Utility Functions and Equity Premium Puzzle: Evidence from the V-4 Economies

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Abstract

The paper introduces the concept of equity premium puzzle within a stochastic discount factor model and then it presents Hansen-Jagannathan bounds as a means of both capturing this phenomena and also testing various utility function specifications, which might help to explain and solve the puzzle. Three utility frameworks are assumed in the paper: constant relative risk aversion, habit formation and Epstein-Zin utility. Data on equity premiums are analyzed for the Czech Republic, Hungary, Poland and Slovakia. The comparison of Hansen--Jagannathan bounds with the restrictions given by the three utility functions shows that it is not possible to expect to employ a universal approach to this issue as the conclusions differ to some extent across the economies examined. Generally the alternative utility frameworks do not seem to be a solution to the equity premium puzzle in case of V-4 economies.

Keywords: *asset pricing, constant relative risk aversion utility, Epstein-Zin utility, equity premium puzzle, habit formation, Hansen-Jagannathan bounds*

JEL Classification: E00, E44, G12

1. Introduction

The paper presents a standard tool for analyzing the relationship between asset prices and macroeconomy, captured by consumption-based asset capital pricing model (CCAPM) as proposed by Lucas (1978) and Breeden (1979). It was well documented (Mehra and Prescott, 1985) that the model has trouble explaining the observed data for the US and other developed economies. The premiums measured as differences between realized equity returns and risk-free rate returns seem to be too high to be explained by the covariance between the stochastic discount factor and equity returns as the main factor. The deficiency

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came to be known as the equity premium puzzle, further extended in Weil (1989) by the so-called risk-free rate puzzle. The puzzle has been challanged by various theories, however, the first ones were concerned with the utility function used to describe the behavior of investors. Traditionally, constant relative risk aversion (CRRA) utility function is preferred due to its characteristics, however, in this case the parameter of risk aversion needs to be extremely high to reconcile the model with data and even then there is the problem with the behavior of risk-free rate. Epstein and Zin (1989) proposed the so-called general expected utility function which distinguishes between the parameters of risk aversion and elasticity of intertemporal substitution and so helps to alleviate the problem of risk-free rate puzzle. Another famous modification of the utility framework is habit formation proposed by Constantinides (1990) and used in a modified version by Campbell and Cochrane (1999). The key aspect of habit formation utility functions is that utility is not derived from current consumption by itself but is based on the relation of current and past consumption. As a result economic agents have strong preference to smooth consumption and economic upturns or downturns automatically cause sufficient changes in the elasticity of intertemporal substitution without the need of setting the CRRA coeffcient at an unreasonably high level.

It is this approach to the problem of equity premium puzzle and risk-free rate puzzle which is dealt with in this paper. The paper is divided into four parts. In the second part the key theoretical results of CCAPM as a starting point to expose the problem of the two puzzles are given. In the third part the concept of Hansen-Jagannathan bounds (Hansen and Jagannathan, 1991) is presented, which will serve as the key method of empirical evaluation of the issues. In the fourth part the empirical analysis both in the form of stylized facts and also by the means of Hansen-Jagannathan bounds estimation is given. The main findings are summarized in the conclusion.

2. Stochastic Discount Factor and Utility Functions

On the most general level, the stochastic discount factor theory states that the price of an asset is given by the expected present value of future pay-off, which is discount by stochastic discount factor:

$$p_t = E_t\left(M_{t+1}x_{t+1}\right) \tag{1}$$

where

p – price of an asset,

M – stochastic discount factor,

x – pay-off a holder of the asset receives at $t+1$,

E – expectation operator.

Any pricing model in economics fits in this framework, probably the most wide-spread capital asset pricing model (CAPM) uses market return as the stochastic discount factor. Dividing (1) through by the current price of an asset yields:

$$1 = E_t\left(M_{t+1}R_{t+1}\right) \tag{2}$$

where

R – gross return on an asset: $1 + r$ with r being real interest rate.

After a few algebraic manipulations, it can be shown that (2) implies for risk--free and risky returns the following:

$$R^f_{t+1} = \frac{1}{E_t\left(M_{t+1}\right)} \tag{3}$$

$$E_t R_{t+1} = R^f_{t+1} - R^f_{t+1}\operatorname{cov}_t\left(M_{t+1}; R_{t+1}\right) \tag{4}$$

where

R^f – gross risk-free return.

