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Quality Assessment and Improvement Methods in Statistics – what Works?[1]

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Abstract

Several methods for quality assessment and assurance in statistics have been developed in a European context. Data Quality Assessment Methods (DatQAM) were considered in a Eurostat handbook in 2007. These methods comprise quality reports and indicators, measurement of process variables, user surveys, self-assessments, audits, labelling and certification. The entry point for the paper is the development of systematic quality work in European statistics with regard to good practices such as those described in the DatQAM handbook. Assessment is one issue, following up recommendations and implementation of improvement actions another. This leads to a discussion on the effect of approaches and tools: Which work well, which have turned out to be more of a challenge, and why? Examples are mainly from Statistics Norway, but these are believed to be representative for several statistical institutes.

Keywords

Quality assurance, quality frameworks, quality reports, user satisfaction studies, labelling of statistics, quality reviews

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INTRODUCTION

During the last decades, both international organisations and National Statistical Institutes (NSIs) have focused on the importance of quality work. A systematic approach to quality has been adopted in many statistical institutes. This has been based on some common principles of quality management. The work has been supported by international initiatives, in Europe in particular the Code of Practice (CoP – Eurostat, 2011) for the production and dissemination of statistics. A second round of peer reviews assessing compliance with CoP has just started.

Several methods for quality assessment and assurance in statistics have been developed in a European context. Data quality assessment methods (DatQAM) were considered in a Eurostat handbook in 2007 (Eurostat, 2007). The entry point for the paper is the development of systematic quality work in European statistics with regard to quality frameworks and good practices, such as those described in the DatQAM handbook. Assessment is one issue, following up recommendations and implementation of improve-

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[2] The paper is based on a presentation at the European Conference on Quality in Official Statistics (Q2014) in Vienna 2–5 June, 2014.

ment actions another. This leads to a discussion on the effect of approaches and tools: Which work well, which have turned out to be more of a challenge, and why?

Examples are mainly from Statistics Norway, but these are believed to be representative for several statistical institutes.

1 QUALITY FRAMEWORKS

A quality framework provides a frame for the identification of quality challenges and actions for their resolution, and it is a prerequisite for systematic quality work. The framework should therefore be reflected upon before considering the use of tools for quality assurance.

1.1 General frameworks

A quality framework or management system consists basically of some definitions, principles and a model linking the principles together. General quality frameworks comprise Total Quality Management (TQM), Six Sigma, European Foundation for Quality Management (EFQM), Common Assessment Framework (CAF), Balanced Scorecard, ISO and Lean or Lean Six Sigma. These systems are to a large extent based on a common set of definitions (e.g. quality as "fit for use") and principles (such as user and process orientation, improvements based on measurements and participation by all), but they differ with respect to main focus and degree of formalisation. In EFQM and ISO emphasis is for example put on rating and certification, whereas Six Sigma focuses on quality control applying statistical methodology. Lean emphasises improved efficiency by the reduction of waste.

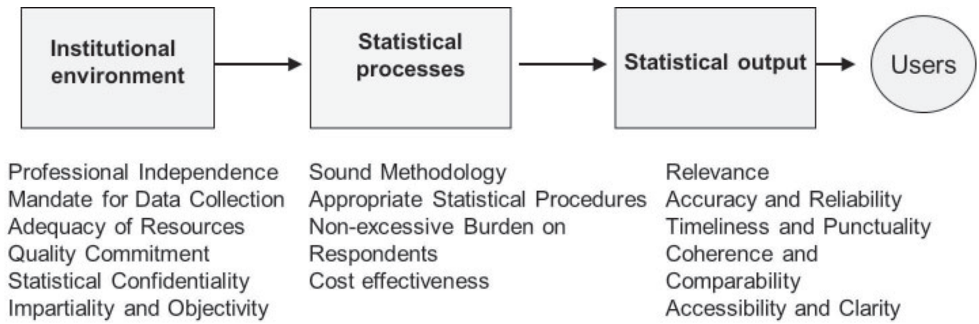
In some sense TQM that was developed in the last century is the mother of all general quality management systems. Concepts and principles developed here constitute a common content of all such systems developed later. However, the variety of systems may complicate comparability of quality work and a just description of strong and weak aspects of such work. Systems have developed, but also changed names over the years. In Norway no one talks about TQM nowadays, but many consultants promote Lean as if this is a completely new system. It is a built-in feature of their business to promote new initiatives, but for a statistical institution that needs continuity, is it important to keep values and principles and also their wording over time. It is crucial that earlier improvement work is recognised, and nothing is more demotivating for staff than being told that the real improvement will start now since earlier work has not succeeded. This is a challenge for management, since implementing something new seems to prove decisive management. Deming's and TQM's principle constancy of purpose is just as valid today as when formulated more than 30 years ago (Deming, 1982). This also points to the fact that quality work is a continuous task. User needs and possibilities (e.g. technology) change, and statistics and production processes must change accordingly.

1.2 Frameworks for official statistics

Some National Statistical Institutes (NSIs) apply one or parts of several of the general quality systems. But there is a set of values and principles of official statistics, and hence the NSIs, which go beyond the principles of these systems. This, in particular, regards independence, impartiality and protection of data on individuals. Such requirements to official statistics were first formulated jointly in the ten UN principles of official statistics adopted in 1992 (UN, 1992). Later, such principles have been incorporated in quality frameworks for statistics.

In Europe, the CoP provides a common quality framework for statistics. It follows a TQM-like model from user needs for products to underlying processes and the institutional environment which is specific for statistical institutions (see Figure 1). The indicators linked to the output represent an agreed definition of the components of quality in statistical products.

Figure 1 Code of Practice as a quality model



Source: Own construction

Other frameworks developed in international statistical cooperation comprise the UN Generic National Quality Assurance Framework (NQAF – UN, 2012) and the African Charter on Statistics (African Union, 2009). Both IMF and OECD have developed quality assurance framework, see (IMF, 2012 and OECD, 2011). In addition, a number of national frameworks or Code of Practices taking the specific requirements to official statistics into account, have been developed.

It is natural that statistical institutions incorporate the specific requirements to official statistics in their quality framework. Extensive discussions on which quality management system is the best should be avoided. What is important is that an organisation has one and goes ahead with implementing it.

1.3 Tools

Different tools can be linked to the elements of the framework which provide the standards for assessing and reporting quality of statistics.

The tools and procedures to assure quality described in the Eurostat handbook on Data Quality Assessment Methods and Tools (2007) comprise:

- Quality reports and indicators,
- Measurement of process variables,
- User surveys,
- Self-assessments and auditing,
- Labelling and certification.

These tools can be applied to a various degree, but to a large extent they build on each other. Audits are for example normally built on self-assessments, and audits or some reviews are a prerequisite for labelling and certification.

Where relevant, tools can be linked to different stages in the production process of statistics, i.e. a business process model (for example quality indicators and process variables). Many statistical institutes, including Statistics Norway have developed a detailed process model based on the international General Statistical Business Process Model (GSBPM), see UNECE (2013) and Statistics Norway (2008). This is a basis for work on standardisation, and documentation is also linked to it.

Together with a quality framework such a business model and an organisation for coordinating quality work constitute a necessary infrastructure for systematic quality work in a statistical institution.

International and European initiatives have supported quality work in the NSIs. On the other hand national work on quality in statistics has influenced European requirements and recommendations, since these have been developed in cooperation with the European NSIs. CoP has been important for the development of systematic quality work in NSIs and constitutes a quality framework for Statistics Norway.

Considerations on different tools reviewed in the DatQAM handbook follow.

2 QUALITY REPORTS

A quality report provides information on the main quality characteristics of a product for its users. Quality reports are normally based on quality indicators describing these characteristics. Quality reports are important for the producers and the management as well. However, the requirements of users and producers are different, but a standard structure is preferable. For European statistics, Eurostat (2014) has developed a handbook for quality reports.

NSIs produce quality reports required by several international organisations and deliver them together with the data. Many NSIs also produce different types of standardised documentation including quality aspects for other and general users. Statistics Norway has a system where "About the statistics" is linked to every statistic on the web, all together about 400 different reports. These contain information on the background for each statistic, production, methodology and definition of concepts in addition to information on product quality such as relevance (use and users), accuracy, timeliness and comparability. Burg (2010) discusses if these types of standard documentation really are quality reports, on the basis of the Austrian Standard Documentation system. The answer is and should be yes, given that the documentation includes the necessary quality aspects.

