globální akciový fond	CZ	CZK	292 113 634	various equity	N	N/A	N/A
ČSOB Akc. stř. a vých. Evropa	CZ	CZK	288 703 329	CEE equity	N	MSCI Eastern Europe	66,73
Partners 7 Stars	CZ	CZK	270 530 000	US, GB, CH, S, AUS, NZ, CDN eq. funds	N	N/A	<28,04
ČSOB Akciový fond - BRIC	BEL	CZK	221 430 343	BRIC equity	N	MSCI BRIC	16,80
KB Privátní správa aktiv 5D-popular A	CZ	CZK	201 419 145	global dividend equity	N	MSCI World High Div. Yield	N/A
AKRO akc. fond nov. ekonomik	CZ	CZK	197 237 277	EM equity	N	MSCI EM	94,23
ČSOB Český Akciový (PX)	BEL	CZK	193 956 871	index fund Czech equity	Y	PX	13,44
ČS Globální akciový FF	CZ	CZK	137 796 834	equity funds	N	N/A	5,46
AKRO fond progresívních společností	CZ	CZK	136 871 535	global mid cap equity	N	MSCI World Small Cap	99,28

Source: own calculations based on data provided by respective asset managers and index providers.

where:

- Dmcl stands for domicile of the fund.
- Curr stands for fund currency.
- NAV in CZK stands for net asset value in local currency according to AKAT or asset manager.
- Focus stands for prevailing investment focus of the fund according to KIID or actual portfolio composition.
- Y/N stands for official presence/absence of fund's benchmark according to KIID or Prospectus.
- AS stands for the active share measure.
- N/A stands for not available, in case it was impossible to set widely recognized or suitable benchmark or its members' weightings were confidential, which in most cases prevented from calculating the active share.
- EM stands for emerging markets.
- V4 stands for Visegrad 4.
- 1 stands for composite index of MSCI World (55,3 %), MSCI Emerging Markets (19,3 %), CECE Composite EUR (10,4 %), Bloomberg Effas Czech 1Y+ (5,0 %), ML Global Government Bond (8,0 %) and a daily interest of geometrical average of 6M PRIBOR minus 0,35 % in respective investment period (2,0 %).
- 2 stands for composite index of MSCI Russia (30 %), PX (17,5 %), BUX (17,5 %), MSCI Turkey (17,5 %), and MSCI Poland (17,5 %).
- 3 stands for composite index of WIG30 (38 %), PX (28 %), BUX (28 %), and SBITOP (6 %).
- 4 stands for composite index of PX (40 %), WIG20 (28 %), BUX (20 %), SAX (2 %), and PRIBID 6M (10 %).
- 5 - adopted from Mikeš 2015: 19.

The table comprises wide variety of funds of several major asset managers. Its main part is consisted of domestic funds denominated in CZK. NAV of each fund denominated in another currency was recalculated to CZK with an exchange rate according to ČNB as at 01/08/2016. However, NAV is only an indicative value and serves to ranking the funds by their size, which is a comparable and unambiguous figure. As shown, nearly ¾ majority of funds is missing any officially stated benchmark. Most of these funds claim they do not track or try to outperform any reference index. Yet an investment focus of almost every fund is defined sufficiently exactly so as to be able to set a benchmark artificially, to which the portfolio holdings can be measured and compared. For the sake of consistency, it was preferred to choose benchmarks of the MSCI family. On the other hand, some funds (e.g. Pioneer All Star or ČS Globální akciový) have so broad investment focus which makes it impossible to set an appropriate benchmark. The final group consists of funds with investment focus so specific (e.g. KB privátní spr. aktiv 5D or AKRO globální akciový fond), that a single benchmark would not represent an appropriate measure. Such funds were not assign any benchmark.

The active share was calculated according to the latest known entire portfolio holdings and benchmark constituents' weightings. These two figures often mismatch in time. Entire portfolio is usually revealed only in annual or semiannual reports which always lag behind up-to-date benchmark portfolio holdings. This can hardly be avoided without exclusive access to historical benchmark weightings, however, weightings do not usually change a lot in the short term. Moreover, some asset managers do not even post the entire portfolio holdings, but only holdings above 1 %. It is important to take these circumstances into account and deem calculated active share approximate.

The lowest values of active share logically appertain to index funds. There are only 3 index funds in the table and active share of the two of them (KBC Multi Track Germany and ČSOB Český Akciový (PX)) lies slightly above 10 %, which reveals these funds are not actually resembling their indices very accurately, since it is desirable to keep the indicator as low as possible within index funds, as shown by KBC Index Fund Euroland with 0,6 %.

Another specific group is represented by funds of funds. These were assessed only with respect to funds they are investing in. Thus a fund of fund can exhibit a very high active share if it does not invest in overtly index funds or ETF or hold just a scarce portion of those. Such examples are Conseq Act. Inv. Dynamický, ČS Global Stocks FF and Pioneer All Star. This high active share lacks its information ability without detailed examination of underlying funds. Another funds of funds (ČS Globální akciový FF, AXA Selection Global Equity, AXA Selection Emerging Equity and to a certain extent also Partners 7 Stars) consist opposites to previous mentioned and might be seen as literally "closet" index funds (of funds). These ones look like ordinary active funds, but in fact most of their portfolios is composed of ETF funds and their active share lies way below 20 % (except Partners). Anyway, all of them charge fees typical for actively managed funds.

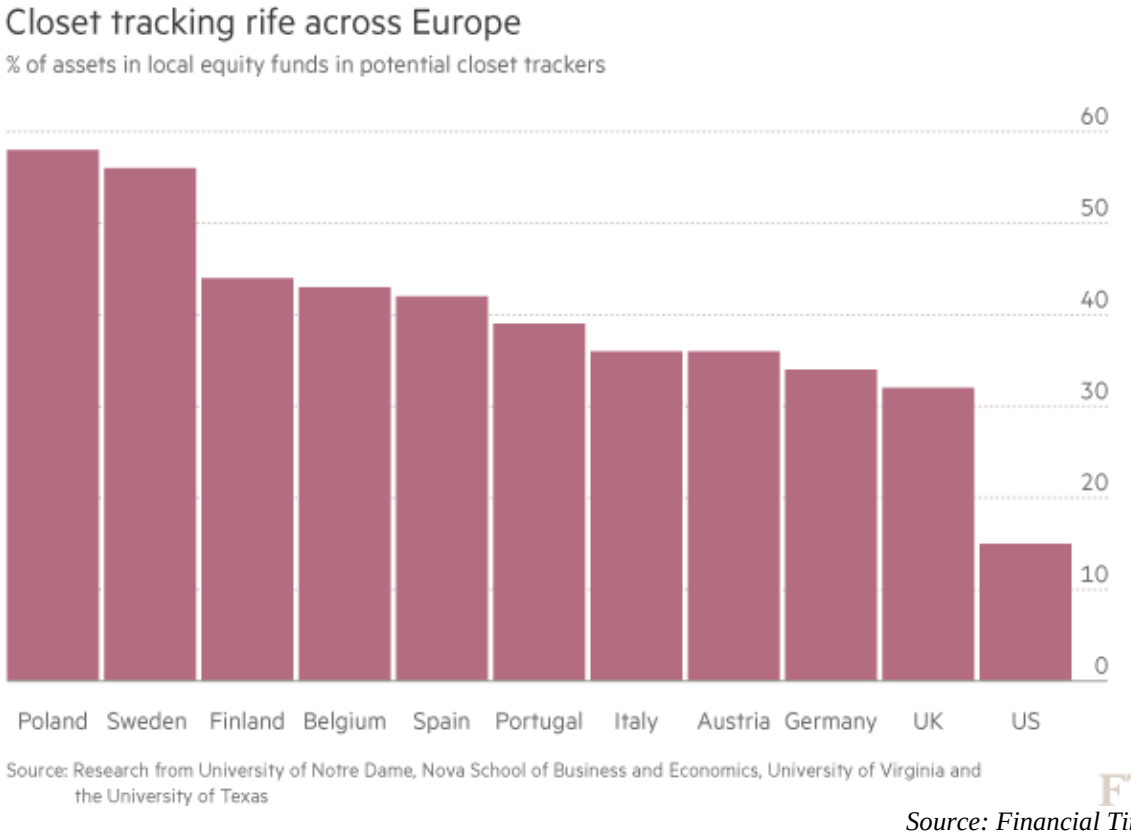
From the rest it is somehow difficult to draw some regularities. It seems that – contrary to previous findings of Fidelity Management – funds investing in small caps show not higher, but lower active share than their large cap counterparts. It can be documented for example by CS EF (Lux) Small and Mid Cap Germany, this is necessary to confirm by more observations, though. Many funds, that have narrow portfolio allocation, either territorial or sectoral, exhibit significantly low active share causing them to fall into closet indexers. The brightest examples of such funds are ČSOB Akciový fond – BRIC, already mentioned CS Small and Mid Cap Germany and AXA CEE Akciový fond. It is a fundamental question, if such funds do make any sense when they are hardly distinguishable from index funds.

It can be noticed that we discovered relatively numerous group of funds with active shares fluctuating between 42 % and 62 %. However, these "index huggers" this group is heterogeneous in terms of manager, investment focus or fund size. Last interesting point we spot is a visible time consistency of the active share, since all funds we investigated, that are at the same time included in Mikeš 2015, have almost the same values of the parameter.

5. Conclusions

Out of 45 funds with calculated active share, 17 (over one third) show values below 60 % (except index funds). Although we analyzed just a representative sample of funds, the figure deserves raised attention. Closet indexing reportedly does not seem to be such a pressing issue like in some other European countries, yet the results of this study are undoubtedly cautionery - unlike the figures of Mikeš. According to joint research of several universities, Czechia oscillates approximately around median of 11 selected European countries as is obvious from the chart below.

Figure 1: Share of assets in local equity funds in potential closet index funds in %



Source: Financial Times.

Despite Czechia being in the middle of the chart, the overall level of potential closet indexing lies worryingly high. Urgency of the matter following the release of the ESMA report on closet indexing earlier this year can be best depicted by extremely high probability of mass legal action took by deceived investors against fund managers. Nonetheless the issue deserves further investigation and optimization of applied methodology. Following research should aim at inclusion of wider scope of funds with a possibility to cover greater portion of equity funds offered in this country in order to disclose the matter in its entirety. Another remaining issue are funds of funds which need deeper insight. A step forward might be a decomposition of assets in holding of such funds and determining weighted active share for each fund in possession which might bring more conclusive results.

References

[1] Braham, L. (2004). How to Spot a Closet Index Fund. *BusinessWeek*, Sep. 6, p. 108 - 109.

[2] Cremers, K.J.M. and Petajisto,A. (2009). How Active Is Your Fund Manager? A New Measure That Predicts Performance. *The Review of Financial Studies*, 22(9), p. 3329 - 3365.

[3] Cremers, M. et al. (2016). Indexing and active fund management: International evidence. *Journal of Financial Economics*, 120 (2016), 539 - 560.

[4] Dráb, T. Performance of actively managed funds in various stages of business cycle. In Krajíček, J and Kajurová, V. (2015). *European Financial Systems 2015*. 1. vyd. Brno: Masarykova univerzita.

[5] ESMA (2016). Supervisory work on potential closet index tracking. ESMA/2016/165.

[6] Laderman, J.M. (1997). Shining a Light on "Closet Index Funds". *BusinessWeek*, Oct. 20, p. 162 - 163.

[7] Mikeš, J. (2013). Řízení fondů je na oko. *Fond Shop*, 25 (2013), p. 16 - 17.

[8] Mikeš, J. (2014). Zastřené indexování. *Fond Shop*, 17 (2014), p. 8 - 9.

[9] Mikeš, J. (2015). „Aktivní řízení" v praxi. *Fond Shop*, 15-16 (2015), p. 18 - 19.

[10] Petajisto, A. (2013). Active Share and Mutual Fund Performance. *Financial Analysts Journal*, 69(4), p. 73 - 93.

[11] Taylor, J. (2004). A note on closet-indexing. *Journal of Economics and Business*, 56 (2004), p. 431 - 441.

The issue of the commercial insurance market and insurance of non-life risks

Barbora Drugdová [1]

Abstract

The article is devoted to insurance of international risks as a part of non-life risks in the Slovak Republic. In the European Union, as well as in developed economies, the economic relations are expanding beyond the borders of the countries. The economic relations that arise between two or more countries generally bring many new risks, but also opportunities to anticipate these risks or eliminate and remove them. Many of the risks are transferred from one country to another. In the Slovak insurance market, the entities dealing with insuring international risks are commercial insurance companies, which are members of the Slovak Association of Insurance Companies (SAP). Until December 31, 2015 there were 21 commercial insurance companies in the Slovak insurance market, of which 20 were associated in SAP. Commercial insurance companies offer a wide range of insurance products including products aimed at insuring international risks.

Key words

insurance non-life risks, international risks, indicators the insurance commerce market, indicators non-life insurance, insurance products

JEL Classification: G 22, G 32

1. Úvod

Poistenie a poisťovníctvo zasahujú priamo alebo nepriamo do všetkých oblastí ľudskej činnosti, preto sa dotýkajú každej firmy, podniku, občana, spoločnosti aj zahraničia. Všetky zmeny v spoločnosti sa určitým spôsobom odrážajú aj v poisťovacej činnosti. Rozvoj vedy, techniky a kultúry prináša na trh nové moderné výrobky, nové činnosti a s tým aj nové riziká. S týmito rizikami musia počítať a kalkulovať podniky a podnikatelia, občania, štát, ale aj poisťovacie spoločnosti. Komerčné poisťovne musia preto stále pripravovať na komerčný poistný trh nové poistné produkty z oblasti životných aj neživotných rizík a zároveň sledovať vývoj v celej ekonomike (napr. vývoj HDP, rast nezamestnanosti, infláciu, rast cien, pohyb peňažných príjmov, demografické údaje a iné makroekonomické a makroekonomické ukazovatele).

Slovenský poistný trh od 1. 5. 2004 je neoddeliteľnou súčasťou jednotného poistného trhu Európskej únie, ktorého súčasťou je viac ako 5000 poisťovacích spoločností. V súvislosti so vstupom do Európskej únie boli prijaté viaceré legislatívne zmeny v oblasti komerčného poisťovníctva, ktoré sa dotýkali aj neživotného poistenia, ako aj poistenia medzinárodných rizík, ako súčasti neživotných rizík. Posledným zákonom v oblasti komerčného poisťovníctva je zákon Národnej rady Slovenskej republiky č. 39/2015 Z. z. o poisťovníctve v ktorom sú zmeny v oblasti životného aj neživotného poistenia oproti zákonu č. 8/2008 Z.z.

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o poisťovníctve. To sa premietlo aj do komerčnéhopoistného trhu životného aj neživotného poistenia.

2 Charakteristika neživotných rizík

Neživotné poistenie je poistenie, ktoré zahŕňa celú škálu rizík neživotného charakteru**.** V odbornej literatúre sa stretneme s rôznym vymedzením podstaty neživotného poistenia.Napr.autori publikácie Poisťovníctvo: Teória a prax vymedzujú podstatu neživotného poistenia v krytí celej škály existujúcich rizík neživotného charakteru, ktoré ohrozujú ľudskú činnosť.[2] V publikácii Praktický pruvodce finanční a pojistnou matematikou T. Cipra medzi neživotné poistenia zaraďuje okrem úrazového, zodpovednostného a majetkového poistenia aj súkromné zdravotné a nemocenské poistenie.[3]

Neživotné riziká môžeme rozdeliť do štyroch skupín: Prvú skupinu tvoria riziká ohrozujúce zdravie a životy osôb – napr. úraz, choroba, invalidita a pod. Druhú skupinu predstavujú riziká, ktoré vyvolávajú finančné straty - úverové riziká, právne riziká, riziká finančných strát, zodpovednostné riziká a pod. Do tretej skupiny zahŕňame riziká vyvolávajúce priame vecné škody – živelné riziká, krádež, vandalizmus a pod. Do štvrtej skupiny zahŕňame medzinárodné riziká. Neživotné postenie je krátkodobé, to znamená, že poistná zmluva sa uzatvára na neurčito s ročným poistným obdobím, resp. kratším ako rok, na rozdiel od životných zmlúv, ktoré sú dlhodobé a uzatvárajú sa na dobu určitú. Poistná zmluva predstavuje dvojstranný právny dokument, ktorý je základom zmluvného vzťahu medzi poisťovateľom a poistníkom. Poisťovateľ sa ňou zaväzuje poskytnúť v dojednanom rozsahu poistné plnenie, ak nastane náhodná poistná udalosť stanovená v poistnej zmluve, a poistník sa zaväzuje platiť poistné.

2.1 Podstata a charakteristika medzinárodných rizík, ako súčasti neživotných rizík

Ekonomické vzťahy, ktoré vznikajú medzi dvoma alebo viacerými štátmi prinášajú veľa nových rizík, ale tiež možnosti ich predvídať, eliminovať a odstraňovať. Mnohé sa prenášajú z jedného štátu do druhého, ako napr. cenové výkyvy, inflačný vývoj a podobne. Na druhej strane niektoré zabraňujú a eliminujú mnohé ťažkosti iného druhu, napr. dovoz nedostatkových surovín a materiálov, napomáhajú rozvoju výroby pri možnostiach rozšírenia exportu a podobne.

V dostupnej domácej a zahraničnej odbornej literatúre sa stretneme s členením medzinárodných rizík ako súčasti neživotných rizík v oblasti neživotného poistenia, ktoré súvisia s medzinárodnými obchodnými vzťahmi. Pod pojmom medzinárodné riziká rozumieme riziká, ktoré vznikajú mimo územia príslušného štátu.

Problematika medzinárodných rizík je veľmi rozsiahla a komplexná z hľadiska: diverzifikácie prípadov podľa, toho či ide o export, reexport alebo import, a taktiež množstva aspektov: komerčných, právnych, technických, bankovo-finančných, ako aj širších politických a ekonomických súvislostí.

Najčastejšie sa rozlišujú tri skupiny medzinárodných rizík, ktorým budeme v príspevku venovať väčšiu pozornosť. Ide o komerčné riziká, politicko-hospodárske riziká a ostatné druhy rizík.

[2] Majtánová, A., Daňhel,J., Ducháčková,E., Kafková,E.:Poisťovníctvo. Teória a prax. Praha: Ekopress, 2006, s. 148. ISBN 80-86929-19-1

[3] Cipra, T.: Praktický průvodce finanční a pojistnou matematikou. Praha: Ekopress, 2005, s. 180 ISBN 80-86119-91-2

3. Členenie medzinárodných rizík, ako súčasti neživotných rizík

Medzinárodné riziká zahŕňame medzi neživotné riziká. Medzinárodné riziká členíme na riziká komerčné alebo obchodné (finančné) riziká, politicko – hospodárske riziká a osobitné druhy rizík.

3.1 Komerčné riziká

Komerčné riziká tvoria systém väčších i menších rizík, týkajúcich sa hlavne medzinárodných obchodno-finančných transferov.

Komerčné riziká pôsobia vo vonkajších vzťahoch intenzívnejšie ako vo vnútornej ekonomike krajín, najmä z nasledujúcich dôvodov:

- Rozdielnosť mentalít, ktorá je spojená s ťažšou komunikatívnosťou, ale aj s ťažkosťami pri hľadaní nových trhov.
- Jazykové bariéry, ktoré prinášajú nebezpečenstvo nedorozumení, prípadne chybných interpretácií pri uzavieraní zmlúv, čo môže mať negatívne následky.
- Rozdielnosť právnych systémov, kde zmluvy môžu byť interpretované podľa práva a obchodných zvyklostí exportéra, ako i importéra. Rozhodnutia daných súdov sa dajú ťažko presadiť v zahraničí a môžu byť drahšie ako samotný predmet sporu.
- Väčšia vzdialenosť medzi zmluvnými stranami, čo má za následok menšie vzájomné poznanie, horší prehľad na cudzom trhu a tiež zvýšené riziko transportu.

Komerčné riziká v zmluvných vzťahoch vyplývajú z nebezpečia, že zahraničný partner nedodrží uzavretú zmluvu buď preto, že k tomu nie je ochotný, alebo preto, že nie je schopný kontrakt dodržať. Neschopnosť partnera splniť záväzky vyplývajúce zo zmluvy má spravidla príčinu v jeho zmenenom ekonomickom postavení alebo v nereálnom odhade jeho ekonomických možností.

Intenzita dopadu komerčných rizík je všeobecne na jednotlivé zahraničné operácie daná dvoma faktormi: Prvým faktorom je spoľahlivosť partnera – z tohto hľadiska treba posudzovať právne postavenie partnera, jeho majetkovú situáciu, komerčnú zdatnosť, technicko-ekonomické predpoklady splnenia záväzku a pod. Druhým faktorom je stupeň právneho zaistenia daného záväzkového vzťahu – ide spravidla o právnu dokonalosť kontraktu, jeho podmienky, právo, podľa ktorého je právny vzťah posudzovaný, charakter sankcií za nesplnenie kontraktu.

Komerčné riziká sú druhmi rizík, ktoré môžu dosahované výsledky ovplyvniť hlavne v negatívnom smere. Sú jedným z málo druhov rizík, u ktorých možno aplikovať ochranu cestou ich minimalizácie. Tieto riziká sú ovplyvnené kvalitou práce subjektov vo sfére zahranično – ekonomických vzťahov, ich schopnosťou odhadovať riziká a voliť vhodné cesty ich obmedzenia. Komerčné riziká môžeme rozdeliť na: trhové riziko, riziko neprevzatia tovaru, riziko z nezaplatenia, tovarové riziko, transportné riziko – dopravné riziko.

3.2 Politicko – hospodárske riziká

Politické riziká vyplývajú z celkovej politickej, ekonomickej a finančnej situácie krajiny, do ktorej smerujú exporty.

S pojmom politické riziko sa vo všeobecnosti spája bezprostredné ohrozenie zisku, v extrémnych prípadoch aj ohrozenie hospodárskej existencie zahraničných spoločností, alebo dokonca i medzinárodných podnikov. Dôvody vzniku politických rizík vyplývajú zo zmien v politickom okolí podniku. Konkretizujú sa opatreniami prijatými štátom a tieto znamenajú pre exportéra riziko neratifikácie alebo zrušenia kontraktu, embargo vývozu, neprevedenie devízového transferu, konfiškáciu materiálov a výrobkov a pod. Z najširšieho hľadiska toto riziko vyplýva z nepriateľských akcií, ako sú vojna, revolúcie, štrajky, nepokoje, občianske vojny a pod. Politicko-hospodárske riziká v medzinárodnom obchode už dávno nemožno

považovať za vedľajšie. Nemožno ich celkovo podceniť. Je nevyhnutné začleniť ich do kalkulácie obchodu. Obrovská zadlženosť rozvojového sveta, rozsiahla platobná neschopnosť, medzinárodná dlžnícka kríza, nebezpečná a výbušná situácia v mnohých regiónoch sveta, všetky tieto faktory len umocňujú a zvýrazňujú nevyhnutnosť znižovania politicko – hospodárskych rizík.

Politicko – hospodárske riziká členíme na tieto druhy, ide o:
a/ výmenné riziko – peniaze, ktoré už importér vyplatil svojej banke, nemožno (dočasne) v dôsledku štátnych opatrení importujúcej krajiny a v dôsledku nedostatku devíz vymeniť za žiadnu cudziu menu.
b/ moratórium – zaplatenie a transfer dlhovej čiastky exportérovi štát nepovolí v čase splatnosti pohľadávky, ale povolí len čiastočné zaplatenie dlhu, prípadne nariadi odklad platby.
c/ transferové riziko – peniaze vyplatené importérom v banke nemožno (čiastočne alebo úplne) previesť do zahraničia, pretože importujúca krajina z hospodársko – politických príčin nechce taký transfer povoliť.

Riziko transferu obsahuje nebezpečenstvo úplného alebo čiastočného obmedzenia transferu zisku, vecných statkov, know-how, licenčných poplatkov, platieb za služby, ako aj kapitálu.

Riziko transferu sa v súčasnosti stalo jedným z najväčších rizík pre medzinárodné finančné inštitúty a banky. Medzi transferové riziká sa spravidla zaraďuje: riziko platobnej neschopnosti krajiny a riziko obchodných prekážok.

3.3 Osobitné druhy rizík

Osobitné druhy rizík tvoria zvláštnu skupinu. Do tejto skupiny patria tieto druhy rizík: kurzové riziko, riziko pohybu úrokových sadzieb, inflačné riziko, právne riziko, riziká pri zahraničných investíciách, riziká prírodných a technických katastrôf, riziká teroristického útoku a špecifické riziká. Do skupiny špecifických rizík možno zahrnúť špecifické riziká istoty, bezpečnosti, pod ktoré patria: udalosti ohrozujúce zdravie, slobodu a život pracovníkov medzinárodných spoločností a inštitúcií, ako aj rodinných príslušníkov a obavy z budúcnosti. Denná realita nás presvedčuje o narastajúcej sa „schopnosti“ človeka alebo skupiny nacionalisticky, ideologicky, nábožensky, sektársky, teroristicky či inak zameraných ľudí, ktorý ohrozujú nielen majetok, ale najmä zdravie a život nevinných ľudí. Rastúci počet teroristických útokov na prelome tisícročia nemôže nechať „s kľudom spať“ všetkých mierumilovných ľudí na tejto zemi. (Uvedieme väčšie teroristické útoky napr. teroristický útok v USA v roku 2001, alebo Španielsku v roku 2004 a ďalšie teroristické útoky najmä v krajinách Európskej únie, uvedieme rok 2016 Belgicku a Francúzsku).

4. Poistenie medzinárodných rizík, ako súčasti neživotných rizík na poistnom trhu

Poisťovaním medzinárodných rizík, ako súčasti neživotných rizík sa na slovenskom komerčnom poistnom trhu zaoberajú komerčné poisťovne, ktoré sú združené v Slovenskej asociácii poisťovní.

Komerčný poistný trh vo vyspelých ekonomikách využíva mnoho komerčných poisťovní. Každá sa zúčastňuje na komerčnom poistnom trhu rôznou ponukou poistných produktov a má záujem pokryť čo najväčší priestor (poistné pole).[4] K 31.12 2015 ako bolo vyššie uvedené,

[4] Poznámka: Na komerčnom poistnom trhu ponúkajú komerčné poisťovne poistné produkty z oblasti životného a neživotného poistenia. V súčasnosti ponúkajú poistné produkty

pôsobilo na slovenskom poistnom trhu 20 komerčných poisťovní, ktoré boli združené v Slovenskej asociácií poisťovní/, ktoré sa podieľali uvedenými ukazovateľmi na technickom poistnom /viď. Tabuľka č. 1. Technické poistné celkom a v neživotnom poistení k 31.12. 2015 v tis. euro. Komerčné poisťovne, ktoré ponúkajú poistné produkty z oblasti medzinárodných rizík sa podieľali na neživotnom poistení nasledujúcimi údajmi.

Tabuľka 1. Technické poistné celkom a v neživotnom poistení v komerčných poisťovniach k 31.12. 2015 v tis. euro [5]

	Technické poistné k 31.12.2015 v tis. euro		
Por. č.	**Komerčná poisťovňa**	**Technické poistné celkom za všetky poistné produkty**	**Technické poistné v neživotnom poistení za poistné produkty**
1	AEGON Životná poisťovňa, a.s.	45 176	0
2	Allianz - Slovenská poisťovňa, a. s.	589 234	341 736
3	AXA životní poisťovňa, pobočka poisťovne z iného členského štátu	56 199	7 432
4	AXA poisťovňa pobočka poisťovne z iného členského štátu	10 640	10 640
5	BASLER Sachversicherung – AG pobočka poisťovne z iného členského štátu	4 506	4 506
6	ČSOB Poisťovňa, a. s.	81 535	29 824
7	D.A.S. Rechtsschutz AG, pobočka poisťovne z iného členského štátu	2 848	2 848
8	ERGO Poisťovňa, a.s.	44 106	1 531
9	Generali Slovensko poisťovňa, a. s.	194 270	110 286
10	Groupama Garancia poisťovňa.a.s. pobočka poisťovne z iného členského štátu	5 093	4 647
11	NN Životná poisťovňa, a. s.	75 560	0
12	KOMUNÁLNA poisťovňa, a. s. VIG	181 110	70 196
13	KOOPERATIVA poisťovňa, a. s., VIG	456 066	254 247
14	MetLife Europe Limited, pobočka poisťovne z iného členského štátu,	121 485	7 075
15	Poisťovňa Cardif Slovakia, a.s.	24 600	17 489
16	Poisťovňa Poštovej banky, a. s.	11 829	1 165
17	Poisťovňa Slovenskej sporiteľne, a. s.	86 804	0
18	UNION poisťovňa, a. s.	44 889	33 809
19	UNIQA poisťovňa, a. s.	115 109	82 496
20	Wüstenrot poisťovňa, a. s..	59 901	22 539
21	Slovenská kancelária poisťovateľov	8	8
	Celkom	**2 210 969**	**1 002 472**

univerzálne komerčné poisťovne a neživotnépoisťovne , ktoré sú zamerané na poisťovanie neživotnźch rizík, ale aj medzinárodnźch rizík.

[5] Zdroj: www.slaspo.sk

Technické poistné celkom za všetky poistné produkty v roku 2015 bolo 2 210 969 tis. euro. Poradie komerčných poisťovní na slovenskom poistnom trhu za rok 2015 je nasledovné, ako vyplýva z tabuľky č. 1. Technické poistné celkom a v neživotnom poistení v komerčných poisťovniach k 31.12. 2015 /v tis. euro/. Prvenstvo na slovenskom poistnom trhu v oblasti neživotného poistenia mala Poisťovňa Allianz - Slovenská poisťovňa, a. s., s 34,09 % čo predstavovalo 341 736 tis. euro. Na druhom mieste sa umiestila poisťovňa KOOPERATÍVA poisťovňa, a.s., VIG v oblasti neživotného poistenia s 25,36 % čo predstavovalo 254 247 tis. euro, tretie miesto obsadila v neživotnom poistení poisťovňa Generali Slovensko, poisťovňa, a.s. s 11,00 % čo predstavovalo 110 286 tis. euro.

Poistné produkty zamerané na poisťovanie medzinárodných rizík sú zahrnuté v neživotnom poistení. Poisťovňa Allianz - Slovenská poisťovňa, a. s. ponúka najširšiu ponuku poistných produktov zameraných na poisťovanie medzinárodných rizík.

Najdôležitejšie poistné produkty z oblasti medzinárodných rizík, ktoré ponúkajú komerčné poisťovne na komerčnom poistnom trhu sú: dopravné poistenie (poistenie zahraničnej prepravy zásielok), poistenie investičných celkov (stavebno-montážne poistenie), poistenie výstav a veľtrhov, poistenie zodpovednosti za výrobok, poistenie medzinárodného cestného dopravcu, havarijné poistenie motorových vozidiel do zahraničia, poistenie z prevádzky lietadla a poistenie špeciálnych rizík. Medzi špeciálne riziká zahŕňame najmä: poistenie zariadení atómového priemyslu a pridružených rizík, poistenie exportných úverov, poistenie kaucie, poistenie včasného dodania tovaru zahraničnému príjemcovi, poistenie ďalšieho dopravného.

Ďalšiu skupinu, ktoré zahŕňame do oblasti medzinárodných rizík a dotýkajú sa poistenia osôb tvoria: poistenie turistických rizík, kde zahŕňame poistenie cestovného ruchu, poistenie liečebných nákladov v cudzine a pod.

V súvislosti s poistením medzinárodných rizík, ako súčasti neživotných rizík treba povedať, že komerčné poisťovne na slovenskom poistnom trhu neponúkajú niektoré poistné produkty, ktoré sú zaužívané v krajinách Európskej únie a v celom vyspelom svete najmä v Japonsku, USA a Kanade. Ide predovšetkým o poistné produkty z oblasti komerčných rizík, (napr. bankovo – finančných rizík) a taktiež z oblasti osobitných druhov rizík (napr. rizík prírodných a technických katastrôf, ako aj rizík teroristických útokov a pod./

5. Záver

Poistenie medzinárodných rizík, ako súčasť neživotných rizík má dôležitý význam ako z hľadiska ekonomického, tak aj politického. Na slovenskom poistnom trhu v súvislosti s poistením medzinárodných rizík, ako súčasti neživotných rizík v procese globalizácie a integrácie možno v budúcnosti predpokladať, že komerčné poisťovne, ktoré pôsobia na slovenskom poistnom budú ponúkať také poistné produkty z oblasti neživotného poistenia zamerané na poistenie medzinárodných rizík, ktoré ponúkajú v súčasnosti komerčné poisťovne v ostatných členských krajinách Európskej únie, najma Veľkej Británie, Nemecka a Francúzska a v celom vyspelom svete najmä v Japonsku, USA a Kanade.

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References

[1] Cipra, T. (2005). *Praktický průvodce finanční a pojistnou matematikou*. Praha: Ekopress, s.180

[2] Daňhel, J. a kol. (2005). *Pojistná teórie*. Praha.Vydavateľstvo: PROFESIONAL PUBLISHING, s. 205

[3] Drugdová, B. (2005). *POISTENIE A POISŤOVNÍCTVO, nemecko – slovenský a slovensko – nemecký slovník***.** Bratislava: Vydavateľstvo: ELITA, s. 183

[4] Drugdová, B. (2013). *Podnikové poistenie. Teória a prax.* Bratislava: Vydavateľstvo ŠEVT, s. 189

[5] Sivák, R. a kol. (2015). Financie. Bratislava, Vydavateľstvo: Wolters Kluver, s. 398

internetové zdroje
www.slaspo.sk
www.cap.cz.
www.nbs.sk
www. europa.eu

Good-Risk/Bad-Risk Model in Automobile Insurance and its Generalization

Jozef Fecenko, Martin Pinda [1]

Abstract

The article deals with the study of a Good-risk/Bad-risk model in automobile insurance. Our considerations are arising from Lemaire's approach for solving mentioned model. The new approach was created, with new type of equations, for solving Good-risk/Bad-risk model using Maxima - open source system.

Key words

Automobile insurance, Good-risk/Bad-risk model, bonus-malus system, mixed Poisson process

JEL Classification: G22, G0

1. Introduction

Factorial moment is a probability quantity defined as the expectation or average of the falling factorial of a random variable. For a natural number *r*, the *r*-th factorial moment of a random variable X (if it exists) is defined as

$$E\left[\left(X\right)_r\right] = E\left[X(X-1)(X-2)\ldots(X-r+1)\right]. \tag{1}$$

If X is a discrete random variable taking values in the non-negative integers, then the probability generating function of X is defined as

$$G_X(t) = E\left(t^X\right) = \sum_{k=0}^{\infty} p_X(k)t^k \tag{2}$$

where p_X is the probability mass function of X. The power series on the right side (2) converges at least for all complex number t with $|t| \leq 1$.

The *r*-th factorial moment of a random variable X (if it exists) can be expressed in the form

$$E\left[\left(X\right)_r\right] = G_X^{(r)}(1) = \left.\frac{\mathrm{d}^r G_X(t)}{\mathrm{d}t^r}\right|_{t=1}. \tag{3}$$

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2. Good-risk/Bad-risk model

2.1 Lemaire's approach for solving Good-risk/Bad-risk model

Lemaire in [7] studies mixed process where the mixing structure function is a simple two-point discrete distribution. The portfolio consists of only two categories of drivers: a fraction a_1 of "good"drivers (Poisson parameter λ_1) and a fraction a_2, $a_1 + a_2 = 1$, of "bad" drivers (parameter λ_2, $\lambda_2 > \lambda_1$). Here is his solution:

$$p_X(k) = a_1 \frac{\mathrm{e}^{-\lambda_1}\lambda_1^{\,k}}{k!} + a_2 \frac{\mathrm{e}^{-\lambda_2}\lambda_2^{\,k}}{k!},\ k = 0,1,2,\ldots \tag{4}$$

with $a_1, a_2, \lambda_1, \lambda_2 > 0$.

The mean

$$E(X) = a_1\lambda_1 + a_2\lambda_2.$$

The variance is

$$\sigma^2 = \alpha_2 - E^2(X)$$

where

$$\alpha_2 = a_1\lambda_1^2 + a_1\lambda_1 + a_2\lambda_2^2 + a_2\lambda_2.$$

The third central moment is

$$\mu_3 = \alpha_3 - 3E(X)\alpha_2 + 2E^3(X),$$

where

$$\alpha_3 = a_1\lambda_1^3 + a_2\lambda_2^3 + 3(a_1\lambda_1^2 + a_2\lambda_2^2) + a_1\lambda_1 + a_2\lambda_2.$$

The moments estimators of the parameters are

$$\hat{a}_1 = \frac{a - \hat{\lambda}_2}{\hat{\lambda}_1 - \hat{\lambda}_2}$$

and

$$\hat{\lambda}_1, \hat{\lambda}_2 = \frac{S \pm \sqrt{S^2 - 4P}}{2}$$

with

$$S = \frac{c - ab}{b - a^2}, \quad P = \frac{ac - b^2}{b - a^2}$$

and

$$a = E(X),\ b = \alpha_2^* - E(X),\ c = \alpha_3^* - 3\alpha_2^* + 2E(X).$$

α_2^* and α_3^* are, respectively, the moment around the origin of order 2 and 3 observed distribution. In our example

$$\hat{a}_1 = 0,9112, \quad \lambda_1 = 0,0762, \quad \hat{a}_2 = 0,0888, \quad \lambda_2 = 0,3567.$$

These results can be interpreted as follows: The portfolio has 91,12 % of good drivers, with claim frequency of 7,62 % (it means that every driver has approximately, on average, 0,762 claims per 10 years), and 8,88 % of bad drivers, with a claim frequency of 35,67 % (here every driver has approximately, on average 3,567 claims per a 10 years).

Table 1: Observed and Fitted Distribution of Number of Claims: Good-risk/Bad-risk model, Moment Methods (Source [7])

k	n_k	np_k
0	96 978	96 975,0
1	9 240	9 252,2
2	704	685,0
3	43	56,9
4	9	4,6
>4	0	0,3
Total	106 974	106 974,0

Table 1 shows that the fit is a good quality, what confirming also χ^2 test, see next.

2.1.1 χ^2 test in R language

Rule A: (i) All $np_k \geq 1$; (ii) At least 80 % of the $np_k \geq 5$.

```
# Rule A
k<-c(0,1,2,3,4)
nk<-c(96978,9240,704,43,9)
npk <-c(96975,9252.2,685,56.9,4.9)
chisq.test(nk, p = npk/sum(nk))
```

```
        Chi-squared test for given probabilities
                    data:  nk
X-squared = 7.3694, df = 4, p-value = 0.1176

    p-value = 0.1176 > 0.05 = α  ⇒ H0
```

Rule B: (i) All $np_k \geq 5$.

```
# Rule B
k<-c(0,1,2,3)
nk<-c(96978,9240,704,52)
npk<-c(96975,9252.2,685,61.8)
chisq.test(nk, p = npk/sum(nk))
```

```
        Chi-squared test for given probabilities

                    data:  nk
X-squared = 2.0972, df = 3, p-value = 0.5525

    p-value = 0.5525  > 0.05 = α  ⇒ H0
```

Computations show that the fit is a good quality.

2.2 Another approach for solving Good-risk/Bad-risk model

As we have seen in previous part, the solution of Good-risk/Bad-risk model is not very easy. The equations are a little complicated.

Let us try deriving a simpler model for the solution of the observed problem. Let us consider (4), where $a_1 + a_2 = 1$.

Then probability generating function is

$$G_X(t)=\sum_{k=0}^{\infty}t^k\cdot p_X(k)=\sum_{k=0}^{\infty}\left(t^k a_1\frac{\mathrm{e}^{-\lambda_1}\lambda_1{}^k}{k!}+t^k a_2\frac{\mathrm{e}^{-\lambda_2}\lambda_2{}^k}{k!}\right)=a_1\mathrm{e}^{\lambda_1(t-1)}+a_2\mathrm{e}^{\lambda_2(t-1)}$$

We have obviously

$$G_X(1)=a_1+a_2=1$$

Let us compute

$$\begin{aligned}G'_X(t)&=a_1\lambda_1\mathrm{e}^{\lambda_1(t-1)}+a_2\lambda_2\mathrm{e}^{\lambda_2(t-1)}\\ G'_X(1)&=a_1\lambda_1+a_2\lambda_2=E\left[(X)_1\right]\\ &\vdots\\ G_X^{(r)}(1)&=a_1\lambda_1{}^r+a_2\lambda_2{}^r=E\left[(X)_r\right].\end{aligned}$$

So we receive an elegant form of Good-risk/Bad-risk model

$$\begin{aligned}a_1+a_2&=1\\ a_1\lambda_1+a_2\lambda_2&=E[(X)_1]\\ a_1\lambda_1^2+a_2\lambda_2^2&=E[(X)_2]\\ a_1\lambda_1^3+a_2\lambda_2^3&=E[(X)_3]\end{aligned}\qquad(5)$$

where $a_1,a_2,\lambda_1,\lambda_2\geq 0.$

In order to solve nonlinear simultaneous equations, we need to create the empirical probability generating function from the values of the table 1.

$$\hat{G}_X(t)=\sum_{k=0}^{4}t^k\cdot\hat{p}_X(k)=\frac{96\,978}{106\,974}+\frac{9\,240}{106\,974}t+\frac{704}{106\,974}t^2+\frac{43}{106\,974}t^3+\frac{9}{106\,974}t^4$$

Then

$$\begin{aligned}\hat{G}'_X(1)&=\hat{E}\left[(X)_1\right]=\frac{10813}{106974}\\ \hat{G}''_X(1)&=\hat{E}\left[(X)_2\right]=\frac{887}{53487}\\ \hat{G}'''_X(1)&=\hat{E}\left[(X)_3\right]=\frac{79}{17829}\end{aligned}$$

The system of nonlinear equation, we need to solve is

$$\begin{aligned}a_1+a_2&=1\\ a_1\lambda_1+a_2\lambda_2&=\frac{10813}{106974}\\ a_1\lambda_1^2+a_2\lambda_2^2&=\frac{887}{53487}\\ a_1\lambda_1^3+a_2\lambda_2^3&=\frac{79}{17829}\end{aligned}\qquad(6)$$

Figure 1: Solving equations (6) using MAXIMA[2]

```
(%i1) float(algsys([a[1]+a[2]=1,
                    a[1]*%lambda[1]+a[2]*%lambda[2]=10813/106974,
                    a[1]*%lambda[1]^2+a[2]*%lambda[2]^2=887/53487,
                    a[1]*%lambda[1]^3+a[2]*%lambda[2]^3=79/17829],
                   [a[1],%lambda[1],a[2],%lambda[2]])),numer;
(%o1) [[a1=0.0888746580304502,λ1=0.3565502836269931,a2=0.9111253419695498,
λ2=0.07616114274237641],[a1=0.9111253417227299,λ1=0.07616114274237644,a2=
0.0888746580304502,λ2=0.3565502836269931]]
```

We receive two solutions, but they are symmetrical. The solution is more precise and is the same as in the path 1.1.

2.3 Generalization of Good-risk/Bad-risk model

Now we consider probability mass function in the form

$$p_X(k) = a_1 \frac{e^{-\lambda_1}\lambda_1^{\,k}}{k!} + a_2 \frac{e^{-\lambda_2}\lambda_2^{\,k}}{k!} + \ldots + a_n \frac{e^{-\lambda_n}\lambda_n^{\,k}}{k!}, \quad k = 0,1,2,\ldots \tag{7}$$

with $a_1, a_2, \ldots, a_n, \lambda_1, \lambda_2, \ldots, \lambda_n > 0$, where $a_1 + a_2 + \ldots + a_n = 1$.

A similar consideration and reasoning as higher we get a system of $2n$ nonlinear equations with $2n$ variables.

$$\begin{aligned}
a_1 \quad &+ a_2 \quad + \ldots + a_n \quad = 1 \\
a_1\lambda_1 &+ a_2\lambda_2 + \ldots + a_n\lambda_n = E\left[(X)_1\right] \\
a_1\lambda_1^2 &+ a_2\lambda_2^2 + \ldots + a_n\lambda_n^2 = E\left[(X)_2\right] \\
a_1\lambda_1^3 &+ a_2\lambda_2^3 + \ldots + a_n\lambda_n^3 = E\left[(X)_3\right] \\
&\vdots \qquad\qquad\qquad\qquad \vdots \\
a_1\lambda_1^{2n-1} &+ a_2\lambda_2^{2n-1} + \ldots + a_n\lambda_n^{2n-1} = E\left[(X)_{2n-1}\right]
\end{aligned} \tag{8}$$

where $E\left[(X)_r\right]$, $r = 1,2,\ldots,2n-1$ will be estimated by empirical factorial moments $\hat{E}\left[(X)_r\right]$.

The system of equations (8) can be expressed in the matrix form

$$\begin{pmatrix} 1 & 1 & \ldots & 1 \\ \lambda_1 & \lambda_2 & \ldots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \ldots & \lambda_n^2 \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_1^{2n-1} & \lambda_2^{2n-1} & \ldots & \lambda_n^{2n-1} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} 1 \\ E\left[(X)_1\right] \\ E\left[(X)_2\right] \\ \vdots \\ E\left[(X)_{2n-1}\right] \end{pmatrix} \tag{9}$$

For the solution of system nonlinear equations (8), (9) we need use some suitable iterative method. Interpretation of the solution is very similar to the Good-risk/Bad-risk model.

[2] MAXIMA is an open source program. It exists for computers and also for smartphones with operation system ANDROID.

3. Conclusion

The article follows the part "Good-risk/Bad-risk model of the [7]". It provides a simple form the system of three nonlinear equations for solving Good-risk/Bad-risk model that can be solution using, for example, open source program Maxima. The contribution also offers a simple form a system of nonlinear equations for solving generalization of mentioned model. It means if we assume that portfolio consists of more than two categories of drivers. For computational solution of the system of equations we need use an iterative solution of nonlinear equations (see for example [8]).

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References

[1] Fecenko, J. (2015) *Calculating of probability generating function of total amount of claims in collective risk model* (in Slovak), Aktuárská veda v teórii a v praxi, EKONÓM Bratislava, ISBN 978-80-225-4156-5.

[2] Fecenko, J. (2011): *Hunger for bonus and its computer realization in open source system* Maxima (in Slovak). Actuarial science in theory and practice, EKONOM, Bratslava, ISBN 978-80-225-3242-6

[3] Skřivánková, V. and Fecenko, J. (2008): *Efektívnosť bonus-malus systému*. Řízení a modelování finančních rizík: sborník ze 4. medzinárodní vedecké konference. Ostrava, ISBN 978-80-248-1846-7

[4] Fecenko, J. and Skřivánková, V. (2008): *Niektoré možnosti aplikácie životného poistenia v bonus-malus systéme.* Řízení a modelování finančních rizík: sborník ze 4. medzinárodní vedecké konference. Ostrava, ISBN 978-80-248-1846-7

[5] Skřivánková, V. and Fecenko, J. (2007): *Rizikové poistné v stabilizovanom systéme bonus-malus*. Aktuárská veda v teórii a v praxi, EKONÓM Bratislava, ISBN 978-80-225-2443-8.

[6] Feller, W (1967) *Introduction to probability theory and its applications*. John Wiley & Sons, Inc.

[7] Lemaire, J. (1995). *Bonus-Malus system in automobile insurance.* Kluwer Academic Publishers Group. USA, ISBN 0-7923-9545-X

[8] Ortega, J. M. and Rheinboldt, W.C (1970) Iterative solution of nonlinear equations in several variables, SIAM Society for Industrial and applied Mathematics, Philadelphia

Models of Regulation as Tool for Risk Elimination in Network Industries Markets Development

Eleonora Fendeková[1]

Abstract

Currently a significant attention in scholarly discussions on various levels is being paid to the subject of network industries. It is understandable as network industries in fact ensure the production and distribution of energy sources which play a key role in an effective operation of the developed economies. The discussions are usually focused on the question of a reasonable profit of the network industries companies and on the other hand on the question of prices which are determined by the reasonable and generally acceptable costs of their production.
In relation with objectification and increases in nationwide operation effectiveness of natural monopolies in network industries2 on the market the role of regulation mechanisms is increasing. Its aim is the closest approximation of proportions between price and level of network industry products supply to the situation which would occur in the conditions of market competition. In the paper we will analyze analytical scheme for monopoly price regulation – rate of return regulation.

Key words

Market structure, network industries, monopoly price regulation, regulatory mechanism, rate of return regulation.

JEL Classification: D42, D43, L51, L13

1. Introduction

Modern market structures in economically developed countries are generally represented by imperfect competition. We need to realize, that monopoly, in consequence of its exclusive market position as a sole supplier of commodity or service on the relevant market, can set the market price and supply any level of production, which allows it to maximize profit above the level of capacity of competitive company.

Blum, Müller and Weiske [1] show that monopoly is, of course, acting within the legitimate market conditions and its position is determined by the real market situation. On the other side it is natural for such means to exist, which allow redistribution of this profit in a way that part of it would support nationwide economy development objectives of the country it is operating in.

In principle, there are two schemes, which can be applied by the state in regulation of monopoly price policy.

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[2] Under network industries in terms of Slovak Republic in accordance with Act No. 276/2001 Coll. on Regulation in Network Industries we understand the following industries

1. generation, transmission, distribution and supply of electricity and the related services
2. production, transport, distribution, storage and supply of gas and the services related to these activities
3. production and distribution of heat
4. water management and mass conduct of sewage by a public sewage system

a) Taxation of monopoly

There is a potent tool at states disposal to regulate the development of market equilibrium conditions on monopolistic market. It is a tax policy. In a simplified way, we can tell that the state is redistributing the profit of monopolistic firm through its taxation. This process of taxation in the imperfect competition environment has, in comparison with perfect competition environment, certain specificity.

Firm operating in perfect competition environment can respond to enforcement of particular scheme of tax burden only by modifying the amount of goods supplied on the market. Fendek and Fendeková [5] show, that monopoly has, however, broader room for decision making because in its competence there is not only setting the amount of goods supplied on the market but also, after taking into account consumer behavior, setting the price of good on relevant market.

b) Application of specific regulatory mechanisms affecting behavior of natural monopolies

In Fendek and Fendeková [4] we can see, that or the regulation of subjects of the chosen industries, which are in light of their market share monopolies or natural monopolies, the state is creating institution, so-called regulator, which task, under the state authority, is to create such legislative environment and regulatory mechanisms, which will ensure market equilibrium for observed commodities while granting fair profit for regulated subject. Among these industries particularly belong network industries, which subjects often share characteristics of monopolies and state has therefore desire to regulate them.

2. Means of state to regulate price policy of monopoly

In Slovak conditions the role of the regulator is represented by Regulatory Office for Network Industries (RONI), which role is to issue licenses, regulate prices and quality norms for network industry products. Regulatory offices in relation with fulfillment of their main mission – technical and price regulation of enterprise in regulated activities of chosen industries – are facing the effective solution of two tasks:

(1) Preparation of working and, this needs to be stressed, competitive market environment for network industries goods while applying standard regulatory mechanisms, mainly in context of Slovak entry into the EU and gradual adaptation of Slovakia to the conditions on energy markets of united Europe;

(2) Preparation of such analytical apparatus for network industries price regulation, which would guarantee effective development of regulated subjects. In the first period it was equally important to eliminate deformations in prices of network industries products.

A need of the particular legislation of regulatory framework for natural monopolies in Slovakia has arisen in the beginning of 1990s. Legislation in this period has not been covering the whole area of regulation therefore in 1993 the Antimonopoly Office of the Slovak Republic prepared a bill for regulation of natural monopolies, which assumed delegation of powers to a single body. This proposal should have supported companies by stimulating competitive environment towards greater effectiveness and reaching better quality levels.

Regulation, according to this proposal, should have been related to generation, transfer and distribution of electricity, heat, gas, water, but also telecommunications. Subjects of regulations should have become prices, tariffs, expenses, trade conditions and entry to the market in form of issuing licenses. Bill on regulation of natural monopolies was worked out in two alternatives from the perspective of founding and status of the regulatory bodies.

First alternative considered founding of four independent regulatory offices, financed from the revenues of regulated subjects. The aim was to create basis for high expertise, objectivity and stability of regulatory system. Second alternative considered that agenda of regulatory office

would be performed by central body of the state administration set by the government. Regulatory office oversees following of obligation to provide a third party with access to the networks and adjudicates disputes. Obligation to provide a third party with network access is also related to full opening of the internal energy market and to consumer option to freely choose energy supplier. According to this, RONI regulations are limited to activities with natural monopoly character only, that is, prices or tariffs set by the office will not include any unregulated items.

In order for Regulatory office to meet this objective it should be financially and personally independent from executive body for reasons of transparency. Additionally, regulatory powers to pass mandatory regulatory statues must be at its disposal. It also needs virtue to enforce them.

RONI is carrying out regulation through the use of standard, in developed economies proven methods. Details about price regulation process in network industries and determination of scope of justified costs and reasonable profit are enacted by the office in office decrees, which are published in Collection of Acts, in Office bulletin and on the website of the office.

Efficient functioning of commodity and service market in network industries in conditions of Slovak republic can be insured only by quality and potent regulatory policy in the context with the Slovak Republic law and respecting the adopted legal acts of European Community and European Union. All consumers, with emphasis on "vulnerable" groups, mainly households, must have an opportunity to make use of high level consumer protection, referring mainly to households, while maintaining adequate guarantees and suitable incentives for regulated subjects.

3. Analytical Methods for network industries regulation

In Waldman, and Jensen [12] we can see, that traditional methodological tool for price regulation applied by price regulators to set maximum price of network industries products is the regulation on the basis of the rate of return – Rate of Return Regulation, by which are the prices of electricity, gas and other companies regulated in the most of the developed countries.

The aim is to ensure that the regulated entity will set the price of commodity or service for its consumers in a way the revenues will cover all its *reasonable and provident costs incurred* as well as regulated return on its *provident*[3] investment.

Let us now analytically derive the allowable rate of cost return for investment, co called *RoR* parameter of the regulated entity. Let us suppose that the firm is producing a homogenous product in production volume *q*, which it realizes on a relevant market for the price *p*. Let us further suppose that the firm uses two production factors, namely labor force with consumption level *L* by labor price *w* and the capital with consumption level *K* by the capital price *r*.

The profit of the firm is generally defined as the difference between the yields and costs

$$\pi(q) = t(q) - n(q)$$

where

$t(q) = p \times q$ – function of revenues of the firm, t: $R \rightarrow R$

$n(q) = nv(q) + nf$ – function of the total costs of the firm, n: $R \rightarrow R$

$nv(q)$ – function of the variable costs of the firm, nv : $R \rightarrow R$

nf – fixed costs of the firm, $nf \in R$

[3] It should be noted, that under term *provident* we understand situations, when certain decision about investment or expenditures has been made in conditions of verified and relevant information available in the time of decision making.

If we substitute general cost function on the basis of consumption of production factors, we get a profit function in a following form

$$\pi(q) = p \times q - w \times L - r \times K$$

If we further express the production volume q on the basis of the production function in the form

$$q = f(K, L)$$

and the production price p on the basis of the price-demand function in the form

$$p = p(q)$$

then we can express the profit function in the form

$$\pi(q) = p(q) \times q - w \times L - r \times K$$

and after further modification in the form

$$\pi(q) = p(f(K, L)) \times f(K, L) - w \times L - r \times K$$

Fendek and Fendeková [8] show, that a non-regulated firm can set its endogenous decision parameters in any way. So it chooses an optimum output volume q^*, an acceptable optimum price p^* and corresponding consumption levels of the production factors labor L and capital K in a way to reach maximum profit. Optimum output and optimum price will be calculated by solving the following mathematical programming task

$$\pi(q) = p(f(K, L)) \times f(K, L) - w \times L - r \times K \rightarrow \max$$
$$K, L \in R_{\geq 0}$$

In this case the non-regulated firm has no formal boundaries for setting the parameters guaranteeing its maximum profit. On the other hand, the regulated firm must respect boundaries given by the regulator. Price regulation regime on the basis of the rate of return lies in a fact, that through the use of exogenously defined control variable *RoR* the allowable level of quotient of the revenues $p \times q$ reduced by its non-capital expenditures $L \times w$ and the volume of consumed capital K is regulated.

In other words, a firm can optimize or freely determine the consumption levels of labor L, capital K by the market prices of production factors w, r and on the other side the level of its production q but also the production price p. However, firm has to respect the rate of return defined by the regulator i.e. the validity of the relation

$$RoR \geq \frac{p \times q - w \times L}{K}$$

Let us further in detail explore the relation between the rate of return of the capital expenditures and the profit of the regulated entity. Profit can be analytically expressed as the difference between the proceeds and the costs of the firm in the form

$$\pi(q) = p \times q - w \times L - r \times K \quad (1)$$

Let us deduct the price of the capital r from the both sides of the relation (1). We get the relation

$$RoR - r \geq \frac{p \times q - w \times L - r \times K}{K} \quad (2)$$

From the comparison of the relation (2) and (3) we get the relation

$$(RoR - r) \times K \geq z(q) \quad (3)$$

From the relation (3) we can see that the regulated entity can set its system parameters only in a way that profit it reaches will not exceed the value of the capital evaluated by the difference between the rate of return *RoR* defined by the regulator and the price of the capital *r*.

Regulated entity can set its controlled (endogenous) decision parameters only in a way respecting regulatory condition. It sets the regulated volume of output q_R, the acceptable regulated price p_R and the corresponding consumptions of production factors labor *L* and capital *K* in a way to reach maximum profit while respecting regulatory conditions (3) about non-exceeding the reasonable level of profit. The regulated output and regulated price are calculated by solving of the following mathematical programming task

$$\pi(q) = p(f(K,L)) \times f(K,L) - w \times L - r \times K \rightarrow \max$$

by the boundaries (4)

$$p(f(K,L)) \times f(K,L) - w \times L - r \times K - (RoR - r) \times K \leq 0$$

$$K, L \in R_{\geq 0}$$

Solution of this optimization task is optimum consumption level of production factors labor L^* and capital K^*. On their basis with the help of production function the regulated optimum level of output q_R^* is quantified according to the following form

$$q_R^* = f(K^*, L^*)$$

and regulated optimum price p_R^* with the help of price-demand and production function on the basis of the relation

$$p_R^* = p(q_R^*) = p(f(K^*, L^*)$$

While respecting the rate of return on capital defined by the *RoR* parameter, i.e. exogenous parameter set by the regulator.

In the situation when the firm would not be regulated and had an exclusive position on the relevant market it would choose such optimum volume of consumption of variable inputs labor L^* and capital K^*, to ensure maximum profit. On the basis of optimum consumption of variable inputs it would set its optimum supply volume and optimum price of production $p^* = p(q^*) = p(f(K^*, L^*)$.

In the case the firm is regulated, it can only choose the combination of input factors in such a way the corresponding production volume and price of production would generate *reasonable profit* i.e. to adhere to this form

$$(RoR - r) \times K \geq p(f(K,L)) \times f(K,L) - w \times L - r \times K$$

$$(RoR - r) \times K \geq \pi(q)$$

Thus the regulated firm can produce in a manner that its *reasonable profit* does not exceeds *RoR* – *r* multiple of the variable capital input volume. This condition is called *reasonable profit margin of the regulated firm*. It is obvious that in case when the regulator sets rate of return on capital *RoR* in a way it will exceed its cost *r*, the firm reaches positive profit for any combination of inputs of production factor *K* with positive volume. This statement predicates the problems of exclusive position of production factor capital on decision making of the regulated firm.

However, this form of price regulation is hiding one serious risk. It often motivates firm to use higher volume of variable input capital than non-regulated firm.

We can show, that firm can produce regulated production volume q^*_R with any combination of variable inputs labor and capital corresponding to isoquant $q^*_R = f(L, K)$. Carlton and Perloff [2] show, that if the regulation on the basis of the rate of return stimulates firm towards using large volumes of capital in order to reach maximum profit allowed, the firm is not motivated to use often more effective and from the point of view of production price and production volume of inputs equivalent combinations of inputs localized on isoquant (e.g. supporting employment) in comparison with purposeless investments into the equipment. However, there are also other, more sophisticated examples of cost structure of the firm.

4. Conclusion

The aim of this paper was to analyze the effects of network industries price regulation from the point of achieved level of reasonable profit of the regulated entities.

On the basis of the firm's behavior in the conditions of the rate of return on capital regulation we show that in this regulatory scheme the firm has the tendency to respond to the tightening of the regulatory conditions by increasing the volume of used capital. Increase of used capital volume is, however, not the goal that the regulatory system is following. Regulation aims to influence others, for the firm and for the economy more important indicators, such as the volume of production, level of sales or the volume of costs.

Therefore, other price regulation forms were developed to influence regulated firm reasonable profit on the basis of its total costs. The aim is, with the help of the regulatory mechanisms, to encourage effective development of the regulated entity.

On the basis of the economic development analysis, we can state that the applied regulatory methods have not only the stabilizing effect in the network industries market environment but also the motivational aspect for the effective development of the regulated entities.

References

[1] Blum, U. - Müller, S. - Weiske, A. (2006). Angewandte Industrieökonomik. Theorien - Modelle – Anwendungen. Berlin: Verlag Gabler.

[2] Carlton, D.W. - Perloff, J. M. (2005). Modern Industrial Organization. Boston: Addison Wesley.

[3] Fendek, M. - Fendeková, E. (2008). Mikroekonomická analýza. IURA Edition. Bratislava,

[4] Fendek, M. - Fendeková, E. (2010). Modely cenovej regulácie sieťových odvetví. Ekonomický časopis: časopis pre ekonomickú teóriu a hospodársku politiku, spoločensko-ekonomické prognózovanie. Bratislava : Ekonomický ústav. vol. 48, no. 10, pp. 1039 – 1055.

[5] Fendek, M. - Fendeková, E. (2009). Models of regulation of network industries in Slovakia. In International journal of economics and business research. - UK : Inderscience Enterprises Ltd, vol. 1, no. 4, pp. 479-495.

[6] Fendek, M. (2008). Natural monopoly cost-oriented price regulation. In: Quantitative methods in economics: multiple criteria decision making XIV. Bratislava, IURA EDITION, pp. 45-53.

[7] Fendeková, E. (2003). Oligopoly v odvetví. Vydavateľstvo EKONÓM.

[8] Fendek, M. – Fendeková, E. (2008). Cost-oriented models of network industries price regulation. In Opera-tions research proceedings, selected papers of the Annual international conference of the German operations research society (GOR) University of Augsburg, September 3-5, 2008. - Berlin: Springer Verlag, pp. 73-78.

[9] Kufelová, I. Analýza cien a cenotvorba vo vybraných sieťových odvetviach. (2015). In: Modely rovnováhy v podmienkach produktovej a cenovej diferenciácie na regulovaných trhoch sieťových odvetví / Eleonora Fendeková a kolektív. Bratislava: EKONÓM, pp 154-169.

[10] Pepall, L. – Richards, D. - Norman , D. (2008). Industrial Organization: Contemporary Theory and Empirical Applications. New York: Wiley-Blackwell.

[11] Sherman, R. (1990).The regulation of monopoly. Cambridge university press. Cambridge.

[12] Waldman, D. E. – Jensen, E. J. (2006). Industrial Organization: Theory and Practice. New York: Addison Wesley

Uncertainty perception in evaluation of long term investments

Monika Foltyn-Zarychta [1]

Abstract

Investment evaluation under uncertainty in case of long-term, intergenerational projects is exposed to a number of biases. Evaluation process is vulnerable due to high level of uncertainty as well as ethical issues as the decision outcomes will influence unborn people.
The paper aims at identifying sources of biases and possible distortions in final evaluation outcome. The results indicate a number of sources, such as loss aversion, myopia, risk aversion and availability effect, that may adversely influence the accuracy of evaluation process via decision makers, individuals as members of general public, or expert judgements attitudes

Key words

investment evaluation, intergenerational, biases, intuitive thinking, Cost Benefit Analysis

JEL Classification: H43

1. Introduction

Long-term, intergenerational investments are difficult to be evaluated. The longevity of such projects, that generate effects over decades or even hundreds of years (i.e. climate change mitigation) is inevitably assisted with high level of uncertainty. The uncertainty in this case may affect not only investing generation, but also future, unborn people, who will benefit or be harmed by investment impacts.

Then, intergenerational investments' evaluation outcome is sensitive to be biased due to lack of precisely described outcomes with probabilities as well as ethical issues related to the scope of rights of future generations affected by uncertain decision results.

Payne et al. (1988) refers to evaluation process that need to consider cognitive and motivational biases and decision rules as well as factors taken into account in deliberative thinking. On the other hand, there are actually made choices, under process information limitations and bounded rationality which can be referred to as intuitive thinking. Viscusi & Gayer (2015) indicate that such shortcomings are present in government policies.

On the other hand, Kunreuther et al. (2014) admits that under many uncertain parameters, such decision processes may be valuable in comparison with other techniques.

The paper aims at identifying sources of biases when intuitive thinking for intergenerational investment evaluation is applied and possible distortions in final evaluation outcome.

The methods of research included critical analysis of relevant literature sources as well as logic abstraction and generalisation.

The paper starts with explaining the concept of intergenerational risk and uncertainty and ethical limitations of analysis due to impacting the wellbeing of future generations. Next, the shortcomings of long-term investment stakeholders decision-making process are identified.

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Finally, the paper presents biases and their possible impact on evaluation of intergenerational investments under uncertainty. The paper ends with conclusions.

2. Investments under uncertainty and future generations perspective

Taking into account the state knowledge about possible future states and their probabilities as well as long-term, Hartzell-Nichols (2012) introduces the term intergenerational risks, which may be understood as threats of harm to future generations or which involve more than one generation and about which we know how probable the possible outcomes are. However, it is important to highlight that the possibility of estimating probabilities of future states of affairs for intergenerational investments may be difficult. Hartzell-Nichols (2012) indicates that given the temporal scale, there almost always will be uncertainties (Hartzell-Nichols, 2012). Then, the concept of intergenerational risk may be enlarged to terms: intergenerational uncertainty with unknown probabilities and known future states and intergenerational radical uncertainty - unknown probabilities, unknown future states due to lack of information. This approach widens the standard economic definition, concentrating on variability, highlighting the concept of harm (cf. Perry, 2007) and forthcoming generations.

Longevity of intergenerational investments, such as climate change or nuclear energy projects is inevitably related to high level of uncertainty.

In case of climate change, uncertainty arise from climate and ecosystem responses to greenhouse gas (GHG) emissions and future concentrations as well as technological systems, market behaviour (aggregated behavioural changes) and public policies, which reach time spans of hundreds of years (cf. Kunreuther et al., 2014, Anthoff *et al*., 2009). Nuclear energy involves long term uncertainties emerging from decommissioning and waste disposal of more than 100 years (cf. Ilg et al., 2016, OXERA, 2002)

Additionally, i.e. Weitzman (2011) and Pindyck (2010) draws attention to the fat-tails problem: the likelihood of an extremely large reduction in wellbeing (cf. Kolstad et al., 2014). Also, IPCC report (2012) indicate that „extreme events are rare, which means there are few data available to make assessments regarding changes in their frequency or intensity".

Additionally, in case of long-term investments, issues of intergenerational justice come into picture. It is due to the fact that these projects and risks are associated with time span of more than one generation. It gives rise to questions whether those unborn people have rights in contemporary decisions and what is the extend of those rights, and how can we incorporate them in the evaluation process. Hartzell-Nichols (2012) highlights that future people cannot participate in the regulation of the risky activities, and, furthermore, risky activities may in part determine who will be affected: the decision may change who will be born leading to an outcome when not the same people will be affected with the investment, than would be affected without it. The last issue is known as non-identity problem formulated by Parfit (1987). Such decisions may also affect the number of people affected. However, this makes future people unable to accuse their ancestors of any harm, due to the fact that past decisions allow them to be conceived (cf. Taebi 2012).

3. Stakeholders in project evaluation under uncertainty and their weaknesses

The method that is probably the most theoretically appropriate for evaluation of intergenerational investments is Cost Benefit Analysis (CBA) that measures changes in welfare due to project implementation. This perspective allows to include welfare of different social groups or even generations as well as non-marketed effects, like public goods changes.

However, CBA is criticized as well. Kunreuther et al. (2014) indicate ethical and methodological critiques concerning the uncertainty about the potential impacts of climate change, possible irreversible and catastrophic effects on ecosystems, and their asymmetric distribution around the planet raising also expected negative utilities from low-probability catastrophic events often referred to as 'fat tails'.

Long term, coupled with uncertainty makes the project assessment vulnerable to prescriptive rather than descriptive parameters and assumptions. Kunreuther et al. (2014), although indicating shortcomings of prescriptive decision-making, admit that taking into account constraints on time and processing capacity of decision-makers, intuitive decisions may still be the best solution for many choices under uncertainty.

However, when the need of prescriptive judgement arises, such decisions, affecting final outcome of the analysis, may be affected by individual attitudes and risk perception, which is vulnerable to a number of biases, making public investment choice resemble individual decisions.

Some researchers give attention to the issue to what extend investment decision making may be affected by individual risk perception.

Florio (2014) indicates that in case of probability analysis in CBA, the appraisers generally relies on personal experience or consultations with other experts. The probability distribution of each critical variable may be then based on empirical data or subjective judgements of probabilities (cf. Anderson & Hardaker, 2003).Kadak (2000) argues that risks to society are judged unevenly even within one agency (i.e. protecting public health).

Kunreuther et al. (2014) turn attention to disadvantages of expert judgements which rests on opinions, not „hard facts" and the help from experts to quantify the risk cannot replace decision-maker duties. Additionally, experts often disagree. Some examples can be given when social and long-term discounting is analysed, and the plethora of parameters are used to calculate social discount rate (Kolstad et al., 2014).