According to (3), gross return on risk-free asset is given by an inversed value of stochastic discount factor. Equation (4) reads that expected gross return on a risky asset is given by the gross risk-free rate which is adjusted for the covariance between stochastic discount factor and return on the risky asset. The last term: $R^f_{t+1}\operatorname{cov}_t\left(M_{t+1}; R_{t+1}\right)$ in (4) is called a risk premium.

The question is: What does the stochastic discount factor depend on? The consumption-based capital asset pricing model is used, which apart from the classic CAPM contains a direct link to the macroeconomic environment.

A representative household (investor) whose preferences are described by bounded, strictly concave and increasing utility function is assumed:

$$U = E_t \sum_{s=t}^{\infty} \beta^s u\left(C_s\right) \tag{5}$$

where

β – subjective discount factor,
C – real consumption,
u – intratemporal utility function,
E – expectation operator.

The derivatives up to second order are assumed to exist and be continuous. The household is constrained by:

$$A_{t+1} - A_t = Y_t + r_t A_t - C_t \tag{6}$$

where

Y – real income given exogenously,

r – real rate of return on asset A.

The household receives income at the beginning of each period, t, and also the return on the stock of asset A. This is used to realize consumption. The difference between the total income and consumption is allocated into the asset (if negative, the asset is used to finance the excess consumption, non-negativity condition does not pose any restriction on the immediate consequences discussed below). According to (5), the household maximizes expected utility because the rate of return on the asset is assumed to be random. Solving this problem in dynamic optimization yields a standard Euler equation:

$$u'(C_t) = \beta E_t\left[(1+r_{t+1})u'(C_{t+1})\right] \tag{7}$$

Subject to transversality condition that the present value of the stock of asset A be zero. In (7) $u'(C)$ denotes marginal utility of consumption and β is subjective discount factor so that $\beta = \frac{1}{1+\theta}$, where θ is marginal rate of time preference. The Euler equation asserts that the decision is optimal when marginal utility of current consumption is proportional to present expected value of marginal utility of next period consumption. The other necessary condition is that the present value of the stock of asset be equal zero. Thus speculative bubbles are ruled out. The Euler equation may be expressed as:

$$1 = E_t\left[\beta \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}\right] \tag{8}$$

The ratio of marginal utilities multiplied by subjectivr discount factor $\beta \frac{u'(C_{t+1})}{u'(C_t)}$ is called stochastic discount factor (or pricing kernel – e.g. Duffie, 2001). This brings attention back to (2), only this time it is clear what affects the stochastic discount factor. It will be shown below which forms the stochastic discount factor can take on depending on the particular utility function used within CCAPM. One of the most frequently used utility functions is CRRA utility function (e.g. Cochrane, 2005):

$$U(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} \tag{9}$$

It can be shown that the parameter σ refers to both coefficient of risk aversion (Arrow-Pratt coefficient of relative risk aversion) and elasticity of intertemporal substitution (σ being the inverse of this). Using (9), the Euler equation (7) takes on the form:

$$1 = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} R_{t+1}\right] \tag{10}$$

According to (10) the stochastic discount factor crucially depends on the growth of consumption. Then from (4) it means that the risk premium tends to increase together with the covariance between consumption growth and returns on the risky asset. The reason is simple enough: the higher the covariance, the more difficult it is to use this asset as a hedge against economic downturns (or upturns). Consumption smoothing is then more difficult to achieve.

As implied above, Mehra and Prescott (1985) found out in the case of the US economy it is impossible for the model prediction to match the observed data when CRRA utility function is used. The risk aversion parameter needs to be calibrated at an extremely high level. If one would accept an unreasonably high σ to match the returns on the risky asset, it would lead to the model predicting an extremely high and volatile risk-free rate, known as the risk-free rate puzzle (Weil, 1989). To show this, joint lognormality of consumption growth and returns is assumed and using log-approximation of the stochastic discount factor, the equations (3) and (4) may be expressed as (for derivation, see, for example, Cuthbertson and Nitzsche, 2005):

$$r_{t+1}^{f} = \theta + \sigma E_t\left(\Delta c_{t+1}\right) - \frac{\sigma^2}{2}\operatorname{var}\left(\Delta c_{t+1}\right) \tag{11}$$

$$r_{t+1} = \theta + \sigma E_t\left(\Delta c_{t+1}\right) - \frac{\sigma^2}{2}\operatorname{var}\left(\Delta c_{t+1}\right) + \sigma\operatorname{cov}\left(\Delta c_{t+1}, r_{t+1}\right) \tag{12}$$

where

r^f – risk-free rate,

$\Delta c_{t+1} - lnC_{t+1} - lnC_t$, *var* means variance.