For an NSI standardised documentation like "About the statistics" going beyond the pure quality aspects are necessary for both users and producers. In Statistics Norway we use this information as a basis for our internal reviews described in Section 7.

The level of detail in these reports is an issue. The extent and complexity of the reports tend to increase over time, and we should realise that the target group in practice is rather expert users. Producers will anyway need more comprehensive documentation linked to from the standard report. For a "normal" user there is a need for simplified information linked to or directly integrated in the text following the release of statistics.

Another issue that came up very clearly during our reviews is the need to update the standardised documentation consecutively. Most of "About the statistics" were not up to date, and there is no reason to believe that the situation is better for statistics not reviewed. That few if any of our users have complained about this may be a sign that this type of documentation is not much used, or is too comprehensive or complicated.

3 QUALITY INDICATORS

Quality indicators are used in the quality reports and in particular by management. In Statistics Norway some aggregated quality indicators are included in a set of performance indicators that are reported to the Ministry of Finance and publicised. This regards indicators on timeliness, punctuality, response burden and response rates (proxy for accuracy). When developing indicators that cover several statistics, weighting and aggregation is an issue. The indicators should therefore be used with some care. Also balancing between different quality aspects substantiates this. It is for example possible to obtain 100 per cent punctuality, but that might be on the cost of timeliness. There is a similar balance between accuracy and timeliness.

Sometimes a quality indicator will show that something is wrong and that there is a need for action. An example can be the steadily decreasing response rates of some surveys. In general naming and shaming works, but not denouncing. In the case with response rates it is obvious that the general development of society with many opinion polls and difficulties to get hold of people (no phonebooks) is the main reason for the decrease, and new sources and ways of collecting data are called for. Management discussions on the development of performance and quality indicators must be constructive by considering and suggesting improvement possibilities.

4 PROCESS VARIABLES

Statistical institutes have always measured some process variables. Examples are measurements of non-response of different types, interviewer performance, costs and use of time for different processes. A method for controlling and improving quality based on such measurements of repetitive processes was introduced in the "classical" paper by Morganstein and Marker (1997), based on Deming's statistical thinking about quality. The DatQAM report considered this and presents some examples of use of such variables, and Sæbø (2007) adds a few more examples. These comprise techniques for mapping processes, supplemented by statistical control methodology monitoring variations in processes (with respect to for example time and errors). The idea is to study how a process described by key process variables varies. If the variability is satisfactory, control limits can be established and used to identify later errors or improving the process by considering the effect of possible actions (checking the significance of these). If the level or variability of a measured process variable is unsatisfactorily, the process should be changed.

However, use of process variables other than resource inputs is still limited in official statistics, often confined to analysing response rates and managing interviewers. Our work with this kind of method has perhaps not been systematic enough, which is a paradox for statisticians familiar with analysing data. Editing and the effect of this is one area where this methodology should be suitable. This process normally counts for a relatively high share of resources used for the production of statistics.

5 USER SATISFACTION STUDIES

A user satisfaction survey is a survey which aims at assessing the satisfaction or the perception of the users, normally as a basis for improvement actions (Eurostat's concepts and definitions database).

The DatQAM handbook (Eurostat, 2007) distinguishes between general surveys directed to diverse known users of products/services (for example all paying customers), image studies directed to unknown users and asking for their perception or confidence in statistics, and specific surveys directed towards target groups such as questionnaires added to printed publications or web questionnaires. Examples of a number of user surveys and recommendations are given. In addition, user satisfaction surveys can be categorised by general surveys covering the satisfaction with the quality of all statistics provided and surveys in specific statistical domains, as carried out on the Internet by Eurostat (Baigorri, Junker, 2010).

In the preparations for the current European peer reviews, Eurostat has surveyed the status of user surveys in European NSIs. It was found that most of them have implemented one kind of user satisfaction study or another, many of them covering both statistics in important fields, quality issues, trust, dissemination and overall evaluation (Eurostat, 2013).

There are several ways of ensuring systematic user feedback, for example by user councils and contact in connection with work on commission. In the quality reviews in Statistics Norway experiences with focus groups are good, revealing new insight in user perceptions and needs (see Section 7). However, these reviews have revealed that user orientation often represents an improvement area for the different subject matter divisions.

User satisfaction studies have some limitations that one should be aware of. When evaluating the quality of statistics, users often emphasise timeliness and coherence (they want to see specific statistics in a broader context). Relevance is normally considered to be good. However, in surveys (or meetings such as focus groups) with known users or target groups using statistics, relevance will almost by definition get a high score. Those who do not find relevant statistics will normally not be included in such surveys. This should not lead to the conclusion that relevance is less important than other quality dimensions, and that it cannot be improved!

Another point is that satisfying the user needs is not always sufficient. Quality assurance and user satisfaction surveys normally answer the question if we do things right, to a less extent if we do the right things. Users do not always know what kind of statistics or solutions for presenting and disseminating them they really would like. Sometimes they should be positively surprised (a good example is Apple and Steve Jobs)! This means that producers of statistics should be pro-active, monitoring and quickly taking the development of society and technological possibilities into account.

The ongoing discussions on new data sources (including "big data") and new communication channels for statistics (such as Facebook and Twitter) could be mentioned in this context. New technology and data and new actors producing and spreading vast quantities of statistics represent both threats and opportunities for statistical institutions and official statistics.

There are a few examples of "paradigm shifts" in statistics in the past, the best is probably the development of the Internet. As one of the first NSIs Statistics Norway started to disseminate statistics on the Internet in February 1995, and this had great significance for our users' satisfaction and trust in the institutions in the years to come.

Timing is crucial regarding user satisfaction. To this end, Statistics Norway has not offered our users any "app" for retrieving statistics on mobile phones and tablets (there have been good reasons for putting priority on modernising the web service as such, also with APIs). A few years ago, this would have been an example of a positive surprise for our users. Now this is rather something they would expect, and since we do not have this it might harm confidence in the institution. However, Statistics Norway is present on social media (Facebook and Twitter which are important on mobile devices).

6 SELF-ASSESSMENTS

Self-assessment is a review of an organisation's activities and results referenced against a model/framework, and carried out by those who are responsible for these activities. Several tools for self-assessment of statistics have been developed, in Europe in particular DESAP for survey managers (Eurostat, 2003), in addition to the self-assessments for statistical institutions conducted as preparations for peer reviews.

In Statistics Norway DESAP has been used to assess all our statistics in 2008 (Næs, 2009). Improvement points comprised systems for more systematic user contacts, better knowledge of quality of administrative data owned by others, more automatic editing, and in general better documentation, including updating of "About the statistics". These measures were reviewed in 2010. There had been some progress, but there were still challenges linked to most of the areas mentioned. There might be a gap between theory and practice in this area – stand-alone self-assessments do not necessarily provide a correct picture (Sæbø, 2006). However, a self-assessment based on a quality framework could be a good start to systematic quality work – to anchor the framework and quality thinking in the organisation and to identify weak points and improvement actions.

Self-assessments are normally used as a part of preparations for reviews and audits. This was the case prior to the European peer reviews in 2006–2008. Here CoP itself constituted the basis for the self-assessments. Together with the underlying Quality Assurance Framework (QAF – Eurostat, 2012) CoP is used as a basis for the self-assessment preceding the current round of peer reviews, and all European NSIs and several other producers of European statistics have filled in comprehensive questionnaires.

