JEL Classification: G1, G23, G28, D8, J11, J26, J32, C15, C32, H55
Keywords: ART, life insurance, life market, longevity risk, mortality risk, pension plans, securitization, tontines

Securitization of Longevity and Mortality Risk*

Tomas CIPRA – Faculty of Mathematics and Physics, Charles University in Prague
(cipra@karlin.mff.cuni.cz)

Abstract
This paper deals with Alternative Risk Transfer (ART) through the securitization of longevity and mortality risks in pension plans and commercial life insurance. Various types of such mortality-linked securities are described (e.g., CATM bonds, longevity bonds, mortality forwards and futures, and mortality swaps). Pricing methods and real examples are given. Hypothetical calculations concerning the pricing of potential mortality forwards that correspond to the evolution of longevity in the Czech Republic are presented.

1. Introduction

This paper deals with an important example of alternative risk transfer, namely, the securitization of longevity and mortality risk, which is a potential solution to the pension and life annuity problem. There is a vast volume of literature devoted to this topic (only a small part of it may be presented here), since it is a truly serious problem for the future. This paper sets out to present the issue in an economic (or financial) way rather than as an actuarial problem (there is no doubt that constructions of future pension systems have economic dimensions above all). As the various ideas and considerations published so far have been only hypothetical, this paper tries to describe some instruments that really exist in practice. Investors, including banks, should be prepared for a brand new type of security engineering motivated by pension systems or the insurance business. Moreover, the paper shows some calculations that enable estimation of the consequences of such approaches if applied in practice in the Czech Republic. First, however, we will explain basic concepts that are important from the point of view of later sections.

The content of this paper is as follows. After introducing the main concepts in section 2 we describe catastrophe bonds (CatBonds) as typical insurance-linked securities (ILS) for non-life insurance in section 3. Moreover, we will present a simple mathematical model of CatBonds, which can serve as a general mathematical scheme for ILS. Section 4 is devoted to ILS for life insurance and pension plans (sometimes called mortality-linked securities): mortality catastrophe bonds (CATM bonds) in section 4.1 (including the practical example of the Vita I bond), mortality swaps (also called survivor swaps) in section 4.2, longevity bonds (LBs) in section 4.3 (including the practical example of the EIB/BNP Paribas bond), and mortality forwards and futures in section 4.4. In section 5 we address some demographic facts and actuarial instruments that are important in the context of securitization of mortality and longevity risks. In particular, we comment on the Cohort Life Tables constructed by Cipra (1998) for Czech pension funds. In section 6 some approaches to the pricing of

* Acknowledgement: The work is a part of research project MSM0021620839 financed by the Ministry of Education of the Czech Republic. The author thanks to the editorial board and reviewers for helpful comments and suggestions improving this text.

mortality-linked securities are briefly mentioned. Finally, section 7 suggests hypothetical calculations concerning the pricing of mortality forwards that correspond to the evolution of longevity in the Czech Republic.

2. Main Concepts

Alternative risk transfer (ART) methods are modern techniques of the insurance industry (both life and non-life) and pension systems which are more appropriate in today's world than the classical cession of insurance risks as, for example, in classical reinsurance (see Cipra, 2004). If one simplifies the problem, many of the ART methods are motivated by the effort to cede huge insurance risks to capital markets, which have many times the capacity of insurance markets. For example, the insurance of oil tankers may be beyond the capacity of big insurance and reinsurance companies even if they collaborate or pool in various ways. To obtain an idea of how this principle works, let us consider, for example, "catastrophe bonds" (see Cat-Bonds below), which mitigate the financial stress within insurance companies in the event of, for example, floods: the coupons from such bonds lie so high above the market standard that investors accede to a substantial reduction of coupons (and principals) if the corresponding insurance event (floods in a given region) occurs. Obviously, this mechanism is really the cession of insurance risk to the capital market. Formally, ART is a product, channel or solution that transfers risk exposures between the insurance industry (including pension funds) and capital markets to achieve stated risk management goals (see Banks, 2004). The ART market is a combined risk management marketplace for innovative insurance and capital market solutions.

One of possible solutions to ART is securitization. Securitization is the process of removing assets, liabilities or cash flows from the balance sheet (of an insurance company, a pension fund, etc.) and conveying them to third parties through tradable securities known as insurance-linked securities (ILS), which include various derivatives. The catastrophe bonds mentioned above are typical ILS. Since ILS trading is a very specialized activity, it usually requires a special organizer established just for this single purpose. Such an organizer is usually called a special purpose vehicle, or SPV (e.g. Vita Capital Ltd. in *Figure 1*).

As far as securitization is concerned, this paper concentrates on securitization of longevity and mortality risks, which play a very important role among other systematic risks in modern finance (see, for example, van Broekhoven, 2002). In particular, longevity risk should be taken into account by providers of pensions (or life annuities) in developed countries, since growing life expectancy can jeopardize the economy of their pension systems (see, for example, OECD, 2006, 2008; Schneider, 2009). Longevity and mortality risks are such serious problems that one can predict the formation of other types of capital markets, usually called life markets (see, for example, Loyes et al., 2007). The annuity markets in the UK and USA are working examples of this phenomenon. In addition, regulators of the commercial insurance industry will address this problem within the Solvency II regulatory system, where the entry denoted as underwriting risk in Pillar 1 will contain longevity and mortality risks as important components (including Solvency II as currently being prepared by the Czech National Bank). Private life insurance linked to pension funds (mainly in contribution-defined pension plans) may play a key role in pension systems of the future (see, for example, CEA, 2006; Cipra, 2002; IAA, 2004; Sandström, 2006).