Decision-makers, due to their lack of expert knowledge, are also vulnerable to make non-rational judgements. Slovic & Weber (2002) give examples of sensitivity of public officials to newly diagnosed sources of danger (overestimating existing dangers or providing too much assurance to a public, irrespectively of real likelihood and level of a threat).

Viscusi & Gayer (2015) indicate that „while such shortcomings have been documented in the behavioural literature, they are also reflected in government policies, both because policymakers are also human and because public pressures incorporate these biases. The result is that government policies often institutionalize rather than overcome behavioural anomalies." Viscusi & Gayer (2015) use the term behavioural public choice theory to refer to biases that investment evaluation may be affected by due to individual attitudes. They highlight that private bad choices leads to higher costs for individual that public bad choices made by a regulator for a citizen, and therefore even fewer errors might be expected among private than among public decision makers.

Individual also play a role in such evaluations as decision makers are impacted by their perceptions of risk and uncertainties, and actions and market decisions undertaken by others.

To summarise, in case of long-term investments, accuracy of investment evaluation and decision-making procedure may be affected by 3 groups of stakeholders:

1. Decision makers, who often have insufficient or imperfect knowledge about climate risks may use intuitive thinking system (cf. Viscusi & Gayer (2015)
2. General public (individuals) that may influence the choice of decision-makes via market behaviour or voting system (cf. Viscusi & Gayer (2015)
3. Experts, who in the situation of lack of knowledge, make some prescriptive judgement, i.e. about the assumptions of a model or some probability parameters (cf. Florio 2014).

4. Biases in managing uncertainty in intergenerational investments and their possible impact on evaluation results

When an investment faces uncertainty, a decision process may incorporate one or some mixture of systems: deliberative and intuitive (Kunreuther, 2014). The former rests on effortful and intentional abstract operations and analytic process including computations or formal logic. Deliberative system uses formal models and tools assessing long-term consequences and their likelihoods. The latter system operates quickly and automatically using emotions and simple rules based on personal past experience and some cultural and social characteristics (cf., Kahneman 2011, Thaler & Sunstein, 2008).

Kunreuther et al. (2014), indicate, in case of high uncertainty, the low-probability high-consequence situations, where decision makers have limited experience with outcomes, alternative decision rules may be applied, that do not need precise probabilities. However, even if lack of knowledge may justify using non-deliberative decision-making, a thread of biases arise.

Tversky and Kahneman (1974) list different kinds of heuristics that may bias the outcome in case of intuitive judgements (cf. Viscusi & Gayer, 2015 Fiedler & von Sydow 2015), of of which availability bias that relates to previous experience is found significant for long-term decision-making (Kunreuther et al., 2014). Additionally, some effects of bounded self-control as loss-aversion is put forward in the literature (cf. Kunreuther et al., 2014) as well as intertemporal trade-offs (cf. Loewenstein & Prelec, 1992) and certainty effects (cf. Kahneman & Tversky, 1979).

Availability bias emerges when people judge the probabilities of future events based on where they have some prior experience or how easy those events are to imagine (cf Camerer & Loewenstein, 2004). Until an event (i.e. flooding) will not occur, the decision-maker may be reluctant to adapt any protective measures (cf. Kunreuther et al., 2014).

In regard to intergenerational investments, where low-probability events may occur, those rare disasters may be overweighed. Viscusi & Gayer, 2015 argue that small risks are overestimated and large risks are underestimated. This effect may be diminished by access to additional information, however Viscusi & Gayer (2015) claim that still government officials may not use this knowledge to produce an unbiased assessment. The example of such behaviour may be nuclear energy which is highly vulnerable to country-specific experiences and public perception of risks (EC, 2014). However, Slovic & Weber (2002) indicate that under repeated choice-making, the opposite effect may be achieved.

Availability bias may also affect far-fetched impacts to future generations, as – simply due to time distance – any event may be perceived as less probable. However, even if probability of any given time period may be very small, it accumulates over time leading to substantial likelihoods when multigenerational time-frame is taken into consideration.

Loss aversion is a part of so called bounded self-control effect (Viscusi & Gayer, 2015) and may lead to various impacts on investment evaluation.

First of all, loss aversion favours status quo due to the fact that any investment involves considerable outlays, whereas „do nothing" option is often (wrongly) perceived as costless (cf. Loewenstein & Mather 1990). Kunreuther et al. (2014) indicate that this is the easy option, „favoured by emotional responses in situations of uncertainty, (*better the devil you know than the devil you don't*), by many proverbs or rules (*when in doubt, do nothing*), and observed biases in the accumulation of arguments for different choice options."

Secondly, in the light of CBA that monetizes non-market effects, losses measured via willingness to accept compensation are valued higher than willingness to pay for gains (cf. Thaler & Sunstein, 2008, Viscusi & Gayer, 2015, Garrod & Willis 1999). It is also related to

endowment effect (Camerer & Loewenstein, 2004). Status-quo is used here as a reference point to calculate possible gains or losses, which are not treated impartially, and measured on an objective scale, but in comparison to a present situation. Furthermore, there may be a discrepancy between public decision-maker and individuals in estimating the value of possible future losses and gains as government officials are not emotionally involved.

In regard to intergenerational justice this involves even further difficulties due to the fact that defining the reference point for future generations: i.e. assuming that future generations are wealthier (or poorer) than the contemporaries, may persuade decision-maker to more hastily reject some climate protective investment. Additionally, judging future gains or losses inflicted to unborn people involves an ethical judgement.

Intertemporal trade-offs in case of intergenerational investments are particularly exposed to inaccuracy. The main reason is the myopia in decision making and focus on short term. Short-sightedness makes investment decisions difficult due to the fact that any investment involves immediate outlays and delayed effects. The preference for immediate effects leads to short-term goals establishing, less affected by uncertainty.

Another possible impact on evaluation would be an issue of hyperbolic discounting, when observed intertemporal choices do not follow constant (as generally used in CBA assessments) but decreasing discount rates (Loewenstein & Prelec 1992, Cropper et al., 1994). Hyperbolic discounting can be also mixed with declining time discounting due to uncertainty (cf. Gollier & Weitzman, 2009, Weikard & Zhu X 2005), and the threat of double risk counting emerges.

Applying constant discount rate lead to undervaluation of distant future effects, whether positive or negative. It also may impact the perception of future generations, although intergenerational justice offers a number of ethical concepts that describe the way of treating unborn people, how much sacrifice is justified on behalf of future generations.

Investing in intergenerational investments may be also affected by myopia due to non-existence of future generations, that do not participate in the decision-making process, as well as ethical issues of non-identity, which may lead to underrepresentation or undervaluation of impacts to future generation.

Finally, certainty effect, following prospect theory, lead to overweighting outcomes considered certain, in comparison with risky outcomes. Kunreuther et al. (2014) refer here to comparing certain investment outlays of mitigation and adaptation to climate change with uncertain future benefits of such actions. Subjective expected utility hypothesis states that the utility is the decision maker's expected utility as a weighted average of the utilities of outcomes and the weights are taken from personal or subjective probabilities of outcomes as weights (Anderson & Hardaker, 2003).

Ignoring risk aversion in evaluation may lead to omitting willingness to pay to avoid uncertainty, and to underestimation of value of an investment effect, which would be higher under risk aversion that when calculating standard expected values. The important factor is also subjectivity of the degree of risk aversion of the decision maker, as higher risk aversion will be associated with higher risk premium. On the other hand, a risk-seeking decision-maker will have negative risk premium (convex utility function), leading to opposite outcomes.

Anderson & Hardaker (2003) highlight also that it is difficult to precisely obtain such functions as they are vulnerable to interviewer bias.

The question arises also how interpret risk aversion in intergenerational utility functions. Some models using discounting procedure, assume risk aversion changes then into intergenerational inequality aversion. However, difficulty remains how to elicit this parameter (cf. Medvecky, 2012).

A closely related is the issue of ambiguity aversion. As Kunreuther et al. (2014) point, most long term, i.e. climate change forecast, exhibit high degree of uncertainty or ambiguity and it is important to consider not only decision makers' risk attitudes, but also attitudes towards ambiguous outcomes, as most decision-makers are ambiguity averse. This aversion lead to preferring choice options with well-specified probabilities over uncertain options. Viscusi and Gayer (2015) add that *g*overnments often reflect ambiguity aversion to novel risks.

5. Conclusions

Descriptive models with objective probabilities are able to produce impartial precise result for decision-making purposes. However, in case of long-term, intergenerational investments such as climate changes or nuclear energy or biodiversity protection, many factors that impact tremendously evaluation process are uncertain. In some cases, even if there are some observed values, applying purely descriptive approach would be ethically arguable due to the fact that projects implemented by contemporaries, affect future unborn generations.

Additionally, decision-makers, as well as experts and general public, are vulnerable to limited capacity of information processing as well as risk perception and may have heterogeneous preferences over ethical attitude toward future generations. All the above parties are involved (directly or indirectly) into decision-making process and sensitive to a number of biases.

Availability bias may influence investment evaluation outcome due to overestimation of low-probability events, i.e. fat tails in climate change policies and underestimation of high-likelihood cases. However, at the same time, unknown event may be underestimated, which may refer to future generations, distant in time.

Loss aversion bias the results by overestimation of losses in comparison with some initial reference point as well as preferring status-quo and inaction policy. Unborn people may be inappropriately regarded depending on their assumed level of wellbeing in comparison with contemporary generation.

Myopia may drift the evaluation outcome to underinvestment, due to favouring immediate benefits and short-term goals preference. Additionally, the decision to apply exponential or hyperbolic discounting assumption, changes the present value of distant outcomes. Future generation welfare is therefore sensitive to short- or far-sightedness of the decision-making agent.

Risk aversion bias the results twofold: first it leads to various outcomes if the assumption is taken into consideration, secondly, the level of risk aversion is not homogenous among individuals. Additionally, the interpretation of risk aversion changes its content when the time frame turns intergenerational into inequality aversion, which reflect ethical rather than risk-based attitude.

The effects described above do not form a complete list of biases that may emerge in long-term project evaluation. Taking into account the rights of future generations or valuation of non-market effects, one may make the list of possible faults much longer.

However, biases related to uncertainty are sometimes neglected in the analysis, resting on deliberative thinking system and due to this fact, it is important to be aware of possible influences on final evaluation outcome. Additionally, when high level of uncertainty exists, coupled with the need to apply normative concepts to unborn people, intuitive processes may be more cost-and-time effective than resting on descriptive models with many uncertain assumptions and lack of knowledge.

References

[1] Anderson, J. and Hardaker, J.B. (2003). Risk Aversion in Economic Decision Making: Pragmatic Guides for Consistent Choice by Natural Resource Managers, In: *Risk and Uncertainty in Environmental and Natural Resource Economics*. Edited by J. Wesseler, H-P. Weikard, R.D. Weaver, Edward Elgar, Cheltenham, Northampton, 2003, pp. 171-187.

[2] Anthoff, D., Tol, R. S.J. and Yohe, G. W. (2009). Discounting for Climate Change. *Economics: The Open-Access, Open-Assessment E-Journal*, 3 (2009-24), pp 1—22.

[3] Camerer, C. F. and Loewenstein ,G. (2004). Behavioral Economics: Past, Present, Future, In: *Advances in Behavioural Economics*. Edited by: C. F. Camerer, G. Loewenstein, M. Rabin, Russel Sage, Princeton University Press, New York, Princeton, pp. 3-51

[4] *Climate Change Adaptation*. Edited by Field, C.B., V. Barros, T.F. Stocker, D. Qin, D.J. Dokken, K.L. Ebi, M.D. Mastrandrea, K.J. Mach, G.-K. Plattner, S.K. Allen, M. Tignor, and P.M. Midgley (eds.)]. A Special Report of Working Groups I and II of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, UK, and New York, NY, USA, pp. 3-21.

[5] Cropper, M., Aydede, S. and Portney, P. (1994). Preferences for Life-Saving Programs: How The Public Discounts Time and Age. *Journal of Risk and Uncertainty* 8, pp. 243–265.

[6] European Commision (2014). *Public risk perception and environmental policy*, Science for Environment Policy http://ec.europa.eu/science-environment-policy

[7] Fiedler, K. and von Sydow, M. (2015). Heuristics and Biases: Beyond Tversky and Kahneman's (1974) Judgment under Uncertainty. In: *Cognitive Psychology: Revisiting the Classical Studies*. Edited by: M. W. Eysenck & D. Groome. Los Angeles, London: Sage, pp. 146-161.

[8] Garrod G. and Willis, K. G. (1999). Economic Valuation of Environment: Methods and Case Studies. Edward Elgar, Cheltenham, Northampton.

[9] Gollier, C. and Weitzman, M. L. (2009). How Should the Distant Future Be Discounted when Discount Rates are Uncertain? *TSE Working Paper Series*, No 09-107.

[10] Ilg P, Gabbert, S, Weikard, H-P. (2016). Nuclear Waste Management under Approaching Disaster: A Comparison of Decommissioning Strategies for the German Repository Asse II. Risk Analysis, Jun 14. doi: 10.1111/risa.12648. [Epub ahead of print]

[11] IPCC. (2012). Summary for Policymakers. In: *Managing the Risks of Extreme Events and Disasters to Advance*

[12] Kadak, A. C. (2000). Intergenerational Risk Decision Making: A Practical Example, *Risk Analysis*, 20 (6), pp. *883-894*

[13] Kahneman, D. (2011). *Thinking, Fast and Slow*. Macmillan.

[14] Kahneman, D. and A. Tversky (1979). Prospect theory: An analysis of decision under risk. *Econometrica* 47, pp. 263 – 291.

[15] Kolstad, C., K. Urama, J. Broome, A. Bruvoll, M. Carino Olvera, D. Fullerton, C. Gollier, W. M. Hanemann, R. Hassan, F. Jotzo, M. R. Khan, L. Meyer, and L. Mundaca, 2014: Social, Economic and Ethical Concepts and Methods. In: *Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*. Edited by: Edenhofer, O., R. Pichs-Madruga, Y. Sokona, E. Farahani, S. Kadner, K. Seyboth, A. Adler, I. Baum, S. Brunner, P. Eickemeier, B. Kriemann, J. Savolainen, S. Schlömer, C. von Stechow, T.

Zwickel and J.C. Minx. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

[16] Kunreuther H., S. Gupta, V. Bosetti, R. Cooke, V. Dutt, M. Ha-Duong, H. Held, J. Llanes-Regueiro, A. Patt, E. Shittu, and E. Weber. (2014). Integrated Risk and Uncertainty Assessment of Climate Change Response Policies. In: *Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*. Edited by: Edenhofer, O., R. Pichs-Madruga, Y. Sokona, E. Farahani, S. Kadner, K. Seyboth, A. Adler, I. Baum, S. Brunner, P. Eickemeier, B. Kriemann, J. Savolainen, S. Schlömer, C. von Stechow, T. Zwickel and J.C. Minx. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

[17] Loewenstein, G. and Mather, J. (1990). Dynamic Processes in Risk Perception. *Journal of Risk and Uncertainty*, 3, pp. 155-175.

[18] Loewenstein, G. and Prelec D. (1992). Anomalies in Intertemporal Choice: Evidence and an Interpretation, In: *Choice over Time*. Edited by: G. Loewenstein and J. Elster, Russel Sage Foundation, New York, pp. 119-145.

[19] OXERA (2002). *A Social Time Preference Rate For Use In Long-Term Discounting*. The Office Of The Deputy Prime Minister, Department For Transport, And Department Of The Environment, Food And Rural Affairs. http://www.oxera.com/Latest-Thinking/Publications/Reports/2002/A-social-time-preference-for-use-in-long-term-disc.aspx.

[20] Parfit, D. (1987). *Reasons and Persons*, Oxford: Clarendon Press

[21] Payne, J. W., Bettman, J. R., & Johnson, E. L. (1988). Adaptive strategy selection in decision making. *Journal of Experimental. Psychology: Learning, Memory, and Cognition*, 14, pp. 534–552.

[22] Perry, S. (2007). Risk, harm, interests, and rights. In: *Risk: philosophical perspectives*. Edited by Lewens T., Routledge, London, pp 190–209

[23] Pindyck, R. S. (2010). Fat Tails, Thin Tails, and Climate Change Policy. *NBER Working Paper* No 16353, http://www.nber.org/papers/w16353

[24] Slovic, P. and Weber, E. U. (2002). Perception of Risk Posed by Extreme Events. *Center for Decision Sciences (CDS) Working Paper*, Columbia University. http://www.rff.org/files/sharepoint/Documents/Events/Workshops%20and%20Conferences/Climate%20Change%20and%20Extreme%20Events/slovic%20extreme%20events%20final%20geneva.pdf.

[25] Taebi B. (2012). Intergenerational Risks of Nuclear Energy, in: *Handbook of Risk Theory Epistemology, Decision Theory, Ethics, and Social Implications of Risk*. Edited by S. Roeser, R. Hillerbrand, P. Sandin, M. Peterson, Springer, Dordrecht, Heidelberg, London, New York, p. 295-318

[26] Thaler, R H. and Sunstein, C R. (2008). *Nudge, Improving Decisions About Health, Wealth, and Happiness*. Yale University Press, New Haven & London.

[27] Tversky, A. and Kahneman, D. (1974). Judgment under Uncertainty: Heuristics and Biases. *Science*, New Series, 185 (4157), pp. 1124-1131.

[28] Viscusi, W. K. and Gayer T. (2015). Behavioral Public Choice: The Behavioral Paradox of Government Policy. *Mercatus Working Paper*, Mercatus Center at George Mason University, Arlington, VA. *http://mercatus.org/publication/behavioral-public-choice-behavioral-paradox-government-policy*.

[29] Weikard H-P. and Zhu X. (2005). Discounting and Environmental Quality: When should dual rates be used? *Economic Modelling*, 22, pp. 868– 878

[30] Weitzman, M. L. (2011). Fat-tailed uncertainty in the economics of catastrophic climate change. *Review of Environmental Economics and Policy*, 5, pp. 275 – 292

Comparison of TOPSIS, COPRAS, ARAS and WASPAS methods in performance evaluation using crisp and interval data

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Abstract

Performance evaluation can have a significant impact on the functioning and development of a company. It is based on a set of possible measures is an important and difficult task for decision makers. This paper considers the performance evaluation constructed from primarily financial measures. In the proposed evaluation scheme the performances are expressed using crisp and interval values available. Criteria weights are calculated using objective approaches based on entropy and CRITIC methods. The goal of this paper is to compare four different MADM methods using crisp and interval data. These methods include TOPSIS, COPRAS, ARAS and WASPAS. Mathematical comparison with test data is used to determine the most appropriate MADM method for this task. The results of this paper will show the differences and applicability of these methods in practice.

Key words

Multiple criteria decision making, objective weighting, TOPSIS, COPRAS-G, ARAS-G, WASPAS.

JEL Classification: C44, G30

1. Introduction

Number of scholars have significantly advanced multiple criteria decision making (MCDM) methods and presented new methods that can be used for specific needs of managerial decision making processes. This growing body of knowledge and methodology also significantly influenced selection of these methods for certain problems. Scholars as well as practitioners should decide which approach is the best for the task at hand. They must investigate the decision-making problem from several perspectives to select the appropriate MCDM method. This creates further difficulty which can go against the need for straight forward decision making process. In this case, there is a desire for knowledge about MCDM selection. This paper deals with a comparison of lately developed methods for selection of best alternative from larger number of choices using quantitative data. This represents frequent problem in ranking and performance appraisal. If the problem is characterized by many alternatives that can be compared using relevant criteria with quantitative information, then the decision maker (DM) can apply several different MCDM methods that can objectively help to rank and select those alternatives. In this paper chosen MCDM methods for such a case are compared and discussed to identify important characteristics of these methods for their future application by other scholars and practitioners alike. The goal of this paper is to apply methods of TOPSIS, ARAS, COPRAS and WASPAS as well as their versions using interval data (grey numbers) TOPSIS-G, ARAS-G and COPRAS-G. The paper is structured into four parts including short summary of references to applied methods and

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their use, application on a real problem, comparison of results and methods and discussion about which MCDM method is more appropriate for different problems.

2. Literature review and summary of applied methods

This section deals with a summary of applied methods. Each method will be described using same characteristics that will be also used for their comparison. Detailed methodology can be found in referenced relevant sources from their respective authors. Presented MCDM methods below are ordered chronologically. In this respective order, they will be applied on a chosen problem.

2.1 Common characteristics of applied methods

All applied methods that are described in this paper share some common characteristics that are related to the practical example that will be used as a mean for their comparison. Following methods will be applied in this paper:

- CRITIC (Criteria Importance Through Intercriteria Correlation) will be used for objective weighting, introduced by Diakoulaki et al. (1995);
- TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) developed by Hwang and Yoon (1981) and TOPSIS Grey as described by Lin et al. (2008);
- COPRAS (Complex Proportional Assessment) developed by Zavadskas et. al (1994) and COPRAS-G (grey) introduced by (Zavadskas et al., 2008);
- ARAS (Additive Ratio Assessment) developed by Zavadskas and Turskis (2010) and ARAS-G (grey) developed by Turskis and Zavadskas (2010);
- WASPAS (Weighted Aggregated Sum Product Assessment) developed by Zavadskas (2012).

The CRITIC method will be only used for estimation of weights. Other methods were used on a same data set using crisp and interval (grey) numbers.

All applied methods share common characteristics which also the main concern when making choice which method to use for certain case. For practitioners, this can bias their choice and appropriate application. The main common characteristic is the nature of the criteria. These methods have been developed for decision making problems involving large number of criteria and alternatives with quantitative data. This approach is different from methods involving more human input (pairwise comparison, allocation of points etc.). However, it is possible to merge the subjective and objective perspectives using subjective weighting. Resulting weights are then used in the final comparison of alternatives. On the other hand, quantitative nature of such decision-making problems enables to use objective weighting which can significantly shorten and computerise the whole decision making process. In this paper CRITIC objective is used for objective weighting to ensure comparability of results for further use. Main differences can then found in criterion function. Another important common characteristic shared by described methods is the mean of normalization which uses primarily max-min type. Also, it is needed to transform minimizing criteria (cost type) to maximizing (benefit type) before application and normalization.

2.2 Description of applied MCDM methods

This section deals with summary of applied methods with references to their original and detailed description in particular sources. Focus will be on criteria function and means of calculation of final scores by which alternatives are ordered. In the end a comparison table is present to summarize methods' characteristics.

Selected methods using grey numbers are characterized and used for comparison. Grey system theory is an approach for analysis and modelling of systems with discrete data and incomplete information, and which may exhibit random uncertainty (Deng, 1989). The major advantage is that it can generate satisfactory outcomes using a relatively small amount of data or with great variability in factors.

Let x denote a closed and bounded set of real numbers. A grey number, $\otimes x$, is defined as an interval with known upper and lower bounds but unknown distribution information for x (Deng, 1989). That is, $\otimes x = [\underline{x}, \overline{x}] = [x' \in x \mid \underline{x} \le x' \le \overline{x}]$ where $\underline{x}$ and $\overline{x}$ are the lower and upper bounds of $\otimes x$, respectively. The importance of each criterion is defined by the grey scale variable vector $\otimes d^k = [\underline{d}^k, \overline{d}^k]$

2.2.1 CRITIC method of weight estimation

The CRITIC (CRiteria Importance Through Intercriteria Correlation) method is a case of statistical estimation of objective weights. This method has been introduced by Diakoulaki et al. (1995). The criteria for weight estimation is based on Pearson's correlation coefficient. Criteria that have lower dependence are given higher importance. The transformation is based on a concept of the ideal point. Each vector x_i is characterized by the standard deviation σ_i, that quantifies the difference in the intensity of a criterion. The standard deviation is a measure of importance of the criterion x_i in each matrix. In this phase, another statistical measure of divergence can be used (entropy or variation). Then a symmetrical matrix $m \times m$ with elements r_{ij} (m = i,j) is created. These elements are characterized by Pearson's correlation coefficient between vectors x_i and x_j. The more are the scores divergent the more is the value of r_{ij} smaller. The information that eventuates from the matrix of alternatives and criteria encapsulates two dimensions: intensity and dependency. Coefficient C_j, then represents an aggregate measure of those dimensions as follows:

$$C_j = \sigma_j \cdot \sum_{k=1}^{m} \left(1 - r_{ij}\right). \tag{1}$$

The higher the Cj the more information is carried by that criterion and thus his relative importance is higher too.

2.2.2 Summary of applied methods for alternative selection

In this section selected MCDM methods are briefly described to have a better understanding of their mathematical properties. This helps the decision maker to choose the right method for different decision problems. This paper also provides a comparison and discussion about the results and their explanation. Following table 1 contains description of normalization, calculation of the utility function and overall score of TOPSIS, COPRAS, ARAS and WASPAS methods. Methods of TOPSIS Grey, CORPAS-G and ARAS-G are described in following sections including some background and references.

In the table 1 all equations are based on their original forms using same notation. It should be mentioned that different researchers use different letters. In this case a particularly generic notation has been used: x_{ij} for values of original decision matrix; r_{ij} for normalized values of the decision matrix; w_j for criteria weights; v_{ij} for weighted normalized values; $S_{(+/-)i}$ for ideal and base values of alternatives; Q_i for values of alternatives' scores. Some other variables, that are methods specific, are described in methodological characteristics.

Table 1: Summary of applied MCDM methods with crisp data

	TOPSIS (Hwang and Yoon, (1981)	COPRAS (Zavadskas and Kaklauskas, 1994)	ARAS (Zavadskas and Turskis, 2010)	WASPAS (Zavadskas et al., 2012)
Normalization of the decision matrix	$r_{ij} = \frac{x_{ij}}{\left(\sum_{j=1}^{p}\left(x_{ij}\right)^2\right)^{1/2}}$	$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$	$r_{ij} = \begin{cases} \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}}; \\ \frac{1/x_{ij}}{\sum_{i=0}^{m} 1/x_{ij}}; \end{cases}$	$r_{kj} = \begin{cases} \frac{x_{ij}}{\max_i x_{ij}}; \\ \frac{\min_i x_{ij}}{x_{ij}}; \end{cases}$
Calculation of the utility function	$S_i^+ = \left(\sum_{j=1}^{k}\left(v_{ij} - H_j\right)^2\right)^{1/2}$ $S_i^- = \left(\sum_{j=1}^{k}\left(v_{ij} - D_j\right)^2\right)^{1/2}$ $i = 1, 2, ..., m.$	$S_{+i} = \sum_{j=1}^{k} r_{ij} \cdot w_j$ $S_{-i} = \sum_{j=k+1}^{n} r_{ij} \cdot w_j$	$v_{ij} = w_j \cdot r_{ij};\ i = 1,2,\ldots,m$ $S_i = \sum_{j=1}^{n} v_{ij};\ i = 0,1,\ldots m$	$Q_i^{(1)} = \sum_{j=1}^{n} r_{ij} w_j$ $Q_i^{(2)} = \prod_{j=1}^{n} (r_{ij})^{w_j}$
Calculation of overall score	$Q_i = \frac{S_i^-}{S_i^+ + S_i^-},\ i = 1, 2, ..., m,$	$Q_i = S_{+i} + \frac{\sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{1}{S_{-i}}}$	$Q_i = \frac{S_i}{S_0};\ i = 1,2,\ldots,m$	$Q_i = \lambda Q_i^{(1)} + (1-\lambda) Q_i^{(2)}$

Source: own elaboration based on referenced publications.

In the table 1 some differences can be see among selected methods. Their common characteristic is that they are used to rank relatively large number of alternatives using a comparison to an ideal alternative based on different means of normalization and overall score calculation. Firstly, when comparing normalization methods, it can be seen, that in TOPSIS generally vector normalization is used, however it is possible to use linear normalization as well (max-min, max or summation) (Salabun, 2013). In this case based on related references the vector normalization is mostly used in TOPSIS calculation. COPRAS and ARAS methods use linear normalization (summation) as well as WASPAS method (max type). The summation type of normalization has a more smoothing effect. Looking at some details of compared methods, it can be seen that up until point of normalization these methods use the same approach. The main difference is in calculation of the distance and criterion function. In TOPSIS the Euclid distance from ideal solution is calculated. This means a that there is a virtual ideal alternative that is used to compare the others. In COPRAS the comparison is made by summation of distances of alternatives according to the normalized values. In this case each alternative is compared with other in all criteria and the final score is normalized towards the best alternative. In ARAS the main difference to COPRAS is that the calculation is simple and the final result is normalized using summation normalization. The difference in order of alternatives is insignificant. The WASPAS method uses a combination of criteria function calculation. Weighted scores of alternatives are calculated using WSM and WPM. Decision-maker has choice to apply different preference towards WSM or WPM when the default is 50:50. This difference to COPRAS and ARAS has an impact on the results and there is significant difference in order of alternatives. Next sections will be focused on grey numbers applications.

2.2.3 TOPSIS and TOPSIS Grey

TOPSIS method is one of the methods which uses the calculation of distance from the ideal variant for criteria's evaluation. For the calculation, it is necessary that all criteria are maximization type, therefore all criteria are modified based on relation $x_{ij} = -x_{ij}$. The

calculation procedure can be summarized into following steps. The columns in this matrix are after transformation created by vectors of unitary length based on Euclid's metrics. Consequently, it is necessary to find weighted criteria matrix W so that every j^{th} column of normalized criteria matrix R is multiplied by relevant weight v_j. Next step is to determine ideal variants $H = (H_1, H_2, ..., H_k)$ and basal variant $D = (D_1, D_2, ..., D_k)$ regarding to values in weighted criteria matrix. Then it is followed with calculating the distance of variant from the ideal variant and distance of individual variants from basal variant. In both cases, Euclid's range of distance is used. Second to last step is to find the relative indicator of the variant's distance from the basal variant.

$$Q_i = \frac{S_i^-}{S_i^+ + S_i^-}, \; i = 1, 2, ..., m, \tag{5}$$

by ranking the variants based on descended values c_i complete ranking of all variants is obtained and there is the possibility to determine the best variant.

Given that grey valuations already have been normalized then there is no need to normalize the decision matrix. When all factors are beneficial measure (the bigger the value the better), and the ideal and base solutions and be determined. First, the most 'ideal' reference solution $S^+(\otimes\tilde{v})$ is determined by selecting the maximum value from amongst each of the factors using equation (6):

$$S^+(\otimes\tilde{v}) = \{ \max(\otimes\tilde{v}_{i1}), \quad \max(\otimes\tilde{v}_{i2}), \quad \ldots, \quad \max(\otimes\tilde{v}_{im}) \} \tag{6}$$

Second, the 'base' reference solution $S^-(\otimes\tilde{v})$ is determined by selecting the minimum value from amongst each of the factors using equation (7):

$$S^-(\otimes\tilde{v}) = \{ \min(\otimes\tilde{v}_{i1}), \quad \min(\otimes\tilde{v}_{i2}), \quad \ldots, \quad \min(\otimes\tilde{v}_{im}) \}. \tag{7}$$

2.2.4 COPRAS and COPRAS-G

The COPRAS (COmplex PRoportional ASsessment) method is presented by Zavadskas et al. (1994). Description of COPRAS methods and possibilities of its application are published in several papers (Zavadskas et al., 2004; Zavadskas et al., 2008).

Ranking alternatives by the COPRAS method assumes direct and proportional dependence of significance and priority of investigated alternatives on a system of criteria (Zavadskas et al. 1994). The determination of significance and priority of alternatives, by using COPRAS method, can be expressed concisely using four stages (Zavadskas et al. 1994; Zavadskas et al. 2008). However, many decisions are made in real-world situations where criterion values are not precisely known. Then criterion values can be expressed in the form of intervals (Zavadskas et al., 2004; Zavadskas et al., 2008). For this reason, a new method of multiple-criteria complex proportional assessment with values determined in intervals the COPRAS-G was developed (Zavadskas et al., 2008). Instead of using crisp values, COPRAS-G uses criterion values determined in intervals. Replacement of the crisp x_{ij} with the interval value $[\underline{x}_{ij}, \overline{x}_{ij}]$, where $\underline{x}_{ij}$ is the lower limit and $\overline{x}_{ij}$ is the upper limit of interval, requires some modifications in the ranking procedure. The detailed procedure for ranking alternatives using COPRAS or COPRAS-G method is shown in (Zavadskas et al., 2009).

2.2.5 ARAS and ARAS-G

The ARAS method (Zavadskas and Turskis, 2010) is based on the argument that phenomena of complicated world could to be understood by using simple relative

comparisons. It is argued that the ratio of the sum of normalized and weighted values of criteria, which describe alternative under consideration, to the sum of the values of normalized and weighted criteria, which describes the optimal alternative, is degree of optimality, which is reached by the alternative under comparison. In the ARAS method, a utility function value determining the complex relative efficiency of a reasonable alternative is directly proportional to the relative effect of values and weights of the main criteria After creating a decision matrix, the next step in the ARAS method is to determine the optimal performance rating for each criterion.

2.2.6 WASPAS

When facing MCDM selection problems, DMs evaluate alternatives using different measurable criteria. Some of these criteria are cost-oriented (non-beneficial) whereas some are beneficial (Stanujkic et al., 2012; Behzadian et al., 2012; Yazdani, 2015). WASPAS is one of the most recently developed methods in MCDM. The WASPAS algorithm is an optimal search algorithm based on two optimization procedures known as the weighted sum model (WSM) and the weighted product model (WPM). The following matrix is used to start the solution procedure:

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{t1} & x_{t2} & \dots & x_{tn} \end{bmatrix} \tag{8}$$

where t is the number of alternative solutions, n is the number of evaluation criteria and, for $i = 1, \dots, t$ and $j = 1, \dots, n$, x_{ij} is the performance rating of the i-th alternative upon the j-th decision criterion. The elements of this matrix are normalized using the following equation:

$$r_{ij} = \begin{cases} \dfrac{x_{ij}}{\max\limits_i x_{ij}}, & \text{for a beneficial criterion} \\ \dfrac{\min\limits_i x_{ij}}{x_{ij}}, & \text{for a non-beneficial criterion} \end{cases} \tag{9}$$

Based on WSM, the total relative importance of the i-th alternative is calculated as follows (Triantaphyllou et al., 1989):

$$Q_i^{(1)} = \sum_{j=1}^{n} r_{ij} w_j \,. \tag{10}$$

At the same time, according to WPM, the total relative importance of the i-th alternative must be computed using the following expression:

$$Q_i^{(2)} = \prod_{j=1}^{n} (r_{ij})^{w_j} \,. \tag{11}$$

Thus, a joint generalized criterion of weighted aggregation is defined as follows:

$$Q_i = 0.5 Q_i^{(1)} + 0.5 Q_i^{(2)} = 0.5 \sum_{j=1}^{n} r_{ij} w_j + 0.5 \prod_{j=1}^{n} (r_{ij})^{w_j} \,. \tag{12}$$

To increase the ranking accuracy and effectiveness of the decision-making process a more generalized equation is employed to determine the total relative importance of the i-th alternative (Zavadskas et al., 2012):

$$Q_i = \lambda Q_i^{(1)} + (1-\lambda) Q_i^{(2)} = \lambda \sum_{j=1}^{n} r_{ij} w_j + (1-\lambda) \prod_{j=1}^{n} (r_{ij})^{w_j} \,. \tag{13}$$

Finally, the alternatives are ranked based on their Q_i values, with the best alternative being the one corresponding to the highest Q_i value. Note that when $\lambda = 0$, WASPAS coincides with WPM, while for $\lambda = 1$, WASPAS reduces to WSM.

3. Performance evaluation example

Presented performance evaluation was done using monthly data for manufacturing plant. Data for interval measures were derived from the actual percentages that particular performance goal was met. This value is considered as a minimum value. The value of the goal for the particular measure in a monthly evaluation was used as a maximum value. Therefore, based on this set of data ranking of monthly performance can be determined.

To compare results from the TOPSIS-Grey, COPRAS-G and ARAS-G model of MADM approach data from the previous research (Franek and Kashi, 2014) were used. The performance evaluation was done using CRITIC based criteria weights.

The performance evaluation was done using 20 measures from 5 different areas assessment in safety, quality, delivery, cost and human resources. The dataset of alternatives (monthly evaluations) consists of 12 items. The CRITIC weights were calculated using approach from described in the section 2.2.1 and equation (1). Weights are shown in the table 2.

Table 2: Objective weight estimated using CRITIC method

Criteria	Weights
Rec Incidents	0,00%
Non-Rec Incidents	5,90%
Audit	7,64%
PPM	0,00%
Complaints	7,18%
Quality Cost	6,75%
First Time Yield	0,00%
Throughput	8,59%
Ontime	5,14%
Invertory	6,84%
Freight Premium	5,63%
Productivity	5,26%
Lean Manufacturing	5,03%
Kaizen	0,00%
Overtime	5,85%
Equipment	7,69%
Training	7,57%
Absence	6,94%
Empl. Turnover	0,00%
Improvement Programme	7,99%

Source: own calculation.

Objective weights form the Table 2 were used for calculation of monthly scores based on evaluation of performance measures. In the following Table 3 final scores for TOPSIS, COPRAS, ARAS and WASPAS methods are compared. In the Table 3 results of monthly evaluation using percentages of performance goals that should be met with criteria weights calculated using CRITIC method. This means that only measured values were used for this evaluation. Calculations were proceeded using equations from the Table 1 and following equations (2) to (13) and detailed procedures from referenced sources. This calculation can be done autonomously using this as a sub-program for decision support system within a company's information system. In Table 3 results of ranking from 1 to 12 are determined.

Tab. 3 Performance evaluation with ranking using TOPSIS, COPRAS, ARAS and WASPAS methods with CRITIC based criteria weights

	TOPSIS		COPRAS		ARAS		WASPAS	
Months	Qi	Rank	Qi	Rank	Qi	Rank	Qi	Rank
January	0,885	2	0,976	2	0,972	2	0,991	3
February	0,615	8	0,887	6	0,884	6	0,591	9
March	0,772	4	0,944	4	0,941	4	0,590	10
April	0,589	9	0,831	10	0,830	10	0,651	6
May	0,985	1	1,000	1	0,996	1	1,000	1
June	0,798	3	0,962	3	0,959	3	0,998	2
July	0,641	7	0,880	8	0,879	8	0,728	5
August	0,586	10	0,848	9	0,845	9	0,620	8
September	0,684	5	0,907	5	0,904	5	0,634	7
October	0,672	6	0,884	7	0,881	7	0,900	4
November	0,515	11	0,757	11	0,758	11	0,479	11
December	0,448	12	0,694	12	0,695	12	0,436	12

Source: own calculation.

In the Table 3 we can see that results for ranking of alternatives are the same for COPRAS and ARAS methods. Differences can be seen when comparing TOPSIS and WASPAS methods. In this example we can suggest that WASPAS has a less differentiating characteristics than TOPSIS, however given the difference in criterion function the distribution of scores is more in favor of tail values rather than mean values. When comparing results for interval data using same CRITIC objective weights there is another difference. This is shown in the Table 4.

Tab. 4 Performance evaluation with ranking using interval data in TOPSIS Grey, COPRAS-G, and ARAS-G methods with objective criteria weights based on CRITIC method

	TOPSIS-G		COPRAS-G		ARAS-G	
Months	Qi	Rank	Qi	Rank	Qi	Rank
January	0,914	2	0,988	2	0,967	2
February	0,729	8	0,946	7	0,879	6
March	0,825	4	0,974	4	0,935	4
April	0,705	9	0,923	10	0,825	10
May	0,987	1	1,000	1	0,990	1
June	0,860	3	0,982	3	0,953	3
July	0,738	7	0,946	6	0,873	8
August	0,700	10	0,925	9	0,840	9
September	0,778	5	0,955	5	0,898	5
October	0,746	6	0,945	8	0,876	7
November	0,635	11	0,888	11	0,753	11
December	0,601	12	0,858	12	0,691	12

Source: own calculation.

In the table Tab. 3 we can see that the main differences among selected methods TOPSIS, COPRAS, ARAS and WASPAS are in January, March, May, August, September and October. The change in the ranking is significant, however the Qi values of these monthly performance is close so the difference is marginal. If we consider interval data for methods

TOPSIS Grey, COPRAS-G and ARAS-G the situation is quite different when considering ranking from 4th to 10th position, however the first three positions remain the same. The interval methods give us results where it does prefer the performance of the plant in January over the performance in May. If we look in the hard data, we can see that the average for goal achievement is better in May. Based on correlation of all rankings with the average percentage of goal achievement the highest overall correlation with the traditional average approach has the COPRAS-G with CRITIC weights (96%) followed by ARAS-G (95%) and TOPSIS (87%). The WASPAS approach produced lower correlation but still quite high (85%).

4. Conclusion

Some scholars can oppose the results of this paper because the comparison was made between quite different methods and datasets (crisp and interval values). On the other hand, the purpose of this paper was to compare the approaches and try to interpret and discus its results. Based on the results the CRITIC objective weighting the resulting performance ranking was similar to the one used by the management of the company which was based on arithmetic average. When using interval values (TOPSIS Grey, COPRAS-G and ARAS-G) and CRITIC based weighting the ranking become even more similar to the average one. This shows that the interval approach (grey numbers) is very close to the results that top managers used to have but with the significant difference and that is the robustness of the approach and its flexibility. It means that changes in monthly targets or priorities of the management can be easily incorporated and produce results that have better explanatory value.

Results of this study suggest more research should be done in interval values in MADM methods and their applicability in selecting performance targets for performance evaluation. Secondly this paper uses theoretical decision making approach that helps to identify and distinguish two decision making perspectives in two levels. In the first level the criteria weighting can be individual or group and subjective or objective (based on hard data). And in the second level the selection of alternatives it can be subjective or objective criteria weights and qualitative or quantitative prioritization of alternatives.

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References

[1] Behzadian, M., Otaghsara, S. K., Yazdani, M. and Ignatius, J. (2012). A state-of the-art survey of TOPSIS applications. *Expert Systems with Applications*. 39(17), pp. 13051-13069.

[2] Deng, J. (1989). Introduction to Grey System Theory. *The Journal of Grey System*. 1(1), pp. 1-24.

[3] Diakoulaki, D., Mavrotas, G. and Papayannakis, L. (1995). Determining Objective Weights in Multiple Criteria Problems: The CRITIC Method. *Computers & Operations Research.* 22(7), pp. 763-770.

[4] Franek, J. and Kashi, K. (2014). MADM Methods: Applying Practical Decision Makong in Business Processes. In Brown, A. and M. Rich, eds. *Conference Proceedings 13th European Conference on Research Methodology for Business and Management.* Eds.: Reading: ACPI. pp. 416-424. ISBN: 978-1-909507-57-9.

[5] Lin, Y-H., Lee, P-C., Chang, T-P. and Ting, H-I. (2008). Multi-attribute group decision making model under the condition of uncertain information. *Automation in Construction.* 17(1), pp. 792-797.

[6] Sałabun, W. (2013). Normalization of attribute values in TOPSIS method. *Nowe Trendy w Naukach Inżynieryjnych 4*. Krakow: Creativetime. pp.180-186. ISBN 978-83-63058-30-2

[7] Stanujkic, D., Magdalinovic, N., Jovanovic, R. and Stojanovic, S. (2012). An objective multi-criteria approach to optimization using MOORA method and interval grey numbers. *Technological and Economic Development of Economy*. 18(2), pp. 331-363.

[8] Turskis, Z. and Zavadskas, E. K. (2010). A novel method for Multiple Criteria Analysis: Grey Additive Ratio Assessment (ARAS-G) method. *Informatica*, 21(4), pp. 597-610.

[9] Tzeng, G-H. and Huang, J.J. (2001). *Multiple attribute decision making: methods and applications.* Boca Raton: CRC Press.

[10] Yazdani, M., and Payam, A.F. (2015). A comparative study on material selection of microelectromechanical systems electrostatic actuators using Ashby, VIKOR and TOPSIS. *Materials & Design*. 65, pp. 328-334.

[11] Zavadskas, E. K. and Turskis, Z. (2010). A new additive ratio assessment (ARAS) method in multicriteria decision-making. *Technological and Economic Development of Economy.* 16(2), pp. 159-172.

[12] Zavadskas, E. K., Kaklauskas, A. and Sarka, V. (1994). The new method of multi-criteria complex proportional assessment of projects. *Technological and Economic Development of Economy.* 1(3), pp. 131-139.

[13] Zavadskas, E.K., Kaklauskas, A., Turskis, Z., and Tamosaitiene, J. (2008). Selection of the effective dwelling house walls applying attributes determined in intervals. *Journal of Civil Engineering and Management*, 14(2): 85-93.

[14] Zavadskas, E.K., Kaklauskas, A., Turskis, Z., and Tamosaitiene, J. (2009). Multi-attribute decision making model by applying grey numbers. *Informatica*. 20(2), pp. 305-320.

[15] Zavadskas, E.K., Turskis, Z., Antucheviciene, J. and Zakarevicius, A. (2012). Optimization of weighted aggregated sum product assessment. *Elektronika ir Elektrotechnika – Electronics and Electrical Engineering.* 122(6), pp. 3–6.

Fitting standard claims by Generalized Linear Models

Ján Gogola [1]

Abstract

This article deals with the issue of creating homogenous tariff classes of non-life insurance and modelling the cost of standard claims of each tariff class. We use a Generalized Linear Models (GLM) for the purpose of finding significant risk factors and also to determine the cost of standard claims of the individual tariff classes. The theoretical part will be completed by application on a typical heterogeneous portfolio of the Motor Third Party Liability (MTPL). All calculations were performed using the R software. The result of the GLM is a multiplicative model where the cost of standard claim of a particular tariff class is given as a product of the cost of standard claim of the reference class and the relativities of the tariff class.

Key words

Generalized linear models, Gamma distribution, R language, likelihood function, risk classification

JEL Classification: C10, C38, G22

1. Introduction

In a homogeneous portfolio, where all policyholders have the same risk level, there is no reason to let the amount of premium vary. In practice, most portfolios are heterogeneous. They mix individuals with different risk levels.

There is a need of risk classification (see Denuit and Charpentier (2005)). Nowadays, it has become extremely difficult for insurance companies to maintain cross subsidies between different risks categories in a competitive setting. Therefore, actuaries have to design a tariff structure that will fairly distribute the burden of claims among policyholders. We apply *Generalized Linear Models* (GLM) to achieve risk classification (see Cox (1972)).

Ratemaking (or risk classification) is essentially about classifying policies according to their risk characteristics. The classification variables are called *a priori variables*, as their values can be determined before the policyholder starts to be covered by the insurance company. In the Motor Third Party Liability (MTPL) insurance, they include the age, gender and occupation of the policyholder, the type and use of their car, etc. These observable characteristics are typically seen as non-random covariates. Even with all the covariates included in price lists, substantial risk differentials remain amongst individual drivers (due to hidden characteristics like temper and skill, aggressiveness behind the wheel, knowledge of the highway, etc.).

The pure premium is the amount the insurance company should charge in order to be able to indemnify all the claims, without loss nor profit. The computation of the pure premium by Jee (1989) relies on a statistical model incorporating all the available information about the risk. The ratemaking aims to evaluate as possible the pure premium for each policyholder.

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Usually, the total claim *S*, generated by a policy of the portfolio, is not the modelling target. Instead, the different components of *S* are modelled separately, such as: frequency claims, standard claims costs, cost of large claims, etc. This allows for a better understanding of the price list, as the risk factors influencing each component of *S* are isolated. The total claim amount S_i generated by policyholder *i* can generally be decomposed as:

$$S_i = \sum_{k=1}^{N_i} C_{ik} + J_i \cdot L_i, \tag{1}$$

where N_i is the number of standard claims filed by policyholder *i*,

C_{ik} is the cost of the *k*-th standard claim filed by policyholder *i*,

J_i indicates whether the policy *i* produced a large claim (at least),

L_i is the cost of this large claims, if any.

If insurance data is subdivided into risk classes determined by many a priori variables, actuaries work with figures which are small in exposure and claim numbers (it is even possible that no observations are available for a particular combination of the rating factors). Hence, simple average will be suspect and a regression model is required.

2. Generalized linear models (GLM)

Generalized Linear Models (GLM), Nelder and McCullagh (1989), are ideally suited to the analysis of non-normal data which insurance analysts typically encounter. The GLM are used to assess and quantify the relationship between a *response variable* (or dependent variable) and a set of possible *explanatory variables* (or independent variables).

GLM is important in insurance applications as:

- the assumption of normality is often not applicable, for example claim counts, claim sizes or claim occurrences on a single policy do not obey the Gaussian distribution;
- the relationship between outcomes and explanatory variables is often multiplicative rather than additive.

With the GLM, the variability in one variable is explained by the changes in one or more other variables. The variable being explained (claim count, claim cost, etc.) is called the *response variable*. The variables that are doing the explaining are the *explanatory variables*, also called *risk factors* or *risk characteristics* in insurance.

GLM describes the connection between the response and the explanatory variables. The explanatory variables may be, and often are, related.

A question arises which explanatory variables are predictive of the response, and what is the appropriate scale for their inclusion in the model?

2.1 Severity model – Gamma regression for standard claims

Our explanatory variables are assumed to be categorical. A categorical variable with κ levels separates the portfolio into κ classes. It can be coded via *k*-1 binary variables being all zero for the reference level. The *linear predictor* (or *score*) for each class is given by the linear combination:

$$\beta_0 + \beta_1 \cdot x_{i1} + \beta_2 \cdot x_{i2} + \cdots + \beta_p \cdot x_{ip} = \boldsymbol{\beta}^{\boldsymbol{T}} \cdot \boldsymbol{X}_{\boldsymbol{i}}, \tag{2}$$

where $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \cdots, \beta_p)^T$ is a vector containing the parameters, β_0 called intercept,

$\boldsymbol{X_i} = (1, x_{i1}, x_{i2}, \cdots, x_{ip})^T$ is a vector containing the explanatory variables (or observable characteristics) for the *i*-th policyholder.

Let C_k be the cost of the *k*-th standard claim. We assume C_1, C_2, … that are independent. Let each C_k conform to the Gamma distribution.

The $\text{Gam}(\mu, \alpha)$ probability density function can be cast into

$$f(x \mid \mu, \alpha) = \frac{1}{x \cdot \Gamma(\alpha)} \cdot \left(\frac{\alpha \cdot x}{\mu} \right)^{\alpha} \cdot \exp\left(-\frac{\alpha \cdot x}{\mu} \right). \tag{3}$$

If $X \sim \text{Gam}(\mu, \alpha)$, then $E(X) = \mu$, $D(X) = \frac{\mu^2}{\alpha}$

and the coefficient of variation $\text{cvar}(X) = \frac{\sigma(X)}{E(X)} = \frac{1}{\sqrt{\alpha}}$ is constant.

The annual expected cost of standard claim (in a Gamma regression we use *log* link function) for each class is given by:

$$\exp\left(\boldsymbol{\beta^T} \cdot \boldsymbol{X}\right) = \exp\left(\beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \cdots + \beta_p \cdot x_p\right), \tag{4}$$

The intercept β_0 representing the risk associated with the reference class (for which $x_1 = x_2 = \cdots = x_p = 0$) and $\exp(\beta_0)$ is the annual expected cost of standard claim for a policy in the reference class. When all explanatory variables are categorical, each policyholder is represented by a vector with components equal to '0' or '1'.

The annual expected cost of standard claim is then equal to:

$$\mu_i = E(C_i) = \exp\left(\boldsymbol{\beta^T} \cdot \boldsymbol{X_i}\right) = \exp(\beta_0) \cdot \prod_{j=1}^{p} \exp(\beta_j \cdot x_{ij}) = \exp(\beta_0) \cdot \prod_{j=1; x_{ij}=1}^{p} \exp(\beta_j), \tag{5}$$

where $\exp(\beta_j)$ models the effect of the *j*-th ratemaking variable.

If $\beta_j > 0$ then $\exp(\beta_j)$ increases the annual expected cost of standard claim. The $\exp(\beta_j)$ is the multiplicative effect on the annual expected cost of standard claim due to the covariate associated with β_j, while holding the other explanatory variables constant. On contrary, if $\beta_j < 0$ then $\exp(\beta_j)$ decreases the annual expected cost of standard claim.

We are estimating the parameters β_j by the maximum likelihood approach.

Let n_i be the number of claims reported by policyholder *i*, and let $c_{i1}, c_{i2}, \ldots, c_{in_i}$ be the corresponding claim costs.

The likelihood function associated with the observations writes

$$L(\boldsymbol{\beta}) = \prod_{i|n_i>0} \prod_{j=1}^{n_i} f(c_{ij} \mid \mu_i, \alpha) = \prod_{i|n_i>0} \prod_{j=1}^{n_i} \left(\frac{1}{c_{ij} \cdot \Gamma(\alpha)} \cdot \left(\frac{\alpha \cdot c_{ij}}{\mu_i} \right)^{\alpha} \cdot \exp\left(-\frac{\alpha \cdot c_{ij}}{\mu_i} \right) \right), \tag{6}$$

where $\mu_i = \exp\left(\boldsymbol{\beta^T} \cdot \boldsymbol{X_i}\right)$.

The log-likelihood equations are

$$\frac{\partial l(\boldsymbol{\beta})}{\partial \beta_k} = 0 \Leftrightarrow \sum_{i|n_i>0} \sum_{j=1}^{n_i} x_{ik} \left(1 - \frac{c_{ij}}{\mu_i} \right) = 0 \tag{7}$$

The goodness of fit is measured by *deviance* (the smaller, the better). The deviance can also be used to compare the fit of two models by taking the difference in the deviances. The difference in the deviance, between the more complex model (*full model*) D_F and the

deviance of the simpler model (*reduced model*) D_R with some parameters dropped out, can also be used to test the null hypothesis that the additional parameters in the full model are equal to zero. Let us test the null hypothesis:

$$H_0: \boldsymbol{\beta} = \boldsymbol{\beta_0} = (\beta_0, \beta_1, \beta_2, \cdots, \beta_q)^T$$
$$H_1: \boldsymbol{\beta} = \boldsymbol{\beta_1} = (\beta_0, \beta_1, \beta_2, \cdots, \beta_p)^T$$

That is, H_0: $\beta_{q+1} = \beta_{q+2} = \cdots = \beta_p = 0$.

The difference in the deviances is a χ^2 distributed variable (under the null hypothesis that the additional regression parameters are equal to zero) with degrees of freedom equal to the difference in the number of regression parameters between the full and the reduced model, or equivalently the number of additional parameters in the full model:

$$D_R - D_F \sim \chi^2_{p-q}. \tag{8}$$

We can formally test the hypothesis that the additional parameters in the full model are zero, by using the difference of the deviances in a formal statistical test. H_0 is rejected if $D_R - D_F$ is "too large", that is if $D_R - D_F > \chi^2_{p-q}(1-\alpha)$. If the χ^2 is statistically significant, then we accept the full model. If it is not significant, we accept the reduced model. The goal of our regression analysis is to find a set of explanatory variables that have high explanatory power as measured through goodness of fit.

3. Results

Our data set is based on one-year vehicle insurance policies of the Motor TPL (Third Party Liability) portfolio of one Belgian insurance company . There are 163 657 policies, of which 18 345 produced at least one claim. The analysis is performed with the help of the *GLM* procedure of R language [R Core Team 2015] and R packages "car" (Fox, Weisberg (2011)), "epicalc" (Chongsuvivatwong (2012)) and "gmodels" (Warnes (2013)).

Table 1. Description of variables with their modalities

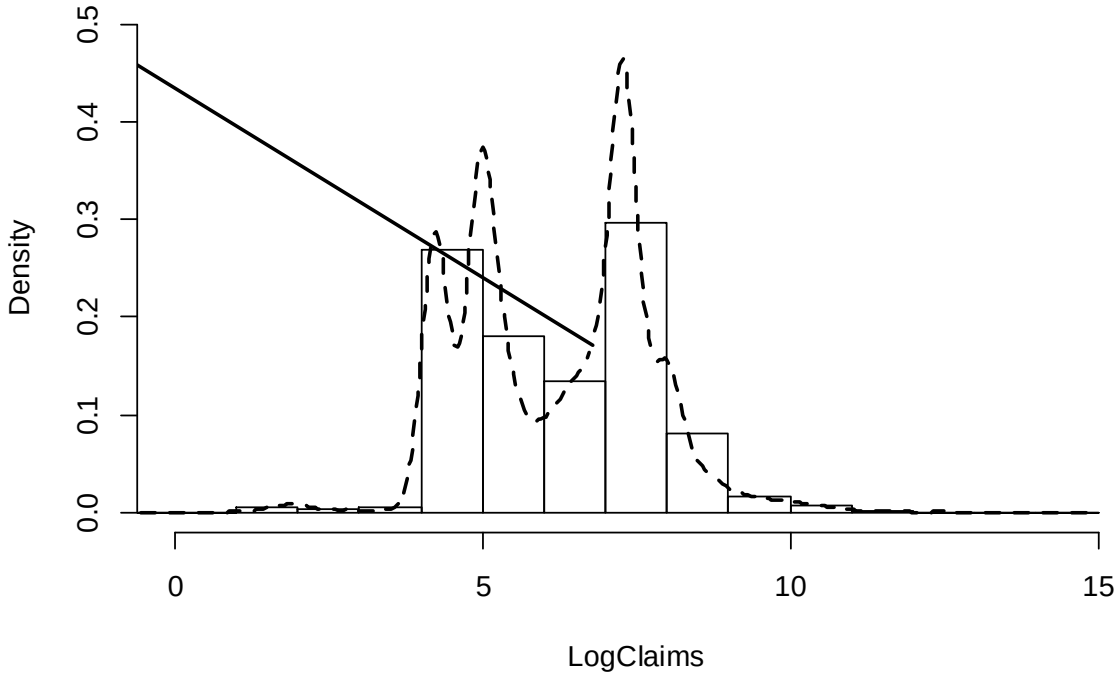
Variable	Description with modalities
duree	Length of the coverage period (or exposure to risk)
nbrtotc	Number of claims
chargtot	Total claim amount
agecar	Age of the vehicle: 0-1, 2-5, 6-10, >10
sexp	Sex of the driver: Male or Female
fuelc	Type of fuel: Petrol or Gasoil
split	Split of the premium: Monthly, Once, Thrice, Twice
usec	Use of the vehicle: Private or Professional
fleetc	Vehicle belonging to a fleet: Yes or No
sportc	Sports car: Yes or No
coverp	Coverage: MTPL, MPTL+, MPTL+++
powerc	Power of the vehicle: <66 kW, 66-110 kW, >110 kW

Source: Author's own study.

First we eliminate atypical extreme losses. By the look at the histogram or pdf of costs (Figure1.) we have chosen a threshold of 50 000 €. Out of the 163 657 data 18 305 represents standard costs (less than 50 000). It means we omit 40 largest costs. The reference class is

composed of the modalities of the variables with the largest risk-exposure. In our case it is: Male, agecar 6-10, Petrol, split Once, Private use, No fleet, No sportcar, MTPL cover and power <66 kW. Often only the total claim amount $C_{i\bullet}$ is available and not the individual c_{ij}´s. In this case, it is convenient to work with the mean claim amount or mean cost $\overline{C_i} = \frac{C_{i\bullet}}{n_i}$, where n_i is the number of claims reported by policyholder *i*.

Figure 1. Pdf of log-costs



In R, the GLM analysis is performed via Gamma regression:

glm(*mean_cost ~ agecar + sexp +…+ powerc*, *family = Gamma*(*link=log*), *weigths=nbrtotc*)

In this case the response variable is the mean cost.

Then we test whether a particular category is significant or not. We start for a model incorporating all the available information and then exclude the irrelevant explanatory variables. The *p*-value tests the relevance of the variable. The limit of 5% is usually used to decide on this relevance.

With the help of **Anova** analysis, we observe which variables are significant.

```
> Anova(GLM_AnalysisCosts, type='III', test.statistic='F')

  Response: DataCosts[["MeanCharge"]]
                            SS    Df       F    Pr(>F)
  DataCosts[["sexp"]]        1     1  0.1189  0.730271
  DataCosts[["usec"]]        0     1  0.0011  0.973738
  DataCosts[["fleetc"]]      3     1  0.5919  0.441697
  DataCosts[["sportc"]]      0     1  0.0786  0.779140
  DataCosts[["coverp"]]    273     2 24.3595 2.722e-11 ***
  DataCosts[["split"]]       7     3  0.3999  0.753046
  DataCosts[["fuelc"]]       1     1  0.1695  0.680581
  DataCosts[["agecar"]]     72     3  4.2983  0.004879 **
  DataCosts[["powerc"]]      3     2  0.3086  0.734473
```

```
  Residuals             102353 18289
  ---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

We remove ”usec” (*p*-value equals to 0.973738). We repeat this process until obtain only significant variables.

```
> Anova(GLM_AnalysisCosts, type='III', test.statistic='F')
  Response: DataCosts[["MeanCharge"]]
                             SS    Df       F    Pr(>F)
  DataCosts[["coverp"]]    275     2 24.4359 2.522e-11 ***
  DataCosts[["agecar"]]     73     3  4.3501  0.004538 **
  Residuals             102786 18299
  ---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

At the end of the process the significant variables are: the type of **cover** and **age** of the vehicle.

However, all the modalities do not need to be significant. Therefore, we try to gather the modalities using the **`fit.contrast`** procedure.

We first look at the confidence interval of the predictions:

```
> confint.default(GLM_AnalysisCosts,level = 0.95)
                                        2.5 %     97.5 %
  (Intercept)                        7.10008164  7.2133231
  DataCosts[["coverp"]]B/MTPL+      -0.30591194 -0.1461062
  DataCosts[["coverp"]]B/MTPL+++     0.02844816  0.2530795
  DataCosts[["agecar"]]B/0-1         0.05554839  0.3699192
  DataCosts[["agecar"]]B/2-5        -0.13953754  0.0269593
  DataCosts[["agecar"]]C/>10        -0.06202141  0.1105743
```

The process of gathering can be summarized as follows: Variable “agecar”: 2-5 and >10 are overlapping and they overlapping “0” the value of the reference class 6-10 of this variable. This means we try to gather them.

```
> fit.contrast(GLM_AnalysisCosts,DataCosts[["agecar"]],c(-1,1,0,0))
                                     Estimate Std. Error  t value    Pr(>|t|)
DataCosts[["agecar"]] c=( -1 1 0 0 ) 0.2127338  0.0801981 2.652604 0.007994252
> fit.contrast(GLM_AnalysisCosts,DataCosts[["agecar"]],c(-1,0,1,0))
                                      Estimate Std. Error   t value Pr(>|t|)
DataCosts[["agecar"]] c=( -1 0 1 0 ) -0.05628912 0.04247446 -1.325246 0.185106
> fit.contrast(GLM_AnalysisCosts,DataCosts[["agecar"]],c(-1,0,0,1))
                                     Estimate Std. Error   t value  Pr(>|t|)
DataCosts[["agecar"]] c=( -1 0 0 1 ) 0.02427644 0.04403032 0.5513572 0.5813955
> fit.contrast(GLM_AnalysisCosts,DataCosts[["agecar"]],c(0,-1,1,0))
                                     Estimate Std. Error   t value     Pr(>|t|)
DataCosts[["agecar"]] c=( 0 -1 1 0 ) -0.2690229 0.07848303 -3.427784 0.0006098684
> fit.contrast(GLM_AnalysisCosts,DataCosts[["agecar"]],c(0,-1,0,1))
                                     Estimate Std. Error   t value   Pr(>|t|)
DataCosts[["agecar"]] c=( 0 -1 0 1 ) -0.1884573 0.08500623 -2.216983 0.02663652
> fit.contrast(GLM_AnalysisCosts,DataCosts[["agecar"]],c(0,0,-1,1))
                                     Estimate Std. Error  t value  Pr(>|t|)
DataCosts[["agecar"]] c=( 0 0 -1 1 ) 0.08056556 0.05060952 1.591905 0.1114233
```

The modalities ">10" and "6-10" of "agecar" should be gathered (as ">5"), as the *p*-value equals 0.5814. We repeat the previous process with other modalities.
Consequently, we gather modalities "B/2-5" and "C/>5" with the reference modality of "agecar" (as "A/>1").

