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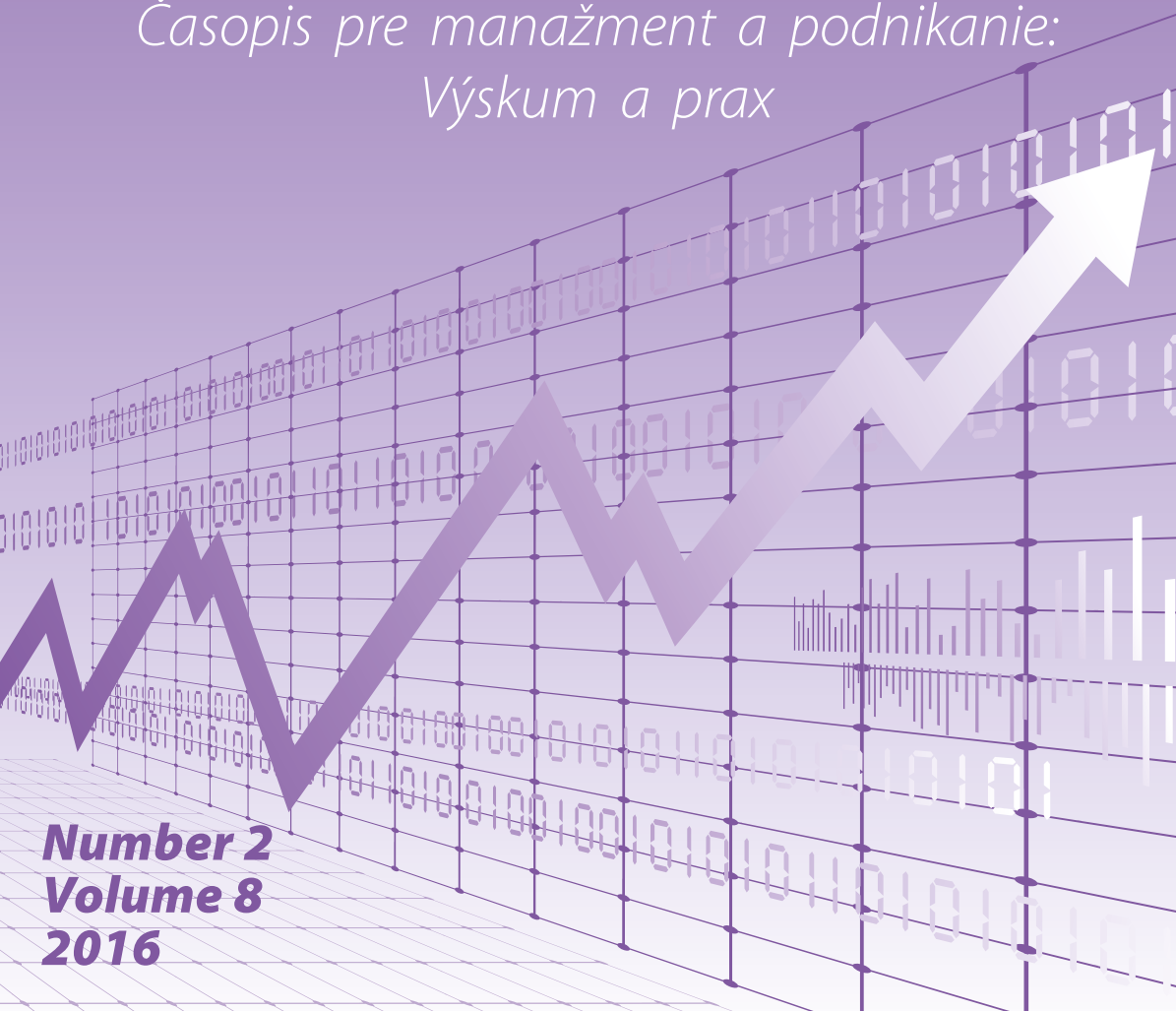
Časopis pre manažment a podnikanie: Výskum a prax



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APROXIMÁCIA PRVEJ DERIVÁCIE DIFERENCIAMI A JEJ APLIKÁCIE V MANAŽMENTE

Abstract: *This paper deals with the possibility to approximate the first derivative by means of the differences. This approximation represents a method for the determination of the differential equations solutions. These equations describe a wide variety of phenomena in the different areas of the scientific research through the mathematical facilities. A main part of the paper is based on the undetermined coefficient method that is used for the approximation of the first derivative by the function values which are lying to the right of the given point. The method is generalized to the arbitrary natural* n *function values. In this paper, we also describe a possibility to increase an approximation degree of the derivative by the combination of the values with the different signs, an influence of the distance between two adjacent abscissa values, as well as an influence of the derivative degree on the accuracy of the obtained results.*
A practical meaning of the paper consists in the possibility to make a sufficiently precise approach of the first derivative that can appear in the various differential equations. By the way, the application of such differential equations is also in the areas of the financial, economic or environmental models.
Key words: *approximation, derivative, Taylor theorem, Cramer's rule, determinant.*

Kľúčové slová: *aproximácia, derivácie, Taylorova veta, Cramerovo pravidlo, determinant.*

JEL: C02, C63, C65

Introduction

The differential equations are used for the phenomena descriptions in many areas like management, finance, economy (the Black-Scholes model of the options pricing, the elasticity of demand model, the environmental management models, etc.). The differential equations are mathematical equations that contain the derivatives of the unknown functions. The direct solutions of such equations can be often very difficult due to a complexity of the analytic expression. Therefore, it is important to find an appropriate way for the obtaining of more accurate solutions that describe the studied phenomenon.
The special methods are often used for the solution of these equations and they consist in the derivatives substitution.

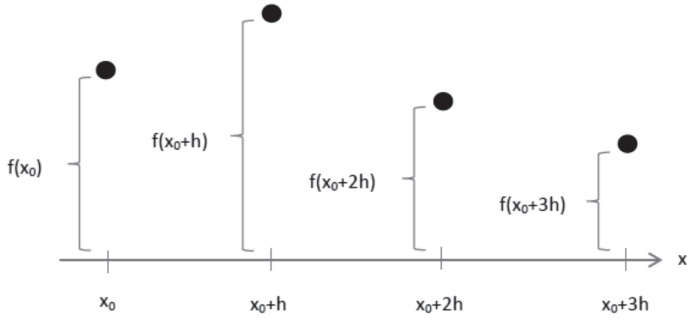
It is possible to approximate the first derivative by several ways. One way is to express a complex derivative function by means of a polynomial. Just polynomials are the simplest functions that can be directly calculated on PC, they are easy handled, and they are easy differentiated and integrated. The finite element method [4] is often used as well.
In this paper, we present a method of undetermined coefficients that provides a relationship for the approximation of the first derivative by means of the n given function values and the applications of this method in management.

Methods

1 Derivative approximation with differences by undetermined coefficients method

Let us consider on x axis the points $x_0, x_0 + h, x_0 + 2h, x_0 + 3h$ and the corresponding function values $f(x_0), f(x_0 + h), f(x_0 + 2h), f(x_0 + 3h)$. We assume that h is small positive real number and x_0 is an arbitrary real number. The considered points and their function values are shown in the Fig. 1.

Fig. 1 System for three point approximation



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Our main goal is to approximate the first derivative $f'(x_0)$ of the function f in the point x_0, by means of these above function values.
Therefore, we require the following $f'(x_0) \approx a_0 \cdot f(x_0) + a_1 \cdot f(x_0 + h) + a_2 \cdot f(x_0 + 2h) + a_3 \cdot f(x_0 + 3h)$ to be valid where a_0, a_1, a_2, a_3 are unknown coefficients that must be determined.

The previous statement can be rewritten in the form

$$
\begin{aligned}
f'(x_0) \approx a_0 \cdot f(x_0) + a_1 \cdot f(x_0 + h) + a_2 \cdot f(x_0 + 2h) + a_3 \cdot f(x_0 + 3h) = \\
= a_0 \cdot f(x_0) + \\
+ a_1 \cdot \left[f(x_0) + \frac{h}{1!} f'(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^3}{3!} f'''(x_0) + \frac{h^4}{4!} f^{IV}(x_0) + \frac{h^5}{5!} f^{V}(x_0) + \ldots \right] + \\
+ a_2 \cdot \left[f(x_0) + \frac{2h}{1!} f'(x_0) + \frac{(2h)^2}{2!} f''(x_0) + \frac{(2h)^3}{3!} f'''(x_0) + \frac{(2h)^4}{4!} f^{IV}(x_0) + \frac{(2h)^5}{5!} f^{V}(x_0) + \ldots \right] + \\
+ a_3 \cdot \left[f(x_0) + \frac{3h}{1!} f'(x_0) + \frac{(3h)^2}{2!} f''(x_0) + \frac{(3h)^3}{3!} f'''(x_0) + \frac{(3h)^4}{4!} f^{IV}(x_0) + \frac{(3h)^5}{5!} f^{V}(x_0) + \ldots \right] = \\
= (a_0 + a_1 + a_2 + a_3) \cdot f(x_0) + (a_1 + 2a_2 + 3a_3) \cdot \frac{h}{1!} f'(x_0) + (a_1 + 2^2 a_2 + 3^2 a_3) \cdot \frac{h^2}{2!} f''(x_0) + \\
+ (a_1 + 2^3 a_2 + 3^3 a_3) \cdot \frac{h^3}{3!} f'''(x_0) + \ldots
\end{aligned}
$$

where Taylor theorem was used.

Since the first derivative is approximated, we conclude from the last relation that the expressions near the values $f(x_0), f''(x_0), f'''(x_0)$ must be equal to zero and the expression near the first derivative $f'(x_0)$ must be equal to one. Hence, we deduce the following system of equations

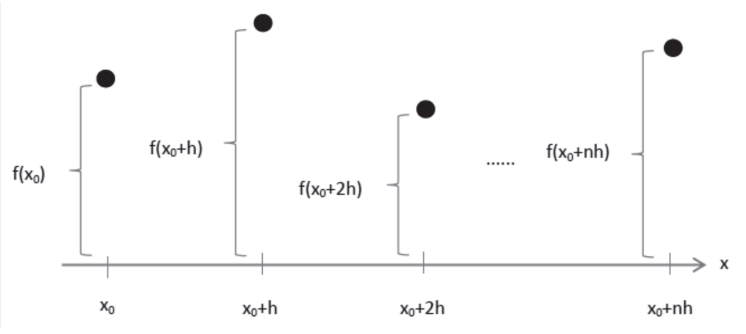
$$
\begin{aligned}
a_0 + a_1 + a_2 + a_3 &= 0 \\
a_1 + 2a_2 + 3a_3 &= 1/h \\
a_1 + 2^2 a_2 + 3^2 a_3 &= 0 \\
a_1 + 2^3 a_2 + 3^3 a_3 &= 0
\end{aligned}
$$

for unknown coefficients a_0, a_1, a_2, a_3. This system of equations can be rewritten to the matrix form as follows

$$
\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2^2 & 3^2 \\ 0 & 1 & 2^3 & 3^3 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1/h \\ 0 \\ 0 \end{pmatrix}.
$$

By analogy, the generalized system of equations for points $x_0, x_0 + h, x_0 + 2h, \ldots, x_0 + nh$ can be deduced (see Fig. 2)

Fig. 2 The system for n-point approximation



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while the desired linear combination has the form

$$f'(x_0) \approx a_0 \cdot f(x_0) + a_1 \cdot f(x_0 + h) + a_2 \cdot f(x_0 + 2h) + \ldots + a_n \cdot f(x_0 + nh) \qquad (1)$$

and the matrix form of the system is the following

$$\begin{pmatrix} 1 & 1 & 1 & 1 & \ldots & 1 & 1 \\ 0 & 1 & 2 & 3 & \ldots & (n-1) & n \\ 0 & 1^2 & 2^2 & 3^2 & \ldots & (n-1)^2 & n^2 \\ 0 & 1^3 & 2^3 & 3^3 & \ldots & (n-1)^3 & n^3 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1^n & 2^n & 3^n & \ldots & (n-1)^n & n^n \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} 0 \\ 1/h \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad (2)$$

We solve the generalized system (2) by means of the Cramer rule.

2 Calculation of the system determinant

The system determinant $V_{n\times n}$ in the form

$$V_{n\times n} = \begin{vmatrix} 1 & 2 & 3 & \ldots & (n-1) & n \\ 1^2 & 2^2 & 3^2 & \ldots & (n-1)^2 & n^2 \\ 1^3 & 2^3 & 3^3 & \ldots & (n-1)^3 & n^3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1^{n-1} & 2^{n-1} & 3^{n-1} & \ldots & (n-1)^{n-1} & n^{n-1} \\ 1^n & 2^n & 3^n & \ldots & (n-1)^n & n^n \end{vmatrix},$$

we adjust according to the following steps

1.1 we transpose the determinant

$$V_{n\times n}=\begin{vmatrix} 1 & 2 & 3 & \dots & (n-1) & n \\ 1^2 & 2^2 & 3^2 & \dots & (n-1)^2 & n^2 \\ 1^3 & 2^3 & 3^3 & \dots & (n-1)^3 & n^3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1^{n-1} & 2^{n-1} & 3^{n-1} & \dots & (n-1)^{n-1} & n^{n-1} \\ 1^n & 2^n & 3^n & \dots & (n-1)^n & n^n \end{vmatrix} = \begin{vmatrix} 1^1 & 1^2 & 1^3 & \dots & 1^{n-1} & 1^n \\ 2^1 & 2^2 & 2^3 & \dots & 2^{n-1} & 2^n \\ 3^1 & 3^2 & 3^3 & \dots & 3^{n-1} & 3^n \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ (n-1)^1 & (n-1)^2 & (n-1)^3 & \dots & (n-1)^{n-1} & (n-1)^n \\ n^1 & n^2 & n^3 & \dots & n^{n-1} & n^n \end{vmatrix} =$$

(the transposed matrix)

1.2 then, we take out the number in the first column from each row, it means, we take out number 1 from the first row, number 2 from the second row, and so on to n, to form the following product

$$= 1\cdot 2\cdot 3\cdot \ldots (n-1)\cdot n\cdot \underbrace{\begin{vmatrix} 1 & 1^1 & 1^2 & \dots & 1^{n-2} & 1^{n-1} \\ 1 & 2^1 & 2^2 & \dots & 2^{n-2} & 2^{n-1} \\ 1 & 3^1 & 3^2 & \dots & 3^{n-2} & 3^{n-1} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & (n-1)^1 & (n-1)^2 & \dots & (n-1)^{n-2} & (n-1)^{n-1} \\ 1 & n^1 & n^2 & \dots & n^{n-2} & n^{n-1} \end{vmatrix}}_{D_{n\times n}} =$$

1.3 next, we "make" as much as possible the zero elements in the first column by the row operations. In this case, we add the *(-1)*-multiple of the first row to each row

1.4

$$= n!\cdot \underbrace{\begin{vmatrix} 1 & 1^1 & 1^2 & \dots & 1^{n-2} & 1^{n-1} \\ 1 & 2^1 & 2^2 & \dots & 2^{n-2} & 2^{n-1} \\ 1 & 3^1 & 3^2 & \dots & 3^{n-2} & 3^{n-1} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & (n-1)^1 & (n-1)^2 & \dots & (n-1)^{n-2} & (n-1)^{n-1} \\ 1 & n^1 & n^2 & \dots & n^{n-2} & n^{n-1} \end{vmatrix}}_{D_{n\times n}} \begin{matrix} (-1) \\ \hookleftarrow \\ \hookleftarrow \\ \\ \hookleftarrow \\ \hookleftarrow \end{matrix} =$$

$$= n!\cdot \begin{vmatrix} 1 & 1^1 & 1^2 & \dots & 1^{n-2} & 1^{n-1} \\ 0 & 2^1-1^1 & 2^2-1^2 & \dots & 2^{n-2}-1^{n-2} & 2^{n-1}-1^{n-1} \\ 0 & 3^1-1^1 & 3^2-1^2 & \dots & 3^{n-2}-1^{n-2} & 3^{n-1}-1^{n-1} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & (n-1)^1-1^1 & (n-1)^2-1^2 & \dots & (n-1)^{n-2}-1^{n-2} & (n-1)^{n-1}-1^{n-1} \\ 0 & n^1-1^1 & n^2-1^2 & \dots & n^{n-2}-1^{n-2} & n^{n-1}-1^{n-1} \end{vmatrix} =$$

1.5 subsequently, we add the negative value of the previous column to each column; the final modification looks like

$$\begin{array}{c} (-1) \longrightarrow (-1) \longrightarrow (-1) \longrightarrow (-1) \longrightarrow (-1) \longrightarrow \\ = n! \cdot \begin{vmatrix} 1 & 1^1 & 1^2 & \dots & 1^{n-2} & 1^{n-1} \\ 0 & 2^1 - 1^1 & 2^2 - 1^2 & \dots & 2^{n-2} - 1^{n-2} & 2^{n-1} - 1^{n-1} \\ 0 & 3^1 - 1^1 & 3^2 - 1^2 & \dots & 3^{n-2} - 1^{n-2} & 3^{n-1} - 1^{n-1} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & (n-1)^1 - 1^1 & (n-1)^2 - 1^2 & \dots & (n-1)^{n-2} - 1^{n-2} & (n-1)^{n-1} - 1^{n-1} \\ 0 & n^1 - 1^1 & n^2 - 1^2 & \dots & n^{n-2} - 1^{n-2} & n^{n-1} - 1^{n-1} \end{vmatrix} = \end{array}$$

$$= n! \cdot \begin{vmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 \cdot 2 & \dots & 1 \cdot 2^{n-3} & 1 \cdot 2^{n-2} \\ 0 & 2 & 2 \cdot 3 & \dots & 2 \cdot 3^{n-3} & 2 \cdot 3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & n-2 & (n-2) \cdot (n-1) & \dots & (n-2) \cdot (n-1)^{n-3} & (n-2) \cdot (n-1)^{n-2} \\ 0 & n-1 & (n-1) \cdot n & \dots & (n-1) \cdot n^{n-3} & (n-1) \cdot n^{n-2} \end{vmatrix} =$$

1.6 we reduce the determinant degree by the Laplace expansion according to the first row and we have

$$= n! \cdot \begin{vmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 \cdot 2 & \dots & 1 \cdot 2^{n-3} & 1 \cdot 2^{n-2} \\ 0 & 2 & 2 \cdot 3 & \dots & 2 \cdot 3^{n-3} & 2 \cdot 3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & n-2 & (n-2) \cdot (n-1) & \dots & (n-2) \cdot (n-1)^{n-3} & (n-2) \cdot (n-1)^{n-2} \\ 0 & n-1 & (n-1) \cdot n & \dots & (n-1) \cdot n^{n-3} & (n-1) \cdot n^{n-2} \end{vmatrix} =$$

$$= n! \cdot 1 \cdot (-1)^{1+1} \cdot \underbrace{\begin{vmatrix} 1 & 1 \cdot 2 & \dots & 1 \cdot 2^{n-3} & 1 \cdot 2^{n-2} \\ 2 & 2 \cdot 3 & \dots & 2 \cdot 3^{n-3} & 2 \cdot 3^{n-3} \\ \vdots & \vdots & & \vdots & \vdots \\ n-2 & (n-2) \cdot (n-1) & \dots & (n-2) \cdot (n-1)^{n-3} & (n-2) \cdot (n-1)^{n-2} \\ n-1 & (n-1) \cdot n & \dots & (n-1) \cdot n^{n-3} & (n-1) \cdot n^{n-2} \end{vmatrix}}_{D_{(n-1)\times(n-1)}}.$$

Thus, the new determinant appears that, to analogy to the step 1.2, allows to take out the factors 1, 2, 3, ..., $n-2$, $n-1$ from the rows. We continue in the similar arrangements as in the steps 1.2 – 1.5, i.e.