Again, to get an idea of longevity risk securitization let us consider "longevity bonds" (see LBs below). While a classical (nominal) bond pays annual or semi-annual coupons of a fixed amount and the principal is repaid at term (maturity), an LB provides regular floating payments according to the proportion of an initial population surviving to a future time. This mechanism obviously allows longevity risk to be ceded from insurance companies or pension funds (investing in these securities) to LB issuers, i.e., from insurance markets to capital markets. In particular, tontines can be mentioned in this context, since formally they are one-year zero-coupon LBs. Milevsky (2006) explains the principle of tontines by means of a very nice (though rather naive) example.[1] There is virtually no other financial product that guarantees such high rates of return, even if they are conditional on survival.

3. Insurance-Linked Securities for Non-Life Insurance

In this section we describe catastrophe bonds (CatBonds) as typical ILS for non-life insurance (see, for example, Cox and Pedersen, 1998; Cummins, 2008; Swiss Re New Markets, 1999) and present a simple mathematical model of how they work.

CatBonds are bonds with a coupon rate usually much higher than the market average for which the suspension of coupons and/or principal occurs in the event of pre-defined natural catastrophes (earthquakes, hailstorms, pandemic events, and the like). For example, an annual reinsurance contract under which a reinsurer reimburses an insured sum S at the end of the contract year if a catastrophe has occurred can be replaced by a 1-year catastrophe bond with an annual coupon. *Table 1* contains the appropriate cash flows that comply with the requirements of all participating sides: q_{cat} is the probability of the natural catastrophe, i is the annual coupon rate, F is the principal of the bond, and P is the reinsurance premium. Moreover, one can use the market price (market quotation) of such a bond to price the reinsurance premium

$$P_b = \frac{1}{1+i} \cdot q_b \cdot S = \frac{S}{1+i} - F^*$$

where P_b is the reinsurance premium priced by the bond market, F^* is the market price of the catastrophe bond, and

$$q_b = \frac{S - F^* \cdot (1+i)}{S}$$

is the probability of catastrophe priced by the bond market (unlike the estimate q_{cat} priced by the reinsurance market).

[1] An 85-year-old grandmother meets her four best friends of the same age every year on December 31. She proposes to juice up their meetings in such a way that each of the five participants deposits $1,000 with 5% interest p.a. and with the guarantee that whoever survives till the end of next year gets to split the $5,250 pot. There is a 20% chance that any given member of this club will die during the next year. Therefore, the odds are that on average each of the four 86-year-old survivors will receive $1,312.50 as the total return on the original investment of $1,000. The 31.25% investment return contains 5% of the bank's money and 26.25% of "mortality credits". These credits represent the capital and interest "lost" by the deceased and "gained" by the survivors.

Table 1 Cash Flows in a 1-year Catastrophe Bond

	Time t = 0	Time t = 1	
		Occurrence of cat. (with prob. q_{cat})	Non-occurrence of cat. (with prob. $1-q_{cat}$)
Insurer	$-P=-\frac{1}{1+i}\cdot q_{cat}\cdot S$	S	0
Reinsurer = Issuer (CatBond)	$P+F$	$-S$	$-S$
Investor (CatBond)	$-F=-\frac{1}{1+i}\cdot(1-q_{cat})\cdot S$	0	S

4. Insurance-Linked Securities for Life Insurance and Pension Plans

This section deals with ILS for life insurance and pension plans, which may be denoted generally as mortality-linked securities (such terminology does not distinguish between mortality-linked and longevity-linked securities). We will start with an introduction to life markets in general.

The modern practice of risk management requires companies (or governments) to manage mortality and longevity risks as effectively as possible as a part of enterprise risk management, rather than to accept it as inevitable. Blake at al. (2006a) and Cairns et al. (2008) mention possible ways of managing mortality and longevity risks:

– insurers can retain these risks as a legitimate business risk;
– insurers can diversify these risks across product ranges, regions, and socio-economic groups (an example of how to hedge through such a balance of gains and losses on the life and the annuity book is given, for example, in Cox and Lin, 2007);
– insurers can enter into various forms of reinsurance (and then the reinsurers can use, for example, securitization, as is the case in *Table 1*);
– pension plans can arrange a full or partial buyout of their liabilities by a specialist insurer;
– insurers can securitize a line of business (see, for example, Cowley and Cummins, 2005);
– mortality and longevity risks can be managed through the application of mortality-linked securities and derivatives (this approach differs from the securitization of a line of business in the previous point since such securities have cash flows that are purely linked to the future value of a mortality index, rather than being a complex package of business risks).

To establish a new flourishing capital market (a life market in our case) several conditions need to be fulfilled (see Corkish et al., 1997, and Loyes et al., 2007). First, the market must provide effective exposure, or hedging, to a state of the world. This state of the world must be economically important and cannot be hedged sufficiently through existing market instruments. Further, the market must use a homogeneous and transparent contract to permit exchange between agents.

Let's give some examples of successful and unsuccessful capital markets for product innovations in the framework of financial risks:

- successful products: credit default swaps (CDS), inflation-linked bonds, interest rate swaps (IRS), mortgage-backed securities (MBS), and real estate investment trusts (REIT);
- unsuccessful products: GDP derivatives (the market for which was meant to be analogous to the markets for inflation-linked bonds) and residential real estate derivatives (which were intended to diversify the risk of the tremendous financial wealth concentrated in family dwellings).

