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# ECONOMETRIC ANALYSIS OF MONETARY AND FISCAL POLICY IMPACTS USING DSGE MODEL

**Master's thesis**

**2021** **Bc. Ádám Csápai**

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**Master’s thesis**

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# ZADANIE ZÁVEREČNEJ PRÁCE

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**Anotácia:** Ekonómia prináša široké spektrum problémov, ktorými sa zaoberajú mnohí vedci či ekonomickí analytici v rôznych inštitúciách. Cieľom tejto práce je za pomoci zvoleného DSGE modelu analyzovať vplyv rôznych šokov reprezentujúcich monetárnu a fiškálnu politiku na ekonomiky.

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**Declaration**

**I hereby declare that I prepared this thesis independently and I listed all the literaty sources I used.**

**Date: 11.5.2021** ........................................

(signature)

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**ABSTRAKT**

CSÁPAI, Ádám: *Ekonmetrická analýza vplyvu monetárnej a fiškálnej politiky pomocou DSGE modelu*. – Ekonomická univerzita v Bratislave. Fakulta hospodárskej informatiky; Katedra operačného výskumu a ekonometrie. – Vedúci záverečnej práce: doc. Ing. Martin Lukáčik, PhD. – Bratislava: FHI EU, 2021, 52s.

Cieľom záverečnej práce bol zostaviť a odhadnúť malý otvorený DSGE model s monetárnou a fiškálnou politikou a analyzovať interakciu týchto politík v podmienkach Maďarska. Práca pozostáva zo 6 kapitol. Obsahuje 16 obrázok, 4 tabuľky a 1 prílohu. V prvej kapitole sme characterizovali historický vývoj DSGE modelovania a popísali sme základné vlastnosti DSGE modelu. V druhej kapitole sme sformulovali náš hlavný cieľ a stanovili sme aj parciálne ciele, ktoré sme mali dosiahnuť, aby sme dosiahli aj hlavný cieľ práce. V tretej kapitole sme charakterizovali metodiku práce a metódy skúmania. V tejto kapitole sme popísali postup prípravu dát, postup odhadu modelu a characterizovali sme nástroje potrebné k interpretácii výsledkov a porovnaniu rôznych modelov. V štvrtej kapitole sme odvodili model, pomocou ktorého sme skúmali interakcie monetárnej a fiškálnej politiky v podmienkach Maďarska. V piatej kapitole sme prezentovali a interpretovali odhadnutých výsledkov. V diskusii sme porovnali štyri varianty modelu a pomocou Bayesianskeho pomeru sme si vybrali najlepší model. Hodnoty parametrov najlepšieho modelu sme následne porovnali s hodnotami parametrov publikovaného DSGE modelu. Náš hlavný cieľ sme dosiahli, a v tejto diplomovej práci prezentujeme model, pomocou ktorého môžeme analyzovať interakcie monetárnej a fiškálnej politiky v podmienkach Maďarska.

**Kľúčové slová:**

DSGE model, fiškálna politika, monetárna politika, Dynamický stochastický model všeobecnej rovnováhy, model malej otvorenej ekonomiky

## ABSTRACT

CSÁPAI, Ádám: *Econometric analysis of monetary and fiscal policy impacts using DSGE model*. – University of Economics in Bratislava. Faculty of Economic Informatics; Department of operations research and econometrics. – Supervisor: doc. Ing. Martin Lukáčik, PhD. – Bratislava: FHI EU, 2021, 52 p.

The principal aim of this thesis was to estimate a small open economy DSGE model with monetary and fiscal policy and analyze the interaction of these policies in Hungary. The thesis consists of 6 chapters. It contains 16 figures, 4 tables and 1 appendix. In the first chapter we characterized the historical development of DSGE modeling and described the basic properties of the DSGE model. In the second chapter, we formulated our main goal and set the partial goals that we had to achieve in order to achieve the main goal of the work. In the third chapter we characterized the methodology and methods. In this chapter, we described the data preparation procedure, the model estimation procedure, and characterized the tools needed to interpret the results and compare different models. In the fourth chapter, we derived a model by which we examined the interactions of monetary and fiscal policy in Hungary. In the fifth chapter, we presented and interpreted the estimated results. In the discussion, we compared four variants of the model and using the Bayesian ratio we chose the best model. We then compared the parameter values of the best model with the parameter values of a published DSGE model. We have achieved our main goal, and in this thesis we present a model by which we can analyze the interactions of monetary and fiscal policy in Hungary.

**Keywords:**

DSGE model, fiscal policy, monetary policy, Dynamic stochastic general equilibrium model, small open economy model

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# Introduction

Dynamic stochastic general equilibrium models are used extensively in both central banks and academic research. With the help of these models we can analyze how selected macroeconomic variables react to unexpected shocks. These models are also useful tools for macroeconomic forecasting. In addition, they provide valuable information for policy debates. They are popular since the model relationships are founded in microeconomic theory. Because of this, the usual problems with structural models can be avoided by using DSGE models.

The first DSGE model was estimated by Kydland and Prescott in a paper published 1982. The authors later received the Nobel Prize in Economics for this paper. They calibrated the model for the U.S. economy and found that productivity was the main driver of the fluctuations in the economy. Their paper served as a foundation for later developments in DSGE modelling. Initially, these models consisted of three agents. These were the representative households, firms and the monetary authority. Later, mainly in the 1990s, small open economy DSGE models gained popularity, and the number of agents increased to four, taking the world economy into consideration. Fiscal policy, however, had a marginal role in these models up until very recently. Originally, fiscal policy was considered to be analogous to government spending, which was modelled as an exogenous first order autoregressive process. In recent years, however, fiscal authority became an active participant in a fair number of published models. Because of this newly gained popularity, we decided to include fiscal policy in our model, which leads us to the principal aim of this thesis.

The principal aim of this thesis is to estimate a small open economy DSGE model with monetary and fiscal policy and analyze the interaction of these policies in Hungary. To achieve this, we first present the theoretical background of DSGE models in the first chapter. In the second chapter we again present the principal aim of the thesis and formulate partial objectives which help us achieve our main goal. In the third chapter we present the methodology and methods needed to prepare the data, estimate the model and interpret the results. In the fourth chapter we derive the DSGE model. We then estimate this model and present the estimation results in the fifth chapter. After interpreting the results, we compare the model to alternative specifications. After selecting the best model, we compare our results to a published DSGE model. In the last section we formulate our conclusions.

# 1 The current state of the field at home and abroad

In chapter 1 we first describe the historical development of DSGE models. To begin with, we describe the criticism faced by new classical models. Then we describe the evolution of Real Business Cycle theory and New Keynesian economics. To continue with, we briefly explain what DSGE models are and how the assumptions of New Keynesian economics are included in the models. To end with, we describe monetary and fiscal policy in the context of DSGE models. To sum up, in this chapter we present a brief summary of the theory behind DSGE models. We start by describing the historical development.

The development of DSGE models started with a critique of mainstream new classical macroeconomic models. These models were criticized because they assumed that estimated model parameters were time invariant. This assumption, however, was proven to be false. The values of the model parameters usually changed as a result of an economic policy change. After this critique gained widespread acceptance, the development of a new class of models begun. These new economic models were based on microfoundations. In addition, agents in these models were expected to formulate rational expectations about the future of the economy.[1] These theoretical and methodological innovations resulted in the development of Real Business Cycle (RBC) theory, which became the precursor to New Keynesian economics.

The foundation of Real Business Cycle theory can be dated to 1982, when Kydland and Prescott first published their influential paper on this subject. In that paper, these authors combine growth theory with business cycles and conclude that long-term technological development can cause short term fluctuations. They reached this conclusion in the following way. The authors set up a macroeconomic model which they calibrated using data on the US economy. Their results indicated that during the examined period investments in the USA were volatile, but consumption was not. In addition, labor productivity was less volatile than hours worked, and both labor productivity and hours worked were correlated to output. What is more, this correlation was stronger than the correlation between output and investments. This showed that changes in labor productivity and hours worked were the driving forces behind output fluctuations. Labor

[1] LUCAS, Robert. Econometric Policy Evaluation: A Critique. In: *Carnegie-Rochester Conference Series on Public Policy* [online]. New York: Elsevier, 1976, vol. 1, no.1, pp. 19–46. [cit 6.5.2021]. ISSN 04-4411-007-0. Available at: https://web.sgh.waw.pl/~atoroj/makroekonomia_zaawansowana/lucas76.pdf

productivity and hours worked were determined by technological development. Based on these findings the authors concluded that business cycles – which can be interpreted as the trajectory of the deviation and return of the economy to its equilibrium – are the result of productivity shocks.[2]

The paper discussed in the previous paragraph is the foundation of modern macroeconomic business cycle research, both empirical and academic. Despite this the methodology had its own fair share of criticism. Firstly, the response of output and labor to technological advancement might have a different trajectory. Technological shocks – at least in the short run – can decrease output and result in an increase of unemployment levels. In addition, the effect of technological development on output can be smaller than what was shown by Kydland and Prescott. What is more, RBC theory ignores the effect monetary policy might have on the economy and its potential to cause economic fluctuation.[3] Because of this criticism RBC theory was further developed and New Keynesian (NK) economics was born.

New Keynesian economics does not completely discard RBC theory, but rather improves it. On the one hand, there are some key commonalities between RBC theory and NK economics. Firstly, households in both frameworks are infinitely lived. These households maximize their utility. This maximization problem is subject to their budget constraint. Secondly, firms have access to homogenous technology and technological development is exogenous. Thirdly, the steady state values of the parameters are the results of stochastic processes of endogenous variables. These processes characterize the optimal behavior of agents subject to their budget constraints. On the other hand, New Keynesian economics – which provides the theoretical background for the model we present in chapter 4 – has two key assumptions which makes it different from RBC theory. Firstly, it introduces the concept of nominal rigidities into economic models. At the beginning it was assumed that due to long-term employment contracts wage stickiness can cause economic fluctuations, but later more attention was paid to prices. Price stickiness means that in the short run, prices are not completely flexible, and firms can only adjust them with a delay. In addition, the existence of nominal rigidities also implies that in the short run, monetary policy is not neutral.

[2] KYDLAND, Finn E. - PRESCOTT, Edward C. Time to Build and Aggregate Fluctuations. In: *Econometrica: Journal of the Econometric Society* [online]. USA: Wiley-Blackwell, 1982, vol. 50, no.6, pp. 1-27 [cit. 20.4.2012]. ISSN 1468-0262-1. Available at: http://www.finnkydland.com/papers/Time%20to%20Build%20and%20Aggregate%20Fluctuations.pdf

[3] GALÍ, Jordi - RABANAL, Pau. Technology Shocks and Aggregate Fluctuations: How Well Does the Real Business Cycle Model Fit Postwar U.S. Data? In: *NBER Macroeconomics Annual* [online]. Massachusets: MIT Press, 2005, vol. 19, pp. 225-317. [cit. 6.5.2021]. Available at: https://www.journals.uchicago.edu/doi/pdfplus/10.1086/ma.19.3585339

An adjustment in interest rates does not result in the same magnitude of adjustment in prices. As a result of this, real interest rates change. The changes in real interest rates affect output, investment and employment. Firms adapt to changing market conditions and react to the change in demand by adjusting their production. In the long run, however, the economy returns to the steady state. Secondly, under the assumption of perfect competition individual agents cannot set prices. As discussed before in this paragraph, this is not the actual case. Because of this, New Keynesian economics assumes the existence of monopolistically competitive firms. These kinds of firms can set and adjust both their prices and the quantity of production inputs to achieve their goals.[4] Based on this paragraph we can conclude that although there are some commonalities between NK economics and RBC theory, there are some key differences as well.

Now that we formulated the theoretical foundations of New Keynesian economics, we need to discuss what DSGE models are. DSGE stands for dynamic stochastic general equilibrium models. In this paragraph we discuss what each of these expressions mean for the model in general. Firstly, since the models are dynamic, they represent a useful tool for analyzing real time dynamic decisions of agents. Secondly, the stochastic nature of these models allows us to examine the effects of macroeconomic shocks and volatility on the modelled macroeconomic variables. This way we can analyze how business cycles affect the economy. Thirdly, by maintaining the general equilibrium assumption, we can analyze how agents affect each other. Because of the microfoundations of these models and the assumption of rational expectations, the behavior of agents can be represented as an optimization problem, which is subject to certain constraints. The behavior of agents and the resulting decisions and actions are not isolated events in the model. On the contrary, they have certain implications for other agents. For example, households allocate their limited resources between consumption, investment and hours worked, in order to maximize their utility while taking into consideration their budget constraint. Firms decide on supply of goods and services and represent the demand for production factors. Their activity is also subject to certain constraints. These decisions and actions are intertwined via price, which ensures that the modelled

[4] MANKIW, Gregory N. - ROMER, David. *New Keynesian Economics, Volume 1: Imperfect Competition and Sticky Prices.* 6th Print. Massachusetts: MIT Press, 1998. p. 430. ISBN 0-262-13266-4. and GALÍ, Jordi. *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework.* Princeton: Princeton University Press, 2008. p. 203. ISBN 978-0-691-133316-4.

economic system moves towards equilibrium.[5] Now that we understand the general attributes of DSGE models, we can discuss how they are used in New Keynesian economics.

New Keynesian DSGE models, which are the main analytical tools of New Keynesian economics, represent the mathematical formulation of the assumptions discussed in the previous paragraphs. These models are built on the assumptions of monopolistically competitive firms and nominal rigidities. Price stickiness is represented in the model by firms, who adjust their prices according to the mechanism introduced by Calvo.[6] By introducing Calvo's price setting mechanism into the model and solving the optimization problems of agents we can derive the New-Keynesian Phillips curve. In addition to Calvo's forward looking price adjustment mechanism, other models also include backward looking firms. These backward looking firms do not take into consideration future inflation expectations when adjusting prices. They adjust their prices based on historical price movements instead.[7] It is clear from this paragraph, that price stickiness affects the behavior of firms in the model. In addition to this, nominal rigidities also have implications for monetary policy.

If nominal rigidities are present in the model, then the movement of nominal interest rates is not accompanied by an immediate change in prices. This results in changed real interest rates. As a result, monetary policy can have a substantial effect on the trajectory of real macroeconomic variables, such as output. For this reason, DSGE models also include the monetary authority reaction function in the form of the Taylor rule, in addition to firms and households. The Taylor rule can have multiple formulations and it can be amended to suit the researcher's needs. Generally, this rule implies that monetary policy is not neutral and so it can be used for stabilizing fluctuations in inflation and economic activity. For example, in the short term, output and inflation are positively affect each other while output and interest rates affect each other negatively. The former relationship is represented via the Phillips curve and the latter via the IS curve in the model.

[5] BILBIIE, Florin O. Dynamic Stochastic General Equilibrium and Business Cycles [online]. University of Oxford, 2005. pp. 83 [cit. 6.5.2021]. Available at: http://www.nuff.ox.ac.uk/Users/Bilbiie/teaching_files/notes_oxford_final.pdf

[6] CALVO, Guillermo A. Staggered prices in an utility-maximizing framework. In: *Journal of Monetary Economics* [online]. United States of America: Elsevier, 1983, vol. 12, no. 3, pp. 1-16 [cit. 4.15.2021]. ISSN 0304-3932. Available at: https://www.sciencedirect.com/science/article/abs/pii/0304393283900600

[7] GALÍ, Jordi - GERTLER, Mark. Inflation Dynamics: a Structural Economic Analysis. In: *NBER Working Paper 7551* [online]. Cambridge: National Bureau of Economic Research, 2000, pp. 1-31 [cit. 4.15.2021]. Available at: https://www.nber.org/system/files/working_papers/w7551/w7551.pdf

Because of this, monetary policy can have either stabilizing or destabilizing effects on the economy, depending on the parameter values of the Taylor rule.[8] Based on this paragraph we can conclude that monetary policy can affect the economy if nominal rigidities are present.