2.1. we take out the number that is in the first column from each row. In our case, these numbers are $1, 2, 3, \ldots, n-2, n-1$. Hence, the previous determinant can be rewritten to the form

$$= n! \cdot 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-2) \cdot (n-1) \cdot \underbrace{\begin{vmatrix} 1 & 2^1 & 2^2 & \ldots & 2^{n-3} & 2^{n-2} \\ 1 & 3^1 & 3^2 & \ldots & 3^{n-3} & 3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & n-1 & (n-1)^2 & \ldots & (n-1)^{n-3} & (n-1)^{n-2} \\ 1 & n & n^2 & \ldots & n^{n-3} & n^{n-2} \end{vmatrix}}_{D_{(n-1)\times(n-1)}} =$$

2.2. or

$$= n! \cdot (n-1)! \cdot \begin{vmatrix} 1 & 2^1 & 2^2 & \ldots & 2^{n-3} & 2^{n-2} \\ 1 & 3^1 & 3^2 & \ldots & 3^{n-3} & 3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & n-1 & (n-1)^2 & \ldots & (n-1)^{n-3} & (n-1)^{n-2} \\ 1 & n & n^2 & \ldots & n^{n-3} & n^{n-2} \end{vmatrix} =$$

2.3. next, we "make" the zero elements in the first column by subtraction the first row from each row

$$= n! \cdot (n-1)! \cdot \begin{vmatrix} 1 & 2^1 & 2^2 & \ldots & 2^{n-3} & 2^{n-2} \\ 1 & 3^1 & 3^2 & \ldots & 3^{n-3} & 3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & n-1 & (n-1)^2 & \ldots & (n-1)^{n-3} & (n-1)^{n-2} \\ 1 & n & n^2 & \ldots & n^{n-3} & n^{n-2} \end{vmatrix} \begin{matrix} (-1) \\ \hookleftarrow \\ \hookleftarrow \\ \\ \hookleftarrow \\ \hookleftarrow \end{matrix} =$$

$$= n! \cdot (n-1)! \cdot \begin{vmatrix} 1 & 2^1 & 2^2 & \ldots & 2^{n-3} & 2^{n-2} \\ 0 & 3^1 - 2^1 & 3^2 - 2^2 & \ldots & 3^{n-3} - 2^{n-3} & 3^{n-2} - 2^{n-2} \\ 0 & 4^1 - 2^1 & 4^2 - 2^2 & \ldots & 4^{n-3} - 2^{n-3} & 4^{n-2} - 2^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & (n-1)^1 - 2^1 & (n-1)^2 - 2^2 & \ldots & (n-1)^{n-3} - 2^{n-3} & (n-1)^{n-2} - 2^{n-2} \\ 0 & n^1 - 2^1 & n^2 - 2^2 & \ldots & n^{n-3} - 2^{n-3} & n^{n-2} - 2^{n-2} \end{vmatrix} =$$

2.4. we create the zero elements in the first row by the following arrangement: we sequentially add the double negative value of the previous column

$$\begin{array}{c} \text{(-2)} \quad \text{(-2)} \quad\quad\quad \text{(-2)} \quad \text{(-2)} \quad \text{(-2)} \quad\quad\quad \\ = n! \cdot (n-1)! \cdot \begin{vmatrix} 1 & 2^1 & 2^2 & \dots & 2^{n-3} & 2^{n-2} \\ 0 & 3^1 - 2^1 & 3^2 - 2^2 & \dots & 3^{n-3} - 2^{n-3} & 3^{n-2} - 2^{n-2} \\ 0 & 4^1 - 2^1 & 4^2 - 2^2 & \dots & 4^{n-3} - 2^{n-3} & 4^{n-2} - 2^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & (n-1)^1 - 2^1 & (n-1)^2 - 2^2 & \dots & (n-1)^{n-3} - 2^{n-3} & (n-1)^{n-2} - 2^{n-2} \\ 0 & n^1 - 2^1 & n^2 - 2^2 & \dots & n^{n-3} - 2^{n-3} & n^{n-2} - 2^{n-2} \end{vmatrix} = \end{array}$$

$$= n! \cdot (n-1)! \cdot \begin{vmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 \cdot 3^1 & \dots & 1 \cdot 3^{n-4} & 1 \cdot 3^{n-3} \\ 0 & 2 & 2 \cdot 4^1 & \dots & 2 \cdot 4^{n-4} & 2 \cdot 4^{n-3} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & n-3 & (n-3) \cdot (n-1) & \dots & (n-3) \cdot (n-1)^{n-4} & (n-3) \cdot (n-1)^{n-3} \\ 0 & n-2 & (n-2) \cdot n & \dots & (n-2) \cdot n^{n-4} & (n-2) \cdot n^{n-3} \end{vmatrix} =$$

2.5. we reduce the determinant degree by means of the Laplace expansion according to the first row. This provide us

$$= n! \cdot (n-1)! \cdot 1 \cdot (-1)^{1+1} \cdot \underbrace{\begin{vmatrix} 1 \cdot 3^0 & 1 \cdot 3^1 & 1 \cdot 3^2 & \dots & 1 \cdot 3^{n-4} & 1 \cdot 3^{n-3} \\ 2 \cdot 4^0 & 2 \cdot 4^1 & 2 \cdot 4^2 & \dots & 2 \cdot 4^{n-4} & 2 \cdot 4^{n-3} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ (n-3) \cdot (n-1)^0 & (n-3) \cdot (n-1)^1 & (n-3) \cdot (n-1)^2 & \dots & (n-3) \cdot (n-1)^{n-4} & (n-3) \cdot (n-1)^{n-3} \\ (n-2) \cdot n^0 & (n-2) \cdot n^1 & (n-2) \cdot n^2 & \dots & (n-2) \cdot n^{n-4} & (n-2) \cdot n^{n-3} \end{vmatrix}}_{D_{(n-2)\times(n-2)}} \quad (3)$$

One can see a certain similarity between the individual arrangement steps during the modification of the determinant of the system when the partial determinants are quite similar but they differ in the degree and in the multiples.

We can use an equal arrangement for the expression (3) in further. The individual modifications are presented in detail in the work [8]. One can come to the determinant $V_{n\times n}$, that can be generalized as follows

$$= n! \cdot (n-1)!\,(n-2)! \cdot \ldots \cdot (n-k)! \cdot \begin{vmatrix} 1 & (k+1)^1 & (k+1)^2 & \ldots & (k+1)^{n-(k+2)} & (k+1)^{n-(k+1)} \\ 1 & (k+2)^1 & (k+2)^2 & \ldots & (k+2)^{n-(k+2)} & (k+2)^{n-(k+1)} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & n-1 & (n-1)^2 & \ldots & (n-1)^{n-(k+2)} & (n-1)^{n-(k+1)} \\ 1 & n & n^2 & \ldots & n^{n-(k+2)} & n^{n-(k+1)} \end{vmatrix} =$$

If $k = n-2$ we obtain the final result of the determinant $V_{n\times n}$

$$= n! \cdot (n-1)!\,(n-2)! \cdot \ldots 3!\,2! \cdot \begin{vmatrix} (n-1)^0 & (n-1)^1 \\ n^0 & n^1 \end{vmatrix} =$$

$$= \boldsymbol{n!\,(n-1)!\,(n-2)! \cdot \ldots \cdot 3!\,2!\,1} = n^1(n-1)^2(n-2)^3 \ldots 3^{n-2}2^{n-1}1^n.$$

As the determinant $V_{n\times n}$ of the matrix of the system is different from zero, one can use the Cramer rule and can follow in the calculation of needed determinants. During the calculations the similar arrangement is applied as the arrangement of the determinant $V_{n\times n}$ of the matrix of the system. The specific steps of the arrangement with the calculated final coefficient values one can find in the work [8]. On the base of these calculations, the following results for the individual coefficients can be summarized

$$\begin{aligned}
a_0 &= (-1)^{0+1} \cdot \tfrac{1}{h}\left(1 + \tfrac{1}{2} + \tfrac{1}{3} + \cdots + \tfrac{1}{n-1} + \tfrac{1}{n}\right) && f(x_0) \\
a_1 &= (-1)^{1+1} \cdot \tfrac{1}{1\cdot h} \cdot \binom{n}{1} && f(x_0 + 1\cdot h) \\
a_2 &= (-1)^{2+1} \cdot \tfrac{1}{2\cdot h} \cdot \binom{n}{2} && f(x_0 + 2\cdot h) \\
a_3 &= (-1)^{3+1} \cdot \tfrac{1}{3\cdot h} \cdot \binom{n}{3} && f(x_0 + 3\cdot h) \\
a_4 &= (-1)^{4+1} \cdot \tfrac{1}{4\cdot h} \cdot \binom{n}{4} && f(x_0 + 4\cdot h) \\
&\vdots && \vdots \\
a_k &= (-1)^{k+1} \cdot \tfrac{1}{k\cdot h} \cdot \binom{n}{k} && f(x_0 + k\cdot h) \\
&\vdots && \vdots \\
a_{n-1} &= (-1)^{n-1+1} \cdot \tfrac{1}{(n-1)\cdot h} \cdot \binom{n}{n-1} && f(x_0 + (n-1)\cdot h) \\
a_n &= (-1)^{n+1} \cdot \tfrac{1}{n\cdot h} \cdot \binom{n}{n} && f(x_0 + n\cdot h)
\end{aligned} \qquad (4)$$

After substituting these coefficients to the relation (1), we will have

$$f'(x_0) \approx \frac{1}{h} \cdot \left[-f(x_0) \cdot \sum_{k=1}^{n} \frac{1}{k} + \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} \binom{n}{k} \cdot f(x_0 + k \cdot h) \right]$$

or, if one takes to the account the reminder terms in expansion, we have the estimation

$$\left| f'(x_0) - \frac{1}{h} \left[-f(x_0) \cdot \sum_{k=1}^{n} \frac{1}{k} + \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} \cdot \binom{n}{k} \cdot f(x_0 + k \cdot h) \right] \right| \leq c \cdot h^n$$

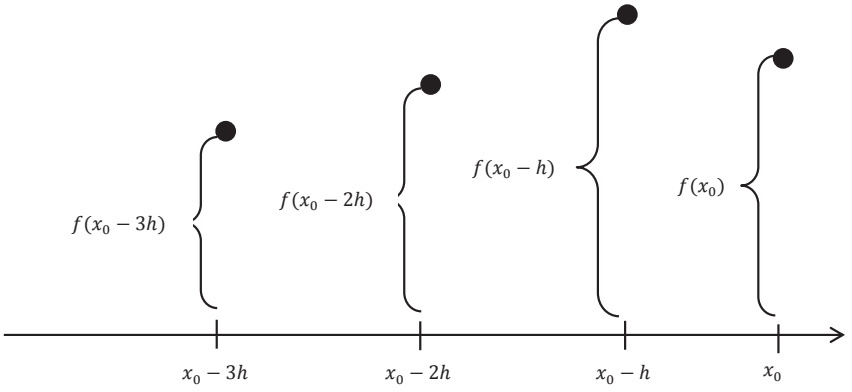
This relation expresses the approximation of the first derivative of the function f by means of the difference values and provides more accurate approximation. The derivation $f'(x_0)$ does not depend on h. Only the subtrahend on the left hand side depends on h. If h is a small positive value then the right hand side of the estimation is small as well and this means that the derivative $f'(x_0)$ can be approximated by means of the subtrahend. It holds, the smaller is h (that represents the distance between two adjacent values on the axis x) the more accurate is approximation.
The accuracy of the approximation of the first derivative depends also on the number n of the terms in the Taylor expansion. The larger is n, the higher is approximation accuracy. Thus, this relation is opposite than to the h number. It is clear that one can adapt the accuracy of the approximation to the practical needs by means of the suitable choice of the numbers n and h.

Discussion

1 Estimation of the first derivative approximation by the function values defined to the left of the given point

The approximation formula in the previous part of this paper expresses the estimation for the first derivative approximation by means of the function values that are defined to the right of the given point x_0. In the case when the function values are defined to the left of the given point x_0, we can define the similar system where the points on the x axis are $x_0, x_0 - h, x_0 - 2h, x_0 - 3h$ with appropriate function values $f(x_0), f(x_0 - h), f(x_0 - 2h), f(x_0 - 3h)$, as can be seen on the Fig. 3

Fig. 3 Three point approximation by the defined points lying to the left of the given point x_0



Source: own processing

The goal is again to approximate the first derivative *f'(x₀)* of the function *f* in the point x_0 by the upper mentioned function values. Therefore, we require to be valid

$$f'(x_0) \approx a_0 \cdot f(x_0) + a_1 \cdot f(x_0 - h) + a_2 \cdot f(x_0 - 2h) + a_3 \cdot f(x_0 - 3h),$$

By the Taylor theorem, we gradually come to the system in the matrix form

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2^2 & 3^2 \\ 0 & 1 & 2^3 & 3^3 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1/h \\ 0 \\ 0 \end{pmatrix}.$$

Once again, it is possible to generalize this system for *n* terms when the approximation relation will have the form

$$f'(x_0) \approx a_0 \cdot f(x_0) + a_1 \cdot f(x_0 - h) + a_2 \cdot f(x_0 - 2h) + \ldots + a_n \cdot f(x_0 - nh)$$

and corresponding matrix form of the system is like this

$$\begin{pmatrix} 1 & 1 & 1 & 1 & \ldots & 1 & 1 \\ 0 & 1 & 2 & 3 & \ldots & (n-1) & n \\ 0 & 1^2 & 2^2 & 3^2 & \ldots & (n-1)^2 & n^2 \\ 0 & 1^3 & 2^3 & 3^3 & \ldots & (n-1)^3 & n^3 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1^n & 2^n & 3^n & \ldots & (n-1)^n & n^n \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} 0 \\ -1/h \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

It is possible to solve the system by the Cramer rule by the same way, as we made it in previous considerations and calculations while on the base of these considerations and calculations, we can come to the final resulted estimation, i.e.

$$\left| f'(x_0) - \frac{1}{h}\left[f(x_0)\cdot \sum_{k=1}^{n} \frac{1}{k} + \sum_{k=1}^{n} \frac{(-1)^k}{k} \cdot \binom{n}{k} \cdot f(x_0 - k\cdot h) \right] \right| \leq c \cdot h^n .$$

2 Increasing of the approximation degree of the first derivative by the combination of the function values to the left and right of the given point

Let us consider the points $x-h, x, x+h$ and the corresponding function values $f(x-h), f(x), f(x+h)$. By the Taylor expansion, we receive

$$f(x+h) = f(x) + \frac{f'(x)}{1!}h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \frac{f^{IV}(x)}{4!}h^4 + \frac{f^{V}(x)}{5!}h^5 + \cdots$$

$$f(x-h) = f(x) - \frac{f'(x)}{1!}h + \frac{f''(x)}{2!}h^2 - \frac{f'''(x)}{3!}h^3 + \frac{f^{IV}(x)}{4!}h^4 - \frac{f^{V}(x)}{5!}h^5 + \cdots$$

After multiplication of the last equation by (-1) and after adding of this multiple to the first equation, we obtain

$$f(x+h) - f(x-h) = 2\frac{f'(x)}{1!}h + 2\frac{f'''(x)}{3!}h^3 + 2\frac{f^{V}(x)}{5!}h^5 + \cdots$$

The next arrangement lead to the goal to secede the first derivative $f'(x)$ and thus to have an approximation of this derivative. Now, the most suitable step appears to divide the last equation by $2h$

$$\frac{f(x+h) - f(x-h)}{2h} = f'(x) + \frac{f'''(x)}{3!}h^2 + \frac{f^{V}(x)}{5!}h^4 + \frac{f^{VII}(x)}{7!}h^6 + \cdots$$

and subtraction the first derivative $f'(x)$ from both sides of the equation. At the same time, we interchange the equation sides, hence

$$f'(x) - \frac{f(x+h) - f(x-h)}{2h} = -\frac{f'''(x)}{3!}h^2 - \frac{f^{V}(x)}{5!}h^4 - \frac{f^{VII}(x)}{7!}h^6 + \cdots$$

The obtained form of the equation enables us to take out h^2 from brackets and by this way to come to the equality

$$\left| f'(x) - \frac{f(x+h)-f(x-h)}{2h} \right| = h^2 \left| -\frac{f'''(x)}{3!} - \frac{f^{V}(x)}{5!}h^2 - \frac{f^{VII}(x)}{7!}h^4 + \cdots \right| .$$

If the expression in the absolute value near h^2 is denoted as a positive constant c then we can write the general expression of the first derivative approximation by the combination of the function values $f(x+h)$ and $f(x-h)$ in the form of the following estimation

$$\left| f'(x) - \frac{f(x+h)-f(x-h)}{2h} \right| \leq c \cdot h^2 .$$

3 Examples of application of differential equations with differences in environmental management, economics and finance

The differential equations with derivatives are used in the different environmental but also in the finance and economic models. One of such models is **the model of the moisture transfer in the porous material** that is expressed by this diffusion equation

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial x}\left[D(w,t) \frac{\partial w}{\partial x} \right], t > 0, 0 < x < 1.$$

Beside to this equation, the boundary condition for $x = 0$ is adding in the form

$$\frac{\partial w}{\partial x}(0,t) = 0.$$

The derivative on the left hand side of the condition can be replaced by the difference and thus the following difference equation can be received [1]

$$\frac{3w_0^{j+1} - 4w_1^{j+1} + w_2^{j+1}}{2h} = 0.$$

This substitution corresponds with the approximation of the derivative by means of the function values defined to the right of the given point (see formula (3)) and one can use the derived approximation estimation for $n = 2$

$$\left| f'(x_0) - \frac{1}{h}\left[-f(x_0) \cdot \sum_{k=1}^{2} \frac{1}{k} + \sum_{k=1}^{2} \frac{(-1)^{k+1}}{k} \cdot \binom{2}{k} \cdot f(x_0 + k \cdot h) \right] \right| \leq c \cdot h^2 .$$

The next example for application of the derivative in the practice is the equation of **elasticity of demand**. The price elasticity of demand (or briefly elasticity of demand) is a notion which provides a measurement how much is changing the demand quantity q of a good if its price p is changing. The demands for goods vary their elasticities. The demand for victuals reacts generally a little to price changes and due to this is non-elastic while, for example, the air transport is extremely price sensitive and due to this is elastic [6].
Let us suppose that $p > 0$ is a unite price of a reference product and $q = q(p) > 0$ is a demand (required amount) on the market for this product depending on the product price. Moreover, we will suppose that the demand function q is a decreasing function (the higher the price, the lower demand, i.e. $q'(p) < 0$) [7].