The market trading mortality or longevity risks (via new life markets) meets these criteria if one considers the systematic parts of these risks. Systematic mortality or longevity risks are undiversifiable, since they affect all individuals in the same way. In particular, systematic mortality risk consists in increased exposure to a catastrophic mortality deterioration (e.g. in whole life insurance or in term insurance). On the contrary, systematic longevity risk consists in the growing costs of meeting increasing life expectancy due to improvements in health conditions across the word (e.g. in pension funds). Unsystematic mortality or longevity risks can be diversified by pooling individuals in large portfolios (the larger the portfolio, the smaller the unsystematic risk).

This paper deals only with systematic mortality or longevity risks, since unsystematic ones can be managed (at least for the time being) by classical insurance instruments. In the remaining part of this section we will describe typical representatives of mortality and longevity-linked securities.

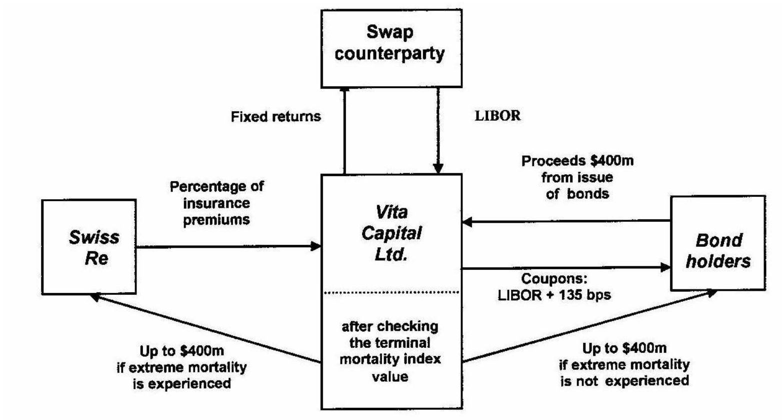
4.1 Mortality Catastrophe Bonds

Mortality catastrophe bonds (CATM bonds) are similar to the CatBonds described in Section 3 – see, for example, Bauer and Kramer (2007), Cairns et al. (2008), Cowley and Cummins (2005), Krutov (2006), and Lin and Cox (2008). They help to reduce exposure to a catastrophic mortality deterioration (i.e., to extreme mortality). Catastrophes pose a big potential problem for life insurers, since fatalities from natural and man-made disasters (such as a repeat of the 1918 Spanish Flu pandemic, a major terrorist attack using weapons of mass destruction, and the earthquake and tsunami in southern Asia and eastern Africa in 2004) can be enormous.

CATM bonds are market-traded securities whose payments are linked to a mortality index. The CATM bonds issued to date have been structured as principal-at--risk notes with a fixed tenor, where the principal repayment is contingent on a catastrophic outcome for the value of a customized mortality index. Such a catastrophic outcome is defined as an extreme rise in mortality beyond a particular baseline. CATM bonds have been issued mostly by reinsurers looking to free up capital related to the extreme mortality risk they face in their life insurance book.

The first bond of this type was the three-year life catastrophe bond Vita I, which came to market in December 2003 and matured on January 1, 2007. It was designed to securitize the exposure of Swiss Re (one of the world's leading reinsurers) to certain catastrophic mortality events: a severe outbreak of influenza, a terrorist attack, or a natural catastrophe. To carry out the transaction, Swiss Re set up a special purpose vehicle called Vita Capital Ltd. This enabled the corresponding cash flows to be kept off Swiss Re's balance sheet. The principal of $400m was at risk if during any single calendar year the mortality index exceeded 130% of the 2002

Figure 1 Scheme of CATM Bond Vita I



base level, and would be exhausted if the index exceeded 150%. In return for having their principal at risk, investors received quarterly coupons equal to the three-month U.S. LIBOR plus 135 basis points. This meant that only the principal was unprotected, and the principal repayment depended on what happened to a specifically constructed mortality index. This mortality index was constructed as a weighted average of mortality rates (deaths per 100,000) over age, sex (male 65% and female 35%), and nationality (USA 70%, UK 15%, France 7.5%, Italy 5%, and Switzerland 2.5%). The Vita I bonds were successful, and soon further CATM bonds followed due to strong investor demand – Vita II and Vita III by Swiss Re, Tartan by Scottish Re, and OSIRIS by AXA. For example, the last-mentioned one – issued in 2006 – was intended to cover extreme mortality in France, Japan, and the USA. In 2008, Munich Re (another leading reinsurer) established a $1.5 billion bond program (with an SPV managed by JPMorgan) for the transfer of catastrophic mortality risk to capital markets (see www.artemis.bm).

The scheme of Vita I is given in *Figure 1*. Usually the SPV (i.e., Vita Capital Ltd. in this case) makes use of a swap counterparty to exchange fixed returns for LIBOR returns necessary for bond holders as coupons (see Figure 1). The payoff function $f_t(\cdot)$ $(t = 1,2,3)$ for bond holders depends on the extreme mortality experienced:

$$f_t(\cdot) = \begin{cases} LIBOR + 1.35\%\,, & t = 1, 2 \\ LIBOR + 1.35\% + \max(0\,; 100\% - \sum_{s=1}^{3} L_s)\,, & t = 3 \end{cases}$$

where

$$L_t = \begin{cases} 0\%\,, & M_t < 1.3M_0 \\ [(M_t - 1.3M_0)/0.2M_0]\cdot 100\%\,, & 1.3M_0 \le M_t \le 1.5M_0 \\ 100\%\,, & 1.5M_0 < M_t \end{cases} \qquad \text{for } t = 1, 2, 3$$

and M_0 is the 2002 base level of the mortality index and M_t is the mortality index for year t.