The Jensen effect is neglected in (12). Following Cochrane (2005), the average postwar real return on US capital market is estimated at 9% (It is important to stress the fact that it depends on the index used and it may be down to 6%) and the average real risk-free rate based on T-bills at 1%. That means that equity premium is approaximately 8%. From (11) and (12) it follows that equity premium should be equal to:

$$r_{t+1} - r_{t+1}^{f} = \sigma\operatorname{cov}\left(\Delta c_{t+1}, r_{t+1}\right) \tag{13}$$

The covariance is estimated at less than 0.2 for US economy data, therefore it requires a cofficient of relative risk aversion of more than 50, which does not seem reasonable. (A reasonable calibration of coefficient of relative risk aversion lies between 1 and 5.) If accepted, (11) then implies high and volatile real risk-free rate.

Another possible utility framework is Epstein and Zin preferences. The so--called general expected utility function may be expressed as:

$$U_t = \left[(1-\delta) C_t^{1-\frac{1}{\psi}} + \delta E_t \left(U_{t+1}^{1-\sigma} \right)^{\frac{1-\frac{1}{\psi}}{1-\sigma}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \tag{14}$$

where

ψ – elasticity of intertemporal substitution.

The biggest issue is the estimation of such a utility function as the expected next period utility is unobservable. On the assumption that the next period consumption (and utility) is given by the return on equity (assets), the stochastic discount factor may be expressed as (e.g. Smith and Wickens, 2002):

$$M_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right]^{\frac{1-\sigma}{1-\frac{1}{\psi}}} \left(R_{t+1} \right)^{\frac{1-\sigma}{1-\frac{1}{\psi}}-1} \tag{15}$$

Finally, assuming habit formation, the utility function may be stated as:

$$U_t = \frac{\left(C_t - \lambda X_t \right)^{1-\sigma} - 1}{1-\sigma} \tag{16}$$

where

X – past consumption,

λ – sensitivity parameter.

Now assuming $\lambda = 1$ and $X_t = C_{t-1}$, the stochastic discount factor is:

$$M_{t+1} = \beta \left(\frac{C_{t+1} - C_t}{C_t - C_{t-1}} \right)^{-\sigma} \tag{17}$$

While it is obvious that Epstein and Zin preferences divide the characteristics of risk aversion and intertemporal substitution apart, the habit formation utility framework does so in an indirect way. Applying the notation of Campbell and

Cochrane (1999), S denotes the so-called surplus consumption: $S_t = \frac{C_t - C_{t-1}}{C_t}$, still assuming λ = 1 and $X_t = C_{t-1}$. Then the Arrow-Pratt coefficient of relative risk aversion is: σ/S_t . Thus the coefficient of relative risk aversion is variable in this case even though σ is kept constant. In recession S decreases and so the time-varying coefficient of relative risk aversion increases, which may help to bring the model closer to the data.

3. Equity Premium and Hansen-Jagannathan Bounds

In this part the principles of Hansen-Jagannathan bounds and its relation to econometrics will be presented, i.e. how the estimates given in the following part were made.

Equation (2) may restated as (e.g. Cochrane, 2005):

$$\frac{\left|E_t\left(R_{t+1}\right)-R_{t+1}^f\right|}{sd\left(R_{t+1}-R_{t+1}^f\right)} = corr\left(M_{t+1}, R_{t+1}\right)\frac{sd\left(M_{t+1}\right)}{E_t\left(M_{t+1}\right)} \tag{18}$$

where

sd denotes standard deviation,
corr stands for correlation.

On the left hand side of (18) is the Sharpe ratio expressing the excess return (or equity premium) per unit of risk. Interestingly, equation (18) represents a set in which all combinations of returns and risk (standard deviations) must lie. This set is called the mean-variance set. If the correlation is 1 in absolute terms, then the set is concerned of those assets whose returns are perfectly correlated with the stochastic discount factors. Those combinations lie on the mean-variance frontier and if the correlation is –1 and risk-free rate is considered, the term capital market line coined by Sharpe (1964) is used. This in turn means that the returns lying on the frontier are also perfectly correlated among themselves and therefore can price other assets equally. Realizing that the correlation coefficient in (18) cannot be higher than 1, one can rewrite (18) as:

$$\frac{\left|E_t\left(R_{t+1}\right)-R_{t+1}^f\right|}{sd\left(R_{t+1}-R_{t+1}^f\right)} \leq \frac{sd\left(M_{t+1}\right)}{E_t\left(M_{t+1}\right)} \tag{19}$$

The relationship given by (19) may be considered in a little different way. Given the market Sharpe ratio, a limit is set for the relation of volatility and expected value of the stochastic discount factor. This limit is independent of any utility framework and may be used to test whether or not a given utility function may be used in the analysis in the particular capital market. In other words, it tests whether or not the given stochastic discount factor based on a particular utility function may really serve as a reasonable stochastic discount factor given the capital market conditions and also the conditions of the real economy under examination. The limits given by (19) are called Hansen-Jagannathan bounds (Hansen and Jagannathan, 1991). Other restrictions on stochastic discount factors are discussed by Cochrane and Hanson (1992).