The last part of our model - presented in chapter 4 - is fiscal policy. Standard New Keynesian DSGE models lack an active fiscal sector. Usually, fiscal policy is only represented in the model in the form of government expenditures. Based on the assumption that fiscal policy only plays a passive role, government expenditure is usually an exogenous process in these models. As a result of this, the possible interactions between monetary and fiscal policy are ignored. However, a recently published model of the euro area includes a comprehensive fiscal sector.[9] Because active fiscal policy becomes increasingly popular, we decided to include it in our model. Now that we understand DSGE models, we can now formulate our aims, present the methodology, estimate the model and interpret the results.

[8] CHRISTIANO, Lawrence J. - EICHENBAUM, Martin - EVANS, Charles L. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. In: *Journal of Political Economy* [online]. Chicago: The University of Chicago, 2005, vol. 113, no. 1, pp. 1-45 [cit. 6.5.2021]. Available at: https://faculty.wcas.northwestern.edu/~yona/research/CEE2005.pdf

[9] RATTO, Marco - ROEGER, Werner - VELD, Jan in't. QUEST III: An estimated open-economy DSGE model of the euro area with fiscal and monetary policy. In: Economic Modelling [online]. Netherlands: Elsevier, 2012, vol. 26, pp. 222-233 [cit. 6.5.2021]. ISSN 0264-9993. Available at: https://www.sciencedirect.com/science/article/abs/pii/S026499930800076X

# 2 Aim of the thesis

The principal aim of this thesis is to estimate a small open economy DSGE model with monetary and fiscal policy and analyze the interaction of these policies in Hungary. We formulated the following partial objectives which help us achieve our main goal:

1. Review the existing literature on DSGE models and New Keynesian economics. We do this in chapter 1.
2. Summarize the methodology and methods which are needed for preparing the data, log-linearizing the model, estimating the parameters and interpreting the results. We present these methods in chapter 3.
3. Formulate the non-linear model and derive the log-linearized steady state equations. We present the model in chapter 4.
4. Compare different models with different fiscal policies and decide which model is the best fit for the data.
5. Compare our results to the only published DSGE model estimated for the Hungarian economy which includes fiscal policy.

Our success in achieving these goals is discussed in the conclusion.

# 3 Methodology and methods

In this chapter we present the methodology and methods we use to obtain the results. These results are discussed in chapter 5 and chapter 6. In chapter 3 we first describe how to log-linearize the model around the steady state. Then we explain how to prepare the data. The data preparation consists of two steps. The first step is seasonal adjustment and the second step is detrending. Then we explain how to obtain values for the DSGE model parameters. There are two methods, namely calibration and Bayesian estimation. These two methods can also be combined. In the following subchapters we explain how to compare multiple model estimates, how to interpret impulse response functions and in the last subchapter we characterize the software used to estimate the model and obtain the results. It is easy to see from this summary that chapter 3 provides a comprehensive overview of data preparation and model estimation.

## 3.1 Log-linearization[10]

In this subchapter we present a simplified method of log-linearization, which involves the use of the Taylor approximation. Using this method we can replace the model equations by their approximations around the steady state. This way we can transform the non-linear model and get linear functions, which are expressed as log-deviations of each variable from its respective steady state.

If $X_t$ is a vector of variables and $\bar{X}$ is the steady state then we can write

$$x_t = logX_t - log\bar{X}$$

where $x_t$ represent the vector of log-deviations. If we multiply $x_t$ by 100 we get the percentage deviations of variables from their steady states at time $t$. To continue with we can write the equilibrium equations as

[10] HARALD, Uhlig. A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily. In: *Discussion Paper 101* [online]. Minneapolis: Federal Reserve Bank of Minneapolis - Institute for Empirical Macroeconomics, 06.1995. pp. 1-50. [cit. 2.3.2021]. Available at: https://www.researchgate.net/profile/Harald-Uhlig-2/publication/4754697_A_Toolkit_for_Analyzing_Nonlinear_Dynamic_Stochastic_Models_Easily/links/0deec520d02f78c45d000000/A-Toolkit-for-Analyzing-Nonlinear-Dynamic-Stochastic-Models-Easily.pdf

$$1 = f(x_t, x_{t-1}) \tag{3.1.1}$$

$$1 = E_t[g(x_{t+1}, x_t)] \tag{3.1.2}$$

where $g(0,0) = 1$ and $f(0,0) = 1$.

The first-order approximation around $(x_t, x_{t-1}) = (0,0)$ results in

$$0 \approx f_1 * x_t + f_2 * x_{t-1}$$

$$0 \approx E_t[g_1 * x_{t+1} + g_2 * x_t]$$

from which in $x_t$ and $x_{t-1}$ we get a linear system in the deterministic equations and in $x_{t+1}$ and $x_t$ we get a linear system in the expectational equations.

We could solve the linear system using the method of undetermined coefficients, but most of the time we do not have to explicitly differentiate *f* and $g$. We can obtain the solution in the following way. Firstly, we multiply everything and let $X_t = \bar{X}\epsilon^{x_t}$. Secondly, let $x_t, y_t \approx 0$ be real numbers. Thirdly, use the following building blocks where *a* is a constant

$$\epsilon^{x_t + a y_t} \approx 1 + x_t + a y_t$$

$$x_t, y_t \approx 0$$

$$E_t[a\epsilon^{x_{t+1}}] \approx E_t[a x_{t+1}] \text{ up to a constant.}$$

As an example

$$\epsilon^{x_t} \approx 1 + x_t$$

$$a X_t \approx a\bar{X} x_t \text{ up to a constant}$$

$$(X_t + a)Y_t \approx \bar{X}\bar{Y} x_t + (\bar{X} + a)\bar{Y} y_t \text{ up to a constant}$$

Since the constant satisfies the steady state relationship, we drop it out in the end. They are, however, needed in order to complete the intermediate steps.

## 3.2 Seasonal adjustment[11]

Data obtained from databases is usually seasonally unadjusted. To adjust the series, we use the STL decomposition, which decomposes the time series with a filtering algorithm based on a LOESS regression. Firstly, there is an additive relationship between the main components of the time series, which is represented as

$$Y_t = S_t + T_t + R_t \tag{3.2.1}$$

The algorithm consists of outer and inner loops and the components are calculated using a double recursive procedure.

First, we characterize the inner loop. *k*, *k=1,2,..,N* is the iteration parameter, *np* gives us the number of periods in a cycle, *N* represents years, *N*np* is the number of observations, and *RW* is a weighting series. The inner loop is then consists of the following six steps:

1. Create $DT_t^k = Y_t^k - T_t^{k-1}$ , where $T_t^{k-1}$ is the previous trend estimate and $DT_t^k$ represents detrended values
2. Split $DT_t^k$ into *np* sub-series, whose length is *NI*. Using LOESS we get the smoothed values. LOESS weights in computation are also weighted by $RW$. As a result of the LOESS regressions we also get smoothed values for missing values and for two cycles, one prior and one after the sample. $C_t^k$ stores the results.
3. Recursively calculate three moving averages for the smoothed sub-series and smooth the new series with another LOESS. $L_t^k$ stores the results.
4. Seasonal values are estimated and calculated as $S_t^k = C_t^k - L_t^k$
5. Deseasonalise, $DS_t^k = Y_t^k - S_t^k$
6. $T_t^k$ is computed from the smoothed $DS_t^k$. LOESS weights in computation are also weighted by $RW$. $T_t^0$ equals to zero.

To continue with, we characterize the outer loop. The inner loop iterations result in the estimate of the trend $T_t^{NI}$ and seasonal component $S_t^{NI}$. The remainder component is estimated as

[11] CLEVELAND, Robert B. et al. STL: A Seasonal-Trend Decomposition Procedure Based on Loess. In: *Journal of Official Statistics* [online]. Sweden: Statistics Sweden, 1990, vol. 6, no. 1, pp. 3-73 [cit. 2.3.2021]. ISSN 0282-423X. Available at: http://www.nniiem.ru/file/news/2016/stl-statistical-model.pdf

$$R_t^{NI} = Y_t - S_t^{NI} - T_t^{NI} \tag{3.2.2}$$

The remainder component gives the basis for calculating robustness weights. With these weights we can correct the outliers. If $h = 6 * median(|\, R_t^{NI}|)$, then $RW_t = B(|\, R_t^{NI}|/h)$. B represents a bisquare function represented as

$$B(x) = (1 - x^2)^2 \; for \; 0 \leq x < 1 \tag{3.2.3}$$

$$B(x) = 0 \; for \; x > 1 \tag{3.2.3}$$

*RW* is then used for the next iteration in the inner loop. Initially *RW* are set at 1.

Finally, we characterize the LOESS estimation. LOESS in STL is used with a tri-cube function and a locally quadratic or locally linear polynomial. Let *A* be the time series which we want to smooth. Let *T* be the number of observations in *A*. Let T, T=*1,2,…,T* denote a trend variable. We smooth the values of *A* one at a time. Every value of *A*, $A_t$ has its own trend $\tau$. To smooth $A_t$ we take *k* observations of *A* and *T* around ($A_t$, $\tau$) and get a smaller series $A^k$ and $T^k$. The weights of the LOESS estimation can be calculated as

$$\omega_t = W\left(\frac{|T_t^k - \tau|}{h_\tau}\right) \tag{3.2.4}$$

where *W* is a tri-cube function represented as

$$W(x) = (1 - x^3)^3 \; for \; 0 \leq x < 1 \tag{3.2.5}$$

$$W(x) = 0 \; for \; x > 1 \tag{3.2.5}$$

and $h_\tau$ is the largest distance of $\tau$ from the $T^k$ values represented as

$$h_\tau = \max\,(\tau - min(T^k), max(T^k) - \tau) \tag{3.2.6}$$

The weights can be used for two things. On the one hand, if we use a locally linear polynomial in the LOESS estimation, we can calculate the weighted average of $A^k$. On the other hand, if we use a locally quadratic polynomial, we can estimate weighted least squares of $A^k$ against $T^k$. The resulting coefficients produce a fitted value of $A_t$.

## 3.3 Detrending the data[12]

The log-linearization procedure described in the previous subchapter results in a model solution which is expressed in terms of stationary variables. The stochastic behavior of these variables is represented as a temporary deviation from their steady state. Because of this we need to represent the data in a similar form. To get stationary variables we need to difference the logarithm values of a given variable, but sometimes differencing might not be enough, and the series can still have a unit root. In that case we need to detrend the series using the Hodrick-Prescott (H-P) filter. To begin with, we decompose $log y_t$ as

$$log y_t = g_t + c_t \tag{3.3.1}$$

where $g_t$ represents the growth component and $c_t$ is the cyclical component. We need to estimate both components so we can solve the minimization problem represented below

$$\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=3}^{T} [(1-L)^2 g_t]^2 \tag{3.3.2}$$

where $\lambda$ represents the weight given to smoothness. The value of $\lambda$ is set to 1600, which is the standard for quarterly data. After the (3.2.2) is solved we can remove the trend as

$$\widehat{y_t} = log y_t - \widehat{g_t} = \widehat{c_t} \qquad (3.3.2)$$

## 3.4 Calibration

In this thesis we want to correctly model the reaction of macroeconomic variables to unexpected shocks. To do this we need to obtain the values of the structural parameters of the DSGE model. There are two ways of doing this. We either use econometric estimation procedures, such as Bayesian estimation, or we calibrate the parameters. In addition, we can also combine these two methods. If the parameters are estimated and calibrated correctly, the reaction of macroeconomic variables to unexpected shocks will be in accordance with economic theory.

In this subchapter we characterize calibration. Calibration is an algorithm and with its help we can answer different empirical economic questions even when we lack reliable data. Calibration

[12] DEJONG, David N. – DAVE, Chetan. *Structural Macroeconometrics*. Princeton: Princeton University Press, 2007. p. 351. ISBN 0-691-12648-8.

consists of six steps, which are as follows. In the first step we formulate the economic question. In the second step we select a relevant model which characterizes the economy. In the third step we log-linearize the model around the steady state using the technique presented in subchapter 3.1. The steady state values of endogenous variables must correspond to the true long run values of these variables. In the fourth step we assign a predefined exact value with zero standard deviation to a set of pre-selected parameters. We can select the assigned values based on a priori information from the real economy and previous studies and specify the exogenous processes which are needed for the simulations. In the fifth step we compare our results to real data and economic theory. In the sixth and final step we interpret the results and answer the economic question formulated in step one.[13] This is how calibration works.

To sum up, calibration is an algorithm which allows us to provide empirical answers to certain economic questions even though we do not have the necessary data. If data is available, a simple Bayesian estimation or a combination of these two methods is preferred. We characterize Bayesian estimation in the following subchapter.

## 3.5 Bayesian estimation[14]

In the previous section we calibrated … parameters. The obtain parameter values for the remaining parameters we use Bayesian estimation techniques. When using Bayesian estimation techniques the parameters are treated as random variables with their own probability density.

Firstly, we write

$$\pi(\theta/y) = \frac{f(y/\theta)\pi(\theta)}{f(y)} \tag{3.5.1}$$

where $y$ is the vector of observable data and $\theta$ is the vector of parameters. $\pi(\theta/y)$ is the posterior probability density function, which represents the information gained from the data about the

[13] CANOVA, Fabio. *Methods for Applied Macroeconomic Research*. Princeton: Princeton University Press, 2007. p. 512. ISBN 978-1-107-01473-2.
[14] This subchapter is based on GREENBERG, Edward. *Introduction to Bayesian Econometrics*. Cambridge: Cambridge University Press, 2008. p. 219. ISBN 978-0-511-50021-3 and FEJEŠ, Martin. Monetárna politika, inflácia a hospodárske cykly v novej keynesiánskej koncepcii : doctoral thesis. Supervisor: Jaroslav Husár. Bratislava, 2016. p. 122.

values of $\theta$. $f(y/\theta)$ is the maximum likelihood function. $\pi(\theta)$ represents the a priori expectations regarding parameter values. $f(y)$ represents the marginal probability of $y$.

Secondly, we can write the posterior mean as

$$E[\theta_i/y] = \int \theta_i \pi(\theta/y) \mathrm{d}\theta \tag{3.5.2}$$

Using (3.5.2) we can estimate $\theta_i$. Since $\pi(\theta/y)$ contains all the information we need to estimate $\theta$, we can also estimate the mean of the continuous parameter likelihood function $h(\theta)$ as

$$E[h(\theta)/y] = \int h(\theta)\pi(\theta/y) \mathrm{d}\theta \tag{3.5.3}$$

To continue with, we characterize the Markov Chain Monte Carlo (MCMC) simulation algorithm. The aim of this algorithm is to construct a transition kernel *p(x,y)* with an invariant density which is the same as the target density. Then we can start the simulation at $x_0$ and from $p(x_0,x_1)$ we draw $x_1$ and from $p(x_{G-1},x_G)$ we draw $x_G$ where G represents the number of simulations. If we do not consider the transient period, we see that the target distribution is almost equal to the distribution of $x_g$. We can find *p(x,y)* using the Metropolis-Hastings (MH) algorithm, which randomly draws from the candidate values of the parameters $\theta^*$. These draws are made from the highly probable part of an unknown posterior distribution.

To generate the random variables, we need the following distribution function

$$q\left(\theta^{(x-1)}, \theta\right) \tag{3.5.4}$$

where $\theta^*$ is drawn from $\theta$ which represents a random value whose probability depends on $\theta^{(x-1)}$. Using these values and the distribution function we can characterize each step of the MH algorithm. At first an initial value $\theta^0$ is selected. Next, the candidate value $\theta^*$ is generated using the distribution function. Afterwards, $\theta^x = \theta^*$ with probability $a\left(\theta^{(x-1)}, \theta^*\right)$ and $\theta^x = \theta^{(x-1)}$ with probability $1 - a\left(\theta^{(x-1)}, \theta^*\right)$ are generated. These steps are replicated X times. Finally, the MH algorithm calculates the mean of X draws and generates $E[h(\theta)/y]$.[15]

[15] SLANICAY, Martin. Asymmetric Shocks and Structural Differences between the Czech Economy and the Euro Area : doctoral thesis. Supervisor: Antonín Slaný. Brno, 2014. p. 226.