4.2 Mortality Swaps

Mortality swaps (also called survivor swaps) are derivative securities where counterparties swap a fixed series of payments in return for a series of payments linked to the number of survivors in a given cohort or linked to the outcome of a mortality index – see, for example, Blake et al. (2006a), Cairns et al. (2008), Dowd et al. (2006), and Lin and Cox (2005). It is the random leg (i.e., the number of survivors or the outcome of the mortality index) that distinguishes mortality swaps from classical swaps (e.g., from the interest rate swaps – IRS – used in *Figure 1*). Although mortality swaps bear a similarity to reinsurance contracts (both of them exchange anticipated for actual payments), they are not insurance contracts in the legal sense (e.g., they may be used for speculative purposes without the existence of an insurable interest).

In 2007, for example, Goldman Sachs launched a monthly index called *QxX.LS* (www.qxx-index.com) in combination with standardized 5 and 10-year mortality swaps. The index was based on pools of approximately 46,000 lives of individuals aged 65 or older with a primary impairment other than AIDS or HIV. A second index – *QxX.LS2* – was launched in 2008 starting with a pool of 65,655 individuals over the age of 65 with impairments that included cancer, cardiovascular conditions, and diabetes.

4.3 Longevity Bonds

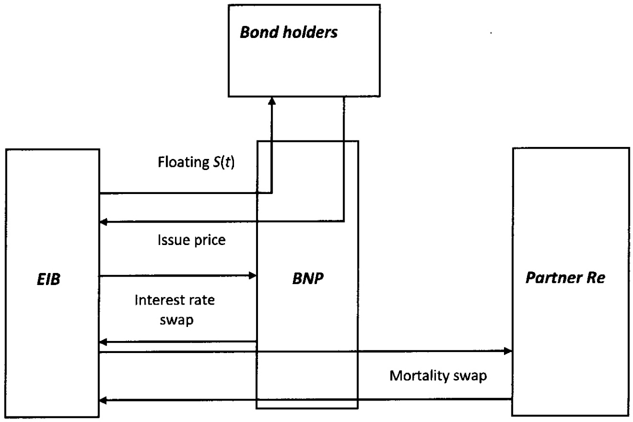
There are various types of longevity bonds (or survivor bonds) – see, for example, Antolin and Blommestein (2007), Blake and Burrows (2001), Blake et al. (2006a, 2006b, 2010), Brown and Orszag (2006), Collet-Hirth and Haas (2007), Kabbaj and Coughlan (2007), Krutov (2006), Leppisaari (2008), Levantesi and Torri (2008), Lin and Cox (2005), Reuters (2010), Richards and Jones (2004), and Thomsen and Andersen (2007). In general, these bonds are designed to protect companies (or governments) from an unexpected increase in the life span of their annuitants, i.e., from systematic longevity risk.

LBs are bonds whose payoffs $f_t(\cdot)$ ($t = 1, ..., T$) depend on a survivor index S_t. This index represents the proportion of the initial population surviving to a future time. While a classical (nominal) bond pays annual or semi-annual coupons of a fixed amount and the principal is repaid at term, an LB provides regular floating payments that depend on the number of cohort survivors, translated again via a selected survivor index (survivor indices may be obtained similarly as the mortality indices described in section 4.1 and section 4.2).

LBs can be divided into several categories:

- standard LBs, i.e., coupon-bearing bonds whose coupon payments fall over time proportionally to a survivor index, i.e., $f_t(\cdot) = k \cdot S_t$ for a positive constant k;
- inverse LBs, i.e., bonds whose coupons are inversely related to a survivor index, i.e., rising over time instead of falling with $f_t(\cdot) = k \cdot (1 - S_t)$;
- longevity zero bonds, i.e., zero-coupon bonds (see, for example, Cipra, 2010) where the principal is a function of a survivor index;

Figure 2 Scheme of EIB/BNP Paribas Longevity Bond



- principal-at-risk LBs, i.e., bonds whose principal, not coupons (fixed or floating), is linked to a survivor index;
- survivor bonds, which, unlike standard LBs, have no specified maturity but continue to pay coupons as long as the last member of the reference population is alive (in particular, they have no principal payment).

Other types of LBs exist but are not mentioned here.

The first LB was the EIB/BNP Paribas bond issued in 2004 (see, for example, Collet-Hirth and Haas, 2007). This bond was issued by the European Investment Bank (EIB), with commercial bank BNP Paribas as its structurer and manager, and Partner Re (Bermuda) as the longevity risk reinsurer (see *Figure 2*). The issue size was £540m, the initial coupon £50m, and the maturity 25 years. The corresponding survivor index was based on the realized mortality experience of the population of English and Welsh males aged 65 in 2003. If $m(t,x)$ denotes the age-specific death rate at age x in year t (see section 5), then

$$S(0)=1 \ ,$$

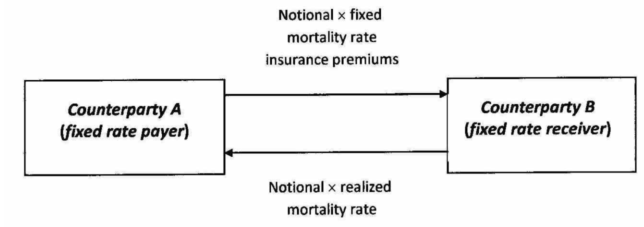
$$S(1)=S(0)\cdot\left(1-m(2003,65)\right) \ ,$$

$$S(t)=S(0)\cdot\left(1-m(2003,65)\right)\cdot\left(1-m(2004,66)\right)\cdot\ldots\cdot\left(1-m(2002+t,64+t)\right) \qquad (1)$$

and at times t = 1,2, ...,25 the bond pays coupon payments of £50m × $S(t)$. This means that the bond was an annuity bond with floating coupon payments linked to the realized mortality rates of English and Welsh males aged 65 in 2002 and with an initial coupon set at £50m.