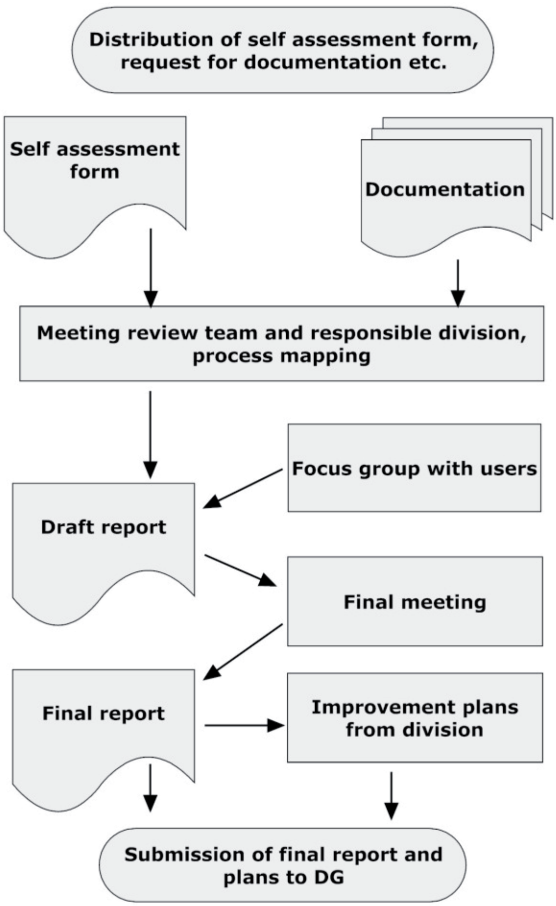
The UN NQAF (UN, 2012) has also been supplemented by a checklist that is suitable for and used for self-assessments in several countries in different parts of the world.

7 REVIEWS AND AUDITS

Statistics Norway started work with systematic internal quality reviews or audits of selected statistics in 2011(Sæbø et al., 2012; Sæbø, Byfuglien, 2013). The CoP and tools linked to this have guided the reviews. The reviewing system has been integrated with our internal control to form a system that covers all aspects of work in the institution. The reviewing process is illustrated in Figure 2. It is performed very much like the European peer reviews, with the exception that specific statistics or subject matter areas are reviewed and not the institution as such.

In the period 2011–2013, 21 different statistics or clusters of statistics have been reviewed; at least one in each division producing statistics. Together they represent almost 30 percent of the working hours used for statistics production in Statistics Norway.

Figure 2 The reviewing process



Source: Own construction

The reviews have been based on three elements: Self-assessments on the compliance with the principles and indicators in the Code of Practice and other documentation, process mapping using Lean techniques (Value Stream Mapping) and focus groups to evaluate user needs. A team of 4 persons has conducted the review. The team members have a background from quality management, statistics production, dissemination and survey methodology. One of the members is a methodologist. The team has been assisted by experts in conducting focus groups.

Statistics reviewed were selected in cooperation with the producers following proposals from the reviewing team, among others based on preferences from the National Accounts and experiences from earlier self-assessments using DESAP.

The reviews were "audit-like" even if they were carried out by an internal team. This implies focus on evidence. Findings are presented objectively in a report that is the sole responsibility of the team. The reports follow a standardised structure, also including a consideration of strengths and weaknesses. There is no ranking, but each report ends up with a set recommendations based on the findings. The division responsible for the relevant statistics reviewed can correct factual errors, but makes a separate action list on the basis of the recommendations. If they disagree with some of these they can express this here. Reports and action plans have been sent to the Director General and are followed up later. They are published on the Statistics Norway Intranet. The different steps in the reviews and experiences are described in more detail in Sæbø and Byfuglien (2013).

The reviews have resulted in more than 170 proposals for improvements. Many of the improvement points concern several statistics, and there is reason to believe that they are valid generally in Statistics Norway.

The most important improvement points concern the need for:

- More focus on user needs and the relevance of statistics, in particular users want to see the statistics in a broader context.
- Better dissemination with more visualisation (graphs, maps, etc.).
- Improved documentation, in particular a need to update "About the statistics" which provides metadata for the users on <www.ssb.no>.
- Improved production processes.
- Increased understanding for and use of statistical methods, in particular in editing which requires relatively large resources.
- Increased knowledge of formalities (such as the basis for data collection).

8 FOLLOWING UP

The European peer reviews have been and will be followed up by monitoring the resulting action plans. This is necessary and often provides support to internal improvement efforts.

Some of the improvements points from the internal reviews in Statistics Norway were the same as those revealed already in the DESAP self-assessments in 2008. This illustrates that self-assessments alone might not be that effective, but foremost that both assessments and reviews must be followed up. This is a responsibility for management on all levels, but monitoring can be performed centrally.

In Statistics Norway, the reviewing team gathers information on status for planned actions annually. Most of the actions planned in 2011–2013 were fulfilled in 2014. Statistics Norway is currently carrying out a Lean programme, and there is a break in the reviews since these have to be adapted to this. However, they have given input to this program, and also provide a basis for the European peer reviews in Statistics Norway.

Measures implemented are primarily linked to improved documentation and metadata, improved dissemination, evaluation and balancing of quality and efficiency in the production, international cooperation (on good practices), and better coordination and collaboration within Statistics Norway. Measures have been carried out to assure confidentiality. There are examples of transitions to use of common and standardised IT solutions.

In general, these reviews have been considered to be useful, and that there are several general observations that can be useful also for improving areas not reviewed.

In line with its responsibility to follow up improvement proposals, management should ensure that identified best practices or "current best methodology" are not only documented, but known and taken aboard in the organization.

9 LABELLING

A discussion on labelling has been on the international agenda. As mentioned in Section 5, developments in technology, data sources and user needs represent both a threat and opportunity for official statistics, and communicating the value of such statistics is important regardless of a labelling system.

It is normally agreed that official statistics shall serve the whole spectrum of society, and hence be easily available and be based on quality criteria such as those formulated in CoP, including professional independence and impartiality. Official statistics should be distinguished from analyses/research and pilot studies. Some of the criteria are absolute and measurable, but most of them are subject to judgements (for example balancing accuracy and timeliness). Definitive requirements could include the use of a release calendar, non-disclosure of information about individuals, use of statistical standards (internationally agreed definitions of units, variables and classifications ensuring coherence and comparability) and transparency by providing documentation on data sources, production processes, methods and quality. Use of best practices in the production can then be judged. Being an active part of the international statistical society contributes to such use even if it does not give any guarantee.

Few NSIs have a system with labelling in the form of marking statistics and statistical tables, but examples from UK and Sweden are mentioned in the DatQAM report. In the UK there is a separate institution, the Statistical Authority, that is responsible for approving national statistics from ONS and other producers, following a system of quality reviews similar to the internal reviews in Statistics Norway, but much more comprehensive. In Sweden the production of statistics is even more decentralised, with 25 producers of official statistics. Here these institutions themselves decide which statistics that fulfil quality criteria and can be marked as official within the specific subject matter area under their mandate.

Labelling can be a tool for increasing trust in statistics if needed, improving quality and to avoid misuse (of statistics that are not approved/labelled). On the other hand it would require more bureaucracy, and there might be problems with how to apply labelling in practice (linked to tables, figures, databases and different technical solutions for dissemination). Eventually, will users really distinguish between labelled and non-labelled statistics? This will vary from country to country, and a general recommendation cannot be given.

The level of centralisation of the statistical system in a country is also a factor that may affect the need for labelling. Norway has a relatively centralised system, with Statistics Norway producing at least 85 per cent of such statistics. Even if we do not apply labelling, statistics presented on ssb.no with our logo are perceived as official. However, for other national producers of statistics it is more unclear what could be regarded as such statistics.

10 CERTIFICATION

Several of the considerations given on labelling of statistics also concern certification. Examples of certification efforts in NSIs comprise the Greek NSI Elstat who is in a process of certifying other national statistics producers according to CoP. Statistics Sweden has recently been certified according to ISO 20252 (quality standard for market, opinion and social research). Certification can be useful to improve trust and in a situation with competition for resources, but it has its costs. Statistics Norway has not considered certification. However, investing in assessments and reviews is a necessary prerequisite for both labelling and certification.

CONCLUSIONS

The basis for a systematic work on quality is a quality framework. In Europe, the Code of Practice together with general quality management principles represents a common quality framework. In addition to a quality framework a business process model and an organisation for coordinating quality work constitute a necessary infrastructure for a systematic quality work in a statistical institution.

Quality assurance by help of tools linked to such a framework should be implemented step by step, from the use of simple tools such as quality reporting and indicators. A self-assessment itself could represent a good starting point for a systematic work on quality. But reviews and audits make a difference. Labelling or certification presupposes a thorough cost benefit analysis – the need for these activities will vary from country to country. However, clarifying and communicating the value of official statistics based on quality criteria is important.