The posterior density is slightly different than the density of $\theta^*$. Because of this the MH algorithm sets an acceptance rate and rejects a portion of candidate values. The probability of acceptance can be represented as

$$a\left(\theta^{(x-1)},\theta^*\right) = min\left[\frac{\pi(\theta=\theta^*/y)q\left(\theta^*,\theta=\theta^{(x-1)}\right)}{\pi\left(\theta=\theta^{(x-1)}/y\right)q\left(\theta^{(x-1)},\theta=\theta^*\right)},1\right] \tag{3.5.5}$$

Above we present the MH algorithm. In this thesis, however, we do not use the simple MH algorithm. We use the Random Walk Chain Metropolis Hastings algorithm (RWCMH) instead, where we write the random walk process as

$$\theta^* = \theta^{(x-1)} + \xi \tag{3.5.6}$$

where $\xi$ represents an incremental random variable. For the distribution the following equality holds

$$q\left(\theta^*,\theta = \theta^{(x-1)}\right) = q\left(\theta^{(x-1)},\theta = \theta^*\right) \tag{3.5.7}$$

Finally, we can rewrite the probability of acceptance (3.5.5) as

$$a\left(\theta^{(x-1)},\theta^*\right) = min\left[\frac{\pi(\theta=\theta^*/y)}{\pi\left(\theta=\theta^{(x-1)}/y\right)},1\right] \tag{3.5.8}$$

The probability distribution from which the algorithm draws is given by the probability density of $\xi$, which is a zero mean multidimensional normal distribution with a variance covariance matrix of K. K affects the acceptance rate, which should be between 25% and 50%. We can then write

$$q\left(\theta^{(x-1)},\theta\right) \rightarrow N\left(\theta/\theta^{(x-1)},K\right) \tag{3.5.9}$$

To sum up, in this subchapter we present the Bayesian technique used for the estimation. We first explain what Markov Chain Monte Carlo methods are used for, then present the MH algorithm. This algorithm is then modified. As a result, we get the Random Walk Chain Metropolis Hastings algorithm, which is used in our estimation. In the last part of the subchapter we present the acceptance ratio and the probability distribution. Since we estimate multiple models using this algorithm, these estimates need to be compared in order to select the best model. The method of comparison is presented in the following subchapter.

## 3.6 Comparing Bayesian estimates[16]

We can compare different DSGE model estimates using the characteristics of the posterior distribution. Let *p(A)* denote the prior distribution of model *A* and let *p(B)* denote the prior distribution of model *B*. The posterior distribution can be calculated over multiple models. To begin with, let *I=A,B*, then we can write the formula

$$p(I/Y_T) = \frac{p(I)p(Y_T/I)}{\sum_{I=A,B} p(I)p(Y_T/I)} \tag{3.6.1}$$

which can be generalized to *I=A,B,..,N* where N is the number of DSGE models.

We can compare two models through the posterior odds ratio, which compares the two posterior distributions. We can write the posterior odds ratio as

$$\frac{p(A/Y_T)}{p(B/Y_T)} = \frac{p(A)p(Y_T/A)}{p(B)p(Y_T/B)} \tag{3.6.2}$$

To compute (3.6.2) we need to find the *"the marginal density of the data conditional on the model,* $p(Y_T/I)$*."*[17] $p(Y_T/I)$ can be evaluated for every model *I=A,B* using the posterior kernel, which we discussed in the previous subchapter. We can write

$$p(Y_T/I) = \int p(\theta_I/I) * p(Y_t/\theta_i, I)\mathrm{d}\theta_t \tag{3.6.3}$$

To continue with, we can estimate $p(Y_T/I)$ using the Harmonic Mean Estimator. The Harmonic Mean estimator can be presented as

$$\hat{p}(Y_T/I) = \left[\frac{1}{B}\sum_{b=1}^{B} \frac{f\left(\theta_I^{(b)}\right)}{p\left(\theta_I^{(b)}/I\right)p\left(Y_T/\theta_I^{(b)}, I\right)}\right]^{-1} \tag{3.6.4}$$

where the drawn vectors $\theta_I^{(b)}$ are the results of MH iterations. *f* represents the probability density function, which weights the posterior kernel. This way the extreme values of $\theta$ are of lesser importance.

[16] GRIFFOLI, Tomasso M. DYNARE User Guide: An introduction to the solution & estimation of DSGE models. [online]. 2010. [cit. 2.3.2021]. Available at: https://www.sfu.ca/~kkasa/UserGuide.pdf

[17] Ibid.

## 3.7 Impulse response functions

We use impulse response functions to examine the effects of exogenous shocks on the selected macroeconomic variables. These functions graphically present the reactions of the selected macroeconomic variables to these exogenous shocks. Because of this we can derive conclusions regarding the behavior of the economy based on these impulse response functions. In this thesis we examine the reactions of output, inflation, the nominal interest rate, government spending and taxes to an exogenous cost push shock, world economy shock, interest rate shock, government spending shock and tax shock. The conclusions drawn from the impulse response functions are presented in chapter 5.

## 3.8 Dynare

We estimate the DSGE model using the Dynare addon for Matlab, which is available on the internet for free.[18] There is also an extensive user guide[19] and a reference manual[20], both of which give valuable information on how to program DSGE models. The main benefit of Dynare is that we do not need to enter the model into Matlab in matrix form. We can use Dynare and directly enter the log-linearized equations instead. In Dynare we first **set** the endogenous and exogenous variables, as well as the predetermined variables. Then we enter the log-linearized equations, assign steady state values to the endogenous variables, calibrate the pre-selected set of parameters and estimate the rest using the Bayesian estimation technique.

To sum up, in chapter 3 we present the methodology and methods used to obtain the results. We present the estimated model in chapter 4 and analyze the results in chapter 5.

[18] https://www.dynare.org/
[19] GRIFFOLI, Tomasso M. DYNARE User Guide: An introduction to the solution & estimation of DSGE models. [online]. 2010. [cit. 2.3.2021]. Available at: https://www.sfu.ca/~kkasa/UserGuide.pdf
[20] https://www.dynare.org/manual/

# 4 The model

In this chapter we present the theoretical formulation of the selected DSGE model. As mentioned before, in this thesis we specify the model to fit the data from Hungary. We selected Hungary because of the length and quality of the required time series available for this country. Since Hungary's economy is small and export oriented, we calibrate and estimate a small open economy DSGE model. The model consists of households, firms, monetary policy and fiscal policy. Multiple research papers are taken into consideration when setting up each of the four sectors of the economy. The selected authors are referenced in the following subchapters.

## 4.1 Households[21]

This subchapter is mainly based on two research papers.[22] To begin with, in this thesis the representative household of a small open economy is infinitely lived and allocates resources between consumption and investment. It has access to international financial markets. Because of this it can invest in both domestic and foreign bonds with a one-period maturity. To finance consumption and investment, it offers labor and receives wages. The aim of the representative household is to maximize the discounted utility function, based on which we can formulate their optimization problem

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, G_t) \tag{4.1.1}$$

where $C_t$ represents composite index of private consumption, $N_t$ represents hours worked and $G_t$ is the public consumption index. We define $C_t$ as

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta-1}{\eta}} \tag{4.1.2}$$

[21] This chapter is mainly based on GALÍ, Jordi - MONACELLI, Tommaso. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. In: *Review of Economic Studies* [online]. Oxford: Oxford University Press, 2005, vol. 70, no. 3, pp. 707-734 [cit. 4.15.2021]. ISSN 1467-937X. Available at: https://academic.oup.com/restud/article-abstract/72/3/707/1553469?redirectedFrom=fulltext

where $\alpha \in [0, 1]$ is the effect of imported goods on private consumption utility, which represents the degree of openness. If $\alpha < 1$, the value of this parameter reflects the presence of home bias in preferences. $\eta > 1$ represents the substitution between foreign and domestic goods.

$C_{H,t}$ is the index representing the consumption of domestic goods in country *i*, which we can formulate as the constant elasticity of substitution (CES) function

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.1.3}$$

where $j \in [0, 1]$ represents the different kinds of goods produced in country *i*. $\varepsilon > 1$ gives us the elasticity of substitution between different varieties of goods produced in any country. Imported goods consumed in country *i* can be represented as

$$C_{F,t} = \left(\int_0^1 \left(C_{i,t}\right)^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} \tag{4.1.4}$$

where $\gamma$ is the elasticity of substitution between imported goods. $C_{i,t}$ represents the quantity of goods which are imported from a foreign country *i* into the domestic economy and which are intended for final consumption. We can write $C_{i,t}$ as

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.1.5}$$

Below we present the demand functions, which represent the optimal expenditure allocation within every single category of goods:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}$$

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t}$$

for each $j,i \in [0, 1$. Here $P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ represents the prices of goods produced domestically, $P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ represent import prices of goods from country *i* for each *i*.

Next, the utility maximization function of the household (4.1.1) is subject to the following intertemporal budget constraint

$$P_tC_t + E_t\big(Q_{t,t+1}D_{t+1}\big) + T \leq D_t + (1-\chi_t)N_tW_t \quad (4.1.6)$$

where $D_t$ represents the value of both domestic and foreign bonds. $Q_{t,t+1}$ is a time variant stochastic discount factor, representing one-period ahead nominal investment returns. $W_t$ is the wage and $N_t$ is hours worked. $T$ stands for constant lump sum taxes and $\chi_t$ is the income tax rate. $P_t$ represents the price level and $C_t$ is the composite index of private consumption.

To continue with, we write the period utility as

$$U(C, N, G) = (1-\chi)logC - \chi logG - \frac{N^{1+\varphi}}{1+\varphi} \quad (4.1.7)$$

where $\chi \in [0, 1]$ gives the relative importance of public consumption in comparison to private. The two optimality conditions for the representative household are written below

$$\frac{(N_t)^{\varphi}}{(C_t)^{-\sigma}} = (1-\chi)\frac{W_t}{P_t} \quad (4.1.8)$$

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \quad (4.1.9)$$

We assume that these conditions are met in all states of nature and each period. By taking into consideration conditional expectations we get from (4.1.8) the Euler equation below

$$\beta R_t^* E_t\left\{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\} = 1 \quad (4.1.10)$$

where $R_t^* = \frac{1}{E_t\{Q_{t,t+1}\}}$. This represents the gross nominal interest rate or, more precisely, the riskless return on one-period bond.

To continue with, (4.1.8) holds for the representative household not just in country *i* but in any country *f* under the assumption that the markets for state-contingent securities are complete. This is represented below

$$\beta\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\left(\frac{P_t^i}{P_{t+1}^i}\right)\left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right) = Q_{t,t+1} \quad (4.1.11)$$

Combining (4.1.8) with (4.1.10) we get the real exchange rate

$$C_t^i = \vartheta_i C_t^i \varrho_{i,t}{}^{\frac{1}{\sigma}} \tag{4.1.12}$$

for each $t$. $\varrho_{i,t} = \frac{\varepsilon_{i,t} P_t^i}{P_t}$ represents the bilateral exchange rate, which expresses the ratio of CPIs of the two countries in domestic currency. $\vartheta_i$ is a constant with symmetric initial conditions, which means there are no net foreign asset holdings and so $\vartheta_i = \vartheta = 1$ for all *i*. In (4.1.12) purchasing parity holds and in the steady state we have perfect foresight if the effective terms of trade $\varrho_{i,t} = S_i = \int_0^1 S_{i,t}^{1-\gamma} \mathrm{d}i = 1$ for all *i*.

We integrate (4.1.12) over *i* and take the logarithms of each side to get

$$c_t = c_t^* + \frac{1}{\sigma} q_t = c_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t \tag{4.1.13}$$

where $s_t = \int_0^1 s_{i,t} \mathrm{d}i$ and $c_t^* = \int_0^1 c_t^i \,\mathrm{d}i$ represents the world consumption index. If $\eta =/1$, *"the second equality holds only up to a first order approximation."* From this we can conclude that if international markets are complete, the terms of trade and world consumption are linked to domestic consumption.

Government purchases are also part of our model. First and foremost, we assume that the government only purchases goods produced in the domestic economy. In such a case the index of public consumption of country *i* is represented as[23]

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

When allocating expenditures across different goods the government aims to minimize its total cost. Based on this the demand schedules of the government can be represented as below

$$G_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} G_t$$

The revenue needed to finance these government expenditures comes from domestic residents in the form of lump sum taxes. This assumption allows us to avoid distortions caused by

[23] GALÍ, Jordi - MONACELLI, Tommaso. Optimal monetary and fiscal policy in a currency union. In: *Journal of International Economics* [online]. Netherlands: Elsevier, 2008, vol. 76, pp. 116-132 [cit. 4.15.2021]. ISSN 0022-1996. Available at: https://joseordinolaboyer.files.wordpress.com/2011/01/gali_y_monacelli_politica_monetaria_y_fiscal_optimas.pdf

different types of financing, so we can focus on analyzing the effects of these government purchases.

Before ending this subchapter, we need to determine the equilibrium dynamics and derive the log-linearized representation of the model. Firstly, the domestic market clearing condition is represented as

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)\mathrm{d}f + G_t(j)$$

or as

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1\left(\frac{P_{H,t}}{\varepsilon_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t \mathrm{d}i + \ G_t\right] \quad (4.1.14)$$

Government demand for domestically produced goods can also be written as $G^i_t(j) = \kappa_t Y_t(j)$. Based on (4.1.14) we can conclude that the good *j* produced domestically is consumed by the government, foreign households and domestic households.

If we plug (4.1.14) into the equation of the domestic aggregate output $Y_t = \left(\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}}\mathrm{d}j\right)^{\frac{\varepsilon}{\varepsilon-1}}$ we get[24]

$$Y_t = \frac{1}{1-\kappa_t}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\left[(1-\alpha) + \alpha\int_0^1\left(S^i_t\right)^{\gamma-\eta}\varrho_{i,t}^{\eta-\frac{1}{\sigma}}\mathrm{d}i\right] \quad (4.1.15)$$

To end with, we present the log linearized form of (4.1.15), which gives us the open economy IS curve. It can be presented as[25]

$$y_t = E_t\{y_{t+1}\} - E_t\{\Delta g_{t+1}\} + \alpha(\omega-1)(\rho_{c^*}-1)c^*_t - \frac{1}{\sigma_\alpha}\left(r_t - E_t\{\pi_{H,t+1}\}\right) \quad (4.1.16)$$

where $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$ and $\sigma_\alpha = \frac{\sigma}{(1-\alpha)+\alpha\omega}$. Besides these parameters it is also important to define the endogenous variables. Firstly, output $y_t$ is defined as $y_t - \bar{y}$, where $\bar{y}$ represents the steady state value of output. Secondly, government spending is given as $g_t =$

[24] ÇEBI, Cem. The interaction between monetary and fiscal policies in Turkey: An estimated New Keynesian DSGE model. In: *Economic Modelling* [online]. Netherlands: Elsevier, 2012, vol. 29, pp. 1258-1267 [cit. 4.15.2021]. ISSN 0264-9993. Available at: https://www.sciencedirect.com/science/article/abs/pii/S0264999312001071
[25] Ibid.

$-ln\left(1-\frac{G_t}{Y_t}\right)$. Thirdly, the nominal interest rate is given by $r_t$. Fourthly, domestic inflation is represented as $\pi_{H,t} = ln\left(\frac{P_{H,t}}{P_{H,t-1}}\right)$, where $P_{H,t}$ is the CPI. Fifthly, $c_t^*$ is an exogenous AR(1) process representing world output.