Let us look at the summary of our fitting (output of *R*):

```
> summary(GLM_AnalysisCosts)
  Call: glm(formula = DataCosts[["MeanCharge"]] ~ DataCosts[["coverp"]] +
        DataCosts[["agecar"]], family = Gamma(link = log), weights =
        DataCosts[["nbrtotc"]])

  Coefficients:
                                Estimate Std. Error t value Pr(>|t|)
  (Intercept)                    7.15477    0.02150 332.831  < 2e-16 ***
  DataCosts[["coverp"]]B/MTPL+  -0.24636    0.03895  -6.325 2.59e-10 ***
  DataCosts[["coverp"]]B/MTPL+++ 0.10340    0.05269   1.962  0.04973 *
  DataCosts[["agecar"]]B/0-1     0.24007    0.07618   3.151  0.00163 **
  ---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Table 2. Estimated parameters and their relativities.

		Costs	
Variable	Modality	Estimate $\vec{\beta}$	Relativities
Intercept		7,15477	**1 280,20** €
sexp	Female	0	100 %
fleetc	Yes	0	100 %
sportc	Yes	0	100 %
coverp	MTPL+	– 0.24636	78,16 %
	MTPL+++	0,10340	110,89 %
fuelc	Gasoil	0	100 %
agecar	0-1	0.24007	127,13 %
powerc	>=66 kW	0	100 %
split	Other	0	100 %

Source: Author’s own calculation.

4. Conclusion

We have applied the GLM model to achieve risk classification of a MTPL portfolio. We used a particular portfolio of policies with variables determined a priori. Different insurance companies could collect different explanatory variables. Portfolio of other insurance company could include different variables such as the age, the period a driver has held a driving licence, marital status, etc. Still they can use the same approach as we propose to find relevant explanatory variables and modalities. There are a number of software programs that insurance industry has developed, for instance SAS GENMOD is used in Denuit et al. (2007). We decided to use ‘R’ software, which is free available software.

The result of the GLM is a multiplicative model where the cost of standard claim of a category is given by the cost of standard claim of the reference class * the relativities exp($\hat{\beta}_j$) of the category. The relativities measure the relative difference with respect to the reference

class. The Gamma regression model for standard claims can be replaced by Inverse-Gaussian or Lognormal distribution. For extreme claims we would suggest Generalized Pareto distribution which provides good approximation to the excess distribution over large threshold (for instance 50 000 €).

We can see how to partition a heterogeneous portfolio into more homogeneous classes with all policyholders belonging to the same class paying the same premium. However, tariff cells are still quite heterogeneous (some risk characteristics are unobservable) despite the use of many *a priori* variables. So, there is a need of the *a posterior* corrections. In *a priori* ratemaking, the actuaries aim to identify the best predictors and to compute the risk premium. In *a posterior* ratemaking, they aim to compute premium corrections according to past claims history. This experience rating is based on a 'crime and punishment' mechanism: claim-free policyholders are rewarded by premium discounts (bonus) and others (who report one or more claims) are penalized by premium surcharges (malus). Past claims experience can reveal the hidden features.

References

[1] Cox D. R. (1972). *Regression Models and Life-tables*, Journal of the Royal Statistical Society. Series B (methodological), no. 34 (2), pp. 187–220.

[2] Chongsuvivatwong V. (2012). *Epicalc: Epidemiological calculator. R package version 2.15.1.0*., http://CRAN.R-project.org/package=epicalc (accessed).

[3] Denuit M., Charpentier A. (2005). *Mathématiques de l'Assurance Non-Vie. Tome II: Tarification et Provisionnement. Collection Economie et Statistique Avancées*, Economica, Paris.

[4] Denuit M., Maréchal X., Pitrebois S., Walhin J.-F. (2007). *Actuarial Modeling of Claim Counts: Risk Classification, Credibility and Bonus Malus Systems*, John Wiley & Sons, New York.

[5] Fox J., Weisberg S. (2011). *An {R} Companion to Applied Regression, Second Edition*, Thousand Oaks CA, Sage, http://socserv.socsci.mcmaster.ca/ jfox/Books/Companion (accessed).

[6] Jee B. (1989). *A comparative analysis of alternative pure premium models in the automobile risk classification system*, Journal of Risk and Insurance, no. 56, pp. 434–459.

[7] Nelder J.A., McCullagh P. (1989). *Generalized linear models (Second edition)*, Chapman & Hall, London.

[8] R Core Team. (2015). *R: A language and environment for statistical computing*, R Foundation for Statistical Computing, Vienna, http://www.R-project.org/ (accessed).

Selected methods of business diagnostics for managerial practice

Barbora Gondzarova [1], Viera Barosova[2]

Abstract

The paper is focused on the summary of the available methods of business diagnostics. There are mentioned the advantages and disadvantages of each method with an emphasis on the benefits and opportunities offered in managerial practice. The result of this paper is the evaluation of the methods by criteria from managerial practice and the propose of the most appropriate methods.

Key words

Management. Methods. Diagnostics. Risk. Crisis.

JEL Classification: G30

1 Úvod

Každá ľudská činnosť je spojená s určitým rizikom. Podľa Damodarana pojem riziko najlepšie vystihuje čínsky výraz pre riziko, ktorý je vyjadrením dvoch pojmov a to nebezpečenstvo a príležitosť(Cisko-Kliestik, 2013). Z tejto definície je zrejmé, že príležitosť neprichádza bez rizika a rovnako, že ak podstupujeme riziko, získavame zároveň príležitosť. Rovnakú paralelu môžeme uplatniť aj v podniku. Aby podnik mohol napredovať a dostávať sa do príležitostí, ktoré môže využiť, tak musí podstúpiť riziko(Majercak,Bartosova,Hraskova,2015). Podstúpenie rizika však neprináša len pozitívne dôsledky, ale môže mať aj negatívne následky v rôznej podobe, najčastejšie v podobe finančných problémov či zániku (Frajtova-Michalikova, Kliestik, Musa,2015). Preto je potrebné riziko riadiť, resp. predchádzať negatívnym následkom a prípadnej kríze. Jednou z možností je uplatnenie vybraných metód podnikovej diagnostiky.

2 Riziko a možnosti jeho riadenia

Riziko je všade prítomným a charakteristickým sprievodným javom fungovania organizácií v súčasnom turbulentnom prostredí. Existuje celý rad druhov rizík v rôznych oblastiach. Riadenie rizík je sústavná, opakujúce sa séria navzájom prepojených aktivít, ktorých cieľom je riadiť potenciálne riziká, teda obmedziť pravdepodobnosť ich výskytu alebo znížiť ich negatívny vplyv. Účelom riadenia rizík je predísť problémom či negatívnym javom, vyhnúť sa krízovému manažmentu a zamedziť vzniku problémov. Vidíme tu teda možnosť prepojenia s metódami podnikovej diagnostiky.

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3 Podniková diagnostika

Vo všeobecnosti je diagnostika definovaná ako náuka o rozpoznávaní a jeho metódach, teda ku správnej diagnostike je potrebné zvoliť a využiť vhodné metódy. Existuje veľké množstvo čiastkových oblastí, ktoré môžu byť v rámci podnikovej diagnostiky skúmané a z toho vyplývajúce veľké množstvo možných využiteľných metód a ich klasifikácií. Pomocou rôznych metód je možné vykonať diagnostiku okolia podniku, diagnostiku životného cyklu a tiež jednotlivých výrobných faktorov a podnikových činností. Kašík a Michalko definujú podnikovú diagnostiku podľa predmetu jej skúmania ako náuku, ktorá sa zaoberá rozpoznávaním a vyhodnocovaním :

- úrovne fungovania podniku ako systému,
- celkovej hodnoty (bonity) podniku,
- silných a slabých stránok podniku,
- problémov a krízových javov podniku vrátane jeho prípadného zániku
- nevyužitých príležitostí a potenciálov podniku. (1998)

Vidíme, že podniková diagnostika sa zaoberá tiež krízovými javmi podniku. Ich prevencia, teda predchádzanie rizika má pozitívne následky pre podnik a môže predísť nielen problémom ale v mnohých prípadoch i zániku podniku.

4 Metódy podnikovej diagnostiky

Neumanová (2013) za v praxi najpoužívanejšie metódy považuje passport, finančno-ekonomickú analýzu a tiež metódy multikriteriálneho vyhodnocovania variantov. Síbertová túto trojicu dopĺňa o strategickú a marketingovú analýzu a o metódy posudzovania rizika vzniku krízy. Samostatnou kategóriou sú podľa Neumanovej metódy diagnostiky okolia podniku. Ďalší autori v literatúre za metódy podnikovej diagnostiky označujú tiež finančno-ekonomickú analýzu, ukazovateľ EVA pre stanovenie hodnoty podniku (Berzakova,Bartosova, Kicova,2015), bankrotné a bonitné modely a iné.
Metódy podnikovej diagnostiky môžeme pre väčšiu prehľadnosť zatriediť do skupín podľa zamerania.

Členenie podľa skúmaných oblastí

Toto členenie vychádza z definície podnikovej diagnostiky podľa Kašíka-Michalka uvedenej v predchádzajúcej časti tohoto príspevku.

Úroveň fungovania podniku ako celku

- Finančno-ekonomická analýza
- Strategická a marketingová analýza
- Bonitné modely

Hodnota podniku

- Výpočet pomocou ukazovateľa EVA

Silné a slabé stránky podniku a nevyužité príležitosti a potenciály podniku

- SW analýza
- O-T analýza
- PEST analýza
- SPACE analýza

- Metódy diagnostiky okolia podniku

Problémy a krízové javy podniku a hrozba jeho zániku

- krízový barometer,
- štruktúrovaný brainstorming,
- analýza scenárov,
- analýza úloh,
- analýza príčin a následkov,
- analýza „čo sa stane ak..“,
- multikriteriálne vyhodnocovania variantov,
- bankrotné modely(Kollar, Kramarova, Valaskova,2015).

Metódy diagnostiky okolia podniku

Diagnostiku okolia podniku a jej metódy môžeme označiť za samostatnú kategóriu. Tak ako sme už uviedli, podniková diagnostika skúma podnik ako celok. To znamená so všetkými vzťahmi a vplyvmi okolia. Diagnostiku okolia podniku je vhodné vykonať pri zakladaní podniku rovnako ako pri riešení krízových situácií. Neumanová (2007) medzi metódy diagnostiky okolia podniku zaraďujú predovšetkým :

- passport okolia podniku,
- diagnostickú misiu,
- monitoring,
- štúdiu okolia podniku.

Charakteristika vybraných metód

Passport okolia podniku

Táto metóda predstavuje prehľadné spracovanie informácií o určitej záujmovej zóne, lokalite, obci, regióne či krajine, kde by mal byť podnik vybudovaný, rozšírený či presťahovaný. Passport poskytuje číselné a slovné informácie o rôznych oblastiach.

Diagnostická misia

Vykonanie diagnostickej misie spočíva v spracovaní rýchleho diagnostického testu, pomocou ktorého sa určia základné charakteristiky okolia podniku v oblasti kapitálových, trhových a ľudských zdrojov. Jeho vypracovanie je v kompetencii špeciálne vyškolených manažérov, Ide o komplexné zhodnotenie postavenia podniku na trhu, jeho postavenie voči konkurencii ako aj postavenie daného odvetvia v rámci národného hospodárstva. Je teda zrejmé, že kvalitne vykonaná diagnostická misia môže byť pre podnik veľkým prínosom, no zároveň zlý odhad a zle zhodnotená situácia môžu byť pre podnik zavádzajúce. Je preto naozaj dôležité, aby pri využití tejto metódy diagnostiky okolia bola táto vykonaná odborne vyškoleným pracovníkom, ktorý má dostatok skúseností.
Táto metóda je komplexná, s dobrou vypovedacou schopnosťou a poskytuje dobrý základ pre posúdenie situácie a následné rozhodovanie. Jej nevýhodou je predovšetkým náročnosť jej vypracovania.

Monitoring

Ide o diagnostickú metódu, ktorú pre podnik nevykonávajú interní zamestnanci, ako tomu bolo napr. pri diagnostickej misii, ale špecializované firmy. Monitoring sa niekedy stotožňuje s marketingovým prieskumom(Hraskova, Bartosova, 2014). Monitoring predstavuje zber

a analýzu informácií zverejňovaných štatistickými úradmi a špecializovanými inštitúciami. Cieľom je získanie požadovaných informácií o okolí podniku.
Táto metóda je taktiež komplexná, zber dát z rôznych zdrojov a realizácia monitoringu externou firmou zabezpečuje objektívnosť výsledkov. Je však časovo i finančne náročná a nemožno ju využívať operatívne pri vzniknutých problémoch vzhľadom na dlhšiu dobu jej vypracovania.

Štúdia okolia podniku

Túto metódu je možné považovať v rámci diagnostiky okolia podniku za najrozsiahlejšiu. Okrem historických údajov týkajúcich sa vybraného územného celku zahŕňa tiež SWOT analýzu podniku, pričom sa zameriava nielen na analýzu súčasného stavu, ale aj na hypotézy možného vývoja v budúcnosti. Výsledkom takejto štúdie okolia podniku je program harmonizácie podniku a jeho okolia(Neumannova, 2007).
Ako sme uviedli, táto metóda je najkomplexnejšia a je vhodné ju v podniku v určitých časových intervaloch vykonávať. Jej vypracovanie je však veľmi náročné a preto sa v manažérskej praxi veľmi nevyužíva.

Finančno-ekonomická analýza

Finančno-ekonomickú analýzu by sme mohli označiť za samostatnú kapitolu. Je možné ju rozdeliť na dve samostatné časti a to finančnú a ekonomickú analýzu. Ale až vykonanie komplexnej finančno-ekonomickej analýzy, teda výpočty doplnené o odbornú interpretáciu výsledkov na základe poznania podniku, prináša požadované výsledky a je teda vhodným nástrojom pri prevencii krízy podniku. Finančno-ekonomická analýza rekapituluje a hodnotí výsledky podniku za analyzované obdobie. Identifikuje a kvantifikuje okolnosti, ktoré ich determinovali. Na základe doterajšieho vývoja a dosiahnutých výsledkov sa snaží načrtnúť budúci možný vývoj. Výsledkom je návrh opatrení za účelom dosiahnutia podnikových cieľov.

Finančno-ekonomická analýza sa spracováva v 2 okruhoch:

1.primárna analýza- analýza súhrnných výsledkov
2. sekundárna analýza -analýza parciálnych výsledkov –v nej sa podrobne venuje „chorým“ miestam zisteným v prvom okruhu

Na vykonanie kvalitnej finančno-ekonomickej analýzy je potrebná určitá zručnosť a tiež skúsenosť. Finančno-ekonomická analýza vykonávaná pre potreby diagnostiky podniku sa nelíši od tej vykonávanej v rámci iného skúmania či kontroly v podniku. Rozdiel je len v interpretácii výsledkov analýzy, ktoré pri diagnostike podniku treba dávať do súvisu s ostatnými výsledkami získanými inými metódami s cieľom dosiahnutia komplexného obrazu podniku, teda jeho zdravia, či prípadnej choroby. Finančno-ekonomická analýza môže mať rôzne formy od rýchlych a jednoduchých po zložitejšie, no komplexnejšie(Bartosova,Hraskova,Paliderova,2014). Jej výsledky môžu byť podkladom pre rozhodovanie a posúdenie situácie, ale tiež základom pre použitie inej metódy. Finančno-ekonomická analýza je v literatúre označovaná za metódu najčastejšie využívanú v manažérskej praxi.(Kicova, Kramarova, 2013)

Metódy multikriteriálneho vyhodnocovania variantov

Úlohou multikriteriálneho vyhodnocovania variantov je objektívne popísať reálny stav pri rozhodovaní a to pomocou štandardných postupov a tým previesť rozhodovací proces na matematický model viackriteriálnej rozhodovacej situácie(Cisko,Kliestik,2013). Na základe tejto metódy získa podnik integrálny ukazovateľ komplexného hodnotenia.

Strategická a marketingová analýza

Táto analýza pozostáva z analýzy interného, externého prostredia a strategickej syntézy. Poskytuje informácie o štruktúre trhu, stave odvetvia či konkurencie. Jej výsledky sú dôležité pri rozhodovaní o pôsobení v odvetví či ďalšom smerovaní podniku. Pomáha tiež identifikovať externé a interné vplyvy na podnik a v podniku a odstrániť tak tie negatívne a využiť tie pozitívne z nich. V rámci externej analýzy je vhodné využiť PEST analýzu. Jej pomocou je možné analyzovať a zhodnotiť stav a postavenie v odvetví a tiež konkurenciu a z toho vyplývajúce príležitosti a ohrozenie pre podnik(Kral,Misankova,2015).Interná analýza, tak ako vyplýva už z jej názvu, je zameraná na interné prostredie, teda na konkrétny podnik. Vhodnými metódami sú :

- posúdenie konkurenčnej výhody,
- zostavenie nákladového reťazca,
- analýza interného profilu schopností,
- analýza zraniteľnosti,
- analýza nákupného správania a preferencií zákazníkov,
- analýza nástrojov a marketingovej komunikácie,
- analýza distribučnej stratégie. (Slavik,2005)

Táto metóda ako celok je značne komplikovaná, avšak v praxi sa využívajú čiastkové analýzy, ktoré však nemajú požadovanú vypovedaciu schopnosť.

Metódy posudzovania rizika vzniku krízy

Diagnostika podniku má predovšetkým profylaktický charakter a práve metódy posudzovania rizika umožňujú do istej miery predpovedať možné riziko vzniku krízy a tak mu predchádzať. Medzi metódy posudzovania rizika patria:

- krízový barometer,
- štruktúrovaný brainstorming.
- analýza scenárov,
- analýza úloh,
- analýza príčin a následkov,
- analýza „čo sa stane ak..“,

multikriteriálne vyhodnocovania variantov. (Rybarova,Grisakova,2010)

Hodnotenie metód podnikovej diagnostiky

Na základe prieskumu bolo zistené, že pre manažérov sú pri výbere metód dôležité predovšetkým nasledujúce kritériá :

- jednoduchosť použitia,
- presnosť výsledku.

Pri väčšine metód sú tieto požiadavky protichodné. Rýchle riešenie prináša nepresný odhad, presné výsledky vyžadujú komplikovanejší a zdĺhavejší postup. Výsledky prieskumu (Neumannova,2014) ukázali, že z metód podnikovej diagnostiky je najpoužívanejšia metóda finančno-ekonomickej analýzy, ktorú využíva väčšina respondentov. Tá vzhľadom na použitie vzorcov a určených postupov zaručuje presnosť výsledku, no jej použitie je relatívne rýchle a pre skúseného manažéra i jednoduché. Ich výber však závisí nielen od uvedených kritérií zistených prieskumom, ale tiež od oblasti, ktorú je potrebné analyzovať. Preto odporúčame, pokiaľ je to možné, vykonať globálnu podnikovú diagnostiku aplikáciou viacerých metód na posúdenie komplexného stavu podniku. Ak to však nie je možné, je vhodné použiť aspoň niektoré z uvedených metód.

5 Záver

Pri skúmaní podnikovej diagnostiky a možnostiach jej využitia a začlenenia v podniku sme prišli ku záveru, že vzhľadom na jej profylaktický charakter je vhodné využiť jej metódy aj v kontexte rizika a krízy podniku. Detekcia rizík a ohrození a včasné nápravné opatrenia môžu zabrániť problémom, prípadne až zániku podniku. Podniková diagnostika ponúka množstvo metód, ktoré môžu slúžiť na prevenciu krízy pri podstúpení určitého rizika. Odporúčame preto ich zaradenie do manažérskej praxe. Uvedený príspevok má za cieľ opísať dostupné metódy so špecifikáciou výhod i nevýhod a rozdelením do jednotlivých kategórií pre ich lepšie použitie v manažérskej praxi.

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References

[1] Kasik, J. and Michalko, M., (1998) Podniková diagnostika. Ostrava : TANDEM, 340s.

[2] Neumanova, A., (2007). Podniková diagnostika. Bratislava: Ekonóm, 114s

[3] Kollar, B. and Kramarová,K. and Valaskova, K., (2015) The Financial Distress Analysis of the Stated-Own Company by Applying Chosen Default Prediction Models in Condition of Slovakia.3rd International Conference on Economics and Social Science (ICESS 2015), Advances in Education Research, Vol. 86, pp. 160 – 167.

[4] Cisko, S. and Kliestik,T., (2013) *Finančný manažment podniku II*. Žilina: EDIS, 759s.

[5] Slavik, S., (2005). Strategický manažment. Bratislava: Sprint vfra, 2005. 403s.

[6] Rybarova, D- Grisakova, N. 2010. Podnikateľské riziko. Bratislava: Iura Edition, 2010. 179s.

[7] Hraskova, D. and Bartosova, V., (2014) Managerial Reporting: A Flexible Tool for Managing Transport Company, 2nd International Conference on Social Sciences Research (SSR 2014), Vol. 5, pp. 97 – 100.

[8] Frajtova-Michalikova, K. and Kliestik, T. and Musa, H., (2015) *Comparison of nonparametric methods for estimating the level of risk in finance*, *Procedia Economics and finance*, Vol 24, pp. 228 – 236.

[9] Bartosova, V. and Hraskova, D. and Paliderova, M., (2014) *Basic Rules and Methodological Framework of the Financial Analysis of the Group of Companies in the Slovak Republic*, 2nd International Conference on Economics and Social Science (ICESS), Advances in Education Research, Vol. 61, pp. 86-91.

[10]]Majercak, P. and Bartosova, V. and Hraskova, D., (2015) Taking Risk into Account in the Evaluation of Economic Efficiency of Investment Projects: Traditional Methods, Procedia Economics and finance, Vol 24, pp. 68 – 75.

[11] Berzakova, V. and Bartosova, V. and Kicova, E., (2015) *Modification of EVA in Value Based Management*, *Procedia Economics and finance*, Vol 26, pp. 317-324.

[12] Kicova, E. and Kramarova, K., (2013) *Possibilities of using financial analysis in the bus transport companies*. 9th International Scientific Conference on Financial Management of Firms and Financial Institutions, Financial Management of Firms and Financial Institutions-Ostrava, CZECH REPUBLIC, pp. 332-341.

[13] Kral, P. and Misankova, M., (2015) *Role of the Project Resource Analysis and Planning*. 3rd International Conference on Economics and Social Science (ICESS 2015), Advances in Education Research, Vol. 86, pp. 195 – 200.

[14] Neumannova, A. and Hutera, M., (2014), Uplatnenie diagnostických metód v malých a stredných podnikoch. Ekonóm, 168p.

Comparative analysis on the importance of the expenses that is given to the students the Economic-Administrative Area, Mexico – Spain

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Abstract

This research will show the variables and the results of the comparison of the costs of students in the career of Administration of the University Center of the Highlands of the University of Guadalajara, Jalisco, Mexico and students of the career of Business Administration from the Loyola University of Andalucía, Cordoba, Spain, in a matrix of classification of expenses of four quadrants based on monthly expenses on the importance and urgency that students consider it. Discusses how it is administered and how does your expenses with planning from its own nature. In 74% of the questions there is agreement on the assigned weight, and the remaining 26% distributed in three quadrants. This categorization indicates how students from two different institutions countries distribute their finances.

Key Words:

Administration, Expenses, Finance, Personal Finances.

JEL Classification: D, D1, D14, R, R1, R10, R13, R2

1. Introduction

Expenses now days is a controversial issue that affects everyone monetarily in general, have control of these can facilitate routines and needs, being the relevance of the changing world is necessary for subsistence. The financial well-being is formed through the creation of skills and knowledge of the risks and financial opportunities that may be, you be, prevented, or exploited. [1]

The organization is the first element that we must take into account to manage everyday expenses like household necessities, food and personal expenses acquisitions to make decisions to know what to spend our money and improve our economy. We must take note of what we earn and what we spend to save, as it is an effective means of preventing mismanagement of our personal finances. [2]

For people it be provided with managing their finances and know how to use finance services for institutions of banks and universities seek to sign agreements to promote this type of

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program which aims people to obtain necessary financial education to thereby obtain Benefits. Significantly, these programs are free. [3]

To develop a budget first thing to do is identify the income and expenses that are usually separate the mandatory fixed charges are from housing, insurance, debt, financial expenses and after expenses can be variables such as light, telephone, internet, gas, transportation. Without ignoring the expenses that occur in recreational activities.

2. Justification

Perform a description of the expenses that are made every day, it can be a difficult task to use however necessary for good supply of entrance, due to the need for self-management needs. Money means different concepts for each of the individuals; It is why we have personal financial goals related to our needs, values and desires. [4]

The accounting treatment of income and expenses is a key point, referring that all people should have it, this due to the daily needs, in order to evaluate and create criteria prioritized needs to place certain amount of money each and there are no missing trying this for a charity itself and good governance. [5]

There are creating opportunities in universities which are generally public in an effort to support students who wish to train and can not do it in different universities due to inflation costs involved, talk about methodologies and tools applied to these needs that can help steady growth without large sums of money, they call student opportunities. [6]

There are other types of expenses that abound within a family which does not go unnoticed as is the education that although not a single direct spending, this happens for many years and directly affects the capital inflows daily, because now day expenses are covered not only in school subjects if not in all that this implies as materials, clothing, school etc. [7] This requires a search for greater economic stability and a need for capital increase for some subsistence. [8]

3. Objective

Analizar y comparar los gastos personales de los estudiantes de la carrera de Administración del Centro Universitario de los Altos de la Universidad de Guadalajara (México) y los alumnos de la carrera de Administración de Empresas de la Universidad de Loyola Andalucía, Córdoba (España) mediante una matriz de gastos que identifique lo que es importante y urgente en sus finanzas personales.

4. Methodology

It has been made a survey as a pilot for validation testing of the measuring instrument with 23 questions generating costs, expenses, savings, budget, control expenses, incidentals and school student performance was conducted.

The application of fieldwork was conducted with students' enrolled Career Management University Center of Los Altos in calendar 2016-A and active students in the career of Business Administration from Loyola University Andalucía, Cordoba, Spain in calendar 2015-B.

The research design is descriptive-type cross.

Sample Size Determination

The student population of the University Center of the Highlands in 14 races, was 2932 students. The population of students of Loyola University Andalucia was the assets of the career of Business Administration in 2015-B cycle.

To determine the sample size was used the following formula: [9]

$$n = \frac{N\,Z^2 p\,q}{(\pm)^2(N-1) + Z^2\,p\,q}$$

n= Sample size
N= Population size
Z= 1.96 (Standardized value corresponding to a confidence level of 95%)
±= Margin of error of 5%
p= Probability of Success
q= Probability of failure
pq= Multiplying the odds ultimate combination (0.5) (0.5)= 0.25

Result as sample size of 340 students representative of the population as a minimum. stratified sampling and sample size for the race of Administration of the University Center of Los Altos is 33 and a total of 80 surveys considered in the investigation were collected was applied. For the race of Business Administration from Loyola University Andalucia a total of 104 surveys were collected, considering the active students at the time of application.

5. Results obtained

In the matrix of classification of expenditures in each of the four categories defined what is urgent and important (I), urgent and not important (II), non-urgent and important (III) and not urgent and not important (IV).

In the results of 80 students in the career of Administration of the University Center of the Highlands and 104 students in the career of Business Administration from Loyola University Andalucia in the survey of 23 questions, the question is classified for each of the quadrants according to the response of further observation. Table 1

Table 1. Quadrants of the Matrix classification of expenses

	Urgent	Not-urgente
Important	I	III
Not important	II	IV

According to revenue with students in their school career, the classification matrix expenditure is obtained, considering the urgency and importance that they consider both students of Management at the University of Guadalajara Mexico as students of Business Administration Loyola University Andalucia Spain. This is shown in the following **Table 2 and Table 3.**

Table 2. Classification Matrix Expense Students of the Centro Universitario de Los Altos.

Classification Matrix Expense	Urgent	Not- urgent
Important	(I) Payment of public transport * Medical expenses Save for medical expenses Make savings of your income Eating at home Buy books, notebooks, school supplies Pay debts Perform control of your income Get an estimate of your expenses * Perform control your expenses Sensitize your personal expenses	(III) Buy clothes, shoes Expenditure on personal care * Telephony and technology expenses Buying a car own Going on holiday Get an estimate of your income
Not- important	(II)	(IV) Payment of rent * Payment gasoline * Make payments with credit cards Spend on food outside the home Spend money on entertainment * Borrow money

Students at the University of Guadalajara, Mexico describe the importance and urgency of your expenses as shown in this table, not considering as urgent and not important option as represented by the quadrant II, * it represents have no coincidence with the results of Loyola University Andalucia, Spain.

Table 3. Matrix Classification Expenditure of the Loyola University Andalucia students.

Classification Matrix Expense	Urgent	Not- urgent
Important	(I) Payment of rent * Gasoline payment * Expenditure on personal care * Medical expenses Saving for your medical expenses Make savings of your income Eating at home Buy books, notebooks, school supplies Pay your debts Perform control of your income Perform control your expenses Sensitize your personal expenses	(III) Buy clothes, shoes Telephony and technology expenses Buying a car own Going on holiday Spend money on entertainment * Get an estimate of your income Get an estimate of your expenses *
Not- important	(II)	(IV) Payment of public transport * Make payments with credit cards Spend on food outside the home Borrow money

The results of this table indicates the importance and urgency that give their expenses, not considered as urgent and not important option as represented by the quadrant II, * it represents have no coincidence with the results of the University of Guadalajara, Mexico.

In this research, in which 80 students of the Bachelor of Administration of the University Center of the Highlands and 104 students in the Bachelor of Business Administration from Loyola University Andalucia, of the 23 questions asked were surveyed for each of 80 students and 104 students respectively, in neither case occupied the variable matrix classification expense urgent and not important, corresponding to the first quadrant II.

Each answer to be classified in each of the areas of the matrix cost classification to observe and analyze the behavior according to their needs importance regarding the personal expenses of the students.

Table 4 Shows the overlap in each of the following quadrants on the urgency and importance they give to their expenses students of universities of Mexico and Spain

Table 4. Classification matrix comparative costs of Students of the University of Guadalajara, Mexico and Loyola University Andalucía, Spain.

Classification Matrix Expense	Urgent	Not- urgent
Important	(I) Medical expenses Saving for your medical expenses Make savings of your income Eating at home Buy books, notebooks, school supplies Pay your debts Perform control of your income Perform control your expenses Sensitize your personal expenses	(III) Buy clothes, shoes Telephony and technology expenses Buying a car own Going on holiday Get an estimate of your income
Not- important	(II)	(IV) Make payments with credit cards Spend on food outside the home Borrow money

Note that the comparative analysis between the students of the Bachelor of Administration of the University Center of the Highlands with students of the Bachelor of Business Administration from the University of Loyola Andalucía matches over 76% of the questions are, only the difference between 6 of them representing 24%, such as: the payment of rent, Payment of public transport, payment of gasoline, spending on personal care, spending money on entertainment, and making a budget your expenses. In quadrant II no Expense Ratio consider urgent and not important for students of both universities.

6. ConclusionsS

With regard to the heading of urgent and important (I) of the students in the Bachelor of Administration of the University Center of Los Altos ranked first 11 responses from 23 in the survey, while students of Business Administration Loyola University Andalucia category I ranked first answers only 12 more students of directors of the University Center of Los Altos. Of which there is agreement in 9 of them representing 39% of the questions (medical expenses, Saving for your medical expenses, make savings of your income, Eat at home, buy books,

notebooks, school supplies, pay your debts, perform control your income, make a check of your expenses, Educate your personal expenses).

In the area of urgent and not important (II) it was not obtained as the primary response, as well as the 80 students of the Bachelor of Administration of the University Center of the Highlands as 104 students in the career of Business Administration from Loyola University Andalucia chose none with more answers.

Regarding what is not urgent and important (III) were obtained as the main response in comparative 5 matches of the 23 questions, which accounts for 22% of the questions (Buying clothes, shoes, costs in telephony and technology, buying a car itself, going on holiday, an estimate of your income)

From the results in the matrix of expenses that are not urgent and not important (IV), were obtained as the main response in the comparative 3 of the 23 questions, this represents a 13% match (Make payments with credit cards, Spending on food outside the home, borrow money).

There is a greater percentage 76% similarity in behavior according to the importance they give to their expenses undergraduate students at the University of Guadalajara, Mexico and Loyola University Andalucía, Spain in the comparative responses according to the result of the Expenditure classification matrix.

If college students, many expenses that can not miss, such as buying school supplies and tuition payments over the course of his professional studies arise. Therefore, students must have a savings base to afford such expenses without feeling pressure from discomfort can lead to stress-induced diseases.

It is effective for the student to make a budget to keep track of income and expenses. Today, thanks to technological advances there are different tools that can help this activity, such as applications that can be even on your smartphone.

References

[1] Blanco Hernández, M. T., 2015. Financial education and the influence in Business Administration students. *Anuario Jurídico y Económico Escurialense,* Svazek 48, pp. 381-400.

[2] OCU Ediciones, 2015. Gastos e ingresos bajo control. *Dinero y Derechos,* Septiembre/Octubre , Issue 150, pp. 6-8.

[3] Sánchez, C., 2015. Enseñarán gratis finanzas personales. *El Norte,* Noviembre .p. 2.

[4] Program, O. o. c. a., 2014. Su dinero cuenta: una guia para adultos sobre las finanzas personales. *Finanzas personales,* pp. 07-08.

[5] Fernández Rodríguez, M. d. M. & López Castante, M. A., 2011. *Cambios en el tratamiento contable de los ingresos y gastos.* místo neznámé:autor neznámý

[6] Larrán-Jorge, M. & Ángel, G.-C., 2015. ¿Influyen los modelos de financiación autonómicos en la eficiencia de las universidades públicas españolas?. *REVISTA DE CONTABILIDAD,* Svazek 18, p. 162.

[7] Rogero García, J. & Andrés Candelas, M., 2013. *Gasto público y de las familias en educación en España.* Madrid: autor neznámý

[8] Martín Mayoral, F. & Nabernegg, M., 2014. GASTO SOCIAL EN AMÉRICA LATINA. *Revista de Economía Mundial ,* pp. 67-89.

[9] Reus González, N. N., 2016. El Ahorro y Control Financiero en Estudiantes del Área Económico-Administrativa de le Región Altos Sur de Jalisco, México. *Revista Global de Negocios,* 4(5), pp. 85-96.

The use of financial analysis indicators for the monitoring of company financial situation and elimination of financial risk

Janka Grofčíková [1]

Abstract

Risk management is part of decision-making processes, which are based on risk analysis. In managing the risks associated with the financial condition of enterprise, it is necessary to evaluate the information obtained from financial analysis. Financial indicators enable to signal critical moments in financial management. The paper examines, which of the financial analysis indicators are actually used in financial management and control in the business praxis. We will test the assumption about the growth of the extent of using financial indicators with increase the size of the company. We assume, that the larger the enterprise, the more it is exposed to risk. Greater extent the use of indicators allows timely to identify a creation of risk situations. We determine, whether this fact is also reflected in the practical application of financial management and control tools.

Key words

Financial management, Financial risk, Financial analysis, Financial indicators.

JEL Classification: G39, M19, M40.

1. Teoretické východiská

Finančné riadenie a rozhodovanie v podniku predstavuje v trhovej ekonomike neoddeliteľnú a dominantnú stránku ekonomickej činnosti. Základom finančného riadenia je rozhodovanie o výbere najvhodnejších variant pohybu peňazí, kapitálu a finančných zdrojov a kontrola realizácie týchto rozhodnutí. Všetky oblasti finančného riadenia musia byť súčasťou kontinuálneho procesu riadenia. Pre dosiahnutie požadovanej kvality tohto procesu je potrebné skúmať jednotlivé oblasti riadenia zo statického aj dynamického pohľadu a skúmať vzťahy medzi nimi (Grofčíková, 2010).

Nevyhnutnou súčasťou riadiaceho procesu je priebežné vyhodnocovanie rizík, ktorým je podnikateľský subjekt vystavený. Riadenie rizík je proces, pri ktorom sa podnikateľský subjekt snaží zamedziť pôsobeniu existujúcich alebo budúcich rizík a navrhuje riešenia, ktoré pomáhajú eliminovať účinok nežiaducich vplyvov a umožňujú využívať príležitosti vytvárajúce pozitívne vplyvy (Smejkal a Rais, 2003). Riadenie rizík je súčasťou rozhodovacích procesov, ktoré vychádzajú z analýzy rizika. Okrem analýzy rizika sú súčasťou riadenia rizík aj ďalšie činnosti. Kritickou fázou procesu riadenia rizík je výber optimálnych riešení rizikových situácií, súčasťou ktorého je učenie úrovne rizika a zhodnotenie nákladov a prínosov zvolených variant opatrení na zníženie rizika. Výsledkom každej fázy riadenia rizika je samotné rozhodnutie o realizácii opatrení na zníženie rizika, resp. o ďalšom monitorovaní rizikových situácií.

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Riziká ovplyvňujúce finančnú situáciu a výkonnosť podniku patria medzi najvýznamnejšie a je potrebné im venovať vysokú pozornosť. Dosahovanie pozitívnych výsledkov hospodárenia je jedným z prioritných cieľov umožňujúci z dlhodobého hľadiska rast hodnoty podniku na trhu.

Informácie o priebehu podnikateľskej činnosti a o jej výsledkoch potrebné pre monitorovanie finančnej situácie a vyhodnocovanie rizík poskytuje účtovníctvo podniku. Účtovníctvo zabezpečuje zber, triedenie, transformáciu a poskytovanie informácií podstatných pre jednotlivé rozhodovania na všetkých úrovniach riadenia podniku. Informácie, ktoré môžeme získať z účtovníctva podniku zachytávajú v zmysle paragrafu 2 zákona č. 431/2002 Zb. z. SR o účtovníctve skutočnosti o stave a pohybe majetku, stave a pohybe záväzkov, rozdiele majetku a záväzkov, výnosoch, nákladoch, príjmoch, výdavkoch, výsledku hospodárenia účtovnej jednotky. Predmetom účtovníctva je vykazovanie skutočností o týchto položkách. Účtovná jednotka je povinná viesť účtovníctvo správne, úplne, preukázateľne, zrozumiteľne a spôsobom zaručujúcim trvalosť účtovných záznamov. Plnenie týchto požiadaviek umožňuje získanie informácií, ktoré korektne, verne a úplne prezentujú skutočný stav, potrebný na meranie skutočnej činnosti podniku a vyhodnocovanie vznikajúcich rizík. Výstupy z účtovníctva vedeného systémom finančného účtovníctva slúžia prevažne pre potreby externých subjektov podniku ako sú napr. veritelia, externé kontrolné orgány, investori. Informácie poskytované manažérskym účtovníctvom slúžia bezprostredne potrebám riadiacich pracovníkov podniku a majiteľov. Manažérske účtovníctvo nie je nijako legislatívne upravené, čím sa vytvára priestor pre vytvorenie vlastnej podoby tohto účtovného okruhu.

Výstupy z účtovníctva majú podobu absolútnych ukazovateľov, ktoré vychádzajú z výkazu ziskov a strát a zo súvahy a vyjadrujú objem, výšku a rozsah finančnej činnosti. Sú charakteristické tým, že prevažuje ich kvantitatívna stránka a závisia na veľkosti účtovnej jednotky, ktorú popisujú. Vo všeobecnosti môžu byť vyjadrované v naturálnych, peňažných, časových, ale i rôznych špeciálnych jednotkách. Mernou jednotkou pre potreby finančného riadenia sú prevažne peňažné jednotky. Vo vzťahu k časovému obdobiu absolútne ukazovatele ďalej členíme na okamžité a intervalové. Tieto ukazovatele však samé o sebe nemajú dostatočnú vypovedaciu schopnosť, a možnosti odhaliť pomocou nich riziká ovplyvňujúce finančné zdravie a výkonnosť podniku sú obmedzené.

Rozpoznať pôsobenie rizikových situácií na finančnú situáciu a výkonnosť podniku umožňuje súbor paralelných finančných ukazovateľov, bonitné a bankrotné modely a pyramídové sústavy ukazovateľov využívané pri finančnej analýze podniku (Neumaierová a Neumaier, 2002).

Základný okruh paralelných finančných ukazovateľov využívaných pri finančnej analýze podniku tvoria ukazovatele likvidity, aktivity, rentability, zadlženosti, trhovej hodnoty, prevádzkové ukazovatele a ukazovatele počítané na báze peňažných fondov a cash flow (Lesáková, 2004; Sedláček, 2007; Lesáková a Grofčíková, 2013). Ide o pomerové ukazovatele, ktoré patria pre svoju jednoduchosť a nízku nákladovosť medzi najobľúbenejšie a najrozšírenejšie nástroje finančnej analýzy. Poskytujú základný obraz o finančnej situácii podniku, umožňujú realizovať trendovú aj komparatívnu analýzu, a používajú sa ako vstupné údaje pre matematické modely, ktoré umožňujú popísať závislosti medzi javmi, klasifikovať stavy a hodnotiť riziká, prípadne predvídať budúci vývoj.

Tento základný okruh ukazovateľov dopĺňajú hodnotovo orientované ukazovatele, ktoré sú založené na aplikácii ukazovateľov pridanej hodnoty pre akcionárov, ktorá je daná zmenou hodnoty investovaného kapitálu za konkrétne obdobie, zvýšenej o vyplatené dividendy. Hodnotovo orientované ukazovatele kladú dôraz na trhovú pridanú hodnotu (MVA - Market Value Added) a ekonomickú pridanú hodnotu (EVA – Economic Value Added). Nástroje

merania a koncepcia jednotlivých indikátorov je v hodnotovo orientovanom manažmente konštruovaná vždy tak, aby bol vyjadrený celkový dosiahnutý efekt vo vzťahu k vloženému kapitálu vyjadrenému v rôznej forme. Vo svete sú známe a používané nasledujúce ukazovatele a systémy: Economic Value Added alebo Cash Value Added, EBIT, EBITDA, NOPAT, E/P, ROIC, ROE, Return on Net Assets, Managing for Value Total, Shareholder Return, Total Business Return, Market Value Added, Excess Return Cash Flow, Real Option, RAROC (Marinič, 2008).

Hodnotením finančnej situácie podniku pomocou paralelných finančných ukazovateľov dostaneme izolované informácie o jednotlivých oblastiach hospodárenia podniku. Pre získanie komplexných informácií sa potrebujeme zaujímať o vzájomné vzťahy medzi ukazovateľmi, prípadne o agregované ukazovatele finančnej výkonnosti podniku. Pyramídové sústavy ukazovateľov umožňujú zachytiť súvislosti a väzby medzi ukazovateľmi a vrcholový ukazovateľ rozložiť na ukazovatele stojace v pozícii príčinných faktorov, čím vzniká účelová hierarchia ukazovateľov. Agregáciu ukazovateľov umožňujú teoretické modely založené na matematicko-štatistických metódach, ktoré formulujú funkcie obsahujúce optimálne kombinácie ukazovateľov vrátane ich váh (Boďa a Úradníček, 2016). Ide o bonitné a bankrotné modely, ktoré slúžia pre rýchlu orientáciu investorov a veriteľov, resp. pre roztriedenie podnikov podľa ich výkonnosti a dôveryhodnosti. Dôležitým prínosom týchto modelov je snaha o obmedzenie subjektivity pri výbere ťažiskových ukazovateľov a ich významnosti. Majú však aj svoje obmedzenia, o ktorých pojednávajú Hiadlovský a Kráľ (2014), Dluhošová a Zmeškal (2014), Karas a Režňáková (2014) alebo napr. Hálek (2012).

Je nepopierateľné, že informácie, ktoré môžeme využitím týchto sústav ukazovateľov a modelov získať sú dôležité, pre efektívne finančné riadenie nevyhnutné a umožňujú v predstihu identifikovať vznikajúce riziká. Otázne je, do akej miery sú však v bežnej podnikateľskej praxi podnikov skutočne využívané. Mikro, malé a stredné podniky sú vystavené mnohým bariéram (Marková a Lesníková, 2015), často bojujú o každého zákazníka a všetku energiu smerujú k dosiahnutiu zisku a zabezpečeniu platobnej schopnosti. Finančnú analýzu mnohé z nich považujú za neproduktívnu činnosť, na ktorú im nezostáva časový priestor. Existuje samozrejme možnosť využiť služby externých špecialistov, na to je však potrebné vynaložiť dodatočné finančné prostriedky, ktoré zaťažia rozpočet týchto podnikov. Určitou náhradou za komplexnú analýzu finančnej situácie je pre tieto podniky výpočet len vybraných finančných ukazovateľov prípadne hodnotenie bonity realizované finančnou inštitúciou v prípade, že podnik žiada o úverové zdroje.

Naproti tomu veľké podniky disponujú väčším objemom kapitálu a dostatok informácií pre efektívne riadenie je pre nich nevyhnutnosťou. Uplatňovanie systémového prístupu k riadeniu týchto podnikov má podľa Musu, Krištofíka a Debnárovej (2014) významný vplyv na riziko, ktoré veľké podniky podstupujú na trhu.

2. Cieľ, materiál a metodika

Cieľom príspevku je prezentovať výsledky výskumu zameraného na využívanie ukazovateľov finančnej analýzy pri riadení a kontrole finančnej situácie podniku. Zaujíma nás, ktoré z ukazovateľov finančnej analýzy podniky pri finančnom riadení a kontrole skutočne využívajú. V príspevku overujeme predpoklad o raste rozsahu využívania finančných ukazovateľov v závislosti od veľkosti podniku. Predpokladáme, že čím je podnik väčší, tým viac si uvedomuje vplyv množstva rizikových faktorov na priebeh svojho finančného hospodárenia a svojej finančnej situácie, a má k dispozícii viac potrebných zdrojov, čo sa premieta aj do rozsahu uplatňovania nástrojov finančnej analýzy pri riadení a kontrole finančného hospodárenia.

Primárnym informačným zdrojov sú údaje získané metódou opytovania formou dotazníka. Dotazníkový prieskum sme uskutočnili na výberovej vzorke podnikov so sídlom v Slovenskej republike. Vzorku podnikov sme vybrali jednoduchým náhodným výberom. Prieskumu sa zúčastnilo 227 respondentov všetkých veľkostných skupín s domácou (89,29 %) aj zahraničnou (10,71 %) majetkovou účasťou. Najvyšší podiel na úhrne respondentov tvorili mikro podniky (41,96 %). Malé podniky mali vo vzorke 32,14-percentné zastúpenie, stredné podniky mali 17,41-percentné zastúpenie, veľkých podnikov bolo 8,48 %.

Využívanie finančných ukazovateľov v podnikovej praxi sme sledovali pomocou podielu podnikov, ktoré označili, že daný ukazovateľ využívajú na celkovom počte podnikov, ktoré odpovedali na príslušnú otázku v členení podľa jednotlivých veľkostných skupín v súlade s odporúčaním EK 2003/361/ES o definícii malých a stredných podnikov. Pri odpovediach na otázky mohli respondenti vybrať jednu alebo viac možností z ponúkaných odpovedí, prípadne odpovedať otvorenou formou. Zisťovali sme využívanie 70 finančných ukazovateľov vrátane frekvencie ich využívania. Prehľad ukazovateľov uvádzame v tabuľke 1.

Tabuľka 1: Prehľad skúmaných ukazovateľov

Ukazovatele likvidity
okamžitá platobná schopnosť, likvidita 1. stupňa, likvidita 2. stupňa, likvidita 3. stupňa, fond pohotových prostriedkov, fond peňažného majetku, čistý pracovný kapitál, objem voľných peňazí - splatné záväzky.
Ukazovatele rentability a výnosnosti
EVA, MVA, miera prevádzkového rizika, prevádzková marža, priem. cena cudzích zdrojov, príspevková marža, rentabilita aktív, rentabilita dlhodobých zdrojov, rentabilita tržieb, stupeň prevádzkovej páky, výnosnosť celková zdrojov, výnosnosť vlastných zdrojov.
Ukazovatele aktivity a produktivity
doba obratu pohľadávok, doba obratu zásob, doba obratu záväzkov, finančný bod zlomu, finančný majetok/pohľadávky, obrat ČPK, opotrebovanosť aktív, pohľadávky/záväzky, produktivita aktív z pridanej hodnoty, produktivita aktív z tržieb, produktivita práce na pracovníka, produktivita práce z osobných nákladov, produktivita výrobných fondov, rýchlosť obratu pohľadávok, rýchlosť obratu zásob, rýchlosť obratu záväzkov, účinnosť aktív, účtovný bod zlomu, nákladovosť, viazanosť aktív v tržbách.
Ukazovatele zadlženosti
dlhodobá zadlženosť, finančná páka, koeficient samofinancovania, koeficient zadlženosti, krátkodobá zadlženosť, stupeň fin. samostatnosti, úrokové krytie, zadlženosť z obchodného styku.
Ukazovatele štruktúry majetku a zdrojov financovania
dlhodobé pasíva/dlhodobé aktíva, krátkodobé aktíva/krátkodobé pasíva, dlhodobé aktíva/aktíva spolu, krátkodobé aktíva/aktíva spolu, obežné aktíva/aktíva spolu, stále aktíva/aktíva spolu, nákladový druh/náklady spolu.
Ukazovatele trhovej hodnoty
čistý zisk na akciu, dividenda na akciu, dividendový výnos, P/E ratio, trhová cena k účtovnej hodnote, účtovná hodnota akcií, udržateľná miera rastu, výplatný pomer, ziskový výnos.
Ukazovatele kontroly investícií
NPV z investície, doby návratnosti investície, EVA projektu, index rentability, rentabilita investície, IRR projektu.

Prameň: Vlastné spracovanie

Rozsah využívania ukazovateľov sme stanovili na základe počtu bodov, ktorý sa rovnal frekvencii využívania príslušného ukazovateľa počas roka. Celkový rozsah využívania ukazovateľov v príslušnom podniku resp. v skupine ukazovateľov bol tvorený súčtom rozsahov využívania jednotlivých skúmaných ukazovateľov.

Na základe našich predpokladov a vyššie uvedených teoretických východísk formulujeme nasledujúcu hypotézu.

H: Predpokladáme existenciu pozitívnej závislosti medzi veľkosťou podniku (meranou počtom zamestnancov) a rozsahom využívania vybraných ukazovateľov finančnej analýzy.

Hypotézu (H_0: $\rho = 0$, H_A: $\rho \neq 0$) overujeme pre každú skupinu ukazovateľov uvedenú v tabuľke 1 samostatne, a za všetky ukazovatele spolu, pomocou Spearmanovho koeficientu poradovej korelácie a Somersovho d, na hladine významnosti $\alpha = 0,05$.

3. Rozsah využívania ukazovateľov finančnej analýzy pri finančnom riadení

Komplexnú finančno-ekonomickú analýzu realizuje 75,6 % podnikateľských subjektov. Podiel podnikov, ktoré robia komplexnú finančno-ekonomickú analýzu podniku, rastie s rastom veľkostnej kategórie podniku (57,8 % mikro, 84,3 % malé, 89,5 % stredné, 100 % veľké). Najvyšší podiel respondentov (32,3 %) analyzuje finančnú situáciu podniku s ročnou periodicitou, 26,8 % podľa potreby, 17,7 % mesačne, 14,0 % štvrťročne a 9,2 % respondentov polročne.

Pri sledovaní využívania ukazovateľov podľa skupín, uvedených v tabuľke 1, sme zistili, že najvyšší podiel respondentov (52,9 %) využíva ukazovatele likvidity. Svoju likviditu monitoruje 46,4 % mikro, 55,6 % malých, 58 % stredných a 57,9 % veľkých podnikov. Z tejto skupiny ukazovateľov je najviac využívaný ukazovateľ okamžitej platobnej schopnosti (67,7 % respondentov), 61,4 % respondentov monitoruje rozdiel objemu voľných peňažných prostriedkov a splatných záväzkov, 57,1 % likviditu 1. stupňa. Ukazovatele likvidity respondenti sledujú prevažne s mesačnou periodicitou.

Skupina ukazovateľov na kontrolu a hodnotenie investícií patrí k druhým v poradí, ktoré respondenti využívajú hneď po ukazovateľoch likvidity. Využíva ich priemerne 46,2 % respondentov, z toho 48,9 % mikro, 45,7 % malých, 43,5 % stredných a 46,1 % veľkých podnikov. Najviac respondentov venuje pozornosť kontrole rentability investície (80,9 %) a doby návratnosti investície (77,1 %). Tieto ukazovatele počítajú nepravidelne, podľa potreby resp. s ročným opakovaním (rentabilitu 21,2 % a návratnosť 22,1 % respondentov).

Ukazovatele majetkovej a finančnej štruktúry pomáhajú pri vertikálnej a horizontálnej analýze finančných výkazov a pri kontrole dodržiavania „zlatých" pravidiel financovania. Pri finančnom riadení tieto ukazovatele využíva priemerne 44,5 % respondentov, z toho 33,3 % mikro, 47,7 % malých, 51,2 % stredných a 61,7 % veľkých podnikov. Z týchto ukazovateľov respondenti preferujú podiel obežných aktív na aktívach spolu (50,8 % respondentov), ktorý vyhodnocujú ročne (21,8 % respondentov) prípadne mesačne (12,2 % respondentov), a podiel dlhodobých pasív a dlhodobých aktív (45,0 %), ktorý 33,3 % respondentov počíta mesačne a 22,2 % ročne.

Vertikálnu a horizontálnu analýzu finančných výkazov, ako samostatný nástroj kontroly výstupov finančného hospodárenia realizuje 47,8 % respondentov, z toho 28,72 % mikropodnikov, 55,56 % malých podnikov, 69,23 % stredne veľkých podnikov a 68,42 % veľkých podnikov. Najviac, 68,22 % respondentov, sa zameriava na horizontálnu analýzu výnosov, 67,29 % respondentov kontroluje vývoj nákladov a štruktúru výnosov a nákladov, 61,68 % robí vertikálnu analýzu svojich aktív a pasív, 60,75 % horizontálnu analýzu pasív a 58,88 % horizontálnu analýzu aktív. Najčastejšie uvádzanou periodicitou, s akou realizujú podniky tieto analýzy, je ročná (43,93 % respondentov) a mesačná (34,58 % respondentov).

Priemerne 42,7 % respondentov (36,8 % mikro, 41,8 % malých, 50,1 % stredných a 52,9 % veľkých podnikov) využíva ukazovatele aktivity a produktivity. Až 75,7 % respondentov monitoruje vzťah medzi pohľadávkami a záväzkami, 54,5 % kontroluje podiel finančného

majetku na pohľadávkach a 57,0 % respondentov ukazovateľ nákladovosti. Stav a vývoj týchto ukazovateľov kontroluje najvyšší podiel respondentov s mesačnou periodicitou.

Ukazovatele zadlženosti patria k ďalším v poradí, ktorých využívanie, pri kontrole finančnej situácie v podnikovej praxi, sme zisťovali. Zadlženosť podniku kontroluje priemerne 35,7 % respondentov, z toho 27,7 % mikro, 37,5 % malých, 38,9 % stredných a 54,6 % veľkých podnikov. Ukazovateľmi, ktoré monitoruje najvyšší podiel respondentov sú zadlženosť z obchodného styku (47,6 %) a koeficient zadlženosti ako podiel záväzkov spolu na aktívach podniku, ktorý sleduje 18,1 % respondentov ročne a 8,5 % mesačne.

Pomocou ukazovateľov rentability a výnosnosti kontroluje svoje hospodárenie len 33,4 % podnikov (28,4 % mikro, 34,6 % malých, 33,6 % stredných a 47,4 % veľkých podnikov). Pri kontrole rentability a výnosnosti počítajú respondenti prevádzkovú maržu, ako podiel prevádzkového zisku na tržbách (52,4 %), rentabilitu tržieb a výnosnosť vlastných zdrojov (51,8 %) a rentabilitu aktív (47,1 %). Ročne kontrolujú svoje hospodárenie pomocou výnosnosti vlastných zdrojov (25,5 %) a rentability aktív (23,4 %), mesačne kontrolujú rentabilitu tržieb (16,0 %) a prevádzkovú maržu (21,8 % podnikov).

Ukazovatele trhovej hodnoty využíva najnižší podiel podnikov(13,4 %). Z hľadiska veľkostnej štruktúry je to 11 % mikro, 8,2 % malých, 22,2 % stredných a 23,4 % veľkých podnikov. Najvyšší podiel respondentov kontroluje výšku čistého zisku na akciu (17,5 %), účtovnú hodnotu akcií (15,3 %) a P/E ratio (14,8 %). Tieto ukazovatele sleduje najvyšší počet respondentov s ročnou periodicitou.

Predpovedné modely pri finančnej kontrole využíva 25,5 % mikropodnikov, 25 % malých podnikov, 38,5 % stredných a 47,4 % veľkých podnikov. Najvyšší podiel podnikov (71,2 %), ktoré tieto modely využívajú, pomocou nich kontroluje aktuálnu finančnú situáciu. Až 30,30 % podnikov ich využíva aj pri kontrole finančnej situácie minulých období a 25,8 % podnikov nimi kontroluje vhodnosť zostavenia svojich finančných plánov.

Vysvetliť vzťahy medzi finančnými ukazovateľmi, zdôvodniť ich veľkosť, zmeny a príčiny odchýlok, je jednou z fáz procesu odhaľovania vplyvu rizikových faktorov na hospodárenie podniku. V tejto fáze majú podniky možnosť využiť viaceré metódy rozkladu finančných ukazovateľov, ku ktorým patria pyramidálny rozklad, aditívny rozklad, multiplikatívny rozklad syntetického ukazovateľa na analytické ukazovatele, metóda reťazového dosadzovania a logaritmická metóda. Zisťovali sme, ktorú z týchto základných metód podniky používajú. Najvyšší podiel podnikov (31,6 %) používa pyramidálne rozklady syntetických finančných ukazovateľov na analytické. Ostatné metódy využíva 11,4 % a menej respondentov. Najvyšší priemerný podiel podnikov, ktoré využívajú tieto metódy, patrí do veľkostnej kategórie stredne veľkých podnikov (61,54 %), nasledujú veľké podniky (priemerne 57,9 %), malé podniky (48,6 %) a mikropodniky (26,6 %).

Zo štatisticko-matematických metód respondenti využívajú pri finančnom riadení najviac grafické metódy (60,42 % respondentov), výpočty priemerov (39,58 %), bodové odhady (18,75 %) a rozklady ukazovateľov (11,81 % respondentov). Využívanie aj týchto metód rastie s rastom veľkostnej kategórie podniku (56,38 % mikro, 59,72 % malé, 79,49 % stredné a 89,47 % veľké podniky).

Rast podielu respondentov, využívajúcich nástroje finančno-ekonomickej analýzy pri riadení finančného hospodárenia a odhaľovaní vznikajúcich rizík sme zaznamenali pri vyhodnocovaní viacerých ukazovateľov. Z uvedeného dôvodu testujeme vyššie formulovanú hypotézu.

Výsledky testu formulovanej hypotézy uvádzame v tabuľke 2, ktorá obsahuje hodnotu symetrickej závislosti (Spearman Correlation) a hodnotu asymetrickej závislosti (Somers´d), pri ktorej závislou premennou je rozsah využívania ukazovateľov pri finančnom riadení a nezávislou premennou veľkosť podniku. O platnosti nulovej hypotézy budeme rozhodovať

na základe p – hodnoty (Sig. (2-tailed)), ktorú sme merali obojstranným testom metódou Monte Carlo.

Tabuľka 2: Korelácia rozsahu využívania ukazovateľov a veľkosti podniku

	Somers' d		Spearman Correlation	
	d	Monte Carlo Sig. (2-tailed)	r_s	Monte Carlo Sig. (2-tailed)
Ukazovatele aktivity a produktivity	0,297	0,000[a]	0,362	0,000[a]
Ukazovatele hodnotenia investícií	0,275	0,000[a]	0,359	0,000[a]
Ukazovatele likvidity	0,305	0,000[a]	0,345	0,000[a]
Ukazovatele rentability a výnosnosti	0,289	0,000[a]	0,362	0,000[a]
Ukazovatele štruktúry	0,304	0,000[a]	0,384	0,000[a]
Ukazovatele trhovej hodnoty	0,140	0,000[a]	0,264	0,000[a]
Ukazovatele zadlženosti	0,275	0,000[a]	0,355	0,000[a]
Vlastné ukazovatele	0,025	0,000[a]	0,147	0,000[a]
Netriedené ukazovatele spolu	0,244	0,000[a]	0,324	0,000[a]

a. Based on 10000 sampled tables with starting seed 1660843777.
Prameň: Vlastné spracovanie

Z dosiahnutých výsledkov vidíme, že vo všetkých prípadoch môžeme na zvolenej hladine významnosti $\alpha = 0,05$ zamietnuť nulovú hypotézu a prijať alternatívnu hypotézu, a môžeme tvrdiť, že existuje pozitívna závislosť medzi veľkosťou podniku (meranou počtom zamestnancov) a rozsahom využívania vybraných ukazovateľov finančnej analýzy. Intenzitu závislosti interpretujeme podľa Cohena (1988) ako slabú, pri ktorej sa absolútna hodnota d resp. r_s pohybuje v intervale od 0,1 do 0,3 až strednú závislosť (pri ktorej sa absolútna hodnota d resp. r_s pohybuje v intervale od 0,3 do 0,5).

4. Záver

Z výsledku výskumu vyplynulo, že rozsah využívania nástrojov a metód finančno-ekonomickej analýzy, v podobe finančných ukazovateľov a rôznych matematicko-štatistických postupov, rastie s rastom veľkostnej kategórie podnikov meranej počtom zamestnancov.

Pri výskume využívania jednotlivých ukazovateľov pri finančnom riadení a kontrole sme zistili, že medzi päť ukazovateľov, ktoré využíva najvyšší podiel respondentov patrí rentabilita investícií, ktorú vyhodnocuje 80,95 % respondentov, doba návratnosti investícií (77,14 %), podiel pohľadávok a záväzkov (75,66 %), okamžitá platobná schopnosť (67,72 %) a rozdiel voľných peňažných prostriedkov a splatných záväzkov (61,38 %).

References

[1] Boďa, M. a Úradníček, V., (2016). Zahrnutie váh a ich neistoty do kvantifikácie v rámci pyramidálneho rozkladu finančného ukazovateľa. *Ekonomický časopis: časopis pre ekonomickú teóriu, hospodársku politiku, spoločensko-ekonomické prognózovanie.* Bratislava: Ekonomický ústav SAV a Prognostický ústav SAV. 2016, 64(1), pp. 70-92.

[2] Cohen, J., (1988). Statistical power analysis for the behavioral sciences. 2nd ed. Lawrence Erlbaum Associates, Inc..

[3] Dluhošová, D. a Zmeškal, Z., (2014). Company financial performance determination by applying the data envelopment analysis (DEA) method. *Managing and Modelling Of Financial Risks: 7th International Scientific Conference, PTS I-III*. Ostrava: VŠB - Technická univerzita Ostrava, pp. 159-166.

[4] Grofčíková, J., (2010). Nástroje hodnotenia finančnej situácie podniku. *Zborník z medzinárodnej vedeckej konferencie Finance a management v teorii a praxi*. Ústí nad Labem: Universita Jana Evangelisty Purkyně v Ústí nad Labem, Fakulta sociálně ekonomická, pp. 23-29.

[5] Hálek, V., (2012). Are we able to predict the financial status of the company via the CCB module? *Managing and Modelling Of Financial Risks - 6th International Scientific Conference Proceedings, Pts 1 And 2*. Ostrava: VŠB - Technická univerzita Ostrava, pp. 229-239.

[6] Hiadlovský, V. a Kráľ, P., (2014). A few notes to business financial health prediction. *Managing and modelling of financial risks: 7th international scientific conference*. Ostrava: VŠB - Technická univerzita Ostrava, pp. 248-255.

[7] Karas, M. a Režňáková, M., (2014). Can the same indicators serve for the prediction of bankruptcy in a different environment? *Managing and Modelling Of Financial Risks: 7th International Scientific Conference, PTS I-III*. Ostrava: VŠB - Technická univerzita Ostrava, pp. 335-343.

[8] Lesáková, Ľ., (2004). Metódy hodnotenia výkonnosti malých a stredných podnikov. Banská Bystrica: Univerzita Mateja Bela.

[9] Lesáková, Ľ. a Grofčíková, J., (2013). Využívanie výsledkov finančnej analýzy ako nástroja finančnej kontroly. *Ekonomika a spoločnosť: journal of economics and social research, vedecký časopis Ekonomickej fakulty Univerzity Mateja Bela v Banskej Bystrici*. Banská Bystrica: Vydavateľstvo Univerzity Mateja Bela - Belianum. 2013, 14(2), pp. 172-182.

[10] Marinič, P., (2008). *Plánovaní a tvorba hodnoty firmy*. Praha: Grada Publishing.

[11] Marková, V. a Lesníková, P., (2015). Motívy a bariéry uplatňovania trvalo udržateľného rozvoja v podnikoch vybraných odvetví na Slovensku. *Ekonomika a spoločnosť: vedecký časopis Ekonomickej fakulty Univerzity Mateja Bela v Banskej Bystrici*. Banská Bystrica: Univerzita Mateja Bela, Ekonomická fakulta. 2015, 16(1), pp. 97-105.

[12] Musa, H., Krištofík, P. a Debnárová, L., (2014). Corporate governance and its impact on financial risk. *Managing and modelling of financial risks: 7th international scientific conference*. Ostrava: VŠB - Technická univerzita Ostrava, pp. 529-535.

[13] Neumaierová, I. a Neumaier, I., (2002). Výkonnost a tržní hodnota firmy. Praha: Grada Publishing.

[14] Odporúčanie Komisie 2003/361/ES zo 6. mája 2003 o definícii malých a stredných podnikov.

[15] Sedláček, J., (2007). Finanční analýza podniku. Brno: Computer Press.

[16] Smejkal, V. a Rais, K., (2003). Řízení rizik. Praha: Grada Publishing.

[17] Zákon NR SR č. 431/2002 Zb. z. o účtovníctve.

Analyzing Hedging Exchange Rate Risk in Gold Market

Haochen Guo [1]

Abstract

Gold acted as a variety of roles in portfolio investment. Why invest in gold? Gold can achieve wealth preservation, as tool to against inflation, it can also be used as a currency hedge. Gold has been utilized as an all-around acknowledged currency all through history. Since it can't be corrupted and it is nobody's obligation, gold can help investors hedge some of the risks associated with foreign assets. The paper proposes a multi-currency cross gold hedging strategy that minimizes the exchange risk. The study examines whether gold is used to hedge against exchange rate risks globally.

Key words

Hedging Strategy, Gold, Exchange Rate Risk

JEL Classification: G11, G32

1. Introduction

Contribute Gold, for people holding gold, such investment means a putting something aside for the future–either a little sparing or a more considerable long–term investment–and they purchase gold or gold–backed budgetary items to ensure their riches and increment their risk adjusted returns. Having measure of gold in an adjusted venture portfolio may decrease their general speculation hazard and secure against business sector stuns. For institutional financial specialists, putting resources into gold is a benefit portion procedure. Be that as it may, as indicated by the hypothesis of portfolio danger administration, the most convincing advantages of gold get from its absence of connection with other resource classes, making it a profitable apparatus for portfolio broadening and hazard administration in a benefit distribution procedure. For speculation directors in banks, gold can convey a material descending effect on instability and increment their danger balanced returns. What's more, gold has some one of a kind attributes as a portfolio diversifier: it is a hedge against inflation and an effective balance against currency risks.

How to contribute Gold? There are numerous approaches to put resources into gold. Diverse items can be utilized to accomplish an assortment of speculation destinations. Financial specialists ought to consider the choices accessible in their business sector, the type of venture that is suitable to their conditions, and the way of expert guidance they will require. Investors can purchase physical gold through coins or bars; they can purchase items upheld by physical gold, which offer direct presentation to the gold; or they can purchase other gold-connected items, which are straightforwardly identified with the gold however do excluded responsibility for. As of late, development has prompted items that offer more prominent adaptability and availability to investors, for example, ETFs and extra hazard administration

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devices for complex financial specialists, including subordinates and organized items. See Figure 1. The different gold-related venture items have distinctive hazard and return profiles, liquidity attributes and expenses. Ordinarily, an advantage distribution procedure will consider long haul versus medium-term returns, and how gold venture items perform in positive or negative relationship with different resources. [31] [32] [33].

Figure 1: 10 years' commodity futures price quotes for gold



Source: Nasdaq

Since gold is utilized as a hedging tool against different other currencies, adding gold to a benefit portfolio makes the portfolio more effective. The capacity of gold to hedge against exchange rate fluctuations and offset changes in the domestic purchasing power of a country's currency has two implications. In the first place, First, when the domestic price index of a country goes up, the price of gold as measured in the local currency increases at the same time. In it increases proportionately, it is a perfect internal hedge. If the ratio is less than 1, it is a partial internal hedge. Second, when an increase in gold price (as measured in the local currency) is equivalent to a drop in the exchange rate of the local currency compared with a foreign currency, it is a perfect external hedge. When the ratio is less than 1, it is a partial external hedge.