Quality work is a continuous effort. User needs change over time, so do the environment for producing statistics including the technological possibilities. Constancy of purpose and management support on all levels are important. Too high ambitions in the short run could be counterproductive.

Quality assurance by monitoring, reviewing and formulation of improvement actions are not enough – following up the implementation of planned actions is crucial.

Finally one should bear in mind that quality assurance should not only apply to doing things right. Doing the right things is just as important, and some resources should be invested to ensure this.

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Analysis of Long-Run GDP Development in the USA, the EU15, China and the USSR/Russia

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Abstract

The paper analyses how changes in GDP in China, the USA, the EU15 and the USSR/Russia over a period of 50 years (1961–2011) were affected by change of intensive factors and change of extensive factors. Intensive factors consist primarily of technological progress. Extensive factors are the amounts of labour and capital. The analysis does not use growth accounting, but instead works with the 'dynamic parameters' of intensity and extensity. Contrary to the values of growth accounting, these parameters can be calculated not only for situations of GDP growth, but also for situations of GDP contraction and stagnation. They thus provide a complete picture of GDP development. The paper briefly explains the methodology for deriving the parameters. Their values for each territory are then analysed. The results show that the parameters are able to describe the real development of GDP and their information value is very high.

Keywords

Intensive and extensive economic growth, dynamic parameter of intensity, dynamic parameter of extensity, measurement of intensity

JEL code

C22, C43, O33, O47

INTRODUCTION

Economic theory solves from its beginning many questions. Which factors affect the development of production at various levels of the social system, belongs to the most important ones, especially the whole-economy level and the company level (Barro and Sala-I-Martin, 1999). Before the start of the Industrial Revolution, society had been developing with only slow application of technical progress, so the key factors of development appeared to be soil, labour and capital. Such development was largely of an extensive nature, characterized mainly by change in inputs, while technology remained basically the same. Starting in the 19th century, the expansion of innovations, which resulted from qualitative (intensive) changes in the production process (Varadzin et al., 2004), gave rise to a need to compare the effects of quantity and

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quality, i.e. extensive and intensive factors. This issue was formulated more precisely by Solow (1957, p. 312) who introduced a special form of the production function considering both extensive and intensive factors.[3] Solow's function reads: $Q = F(K,L;t)$, where Q represents output and K and L represent capital and labour inputs in 'physical' units. As for symbol t, Solow adds: 'The variable t for time appears in F to allow for technical change. It will be seen that I am using the phrase "technical change" as a short-hand expression for *any kind of shift* in the production function. Thus slowdowns, speed-ups, improvements in the education of the labor force, and all sorts of things will appear as "technical change".'

Growth accounting was developed at the end of the first half of the 20th century to measure the impact of qualitative changes (technological progress) and quantitative changes (changes in the volume of labour and capital – generally inputs) on the change in output. However, growth accounting is based on many special conditions. It turned out that the influence of technical progress could be assessed only for growth of production induced by current growth of labour, capital and technical progress, and only roughly for slow rates of growth.[4] To overcome this problem with growth accounting it was proposed (Mihola, 2007a; Mihola, 2007b; Hájek, 2006; Hájek and Mihola, 2009) to use an alternative solution in the form of dynamic parameters of intensity and extensity that quantify the influence of intensive factors (innovations) and extensive factors (input changes) on the change in output. The advantage of these dynamic parameters lies mainly in the fact that they can be used for any rate of growth or decline in GDP. Another advantage consists in their ability to quantify the influence of intensive and extensive factors for any development of these factors. The indicators can thus be used for present growth in intensive and extensive factors, for a present decline in intensive and extensive factors and for the situation of total or partial compensation, i.e. where one factor is increasing and the other one is decreasing. Applications (e.g. Cyhelský, Mihola and Wawrosz, 2012; Mihola and Wawrosz, 2013) of these dynamic parameters at the whole-economy and company level have so far indicated that the results are very easy to interpret.

The aim of this paper is to apply this methodology to compare the quality of the dynamics of development of big countries (China, the USA and the USSR/Russia) and the 15-country European Union[5] (EU15). Fifty-year-old time series of initial data (1960–2011) were collected for each territory under scrutiny to enable us to carry out an annual quality analysis of their development. The paper initially deals with the question of how to set the weights of labour and capital in the total input. Growth accounting sets the weights for labour and capital in each evaluated year based on real values under the condition that the sum of the scales equals 1. Based on an analysis of real isoquants, the present paper suggests a simplified option, setting both the weight on labour and the weight on capital equal to 0.5. The paper demonstrates that this relatively simple application methodology[6] provides us with information that is consistent with the results obtained by using growth accounting or other methods based on more complex tools. The article is organized as follows. First, we explain how to quantify the impact of a change in extensive or intensive factors on the change in GDP. The outputs of the explanation are the 'dynamic parameters' of intensity and extensity. Section 2 presents the methodology for comparing the territories under analysis, and especially the way of acquiring input data. Sections 3 and 4 represent the core of

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[3] The same applies if one wants to find out how time and velocity change, i.e. how acceleration affects the distance travelled. If the acceleration is zero, the velocity is constant and the distance travelled depends only on the time for which you are in motion.

[4] Details about growth accounting and the aforementioned problem with it can be found, for example, in Barro (1999) and Čadil (2007).

[5] The EU15 consists of the following countries: Belgium, Denmark, Finland, France, Italy, Ireland, Luxembourg, the Netherlands, Germany, Portugal, Austria, Greece, Spain, Sweden and the UK. The European Union made up of these 15 countries existed from 1 January 1995 to 30 April 2004.

[6] The issue of the weight levels for labour and capital is analysed, for example, by Mihola and Wawrosz (2013). The main condition is that no input factor (neither labour nor capital) can equal 0, and so the isoquants cannot cross any axis. Our function satisfies this condition.

the article. Section 3 presents the input data for China, the USA, the EU15 and the USSR/Russia, i.e. $G(Y)$, $G(L)$ and $G(K)$, the values calculated from these data, i.e. $G(K/L)$, $G(TIF)$ and $G(TFP)$, and the dynamic parameters of intensity and extensity for each territory. Section 4 analyses the evolution of the dynamic parameters of intensity and extensity in each year for each territory analysed, focusing primarily on years in which the dynamic parameter of intensity is negative, and links these negative values with relevant real events. The conclusion summarizes the main findings.

1 HOW TO MEASURE THE IMPACT OF A CHANGE IN EXTENSIVE OR INTENSIVE FACTORS ON THE CHANGE IN GDP

Let's start with an aggregate economic production function expressing GDP as a product of total factor productivity[7] *TFP* and the total input factor *TIF*:

$$GDP = TFP \cdot TIF. \tag{1}$$

Qualitative development is reflected in changes in total factor productivity *TFP*, whereas quantitative development is reflected in changes in the total input factor *TIF*. Their development is based on the specific structure of production and the technologies applied. The total input factor *TIF* (Mihola and Wawrosz, 2013, p. 32) is obtained as the geometric mean of two basic production factors[8] – labour L and capital K. We thus apply the production function with technical progress[9] for $\alpha = 0.5$

$$TIF = \sqrt{L \cdot K}. \tag{2}$$

This function by definition has constant returns to scale, because, as a result of the scale sum 1, if each production factor is scaled up by a factor of t, *TIF* will also be scaled up by a factor of t (Soukup, 2010)

$$t \cdot TIF = \sqrt{(t \cdot L) \cdot (t \cdot K)}. \tag{3}$$

If we insert expression (2) into expression (1) we get:

$$GDP = TFP \cdot \sqrt{L} \cdot \sqrt{K}. \tag{4}$$

Whether this function has constant returns to scale is determined by the size of *TFP*, which reflects the qualitative element of development. If *TFP* does not change and L and K increase by a factor of t, the growth is 'strictly extensive', corresponding to constant returns to scale. Growth of Y resulting solely from changes in *TFP* represents 'strictly intensive' growth. In order to be able to better quantify the influence of *TFP* and *TIF*, it is better to dynamize the production functions. The dynamic version of the aggregate production function (1) can be written either as follows (in terms of indexes of change):

$$I(GDP) = I(TFP) \cdot (TIF), \tag{5}$$

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7 Robert M. Solow (see Solow, 1957) examines steady state growth as characterized by stabilization of the rate of growth of capital and labour. Growth in output per capita is conditional on technological progress, which is regarded as an exogenous factor. Further elaboration of this idea revealed that such growth is due not to technological progress alone, but to the overall effect of all intensive factors.