The price elasticity of demand η is defined as a ration of the percentage change of the required amount $\frac{dq}{q} \cdot 100\%$ to the percentage change of the price $\frac{dp}{p} \cdot 100\%$

$$\eta = \frac{percentage\ change\ \ of\ required\ \ amount}{percentage\ change\ of\ price}$$

which can be rewritten in the form [5]

$$\eta = \eta_q(p) = -\frac{\frac{dq}{q} \cdot 100\%}{\frac{dp}{p} \cdot 100\%} = -\frac{dq}{dp} \cdot \frac{p}{q} = -q'(p) \cdot \frac{p}{q} = -p\frac{q'(p)}{q(p)}$$

provided that the function $q(p)$ expresses the required amount at price $p > 0$, the change of price is dp, the change of required demand is dq and the derivative is $q'(p)$.
Three cases of the price elasticities are distinguishing according to the required amount sensitivity to the price change. We say that the demand q at the price p

1. has the unite elasticity if $\eta_q(p) = 1$, i.e. if one percent increase in the price leads to the one percent decrease of the required amount (the total income is not changed);
2. is elastic if $\eta_q(p) > 1$, i.e. if one percent increase in the price leads to the higher than one percent decrease of the required amount;
3. is non-elastic if $\eta_q(p) < 1$, i.e. if one percent increase in the price leads to the lower than one percent decrease of the required amount [6].

Moreover, one can meet with two specific cases. In the first case, this is perfectly non-elastic demand if $\eta_q(p) = 0$, i.e. if the required amount is not changing at any change in prices. In the second case, this is the perfectly elastic demand if $\eta_q(p) = \infty$, that means a minimal change in price leads to the infinite change in the required amount [5].

For example, if it holds $\eta_q(p_0) = -p_0 \frac{q'(p_0)}{q(p_0)} = 1$ for same $p = p_0$ then we can substitute the derivative $q'(p_0)$ by the corresponding difference. According to the approximation formula, we obtain for $n = 3$

$$-\frac{p_0}{q(p_0)} \cdot \left[\frac{1}{h} q(p_0) \cdot \sum_{k=1}^{3} \frac{1}{k} + \sum_{k=1}^{3} \frac{(-1)^k}{k} \cdot \binom{3}{k} \cdot q(p_0 - k \cdot h) \right] = 1$$

It has, however, a sense to approximate the elasticity of demand only if the function $q(p)$ is unknown.

As the next example, we can present the enforcement of the differential equations in finance area which is represented by the **Black-Scholes model**. This model presents a model of the option pricing that provides an analytic solution for the price specification of selected types of options. The Black-Scholes model can be applied to the European call options to shares (vanilla options) which are not paid dividends. At the same time, the model can be applied to the American call options (do not carry dividends to share) because it is not possible to realize an optimal option before the date of expiration [3].
The solution of the partial differential Black-Scholes equation was studied by many authors in the different views. For example, Tagliani and Milev in their work [9] dealt with the method that connects the Laplace transformation and the method of finite differences. They solved the partial differential Black-Scholes equation

$$-\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0$$

by the mentioned method where t – is the time to expiry T, S – is the current value of the underlying asset (option price in Euro), r – risk-free interest rate, σ – volatility of the option price, $V(S,t)$ – is derivate price as a function of time and option price. According to the derivative type, a terminal condition $V(S,T)$ is adding to the Black-Scholes equation at the terminated time T of expiration. In the work [9], authors study two different approaches based both on the difference method.

In the work [2], the different effective methods are presented for an enhanced Black-Scholes model where is shown, at the same time, a possibility for approximation of the derivative $\frac{\partial V}{\partial S}$. This derivative is replaced by the difference $\frac{V(S+h)-V(S-h)}{2h}$ in the work. This replacement corresponds to the approximation of the derivative by combination of function values to the left and to the right of the given point *S*.

Summary

The paper deals with the possibility to approximate the first derivative by differences and describes, at the same time, an individual possibility to apply this approximation to areas of management, finance and economy. The main asset of the paper is a derivation of the approximation formula for the derivative of the first order as a linear combination of the function values in several equidistant points on the axis x. Even if there exist many models in different areas that are described by the differential equations, their solutions are oftentimes rather difficult. In these cases, we recommend to use a possibility to approximate the derivatives by the differences because even if the model has an exact solution just the approximation can provide sufficiently close solution and, at the same time, to reduce the complexity and difficulty of the solution. Another reason for application of differences is that the problems in which we replace the derivatives by differences are better algorithmic. Hence, the wider is applicability of the computation technique.

An important asset is a finding that the accuracy of the approach of the first derivative by a difference depends on the selected number *n* of the terms in the approximation formula, namely, proportionally (the higher *n,* the higher accuracy) while the constant *h,* which expresses a distance between two adjacent function values on the axis *x*, influences the approximation only inversely (the smaller is positive constant *h*, the higher accuracy). By means of the suitable selection of the number *n* and the constant *h*, one can adapt the accuracy of the approximation to the practical requirements [8].

Súhrn

Článok sa zaoberal možnosťou aproximácie derivácií diferenciami a zároveň popisoval jednotlivé možnosti jej využitia v oblastiach manažmentu, financií, či ekonómie. Hlavným prínosom práce bolo odvodenie aproximačného vzorca pre deriváciu prvého rádu funkcie ako lineárnu kombináciu hodnôt tejto funkcie vo viacerých bodoch s rovnakou vzdialenosťou. Aj keď existuje množstvo modelov v rôznych oblastiach, ktoré sú opísané diferenciálnymi rovnicami obsahujúcimi derivácie ich riešenie býva častokrát príliš náročné. V takomto prípade preto odporúčame využiť možnosť priblíženia derivácie diferenciami, pretože aj keď je presné vyjadrenie známe, môže práve aproximácia poskytnúť dostatočne presné riešenie a zároveň podstatne znížiť zložitosť a náročnosť problému. Dôvodom na využívania aproximácie je aj to, že úlohy, v ktorých zameníme derivácie na diferencie sa omnoho lepšie algoritmizujú. Následné sa dá lepšie využiť aj výpočtová technika.

Dôležitým prínosom, bolo aj zistenie, že presnosť priblíženia prvej derivácie diferenciou závisí od zvoleného počtu členov *n* v aproximačnom vzorci a to priamo úmerne (čím väčšie *n*, tým väčšia presnosť), zatiaľ čo konštanta *h*, vyjadrujúca vzdialenosť medzi dvoma susednými hodnotami funkcie na osi *x* ovplyvňuje aproximáciu nepriamo úmerne (čím menšia kladná hodnota konštanty *h*, tým väčšia presnosť). Vhodnou voľbou čísla *n* a v prípade možnosti aj konštanty *h*, môžeme potom presnosť aproximácie prispôsobiť praktickým požiadavkám [8].

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Tomáš Valentiny

ESTIMATION OF DIFFERENT ETS MODELS PARTICIPATION EFFICIENCY AND RELATIONSHIP BETWEEN ETS SYSTEMS AND ENVIRONMENTAL TAXATION

URČENIE EFEKTÍVNOSTI PARTICIPÁCIE NA RÔZNYCH ETS MODELOCH A VZŤAH MEDZI NIMI A ENIVORNMENTÁLNYM ZDAŇOVANÍM

Abstract: *Communities across the world tries reduce impact of economic activities to status of environment. This ambition leads to understanding the aspects of barriers between sphere of nature protection and economic development, so representatives of countries creates various tools to reach optimal state between both, environment and development. In case of air pollution is commonly used indicator of green house gas (GHG) emissions, whereby we decide understand relations between GHG and some elements of nature protection tools, like environmental taxation and participation in various versions of ETS. Data samples are represents by times series from 2000 to 2014 in 31 OECD countries and 4 non OECD countries. There are also three different datasets represents all observed countries, observations with participation in ETS and observations of ETS, except pilot phase. To analysis of relations between indicators is using regression analysis, exactly pooled regression, random and fixed effects model (because of dataset characteristics). Results of relations based on analysis are presented by tabulated form with following interpretation, which shows existence of synergic effect (efficiency is related to scale of area) in ETS participation and substitution effect between ETS participation and environmental taxation.*
Key words: *green house gas emissions, ETS, CAP-and trade, environmental taxation*

Kľúčové slová: *emisie skleníkových plynov, ETS, CAP-and trade, environmentálne zdanenie*

JEL: Q51, Q58

Introduction

Constantly increasing amounts of GHG emissions producing by humankind became global threat, so first reaction of authorities leaded to new kind of supply distortion taxes, environmental taxes. In additional, several governments launched specific mechanism, called emission trade system (ETS) or cap-and-trade scheme. In general, we can divide this mechanism to 3 major types based on area size – regional, national and international.

Although direct taxes on emissions are efficient than indirect taxes, such as taxes on related goods or alternative policies (mandated technology standards), Strand [33], Karp and Zhang [19], Kiyono and Ishikawa [20] and others realize difference between various established emissions and taxation systems or mix of these systems. Levinson [23] and O`Riordan [27] said that effectiveness of environmental taxes consist of incentive for polluters to reduce their pollution up to the point where further reduction would cost more than paying the tax, and to do so in the least costly way. Direct taxes to correct market imperfection are quite another matter, but the use of the revenue is generally not discussed. Some authors [7][15][25] define main advantage of direct tax as fact, that it provides public revenues which can be recycled. That leads to situation of direct environmental tax preference to subsidies or emission quotas, because government use these revenues to decrease other distortion taxes, an environmental tax may simultaneously improve the environmental quality and achieve a less distortion tax system. This can cause double dividend, but it essentially depends on the possibility of transferring the global tax burden from wage earners to some fixed production factors. On the other hand some authors [5][8] find that direct emission taxes are equal to any consumption tax, they often appear to be regressive, i.e. more harmful for the welfare of the poorest households than for the richest ones. In particular, a tax on energy or transport consumption harms the lowest wage households few times more than the highest wage households. Moreover, the usual recycling of the environmental tax revenues through a decrease in the labour tax rate could also be regressive.

After 2 years of attempting to establish a harmonized carbon tax within the EU, policy makers instead initiated the EU ETS. Following its implementation, other diverse schemes have been developed or discussed[1]. There are two schemes within North America, the Regional GHG Initiative cap-and-trade scheme and Western Climate Initiative [6][29][31][34]. As Delbeke [11], Ellerman and Joskow [14] said, The EU ETS would not exist if it were not for the Kyoto Protocol and it is the ”flagship measure” by which the member states of the EU will meet their obligations under the Kyoto Protocol.

European system can be described as classic cap-and-trade system launched in 2005. EU ETS also contains some significant design relate to how the cap is set, the process for allocating emission allowances, banking and borrowing provisions, the monitoring, reporting, and verification procedures, and the linking or off-system provisions. Next countries involved in this kind of restriction to polluters became USA and Canada. In 2007 start pilot regional project of ETS in five U.S. States and four Canadian provinces by creating Western Climate Initiative.

[1] Australia, New Zealand, Japan, RCGI, WCI, Switzerland, etc.

After two years the House of Representatives passed the American Clean Energy and Security Act, which sought to place nationwide caps on GHG, contains targeted a 17% reduction of emissions by 2020 and also mandated rise share of electricity created by renewable sources to 20% [21][22].
In Australia, an ETS was officially proposed in late 2008 to provide economic incentives for emission mitigation, known as the Carbon Pollution Reduction Scheme (CPRS) [12]. However Curran [9] and Spash [32] considered the amount of compensation and subsidies available to major polluters to be considered excessively generous. After years of political stagnation on climate change policy, in 2008 New Zealand established an ETS featuring ambitious target encompassing all sectors of economy and all six Kyoto GHG. The system is highly linked to international carbon markets as it allows the importing of most of the Kyoto Protocol emission units. However, in present, the scheme will effectively transition into a domestic scheme, with restricted access to international Kyoto units [4][17][26]. In September 2009, local government proposed the 25% reduction GHG emissions relative to the 1990 national level. Arimura et al. [2] describe that representatives of country proposed several policy instruments to achieve this target. One of these tools was a domestic emission trading scheme. A scheme to limit carbon emissions launched in April 2010 covers the top 1,400 emitters in Tokyo, and is enforced and overseen by the Tokyo Metropolitan Government [16].
We should enlighten Kiyono and Ishikawa [20] statement, which identify a carbon-leakage effect that works under tax regulation only and causes the global emissions to be larger with taxes than with ETS. So, in general, direct taxation of emission is more effective than indirect, but if ETS is implemented, emissions become even lower. Except combination of different tools, we assume existence dependency of effectiveness on integrated system variations based on synergy effect. These variations are international[2], national[3] and regional[4] ETS.

Framework, methodology and research

Many authorities discussed about efficiency of ETS[5], although most of them are concentrate only to comparisons different models efficiency or description of important GHG emissions determinants. Also, no one consider about influence of temperature effect, which implements impact of temperatures to generating GHG emissions.
Impact factors of carbon dioxide production illustrated by Jeffrey and Perkins [18]. They assumed mix of determinants, like taxation, EUETS participation, amount of GDP and amount of energy from renewable resources, included combination of influence taxation and EUETS participation.

[2] EU ETS
[3] Australia, New Zealand, etc.
[4] RCGI (USA and Canada), Japan, etc.
[5] ETS efficiency measured by decreasing GHG emissions

Another approach of carbon dioxide pollutant factors (positive and negative) consist in directional output distance function [3] contains real GDP and technological influence represent by labour force and capital stock. Abrell [1] used labour and capital like primary factor in GCE-analysis model too.
Another work enlightened fact, that some forms of ecological payments systems doesn't allow regulation function of ecological externalities, especially if environmental taxation income exceeds public expenditures on environmental protection [30].

These fact results from Orlov and Grethe [28] statement, that under the assumption of international capital immobility, the burden of carbon taxes is partially borne by capital in terms of decreasing capital income. In other words, increasing energy costs do not fully pass on to final consumers. This indicates the so-called tax-shifting effect [10]. Doytch and Narayan [13] described influence of FDI (in four different variations)[6] on non-renewable and renewable industrial energy consumption.
They computed full 76 countries sample, but also made sub-sample analysis of high-income, upper middle-income, lower middle-income and low-income countries.

Aim

The main purpose of this paper can be described as identifying efficiency of different ETS participation models, which are divided into three main categories based on scale of area, exactly regional, national and international. In general, we believe in fact, that synergy effect should bring additional reduction of GHG emissions, so international model is more effective than national, but efficiency of national model is higher than regional. The next aim of paper is verifying existence of substitution effect between environmental taxation and ETS participation, so if country participate in ETS scheme, this should leads to reducing of insignificancy (or reducing of impact) of environmental taxation to capability of generating GHG emission.

Hypotheses

We assume that efficiency of ETS depends on number of involved regions and countries, so research question contains statement about indirect proportionality between amount of emissions and scale of area implement ETS or CAP-and trade mechanism. We suppose existence of relation between pilot phase of ETS implementation and efficiency of this system or scheme, because of looking proper setting of model in early phase. And we postulated compatibility of ETS and taxation. Assign these premises leads to following hypothesis:

[6] Mining FDI, Manufacturing FDI, Service FDI and Finance FDI

H1: Significance of ETS implementation effectiveness depends on scale of region participates in mechanism.

H2: Conjunction of environmental taxation and ETS participation brings taxation effect (to reducing GHG) more insignificant and this insignificancy become stronger after 1st trading (pilot) phase.

H3: Pilot (1st) phase of ETS (or CAP-and trade) is related with reduction of efficiency this system.

Models, variables and data structure

We use three different regression models (pooled regression OLS, random and fixed effects model) to estimate impact of relationship between environmental taxation, scale and phase of ETS in time series. This method leads to construction of following formulas:

$$\textbf{PR: } GHG_{tc} = \alpha + \beta_1 * TAX_{tci} + \beta_2 * ETS_{tci} + \beta_3 * PHASE_{tci} + \varepsilon_{tc}$$

$$\textbf{FEM: } GHG_{tc} = \alpha_t + \beta_1 * TAX_{tci} + \beta_2 * ETS_{tci} + \beta_3 * PHASE_{tci} + \varepsilon_{tc} \; ; \; \alpha_t = \alpha_{t1} + \alpha_{t2} + \ldots + \alpha_{te}$$

$$\textbf{REM: } GHG_{tc} = \beta_1 * TAX_{tci} + \beta_2 * ETS_{tci} + \beta_3 * PHASE_{tci} + (\alpha + u_c) + \varepsilon_{tc}$$

Emission trade system, or CAP-and trade system variable has to be split to three main variant (regional, national and international), because of efficiency evaluation. Scale assumption consists in synergic effect. Although, scale assumption can be disrupt by fact, that ETS participation and environmental taxation are mutual or convertible to impact on GHG emissions. That is reason of implementation taxation factor to regression model. Last independent variable in basic model is phase. We expect some penalty in efficiency during first trading phase related to lack of knowledge in implementation of rules and mechanism. After corrections system should be superior and next trading season brings better results.