The hedging capacity of gold is not constrained to particular currencies; its hedging viewpoints can likewise vary between gold supplier and demander. Gold creating and gold expending nations are prone to consider the instabilities emerging from future changes in return rates, which could bring about gold costs to vary. investors, hedgers, and central banks an account with national stores should likewise consider the communication amongst gold and the key currencies. Swapping scale changes impacts affect the choices of gold makers, gold purchasers, and financial specialists to hold gold. Since the thankfulness and devaluation of a nation' cash lead to changes in the costs of outside contrasted and local merchandise, the monetary, creation and money related speculation conduct of the nation is straightforwardly invigorated. In this manner influencing the costs of monetary instruments and the connection of different resources. At the point when a nation is influenced by inside and outer effects, the asymmetries activated by cash changes specifically influence its different markers of monetary and money related execution, and gold makers, shoppers, and financial specialists change their practices properly.

From the above discourse, we construe two key focuses. First, although the use of gold as a currency hedge is important, the effectiveness of this practice varies between the major gold-

producing, gold-consuming, and key currency countries. Second, the uncertainties due to exchange rate fluctuations need to be considered. The paper proposes a multi-currency cross gold hedging strategy that minimizes the exchange risk. The study examines whether gold is used to hedge against exchange rate risks internationally.

2. Literature review

The main hedging motives are the minimization of the impact of foreign exchange rate fluctuations on the variability of the firm's operational cash flow and the reduction of the probability of financial distress and bankruptcy [15] [29]. Gold prices and related models have centered three main issues:

- The relationship between gold prices and macroeconomic variables such as the currency exchange rate, interest rate, and income [4] [13] [14] [24] [25] [26] [27] [28];
- The factors affecting fluctuations in gold prices [9] [12] [18] [21];
- The short and long postion relationships between gold prices and commodity price indices and the effectiveness of using gold as a hedge against inflation [10] [17] [19] [20]. [1] [30].

3. Hedging the foreign exchange risk

The hedge decision requires to establish both the optimal hedge ratio and the risk measure that needs to be abbreviated. [1].

3.1 Optimal hedge ratio

Suppose that there is a set of two currencies with return r_0 and r_1. Cross hedging implies that a short (long) position in a currency is used to hedge a long (shot) position in the other, assuming that both currencies are positively correlated. Cross hedging could exploit the correlation with more than one currency in order to reduce the hedge portfolio risk. in the case of n currencies the hedge portfolio return can be expressed as,

$$r_h = r_0 + \sum_{i=1}^{n} h_i r_i \text{ ,} \tag{1}$$

where r_0 represents a long or short position in a currency, r_i represents a long 0 short position in a currency i in order to hedge and r_h represents the hedge portfolio return. One of the most important issues in hedging refers to the determination of the optimal hedge ratios, h_i. The optimal hedge ratio depends critically on the particular objective function to be optimized and the measure of risk considered. Depending on the objective function the problem to solve is single-objective, since the risk of the hedge portfolio return is minimized, or multi objective, because the risk and the expected return of the hedge portfolio are minimized/maximized simultaneously.

The most widely used optimal hedge ratio is the so-called minimum-variance (MV) hedge ratio. This is a solitary target issue where the risk, measured with the variance, is minimized. This MV fence proportion is determined by minimizing the variance of the hedged portfolio and it is very easy to comprehend and evaluate. In any case, the MV hedge ratio overlooks the normal return of the hedged portfolio thus as a rule, the MV hedge ratio is not reliable with the mean-variance structure. To make this hedge ratio steady with the mean-variance structure, it is important to incorporate the normal profit for the hedged portfolio in the target work unequivocally. [11]. Under return-risk hedge ratios, expected return and risk of the hedged portfolio are considered. Companies determine the expected returns and risk, and as a

consequence, the optimal hedging is obtained. When variance is used to measure risk, this approach is called mean-variance hedge ratio.

3.2 Hedge risk measurement

The diverse measures of portfolio risk can be portrayed in a few ways. The most essential attributes allude to the cognizance of the proposed measure and to its capacity to manage the asymmetry of the returns capacity dispersion.

Risk measurement can be viewed as a single number $p(r)$ assigned to the distribution of the portfolio return r. It is said that a risk measurement is coherent if it satisfies four properties: monotonicity, translation invariance, homogeneity and subadditivity. [6]. Standard deviation and CVaR satisfy the four properties while VaR satisfies three of them but it does not satisfy subadditivity under certain conditions. [2]. An optimization problem that includes non-coherent measures is usually ill-posed, which means that it does not have a single and exact solution.

Risk measures can be also classified in symmetric and asymmetric measures. Symmetric measures are those that do not take into consideration the asymmetry of the return function distribution, such as variance or standard deviation. Their use is only appropriate when those functions are normally distributed or, at least, symmetric. Asymmetric risk measures are those that take into consideration the skewness and kurtosis of the return function distribution. [16].

Some of the advantages of VaR are that it takes into account the asymmetric risk, a temporal period and a confidence level. VaR can be defined as the maximum expected loss within an investment horizon of n days with an error probability of $\alpha\%$. By definition, VaR is a quantile of the probability distribution of the portfolio value. Let $f(rh)$ be the probability distribution function of the future portfolio return and α the significant level (usually 1% or 5%), VaR is implicitly defined in the following equation,

$$\int_{-\infty}^{VaR} f(r_h)dr = \alpha \,. \tag{2}$$

VaR can be computed by using an analytic method or Delta Normal, a Montecarlo method or a historical simulation method. The analytic method assumes that returns are normally distributed and that VaR is proportional to the variance. In the Montecarlo method, simulations are carried out to generate returns assuming that the return distribution function is known and not necessarily symmetric. Finally, the historical simulation method does not make any assumption regarding the return distribution function. It is based on the idea that past behavior is a good predictor of future behavior. In this work VaR si computed by using a historical simulation method.

It uses a multi objective genetic algorithm to obtain the optimal hedge ratios when VaR is considered as objective function to minimize. CVaR is a risk measure that has many of the advantages of VaR and quite a few less disadvantages. [3] [7]. CVaR takes into account asymmetric risk, a temporal period and a confidence level. Propose CVaR as a coherent risk measure, it deals with the kurtosis and skewness of the return distribution function and exact solutions may be found for an optimization problem. [22]. CVaR is defined as the average of all losses exceeding the VaR and it is computed as the expected value of r conditional on exceeding the VaR,

$$CVaR_{\alpha} = \frac{\int_{-\infty}^{VaR} r_h f(r_h)dr}{\int_{-\infty}^{VaR} f(r_h)dr} \,. \tag{3}$$

CVaR is closely linked to VaR. it computes CVaR by using a historical simulation method, as in VaR case. That is, we obtain CVaR from the entire distribution of historical returns, as the sample mean of $r_{h,j}$ lower than VaR.

$$CVaR_{\alpha}(r_h) = (\alpha T)^{-1} \sum_{j=1}^{\alpha T} r_{h,j} \,. \tag{4}$$

Under the historical simulation the CVaR and VaR hedge ratios may be different. Given that risk minimization turns the optimization problem in a well-posed problem if CVaR is used, or ill-posed problem if VaR is used, uses a linear programming or a multi objective genetic algorithm to obtain optimal hedge ratios depending on whether CVaR or VaR is used to measure risk.

3.3 Mean risk hedge ratio problems

The mean-VaR hedge ratio for cross-hedging a long position with long or short positions in other currencies can be obtained by solving the following multi objective problem:

$$\min_h \quad VaR_\alpha(h) = \inf\{ r_h / F(r_h) \geq \alpha \}$$

$$\max_h \quad r_h = r_0 + \sum_{i=1}^{n} h_i r_i \tag{5}$$

Portfolio optimization problems attempt to obtain the smallest risk value for a given return, or the highest return for a certain risk level. Logically, using VaR as a risk measure, the optimization problem is a typical ill-posed problem, because VaR is difficult to optimize for discrete distributions since is non-convex and has multiple local extrema. [2].

In demarcation to VaR, in the case of discrete random variables with T possible outcomes, it is possible to linearize CVaR by introducing a vector of auxiliary variables. CVaR is replaced by a linear function in the objective function. Then, the mean-CVaR hedge ratio for cross-hedging a long or short position with long or short positions in other currencies can be obtained solving the following linear problem. [22] [23].

$$\min \quad CVaR_\alpha(h) = \min_{VaR,h,z} -VaR + \frac{1}{\alpha T} \sum_{j=1}^{T} z_j$$

s.t.

$$z_j \geq -r_0 + \sum_{j=1}^{T} h_i r_{ij} + VaR \qquad \forall j = 1,\ldots,T$$

$$\bar{r}_0 + \sum_{i=1}^{n} h_i \bar{r}_i \geq r_h^* \tag{6}$$

4. Conclusion

Developments in exchange rates are a major risk for companies with foreign currency based activities. Different approaches, such as hedging via forwards, currency swaps, futures options and many other complex financial instruments, have been utilized in order to effectively manage risk. Multi-currency cross hedging is relevant because it greatly expands the opportunity set of risk reducing alternatives. Optimal hedge strategy that minimizes VaR is different from the minimum CVaR hedge strategy. The methodology depicted could help to hedgers to obtain the optimal minimum VaR hedge strategy through a multi objective genetic algorithm. Portfolio diversification has the potential to substitute the hedged portfolios with derivatives, which can help small and medium-sized enterprises, but one that remains relatively neglected by many investors. While not hedging is understandable for investors who wish to take a view on currencies, many investors are not equipped or willing to do as such. Actually, investigate proposes that hedging exchange rate risk is generally a superior alternative. Foreign exchange hedging has been proven to lower portfolio risk.

References

[1] Aivarez-Diez. S., Alfaro-Cid. E., Fernandez-Blanco. M.O. (2016). Hedging foreign exchange rate risk: multi-currency diversification. *European Journal of Management and Business Economics*, 25, pp. 2-7.

[2] Alexander. S., Coleman. T.F., Li. Y. (2006). Minimizing CVaR and VaR for a portfolio of derivatives. *Journal of Banking and Finance*, 30, pp. 583–605.

[3] Alfaro-Cid. E., Baixauli-Soler. J.S., Fernández-Blanco. M.O. (2011). Minimising value-at-risk in a portfolio optimisation problem using a multi-objective genetic algorithm. *International Journal of Risk Assessment and Management*, 5 (5/6), pp. 453–477.

[4] Ariovich, G. (1983). The impact of political tension on the price of gold. *J. Stud. Econ. Econom.*, 16, pp. 17–37.

[5] Arouri. M.E.H., Lahiani. A., Nguyen. D.K. (2015). World gold prices and stock returns in China: insights for hedging and diversification strategies. *Economic Modelling*. 44, pp. 273-282.

[6] Artzner. P., Fl Delbaen., Erber. J.M., Heath. D. (1999). Coherent measures of risk. *Mathematical Finance*, 9, pp. 203–228.

[7] Baixauli-Soler. J.S., Alfaro-Cid. E., Fernández-Blanco. M.O. (2011). Mean-VaR portfolio selection under real constrains. *Computational Economics*, 37 (2), pp. 113–131.

[8] Baixauli-Soler. J.S., Alfaro-Cid. E., Fernández-Blanco. M.O. (2010). Several risk measures in portfolio selection: Is it worthwhile? *Spanish Journal of Finance and Accounting*, 39 (147), pp. 421–444

[9] Baker. S.A., Tassel van. R.C. (1985). Forecasting the price of gold: a fundamentalist approach. *Atl. Econ. J.*, 13, pp. 43–51.

[10] Chappell. D., Dowd. K. (1997). A simple model of the gold standard. J. *Money Credit Banking*, 29, pp. 94–105.

[11] Chen. S.S., Lee. C.F., Shrestha. K. (2008). Do the pure martingale and joint normality hypotheses hold for futures contracts? Implications for the optimal hedge ratios. *The Quarterly Review of Economics and Finance*, 48, pp. 153–174.

[12] Diba. B., Grossman. H. (1984) Rational Bubbles in the Price of Gold, NBER Working Paper 1300. *National Bureau of Economic Research*, Cambridge, MA.

[13] Dooley. M.P., Isard. P., Taylor. M.P. (1995). Exchange rates, country-specific shocks and gold. *Appl. Financ. Econ.*, 5, pp. 121–129.

[14] Fortune, J.N. (1987). The inflation rate of the price of gold expected prices and interest rates. *J. Macroecon*., 9 (1), pp. 71–82.

[15] Hagelin, N. (2003). Why firms hedge with currency derivatives: An examination of transaction and translation exposure. *Applied Financial Economics*, 13, pp.55-69.

[16] Harris. R.D.F., Shen. J. (2006). Hedging and value at risk. *The Journal of Futures Markets*, 26 (4), pp. 369–390.

[17] Kolluri. B.R. (1981). Gold as a hedge against inflation: an empirical investigation. *Q. Rev. Econ. Bus*., 21, pp. 13–24.

[18] Koutsoyiannis. A. (1983). A short-run pricing model for a speculative asset, tested with data from the gold bullion market. *Appl. Econ*., 15, pp. 563–581.

[19] Laurent. R.D. (1994). Is there a role for gold in monetary policy? *Econ. Perspect*., 18, pp. 2–14.

[20] Moore. G. (1990). Gold prices and a leading index of inflation. *Challenge*, 33, pp. 52–56.

[21] Pindyck. R.S. (1993). The present value model of rational commodity pricing. *Econ. J.*, 103, pp. 511–530.

[22] Rockafellar. R.T., Uryasev. S. (2000). Optimization of conditional value at risk. *Journal of Risk*, 2, pp. 21–41.

[23] Rockafellar. R.T., Uryasev. S. (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking and Finance*, 26, pp. 1443–1471.

[24] Sherman E. (1986). Gold Investment: Theory and Application. *Prentice Hall*, New York.

[25] Sherman. E. (1982). New gold model explains variations. *Commodity J., 17*, pp. 16–20.

[26] Sherman. E. (1983). A gold pricing model. *J. Portf. Manag*., 9, pp. 68–70.

[27] Sjaastad. L.A. (2008). The price of gold and the exchange rates: once again. *Resour. Policy*, 33, pp. 118–124.

[28] Sjaastad. L.A., Scacciallani. F. (1996). The price of gold and the exchange rate. *J. Money Finance*, 15, pp. 879–897.

[29] Solomon, J.F., Joseph, N.L. (2000). Which corporate hedging motives are appropriate? An institutional shareholders' perspective. *International Journal of Finance and Economics*, 5, pp339-347.

[30] Wang. K.M., LEE. Y.M. (2016). Hedging exchange rate risk in the gold market: a panel data analysis. *J. of Multi. Fin. Manag.* 35, pp. 1-23.

[31] World Gold Council. (2013). Gold Investor: Risk management and capital preservation, Vol. 4, Why invest in gold? Gold's role in long-term strategies.

[32] World Gold Council. (2013). Gold Investor: Risk management and capital preservation, Vol.4, Gold and currencies: the evolving relationship with the US dollar.

[33] World Gold Council. (2013). Gold Investor: Risk management and capital preservation, Vol.1, Gold and currencies: hedging foreign-exchange risk.

Optimization of Deliveries within BM under the Non-uniform Consumption's Assumption[1]

Petr Gurný[2]

Abstract

The paper is devoted to the optimization within management of current assets under the assumption of violation of uniform consumption. Quadratic consumption is considered, particularly. The goal of the paper is to derive the general formula for calculation of the optimum quantity for Baumol's model under the assumption of quadratic consumption and to analyse relationship between evolution of the consumption and optimum quantity of current assets order. It was found that the slower growth of consumption leads to the lower total cost at higher optimal quantity.

Key words

Current assets management, Baumol model, nonlinear optimization.

JEL Classification: C01, C61, G3.

1. Úvod

Řízení pracovního kapitálu lze obecně zařadit mezi jedny z klíčových operativních rozhodnutí firmy ovlivňujících jak její výkonnost, tak zejména její likviditu. Pokud pomineme řízení krátkodobých závazků a pohledávek, je při řízení pracovního kapitálu pozornost věnována zejména řízení zásob a peněžních prostředků (dále v článku jen oběžná aktiva). K řízení těchto krátkodobých výrobních faktorů existuje řada přístupů a modelů, od intuitivního expertního řízení po modely JIC, JIT a ABC pro řízení zásob a Millerův-Orrův model (Miller a Orr, 1966) pro řízení peněžních prostředků. Zvláštní kapitolou jsou pak tzv. optimalizační modely, reprezentované zejména Baumolovým model (Baumol, 1952), které pro operativní řízení oběžných aktiv (zejména výši a čas pořízení) využívají optimalizaci vybraného kritéria. Tyto optimalizační modely v praxi nejsou příliš využívány, a to zejména pro jejich zjednodušující předpoklady. Jedním z těchto předpokladů je předpoklad rovnoměrné spotřeby daného oběžného aktiva v čase.

Cílem tohoto příspěvku je odvodit obecný vztah pro určení optimální dodávky oběžného aktiva při předpokladu nelineární spotřeby, konkrétně bude v příspěvku uvažována kvadratická spotřeba. Dále bude analyzován vztah mezi vývojem spotřeby a hodnoceným kritériem.

Odvozený vztah je pak možno kombinovat s dalšími pracemi zabývajícími se zreálněním zjednodušujících předpokladů optimalizačního Baumolova modelu. Jedna ze zdařilých prací na toto téma zahrnuje například zařazení mimořádných výdajů a náklady nedostatku hotovosti, viz Valecký (2011), nebo zahrnutí náhodné potřeby hotovosti pro uvažované období, viz Valecký (2016).

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2. Baumolův optimalizační model

Baumolův optimalizační model je model využívaný k nalezení optimální velikosti dodávky oběžného aktiva (ať už zásob nebo peněžních prostředků), a to pomocí minimalizace kritéria celkových nákladů. Zde se předpokládá, že celkové náklady zahrnují náklady na pořízení daného aktiva (ve formě akvizičních (transakčních) nákladů) a náklady na skladování tohoto aktiva (ať už například reálné náklady na skladování materiálu, nebo alternativní náklady držby peněžních prostředků v případě řízení hotovosti).

Tento jednoduchý princip optimalizace nicméně vychází z řady omezujících předpokladů:

- je známa celková potřeba daného oběžného aktiva pro následující období;
- jsou známy náklady na skladování jedné jednotky daného oběžného aktiva;
- je známa výše nákladů na pořízení jedné dodávky daného oběžného aktiva;[3]
- spotřeba daného oběžného aktiva je lineární (rovnoměrná).

2.1 Baumolův optimalizační model s lineární spotřebou

Pokud budeme uvažovat obecné předpoklady Baumolova optimalizačního modelu včetně lineární spotřeby daného oběžného aktiva, budou celkové náklady, *TC (Total Costs)* spojené s řízením určeny jako

$$TC = C_A \cdot \frac{R}{Q} + C_S \cdot \frac{Q}{2} \tag{1}$$

kde C_A jsou akviziční náklady *(Acquisition Costs)*, R je celková potřeba daného aktiva *(Requirements)*, C_S jsou náklady na skladování jednotky daného aktiva *(Storage Costs)* a Q je velikost dané akvizice *(Quantity)*. Výraz $\frac{Q}{2}$ ve druhém členu pravé strany rovnice (1) vyjadřuje průměrnou výši daného oběžného aktiva v uvažovaném období při lineární spotřebě. Optimální výší akvizice, Q_{opt}, pak nalezneme minimalizací (1), tedy řešením rovnice $\frac{\partial TC}{\partial Q} = 0$. Výsledný vztah pro optimální výši akvizice je tedy

$$Q_{opt} = \sqrt{\frac{2 \cdot C_A \cdot R}{C_S}}\,. \tag{2}$$

2.2 Baumolův optimalizační model s nelineární spotřebou

Pokud budeme uvažovat výše uvedené předpoklady, ale nebudeme uvažovat s lineární spotřebou daného aktiva, lze rovnice (1) obecně přepsat jako

$$TC = C_A \cdot \frac{R}{Q} + C_S \cdot z\,, \tag{3}$$

kde z je průměrná výše daného aktiva v uvažovaném období, což můžeme pro jakoukoliv funkci spotřeby vyjádřit jako

$$z = \frac{1}{T-0}\int_0^T f(t)dt\,, \tag{4}$$

[3] Tyto náklady mohou být jak fixního, tak variabilního charakteru, což ve výsledku neovlivňuje velikost optimální výše dodávky, viz Baumol (1952).

kde $f(t)$ je libovolná funkce vývoje spotřeby daného oběžného aktiva, přičemž pro jednotkový interval (uvažujeme jedno období), $T=1$, lze rovnici (4) přepsat jako

$$z=\int_0^1 f(t)dt\,. \tag{5}$$

Dosazením (5) do (3) získáme obecnou rovnici pro výši celkových nákladů při nelineární spotřebě

$$TC=C_A\cdot\frac{R}{Q}+C_S\cdot\int_0^1 f(t)dt\,. \tag{6}$$

Pokud budeme za $f(t)$ uvažovat kvadratickou funkci, tedy

$$f(t)=at^2+bt+c \tag{7}$$

pak pro $t=0$ musí platit, že $f(t)=Q$, tedy že $f(t)=at^2+bt+Q$, a pro $t=1$ musí platit, že $f(t)=0$, tedy že $0=a+b+Q$. Dosazením do (7) získáváme

$$f(t)=at^2+(-a-Q)t+Q\,. \tag{8}$$

Dále dosazením (8) do (5)

$$z=\int_0^1 (at^2-at-Qt+Q)dt$$

a řešením uvedeného integrálu získáme obecné vyjádření průměrné výše daného aktiva v uvažovaném období pro kvadratický průběh spotřeby

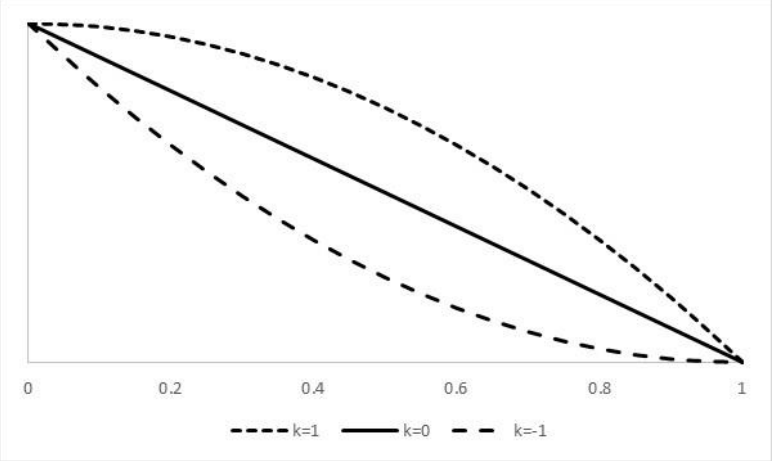
$$z=-\frac{1}{6}a+\frac{1}{2}Q\,. \tag{9}$$

A jelikož $f(t)\le Q$ (hodnota daného aktiva nemůže být za daných podmínek logicky vyšší, než hodnota nakoupené množství), tak $a\in\langle -Q;Q\rangle$. Tedy $a=f(Q)$, kde logicky $a=kQ$ pro $k\in\langle -1;1\rangle$. Tedy můžeme (9) upravit jako

$$z=\left(\frac{1}{2}-\frac{1}{6}k\right)\cdot Q \text{ pro } k\in\langle -1;1\rangle\,. \tag{10}$$

Parametr k v (10) můžeme označit jako koeficient spotřeby, přičemž pro $k>0$ platí, že spotřeba je na začátku období pomalejší a následně se zrychluje, pro $k<0$ platí opak a pro $k=0$ je spotřeba rovnoměrná, viz Obrázek 1.

Obrázek 1: Vývoj spotřeby oběžného aktiva při rozdílných koeficientech spotřeby



Dosazením (10) do (3) získáme vztah pro určení celkových nákladů při kvadratické spotřebě

$$TC = C_A \cdot \frac{R}{Q} + C_S \cdot \left(\frac{1}{2} - \frac{1}{6}k\right) \cdot Q, \tag{11}$$

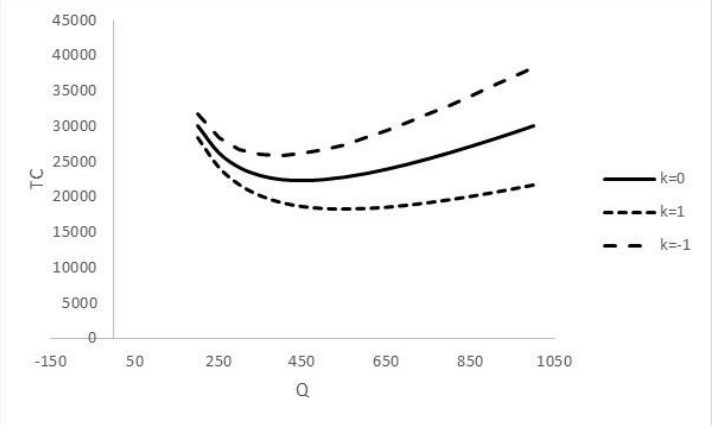
což pro $\frac{\partial TC}{\partial Q} = 0$ znamená, že můžeme vyjádřit vztah pro určení optimální velikosti dodávky daného oběžného aktiva jako

$$Q_{opt} = \sqrt{\frac{C_A \cdot R}{\left(\frac{1}{2} - \frac{1}{6}k\right) \cdot C_S}} \text{ pro } k \in \langle -1;1 \rangle. \tag{12}$$

3. Ilustrativní příklad

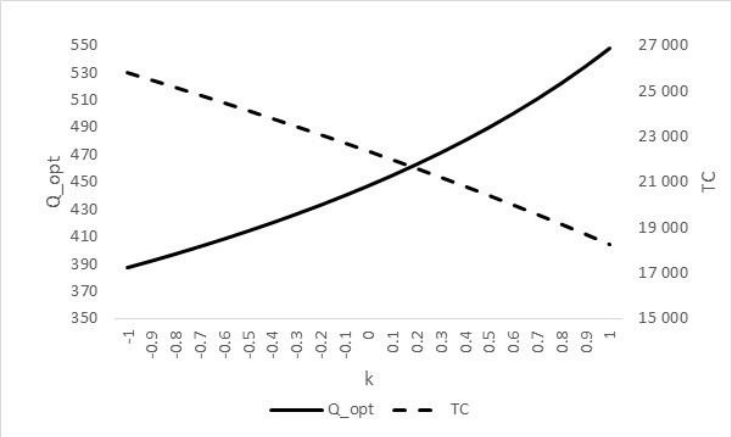
Předpokládejme, že chceme optimalizovat dodávku oběžného aktiva (například materiálu) v příštím období, jestliže víme, že celková potřeba materiálu je $R = 10000$, akviziční náklady jsou $C_A = 500$ a náklady na skladování jedné jednotky jsou $C_S = 50$. Celkové náklady na řízení tohoto materiálu se budou vyvíjet dle dvou parametrů: množství materiálu v jedné dodávce a průběhu spotřeby materiálu. Graficky je vztah mezi celkovými náklady, viz vzorec (11), a zmíněnými parametry zachycen v Obrázku 2.

Obrázek 2: Vztah mezi TC, Q a k



Následně můžeme spočítat velikost optimální dodávky pro různé koeficienty spotřeby dle (12). Vztah mezi koeficientem spotřeby a optimální dodávkou, stejně tak jako velikost celkových nákladů pro optimální dodávky v rámci různých koeficientů spotřeby jsou zachyceny v Obrázku 3.

Obrázek 3: Vztah mezi TC, Q opt a k



Z Obrázku 3 je zřejmé, že při pomalejší spotřebě na začátku období, situace kdy $k > 0$, je výhodnější zvětšovat množství jedné dodávky (a tím zmenšovat počet dodávek), přičemž při pomalejším nárůstu spotřeby, $k < 0$, je lepší dovážet častěji méně aktiv. Dalším zjevným závěrem je, že čím pomalejší je spotřeba, tím jsou při optimálních dodávkách celkové náklady nižší.

4. Závěr

V příspěvku byl upraven klasický Baumolův model řízení oběžných aktiv (zásoby a hotovost) pro předpoklad nerovnoměrné spotřeby. Byl odvozen vzorec pro výpočet optimální velikosti dodávky pro kvadratickou spotřebu, tedy pro různou hodnotu koeficientu spotřeby. Na ilustrativním příkladu bylo ukázáno, že při pomalejší spotřebě je výhodnější zvětšovat velikost jedné dodávky a vice versa.

References

[1] Baumol, W. J. (1952): The Transactions Demand for Cash: An Inventory Theoretic Approach. *Quarterly Journal of Economics 66,* p. 545-556.

[2] Miller, M. H.; Orr, D. (1966): A Model of the Demand for Money by Firms. *Quarterly Journal of Economics* 80, p. 413-435.

[3] Valecký, J. (2011): Aplikace Baumolova modelu řízení peněžních prostředků s mimořádnými výdaji. In conference proceedings: *8th International Scientific Conference on Financial Management of Firms and Financial Institutions*. Ostrava, pp: 40 – 48.

[4] Valecký, J. (2016). Baumol's model for cash management with random value of transactions. In conference proceedings: *8th International Scientific Conference on Managing and Modelling of Financial Risks*. Ostrava, pp: 1047 - 1054.

Verification of the Selected Bankruptcy Models' Prediction Power: Case of Czech Republic[1]

Petr Gurný[2], Marlene Brišová

Abstract

The paper is focused on the evaluation and the comparison of predicative ability of the selected bankruptcy prediction models in the condition of Czech Republic. The bankruptcy prediction models are applied on the sample of Czech companies in the construction and manufacturing industry. The goal of the paper is to verify suitability of using these models in the terms of Czech Republic in the chosen industry.

Key words

Credit risk; bankruptcy models; verification of the bankruptcy models.

JEL Classification: C52; G03.

1. Úvod

Hodnocení finanční výkonnosti a stability firem je jedním z klíčových úkolů v rámci finančního řízení a rozhodování. Zejména hodnocení finanční stability a z toho vyplývající možnosti defaultu firem je nezbytnou součástí řízení kreditního rizika a tedy správného ocenění alokovaného kapitálu. Pro odhad pravděpodobnosti defaultu je možno využít řady různých modelů. Jedním z nejvíce využívaných, zejména pro svou relativní aplikační jednoduchost a interpretovatelnost, jsou tzv. souhrnné modely hodnocení finanční úrovně (často také bankrotní modely), které kombinací základních poměrových ukazatelů finanční analýzy klasifikují firmy do oblastí s příslušnou pravděpodobností defaultu. Tyto modely byly odvozeny převážně pomocí diskriminační analýzy nebo logistické regrese a v rámci finanční teorie jich byla vyvinuta celá řada. Navzájem se odlišují především původem vstupních dat, na kterých byly dané modely odhadnuty (typ společností, geografické omezení, oblast podnikání apod.).

Cílem tohoto příspěvku je ověřit možnost aplikace vybraných bankrotních modelů v České republice v rámci stavebního a zpracovatelského průmyslu a statisticky otestovat jejich významnost.

2. Charakteristika vybraných bankrotních modelů

Rostoucí počet podniků a s ním spojená možnost vzniku bankrotu se stala předmětem zkoumání mnoha vědců, jejichž cílem bylo nalezení způsobu, kterým by bylo možné ohodnotit finanční situaci podniku pomoci jednoho čísla. Vznikly tak bankrotní modely, které jsou založeny na předpokladu, že nezdravý podnik vykazuje jisté symptomy, které jsou typické pro bankrotem ohrožené podniky. Konstrukce bankrotních modelů je založena na

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skutečných datech společností, které se v minulosti ocitly v úpadku a na datech společností, které jsou finančně zdravé a prosperující. Na základě těchto informací jsou pomocí statistických nástrojů odhadovány vztahy, jejichž výsledkem je hodnota, která vypovídá o míře ohrožení podniku bankrotem.

1.1 Altmanův model

Altmanův model patří mezi nejvýznamnější a světově nejpoužívanější bankrotní modely. Tento model má několik verzí, přičemž v tomto příspěvku bude analyzován model vytvořený pro společnosti veřejně neobchodované na kapitálových trzích. V rámci tohoto modelu se odhaduje tzv. Z'-Score, podrobněji viz Altman (2000), a to následovně

$$Z' = 0{,}717 \cdot X_1 + 0{,}847 \cdot X_2 + 3{,}107 \cdot X_3 + 0{,}420 \cdot X_4 + 0{,}998 \cdot X_5, \quad (1)$$

kde X_1 – čistý pracovní kapitál/celková aktiva, X_2 – nerozdělený zisk/celková aktiva, X_3 – zisk před úroky a daněmi /celková aktiva, X4 – účetní hodnota vlastního kapitálu/dluhy celkem, X_5 – tržby/celková aktiva. V případě, že je hodnota souhrnného indexu větší než 2,90, je finanční situace firmy velmi dobrá. Hodnoty výsledků v rozmezí 1,23 až 2,90 značí šedou zónu. Kandidáty bankrotu jsou pak ty firmy, jejichž výsledek Z'-Score je menší než je hodnota 1,23. (Altman a Hotchkiss 2006).

1.2 Tafflerův model

Tafflerův model se svou konstrukcí podobá Altmanovu Z-Score, avšak při jeho sestrojení se vycházelo z podmínek charakteristických pro Anglii. Výsledný vztah má následující podobu

$$T = 0{,}53 \cdot X_1 + 0{,}13 \cdot X_2 + 0{,}18 \cdot X_3 + 0{,}16 \cdot X_4, \quad (2)$$

kde X_1 – zisk před zdaněním/krátkodobé závazky, X_2 – oběžná aktiva/cizí zdroje, X_3 – krátkodobé závazky/aktiva, X_4 – tržby/celková aktiva. Výsledné hodnoty větší než 0,3 vypovídají o bonitním podniku. Šedá zóna je vymezena intervalem 0,2 až 0,3. K bankrotu spějí ty podniky, jejichž hodnota souhrnného indexu je menší než 0,2. Zvláštnosti Tafflerova modelu je, že ve srovnání s ostatními bankrotní modely se skládá pouze ze čtyř poměrových ukazatelů. Další zajímavosti je skutečnost, že nevyužívá ukazatele rentability aktiv, na kterou je u ostatních modelů kladen důraz.

1.3 Index IN95

Tento model byl odhadnut manžely Neumaierovými, viz Neumaierová a Neumaier (2002), a patří mezi jeden z řady modelů odhadnutých speciálně pro český trh. Index IN95 se skládá se ze šesti poměrových ukazatelů, které na rozdíl od Altmanova modelu, zahrnují podíl závazků po lhůtě splatnosti na celkových výnosech. Zavedením tohoto ukazatele byla vyřešena problematika platební neschopnosti, která představovala slabé místo tehdejší ekonomiky. Důležitými prvky indexu IN95 jsou váhy jednotlivých ukazatelů, které jsou stanoveny pro každé odvětví zvlášť, a které tak umožňují postihnout odvětvová specifika. Výsledný model má tvar

$$IN95 = V_1 \cdot X_1 + 0{,}11 \cdot X_2 + V_3 \cdot X_3 + V_4 \cdot X_4 + 0{,}10 \cdot X_5 - V_6 \cdot X_6, \quad (3)$$

kde X_1 – celková aktiva/cizí zdroje, X_2 – zisk před úroky a daněmi/nákladové úroky, X_3 – zisk před úroky a daněmi/celková aktiva, X_4 – výnosy/celková aktiva, X_5 – oběžná

aktiva/(krátkodobé závazky + krátkodobé bankovní úvěry), X_6 – závazky po splatnosti/výnosy. Váhy pro jednotlivá odvětví, stejně tak jako váhy pro ekonomiku jako celek (dále označováno jako IN95E) viz Neumaierová a Neumaier (2002). Pokud hodnota indexu IN95 je větší než hodnota 2, je podnik zařazen mezi finančně zdravé. Hodnoty v rozmezí 1 až 2 značí šedou zónu. Ve špatné finanční situaci se pak nacházejí ty podniky, jejichž hodnota indexu IN95 je menší než 1.

1.4 Index IN01

Tuto další z modifikací modelů IN můžeme označit jako komplexní variantu. Byl vytvořen v roce 2002 a vycházel z dat 1915 průmyslových podniků. Zkoumaný vzorek se skládal ze skupiny 583 podniků, které tvořily ekonomickou hodnotu. Dále ze skupiny 503 podniků v bankrotu nebo těsně před bankrotem a z 829 ostatních podniků, (Neumaierová a Neumaier 2002).

$$IN01 = 0{,}13 \cdot X_1 + 0{,}04 \cdot X_2 + 3{,}92 \cdot X_3 + 0{,}21 \cdot X_4 + 0{,}09 \cdot X_5, \tag{4}$$

kde X_1 – celková aktiva/cizí zdroje, X_2 – zisk před úroky a daněmi/nákladové úroky, X_3 – zisk před úroky a daněmi/celková aktiva, X_4 – výnosy/celková aktiva, X_5 – oběžná aktiva/(krátkodobé závazky + krátkodobé bankovní úvěry). V případě, že výsledná hodnota indexu IN01 je vyšší než 1,77 potom podnik tvoří hodnotu s pravděpodobností 67 %. Hodnoty v rozmezí 0,75 až 1,77 značí šedou zónu. S pravděpodobností 86 % spějí k bankrotu ty podniky, jejichž hodnota indexu IN je menší než 0,75.

1.5 Index IN05

Tato verze modelu je aktualizací modelu IN01 na datech z roku 2004, (Neumaierová a Neumaier 2005). Výsledný vztah získal tvar

$$IN01 = 0{,}13 \cdot X_1 + 0{,}04 \cdot X_2 + 3{,}97 \cdot X_3 + 0{,}21 \cdot X_4 + 0{,}09 \cdot X_5, \tag{5}$$

kde jsou jednotlivé ukazatele totožné jako u IN01, drobnou úpravou váhy u X_3 došlo rovněž k úpravě hodnotících intervalů. V případě, že je výsledná hodnota indexu IN05 vetší než 1,6, tvoří podnik hodnotu s pravděpodobností 67 %. Pokud se výsledky nachází v rozmezí hodnot 0,9, až 1,6 jedná se o tzv. šedou zónu. Hodnota menší než 0,9 znamená, že podnik spěje k bankrotu s pravděpodobností 86 %.

2 Aplikace bankrotních modelů na vybraný vzorek firem

V této subkapitole bude nejprve představen vzorek vstupních dat a následně budou pro vybrané společnosti kalkulovány hodnoty představených bankrotních modelů.

Pro analýzu bylo vybráno celkem 32 náhodně zvolených firem, z nichž 16 patří do oblasti stavebního a 16 do oblasti zpracovatelského průmyslu, přičemž poměr mezi defaultními a zdravými společnosti byl u obou oblastí 50:50. Tato odvětví byla zvolena z toho důvodu, že představují důležité složky národního hospodářství a mají významný vliv na vývoj tuzemské ekonomiky. Rozhodným dnem (*t*) byl zvolen rok 2013 pro defaultní společnosti, jelikož bylo v tomto roce podáno nejvíce insolvenčních návrhů, a rok 2015 pro nedefaultní společnosti. Vzhledem k charakteristice bankrotních modelů představených v kapitole 2 byly vstupní data pro výpočet jednotlivých modelů brána jeden rok před rozhodnou událostí (*t-1*), tedy z roku 2012.

Konkrétně pak byly analyzovány tyto ***defaultní firmy***:

Stavební průmysl: SKD Průmstav - stavby, a.s., Toužimská stavební společnost s.r.o., JC STAV s.r.o., THERMIA-BAU a.s., CESIS s.r.o., BORSTAV s.r.o., GEDOS spol. s.r.o., INTERMONT Karlovy Vary spol. s.r.o.,

Zpracovatelský průmysl: OTROKOVICKÉ PAPÍRNY, a.s., JIP - Papírny Větřní, a.s., DOTEX spol. s.r.o. Nový Jičín, ELEKTROSPOLEČNOST B.D. spol. s.r.o., ITS HK s.r.o., ALMAROS, s.r.o., ASTIN Catalysts and Chemicals, s.r.o., EPIAG LOFIDA – PORCELÁN CZ, s.r.o.

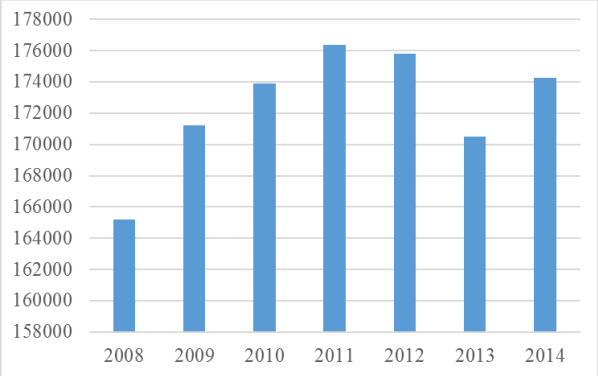
A tyto ***nedefaultní firmy***:

Stavební průmysl: STAMONT – POZEMNÍ STAVITELSTVÍ s.r.o., Barstav spol. s.r.o., Iskom s.r.o., STAVO – ČESTAV, s.r.o., JKNstavby s.r.o., STAKO – ČERVENÝ KOSTELEC s.r.o., HMS. spol. s.r.o., Stavitelství Šmíd s.r.o.

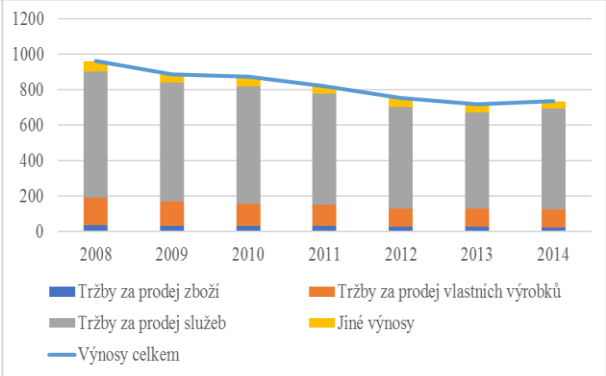
Zpracovatelský průmysl: JH GROUP, spol. s.r.o., MLÝNY J. VOŽENÍLEK, spol. s.r.o., BROUSÍCÍ STROJE KUČERA s.r.o., DESTILA, s.r.o., LUCID spol. s.r.o., STEPA s.r.o., KRPA FORM, a.s., EVERSTAR s.r.o.

Vývoj základních charakteristik pro stavební a zpracovatelský průmysl je uveden v Obr. 1 - 4.

Obr. 1: Vývoj počtu podniků ve stavebnictví

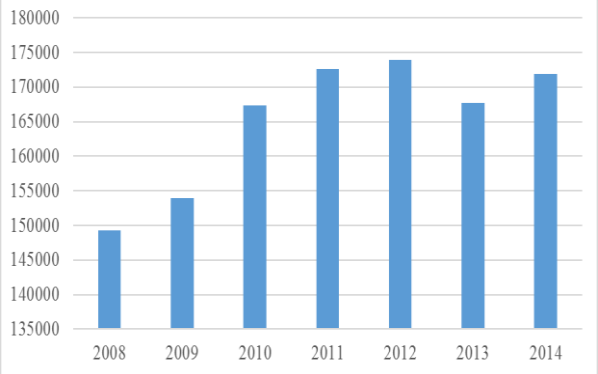


Obr. 2: Vývoj výnosů stavebních subjektů

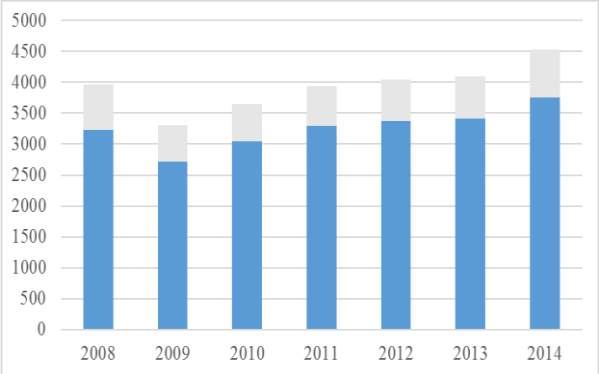


Zdroj: Stavebnictví České republiky (2015), http://www.mpo.cz/dokument172456.html

Obr. 3: Vývoj počtu podniků ve zpracovatelském prům.



Obr. 4 Vývoj výnosů ve zpracovatelském prům.



Zdroj: Panorama zpracovatelského průmyslu ČR (2014), http://www.mpo.cz/dokument162071.html

Aplikacemi vzorců (1) – (5) na vstupní data pak byly určeny následující hodnoty jednotlivých modelů pro čas *t-1*.

Tab. 1: Vývoj výsledků bankrotních modelů na datech z roku t-1 u defaultních stavebních společností

Stavební společnosti	*t-1*					
	Z'-Score	IN95 E	IN95	Tafflerův model	IN01	IN05
INTERMONT Karlovy Vary spol. s.r.o.	-0,568	-8,912	-8,418	0,032	-0,538	-0,544
SKD Průmstav - stavby, a.s.	1,648	-1,092	-1,236	0,525	0,722	0,723
BORSTAV s.r.o.	-0,489	-12,913	-10,994	0,173	-3,231	-3,272
CESIS s.r.o.	-2,513	-78,251	-74,402	0,223	-5,783	-5,838
GEDOS spol. s.r.o.	-0,095	-16,109	-14,335	0,003	-4,916	-4,953
Toužimská stavební společnost s.r.o.	-0,076	-16,587	-15,771	0,410	-1,452	-1,467
JC STAV s.r.o.	1,093	-3,365	-3,237	0,354	0,029	0,026
THERMIA-BAU a.s.	2,064	-1,966	-2,080	0,615	0,504	0,504

Tab. 2: Vývoj výsledků bankrotních modelů na datech z roku t-1 u nedefaultních stavebních společností

Stavební společnosti	*t-1*					
	Z'-Score	IN95 E	IN95	Tafflerův model	IN01	IN05
Stavitelství Šmíd s.r.o.	3,687	6,496	6,195	0,838	2,720	2,726
STAMONT s.r.o.	1,528	-0,042	-0,206	0,415	0,782	0,784
STAVO – ČESTAV, s.r.o.	4,882	6,273	6,640	0,780	2,810	2,812
HMS. spol. s.r.o.	5,230	3,882	3,784	1,012	1,916	1,917
Barstav spol. s.r.o.	3,229	2,370	2,339	0,739	1,722	1,726
Iskom s.r.o.	3,245	2,349	2,422	0,600	1,274	1,274
JKNstavby s.r.o.	4,422	4,652	3,951	0,943	1,970	1,974
STAKO Červený Kostelec s.r.o.	2,955	2,150	1,732	0,703	0,957	0,958

Tab. 3: Vývoj výsledků bankrotních modelů na datech z roku t-1 u defaultních společností zpracovatelského průmyslu

Společnosti zpracovatelského průmyslu	*t-1*					
	Z'-Score	IN95 E	IN95	Tafflerův model	IN01	IN05
ELEKTROSPOLEČNOST B.D. s.r.o.	-0,685	-23,154	-20,866	0,335	-6,573	-6,606
ASTIN Catalysts and Chemicals, s.r.o.	-0,216	-6,659	-5,640	-0,742	-1,760	-1,781
ITS HK s.r.o.	-13,296	-30,780	-26,571	5,702	-11,152	-11,322
DOTEX spol. s.r.o. Nový Jičín	2,163	0,704	1,095	0,580	1,085	1,087
OTROKOVICKÉ PAPÍRNY, a.s.	-0,840	-16,771	-12,681	0,442	-1,162	-1,170
ALMAROS, s.r.o.	-9,159	-46,818	-40,412	-0,202	-14,944	-15,116
JIP - Papírny Větřní, a.s.	0,531	-4,169	-3,372	0,270	-0,473	-0,476
EPIAG LOFIDA – PORCELÁN, s.r.o.	0,422	-3,924	-3,297	-0,459	-0,860	-0,871

Tab. 4: Vývoj výsledků bankrotních modelů na datech z roku t-1 u nedefaultních společností zpracovatelského průmyslu

Společnosti zpracovatelského průmyslu	*t-1*					
	Z'-Score	IN95 E	IN95	Tafflerův model	IN01	IN05
DESTILA, s.r.o.	2,135	47,159	47,224	0,416	17,502	17,503
EVERSTAR s.r.o.	4,191	9,061	8,969	1,131	3,906	3,915
JH GROUP, spol. s.r.o.	1,696	1,657	1,659	0,312	0,808	0,808
STEPA s.r.o.	3,012	4,443	4,383	0,919	2,107	2,112
BROUSÍCÍ STROJE KUČERA s.r.o.	5,283	6,226	6,039	1,994	3,147	3,163
LUCID spol. s.r.o.	3,280	2,502	2,416	0,610	1,223	1,225
KRPA FORM, a.s.	2,320	0,956	0,934	0,425	0,512	0,511
MLÝNY J. VOŽENÍLEK, spol. s.r.o.	3,951	5,999	5,939	0,735	2,527	2,529

Z Tab. 1 – 4 je porovnáním s hodnotícími kritérii pro jednotlivé modely zřejmá rozdílná vypovídací schopnost jednotlivých modelů. Zejména je možné si povšimnou výsledků u Tafflerova modelu, který 100% pasuje na nedefaultní společnosti, nicméně má mnohem horší schopnost predikovat default společností. Rovněž je zřejmé relativně časté zařazení společností do šedých zón, kdy není možné jednoznačně určit, zda se jedná o defaultní, nebo nedefaultní společnost. Souhrnná úspěšnost predikce aplikovaných modelů na celý analyzovaný vzorek je pro defaultní společnosti uvedena v Tab. 5 a pro nedefaultní společnosti v Tab. 6.

Tab. 5: Vyhodnocení úspěšnosti predikce pro rok t-1 u vzorku defaultních společností

modely	default		nedefault		šedá zóna		celkem	
	počet	%	počet	%	počet	%	počet	%
Z'-Score	13	81,25%	0	0,00%	3	18,75%	16	100,00%
IN95 E	16	100,00%	0	0,00%	0	0,00%	16	100,00%
IN95	15	93,75%	0	0,00%	1	6,25%	16	100,00%
T	6	37,50%	8	50,00%	2	12,50%	16	100,00%
IN01	15	93,75%	0	0,00%	1	6,25%	16	100,00%
IN05	15	93,75%	0	0,00%	1	6,25%	16	100,00%

Tab. 6: Vyhodnocení úspěšnosti predikce pro rok t-1 u vzorku nedefaultních společností

modely	default		nedefault		šedá zóna		celkem	
	počet	%	počet	%	počet	%	počet	%
Z'-Score	1	6,25%	11	68,75%	4	25,00%	16	100,00%
IN95 E	3	18,75%	13	81,25%	0	0,00%	16	100,00%
IN95	2	12,50%	12	75,00%	2	12,50%	16	100,00%
T	0	0,00%	14	87,50%	2	12,50%	16	100,00%
IN01	2	12,50%	6	37,50%	8	50,00%	16	100,00%
IN05	3	18,75%	7	43,75%	6	37,50%	16	100,00%

3 Statistická verifikace predikční schopnosti modelů

Dále se zaměříme na zkoumání statistické rozdílnosti výsledků aplikovaných modelů při jejich aplikaci na celý vzorek společností. K testování jsou použity metody *čistý test významnosti*, více viz Litschmannová (2011), a *ROC křivka*, viz Hanley and McNeil (1982).

Cílem aplikace čistého testu významnosti je posoudit, zda se od sebe statisticky liší výsledky jednotlivých bankrotních modelů. K ověření této hypotézy je použit test homogenity dvou binomických rozdělení, který má prvky čistého testu významnosti. Nulová hypotéza představuje tvrzení, že se výsledky bankrotních modelů neliší. Testová statistika se určí jako $x = \frac{(p_s - p_p) - (\pi_s - \pi_p)}{\sqrt{\frac{p_s \cdot (1-p_s)}{n_s} + \frac{p_p \cdot (1-p_p)}{n_p}}}$, kde π_s je pravděpodobnost prvního vzorku, π_p pravděpodobnost druhého vzorku, p_s relativní četnost prvního vzorku, p_p relativní četnost druhého vzorku, n_s rozsah prvního vzorku a n_p rozsah druhého vzorku. Pro určení *p-value* je pak využito vzorce $p - value = 2 \cdot \min\{F_0 \cdot (x); 1 - F_0 \cdot (x)\}$, kde F_0 je distribuční funkce normálního rozdělení. Hladina významnosti byla zvolena jako $\alpha = 0{,}05$. V Tab. 7 pak můžeme pozorovat výsledné hodnoty *p-value* pro jednotlivé modely u defaultních společností, v Tab. 8 u nedefaultních společností.

Tab. 7: Výsledné hodnoty p-value pro modely u vzorku defaultních společností

modely	Z'-Score	IN95 E	IN95	T	IN01	IN05
Z'-Score		0,449	0,614	0,049	0,614	0,614
IN95 E	0,449		0,802	0,005	0,802	0,802
IN95	0,614	0,802		0,012	1,000	1,000
T	0,049	0,005	0,012		0,012	0,012
IN01	0,614	0,802	1,000	0,012		1,000
IN05	0,614	0,802	1,000	0,012	1,000	

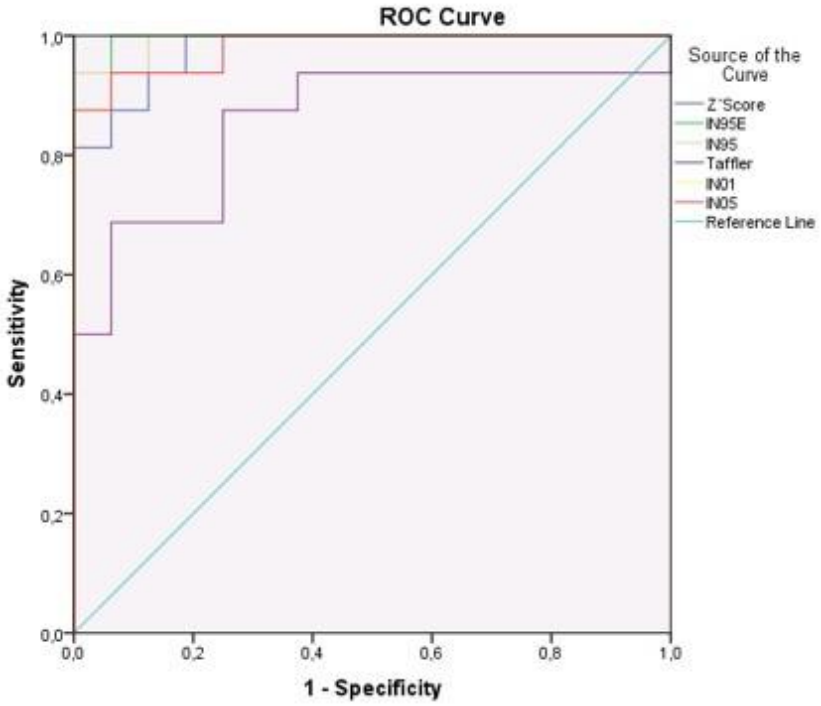
Tab. 8: Výsledné hodnoty p-value pro modely u vzorku nedefaultních společností

Bankrotní modely	Z'-Score	IN95 E	IN95	T	IN01	IN05
Z'-Score		0,605	0,794	0,440	0,150	0,261
IN95 E	0,605		0,798	0,800	0,049	0,098
IN95	0,794	0,798		0,610	0,088	0,165
T	0,440	0,800	0,610		0,025	0,055
IN01	0,150	0,049	0,088	0,025		0,756
IN05	0,261	0,098	0,165	0,055	0,756	

Z Tab. 7 je zřejmé, že na zvolené hladině pravděpodobnosti nejsou vyjma Tafflerova modelu jednotlivé modely statisticky odlišné. Z Tab. 8 pak vyplývá, že vypovídající schopnost většiny bankrotních modelů se při predikci prosperity u finančně zdravých společností jeví jako srovnatelná. Významný statistický rozdíl je zaznamenán pouze mezi výslednými hodnotami Tafflerova modelu a českého indexu důvěryhodnosti IN01 a zároveň mezi výslednými hodnotami indexů důvěryhodnosti IN01 a IN95E, jehož konstrukce zahrnuje váhy určené pro celou ekonomiku ČR.

Pro posouzení vypovídající schopnosti bankrotních modelů při predikci byla dále využita *ROC analýza*, jejíž výsledky jsou shrnuty v Obr. 4 a Tab. 9.

Obr. 4: Vývoj ROC křivek pro vybrané bankrotních modely v čase t-1



Tab. 8: Velikost AUC pro vybrané bankrotní modely v čase t-1

Area Under the Curve

Test Result Variable(s)	Area	Std. Error[a]	Asymptotic Sig.[b]	Asymptotic 95% Confidence Interval	
				Lower Bound	Upper Bound
Z'Score	,977	,021	,000	,936	1,000
IN95E	,996	,006	,000	,983	1,000
IN95	,992	,010	,000	,972	1,000
Taffler	,855	,072	,001	,714	,997
IN01	,980	,019	,000	,943	1,000
IN05	,980	,019	,000	,943	1,000

a. Under the nonparametric assumption

b. Null hypothesis: true area = 0.5

Z Obr. 4 a Tab. 8 pak lze vyvodit, že nejlepších výsledků při predikci u vybraného vzorku společností je dosaženo aplikací indexů důvěryhodnosti IN95E. Za nejméně přesný se jeví Tafflerův model, neboť jeho plocha pod křivkou je ve srovnání s ostatními bankrotními modely nejmenší.

4 Závěr

V příspěvku byla diskutována možnost aplikace vybraných bankrotních modelů na podmínky České republiky, konkrétně na stavební a zpracovatelský průmysl, a byla statisticky testována jejich predikční schopnost. K analýze byly vybrány dva zahraniční a čtyři tuzemské modely. Na náhodně vybraném vzorku 32 společností, v němž bylo zastoupení defaultních a nedefaultních společností 50:50, bylo zjištěno, že pro predikci defaultu se jeví jako nevhodný

Tafflerův model, u zbylých modelů nebyla prokázána statistická odlišnost. Nejlépe hodnoceným modelem se pak pro daný vzorek překvapivě jeví index IN95E, jehož konstrukce zahrnuje váhy určené pro celou ekonomiku ČR. Tento model vykázal dokonce lepší výsledky, než model IN95 s vahami odvozenými pro dané odvětví. Tento rozdíl je ovšem v rámci statistické chyby zanedbatelný.

References

[1] Altman, E. I. (2000). *Predicting financial distress of companies: revisiting the Z-score and ZETA models* [online]. Stern School of Business, New York University.

[2] Altman, E I. and Hotchkiss, E. (2006). *Corporate financial distress and bankruptcy: predict and avoid bankruptcy, analyze and invest in distressed debt*. 3rd ed. Hoboken, N.J.: Wiley.

[3] Hanley, J. A. and Barbara McNeil, J. (1982) The Meaning and Use of the Area Under a Receiver Operating Characteristics (ROC) Curve. *Radiology*. 1982, p. 561-557.

[4] Litschmannová, M. (2016) *Úvod do statistiky* [online]. Ostrava, 2011 [cit. 10. 4. 2016]. Dostupné z: http://mi21.vsb.cz/sites/mi21.vsb.cz/files/unit/uvod_do_statistiky.pdf

[5] Neumaierová, I. a Neumaier, I. (2005). Index IN05. In: *Evropské finanční systémy*. Brno: Masarykova univerzita.

[6] Neumaierová, I. a Neumaier, I. (2002). *Výkonnost a tržní hodnota firmy*. 1. vyd. Praha: Grada Publishing.

Management of the liquidity risk in businesses operating in the tourism industry

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Abstract

The article explores a mutual relation between the liquidity ratios and selected activity ratios, namely the average settlement period for trade receivables and average settlement period for trade payables. To maintain their sufficient level, the appropriate measures within liquidity and involved risk management should be taken. The aim of the article is to verify the relationship between financial ratios of liquidity and activity and identify the rate of their dependence. The research sample includes microenterprises operating in the tourism industry with the established domicile in the Prešov municipal region. In accordance with the SK NACE classification these businesses belong to the category of 55 Accommodation. On the basis of the results of the realised research the article points out to potential problems resulting from the liquidity risk and briefly outlines possible solutions.

Key words

Liquidity Risk, Liquidity Ratios, Activity Ratios, Tourism Industry

JEL Classification: G30, M21

1. Úvod

Podnikateľské riziko v dôsledku turbulencií v ekonomickom systéme narastá. Malé a stredné podniky uskutočňujú svoju činnosť v náročnejšom ekonomickom prostredí, pričom mnohé z nich bojujú o vlastné prežitie. Je zjavné, že všetky tieto riziká sa odrážajú vo finančnom riadení podniku (Belás, Macháček, Bartoš, Hlawiczka a Hudáková, 2014, s. 30). Prejavujú sa ako finančné riziká, ktoré sú spojené s financovaním podniku a s využívaním rôznych finančných nástrojov. Jedným z bežných finančných rizík je aj riziko likvidity, ktorému venujeme osobitnú pozornosť v predloženom príspevku. Zameriavame sa na charakteristiku rizika likvidity a jeho riadenia v podniku. Cieľom príspevku je verifikovať závislosť medzi finančnými ukazovateľmi likvidity a ukazovateľmi aktivity a identifikovať mieru ich vzájomnej závislosti. Keďže našu výskumnú vzorku tvoria mikropodniky pôsobiace v cestovnom ruchu, poukazujeme tiež na odlišnosť riadenia rizika likvidity v malých podnikoch v porovnaní s veľkými podnikmi.

2. Riziko likvidity v podniku

Riziko je neoddeliteľnou súčasťou rozhodovania pri uskutočňovaní podnikateľskej činnosti. V reálnom podnikateľskom prostredí nedokonalého trhu je v záujme tvorby hodnoty a zabezpečenia jej udržateľnosti potrebné riadiť riziká s cieľom ich eliminácie a zníženia

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transakčných nákladov a nákladov spojených s finančnými ťažkosťami alebo dokonca bankrotom podniku. Podnik je vystavený komplexnému podnikateľskému riziku, ktorého súčasťou je široká škála finančných rizík. Podnikateľská činnosť je vždy spojená s určitými finančnými rizikami, ktoré nie je možné úplne odstrániť, ale prostredníctvom rôznych nástrojov a preventívnych opatrení je možné ich minimalizovať a znížiť potenciálnu stratu (Virglerová, Kozubíková a Vojtovič, 2016).

2.1. Riadenie rizika likvidity v podniku

Finančné riziká sú riziká spojené s financovaním podniku a s využívaním rôznych finančných nástrojov. Jedným z bežných finančných rizík je riziko likvidity. Ide o riziko neschopnosti podniku v pozícii dlžníka uhradiť svoje aktuálne splatné záväzky prostredníctvom dostupných likvidných prostriedkov. Riadenie rizika likvidity znamená predovšetkým riadenie prevádzkového kapitálu takým spôsobom, aby sa zabezpečila každodenná bezproblémová prevádzka podniku. To však nie je jednoduchá úloha, pretože manažéri musia zabezpečiť, aby podnik fungoval efektívne. Ak vzniknú nezrovnalosti medzi zložkami obežného majetku a krátkodobými pasívami, negatívne to ovplyvní rast a ziskovosť podniku, môže to viesť k vzniku finančných ťažkostí alebo dokonca k jeho bankrotu (Khatik a Varghese, 2015, s. 285). Efektívne riadenie likvidity teda zohráva pre udržanie prosperity, ako aj pre samotné prežitie podniku kľúčovú úlohu. Riadenie rizika likvidity (najmä bežnej) zahrňuje predovšetkým riadenie pohotových prostriedkov a riadenie pohľadávok. Kým hlavným cieľom riadenia pohotových prostriedkov je snaha vyhnúť sa nedostatku hotovosti potrebnej na zabezpečenie bežného chodu podniku, manažment pohľadávok zahrňuje nielen poskytovanie obchodného úveru, ale aj hodnotenie jednotlivých zákazníkov, rozhodovanie o dobe splatnosti pohľadávok a o opatreniach na zabezpečenie ich včasného inkasa. Na druhej strane likviditu podniku významne ovplyvňuje aj využívanie obchodného úveru poskytnutého dodávateľmi. V rámci jej riadenia je potrebné venovať pozornosť vzájomnému vzťahu medzi dobou inkasa pohľadávok a dobou úhrady záväzkov a ich vplyvu na likviditu podniku.

Zabezpečenie dostatočnej likvidity je najmä pre malé podniky dôležité. Ekanem (2010, s. 124-125) uvádza, že likvidita je v týchto podnikoch záležitosťou života alebo smrti, pretože malý podnik môže dlhodobo prežiť bez dosahovania zisku, ale môže skrachovať za deň, ak nie je schopný uhradiť kľúčový záväzok. Menšie podniky sú pritom v porovnaní s veľkými citlivejšie najmä na existenciu nedobytných pohľadávok, pretože majú spravidla menší počet odberateľov. Nástroje, ktoré pri riadení rizika využívajú veľké podniky, nie je pritom väčšinou možné rovnako aplikovať aj v malých a stredných podnikoch. Malé a stredné podniky nečelia menšiemu počtu rizík než veľké podniky, ich manažment rizika sa od veľkých podnikov odlišuje, pretože nemajú dostatok zdrojov, aby mohli zamestnať samostatného odborníka v pozícii manažéra rizika (Sharifi, 2014, s. 84-85). Napriek tomu je hodnotenie likvidity, jej riadenia a súvisiacich rizík v malých podnikoch založené na princípoch a praktikách uplatňovaných vo veľkých podnikoch, pričom sa len malá pozornosť venuje reálnym praktickým potrebám malých podnikov, kde vlastník je zvyčajne súčasne aj manažérom.

2.2 Teoretické východiská ukazovateľov likvidity a aktivity

Ako uvádzajú Lesáková et al. (2007, s. 57), likvidita podniku predstavuje všeobecnú schopnosť daného podniku hradiť svoje krátkodobé záväzky. Pod analýzou likvidity rozumieme posudzovanie miery schopnosti vyrovnávať svoje platobné povinnosti v peňažných prostriedkoch, ktoré plynú z činnosti podniku. Vo všeobecnosti môžeme tvrdiť, že finančne stabilný podnik je schopný uhrádzať svoje záväzky v stanovenom termíne a podnik s finančnými ťažkosťami s tým má problémy. Daná schopnosť je ovplyvnená niekoľkými skutočnosťami, pričom medzi najdôležitejšie patrí štruktúra majetku a primeraný a pravidelný prítok peňažných prostriedkov do podniku.

Práve efektívne riadenie likvidity podniku predstavuje rozhodujúci prvok pre zabezpečenie celkovej schopnosti podniku uhrádzať svoje krátkodobé záväzky prostredníctvom ľahko speňažiteľných aktív. Riadenie likvidity má dôležité postavenie v rámci finančného manažmentu najmä v období hospodárskej a finančnej krízy, ktoré je charakteristické znížením peňažných tokov a zhoršujúcimi sa podmienkami na trhu. Na riadenie a vyhodnocovanie likvidity podnik využíva rôzne špecifické a analytické nástroje.

V rámci analýzy likvidity môžeme ukazovatele likvidity rozdeliť na statické a dynamické ukazovatele v závislosti od toho, na základe akého účtovného výkazu sú skonštruované. V predloženom príspevku sa zameriame iba na statické ukazovatele, a to konkrétne na tri pomerové ukazovatele – likvidita I. stupňa (pohotová), likvidita II. stupňa (bežná) a likvidita III. stupňa (celková). Ukazovateľ likvidity I. stupňa uvádza, aká časť celkových krátkodobých záväzkov je krytá pohotovými peňažnými prostriedkami. V odbornej literatúre sa môžeme stretnúť s odporúčaným intervalom 0,2 – 0,6. V prípade likvidity II. stupňa sa na krytie celkových krátkodobých záväzkov používajú okrem pohotových peňažných prostriedkov aj krátkodobé pohľadávky, pričom ukazovateľ by mal presahovať hodnotu 1 (1 – 1,5). Likvidita III. stupňa vypovedá o schopnosti podniku uhrádzať svoje krátkodobé záväzky prostredníctvom obežného majetku. Vierohodnosť obrazu o likvidite preto veľmi závisí od štruktúry obežného majetku. Výpoveď môže byť skreslená nepotrebnými zásobami a nedobytnými pohľadávkami. Odporúčaná hodnota pre celkovú likviditu je v rozmedzí 2 – 2,5.

Okrem toho, že ukazovatele likvidity ovplyvňujú ukazovatele rentability (zvyšovaním likvidity by mohol manažment ohroziť investovanie, a tým aj rentabilitu podniku), môžeme vidieť vzájomný vzťah aj medzi ukazovateľmi likvidity a ukazovateľmi aktivity, a to konkrétne medzi dobou inkasa pohľadávok a dobou úhrady záväzkov. Doba inkasa pohľadávok vypovedá o platobnej disciplíne odberateľov podniku, pričom vyjadruje čas, ktorý uplynie od vystavenia faktúry po jej úhradu. Doba úhrady záväzkov predstavuje ukazovateľ, ktorý reflektuje platobnú schopnosť daného podnikateľského subjektu, pričom z pohľadu podniku je priaznivé, ak je doba inkasa pohľadávok kratšia ako doba úhrady záväzkov. V takomto prípade podnik skôr inkasuje peňažné prostriedky ako uhrádza záväzky. Ukazovatele doby inkasa pohľadávok a doby úhrady záväzkov sú dôležité pre posudzovanie časového nesúladu od vzniku pohľadávky do doby jej inkasa a od vzniku záväzku do doby jeho úhrady. Tento nesúlad priamo ovplyvňuje likviditu podnikateľského subjektu. Knápková, Pavelková a Šteker (2013) uvádzajú, že v prípade, ak je doba úhrady záväzkov vyššia ako súčet obratu zásob a pohľadávok, obchodný úver financujú pohľadávky a zásoby, čo je výhodné. Daná situácia sa však môže prejaviť v nízkych hodnotách likvidity. Medzi výškou likvidity a aktivity je vzájomná väzba, pričom je potrebné medzi nimi hľadať kompromis.

3 Cieľ, materiál a metodika skúmania

Cieľom predloženého príspevku je verifikovať závislosť medzi finančnými ukazovateľmi likvidity a vybranými ukazovateľmi aktivity (doba inkasa pohľadávok a doba úhrady záväzkov) a identifikovať mieru ich vzájomnej závislosti. Vstupnými zdrojmi dát pre zostavenie výberovej vzorky podnikov bola databáza portálu indexpodnikatela.sk. Výberový súbor zahŕňa 72 mikropodnikov pôsobiacich v odvetví cestovného ruchu so sídlom v Prešovskom samosprávnom kraji Slovenskej republiky. Na základe klasifikácie SK NACE sú podniky zaradené do kategórie 55 Ubytovanie. Podniky zaradené do výberového súboru vykazovali aktívnu podnikateľskú činnosť za rok 2015, ich tržby za analyzovaný rok predstavovali maximálne 2 milióny euro a počet zamestnancov sa pohyboval v rozmedzí 0-9.

Pre nami analyzovaných 72 podnikov boli vyčíslené hodnoty ukazovateľov likvidity (pohotová, bežná a celková likvidita) a vybraných ukazovateľov aktivity (doba inkasa

pohľadávok a doba úhrady záväzkov) za rok 2015. Metodika výpočtu vybraných pomerových finančných ukazovateľov bola stanovená na základe metodiky portálu indexpodnikatela.sk.

Výber mikropodnikov pôsobiacich v oblasti cestovného ruchu bol determinovaný tlačovou správou Zväzu cestovného ruchu SR. Správa zdôrazňuje, že v odvetví cestovného ruchu tvoria mikro a malé podniky viac ako 90 % všetkých poskytovateľov služieb v cestovnom ruchu. Dôvodom, prečo sme sa zamerali na rok 2015, bola nielen jeho aktuálnosť z hľadiska účtovných údajov, ale aj skutočnosť, že si cestovný ruch v roku 2015 výrazne polepšil a stal sa najúspešnejším rokom v histórii samostatnej Slovenskej republiky (Tlačová správa zo 6. valného zhromaždenia ZCR SR 02.02.2016, 2016). V roku 2015 si Slovensko ako dovolenkovú destináciu vybralo viac ako 4,3 mil. návštevníkov. Počet prenocovaní vzrástol oproti predchádzajúcemu roku o 13,3 %, t. j. nadobudol hodnotu 12,35 mil. prenocovaní. Podiel cestovného ruchu na HDP v roku 2015 predstavoval 2,7 % a cestovný ruch sa na exporte služieb podieľal 29,4 %. Na základe výsledkov daných štatistík môžeme potvrdiť, že cestovný ruch tvorí významnú súčasť ekonomiky Slovenskej republiky (Ubytovacia štatistika CR na Slovensku. Štatistika CR-vývoj platobnej bilancie, 2016). Prešovský kraj sa v počte návštevníkov v ubytovacích zariadeniach CR SR za rok 2015 umiestnil na tretej pozícii s počtom 740 698 návštevníkov po Bratislavskom (1 194 479) a Žilinskom (846 508) kraji. Pri počte zahraničných návštevníkov (232 363) a počte prenocovaní (715 081) sa Prešovský kraj aj týmto zaradil medzi tri najúspešnejšie kraje Slovenskej republiky za rok 2015 (Základné ukazovatele za ubytovacie zariadenia cestovného ruchu SR za rok 2015, 2016).

V nadväznosti na vyššie uvedený hlavný cieľ príspevku sme formulovali dve hypotézy.

H1: Predpokladáme, že existuje štatisticky významná závislosť medzi finančnými ukazovateľmi likvidity (likvidita I., II. a III. stupňa) a dobou úhrady záväzkov.

V predloženom príspevku vychádzame z predpokladu, že čím je hodnota ukazovateľov likvidity menšia, tým je hodnota ukazovateľa doba úhrady krátkodobých záväzkov dlhšia. Z formulácie daného predpokladu vyplýva, že predpokladáme nepriamu závislosť medzi ukazovateľmi likvidity a dobou úhrady záväzkov.

H2: Predpokladáme, že existuje štatisticky významná závislosť medzi likviditou I. stupňa a dobou inkasa pohľadávok.

Keďže sa vo výberovom súbore nachádzajú podnikateľské subjekty, ktoré pôsobia v oblasti cestovného ruchu (oblasť služieb), pre ktoré je typické, že inkasujú väčšinu svojich úhrad v hotovosti, zameriame sa iba na vzťah medzi likviditou I. stupňa a dobou inkasa pohľadávok. Predpokladáme, že čím nižšiu hodnotu doby inkasa pohľadávok podnik dosahuje, t. j. čím častejšie inkasuje svoje peňažné prostriedky, tým je hodnota likvidity I. stupňa vyššia. Medzi ukazovateľom likvidity I. stupňa a dobou inkasa pohľadávok predpokladáme nepriamu závislosť.

Platnosť stanovených hypotéz budeme overovať korelačnou analýzou. Primárnym cieľom korelačnej analýzy je zistiť silu, intenzitu (tesnosť) korelačnej závislosti charakterizovanú korelačnou charakteristikou. Prostredníctvom štatistického programu SPSS sme verifikovali normalitu vybraných finančných pomerových ukazovateľov. Stanovili sme nulovú a alternatívnu hypotézu (H0=normálne rozdelenie analyzovaných finančných ukazovateľov, H1=Non H0). Na základe výsledku Kolmogorov-Smirnovho testu môžeme zamietnuť nulovú hypotézu o normálnom rozdelení analyzovaných finančných ukazovateľov. Ak platí, že výber nemá normálne rozdelenie, je vhodné použiť Spearmanov korelačný koeficient. Predpokladajme, že $(X_1, Y_1)´$, ..., $(X_n, Y_n)´$ je výber zo spojitého dvojrozmerného rozdelenia. Nech R_1, ..., R_n sú poradia veličín X_1, ..., X_n a nech Q_1, ..., Q_n sú poradia veličín Y_1, ..., Y_n. Veľmi často sa dvojice $(X_1, Y_1)´$, .., $(X_n, Y_n)´$ už dopredu usporiadajú podľa rastúcich hodnôt X_1, ..., X_n. V takomto prípade $R_i = i$, $i = 1$, ..., n.

Spearmanov korelačný koeficient sa definuje ako výberový korelačný koeficient počítaný z dvojíc $(R_1, Q_1)´$, ..., $(R_n, Q_n)´$. Podľa Anděla (2003) platí:

$$r_s = 1 - \frac{6}{n.\,(n^2-1)} \sum_{i=1}^{n} (R_i - Q_i)^2 .$$

Koeficient poradovej korelácie môže nadobúdať hodnoty v intervale <-1; 1>, pričom kladné hodnoty znamenajú priamu závislosť a záporné hodnoty znamenajú nepriamu závislosť. Hodnoty pohybujúce sa okolo 0 nenaznačujú žiadnu závislosť medzi premennými. Po vypočítaní testovacích charakteristík musíme rozhodnúť o výsledku testu, pričom existujú dva základné spôsoby:

1. veľkosť testovacej štatistiky je porovnaná s kritickou hodnotou, ktorá vymedzuje oblasť zamietnutia nulovej hypotézy,
2. veľkosť p-hodnoty je porovnaná s hladinou významnosti α (v našom prípade hladina významnosti α = 0,01), pričom H0 je zamietnutá, ak je p-hodnota menšia ako hladina významnosti, a H0 nie je zamietnutá, ak je p-hodnota väčšia ako hladina významnosti. Rozhodli sme sa použiť druhý spôsob rozhodnutia o výsledku.

4 Výsledky a diskusia

Prostredníctvom štatistického programu SPSS sme vyčíslili priemer, štandardnú odchýlku a medián ukazovateľov likvidity (pohotová, bežná a celková likvidita) a ukazovateľov aktivity (doba inkasa pohľadávok a doba úhrady záväzkov) analyzovaných podnikov za rok 2015.

Tabuľka 1: Popisná štatistika finančných pomerových ukazovateľov

	L1	L2	L3	DIP	DÚZ
Priemer	1,6036	2,2937	2,3713	101,6111	507,5833
Štandardná odchýlka	0,3961	0,4436	0,4446	48,0579	95,6578
Medián	0,2810	0,8765	1,0300	18,0000	129,0000
Medián odvetvia (r.2014)	-	0,6400	0,7400	43,7600	350,9900

Prameň: Vlastné spracovanie na základe SPSS a univerzálneho registra Cribis.

Z výsledkov prezentovaných v tabuľke 1 vyplýva, že priemerná hodnota likvidity I. stupňa bola na úrovni 1,60, avšak polovica analyzovaných podnikov vykázala hodnotu len 0,28. Priemerná hodnota likvidity II. stupňa bola 2,29, no medián týchto podnikov vykázal hodnotu 0,87. Hodnota mediánu analyzovanej vzorky je v porovnaní s mediánom všetkých podnikov nachádzajúcich sa v kategórii 55 Ubytovanie z roku 2014 (0,64) vyššia. Likvidita III. stupňa vykázala priemernú hodnotu 2,37, pričom medián podnikov nachádzajúci sa vo výberovom súbore vykazuje hodnotu 1,03, čo znamená, že polovica podnikov po speňažení celého svojho obežného majetku splatí všetky svoje krátkodobé záväzky, avšak druhá polovica môže mať s touto úhradou veľké problémy.

Ak by sme tieto ukazovatele porovnali na základe mediánov všetkých podnikov zaradených do odvetvia Ubytovacie a stravovacie služby podľa krajov v SR za rok 2014, zistili by sme, že medián ukazovateľov likvidity II. (0,85) a III. stupňa (1,04) v Prešovskom kraji vykazuje druhú najvyššiu hodnotu v porovnaní s ostatnými krajmi na Slovensku. Ak porovnáme mediány likvidít všetkých podnikov z odvetvia Ubytovacie a stravovacie služby v Prešovskom kraji a nami vybranej vzorky podnikov z tohto kraja, zistíme, že tieto hodnoty sú si veľmi podobné, čím potvrdzujeme správnosť a vhodnosť vybranej vzorky podnikov.

Ukazovateľ doba inkasa pohľadávok alebo platobná disciplína odberateľov vykázal v roku 2015 priemernú hodnotu 101,61 dní, t. j. počet dní, kým sa peniaze viazané v pohľadávkach zinkasujú. 50 % analyzovaných podnikov vykázalo 18 dní a viac, kým zinkasuje peniaze od svojich odberateľov a 50 % podnikov malo túto dobu 18 dní a menej.

Priemerná hodnota doby úhrady záväzkov bola 507,58 dní, čo je oveľa vyššia hodnota ako doba inkasa pohľadávok. Tento trend môže byť na jednej strane vnímaný ako pozitívny, pretože priemerná doba úhrady záväzkov je oveľa dlhšia ako doba inkasa pohľadávok. Na druhej strane príliš vysoká hodnota doby úhrady záväzkov ovplyvňuje dobré meno a imidž podniku voči svojmu okoliu a voči svojim súčasným a potenciálnym dodávateľom. Z výsledkov analýzy ukazovateľov doby obratu vyplýva, že polovica podnikov dosiahla dobu inkasa pohľadávok 18 dní a viac a dobu úhrady záväzkov 129 dní a viac, čo znamená, že podniky využívali disponibilné, relatívne lacné zdroje financovania.

Medián doby inkasa pohľadávok celého odvetvia za rok 2014 bol 43,76 dní, čo je v porovnaní s mediánom vybranej vzorky podnikov dvojnásobne vyššia hodnota. Medián doby úhrady záväzkov v nami analyzovanej vzorke podnikov bol vo výške 129 dní. V porovnaní s mediánom všetkých podnikov podľa klasifikácie SK NACE zaradených do kategórie 55 z roku 2014 v hodnote 350,99 dní je približne 2,5 - krát menší. Túto skutočnosť hodnotíme pozitívne, avšak pri priemernej hodnote viac ako 500 dní môžeme predpokladať v podnikoch finančné problémy pri uhrádzaní svojich záväzkov.

Ak porovnáme mediány týchto ukazovateľov všetkých podnikov zaradených do odvetvia Ubytovacie a stravovacie služby medzi krajmi v SR, zistíme, že medián doby inkasa pohľadávok bol za Prešovský kraj tretí najvyšší v hodnote 23,57 dní, čo je aj v porovnaní s mediánom všetkých podnikov za všetky kraje (22,64) vyššia hodnota. Avšak medián doby úhrady záväzkov v hodnote 168,54 dní za podniky v Prešovskom kraji je druhým najnižším v porovnaní s ostatnými krajmi SR za rok 2014. Nami vybraná vzorka podnikov vykázala túto hodnotu ešte nižšiu (129 dní), čo hodnotíme pozitívne a aj pri porovnaní s mediánom všetkých podnikov za všetky kraje je hodnota takmer dvakrát nižšia.

Tabuľka 2: Spearmanovo rho

		DIP	DÚZ
L1	Korelačný koeficient	-0,101	-,764**
	Významnosť (obojstranná)	0,397	0,000
	N	72	72
L2	Korelačný koeficient	0,143	-,806**
	Významnosť (obojstranná)	0,231	0,000
	N	72	72
L3	Korelačný koeficient	0,130	-,821**
	Významnosť (obojstranná)	0,278	0,000
	N	72	72
** Korelácia je významná na hladine významnosti 0,01 (obojstranná).			

Prameň: Vlastné spracovanie na základe SPSS.

V ďalšom kroku sme verifikovali vzájomný vzťah medzi ukazovateľmi likvidity a ukazovateľmi aktivity. Pri formulovaní záverov sme vychádzali z „voľnejšej“ škály hodnôt korelačných koeficientov, keďže finančný manažment nemožno radiť medzi exaktné vedy. Ak je hodnota korelačného koeficientu v rozmedzí 0,7-1,0, ide o silnú koreláciu; hodnota v rozmedzí 0,5-0,7 znamená strednú koreláciu; hodnota v rozmedzí 0,3-0,5 znamená miernu koreláciu a hodnota v rozmedzí 0-0,3 vyjadruje slabú koreláciu (Závarská, 2011, s. 198).

Na základe vypočítaných hodnôt korelačných koeficientov sme zistili, že medzi ukazovateľmi likvidity a ukazovateľmi aktivity existuje závislosť. Medzi všetkými tromi ukazovateľmi likvidity existuje štatisticky významná (Sig. = 0,000 < 0,01) silná nepriama závislosť s finančným ukazovateľom doba úhrady záväzkov. Pre likviditu I. stupňa je hodnota rho = - 0,764, pre likviditu II. stupňa je rho = - 0,806 a pre likviditu III. stupňa je hodnota rho