8 We do not intend to carry out a detailed analysis of the measurement of L and K. The domains of definition of all the quantities used result from the domains of definition of labour and capital $L > 0$ and $K > 0$.

9 A comprehensive study of the multiplicative Cobb-Douglas production function, with labour, capital and technological progress as factors ($Y = AK^{\alpha}L^{(1-\alpha)}$), is presented in Barro and Sala-I-Martin (1999, p. 29). The authors also describe the production functions proposed by Leontief in 1941 ($Y = F(K, L) = \min(AK, BL)$), Harrod in 1939, Domar in 1946 and Solow in 1969, among many others. For a production function relevant to the Czech Republic, see, for example, Hájková and Hurník (2007).

or as follows (in terms of rates of growth):

$$G(GDP) = (G(TFP) + 1) \cdot (G(TIF) + 1) - 1. \tag{6}$$

Similarly, expression (2) can be expressed dynamically:

$$I(TIF) = I(\sqrt{L}) \cdot I(\sqrt{K}), \tag{7}$$

where the rates of growth follow:

$$G(TIF) = G(\sqrt{L}) + 1) \cdot (G(\sqrt{K}) + 1) - 1. \tag{8}$$

If we insert expression (7) into expression (5), we get the dynamic aggregate production function:

$$I(GDP) = I(TFP) \cdot I(\sqrt{L}) \cdot I(\sqrt{K}). \tag{9}$$

After calculating the logarithm of expression (5), we get the following formula:

$$\ln I(GDP) = \ln I(TFP) + \ln I(TIF). \tag{10}$$

Expression (10) is the basis for the dynamic parameters of intensity and extensity. Their detailed derivation is described in Mihola (2007a, pp. 123–124). The dynamic parameter of intensity is determined by the relation:

$$i = \frac{\ln I(TFP)}{|\ln I(TFP)| + |\ln I(TIF)|}. \tag{11}$$

The dynamic parameter of extensity is then determined by the relation:

$$e = \frac{\ln I(TIF)}{|\ln I(TFP)| + |\ln I(TIF)|}. \tag{12}$$

The analysis of countries (or economic unions such as the EU15) in sections 3 and 4 uses an algorithm which (based on familiar data such as the rate of GDP growth $G(Y)$, the rate of labour growth $G(L)$ and the rate of capital growth $G(K)$) first computes $G(TIF)$ by means of expression (8) and subsequently calculates $G(TFP)$ with the aid of expression (13) based on expression (6).

$$G(TFP) = \frac{G(Y) + 1}{G(TIF) + 1} - 1. \tag{13}$$

The following relation is applied to calculate the index of change of labour over capital $I(K/L)$

$$I = \left(\frac{K}{L}\right) = \frac{I(K)}{I(L)}. \tag{14}$$

The rate of growth of the change in labour over capital G(K/L) follows:

$$G = \left(\frac{K}{L}\right) = \frac{G(K) + 1}{G(L) + 1} - 1. \tag{15}$$

2 INTERNATIONAL COMPARISON METHODOLOGY

The quality of the development dynamics of China, the USA, the EU15 and Russia (until 1992 the USSR) over the last fifty years (1961–2011) will be assessed on the basis of data on annual rates of growth of output, labour and capital, i.e. $G(GDP)$, $G(L)$ and $G(K)$. How were the data obtained? For the USA and the EU15, the main source for $G(GDP)$, $G(L)$ and $G(K)$ was the Statistical Annex of European Economy, which is released by the European Commission every year. $G(GDP)$ is available for the USA and the EU15 for each year of the whole period since 1961. As for determining the rate of capital growth, the data were obtained using the perpetual inventory method (for details see Sixta, 2007). The method is based on adding gross investment to the capital reserve and subtracting depreciated capital, with the value of the depreciation coming from the estimated rate of depreciation.

As for China, the rates of GDP growth were taken from Chinese Statistical Yearbooks and from the website of the National Bureau of Statistics of China. The data on the rate of labour growth for China were obtained from the International Labour Organization (ILO). The rate of capital growth of China for the first half of the period is mentioned in the literature only as the contribution of capital to GDP growth, calculated as the capital income share multiplied by the rate of capital growth. By reverse division by the capital income share, we get the growth rate of the capital stock. The rate of capital growth $G(K)$ in the second period was taken from the literature (e.g. Chong-En et al., 2006), which also uses the perpetual inventory method.

Difficulties emerge in the case of Russia, specifically for the period of 1961–1991, as Russia was part of the Soviet Union and the available data refer to the USSR, not to Russia. We therefore decided to make the USSR identical to Russia, because Russia as the biggest part of the USSR had great significance in terms of all three input indicators – GDP growth, labour growth and capital growth. The rates of GDP growth since 1992 are taken from the web page of the International Monetary Fund.[10] For the Soviet Union (1961–1991), the rates of output growth refer to real gross national product GNP, the dynamics of which do not significantly differ from those of GDP. The rates of GNP growth for the Soviet Union were obtained from the literature (e.g. Christian Science Monitor, 1982; Shanker, 1986; Bergson 1997, BBC 1998, Kontorovich, 1999) and they represent estimates, because the former Soviet Union did not publish these data. Where annual data were missing but five-year averages were available, the missing annual data were supplemented in order to maintain the average five-year rate of growth. The rates of labour growth for Russia since 1992 are taken from the ILO. For the former Soviet Union, the data for 1961–1991 were obtained from journal articles (see above) and the missing annual rates of growth were supplemented in order to correspond to the average rates of growth for the five-year periods. As for the rate of capital growth, the data for 1992–2011 were obtained from a UN study and the IMF's World Economic Outlook. The rates of capital growth were derived from the contribution of capital to GDP growth. The rates of capital growth for the Soviet Union for the period of 1961–1991 were obtained from the literature (see above) and the missing annual data were supplemented in order to correspond to the five-year average rate of growth described in the literature.

With the aid of expression (8), the rate of growth of the total input factor $G(TIF)$ was calculated for each territory under analysis. Expression (13) was used to calculate the rate of growth of the total productivity factor $G(TFP)$. The rates of growth determined in this way allow us to calculate both dynamic parameters i and e with the aid of expressions (11) and (12). Expression (15) was used to calculate the rate of growth of labour over capital $G(K/L)$.

3 ANALYSIS OF THE DEVELOPMENT DYNAMICS OF THE USA, CHINA, THE EU15 AND THE USSR/RUSSIA

The initial average data obtained by the means described in the previous section, together with all the calculated average rates for the whole period, are presented in Table 1 and depicted in Figure 1 and 2.

[10] From World Economic Outlook.

[11] The calculation of the average annual rates of growth is based on the geometric mean of the indexes.

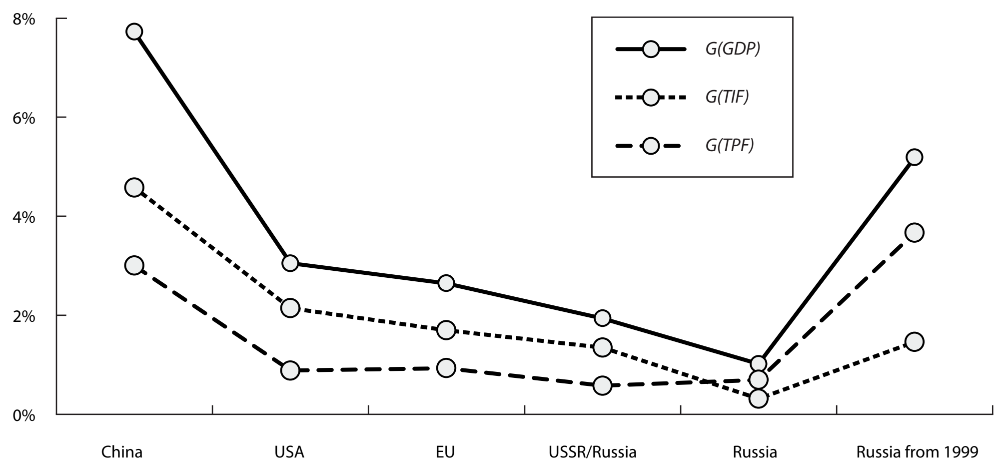
The highest average annual rate of GDP growth, 7.7%, is shown by China. It is followed by the USA with an average rate of growth of 3.1%. The EU15 shows an average rate of GDP growth of 2.6%. The lowest rate – 1.9% – is recorded by the USSR/Russia.[12] Russia alone (1992–2011) has an even lower rate of output growth of 1%. However, since consolidating and overcoming the negative effects of the transformation period after the break-up of the USSR, i.e. since 1999, Russia has been showing an annual average rate of output growth of 5.2%.