In practice, this LB was made up of three components (see *Figure 2*). The first was a floating-rate (annuity) bond issued by the EIB with a commitment to pay floating coupons in €s. The second was a (cross-currency) interest rate swap (see also section 4.2) between the EIB and BNP Paribas in which the EIB paid floating €s and received fixed £s. The third component was the key one, since it was a mortality

Figure 3 Scheme of *q*-Forwards



swap (see section 4.2) between the EIB and Partner Re in which the EIB exchanged fixed payments in £s for floating £50m × $S(t)$ payments. In particular, the first and third components were structured and organized via BNP Paribas (see *Figure 1*). Unfortunately, the EIB/BNP Paribas bond was only partially subscribed and was later withdrawn due to inadequate design.

4.4 Mortality Forwards and Futures

Mortality forwards (q-forwards) resemble interest rate forwards (see, for example, Cipra, 2010). They are forward contracts linked to a future mortality rate (the standard actuarial notation in section 5 uses the symbol q for the mortality rate) – see, for example, Cairns et al. (2008), Coughlan et al. (2007a, 2007b), and Loyes et al. (2007). The q-forward exchanges at time T a realized (i.e., "delivered") mortality rate $q\ (T-1, x)$ in return for a fixed mortality rate which is agreed at the beginning of the contract at time $T-1$ (of course, this exchange is made in financial terms – see *Figure 3)*. In practice, mortality forwards can be used to hedge the mortality swaps referred to in section 4.2, which are also important for the financial engineering of LBs (see, for example, *Figure 2*). For instance, JPMorgan announced the launch of q-forwards in 2007 (see also the corresponding business system called *LifeMetrics* in Coughlan et al., 2007a).

Mortality futures (q-futures) are mortality forward contracts standardized to be marketable on exchanges (see, for example, Blake et al., 2006a).

5. Population and Actuarial Instruments and Methods

In this section we review some basic concepts of population mathematics that are important in the context of mortality-linked securities (see, for example, Cipra, 2010).

The age-specific death rate $m(t, x)$ mentioned in section 4.3 is defined as the relative number of deaths at age x and time t in the mid-population of this period

$$m(t,x) = \frac{D(t,x)}{E(t,x)} = \frac{\text{number of deaths during calendar year } t \text{ aged } x}{\text{mid-population during calendar year } t \text{ aged } x} \tag{2}$$

The (age-specific) mortality rate $q(t, x)$ is the probability that a person aged x at time t will die within one year. It can be calculated approximately (for forces of mortality remaining constant in particular years) as

$$q(t,x) = 1 - \mathrm{e}^{m(x,t)} \tag{3}$$

(the approximate relation (3) can be compared with the exact relation (5) using the concept of force of mortality). The corresponding survival probability $p(t, x) = 1 - q(t, x)$ can be generalized over n years by chain relation

$$_{n}p(t,x) = p(t,x) \cdot p(t+1,x+1) \cdot \ldots \cdot p(t+n-1,x+n-1) \tag{4}$$

The survivor index $S(t)$ in (1) may be taken as the estimated survival probability $_{t}p(2003, 65)$.

The force of mortality $\mu(x, t)$ is the instantaneous death rate for persons aged x at time t. The rigorous form of the relation (3) is then

$$q(t,x) = 1 - \exp\left\{-\int_{0}^{1} \mu(t+\tau, x+\tau)d\tau\right\} \tag{5}$$

Another important concept is life expectancy $e(t, x)$ for persons aged x at time t

$$e(t,x) = \int_{0}^{\infty} \tau \cdot {}_{\tau}p(t,x) \cdot \mu(t+\tau,x+\tau)d\tau \tag{6}$$

In practice, the observed values of these variables are arranged in life tables (LTs). In particular, so-called cohort (or generation) LTs are suitable if one needs to do calculations over long time horizons, as is usual, for example, in pension calculations. Cohort LTs can be used as records of the actual lifetimes of particular generations or cohorts (while so-called period LTs display mortality for people of different ages at one point in time so that they include people born in different years, i.e., belonging to different cohorts). Moreover, cohort LTs enable projections of mortalities and life expectancies over long time horizons (see, for example, Lee and Carter, 1992) and can be adjusted to respect the corresponding selection principles. For example, the cohort LTs constructed by Cipra (1998) are suitable for pension annuities since they take into account the selection approach by potential annuitants. Some results due to these LTs (including the volatility of survival projections – see also Blake et al., 2008) are applied in the context of longevity securitization in Section 7.

Pension annuities (or life annuities) are mentioned above. For example, the (fair) value of such an annuity with unit payments in arrears for persons aged x at time t is

$$a(t,x) = \sum_{n=1}^{\infty} d(0,n) \cdot {}_{n}p(t,x) \tag{7}$$

where $_{1}p(t, x) = p(t, x)$ and $d(0, t)$ is the corresponding discount factor (i.e., the price at time 0 for a unit payment payable with certainty at time t).

6. Pricing of Mortality-Linked Securities

Mortality-linked securities involve significant valuation problems that are mostly solved using stochastic modeling – see, for example, Barbarin (2007), Bauer and Kramer (2007), Bauer and Russ (2006), Blake et al. (2006b), Cairns et al. (2006), Cox and Lin (2007), Cox and Pedersen (1998), Dahl (2004), Dahl and Møller (2006), Denuit et al. (2007), Hári et al. (2008), Leppisaari (2008), Levantesi and Torri (2008), Lin and Cox (2005, 2008), and Wang (2002).