To employ this relationship it is necessary to assert a relation between the variability and observable variables (ie returns) properly; loosely following Cochrane (2005).

The idea expressed in (19) may be formulated as a projection of the stochastic discount factor on a set of returns:

$$M_t - E(M) = \left[R_t - E(R)\right]^T \alpha + \varepsilon_t \tag{20}$$

where

α – a regression coefficient,
ε – error assumed to be idd,
^{T} – transpose.

The error term is not correlated with returns. Multiplying both sides of (18) by $R_t - E(R)$, expression (20) becomes:

$$E(MR) = E(M)E(R) + \Sigma\alpha \tag{21}$$

where

Σ – variance-covariance matrix of returns.

Applying (2) to (21), one readily obtains:

$$\alpha = \Sigma^{-1}\left[1 - E(M)E(R)\right] \tag{22}$$

Now expressing variance of (20):

$$\operatorname{var}(M) = \operatorname{var}\left\{\left[R - E(R)\right]^T \alpha\right\} + \operatorname{var}(\varepsilon) \tag{23}$$

and using (22), one obtains an operational expression for Hansen-Jagannathan bounds:

$$\operatorname{var}(M) \geq \left[1 - E(M)E(R)\right]^T \Sigma^{-1}\left[1 - E(M)E(R)\right] \tag{24}$$

Substituing sample mean of set of returns and sample variance-covariance matrix into (24) yields a quadratic relationship between variance of stochastic discount factor and its mean. In the empirical part of the paper market return and risk-free rate as assets are used and Hansen-Jagannathan bounds are constructed for a set of possible means of stochastic discount factor. The estimation of the given relationship is performed using general method of moments (GMM).

4. Empirical Analysis

First some stylized facts concerning equity premiums in the V-4 economies will be presented. The approximative formulas presented in the theoretical part of the paper will be used here. Then the estimates of Hansen-Jagannathan bounds will be given and compared to the characteristics of stochastic discount factors under the three utility frameworks mentioned above.

T a b l e 1

Descriptive Statistics

	CZ	HU	PL	SK
Consumption growth				
mean	2.864	2.518	4.336	4.378
st. deviation	2.479	4.768	1.942	2.842
JB	0.716	0.309	2.068	0.842
ADF	–3.611**	–1.735	–2.996*	–1.990
Nominal interest rate				
mean	5.864	11.906	11.925	7.159
st. deviation	4.782	7.988	8.107	4.809
JB	2.937	3.938	1.759	1.378
ADF	–3.302**	–4.439***	–1.923	–0.478
Inflation				
mean	4.297	10.026	7.447	5.919
st. deviation	3.326	7.407	7.692	3.283
JB	1.823	5.186*	6.885**	0.723
ADF	–2.217	–5.537***	–5.127***	–1.949
Nominal capital market index growth				
mean	7.763	26.315	14.257	5.576
st. deviation	27.324	42.468	28.568	36.027
JB	0.534	2.95	1.018	10.263**
ADF	–3.342**	–2.949*	–3.853**	–2.643

Note: All data is expressed in per cent (consumption growth, nominal capital market index growth and inflation are calculated as yearly relative changes of consumption, capital market index and HICP, respectively). JB stands for Jarque-Bera statistic with null of normal distribution. ADF stands for t-statistic of Augmented Dickey-Fuller test with null of unit root. *, **, *** denotes rejection of the null at 10%, 5%, 1% level of significance, respectively.

Source: Eurostat; own computation.

Data from the Eurostat database are used. Data on real consumption, HICP index, capital market indices and short-term risk-free rate approximated by 3-months money market rates were retrieved to compute the variables needed for the analysis. Table 1 summarizes the key characteristics of the series. The analysis is carried out on annual basis on the sample 1996 – 2010. Annual data is typically used for this type of analysis. Using quarterly data would have no qualitative influence on the results presented below. It is necessary to note the fact that the estimated average return on capital market may differ according to data used, and due to the limited time span of the series it is susceptible to the sample chosen for an analysis. Again this would hold tru efor quarterly frequency as well. Of course, this mere fact renders the results of the analysis tentative as far as the exact quantitative output is concerned. It is a fact that at the beginning of the sample the economies were in transition, thus the data is influenced by the process. However, regarding the already short sample of data used, the data is kept within the sample. Again the results given below would not be qualitatively different.