Table 1 Rates of growth of inputs and output parameters *i* and *e*, 1960–2011

	G(GDP)	*G(L)*	*G(K)*	*G(K/L)*	*G(TIF)*	*G(TFP)*	*i*	*e*
China	7.7%	2.2%	7.1%	4.8%	4.6%	3.0%	40%	60%
USA	3.1%	1.5%	2.8%	1.3%	2.1%	0.9%	29%	71%
EU	2.6%	0.4%	3.0%	2.5%	1.7%	0.9%	35%	65%
USSR/Russia	1.9%	0.5%	2.2%	1.6%	1.3%	0.6%	30%	70%
Russia since 1992	1.0%	–0.3%	0.9%	1.3%	0.3%	0.7%	69%	31%
Russia since 1999	5.2%	0.7%	2.3%	1.6%	1.5%	3.7%	71%	29%

Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. *G*(*Y*), *G*(*L*) and *G*(*K*)

Figure 1 Average rates of growth *G*(*GDP*)



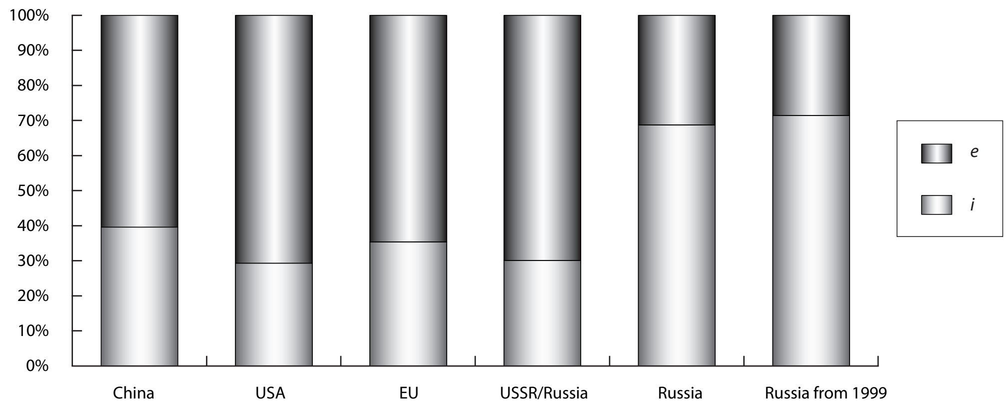
Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. *G*(*Y*), *G*(*L*) and *G*(*K*)

Let's look at Figure 1. China shows the highest values of all six rates of growth analysed. It has the highest rates of GDP growth (7.7%), labour growth (2.2%) and especially capital growth (7.1%). This is reflected in a high rate of growth of TIF (4.6%). China uses its factors of production with the highest rate of growth of TFP (3.0%), as reflected in the highest rate of growth of labour over capital (4.8%). The second highest rates of growth of GDP, labour and TIF are shown by the USA. However, its rate of TFP growth is the same as that in the EU15, i.e. 0.9%. The EU15 shows a significantly higher rate of growth of labour over capital (2.5%) than the USSR and Russia (1.6%), Russia since 1992 (1.3%), Russia

[12] The Russian data are a follow-up to the USSR data.

since 1999 (1.6%) and the USA (1.3%.) However, we cannot simply assume that the USA is at a lower technical level, as it may be that the USA already achieved this higher level before 1960. The lowest rate of GDP growth for the whole period of 1961–1991 is shown by the USSR/Russia. This is, however, substantially influenced by the break-up of the USSR. Russia alone (i.e. since 1992) shows the lowest rates of growth of both GDP (only 1%) and capital 0.9%, along with a negative rate of labour growth –0.3%. This is reflected in the lowest rate of growth of $G(TIF)$ and a modest rate of growth of $G(TFP)$ (0.7%). At the same time, it shows an extremely high intensity of 69%. If we study the consolidated Russia since 1999, we find that Russia shows the second highest (behind China) annual average rate of GDP growth (5.2%) and the highest rate of TFP growth (3.7%).

Figure 2 Intensity and extensity of development 1961–2011



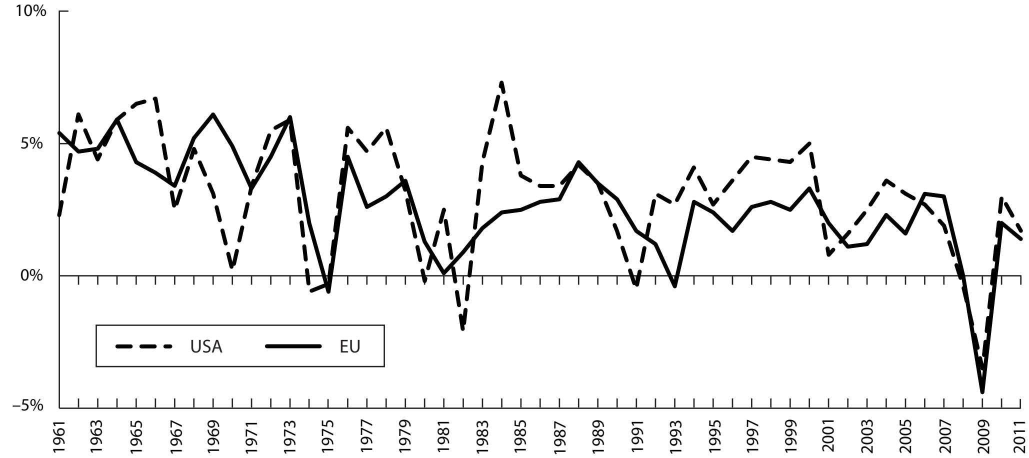
Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. $G(Y)$, $G(L)$ and $G(K)$

Figure 2 compares the quality of development of the territories analysed. It contains the average values of the dynamic parameter of intensity and the dynamic parameter of extensity for the whole period of 1961–2011. In the case of Russia (i.e. Russia excluding the USSR) the values of the parameters are calculated separately for 1992–2011 and 1999–2011. Extensive development prevails in all the economies studied (except for Russia since 1992 and Russia since 1999). China achieves the highest intensity (40%), followed by the EU15 (35%), the USSR/Russia and the USA (the USA – 29 %, %), the USSR/Russia – 30%). Russia has been showing a high share of intensive factors – 69% since 1992 and 71% since 1999.

4 ANALYSIS OF THE ANNUAL DYNAMICS OF THE QUALITY OF DEVELOPMENT OF THE USA, CHINA, THE EU15 AND THE USSR/RUSSIA

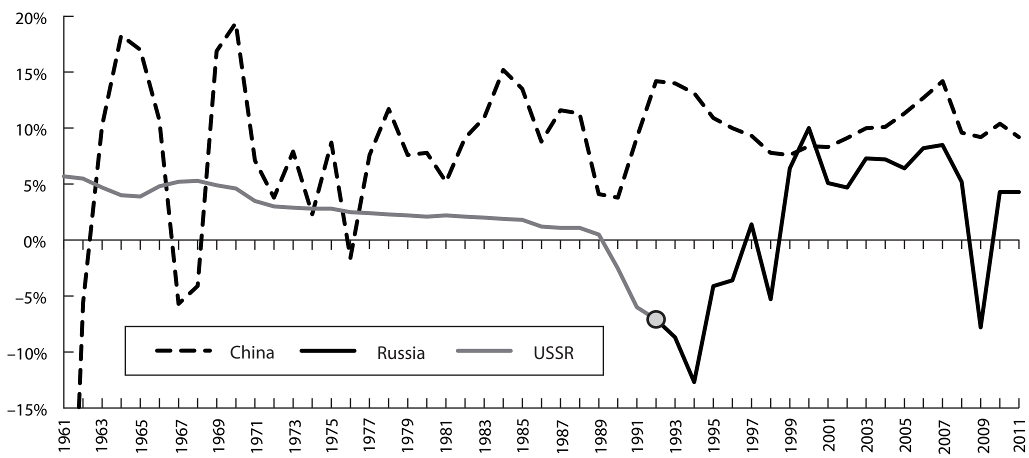
Figure 3 and 4 allow us to compare the annual $G(GDP)$ rates of the territories analysed. The rates of growth of the USA and the EU show lower volatility than the sustained high rates of growth of China. The lowest volatility is shown by the USSR with its continuously slowly decreasing rates of GDP growth. The development of Russia after the break-up of the USSR is very interesting. The period of 1992–1999 was one of chaos following the dissolution of the USSR, with unsuccessful reforms and privatization. Boris Yeltsin was president at that time. After this period, Russia shows stably high rates of growth interrupted only by the world crisis in 2009. This global economic crisis was hardly reflected at all in the rate of development of China, which shows a significant home market and turnover despite its growing openness.