This section describes very briefly and without any technical details two approaches to pricing, for example, the standard LBs described in section 4.3 (a more practical approach to pricing systematic longevity risks is shown in section 7).

The first of them is the distortion approach by Wang (see, for example, Wang, 2002), which distorts the distribution of the survivor index to obtain suitable risk-adjusted expected values of this index. For a distribution function $F(t)$ the corresponding Wang transform is

$$F^*(t) = \Phi[\Phi^{-1}\left(F(t)\right) - \lambda] \tag{8}$$

where $\Phi(\cdot)$ is the standard normal distribution function and λ is the market price of risk. After such a transform, the survivor index can be discounted at the risk-free rate, assuming that mortality and interest rate risk are independent. This means that the (fair) value V(LB) of a standard LB with a unit initial coupon can be obtained as

$$V(LB) = \sum_{t=1}^{T} d(0,t) \cdot \mathrm{E}^*\left(S(t)\right) \tag{9}$$

where $E^*(S(t))$ is the expected cash flow under the transformed distribution $F^*(t)$ of the corresponding survival index $S(t)$ starting at age x (see (1)) and $d(0, t)$ is the risk-free discount factor (i.e., the price at time 0 for a unit payment payable with certainty at time t – see also (7)). Moreover, parameter λ, reflecting the level of systematic longevity risk, can be calibrated by means of market prices of this risk for corresponding assets existing in the marketplace, i.e., one looks for λ solving equations of the type

$$a^{market}(t,x) = \sum_{n=1}^{\infty} d(0,n) \cdot \Phi\left[\Phi^{-1}\left(S(t)\right) - \lambda\right] \tag{10}$$

for quoted annuity values on the market.

The second approach is based on risk-neutral pricing, which is popular in finance in general. Assuming an arbitrage-free environment there exists a risk-neutral measure Q allowing risk-free discounting using the same discount factor $d(t,0)$ as in (9):

$$V(LB) = \sum_{t=1}^{T} d(0,t) \cdot \mathrm{E}_Q\left(S(t) \mid \Omega_0\right) \tag{11}$$

where $\mathrm{E}_Q(S(t) \mid \Omega_0)$ is the expected value of $S(t)$ under the risk-neutral measure Q conditional on the information Ω_0 available at time 0. Currently, however, due to the non-existence of regular *LB* quotations in the markets, the corresponding Q measures cannot be calibrated.

7. Practical Pricing of Mortality Forwards

Mortality forwards are described in Section 4.4 as contracts linked to a future mortality rate in such a way that they exchange a realized (delivered) mortality rate q in return for a fixed mortality rate which is agreed at the beginning of the contract.

As an example of a possible practical approach to pricing such securities (see Loyes et al., 2007), let us consider a 10-year forward for the 75-year-old cohort of males in the Czech Republic that are aged 65 at the beginning of the contract in 2010. *Table 2* shows the male and female mortality rates $q(t, x)$, $t = 2010, ..., x = 65, ...$ (see Section 5) for the corresponding male and female cohorts born in 1945 according to the cohort life tables constructed by Cipra (1998). These LTs respect the corresponding selection principle in the framework of pension systems and life annuity markets, i.e., they take into account the selection approach by potential annuitants.

Table 2 Male and Female Mortality Rates for the Corresponding Male (*x*) and Female (*y*) Cohort Born in 1945 (Czech Republic) i.e. aged *x, y* = 65, … in *t* = 2010, …

x	*q*(*x*, *t*)	*y*	*q*(*y*, *t*)
65	0.014425	65	0.005139
66	0.015771	66	0.005692
67	0.017345	67	0.006345
68	0.019146	68	0.007109
69	0.021134	69	0.007999
70	0.023320	70	0.009047
71	0.025659	71	0.010244
72	0.028102	72	0.011560
73	0.030615	73	0.012968
74	0.033220	74	0.014438
75	0.035828	75	0.015929

Source: Cipra (1998, Table 3 and 4)

The mortality forward can be practically implemented in such a way that an investor buys a 10-year zero-coupon bond with a principal of 100 monetary units and simultaneously enters into a mortality forward contract of notional value 100. This investment may earn 100 + 100·($q_{index} - q_{forward}$) at maturity, where q_{index} is the mortality index (see Section 4.1) delivered at maturity by a suitable agency (similarly to security indices of the S&P100 type) and $q_{forward}$ is the contracted forward price (a more general payoff may be $100 + 100 \cdot k \cdot (q_{index} - q_{forward})$, where k is a suitable leverage coefficient). This means that the investor makes a profit in this forward contract when $q_{index} - q_{forward} > 0$ (i.e., when the longevity risk does not occur) and suffers a loss when $q_{index} - q_{forward} < 0$ (i.e., when the counterparty of the issuer faces the longevity risk).

In order to find $q_{forward}$ (i.e., to price this mortality forward) and at the same time to take into account the volatility of future mortality rates, one can make use of the Sharpe ratio (excess return divided by volatility), which should attain a reasonable value for such investments (Loyes et al., 2007, recommend a value of 0.25 in view of the longer-term returns of bonds and equities). Hence, the calibrated value $q_{forward}$ should fulfill

$$\frac{\left(q_{projection} - q_{forward}\right)/10}{volatility} = 0.25 \tag{12}$$

where $q_{projection}$ is the mortality rate (in our case $q(2020, 75)$) projected by means of the cohort LT (see *Table 2*), the numerator in (12) is the annualized excess return (ignoring compounding effects), and the denominator of (12) is the annualized risk (i.e., the annual volatility of projections of mortality rates). From (12) one obtains a simple formula

$$q_{forward} = q_{projection} - 10 \cdot 0.25 \cdot volatility \tag{13}$$

The numerical value corresponding to our example can be obtained using *Table 2* for mortality rate projections and *Table 3* for volatilities. The annual vola-