4.1. Stylized Facts

Czech Republic

The average real return on the Czech capital market was app. 0.035. The average real risk-free rate was app. 0.015. This amounts to equity premium of app. 0.02. This is relatively low compared to most advanced economies. Taking account of the variability of real return on market measured by standard deviation, which was app. 0.291, the average Sharpe ratio amounts to 0.07. This is very low compared to, for example, the US capital market, where it is estimated at app. 0.5 in the postwar data. The average real consumption growth was app. 0.029 with standard deviation of 0.025.

Now assuming CRRA utility and log-normal approximation as expressed in (13), there is app. 0.02 equity premium on the left side of (13) and covariance between real market returns and real consumption growths of app. 0.0018 on the right side. This means that the parameter of risk aversion would need to amount to at least 11, which is rather high. Now using this parameter of risk aversion and substituing into (11), the real risk-free rate would amount to app. 0.297; an extremely high figure. Going from the other end, the coefficient of relative risk aversion needed for the model to fit the data on the real risk-free rate would, according to the approximate formula (11), need to be at app. 95. This by itself is an extreme value of the parameter. When substituted into (13), it would yield an equity premium of app. 0.172; clearly not the case.

Hungary

The average real return on the Hungarian capital market reached app. 0.163. This is high in comparison with the case of the Czech economy and it is caused by high returns in 1996 and 1997. Leaving these years out would result in much lower average real return. This just further stresses the fact that due to the relatively short span of the data, the quantitative results cannot be taken as definite. The average real risk-free rate was app. 0.018 leading to an equity premium of app. 0.145. Taking account of the volatility of real returns measured by standard deviation at the level of app. 0.409; the Sharpe ratio reached app. 0.36.

The average consumption growth was app. 0.025 with standard deviation of 0.048; i.e. much more volatile than in the case of the Czech economy. The covariance between real consumption growth and real returns was very low: app. 0.00002. This resulted in a coefficient of relative risk aversion at app. 7 250 according to (13). A ridiculously high number. Applying this value of coefficient of relative risk aversion to (11) leads to the real risk-free rate being app. –344.357, which is out of touch with reality. Using (11) autonomously, the required coefficient of relative risk aversion would be app. 80, but that would result in equity premium at just app. 0.008, which is not supported by the data.

Poland

The data for Poland give very similar picture to that of Hungary. The reason is the same: relatively high equity premium caused by unsustainably high real returns on the market in 1996 and 1997 with respect to low covariance between real returns and real consumption growths.

To be more precise, the average real return on the capital market reached app. 0.068 with standard deviation of app. 0.305. The real risk-free rate was app. 0.048, by far the highest in comparison with the other three economies. This leads to equity premium of app. 0.02 or the Sharpe ratio at the level of app. 0.07.

The real consumption grew by 0.043 on average and its volatility as measured by standard deviation reached app. 0.019. This means the highest average growth of consumption in the sample and also the least risky one. The covariance between real returns and real consumption growth was again very low: app. 0.00006.

Applying these data to (13), a coefficient of relative risk aversion of app. 333 is required to bring the model to the data. Plugging this figure into (11) yields real risk-free rate of app. –5.648. To match the model and the data from the point of view of the risk-free rate, a coefficient of relative risk aversion at app. 238 would be needed according to (11). However, this would imply equity premium of app. 0.014, which is not supported by the data.

Slovakia

The average real return on the capital market was app. 0.0398, which is similar to the case of the Czech economy. The average real risk-free rate reached app. 0.013, again very close to the first case. This together leads to a realized equity premium at the level of app. 0.03, a little higher than in the Czech economy.

The volatility of the real returns on the capital market as measured by standard deviation reached app. 0.366, which is higher than in the Czech Republic and thus the resulting Sharpe ratio is almost the same: app. 0.07. The covariance between real consumption and real returns is the highest as compared to the other economies: app. 0.00355.

Real consumption grew by app. 0.044 with standard deviation of app. 0.028. Applying these data to (13) yields a coefficient of relative risk aversion of app. 8, which is by far the lowest figure. This is just a result of a relatively low equity premium with respect to the covariance term.