To sum up, in this subchapter we present the behavior of a representative household. We take into consideration the household's optimization problem, intertemporal budget constraint, international risk sharing, and the market clearing condition. At the end of the subchapter we present the log-linearized representation of the open economy IS curve.

## 4.2 Firms[26]

In this subchapter we present the behavior of the firms in the economy. The economy consists of monopolistically competitive firms. These firms employ households to produce a differentiated good. In this work we assume linear production technology, which is represented below by the production function (4.2.1)

$$Y_t(j) = A_t N_t(j) \tag{4.2.1}$$

where $a_t = log(A_t)$ is represented by an AR(1) process $a_t = p_a a_{t-1} + \varepsilon_t$ and $j \in [0, 1]$ is the index of firms.

The firms set their prices according to the specification formulated by Calvo.[27] This implies that since prices are sticky, a given firm has the ability to change its prices at time $t$ with a probability of $1-\theta$, while the remaining $\theta$ fraction of firms cannot adjust their prices. No firm knows exactly when it is going be able to change prices, so the firms want to maximize their profits at time $t$. Based on this we can formulate the aggregated price index of domestic producers as

[26] This chapter is mainly based on GALÍ, Jordi - MONACELLI, Tommaso. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. In: *NBER Working Paper 8905* [online]. Cambridge: National Bureau of Economic Research, 2002, pp. 1-45 [cit. 4.15.2021]. Available at: https://www.nber.org/system/files/working_papers/w8905/w8905.pdf

[27] CALVO, Guillermo A. Staggered prices in an utility-maximizing framework. In: *Journal of Monetary Economics* [online]. United States of America: Elsevier, 1983, vol. 12, no. 3, pp. 1-16 [cit. 4.15.2021]. ISSN 0304-3932. Available at: https://www.sciencedirect.com/science/article/abs/pii/0304393283900600

$$P_{H,t} = \left[\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta)\left(\hat{P}_{H,t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{4.2.2}$$

The optimization problem of those firms which can change their prices at time *t* can be represented as

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{Q_{t,t+k}\left[Y_{t+k}\left(\bar{P}_{H,t} - MC_{t+k}^n\right)\right]\right\} \tag{4.2.3}$$

where $MC_{t+k}^n = \frac{W_t}{A_t}$ represents the nominal marginal cost and $\bar{P}_{H,t}$ represents the price of a good set by firm *j* at time *t*. Under the assumption of the single price rule the final price is optimal for all firms which can adjust their prices.

When setting the new price firms have to consider the following demand constraint

$$Y_{t+k}(j) \leq \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \left(C_{H,t+k} + C_{H,t+k}^*\right) = Y_{t+k}^d\left(\bar{P}_{H,t}\right)$$

(4.2.4)

from which we get the following first order condition

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k}\left(\bar{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1} MC_{t+k}^n\right)\right] = 0 \tag{4.2.5}$$

After combining (1.1.9) and (4.2.5) we get

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[P_{t+k}^{-1} C_{t+k}^{-\frac{1}{\tau}} Y_{t+k}\left(\bar{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1} MC_{t+k}^n\right)\right] = 0 \tag{4.2.6}$$

we further add inflation $\Pi_{t-1,t+k}^H = \frac{P_{H,t+k}}{P_{H,t-1}}$ and marginal cost $MC_{t+k} = \frac{MC_{t+k}^n}{P_{H,t+k}}$ so we can get the stationary model

$$\sum_{k=0}^{\infty} \beta\theta^k E_t \left[C_{t+k}^{-\frac{1}{\tau}} Y_{t+k} \frac{P_{H,t-1}}{P_{t+k}}\left(\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon-1} \Pi_{t-1,t+k}^H MC_{t+k}\right)\right] = 0 \tag{4.2.7}$$

The steady state form of the model does not include inflation and it is assumed that trade is balanced, therefore we derive the steady state log-linearized form get the following equation

$$p_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\pi_{H,t+k}\} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t(mc_{t+k})$$

(4.2.8)

where $mc_{t+k} = mc_t - \overline{mc}$ is the logarithm of the devation of real marginal cost from its steady state value, represented as $\overline{mc} = -\,log\frac{\varepsilon}{\varepsilon-1} = -\mu$.

Since we are interested in expected inflation and not expected marginal cost, we assume that if *k=1* has a value of 1 and *k=2* has a value of 0, and that domestic inflation is $\pi_{H,t+1} = \bar{p}_{H,t+1} - p_{H,t}$, then we get

$$\bar{p}_{H,t} - p_{H,t-1} = \beta\theta E_t(\bar{p}_{H,t+1} - p_{H,t}) + \pi_{H,t} + (1-\beta\theta)mc_t$$

(4.2.9)

Then we take the logarithm of the steady state value of the aggregated price index and write

$$\pi_{H,t} = (1-\theta)(\bar{p}_{H,t} - p_{H,t-1})$$

(4.2.10)

Finally, combining (4.2.9) and (4.2.10) we can derive the log-linearized New-Keynesian Phillips curve as

$$\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \kappa mc_t \qquad (4.2.11)$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. This relationship represents the domestic inflation dynamics of the small open economy expressed in terms of real marginal costs.

The New-Keynesian Phillips curve derived above can be extended. From now on this chapter is based on another research paper.[28] In this part of the subchapter we include in this curve firms with backward looking price setting behavior. We assume that in our model $1-\xi$ fraction of the firms is forward looking and follows the price setting behavior of Calvo. These firms optimize their price setting behavior using the information available to formulate their expectations on the future value of real marginal cost. This means they adjust their prices with *1-θ* probability. $\xi$ fraction of the firms is backward looking for they adjust their prices based on historical price

[28] GALÍ, Jordi - GERTLER, Mark. Inflation Dynamics: a Structural Economic Analysis. In: *NBER Working Paper 7551* [online]. Cambridge: National Bureau of Economic Research, 2000, pp. 1-31 [cit. 4.15.2021]. Available at: https://www.nber.org/system/files/working_papers/w7551/w7551.pdf

behavior. No firm knows whether any of its competitors is forward or backward looking. It is clear from this paragraph that the New-Keynesian Phillips curve can be extended.

By taking into consideration the information from the previous paragraph we can formulate the aggregate level of prices as

$$p_t = \theta p_{t-1} + (1-\theta\,)p'_t \tag{4.2.12}$$

where $p'_t$ represents an index of the prices adjusted in period *t* and is given as

$$p'_t = (1-\xi)p^f_t + \xi p^b_t \tag{4.2.13}$$

where $p^f_t$ represents the price adjusted at time *t* by the forward looking firm and $p^b_t$ is the price adjusted by the backward looking firm. Because forward looking firms adjust their prices as in the Calvo model we can write $p^f_t$ as

$$p^f_t = (1-\beta\theta\,)\sum_{k=0}^{\infty}(\beta\theta)^k E_t(mc_{t+k}) \tag{4.2.14}$$

The rule of thumb obeyed by backward looking firms has two distinct features. Firstly, in the steady state the rule does not deviate from optimal behavior. Secondly, the price adjusted according to this rule of thumb in period *t* depends on the information set available at *t-1*. Using the information in this paragraph we can formulate $p^b_t$ as

$$p^b_t = p'_{t-1} - \pi_{t-1} \tag{4.2.15}$$

(4.2.15) means that backward looking price setting behavior at *t* depends on the average price adjustment at *t-1* and on lagged inflation, which is used to forecast inflation at *t*.

We combine the previous four equations and get the log-linearized open economy hybrid New-Keynesian Phillips curve represented as deviations from the steady state.[29] This curve is presented below

$$\pi_{H,t} = \lambda^b \pi_{H,t-1} + \lambda^f E_t\{\pi_{H,t+1}\} + \kappa mc_t + \epsilon^\pi_t \tag{4.2.16}$$

[29] ÇEBI, Cem. The interaction between monetary and fiscal policies in Turkey: An estimated New Keynesian DSGE model. In: *Economic Modelling* [online]. Netherlands: Elsevier, 2012, vol. 29, pp. 1258-1267 [cit. 4.15.2021]. ISSN 0264-9993. Available at: https://www.sciencedirect.com/science/article/abs/pii/S0264999312001071

where $mc_t = (\sigma_\alpha + \varphi)(y_t - y_t^n) - \sigma_\alpha g_t + \tau$ represents real marginal cost, $\tau$ is the log-linearized government revenue equation and $\epsilon_t^\pi$ is a i.i.d cost push shock[30]. The remaining parameters are given as follows: $\lambda^b = \frac{\xi}{\theta+\xi(1-\theta(1-\beta))}$, $\lambda^f = \frac{\beta\theta}{\theta+\xi(1-\theta(1-\beta))}$, $\kappa = \frac{(1-\beta\theta)(1-\theta)(1-\xi)}{\theta+\xi(1-\theta(1-\beta))}$.

In this paragraph we characterize (4.2.16). Firstly, it is clear that the output gap, government spending and taxation indirectly affects inflation via the real marginal cost. The sensitivity of inflation to real marginal cost is represented by the slope coefficient $\kappa$. Secondly, $\kappa$ and the remaining two structural form parameters of the Phillips curve are represented by three deep model parameters, namely $\beta$, $\theta$ and $\xi$. If $\xi = 0$ then we have a forward looking New-Keynesian Phillips curve, otherwise the Phillips curve is hybrid. If $\beta = 1$, the sum of the parameters of forward and backward looking inflation equals to 1. What is more, the value of $\lambda^b$ and $\lambda^f$ falls between $\beta$ (if $\xi = 0$) and 1 (if $\xi = 1$). Because $\beta$ is always close to 1, $\lambda^b$ and $\lambda^f$ represent the relative weights given to past and expected inflation. From this we can conclude that if the number of backward looking firms increases and price stickiness is high then current inflation is less sensitive to current real marginal cost.

To sum up, at the beginning of this subchapter we derive the New-Keynesian Phillips curve using expected inflation and real marginal cost. In the second part of this subchapter we extend the derived Phillips curve and include backward looking firms, which adjust their prices by taking into consideration historical prices. At the end of this subchapter we present the log-linearized version of the hybrid New-Keynesian Phillips curve and characterize the structural parameters.

## 4.3 Monetary policy

In this subchapter we present the monetary policy rule formulated by Taylor.[31] We assume that the monetary authority follows an inflation targeting regime and tries to keep inflation slightly under 2%. To achieve this goal, the monetary authority gradually adjusts the nominal policy rate $r_t$

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[30] SMETS, Frank - WOUTERS, Rafael. Shocks and Frictions in US Business Cycles: a Bayesian DSGE approach. In: *Working paper no 722* [online]. Frakfurt am Main: European Central Bank, 2007. pp. 1-57 [cit. 4.15.2021]. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=958687

[31] TAYLOR, John B. Discretion versus policy rules in practice. In: *Carnegie-Rochester Conference series on public policy* [online]. Netherlands: Elsevier, 1993, vol. 39, no. 1, pp. 195-214 [cit. 4.15.2021]. ISSN 0167-2231. Available at: http://opendata.dspace.ceu.es/bitstream/10637/2345/1/p%20195_214.pdf

in response to a deviation of output from potential output and inflation from its value in the steady state. In addition, the monetary authority takes into consideration the past value of the policy rate as well.[32] The log-linearized monetary policy rule is represented as a deviation from the steady state and it is shown below

$$r_t = \rho_r(r_{t-1} - r_t^n) + (1 - \rho_r)\left[r_\pi \pi_{H,t} + r_y(y_t - y_t^n)\right] + r_t^n + \epsilon_t^r \qquad (4.3.1)$$

where $y_t^n = \frac{(1+\varphi)}{(\sigma_\alpha+\varphi)} a_t - \frac{(\sigma-\sigma_\alpha)}{(\sigma_\alpha+\varphi)} c_t^*$ is the potential output with $a_t$ being an exogenous AR(1) technology process. $r_t^n = \sigma_\alpha(E_t\{y_{t+1}^n\} - y_t^n) + \sigma_\alpha \alpha(\omega - 1)(\rho_{c^*} - 1)c_t^*$ represents the natural level of the interest rate. $\rho_r$ represents the degree of interest rate smoothing, $r_\pi$ represents the monetary authority's reaction to inflation, $r_\pi$ represents the monetary authority's reaction to the output gap and $\epsilon_t^r$ is an i.i.d non-systematic policy rate shock.[33] To sum up, in this subchapter we present the monetary policy rule and interpret the parameters of the log-linearized equation (4.3.1).

## 4.4 Fiscal policy

This subchapter is based on one research paper.[34] In this subchapter we characterize the behavior of the fiscal authority. In the framework presented below, the fiscal authority aims to stabilize output and debt. Assumingly fiscal policy cannot instantaneously react to changes in economic activity, so in this thesis we take this lag into consideration when formulating the reaction functions. The log-linearized reaction functions of backward looking fiscal policy are represented as deviations from the steady state and are shown below

$$g_t = \rho_g g_{t-1} + \left(1 - \rho_g\right)\left[g_y(y_{t-1} - y_{t-1}^n) + g_b b_t\right] + \epsilon_t^g \qquad (4.4.1)$$

$$\tau_t = \rho_\tau \tau_{t-1} + (1 - \rho_\tau)\left[\tau_y(y_{t-1} - y_{t-1}^n) + \tau_b b_t\right] + \epsilon_t^\tau \qquad (4.4.2)$$

[32] SMETS, Frank - WOUTERS, Rafael. Shocks and Frictions in US Business Cycles: a Bayesian DSGE approach. In: *Working paper no 722* [online]. Frakfurt am Main: European Central Bank, 2007. pp. 1-57 [cit. 4.15.2021]. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=958687

[33] ÇEBI, Cem. The interaction between monetary and fiscal policies in Turkey: An estimated New Keynesian DSGE model. In: *Economic Modelling* [online]. Netherlands: Elsevier, 2012, vol. 29, pp. 1258-1267 [cit. 4.15.2021]. ISSN 0264-9993. Available at: https://www.sciencedirect.com/science/article/abs/pii/S0264999312001071

[34] Ibid.

In this paragraph we characterize the parameters of (4.4.1) and (4.4.2). Firstly, parameters $\rho_g$ and $\rho_\tau$ represent the fiscal spending smoothing and tax smoothing parameters, respectively. Secondly, parameters $g_y$ and $\tau_y$ represent the reaction of government spending and the lump sum tax to changes in the lagged output gap. Thirdly, parameters $g_b$ and $\tau_b$ represent the reaction of government spending and the lump sum tax to changes in the debt stock. Lastly, we have the exogenous i.i.d fiscal shocks, namely $\epsilon_t^g$ and $\epsilon_t^\tau$, which represent the non-systematic changes in government spending and the lump sum tax. From (4.4.1) and (4.4.2) we can see that in the presence of a high degree of fiscal smoothing the reactions of government spending and tax to lagged output gap and debt are smaller.

The second component of fiscal policy and the last component of our model is the fiscal constraint, which can be represented as

$$B_{t+1} = (1+r)\left(B_t \frac{P_{H,t-1}}{P_{H,t}} + G_t - \chi_t Y_t\right) \qquad (4.4.3)$$

where $B_t = \frac{Debt_t}{P_{H,t-1}}$. The log-linearized fiscal constraint is represented as a deviation from the steady state and it is shown below

$$b_{t+1} = r_t + \frac{1}{\beta}\left[b_t - \pi_{H,t} + (1-\beta)(\tau_t - y_t) + \frac{\bar{C}}{\bar{B}}(g_t - \tau_t)\right] \qquad (4.4.4)$$

where $b_t = ln\left(\frac{B_t}{P_{H,t-1}}\right)$, $B_t$ is nominal debt, $\bar{C}$ is the steady state value of private consumption to GDP ratio and $\bar{B}$ is the steady state value of the debt to GDP ratio.