Table 3 Annual Volatilities for Selected Ages as the Percentage of the Corresponding Mortality Rates (England & Wales, US, Czech Republic)

Male volatility (%)				Female volatility (%)			
x	E & W	US	CZ	*y*	E & W	US	CZ
45	2.96	2.31	3.10	45	2.82	2.41	2.93
55	2.57	1.53	2.69	55	2.90	1.61	3.01
65	2.64	1.01	2.78	65	2.36	1.52	2.45
75	3.03	1.47	3.15	75	2.81	1.66	2.90

tilities in *Table 3* following from the construction of projections in the framework of the cohort LT are given as the percentage of the corresponding mortality rate; they are slightly higher than the ones presented in Loyes et al. (2007) for the population in England and Wales and in the USA (see *Table 3*).

Numerically, according to (13) and *Tables 2* and *3* (for the Czech Republic) we will obtain for males

$$q_{forward} = (1 - 10 \cdot 0.25 \cdot 0.0315) \cdot 0.035828 = 0.03301 \approx 3.30\ \%$$

This means that the forward needs to be 0.28% below the projected future mortality of 3.58% (3.30 – 3.58 = – 0.28%), which is a discount of 0.28/3.58 ≈ 7.82% on the projected mortality. What does this mean numerically? Let the corresponding forward contract with a volume of CZK 5 billion be negotiated with $q_{forward}$ = 3.30% but the mortality index achieve the real value q_{index} = 3.52% (i.e., 6 basis points below the projected value $q_{projection}$ = 3.58%). Then the profit margin of investors is $(0.0352 - 0.0330) \cdot 5 \cdot 10^9 = 11 \cdot 10^6$ = CZK 11 million. Obviously, the investors' profit decreases with declining mortality index q_{index}, i.e., with growing longevity of the population, since they are not averse to longevity risk.

8. Conclusions

This paper shows that some risks – natural disasters, ecological damage, and terrorism, but also the "positive" risks of longevity – cannot be covered by classical insurance instruments. Therefore, alternative risk transfer methods are being developed and tested to mitigate these risks. The markets have already tested several methods for managing risk via securitization, some more successfully than others.

Asset-backed securities based on low quality mortgages are one – spectacular – example of overly aggressive application of risk securitization. A more promising avenue for the securitization process is the transfer of longevity risk from existing pension systems to willing market participants. Institutional investors, including banks, can expect a new generation of financial instruments (securities, financial derivatives, annuities, credits, and others) linked to insurance or pension systems. Naturally, responsible risk evaluation will be the key assumption of such investing, which on the other hand can make for lucrative profits (see, for example, footnote 1).

A very promising area for the application of these approaches seems to be future pension systems with a substantial risk of longevity (in addition to demographic, migration, labor, tax, and other problems). So far, such applications are only experimental and confined to countries with "effective" annuity markets (mainly the UK and the USA, but also Australia, Chile, Singapore, and Switzerland – see, for example, Cannon and Tonks, 2008). On the other hand, some alternative risk transfer ideas and principles may be instructive even for pension reforms in Central Europe, with the expected transfer of responsibility from governments to other entities.

REFERENCES

Antolin P, Blommestein H (2007): Governments and the Market for Longevity-Indexed Bonds. *OECD Working Paper on Insurance and Private Pensions*, no. 4.

Banks E (2004): *Alternative Risk Transfer*. Wiley, Chichester.

Barbarin J (2007): Heath-Jarrow-Morton Modelling of Longevity Bonds and the Risk Minimization of Life Insurance Portfolios. *Working Paper, Université Catholique de Louvain*.

Bauer D, Kramer FW (2007): Risk and Valuation of Mortality Contingent Catastrophe Bonds. *Working Paper, Ulm University*.

Bauer D, Russ J (2006): Pricing Longevity Bonds using Implied Survival Probabilities. *Working Paper, Ulm University*.

Blake D, Boardman T, Cairns A (2010): Sharing Longevity Risk: Why Governments Should Issue Longevity Bonds. *Discussion Paper PI-1002, The Pension Institute*, *Cass Business School*, City University, London.

Blake D, Burrows W (2001): Survivor Bonds: Helping to Hedge Mortality Risk. *Journal of Risk and Insurance*, 68:339–348.

Blake D, Cairns A, Dowd K (2008): Longevity Risk and the Grim Reaper's Toxic Tail: The Survivor Fan Charts. *Insurance: Mathematics & Economics*, 42:1062–1066.

Blake D, Cairns AJG, Dowd K (2006a): Living with Mortality: Longevity Bonds and other Mortality-Linked Securities. *British Actuarial Journal*, 12:153–228.

Blake D, Cairns AJG, Dowd K, MacMinn R (2006b): Longevity Bonds: Financial Engineering, Valuation, and Hedging. *Journal of Risk and Insurance*, 73:647–672.

Broekhoven H van (2002): Market Value of Liabilities Mortality Risk: A Practical Model. *North American Actuarial Journal*, 6:95–106.

Brown JR, Orszag PR (2006): The Political Economy of Government Issued Longevity Bonds. *Journal of Risk and Insurance*, 73: 611–631.

Cairns AJG, Blake D, Dowd K (2006): Pricing Death: Frameworks for the Valuation and Securitization of Mortality Risk. *ASTIN Bulletin*, 36:79–120.

Cairns AJG, Blake D, Dowd K (2008): Modelling and Management of Mortality Risk: A Review. *Scandinavian Actuarial Journal*, 2008(23):79–113.