However, applying this value of the coefficient to (11) results in average real risk-free rate at the level of app. 0.339, which is still very high. On the other hand, using (11) autonomously leads to a coefficient of relative risk aversion of app. 238, which in turn results in equity premium of app. 0.014, which is not supported by the data.

Table 2

Stylized Facts

	CZ	HU	PL	SK
Average reak return on market index	**0.035**	**0.163**	**0.068**	**0.040**
Average real risk-free return	**0.015**	**0.017**	**0.048**	**0.013**
Equity premium	**0.020**	**0.145**	**0.020**	**0.030**
St. dev. Of real market return	0.291	0.409	0.305	0.366
Sharpe ratio	**0.069**	**0.355**	**0.066**	**0.073**
Average real consumption growth	0.029	0.025	0.043	0.044
St. dev. Of consumption growth	0.025	0.048	0.019	0.028
Covariance between real returns and consumption	0.002	0.00002	0.00006	0.004

Source: Own computation.

4.2. Hansen-Jagannathan Bounds, CRRA Utility, Habit Formation and Epstein-Zin Preferences

The previous analysis showed that CRRA utility function results within CCAPM framework cannot be expected to comply with the empirical data. This result will be now supported by comparing mean and standard deviation of the stochastic discount factor implied by CRRA with those set by Hansen-Jagannathan bounds. This comparison will further be used for the case of habit formation in the utility function and Epstein-Zin preferences.

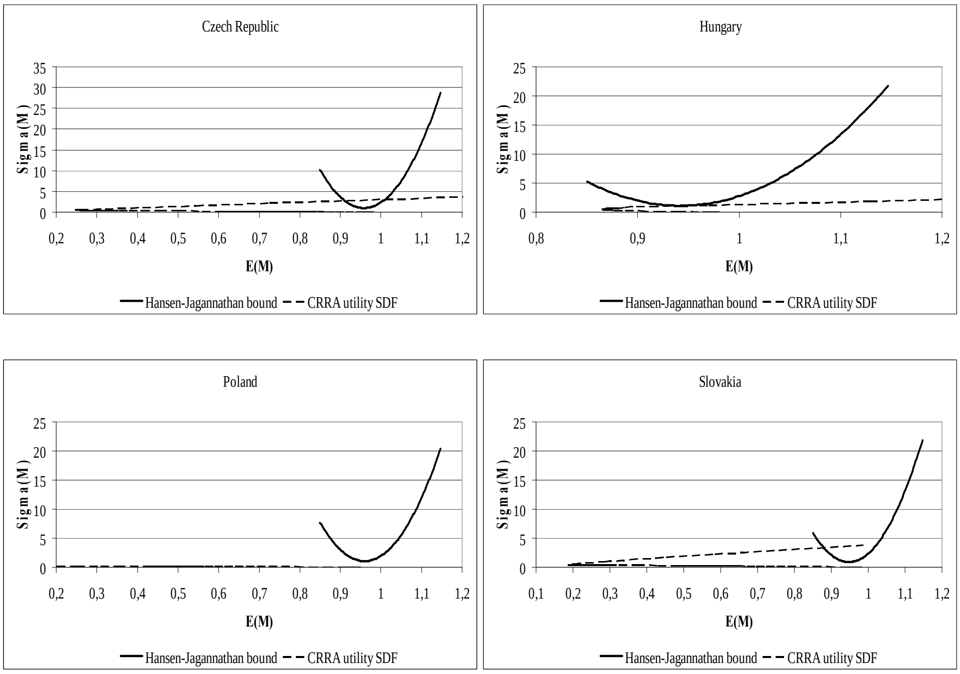
CRRA Utility Function

Figures 1 – 4 show the estimated Hansen-Jagannathan bounds, which set limits on the minimum variability (standard deviation) of the stochastic discount factor given its mean. The dashed lines represent combinations of means and standard deviations given the stochastic discount factor based on CRRA utility function.

The stochastic discount factors were estimated for a set of coefficient of relative risk aversion running from 0.1 to 1000. The computations were carried out using „raw“ formulas, not lognormal approximations.

F i g u r e 1 – 4

Hansen-Jagannathan Bounds and CRRA Utility Function Stochastic Discount Factor



Source: Own construction.

There seems to be some support in the cases of the Czech Republic and Slovakia (in the case of Hungary the SDF (stochastic discount factor) falls just a little short of the limit). However, the limits are met only with extremely high coefficients of relative risk aversion: 700 and 900, respectively. Problems related to such high values of the parameter were presented in the previous part. All in all the CRRA utility function is not suitable for this kind of analysis.