= - 0,821 na hladine významnosti 0,01, čo potvrdzuje silnú závislosť (0,700 < rho < 1,000). Na základe výsledkov môžeme zamietnuť nulovú hypotézu (rho = 0) a prijať výskumnú hypotézu H1. Na 1 % - nej hladine významnosti môžeme potvrdiť, že existuje silná nepriama závislosť medzi všetkými tromi ukazovateľmi likvidity a ukazovateľom doby úhrady záväzkov (Sig. = 0,000 < 0,01). Výsledky korelačnej analýzy nepotvrdili štatisticky významnú závislosť likviditou I. stupňa a dobou inkasa pohľadávok (Sig. = 0,278 – 0,397 > 0,01), preto na 1 %-nej hladine významnosti zamietame výskumnú hypotézu H2.

Pre mikropodniky pôsobiace v odvetví cestovného ruchu a v rámci neho v oblasti ubytovania je charakteristické, že v súvahe evidujú nižšie sumy pohľadávok, pretože za svoje služby spravidla inkasujú platby v hotovosti. Na druhej strane na úhradu záväzkov majú dohodnutú určitú dobu splatnosti, čiže využívajú obchodný úver ako pomerne lacný zdroj financovania, čo má pozitívny vplyv na rentabilitu. Tieto podniky sa tiež vyznačujú nízkym podielom zásob na obežnom majetku. Pri riadení likvidity je preto potrebné upriamiť pozornosť najmä na zabezpečenie dostatočného množstva pohotových prostriedkov vo vzťahu k potrebe uhrádzať bežné výdavky a aktuálne splatné záväzky. Z výsledkov nášho výskumu vyplýva existencia významného nepriameho vzťahu medzi dobou úhrady záväzkov a likviditou podniku. Je však nutné zdôrazniť, že mikropodniky poskytujúce ubytovacie služby v rámci cestovného ruchu musia udržiavať korektné dodávateľsko-odberateľské vzťahy, pričom väčšina podnikov má snahu dohodnúť si čo najdlhšiu dobu splatnosti so svojimi dodávateľmi. Samozrejme stanovenú dobou splatnosti by mali podnikateľské subjekty dodržať. V rámci finančného riadenia by podniky mali dbať na dodržiavanie základných pravidiel financovania a krátkodobé zdroje v podobe obchodného úveru využívať len na financovanie prevádzkových potrieb. Aj keď sme uviedli, že objem pohľadávok v nami analyzovaných podnikoch je spravidla pomerne nízky, je nutné venovať primeranú pozornosť aj ich riadeniu, sledovať finančnú situáciu svojich odberateľov, aby sa podnik nedostal do druhotnej platobnej neschopnosti. Vyššie sme poukázali na odlišnosť riadenia prevádzkového kapitálu v malých podnikoch v porovnaní s veľkými a vychádzajúc z nej odporúčame malým podnikom (vrátane mikropodnikov) využívať pri riadení likvidity a s ňou spojeného rizika najmä jednoduché nástroje, napríklad pravidelné zostavovanie platobných kalendárov, pomocou ktorých sa podniky môžu vyhnúť neočakávanému prevýšeniu bežných výdavkov nad disponibilnými príjmami.

5 Záver

V príspevku sme sa zaoberali riadením rizika likvidity, analyzovali sme vzťah medzi likviditou a vybranými ukazovateľmi aktivity na vzorke mikropodnikov pôsobiacich v oblasti poskytovania ubytovacích služieb so sídlom v Prešovskom kraji. Výsledky nášho výskumu potvrdili existenciu štatisticky významnej závislosti medzi dobou úhrady záväzkov a likviditou podniku. Keďže analyzované podniky sa vyznačujú nízkym objemom pohľadávok a zásob, pri riadení ich likvidity je potrebné venovať sa hlavne riadeniu pohotových peňažných prostriedkov a efektívnemu využívaniu obchodného úveru, avšak v súlade s dohodnutými zmluvnými podmienkami.

Manažment prevádzkového kapitálu, a v rámci neho najmä riadenie likvidity vrátane riadenia súvisiaceho rizika, sa musí dostať do pozornosti aj v malých podnikoch. Tieto podniky nemožno chápať ako zmenšené verzie veľkých podnikov, ale ako podniky, ktoré sa od veľkých v mnohých aspektoch líšia. Rovnako sa líšia aj nároky na riadenie likvidity a s ňou spojeného rizika. Je nutné zamerať sa na nástroje a metódy riadenia likvidity, ktoré sú vhodné práve pre malé podniky. Nevhodné riadenie prevádzkového kapitálu vedie k neefektívnemu viazaniu zdrojov v neproduktívnych zložkách aktív. Poskytovanie obchodného úveru

odberateľom je vo väčšine odvetví štandardom a môže viesť k podpore predaja. Ak sú však vzniknuté pohľadávky problémové, podniku vznikajú náklady spojené s ich vymáhaním, prípadne až straty spojené so vznikom nedobytných pohľadávok. Na druhej strane využívaním obchodného úveru poskytnutého dodávateľmi podnik získava lacné zdroje financovania, čím dokáže znížiť svoje náklady na kapitál. V prípade nedodržania dohodnutých podmienok však podniku môžu vzniknúť náklady finančných ťažkostí. Je preto potrebné riadiť prevádzkový kapitál spôsobom, ktorý bude viesť k efektívnemu využívaniu jednotlivých zložiek obežného majetku, k zabezpečeniu dostatočnej likvidity a zároveň primeranej rentability podniku.

References

[1] Anděl, J. (2003). *Statistické metody.* 3. vyd. Praha: MATFYZPRESS.

[2] Belás, J., Macháček, J., Bartoš, P., Hlawiczka, R., Hudáková, M. (2014). Business Risks and the Level of Entrepreneurial Optimism among SME in the Czech and Slovak Republic. *Journal of Competitiveness,* 6(2), s. 30-41.

[3] Ekanem, I. (2010). Liquidity management in small firms: a learning perspective. *Journal of Small Business and Enterprise Development,* 17(1), s. 123-138.

[4] Khatik, S., Varghese, T. (2015). Impact of Working Capital Management on Firm Profitability: An Empirical Study of ITC Ltd. *International Journal of Information, Business and Management,* 7(4), s. 284-306.

[5] Knápková, A., Pavelková, D., Štektr K. (2013). *Finanční analýza. Komplexní prúvodce s příklady.* Praha: Grada Publishing.

[6] Lesáková, Ľ. a kol. (2007). *Finančno-ekonomická analýza podniku.* Banská Bystica: Ekonmická fakulta UMB.

[7] Sharifi, O. (2014). Financial Risk Management for Small and Medium Sized Enterprises. *International Journal of Information, Business and Management,* 6(2), s. 82-94.

[8] *Tlačová správa zo 6.valného zhromaždenia ZCR SR 02.02.2016*. (2016). Bratislava: Zväz cestovného ruchu Slovenskej republiky. Dostupné na: <http://www.zcrsr.sk/tlacova-sprava-zo-6-valneho-zhromazdenia-zcr-sr-02-02-2016/>.

[9] *Ubytovacia štatistika CR na Slovensku. Štatistika CR - vývoj platobnej bilancie.* (2016). Bratislava: Slovenská agentúra pre cestovný ruch. Dostupné na: <http://www.sacr.sk/odborna-verejnost/analyzy-a-statistiky/statistiky/?no_cache=1&cid=1163&did=9410&sechash=5f4e9ddf>.

[10] *Univerzálny register Cribis.* (2016). Dostupné na: <https://www2.cribis.sk/Home.aspx>.

[11] Virglerová, Z., Kozubíková, L., Vojtovič, S. (2016). Influence of selected factors on financial risk management in SMEs in the Czech Republic. *Montenegrin Journal of Economics,* 12(1), s. 21-36.

[12] *Základné ukazovatele za ubytovacie zariadenia cestovného ruchu SR za rok 2015.* (2016). Bratislava: Štatistický úrad SR. Dostupné na: <https://slovak.statistics.sk/PortalTraffic/fileServlet?Dokument=10376db0-af20-40cd-b792-7327e994148e>.

[13] Závarská, Z. (2011). Zostavenie súboru kľúčových indikátorov finančnej výkonnosti podniku. *Acta Academica Karviniensia,* 3, s.194-2017.

Monetary policy, indebtedness and debt service costs

Martin Hodula[1],Ales Melecky [2]

Abstract

Nowadays, there is a broad agreement on the need to separate monetary policy and fiscal policy from the debt management due to different goals and instruments of these policies. This paper examines how the change of monetary policy influences government debt-to-GDP ratio and its effective interest rate in the Czech Republic. The primary objective of monetary policy is price stability (inflation targeting) which might be in conflict with the debt management goal to minimize debt servicing cost with respect to risk objectives. We use the Czech PRIBOR rate to proxy for the CNB's key monetary policy rate, i.e. the two-week repo, in a similar way to which it is done in CNB's forecasting system. Preliminary results from the impulse response analysis based on estimated FAVAR model suggest that the change in PRIBOR rate significantly influences indebtedness and debt servicing costs in the Czech Republic.

Key words

Government debt, Monetary policy, FAVAR model, Czech Republic

JEL Classification: E52, H63, C3

1. Introduction

Historically, debt management has not been separated as a stand-along policy, but was part of fiscal or monetary policy. In 1980s, indebtedness of the OECD countries together with rising riskiness of their debt portfolios led to the modernization and reform of the government debt management in many countries (Togo, 2003). The IMF and the World Bank published, in cooperation with national debt management experts, a set of guidelines on public debt management for policy makers (IMF and the World Bank, 2001). These guidelines stress the importance of formulating a sound debt management strategy for optimally allocating government debt in view of the government objectives and constraints as well as the need to separate individual policies. In 2014 the IMF and The World Bank released revised version of public debt management guidelines in response to financial sector regulatory changes and macroeconomic policy developments (IMF and the World Bank, 2014).

Institutional arrangements for government debt management can be classified into four categories: an autonomous debt management office, a debt management department under the central bank, and an independent unit under the state treasury or the ministry of finance (see e.g. Wheeler, 2005). In the Czech Republic, the division for management of government debt and financial assets was established in 2005 under the ministry of finance (Melecky and Melecky, 2015). Because of potential policy conflicts, separating the debt management from fiscal and monetary policies is recommended, which may strengthen the credibility of the

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policies and help achieve better results. This paper focuses on potential conflicts between debt management and monetary policy, concretely how different setting of monetary policy may influence debt to GDP and debt servicing costs in the Czech Republic.

The primary objective of monetary policy is price stability (inflation targeting) which might be in conflict with the debt management goal to minimize debt servicing cost with respect to risk objectives. Generally, conflicts between monetary policy and debt management may arise because of the shortage of policy instruments which are independent of one another. This can happen, if both the monetary authority and the debt manager operate in the primary market to pursue their respective policy objectives. If the central bank issues its own debt with direct interest cost, the public will perceive that the central bank would have less incentive to raise interest rates to reduce inflation, because lower rates mean lower borrowing costs. The shortage of policy instruments may force the authorities into potential conflict and weakens the credibility of their ability to achieve their goals (see e.g. Togo, 2007).

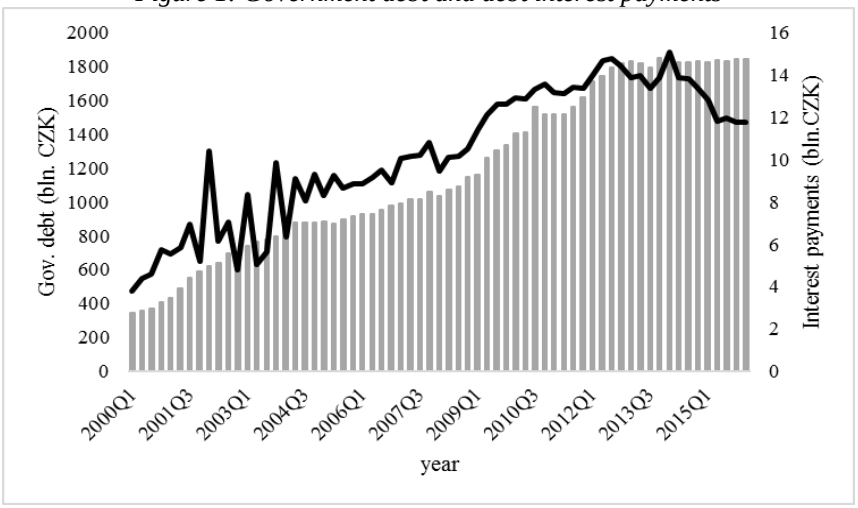
To achieve inflation targets, the central banks created the environment of low or even negative interest rates and applied various unconventional monetary policy tools, such as quantitative easing which influences yield curves of the government bonds. In November 2013 the Czech National Bank (CNB) adopted an exchange rate commitment to intervene on foreign exchange market. The CNB has weaken the koruna exchange rate close to CZK 27 to the euro, in order to increase low inflation and avoid deflation in the Czech Republic. Such actions influence the cost of unhedged foreign currency denominated debt. In the Czech Republic, inflation-linked bonds are issued only within retail program and make up a negligible part of the government debt portfolio. Inflation-linked bonds however, may be at least partially substituted by bills indexed to the short term interest rate. A change in inflation may affect short-term interest rates and therefore the cost of debt portfolio and government debt.

This paper examines how a change of monetary policy influences government debt-to-GDP ratio and its effective interest rate in the Czech Republic. We use the Czech PRIBOR rate to proxy for the CNB's key monetary policy rate, i.e. the two-week repo, in a similar way to which it is done in CNB's forecasting system[3].

Figure 1 presents the evolution of government debt (left axis) and debt interest payments (right axis) in the Czech Republic over the period 2001-2016. During the period 2000-2012, obvious upward trend in government debt in the Czech Republic was present, accompanied by rising debt interest payments. After 2012, the government debt dynamics was significantly reduced. Debt interest payments reached its peak in 2014 and from that time on were declining. This downward trend is caused by an escape from risk at which investors buy bonds of the relatively "safe" countries with stable currency. This environment led to negative yields of bonds which were further deepened by the Czech koruna interventions. The Czech Ministry of Finance used the environment of negative yields of the medium- and long-term government bonds together with the timing at the end of the calendar year and executed the auction of the government bond with maturity in 2017 in December 2015 with the all-time low yield of -0.35% p.a. The total state budget revenue from this investment activity, lending facilities with government bonds and from the negative yields of government bonds amounted to CZK 524.9 million (Ministry of Finance, 2016).

[3] It should be noted that the repo rate does not change continuously and it is strongly correlated with the PRIBOR rate (0,982).

Figure 1: Government debt and debt interest payments



Note: Grey slopes – Government debt (in bln. CZK), black line – Debt interest payments
Source: Czech Statistical Office

2. Estimation methodology

In this section, we discuss our empirical framework. To analyze how the change of monetary policy influences government debt-to-GDP ratio and its effective interest rate, we suggest employing vector autoregression (VAR) model. The VAR approach to measuring the effects of monetary policy shocks on various variables has proven effective and it delivered a great deal of useful structural information. Yet this method does not lack for criticism.

A simple VAR model comprising of at most representative set of fiscal policy, monetary policy and macroeconomic variables would be far too large to be estimated using classical methods. To retain the degrees of freedom, standard VAR models usually employ at most 6 to 8 variables. As stated by Bernanke et al. (2005, p. 388) such small number of variables is unlikely to span the information sets used by actual central banks, who are known to follow literally hundreds of data series, or by the financial market participants and other observers. Hence, to solve the dimensionality problem, we employ a factor-augmented VAR model (FAVAR) based on Stock a Watson (2002) and Bernanke et al. (2005).

The basic idea of a FAVAR models rests on incorporating the large amount of macroeconomic data into a small number of factors which are used for the estimation of a VAR model. We specify a $M \times 1$ vector of macroeconomic time series denoted Y_t and $K \times 1$ vector of unobserved factors F_t. Following Bernanke et al. (2005), we assume that the joint dynamics of F_t', Y_t' are given by the following equation:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \varepsilon_t , \tag{1}$$

where $\Phi(L)$ is lag polynomial and ε is mean zero error term with covariance matrix Q. Equation (1) is a standard VAR model which can be interpreted as reduced form of a linear rational-expectations model with both observed and unobserved variables. However, the unobserved variables are making the model impossible to estimate. In order to get around this fact, we can assume that additional informational time series X_t (capturing for example economic activity, market conditions etc.) are somehow linked to the originally unobservable factors F_t and the observable factors Y_t by:

$$X_t' = \Lambda^f F_t' + \Lambda^y Y_t' + e_t' , \tag{2}$$

where Λ^f and Λ^y are matrices of factor loadings and e_t' is a mean zero serially uncorrelated error term, also referred to as an innovation shock. Equation (2) captures the idea that both vectors Y_t and F_t are pervasive forces that might drive the common dynamics of X_t. We use a two-step principal components approach, which is a nonparametric way of estimating the space spanned by the common components $C_t' = (F_t', Y_t')$ in (1). For details on estimation using principal components method see Stock and Watson (2002) or Bernanke et al. (2005).

We assume that the interest rate is the only observable factor in Y_t and treat it as a factor that has pervasive effect on the economy X_t. The increase in short-term interest rates (a positive shock) is identified by recursively applying the Cholesky decomposition to the covariance matrix. To implement defined identification scheme, it is useful to define two categories of variables: slow/fast moving. Slow-moving variables display a lagged response to a shock while fast-moving ones react contemporaneously. In the monetary policy shock identification, blocks describing the real economy, fiscal variables, prices and the external environment are classed as slow-moving.

3. Data

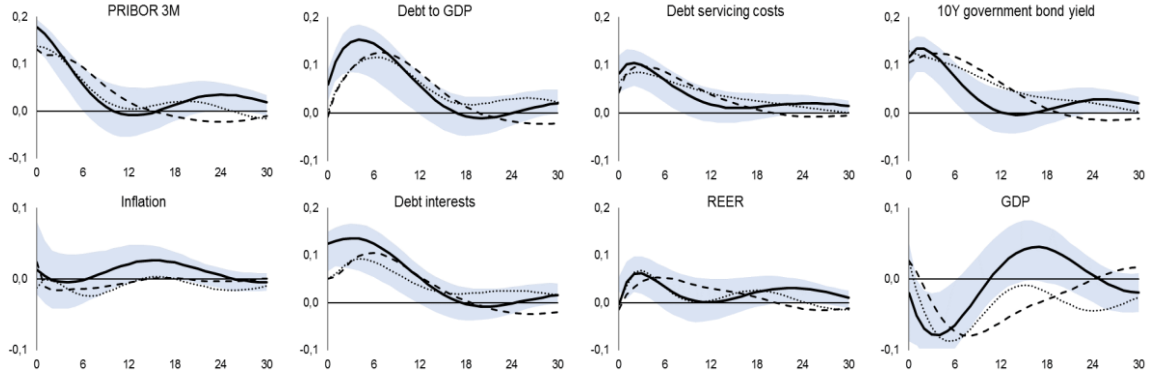
In our application, economy vector X_t consists of a balanced panel of 130 quarterly macroeconomic time series representing the Czech economy and outer world drawn mainly from CNB database ARAD, Czech statistical office and ECB database, respectively. The data span the period from the first quarter of 2001 to the first quarter of 2016 (61 observations). The set of variables can be divided into six blocks: (i) the real economy variables (industrial production index, construction production index, GDP, retail sales, housing market, labour market); (ii) fiscal variables (government debt and its financing, data on primary balance); (iii) prices (consumer price indexes, industrial producer price indexes); (iv) credit and interest rates; (v) financial sector and (vi) the external environment. Prior estimates, data were transformed to assure stationarity of time series using natural logarithm and first differences. The description of the series in the dataset and their transformation is available upon request.

4. Estimation results

The effects of monetary policy shock are presented graphically using impulse response functions (IRFs). We use one standard deviation shock, i.e. the shock of the same probability, not the same size. IRFs show the response of selected variables to identified shock and the propagation of the shock over time. Accurate confidence intervals are calculated via bootstrap procedure as in Kilian (1998) that accounts for the uncertainty in the factor estimation. One of the main advantage of the FAVAR approach is that IRFs can be constructed for any variable in the informational dataset. However, it is not needed or possible to present here all IRFs. We have selected only those relevant for our analysis.

The specification of model framework is based on Schwartz information criteria. The baseline model employs 1 lag of explanatory variables (FAVAR(1)) and 3 factors. Figure 2 presents impulse responses of selected variables to one standard deviation contractionary monetary policy shock. Results of the baseline model are depicted with solid line surrounded by grey area representing 90% confidence interval. To check the robustness of our results, with respect to number of factors included in the FAVAR model, we also present the IRFs for FAVAR(1) model with 5 and 7 factors, respectively. The findings are further discussed in detail.

Figure 2: Impulse responses generated from FAVAR(1) with 3, 5 and 7 factors



Note: Quarters after shock are put on the horizontal axis, vertical axis describes the strength of the response to one standard deviation contractionary monetary policy shock. We are presenting here estimated FAVAR model with 3 factors (solid line), 5 factors (dotted line) and 7 factors (dashed line).
Source: Own computation

After contractionary monetary policy shock the yields of government bonds increase significantly affecting the yield curve of the Czech government debt portfolio (see for instance reaction of the Czech 10-year government bond yield in the first row of Figure 2). Due to increased yields of the government bods the debt servicing costs increase with the maximal reaction in 2 quarters after the shock. Higher debt servicing costs are also reflected in debt interest payments. Increased cost of financing together with declining GDP, which means higher borrowing needs of the Czech government and decrease of the denominator in debt to GDP ratio, negatively affect indebtedness measured by the debt to GDP ratio. The strongest effect of the monetary policy shock on the indebtedness occurs after 4 quarters. Reaction of real effective exchange rate to monetary policy shock is in line with economic theory when higher interest rate differential leads to appreciation of the domestic currency. Reaction of inflation is not statistically significant.

The FAVAR model with 5 and 7 factors tell the same story. The results are robust with respect to the number of factors included into the model as the most of the IFRs from the models with 5 and 7 factors lie inside the 90% confidence interval of the FAVAR model with 3 factors. The only difference is slightly slower reaction of the variables to monetary policy shock in the case of models with larger number of factors.

5. Conclusion

This paper estimated the impact of contractionary monetary policy shock on the government debt-to-GDP ratio and its effective interest rate in the Czech Republic. The preliminary results from the impulse response analysis based on an estimated FAVAR model suggest that the changes in monetary policy, approximated by the changes in PRIBOR rate, significantly influence debt to GDP ratio and debt servicing cost in the Czech Republic. The results are robust with respect to a number of factors included in the model.

The results of our analysis have important policy implications suggesting that changes in monetary policy significantly affect debt management policy a therefore there is the need for intensive communication between monetary authority and the debt manager in order to avoid unintended increase the cost of financing government debt.

In future research we plan to check the robustness of our results with respect to the number of lags included in the model and examine the impact of other policies on indebtedness and debt servicing costs.

Acknowledgements

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References

[1] Bernanke, S. B., Boivin, J. and Eliasz, P. (2005). Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach. *Quarterly Journal of Economics*, vol. 120, pp. 387-422.

[2] Currie, E., Dethier, J-J and Togo, E. (2003). Institutional arrangements for public debt management *World Bank Policy Research Working Paper,* No. 3021, Washington: The World Bank.

[3] IMF and the World Bank (2014). *Revised guidelines for public debt management.* Washington: International Monetary Fund and The World Bank. Available at www: < https://www.imf.org/external/np/pp/eng/2014/040114.pdf >.

[4] IMF and the World Bank (2001). *Guidelines for Public Debt Management*. Washington: International Monetary Fund and The World Bank. Available at www: <http://www.imf.org/external/np/ mae/pdebt/2000/eng/guide.pdf>.

[5] Kilian, L. (1998). Small-sample Confidence Intervals for Impulse Response Functions. *Review of Economics and Statistics*, vol. 80, No. 2, pp. 218-230.

[6] Melecký, A., Melecký, M. (2015). *Managing government debt: Strategy, risk, allocation*. SAEI, Ostrava: VSB-TU Ostrava.

[7] Ministry of Finance (2016). *The Czech Republic Government Debt Management Annual Report for 2015*. Prague: Ministry of Finance. Available at www: <http://www.mfcr.cz/en/themes/state-debt/publications-and-presentations/government-debt-management-annual-report/2015/the-czech-republic-government-debt-manag-23987>

[8] Stock, H. J. and Watson, W. M. (2002). Forecasting Using Principal Components From a Large Number of Predictors. *Journal of the American Statistical Association*, vol. 97, pp. 1167-1178.

[9] Togo, E. (2007). Coordinating public debt management with fiscal and monetary policies: an analytical framework. *World Bank Policy Research Working Paper,* No. 4369, Washington: The World Bank.

[10] Wheeler, G. (2004). *Sound Practice in Government Debt Management*. Washington: The World Bank.

F-transform Method for Option Pricing

Michal Holčapek, Petra Števuliáková, Irina Perfilieva[1]

Abstract

The aim of this contribution is to develop a numerical method based on the Crank-Nicolson scheme for time discretization and the F-transform to solve one-factor Black-Scholes partial differential equation (PDE). The F-transform is an efficient method for the approximation of (multivariate) functions, where a transformation of a space of locally square integrable functions into a simple vector space of F-transform components is used. We show how the PDE after the application of the Crank-Nicolson scheme for time discretization can be transformed into a simple vector space of F-transform components in order to find a solution in terms of the F-transform components. An approximate solution of the PDE is derived by the inverse F-transform.

Keywords

Option pricing, Black-Scholes model, Crank-Nicolson scheme, partial differential equation, fuzzy partition, F-transform.

JEL Classification: G22.

1. Introduction

Option prising is an important part of market practice. In literature, one can find several models and methodologies for option pricing, e.g., the Black-Scholes models, Heston or Lévy model. It is well-known that no analytic solution can be found for certain classes of more complex models; therefore, instead of an exact solution one seeks an approximate solution by means of classical numerical methods such as finite difference or element method and Monte Carlo method or modern methods like discontinuous Galerkin method (see, e.g., [6]) or wavelet methods (see, e.g., [2, 3]).

In this contribution, we focus on the one-factor Black-Scholes partial differential equation (PDF) describing a single plain vanilla option pricing problem and derive a procedure for its numerical solution with the help of the fuzzy transform technique. The fuzzy transform (F-transform, for short) was introduce by Perfilieva in [8] (see also [7]) as a soft computing method that is used for the approximation of functions. A generalization to two dimensional case was proposed in [12]. The F-transform has two phases: direct and inverse. The direct F-transform transforms a bounded real function to a finite vector

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of real numbers (components of F-transform). The inverse F-transform sends the latter vector back. The result of the inverse F-transform is a function that approximates the original one.

The F-transform technique has been already applied, among others, in solving differential equations. In [8], a numerical method based on the F-transform has been proposed for an ordinary Cauchy problem. The proposed technique which generalized the Euler method showed its potential for solving differential equations in comparison with numerous classical techniques. In [12], the F-transform technique was used to solve special types of partial differential equations. In [9], a novel algorithm based on the F-transform has been proposed to obtain an approximate solution for a class of second-order ordinary differential equations with classical initial conditions. In [10], new numerical methods based on the higher degree F-transform for solving the Cauchy problem have been presented. The results of the proposed methods outperformed the second order Runge-Kutta method.

The numerical solution of the one-factor Black-Scholes partial differential equation, however, needs a novel approach. With the help of the Crank-Nicolson scheme for time discretization, we provide a theoretical background for the numerical solution of the mentioned PDF, which is the first step in our research concerning the application of the F-transform technique in option pricing.

The paper is structured as follows. The next section provide a necessary background of the F-transform theory. Section 3 is devoted to a one factor Black-Scholes partial differential equation (PDE). Section 4 provides a numerical solution of the PDE presented in Section 3 with help of the Crank-Nicolson scheme for time discretization and the 2D F-transform. Section 5 is a conclusion.

2. Preliminaries: two dimensional F-transform

In this section, we recall the main definitions and properties of the F-transform that are used in this paper. Generally, the F-transform depends on a chosen fuzzy partition of a real interval by means of fuzzy sets (see [7]). It is well-known (see, e.g. [4]) that a uniform fuzzy partition can be defined using a generating function. Each fuzzy set of the uniform fuzzy partition, which is called the basic function, is then obtained by a shift of the modified (scaled) generating function. The generating function can be defined as follows.

A function $K : \mathbb{R} \to [0, 1]$ is said to be a *generating function* if K is even Lebesgue integrable function (fuzzy set) which is non-increasing in $[0; \infty)$ and

$$K(x) \begin{cases} > 0, & \text{if } x \in (-1, 1); \\ = 0, & \text{otherwise.} \end{cases}$$

A generating function K is said to be normal if $K(0) = 1$. The following two functions

$$K^{tr}(x) = \max(1 - |x|, 0) \quad \text{and} \quad K^{rc}(x) = \begin{cases} \frac{1}{2}(1 + \cos(\pi x), & -1 \leq x \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

belong among the most important generating functions in applications of the F-transform technique. The function K^{tr} is referred to the *triangle* generating function and K^{rc} to the *raised cosine* generating function (see Fig. 1).

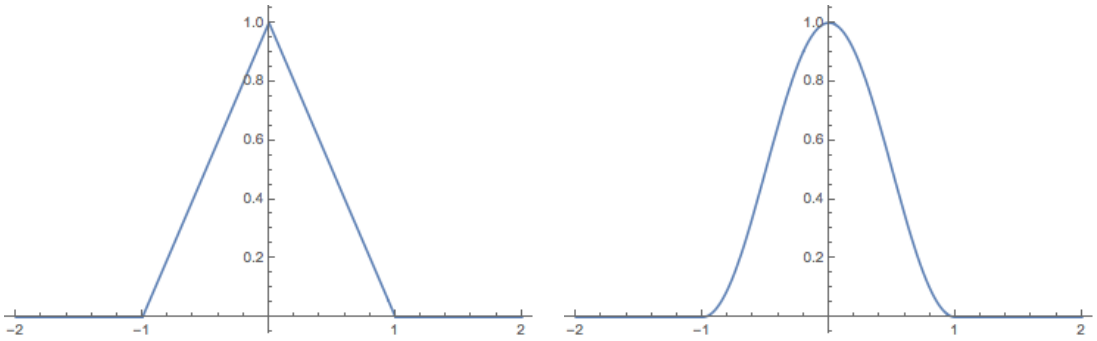


Figure 1: Plots display the triangle univariate generating function (left) and the raised cosine univariate generating function (right)

A *two dimensional (2D) generating function* $\mathcal{K}$ can be defined as a product of two univariate generating functions K_1 and K_2 as follows::

$$\mathcal{K}(x,y) = K_1(y)K_2(y), \quad (x,y) \in \mathbb{R}^2.$$

For a general definition of multivariate generating function, we refer to [5]. Let $\mathcal{K}$ be a two dimensional generating function, and let $h_1, h_2 > 0$. A *scaled* 2D generating function $\mathcal{K}$ is the function $\mathcal{K}_{h_1,h_2} : \mathbb{R}^2 \to [0,1]$ defined by

$$\mathcal{K}_{h_1,h_2}(x,y) = \mathcal{K}\left(\frac{x}{h_1}, \frac{y}{h_2}\right).$$

The parameters h_1 and h_2 are called the *bandwidth*. Now, we can provide the definition of 2D uniform fuzzy partition of $\mathbb{R}^2$ that forms a core of the 2D F-transform technique.

Definition 1 Let $\mathcal{K}$ be a 2D generating function, let $h_1, h_2, r_1, r_2 > 0$ and $\boldsymbol{t} = (t_1, t_2) \in \mathbb{R}^2$. A family of fuzzy relations $\{A_{i,j} \mid (i,j) \in \mathbb{Z}^2\}$ defined by

$$A_{i,j}(x,y) = \mathcal{K}_{h_1,h_2}(x - t_1 - ir_1, y - t_2 - jr_2), \quad (x,y) \in \mathbb{R}^2,$$

is said to be a *2D uniform fuzzy partition of* $\mathbb{R}^2$ determined by the quadruplet $(\mathcal{K}, \mathbf{h}, \mathbf{r}, \boldsymbol{t})$, where $\mathbf{h} = (h_1, h_2)$ and $\mathbf{r} = (r_1, r_2)$, provided that a la Ruspini condition is satisfied, i.e.,

$$\sum_{i\in\mathbb{Z}} \sum_{j\in\mathbb{Z}} A_{i,j}(x,y) = 1$$

for any $(x,y) \in \mathbb{R}^2$. The parameters $\mathbf{r}$ and $\boldsymbol{t}$ are called the *shift vector* and the *central node*, respectively. The function $A_{i,j}$ is called the *basic function*.

In this paper, we are interested in the partitioning of $[a,b] \times [c,d]$. For simplicity, we consider 2D uniform fuzzy partition, where $\mathbf{h} = \mathbf{r}$, i.e., at most four fuzzy set (or better fuzzy relations) can be overlapped. More specifically, we put $h_1 = \frac{b-a}{N-1}$ and $h_2 = \frac{d-c}{M-1}$, where $M, N \in \mathbb{N}$. Let $\mathcal{K}$ be a uniform generating function with $\mathcal{K}(0) = 1$. For $i = 1, \ldots, N$, $j = 1, \ldots, M$, we set

$$A_{i,j}(x,y) = K\left(\frac{x - t_1 - (i-1)h_1}{h_1}\right) K\left(\frac{y - t_2 - (j-1)h_2}{h_2}\right)$$

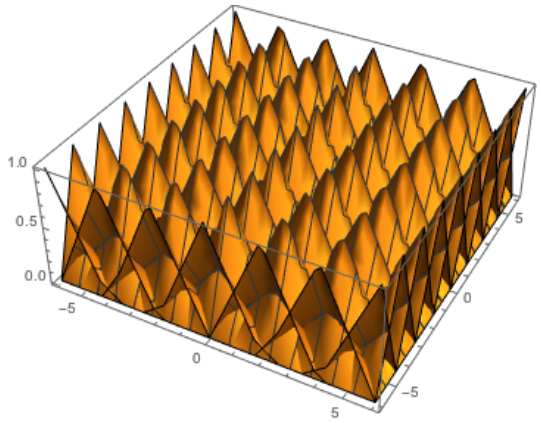


Figure 2: Simple triangle uniform fuzzy partition of $[-6,6] \times [-6,6]$.

The family $\{A_{i,j} \mid i = 1,\dots,N, j = 1,\dots,M\}$ is a two dimensional uniform fuzzy partition of $[a,b] \times [c,d]$ if

$$\sum_{i=1}^{N}\sum_{j=1}^{M} A_{i,j}(x,y) = 1, \quad \text{for all} \quad (x,y) \in [a,b] \times [c,d].$$

This type of the two dimensional uniform fuzzy partition is referred to *simple*. In Fig. 2, we depict an example of a simple triangle uniform fuzzy partition of $[-6,6] \times [-6,6]$ specified by $\mathbf{h} = (2, 1.5)$ and $\boldsymbol{t} = (0,0)$. The basic functions are then defined as follows

$$A_{ij}(x,y) = K^{tr}\left(\frac{x - 2(i-1)}{2}\right) K^{tr}\left(\frac{y - 1.5(j-1)}{1.5}\right), \quad (x,y) \in [-6,6] \times [-6,6],$$

for $i = 1,\dots,7$, $j = 1,\dots,9$.

The direct (two dimensional) F-transform with respect to a (simple) uniform fuzzy partition is defined as the weighted mean of a given function, where the weights are determined from the basic functions of the uniform fuzzy partition. In this paper, we consider the following definition (cf. [7, 12]).

Definition 2 Let $\mathbb{A} = \{A_{i,j} \mid i = 1,\dots,M,\ j = 1,\dots,N\}$ be a simple uniform fuzzy partition of $[a,b] \times [c,d]$, and let f be a continuous function on $[a,b] \times [c,d]$. We say that the $N \times M$-matrix of real numbers

$$\mathbf{F} = \begin{pmatrix} F_{1,1} & \dots & F_{1,N} \\ \vdots & \vdots & \vdots \\ F_{M,1} & \dots & F_{M,N} \end{pmatrix}$$

given by

$$F_{ij} = \frac{\int_a^b \int_c^d f(x,y) A_{i,j}(x,y)\,dxdy}{\int_a^b \int_c^d A_{i,j}(x,y)\,dxdy}, \quad i = 1,\dots,m,\ j = 1,\dots,N,$$

is the *direct (integral) 2D F-transform* of f w.r.t. $\mathbb{A}$. The value $F_{i,j}$ of $\mathbf{F}$ is called the (i,j)-th *component of the direct 2D F-transform w.r.t.* $\mathbb{A}$.

One can see that the original continuous function f is now expressed by F-transform components in a simple space of matrices. The following definition provides the way how the original function can be approximated from the F-transform components. This phase is called the inverse (two dimensional) F-transform with respect to a (simple) uniform fuzzy partition.

Definition 3 Let $\mathbb{A} = \{A_{i,j} \mid i = 1, \ldots, N, j = 1, \ldots, M\}$ be a simple uniform fuzzy partition of $[a, b] \times [c, d]$ and f be a continuous function on $[a, b] \times [c, d]$. Let $\mathbf{F} = (F_{i,j})$ be the direct 2D F-transform of f w.r.t. $\mathbb{A}$. The *inverse 2D F-transform* of f w.r.t. $\mathbb{A}$ is

$$\hat{f}(x, y) = \mathbf{F}[f](x, y) = \sum_{i=1}^{N} \sum_{j=1}^{M} F_{i,j} A_{i,j}(x, y), \quad (x, y) \in [a, b] \times [c, d],$$

3. Black-Scholes model

A one-factor Black-Scholes partial differential equation (PDE) describes a single plain vanilla option pricing problem. Let σ be a constant volatility of the underlying asset S and r be a constant risk-free interest rate. Then the price of the option $V(S, t)$ is obtained by the following backward PDE (see [1, 11]).

$$\frac{\partial V}{\partial t} - \mathcal{L}_{BS}(V) = 0,$$

where $V : (0, S^{\max}) \times (0, T) \to \mathbb{R}$ and

$$\mathcal{L}_{BS}(V) = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV$$

with the boundary and initial conditions:

$$V(0, t) = V(S^{\max}, t) = h(S), \quad t \in (0, T),$$
$$V(S, 0) = g(S), \quad S \in (0, S^{\max}).$$

Let us consider $W \in H^1((0, S^{\max}) \times (0, T))$ such that $W(S, 0) = h(S)$ and put

$$\tilde{V} = V - W.$$

Then the Black-Scholes equation can be rewritten as follows:

$$\frac{\partial \tilde{V}}{\partial t} = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \tilde{V}}{\partial S^2} + rS\frac{\partial \tilde{V}}{\partial S} - r\tilde{V} + g \tag{1}$$

where $g = -\frac{\partial W}{\partial t} + \mathcal{L}_{BS}(W)$ with the homogeneous Dirichlet boundary and initial conditions:

$$\tilde{V}(0, t) = \tilde{V}(S^{\max}, t) = 0, \quad t \in (0, T),$$
$$\tilde{V}(S, 0) = g(S) - W(S, 0), \quad S \in (0, S^{\max}).$$

Note that if $\tilde{V}$ is a solution of (1), then the solution of the original Black-Scholes partial differential equation is $V = \hat{V} + W$.

In the next section, we provide a numerical solution of the previously specified problem with the help of the Crank-Nicolson scheme for time discretization and the 2D F-transform technique.

4. F-transform based solution of PDE

The aim of this section is to derive a numerical solution of the problem (1) the 2D F-transform. For a simplification of notation, we consider the following form of equation (1):

$$\frac{\partial f}{\partial t} = \alpha \frac{\partial^2 f}{\partial s^2} + \beta \frac{\partial f}{\partial s} + \gamma f + g, \tag{2}$$

where

$$f, g : (0, S^{\max}) \times (0, T) \to \mathbb{R}$$
$$\alpha, \beta, \gamma : (0, S^{\max}) \to \mathbb{R}$$

and which satisfies the homogeneous Dirichlet boundary and initial conditions:

$$f(0,t) = f(S^{\max}, t) = 0, \quad t \in (0,T);$$
$$f(s,0) = \varphi(s), \quad s \in (0, S^{\max}).$$

One can see that equation (2) is slightly more general than equation (1).

Let us choose $M, N > 2$ and create a simple uniform fuzzy partition of $[0, S^{\max}] \times [0, T]$, where $h_s = \frac{S^{\max}}{M-1}$ and $h_t = \frac{T}{N-1}$. Denote $t_j = j \cdot h_t$ for $j = 1, \ldots, N$. Using the Crank-Nicolson method for finite differences in time, we rewrite equation (2) into the system of equations in the following form ($j = 1, \ldots, N-1$):

$$\frac{f(s,t_{j+1}) - f(s,t_j)}{h_t} = \frac{1}{2}\Bigg(\alpha(s)\frac{\partial^2 f}{\partial s^2}(s,t_{j+1}) + \beta(s)\frac{\partial f}{\partial s}(s,t_{j+1}) + \gamma(s) f(s,t_{j+1}) + g(s,t_{j+1})$$
$$+ \alpha(s)\frac{\partial^2 f}{\partial s^2}(s,t_j) + \beta(s)\frac{\partial f}{\partial s}(s,t_j) + \gamma(s) f(s,t_j) + g(s,t_j) \Bigg). \tag{3}$$

Denote $s_i = i \cdot h_s$ for $i = 1, \ldots, M$. Let $\mathbb{A}$ be a simple uniform fuzzy partition of $[0, S^{\max}] \times [0, T]$ determined by a generating function $\mathcal{K}$ and the pair of bandwidths $\mathbf{h} = (h_s, h_t)$. The idea how to solve the previous system (3) by means of the F-transform consists in the replacement of functions and their derivatives by F-transform components as follows:

$$\frac{F_{i,j+1} - F_{i,j}}{h_t} = \frac{1}{2}\Bigg(\alpha(s_i) F^{ss}_{i,j+1} + \beta(s_i) F^{s}_{i,j+1} + \gamma(s_i) F_{i,j+1} + G_{i,j+1}$$
$$+ \alpha(s_i) F^{ss}_{i,j} + \beta(s_i) F^{s}_{i,j} + \gamma(s_i) F_{i,j} + G_{i,j} \Bigg),$$

where $F^{ss}_{i,j}$ (similarly $F^{s}_{i,j}$, $F_{i,j}$) and $G_{i,j}$ denote the (i,j)-th F-transform component of the partial derivative $\frac{\partial^2 f}{\partial s^2}$ and $g$respectively, with regard to the simple uniform fuzzy partition $\mathbb{A}$. One can see that we did not replace the functions α, β and γ by the respective F-transform components, but we fixed their function values at the point s_i. The reason is that the F-transform is a linear map, but it does not preserve the multiplication. The mentioned suggestion seems to be as a reasonable solution of this issue. Now, let us express the respective derivatives numerically as follows:

- $F^{ss}_{i,j} = \frac{F_{i+1,j} - 2F_{i,j} + F_{i-1,j}}{h_s^2}$,

- $F^s_{i,j} = \frac{F_{i+1,j}-F_{i,j}}{h_s}$

and put $\alpha_i = \alpha(s_i)$, $\beta_i = \beta(s_i)$ and $\gamma_i = \gamma(s_i)$. Then, we obtain

$$\frac{F_{i,j+1}-F_{i,j}}{h_t} =$$
$$\frac{1}{2}\Bigg(\alpha_i\frac{F_{i+1,j+1}-2F_{i,j+1}+F_{i-1,j+1}}{h_s^2}+\beta_i\frac{F_{i+1,j+1}-F_{i,j+1}}{h_s}+\gamma_i F_{i,j+1}+G_{i,j+1}$$
$$+\alpha_i\frac{F_{i+1,j}-2F_{i,j}+F_{i-1,j}}{h_s^2}+\beta_i\frac{F_{i+1,j}-F_{i,j}}{h_s}+\gamma_i F_{i,j}+G_{i,j}\Bigg),$$

From the boundary condition, we assume that $F_{1,j} = 0 = F_{N,j}$. Then, for any $j = 1,\dots,N-1$, one can simply find the following system of linear equations:

$$\begin{aligned}
B_2F_{2,j+1}+C_2F_{3,j+1} &= B_2^*F_{2,j}-C_2F_{3,j}+D_{2,j}\\
A_3F_{2,j+1}+B_3F_{3,j+1}+C_3F_{4,j+1} &= -A_3F_{2,j}+B_3^*F_{3,j}-C_3F_{4,j}+D_{3,j}\\
&\vdots\\
A_{N-1}F_{N-2,j+1}+B_{N-1}F_{N-1,j+1} &= -A_{N-1}F_{N-2,j}+B_{N-1}^*F_{N-1,j}+D_{N-1,j},
\end{aligned}$$

where

$$\begin{aligned}
A_i &= -\frac{\alpha_i h_t}{2h_s^2}\\
B_i &= 1+\left(\frac{2\alpha_i}{h_s^2}+\frac{\beta_i}{h_s}-\gamma_i\right)\frac{h_t}{2}\\
C_i &= -\left(\frac{\alpha_i}{h_s^2}+\frac{\beta_i}{h_s}\right)\frac{h_t}{2},\quad D_{i,j} = (G_{i,j+1}+G_{i,j})\frac{h_t}{2}
\end{aligned}$$

for $i = 1,\dots,M$. To express the previous system of linear equations in a matrix form, put $B_i^* = 2-B_i$ and define

$$\mathbf{K} = \begin{pmatrix}
B_2 & C_2 & 0 & 0 & 0 & \cdots & 0\\
A_3 & B_3 & C_3 & 0 & 0 & \cdots & 0\\
 & & & \ddots & & & \\
0 & 0 & 0 & 0 & \cdots & A_{M-1} & B_{M-1}
\end{pmatrix}$$

$$\mathbf{L} = \begin{pmatrix}
B_2^* & -C_2 & 0 & 0 & 0 & \cdots & 0\\
-A_3 & B_3^* & -C_3 & 0 & 0 & \cdots & 0\\
 & & & \ddots & & & \\
0 & 0 & 0 & 0 & \cdots & -A_{M-1} & B_{M-1}^*
\end{pmatrix}$$

$$\mathbf{F}_j^- = (F_{2,j},\dots,F_{M-1,j})^T$$

$$\mathbf{D}_j = \left(\frac{(G_{2,j+1}+G_{2,j})h_t}{2},\dots,\frac{(G_{M-1,j+1}+G_{M-1,j})h_t}{2}\right)^T,$$

where $\mathbf{F}_j^-$ denotes the vector of 2D F-transform components in the j-th column of the matrix $\mathbf{F}$ representing the direct 2D F-transform w.r.t $\mathbb{A}$. Recall that $G_{i,j}$ is the (i,j)-th

2D F-transform component of the function g. Now, the solution can be found by solving the following system of linear equations ($j = 1, \ldots, N-1$):

$$\mathbf{K}\mathbf{F}^-_{j+1} = \mathbf{L}\mathbf{F}^-_j + \mathbf{D}_j. \tag{4}$$

From the initial condition, we know that

$$F_{2,1} = \varphi(s_2), \ldots, F_{M-1,1} = \varphi(s_{M-1}),$$

and from (4) for $j = 1$, we get the equation

$$\mathbf{K}\mathbf{F}^-_2 = \mathbf{L}\mathbf{F}^-_1 + \mathbf{D}_1,$$

which solution provides the vector $\mathbf{F}^-_2$ of the 2D F-transform components $F_{2,2}, \ldots, F_{N-1,2}$. Repeating the procedure and using the boundary condition, i.e., $F_{1,j} = F_{M,j} = 0$ for $j = 1, \ldots, N$, we obtain the matrix $\mathbf{F}$ of all the 2D F-transform components of the desired function f:

$$\mathbf{F} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \varphi(s_2) & F_{2,2} & \cdots & F_{2,N} \\ \varphi(s_3) & F_{3,2} & \ldots & F_{3,N} \\ \cdots & \cdots & \ddots & \cdots \\ \varphi(s_{M-1}) & F_{M-1,2} & \cdots & F_{M-1,N} \\ 0 & 0 & \cdots & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \mathbf{F}^-_1 & \mathbf{F}^-_2 & \cdots & \mathbf{F}^-_N \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

The approximate solution of equation (2) is expressed by the inverse F-transform of f w.r.t. $\mathbb{A}$, i.e.,

$$\hat{f}(x, y) = \mathbf{F}[f](x, y) = \sum_{i=1}^{M} \sum_{j=1}^{N} F_{i,j} A_{i,j}(x, y)$$

for $(x, y) \in (0, S^{\max}) \times (0, T)$.

5. Conclusions

In this paper, we proposed a numerical method based on the Crank-Nicolson scheme for time discretization and F-transform to solve one-factor Black-Scholes partial differential equation (PDE) describing a single plain vanilla option pricing problem. The method was explained in details to allow users to prepare an efficient algorithm for solving this type of PDE. In our future work, we plan to develop an algorithm and compare the results of the proposed method with the analytic solution of the Black-Scholes equation under different initial and boundary conditions. Moreover, we plan to compare our results with results achieved by other numerical methods like discontinuous Galerkin or wavelet (Galerkin-wavelet) methods. Finally, we want to extend the proposed method to more general PDEs whose solutions are related to solutions of more complex option pricing models.

References

[1] Black, F. and Scholes, M. (1973). The Valuation of Options and Corporate Liabilities. *Journal of Political Economy* **81**, pp. 637–654.

[2] Černá, D. (2016). Numerical Solution of the Black-Scholes Equation Using Cubic Spline Wavelets. In: *Proc. of the 42st International Conference Applications of Mathematics in Engineering and Economics*, AIP Conference Proceedings, pp. 1–8.

[3] Finěk, V. (2016). Fractional Step Method for Wavelet Based Solution of Black-Scholes Equation. In: *In proc. of the 42st International Conference Applications of Mathematics in Engineering and Economics*, AIP Conference Proceedings, pp. 1–4.

[4] Holčapek, M., Perfilieva, I., Novák, V., Kreinovich, V. (2015). Necessary and sufficient conditions for generalized uniform fuzzy partitions. *Fuzzy Sets Syst.* **277**, pp. 97–121.

[5] M. Holčapek, T. Tichý (2015). Discrete multivariate F-transform of higher degree, in: Proc. of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2015), IEEE, pp. 1–8.

[6] Hozman, J., and Tichý, T. (2015). A discontinuous Galerkin method for pricing of two-asset options. In: 33rd International Conference Mathematical Methods in Economics, Cheb, ZČU Plzeň, pp. 273–278.

[7] Perfilieva, I. (2006). Fuzzy transforms: Theory and applications. *Fuzzy Sets Syst.* **157**, pp. 993–1023.

[8] Perfilieva, I. (2003). Fuzzy transform: application to the Reef growth problem. In: *R.V. Demicco, G.J. Klir (Eds.), Fuzzy Logic in Geology, Academic Press*, Amsterdam, pp. 275–300, Chap. 9.

[9] Chen, W., Schen, Y. (2014). Approximate solution for a class of second-order ordinary differential equations by the fuzzy transform. *Intelligent and Fuzzy Systems* **27**, pp. 73–72.

[10] Khastan, A., Perfilieva, I., Alijani, Z. (2016). A new fuzzy approximation method to Cauchy problems by fuzzy transform. *Fuzzy Sets Syst.* **288**, pp. 75–89.

[11] Merton, R. (1973). Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science* **4**, pp. 141–183.

[12] Štěpnička, M., Valasek, R. (2005). Numerical solution of partial differential equations with help of fuzzy transform. *Proceedings of the 2005 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2015)*, 1153–1162.

Business risks connected with Industry 4.0

Josef Horák[1], Jiřina Bokšová [2]

Abstract

The paper deals with the phenomenon Industry 4.0 and characterizes the business risks connected with the implementation of Industry 4.0 in the corporate environment. This concept is based on the computerization of production and interconnection of machines. The products will be produced in smart factories with the minimal share of human labour. The companies will have to invest enormous funds in the new technologies that will secure the faster, cheaper and more effective production with the comparison of the present systems of production. On the other hand, this concept is associated with a lot of problems and possible risk which companies will face. The main aim of the submitted paper is to highlight the selected risks connected with the financial management, controlling and the financial accounting.

Key words

business risks, financial accounting, Industry 4.0, industrial companies, new technologies

JEL Classification: M11, O31, O33

1. Introduction

The current market environment is forcing companies to innovate and streamline their production processes. Only a company that produces effectively can be successful on the market and it can survive in the hard competitive environment. Companies must constantly invest in the new technologies that provide higher performance of the company and also affect the cost of the production. In the industry, there is a progressive automation of all production processes. The most of the production will be produced by machines without the use of human factor in the near future. This is the basic idea of the 4th Industrial revolution often called Industry 4.0.

Industry 4.0 has been discussed since 2011 in Germany and later all over the European Union. The government of the Czech Republic is preparing the strategy and it is investing in the research and other activities that can help companies to prepare for the upcoming changes. This situation is not analysed only from the view of the companies, but it is analysed from the view of trade unions, labours and the state too because these changes will influence the whole economy of each industrial state in the near future. The above mentioned concept is connected with micro and macroeconomic challenges but on the other hand, with the micro and macroeconomic disadvantages and problems.

It is clear from the foregoing text that the Industry 4.0 is a big challenge for the companies and customers but it will be connected with a lot of disadvantages, risks and problems. The main aim of the paper is to find and highlight the main business risks that the company could face in the near future. The purpose of this paper is not to analyse all risks connected with the Industry 4.0 and the company. The paper will be mainly focused on the selected risks from the

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view of the financial management, controlling and financial accounting. Finally, many of these risks will be interconnected with the IT technologies. [1], [2]

2. Methodology

The phenomenon Industry 4.0 is based on the cyber- physical systems. These systems will be part of the companies and the traditional production process will be fundamentally changed. All parts of the production – machines, technologies, components, material, semi-finished products, work in progress, final products will share information each other and the production process will be much faster, effective, operative, able to respond to changing demand of the customers and dynamic. [3], [4]

IT technologies will play the fundamental role in the concept Industry 4.0. Every company will be forced to employ more IT specialists, invest more money in IT technologies in comparison with the present situation. IT specialists will be one of the most required employees and there will be probably their shortage in the labour market. Everything will be connected with the ensuring and the securing of the production process. The data protection will be a crucial question. [5]

In the manufacturing process, the production company will have initially employ algorithms, through which the product can be produced. The algorithms will be owned by the company or purchased as a service from an external entity. Subsequently, it will be possible to produce production based on the data stored in the cloud storage service. There will be kept the whole documentation of the produced products, 3-D Models, Process Data etc. In the end, there will be produced a product that will meet customers' requirements and wishes. [6]

The cyber-physical systems require 3 main levels that must be achieved, it means that services will be based on the available data, the data models of the physical models will be available in the network infrastructure and the physical objects will be available for the production. [7]

Industry 4.0 will cause the shorter innovation cycles of the products, enable larger data volumes and it will be possible to produce more complex products. Products and production will be integrated. The second advantage is the flexibility of the production process. This concept is based on the individualized mass production with achieved high productivity. The production will be able to reflect the volatile markets. The quality is the third important area of the production. [8] The production process will be controlled from the beginning to the end by machines and these machines will not allow install faulty components in the final product. It will be closed loop quality process with the traceable and integrated genealogy of the quality. The production process will be transparent from the beginning to its end with high efficiency because there will be savings in consumption of energy. There will be optimized production resources due to the new conditions in the production process.

Mařík (2016) found that the concept Industry 4.0 is based on the 6 important fundamental principles:

1. Interoperability is the ability to communicate among individual agents using the Internet of Things and the Internet of the Services.
2. Virtualization is the ability to interlink physical systems with virtual models and simulation tool. It is predicted that the simulations will be very important for the whole production process.
3. Decentralization is based on the autonomously and parallel decision-making and the control.
4. Real-time performance continuously identifies the goals and the targets that will ensure the success of the production process.

5. Service orientation is based on the situation that the agents are expected to explore the standard services of the others.
6. Modularity and reconfiguration are characteristic by the flexible reaction to any structural changes such as problems in the production systems – broken components, new machines added into the production line, failures of the machines etc. [9]

It will lead to the savings in the production costs of finished products because the labour costs will be dramatically decreased. [10], [11] On the other hand, the fixed costs connected with the new technologies will negatively influence the final price of the products. These irreversible changes will take place in the industry at first and then the human factor will be very often replaced by machines in the sector of services.

Due to the upcoming changes, the financial management of the company, financial accounting and controlling will be influenced by the concept Industry 4.0. Due to computerization, there will be a possibility to obtain required information faster. [12], [13]

The administrative task will be provided by the machines and it will lead to the decrease of expenses connected with wages. On the other hand, the company will have to invest money in the new technologies that will be able to ensure the preparation of required information. However, it will lead to the increase of the total amount of amortization from the view of operational expenses and the companies will ensure the security and the accuracy of provided and published information.

The authors of the submitted paper found following selected impacts affecting the financial management, controlling and financial accounting based on the expert discussion and the analysis as it is presented in the Table 1. Many of these impacts connected with the implementation of the Industry 4.0 in the company represent the business risks to the company.

Table 1: Impacts of selected factors on the financial management, the financial accounting and the controlling

IMPACTS ON FINANCIAL MANAGEMENT AND FINANCIAL ACCOUNTING	IMPACTS ON CONTROLLING
The need to filter data from the Big Data	Online implementation of budgets
Establishment of new models for effective filtration of data used for the online management	The risk of incorrectly configured IT systems (enormous impact on information provided by the financial accounting)
Establishment of new methods used for the analysis of the company	Controlling will be mapping new possible risks
IT interconnection of the company and the bank in case of negative cash flow	Control of the price versus price request of the customer
Radical decrease of the employees in the area of financial management and financial accounting	Question how to measure risks connected with labour (mental disorders of the employees, their inadaptability)
Online monitoring of the financial performance of the company	Risk of access of the labour to the Big Data of the company
Reduction of unnecessary administrative tasks	Misuse of the data by external entities
Termination of paper accounting documents	The need for manual intervention in the system
New way of calculation of wages	Online communication with the auditor
Crucial change in the structure of expenses	The link between online ordering and setting prices, confirmation of the price, production and transfer of the product to the customer in the shortest possible time
Direct interconnection between the company and the financial authority	Enormous increase of data mining
New way of measurement of own produced inventories	Reduction of unnecessary administrative tasks in controlling

Reporting period could be shorter than 1 year	Radical decrease of the employees in the controlling
Absolute computerization of the financial accounting	Process of implementation of online entries of financial authority to the information system of the company

Source: own elaboration

3. Discussion

However, the role of IT is crucial in the concept Industry 4.0, all companies will have to be well prepared if they want increase their financial performance. IT technologies are the foundation stone of the production process and they influence all phases of the production process. This chapter is focused on the discussion about identified and analyzed crucial opportunities and business risks connected with the Industry 4.0 from the view of the financial management, financial accounting and controlling.

3.1 Impacts of selected factors on the financial management and the financial accounting

A lot of data will be created during the transformation of material to the final product that will be sold to the customer. The company will have to filter the basic and important data and it can cause the risk that some important information will be lost in a huge amount of data. Everything will be connected with the establishment of the new models that will effectively filter data for the management of the company. The interconnection between the company, its suppliers, customers, banks, state authorities etc. will create the effective way of the communication but it can cause some problems e. g. the bank will be informed if the company has the negative cash flow, all changes of once accounted transactions in the information system will be probably sent to the financial authority etc. The employees, who prepare now the documents for accounting, clearing transactions in financial accounting will be replaced by the machines. It will bring the savings in personnel costs but on the other hand, here comes a question who will be responsible for the data created in the information systems. The reduction of employees working for the purposes of the financial accounting can decrease the reliability of the presented accounting information to its users. The same situation will be in the case of the staff processing the documents for the financial management.

On the other hand, there will be new possibilities to obtain online data from financial accounting for the purposes of financial management or other users of the accounting information. The whole production process will be accounted by itself. All components will be equipped with the chips that will automatically record the transaction such as consumption of material, depreciation of the machines, increase or decrease of the amount of the work in progress, semi-finished products, products, goods. The invoices will be automatically sent to the customers and the whole process will be much faster and probably much cheaper in comparison with the present situation.

The company will decrease the costs connected with the reduction of the unnecessary administrative task, all paper documents will be terminated, because the whole accounting process will be in electronic form only. On the other hand, here comes a question of protection of all information against abuse and the data security.

The employment of the employees will be probably in another form in comparison with the present situation. It is predicted that one employee will work for more than one company, a lot of employees will work from their home offices. It will be necessary to change the current labor legislation because this situation will change the system of the computing the wages. Due to the radical changes in the recording of accounting transactions in the new production

models, the structure of expenses will be probably modified in accordance with the new requirements. Because of the total computerization of the production process, the new ways of measurement of the own inventories will have to be developed.

All these changes can lead to the question if it is possible to have a shorter accounting period than one year. The authors of the submitted paper suppose that the accounting period will be a quarter or one month in the future. All users of accounting information will obtain financial statements periodically. The costs connected with the preparing financial statements will be close to zero and the state authorities will obtain income taxes quickly and regularly. For information of the reader of this paper, the same situation is in reporting of Value Added Tax and Consumption Tax now in the Czech Republic.

3.2 Impacts of selected factors on the financial management and the financial accounting

IT technologies allow creating online budgets for the purposes of the controlling. The systems will be implemented in the company. All budgets will be prepared by machines from the information obtained from the financial and managerial accounting. There can occur a problem when some of the entered data will be incorrect. There must be prepared some control that will be able to find problems and correct them immediately. Incorrectly configured IT systems or wrongly prepared budgets can dramatically influence the financial performance of the company.

The authors of this paper suppose that the new IT technologies will be able to identify new possible business risks from the filtered Big Data in advance and it will lead to the adequate actions against these risks.

Selling prices of the product will be calculated automatically from entered data without the intervention of human factor. Here come following questions. Will the customer accept this price? Will be there any possibilities to improve the price in accordance with the requests of the customer? Is it possible to set the price of the product only by machines? Will be possible to influence the entered data in systems manually? The companies will have to prepare the link between online ordering and setting prices, confirmation of the price, production and transfer of the product to the customer in the shortest possible time.

The next important problematic area is the area of the human resources. It is clear that the nature of work of the employees will different than it is now. It is expected that employees will be employed by several companies at once. The next problems can occur when the companies reduce the unnecessary administrative task prepared by employees. Their work will be made by machines and these employees will be no longer employed. The demand for labor will be decreased in the future. Only well-educated employees will have the possibility to be employed by the companies. Manual and administrative work will be made mostly by robots. Employees will be under considerable psychological pressure and it can cause them a lot of psychic problems. The companies will have to solve the occupational diseases. Here comes a question how to measure risks connected with labour (mental disorders of the employees, their inadaptability etc.).

There will be the possibility of very strong data mining by the state due to the computerization and the interconnection of the used systems with the state authorities (mostly financial authority). If the state authorities have the access to the complete information system, it can cause big problems with the security of all available information that could be misused. The valid legislation of each state will have to be prepared for these possible problems.

4. Conclusion

Industry 4.0 is a big challenge for the companies, society and the economy of the developed market countries. The production process will be much faster and effective. The modified production process will lead to the reduction of the costs in the long term. If the company does not prepare for the upcoming changes, it will lose probably its competitiveness. Only digitalized companies will be able to cooperate. The authors of the paper suppose that a lot of small companies will go to the bankruptcy or they will be sold or merged with other big companies, because they will not be able to invest in new IT technologies.

Computerization in the industry and later in services will lead to the big changes in the labor market. Only well-educated employees will have a chance to be employed and a lot of people will be probably unemployed. On the other hand, there will be created new jobs connected with the new important services used for the production in the economy.

The financial management of the company, financial accounting and internal control system will be influenced by the concept Industry 4.0. Due to computerization, there will be a possibility to obtain required information faster. The administrative task will be provided by the machines and it will lead to the decrease of expenses connected with wages. Nevertheless, the company will have to invest money in the new technologies that will be able to ensure the preparation of required information. It will lead to the increase of the total amount of amortization from the view of operational expenses.

The authors of the paper suppose that the impacts on internal control system will be mainly positive. The whole process will be supervised by the state authorities (probably by the financial authorities). They will have online data and there will be more difficult to implement creative accounting to the corporate environment in comparison with the present situation. The users of accounting information will obtain true and fair information about the company and they will be operatively able to accept and take appropriate actions.

Companies will face considerable business risk and must be prepared to adapt to the new conditions and increase their competitiveness and financial performance. The business risks in terms of financial management, financial accounting and controlling connected with the Industry 4.0 were identified and described in the above text.

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References

[1] Kagermann, H. (2015). Change Through Digitization—Value Creation in the Age of Industry 4.0. *Management of Permanent Change*, pp. 23-45.

[2] Brettel, M., Friederichsen, N., Keller, M. and Rosenberg, M. (2014). How virtualization, decentralization and network building change the manufacturing landscape: An industry 4.0 perspective. *International Journal of Mechanical, Industrial Science and Engineering*, 8(1), pp. 37-44.

[3] Lee, J., Bagheri, B., and Kao, H. A. (2015). A cyber-physical systems architecture for industry 4.0-based manufacturing systems. *Manufacturing Letters*, 3, pp. 18-23.