Cannon E, Tonks I (2008): *Annuity Markets*. Oxford University Press, Oxford.

CEA (2006): *Working Document on the Standard Approach for Calculating Solvency Capital Requirement*. European Insurance and Reinsurance Federation.

Cipra T (1998): Cohort Life Tables for Pension Insurance and Pension Funds. *Pojistné rozpravy*, 3:31–57 (in Czech).

Cipra T (2002): *Capital Adequacy in Finance and Solvency in Insurance*. Ekopress, Prague (in Czech).

Cipra T (2004): *Reinsurance and Alternative Risk Transfer*. Grada Publishing, Prague (in Czech).

Cipra T (2010): *Financial and Insurance Formulas*. Physica Verlag/Springer, Heidelberg.

Collet-Hirth O, Haas S (2007): *Longevity Risk. The Longevity bond. Technical Report* (November 2007), Patner Re, Bermuda.

Corkish J, Holland A, Vila AF (1997): *The Determinants of Successful Financial Innovations: An Empirical Analysis of Future Innovations on LIFFE*. Bank of England, London.

Coughlan G et al. (2007a): *LifeMetrics: A Toolkit for Measuring and Managing Longevity and Mortality Risks*. Technical Document. JPMorgan Pension Advisory Group (March 2007).

Coughlan G, Epstein D, Sinha A, Honig P (2007b): *q-Forwards: Derivatives for Transfering Longevity and Mortality Risks*. JPMorgan Pension Advisory Group (July 2007).

Cowley A, Cummins JD (2005): Securitization of Life Insurance Assets and Liabilities. *Journal of Risk and Insurance*, 72:193–226.

Cox SH, Lin Y (2007): Natural Hedging of Life and Annuity Risks. *North American Actuarial Journal*, 11:115.

Cox SH, Pedersen HW (1998): Catastrophe Risk Bonds. *Actuarial Research Clearing House,* 1998(1):421–452.

Cummins JD (2008): CAT Bonds and other Risk-Linked Securities: State of the Market and Recent Developments. *Risk Management and Insurance Review*, 11:23–47.

Dahl M (2004): Stochastic Mortality in Life Insurance: Market Reserves and Mortality-Linked Insurance Contracts. *Insurance: Mathematics & Economics*, 35:113–136.

Dahl M, Møller T (2006): Valuation and Hedging of Life Insurance Risks with Systematic Mortality Risk. I*nsurance: Mathematics & Economics*, 39:193–217.

Denuit M, Devolder P, Goderniaux A-C (2007): Securitization of Longevity Risk: Pricing Survival Bonds with Wang Transform in the Lee-Carter Framework. *Journal of Risk and Insurance*, 74: 87–113.

Dowd K, Blake D, Cairns AGJ, Dawson P (2006): Survivor Swaps. *Journal of Risk and Insurance*, 73:1–17.

Hári N, De Waegenaere A, Melenberg B, Nijman TE (2008): Longevity Risk in Portfolios of Pension Annuities. *Insurance: Mathematics & Economics*, 42:505–519.

IAA (2004): *Global Framework for Insurer Solvency Assessment.* International Actuarial Association.

Kabbaj F, Coughlan G (2007): Managing Longevity Risk through Capital Markets. *De Actuaris* (September 2007):26–29.

Krutov A (2006): Insurance-Linked Securities: An Emerging Class of Financial Instruments. *Financial Engineering News*, 48:7–16.

Lee RD, Carter LR (1992): Modelling and Forecasting U.S. Mortality. *JASA*, 87(419):659–675.

Leppisaari M (2008): *Managing Longevity Risk with Longevity Bonds*. Helsinki University of Technology (Mat-2.4108, August 2008).

Levantesi S, Torri T (2008): *Setting the hedge of Longevity Risk through Securitization.* Proceedings of the 10th Italian-Spanish Congress of Financial and Actuarial Mathematics, Cagliari.

Lin Y, Cox SH (2005): Securitization of Mortality Risks in Life Annuities. *Journal of Risk and Insurance*, 72:227–252.

Lin Y, Cox SH (2008): Securitization of Catastrophe Mortality Risks. *Insurance: Mathematics & Economics*, 42:628–637.

Loyes J, Panigirtzoglou N, Ribeiro RM (2007): Longevity: A Market in the Making. JPMorgan Technical Report (*Global Market Strategy*, July 2007).

Milevsky MA (2006): The *Calculus of Retirement Income.* Cambridge University Press, Cambridge.

OECD (2006): *Pension Markets in Focus. Issue 3* (October 2006), Paris.

OECD (2008): *Pension Markets in Focus. Issue 5* (December 2008), Paris.

Reuters (2010): *Factbox: How Longevity Bonds May Work.* Thomson Reuters (Wed, Apr 7, 2010).

Richards S, Jones G (2004): Financial Aspects of Longevity Risk. *Staple Inn Actuarial Society* (October 2004), London.

Sandström A (2006): *Solvency. Models, Assessment and Regulation*. Chapman & Hall/CRC, London.

Schneider O (2009): Reforming Pensions in Europe: Economic Fundamentals and Political Factors. *Finance a úvěr-Czech Journal of Economics and Finance*, 59(4):292–308.

Swiss Re New Markets (1999): *Insurance Linked Securities*. Zürich 1999.

Thomsen GJ, Andersen JV (2007): Longevity Bonds: A Financial Market Instrument to Manage Longevity Risk. *Monetary Review*, 4th Quarter: 29–44.

Wang SS (2002): A Universal Framework for Pricing Financial and Insurance Risks. *ASTIN Bulletin*, 32:213–234.