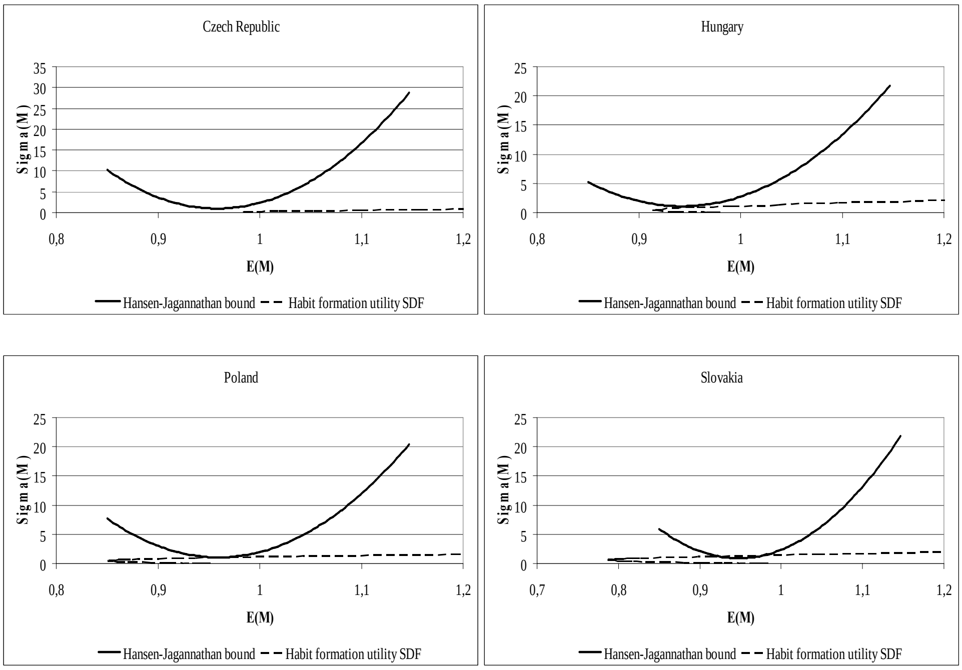
Habit Formation in the Utility Function

Figures 5 – 8 present the same Hansen-Jagannathan bounds and SDF parameters for power utility function with habit formation as described in the theoretical part of the paper. The „habitual“ level is given by the previous year consumption, however, the sensitivity parameter was set at 0.8 (as compared to 1 in the theoretical part). This was done based on simulations, where the volatility of „consumption“ growth for λ above 0.8 was too high.

Only for Slovakia do the characteristics of the SDF meet the limits set by Hansen-Jagannathan bounds. However, relative risk aversion of 29 is needed. This results in lower estimate of real market return (app. 0.015) for an acceptable estimate of real risk-free rate. Therefore, habit formation in power utility function does not alleviate the problem at all.

F i g u r e 5 – 8

Hansen-Jagannathan Bounds and Habit Formation Utility Function Stochastic Discount Factor



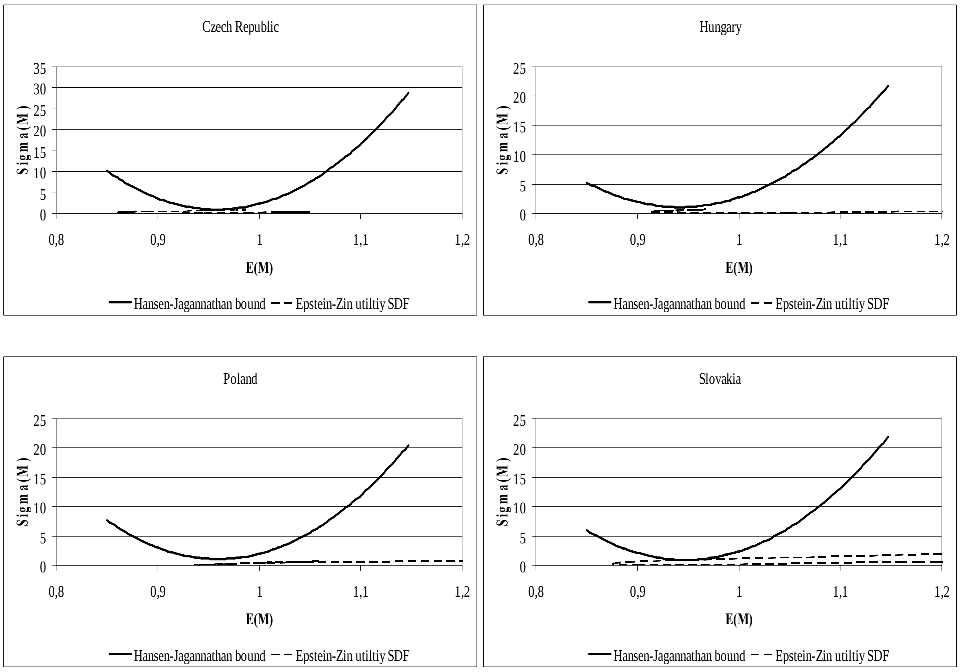
Source: Own construction.