[4] Hřib, K. (2016). Internet věcí a Asset Management. *PPP 4.0 – Připraveno pro Průmysl 4.0*, pp 50-50.

[5] Varghese, A. and Tandur, D. (2014) Wireless requirements and challenges in industry 4.0. *Contemporary Computing and Informatics (IC3I)*, pp. 634-638.

[6] Mosterman, P.J. and Zander, J. (2016). Industry 4. 0 as a Cyber-Physical System study. *Software and Systems Modeling*, 15(1), pp 17-29.

[7] Drath, R. and Horch, A. (2014). Industrie 4.0: Hit or Hype? [Industry Forum]. *IEEE Industrial Electronics Magazine*, 8(2), pp 56-58.

[8] Havle, O. (2016). Průmysl 4.0 z pohledu MSP. *PPP 4.0 – Připraveno pro Průmysl 4.0*, pp. 16-22.

[9] Mařík, V. (2016). *Průmysl 4.0 – Výzva pro Českou republiku*. Brno: Management Press.

[10] Shrouf, F. and Miragliotta, G. (2015). Energy management based on Internet of Things: Practices and framework for adoption in production management. *Journal of Cleaner Production*, pp 235-246.

[11] Mařík, V. et al. (2016). *Národní iniciativa Průmysl 4.0*. Praha: ČVUT.

[12] Gorecky, D., Schmitt, M., Loskyll, M., and Zuhlke, D. (2014). Human-machine-interaction in the industry 4.0 era. *2014 12th IEEE International Conference on Industrial Informatics (INDIN)*.

[13] Havelka, Z. (2016). Datová architektura, syntetická inteligence – průmysl 2025 aneb kolik andělů se vejde na špičku jehly? *PPP 4.0 – Připraveno pro Průmysl 4.0*, pp 16-22.

Estimation of IBNR reserves with the help of a random shift

Galina Horáková [1]

Abstract

Stochastic methods are used in particular with the aim of modelling the capital needed to cover expected and unexpected claims, but not usually in connection with estimating reserves in respect of incurred but not reported claims (*IBNR*). As opposed to the often used deterministic models, which estimate the expected value of the claim amounts, a stochastic model calculates the possible claim amounts. The question of how to estimate reserves in respect of claims incurred in the reporting period, but not yet reported, is covered in the tendered paper, in which we represent the *IBNR* claims as a random variable.

Key words

IBNR claims, *RBNS* claims, random variable of the delay in reporting claims, individual model, collective model, reserves

JEL Classification: G 22

1. Technical claim reserve

The technical claim reserve represents in both life and non-life insurance an estimate of the total claim amounts arisen up to the end of the accounting period reduced by those relevant amounts already paid. It is based on the claim amounts in respect of the claims which have been incurred during the accounting period, have been reported but not yet settled (reported but settled claims) and the claims which have been incurred, but have not yet been reported by the end of the accounting period (*incurred but not reported*). The definition of the *IBNR* reserve allows us to justify what can be considered as an *IBNR* claim.

Given that the aim of the paper is to describe a stochastic approach to estimating the *IBNR* reserve we will only mention in passing that the most common approach used by insurers is the deterministic one, whereby on the basis of historical data relating to the number of notified and settled claims, the year and time of the claim event, and the claim amount reserves are estimated by classifying and aggregating the data into development triangles. Amongst such approaches belong for example the Chain Ladder and Bornhuetter-Ferguson methods. The bootstrap method can also be applied to estimate deterministically the reserves, but this then falls into the stochastic methods. As opposed to the deterministic method, where *IBNR* reserves are estimated on the basis of the expected levels of future claim amounts, making use solely of the claim development in previous years, by using the bootstrap method we gain comparable results, but also information as to how the data diverge from the assumed model. Thus this stochastic method is simple and is founded on a correctly set up model for estimating the reserves.

In the paper we will deal with a method, where the delay in notifying a claim is expressed as a random variable and the results obtained are used in the context of an individual risk

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model. This allows us, besides the variability, also to gain information for example about the economic capital needed, i.e. the capital required to cover unexpected claims, by means of the *CVaR* risk measure. An *IBNR* reserve in life insurance can be set up in respect of mortality risks or for example in respect of risk rider benefits. The total reserve is calculated as the sum of the unearned premiums for each relevant insurance contract.

2. Estimating the IBNR claim distribution

The method we set out is more onerous but can be used for example also in the case that the insurer does not have available data for previous years, or there has been a change in contract conditions which can affect the level of claims. We will consider the date of the claim event and the date it was notified. We will construct a model with the following assumptions and notation:

- the portfolio examined is made up of *m* independent insurance contracts,
- a claim is equally likely to happen at any time in the period of insurance and the probability of a claim arising for an individual risk we will denote as *q*. (This assumption can be extended to cover types of insurance such as travel insurance and agricultural insurance, where there is a seasonal effect),
- the level of an individual claim *Y* may also be a degenerate random variable independent of the random variable *D* describing the delay in reporting a claim,
- for each risk there can arise at most one insured event,
- we will take the period of insurance as being 365 days and for the purpose of record assume that claim events are notified at time 23:59:59,
- the probability that a claim event occurs during a particular day of the insured period is $q_{1/365}$,
- the probability of a claim notified up to the time of calculation of the IBNR reserve is q_{RBNS}
- the probability that a claim will be notified in a later insurance period we will denote as q_{IBNR}.

The first intermediate step will be to express the probabilities that a claim arises, i.e. q_{RBNS} and q_{IBNR}. We will then combine these probabilities into a model typical for life insurance, the individual model. Using the basic properties of this model we will express relationships which allow us not only to estimate the reserves needed to cover the total claims from the portfolio of life insurance contracts, but also to use the well-established approach to determining the capital to cover unexpected claims set out in [3].

2.1 Probability of a claim arising

Given the assumption that during the period of insurance at most one claim event can occur for a particular risk, the probability a claim occurs on a particular day in the insured period is $q_{1/365} = \frac{q}{365}$. We will denote $q_{c>b}$ as the probability that a claim will be notified at the earliest *b* days from the date it occurred. Then the probability of a claim arising which will be notified at the earliest *b* days from the date it occurred, in the case where we are considering an insurance period of 365 days is

$$q_{c>b} = \frac{q}{365} \cdot \left(1 - P\left(D \le \frac{b}{365}\right)\right) \tag{2.1}$$

and on the basis of the expression for the probability that the claim will be notified on the *i*th day of the insured period, where $i = 1, 2, \cdots, 365$, we get

$$q_{IBNR} = \frac{q}{365} \cdot \left(1 - P\left(D \le \frac{365}{365}\right)\right) + \frac{q}{365} \cdot \left(1 - P\left(D \le \frac{364}{365}\right)\right) + \cdots + \frac{q}{365} \cdot \left(1 - P\left(D \le \frac{1}{365}\right)\right)$$
$$= \sum_{i=1}^{365} \frac{q}{365} \cdot \left(1 - P\left(D \le \frac{366 - i}{365}\right)\right) = q - \frac{q}{365} \sum_{i=1}^{365} F_D\left(\frac{i}{365}\right) \tag{2.2}$$

The function $F_D(x)$ is the distribution function for the continuous random variable describing the delay in claim notification. We can estimate the distribution on the basis of data relating to the delay in claim notification for previous insurance periods. The probability that a claim will arise in a given insurance period is the sum of the probability that a claim arose in the insurance period in which it was notified and the probability that a claim will occur in the considered period of insurance but will be notified only in a later period. Thus

$$q = q_{RBNS} + q_{IBNR} \tag{2.3}$$

By putting (2.2) into (2.3) we get

$$q = q_{RBNS} + q - \frac{q}{365} \sum_{i=1}^{365} F_D\left(\frac{i}{365}\right)$$

Hence by making simple adjustments

$$q = \frac{q_{RBNS}}{\frac{1}{365} \sum_{i=1}^{365} F_D\left(\frac{i}{365}\right)} \tag{2.4}$$

In the case that the distribution of claim notification delays is exponential with the distribution function

$$F_D(x) = 1 - e^{-\delta x}$$

the relationship for estimating the probability q_{IBNR} is the following

$$q_{IBNR} = q - \frac{q}{365} \sum_{i=1}^{365} \left(1 - e^{-\delta \frac{i}{365}}\right) = q\left(e^{-\delta \frac{1}{365}} + e^{-\delta \frac{2}{365}} + \cdots + e^{-\delta \frac{365}{365}}\right)$$
$$= \frac{q}{365}\left(e^{-\delta \frac{1}{365}} \frac{1 - e^{-\delta}}{1 - e^{-\delta \frac{1}{365}}}\right) \tag{2.5}$$

Our task is to determine the probability *q* that a claim arises in a particular period of insurance for a delay expressed by the exponential distribution, i.e. by inserting the relationship (2.5) into (2.3).

$$q_{RBNS} = q - q_{IBNR} = q - \frac{q}{365}\left(e^{-\delta\frac{1}{365}}\frac{1-e^{-\delta}}{1-e^{-\delta\frac{1}{365}}}\right) = q\left(1-\frac{1}{365}e^{-\delta\frac{1}{365}}\frac{1-e^{-\delta}}{1-e^{-\delta\frac{1}{365}}}\right)$$

and thus

$$q = \frac{q_{RBNS}}{\left(1-\frac{1}{365}e^{-\delta\frac{1}{365}}\frac{1-e^{-\delta}}{1-e^{-\delta\frac{1}{365}}}\right)} \tag{2.6}$$

The approach can be used for any type of distribution that can be applied to the delay. For example in the case that the random variable representing the delay in notifying a claim has a Pareto distribution with distribution function

$$F_D(x) = 1-\left(\frac{\delta}{x}\right)^{\alpha},\ x \geq \delta$$

$$\begin{aligned} q_{IBNR} &= q - \frac{q}{365}\sum_{i=1}^{365}\left(1-\left(\frac{365\cdot\delta}{i}\right)^{\alpha}\right) = \frac{q}{365}\left(\left(\frac{365\cdot\delta}{1}\right)^{\alpha}+\left(\frac{365\cdot\delta}{2}\right)^{\alpha}+\cdots+\left(\frac{365\cdot\delta}{365}\right)^{\alpha}\right) \\ &= \frac{q}{365}\left(365\cdot\delta^{\alpha}\left(\sum_{i=1}^{365}\frac{1}{i^{\alpha}}\right)\right) \end{aligned} \tag{2.7}$$

and thus the probability that a claim arises expressed in terms of the parameters of the Pareto distribution i

$$q = \frac{q_{RBNS}}{365\cdot\delta^{\alpha}\cdot\sum_{i=1}^{365}\frac{1}{i^{\alpha}}} \tag{2.8}$$

We can extend these thoughts to the situation where the claim event will be notified in an arbitrary future insurance period. Instead of dividing the insurance period into 365 days we can divide it into *m* parts, whereby we will consider the situation where $m \to \infty$ and the already calculated equation applies $q_{1/m} = \frac{q}{m}$, which is the probability a claim arises in a particular part of the insurance period. Then again the probability of a claim arising which is notified at the earliest *b* days after the claim event, where *r* is the number of completed accounting periods from the claim event date can be expressed using

$$q_{c>b} = \frac{q}{m}\cdot\left(1-P\left(D \leq \frac{b}{m}\right)\right)$$

and by modifying equation (2.2) we get

$$q_{IBNR} = \lim_{m\to\infty}\left[\frac{q}{m}\cdot\left(1-P\left(D\le\frac{rm}{m}\right)\right)+\frac{q}{m}\cdot\left(1-P\left(D\le\frac{rm-1}{m}\right)\right)+\cdots+\frac{q}{m}\cdot\left(1-P\left(D\le\frac{rm-m+1}{m}\right)\right)\right]$$
$$= \lim_{m\to\infty}\frac{q}{m}\cdot\sum_{i=0}^{m-1}\left(1-P\left(D\le\frac{rm-i}{m}\right)\right) = q-\lim_{m\to\infty}\frac{q}{m}\cdot\sum_{i=0}^{m-1}\left(1-P\left(D\le\frac{rm-i}{m}\right)\right)$$

In the case that the delay in notification has an exponential distribution

$$q_{IBNR} = \lim_{m\to\infty}\left[\sum_{i=0}^{m-1}\frac{q}{m}\left(e^{-\frac{\delta}{m}(rm-1)}\right)\right]$$
$$= \lim_{m\to\infty}\frac{q}{m}\left(e^{-\delta\frac{rm}{m}}\,\frac{1-e^{-\delta}}{1-e^{-\delta\frac{1}{m}}}\right) = q\;1-e^{-\delta}\;\;e^{-\delta r}\lim_{m\to\infty}\frac{1}{m\left(e^{\frac{\delta}{m}}-1\right)}$$

we then have

$$q_{IBNR} = \frac{q}{\delta}\;1-e^{-\delta}\;\;e^{-\delta r} \tag{2.9}$$

and

$$q = \frac{q_{RBNS}}{1-\frac{1}{\delta}\;e^{\delta\,1-r}\;-e^{\delta r}} \tag{2.10}$$

The given approach zooms in on the estimate of the probability of a claim arising for various cases of the distribution of the delay in notification from the date of the claim event. We will set out how all this affects the level of the reserves in the next section.

2.2 Incorporating IBNR claims into the individual risk model

The assumptions with which we have been working are those of the individual risk model and we can therefore use this model to estimate the characteristics of the random variable describing the total claim amounts from a given portfolio of insurance contracts. It also however permits the calculation of the reserves needed as well as the capital for unexpected claims and thus extends the amount of data about the risk as compared with the triangle methods and the use of a bootstrap.

Let the random variable

$$S^{ind} = X_1 + X_2 + \cdots X_m \tag{2.11}$$

represent the total claim amounts from *m* risks, whereby in respect of each risk at most one claim event can arise. The random variables $X_i,\ i=1,2,\cdots,m$ represent the size of each claim amount from a given risk and are mutually independent with a compound alternative distribution, i.e. $X_i \sim CoA(q_i; F_{Y_i}(x))$, where q_i is the probability of an *RBNS* claim event for the *i*th risk and $Y_i, i=1,2,\cdots,m$ correspond to the levels of the claim amount if a claim arose. On the basis of the theorem for expressing the distribution law of a sum of independent compound distributions, we can in the case that the variables X_i have an identical distribution law work with the random variable

$$S_m^{ind} \sim CoBi(m; q_{RBNS}; F_Y(x)), \tag{2.12}$$

in the case of non-identical random variable distributions

$$S_m^{ind} \approx CoPo\left[\sum_{i=1}^{m} q_i; F_Y(x) = \sum_{i=1}^{m} \frac{q_i}{\sum_{i=1}^{m} q_i} F_{Y_i}(x)\right] \tag{2.13}$$

The distribution of the total claim amounts is expressed using the collective model. A specific expression for the reserves and economic capital will not be given here, as for the collective model the approach is covered in detail in [5]. Our specific task is to specify in equations (2.11) and (2.12) the probability that a claim event occurs in connection with q_{RBNS} a q_{IBNR}. We will therefore devote our attention to the composition of proven equations for expressing the distributions of the total claim amounts in a portfolio of *m* independent life insurance contracts.

Let the number of risks, for which a claim can arise which will be notified in the same insurance period, in a portfolio of *m* risks be *k* with $k = m$. Then the number of *IBNR* claims is zero and we get the following relationship

$$S_m^{ind} \sim CoBi(m; q_{RBNS}; F_Y(x)),$$
$$E\ S_m^{ind} = m \cdot q_{RBNS} \cdot E\ Y\ , D\ S_m^{ind} = m \cdot q_{RBNS} \cdot D\ Y\ + m \cdot q_{RBNS} \cdot 1 - q_{RBNS}\ \cdot E^2\ Y \tag{2.14}$$

If we now let *k*, the number of risks where a claim is notified in the period in which the claim event arises, equal $k = m-1$, we can express the distribution of the total claim amounts as follows

$$S_m^{ind} = S_{m-1\ RBNS}^{kol} + S_{1IBNR}^{kol}$$

whereby

$$S_{(m-1)RBNS}^{ind} \sim CoBi\ \ m-1; q_{IBNR}; F_Y(x)\ ; S_{1IBNR}^{ind} \sim CoBi\ \ 1; q_{IBNR}; F_Y(x) \tag{2.15}$$

The most important characteristics are then in this case

$$E\ S_m^{ind} = m-1 \cdot q_{RBNS} \cdot E\ Y\ + q_{IBNR} \cdot E\ Y\ ,$$
$$D\ S_m^{ind} = m-1 \cdot q_{RBNS} \cdot \bar{q}_{RBNS} \cdot E\ Y\ + m-1 \cdot q_{RBNS} \cdot D\ Y\ + q_{IBNR} \cdot \bar{q}_{IBNR} \cdot E^2\ Y\ + q_{IBNR} \cdot D\ Y \tag{2.16}$$

where $\bar{q}_{IBNR} = 1 - q_{IBNR}, . \bar{q}_{RBNS} = 1 - q_{RBNS}$.

We repeat this process, whereby we finish up with the case where no claim is notified in the considered insurance period to get

$$S_m^{ind} = S_{mIBNR}^{kol}, \text{ where } S_{mIBNR}^{kol} \sim CoBi\ \ m; q_{IBNR}; F_Y(x) \tag{2.17}$$

The end result is then that we can express the total claim amounts from the given portfolio of *m* insurance contracts using the variable

$$S_m^{ind} = S_{m-k}^{kol}\ \ m-k; q_{RBNS}; F_Y(x)\ + S_k^{kol}\ \ k; q_{IBNR}; F_Y(x) \tag{2.18}$$

where $k = 0,1,...,m$, and using the basic properties of means we can express its mean value as

$$E\ S_m^{ind} = E\ S_{(m-k)RBNS}^{kol} + E\ S_{klBNR}^{kol} =$$

$$= E\ Y \cdot m \cdot q_{RBNS} + k\ q_{IBNR} - q_{RBNS} \tag{2.19}$$

Analogously using more complicated adjustments we can express the dispersion as

$$D\ S_m^{ind} = E\ Y^2 \cdot m-k \cdot q_{RBNS} + k \cdot q_{IBNR} - E^2\ Y \cdot m-k \cdot q_{RBNS}^2 + k \cdot q_{IBNR}^2 \tag{2.20}$$

Given that we have the basic information regarding claims which have been notified, we can without problem estimate the relevant reserves. Estimation of the *IBNR* reserves represents a more complex problem, as we will only have information regarding them in some future insurance period. We can solve this also on the basis of the characteristics set out in (2.19), (2.20). We can also express the random variable for the total claim amounts in a portfolio of *m* life insurance contracts in the case that we also consider *IBNR* claims by making use of the random variable $S_k^{kol}\ k; q_{IBNR}; F_Y(x)$ in the case that a claim can be notified at the most for *m-k* risks in the insurance period. This random variable enables us to estimate the IBNR claim amounts by an approach which can also be used to estimate the total reserves. We adduce the simplest case whereby we use the normal distribution, but note that we can also use for example an approximation using the gamma distribution with a shift or the inverse Gauss distribution with a shift, which also make available expressions for the values $CVaR_{0,995}(S_m^{kol})$.

We can then estimate the reserves *U* for the total claim amounts in the case that in equation (2.12) we consider the probability that an occurred claim will be notified in the followed insurance period as well as the probability that a claim has in fact occurred in that period, but will only be notified in the following period by adjusting the equation

$$P\ U + 1+\theta \cdot E\ Y \cdot m-k \cdot q_{RBNS} + k \cdot q_{IBNR} - S_m^{ind} < 0 = \varepsilon$$

$$\Phi\left(\frac{U + \theta \cdot E\ Y \cdot m-k \cdot q_{RBNS} + k \cdot q_{IBNR}}{\sqrt{E\ Y^2 \cdot m-k \cdot q_{RBNS} + k \cdot q_{IBNR} - E^2\ Y \cdot m-k \cdot q_{RBNS}^2 + k \cdot q_{IBNR}^2}} \right) \approx 1-\varepsilon$$

$$U \approx u_{1-\varepsilon} \cdot \sqrt{E\ Y^2 \cdot m-k \cdot q_{RBNS} + k \cdot q_{IBNR} - E^2\ Y \cdot m-k \cdot q_{RBNS}^2 + k \cdot q_{IBNR}^2} - \tag{2.21}$$
$$- E\ Y \cdot m-k \cdot q_{RBNS} + k \cdot q_{IBNR}$$

3. Conclusion

The aim of the paper has been to present an approach whereby, on the basis of a random variable representing the delay in notifying an insurance claim from the date of its happening, one can modify the estimate of the reserves required to cover the given risks taken on. The approach is based on individual claims data together with data on the delay in the notification of claims. An assumption, which needs to be tested, is whether the delay in notification of a claim depends on the size of the individual claim amount. An estimate of the probability that a

claim arises which is notified in the followed insurance period and an expression for the probability that a claim event arises in the followed period, but only notified in the next period, are incorporated into the construction of an individual model and it was shown that we can consider the total claim amount as a random variable with a compound distribution specified by equation (2.21), which allows us to estimate the economic capital using the approach set out in detail in [5]. We can therefore assert that as a final result it is possible to analyse the risk in a simple way by inserting given data into the equations which we have derived here. After their derivation the most laborious task is to estimate the random variable for the delay in notification, for which one can use the Kolmogorov-Smirnov test. The processing of a significant amount of statistical data very often leads to a rejection of the hypothesis as the significance level comes very close to zero. This comes from the fact that the quantities used to compare the deviations of the empirical distribution from the assumed converge very quickly to zero as the number of observations increases. It is therefore advisable to choose a distribution with the highest value for the parameter "significance level", for example by using the Statgraphics program.

References

[1] Klugman, S. A. – Panjer, H. H. – Willmot, G. E. (2012). *Loss Models* (From Data to Decision). 4. ed. New York : John Wiley & Sons, 2012. ISBN 978-1-118-31532-3 .

[2] Panjer, H., H. (2006). *Operational Risk* (Modeling Analytics). John Wiley &Sons. ISBN -13 978-0-471-76089-4.

[3] Horáková,G. (2014). *Stochastic approach to determining reservves in non –life insurance*. 7. mezinárodní konference Řízení a modelování finančních rizik, KF Ekonomická fakulta VŠB-TU Ostrava, 2014.

[4] Horáková, G. (2013). *Brownov pohyb pre odhad pravdepepodobnosti krachu procesu prebytku.* Aktuárska veda v teórii a v praxi. 9. medzinárodná vedecká konferencia. Vydavateľstvo EKONÓM, 2013. - ISBN 978-80-225-3639-4.

[5] Horáková,G. – Páleš, M. – Slaninka, F. (2015). *Teória rizika v poistení*. Wolters Kluwer. ISBN 978-80-8168-273-59.

[6] Horáková,G. – Slaninka, F. – Simanová, B. (2013). *Funkcia strednej nadmernej škody a jej praktické využitie v aktuárstve*. In Ekonomika a informatika. roč. 11, č. 1 : vedecký časopis FHI EU v Bratislave a SSHI. Bratislava : Fakulta hospodárskej informatiky : Slovenská spoločnosť pre hospodársku informatiku, ISSN 1336-3514.

[7] Gašpar, P.(2004). Metódy výpočtu rozdelenia celkových škôd a ich aplikácia na odhad rezerv na poistné plnenia. Dizertačná práca, EUBA, FHI, KM.

[8] Sakálova, K. –Bilíková, M. –Mihalechová, J. (2015). *Interest rate risk, its management and the effect on actuarial calculations in respect of annuities from Pillar II*. Conference:10th International Scientific Conference on Financial Management of Firms and Financial VŠB-TU Ostrava.

[9] Skřivánková V.(2010). *The influence of extreme claims on the risk of insolvency* In Managing and Modelling of Financial Risks, 2010, VŠB-TU Ostrava, pp. 357-363.

[10] Zákon č. 39/2015 Z.z. o poisťovníctve a o zmene a doplnení niektorých zákonov.

Príspevok vznikol v rámci projektu VEGA 1/0806/14: Kalkulácia SCR na krytie rizík neživotného poistenia v súlade s potrebami praxe.

Accountig measurement of the acquired assets and assumed liabilities in the business combination[1]

Renáta Hornická [2]

Abstract

The paper deals with accounting of business combinations which are regulated by IFRS 3 Business combinations (2008). An acquirer recognises the assets acquired and liabilities assumed at their acquisition-date fair values and discloses information that enables users to evaluate the nature and financial effects of the acquisition. IFRS 13 defines fair value as the exit price, that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at measurement date. The objective of the paper is to present and explain requirements related to measurement of acquired assets and assumed liabilities in the business combinations and explain impact of the initially measurement of financial effects of business combination.

Key words: business combinations, acquired assets, assumed liabilities, fair value

JEL Classification: M40, M41

1. Úvod

Uplatenenie IFRS 3 *Podnikové kombinácie*, ktorý bol vydaný v roku 2008 [ďalej aj „IFRS 3 (2008)“][3] sa v podmienkach Slovenskej republiky týka predovšetkým materských účtovných jednotiek, ktoré zostavujú konsolidovanú účtovnú závierku. Konsolidovaná účtovná závierka podnikateľských subjektov sa v Slovenskej republike zostavuje výlučne podľa IFRS prevzatých Európskou úniou (od 1. januára 2005). Uplatenie IFRS 3 (2008) sa môže týkať aj účtovných jednotiek, ktoré zostavujú individuálnu účtovnú závierku podľa IFRS, ak uskutočnili podnikovú kombináciu, ktorá nevedie k vzniku povinnosti zostaviť konoslidovanú účtovnú závierku. Povinnosť zostaviť individuálnu účtovnú závierku podľa IFRS majú účtovné jednotky upravené zákonom č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov.

Podnikové kombinácie predstavujú podľa IFRS 3 (2008) transakciu alebo inú udalosť, pri ktorej nadobúdateľ získa kontrolu nad nadobúdaným podnikom (nadobúdanými podnikmi). Hlavnou zásadou je, aby **nadobúdateľ vykázal nadobudnuté identifikovateľné aktíva a prevzaté záväzky ocenené ku dňu akvizície v reálnej hodnote** a zverejnil informácie, ktoré umožňujú používateľom zhodnotiť charakter a finančné dopady akvizície.

[1] Príspevok bol spracovaný ako výstup výskumného projektu KEGA č. 026 EU-4/2016 *Potreba skvalitnenia účtovných informácií s využitím nadnárodných úprav, osobitne IFRS, v legislatívnych podmienkach Slovenskej republiky.*

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[3] Tento štandard bol vydaný v januári 2008 s účinnosťou od 1. 7. 2009 a je výsledkom postupného vývoja v rámci realizovaného projektu zameraného na podnikové kombinácie, ktorý viedla IASB v rokoch 2001-2008.

Cieľom príspevku je prezentovať požiadavky týkajúce sa oceňovania nadobudnutých aktív a prevzatých záväzkov v podnikovej kombinácii ku dňu akvizície podľa IFRS 3 (2008) a poukázať na dôsledky prvotného ocenenia nadobudnutých aktív a prevzatých záväzkov v podnikovej kombinácii.

2. Identifikácia, oceňovanie a vykazovanie nadobudnutých aktív a prevzatých záväzkov v podnikovej kombinácii podľa IFRS 3 (2008)

Oceňovanie nadobudnutého identifikovateľného majetku a prevzatých záväzkov v podnikovej kombinácii je samostatným krokom akvizičnej metódy účtovania[4]. V účtovníctve nadobúdateľa je potrebné zaznamenať (t. j. zaúčtovať a vykázať) jednotlivé nadobudnuté položky majetku a prevzatých záväzkov patriace nadobúdanému podniku, ktorý je predmetom transakcie podnikovej kombinácie. Ocenenie majetku a záväzkov nadobúdateľa nie je podnikovou kombináciou dotknuté, pretože majetok a záväzky nadobúdateľa nie sú predmetom podnikovej kombinácie[5].

V súvislosti s oceňovaním nadobudnutých aktív a prevzatých záväzkov v podnikovej kombinácii je dôležité oceňovanie ku dňu akvizície a oceňovanie po dni akvizície (tabuľka 1).

Tabuľka 1 : Oceňovanie nadobudnutých aktív a prevzatých záväzkov v podnikovej kombinácii

Oceňovanie	Postup
Oceňovanie ku dňu akvizície	Postup podľa IFRS 3 (2008) – v reálnej hodnote okrem stanovených výnimiek
Oceňovanie po dni akvizície	Postup podľa IFRS, do pôsobnosti ktorých nadobudnutý identifikovateľný majetok a prevzaté záväzky patria okrem stanovených výnimiek v IFRS 3 (2008)

Zdroj: Vlastné spracovanie

Nadobúdateľ má vykázať **ku dňu akvizície** všetky nadobudnuté identifikovateľné aktíva a prevzaté záväzky ocenené podľa IFRS 3 (2008) **v reálnej hodnote.** Táto požiadavka vyplýva z aplikácie dvoch základných princípov IFRS 3 (2008), a to princípu vykazovania (recognition principle) a princípu oceňovania (measurement principle). Tam kde nie je možné tieto princípy uplatniť, stanovuje IFRS 3 (2008) výnimky z ich uplatnenia.

Vykázané nadobudnuté identifikovateľné aktíva a prevzaté záväzky musia spĺňať **podmienky vykazovania** (recognition conditions): musia spĺňať **definíciu aktív a záväzkov** stanovenú Koncepčným rámcom pre finančné vykazovanie IFRS a musia **byť súčasťou transakcie podnikovej kombinácie,** nesmú byť súčasťou samostatnej transakcie realizovanej mimo podnikovej kombinácie[6], ktoré sa účtujú osobitne.

[4] Akvizičná metóda pozostáva podľa IFRS 3 (2008) z nasledujúcich krokov: identifikácia nadobúdateľa, určenie dňa akvizície, vykazovanie a oceňovanie nadobudnutých aktív a prevzatých záväzkov nadobúdaného podniku a nekontrolujúcich podielov, vykazovanie a oceňovanie goodwillu alebo zisku z výhodnej kúpy.

[5] S výnimkou vzniku odloženej daňovej pohľadávky nadobúdateľa, ktorá môže vzniknúť v situácii, keď bude schopný uplatniť svoje nevyužité daňové straty oproti zdaniteľnému zisku nadobúdaného podniku. Táto odložená daňová pohľadávka však nie je súčasťou prvotného zaúčtovania podnikovej kombinácie.

[6] Samostatnou transakciou transakcie uzatvorené pred podnikovou kombináciou nadobúdateľom alebo v mene nadobúdateľa, ktoré sa zadali skôr na prospech alebo primárne na prospech

Dôsledkom uplatnenia princípu vykazovania a podmienok vykazovania môže byť skutočnosť, že **nadobúdateľ vykazuje aj také aktíva a záväzky, ktoré predtým nadobúdaný podnik nevykazoval**, napríklad nehmotný majetok vytvorený nadobúdaným podnikom vlastnou činnosťou (interne) alebo podmienené záväzky, ak sú splnené kritériá stanovené IFRS 3 (2008).

Nehmotný majetok nadobúdaného podniku má byť vykázaný samostatne (t. j. oddelene od goodwillu), ak je identifikovateľný. Identifikovateľnosť nehmotného majetku znamená, že ho možno oddeliť od nadobúdaného podniku alebo spĺňa zmluvno-právne kritérium (bez ohľadu na to, či tieto práva sú oddeliteľné od nadobúdaného podniku). IAS 38 *Nehmotné aktíva* (v súčasnom platnom znení) rieši obstaranie nehmotného majetku v podnikovej kombinácii samostatne (IAS 38.33-38.43). Kritérium pravdepodobnosti plynutia ekonomických úžitkov a kritérium spoľahlivosti ocenenia reálnou hodnotou sa pre identifikovateľný nehmotný majetok nadobudnutý v podnikovej kombinácii považujú vždy za splnené.

Nadobúdateľ môže vykázať podľa IAS 38 samostatne ku dňu akvizície aj výskumný a vývojový projekt nadobúdaného podniku, ak spĺňa definíciu majetku a je identifikovateľný. IFRS 3 (2008) rozdeľuje v ilustratívnom zozname nehmotný majetok do viacerých skupín, a to súvisiaci s marketingom, súvisiaci so zákazníkmi, súvisiaci s umeleckými dielami, na zmluvnom základe a súvisiaci s technológiami.

Ak nehmotný majetok nadobúdaného podniku nie je identifikovateľný, nemôže byť v podnikovej kombinácii vykázaný samostatne, napríklad ekonomické úžitky z pracovného kolektívu nie je možné identifikovať ako nehmotný majetok. Neidentifikovateľný nehmotný majetok zostáva súčasťou sumy vykázanej ako goodwill.

Výnimku z princípu vykazovania predstavujú **podmienené záväzky**. Podmienené záväzky sa podľa IAS 37 *Rezervy, podmienené záväzky a podmienené aktíva* nesmú vykazovať v súvahe (IAS 37.27), informácie o podmienených záväzkoch sa zverejňujú v poznámkach. Podľa pravidiel IFRS 3 (2008) vykáže nadobúdateľ samostatne ku dňu akvizície podmienený záväzok nadobúdaného podniku prevzatý v podnikovej kombinácii, keď je súčasnou povinnosťou, ktorá vyplýva z minulých udalostí a je možné jeho reálnu hodnotu spoľahlivo stanoviť. Podmienený záväzok sa vykáže, aj keď nie je splnené kritérium pravdepodobnosti úbytku ekonomických zdrojov stelesňujúcich ekonomické úžitky na úhradu záväzku.

IFRS 3 (2008) obsahuje **špeciálne pravidlá** týkajúce sa vykazovania ďalších položiek nadobudnutých aktív a prevzatých záväzkov, napríklad pravidlá týkajúce sa operatívneho lízingu, v ktorom je nadobúdaný podnik nájomcom, nadobúdateľ nevykáže žiadny majetok alebo záväzky spojené s týmto operatívnym lízingom, ale vykáže samostatne nehmotný majetok, ak sú podmienky zmluvy výhodné oproti trhovým podmienkam alebo vykáže záväzok, ak sú podmienky zmluvy nevýhodné oproti trhovým podmienkam; pravidlá týkajúce sa znovunadobudnutých práv.

IFRS 3 (2008) upravuje **výnimky z princípu vykazovania a oceňovania** (odložené dane, zamestnanecké požitky a majetok z odškodnenia) a **výnimky z princípu oceňovania** (znovunadobudnuté práva, platby na základe podielov a aktíva držané na predaj). Výnimky z princípu oceňovania znamenajú, že položky nadobudnutého majetku a prevzatých záväzkov zaradené do uvedených výnimiek nebudú ocenené v reálnej hodnote, ale IFRS 3 (2008) upravuje spôsob ich ocenenia samostatne, a to buď uvedením príslušného štandardu, podľa ktorého majú byť ocenené (napríklad zamestnanecké požitky (IAS 19), platby viazané

nadobúdateľa alebo kombinovaného podniku a nie primárne na prospech nadobúdaného podniku (alebo jeho predchádzajúcich vlastníkov) pred kombináciou.

na akcie (IFRS 2), odložené dane (IAS 12) alebo priamo stanovuje spôsob ich ocenenia (znovunadobudnuté práva, majetok z odškodnenia, aktíva držané na predaj).

Nadobúdateľ musí ku dňu akvizície **klasifikovať (určiť)** na základe zmluvných podmienok, ekonomických podmienok, svojej prevádzkovej politiky alebo účtovnej politiky **nadobudnutý majetok** (určiť jednotlivé druhy majetku, napríklad nehmotný majetok, nehnuteľnosti, stroje a zariadenia, zásoby a podobne) a prevzaté záväzky (určiť jednotlivé finančné alebo nefinančné záväzky). Výnimku z tejto klasifikácie predstavujú podľa IFRS 3 (2008) lízingové zmluvy a poistné zmluvy (IFRS 3.17). Lízingové zmluvy, ktoré upravuje IAS 17 *Lízingy* a poistné zmluvy upravené IFRS 4 *Poistné zmluvy*, zostávajú klasifikované na základe podmienok ku dňu účinnosti zmluvy (iba ak by sa menili podmienky zmluvy spôsobom, ktorý by viedol k zmene klasifikácii zmluvy a dňom tejto úpravy by bol deň akvizície).

Klasifikácia nadobudnutého identifikovateľného majetku a prevzatých záväzkov je dôležitá pre ich oceňovanie a vykazovanie po dni akvizície, pretože na jej základe sa stanovia IFRS, do pôsobnosti ktorých bude nadobudnutý majetok a prevzaté záväzky patriť. Špeciálne pravidlá pre vykazovanie po dni akvizície stanovuje IFRS 3 (2008) pre znovunadobudnuté práva, podmienené záväzky a majetok z odškodnenia.

2.1.1 Uplatnenie reálnej hodnoty ku dňu akvizície

Uplatnenie reálnej hodnoty sa spája s oceňovaním nadobudnutých identifikovateľných aktív a prevzatých záväzkov ku dňu akvizície. Aj keď majetok a záväzky existovali pred podnikovou kombináciou v nadobúdanom podniku (vznikli v minulosti), z pohľadu nadobúdateľa dochádza k získaniu kontroly nad majetkom a vzniku povinnosti súvisiacej so záväzkami ku dňu akvizície. Ocenenie nadobudnutých aktív a prevzatých záväzkov v reálnej hodnote má vyjadriť očakávané ekonomické úžitky spojené s majetkom a očakávaný úbytok ekonomických úžitkov spojený so záväzkami v momente dňa akvizície, a preto nie je možné vychádzať z historického (pôvodného) ocenenia tohto majetku a záväzkov. V ocenení nadobudnutého majetku a prevzatých záväzkov nadobúdaného podniku reálnou hodnotou sa zohľadňuje aj neistota spojená s očakávanými peňažnými tokmi.

V súvislosti s uplatnením reálnej hodnoty je dôležité vymedzenie reálnej hodnoty a spôsob jej stanovenia. IFRS 3 (2008) neobsahuje samostatný návod na stanovenie reálnej hodnoty, pretože v čase jeho tvorby a prijatia viedla IASB samostatný projekt zameraný na oceňovanie reálnou hodnotou. Tento projekt bol súčasťou procesu konvergencie medzi IFRS a US GAAP v oblasti oceňovania reálnou hodnotou (Mokošová, D., Bednárová B., Tkáčová L., 2013, s. 592) a výsledkom projektu bolo vydanie IFRS 13 *Oceňovanie reálnou hodnotou*. IFRS 3 (2008) zaviedol princíp oceňovania v reálnej hodnote v podnikovej kombinácii ako všeobecný princíp. Táto skutočnosť znamená, že oceňovanie v reálnej hodnote sa má aplikovať aj na ďalšie (všetky) zložky podnikovej kombinácie (napríklad poskytnutú protihodnotu, nekontrolujúci podiel a podobne) okrem stanovených výnimiek[7].

Podľa IFRS 13 (IFRS 13.9) sa reálna hodnota definuje ako cena, ktorá by sa získala za predaj aktíva alebo bola zaplatená za prevod záväzku pri riadnej transakcii medzi účastníkmi trhu k dátumu ocenenia. Oceňovanie reálnou hodnotou podľa IFRS 13 je **trhové oceňovanie a nie oceňovanie na úrovni účtovnej jednotky.** Ak podľa IFRS 13 nie je možné stanoviť trhovú cenu, má sa reálna hodnota stanoviť použitím inej oceňovacej techniky. Reálna hodnota podľa IFRS 13 je stanovená na základe výstupnej ceny. Pri oceňovaní reálnou hodnotou je podľa IFRS 13 dôležité (IFRS 13.B2):

[7] V príspevku sa venujeme iba oceňovaniu nadobudnutých identifikovateľných aktív a prevzatých záväzkov nadobúdaného podniku. Oceňovaniu ostatných zložiek podnikovej kombinácie sa nebudeme venovať.

- určiť konkrétny majetok alebo záväzok, ktorý je predmetom oceňovania (v súlade so zúčtovacou jednotkou),
- pre nefinančné aktívum určiť predpoklad oceňovania, ktorý je vhodný pre ocenenie (v súlade s maximálnym a najlepším využitím),
- určiť hlavný (alebo najvýhodnejší) trh pre toto aktívum alebo záväzok,
- stanoviť techniku (techniky) oceňovania vhodné pre stanovenie reálnej hodnoty.

Určenie konkrétneho aktíva alebo záväzku závisí od ich **zúčtovacej jednotky**. Pri oceňovaní konkrétneho aktíva alebo záväzku sa musia podľa IFRS 13 (odsek13.11) brať do úvahy vlastnosti aktíva (stav a umiestnenie aktíva a prípadné obmedzenia predaja alebo používania aktíva) alebo záväzku k dátumu ocenenia.

Pri stanovení reálnej hodnoty sa predpokladá, že sa riadna transakcia **realizuje za bežných trhových podmienok** a uskutočňuje na **hlavnom trhu** pre aktívum alebo záväzok, alebo ak hlavný trh neexistuje, na **najvýhodnejšom trhu** pre aktívum alebo záväzok. Účastníci trhu (kupujúci alebo predávajúci) by pri tejto transakcii mali konať **vo svojom najlepšom ekonomickom záujme**[8]**.** Reálna hodnota sa neupravuje o transakčné náklady, ktoré sa zaúčtujú v súlade s príslušnými IFRS (závisia od spôsobu uskutočnenia transakcie). Transakčné náklady nezahŕňajú náklady na dopravu, ktoré vzniknú za prepravu aktíva z jeho súčasného miesta na jeho hlavný (alebo najvýhodnejší) trh. IFRS 13 obsahuje zásady stanovenia reálnej hodnoty nefinančného majetku, záväzkov a nástrojov vlastného imania účtovnej jednotky, ktoré uvádzame v tabuľke 2.

Tabuľka 2 : Stanovenie reálnej hodnoty podľa IFRS 13

	Zásady oceňovania
Nefinančný majetok: pozemky, budovy a zariadenia nehmotný majetok	Reálna hodnota by mala byť cena, v ktorej bude zohľadnená schopnosť účastníka trhu vytvárať ekonomické úžitky **maximálnym a najlepším využitím aktíva alebo jeho predajom** inému účastníkovi trhu, ktorý by aktívum maximálne a čo najlepšie využil.
Finančné záväzky a nefinančné záväzky	Reálna hodnota by mala byť **cena, v ktorej by bol záväzok k dátumu ocenenia prevedený na účastníka trhu** za predpokladu, že záväzok zostal nesplatený a od nadobúdaného účastníka trhu by sa požadovalo splnenie záväzku, k dátumu ocenenia by záväzok nebol vysporiadaný s protistranou, ani by inak nezanikol.
Nástroje vlastného imania	Reálna hodnota **vyjadruje cenu, v ktorej by boli nástroje vlastného imania prevedené na účastníka trhu**, pričom sa predpokladá, že by zostali nesplatené a nadobúdajúci účastník trhu by prevzal práva a zodpovednosti spojené s nástrojom vlastného imania. K dátumu ocenenia by nástroj vlastného imania nebol zrušený, ani by nezanikol.

Zdroj: Spracované podľa IFRS 13 *Oceňovanie reálnou hodnotou*, odseky 27-37.

Pri stanovení reálnej hodnoty uplatňuje účtovná jednotka **techniky oceňovania**. Výber techniky oceňovania závisí od účtovnej jednotky. Techniky oceňovania by podľa IFRS 13 mali byť založené na maximalizácii používania zistiteľných vstupov a minimalizácii používania nezistiteľných vstupov. IFRS 13 rozlišuje tri základné techniky oceňovania, a to **trhovú metódu, nákladovú metódu a výnosovú metódu**.

[8] IFRS 13 vo svojich ustanoveniach obsahuje predpoklady, podľa ktorých je potrebné posúdiť existenciu hlavného trhu alebo najvýhodnejšieho trhu a posúdenie riadnej transakcie, ktorá je vo svojej podstate hypotetickou transakciou a účastníkov trhu. V dodatku A štandardu IFRS 13 sú definované všetky základné pojmy spojené s oceňovaním reálnou hodnotou.

IFRS 13 zavádza **hierarchiu reálnych hodnôt** na základe kategorizácie vstupov techník oceňovania používaných na oceňovanie reálnou hodnotou do troch kategórii (vstupy úrovne 1, vstupy úrovne 2 a vstupy úrovne 3). Najvyššiu prioritu majú kótované ceny (neupravené) za rovnaké aktíva alebo rovnaké záväzky na aktívnych trhoch (vstupy úrovne 1) a najnižšiu prioritu nezistiteľné vstupy (vstupy úrovne 3).

Prax účtovných jednotiek v Slovenskej republike (R. Farkaš, 2013, s. 119) ukazuje, že pri oceňovaní majetku a záväzkov v reálnej hodnote pre účely zostavovania konsolidovanej účtovnej závierky dochádza k využívaniu služieb súdnych znalcov alebo odborných poradcov, aj keď to IFRS priamo nevyžadujú, napríklad pri dlhodobom hmotnom majetku, špecifických položkách nehmotného majetku, zložitejších rezervách, zamestnaneckých požitkoch. V prípade stanovenia reálnej hodnoty zásob, pohľadávok a záväzkov z obchodného styku, jednoduchších rezerv dokážu reálnu hodnotu stanoviť aj účtovné jednotky, pretože vychádzajú z vlastných podkladov (napríklad použiteľnosť zásob, vymožiteľnosť pohľadávok).

2. 1. 2 Dôsledky prvotného ocenenia nadobudnutých aktív a prevzatých záväzkov v podnikovej kombinácii

Suma čistých aktív nadobúdaného podniku je rozdiel medzi sumou nadobudnutého identifikovateľného majetku oceneného v reálnej hodnote a sumou prevzatých záväzkov ocenených v reálnej hodnote ku dňu akvizície. Suma čistých aktív nadobúdaného podniku je súčasťou vzťahu pre výpočet goodwillu alebo zisku z výhodnej kúpy. Vplyv pravidiel uplatnených podľa IFRS 3 (2008) na vykazovanie a oceňovanie čistých aktív nadobúdaného podniku ku dňu akvizície ilustrujeme na nasledujúcich modelových situáciách.

Účtovná jednotka A nadobudla akvizíciou 100 % podiel v účtovnej jednotke B, za nadobudnutie 100 % podielu v účtovnej jednotke B poskytla protihodnotu v reálnej hodnote vo výške 3 000. Predpokladajme, že by mohla nastať niektorá z nasledujúcich troch alternatív:

a) účtovná jednotka A identifikovala v nadobúdanom podniku pri položke nehnuteľnosti, stroje a zariadenia reálnu hodnotu vo výške 1 300 a pri položke zásoby reálnu hodnotu vo výške 1 000.

b) účtovná jednotka A identifikovala v nadobúdanom podniku účtovnej jednotky B okrem rozdielov medzi reálnou a účtovnou hodnotou nehnuteľností strojov a zariadení a zásob aj interne vytvorenú obchodnú značku, ktorú ocenila reálnou hodnotou vo výške 400.

c) účtovná jednotka A identifikovala v nadobúdanom podniku okrem rozdielov medzi reálnou účtovnou hodnotou nehnuteľností, strojov a zariadení a zásob, obchodnej značky aj podmienený záväzok nadobúdaného podniku, ktorý je súčasnou povinnosťou a jeho reálnu hodnotu je možné spoľahlivo stanoviť vo výške 100.

Položky čistých aktív nadobúdaného podniku	**Pôvodná účtovná hodnota** pred uplatnením IFRS 3 (2008)	**Alternatíva a):**	**Alternatíva b):**	**Alternatíva c):**
Nehmotný majetok	x	x	400	400
Nehnuteľnosti, stroje a zariadenia	1 100	1 300	1 300	1 300
Zásoby	700	1 000	1 000	1 000
Záväzky	300	300	300	300
Podmienený záväzok	x	x	x	100
Čisté aktíva	**1 500**	**2 000**	**2 400**	**2 300**
Protihodnota-čisté atíva	1 500	1 000	600	700

Zdroj: Vlastné spracovanie

Úpravou ocenenia nehnuteľnosti, strojov a zariadení a zásob na reálnu hodnotu *(alternatíva a)* sa zvýšila suma čistých aktív nadobúdaného podniku a znížil sa rozdiel medzi poskytnutou protihodnotou a sumou čistých aktív nadobúdaného podniku. Samostatnou identifikáciou nehmotného majetku *(alternatíva b)* sa zvýšila suma čistých aktív nadobúdaného podniku, čím došlo k zníženiu rozdielu medzi poskytnutou protihodnotou a sumou čistých aktív. Identifikáciou podmieneného záväzku nadobúdaného podniku *(alternatíva c)* sa znížila suma čistých aktív nadobúdaného podniku a zvýšil sa rozdiel medzi poskytnutou protihodnotou a sumou čistých aktív.

V tomto prípade by šlo o podnikovú kombináciu, ktorá viedla k vzniku skupiny podnikov, pretože vznikol vzťah materská účtovná jednotka (účtovná jednotka A) a dcérska účtovná jednotka (účtovná jednotka B). Podniková kombinácia by sa vykázala v konsolidovanej účtovnej závierke účtovnej jednotky A. Výška vykázaného goodwillu by závisela od toho, ktorá alternatíva by v skutočnosti nastala. Goodwill[9] sa vykáže ku dňu akvizície ako aktívum a následne po dni akvizície sa testuje na zníženie hodnoty v súlade s IAS 36 *Zníženie hodnoty aktív*.

Nadobúdateľ nemôže **pri prvotnom zaúčtovaní podnikovej kombinácie ku dňu akvizície vychádzať z účtovnej hodnoty čistých aktív nadobúdaného podniku**, ale **musí vychádzať z ich reálnej hodnoty.** Len pri dodržaní pravidiel vykazovania a oceňovania podľa IFRS 3 (2008) sa môže v podnikovej kombinácii vykázať a oceniť goodwill alebo zisk z výhodnej kúpy.

Úpravou ocenenia aktív a záväzkov nadobúdaného podniku na reálnu hodnotu, dochádza v praxi aj k vzniku dočasných rozdielov medzi ich účtovnou hodnotou a daňovou hodnotou, ktoré sú predmetom odložených daní v súlade s IAS 12 *Dane z príjmu.* Takto vzniknuté odložené dane[10] z príjmu (odložený daňový záväzok alebo odložená daňová pohľadávka) ovplyvňujú výšku vykázaných čistých aktív nadobúdaného podniku ku dňu akvizície, a teda aj výšku goodwillu alebo zisku z výhodnej kúpy.

Vykázanie nadobudnutého majetku a prevzatých záväzkov v podnikovej kombinácii ku dňu akvizície ovplyvní súvahu nadobúdateľa. Náklady a výnosy, komplexný výsledok nadobúdateľa nie sú ku dňu akvizície ovplyvnené.

Vzniknuté náklady a výnosy súvisiace s nadobudnutým majetkom a prevzatým záväzkami po dni akvizície budú vychádzať z ich reálnej hodnoty ku dňu akvizície, napríklad odpisy nehnuteľností, strojov a zariadení, prípadné straty zo zníženia hodnoty nehmotného majetku (v závislosti od posúdenia doby jej použiteľnosti). Goodwill sa testuje na zníženie hodnoty podľa IAS 36 a straty zo zníženia hodnoty goodwillu sa účtujú do nákladov.

3. Záver

Oceňovanie nadobudnutého majetku a prevzatých záväzkov v podnikovej kombinácii je náročnou oblasťou aplikácie akvizičnej metódy. Podľa IFRS 3 *Podnikové kombinácie* (2008) sa ku dňu akvizície aplikuje pri oceňovaní nadobudnutého majetku a prevzatých záväzkov reálna hodnota.

Definícia reálnej hodnoty a spôsob jej stanovenia je upravený v IFRS 13 *Oceňovanie reálnou hodnotou*. Oceňovanie reálnou hodnotou podľa IFRS 13 vychádza z výstupnej ceny, pričom sa vychádza z trhového ocenenia, nie z pohľadu účtovnej jednotky. Zisťovanie reálnej

[9] Opakom goodwillu v podnikovej kombinácii je zisk z výhodnej kúpy. Zisk z výhodnej kúpy sa po prehodnotení zaúčtuje do výnosov nadobúdateľa (ovplyvní výsledok hospodárenia).

[10] Odložené dane podľa IFRS 3 sú výnimkou z princípu vykazovania a aj princípu oceňovania.

hodnoty je problematické pri neexistencii aktívneho trhu, v takomto prípade musí účtovná jednotka uplatniť vlastné predpoklady a úsudky.

Nadobúdateľ vychádza pri výpočte goodwillu alebo zisku z výhodnej kúpy z reálnej hodnoty nadobudnutých čistých aktív. Vykázané nadobudnuté čisté aktíva ovplyvnia celkovú výšku čistých aktív nadobúdateľa (t. j. sumu vykázaného majetku a vykázaných záväzkov nadobúdateľa). Náklady a výnosy súvisiace s vykázanými čistými aktívami nadobúdaného podniku po dni akvizície, budú súčasťou celkových nákladov a výnosov nadobúdateľa a ovplyvnia jeho komplexný výsledok po dni akvizície, čím budú mať vplyv na formovanie finančnej situácie nadobúdateľa.

V tejto súvislosti je dôležité zdôrazniť, že aplikácia požiadaviek IFRS 3 vyžaduje, aby zostavovateľ účtovnej závierky (účtovný odborník) s dostatočnými vedomosťami a skúsenosťami starostlivo pristupoval k identifikácii a oceneniu jednotlivých položiek nadobudnutého majetku a prevzatých záväzkov v podnikovej kombinácii s cieľom poskytnúť používateľovi informácií z účtovnej závierky spoľahlivé informácie, ktoré budú verne prezentovať podstatu transakcie, budú neutrálne a bez chýb.

References

[1] Deloitte (2013). *iGAAP 2013. A quide to IFRS reporting*. Volume A, Part 2. Sixth edition. LexisNexis.2013, p. 2939. ISBN 978-0-7545-4772-3.

[2] Farkaš, R. (2013). *Konsolidovaná účtovná závierka v Slovenskej republike*. Prvé vydanie. Bratislava : Iura Edition, 2013, p. 309, ISBN 978-80-8078-572-7.

[3] Meluchová, J. (2013). Fixed assets of the company. *In Journal of East European Science and Research : scientific peer-reviewed journal*. Bratislava: European Institute for Development and Education, 2013. ISSN 1338-3302. - Vol. 4, no. 1. p. 121-130.

[4] Mokošová, D., Bednárová, B., Tkáčová, L. (2013). Impact of fair value adoption in national and international frameworks for the business accounting and reporting In *Financial management of firms and financial institutions* [elektronický zdroj] : proceedings : 9th international scientific conference : 9th - 10th september 2013, Ostrava VŠB - Technical university of Ostrava, 2013. - ISBN 978-80-248-3172-5. -ISSN 2336-162X. – p. 589-596.

[5] Parišová, R. (2010). *Podnikové kombinácie podľa IFRS*. Prvé vydanie. Bratislava : IURA Editon, 2010, p.173, ISBN 978-80-8078-330-3.

[6] Vašeková, M. (2015) Reforma vykazovania lízingov. In *AIESA – budovanie spoločnosti založenej na vedomostiach* [elektronický zdroj] : zborník : 16. medzinárodná vedecká konferencia : Bratislava, 5. - 6. november 2015. Bratislava : Vydavateľstvo EKONÓM, 2015. - ISBN 978-80-225-4151-0. – p. 397-402.

[7] Zákon č. 431/2002 Z. z. o účtovníctve v znení neskorších predpisov.

[8] IFRS 3 *Podnikové kombinácie*.

[9] IAS 36 *Zníženie hodnoty aktív*.

[10] IAS 38 *Nehmotné aktíva*.

[11] IAS 37 *Rezervy, podmienené záväzky a podmienené aktíva*.

DG solver for one-factor and two-factor Black-Scholes models

Jiří Hozman[1], Tomáš Tichý[2]

Abstract

Option pricing theory is a very important discipline which has a lot of applications in financial engineering. In this paper we focus on a single plain vanilla option pricing problem and its generalization for multi-asset options, especially in two dimensions. The main aim is to present a practical numerical scheme to solve a nonstationary PDE model arising from a classical Black-Scholes framework. This scheme is based on the discontinuous Galerkin (DG) approximation in spatial domain together with the θ-method for the time discretization, and leads to a sparse matrix system at each time level. One of the advantages of DG approach is a piecewise polynomial approximation that can better resolve possible discontinuities in solutions, the second one is a relatively easy implementation of *hp*-refinement. The presented DG solver is tested on several numerical benchmarks with real market data.

Keywords

Option pricing, discontinuous Galerkin method, Black-Scholes model, single vanilla option, two-asset option.

JEL Classification: G22

1. Introduction

Pricing of options is a challenging academic discipline as well as important part of market practice. Since introduction of the Black-Scholes (BS) model in [2, 12], the options become commonly traded not only at OTC markets, but even at organized exchange markets. During the decades, a large selection of options with various types of payoff functions has been introduced to the market to meet specific needs of the clients. Obviously, various payoff functions might need different models for efficient pricing. This fact is reflected also in the literature devoted to the numerical studies of the option pricing problems, from binomial models, over finite difference methods [4], to finite element approaches [1, 14], and in references cited therein.

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In this contribution we continue in our previous research on numerical option pricing via discontinuous Galerkin (DG) approximation technique, see e.g. [8, 9, 10], and focus on payoff depending on more potentially dependent factors. If we consider a single asset option with stochastic volatility, we mostly assume independence of both factors (i.e. underlying asset price and volatility of its returns) so that the solution of the option pricing problem can be simplified substantially. On the other hand if one consider an option involving more than one asset, such basket options, spread options, or exchange options, the dependency function must be taken into account and various features which make the analytical approach problematic can be present. They can include the following – Is the dependency linear or at least symmetric function? Is the option price a monotone function of the dependency? What is the marginal effect of particular volatilities? How to calibrate it to market date?

In this paper we specifically focus on a relatively simple problem of an option with payoff given by the maximum of two risky assets under Black-Scholes setting with linear dependency measure (i.e. Pearson correlation) – since in such case a closed-form solution is available – and compare it to single asset option. We proceed as follows. Section 2 is devoted to the formulation of relevant pricing equations. In Section 3 we develop discontinuous Galerkin approach for both cases (single- and two- asset option). Finally, in Section 4 numerical examples based on real market data are provided.

2. Governing equations

We start from a one-factor Black-Scholes partial differential equation (PDE) describing a single plain vanilla option pricing problem. Let σ_1 be the constant volatility of the underlying asset S_1 and r be the constant risk-free interest rate, then the price of the option $V(S_1, t)$ is driven by the following backward PDE (see [2, 12])

$$\frac{\partial V}{\partial t} - \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} - rS_1 \frac{\partial V}{\partial S_1} + rV = 0 \quad \text{in} \quad I\!R_+ \times (0, T), \tag{1}$$

where t denotes the time to maturity T.

The second problem treats the two-asset options with the constant volatilities σ_1 and σ_2 of the corresponding two assets, which are correlated by factor $\rho \in (-1, 1)$. The resulting governing equation for a two-factor BS model reads as

$$\frac{\partial V}{\partial t} - \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} - \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} - \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} - rS_1 \frac{\partial V}{\partial S_1} - rS_2 \frac{\partial V}{\partial S_2} + rV = 0, \tag{2}$$

where $V(S_1, S_2, t) : I\!R_+^2 \times (0, T) \to [0, +\infty)$ is the price function of the two-asset option.

In order to unify both problems (1) and (2) the price of the multi-asset option $V(S_1, \ldots, S_d, t)$ has to satisfy the multidimensional version of BS equation, i.e.

$$\frac{\partial V}{\partial t} - \frac{1}{2}\sum_{k,l=1}^{d} \rho_{kl}\sigma_k\sigma_l S_k S_l \frac{\partial^2 V}{\partial S_k \partial S_l} - r\sum_{k=1}^{d} S_k \frac{\partial V}{\partial S_k} + rV = 0 \quad \text{in} \quad I\!R_+^d \times (0, T),\ d = 1, 2, \tag{3}$$

where d denotes the spatial dimension and $\rho_{kl} \in (-1, 1), 1 \le k, l \le d$, stands for Pearson correlation coefficient between k-th and l-th assets. Obviously, $\rho_{kk} = 1$.

To numerically solve the problem (3), an unbounded domain in the variables $S_1, \ldots, S_d$ has to be truncated as $\Omega = \times_{k=1}^{d}(0, S_k^{max})$, where S_k^{max} stands for the maximal price of

k-th asset. Next, this problem should be equipped with the initial conditions given by the payoff function with respect to the type of the option from the holder's point of view. Concurrently, the BS model should be supplied with suitable choices of additional boundary conditions on appropriate parts of the boundary of the computational domain Ω.

In what follows, we consider only put options for a simplicity. The generalization of all conditions for call options can be done straightforwardly with the aid of the so-called put-call parity, see [5]. In the case of puts, we prescribe the initial condition as

$$V(S_1,\dots,S_d,0) = V_0(S_1,\dots,S_d;K) = \max(K-\max(S_1,\dots,S_d),0), \quad d=1,2, \tag{4}$$

where K is the exercise price. Let us note that payoff function (4) represents the standard profit of the put option in the case $d=1$, but for $d=2$ it is the expected value of the put option on the maximum of two risky assets.

Next, the equation (3) is complemented by the boundary conditions set according to the problem dimension. For a one-factor model, it is usually chosen a couple of Dirichlet boundary conditions at both endpoints of the computational domain, i.e.

$$V(0,t) = Ke^{-rt} \qquad \text{and} \qquad V(S_1^{max},t) \approx 0, \quad t\in(0,T), \tag{5}$$

where the option value at the right-boundary node S_1^{max} comes from the asymptotic behaviour of the exact solution at infinity. To suppress the undesirable effect of this approximation, one has to use a sufficiently large computational domain, usually $S_1^{max} > 2K$. The alternative approach of the treatment of boundary conditions for the one-factor BS model is studied in [8]. In the case $d=2$, we distinguish three parts of the rectangular boundary $\partial\Omega$ defined as $\Gamma_1 = (0,S_1^{max})\times\{0\}$, $\Gamma_2 = \{0\}\times(0,S_2^{max})$ and $\Gamma_3 = \partial\Omega \cap I\!R^2_+$. Then boundary conditions are usually prescribed as a mixed type in the following sense

$$\Big(I\!D(S_1,S_2)\nabla V(S_1,S_2,t)\Big)\cdot\vec{n} = 0 \text{ on } \Gamma_i,\ i=1,2, \quad \text{and} \quad V(S_1,S_2,t) = 0 \text{ on } \Gamma_3, \tag{6}$$

where $I\!D(S_1,S_2) = (D_{kl})_{k,l=1}^2 = \frac{1}{2}\rho_{kl}\sigma_k\sigma_l S_k S_l$ denotes the symmetric positive semi-definite matrix and $\vec{n}$ is the outer unit normal to Γ_i. Note that homogeneous Neumann boundary conditions from (6) prescribed on axes $S_1=0$ and $S_2=0$ are a priori fulfilled and correspond to the do-nothing boundary condition. On the other hand homogeneous Dirichlet condition from (6) follows from the behaviour of puts for large values of underlying assets.

In order to derive the numerical scheme more easily, it is convenient to treat the equation (3) in a divergence-free version. To this end, we introduce the new independent variable $x=[x_1,\dots,x_d]$, with $x_k = S_k/K$, $k=1,\dots,d$, and the option price transforms into $u(x_1,\dots,x_d,t) = V(x_1,\dots,x_d,t)/K$. Then we obtain the new pricing equation in terms of the value function $u(x,t): Q_T = \Omega\times(0,T)\to[0,+\infty)$, written in the divergence-free form as

$$\frac{\partial u}{\partial t} - \sum_{k=1}^{d}\frac{\partial}{\partial x_k}\Big(\sum_{l=1}^{d} D_{kl}(x)\frac{\partial u}{\partial x_l}\Big) + \sum_{k=1}^{d}\frac{\partial}{\partial x_k}\big(a_k(x)u\big) + \kappa(x)u = 0 \quad \text{in } Q_T, \tag{7}$$

where

$$D_{kl}(x) = \frac{\rho_{kl}\sigma_k\sigma_l x_k x_l}{2},\ a_k(x) = \sum_{l=1}^{d}\frac{\partial D_{kl}(x)}{\partial x_l} - r x_k,\ \kappa(x) = (d+1)r - \sum_{k,l=1}^{d}\frac{\partial^2 D_{kl}(x)}{\partial x_k \partial x_l}. \tag{8}$$

Next, the system is closed by the set of the initial condition and mixed boundary conditions, transformed in the same way as the option price $u(x,t)$. Finally, let us comment that the equation (7)–(8) represents a parabolic problem that exhibits a convection-diffusion character. This fact is the source of many numerical difficulties, which are overcome by the suitable numerical technique based on the DG method.

3. Discontinuous Galerkin solver

Since the exact solution of the problem (3)–(6) is usually difficult to find, we use its approximation using the DG technique, for survey see [3, 13]. This method uses piecewise polynomial, generally discontinuous, approximation of the p-th order describing a global solution u_{DG} in the computational domain Ω.

First, we construct a partition $\mathcal{T}_h = \{\mathcal{K}\}_{\mathcal{K}\in\mathcal{T}_h}$ of domain Ω (for more details see [7] for case $d = 1$ and [10] for $d = 2$) and define a finite dimensional space

$$S_h^p = \left\{ v_h \in L_w^2(\Omega);\ v_h\big|_{\mathcal{K}} \in P_p(\mathcal{K})\ \forall\, \mathcal{K} \in \mathcal{T}_h \right\} \subset H_w^1(\Omega), \tag{9}$$

which approximates the corresponding weighted Sobolev space $H_w^1(\Omega)$, see [11]. The symbols $L_w^2(\Omega)$ and $P_p(\mathcal{K})$ stand for the corresponding weighted Lebesgue space and the space of all polynomials of order less or equal to p defined on the element $\mathcal{K}$, respectively.

Next, we recall the space semi-discrete DG formulation of the option pricing problem (3)–(6). The complete derivation for the one-factor BS model can be found in [7] and the detailed space semi-discrete DG scheme for two-asset options is presented in [9]. Therefore, we introduce the DG semi-discrete solution $u_{DG}(t) \in S_h^p$ represented by the system of the ordinary differential equations (ODEs)

$$\begin{aligned}\left(\frac{\partial u_{DG}(t)}{\partial t}, v_h\right) + a_h(u_{DG}(t), v_h) + b_h(u_{DG}(t), v_h) + J_h(u_{DG}(t), v_h) \\ +(\kappa u_{DG}(t), v_h) = l_h(v_h)\,(t), \quad \forall\, v_h \in S_h^p,\ \forall\, t \in (0,T)\end{aligned} \tag{10}$$

with the initial state $u_{DG}(0)$ as S_h^p-approximation of the initial condition. The symbol $(\cdot,\cdot)$ stands for the L^2-scalar product and the remaining bilinear (linear) forms in (10) have the following meaning: a_h - diffusion form including nonsymmetric variant of interior penalty Galerkin stabilization, b_h - convection form with the numerical flux based on the upwinding technique, J_h - penalty form compensating the inter-element discontinuities in the semi-discrete solution and l_h - the right-hand side form enforcing the fulfillment of boundary conditions.

Further, to obtain fully time-space discrete DG formulation, we has to discretize also in the temporal variable t. There exists a wide range of approaches for the time discretization of ODE systems. In practical computations, the simplest time discretization is via an explicit scheme (e.g. forward Euler), which suffers from a limitation on the time step due to a CFL-stability condition. On the other hand, to avoid this time step restriction, it is advantageous to use an implicit time discretization (e.g. backward Euler), which does not require any additional linearization for linear problems, see [7].

Here, we consider the so-called θ-scheme, which can be equivalently written as the weighted average of forward Euler and backward Euler methods. Let $0 = t_0 < t_1 < \cdots < t_s = T$ be a partition of the interval $(0,T)$ with the constant time step τ. We define

the DG approximate solution of problem (10) as functions $u_{DG}^m \approx u_{DG}(t_m)$, $t_m \in (0,T)$, satisfying the following numerical scheme

$$\begin{aligned}
&\left(\frac{1}{\tau}+\theta\kappa\right)\left(u_{DG}^{m+1},v_h\right)+\theta\Big(a_h\big(u_{DG}^{m+1},v_h\big)+b_h\big(u_{DG}^{m+1},v_h\big)+J_h\big(u_{DG}^{m+1},v_h\big)\Big)=\\
&\left(\frac{1}{\tau}-(1-\theta)\kappa\right)\left(u_{DG}^{m},v_h\right)-(1-\theta)\Big(a_h(u_{DG}^{m},v_h)+b_h(u_{DG}^{m},v_h)+J_h(u_{DG}^{m},v_h)\Big)\\
&+\theta l_h(v_h)\left(t_{m+1}\right)+(1-\theta)l_h(v_h)\left(t_m\right)\quad \forall\, v_h\in S_h^p,\ m=0,\dots,s-1
\end{aligned}\tag{11}$$

with the starting data $u_{DG}^0 \approx u_{DG}(0)$. Note that the function κ has the constant value from (8) and the value of parameter θ lies in interval $[0,1]$. The case $\theta=\frac{1}{2}$ is well-known as Crank-Nicolson method, which is practically unconditionally stable and gives the second order convergence in time.

Moreover, one can easily identify that the discrete scheme (11) corresponds to the system of linear equations. More precisely, we rewrite the discrete DG solution as a linear combination of basis functions, i.e.

$$u_{DG}^m(x)=\sum_{k=1}^{DOF}\beta_k^m\varphi_k(x),\ x\in\Omega,\qquad \text{where}\ \ S_h^p=\mathcal{L}\left(\varphi_1,\dots,\varphi_{DOF}\right)\tag{12}$$

and set the vector of real coefficients $U_m=\left\{\beta_k^m\right\}_{k=1}^{DOF}\in I\!R^{DOF}$, then we receive the sparse matrix equation

$$\begin{aligned}
\underbrace{\Big((1/\tau+\theta\kappa)\mathbf{M}+\theta(\mathbf{A}+\mathbf{B}+\mathbf{J})\Big)}_{\mathbf{C}}U_{m+1}=&\Big((1/\tau-(1-\theta)\kappa)\mathbf{M}-(1-\theta)(\mathbf{A}+\mathbf{B}+\mathbf{J})\Big)U_m\\
&+\theta F_{m+1}+(1-\theta)F_m,
\end{aligned}\tag{13}$$

where the matrix $\mathbf{M}$ is related to L^2-scalar product, the matrix $\mathbf{A}$ to the form a_h, the matrix $\mathbf{B}$ to the form b_h, the matrix $\mathbf{J}$ to the penalties J_h and the vector F_m represents the right-hand side form l_h, respectively. The system matrix $\mathbf{C}$ is a composition of particular sparse matrices and has a matrix inverse, which implies the solvability of the discrete problem (11), i.e. the existence and uniqueness of the DG solution u_{DG}^m at each time level t_m. For illustration, the sparsity patterns of the system matrices for different settings are depicted in Figure 1.

4. Numerical simulations

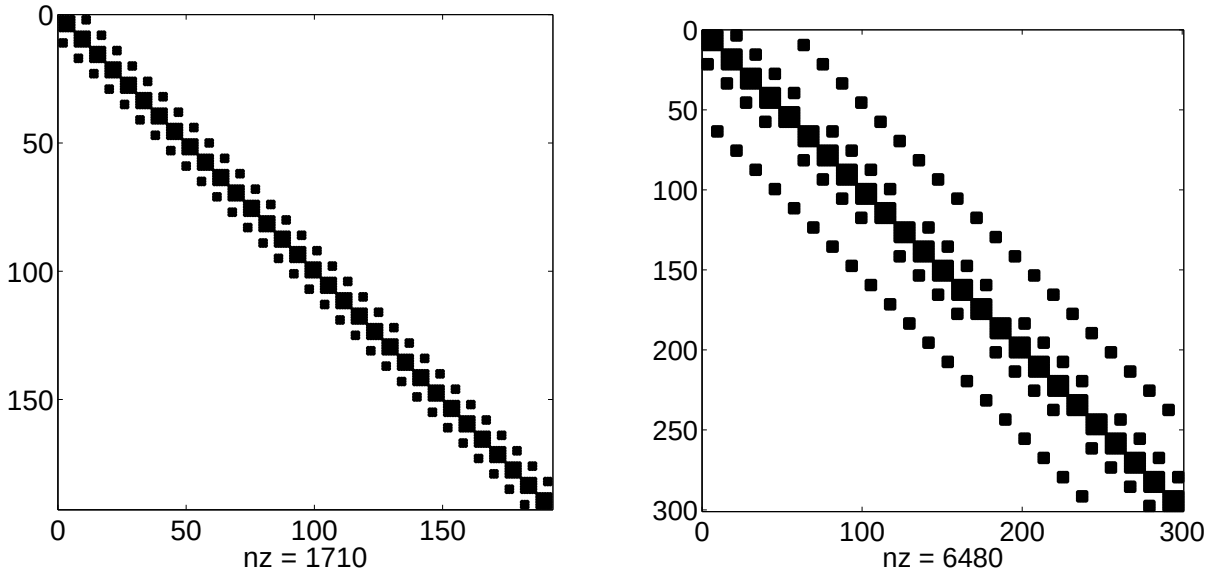
In this section we shall numerically study the dependency of the discrete option price u_{DG} and Greeks (sensitivity measures) $\frac{\partial u_{DG}}{\partial x_k}$, $=1,\dots,d$, on the order of polynomial approximation with respect to the number of degrees of freedom and the spatial dimension.

Therefore, the relative errors are evaluated at final state in L^2-norms

$$e_2=\frac{\|u_{DG}^s-u(T)\|}{\|u(T)\|}\qquad\text{and}\qquad e_1=\frac{\|\nabla u_{DG}^s-\nabla u(T)\|}{\|\nabla u(T)\|},\tag{14}$$

where $u(T)$ is the exact solution at time T and u_{DG}^s the numerical one at the same time level obtained by the scheme (11). Since the mesh step is proportional to the amount $1/\sqrt[d]{DOF}$, we suppose that errors behave according to the formula $e_i\approx C_i\left(1/\sqrt[d]{DOF}\right)^{r_i}$,

Figure 1: The visualization of the sparsity pattern of the system matrices $\mathbf{C}$ *(together with the number of non-zero elements) for piecewise quadratic approximations on different meshes:* $\#\mathcal{T}_h = 64$ *in 1D (left) and* $\#\mathcal{T}_h = 50$ *in 2D (right)*



$i = 1, 2$, $d = 1, 2$, where DOF denotes the number of degrees of freedom, the constants C_i are independent of DOF and the values r_i are the experimental orders of convergence, defined by

$$r_i \approx \frac{\log\left(e_i(DOF_1)/e_i(DOF_2)\right)}{\log\left(\sqrt[d]{DOF_2}/\sqrt[d]{DOF_1}\right)}, \quad i = 1, 2. \tag{15}$$

The convergence of the method (11) is investigated on the idealized problems arising from the real stock market. All computations are carried out with an algorithm implemented in the solver Freefem++, i.e. a mesh generation/adaptation, the DG discretization, assembly of a linear algebraic problem and its solving. The detailed description can be found in [6]. The remaining post-processing is performed in MATLAB.