Table 1 List of variables, descriptions and data sources

Variable	Description	Source	Usage
GHG	Greenhouse Gas Emissions per capita (in tones of CO_2 equivalent). This variable consists of all kind of greenhouse gas emission excluding Land Use, Land Use Change and Forestry (LULUCF) and population ratio. Coefficient used in regression model is expressed by natural logarithm.	OECD	All panels
GDP	Gross Domestic Product per capita (in purchasing parity standards unit measured in Euros). This variable includes ratio of all country`s production in recent year to population, contains production of goods and services expressed in form of natural logarithm.	OECD	All panels
PES	Primary Energy Supply (in tonnes of oil equivalent). This variable represents ratio of primary energy supply and gross domestic product, but primary energy supply is adjusted by temperature factor[1] and price of energy[2].	OECD	All panels
FDI	Foreign Direct Investment (in Euros). This variable represents net flows investment, difference between inflows and outflows investment. Coefficient compute by ratio of foreign direct investment and gross domestic product.	OECD	All panels
RES	Renewable Energy Sources (in tonnes of oil equivalent). This variable consists of energy supply by renewable sources and total energy supply (aka primary energy supply) ratio.	OECD	All panels
ETAX	Environmental Taxation (in Euros). This variable includes aggregate collected taxes in energy, transport, pollution and resource area. Construction of this coefficient based on aggregate taxes and gross domestic product ratio.	OECD	All panels
REG	Regional ETS (or CAP-and-trade) scheme (boolean value). This dummy variable represents participation in regional ETS scheme (1- true, 0 - false).		All panels
NAT	National ETS (or CAP-and-trade) scheme (boolean value). This dummy variable represents participation in national ETS scheme (1- true, 0 - false).		All panels
EU	EU ETS (or CAP-and-trade) scheme (boolean value). This dummy variable represents participation in EU ETS (international ETS) scheme (1- true, 0 - false).		All panels
PIL	Pilot (1st) phase of ETS implementation (boolean value). This dummy variable represents starter phase (tools and mechanisms are not properly set) of country participation in ETS.		Panel A and panel B

[1] We create temperature factor: $t.factor_{tc} = \left(1 + \frac{(temp_{tc} - min_t)/(max_t - min_t)}{(max_t - min_t)/(avg_t - min_t)}\right)$; t - year c – country
Source: Global temperature Database at: http://www.rimfrost.no/, country`s temperature in year *"t"* equals average temperature of observed country`s weather stations values in that year

[2] Price of energy is construct represents mix of different source`s energy price adjusts by rate of this source

Analysis focuses to 35 counties in 15 periods with dataset divides to three main variations. First version "total" include all observations, next variation "ETS" contains only observations of counties using ETS system in recent year and last modification "ETS without pilot" remove observations without ETS system or with ETS system in 1st phase of trading.

Additionally, we consider some other independent variables involved in computation based on previous research in this field. GDP indicator represents predominant connection between GHG emission and economic performance of observed country and GDP is also related with labour force and capital stock. Next independent variable, ratio of primary energy supply and gross domestic product, can be described as expression of energy consumption influence to emissions, but we assume existence of temperature impact on energy supply. Building heating and less effective energy using[7] connect with higher energy demand intensity by households and organisations in colder environment. Foreign direct investment factor is one of the most confusable factors. Investments should boost up economy and that leads to increasing GHG emissions. But investments are also capable to increase producing process efficiency or can help to adopt new methods or technologies of environment protection (catalytic converter, etc.), which can bring lesser amount of emissions produced by current economy. Last involved independent indicator represents share of renewable sources of energy. It`s obvious, that increasing fossil fuel free sources of energy moves towards lesser emissions, but impact of renewable sources is not mutual to emission trade system participation (or environmental taxation) effect, so that is reason of addition of this factor to estimation model. All indicators used in regression model represents table 1.

Construction of regression model in three different variations based on participation in ETS and phase of ETS by set of indicators noted above. Because of dataset panel specification, we must analyse dataset by pooled regression (PR) and by random effects model (REM) or fixed effects model (FEM). We also test stationary of all (dependant and independent) indicators by Augmented Dickey-Fuller (ADF) test. Decision about application pooled regression or application effects model accomplished by Breusch-Pagan test. Preference of effect model (REM or FEM) based on Hausman specification test. Only for completeness we carry out all three models (PR, REM and FEM) comparison. We also test stationary of all indicators, even dependant, even independent. At the first, we perform descriptive statistics and correlations, allocates in table 2 and table 3.

[7] Less efficiency, because of harder optimalization some producing processes (especially in agricultural industry – fermentation, wine making, cheese and milk producing etc.), like higher need of energy to accomplish process.

Table 2 Descriptive statistics

	GHG	*GDP*	*SUP*	*FDI*	*RES*	*TAX*	*GHG*	*GDP*	*SUP*	*FDI*	*RES*	*TAX*
	2014						**2006**					
Mean	11,970861	20,029976	0,0201311	1,0388527	0,1891318	0,1789841	12,031295	19,70327	0,032048	1,0906716	0,1426453	0,1616649
Standard Error	0,3238915	0,2882021	0,0047428	0,0111115	0,0287866	0,0186	0,3008364	0,2815523	0,0067526	0,0250497	0,0271934	0,0203252
Median	11,526861	20,090715	0,0125219	1,0247824	0,1156722	0,1649732	11,791899	19,660285	0,0210296	1,0409049	0,0829942	0,1513885
Standard Deviation	1,9161679	1,7050265	0,0280589	0,0657365	0,1703039	0,1100391	1,7797722	1,6656856	0,0399489	0,1481959	0,1608785	0,1202454
Sample Variance	3,6716995	2,9071155	0,0007873	0,0043213	0,0290034	0,0121086	3,1675891	2,7745086	0,0015959	0,021962	0,0258819	0,0144589
Kurtosis	-0,093617	-0,482689	16,111449	14,45674	7,3901753	0,6279211	-0,163065	-0,418499	9,5826791	14,518619	7,5666498	3,4934071
Skewness	0,5151822	-0,068245	3,7371673	3,4787077	2,3109691	0,8734855	0,2698897	-0,106946	3,0391471	3,6869696	2,4653854	1,4793946
Range	8,1324308	6,9259022	0,1526661	0,3768237	0,8647512	0,4563713	7,5626076	6,7053939	0,1807434	0,769264	0,7847167	0,5877383
Minimum	8,4331266	16,650845	0,0021629	0,9691129	0,0252791	0,0052001	8,4015668	16,646582	0,0047408	0,9994498	0,0175931	0,0011488
Maximum	16,565557	23,576747	0,154829	1,3459366	0,8900303	0,4615715	15,964174	23,351976	0,1854842	1,7687138	0,8023098	0,5888871
Confidence Level	0,6582267	0,5856971	0,0096386	0,0225813	0,0585014	0,0377998	0,6113731	0,572183	0,0137229	0,0509071	0,0552637	0,0413057
	2013						**2005**					
Mean	11,962705	20,015551	0,0215683	1,0355623	0,1828682	0,2044101	12,020261	19,611152	0,0366673	1,065288	0,1420086	0,1586281
Standard Error	0,3131109	0,2905181	0,0052685	0,0148209	0,0284386	0,0239153	0,3014862	0,2840617	0,0082352	0,0133354	0,0271514	0,0203405
Median	11,558561	20,074607	0,012815	1,0152589	0,1121051	0,1889801	11,82068	19,566968	0,0210269	1,0362701	0,0756928	0,1505464
Standard Deviation	1,852389	1,7187283	0,0311688	0,0876815	0,168245	0,1414851	1,7836162	1,6805314	0,0487202	0,0788933	0,1606297	0,1203362
Sample Variance	3,4313448	2,954027	0,0009715	0,007688	0,0283064	0,020018	3,1812868	2,8241858	0,0023737	0,0062242	0,0258019	0,0144808
Kurtosis	-0,07728	-0,509031	18,353865	11,692578	8,4919691	1,2114668	-0,160655	-0,349505	10,561266	2,579915	5,9273511	4,9005371
Skewness	0,4423431	-0,108714	3,9912138	3,2329025	2,4606158	0,9516061	0,2288553	-0,113811	3,2080489	1,7574442	2,2552822	1,7015222
Range	8,0314462	6,9881084	0,173497	0,4896305	0,8695829	0,6414311	7,6019481	6,840396	0,2214903	0,3456684	0,7475446	0,6144021
Minimum	8,4195054	16,548358	0,0025648	0,9313059	0,0256798	0,0008955	8,2681393	16,455003	0,0047808	0,9637945	0,0163538	0,0007979
Maximum	16,450952	23,536466	0,1760619	1,4209363	0,8952626	0,6423266	15,870087	23,295399	0,2262711	1,3094629	0,7638984	0,6152
Confidence Level	0,6363179	0,5904038	0,0107069	0,0301196	0,0577942	0,0486018	0,6126936	0,5772827	0,016736	0,0271008	0,0551782	0,0413369

	GHG	*GDP*	*SUP*	*FDI*	*RES*	*TAX*	*GHG*	*GDP*	*SUP*	*FDI*	*RES*	*TAX*
	2012						**2004**					
Mean	11,958192	19,977048	0,0227397	1,1107053	0,1798405	0,1947614	12,01886	19,514533	0,0435877	1,0363544	0,1373721	0,151203
Standard Error	0,3089854	0,293808	0,0055002	0,0724846	0,0294305	0,0223724	0,3000386	0,2873683	0,010263	0,008269	0,0261762	0,0195023
Median	11,627965	20,030574	0,0121891	1,0190947	0,1063602	0,1879057	11,793585	19,518817	0,0248063	1,0260619	0,0708872	0,1507816
Standard Deviation	1,8279821	1,7381915	0,0325395	0,4288245	0,174113	0,1323571	1,7750525	1,7000938	0,0607168	0,04892	0,1548607	0,1153774
Sample Variance	3,3415186	3,0213096	0,0010588	0,1838905	0,0303153	0,0175184	3,1508115	2,890319	0,0036865	0,0023932	0,0239818	0,0133119
Kurtosis	-0,097314	-0,524575	17,687701	33,717236	7,5071767	1,0672737	-0,173994	-0,285598	11,937418	5,0412163	6,8430313	4,5796136
Skewness	0,4115271	-0,119214	3,9237991	5,7648373	2,3722855	0,9157023	0,2092011	-0,114403	3,3159724	1,7459426	2,3370858	1,6049825
Range	7,9146391	7,0371448	0,1800539	2,5542159	0,8732333	0,5965391	7,5339577	6,9254858	0,3018257	0,2716264	0,7481139	0,5841366
Minimum	8,4229704	16,468366	0,0024893	1,0000168	0,0241336	0,001146	8,2789665	16,305339	0,0051869	0,9433028	0,0116744	0,0007985
Maximum	16,337609	23,505511	0,1825432	3,5542327	0,8973669	0,5976851	15,812924	23,230825	0,3070126	1,2149292	0,7597883	0,584935
Confidence Level	0,6279338	0,5970897	0,0111777	0,1473064	0,0598099	0,0454663	0,6097519	0,5840027	0,020857	0,0168046	0,0531965	0,0396335
	2011						**2003**					
Mean	11,971748	20,010475	0,0231792	1,0501024	0,1709325	0,2047122	12,006377	19,349727	0,0535844	1,0326799	0,1342607	0,1316995
Standard Error	0,3066785	0,2904238	0,0057567	0,0131658	0,0290984	0,0235403	0,2990729	0,2906912	0,0136915	0,0064097	0,0260315	0,0165766
Median	11,658603	20,082664	0,0134993	1,0271111	0,1034392	0,2017773	11,788827	19,378963	0,0291175	1,0207397	0,0710795	0,1361831
Standard Deviation	1,8143343	1,7181707	0,0340569	0,0778899	0,1721487	0,1392665	1,7693391	1,7197522	0,0809999	0,0379203	0,1540046	0,0980687
Sample Variance	3,2918089	2,9521104	0,0011599	0,0060668	0,0296352	0,0193951	3,1305609	2,9575478	0,006561	0,001438	0,0237174	0,0096175
Kurtosis	-0,055709	-0,503473	16,453543	10,734855	8,510684	1,1023364	-0,203488	-0,231506	17,340506	3,1022421	7,7975922	3,2909428
Skewness	0,4018969	-0,167673	3,7917811	3,2132202	2,5410339	0,9293674	0,1732949	-0,087531	3,8734105	1,6844774	2,4651922	1,332706
Range	7,889442	6,964263	0,1852003	0,3932629	0,8732761	0,6276316	7,519567	7,0651966	0,4428726	0,185039	0,7595907	0,4812784
Minimum	8,4163159	16,500999	0,0023556	0,9780449	0,0245664	0,0008245	8,2734872	16,101344	0,0059334	0,9618821	0,0106453	0,0005198
Maximum	16,305758	23,465262	0,1875559	1,3713079	0,8978425	0,6284561	15,793054	23,16654	0,448806	1,146921	0,770236	0,4817982
Confidence Level	0,6232456	0,5902123	0,0116989	0,0267561	0,0591351	0,0478397	0,6077892	0,5907556	0,0278245	0,0130261	0,0529024	0,0336878

	GHG	*GDP*	*SUP*	*FDI*	*RES*	*TAX*	*GHG*	*GDP*	*SUP*	*FDI*	*RES*	*TAX*
	2010						**2002**					
Mean	11,992941	19,903342	0,0266972	1,0509565	0,1668426	0,1812532	11,981746	19,15023	0,0636436	1,0463374	0,1376587	0,1093138
Standard Error	0,3038161	0,2908925	0,0060444	0,0231631	0,0283819	0,0204805	0,2984188	0,2971497	0,0165348	0,0083467	0,0270787	0,0144207
Median	11,684635	19,987717	0,0141439	1,0251818	0,1096502	0,1731294	11,76196	19,176665	0,0339623	1,0343211	0,0628303	0,104499
Standard Deviation	1,7974002	1,720943	0,035759	0,1370348	0,1679096	0,1211645	1,7654694	1,7579614	0,097821	0,0493795	0,1601995	0,0853138
Sample Variance	3,2306475	2,9616448	0,0012787	0,0187785	0,0281936	0,0146808	3,1168823	3,0904282	0,0095689	0,0024383	0,0256639	0,0072785
Kurtosis	-0,076591	-0,440491	11,737409	20,490495	9,0674736	1,2300642	-0,243473	-0,201460	20,274552	5,9085974	6,4349534	5,2977031
Skewness	0,3747459	-0,184901	3,249625	4,0488196	2,5938808	0,9114306	0,1643206	-0,056712	4,1639078	2,2472884	2,3061031	1,7260195
Range	7,7682233	7,03042	0,1822776	0,9083282	0,8596825	0,5560983	7,5094029	7,3127114	0,5534483	0,2301826	0,7576398	0,4392436
Minimum	8,4617545	16,398518	0,0030555	0,8390891	0,0257051	0,0008641	8,2781489	15,806403	0,0068164	1,0002223	0,0103869	0,0004848
Maximum	16,229978	23,428938	0,1853331	1,7474172	0,8853876	0,5569624	15,787552	23,119115	0,5602647	1,2304049	0,7680267	0,4397284
Confidence Level	0,6174286	0,5911646	0,0122837	0,0470731	0,057679	0,0416215	0,60646	0,6038809	0,0336027	0,0169625	0,0550304	0,0293063
	2009						**2001**					
Mean	11,95972	19,858712	0,0270264	1,0440017	0,1634217	0,1818395	11,977964	19,051362	0,0727875	1,0487581	0,1371339	0,0983075
Standard Error	0,304405	0,2851418	0,0062175	0,0171528	0,0288899	0,0214145	0,2980115	0,3033745	0,0195757	0,0106483	0,0264236	0,0131543
Median	11,73451	19,878498	0,0154283	1,0165795	0,1013947	0,1828655	11,762644	19,093652	0,0366771	1,0353189	0,0652314	0,0918066
Standard Deviation	1,8008843	1,6869215	0,036783	0,1014775	0,1709147	0,1266899	1,76306	1,7947877	0,1158115	0,0629963	0,1563239	0,0778218
Sample Variance	3,2431842	2,8457041	0,001353	0,0102977	0,0292118	0,0160503	3,1083807	3,2212628	0,0134123	0,0039685	0,0244372	0,0060562
Kurtosis	-0,119161	-0,376836	10,183222	17,443888	8,1153437	2,1651457	-0,249739	-0,239338	21,089534	21,681171	7,0123699	4,4376782
Skewness	0,358004	-0,170523	3,1124109	3,8773668	2,4966889	1,1286109	0,1475732	-0,041237	4,2541674	4,2801496	2,3224273	1,5998113
Range	7,690069	7,0225301	0,1787721	0,5760174	0,8493292	0,6006794	7,5097114	7,4388733	0,6585101	0,3736353	0,7599198	0,3901016
Minimum	8,487923	16,369264	0,0035968	0,9649044	0,0282533	0,0007353	8,2716379	15,647303	0,0077564	1,0000147	0,0103497	0,0003804
Maximum	16,177992	23,391795	0,1823689	1,5409218	0,8775826	0,6014147	15,781349	23,086177	0,6662664	1,37365	0,7702695	0,3904821
Confidence Level	0,6186254	0,5794778	0,0126354	0,0348587	0,0587113	0,0435195	0,6056323	0,6165311	0,0397826	0,02164	0,0536991	0,0267327