To sum up, in the first part of this subchapter we present the reaction functions of government spending and lump sum taxes. These reaction functions consist of a smoothing parameter and the reaction of fiscal authorities to the output gap and debt. In addition, the non-systematic component of fiscal policy is also represented. In the second part of the subchapter we present the fiscal constraint, which is the last component of the DSGE model. The log-linearized equations of the estimated model are presented in the next subchapter.

## 4.5 Log-linearized model and the steady state

In this subchapter we summarize the 16 log-linearized equations and model parameters which we estimate using Dynare. The steady state value of each variable below is zero, because they are defined as deviations from their respective steady state values. These equations and parameters are derived and presented in subchapters 4.1 – 4.4.

IS curve:

$$y_t = E_t\{y_{t+1}\} - E_t\{\Delta g_{t+1}\} + \alpha(\omega - 1)(\rho_{c^*} - 1)c_t^* - \frac{1}{\sigma_\alpha}\left(r_t - E_t\left\{\pi_{H,t+1}\right\}\right)$$

where $c_t^*$ represents the world, while $\omega$ and $\sigma_\alpha$ are parameters defined as

$$\omega = \sigma\gamma + (1 - \alpha)(\sigma\eta - 1)$$

$$\sigma_\alpha = \frac{\sigma}{(1 - \alpha) + \alpha\omega}$$

$$c_t^* = \rho_{c^*} c_{t-1}^* + \epsilon_t^{c^*}$$

Hybrid Phillips curve:

$$\pi_{H,t} = \lambda^b \pi_{H,t-1} + \lambda^f E_t\left\{\pi_{H,t+1}\right\} + \kappa mc_t + \epsilon_t^\pi$$

where $mc_t$ represents real marginal cost, $\kappa$ is the slope coefficient and $\lambda^b$ and $\lambda^f$ are parameters defined as

$$mc_t = (\sigma_\alpha + \varphi)(y_t - y_t^n) - \sigma_\alpha g_t + \tau$$

$$\lambda^b = \frac{\xi}{\theta + \xi\big(1 - \theta(1 - \beta)\big)}$$

$$\lambda^f = \frac{\beta\theta}{\theta + \xi\big(1 - \theta(1 - \beta)\big)}$$

$$\kappa = \frac{(1 - \beta\theta)(1 - \theta)(1 - \xi)}{\theta + \xi\big(1 - \theta(1 - \beta)\big)}$$

Monetary policy rule:

$$r_t = \rho_r(r_{t-1} - r_t^n) + (1 - \rho_r)\left[r_\pi \pi_{H,t} + r_y(y_t - y_t^n)\right] + r_t^n + \epsilon_t^r$$

where $y_t^n$ represents potential output, $r_t^n$ is the natural interest rate and $a_t$ represents technology

$$y_t^n = \frac{(1 + \varphi)}{(\sigma_\alpha + \varphi)} a_t - \frac{(\sigma - \sigma_\alpha)}{(\sigma_\alpha + \varphi)} c_t^*$$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a$$

$$r_t^n = \sigma_\alpha(E_t\{y_{t+1}^n\} - y_t^n) + \sigma_\alpha \alpha(\omega - 1)(\rho_{c^*} - 1)c_t^*$$

Government spending:

$$g_t = \rho_g g_{t-1} + \left(1 - \rho_g\right)\left[g_y(y_{t-1} - y_{t-1}^n) + g_b b_t\right] + \epsilon_t^g$$

Tax:

$$\tau_t = \rho_\tau \tau_{t-1} + (1 - \rho_\tau)\left[\tau_y(y_{t-1} - y_{t-1}^n) + \tau_b b_t\right] + \epsilon_t^\tau$$

Fiscal constraint:

$$b_{t+1} = r_t + \frac{1}{\beta}\left[b_t - \pi_{H,t} + (1 - \beta)(\tau_t - y_t) + \frac{\bar{C}}{\bar{B}}(g_t - \tau_t)\right]$$

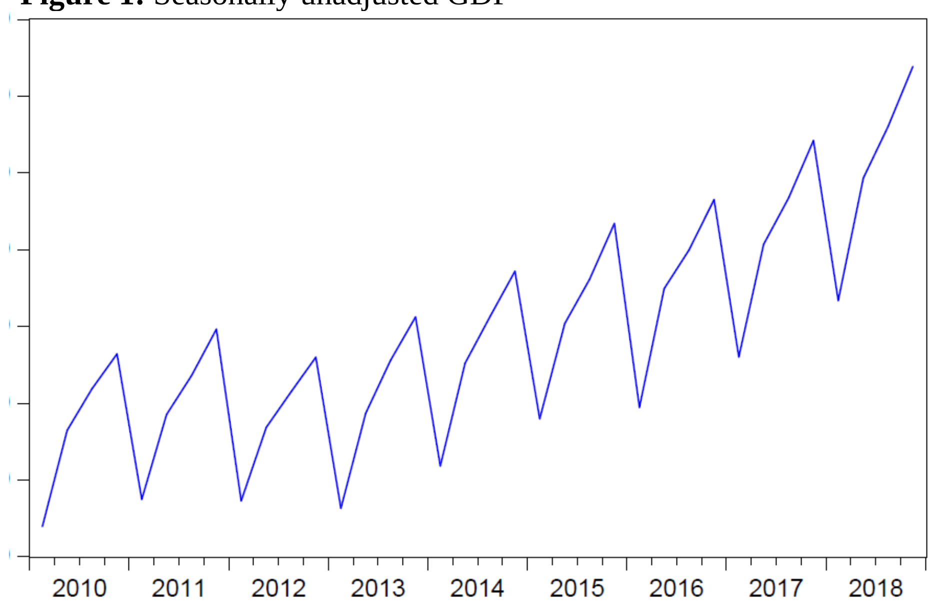
where $b_t$ is a predetermined variable.

# 5 Results

## 5.1 Data description

In this thesis we estimated the DSGE model of the Hungarian economy presented in chapter 4. In this paragraph we summarize the information about the data. As stated in subchapter 3.7, we have five observable endogenous variables in the model, namely output, inflation, government spending, tax and the nominal interest rate. Real GDP is the measure of output, CPI is the measure inflation, the three-month T-bill rate is the measure of the nominal interest rate, government spending to GDP ratio is the measure of government spending and tax to GDP ratio is the measure of tax. The seasonally unadjusted times series of these observable variables are available in the International Financial Statistics database of the International Monetary Fund.[35] The data covers the period of 2010Q1:2018Q4. Since the variables were nonstationary, we had to difference them, thus the first observation was lost, and we estimated the model using a dataset covering the period of 2010Q2:2018Q4. After briefly summarizing the data in this paragraph we continue with by explaining how we prepared the data for estimation.

**Figure 1:** Seasonally unadjusted GDP

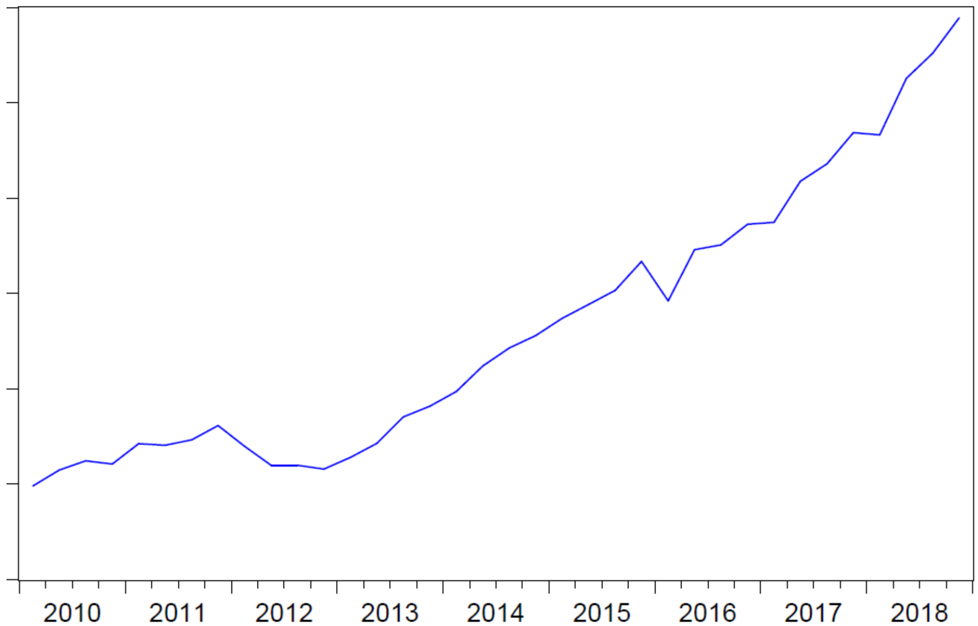



Source: author's own calculations in EViews

[35] https://data.imf.org/?sk=4c514d48-b6ba-49ed-8ab9-52b0c1a0179b&sId=1409151240976

Firstly, since the data was obtained in a seasonally unadjusted form, we plotted the graph of the time series to search for seasonal patterns. The time series of GDP is presented on figure 1. It is clear from figure 1 that GDP had a seasonal pattern. Because of this we used the STL decomposition seasonal adjustment procedure characterized in subchapter 3.2. The seasonal adjustment was done in EViews. The results are presented on figure 2. Based on figure 2 we concluded that the data was sufficiently smoothed, so we proceeded to make the series stationary.

**Figure 2:** Seasonally adjusted GDP

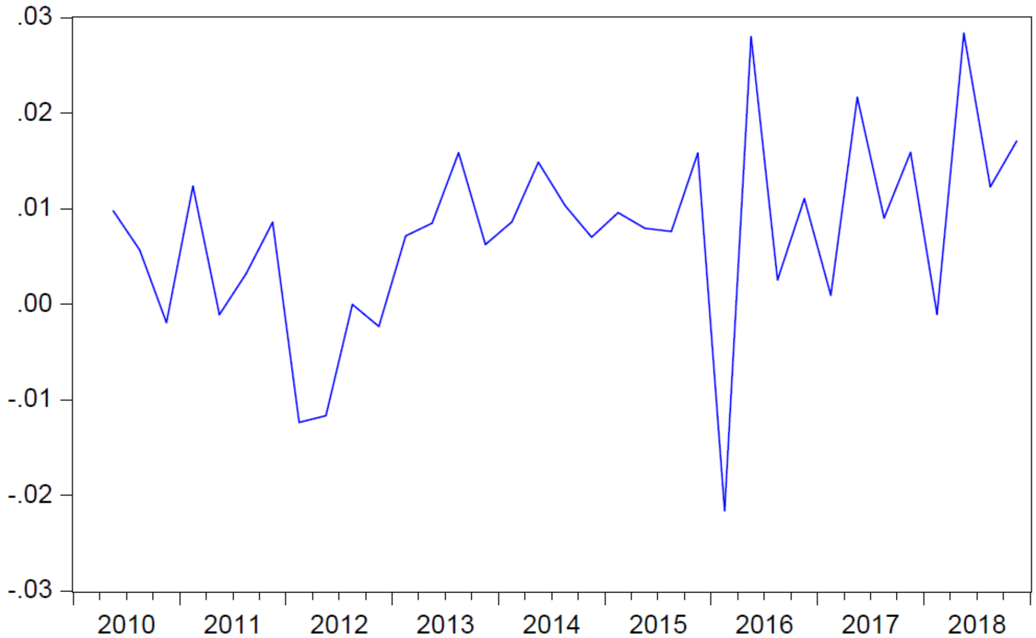



Source: author's own calculations in EViews

Secondly, according to chapter 3.3, the data used for DSGE modelling must be stationary. On figure 2 we can clearly see an upward trend. Because of this we concluded that the series of GDP had a unit root. At first, we differenced the natural logarithm of the data to obtain the stationary series. The differenced series is presented on figure 3. On figure 3 we can still see a slight upward trend. Since differencing did not make the series stationary, we applied the H-P filter from subchapter 3.3. The filtered series is presented on figure 4. Based on figure 4 we concluded that the trend was successfully eliminated from the data and the series became stationary. We, therefore, used it in the estimation.

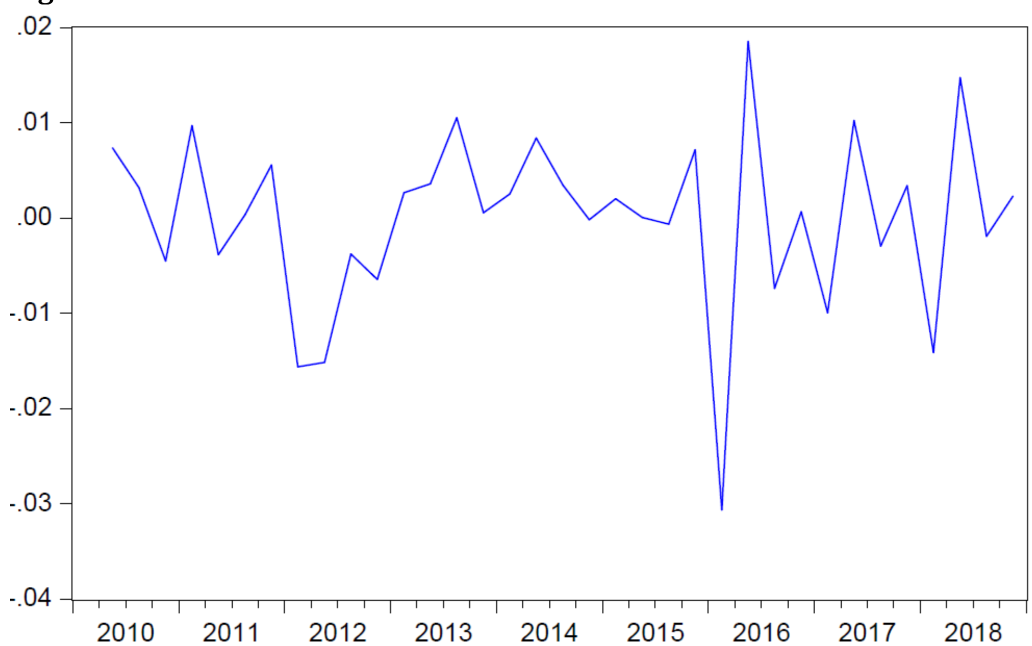
In case of the remaining four time series we proceeded as follows. Firstly, CPI and the T-bill rate were also nonstationary. We differenced their natural logarithms and obtained the stationary series without resorting to the H-P filter.

**Figure 3:** Nonstationary GDP




Source: author's own calculations in EViews

**Figure 4:** Detrended GDP




Source: author's own calculations in EViews

Secondly, the government spending to GDP and taxes to GDP ratios were obtained in the following way. At the beginning we subtracted the interest payments from the nominal government spending and added up all tax revenue. In the next step we calculated the government Afterwards we calculated the real values using CPI, seasonally adjusted the data using the STL decomposition and calculated the aforementioned ratios. At the end, the stationary series were obtained by differencing the natural logarithms of these nonstationary variables. This way every variable became appropriate for use in the estimation.

To sum up, in this chapter we present the method of data preparation we used. This way we obtained the data, which we needed for the estimation. The next step in the modelling process was the calibration of parameters and setting of priors. We describe this in detail in the next subchapter.

## 5.2 Calibration and priors

In this subchapter we present the calibrated parameter values in table 1, then in table 2 we present the priors and standard deviations we used for the Bayesian estimates. The theory behind this chapter is explained in subchapters 3.4 and 3.5.