Figure 3 Average annual rates of growth *G*(*GDP*) – the USA and the EU15



Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. *G*(*Y*), *G*(*L*) and *G*(*K*)

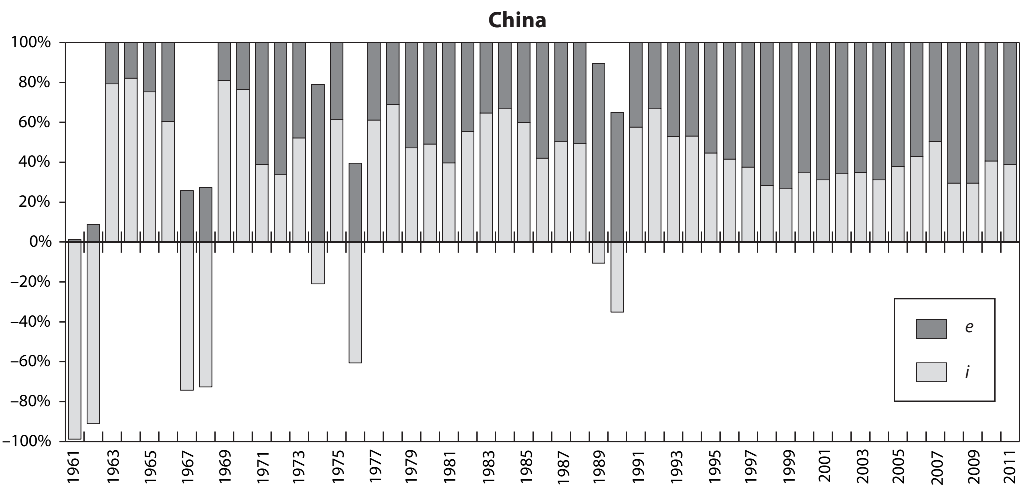
Figure 4 Average annual rates of growth *G*(*GDP*) – China and the USSR/Russia



Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. *G*(*Y*), *G*(*L*) and *G*(*K*)

Information on the intensity of GDP growth, as described in Figure 3 and 4, in the territories analysed over the whole period is presented in the following four figures (Figures 5–8). The figures clearly demonstrate that the GDP growth of all the territories analysed was mostly due to both extensive and intensive factors. Figure 5 presents information on the influence of intensive and extensive factors on GDP development in China. The years in which China achieves high *G*(*GDP*) rates simultaneously show high intensity. Each recession or sharp decrease in the rate of GDP growth shows dis-intensive development with decreasing efficiency and thus negative intensity. Such development occurred in 1961, 1962, 1967, 1968, 1974, 1976, 1989 and 1990. In all these years, the fluctuation is a result of some significant event. Briefly, 1961 and 1962 fall within the period of the 'Great Leap Forward' (usually dated as lasting from 1958 to 1962), a set of measures introduced by the Chinese Communist leader Mao Zedong, who aimed to rapidly transform the country. Mao's collectivization measures divided China into communes, which were supposed to be self-sufficient and responsible for their achievements. However, the measures had the opposite result to what Mao intended – GDP decreased and tens of millions of people died.[13] After this policy was abandoned, the country recovered and its rates of GDP growth rose to 18%, though the base for this growth was evidently low. The growth recorded in the 1960s, however, was stopped in 1966 by the 'Cultural Revolution', which caused further chaos as Red Guard groups[14] went on the rampage and campaigns were launched against intellectuals and others. This resulted again in shrinking GDP (especially in 1967 and 1968), the deaths of many inhabitants[15] and other negative consequences. The negative aspects of the Cultural Revolution began to be gradually eliminated in 1969. Slow progress continued to be made in the early 1970s – diplomatic relations with the USA were restored (including President Nixon's official visit to China in 1972) and China joined the United Nations and became more involved in international trade. All this was positively reflected in the country's economic development, although many of the negative aspects and consequences of the Cultural Revolution persisted. Zhou Enlai, the Chinese premier, and Mao Zedong, the Communist leader, both died in 1976 (in January and September respectively). A struggle for power ensued. 1989 saw the suppression of student movements. The high intensity of development achieved between 1977 and 1988 reflects China's policy of opening up to the outside world and partial economic and political liberalization. The 1990s saw high rates of GDP growth and intensity, although with a falling tendency. Nonetheless, other reforms implemented at that time and in the early 21st century (during the presidencies of Jiang Zemin and Hu Jintao) resulted in increased rates of GDP growth and growth of the dynamic parameter of intensity, especially after 2005. The slight deceleration in GDP growth and the decrease in the dynamic parameter of intensity after 2008 are both manifestations of the global economic crisis, which inevitably hit China because of its increased involvement in international trade. The main reason for the lower GDP growth rate and lower dynamic parameter of intensity is lower foreign demand for Chinese goods.

Figure 5 Intensity and extensity of development of China, 1961–2011

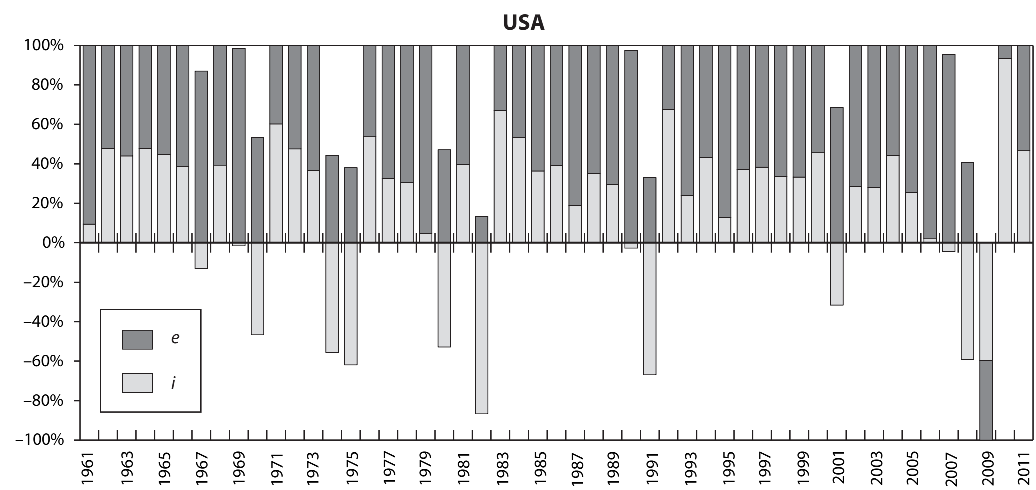


Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. $G(Y)$, $G(L)$ and $G(K)$

[13] It is estimated that 20–40 million died during the Great Leap Forward. See Fairbank (2010).
[14] Red Guard groups were formed mainly of young people. For more details see e.g. Walder (2009).
[15] The number of victims of the Cultural Revolution is estimated at around 8 million. See Fairbank (2010).