	GHG	*GDP*	*SUP*	*FDI*	*RES*	*TAX*	*GHG*	*GDP*	*SUP*	*FDI*	*RES*	*TAX*
	2008						**2000**					
Mean	12,023001	19,981191	0,0233773	1,0643108	0,1528705	0,1922002	11,962839	19,040468	0,077559	1,0836806	0,1415228	0,1009932
Standard Error	0,3011493	0,276396	0,0047031	0,0167858	0,0285281	0,0232759	0,3001233	0,306451	0,0234912	0,0234196	0,0278502	0,0137725
Median	11,785311	20,057667	0,0140162	1,0376302	0,0943288	0,1878641	11,757343	19,095774	0,0307655	1,0503412	0,0624692	0,0964015
Standard Deviation	1,7816235	1,6351806	0,0278238	0,0993062	0,1687745	0,1377018	1,7755535	1,8129886	0,1389756	0,138552	0,1647638	0,0814793
Sample Variance	3,1741822	2,6738154	0,0007742	0,0098617	0,0284848	0,0189618	3,1525901	3,2869277	0,0193142	0,0191967	0,0271471	0,0066389
Kurtosis	-0,136500	-0,468737	6,3369155	10,490633	8,7561252	3,0051671	-0,288940	-0,27992	23,785651	22,742295	5,6598842	4,309126
Skewness	0,3493573	-0,162384	2,6149075	3,162431	2,6125842	1,3260445	0,1461755	-0,041079	4,5888187	4,4416942	2,1718309	1,6043398
Range	7,5754212	6,7329152	0,1147223	0,4900971	0,8423811	0,6679359	7,5130504	7,5004531	0,8019405	0,8342529	0,7631625	0,4047139
Minimum	8,5448745	16,679461	0,0033924	0,9874042	0,0257505	0,0014116	8,2846986	15,553478	0,007203	0,9713471	0,0101526	0,0003668
Maximum	16,120296	23,412377	0,1181147	1,4775013	0,8681316	0,6693474	15,797749	23,053931	0,8091434	1,8056	0,7733151	0,4050807
Confidence Level	0,6120091	0,5617042	0,0095578	0,0341129	0,0579761	0,0473022	0,6099239	0,6227834	0,0477398	0,0475943	0,0565984	0,0279891
	2007											
Mean	12,036814	19,875553	0,0259037	1,0856779	0,1488909	0,1826404						
Standard Error	0,3005703	0,2744718	0,0052225	0,0338374	0,0283262	0,0230719						
Median	11,812761	19,876415	0,0168883	1,0561708	0,0920565	0,17561						
Standard Deviation	1,7781978	1,6237971	0,0308969	0,2001849	0,1675803	0,1364955						
Sample Variance	3,1619875	2,6367171	0,0009546	0,040074	0,0280832	0,018631						
Kurtosis	-0,154329	-0,48681	8,4807333	10,435529	7,753266	3,8128603						
Skewness	0,3094052	-0,096114	2,8448416	1,0111481	2,5039584	1,5418011						
Range	7,5679579	6,5219424	0,1434052	1,4642026	0,816797	0,673304						
Minimum	8,458466	16,873928	0,003715	0,4102233	0,0218316	0,0011271						
Maximum	16,026424	23,395871	0,1471203	1,8744259	0,8386285	0,6744311						
Confidence Level	0,6108323	0,5577938	0,0106134	0,0687659	0,0575658	0,0468878						

Source: own processing based on OECD dataset

Complete regression model formulas consist of three main indicators (environmental taxation, phase of ETS and form of participation in ETS divided to three variables, regional, national and international-EU) and mix of other independent indicators (located in table 1):

$$\textbf{PR: } GHG_{tc} = \alpha + \beta_1 * GDP_{tci} + \beta_2 * PES_{tci} + \beta_3 * FDI_{tci} + \beta_4 * RES_{tci} + \beta_5 * ETAX_{tci} + \\ + \beta_6 * NAT_{tci} + \beta_7 * REG_{tci} + \beta_8 * EU_{tci} + \beta_9 * PIL_{tci} + \varepsilon_{tc}$$

$$\textbf{FEM: } GHG_{tc} = \alpha_c + \beta_1 * GDP_{tci} + \beta_2 * PES_{tci} + \beta_3 * FDI_{tci} + \beta_4 * RES_{tci} + \\ + \beta_5 * ETAX_{tci} + \beta_6 * NAT_{tci} + \beta_7 * REG_{tci} + \beta_8 * EU_{tci} + \beta_9 * PIL_{tci} + \varepsilon_{tc} \; ; \; \alpha_c \\ = \alpha_1 * Z_{c1} + \alpha_2 * Z_{c2} + . + \alpha_e * Z_{ce}$$

$$\textbf{REM: } GHG_{tc} = \beta_1 * GDP_{tci} + \beta_2 * PES_{tci} + \beta_3 * FDI_{tci} + \beta_4 * RES_{tci} + \beta_5 * ETAX_{tci} + \\ + \beta_6 * NAT_{tci} + + \beta_7 * REG_{tci} + \beta_8 * EU_{tci} + \beta_9 * PIL_{tci} + (\alpha + u_c) + \varepsilon_{tc}$$

where index-t represents time (recent year), index-c is country, index-i is considered as vector of indicator`s mix, Z_{ce} variable means individual effects and (ε_{tc}+u_c) represents composed residual estimates origin and specific residual.

Results

Only if we consider datasets of indicators as stationary, we are capable continued to estimate regression model, so all indicators should be controlled by ADF test. ADF tests of dependant and independent variables shows stationary condition, whereby we expect existence of one timeframe lag with constant and trend. Records of stationary test are presented by table 4. Because all variables stationary condition acceptance, we can carry out regression models, which are designated to panel data analysis. Breusch-Pagan test reveals information about favour of pooled (null hypothesis) or effects regression model. We also execute Hausman test, which informs about preference of REM (null hypothesis) or FEM regression model. Both data test can be found in table 5.

Table 3 Correlation

All observations										
	GHG	***GDP***	***PES***	***FDI***	***RES***	***ETAX***	***REG***	***NAT***	***EU***	***PIL***
GHG	1									
GDP	0,444812	1								
PES	0,074702	-0,53962	1							
FDI	0,146516	0,137801	-0,05268	1						
RES	-0,015604	0,100008	0,076545	-0,12729	1					
ETAX	-0,013181	0,556769	-0,43538	0,146821	-0,21868	1				
REG	0,226888	0,096656	-0,02807	-0,05601	-0,04264	-0,16258	1			
NAT	0,134261	0,193104	-0,07812	-0,04316	0,027181	0,071423	-0,04659	1		
EU	-0,14805	0,311294	-0,23346	0,119209	0,015759	0,40483	-0,21352	-0,19094	1	
PIL	0,014903	0,212023	-0,14585	0,056189	-0,09474	0,117435	0,305892	0,278663	0,283519	1

Source: own processing based on OECD dataset

The interactions between dependant and independent variable are identical with expectations written above. Correlation coefficient between GHG and ETS participation variables (REG, NAT and EU) confirms 1st hypothesis about scale of implementation effect. We found significantly inverse relation with participation in EU ETS, additionally regional and national ETS participation variations even shows direct correlation with amount of emitted pollution in reduced database models. This is related to fact, that although region implement ETS reduce GHG emissions, but rest of the country still constantly increasing total emissions, so ETS participation effect became overcome.

We also found existence of direct correlation with FDI net inflows factor to emission production, so this indicates outgrowing of GDP increase effect over technology change (benefit) effect. Direct correlation coefficient of PIL variable to GHG represents verification of 3rd hypothesis about negative influence of pilot phase trading sequence to total ETS mechanism efficiency. In general, correlation matrix confirms our expectations about potential relations between variables, except REG indicator.

Table 4 ADF tests of stationary

ADF test for GHG (including 1 lag with constant and trend)				**ADF test for FDI (including 1 lag with constant and trend)**			
t_{δ}	-7,6493	*Inverse* χ^2	314,759	t_{δ}	-5,0348	*Inverse* χ^2	186,197
	[0,0000]		[0,0000]		[0,0000]		[0,0000]
		Inverse normal test	-10,5055			*Inverse normal test*	-7,1407
			[0,0000]				[0,0000]
		Inverse Logit test	-14,0027			*Inverse Logit test*	-7,5778
			[0,0000]				[0,0000]
ADF test for GDP (including 1 lag with constant and trend)				**ADF test for RES (including 1 lag with constant and trend)**			
t_{δ}	-2,59089	*Inverse* χ^2	103,636	t_{δ}	-5,5778	*Inverse* χ^2	177,968
	[0,0048]		[0,0056]		[0,0000]		[0,0000]
		Inverse normal test	-3,71408			*Inverse normal test*	-8,0089
			[0,0001]				[0,0000]
		Inverse Logit test	-3,57526			*Inverse Logit test*	-7,9359
			[0,0002]				[0,0000]
ADF test for PES (including 1 lag with constant and trend)				**ADF test for ETAX (including 1 lag with constant and trend)**			
t_{δ}	-3,8057	*Inverse* χ^2	179,997	t_{δ}	-4,5261	*Inverse* χ^2	189,572
	[0,0001]		[0,0000]		[0,0000]		[0,0000]
		Inverse normal test	-5,3628			*Inverse normal test*	-6,253
			[0,0000]				[0,0000]
		Inverse Logit test	-5,8482			*Inverse Logit test*	-7,0811

Source: own processing based on OECD dataset

As mentioned above, we accomplish all there regression model, but because of Breusch-Pagan test and Hausman test, we consider as relevant only one model, pooled regression.

Table 5 Breusch-Pagan and Hausman test statistics

Breusch-Pagan test statistic			
LM value	2,35559	*P-value*	[0,1248]
Hausman test statistic			
H value	37,2166	*P-value*	[2,40764e-005]

Source: own processing based on OECD datasets

Execution of pooled regression model brings omission of REG variable, due to insignificancy issue (insignificant even if predetermined significance level α=0,1). However FDI and GHG correlation indicates inverse proportionality, while FDI regression slope isn`t. This can be related to double effect of FDI to emission production, which is mentioned above. Independent variable`s slopes (GDP and PES), which represents production and consumption of country output denotes direct proportionality to GHG emissions indicator. Connection between energy produced by renewable sources and emissions level is notable. This statement is confirmed by pooled regression model, because of inverse correlation of GHG and RES indicators. Explanation of environmental taxation and ETS model substitution, due to similar impact on company`s production cost, should leads to not relevant (insignificant) ETAX variable, actually ETS factor is capable to change (or cover) impact of environmental taxation on measurement. Issue about insignificance of regional ETS mechanism effect may be connected with insufficient area involved to mechanism, which leads to comparable effect of regional ETS and non ETS countries. Different regression models analysis, which are enlighten changes in factor impacts represent following table 6.

1st hypothesis (H1) about dependency of scale to ETS efficiency is supported by results of correlation and regression analysis, although we must mention fact about overcoming of EU ETS by national ETS (only in last dataset variation excluding non ETS and pilot ETS observation), but this regression coefficient seems incomparable, due lack of data (in national variable). ETAX slope differences in comparison of dataset variations confirms

2nd hypothesis (H2) about insignificancy of environmental taxation after ETS implementation. So, environmental taxation extensively and positively affects GHG producing (shown by ETAX slopes), but this support is consumed by (became part of) mix of ETS factors. Pilot phase ETS participation brings expected penalty to estimation (direct correlation of PIL slope) and this revelation boosts 3rd hypothesis (H3) about pilot phase reduction effect.

Table 6 Regression models

Comparison of regression methods			
	PR	***REM ●***	***FEMΔ ●***
const	-2,38877 [1,93e-026] ***	-2,26805 [1,21e-027] ***	-2,1972 [9,97e-023] ***
GDP	0,487330 [8,24e-080] ***	0,477280 [1,54e-115] ***	0,471675 [6,22e-075] ***
PES	2,79523 [4,29e-029] ***	2,74898 [5,15e-033] ***	2,72718 [1,45e-027] ***
FDI	0,198057 [0,0269] **	0,182548 [0,0371] **	0,173231 [0,0538] *
RES	-0,935645 [3,38e-024] ***	-0,934914 [5,77e-028] ***	-0,937195 [3,26e-024] ***
ETAX	-1,33282 [5,86e-018] ***	-1,35147 [1,18e-020] ***	-1,36244 [1,15e-018] ***
REG	-0,0238807 [0,7409]	-0,0143675 [0,8651]	-0,00677204 [0,9520]
NAT	-0,133706 [0,0493] **	-0,0968476 [0,2727]	-0,0753016 [0,5051]
EU	-0,224909 [1,39e-010] ***	-0,222676 [3,43e-07] ***	-0,223695 [0,0003] ***
PIL	0,105149 [0,0052] **	0,0996462 [0,0427] **	0,0907004 [0,1868]
NON ETS	[reference variable]	[reference variable]	[reference variable]
Adjusted R^2 / corr (y,yhat)^2	0,562777	0,569818	0,579370

Comparison of dataset variation			
	all observations	***with ETS***	***with ETS, except Pilot***
const	-2,38877 [1,93e-026] ***	-2,19412 [5,91e-012] ***	-1,13651 [0,0047] ***
GDP	0,487330 [8,24e-080] ***	0,429252 [5,02e-032] ***	0,363057 [3,15e-017] ***
PES	2,79523 [4,29e-029] ***	8,70705 [2,08e-022] ***	9,58105 [1,83e-021] ***
FDI	0,198057 [0,0269] **	0,144704 [0,0826] *	0,204618 [0,0131] **
RES	-0,935645 [3,38e-024] ***	-1,40035 [1,08e-020] ***	-1,55398 [2,82e-017] ***
ETAX	-1,33282 [5,86e-018] ***	-0,415147 [0,0135] **	-0,00560 [0,9759]
REG	-0,0238807 [0,7409]	0,202210 [0,0016] ***	0,50902 [2,06e-05] ***
NAT	-0,133706 [0,0493] **	0,27504 [8,31e-06] ***	-0,159068 [0,2049]
EU	-0,224909 [1,39e-010] ***	[reference variable]	[reference variable]
PIL	0,105149 [0,0052] **	0,082016 [0,0124] **	[irrelevant]
NON ETS	[reference variable]	[irrelevant]	[irrelevant]
Adjusted R^2	0,562777	0,582484	0,653572

*, **, *** – variable is significant on 10%, 5% and 1% level

● – main preference of pooled regression model, because of test statistics (only informative purpose)

Δ –fixed effect model unit constants are located in appendix in table 7

Source: own processing based on OECD dataset

Discussion and conclusion

Complex analysis of results brings enlightenment to formulated hypothesis concerns relation of GHG emissions amount with participation and environmental taxation. But estimation of influence was computed not only for testing variables. Constructed mix of autonomous variables (based on previous studies, mentioned above) clarified direct impact of taxation and ETS (variation and phase) to emissions reduction and this mix consist of gross domestic product, primary energy supply, foreign direct investment and share of renewable energy sources. No one author involved temperature factor before and decision to implement it in our computation was based on two major factors, building heating and less effective energy using in colder environment. Both of them lead to higher energy demand, respectively higher energy consumption and that brings higher GHG emissions. So, we decide make "penalty" factor for countries in warmer environment (location), because of non relevant comparison of countries (or periods) in colder and warmer area (period). In fact, this "temperature weight" penalised counties growing their energy consumption, while their clime became colder and vice versa. This temperature factor can be characterised as weight for primary energy supply variable.

At the first, we should explain dependency of ETS efficiency and size of the area. Our expectation of better performance in international ETS variation consists in synergy effect premise. We also assume essential influence of technocratic approach in countries implemented international ETS, because of fact, that all these countries are members of European Union. So, technocratic approach leads to suppression of sovereign state`s interests (primary protection of economic development, etc.) and concentrates more endeavour to reach target sets by Kyoto protocol. Expectation confirms fact, that implementation of any ETS variations brings lower production of GHG emission, but regional ETS causes weaker impact than national ETS. This is related to incapability of regional ETS to reduce air pollution in whole country.
Analysis of connection between environmental taxation and ETS participation discovers substitution ability of both emission reduction methods, thus leads to decreasing significance of environmental taxation in regions, which participated in ETS mechanism. This effect is strengthened even by later phase factor of ETS implementation, which demonstrates ETAX slopes (and also significance criteria) changing in comparison of dataset variations. This result also supports by work of Jeffrey and Perkins (2015). However, in defence of environmental taxation irreplaceable role, we must mentioned fact, that any ETS mechanism takes no care of reduction pollutants made by transport (non participated in enterprise) or (directly by) households, when ETS systems cover only around 40-60% of all GHG emissions.

We also confirmed hypothesis about bond between first trading season and lesser efficiency of ETS. This is connected with lack of knowledge in setting of rules and another relevant factor is small numbers of companies participated in trading emission mechanism. Authorities have enough time and lot of information to set rules properly and more effective during first sequence. And also more companies involved to system bring wider reduction of pollutants connected with production of goods and services.