**Table 1:** Calibrated parameter values

| Parameter | Calibrated value |
|---|---|
| $\boldsymbol{\alpha}$ | 0.69 |
| $\boldsymbol{\eta}$ | 1.00 |
| $\boldsymbol{\gamma}$ | 1.00 |
| $\boldsymbol{\beta}$ | 0.99 |
| $\boldsymbol{\bar{C}}$ | 0.51 |
| $\boldsymbol{\bar{B}}$ | 0.78 |

Source: various publications and the author’s calculations

At first, we present the calibrated parameters in table 1. These parameters were calibrated because their values were almost identical in most studies. Firstly, we borrowed the value of the degree of openness from a DSGE model calibrated for the Hungarian economy. We set this parameter's value at 0.69.[36] Secondly, the parameters representing the elasticity of substitution between domestic and foreign goods and the elasticity of substitution between foreign goods from different countries were set at 1.00. This value was also borrowed from another research paper.[37] Thirdly, the value of the discount factor was calibrated to 0.99 based on two previous DSGE models calibrated and estimated for the Hungarian economy.[38] Finally, the last two parameters, namely the steady state values of the private consumption to GDP ratio and the debt to GDP ratio were set at 0.51 and 0.78, respectively. We obtained these values by calculating the sample means for the estimation period. To sum up, we calibrated these parameters because their values were given in most studies or could be easily calculated. The rest of the parameters were estimated using Bayesian techniques.

In the second part of this chapter we describe how we selected the probability distributions, prior means and prior standard deviations of the remaining model parameters. The probability distribution, the prior means and the standard deviations are presented in table 2. Firstly, we used the same probability distributions as other researchers who estimated this model. We used beta distributions for parameters whose value falls between zero and one. Inverse gamma distributions were used for the shocks so they could not have negative values.[39] Secondly, we obtained the prior means and standard deviations either by borrowing them from other studies or by running regressions in EViews. The latter, namely the OLS regressions, were used to obtain prior means and standard deviations for the autoregressive parameters of world output, government spending and taxation. The former method was used for the rest of parameters. The first four parameters in table 2, namely the Calvo parameter, the inverse elasticity of labour supply, the inverse elasticity

[36] ALGOZHINA, Aliya. Monetary and Fiscal Policy Interactions in an Emerging Open Economy: A Non-Ricardian DSGE approach. In: *FIW Working Paper No. 94* [online]. Vienna: Research Centre International Economics, 2012. pp. 1-34 [cit. 6. 5.2021]. Available at: hdl.handle.net/10419/121099

[37] ÇEBI, Cem. The interaction between monetary and fiscal policies in Turkey: An estimated New Keynesian DSGE model.

[38] JAKAB, Zoltán M. - KÓNYA, István. An open economy DSGE model with search-and-matching frictions: the case of Hungary. In: *Emerging Markets Finance and Trade* [online]. United Kingdom: Taylor & Francis, 2016, vol. 52, n. 7, pp. 1606-1626 [cit 6.5.2021]. ISSN 1558-0938. Available at: http://real.mtak.hu/39369/1/LaborDSGE_final.pdf and JAKAB, Zoltán M. - VILÁGI, Balázs. An estimated DSGE model of the Hungarian economy. In: *MNB Working papers 2008/9* [cit 6.5.2021]. Hungary: National Bank of Hungary, 2008. pp. 1-86 [cit 6.5.2021]. Available at: https://www.econstor.eu/bitstream/10419/83617/1/589207865.pdf

[39] ÇEBI, Cem. The interaction between monetary and fiscal policies in Turkey: An estimated New Keynesian DSGE model.

of substitution in consumption and the interest rate smoothing parameter were borrowed from two studies on the Hungarian economy.

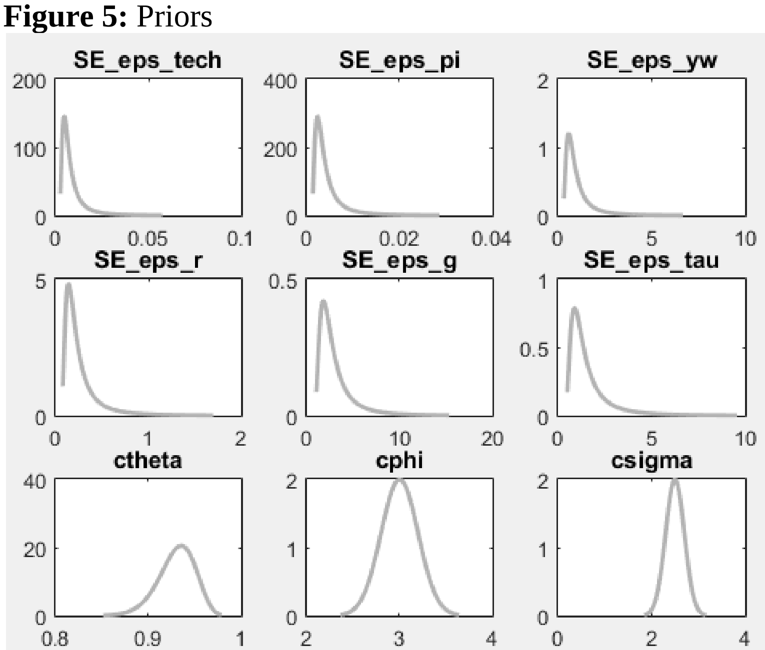
**Table 2:** Prior distributions

| Parameter | Probability distribution | Prior mean | Prior standard deviation |
|---|---|---|---|
| $\boldsymbol{\theta}$ | Beta | 0.93 | 0.02 |
| $\boldsymbol{\varphi}$ | Normal | 3.00 | 0.20 |
| $\boldsymbol{\sigma}$ | Normal | 2.50 | 0.20 |
| $\boldsymbol{\rho_r}$ | Beta | 0.76 | 0.05 |
| $\boldsymbol{r_\pi}$ | Gamma | 1.50 | 0.40 |
| $\boldsymbol{r_y}$ | Gamma | 0.50 | 0.10 |
| $\boldsymbol{\rho_g}$ | Beta | 0.68 | 0.14 |
| $\boldsymbol{g_y}$ | Normal | 0.50 | 0.20 |
| $\boldsymbol{\rho_\tau}$ | Beta | 0.73 | 0.05 |
| $\boldsymbol{\tau_y}$ | Normal | 0.63 | 0.20 |
| $\boldsymbol{g_b}$ | Normal | 0.40 | 0.02 |
| $\boldsymbol{\tau_b}$ | Normal | 0.40 | 0.02 |
| $\boldsymbol{\xi}$ | Beta | 0.75 | 0.05 |
| $\boldsymbol{\rho_a}$ | Beta | 0.50 | 0.15 |
| $\boldsymbol{\rho_{c^*}}$ | Beta | 0.80 | 0.05 |
| $\boldsymbol{\epsilon_t^a}$ | Inverse gamma | 0.01 | 2.00 |
| $\boldsymbol{\epsilon_t^\pi}$ | Inverse gamma | 0.005 | 2.00 |
| $\boldsymbol{\epsilon_t^{c^*}}$ | Inverse gamma | 0.70 | 4.00 |
| $\boldsymbol{\epsilon_t^r}$ | Inverse gamma | 0.30 | 2.00 |
| $\boldsymbol{\epsilon_t^g}$ | Inverse gamma | 3.30 | 4.00 |
| $\boldsymbol{\epsilon_t^\tau}$ | Inverse gamma | 1.80 | 4.00 |

Source: various publications and the author’s estimates

The prior mean and standard deviation of the technology parameter were also obtained from these same studies.[40] The Calvo parameter with its value of 0.93 was especially high compared to the industry standard, which is set between 0.5 and 0.75. The Taylor parameters were set according to the industry standard.[41] We borrowed the parameter representing the portion of backward looking firms from another study and set it 0.05 higher at 0.75.[42] The parameters representing the fiscal responses to the output gap were borrowed from another paper, in which the author estimates a DSGE model for the Hungarian economy.[43] Lastly, we obtained the prior means and standard deviations of the errors from the studies referenced in the previous paragraph. We slightly modified these to better fit the model. We selected some from the priors described in this subchapter and present their distributions on figure 5. After calibrating the selected parameters and selecting priors for the remainder we proceeded to estimate the model.

**Figure 5:** Priors




Source: author’s estimates in Dynare

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[43] ALGOZHINA, Aliya. Monetary and Fiscal Policy Interactions in an Emerging Open Economy: A Non-Ricardian DSGE approach.

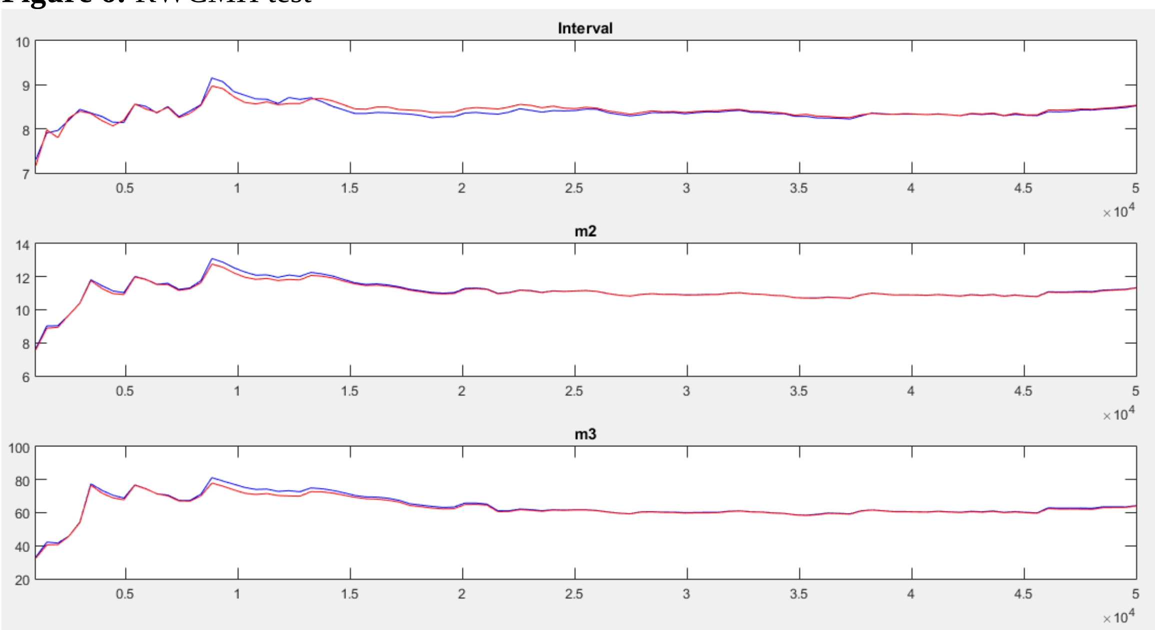
## 5.3 Estimation results

### *5.3.1 Initial tests*

Before we could proceed to interpret the results of the Bayesian estimation described in subchapter 3.6, we had to run diagnostics tests. These showed us that the model was appropriate for further analysis. We tested the Blanchard-Khan condition, the correct course of the RWCMH algorithm and the quality of the parameter estimates. Each of these test results are described below.

Firstly, rational expectations models must satisfy the Blanchard-Kahn condition. This condition requires that the number of unstable eigenvalues of the model system equal to the number of forward looking variables. This test could be done manually, but Dynare controls this B-K condition before beginning to run the RWCMH algorithm. If the condition is not met, Dynare does not start the RWCMH algorithm. Since Dynare explicitly stated that the condition was met and it proceeded to estimate the model, we concluded that the B-K condition was satisfied.

**Figure 6:** RWCMH test

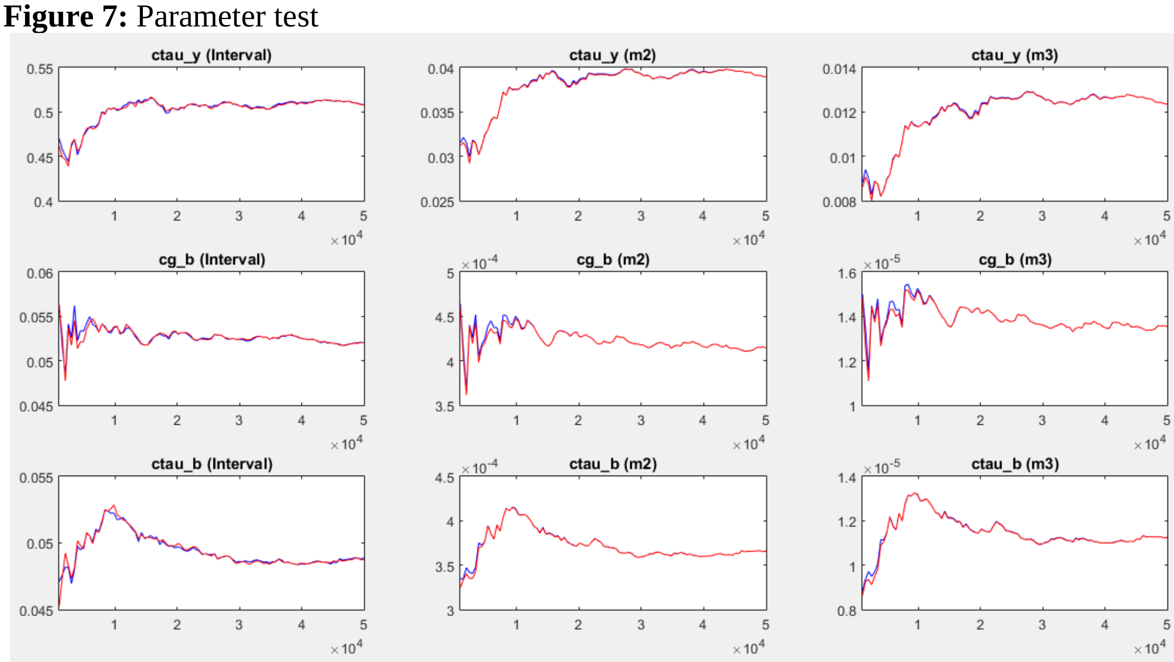



Source: author's estimates in Dynare

Secondly, the second test allowed us to control the course of the RWCMH algorithm. Dynare provided us a visual diagnostic test presented on figure 6. On figure 6 we see two curves, one red and one blue. These curves represent the initial parameter values of the RWCMH algorithm. The first rectangle on figure 6 represents the 80% confidence band for the parameter mean, the second rectangle represents the variance and the third rectangle represents the third moment. If the algorithm runs correctly, the two curves have a similar course and they both converge to the same value. Based on figure 6 we concluded that the RWCMH algorithm run correctly, with an acceptance ratio of 32.066% on the first chain and 31.844% on the second chain.

Thirdly, the third test we used was the Brooks and Gelman diagnostic test, which tests the accuracy of the parameter estimates of the RWCMH algorithm. The test results of the Brooks and Gelman diagnostic test for are presented on figure 7 for a subset of parameters. On figure 7 we see two curves, one red and one blue. These curves represent the initial parameter values. The first column represents the 80% confidence bands for the parameter mean, the second column presents the variance and the third column presents the third moment. If the algorithm runs correctly, the two curves have a similar course and they both converge to the same value. Based on figure 7 we concluded that the RWCMH algorithm correctly estimated the parameter values.

**Figure 7:** Parameter test




Source: author’s estimates in Dynare

After concluding that the algorithm run correctly, we moved on to interpret the estimated posterior values and compare them to the prior values presented in subchapter 5.2.