We consider piecewise linear and quadratic approximations on uniform and adaptively refined grids for both models (1) and (2). In order to restrain the discretization errors with respect to the time discretization, a sufficiently small time step and $\theta = \frac{1}{2}$ in scheme (11) is used. Then the resulting sparse system is solved with the aid of GMRES solver.

Since the problem (7)–(8) belongs to the class of convection–diffusion equations, it is known that DG approximations are optimal in H^1-seminorn (i.e. for e_1) and suboptimal in L^2-norm (i.e. for e_2) in general, see, e.g. [13]. The forthcoming experiments provide the deeper insight to the properties of the presented DG solver in connection with the option pricing problem.

4.1 European vanilla put option

This experiment arises from the numerical example introduced in [8], where the case of a vanilla put option with the real data of the German DAX on 15 September 2011 with implied volatilities is considered. The analytic solution of this one-factor BS model is known and given by the Black–Scholes formula as (see [5])

$$V_P(S_1, T; K) = Ke^{-rT}N(-d_2) - S_1N(-d_1) \tag{16}$$

where

$$d_1 = \frac{\ln(S_1/K) + (r + \sigma_1^2/2)T}{\sigma_1\sqrt{T}}, \quad d_2 = \frac{\ln(S_1/K) + (r - \sigma_1^2/2)T}{\sigma_1\sqrt{T}}$$

and

$$N(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} \exp\left(-\frac{x^2}{2}\right) dx \tag{17}$$

stands for the standardized cumulative normal distribution function.

Similar to [8], we have the put option with the expiration date $T = 1/3$ (i.e. 4 months) and the strike price $K = 4000$ Euro. As the model parameters, the Black–Scholes risk-free interest rate $r = 0.0176$ and implied volatility $\sigma_1 = 0.4594$ are set. The length of a computational domain depends on the maximal stock price. Here, we take $S_1^{max} = 4K$ in order to suppress the influence of Dirichlet boundary conditions and $\tau = 1/3000$ to guarantee the negligible impact of the temporal discretization.

The run of the algorithm is executed on several consecutive (uniformly and adaptively) refined grids with different values of DOF. The relative errors with the corresponding experimental orders are recorded in Tables 1-2.

Table 1: Computational errors and experimental orders of convergence for piecewise linear approximation on a consequence of meshes uniformly and adaptively refined

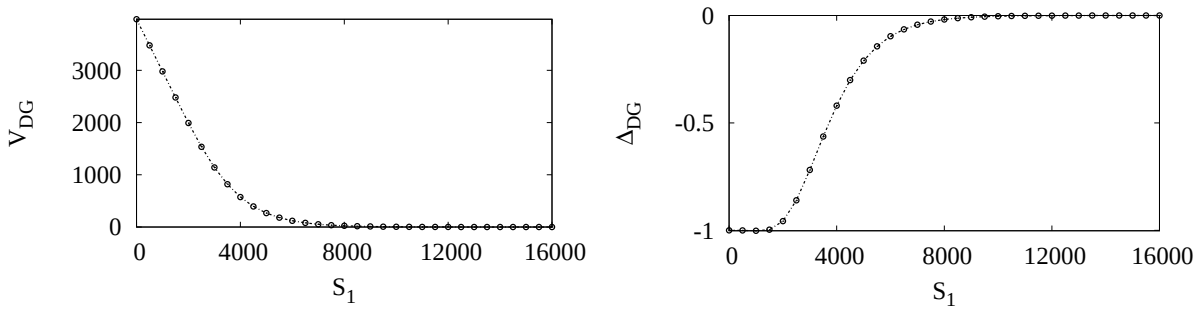
uniform mesh					adaptive mesh				
DOF	e_2	r_2	e_1	r_1	DOF	e_2	r_2	e_1	r_1
16	4.637e-2	—	2.005e-1	—	6	2.627e-1	—	3.310e-1	—
32	1.686e-2	1.46	1.048e-1	0.94	10	7.929e-2	2.35	1.597e-1	1.43
64	6.328e-3	1.41	5.184e-2	1.02	18	1.604e-2	2.72	6.713e-2	1.47
128	2.098e-3	1.59	2.476e-2	1.07	34	6.123e-3	1.51	3.013e-2	1.26
256	5.215e-4	2.01	1.147e-2	1.11	66	1.644e-3	1.98	1.468e-2	1.08
512	3.328e-4	0.65	5.215e-3	1.14	130	6.620e-4	1.34	7.323e-3	1.03

Table 2: Computational errors and experimental orders of convergence for piecewise quadratic approximation on a consequence of meshes uniformly and adaptively refined

uniform mesh					adaptive mesh				
DOF	e_2	r_2	e_1	r_1	DOF	e_2	r_2	e_1	r_1
12	4.690e-2	—	1.305e-1	—	9	5.363e-2	—	1.256e-1	—
24	5.078e-3	3.27	2.757e-2	2.24	15	3.739e-3	5.21	2.232e-2	3.38
48	9.078e-4	2.48	9.100e-3	1.59	27	7.075e-4	2.83	8.242e-3	1.69
96	1.421e-4	2.68	2.211e-3	2.04	51	1.824e-4	2.13	2.845e-3	1.67
192	2.628e-5	2.43	5.451e-4	2.02	99	4.530e-5	2.10	8.222e-4	1.87
384	1.669e-5	0.66	1.431e-4	1.93	195	2.135e-5	1.11	2.495e-4	1.76

One can easily observe that DG technique produces optimal orders of convergence $r_2 \approx 2$ and $r_1 \approx 1$ for piecewise linear approximation and for both types of partitions. However, the adaptively refined computational domains generate the smaller errors for the same number DOF. On the other hand, the piecewise quadratic approximation leads to the optimal order only for H^1-seminorms (i.e. $r_1 \approx 2$) and $r_2 \approx 2$ is suboptimal (because the third order is usually expected). Both results correspond with conclusions in [8]. The particular results of discrete option values V_{DG} and sensitivity measures Δ_{DG} are depicted in Figure 2.

Figure 2: Discrete solutions of option prices (left) and the corresponding sensitivity measures (right) at final state for the case of a uniform partition with $DOF = 32$ and piecewise quadratic approximations



4.2 Put option on the maximum of two risky assets

The second benchmark is chosen as a European put option on the maximum of two risky stocks, because the analytic solution of this problem can be simple written in the closed-form as (from [5])

$$\begin{aligned} V_P^{max}(S_1, S_2, T; K) &= Ke^{-rT} - S_2 - S_1 N(d) + S_2 N(d - \sigma\sqrt{T}) \\ &\quad + S_1 M(y_1, d; \rho_1) + S_2 M(y_2, -d + \sigma\sqrt{T}; \rho_2) \\ &\quad - Ke^{-rT} \times (1 - M(-y_1 + \sigma_1\sqrt{T}, -y_2 + \sigma_2\sqrt{T}; \rho)), \end{aligned} \tag{18}$$

where

$$d = \frac{\ln(S_1/S_2) + \sigma^2 T/2}{\sigma\sqrt{T}},\ y_1 = \frac{\ln(S_1/K) + (r + \sigma_1^2/2)T}{\sigma_1\sqrt{T}},\ y_2 = \frac{\ln(S_2/K) + (r + \sigma_2^2/2)T}{\sigma_2\sqrt{T}},$$
$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}, \qquad \rho_1 = \frac{\sigma_1 - \rho\sigma_2}{\sigma}, \qquad \rho_2 = \frac{\sigma_2 - \rho\sigma_1}{\sigma}$$

and

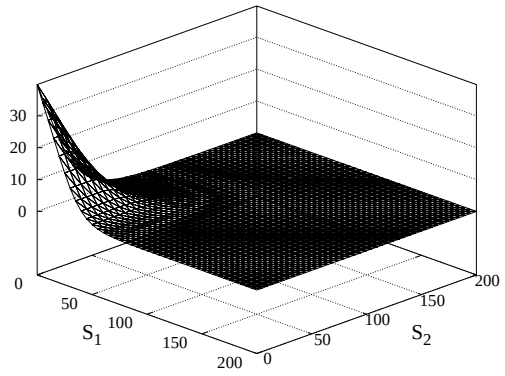
$$M(a, b; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{a} \int_{-\infty}^{b} \exp\left(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right) dxdy \tag{19}$$

stands for the standardized cumulative bivariate normal distribution function. For illustration, the exact solution for option prices is plot in Figure 3.

The real market parameters are chosen in consistency with [9], where we consider two-asset put option with Allianz stock and Deutshce Bank stock, strike at 40 Euro, and maturing in 94 calendar days. Current date is September 13, 2011 and two stocks trades at $S_1 = 59.79$ and $S_2 = 23.40$ Euro, respectively, with last year estimate of the Pearson linear correlation $\rho = 0.88$, constant implied volatilities $\sigma_1 = 0.6392$, $\sigma_2 = 0.9461$ and market implied riskless rate for a given horizon $r = 0.01557$ p.a.

The experiments are performed with piecewise linear and quadratic approximations on a square domain with maximal asset values $S_1^{max} = S_2^{max} = 5K$ and the time step is chosen proportionally to the half of a day (i.e. 1/730). The convergence history for uniformly refined grids is shown in Table 3. Since the exact solution (18) has lower regularity due to the discontinuity of its first derivative, the optimal experimental orders

Figure 3: Analytic solution at final state for the option pricing problem with the put option on the maximum of two risky assets



are not achieved. We have frequently observed rates $r_2 \in (1, 2)$ and $r_1 \approx 1$ for both polynomial approximations. These conclusions comply with numerical analysis of the DG method for the case of PDEs with not sufficiently regular solutions, cf. [13]. The numerical solutions (after 94 days) of Delta sensitivity measures with respect to both assets are displayed in Figure 4.

Table 3: Computational errors and experimental orders of convergence for piecewise linear and quadratic approximation on a consequence of meshes uniformly refined

linear case					quadratic case				
DOF	e_2	r_2	e_1	r_1	DOF	e_2	r_2	e_1	r_1
150	2.181e-1	—	1.125e-0	—	300	5.984e-2	—	8.278e-1	—
600	9.766e-2	1.16	6.498e-1	0.79	1200	1.256e-2	2.25	3.611e-1	1.20
2400	3.119e-2	1.65	3.689e-1	0.82	4800	3.523e-3	1.83	1.768e-1	1.03
9600	1.068e-2	1.55	2.170e-1	0.77	19200	1.008e-3	1.81	9.418e-2	0.91
38400	3.290e-3	1.70	1.259e-1	0.79	76800	3.406e-4	1.56	4.944e-2	0.93

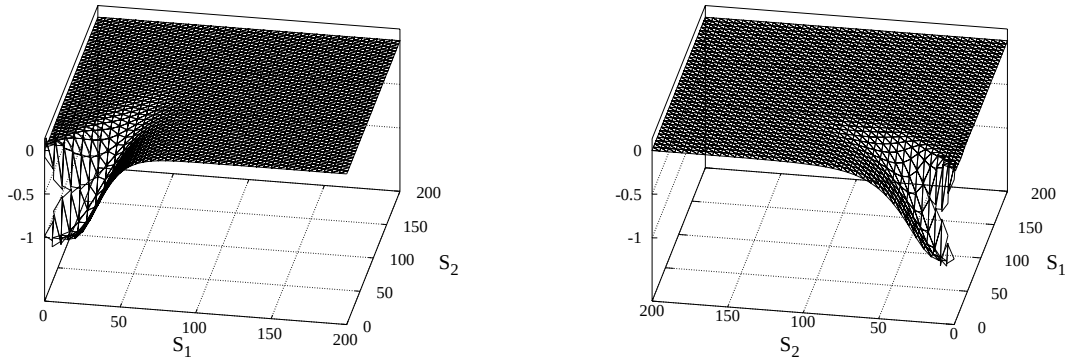
5. Conclusions

The financial markets offer various types of options. In many cases their valuation require usage of advanced methods. This is also the case when dependency of underlying factors is to be taken into account. In this contribution we provide discontinuous Galerkin scheme, which is a technique useful to obtain numerical solutions, for single- as well as two-factor options. Both approaches are thoroughly discussed and compared with analytical solution via numerical experiments with market data. Through these examples, we could reasonably conclude that the two-factor case does not give the same accuracy in errors as the single option pricing problem, but it also illustrates a huge potency of DG method.

References

[1] Achdou, Y. and Pironneau, O. (2005). *Computational Methods for Option Pricing.* Society for Industrial and Applied Mathematics, Philadelphia.

Figure 4: Discrete solutions of Delta sensitivity measures with respect to asset S_1 (left) and asset S_2 (right) at final state for the case of $DOF = 19200$ and piecewise quadratic approximations



[2] Black, F. and Scholes, M. (1973). The Valuation of Options and Corporate Liabilities. *Journal of Political Economy* **81**, pp. 637-654.

[3] Cockburn, B., Karniadakis, G.E. and Shu, C.–W., editors. (2000). *Discontinuous Galerkin Methods*. Springer, Berlin.

[4] Duffy, D.J. (2006). *Finite Difference Methods in Financial Engineering*. Wiley.

[5] Haug, E.G. (2006). *The Complete Guide to Option Pricing Formulas*. McGraw-Hill.

[6] Hecht, F. (2012). New development in FreeFem++. *Journal of Numerical Mathematics* **20**, no. 3-4, pp. 251-265.

[7] Hozman, J. (2012). Discontinous Galerkin method for the numerical solution of option pricig. *Aplimat – Journal of Apllied Mathematics* **5**(2), pp. 271-280.

[8] Hozman, J. and Tichý, T. (2014). Black-Scholes option pricing model: Comparison of h-convergence of DG method with respect to boundary condition treatment. *ECON – Journal of Economics, Management and Business* **24**(4), pp. 141-152.

[9] Hozman, J. and Tichý, T. (2015). A discontinuous Galerkin method for pricing of two-asset options. *Mathematical Methods in Economics*, pp. 273-278, Plzen: UWB.

[10] Hozman, J. and Tichý, T. (2015). Numerical pricing of European basket options with discrete barrier via the discontinuous Galerkin method. *Financial Management and Financial Institutions*, pp. 385-395, Ostrava: VSB-TUO.

[11] Kufner, A. (1980). Weighted Sobolev spaces. *Teubner-Texte zur Mathematik* **31**, BSB B. G. Teubner Verlagsgesellschaft, Leipzig.

[12] Merton, R. (1973). Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science* **4**, pp. 141-183.

[13] Riviére, B. (2008). *Discontinuous Galerkin Methods for Solving Elliptic and Parabolic Equations: Theory and Implementation*. Frontiers in Applied Mathematics. SIAM, Philadelphia.

[14] Topper, J. (2005). *Financial Engineering with Finite Elements*. Wiley.

The risk factors of the phrase true and fair view of the facts in accounting versus European Regulation 2013/34/EU and Slovak accounting legislation

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Abstract

The main aim of contribution is focus on importance true and fair accounting information for management decisions, which are realised based on accounting information from financial statements. Using key accounting information by managers is important for their key managerial decision-making. The principle true and fair view is the priority accounting principle, true and fair financial information from financial statements are important for users of financial statements – for example managers, competitive companies. The partial aim is to analyse the expression true and fair view, particularly Directive 2013/34/EU and Slovak accounting legislation - the Accounting Act no. 431/2002. The next partial aim of the contribution is to describe the risk factors of the phrase true and fair view and their negative impact for practise work of accountants and auditors, too.

Key words:

true and fair view, accounting Act, european regulation, management decision-making

JEL Classification: M40, M41

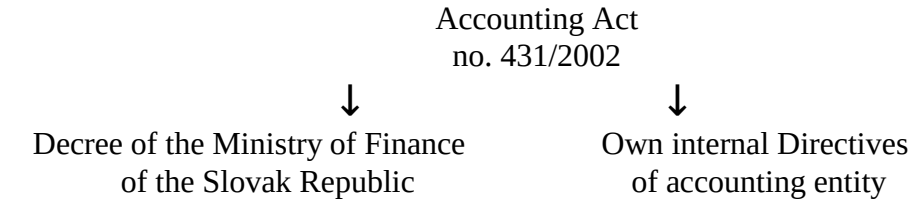
1. Expression true and fair view of facts versus Slovak and European accounting legislation

The financial statements are result the work of accountant and source of accounting information for decision-making of managers. True and fair accounting information could be have, like a part of total set of information for decisions of managers, responsibility for managing of business. What national and international accounting legislation is significant for accountants and guarantees, that true and fair accounting information could to help for manager decisions?

The own aim, respectively mission of accounting is provide all relevant users of the financial statements real view to financial situation of the entity. Is Slovak (accounting act no. 431/2002) and European accounting legislation (Directive 2013/34/EU) guarantor of truthfulness of accounting, isn´t? The figure 1 presents hierarchical level of accounting legislation in the Slovak republic. The Slovak national accounting legislation, as the member state of the European union, realised transfer Directive 2013/34/EU into accounting act no. 431/2002.

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Figure 1: Hierarchical level of accounting legislation in the Slovak republic



Source: own processing

1.1 The expression true and fair view in the Directive 2013/34/EU

The importance of communication between managers (like users of accounting information) and accountants become very significant and more complex. Internal managers have access to all detailed financial information from financial statements (external managers don´t have access to confidential internal accounting information in detail). The accounting information are presented in the financial statements. In Slovakia is public electronic register of the financial statements with access for potential users of accounting information. The finnancial statements, for all users of accounting information, have to be prepare in accordance with actual national and european accounting legislation, too.

In particular managers need a full understanding and knowledge of accounting information in the financial statements (of course there isn´t necessary to have the same comprehensive accounting education, like accountants). The most comprehensive accounting legislation knowledge is a must for profession of accountans. Only perfect knowledge of national and european accounting legislation by accounting is guarantor true and fair information in financial statements. What is actual european accounting legislation related to true and fair view? It is the content of following sentences.

The legislation related to the accounting is issued as a regulation or a directive. The European union regulation is defined as law mandatory and directly applicable in all member states. By regulations is not necessary their direct transfer into the national legislation, although it is not excluded that the adopted regulation need to change national legislation. Directives shall be mandatorily converted into the national legislation, to take direct legislative effect.[2]

The Slovak republic realised transformation national accounting act no. 431/2002 with directive 2013/34/EU. In Directive 2013/34/EU are following „true and fair view“ statements:

(9) Annual financial statements should be prepared on a prudent basis and should give a true and fair view of an undertaking's assets and liabilities, financial position and profit or loss. It is possible that, in exceptional cases, a financial statement does not give such a true and fair view where provisions of this Directive are applied. In such cases, the undertaking should depart from such provisions in order to give a true and fair view.

(38) In order to promote credible financial reporting processes across the Union, members of the body within an undertaking that is responsible for the preparation of the undertaking's financial statements should ensure that the financial information included in the undertaking's

annual financial statement and the group's consolidated financial statement gives a true and fair view.

[2] Mokošová, D. (2015). *Adoption of the accounting regulation by the European union.* In Účtovníctvo a audítorstvo v procese svetovej harmonizácie: Zborník z medzinárodnej vedeckej konferencie VIRT – Vzdelávacie zariadenie EUBA 8. – 10. septembra 2015. Vydavateľstvo Ekonóm, 2015.ISBN 978-80-225-4134-3, s. 103-106

This directive mentions expression true and fair view, but is neccessary to say, that formulation is very abstract and miss particular, respectively clever explanation with real and individual characters for *true* and for *fair*, too. There is not obvios what is particular what. The problem of these statements in the european regulation is, that, both points are related, respectively adjusting only processing financial statements. Paradoxically for the financial statements and the financial situation of entity is very important important tax legislation (and many times is more important like european accounting directive). Really qualified european accounting legislation should by defined comprihensive dynamic information system. The proposed parts of dynamic system could be:

- collection and preparing original information about economic life of entitiy,
- processing and presentation of the information (including safe deposit information).

1.2 The expression true and fair view in Accounting Act no. 431/2002

It´s obvios, that any act is not and will not absulutely perfect. This fact apply to the accounting act. no 431/2002, too. The logical process is that the entity must respect the accounting law.

In accounting act no. 431/2002 are „true and fair view" statements:
§7 (1) An entity is obliged bookkeeping so, that the financial statements present a true and fair view of facts, which are subject of accounting and about the financial position of the entity.
§7 (2) The presentation in the financial statements is fair, if content the items of financial statements in compliance with reality and it is consistent with accounting principles and accounting methods. Presentation in the financial statements is true, if they are prepared using accounting principles and methods, which lead to the achievement of fair presentation of the financial statements.

There is not explicitly explained the expression true and fair view in slovak accounting act, but in accordance with true and fair view is in §25 mentioned a prudent basis. The own aim of accounting is provide a realistic picture the economic life of the entity about. But, is it possible without clever explanation the priority principle "true and fair presentation of the facts"? Every erudite person must be clear that with the absence of a clear explanation is accounting act ineffective. In accordance with inadequate explanation of priority principle true and fair view of the facts shown the next partial problems with provisions, valuation, value adjustments, too.

Entity should make accounting (in accordance with imperfect accounting act no. 431/2002) so, that financial statements present true and fair view of facts, which are object of accounting the financial situation of entity. Imperfect national accounting act is source of other financial uncertainties of accountants and auditors in practice, and the antagonism with true and fair view of the financial statements, too.

2. The risk factors of the expression true and fair view of facts

Accountants use working with national or international accounting legislation, by compilation of the financial statements. There is neccessary to say, that the accounting legislation is guarantor of the principle true and fair view, but there are some types of risk factors. Alternative of untrue financial statements impacts as the wrong source of information for correct management decisions. Accountants by the preparing financial statements have to make confrontation accounting facts with comparable variables to accounting, but slovak accounting legislation (including european legislation, too) miss the particular variables.

The biggest risk factors accounting legislation, related to compliance priority principle, are valuation and depreciation, amortization. For prefession of accountants is very complicated to

prepare true and fair financial statements (components of financial statements are balance sheet, income statement, notes to the financial statements), if is our legislation imperfect. §25 is result the Ministry of finance of the slovak republic, and §25 like valuation article has very different understanding accountants (entities) by the accounting for accounting transactions (bookkeeping). Reason is the antagonism §25 with §2. There is defeined the subject of accounting in §2 Accounting Act. The accounting act no. 431/2002 in §2 is follows:[3]

The subject of the accounting shall by accounting for the following:[4]

a) the balance and movement of assets,
b) the balance and movements of liabilities,
c) the diference between assets and liabilities,
d) income,
e) expenses,
f) cash receipts,
g) cash payments, and
h) net profit/net loss of the accounting entity

There is antagonism between § 2 and § 25 - particularly §25 (valuation) related to § 2 (the subject of accounting). In our opinion, §25 real doesn´t support valuation, but presents information about subject accounting with valuation values. This fact has negative effect for compliance true and fair view of financial statements by practise accounting transactions realise within accountants.

Imperfect of national and international accounting legislation impact as a risk faktor for the preparing the true and fair financial statements by accountans (for example in area of insurance companies) and finally for auditor´s opinion to the financial reporting framework, too. Defect accounting information may have a great influence for managers decision-making and future business plans.

Accountants

What is the role of accountants for managers decisions-making? Accountants through accounting, like a part of total set of complicated information for managers, should providing available true and fair financial information for objective decision making by managers. Good managers decision making depend on good understanding of the financial statements. There is important appropriate communication between management and accountants. Managers have to generate relevant true and fair accounting information for future company´s planning activities. Accounting information available to managers have to be in accordance with principle true and fair view of facts. Compilation the financial statements by accountants is only the one part of total completed information set for managers. All steps by compilation the financial statements in accordance wit International Finance Reporting Standards are usually more complicated. Accountants (by IFRS) have to work with international accounting legislation and one partial accounting step can have influance to the formulation prior accounting step.

The insurance companies must prepare financial statements in accordance with International Finance Reporting Standards (IFRS). Insurance companies do not have a specific structure of the financial statements to the end of accounting period are prepared. Any

[3] Accounting Act no. 431/2002, §2

[4] Farkaš, R. (2013), *Financial statements of Companies in the Slovak Republic*. KPMG Slovensko. Sixth revised edition, Bratislava, ISBN 978-80-971312-0-3, p. 12

insurance company the reports compiled separately, for the purpose of reporting the structure of financial statements chosen individually.[5]

Auditors

Auditor realise a closer „inspection" of the financial statements, verified their accordance with true and fair view, e.g. accordance the financial statements with accounting legislation. Auditor´s opinion is a guarantor true and fair view of facst in verified financial statements. At the same true and fair financial statements are guarantor for good decision of managers about the financial future of the company.

For compilation of financial statements in accordance wih true and fair view of facts is important auditor´s opinion, required by national law and regulation. Auditor uses one from the following formulations:[6]

a) The financial statements present fairly, in all material respects,... in accordance with (the applicable financial reporting framework), or
b) The financial statements give a true and fair view of ... in accordance with (the applicable financial reporting framework).

Opinion doc. Kareš is, that both two formulation could by seen as an equivalent only, if the relevant financial reporting framework was ideal and in this case would provide "ideal state" within the meaning of true and fair view.

3. Summary

In other words, accounting (in accordance with national and international accounting) is for all users of accounting information a „language" – language of financial activity of company. This language, through accounting, provide all relevant users of the financial statements (for example managers) real view to the objective financial situation of the company and managers decision-making. The financial statements (compiled by accountants and verified by auditor), like a part of wide range complicated information for managers, should providing available true and fair financial information for objective decision making and future company plans by managers. Is necessary to stressed, that internal managers have access to all detailed and confidential financial information from financial statements of company (external managers don´t have the same access to completed internal accounting information in detail). However good managers decision making does not depends only on accounting information from financial statements.

References

[1] DIRECTIVE 2013/34/EU OF THE EUROPEAN PARLIAMENT AND OF THE COUNC

[5] Juhaszová, Z. (2015), *Premiums earned in the financial statements.* In Financial management of Firms and Financial Institutions Proceedings (Part II.) : 10 th International Scientific Conference Financial management of Firrms and Financial Institutions: 7th – 8 th September 2015, Ostrava, Czech Republic. ISBN 978-80-248-8365-6. ISSN 2336-162X . P. 479-483

[6] Kareš, L. (2015). Some Aspects of Area True and Fair View (Compliance with Financial Reporting Framework) – in Forming the Audit Opinion (on the other Audit Reports). In Účtovníctvo a audítorstvo v procese svetovej harmonizácie: Zborník z medzinárodnej vedeckej konferencie VIRT – Vzdelávacie zariadenie EUBA 8. – 10. septembra 2015. Vydavateľstvo Ekonóm, 2015.ISBN 978-80-225-4134-3, s. 43-46

[2] IL of 26 June 2013 on the annual financial statements, consolidated financial statements and related reports of certain types of undertakings, amending Directive 2006/43/EC of the European Parliament and of the Council and repealing Council Directives 78/660/EEC and 83/349/EEC

[3] Accounting Act no. 431/2002

[4] Farkaš, R. (2013), *Financial statements of Companies in the Slovak Republic*. KPMG Slovensko. Sixth revised edition, Bratislava, ISBN 978-80-971312-0-3, P. 727

[5] Juhaszová, Z. (2015), *Premiums earned in the financial statements.* In Financial management of Firms and Financial Institutions´Proceedings (Part II.) : 10 th International Scientific Conference Financial management of Firrms and Financial Institutions: 7th – 8 th September 2015, Ostrava, Czech Republic. ISBN 978-80-248-8365-6. ISSN 2336-162X . P. 747

[6] Kareš, L. (2015). *Some Aspects of Area True and Fair View (Compliance with Financial Reporting Framework) – in Forming the Audit Opinion (on the other Audit Reports).* In Účtovníctvo a audítorstvo v procese svetovej harmonizácie: Zborník z medzinárodnej vedeckej konferencie VIRT – Vzdelávacie zariadenie EUBA 8. – 10. septembra 2015. Vydavateľstvo Ekonóm, 2015. ISBN 978-80-225-4134-3, P. 195

[7] Mokošová, D. (2015). *Adoption of the accounting regulation by the European union.* In Účtovníctvo a audítorstvo v procese svetovej harmonizácie: Zborník z medzinárodnej vedeckej konferencie VIRT – Vzdelávacie zariadenie EUBA 8. – 10. septembra 2015. Vydavateľstvo Ekonóm, 2015. ISBN 978-80-225-4134-3, P. 195

An Empirical Study: Financial Literacy of Students of Law in the Czech Republic

Barbora Chmelíková, Martin Svoboda [1]

Abstract

Current complex financial markets demand from consumers of financial products at least a basic knowledge of financial concepts and conscious financial behaviour, therefore this paper focus on financial literacy which includes not only financial knowledge, but also financial behaviour and attitudes. As a target group the students of law was selected. The aim of this paper is to examine whether the law students, who are majoring in non-economic fields, are financially literate. The data was collected at the Faculty of Law at Masaryk University in autumn 2015. The analysis indicates positive results in favour of law students, however, further improvement in financial behaviour is required.

Key words

Financial literacy, university students, financial education

JEL Classification: A10, A22, A23, I23

1. Introduction

Financial education is a necessity in such complex financial market where the consumers are responsible for their own financial decisions. Consumers are exposed to a wide range of different financial products and it is important for the consumers to comprehend at least basic financial principles. The emphasis is put on financial literacy (Atkinson, Messy, 2012) or financial capability (World Bank, 2013) based on which are developed tools for measuring financial literacy of adults (OECD/INFE, 2013), pupils (OECD, 2014) or population in low and middle-income countries (Scott et al., 2013). The focus was also shifted to examine university students' financial literacy and their behaviour related to finances. Studies focused on university students were conducted, for instance in Hungary (Luksander et al., 2014), in Turkey (Özdemir et al., 2015), in USA (Xiao et al., 2014), in Australia (Cull and Whitton, 2011), and in South Africa (Shambare and Rugimbana, 2012).

Research in this areas shows that financial literacy is affected by several factors such as age, gender, living area, income, and education (Brumcrot et al., 2013). Individuals with higher education are more likely to be financially literate than individuals with lower educational level.

The aim of this paper is to examine university students and their level of financial literacy. For a consistency with our previous research (Chmelíková, Svoboda, 2015) we have used students of law as a target group. The tested hypothesis is whether the law students are advanced in financial literacy.

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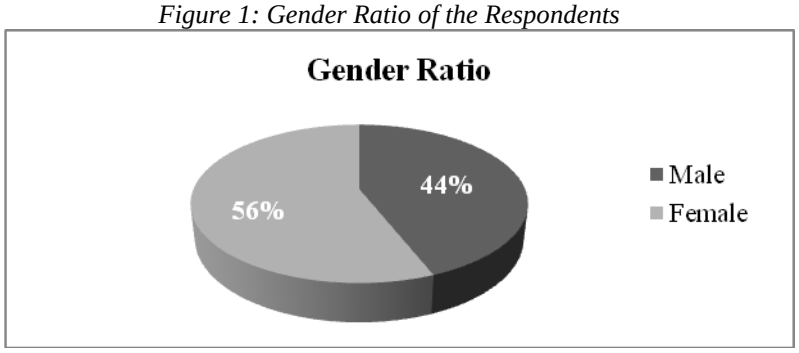
2. Methodology and Data

In relation to the research topic the quantitative analysis was selected. In order to test the research hypothesis the OECD questionnaire (OECD, 2013, 2015) developed to measure financial literacy on the national and international level was used. The OECD methodology aims to test different components of financial literacy, such as financial knowledge, financial behaviour and financial attitudes. All the questions were carefully designed to be used for testing financial literacy of adult population.

The financial knowledge questions included simple division, time value of money, simple and compound interest, risk and return, concept of inflation and diversification. Financial behaviour section tested students' approach and actual positive behaviour regarding their finances, such as management of personal finances, budgeting, loans, planning for the long term. Questions concerning financial attitudes comprised of statements with scaled responses. Each section created separate score (financial knowledge score, financial behaviour score, financial attitudes score) which together formed overall financial literacy score.

The study was presented to the students at the Faculty of Law at Masaryk University in the form of an online questionnaire. The size of the sample was n = 231. The data was collected during the fall semester 2015.

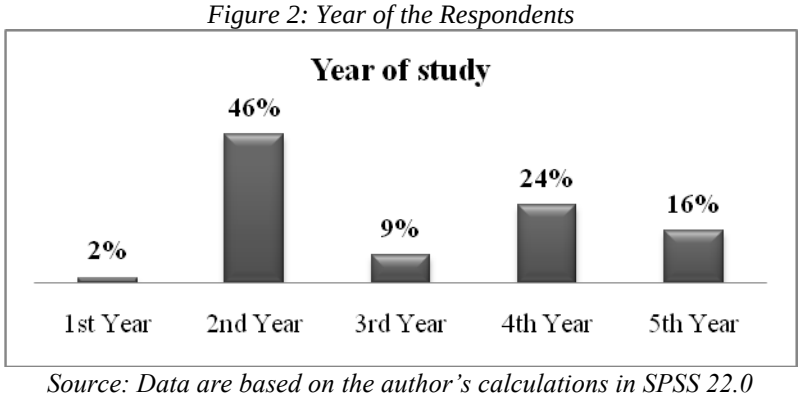
Figure 1: Gender Ratio of the Respondents



Source: Data are based on the author's calculations in SPSS 22.0

The socio-demographic characteristic of the respondents were recorded as well. Of the respondents, 56% were female and 44% were male (see Figure 1). The age of respondents ranged in majority from 19 to 30 years.

Figure 2: Year of the Respondents



Source: Data are based on the author's calculations in SPSS 22.0

Figure 2 shows that 2% were in the first year of the study, 46% of the respondents were in the second year, 9% in the third year, 24% in the fourth year, 16% in the fifth year. The remaining percentage represents students who are studying longer or have already completed their studies.

3. Results

Analysis results present scores of participating law students testing financial knowledge, financial behaviour and attitudes. Questions included in the financial knowledge block are provided in Table 1 where can be observed that 93% of students answered correctly to the division question, 85% of students comprehend time value of money, 92% of students are able to detect the correct percentage of the interest paid on loan. Furthermore, 90% of the law students are capable to calculate the simple interest, while 76% could calculate the compound interest. The law students' knowledge of risk and return is relatively high when 82% of the students knew the correct answer. On the other hand, 59% of the students answered correctly the question about inflation. Of the students, 64% were aware of the diversification.

Table 1: Financial Knowledge of Law Students

Correct answers	Law Students %
Division	93
Time value of money	85
Interest paid on loan	92
Calculation of interest plus principal	90
Compound interest	76
Risk and return	82
Definition of inflation	59
Diversification	64

Source: Data are based on the author's calculations in SPSS 22.0

Financial behaviour of participating law students is shown in Table 2. Students were asked whether they agree or disagree with the statements below and to put their answers on a scale. More than three quarters of students carefully consider their purchases and carefully watch their personal finances. Four fifths of students pay bills on time. Less than a half of students sets long-term financial goals and takes action to accomplish them. Of the students, 35% are responsible for day-to-day decisions about money and have a budget. Less than four fifths of students have been actively saving or investing in the past year. Nearly two fifths of students select a financial product after gathering some information and nearly three fifths choose a financial product after shopping around and using independent advice. More than a half of students needed to borrow money to be able to make ends meet.

Table 2: Financial Behaviour of Law Students

Positive answers	Law Students %
Carefully considers purchases	78
Pay bills on time	81
Keeps close watch on personal financial affairs	78
Sets long term goals and strives to achieve them	47
Responsible and has budget	35
Has been actively saving or investing in the past year	78
Financial product choice after gathering some info	38
Financial product choice after shopping around	56
Has not borrowed to make ends meet	48

Source: Data are based on the author's calculations in SPSS 22.0

Positive financial attitudes of law students are presented in Table 3. Less than a half of the students find it less satisfying to save money for the long-term than to spend the money. Of

the students, 35% tend to live for today, whereas 65% tend to care about future. More than four fifths of students agree that money is here to be spent when 22% think the opposite.

Table 3: Financial Attitudes of Law Students

Positive answers	Law Students %
Tend to save for the long-term	44
Tend to care about future	65
Money is not here to be spent	22

Source: Data are based on the author's calculations in SPSS 22.0

Table 4 provides average scores that students of Faculty of Law achieved in financial literacy scores. Average financial knowledge score was 6.44 where maximum was 8 points. Average financial behaviour score was 5.94 out of 9. Average financial attitudes score was 3.2 out of maximum 5 points. Overall average financial literacy score was 15.09 out of 22 points.

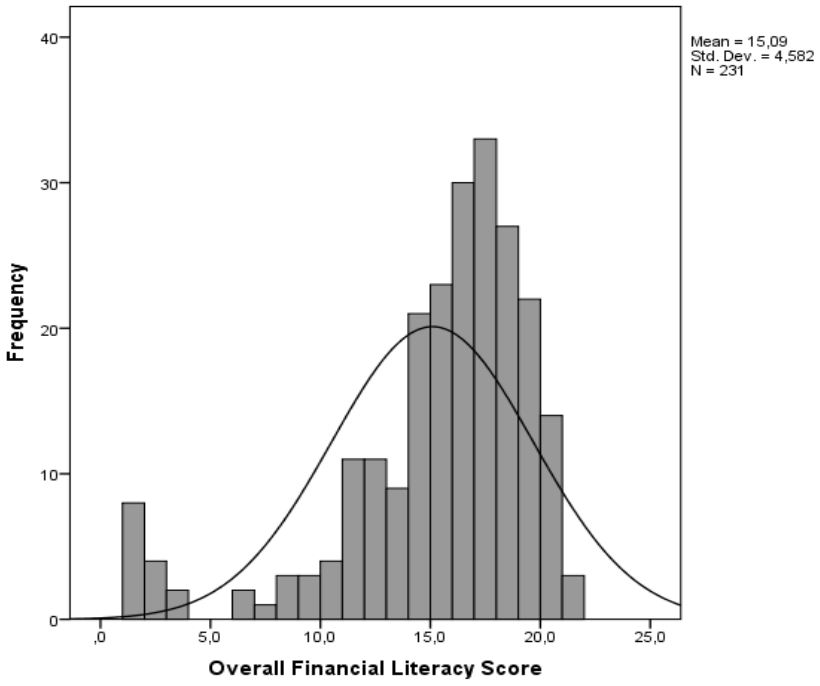
Table 4: Average Financial Literacy Scores of Law Students

Law Students'	Mean
Financial Knowledge	6.44
Financial Behavior	5.94
Financial Attitudes	3.20
Overall Financial Literacy Score	15.09

Source: Data are based on the author's calculations in SPSS 22.0

Figure 3 below represents frequency distribution of overall financial literacy scores achieved by participating students. The presented histogram shows that the distribution is slightly skewed to the right suggesting that students are likely to achieve high overall financial literacy score.

Figure 3: Frequency Distribution of Overall Financial Literacy Scores of Law Students



Source: Data are based on the author's calculations in SPSS 22.0

4. Conclusions

The results of the study demonstrated relatively high financial knowledge of law students who participated in the study. Their comprehension of basic financial concepts is on a high level, however, they might improve their knowledge about inflation and diversification. In terms of financial behaviour, the students could enhance their ability of budgeting and setting long-term financial goals. On the other hand, majority of the the law students pay bills on time, carefully consider purchases and try to keep closer look at personal financial affairs. In contrast, their financial attitudes are rather inclined towards the short term. The overall results suggest that university students should improve their ability to efficiently manage personal finances.

5. Acknowledgement

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References

[1] Bumcrot, C., Lin, J. and Lusardi, A., (2013). The Geography of Financial Literacy. *Numeracy: Advancing Education in Quantitative Literacy*. 6(2), pp. 1-16.

[2] Chmelíková, B. and Svoboda, M., (2015). The Financial Literacy of the Law Students at Masaryk University. *In Proceedings of 10th International Scientific Conference Financial Management of Firms and Financial Institutions*. pp. 429-434.

[3] Luksander, A. et al., (2014). Analysis of the Factors That Influence the Financial Literacy of Young People Studying in Higher Education. *Public Finance Quarterly (0031-496X,)* 59(2), pp. 220–241.

[4] Messy, F. and Atkinson, A. (2012). Measuring Financial Literacy: Results of the OECD / International Network on Financial Education (INFE) Pilot Study. *OECD Working Papers on Finance, Insurance and Private Pensions*.

[5] OECD. (2012). *PISA 2012 Results: Students and Money (Volume VI)*. Paris: Organisation for Economic Co-operation and Development.

[6] OECD/INFE. (2013). *Financial Literacy and Inclusion - Results of OECD/INFE Survey Across Countries and By Gender*. World Bank, OECD.

[7] OECD. (2015). *Toolkit For Measuring Financial Literacy And Financial Inclusion*. Paris: Organisation for Economic Co-operation and Development.

[8] OECD. (2013). *Toolkit For Measuring Financial Literacy And Financial Inclusion*: *Guidance, Core Questionnaire And Supplementary Questions*. Paris: Organisation for Economic Co-operation and Development.

[9] Özdemir, A., et al., (2015). *Financial Literacy of University Students: A Case Study for Anadolu University, Turkey*, 11(24). pp. 97–110.

[10] Scott, K., Kempson, E. and Perotti, V., (2013) *Measuring Financial Capability : A New Instrument and Results from Low-and Middle-Income Countries*. The World Bank.

[11] The World Bank. (2013). *Financial Capability Surveys Around the World: Why Financial Capability Is Important and How Surveys Can Help*. The World Bank.

[12] Xiao, J., et al. (2014). Earlier Financial Literacy and Later Financial Behaviour of College Students." *International Journal of Consumer Studies*, 38(6). pp. 593–601.

Banking efficiency in the Visegrad Group according to Data Envelopment Analysis with the risk component

Lucie Chytilová [1]

Abstract

The nowadays situation in the banking industry is discussed all around the world. The region of Central Europe and especially countries of the Visegrad Group have to keep up with a lot of changes of the recent history – political and economical development, rules of the European Commission or the 2008 crisis. There are many studies which evaluate efficiency of banking industry in the Visegrad Group. Generally, these studies evaluate the efficiency under certainty and for small sample. The problem is missing data or the lack of data which may cause imprecise results. The aim of this paper is to determine the efficiency of the banking industry in Visegrad Group for some of the banks and include into the analysis the risk component. The technical efficiency of banks in the Visegrad Group is estimated for years from 2011 to 2013. The special Slack-based measure type of DEA models with the risk component are used. The development of calculated technical efficiency is analysed in the cross section among the units, in the time and based on size. Also the influence of the risk variable is analysed as well.

Key words

Data Envelopment Analysis, Slack-based model, bank efficiency, risk component;

JEL Classification: C6, G21

1. Introduction

All financial institutions are having very hard time nowadays. The trading activities of the financial industry are rising and the structure of the market is still innovating. Therefore, the risk of investment for financial institutions also increases. The role of banks is getting more difficult. The main goal of them is to make profit but on the same hand, they have to develop a range of investments products which will be attractive for clients. This leads to higher potential of profits which involve more risk and higher probability of a bankruptcy. For this reason, banks are paying more attention to the high risks attached to the high potential profits. This is also seen in general higher activity of the risk management.

Data Envelopment Analysis (DEA) is mathematical method for efficiency measurement. First time this method was introduced by Charnes et al. (1978). The method has been widely applied in many types of industries – banking, health care and so on. In the banking industry, the first applications were mainly upon the economies of scale and scope (McAllister and McManus, 1993), total productivity (Fukuyama et al., 1999), and the efficiency effect (Casu and Molyneux, 2003). Later authors become to model include the risk into the equations. There were two views how to treat the risk: exogenously or endogenously. The results with exogenous treatment (Ataullah et al., 2004; Chang and Chiu, 2006) showed that the efficiency level is significantly correlated with the risk indicators. The second treatment was in order to

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analyse banks' efficiency (Chiu and Chen, 2008; Girardone et al., 2004). However, the majority of studies adopted the overdue loan ratio as the substitute variable for risks. Certainly this does not reflect the characteristic of uncertainty that risks display.

The risk is defined as the uncertainty and the degree of risk varies with the asset value fluctuation and the manager's attitude toward the risk. The risk may bring profit or loss to the asset value. In this context the basic function of capital helps to cover the possible loss incurred by taking risks. There are many ways how to include the risk component into the model. For example, there are studies which employ the Value at Risk (VaR) based on the New Basel Capital Accord (Basel II) (Kao and Liu, 2000; Chen et al., 2013) or there are studies which use so called demography risk, which conclude many variables – age, gender, income and so on (Cooper et al., 2014). According to the situation that there are measured banking industries from four countries, the demography risk variable is used. On the basis of previous works of Kao and Liu (2000) and Chen et al. (2013) this paper seeks to combine the Slack-Based Measure of Efficiency (SBM) as proposed by Tone (2001).

2. Research methodology

DEA is a non-parametric approach for measuring the relative efficiency of decision making units (DMUs) when the data for multiple inputs and outputs are known exactly. The special model which utilizes non-radial estimation method is SBM method and it was proposed by Tone (2001). It takes into consideration he slacks of investment and production items. Due to the employment of the non-radial method to estimate the efficiency value, the issues such as infeasible would not occur.

Slack-Based Measure of Efficiency

Slack-Based Measure of Efficiency (SBM) was proposed by Tone (2001). This method is in a manner of a non-ray efficiency of the estimated value. In this method is not incurring a problem that cannot be estimated.

Suppose there are *n* DMUs, let there are *m* inputs and *s* outputs. The set of production possibility is defined as $P = \{(x,y) | x \geq X\lambda,\ y \leq Y\lambda,\ \lambda \geq 0\}$ in witch $X = (x_{ij}) \in R^{m \times n}$ is the input matrix and $Y = (y_{rj}) \in R^{s \times n}$ is the output matrix. The index ρ_j for the DMU$_j$ is from (x_0, y_0) so as to average distance $(\bar{x}, \bar{y}) \in P(x_0, y_0)$. The SBM is as follows:

$$\min_{\rho,\lambda_1,\ldots,\lambda_n} \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \bar{x}_{ij}}{1 + \frac{1}{s}\sum_{r=1}^{s} \bar{y}_{rj}} \tag{1}$$

$$\bar{x}_j \geq \sum_{k=1}^{n} \lambda_k x_k$$

$$\bar{y}_j \leq \sum_{k=1}^{n} \lambda_k y_k$$

$$\bar{x}_j \geq x_0 \quad \text{and} \quad \bar{y}_j \leq y_0$$

$$\bar{y}_j \geq 0, \lambda_k \geq 0.$$

3. Empirical results

This chapter is looking at required data set of inputs and outputs for the research and the proper risk component. These variables are used for the calculation of empirical results.

3.1 Data sources

The required data set of inputs and outputs have been collected from the Bankscope[2]. This research uses the sample of 25 banks of the V4 countries.

3.1.1 Input and output values for the calculation

The definitions of the production process of banking can use different approaches. There are two basic approaches. The first, production approached, was introduced by Sherman and Gold (1985). This approach perceived banks as producers of deposit and loan accounts. The second approach was introduced by Sealey and Lindley (1997) and is called an intermediation approach. This approach sees banks as institutions that transform money from deposits into funds provided to borrowers in the form of loans.

Analysis of efficiency in this research is primarily based on the intermediation approach. The bank is seen as an institution that accepts deposits from diverse types of depositors and with the help of fixed capital these funds are converted to loans to various economic subjects. In Table 1 is seen the description of inputs and outputs.

Table 1: Description of input and output variables and input prices.

Variables	Description in the balance sheet	Unit of measurement
Inputs		
Labour (x_1 – EMP)	Number of employees	Number
Physical capital (x_2 – FA)	Fixed assets = Tangible + Intangible assets	Thousands of Euro
Loanable funds (x_3 – DEP)	Deposits + short term fundings	Thousands of Euro
Outputs		
Advances (y_1 – ADV)	Loans and advances to banks	Thousands of Euro
Investments (y_2 – SEC)	Other securities	Thousands of Euro
Non-interest income (y_3 – NEA)	Non-earning assets	Thousands of Euro

The banks are going to be classify by the size and the country where they are located. The sample contains 8 banks from the Czech Republic (CZ), 4 bank from Hungary (HU), 8 banks from Poland (PL) and 5 banks from Slovakia (SK)[3]. The classification by size is made on the basis of quartile value of fixed assets:

- micro banks - fixed assets is less than quartile Q1;
- small banks - fixed assets is equal or higher than quartile Q1 and less than median Q2;
- medium banks - fixed assets is between median Q2 and quartile Q3;
- large banks - fixed assets is greater than quartile Q3.

3.1.2 Risk component

The general demography risk consists of many variables, for example: age, gender, marital and family status, investment experience or income. In study by Cooper et al. (2014) the questionnaire is used and all these variables may be used for the demography risk component. In this general case for the Visegrad Group the demography risk component is made just by the average salary for each country and each year.

[2] https://bankscope.bvdinfo.com/

[3] The list of banks on request form the author.

As it is known the relationship between income (wealth) and the financial risk generally correlates. The question is about the self employes, the situation there is not similar. For this research we assume, that the risk component will be put into the side of the output. The unit for this variable is in Euro.

3.2 Empirical results

Table 2 shows the results for model without (SBM) and with the risk component (SBM-risk) for years 2011, 2012 and 2013. The results show that efficiency score for all banks is same or lower if the risk component is not included. This is true for all three years. For example: DMU01 has always efficiency score lower for the model SBM. DMU02, on the other hand, has the efficiency score same for both models. These results are seen for all countries. It does not depend on the origin of the bank.

Most efficient are bank in the Czech Republic, as it was seen in researches before. The least efficient are banks in the Poland. From Table 2, it may be said that banks from the Czech Republic are not so dependent on risk variable as banks from Poland. Poland banks have more banks with different efficiency score in SBM model and SBM-risk model.

Table 2: Results for the banks in the V4

		2011		2012		2013	
DMU	**country**	**SBM**	**SBM-risk**	**SBM**	**SBM-risk**	**SBM**	**SBM-risk**
DMU01	PL	0.8560	0.9252	0.8655	0.8712	0.8439	0.8575
DMU02	PL	0.8386	0.8386	0.6907	0.6907	0.6016	0.6016
DMU03	PL	0.7916	0.7916	0.6551	0.6551	0.7057	0.7057
DMU04	PL	0.7293	0.7307	1.0000	1.0000	0.5540	0.5733
DMU05	CZ	0.8345	0.8345	0.8217	0.8217	0.7662	0.7556
DMU06	CZ	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU07	CZ	0.7963	0.7963	0.8070	0.8070	0.7771	0.7800
DMU08	PL	1.0000	1.0000	0.8416	0.8416	0.7951	0.7951
DMU09	CZ	1.0000	1.0000	1.0000	1.0000	0.9380	0.9380
DMU10	HU	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU11	PL	0.3766	1.0000	0.7025	1.0000	0.7533	1.0000
DMU12	PL	0.6067	0.6067	0.6223	0.6223	0.6488	0.6488
DMU13	PL	0.5244	0.5244	0.4956	0.4956	0.3485	0.3575
DMU14	HU	1.0000	1.0000	0.7336	0.7336	0.7550	0.7550
DMU15	SK	0.6112	0.8743	0.4650	0.5993	0.4200	0.5166
DMU16	CZ	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU17	SK	1.0000	1.0000	0.7250	0.7669	0.6502	0.7056
DMU18	HU	0.9048	0.9048	0.7370	0.7370	0.6463	0.6463
DMU19	CZ	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU20	CZ	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU21	SK	0.7844	0.7924	0.6431	0.6431	0.6194	0.6195
DMU22	CZ	0.7901	0.7901	0.8006	0.8006	0.6270	0.6270
DMU23	HU	0.5118	0.5137	0.6548	0.6657	0.8277	0.8277
DMU24	SK	1.0000	1.0000	0.9120	0.9120	0.5424	0.5738
DMU25	SK	0.7680	0.7680	0.6446	0.6446	0.6706	0.6808
average		0.8290	0.8677	0.7927	0.8123	0.7396	0.7586
min		0.3766	0.5137	0.4650	0.4956	0.3485	0.3575

max	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 3 shows results for the banks according to the country. As it was written before, the most efficient banks are in the Czech Republic. They are followed by banks from Hungary, Slovakia and Poland.

Again, there should be see that the SBM model has lower efficient score then the SBM-risk model (model with risk component).

Table 3: Results for the banks by country in the V4

	2011		**2012**		**2013**	
country	**SBM**	**SBM-risk**	**SBM**	**SBM-risk**	**SBM**	**SBM-risk**
CZ	0.9276	0.9276	0.9287	0.9287	0.8885	0.8876
HU	0.8542	0.8546	0.7814	0.7841	0.8073	0.8073
PL	0.7154	0.8022	0.7342	0.7721	0.6564	0.6924
SK	0.8327	0.8869	0.6779	0.7132	0.5805	0.6193

Table 4 shows results for the banks according to the size. The most efficient banks are the micro banks. They are usually followed by large, medium and small banks. Also, it could be seen that score for SBM model is lower then for the SBM-risk model.

Table 4: Results for the banks by size in the V4

	2011		**2012**		**2013**	
size	**SBM**	**SBM-risk**	**SBM**	**SBM-risk**	**SBM**	**SBM-risk**
micro	0.8961	1.0000	0.9504	1.0000	0.9486	0.9897
small	0.8037	0.8513	0.7429	0.7689	0.5722	0.6044
medium	0.8091	0.8107	0.7641	0.7659	0.7566	0.7584
large	0.8113	0.8113	0.7217	0.7217	0.7091	0.7078

4. Conclusion

The nowadays situation in the banking industry is discussed all around the world. There are many studies which evaluate efficiency of banking industry in the Visegrad Group. Generally, these studies evaluate the efficiency under certainty and for small sample. This paper is including one part of demography risk variable (average salary). The analysis shows that the model without risk variable has generally lower or same efficiency score. If the SBM score is not lower, it should be upper.

For future work, it should be good to introduce new variable which will more look for the risk. Also it would be nice to have bigger.

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References

[1] Ataullah, A., Cockerill, T. and Le, H. (2004). Financial liberalization and bank efficiency: a comparative analysis of India and Pakistan. Applied Economics, 36, p. 1915-1924.

[2] Casu, B. and Molyneux, P. (2003). A comparative study of efficiency in European banking. *Applied Economics*, 35, p. 1865–1876.

[3] Fukuyama, H., Guerra, R. and Weber W.K. (1999). Efficiency and ownership: evidence from Japanese credit cooperatives. *Journal of Economics and Business*, 51, p.473-487.

[4] Girardone, C., Molyneux, P., and Gardener, E. P.M. (2004). Analysing the determinants of bank efficiency: the case of Italian banks. *Applied Economics*, 36, p. 215–227

[5] Chang, T.C. and Chiu, Y.H. (2006). Affecting factors on risk-adjusted efficiency in Taiwan banking industry. *Contemporary Economic Policy*, 24, p. 4.

[6] Charnes, A., Cooper, W.W. and Rhodes, E. (1978). Measuring the efficiency of decisionmaking units. *European Journal of Operational Research*, 2(6), p. 429–444.

[7] Chen Yu-Chuan et al. (2013). The analysis of bank business performance and market risk – Applying Fuzzy DEA. *Economic modelling*, 32, p. 225-232.

[8] Chiu, Y.H. and Chen, Y.C. (2008). The analysis of Taiwanese bank efficiency: incorporating both external environment risk and internal risk. *Economic Modelling*, 26, p. 456–463.

[9] Cooper, W.W., Kingyens, A.T. and Paradi, J.C. (2014) Two-stage financial risk tolerance assessment using data envelopment analysis. *European Journal of the Operational Research*, 233, p. 273–280.

[10] Kao, C. and Liu, S. (2000). Data envelopment analysis with missing data: an application to university libraries in Taiwan. *Journal of the Operational Research Society*, 51, p. 897–905.

[11] McAllister, P.H. and McManus D. (1993). Resolving the scale efficiency puzzle in banking. *Journal of Banking and Finance*, 17, p .389-405.

[12] Sealey, C.W.Jr. and Lindley, T.J. (1977). Inputs, Outputs and a Theory of Production and Cost at Depository Financial Institutions. *Journal of Finance*, 32(4), p. 1251-1266.

[13] Sherman, D.H. and Gold, F. (1985). Bank Branch Operating Efficiency: Evaluation with Data Envelopment Analysis. *Journal of Banking and Finance*, 9(3), p. 297-315.

[14] Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research*, 130, p. 498–509.

The profit or loss reported in the consolidated financial statements of public administration accounting entities of the Slovak Republic

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Abstract

The article focuses on the profit or loss reported in the consolidated financial statements of the entity of public administration in the Slovak Republic in accordance with current applicable legislation. I present a breakdown of the contribution of public administration in accordance with the financial rules and accounting entities and because of that the consolidated financial statements of an entity of the public administration shall be based on the business unit in this breakdown. The contribution is divided into three parts, which examines the particularities of profit, which is recognized in the consolidated financial statements of the entity of public administration, in the individual financial statements of an entity of public administration and in the individual financial statements of an entity - a company involved consolidating entity of public administration.

Key words

Public administration, consolidated financial statements, consolidated group, accounting entity, legal entity

JEL Classification: M41, M48

1. Introduction

Accounting entities of public administration are legal persons registered by the law of budgetary rules written by it into the register and kept by the Statistical Office of the Slovak Republic. The public administration of the Slovak Republic forms the inner essence of the report of the State as a form of organization and regulations by providing executive power to the state. The breakdown of general government of the Slovak Republic (Law no. 523/2004 Coll., on budgetary rules, § 3) is as follows:

- Central Administration - the state budget chapter administrators (ministries, central authorities and other organizations) and they establish state budgetary organizations and state subsidized organizations, public universities, state funds, National Property Fund, the Slovak Land Fund.
- Local government: municipalities, higher territorial units (the HTU) and they establish budgetary or subsidized organizations.
- Special group of entities as the Social Insurance Fund, Health Insurance Funds.

Accounting entity of public administration (hereinafter referred to as AE PA) has the obligation to draw up consolidated accounts if the established budgetary organizations, contributory organizations or established a trading company, has a decisive influence on

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the activity of the company within the meaning of the Accounting Law (Law no. 431/2002 Coll., § 6 par. 4, § 22, § 22. 3, par. 4) at present, these have consolidation levels of AE PA:

- The consolidated financial statements of government units (up: the state budget chapter administrator, town, HTU).
- The consolidated financial statements of the central administration (compiled by the Ministry of Finance SR).
- Aggregate accounts for general government (compiled by the Ministry of Finance SR).

AE PA in its consolidation scope includes subsidiaries of AE PA (state budgetary organizations, state subsidized organizations, financial organizations established by the municipality, the higher territorial units, subsidized organizations established by the municipality, trading companies), joint and associated AE PA usually designed for business purposes (commercial companies).

2. The consolidation financial statement of an accounting entity of public administration

AE PA follows the Slovak Accounting Act (Law no. 431/2002, § 22) and the Decree issued by the Ministry of Finance of Slovak Republic (hereinafter MF SR) no.27526/2008 laying down the details of the methods and procedures of consolidation in the public sector and details of the structure and identification of items in the consolidated financial statements of general government. At the present, the measure has one attachment, which comprise the consolidated balance sheet AE PA, and the consolidated income statement and notes consolidated financial statements AE PA. The Ministry of Finance (hereinafter referred to as the MoF) uses this Annex for preparing consolidated financial statements of the central administration. The Annex is not used for the aggregate of the financial statements of public administration. The conclusion I have reached on the basis of the Precaution of MF SR, which does not duty for the MoF to use this attachment to compile this account. I focus only on AE PA, which prepares the consolidated financial statements of government units.

2.1 Consolidated balance sheet of an accounting entity of public administration

This accounting record provides information about the property and the sources of property coverage AE PA included in the consolidated AE PA. Arrangement, marking the consolidated balance sheet items AE PA based on the balance sheets of individual financial statements AE PA, which was supplemented by items such as goodwill, long term deferred tax assets, shares in other accounting units, deferred tax liability. Gross column does not contain a column for correction of asset, as we know from the balance sheets of individual financial statements AE PA. It is recognized in them the value of assets and liabilities and their carrying amounts in the difference (net value) based on the consolidation methods used in preparing the consolidated financial statements AE PA. Trading income reported in the consolidated financial statements AE PA is actually fiction, it provides potential information, which provides the ability to maintain public assets in the group AE PA. This record result of operations does not serve for fiscal needs and is not reallocated.

2.1.1 Profit or loss reported in the consolidated balance sheet of an accounting entity of public administration

Profit or loss is recognized on the liability side of the capital as the sum of any pending outcomes for the previous years and profit for the current period. The same also applies to the previous years. Profit for the current accounting period is determined by deducting the

carrying amount of the assets of the consolidated group AE PA on the first line on the asset side of the item by the liabilities and the revaluation reserves, funds, unsettled earnings from previous years, shares of other accounting, accounts payable, Accruals, relationship to the accounts of clients of the State Treasury. The same procedure also applies to the profit for the previous financial year. The consolidated balance sheet AE PA is using the term result for the accounting period and its value is recognized in monetary terms for the current year and for the immediately preceding financial year. This is the result after tax.

2.1.2 Consolidated profit and loss accounts of an accounting entity of public administration

This is the accounting record, which provides information on the outturn for the current year and for the immediately preceding financial year. Trading income reflects changes in assets of AE PA included in the consolidated consolidating AE PA**.** Layout and labelled items in the consolidated income statement of AE PA are based on the income statement in the separate individual financial statements of AE PA, which is complemented by lines of proportion of consolidating AE PA for profit or loss of associated AE PA in cost and in income and profit or loss after tax from this, and the share attributable to other accounting.

2.1.3 Profit or loss reported in the consolidated profit and loss accounts of an accounting entity of public administration

Reported profit or loss before and after taxation is not the same as arithmetic sum of the profit or loss in monetary terms before and after tax profit and loss account included in the consolidation AE PA for the current period. It depends on the type of AE PA included in the consolidated group, mutual relations and consolidation method used.

It contains column 1 in which it has the final costs, revenues, income tax, and profit from main activities of the consolidating accounting entity and consolidated AE PA for the current period. If AE PA carries along the main activities also business activity, then the final balances of costs, revenues, income taxes and results of operations are being reported separately for business in column 2 for the current financial year. From experience it points us to that in column 2 for business are being recognized final costs, revenues, income tax, profit before and after taxation of companies included in the consolidated AE PA for the current period.

In column 4 for the immediately preceding accounting period are costs, revenues, income, and profit before and after taxation not divided into core activity or business activity. In the consolidated income statement AE PA is being used:

a) concept of profit or loss before tax. Its value is recognized in monetary terms for the current period reported by type of activity (core activity, entrepreneurial activity, in total) for the immediately preceding accounting period in the same monetary value;
b) concept of profit or loss after tax. Its value is recognized in monetary terms for the current period reported by type of activity (core activity, entrepreneurial activity, in total) for the immediately preceding accounting period in the same monetary value.

2.1.4 Profit or loss reported in the full consolidation method

This method consists of a consolidation of capital, consolidation of commitments, consolidated profit in asset and consolidation costs and revenues. Individual items of assets, liabilities and the difference between assets and liabilities, income and expenses recognized in the carrying amount of the individual financial statements of consolidated AE PA are combined after making the assignment and relaying operations, for example the subsidiaries at AE PA, who shall proceed according to the regulations as consolidating AE PA through the

transitional bridge to the aggregated balance sheet and aggregate income statement, which follows the structure of the Consolidated Financial Statements AE PA through the consolidation package. This excludes the interrelationships that exist between them and the focus will be on mutual relations outside of them.

It focuses on consolidation of commitments, profit or loss in the items of property for which it is eliminated positive and negative earnings from operations between AE PA included in the consolidated group and the consolidation of costs and benefits in the context of the fiction of one legal entity.

If the assets and liabilities between AE PA included in the consolidated AE PA are equal, they are eliminated at the level of the aggregated balance sheet and they have no effect on the consolidated balance sheet of AE PA. If the value is not the same, for example the commitment to consolidating AE PA other than statements Combined AE PA receivable (impaired), a consolidated difference is created, which is entered to the profit of the consolidated balance sheet of the application of the precautionary principle in the context of the consolidation of expenses and revenues. Liability shall be recognized and the asset is adjusted to the amount of the obligation.