Appendix

Table 7 List of observed countries and unit constant in FEM

Country	OECD membership	Unit constant in FEM
Australia	OECD	-4,25524
Austria	OECD	-4,528666
Belgium	OECD	-4,099212
Canada	OECD	-4,224609
Czech Republic	OECD	-4,331557
Denmark	OECD	-4,166712
Estonia	OECD	-4,354339
Finland	OECD	-4,22641
France	OECD	-4,404503
Germany	OECD	-4,241934
Greece	OECD	-4,2419
Hungary	OECD	-4,357454
Iceland	OECD	-4,238738
Ireland	OECD	-4,322218
Italy	OECD	-4,503504
Japan	OECD	-4,658796
Latvia	OECD	-4,376959
Luxembourg	OECD	-4,548213
Netherlands	OECD	-4,501241
New Zealand	OECD	-4,473833
Norway	OECD	-4,495713
Poland	OECD	-4,445533
Portugal	OECD	-4,581281
Slovak Republic	OECD	-4,511108
Slovenia	OECD	-4,714487
Spain	OECD	-4,611744
Sweden	OECD	-4,690447
Switzerland	OECD	-4,607101
Turkey	OECD	-4,586719
United Kingdom	OECD	-4,674696
United States	OECD	-4,479623
Brazil	Non OECD	-4,566074
China (People's Republic of)	Non OECD	-4,559687
Lithuania	Non OECD	-4,586374
Russia	Non OECD	-4,600549

Source: own processing based on OECD dataset

Súhrn
Politické nástroje zabezpečujúce ochranu životného prostredia, akými sú aj environmentálne dane a participácia na systémoch ETS majú viesť k znižovanie antropogénnej záťaže spájanej najmä s ekonomickými aktivitami. Rôzne nástroje však poskytujú rôzne formy ochrany a vytvárajú odlišné bariéry pre podniky nachádzajúce sa na ich území. Preto je dôležité vhodne porovnávať efektivitu týchto nástrojov, čo bude smerovať k optimalizácii ich využitia, resp. nahradenia vhodnejším nástrojom. V článku sme sa primárne venovali vplyvu dvoch vyššie uvedených nástrojov na znečistenie ovzdušia meraného pomocou objemu vypúšťaných emisií skleníkových plynov. Cieľom bolo určiť, ktoré z rôznych variácií ETS vo väčšej miere napomáhajú znižovať emisie, pričom sme brali do úvahy tri špecifiká s tým spojené. Prvým bol synergický efekt spätý z veľkosťou oblasti, na ktorú sa systém aplikoval. Ďalším bola fáza v ktorej sa obchodovanie s emisiami nachádzalo a posledným špecifikom bol substitučný efekt medzi emisnými systémami a environmentálnym zdanením. Všetky tri špecifiká sa na základe vykonanej analýzy potvrdili. Okrem špecifík sme brali do úvahy mix ukazovateľov spájajúci sledovanú problematiku s ekonomickou výkonnosťou krajín. Analýza spočívala v troch regresných modeloch primárne určených na aplikáciu pre panelové dáta. Na základe testovacích štatistík sme určili relevantnosť jedného z modelov. Zároveň sme analyzovali tri rôzne datasety (pomocou relevantného modelu), zohľadňujúce špecifiká skúmanej problematiky. Výsledky sú prezentované v tabuľkách s následnou interpretáciou v texte, ktorá potvrdzuje stanovené hypotézy. Teda, čím väčšia oblasť participácie na ETS, tým väčšia miera redukcie skleníkových plynov na osobu. Zároveň platí, že v pilotnej (prvej) fáze obchodovania s emisiami sa redukuje schopnosť systému znižovať emisie. A v neposlednom rade sme odlišnou metódou potvrdili predchádzajúce výskumy o substitučnom efekte medzi environmentálnym zdanením a participáciou na ETS, pričom platí, že takéto zdanenie stráca na významnosti po aplikácii emisných obchodných systémov.

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STOCK EXCHANGE BONDS AS A SOURCE OF FINANCIAL CAPITAL FOR FIRMS IN POLAND AND IN SELECTED COUNTRIES OF EU

BURZOVÉ DLHOPISY AKO ZDROJ FINANČNÉHO KAPITÁLU PRE FIRMY V POĽSKU A VO VYBRANÝCH KRAJINÁCH EÚ

Abstract: *The article presents the characteristics of the bonds as a source of capital for the companies. Analysis of the stock exchange bond markets in the selected countries is also carried out. Moreover, based on the analysis of the size of capital raised through the stock exchange bond issues, author tries to assess the importance of this form of raising capital in the selected countries. Verification of hypotheses was based on literature studies and analysis of statistical data from 1999 to 2013.*
Key words: *stock exchange, corporate bonds, bank credit.*

JEL: G1, G3

Introduction

Choosing the right source of capital is one of the key decisions for firms, especially as they may use various sources of capital supply, depending on their needs (issue of shares and bonds, debt securities, bank loans, venture capital, leasing, factoring and other).

One of the important sources of capital is a stock exchange where companies can place the issues of shares and/or bonds and increase respectively equity capital or outside capital. Stock Exchange is the main competitive source when it comes to raising capital in relation to the banking sector and loans it offers.

Therefore, it seems worth to analyze the role of the stock exchange bond issues in corporate financing in the countries of the European Union, including Poland. The aim of the study was to investigate the trading volume of the bond issue as a source of corporate financing in Poland and in selected European Union countries (France, Greece, Ireland, Germany, Great Britain). Stock Exchange is an important place through which companies can raise capital in the form of new issues of shares and/or bonds. However, the importance of stock exchange in the economy can vary and may depend on the financial system of the given country, the level of financial development and economic development or alternative methods of raising capital (primarily long-term bank loan). Therefore, the paper proposes the following research hypothesis: Issuance of corporate bonds is less important as a form of raising capital in the countries of continental Europe (including Poland) than in countries with Anglo-Saxon model, which is conditioned by a greater role of bank credit as a source of funding.

The higher the level of financial development and economic development of the country, the tendency of companies to raise capital from the stock exchange is usually greater.

1 Bonds as a financial instrument listed on the stock exchanges

The bonds are widely used as a capital market instrument. Bond issuers offer them to the investors in order to raise capital for investments, infrastructure development, financing the deficit and budget expenditures, or to subsidize the developing countries that are members of economic communities. Polish Act on Bonds defines bond as "a security issued in the series, in which the issuer states that it is indebted to the bond owner (bondholder) and is committed to meet certain performance" (see: Act on Bonds 1995, pos. 1300).

W. Dębski depicts bonds as "security certifying the claim, which is a debt obligation of the issuer to its owner on a certain amount, together with a commitment to pay interest at fixed dates. It may also contain an obligation of the issuer to the specified non-cash benefit. " (see: Dębski 2007, p. 234).

W. Bień specifies bonds as securities in which “the issuer confirms taking out a certain amount of the loan and undertakes to return it to the owner of bonds at a predetermined time and to pay interest" (see: Bien 1999, p. 31).
The economic role of bonds as security is implemented through a set of functions that it meets as an instrument of the capital market. Among the main features of the bond you can list the following functions: credit, investment, circulation, payment and warranty (see: Antkiewicz 2011, p. 44).

The parameters which determine the characteristics of each bond are the nominal value of bonds, maturity, interest. The nominal value of the bonds - face value, assigned to the bond amount representing the amount of credit granted to their issuer. It is the basis for the calculation of bond interest. In the period from the issue of the bond till the maturity the face value doesn’t change and when maturity date comes nominal value shall be returned to the bond holder. Hence, the maturity date is the period after which the debtor should pay the borrowed money represented in bonds. In terms of the issue, the issuer may determine the rules of early redemption of the bonds. The process of redemption may be one-off or in installments. In the latter case, which series of bonds will be subject to a redemption is typically decided by a draw. Interest - represent a benefit for the bond holder for borrowing money to the company that has issued these bonds. They give the bondholder steady income, independent of the performance of the issuer profits. They are regularly paid during the periods and amounts determined in terms of the issue (except for zero-coupon bonds). Principles of charging interest should be available to investors at the moment of issuance of bonds (see: Dębski 2007, pp. 235 - 238).

There are many types of bonds in the exchange-traded and over the counter markets. The criteria of classification can be a type of the issuer, type of interest rates, time to maturity or the legal rights. Taking into account the criterion of the issuer W. Bien distinguished types of bonds such as government bonds called treasury bonds, financial institutions bonds eg. banks, insurance companies, government bodies bonds or municipal bonds, public enterprises bonds - post office, railway bonds and finally bonds issued by other companies and institutions (see: Bień 1999, p. 38).

W. Dębski proposes more concise selection that divides bonds into treasury, municipal and enterprises bonds (see: Dębski 2007, pp. 238 - 240). On account of the way interest rates are calculated bonds can be divided into bonds with a fixed interest rate, which give the holder steady and regular income (when issuer his obligation). Throughout the period the interest rate does not change. Zero-coupon bonds are bonds without interest coupons, interest is not paid on it. Profit for the bondholder lies in the fact that he buys them at a significant discount to their nominal value. Bonds with floating interest rate - the amount of income for the creditor is not pre-determined. Only the principle method of calculating interest is known. The amount to be paid is known only for a short period before the due date. The purpose of issuing interest-bearing bonds is to provide investors with a defined purchasing power of capital they invested, in the case of high inflation and rising interest rates. The interest rate can be determined on the basis of the interbank market interest rates (LIBOR, WIBOR 3 or 6 months), the interest rate of treasury bills (52-week) or profitability of swaps. Bonds with an indexed interest rate - a percentage value is based on the given index or ratio, which must be specified in the issuance contract. This may be the price index in a given period (e.g. 1 year) or the rate of return in the certain period for the chosen goods e.g. gold (see: Antkiewicz 2011, pp. 40 - 41).

Bonds can be also divided into short-term bonds - with a maturity date of up to 5 years, medium-term bonds - with a maturity of from 5 to 10 years, and long-term bonds - maturing over 10 years. It is worth noting that the dividing lines in this classification are floating and it is hard to point out just one correct classification (see: Najlepszy 2000, p. 401). For example, In Poland short-term bonds are considered to be bonds with a maturity of up to one year, while medium - and long-term bonds have maturity over 1 year.

According to the criterion of legal rights linked to bonds, we can divide bonds into ordinary bonds, which do not have special rights and privileges. In dematerialized form they are widely used on the secondary market (e.g. in Poland these type of bonds is traded on the Warsaw Stock Exchange and Over-The-Counter regulated market which has been running since 2009 organized by two institutions in the form of two markets for the specific group of investors - Catalyst and Bondspot). Next type of bonds are convertible bonds.

This type of financial instrument gives the owner of these bonds the right to convert them in the near future into shares of the company. This type of bonds feature all the basic characteristics of classic bonds with the additional option, which gives the right to convert them into shares of the issuer (see Taylor et. Al., pp. 4).

From this type of issue usually benefit developing, early-stages firms, due to profits this issue brings: lower interest rate, conversion of debt into share/equity capital, less stringent conditions of issuance than other bonds (see: Dębski 2007, pp. 242 - 246). Also, companies with a strong financial position use this form of supply of financial capital, precisely because of the lower cost of the convertible bond issue, despite the fact that they easily could issue shares or regular bonds. Moreover, convertible bonds as a form of borrowing financing lowers the total cost of capital due to the tax shield (reduction of the tax base of the deductible, which is interest on liabilities). Convertible bonds may also increase the credibility of the company, and consequently the credit standing of the company, which may result in easier possibilities to raise new funds, if necessary (see: Brealey et. Al. 2001, pp. 507 - 508). In addition, convertible bonds have a low cost of ongoing maintenance of financing, allow the issuer more flexibility in shaping the indicator of capital structure, and if it is incorporated in them - the option of early redemption. Moreover, the conversion into shares reduces the size of the debt and the company may incur new debt (see: Antkiewicz 2011, p. 85).

Among the convertible bond, we can distinguish mandatory bonds (they require bondholder to convert them into shares at maturity, popular in the United States) and reverse convertible bonds (the right to choose how to redeem these bonds - either by returning the nominal amount together with coupons, or by convert them into shares - is entitled to the issuer, not the bondholder. In return, the issuer offers investors a higher interest rate) (see Brown 2006, pp. 9 - 26).

Other types of bonds can include unsecured and secured type of bonds. Unsecured bonds are described by higher risk and should offer a higher rate of return for investors (see: Fabozzi 2002, pp. 251 - 282). Among the covered bonds can be distinguished retail bonds - available to all investors through brokerage houses, internet, banks and other intermediaries; and wholesale bonds available exclusively at auctions in the primary market institutions and to investors who are able to meet certain requirements and who have permissions to participate in auctions (see: Mishkin et. al. 2006, pp. 245 - 256).

2 Corporate bonds issued on stock exchange

Issue of securities by the company whether in the public market, or the private one involves a change in its capital structure, which in turn may also lead to changes in the ownership structure of the company. Company can ordinary shares, preference shares, corporate bonds or convertible bonds.

Issuance of long-term securities contributes to company's need for development capital need, causing at the same time a change in ownership structure (in the case of the issue of shares) or the level of long-term debt (in the case of a bond issue). These changes must be taken into account by the current owners, who must be able to examine not only whether it is worthwhile to go this route in order to gain financial supply, but what the consequences are and what impact will it have on the continued operation of the company.
The public issue of financial instruments requires adequate preparation. The company must has to undertake a series of steps, in order to find buyers for its stocks in the public market and bring the new capital in. The issue of securities is determined by many factors that contribute to the final success. This is related to the functioning of the public market itself, political, economic, organizational, legal and historical framework that exist in a given market. Introduction of financial instruments to public trading, is associated with the fact that they must meet a number of formal and legal requirements in order to be admitted to public trading. The very design of public trading imposes on the issuer a lot of organizational and informative responsibilities it has to meet. Although there are few cases where the public issue of securities is not finalized with listing them on the stock market, however, the vast majority of newly issued financial instruments is traded on an organized regulated market.
Among the reasons that lie behind the public offering of financial instruments it seems necessary to include the desire to raise capital for growth, the prospect for better access to capital funds in the future, ensuring improved liquidity of shares, changes in the management of the company, marketing effects company and its products or services (see: Nawrot, 2010 p. 25), as well as connection with a company already listed on the stock exchange, if that was the plan in the first place (see: Pagano et. al. 1998, pp. 27 - 64).

W. Milo states that the factors of issuance can include, inter alia, the need to obtain new capital (the most common and natural motive of companies deciding to issue securities), the need for impartial valuation of companies, which is on the market, cancellation of debt processes and rebuying of the shares by the issuer (see: Milo 2000, pp. 100 - 105).
Among the reasons behind the issue of capital market instruments can also be mentioned easier access to new sources of capital, low cost of capital, increase in the credibility of the company - the fact of having the status of a public company listed on the regulated market creates some kind of prestige in the eyes of potential investors, which may result in easier access to other sources of financing, previously beyond the reach of the company. Also, the possibility of acquiring a strategic investor may be the factor that will influence the decisions of the public issue of securities, as the status of a public company gives the firm an advantage and relatively eases the search for a strategic investor, when the company is looking for funds to finance its further development and research or is looking for technological support (see: Pośląd et. al. 2006, pp. 27 - 39).

In addition to the above-mentioned reasons of financial instruments issuance W. Nawrot also points out to other factors that company should also consider. She Indicates: investment strategies of holders of shares, the stock market situation, the risks associated with the instruments and the political and macroeconomic situation (see: Nawrot, 2010 p. 27).

Securities market plays an important role in the financial system of free market economies. Through its functions it facilitates allocation of individual's capital from those have a surplus to those which demand it. The stock exchange in its function of concentration of capital facilitates the meeting of the supply side with the demand side amidst the clear conditions. Well developed securities market can provide favorable conditions for economic growth by influencing the level of investment and savings, and the absorption of economic shocks (see: Bukowski 2009, p. 16; Bukowski 2009 (II), pp. 186 - 188; Kosztowniak 2011, pp. 271 - 280; Pszczółka 2013, pp. 75 - 77). The changes in stock market conditions have a significant impact on investors' decisions, and hence on the ability to raise capital on the stock exchange by concerned companies. It seems that during the bull market on the stock exchange it is much easier for companies to obtain financial provisioning, find investors, and thus fulfill the established goal for the issue. Reverse trend occurs during a "bad" stock market situation - a bear market. There are more parties willing to sell securities and to modify its portfolio and to seek alternative sources of investment (see: Mishkin, 2001, pp. 1 - 2).

Issuance of corporate bonds is a source of external financing of enterprises. A bond is a debt instrument of long-term financing and can be considered as a main alternative (along with issuance of stocks) to bank credit. The public issue of bonds by the company is the issue of debt with a nominal value, from which issuer is obliged to pay the bondholder interest and return the borrowed capital at a fixed time. Further details of the issuance like installments or the lack of them (in the case of zero-coupon bonds), the maturity period and the implementation of additional rights arising from holding a bond (e.g. from convertible bonds or warrants), securing a bond issue or not, the amount of interest paid and the frequency of it are key elements of the issue, which may determine its success (see: Fabozzi 2000, pp. 165 - 205). By conducting appropriate analyzes the company can obtain financial empowering relatively cheaply. As in the case of a public issue of shares, so in the case of a public bond issue the company must meet certain requirements and regulations in order to be admitted onto the public market. The company that wishes to offer bonds to the investor in the public market must use the brokerage house or investment firm as a intermediary, which will carry the company through all stages of the issue. In the case of bond issues the expected profits of prospective buyers are of key importance. These yields are equal to bond's interest rate.

The company should establish the level of interest rates at a level that will bring investors a satisfactory rate of return on investment, while not exposing themselves to the risk of inability to pay the debt. Investors who are interested in purchasing the bonds make their valuation and estimate the level of risk (see: Madura 2010, pp. 173 - 203). Therefore, it is important whether the company has a good rating or not. The rating of the company can determine the terms of the issue both for issuer and debtholder. The high rating can significantly reduce the cost of issuance, whereas the credit rating is at the risk level (speculative) then the costs of issuance must rise, as investors expect a premium for an increased level of risk that they are willing to accept (see: Brown 2006, p.118 ; Chisholm 2009, p. 72).

The issue of debt securities can be considered as a main alternative to bank credit. The Characteristics of bonds, which may increase interest in this kind of instrument among issuers are primarily a diversification of funding sources, high rate of getting the funds, optionality of collateral issue. Also there is no need for the issuer to determine the goal funds would be spend on, as well as the ability of the issuer to establish strong relationships with institutional investors, which may result in further cooperation in the future (see: Antkiewicz 2011, p. 141).

The disadvantages associated with the public offering of the bonds may include issuance costs, information obligations and the process being time-consuming. From the point of view of the investor risks associated with investments in bonds may involve credit risk that could lead to bankruptcy of the issuer and, consequently, not to pay off a debt; the risk of reinvesting which reflect unpredictable changes in interest rates; currency risk relevant for foreign investors and the risk of purchasing power to reflect unforeseen changes in the future value of money, which has an impact on the real bond yields (see Bailey 2005, pp. 297 - 298).
In the case of the corporate bond market in analyzed countries it is the most important market in the Irish economy. Its size in the years 2006 - 2011 exceeded 100% of GDP (with the exception of 2008). The development of this market in Ireland appears to be equally impressive. In the analyzed period, the size of the corporate bond market increased by 107,43 pp. (almost 19-fold increase). In the other countries corporate bond market seems to be a market relatively insignificant, except for the economy of France (56,26% of GDP in 2011) and Greece (increase in the market over the period considered by 33,54 pp.).
Data presented in Table 1 gives very clear view on the stock exchange bond market in Poland. Capitalization of corporate bond market in relation to GDP in Poland is the smallest of all the variables studied, and several times lower than the rate of other analyzed countries. This reflects the low level of development of this segment of the financial market in Poland. In 1999 this ratio stood at 0,15% of GDP and in 2011 at the level of 2,07% of GDP. The share of this indicator in the Polish GDP seems negligible. Its growth is the recent years was primarily due to the start-up of organized trading in the form of Catalyst and Bondspot markets on 30th September 2009 by Warsaw Stock Exchange.