*5.3.2 Parameter estimation results*

**Table 3:** Posterior estimates

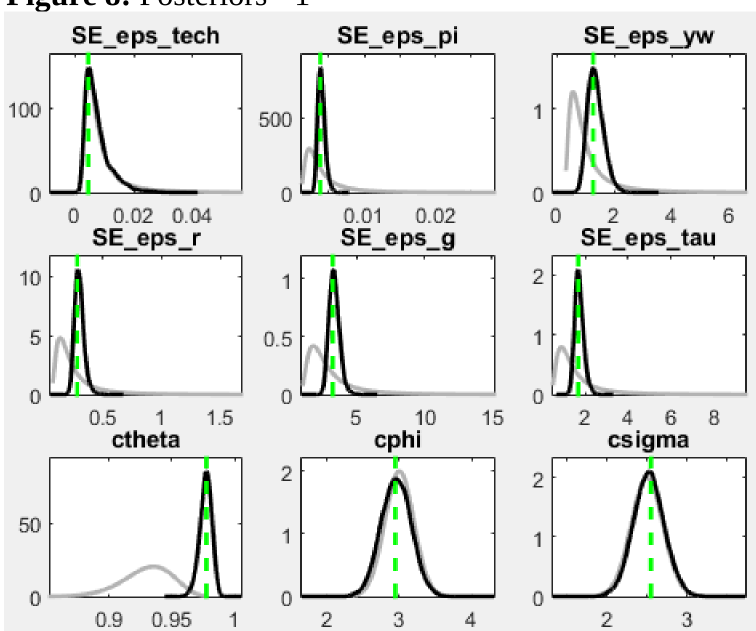
| Parameter | Prior mean | Prior standard deviation | Posterior mode | Posterior standard deviation | Posterior mean | Lower 90% confidence band | Upper 90% confidence band |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{\theta}$ | 0.93 | 0.02 | 0.97 | 0.006 | 0.97 | 0.956 | 0.977 |
| $\boldsymbol{\varphi}$ | 3.00 | 0.20 | 2.98 | 0.20 | 2.98 | 2.659 | 3.304 |
| $\boldsymbol{\sigma}$ | 2.50 | 0.20 | 2.55 | 0.19 | 2.52 | 2.214 | 2.844 |
| $\boldsymbol{\rho_r}$ | 0.76 | 0.05 | 0.73 | 0.05 | 0.72 | 0.642 | 0.804 |
| $\boldsymbol{r_\pi}$ | 1.50 | 0.40 | 1.39 | 0.38 | 1.50 | 0.859 | 2.160 |
| $\boldsymbol{r_y}$ | 0.50 | 0.10 | 0.51 | 0.10 | 0.53 | 0.357 | 0.683 |
| $\boldsymbol{\rho_g}$ | 0.68 | 0.14 | 0.13 | 0.02 | 0.13 | 0.099 | 0.164 |
| $\boldsymbol{g_y}$ | 0.50 | 0.20 | 1.41 | 0.17 | 1.39 | 1.140 | 1.668 |
| $\boldsymbol{\rho_\tau}$ | 0.73 | 0.05 | 0.63 | 0.05 | 0.63 | 0.550 | 0.712 |
| $\boldsymbol{\tau_y}$ | 0.63 | 0.20 | 0.69 | 0.20 | 0.69 | 0.368 | 1.013 |
| $\boldsymbol{g_b}$ | 0.40 | 0.02 | 0.36 | 0.02 | 0.36 | 0.326 | 0.393 |
| $\boldsymbol{\tau_b}$ | 0.40 | 0.02 | 0.40 | 0.02 | 0.40 | 0.371 | 0.433 |
| $\boldsymbol{\xi}$ | 0.75 | 0.05 | 0.74 | 0.05 | 0.74 | 0.657 | 0.819 |
| $\boldsymbol{\rho_a}$ | 0.50 | 0.15 | 0.50 | 0.18 | 0.51 | 0.248 | 0.739 |
| $\boldsymbol{\rho_{c^*}}$ | 0.80 | 0.05 | 0.84 | 0.03 | 0.84 | 0.787 | 0.899 |
| $\boldsymbol{\epsilon_t^a}$ | 0.01 | 2.00 | 0.005 | 0.002 | 0.009 | 0.002 | 0.017 |
| $\boldsymbol{\epsilon_t^\pi}$ | 0.005 | 2.00 | 0.004 | 0.0004 | 0.004 | 0.003 | 0.005 |
| $\boldsymbol{\epsilon_t^{c^*}}$ | 0.70 | 2.00 | 0.63 | 0.12 | 0.69 | 0.467 | 0.919 |
| $\boldsymbol{\epsilon_t^r}$ | 0.30 | 2.00 | 0.28 | 0.03 | 0.29 | 0.239 | 0.350 |
| $\boldsymbol{\epsilon_t^g}$ | 3.30 | 4.00 | 3.34 | 0.38 | 3.45 | 2.802 | 4.104 |
| $\boldsymbol{\epsilon_t^\tau}$ | 1.80 | 4.00 | 1.67 | 0.19 | 1.72 | 1.382 | 2.032 |

Source: author's estimates in Dynare

Table 3 present the estimated posterior mode, standard deviation, posterior mean and the 90% confidence bands. It is clear from table 3 that all estimated values were statistically significant as their values were inside the 90% confidence bands. In addition, the standard deviations of the priors and posteriors were almost identical. What is more, most of the posterior means are close to the prior distribution, except for the fiscal parameters in the government spending equation. Taking everything into consideration, we concluded that the estimates were suitable to estimate the Bayesian impulse response functions.

Figures 8-10 - which compare the posterior and prior distributions - also supported the conclusion we reached in the previous paragraph based on table 3. On figures 8-10 the dark distribution represents the posterior while the grey distribution represents the prior.

**Figure 8:** Posteriors - 1

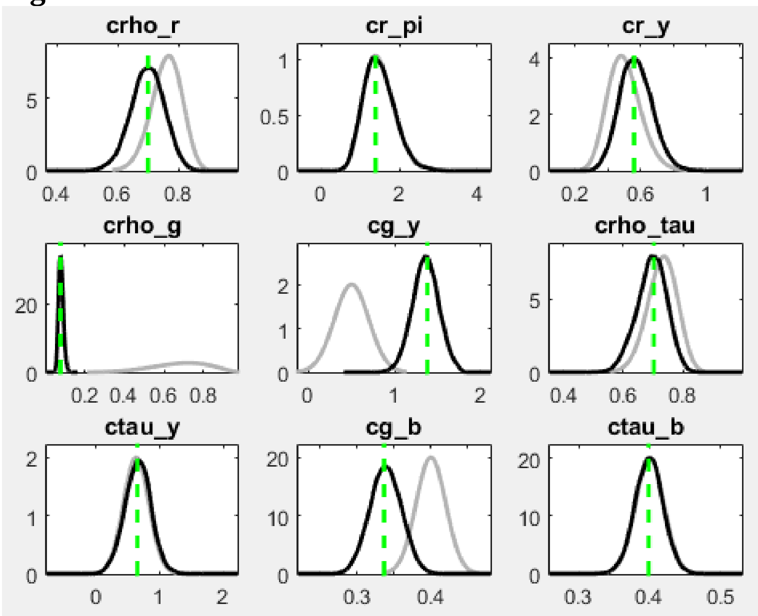



Source: author’s estimates in Dynare

At first glance the posterior distributions of most of the shocks slightly differed from their prior distributions. This was, however, the result of the specific prior distributions we used for these shocks and these kinds of differences in priors and posteriors of shocks often come up in the literature. In addition to this, we also had different prior and posterior distributions in case of the government spending equation. This was expected based on the estimated values in table 3. The estimated parameter values of the government spending equation were, however, in the 90%

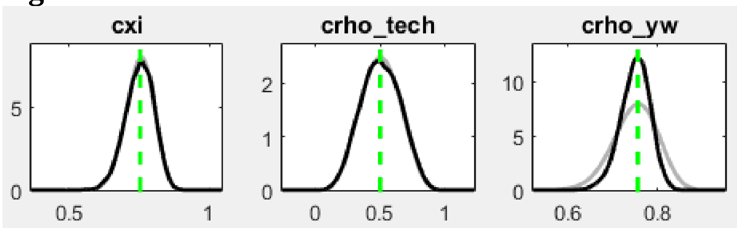
confidence band, so we proceeded to estimate the Bayesian impulse response functions. These impulse response functions are presented in the next subchapter.

**Figure 9:** Posteriors - 2




Source: author’s estimates in Dynare

**Figure 10:** Posteriors - 3



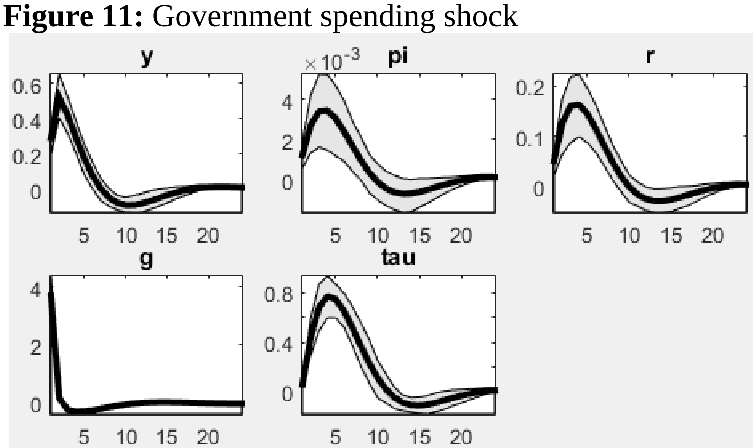

Source: author’s estimates in Dynare

## 5.4 Impulse response functions

In this subchapter we present the Bayesian impulse response functions with 90% confidence bands. There are six shocks in the model, namely the government spending shock, the tax shock, the productivity shock, the interest rate shock, the cost push shock and the world output shock. Five endogenous variables react to these shocks. These endogenous variables are output, inflation, the nominal interest rate, government spending and tax. We begin with analyzing the effects of the government spending shock on the economy.

Firstly, the effects of an unexpected increase in government spending on the economy are presented on figure 11. As a result of an unexpected increase in government spending both output and inflation rise. The rise in output is expected, but at first glance the rise in inflation might seem contradictory. Government spending should result in a decrease in inflation via marginal cost. In our case, however, the increase in output is higher than government spending's effect on the marginal cost of firms. This explains why inflation rises. Because inflation is higher, the monetary authority reacts to it by raising the interest rate. Debt also reacts to these factors and increases, because interest rates are higher and government spending is increased. Because of this the government needs to stabilize debt levels.  It raises taxes to do so. This results in a unique situation, when the expansionary fiscal policy – which means increased government spending – is accompanied by restrictive monetary policy and increased taxation. Based on figure 11 we can conclude that the effects of the shocks are statistically significant, and the variables return to their respective steady states.

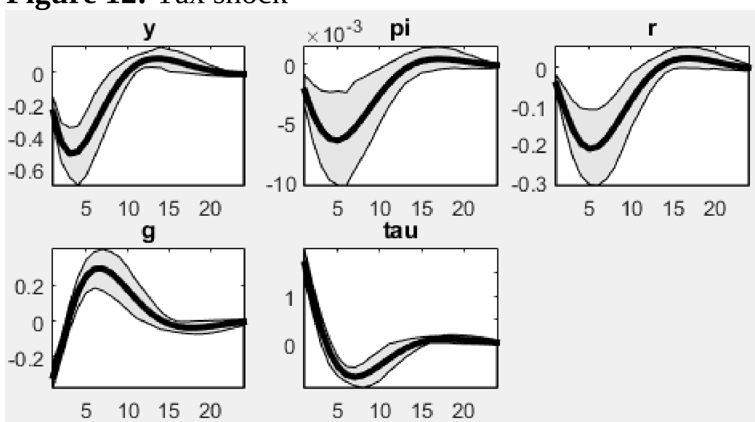
**Figure 11:** Government spending shock




Source: author's estimates in Dynare

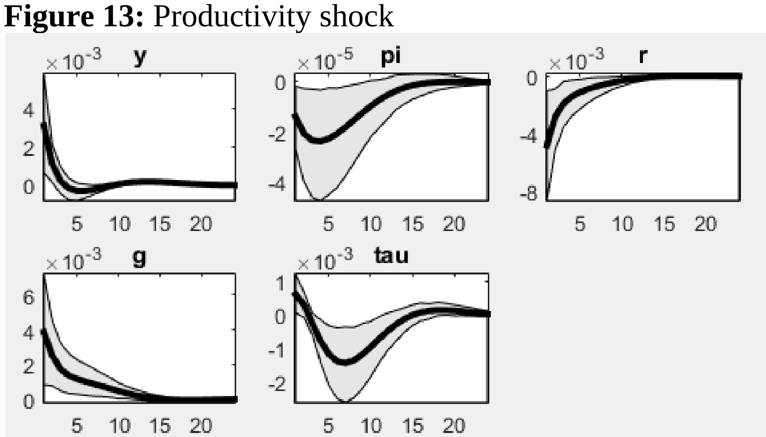
Secondly, the effects of an unexpected increase in taxes are presented on figure 12. As a result of an unexpected increase in taxes output decreases. Taxes affect the economy via two channels. Through the first channel income taxes reduce disposable income and lead to a decrease in output. Through the second channel an increase in payroll taxes leads to an increase in the marginal cost of firms, thereby reducing aggregate supply. Furthermore, an increase in payroll taxes also increases prices, again via the marginal cost. We, however, cannot see an increase in inflation on figure 12. On the contrary, inflation decreases. At the beginning taxes increase and government spending falls, which leads to a decrease in debt. After the government debt was reduced to the appropriate level, taxes return to their steady state and government spending increases, before returning to its own steady state value. Since government spending decreases inflation via marginal cost, it seems that the effects of spending outweigh the effects of taxation. The monetary authority reacts to the decrease in inflation by conducting an expansionary monetary policy and thus it decreases the interest rate to stimulate the economy. Based on figure 12 we can conclude that the effects of the shocks are statistically significant, and the variables return to their respective steady states.

**Figure 12:** Tax shock




Source: author's estimates in Dynare

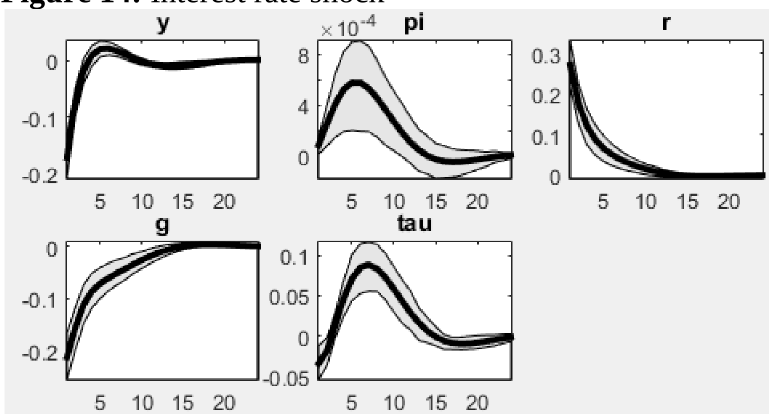
**Figure 13:** Productivity shock




Source: author's estimates in Dynare

Thirdly, the effects of an unexpected increase in productivity are represented on figure 13. As a result of an unexpected productivity shock potential output increases while the natural interest rate decreases. This latter decrease results in the decrease of the nominal interest rate. Because the nominal interest rate decreases real interest rates also fall, but this fall is somewhat mitigated by the decrease in inflation. Since the decrease in inflation is smaller than the decrease in nominal interest rates, economic activity is increased. This is best represented by the reaction of output. The fall of nominal interest rates also results in the decrease of debt. Since debt is decreased, the government conducts expansionary fiscal policy and increases spending while simultaneously reducing taxes. The tax reduction decreases the marginal cost of firms, which results in a decrease in inflation. Based on figure 13 we can conclude that the effects of the shocks are statistically significant, and the variables return to their respective steady states.