Epstein-Zin Preferences

Figures 9 – 12 again present Hansen-Jagannathan bounds now means and standard deviations for stochastic discount factors based on Epstein-Zin preferences are given. Intertemporal elasticity of substitution was, after some simulation exercises, calibrated at 0.5 (which corresponds with σ equal to 2 from the point of view of CRRA utility function). In the cases of the Czech Republic and Hungary the results are much better as compared with the previous cases but still the limits are not met. The SDF for Slovakia meets the limits and the result is much the same as in the habit formation case, i.e. low estimate of average real market return (app. 1.6%) for an acceptable real-risk free rate. However, important difference is that coefficient of relative risk aversion of app. 6 is needed.

Figure 9 – 12

Hansen-Jagannathan Bounds and Epstein-Zin Preferences Stochastic Discount Factor



Source: Own construction.

4.3. Other Publisher Results

To my best knowledge the results presented here cannot be directly compared with some similar previously published studies which would cover the countries in question. However, some comparison can be made with the use of studies published for other economies.

Otrok, Ravikumar and Whiteman (2004) also reports relatively high coefficients of relative risk aversion needed for the stochastic discount factor based on CRRA utility or Epstein-Zin utility to match the data in case of the US market. However, for habit formation in the utility case relatively low figures of relative risk aversion are reported: slightly over 3.

Ki and Kwang (2009) report qualitatively very similar results for Korean economy to those reported in this paper. They conclude that longer time series are necessary to reach firmer conclusions.

Engsted, Mamme and Tanggaard (2000) extend the traditional analysis by assuming short and long investment horizons. They apply the analysis to the US and Danish capital markets and show that the results vary across the markets and across the investment horizons considered. Due to short time series of the data used in this paper, it is not possible to consider longer investment horizons.

Li (2010) offers an interesting application of the problem in an international context and shows that when heterogeneity in the fashion of Constantinides and Duffie (1996) is introduced into the model, the required coefficient of relative risk aversion to match the data is much lower. This seems to be a possible way to address the problem in case of V-4 countries.

Conclusions

Key relationships concerning the real and financial variables of an economy using the consumption-based capital asset pricing model framework were derived. The risk premium for an asset is dependent on the covariance between the asset's returns and consumption growth, therefore is dependent on the behavior of the real economy. However, it has been shown that the CCAPM model has problem reconciling financial and real data, especially when using constant relative risk aversion utility.

One strand of approach to solving this problem rests on using different utility frameworks, especially Epstein-Zin preferences, general expected utility function, and Constantinides's habit formation in the power utility function. These approaches are not able to solve the problem completely, but usually help to alleviate it.

After equity premium puzzle and risk-free rate puzzle were presented, using lognormal approximations of the Euler equations, CRRA utility and the data for the V-4 economies, Hansen-Jagannathan bound was estimated to examine the three utility framework more closely.

In accordance with expectations, the CRRA utility is able to seemingly reconcile the financial and real data in the cases of the Czech and Slovakian economy but only for unreasonably high coefficient of relative risk aversion. However, as opposed to some research results for advanced economies, the habit formation and Epstein-Zin utility frameworks do not help much to lessen the problems which are faced with CRRA utility. The results based on habit formation in power utility function were especially disappointing. Results based on Epstein-Zin utility seemed more promising but still does not help to bring the model to the data.

The quantitative results of the analysis are influenced by relatively short time series. But for example, taking data from 1998 onwards the qualitative results for all the four economies would be the same as regarding the comparison of Hansen-Jagannathan bounds and CRRA utility discount factor. Generally, it is impossible to campare the analysis with the results obtained for advanced economies because the short sample of the data makes the quantitative results rather sensitive to the beginning/end of the sample. However, still the analysis shows that different approaches to solving the problem are needed. Due to data limitations many approaches applied for other countries to the analysis are out of bounds at this time. Models with heterogenous agents and models with information asymmetries might provide better results and be applicable to the current amount of data.

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