Table 1 The capitalization of the corporate bonds market/GDP in % in constant prices in the period 1999-2013.

Year	Germany	Great Britain	France	Ireland	Greece	Poland
1999	58,72	18,68	35,06	6,04	0,54	0,15
2000	57,06	18,26	34,19	7,73	0,23	0,18
2001	53,04	17,28	35,24	9,38	0,17	0,22
2002	48,12	17,30	37,86	10,31	0,24	0,31
2003	46,70	17,00	40,58	30,86	1,06	0,44
2004	41,52	16,35	41,19	62,71	2,32	0,51
2005	34,92	15,08	38,31	80,99	3,89	0,52
2006	32,70	14,85	39,14	96,51	6,18	0,67
2007	35,41	15,81	46,59	102,51	8,41	0,98
2008	35,40	15,56	51,15	95,32	10,94	0,96
2009	36,51	15,93	53,78	103,98	13,43	1,20
2010	31,59	15,05	55,88	119,15	23,52	1,67
2011	24,02	12,32	56,26	113,47	34,09	2,07
2012	N/A	N/A	N/A	N/A	N/A	N/A
2013	N/A	N/A	N/A	N/A	N/A	N/A

Source: own study based on: T. Beck, A. Demigurc-Kunt, R. Levine, *A New Database on Financial Development and* Structure, World Bank Economic Review 2000, nr 14, s. 597-605; T. Beck, A. Demigurc-Kunt, R. Levine, *Financial Institutions and Markets across Countries over Time: Data and Analysis*, World Bank Policy Research, Working Paper 4943, Maj 2009; M. Cihak, A. Demirguc-Kunt, E. Feyen, R. Levine, *Benchmarking Financial Development around the World*, World Bank Policy Research, Working Paper 6175, August 2012; Financial structure dataset September 2015, http://www.worldbank.org/en/publication/gfdr/data/financial-structure-database.

The issuance of corporate bonds is an important source of raising capital on the stock exchange but in a small group of countries based on the available data for each year. You will notice that the stock market bonds in the Great Britain and France dominate in terms of development and the amounts of capital obtained from the exchange. It seems that the main reason behind it may be that the internationalization of both markets, an appropriate level of liquidity of both markets, traditions of London Stock Exchange market, which in its early days traded in debt securities and a higher degree of confidence which investors have for the London market. In the case of the Euronext (one of its founders was Paris Stock Exchange), it seems that big significance at the very beginning of its functioning was mainly due to multi-quotation of instruments on the combined exchanges (Paris, Lisbon, Brussels and Amsterdam), greatly expanding a group of investors and increasing the level of market liquidity.

The amount of capital raised by the issuance of bonds on the Frankfurt Stock Exchange in Germany showed a rising trend since 2008 and reached 6,53 bln USD in 2011. The highest value of capital raised by German companies occurred in 2003 and had a value of 355,81 bln USD.

The amount of capital raised by the issuance of corporate bonds on the London Stock Exchange in the period 1999 - 2008 increased almost 5-fold and reached 629,1 bln USD at the end of 2014. compared to 132,8 bln in 1999. During this period, the value of the bond issuance showed an increasing trend with an exception of 2001 and 2007.

French stock market Raising capital for the company by issuing corporate bonds has reached significant proportions on the French stock market. The size of capital raised in 2010 was almost 13 times higher than the amount of capital companies acquired 1999 in the form of corporate bonds. Amount of capital raised reached 2,25 trillion USD in 2003, which represents the highest value from all analyzed exchanges in the adopted timeframe.

Table 2 The value of capital raised through the issuance of domestic corporate bonds on the respective stock exchanges (main market) in bln USD in the period 1999-2013.

Year	Germany	Great Britain*	France*	Ireland	Greece	Poland
1999	205,01	132,80	36,51	1,30	N/A	0,00
2000	N/A	144,95	96,05	1,93	N/A	0,00
2001	N/A	121,29	79,82	0,88	N/A	0,00
2002	N/A	138,90	223,85	N/A	N/A	0,01
2003	320,78	195,11	2 251,26	N/A	N/A	0,01
2004	131,69	234,05	143,72	N/A	N/A	0,14
2005	N/A	265,74	165,11	N/A	0,00	0,02
2006	N/A	430,17	154,26	N/A	0,01	0,07
2007	N/A	343,49	247,24	N/A	0,00	0,31
2008	1,83	629,09	548,84	N/A	0,44	0,37
2009	4,41	399,07	518,22	0,00	0,90	0,04
2010	5,17	N/A	456,85	0,00	0,33	0,72
2011	6,53	N/A	N/A	N/A	0,00	0,75
2012	N/A	N/A	N/A	N/A	0,00	0,86
2013	N/A	N/A	N/A	N/A	0,00	1,27

Source: www.world-exchanges.org, http://bse.hu/topmenu., http://www.helex.gr, * issues relate to the entire market, the private sector and the domestic, foreign and international issues of bonds.

In the case of Greece, the issuance of corporate bonds on stock exchange is not a significant way of raising capital for companies in the analyzed period. With the exception of 2009 the amount of capital raised from the stock market was relatively small or there was no new bonds issues. The main reason behind it seems to be the financial crisis of 2008 and subsequent deep debt crisis of the Greek economy.

As for the stock market corporate bond issues in Poland, the value of capital raised through the issuance of debt securities was incomparably smaller than the issue of shares. A significant increase in the number and value of issues from 2009 and onwards was associated with the start-up run by the Warsaw Stock Exchange and BondSpot of two markets especially intended for trading of treasury bonds, corporate bonds, municipal bonds and mortgage bonds. The first increase in the value of the bond issue occurred in 2007. Enterprises who wanted to take advantage of the prevailing period of the stock market situation decided to raise capital from the stock market, which resulted in an increase in corporate bond issuance compared to previous years. The value of the issue amounted to 307,52 mln USD in 2007 and in 2008 it was 373,56 mln USD with both numbers greater than in year 2006 respectively by 239,9 mln USD (more than 3,5-fold increase in the value of issues) in 2007 and by 306 mln USD (more than 5,5-fold increase in the value of issues) in 2008. The decrease in the value of issues in 2009 was in turn due to the fact that vast majority of issuers went with the issues on Alternative Trading Platform (ATP). However, a considerable increase of interest in the main market for bonds can be seen since 2010, which resulted in a steadily increasing value of capital raised through the issuance of corporate bonds on the regulated market. The number of new corporate bond issues was 63 in 2012. It exceeded the equivalent of 1 bln USD obtained this way in 2013. The growing trend was maintained in 2014, in which the value of capital raised from the issuance of corporate bonds amounted to 1,4 bln USD. In relation to 2009 this is a huge increase in value of capital raised by issuance of corporate bonds in Poland. The value of capital raised in 2014 compared to 2009 increased almost 40-fold.

A thorough analysis of the amount of capital companies acquired from corporate bond issues on the stock exchange is unfortunately hampered by the unavailability of statistical data. There are series of data, but they are incomplete or, as in the case of Ireland data series is very short, which makes it impossible to make a full analysis for these exchanges and only a partial analysis for given exchanges can be carried out.

At the same time it must be noted that the corporate bond market in Poland is mainly based on private market bond issues organized on the interbank market. Whereas the public corporate bond market had its beginning in 2000, when the Warsaw Stock Exchange carried out the first issue of corporate bonds of the Centrum Leasingu i Finansów Clif S.A. Company.

In the following years, the number and size of corporate bonds on the Warsaw Stock Exchange did not undergo significant changes. The exception is 2004 during which the 18 issues amounted for a total of 136,73 mln USD. It was 11-fold increase compared with the previous year. This resulted primarily from the Polish accession to the European Union. However, in the next two years, the volume of capital raised from the stock capital through the issuance of corporate bonds was significantly reduced to 22 mln USD in 2005 and 67 mln USD 2006. In 2007 and 2008 the value of issuance exceeded in both years 300 mln USD making good use of ending bull market. It is also worth noting the 2008 issuance which amounted to over 373 mln USD. This could be due to the fact that the stock market was uncertain and unstable at the time. A significant decrease in issues in the bond market occurred only in 2009 with the total amount raised of 35,96 mln USD. The number of new corporate issues carried out on regulated market in Poland also differ from year to year. There was 37 issues in 2010 resulting in acquiring capital equivalent to 722,98 mln USD. The largest number of issues was carried out in 2008 with 63 issuers. In turn in 2013 value of the issue for the first time exceeded 1 billion USD (1,27 bln USD from 41 issues). The largest percentage of issued bonds represents corporate bonds issues. In 2013 41 out of 47 issues were corporate bonds issues and in 2014 51 out of 53 of total issues were carried out by firms. The increase in issuance in recent years seems to be affected by increased interest in this form of raising capital, as well as, keeping the main interest rates at a low level (which leads to a reduction in the cost of capita) and the desire to diversify sources of funding.

However, the positive picture of the changes taking place in the corporate bond public market in Poland does not seem to be devoid of defects. Most corporate bonds are issues ranging from a few to several tens of millions PLN. A relatively small percentage of issues are comparable to the size of the share issue. The highest issues are also the domain of the banking sector, and then the energy sector, construction and development sector and telecommunications. Furthermore, issues of high value are the domain of large, well-known enterprises[*]. In addition, the vast majority of issuers do not use the services of credit rating agencies because this is an additional cost. Finally, the secondary market of corporate bonds is a market with negligible liquidity, and the maturities range only from few to several years and rarely exceed a period of 10 years. Moreover (especially in 2014 alone with 5 notices of bankruptcy and 16 issuers failed to meet maturity payments to bondholders), issuers are failing to meet the issuance conditions which is causing unfavorable environment for further development of corporate bond public market in Poland (see: Rozwój 2014, p. 260).

[*] In 2012 the biggest issuers: Bank Gospodarstwa Krajowego, PGNiG, PKO BP, Energa, PKN Orlen - all issues over 1 bln PLN. In 2013 in the biggest issues were involved PGE, ING Bank Śląski, GNB Auto and Ciech. In turn, the biggest issues (each 500 mln PLN) of 2014 were attributed to 2 banks: mBank and Bank Zachodni WBK.

All mentioned above factors reflect a small share of corporate bonds in acquiring capital in Poland. Against the obtaining funds from the public issue of shares and long-term bank loans, the role of the public issue of corporate bonds seems to be negligible. Even if the value of the issue in recent years showed rapid growth, still for many companies the main source of capital in the form of external financing is bank loan. The reasons behind that little use of corporate bonds as a source of capital might be relatively high cost of raising capital on the stock market - 5% of the value of the issue (see: Antkiewicz 2011, pp. 142 - 143), which can be a disincentive for companies wanting to place an issue on the stock market. P. Niedziółka indicates also the dispersion of the Polish economy, which is dominated by small and medium-sized enterprises (consequently the amount of capital they want to obtain is too low achieve the ranks of the stock market issuance), little interest on the part of investors to take on the risk of the issuers, relatively high requirements for admittance and information , costly rating institutions and the instability of the demand for the new bond issues (see: Niedziółka 2005, p. 297).

3 Long-term credit for firms and raising capital through the issuance of corporate bonds

Market of long-term loans granted by banks is main competitor of other forms of raising capital (mainly the issue of shares and bonds). However, this market also complements and extends the area of possible ways of getting needed capital within the financial market.

The market of bank loans to the private sector is of paramount importance in Germany, Great Britain, France, Ireland and Greece, where as the Poland is an outsider in the analyzed group of countries with the lowest level of bank credit/GDP ratio. The level of 50% was just exceeded in 2011. In these countries (except Germany) in 2013 the ratio significantly exceeded the level of 100% of GDP as the table 3 shows. The greatest significance of this market is in the economies of Ireland (168,06% in 2013.) and Great Britain (157,96% in 2013.). The dynamics of changes of "bank loans to the private sector / GDP" ratio in the analyzed period indicates that the largest increase in this ratio occurred in the Greek (81,19 pp.) and Irish economy (81,97 pp.). In the Great Britain and France ratio value increased during the analyzed period by respectively 48,53 pp. and 33 pp. In contrast, in German economy value of the ratio has decreased by 18,75 pp. At the same time it is worth noting the growing importance of bank loans to the private sector in the initial period of the financial crisis (since 2008). This can be seen on the example of the Great Britain and Ireland, where the value of the market has exceeded 2 times the GDP peaking in Ireland in 2009. However after 2009 significant drop in ratio can be noted in each country except Poland and Greece. The largest decline in the size of loans in the 2008 - 2013 period falls on Great Britain – 29,19 pp. and Ireland - 42,33 pp. Significant rise of 30,52 pp. reported the Greek economy.

Table 3 Bank credit for private sector/GDP in % in constant prices in the period 1999 – 2013.

Year	Germany	Great Britain	France	Ireland	Greece	Poland
1999	115,55	109,42	bd.	86,09	36,36	22,94
2000	116,33	114,34	81,29	94,07	42,21	24,95
2001	117,40	123,39	85,07	98,99	49,76	26,5
2002	117,17	127,24	85,82	100,58	56,47	26,98
2003	116,26	130,04	86,23	104,72	59,34	27,17
2004	113,64	136,23	88,15	116,06	64,61	26,97
2005	111,83	143,17	90,24	136,24	73,20	27,74
2006	108,72	151,06	93,66	158,74	79,16	30,13
2007	104,56	164,09	99,33	178,30	86,04	34,66
2008	106,46	187,15	106,76	210,39	94,04	43,23
2009	113,03	202,20	111,38	227,53	95,73	47,39
2010	107,41	190,24	111,53	215,88	108,31	48,75
2011	103,37	179,87	113,33	194,32	123,84	51,4
2012	101,47	168,22	115,23	183,12	125,74	50,11
2013	96,81	157,96	114,30	168,06	124,56	N/A

Source: authors own study based on the same source as the Table 1.

As for the value of the long-term loans to companies in the analyzed period it was shaped in different ways. Clearly the visible divide gas been marked between countries where the corporate loans were used to a greater extent in the financial supply of companies and countries in which this financing method was used on a smaller scale. Germany and France are the countries where the value of corporate loans at the end of each year was clearly higher than in other countries, with an exception of Great Britain.

Conclusion can be drawn, based on data series in table 4, that long-term credit has been an important source of financial supply of the firms in analyzed countries and played a greater role than raising funds from the stock market in the form of a bond issue.

Table 4 The value of long-term credit (above 1 year) to enterprises In bln USD In the period of 1999-2013. The end of the year values.

Year	Germany	Great Britain	France	Ireland	Greece	Poland
1999	812,35	384,75	485,21	N/A	N/A	34,28
2000	804,21	337,61	451,99	N/A	N/A	36,33
2001	847,72	369,27	472,01	108,91	27,37	41,27
2002	955,35	411,29	507,53	99,20	32,27	48,25
2003	1 130,18	454,02	570,28	127,67	44,76	56,56
2004	1 201,93	541,87	651,15	140,27	55,69	55,69
2005	1 223,37	674,61	689,61	244,54	64,87	66,31
2006	1 291,63	748,55	772,10	336,77	72,17	87,71
2007	1 448,87	710,68	958,89	409,20	88,04	120,83
2008	1 647,19	774,40	1 154,63	505,44	106,64	199,62
2009	1 629,63	698,70	1 109,24	503,71	120,74	156,67
2010	1 594,67	826,39	1 071,67	514,89	130,87	160,96
2011	1 719,25	880,42	1 174,09	531,27	135,56	191,87
2012	1 615,30	943,21	1 076,49	469,39	113,98	175,84
2013	N/A	934,16	N/A	N/A	108,63	N/A

Source: http://ec.europa.eu/eurostat/web/main/home., access date: 13.12.2015r.

The value of long-term loans granted in 2012 in all of the analyzed countries amounts to 4,4 trillion USD. In 1999 the value of loans in the analyzed group was 1,6 trillion USD which is an increase of 154% and translates into an average annual growth of 190 billion USD.

On the other hand, taking into account the rate of change in the value of loans granted by banks in respective countries in the period 1999 - 2012, in all of the analyzed economies, the value of loans granted to companies has clearly increased. In Ireland, the value of loans increased by 331%, in Greece by 316%, in the Great Britain by 145%, 122% in France and in Germany the increase was 99%. However, in the analyzed period, the value of loans to businesses has experienced also smaller or bigger falls. Those decreases in value of long-term bank credit can be liked to 2008 and onwards. It caused the lending from financial institutions to be subjected to large fluctuations. It was primarily the result of economic situation in the world at that time - financial crisis, debt crisis, tightening prudential norms, the shortage of liquidity in the banking sector.

Conclusions

The raising of capital from the stock market by the companies in the analyzed countries in the given period was influenced by number of factors. It seems that at the forefront lies of the state of the stock market situation in different countries and the importance of bank credit to businesses. Periods of good stock market situation in 1999 - 2000, 2003 - 2007 and 2013 - 2014 have contributed to a significant increase in the number and value of new issues on the stock markets. The period 1999 - 2000, prior to the crisis caused by the market revision of the internet companies (so called dot-coms in the US market, with particular emphasis on the NASDAQ) was represented by a large number of stock exchange issuances of both stocks and bonds.

In the Anglo-Saxon financial system (market-oriented) stock exchange plays a greater role as a source of raising capital than in the German financial system (bank-oriented). However, the importance of stock exchange in raising capital varies significantly depending on the country as the analysis of the sources and scale of capital gain for companies in the Great Britain and Ireland shows. It should also be noted that in Great Britain in the period 1999 - 2013 corporate bond issuances played increasingly important role. The analysis of statistical data concerning France allows to draw the conclusion that after the establishment of the common market Euronext issuance of corporate bonds played important role in raising capital for the companies, and the development of this market segment is larger than in the Great Britain and Germany.

In the countries of continental Europe with bank-oriented financial system (including Poland), a process of raising capital was dominated by bank loans rather than the issuance of shares and bonds. In addition, it should be emphasized that the stock market issue of corporate bonds in Poland is of little importance to businesses, despite the creation and launch of trading platform dedicated primarily to bonds and other debt instruments issued by firms. This is due, among others, to the fact that an important alternative to a regulated market bond issuance in Poland is the bank credit and the OTC market of corporate bonds ran by banks. In addition, the stock exchange corporate bond market in Poland is still a young market with limited liquidity and significant risks for investors. This results in that Poland has one of the lowest rates of raising capital from the stock exchange by issuance of corporate bonds.