**Figure 14:** Interest rate shock

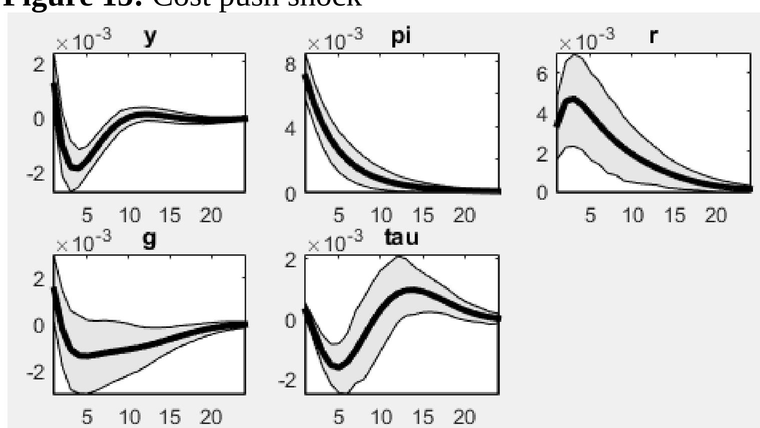



Source: author's estimates in Dynare

Fourthly, the effects of an unexpected increase in the nominal interest rate are presented on figure 14. As a result of an unexpected nominal interest rate shock output decreases along with inflation. The interest paid on government bonds is higher, which leads to higher levels of government indebtedness. To stabilize debt the government implements restrictive fiscal policy, resulting in government spending cuts and an increase in taxes. This kind of fiscal policy has two effects. Firstly, lower government spending further reduces output. Furthermore, larger taxes decrease the purchasing power of households, further decreasing demand. Secondly, these government measures affect the behavior of firms via marginal cost. As these firms now face increased costs, they raise prices and households need to bear the cost burden. This leads to an increase in inflation, which we can see on figure 14. It seems that the decrease in inflation was offset by the fiscal policy reaction, which results in an increase in the price level. According to these reactions both the monetary and fiscal authorities react the same way to an unexpected nominal interest rate shock. Both implement restrictive policies. Based on figure 14 we can conclude that the effects of the shocks are statistically significant, and the variables return to their respective steady states.

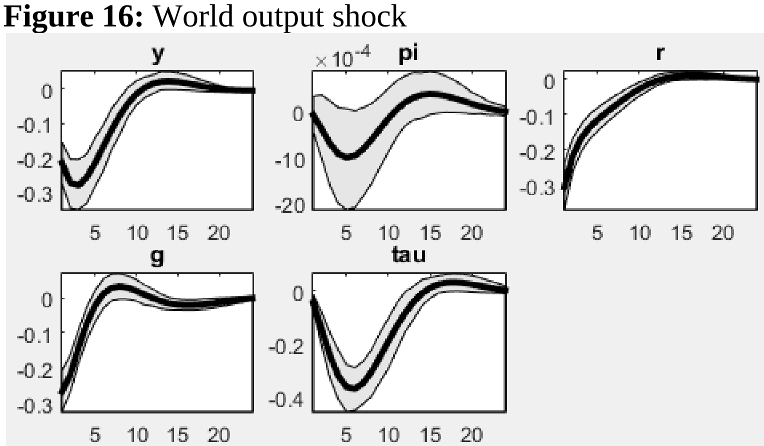
Fifthly, the effects of an unexpected cost push shock are presented on figure 15. As a result of an unexpected cost push shock, inflation increases. This inflationary pressure prompts the monetary authority to raise nominal interest rates to keep prices stable. Since the effect of inflation is bigger than the effect of the following monetary policy measure, the government debt is decreased. To avoid an even greater decrease in output and to return the debt to its steady state value the government intervenes with an expansionary fiscal policy. This results in tax cuts and should result in spending increases as well. As we can see on figure 15, however, the reaction of government spending to this cost push shock is not in accordance with economic theory. The posterior distribution of the parameters in the government spending equation differs from the prior distribution, which might cause this unexpected reaction of government spending. Based on figure 15 we can conclude that the effects of the shocks are statistically significant, and the variables return to their respective steady states.

**Figure 15:** Cost push shock




Source: author's estimates in Dynare

**Figure 16:** World output shock




Source: author's estimates in Dynare

Sixthly, the effects of an unexpected of an unexpected world demand shock are presented on figure 16. As a result of an unexpected world demand shock, both potential output and the natural interest rate decreases. This decrease in the natural interest rate is accompanied by a fall in the nominal interest rate. The lower level of nominal interest rates leads to a decrease in government debt, so the government implements expansionary fiscal policy. Taxes are cut, and government spending should increase. But this is not the case on figure 16. Because of this, the reaction of the government spending does not correspond to economic theory. The posterior distribution of the parameters in the government spending equation differs from the prior distribution, which might cause this unexpected reaction of government spending. The tax cut results in a decrease in marginal cost. As costs decrease for firms, the firms are not pushed to raise prices. Because of this, inflation is also decreased. To offset the deflationary pressure, the monetary authority ensures that interest rates remain low.

To sum up, in this subchapter we present the Bayesian impulse response functions and analyze the reactions given to unexpected shocks by the five observable endogenous variables of the model. Most of the impulse response functions present similar reactions to shocks than the impulse response functions in another study, in which the same model was estimated.[44] The exceptions are the reactions of government spending to a cost push shock and world output shock. In addition to the impulse response analysis, in this chapter we also present the data preparation, model calibration, the results of the diagnostics tests and the posterior distributions of the estimated parameters. Only the model comparison remains. We discuss this topic in the next chapter.

[44] ÇEBI, Cem. The interaction between monetary and fiscal policies in Turkey: An estimated New Keynesian DSGE mode

# 6 Discussion

As mentioned among the aims of the study, in this chapter we compare four different models. Our baseline DSGE model assumes that fiscal policy reacts to the lagged values of the output gap. A recent DSGE model calibrated for the Hungarian economy, however, assumes that fiscal policy in Hungary reacts not to the lagged output gap but to actual output.[45] For this reason we estimated four models and compared them using the Bayesian technique explained in subchapter 3.6. Model 1 represents the model in which the fiscal authority reacts to actual output. Model 2 represents the model in which the fiscal authority reacts to the lagged value of output. Model 3 represents the model in which the fiscal authority reacts to the actual value of the output gap. Model 4 represents the model in which the fiscal authority reacts to the lagged value of the output gap. The results are presented in table 4 below.

**Table 4:** Model comparison

| Model | Prior weights | Log marginal density | Bayesian probability ratio | Posterior model probability |
|---|---|---|---|---|
| **Model 1** | 0.25 | -107.69 | 1.0 | 0.00 |
| **Model 2** | 0.25 | -105.97 | 5.57 | 0.00 |
| **Model 3** | 0.25 | -97.64 | 11552.98 | 0.01 |
| **Model 4** | 0.25 | -93.06 | 1125458.05 | 0.99 |

Source: author's estimates in Dynare

The results presented in table 4 allow us to answer our question formulated in chapter 2. These results can be interpreted as follows. Model 2, 3 and 4 are compared to model 1 in table 4. Table four consists of four columns. In the first column we present the log marginal density of each model. Using the data at our disposal we estimated the likelihood function of the parameter values. As a result, we got the log marginal density of each model. Log marginal density measures how well the model parameters characterize the data. The model with the highest log marginal density the best. In the second column we present the Bayes ratio. If this ratio is larger than one, the model with the higher value better represents the data. Based on the values in the first and second columns

[45] ALGOZHINA, Aliya. Monetary and Fiscal Policy Interactions in an Emerging Open Economy: A Non-Ricardian DSGE approach.

we can conclude that model 4 is the best representation of the data. In the third column we see the posterior probability of the models - which is 0.99 for model 4 - further supporting our conclusion. This means the Hungarian economy can be better modelled if we include a fiscal authority in the model that reacts not to the past or the actual value of output, neither to the current output gap, but to the past output gap.

In the last part of this thesis we compare our results to the only DSGE model calibrated for the Hungarian economy which includes fiscal policy.[46] Firstly, in the referenced model the Calvo parameter is set at 0.90. In this thesis the estimated value of this parameter is 0.95. This means that portion of firms which can adjust their prices is even smaller than expected. Secondly, in the referenced model the degree of interest rate smoothing is 0.76. In this thesis the estimated value of this parameter is 0.73. This means that the monetary authority puts less weight than expected on past interest rates when adjusting the policy rate. Thirdly, it is also clear that the monetary authority considers not just past interest rates but macroeconomic variables as well. In the referenced study the weight on inflation is set at 1.37. In this thesis the estimated value of this parameter is 1.5, which corresponds to the literature standard. In addition, the weight on the output gap is set at 0.125 in the referenced study. In this study the estimated value of this parameter is 0.51, which is close to 0.50, the latter representing the standard value in the literature. Based on this we can conclude that compared to the referenced study, the monetary authority of the model estimated in this thesis puts slightly less weight on the past value of interest rates when setting the policy rates. On the other hand, it puts greater emphasis stimulating the economy and inflation. Fourthly, in the referenced study there are no backward looking firms. In this study the estimated value of the parameter characterizing backward looking price setting behavior is 0.75. This means that a huge portion of firms sets adjust their prices according to historical price behavior. Since the portion of these firms is high in Hungary, the DSGE models estimated in the future for the Hungarian economy should take this parameter into consideration. Fifthly, the reaction of taxes and government spending to debt is set at 0.4 in the referenced study. In this study the estimated values of these parameters are 0.4 and 0.36, respectively. This means that according to the results of our estimation the fiscal authority puts less weight on debt when it considers government spending adjustments. To sum up, when we compare our results to the referenced study, we mostly find

[46] ALGOZHINA, Aliya. Monetary and Fiscal Policy Interactions in an Emerging Open Economy: A Non-Ricardian DSGE approach.

minor differences, with the output gap and the autoregressive parameter of technology being the exception.

When interpreting the results, we need to keep in mind that the posterior distribution of the parameters in the government spending equation differs from the prior distribution. The empirical responses of government spending to the cost push shock and world output shock are different than what we expected. The difference between the prior and posterior distributions might cause this unexpected reaction of government spending. The analysis of this difference might be a subject of further study.

# Conclusion

In this thesis we reviewed the existing literature on DSGE models and New Keynesian economics, described what innovations did New Keynesianism bring to economic modelling, and the differences between New Keynesianism and the original Real Business Cycle framework. Based on the literature review we formulated the main aim of this thesis and set partial goals.

We summarized the methodology and methods, including data preparation, log-linearization, calibration, Bayesian estimation, impulse response functions. We also described the software used for the estimation. After summarizing the methods, we formulated the non-linear model and log-linearized it around the steady state.

In chapter 5 we calibrated the model parameters, selected the priors and estimated the log-linearized equations. At the end of chapter 5 we analyzed the estimated results using impulse response functions. We focused our attention on the type of fiscal policy and monetary policy implemented by the authorities. Sometimes restrictive fiscal policy was accompanied by restrictive monetary policy. At other times the policies adopted by the fiscal and monetary authorities were different from each other. For example, in the case of the government spending shock expansionary fiscal policy was accompanied by restrictive monetary policy. Most of the estimated impulse response functions presented similar reactions to shocks than the impulse response functions in another study, in which the same model was estimated. In this thesis, however, two impulse response functions were not in accordance with economic theory.

In the last part of the thesis we compared four models, each with a different fiscal policy regime. We concluded that our baseline model is the best at describing the available data. The baseline model reacts to past output gap. At the end we compared our estimated parameters to the parameters of the only published Hungarian DSGE model which includes fiscal policy. Most of the parameters differed only slightly, except for the output gap and the autoregressive parameter of the technology shock.

The principal aim of this thesis was to estimate a small open economy DSGE model with monetary and fiscal policy and analyze the interaction of these policies in Hungary. Based on the paragraphs above we can conclude that we achieved the principal aim of this thesis.

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# Appendix

## Appendix A – program for estimating the parameters of model 4

```
// endogenous variables
var y sigmaalpha omega pi lambdab lambdaf kappa mc b r g tau yn rn yw tech;
predetermined_variables b;

// exogenous variables
varexo eps_tech eps_pi eps_yw eps_r eps_g eps_tau;

// parameters
parameters cbeta ceta cgamma calpha ccons cb ctheta cphi csigma crho_r cr_pi
cr_y crho_g cg_y crho_tau ctau_y cg_b ctau_b cxi crho_tech crho_yw;

// calibration
cbeta = 0.99;
ceta = 1.0;
cgamma = 1.0;
calpha = 0.69;
ccons = 0.51;
cb = 0.78;

// model
model;
y=y(+1)-g(+1)+calpha*(omega-1)*(crho_yw-1)*yw-(1/sigmaalpha)*(r-pi(+1));
yw=crho_yw*yw(-1)+eps_yw;
tech=crho_tech*tech(-1)+eps_tech;
yn=((1+cphi)/(sigmaalpha+cphi))*tech-((csigma-
sigmaalpha)/(sigmaalpha+cphi))*yw;
sigmaalpha=(csigma/((1-calpha)+calpha*omega));
omega=csigma*cgamma+(1-calpha)*(csigma*ceta-1);
pi=lambdab*pi(-1)+lambdaf*pi(+1)+kappa*mc+eps_pi;
lambdab=(cxi/(ctheta+cxi*(1-ctheta*(1-cbeta))));
lambdaf=(cbeta*ctheta/(ctheta+cxi*(1-ctheta*(1-cbeta))));
kappa=(((1-cbeta*ctheta)*(1-ctheta)*(1-cxi))/(ctheta+cxi*(1-ctheta*(1-
cbeta))));
mc=(sigmaalpha+cphi)*(y-yn)-sigmaalpha*g+tau;
b(+1)=r+(1/cbeta)*(b-pi+(1-cbeta)*(tau-y)+(ccons/cb)*(g-tau));
rn=sigmaalpha*(yn(+1)-yn)+sigmaalpha*calpha*(omega-1)*(crho_yw-1)*yw;
r=crho_r*(r(-1)-rn(-1))+(1-crho_r)*(cr_pi*pi+cr_y*(y-yn))+rn+eps_r;
g=crho_g*g(-1)+(1-crho_g)*(cg_y*(y(-1)-yn(-1))-cg_b*b)+eps_g;
tau=crho_tau*tau(-1)+(1-crho_tau)*(ctau_y*(y(-1)-yn(-1))+ctau_b*b)+eps_tau;
end;

// specifying the steady state initial values
initval;
y=0;
sigmaalpha=0;
omega=0;
pi=0;
lambdab=0;
lambdaf=0;
kappa=0;
```

```
mc=0;
b=0;
r=0;
g=0;
tau=0;
yn=0;
rn=0;
yw=0;
tech=0;
end;


// observable variables
varobs y pi r g tau;

//priors
estimated_params;
// name      distribution     mean     stdev
ctheta,         beta_pdf,    0.93,    0.02;
cphi,           normal_pdf, 3.0,     0.2;
csigma,         normal_pdf, 2.5,     0.2;
crho_r,         beta_pdf,    0.76,    0.05;
cr_pi,          gamma_pdf,  1.5,     0.4;
cr_y,           gamma_pdf,  0.5,   0.1;
crho_g,         beta_pdf,    0.68,    0.14;
cg_y,           normal_pdf, 0.5,     0.2;
crho_tau,       beta_pdf,    0.73,    0.05;
ctau_y,         normal_pdf, 0.63,     0.2;
cg_b,           normal_pdf, 0.4,    0.02;
ctau_b,         normal_pdf, 0.4,    0.02;
cxi,            beta_pdf,    0.75,     0.05;
crho_tech,      beta_pdf,    0.5,     0.15;
crho_yw,        beta_pdf,    0.75,     0.05;
stderr eps_tech,    inv_gamma_pdf, 0.01, 2.0;
stderr eps_pi,      inv_gamma_pdf, 0.005, 2.0;
stderr eps_yw,      inv_gamma_pdf, 1.2,  4.0;
stderr eps_r,       inv_gamma_pdf, 0.3, 2.0;
stderr eps_g,       inv_gamma_pdf, 3.3, 4.0;
stderr eps_tau,     inv_gamma_pdf, 1.8, 4.0;
end;
// model estimation
estimation(datafile=cchun,mode_compute=4,mh_replic=50000,mh_nblocks=2,mh_drop=
0.15,mh_jscale=0.44,bayesian_irf,irf=2)y pi r g tau;
```