In consolidating the economic result is a so-called elimination of in between result of cross-supplies (inventories, tangible assets, or other transactions) between AE PA, unless those transactions are not consolidated AE PA done outside the consolidated group AE PA. This is done with respect of intra-group transactions and for this reason has not implemented any profit or loss. The result is that in the consolidated balance sheet are the assets acquired as part of a consolidated group AE PA valued at cost price, without the realization of gain or loss.

Basis of consolidation of the cost benefit is the mutual supply of services between consolidating AE PA reflected in the different revenues and expenses in the aggregate income statement. In terms of fictive one entity, these costs and benefits are being considered as unrealized. Consolidating AE PA in income eliminates the dividends received from subsidiaries of AE PA - companies. In the consolidated income statement AE PA are recognized only to the costs and benefits of AE out of the consolidated group.

2.1.5 Profit or loss reported for the method of proportionate consolidation

This method is used by consolidating AE PA for the consolidation of joint AE PA, and since 2015 they can use for common AE PA equity method. This is based on the individual financial statements AE PA included in the consolidated entity done with full consolidation method, which are aggregated pro rata share of equity by the owner – i.e. AE PA in the capital of joint AE PA. In proportionate consolidation method are included the consolidated balance sheet of equity and liabilities of the AE PA, and the consolidated income statement AE PA include its share of the cost, shared revenue of AE PA. Under this method of consolidation are not recognized minority interests of other owners as the Common AE does not have a decisive impact on AE, and therefore they don't decide on these shares (Harumová A., 2011).

2.1.6 Profit or loss reported under the equity method

There is no aggregation of the individual financial statements of AE. Consolidation is done from converting the individual financial statements of the consolidating entity and the associated AE. (Harumová A., 2011). Trading income is consolidated to the extent of shares of consolidating AE PA.

3. Profit or loss reported in the separate financial statements of an entity of public administration and company

AE PA works with the concept of positive economic result or the notion of negative economic results, profit or loss. It does not use the term accounting profit and accounting loss and accounting result in the approval as for the corporate entities. The reason is the different nature of the activity between AE PA and trading company and the reason for their existence. Profit or loss of AE PA is not approved and redistributed by the General Assembly as it is in for a corporate company, which follows the statutes and the Commercial Code. The municipal council approves and decides on the final account, which is linked to the financial management of the village. This is the ultimate balance of funds recognized on the asset side of the balance sheet in current assets. Positive or negative results of operations AE PA gets into an undivided result from previous years. Positive earnings from business activities are used as an internal source of AE PA. Trading company uses the concept of retained earnings or accumulated losses from previous years and the result of loss to be approved. Trading company can use the profit only after approval by the General Assembly on the creation of resources or use them to meet obligations to shareholders (in the form of joint stock company dividends, with the other partners in the form of profit sharing).

3.1 Profit or loss recognized on the balance sheet of an accounting entity of public administration

Profit or loss is recognized on the liability side of the capital as the sum of AE PA any unsettled outcome of previous years and profit for the current period. The same also applies to the previous accounting period. Profit for the current accounting period is determined by deducting the carrying amount of assets AE PA on the first line of the balance sheet on the asset side of the item by the liabilities and revaluation reserves, funds, unsettled earnings from previous years, liabilities, accruals, relationship to the accounts of Treasury clients. The same procedure also applies to the profit for the previous financial year. This is the result after taxation.

3.2 Profit or loss recognized in the income statement of an accounting entity of public administration

AE PA determines income before income tax is deducted from revenues from its operations, which incomes are taxed by tax as the difference between revenues which are recorded in accounting group 6 Income and expenses charged to the same account class (hereinafter AC) 5 Costs excluding accounts 591 Current income and additional income 595 Income tax paid at the date of preparation of the financial statements.

AE PA detects profit or loss after income tax as the difference between revenues recorded in the accounts AC 6 Revenues and expenses in the accounts AC 5 Costs to the date on which the financial statements is due to. Profit or loss of AE PA can be positive or negative but also can have a zero value.

3.3 Particular economic profit or loss of general government entities included in the local government

3.3.1 Municipality, higher territorial units

Trading income arises as the difference between revenues of municipalities, higher territorial units and the corresponding costs charged from the income of municipality / HTU (consumption, energy consumption, labour costs, depreciation and provisions, net book value of tangible and intangible assets of municipalities, HTU) acquired by their own municipal

revenues, HTU and transfers provided by the established public and subsidized organizations. When there is no current or capital transfers received from other government entities or by entities outside the public administration (Farkaš, R., Kordošová A., Kovalčíková A., Kupkovič I., Somogyiová K., 2006).

3.3.2 Budget organization founded by the municipality, higher territorial units

Trading income arises as the difference between revenues from current transfers by the founder and the corresponding costs (material consumption, energy consumption, labour costs, depreciation and provisions, net book value of tangible and intangible assets). It does not occur in capital transfers from the founder, in transfers of current and capital transfers from other general government entities and entities from outside the public administration, when there is no budget revenue (Farkaš, R., Kordošová A., Kovalčíková A., Kupkovič I., Somogyiová K., 2006).

3.3.3 Subsidized organization founded by the municipality, higher territorial units

Trading income arises as the difference between income from the activities subsidized organization and the corresponding costs charged from income from operations of subsidized organization (material consumption, energy consumption, labour costs, depreciation and provisions, net book value of tangible and intangible property of the municipality, higher territorial units acquired from its own income allowance organizations), as well as the difference between revenues from current transfers by the founder and the corresponding costs charged to these transfers (material consumption, energy consumption, labour costs). It does not occur in capital transfers from the founder and current and capital transfers received by subsidized organization from other government entities and entities from outside the public administration (Farkaš, R., Kordošová A., Kovalčíková A., Kupkovič I., Somogyiová K., 2006).

3.4 Particular profit or loss recognized in the separate financial statements of the company

3.4.1 Profit or loss recognized on the balance sheet of the company

Depending on the size of the company, the profit or loss is recognized in the balance sheet as follows:

Table 1: The profit or loss reported on the balance sheet of the company

• **Micro accounting entity**	• **Small accounting entity, large accounting entity, public interest entities:**
Profit for the current accounting period is determined by deducting the carrying amount of assets of the entity on the first line of the balance sheet asset item on the part of the liabilities as follows:	
• capital, • share premium, • other capital funds, • the legal reserve, • other funds from profit, • revaluation reserve, • retained earnings, • commitments	• capital, • share premium, • other capital funds, • the legal reserve, • other funds from profit, • revaluation reserve, • retained earnings, • commitments,

	• time distinction
The same procedure also applies to the profit for the previous financial year.	

Source: own processing on the basis of the MF SR no. MF/15464/2013-74, no. MF/23378/2014-74, no. č. MF/23377/2014-74.

3.4.2 Profit or loss recognized in the income statement of the company

Economic result measures the conversion of assets of the entity - trading company. It is checked before income taxes as the difference between revenues which are recorded in the accounts of accounting class 6 Income and expenses charged to the same account class 5 Costs excluding accounts 591 Income taxes, 592 Deferred income tax and 595 Additional income tax payments and accounts, intra-organizational revenues and intra - organizational costs.

Profit after income tax is determined as the difference between revenues recorded in the accounts of accounting class 6 Income and expenses charged to the same account class 5 Costs and results of operations may be accounting profit or loss accounts. Economic result is also divided depending on from which activities it was created as follows:

- from economic activity as the difference between revenues recorded in the accounts of account class 60 Turnover up to 65 Reversal of certain items from economic activity expenses and charged to the accounts of account class 50 Consumed purchases till 55 Depreciation and impairment losses on fixed assets and,
- from financing activities as the difference between revenues recorded in the accounts of account class 66 Financial income and expenses recorded in account class 56 Financial expenses.

4. Conclusion

Economic result that is recognized in the consolidated financial statements of the entity of public administration is an interesting topic. This profit or loss is not the same from the profit of the individual financial statements of consolidated entities of public administration. This is the result for various transactions linked to a consolidating entity of public administration and the nature of their activities. The profit, which is recognized in the consolidated financial statements of government units do not bind the tax liability to the consolidating entity public administration but presents the result for the economic units to other accounting entities outside the consolidated group entity of public administration. It is a complex number to a specific date to detect differences of costs and benefits for a certain period. It is a fiction performance of an entity of public administration entity including in the consolidation of public administration and profit peculiarity is, that it is the output of the entity's public reports which are very different. Their activities are not identical and therefore this topic is interesting and inspiring.

References

[1] Harumová, A. (2011). *The financial management of the holding*. Bratislava: Publisher Sprint dva, p. 351.

[2] Hvoždarová, J. – Sapara, J. – Užík, J. (2014). *The consolidated financial statements*. Bratislava: Publisher EKONÓM, p. 288.

[3] Kašiarová, L. – Majorová, M. (2008). *A practical guide to consolidation of municipalities – part 1*. Publisher Andra vzdelávacie centrum n.o., p. 128.

[4] Kovalčíková, A.- Kordošová, A. (2014). *Accounting of municipalities*. Bratislava: Publisher EKONÓM, p.173.

[5] Kovalčíková, A. (2010). *Accounting financial organization*. Bratislava: Publisher Iura Edition, spol. s r.o. 314p.

[6] Farkaš, R. – Kordošová, A. – Kovalčíková, A. – Kupkovič, I.- Somogyiová, K. (2006). *The accounts of the State and municipalities according to the principles of international accounting standards for the public sector – part I.* Bratislava: Publisher KPMG Slovensko, spol. s r.o., Pricewaterhouse Coopers Slovensko, s.r.o. , p. 326.

Law reference:

[1] Law no. 431/2002 Coll. on Accounting and on the Amendment of Certain Laws.

[2] Law no. 523/2004 Coll. on Budget Rules of Public Administration and on the Amendment of Certain Laws.

[3] MF Decree no. MF/27526/2008-31, which sets out the details of the methods and procedures of consolidation in the public sector and details of the structure and identification of the items in consolidated financial statements in the public sector as amended.

[4] MF Decree no. MF/25755/2007-31, which sets out the details of the arrangement, description and content of items in individual accounts, date and place of submission of accounts to budget organizations, subsidized organizations, state funds, municipalities and higher territorial units, as amended.

[5] MF Decree no. MF/15464/2013-74, which sets out the details on the arrangement, description and content of items of individual financial statements and the scope of data from individual financial statement for micro entities as later amended.

[6] MF Decree no. MF/23378/2014-74, which sets out the details of the individual financial statements and the scope of data from individual financial statement for small entities as later amended.

[7] MF Decree no. MF/23377/2014-74, which sets out the details of the individual financial statements and the scope of data from individual financial statement for large entities as later amended and public interest entities.

[8] MF Decree no. MF/23054/2002-92, which sets out the details of accounting procedures and framework chart of accounts for entrepreneurs using double-entry bookkeeping as later amended.

The behavioral study: adjustment and anchoring & mental accounting in general economic decisions

Věra Jančurová[1]

Abstract

The purpose of this article is to verify whether there can be identified adjustment and anchoring and mental accounting bias in general economic decisions of Czech population. Consequently, the target is to detect how general economic decisions are done and influenced by studied biases and heuristics.

This goal is reached by conducting own research which is based on an electronic and paper questionnaire. The questionnaire asks respondents to provide data regarding their demographic situation; such as age, gender, education, etc. The main part concentrates on questions concerning the decision making in general economic situation; e.g. shopping, insurance, savings etc. This part is divided theoretically into two sections, one tests the real skills and behavior at financial markets and the second asks the respondents to evaluate them. Based on that could be tested also the overconfidence of the investors. However overconfidence is not the main purpose of this study and thus is mentioned only marginally.

The methods, used for evaluation of the questionnaire evaluation are descriptive statistics; p-test and F-test to detect the significance of the results. Respondent´s knowledge corresponds to subjective assessment of own knowledge.

The results show that anchoring and adjustment are significantly present in general economic decisions as well as mental accounting.

Key words

Mental accounting bias, adjustment and anchoring bias, behavioral finance

JEL Classification: G110, G190, G020, I20

1. Introduction

General economic or financial decision is a complicated process frequently studied by diverse areas of science especially by behavioral finance in the last decades. Decision making itself is influenced by many factors that need to be taken into consideration. Decision making under uncertainty that is typical for financial and economic decisions is extremely complicated because it incorporates many intangibles that need to be considered.

With the uncertainty increases also the complexity and decisions become more difficult. However the capacity of human brain is not unlimited. To be able to handle such complex situation there are used heuristics and biases that simplify the process of decision finding. Over the years of research there were defined numerous heuristics and biases that influence individuals decision making process. However there was shown that besides the benefits of these heuristics and biases in simplification of the decision finding, there are also negatives in form of irrational biased decisions.

This article concentrates on two of these heuristics and biases; how anchoring and adjustment bias and mental accounting bias influence decisions of individuals.

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The study investigates mainly on these questions:
Does anchoring and adjustment influence financial and economic decisions?
Does the studied sample of Czech population suffer from mental accounting?

2. Theoretical background

For the purpose ot this study especially the theoretical background of adjustment and anchoring bias and mental accounting bias are desirable. Both cognitive biases are considered to influence the perception of information and thus influencing the decision making process. Such cognitive psychology drive individual behavior, for the purpose of this study especially influences on perception and attention is crucial.

The anchoring and adjustment bias is cognitive bias described as inclination to irrelevant information as a reference for evaluation or estimation. "Numerous studies demonstrate that regardless of how the initial anchor was chosen, people tend to adjust their anchors insufficiently and produce end approximations that are biased."

Commonly is anchoring and adjustment used by sales, giving very high price at the beginning of the negotiation. In this way the anchor is the high price and the final price will be most probably higher than in case the salesman would start the negotiation at the fair or real price. One of the most famous studies of anchoring and adjustment was performed in 1987 by Gregory Northcraft and Margaret Neale and showed that even professionals at particular area are influenced by anchors. The study analyzed the behavior of real estate agents by evaluating a property after receiving proposed selling price. Northcraft and Neale (1987) clearly showed that anchoring and adjustment bias is present in numerous areas of financial decisions.

This anchoring-and-adjustment heuristic is assumed to underlie many intuitive judgments, and insufficient adjustment is commonly invoked to explain judgmental biases.

Anchoring and adjustment bias is typically tested by giving the respondents divergent anchors to monitor the impact on final decision of them. Very often respondents are asked to evaluate different staff by facing more or less obvious anchors. One of the most famous experiments was using rotating wheel to influence the answers of participants answering questions that obviously could not be anyhow answered by the rotating wheel.

This study focuses always on two types of questions; one of them includes an anchor, the second one does not.

Mental Accounting describes the inclination to categorize, code and evaluate outcomes using mental accounts. Thus various sums of money are treated differently based on their categorization or evaluation, very commonly these demonstrates the origin of the budget. This contradicts the theory of completely rationally deciding individuals, because for rationally handling person the origin of amount is not relevant.

For the purpose of this study the main concentration is given to the mental accounts created based on how the resources were reached.

Richard Thaler described the process of categorizing economic outcomes, known as mental accounting. Based on Thaler theory people have mental accounts for their resources, typically multiple accounts for the same resource.

Shefrin and Thaler extended the study by adding the frame to the resources based on current income, current wealth or future income.

3. Methodology and Data

Methods used for the study of adjustment and anchoring bias and mental accounting bias are based on the knowledge of behavioral finance that individuals use not relevant anchors for their decisions and different mental accounts for evaluation of outcomes.

The survey studies how the respondents perceive information, if they are influenced by given anchors and if they use mental accounts.

These questions are followed by questions asking he participants to evaluate own financial decisions´skills.

The data for the purpose of this study were collected in a controlled field research, based on a questionnaire proving inclination to behavioral biases.

The sample consists of 302 respondents, the age between 18 and 70 years.

Sample characteristics:

- Gender: 70% women, 30% men
- Age: 18-30: 50%; 31-50: 30%; 51-65: 17%, more than 65: 3%
- Education: secondary school: 2%, high school with school leaving exam 45%, high school without school leaving exam 11%, higher school 4%, university 38%
- Marital status: single/divorced 50%, mate 10%, married 39%, widow/widower 1%

4. Research Results

The questionnaire tested mental accounting by presenting two variants of the questionnaire. First half of the questionnaire presented a situation where respondents saved particular amount of resources by restructuring their expenditures. The second half of the questionnaire set the respondents in the situation that they received the same amount unexpectedly – by coincidence.

Further both parts were devided into addition two subsets, each of them saved/won different amount – 500 Kč or 1 200 Kč.

Table 1: Use of money based on how they were gained and on the amount gained.

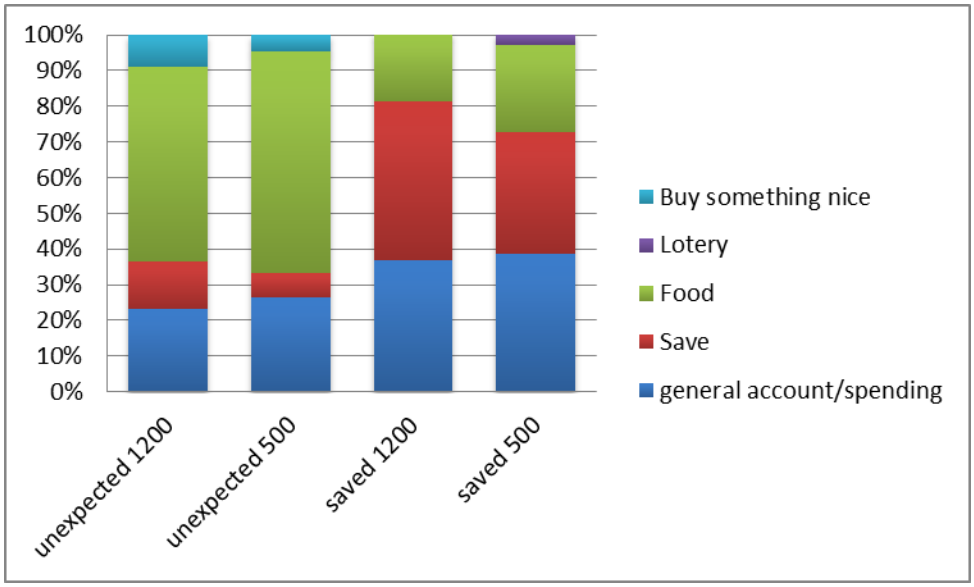
	Coincidence		Saved	
Amount	1200	500	1200	500
Respondents	143	129	147	143
General account/expenditure	23%	26,35%	36,7%	38,5%
Savings	13,3%	6,9%	44,2%	34,3%
Food	54,5%	62%	18,4%	24,5%
Lottery	0%	0%	0%	2,8%
Buy something him/herself	9%	4,7%	0%	0%

The table shows how people use the money differently based on the way they gained them. If the budget is gained by coincidence independently on the amount the majority of respondents answered that they spend this additional amount on food. In contrary if the additional budget is gained by savings, the majority of respondents answered that they save the money. Interestingly we can see that the same set of people behave completely differently based on how the additional budget was gained.

The second tested hypotheses on this part were how the amount saved/coincidently gained influences the decision. The results of the research show that the impact of the amount is less significant. In total lower amount increases the tendency to spend the money on food and

general expenditure and decreases the tendency to save. Interestingly only the respondents gaining the additional amount by coincidence answered that they would by something for themselves. After closer look on the results it is obvious that this answer was given only by young respondents in the age from 18-25.

Graph 1: Use of money based on how they were gained and on the amount gained



The graphical representation of the results shows, that the consumption tendency is obvious in Czech population as about 80 % of respondents gaining the budget by coincidence and above 50% of respondents gaining the budget by savings would spend the additional amount on food and general consumption.

Another way of testing of mental accounting there was used situation with purchase tickets to the concert of favorite band. The research was again used in two variants, the first scenario: a ticket purchased for CZK 1,000; the second variant: a free ticket.

After that the respondents were given following information: The concert takes place 60 km from your place of residence. It's a hard winter and huge snowstorm is expected on the day the concert will be held. The respondents were asked if they attend the concert or not.

Classical economics assume that the respondents should attend the concert in both cases equally happy and equally likely. However, the results of the study contradict classical economics´expectation.

Table 2: Mental Accounting: concert visit

	bought	free ticket
altogether	145	145
visit the concert	135	35
stay home	10	110

Of those who received a free ticket only 35 people would visit the concert of favorite band, the rest of the group would prefer to stay home.

A group of people who have had to buy a ticket for CZK 1,000, 135 people would attend the concert, only 10 respondents would stay home.

This result can be supported by the loss aversion bias as well; the feeling of a loss is stronger when there are own resources spent.

Anchoring and adjustment was tested by giving the respondents different anchors for saving amount.

Table 3: *Anchoring and adjustment*

anchor	*649 Kč*	*1649 Kč*
Average	**626**	**1535**
Men	**629**	**1539**
Women	**621**	**1533**

The result shows very small difference among men and women, in both cases the estimation of men was higher however the difference cannot be considered to be significant.

However the research has two interesting results. First by estimating the optimal saving amount the respondents were given anchors 649/1649, the estimation of the respondents were rounded to hundred/tens. Second by estimating of the ideal saving amount the respondents used tens only in v case of anchor 649Kč, in case of anchor 1649 Kč the answer was rounded to hundred.

Table 4 *Anchoring and adjustment: most common answers*

anchor	*649 Kč*		*1649 Kč*	
	550	**36**	**1500**	**50**
	750	**29**	**1700**	**42**
	600	**27**	**1000/2000**	**22**

The above introduced research confirmed presence of adjustment and anchoring and mental accounting biases in the tested sample of Czech population. The study did not confirm significant difference based on gender or age.

5. Conclusion

The purpose of this article is to present the results of a study concerning anchoring and adjustment and mental accounting bias. The main goal is to show whether above mentioned bias are present in general economic decisions of Czech population.

The results show that anchoring and adjustment bias and mental accounting bias are both significantly present in general economic decisions of the sample.

Interestingly people are influenced by the way how the additional budget was gained however not by the amount itself. Mental accounting is confirmed also by the second part of the study that shows the individuals to decide based on the way the "property", in the case study the ticket, was gained. The impact of gender and age was not confirmed for either of tested biases.

Future studies could investigate more the details of mental accounting bias and anchoring and adjustment bias, especially if there are any links among these biases and some other demographic details, e.g. education.

References

[1] Baker, H.K., and Vctor, R. (2014). Investor Behavior: The Psychology of Financial Planning and Investing. Wiley, New Yersey, USA.

[2] Goldberg, K and Nitzsch, R. (2004). *Behavioral Finance: Gewinnen mit Kompetenz*. FinanzBuch Verlag GmbH, Muenchen, Germany.

[3] Green, H. (1993). *Econometric Analysis*, 2nd ed. New York: Macmillan.

[4] Howard, T.C. (2014). *Behavioral Portfolio Management*. Harriman House, London, UK.

[5] Nofsinger, J.R. (2011). *The Psychology of Investing*. Pearson Education, Inc., Publishing, Boston, USA.

[6] Pompian, M. (2006). *Behavioral Finance and Wealth Management*. John Wiley & Sons, Inc., New Jersey, USA.

[7] Seppala, A., (2009), *Behavioral Finance of Investment Advisors*. Helsinki School of Economics, Master´s thesis. Helsinki, Finland, p.1-72.

[8] Shefrin, H. (2002). *Beyond Greed and Fear*: Understanding Behavioral Finance and the Psychology of Investing. Oxford University Press, New York, USA.

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The Analyzes of Model of Investment Decision Making

Věra Jančurová[1], Zuzana Rakovská [2]

Abstract

The process of investment decision making is one of the most important areas of behavioral finance. The way individuals decide in financial markets and how they should make their investment decisions was a subject of number of studies that introduced numerous models of investment decision making.
The goal of this paper is to analyze existing models of investment decision making and based on conducted analyzes recommend their usage in investment decision making. The study concentrates on psych-graphic models. The study also provides brief overview of behavioral biases influencing particular studied models of investment decision making.
The methods used for the purpose of this study are especially description, enabling to make the first assessment of the models, and comparison.
As a result of conducted analyzes there are introduced suggestions for improvements for the existing models and possible inputs for creation of a new model of investment decision that combines the benefits of studied models and reduces the weaknesses.

Key words

Behaviooral finance, decision making,

JEL Classification: G110, G190, G020, I20

1. Models of decision making

The proces of decision making is as old as the mankind itself. However the complexity and number of information cause the decision making to be nowadays more and more difficult. At the beginning people were used to rely on the help of gods and magic. Nowadays we have to rely on our knowledge and skills. These article concentrates on the theoretical level of analyzing the decision making proces, decision models.

The theory has identified numerous decisions models. To streamline the number of models there is commonly used these three groups: psycho graphical models, models based on believes and models based on preferences. This study concentrates mainly on psycho-graphical models.

The Barnewall two-way model distinquishes between passive and active investors.

Passive investors are considered to be invoestors who did not have to risk their own capital to get their wealth, and thus become rich passively. Commonly these are individuals who has accumulated the wealth for their whole life from employment, or disciplined savings. This group of investors is risk averse, typically their acceptance to risk is lower, requesting more secure investments; certainty. These investors did not risk their property to get wealthy; their property was reached by risk-taking of foreign property or inherited.

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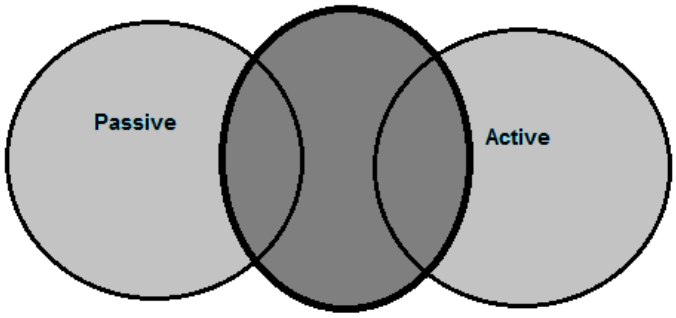
Passive investors suffer from numerous biases during decision making process under uncertainty. Endowment effect, loss aversion, status quo, regret aversion, mental accounting, anchoring and adjustment, availability, hindsight and framing bias were identified to be present by passive investors commonly.

Active investors in contrast got wealthy by risking of their own property. In order to increase revenues these investors are ready to undergo higher risk and typically they participated actively in the own enrichment.

This group of investors is characterized by less aversion to risk-taking than that of a passive group. High confidence in own abilities is common for this group.

Active investors are considered to suffer most from overconfidence and self-attribution, availability, confirmation, representativeness, illusion of control and self-control bias.

Figure 1:Barnewall two way model: brriers



After closer analyzes of the model of passive and active investors, we conclude that the model is very strongly extrapolated. The model completely misses many combinations that are significantly present at the market and thus does not describe the situation.

First there is nothing among getting the wealthy passive or active, however in the reality there is often the combination of both the way how people get their property. Most often it is partially inherited and increased by own whole life savings.

Further the model connects the risk tolerance only to the way how the wealth was gained; in other words passively gained wealth leads to “passive” behavior. However in our opinion even active person can inherit a lot of money and can act more actively/even aggressive than an individual who received the wealth by own initiative or action.

The Bailard, Biehl and Kaiser model is another theoretical model that concentrates on the personality of the investors based on two main characteristics, level of self-confidence and the way of behavior.

Bailard, Biehl and Kaiser constructed a model with two axes; one of them shows the self-confidence and the second activity (impulsiveness).

Self-confidence describes the confidence with which the investor's approach their life, regardless of whether it is professional or family life, health or financial issues.

The way of behavior divides investors based on their attitude towards life; whether they are methodological and analytical or emotional, intuitive or reckless.

Figure 2:The Bailard, biehl and Kaiser model



As the model shows these two charakteristics devide invesotrs into 5 groups; straight arrow, individualist, adventurer, celebrity and guardian.

In the middle of the axes there are the straight arrows, investors that are sensitive and sure. These investors are a combination of all characteristics involved in the model – they are little careful, little impetuous, little confident and little anxious. These investors are willing to take the risk when expecting reasonable yields.

Individualists are independent and self-confident investors, typically these characteristics are present by the owners of smaller businesses. Individualist tend to make own decision based on analysis of all available options and possible impacts. They are considered to be good clients who are willing to listen and make a rational choice based on receiving and processing available information.

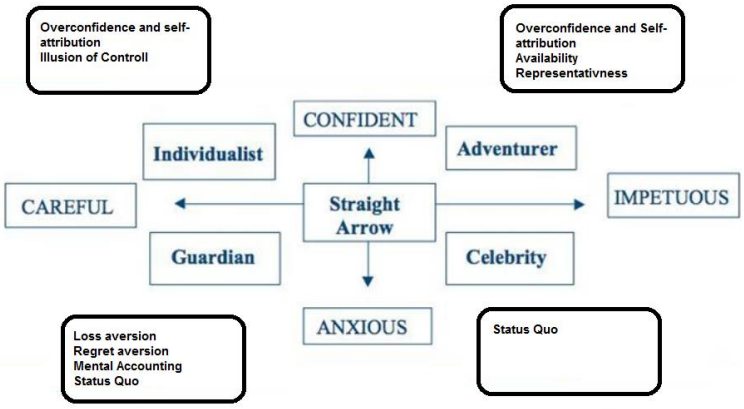
Adventurer is typically used to own a highly undiversified portfolio; it is self-confident person willing to take the risks. Their confidence support the making of own decisions and unwillingness to be open to advice from outside.

Celebrity likes to be center of attention; always want to be where something's happening. Celebrities usually have strong conviction or idea. However at the same time they are able to recognize their limitations and therefore are more willing to seek assistance from the outside and get advice regarding their investments.

Guardian are investors that are cautious and tend to to worry about everything, especially their fortunes and future. It is typical to become guardian over time – by getting older. They are much more concentrated on protecting their acquired funds and also from this reason they are more willing to seek assistance.

The Bailard, Biehl and Kaiser model is strongly concentrated on two characteristics of personality. For each of defined group there can be identified biases that this group inclines to.

Figure 3:Biases in the Bailard, Biehl and Kaiser model



To improve the model there can be added more characteristics however this would lead to increasing complexity. The way the Bailard, Biehl and Kaiser model could be easily extended is his implementation in another model that is missing the personalities, as for example the Barnewall two-way model.

However it has to be noted that another week point of the model is the assumption that investors access all life issues with the same attitude, for this point it would be better to concentrate only on the behavior on financial markets; especially by savings, investment or insurance.

Pompian presented a way how to extend the model of Barnewall in 2006 adding the investors types, *Behavioral Alpha process: A Top- Down model*. Pompian invented a model that is supposed to help especially financial advisors to identify the customer and help him accordingly.

Pompian extention presents four kinds of investors; passive preserver, friendly follower, independent individualist, active accumulator.

Figure 4: Behavioral Alpha process: A Top- Down model

General Type	**PASSIVE**		**ACTIVE**	
Risk Tolerance	Low ◄			► High
Investment Style	Conservative	Moderate	Growth	Aggressive
Bias Types	Primarily emotional	Primarily cognitive	Primarily cognitive	Primarily emotional
BITs	Passive Preserver (PP)	Friendly Follower (FF)	Independent Individualist (II)	Active Accumulator (AA)
Biases				
Emotional	Endownment Loss aversion Staus quo Regret aversion	Regret aversion	Overconfidence and self-attribution	Overconfidence Self-control
Cognitive	Mental accounting Anchoring and adjustment	Availability Hindsight Framing	Conservatism Availability Confirmation Representativeness	Illusion of control

Passive preserver is emotionally biased investor and therefore excessive spammed by cognitive details which leads to the loss of their attention. First it is needed to persuade the general philosophy then they are willing to receive any particular advice and follow them. The most commonly occurring errors are emotional endowment, regret aversion, loss aversion or status quo and cognitive mental accounting or anchoring and adjustment.

Friendly follower predominately suffers from cognitive errors, especially availability, hindsight and framing. However emotional biases cannot be excluded, friendly followerd typically suffer from regret aversion bias.

Independent individualist are from the nature complicated investors, which is caused by their independency, extremely difficult task for an advisor. These people suffer from self-confidence and overconfidence. From cognitive errors independent individualist are characterized by conservatism, availability, confirmation and representativeness.

The last investor type defined by Pompian is active accumulator, extremely aggressive investor, suffering most from overconfidence, self-confidence and illusion of controll.

2. Discussion of presented models

Based on the conducted analyzes the study developed some weak points in the presented models. For Barnewall two ways model there is missing any mixture of actively and passively reached wealth or property; however in the reality it is often that case. Further

The Bailard, Biehl and Kaiser model is strongly concentrated on the personality of the investor which makes the model to be more detailed, however consisting only of two characteristics of the personality seems not to be sufficient.

The Pompian Behavioral Alpha process: A Top- Down model implements parts of both of above mentioned models.

Based on general type aned risk tolerance there are defined four kinds of investors. However following table shows that the the Pompian model does not discuss all possible combinations.

Figure 5: Analyses of Behavioral Alpha process: A Top- Down model

Confidence (confident/anxious)	Risk attitude (tolerance/aversion)	Handling (impetuous/careful)	
confident	tolerance	impetuous	Adventurer
confident	aversion	impetuous	?
confident	tolerance	careful	Individualist
confident	aversion	careful	?
anxious	tolerance	impetuous	Celebrity
anxious	aversion	impetuous	?
anxious	aversion	careful	Guardian
anxious	tolerance	careful	?

Pompian model closely connects the confidence level with risk acceptance. This model completely miss four of above stated combinations that are based of the theoretical research possible.

In the previous parts there were discussed the psycho-graphical models that can be considered as theoretical background for the analyzes of decision making process. The study shows that all of the studied models are simplified and areas for improvement can be found.

3. Conclusion

The goal of this paper is to analyze existing models of investment decision making and based on conducted analyzes recommend suggest improvements for their use. The study discusses the psycho-graphic models, their benefits and weaknesses as well as analyzes from behavioral point of view – discussion of biases.

The conducted analyzes shows that each of studied models suffers from incompleteness. For the Barnewall two-way model completely miss the possibility to get the wealth by combination of passive and active way. As well as forget the possibility that an active individual can gain the wealth by passive way and vice versa. For the Bailard, Biehl and Kaiser model there are used only two axes – characteristics to describe the personality, which sets the limits of this model; more dimensional interpretation of the personality will bring more reliable outcome. The Pompian model brings additional dimension to the model however completely miss four possible outcomes.

The construction of more complicated and detailed model will definitely make the analyzes of the decision making simply.

References

[1] Baker, D., Bridges, D., Hunter, R., Johnson, G., Krupa, J., Murphy, J. and Sorenson, K. (2002) *Guidebook to Decision-Making Methods*, WSRC-IM-2002-00002, Department of Energy, USA.

[2] Barnewall, M.M. (1987): Psychological characteristics of the individual investor. *ICFA Continuing Education Series* 1987.2 62-71.

[3] Bailard, T., Biehl D., and Kaiser R.W. (1986) Personal MoneyManagement, Chicago: *Science Research Associates*.

[4] Pompian, M.M. (2006) "Behavioral finance and wealth management." *How to Build Optimal Portfolios That Account for Investor Biases, New Jersey*.

[5] Thomas, T. C., G. Rajendran. (2012) BB&K Five-way Model and Investment Behavior of Individual Investors: Evidence From India. *Journal of Economics and Management* 6.1: 115-127

[6] BARBERIS, N., SHLEIFER, A., VISHNY, R. (1998). *A Model of Investor Sentiment*. In: Journal of Financial Economics, 49(3), 307–343.

[7] Baker, H. K., Nofsinger, J.R. (2010) eds. *Behavioral finance: investors, corporations, and markets*. Vol. 6. John Wiley & Sons, S. 358.

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Evaluation of the Impact of Selected Variables on the Investment Behavior[1]

Marek Jochec[2], Hana Dvořáčková[3]

Abstract

The paper Evaluation of the Impact of Selected Variables on the Investment Behavior is focused on the behavioral finance and financial markets. The aim of the paper is to evaluate the impact of selected variables on the results of high liquid financial market investments, using a model of logistic regression. The set of variables includes for example gender of investor, length of trades, currency pair used for investment and its price, the volume of trade or an attitude to risk taking of investor. Data set was completed during lectures of Ing. Marek Jochec, PhD during years 2009 and 2014. There were involved 295 students in the research, who made more than 12 000 trades on the OANDA FX Trade Practice platform. Nowadays the data collection is proceeding at the VŠBTUO, Faculty of Economic.

Key words

behavioral finance, investment behavior, logistic regression model

JEL Classification: C12, C13, G02

1. Úvod

Článek je zaměřen na behaviorální finance. Cílem je posouzení vlivu zvolených proměnných na zisk nebo ztrátu při investování na vysoce likvidních trzích, zahrnujících, kromě měnových párů, také CFD kontrakty a komodity, pomocí modelu vytvořeného metodou logistické regrese. Posuzovanými proměnnými jsou například pohlaví investora, délka obchodu, měnový pár, se kterým investor obchodoval, pořizovací cena měnového páru, počet jednotek měnového páru, do nichž je investováno, zda má investor vzdělání zaměřené na finance, atd. Vstupní data byla získána v rámci výuky panem Ing. Markem Jochcem, PhD v letech 2009 až 2014. Téměř tři sta studentů z různých zemí světa uskutečnilo přes dvanáct tisíc obchodních transakcí na platformě OANDA FX Trade Practice, umožňující obchodování s měnovými páry a také některými komoditami (například zlato nebo stříbro) nebo CFD kontrakty. Tyto obchody byly v rámci výzkumu vyhodnoceny a vytvořily základ pro stanovení modelu, určujícího pravděpodobnost dosažení zisku nebo ztráty investora působícího na FX trhu.

V rámci článku je vysvětlen proces získání vstupních dat a zmíněna použitá metodologie. Dále jsou vymezeny a popsány hypotézy vlivu proměnných na dosažení zisku/ztráty obchodujícím studentem a následně je stanoven model logistické regrese na základě výstupu z SPSS, který je dále hodnocen z hlediska statistické významnosti, predikčních schopností a verifikován. V neposlední řadě je zde popsán vliv jednotlivých proměnných na pravděpodobnost dosažení zisku investora dle vytvořeného modelu.

V závěru jsou shrnuty informace o datech a jejich původu a také výsledky výzkumu.

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2. Vstupní data a metodologie

Data zahrnuta v projektu jsou výsledkem výuky v průběhu let 2009 až 2015 Ing. Marka Jochce, Ph.D., v rámci které studenti obchodovali s měnovými páry a páry některých komodit s měnami na platformě OANDA FX Trade Practice. Na začátku investičního období disponovali částkou sto tisíc amerických dolarů, se kterou dle svého uvážení obchodovali po období výuky, tedy tří měsíců. Pro navození atmosféry co nejbližší skutečnému investování byli pro dosažení co nejlepších výsledků motivováni jak možností bodového zisku ke zkoušce, tak finanční odměnou. Celkem bylo zapojeno do výzkumu 292 studentů, kteří ve stanovené době uskutečnili celkem 12 416 obchodů o celkovém objemu více než tři miliardy jednotek. Z celkového počtu studentů jich lze 43% (125 studentů), označit jako Profitmaker, tedy na konci sledovaného období byl stav jejich účtu vyšší než počátečních sto tisíc amerických dolarů. Z celkového datového souboru byly vybrány proměnné, jejichž vliv na pravděpodobnost dosažení zisku bude zkoumán pomocí logistické regrese. Těmito proměnnými jsou:

- počet obchodů, které student za sledovaný časový úsek uskutečnil,
- zda daný obchod student ukončil aktivně nebo pasivně (0 = pasivně; 1 = aktivně),
- zda student obchodoval s měnovým párem EUR/USD nebo jiným (0 = ostatní;1 = EUR/USD),
- cena, kterou student za měnový pár zaplatil,
- množství jednotek, které student v rámci daného obchodu nakoupil,
- délka daného obchodu (ve dnech),
- pohlaví obchodujícího studenta (0 = muž; 1 = žena).

Dále jsou do datového souboru zahrnuty odpovědi na některé otázky z dotazníku, které studenti museli v rámci absolvování předmětu vyplnit. Původní dotazník obsahoval 16 otázek, z čehož bylo vybráno šest otázek pro účely tohoto výzkumu. Pokud student odpověděl na otázku „NE“ v datovém souboru je otázka označena jako 0, pokud odpověděl „ANO“ otázka je označena 1, pokud byla odpověď nejednoznačná, odpověď byla označena 0,5. Zvoleny byly tyto otázky:

Otázka č. 3: Měl jste někdy pocit, že byste se cítil na obchodování závislý?
Otázka č. 5: Považujete se za osobu, která si užívá riziko?
Otázka č. 6: Uskutečnil jste nějakou finanční investici, kromě otevření spořicího účtu (např. nákup akcií, dluhopisů atd.)?
Otázka č. 9: Provozoval jste někdy nějaký adrenalinový sport?

3. Aplikační část výzkumu

Na základě analýzy dat a literatury byly stanoveny následující hypotézy o vlivu proměnných na závislou proměnnou, dosažení zisku investorem:

1. ženy mají větší pravděpodobnost dosažení zisku než muži;
2. čím delší je obchod, tím je větší pravděpodobnost dosažení zisku;
3. při investování do EUR/USD je větší pravděpodobnost dosažení zisku, než při investování do jiných měn;
4. při investování do většího počtu jednotek je vyšší pravděpodobnost dosažení zisku;
5. investor se vzděláním zaměřeným na finance má vyšší pravděpodobnost dosažení zisku;
6. investor, který rád riskuje má větší pravděpodobnost dosažení zisku;
7. investor, který má předchozí zkušenosti s investováním má vyšší pravděpodobnost na dosažení zisku;
8. investor, který se věnuje adrenalinovým sportům má větší pravděpodobnost na dosažení zisku;
9. investor, který uvažuje o investování na profesionální úrovni má vyšší pravděpodobnost dosažení zisku;
10. investor, který zvažuje budoucí investování s vlastními prostředky má vyšší pravděpodobnost dosažení zisku.

První hypotéza byla stanovena na základě grafické analýzy dat, ze které vyplývá, že ženy dosáhly zisku ve více případech, než muži. Hypotéza číslo tři je založena na faktu, že EUR/USD byl studenty nejčastěji obchodovaným párem, přičemž lze předpokládat, že jej studenti volili na základě racionálního rozhodnutí a očekávali, že zisku dosáhnou s větší pravděpodobností, než kdyby volili jiný pár.

U hypotéz číslo devět a deset se předpokládá, že pokud má student k investování proaktivní přístup a zvažuje do budoucna investovat i vlastní prostředky, bude již v současnosti věnovat více pozornosti aktivitám spojeným s analýzou a vývojem trhu. Stejný předpoklad je uplatněn také u hypotézy číslo pět, kde dosažení zisku podporuje vzdělání ve financích a zkušenost s investováním, tedy teoretické a praktické znalosti v oblasti investování na finančních trzích.

3.1 Stanovení modelu logistické regrese metodou ENTER

Pro zjištění pravděpodobnosti, zda člověk obchodující na FX trhu dosáhne zisku nebo ztráty byla použila metoda logistické regrese. Tato metoda patří mezi nejčastěji používané modely pro odhad binárního výstupu dichotomických proměnných. Vysvětlovaná proměnná může nabýt pouze dvou hodnot, nula nebo jedna. V tomto případě nabývá hodnoty nula v případě, že je investor Lossmaker (dosáhl ztráty) a jedna pokud je Profitmaker (dosáhl zisku). Vysvětlující proměnné jsou tvořeny jedenácti ukazateli. Těmi jsou proměnné uvedené v následující Tab. 3.1:

Tab. 3.1 Přehled vysvětlujících proměnných

označení	název	interpretace
x_1	**nr. of deals**	Počet realizovaných obchodů
x_2	**active_passive**	informace zda byl obchod zakončen aktivně nebo pasivně
x_3	**eur_usd_1_others_0**	zda bylo investováno do měnového páru eur/usd, nebo do jiného
x_4	**price**	cena obchodovaného měnového páru
x_5	**units**	počet jednotek měnového páru nakoupených v rámci jednoho obchodu
x_6	**origin_to_link_time_in_days**	doba trvání obchodu
x_7	**gender**	pohlaví obchodujícího studenta
x_8	**@3**	student pociťoval závislost na obchodování
x_9	**@5**	student si o sobě myslí, že má sklon k riziku
x_{10}	**@6**	student prováděl investice na finančním trhu
x_{11}	**@9**	student provozoval/provozuje adrenalinové sporty

3.2 Odhad koeficientů modelu

Odhad modelu logistické regrese byl proveden pomocí programu SPSS. V programu byla zvolena nabídka Analyze - Regression a dále Binary Regression z důvodu binární proměnné. V rámci programu SPSS lze volit ze dvou základních metod logistické regrese. Jedná se o metodu ENTER, kdy do modelu vstupují všechny proměnné a metodu STEPWISE, která do modelu vkládá proměnné postupně, cílem je nalezení nejlepšího modelu.

Pomocí metody ENTER je zjišťováno, jak velký podíl variance závisle proměnné je popsán nezávisle proměnnými a také jaký vliv má každá z nezávisle proměnných na závisle proměnnou. V neposlední řadě je také určena relativní důležitost každé z nezávisle proměnných pomocí standardizovaných koeficientů Beta.

Následující Tab. 3.2 obsahuje informace o statistické významnosti proměnných a jejich vlivu na závisle proměnnou. Ve sloupci Sig. je určena statistická významnost daných proměnných, ty jsou významné pokud je hodnota menší než 0,05. Z tabulky je evidentní, že proměnné x_2 a $x_{9(b)}$ statisticky významné nejsou, většina proměnných však statisticky významná je.

V případě proměnných x_8 a x_9 byly na základě odpovědí vytvořeny dummy proměnné, nabývající pouze hodnot nula nebo jedna. Sloupec B obsahuje koeficienty Beta, které v případě, že nabývají kladných hodnot, zvyšují pravděpodobnost toho, že investor bude Profitmaker. Pokud jsou záporné, pravděpodobnost dosažení zisku snižují. Koeficienty *Exp(B)* působí na závislou proměnnou pozitivně v případě, že je jejich hodnota větší než jedna, pokud je hodnota menší, koeficient působí negativně. Jako příklad pozitivního působení koeficientu *Exp(B)* lze uvést proměnnou x_3 (Eur_USD_Others), u které tento koeficient nabývá hodnoty 1,417, což znamená, že pokud bude investor investovat do měnového páru EUR/USD, šance na to, že se zařadí mezi Profitmakery vzroste o 41,7%. Koeficient 0,989 u proměnné x_2 (Active_Passive) značí, že pokud je obchod uzavřen aktivně, pravděpodobnost dosažení zisku se snižuje o 1,1%.

U proměnných x_8 a x_9 nebyly odpovědi studentů vždy jednoznačné. Kromě odpovědí „ano“ (1) a „ ne“ (0) jsou v databázi zahrnuty také odpovědi „spíše ano“ (0,75), „jak kdy“ (0,5) a „spíše ne“ (0,25). Z tohoto důvodu byly vygenerovány proměnné $x_{8(a\ -\ d)}$ a $x_{9(a\ -\ c)}$ a k nim zvláště jednotlivé koeficienty.

Tab. 3.2 Statistická významnost jednotlivých proměnných

	B	Wald	Sig.	Exp(B)
x_1	-0,004	285,649	0,000	0,996
x_2	-0,011	0,014	0,906	0,989
x_3	0,348	45,616	0,000	1,417
x_4	0,000	177,033	0,000	1,000
x_5	0,000	91,056	0,000	1,000
x_6	0,007	9,914	0,002	1,007
x_7	-0,092	3,896	0,048	0,913
x_8		166,400	0,000	
$x_{8(a)}$	0,097	4,840	0,028	1,102
$x_{8(b)}$	-0,934	25,936	0,000	0,393
$x_{8(c)}$	-0,596	65,730	0,000	0,551
$x_{8(d)}$	-1,050	52,562	0,000	0,350
x_9		210,860	0,000	
$x_{9(a)}$	0,147	8,337	0,004	1,158
$x_{9(b)}$	21,986	0,000	0,996	3,534E9
$x_{9(c)}$	0,851	207,768	0,000	2,341
x_{10}	0,489	130,066	0,000	1,630
x_{11}	0,172	15,276	0,000	1,188
Constant	-0, 903	135,228	0,000	0,405

Následující vztah vyjadřuje vliv odhadnutých koeficientů na proměnné ovlivňující závislou proměnnou

$$ln\left(\frac{P(Profitmaker)}{P(Lossmaker)}\right) = -0{,}903 - 0{,}004x_1 - 0{,}11x_2 + 0{,}348x_3 + 0{,}007x_6 - 0{,}0092x_7 + \ldots$$
$$\ldots + 0{,}97x_{8(a)} - 0{,}934x_{8(b)} - 0{,}596x_{8(c)} - 1{,}05x_{8(d)} + 0{,}147x_{9(a)} + 21{,}986x_{9(b)} + 0{,}489x_{9(c)} + \ldots.$$
$$+ 0{,}489x_{10} + 0{,}172x_{11} \qquad (3.1)$$

3.3 Hodnocení multiokolinearity

V Tab. 3.3 jsou uvedeny Pearsonovy korelační koeficienty pro numerické proměnné. Tento koeficient může nabývat hodnot od -1 do 1, přičemž -1 značí zcela nepřímou závislost a +1 zcela přímou závislost. Pokud je korelační koeficient roven 0, není možné mezi proměnnými najít statisticky zjistitelnou lineární závislost, což však zcela nevylučuje závislosti, které není možné vyjádřit lineární funkcí.

Tab. 3.3 Pearsonův korelační koeficient

Pearson Correlation	x_1	x_4	x_5	x_6
x_1		0,79	-0,13	-0,171
x_4	0,079		-0,66	-0,042
x_5	-0,013	-0,066		-0,094
x_6	-0,171	-0,42	-0,094	

Následující *Tab. 3.4* vyjadřuje korelaci mezi kvantitativními proměnnými. Pokud je mezi proměnnými korelační koeficient větší než 0,8, jedná se o velmi silnou lineární závislost. Takové proměnné nejsou pro modelování vhodné. V případě tohoto souboru dat tento problém nevzniknul, maximální korelace je mezi x_1 a x_5, přičemž dosahuje hodnoty 0,15.

Tab. 3.4 Sigma oboustranný test

Signifikace	x_1	x_4	x_5	x_6
x_1		0,000	0,150	0,000
x_4	0,000		0,000	0,000
x_5	0,150	0,000		0,000
x_6	0,000	0,000	0,000	

Funkční závislosti proměnných lze obecně zapsat následovně.

$$Profitmaker = f(x_1; x_2; x_3; x_4; x_5; x_6; x_7; x_8; x_9; x_{10}; x_{11}). \quad (3.2)$$

3.4 Vyjádření ekonomického modelu pomocí lineární matematické funkce

Chceme li zjistit pravděpodobnost, s jakou bude investor Profitmaker, je potřeba dosadit vypočtené koeficienty beta do následující rovnice

P(Profitmaker) =

$$\frac{1}{1+e^{-\beta_0-\beta_1x_1-\beta_2x_2-\beta_3x_3-\beta_4x_4-\beta_5x_5-\beta_6x_6-\beta_7x_7-\beta_8x_{8(a)}-\beta_8x_{8(b)}-\beta_8x_{8(c)}-\beta_8x_{8(d)}-\beta_9x_{9(a)}-\beta_9x_{9(b)}-\beta_9x_{9(c)}-\beta_{10}x_{10}-\beta_{11}x_{11}}} \quad (3.3)$$

Pro zjištění šance, která je vyjádřena jako podíl pravděpodobnosti toho, že bude investor Profitmaker a pravděpodobnosti toho, že bude Lossmaker je potřeba dosadit do této rovnice

$$\frac{P(Profitmaker)}{P(Lossmaker)} = e^{\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4+\beta_5x_5+\beta_6x_6+\beta_7x_7+\beta_8x_{8(a)}+\beta_8x_{8(b)}+\beta_8x_{8(c)}+\beta_8x_{8(d)}+\beta_9x_{9(a)}+\beta_9x_{9(b)}+\beta_9x_{9(c)}+\beta_{10}x_{10}+\beta_{11}x_{11}} \quad (3.4)$$

Šance nemá žádnou stanovenou maximální hodnotu, avšak její minimální hodnota je nula, při interpretaci je důležité nezaměňovat hodnoty pravděpodobnosti a šance.

Zlogaritmováním předchozí rovnice dostaneme logit $ln\left(\frac{P(Profitmaker)}{P(Lossmaker)}\right)$, neboli přirozený logaritmus šance, jehož hodnoty se pohybují od mínus nekonečna do plus nekonečna, jak již bylo zmíněno v kapitole 2

$$ln\left(\frac{P(Profitmaker)}{P(Lossmaker)}\right) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_5x_5 + \beta_6x_6 + \beta_7x_7 + \ldots$$
$$\ldots + \beta_8x_{8_{(a)}} + \beta_8x_{8_{(b)}} + \beta_8x_{8_{(c)}} + \beta_8x_{8_{(d)}} + \beta_9x_{9_{(a)}} + \beta_9x_{9_{(b)}} + \beta_9x_{9_{(c)}} + \beta_{10}x_{10} + \beta_{11}x_{11}. \quad (3.5)$$

Tento deterministický model popisuje ve funkčním vztahu vysvětlovanou proměnnou *Profitmaker* v závislosti na hodnotách vysvětlujících proměnných x_1 - x_{11}. V tomto modelu není zahrnuta náhodná složka.

Popis regresních koeficientů:

β_o Odhadnutá úrovňová konstanta;

β_1 odhadnutý regresní koeficient, popisující vliv počtu obchodů na to, zda je investor Profitmaker;

β_2 odhadnutý regresní koeficient, vyjadřující vliv aktivního/pasivního ukončení obchodu na to, zda je investor Profitmaker;

β_3 odhadnutý regresní koeficient, který určuje, jaký má vliv na dosažení zisku investice do měnového páru EUR/USD;

β_4 odhadnutý regresní koeficient, vyjadřující vliv nákupní ceny obchodovaného měnového páru na dosažení zisku;

β_5 odhadnutý regresní koeficient, který vyjadřuje, jaký má vliv na vysvětlovanou proměnnou počet nakoupených jednotek měnového páru;

β_6 odhadnutý regresní koeficient popisující vliv délky obchodu na zařazení mezi Profitmakers;

β_7 odhadnutý regresní koeficient vyjadřující vliv pohlaví investora na dosažení zisku;

β_8 odhadnutý regresní koeficient, vysvětlující vliv pociťované závislosti na obchodování na případné dosažení zisku;

β_9 odhadnutý regresní koeficient, který popisuje vztah pociťovaného sklonu k riziku a dosažení zisku;

β_{10} odhadnutý regresní koeficient, který vyjadřuje souvislost mezi zařazením investora mezi Profitmakers a jeho předchozími zkušenostmi s obchodováním na finančních trzích;

β_{11} odhadnutý regresní koeficient, určující vliv mezi provozováním adrenalinových sportů a dosažením zisku při obchodování na FX trzích.

3.5 Statistická verifikace modelu

V rámci statistické verifikace modelu je posuzována statistická reálnost parametrů i celého modelu. První z charakteristik pro posouzení správnosti modelu je *míra těsnosti proložení logistickým modelem (D)*, která odpovídá sumě čtverců v regresní analýze a má asymptoticky rozložení χ^2.

Minimální hodnotou, kterou může tato charakteristika nabýt je 0. V takovémto případě se jedná o nejlepší možnou těsnost proložení. Obesně řečeno tedy čím je hodnota ukazatele nižší, tím je těsnost proložení modelu lepší naopak vyšší hodnoty indikují horší predikční vlastnosti modelu. Ukazatel je vhodný pro porovnání modelů odhadnutými pomocí různých metod. V případě tohoto modelu je hodnota *-2LL* 1535,2, jak je uvedeno v Tab. 3.5.

Dalším možným způsobem verifikace modelu je koeficient R^2, vysvětlující predikční schopnosti modelu. V případě tohoto modelu je hodnota 0,101, model tedy vysvětluje 10,1% změn závisle proměnné. Tato hodnota je poměrně nízká, u predikce je však vidět, že model je schopen dobrých odhadů, zda jde o Profitmakera/Lossmakera na základě dat.

Tento R^2_{NA} je svým chováním blízký koeficientu lineárního modelu.

Poslední vypočtenou hodnotou je R^2 dle Nagelkerke. Tento R^2_{NA} je svým chováním blízký koeficientu lineárního modelu. Podle něj tento model vysvětluje 13,7% závisle proměnných.

V následující Tab. 3.5 jsou uvedeny koeficienty -2LL, $R^2_{C\&S}$ a R^2_{NA} vypočteny programem SPSS.

Tab. 3.5 Model Summary

Step	-2 Log Likelihood	Cox&Snell R^2	Nagelkerke R^2
1	15353,200	0,101	0,137

3.6 Predikce

Tab. 3.6 obsahuje informace o pozorovaných proměnných a predikovaných proměnných daným modelem logistické regrese. Rozlišuje, zda byly predikce True positive (*TP*), False Positive (*FP*), True Negative (*TN*) a False Negative (*FN*). Z následující Tab. 2.7 vyplývá, že v 6360 případech byl Lossmaker vyhodnocen jako Lossmaker a v 1135 případech byl chybně vyhodnocen jako Profitmaker, úspěšnost predikce tedy byla 84,9%. Z celkového počtu 4921 Profitmakerů bylo 3313 chybně vyhodnoceno jako Lossmaker a v 1608 byli vyhodnoceni správně jako Profitmaker, úspěšnost v tomto případě je 32,7%. Celková úspěšnost predikcí modelu je 64,2%.

The cut value 0,500 znamená, že když je predikovaná pravděpodobnost zařazení jako Profitmaker větší rovno 0.5, investor se klasifikuje jako Profitmaker. Pokud je menší než 0.5, je klasifikován jako Lossmaker.

Tab. 3.6 Klasifikační tabulka

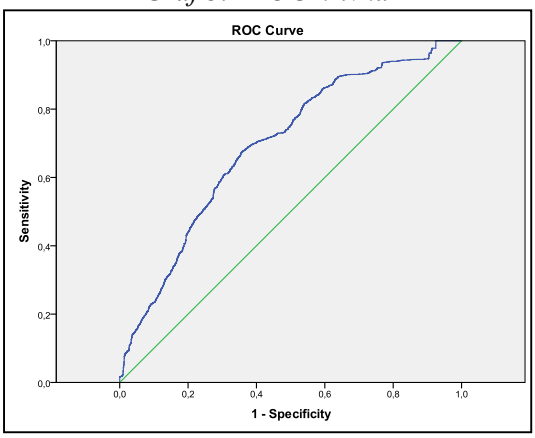
Pozorování	Predikce		
	Lossmaker	Profitmaker	Správnost odhadu (%)
Lossmaker	6360	1135	84,9
Profitmaker	3313	1608	32,7
Celkové procento správnosti odhadu			64,2
Cut Value: 0,500			

Lze říct, že model má tím lepší vypovídající schopnost, čím jsou specificita a senzitivita vyšší.

Receiving Operating Characteristic, neboli ROC křivka je nástrojem hodnocení klasifikačního systému, hodnotícím vztah mezi specificitou a senzitivitou daného testu pro všechno přípustné hodnoty prahu.

Na svislé ose grafu se nachází relativní četnost skutečně pozitivních případů, True Positive, neboli pravděpodobnost, že Profitmaker bude vyhodnocen jako Profitmaker. Na vodorovné ose se pak nachází relativní četnost falešně pozitivních případů, False Positive, vyjadřující pravděpodobnost, že Lossmaker bude vyhodnocen jako Profitmaker. Následující graf 3.1 vyjadřuje ROC křivku spjatou s řešeným modelem.

Graf 3.1 ROC křivka



Z následující Tab. 3.7 vyplývá, že oblast pod křivkou odpovídá hodnotě 0,693, na hladině významnosti 95% pak intervalu (0,693;0,702) Asymptotic Sigma, mající hodnotu 0,000 vyjadřuje, že daný model logistické regrese klasifikuje investory jako Profitmaker/Lossmaker statisticky významně lépe, než náhodná klasifikace.

Tab. 3.7 Area Under the Curve

Výsledky testu pro predikovanou pravděpodobnost				
Oblast	Standardní chyba	Asymptotic. Sigma*	Asymptotický 95% interval spolehlivosti	
			Spodní hranice	Horní hranice
0,693	0,005	0,000	0,683	0,702
*nulová hypotéza: true area=0,5				

4. Závěr

Cílem výzkumu bylo stanovení vlivu zvolených proměnných na zisk nebo ztrátu při investování na vysoce likvidních trzích pomocí modelu vytvořeného metodou Enter logistické regrese. Závisle proměnnou byla schopnost investora, v tomto případě studenta účastnícího se projektu, dosáhnout zisku po určité době investování a tímto se zařadit mezi Profitmakers. V opačném případě je student označen jako Lossmaker. Studenti pro uskutečnění investic využívali internetovou platformu OANDA FX Trade Practice a na začátku investičního období disponovali částkou sto tisíc amerických dolarů, se kterou dle svého uvážení obchodovali po období tří měsíců. Pro navození atmosféry co nejbližší skutečnému investování byli pro dosažení co nejlepších výsledků motivováni jak možností bodového zisku ke zkoušce, tak finanční odměnou. Celkem bylo zapojeno do výzkumu 292 studentů, kteří ve stanovené době uskutečnili celkem 12 416 obchodů. Z celkového počtu studentů jich lze 43% (125 studentů), označit jako Profitmaker, tedy na konci sledovaného období byl stav jejich účtu vyšší než sto tisíc amerických dolarů. V případě že ke konci období byli ve ztrátě oproti počátečnímu stavu konta, zařadili se mezi Lossmakers.

Z výsledků logistické regrese vyplývá, že proměnné x_1 *(počet obchodů),* x_2 *(aktivní ukončení obchodu),* x_7 *(žena)* a x_8 *(pociťovaná závislost)* snižují pravděpodobnost, že investor dosáhne zisku. Ostatní proměnné x_3 *(obchodování s EUR/USD),* x_4 *(vyšší cena obchodovaného páru),* x_5 *(vyšší počet jednotek v rámci),* x_6 *(délka obchodu),* x_9 *(pozitivní vztah k riziku),* x_{10} *(předchozí zkušenost s investováním)* a x_{11} *(záliba v adrenalinových sportech)* tuto pravděpodobnost zvyšují.

Na základě rozboru lze říci, že výsledný model dokáže klasifikovat investora jako Profitmakera statisticky významně lépe, než náhodný výběr. Lépe než to, zda investor zisku dosáhne, však dokáže určit, zda jej nedosáhne. Pravděpodobnost, že model správně odhadne budoucího Lossmakera je 84,9%, u Profitmakera se jedná o 32,7%, celková úspěšnost předpovědí modelu pak je 64,2%. Predikční schopnosti modelu se zvyšují spolu s rostoucím objemem uskutečněných obchodů.

Za přínos tohoto modelu lze považovat fakt, že pomohl určit, které proměnné jsou podstatné pro dosažení zisku a které šanci na zisk při investování snižují. Dle vytvořeného modelu je úspěšným investorem muž, který rád riskuje, vyžívá se v adrenalinových sportech, má finanční vzdělání a zkušenosti s obchodováním, nepociťuje na investování závislost, uzavře méně obchodů, ale s vysokým počtem jednotek na měnový pár EUR/USD, s delší dobou vypořádání a ukončí je pasivně. Tato znalost může pomoci investorovi při investičním rozhodování spolu s klasickými metodami analýzy trhu. Pro rozšíření datového souboru byli zapojeni do projektu také studenti ekonomické fakulty VŠ-TUO v rámci projektu modelování behaviorálních faktorů na finančních trzích.

References

[1] BIGALOW, S.W. (2011). *Profitable Candlestick Trading: Pinpointing Market Opportunities to Maximize Profits*. New Jersey: John Wiley & Sons.

[2] BURTON, E., SHAH S. (2013). *Behavioural Finance: Understanding the Social, Cognitive, and Economic Debates.* New Jersey: John Wiley & Sons.

[3] CIPRA, T. (2008). *Finanční ekonometrie.* 1st ed. Praha: Ekopress.

[4] DIAMOND,P., VARTIAINEN H. (2007). *Behavioural Economics and it´s Applications*. New Jersey: Princeton University Press.

[5] FIELD, A. (2013). *Discovering Statistics Using SPSS*. 4th edition. London: Sage.

[6] FREES, E. W. (2010). *Regression Modeling with Actuarial and Financial Applications.* New York: Cambridge University Press.

[7] HAIR, J.F. (2009). *Multivariate Data Analysis*. 7th ed. New Jersey: Prentice Hall.

[8] HOSMER, D.W., LEMESHOW S., STURDIVANT R.X. (2013). *Applied Logistic Regression.* 3rd ed. New Jersey: John Wiley & Sons.

[9] KEVIN.S. (2015). *Security Analysis and Portfolio Management*. 2nd Ed. New Delhi: PHI Learning.

[10] ŘEHÁKOVÁ, B. (2000). Nebojte se logistické regrese. *Sociologický časopis* [online], 36(4), p. 475.

[11] Statisticalhorizons.com. (2016). *What's the Best R-Squared for Logistic Regression | Statistical Horizons*. [online] Available at: http://statisticalhorizons.com/r2logistic [Accessed 10.4. 2016].