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QUALITY FEATURES OF HUMAN CAPITAL IN THE PRESENT STAGE OF DEVELOPMENT OF INNOVATIVE ECONOMY EVIDENCE FROM FEDERAL SUBJECT OF RUSSIAN FEDERATION - REPUBLIC OF TATARSTAN

SÚČASNÝ STAV KVALITATÍVNYCH VLASTNOSTÍ ĽUDSKÉHO KAPITÁLU INOVATÍVNEJ EKONOMIKY, NA PRÍKLADE SUBJEKTU RUSKEJ FEDERÁCIE – REPUBLIKA TATARSTAN

***Abstract**: In the modern economic system a key factor in the competitiveness of the economy, the basis of economic growth is productivity. Impact of performance factors are equally important both at the macroeconomic level and at the enterprise level.*

High labor productivity contributes to more efficient use of all resources, which is an important component of the quality of the labor force. At the same time it is an opportunity to produce goods more effectively and provide services to ensure high quality of growth of value added, rather than merely its gross increase. Efficient production contributes to increase the volume and quality of produced goods and services and, ultimately, improve the quality of life.

The accumulated human capital plays significant role in the formation of a high-performance workforce, along with the level of technological development of production. Ascomplex economic category it has qualitative and quantitative characteristics. Human capital is characterized by a certain level of physical development, mental abilities and knowledge, innovative thinking necessary for effective work, adequate current economic environment.

***Key words**: human capital, innovative economy, productivity, human resources, quality of life*

***Kľúčové slová:** ľudský kapitál, inovatívna ekonómia, produktivita, ľudské zdroje, kvalita života*

JEL: J24

Introduction

For a successful economic developmentit is necessary to ensure the appropriate quality of the labor force. This is the policy currently adopted by most developed countries investing in this area their considerable financial resources.

Human capital is the main postulate of the Strategy of the Republic of Tatarstan social and economic development in the Russian Federation until 2030.
Human capital becomes more sensitive factor of production in the innovation economy when the situation related to its under-provision in quality or quantity can be critical for production [1]. Its formation should be carried out through the achievement of high quality of life in general, and in particular, at the micro level - by creating a comfortable and safe working conditions, paid employment, self-realization. In terms of strategy activities - 2030 provides for a number of priority projects on factors like human capital formation (education), and the conditions of its accumulation and use (the labor market and employment support). On the other hand, we note that it is the labor market is the main source of capital accumulation and wealth in the country, and hence the incentive for interest from investors [2]. The main objective is to stimulate the demand for innovation and as a consequence, on human capital. It is necessary to take into account the relationship between the activities of innovative companies, training, inter-regional and international mobility of persons with high levels of human development. [3]

At the level of the individual enterprise increasing productivity will ensure business competitiveness: reduce costs, save investment capital to meet its obligations to shareholders, employees and the state.

Companies actively introducing innovative technologies and producing innovative products, conduct a targeted policy aimed at improving the quality of human resources, as well as seeking to increase the number of its employees in order to gain control over a greater share of the market. At the same time the introduction of new technologies increases labor demand higher skills, which positively affects both the wage level and the company's spending on training and development of its own staff. In addition, in the medium and long term release of the labor force, effectively used in some enterprises, leads to a redistribution of labor between enterprises in the economy.

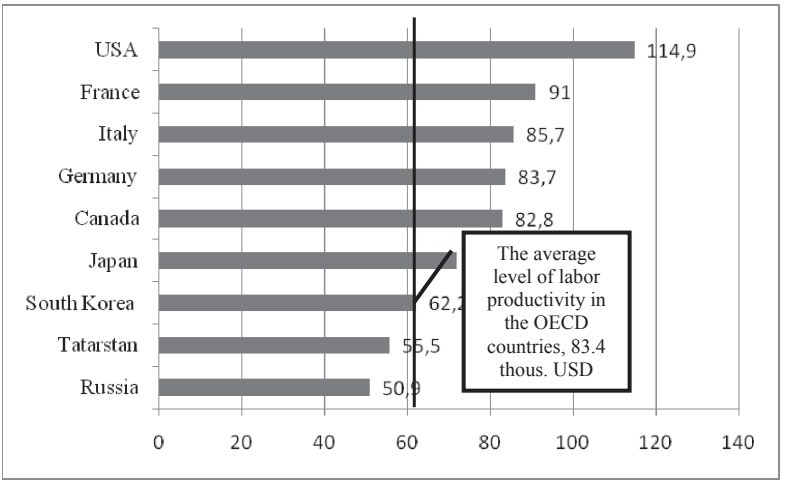
Methods

The research methodology presents the methods of graphical analysis, synthesis, communication of the historical and logical, normative and positive analysis, as well as methods of expert estimations.

Results

Social and economic policies in the Republic of Tatarstan, is aimed at improving the quality of life and ensuring the release of the republic on the path of sustainable innovation development of economy. Largely due to this policy and systematically constructed work, Tatarstan is one of the leading regions of the Russian Federation with a significant economic potential and development of human capital.

Fig. 1 The labor productivity index in the countries of the Organization for Economic Cooperation and Development (OECD) in 2014, in US dollars per employee (in purchasing power parity)



Source: compiled by the author according to Rosstat (Federal State Statistics Service) [4]

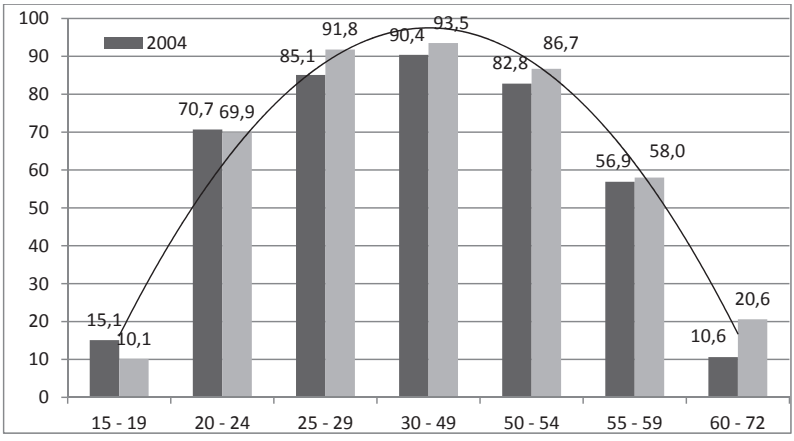
Tatarstan has great potential for the growth of labor productivity of enterprises. In general, labor productivity, calculated in US dollars per person employed in 2014, in the Republic of Tatarstan's economy is 1.5 times lower than the average for the same period of the Organization for Economic Cooperation and Development (OECD).

The group of countries with the highest labor productivity - the US, France, Italy, Germany, Canada refers to the economic leaders. At the same time the average productivity of the countries of the Organization for Economic Cooperation and Development is 83.4 thousand dollars [5].

Over the past five years in the Republic of Tatarstan labor productivity increased by 16 percent in real terms, while real wages - by 25 percent. If directed to the payment of wages, in some enterprises, this ratio exceeds 50% on average from each ruble of added value produced by one worker, about 25 cents. The only reserve for the further development of enterprises and enhance the wages is the growth of labor productivity and production efficiency.

The enterprises within the same activity level of labor productivity can vary significantly, suggesting that there is provision for its growth. On the lack of effective use of reserves and says the dynamics of high-performance jobs, i.e. all workplaces of the company, in which the average monthly salary of employees exceeds a predetermined value or threshold. Before the Republic of Tatarstan the task in 2020 to create and upgrade 750 thousand high-performance workplaces. Thanks to the implementation of investment projects for three years, their number increased by one third and reached by the end of 2014 about 550 thousand. Leader in the number of jobs is a high-performance manufacturing industry, however, the rate of increase is significantly lower.

Fig. 2 The level of economic activity of the Republic of Tatarstan of the population by age group in 2004-2014. (Percentage of the population of the relevant age group)



Source: compiled by the author according to Tatarstanstat [6]

To date, there is a certain structure and quality of labor resourcesat enterprises and organizations of the Republic of Tatarstan economic sectors. Overall, about 50% of the economically active population in the country, that is, undertake an economic activity in a particular area of the economy, as entrepreneurs or employees. Almost 90% of the economically active population - citizens aged 25 to 55 years. In other age categories of population activity does not exceed 50 - 70%.

The distribution of the economically active population by age Tatarstan is close to the optimum. During the period from 2004 to 2014 there have been both quantitative and qualitative changes that put it in a general increase in the proportion of the working population in the main groups of working age. At the same time, it decreased the proportion of economic activity in the age of 15 - 19 years, and mess of increased aged 60 - 72 years. This trend is due to the demographic process of an aging population in the near future will lead to an increase in the demographic load on working, and thus strengthen the role of young people in social and economic development of the republic. In general, in terms of the quality of human capital, its reserves and possibilities age composition of the labor force is a valid, able to function in the innovative economy.

Here it should be noted about the importance of health human capital as a factor of economic growth. This is due to the influence of health on savings, investment in human capital, labor supply, labor productivity growth. Physical strength, endurance, efficiency, immunity to disease, increasing the period of active employment are necessary to every person, in every sphere of professional activity.

The essence of health as a social and economic category is shown in the fact that its deterioration entails additional costs for the restoration, as well as losses due to a decrease in capacity of the economic activity.

In a number of Russian and foreign researches initial health status, as measured by life expectancy was statistically more significant factor in the predicted growth than indicators related to education level. Poor health of the adult population significantly affects the economy through the labor market. According to official statistics in 2014 reported 84 thousand per 100 thousand. Population with newly diagnosed by main groups of diseases. Over the past five years, the positive dynamics is observed. Accordingly, the number of cases of temporary disabilityis not reduced. Due to sickness an average of 10 working days are lost per year per employee. All this leads to a serious loss of working time for the economy and detrimental to human health.
Today in Tatarstan life expectancy exceeds 70 years. The main factors for the increase of this indicator, depending on the person, is the preservation of health and healthy lifestyles. As the results of a sociological survey of the population of the republic, for a large part of the population aged 15 years and over "strong" health is one of the most important values in life (91,7%). The priority values are "children" (88,3%), "good family" (84,3%), "material wealth" (75,2%) and "good work" (60,5%).

In turn, quality of food affects the human health. This further affects the productivity and, in general, on economic growth. Thus, poor nutrition in childhood leads to low productivity in adulthood. The consequence of malnutrition in middle age is a decrease in the effectiveness of both mental and physical labor.
The quality of human capital is largely determined by the level of education. Of course, a person with a certain amount of knowledge is more creative and competitive. In recent years, the trend of young people the desire to obtain a higher qualification or vocational training the second. The reasons for this situation is likely to:

- The priority of higher education to secondaryvovational;
- Availability of higher education associated with having a commercial form of training;
- The desire to have a prestigious high-paying job and have an alternative choice in the workplace;
- The desire to receive higher incomes.

In turn, timely career guidance, targeted training for specialized professions allow businesses to obtain the necessary qualified personnel, reduce the imbalance in the labor market between supply and demand.

Table 1 The structure of employment in the economy of the Republic of Tatarstan by level of education in 2004-2014. (Based on population surveys on employment, as a percentage)

	Year 2004	Year 2014
Busy housekeeper in all	100	100
Including education:		
higher professional	24,0	30,8
secondary vocational	21,9	19,8
initial vocational	18,5	21,3
secondary	29,9	25,5
compulsory	5,3	2,4
have no education	0,4	0,2

Source: compiled by the author according to Tatarstanstat [6]

Developed countries and emerging economiesare characterized by a high percentage of the employed population with higher and secondary vocational education. Despite the fact that developed countries are significantly inferior to developing States for the labor force, they are ahead of the latest in quality of labor - educational level, the number of highly skilled labor force mobility. The low level of literacy of the population of individual developing countries determines the relatively low labor productivity, leading to the preservation of obsolete forms of economic, technological progress slows.

Both in Russia and in Tatarstan, the proportion of persons with higher and secondary education is 51% of the total number of employed. During last decade, the share of employment in the Republic ofTatarstan enterprises with higher education increased from 24% to 31%. As mentioned above, this is due to the development of commercial systems of higher education institutions, as well as employers' requirements to the candidates on the availability of higher education diploma.

In general, there is a positive trend of redistribution of employed in the economy in the group with higher levels of education. However, it is still around 28% or slightly more than a quarter of the labor force remains below the professional education, have only a certificate of completion of secondary school education.

Moreover, among the leaders of enterprises only 75% have a college education, and the remaining 25%, or one in four - only secondary or basic vocational. The level of qualification of engineering job workers is much higher: 85% of specialists with higher education in the field of science and engineering, and 92% - in education and healthcare.

As can be seen, specialists, managers and business leaders have a need in raising the qualifications and skill growth. One reason for the backwardness of Russia from the industrialized countries is a stereotype perception of professional education as an end.

However, the gain knowledge is not always remain their value throughout a professional career. Basic vocational education should not be regarded as sufficient for the entire period of employment. These human knowledge and experience has to be updated and expanded throughout the period of activity. In the current economic situation on the innovative enterprise employee must act at an advanced level, to be able to navigate in today's business environment, atypical, business situations, driving in conditions of permanent change and uncertainty.

Table 2 Distribution of the number of employees of enterprises and organizations of the Republic of Tatarstan by occupation and level of education (as a percentage of the total number of enterprises surveyed employees)

	Total	**including with education**			
		secondary vocational			**compulsory**
		higher	secondary	elementary	
Executives	100	75,6	19,0	3,2	2,3
Higher skill level specialists	100	84,8	12,9	1,0	1,4
Experts in the field of natural sciences and engineering	100	85,1	13,7	0,5	0,8
Experts in biological and agricultural sciences, and health professionals	100	92,0	6,6	0,7	0,7
Teaching professionals	100	90,1	9,3	0,4	0,3

Source: compiled by the author according to Tatarstanstat [6]

Thus, the development of innovative economy currently causes an increase in requirements for human capital. The need to improve the level of education due to the change of working conditions in connection with the application of new technologies and features extensive use of information and communication systems. The employee must have the ability to master new equipment and IT technology, to be ready to implement new works and services, the mastery of related professions, a new workplace, communication with foreign partners.

Summary

The transition from a resource-based economy to an innovative way of development requires substantial investment to modernize economy. However, due to budgetary constraints there is a problem of financial support and development of the innovative kind of projects, as well as mechanisms of investment in innovation [7]. Investigation of the basic characteristics of the development of human capital evidence from the federal subject of the Russian Federation – the Republic of Tatarstan revealed the following trends:

Measures taken by the authorities of the Republic of Tatarstan aimed at improving the quality of human capital, the possibilities of its use in the innovative economy. In general, the analysis of the main qualitative characteristics of human capital in terms of microeconomics has revealed both positive and problematic sides. Basically, this is reflected in the demographic aging of the labor force, increasing the workload on the younger generation, the presence of health problems and physical activity, poor training of employees for the implementation of high economic activity. These factors determining the current level of quality of human capital, do not allow innovative companies to fully realize their intended targets, implement new ideas in business development.

In this regard, along with the measures taken by the government on labor market regulation in the innovative economy significantly increases the role of the corporate incentive system to improve the quality of human capital.

Zhrnutie

Prechod z ekonomiky zdrojov založených na inovatívny spôsob rozvoja si vyžaduje značné investície do modernizácie hospodárstva. Avšak, vzhľadom na rozpočtové obmedzenia sa vyskytuje problém finančnej podpory rozvoja inovatívnych druhov projektov, ako aj mechanizmov investícií do inovácií [7]. Skúmanie základných charakteristík rozvoja ľudského kapitálu na príklade Ruskej federácie – Republiky Tatarstan odhalila tieto trendy:

Opatrenia prijaté orgánmi republiky Tatarstan sa zameriavajú na zlepšenie kvality ľudského kapitálu, a možnosti jeho využitia v inovatívnej ekonomike. Všeobecne platí, že analýza hlavných kvalitatívnych charakteristík ľudských zdrojov, pokiaľ ide o oblasť mikroekonómie odhalila pozitívne i problematické stránky. Problematické stránky sa odrážajú najmä v demografickom starnutí pracovnej sily, čím sa zvyšuje pracovná záťaž mladej generácie, prítomnosť zdravotných problémov a fyzickej aktivity, nedostatočné školenie zamestnancov na realizáciu vysokej ekonomickej aktivity. Tieto faktory určujúce súčasnú úroveň kvality ľudského kapitálu neumožňujú inovatívnym spoločnostiam plne realizovať ich požadované ciele, realizovať nové nápady v oblasti rozvoja podnikania.

V tomto ohľade, spolu s opatreniami prijatými vládou o regulácii trhu práce v ekonomike inovácií, výrazne zvyšuje úlohu firemného motivačného systému k zlepšeniu kvality ľudského kapitálu.

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The text of the article The recommended structure of the text of the article is as follows: **Introduction, Material and Methods, Results and Discussion, Summary**. The summary is always also in the Slovak or Czech. The text is written in Times New Roman, 14 points, justified. Single line spacing, pages are not numbered.
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After each part of the given structure insert a blank line in the font size of the respective part.
Tables and graphs must be numbered and must also be distinguishable in a smaller format.
Reference and the title of the table - 'caption' (Tab. 1: Caption) or graph (Fig. 1: Caption) are written in Times New Roman, 12 points, aligned to the left. The contents of the table are formatted in Times New Roman, 10 points. The source of the table and graph must be provided below the table or graph (Source :), written in Times New Roman, size 12, aligned to the left.
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Length of the article should not exceed 15 standard pages including annexes.
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[1] ROTHBARD, M., N., 2008. Mystery of Banking. Aubur, Alabama: Ludwig von Mises Institute. ISBN 978-1-933550-28-2.
[2] HORVÁT, J. et al., 1999. Anatómia a biológia človeka. 2. vyd. Bratislava: Obzor. ISBN 80-07-00031-5.
[3] STEINEROVÁ, J., 2000. Princípy formovania vzdelania v informačnej vede. In: Pedagogická revue. Roč. 2, č. 3, s. 8-16. ISSN 1335-1982.

The square brackets include the serial number of the work and page(s) of the paraphrased work.

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