Figure 6 shows the influence of intensive and extensive factors on GDP growth in the USA. When the USA achieved *G*(*GDP*) rates exceeding 2.3%, both factors affected the growth. High rates of output growth are always attended by high intensity. Recessions or sharp decreases in GDP growth were in all cases accompanied by dis-intensive development, with intensive factors affecting the decrease in the rate of GDP growth or the decrease in GDP itself. Such development occurred in 1967, 1970, 1974, 1975, 1980, 1982, 1991, 2001, 2007, 2008 and 2009. All these fluctuations correspond to significant events that occurred in the U.S. economy. Briefly, the Caribbean crisis in 1961 is followed by the golden growth of the 1960s, which ended with the first collapse of the Bretton Woods system of fixed exchange rates in 1971. In 1972 and 1973, intensive and extensive quantities are both positive. In 1974 and 1975, however, the rate of GDP growth decreases and there is negative intensity. The slump was caused by the definitive collapse of the Bretton Woods system[16] in 1973, the increase in oil prices following the defeat of the Arab countries by Israel in the Yom Kippur War in the same year, growth in inflation resulting from this oil price increase, high government spending on the war in Vietnam and the de facto defeat of the USA in that war, and even by the Watergate scandal. The GDP slump accompanied by negative intensity in 1980 was caused by the victory of the Islamic Revolution in Iran in 1979, which resulted in another oil price increase. In 1981, Ronald Reagan became president. Reagan's presidency is associated with tax cuts and a decrease in other public budget revenues, which, however, were not matched by a commensurate reduction in public spending. The good entrepreneurial environment created by Reagan's policies was threatened by high inflation in the early part of his presidency. In 1982, restrictive monetary policy succeeded in bringing down inflation. For a short time, however, it induced a recession and negative intensity. Reaganomics continued for a short time after 1989 with George Bush as president. A slump in GDP and intensity occurred in 1991 when war erupted in Iraq. The decrease in intensity in 2001 is connected with the 9/11 attacks in New York and the stagnation around 2001 caused by the bursting of the technological bubble. The period of 2007–2009 was marked by a mortgage-related financial crisis. Years immediately following a crisis are always characterized by high intensity.

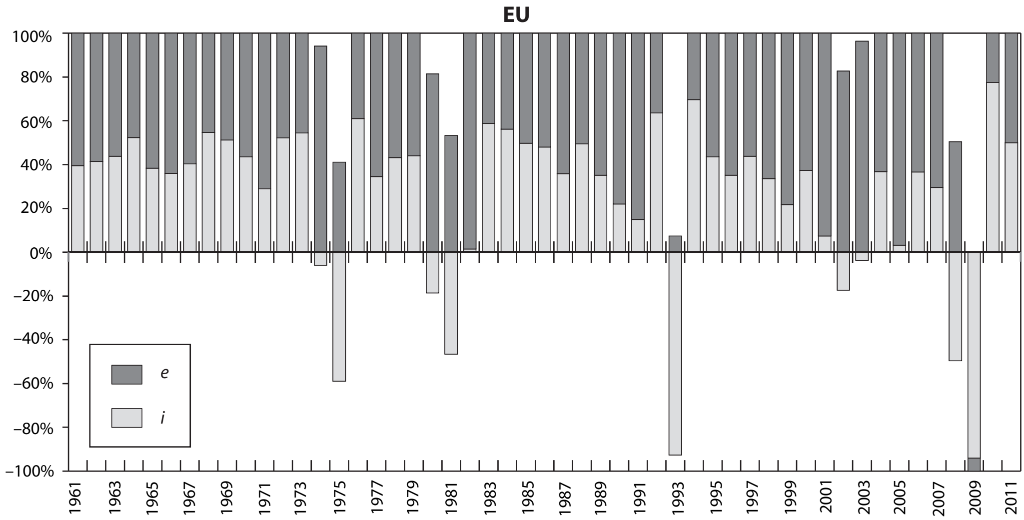
Figure 6 Intensity and extensity of development of the USA, 1961–2011



Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. *G*(*Y*), *G*(*L*) and *G*(*K*)

[16] Some attempts were made to restore the Bretton Woods system between 1971 and 1973, but they failed. For details see e.g. Scammel (1975).

Figure 7 Intensity and extensity of development of the EU15, 1961–2011



Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. *G*(*Y*), *G*(*L*) and *G*(*K*)

Figure 7 illustrates the development of the EU15. A comparison with Figure 6 clearly demonstrates that the development of the EU15 largely copies that of the USA. With the exception of the global crisis in 2009, there is no negative value of the dynamic parameter of extensity. The dynamic parameter of intensity was negative in only nine cases. The average annual *G*(*GDP*) rate of 5% in the 1960s was accompanied by high values of the dynamic parameter of intensity (25%–55%). This period ended with stagflation caused by the oil crisis in 1974 and 1975. As a consequence of the Arab-Israeli War in October 1973, Middle Eastern oil-producing countries increased their prices and restricted oil supplies to some European countries. This caused economic problems throughout the EU. The GDP decrease of 0.6% in 1975 was accompanied by a labour decrease of nearly 1%. The intensity parameter dropped to nearly –60%. The development that year was mainly intensive – intensity achieved 61% while extensity stood at 39%. The crisis in 1980 and 1981 reflects inefficient economic policy in some countries, for example the UK, which resulted in high rates of inflation and unemployment. (In the UK inflation reached 15% and unemployment 8%. These problems contributed to the victory of Margaret Thatcher in the 1979 general election.) A further cause of problems was the victory of the Islamic Revolution in Iran and the subsequent increase in oil prices. The post-crisis years of 1983 and 1984 are marked mainly by intensive growth of 59% and 57% respectively. The recession in 1993 was characterized by a GDP decrease of 0.4% amid almost purely dis-intensive[17] development with an intensity of –93%. This recession was a result of transformation processes in the EU. In 1992, the Maastricht Treaty establishing the European Union was signed. This represents the most important turning point in EU history. It stipulated rules for the future single currency, for foreign and security policy and for closer cooperation in the areas of justice and domestic affairs. Under the Treaty, the name 'European Community' was officially replaced by 'European Union'. In 1993, the single market was created and its four freedoms – free movement of goods, services, persons and capital – became reality. Since 1986, more than 200 legal rules have been issued, aiming to eliminate obstacles especially in the area of tax policy, business activity and profes-

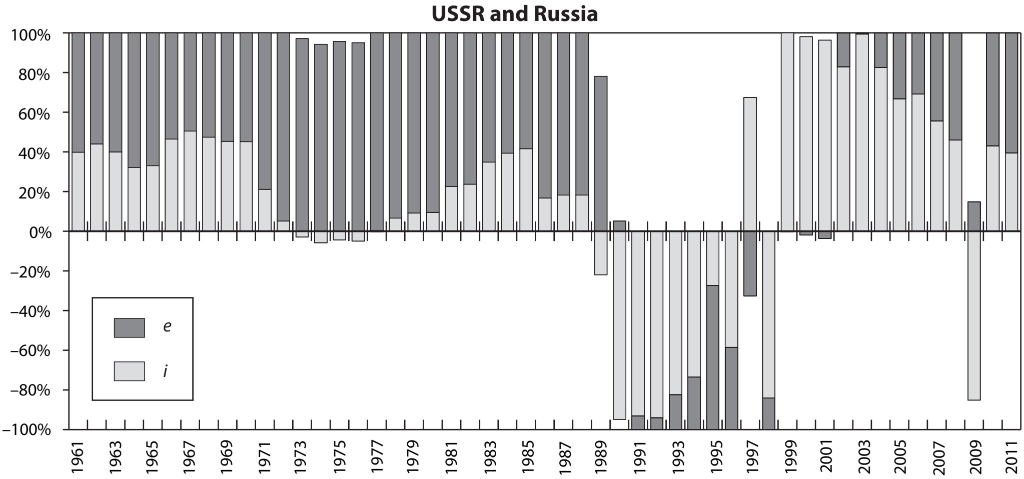
[17] Term "dis-intensive" means that the value of the dynamic parameter of intensity is negative.

sional qualifications. The implementation of free movement of some services, however, was delayed. In 2009, GDP decreased by 4.4%, with negative intensity of −94% and negative extensity of −6%. The world economy was affected by the global financial crisis. Problems started to arise because of bad mortgage loans in the USA. Several European banks also ran into difficulties. The crisis brought about closer economic cooperation between EU countries. It turns out that in the case of the EU15, our analytical tools respond well to the real course of events.

Generally, we observe that the dynamic parameters of intensity and extensity can, in the case of China, the USA and the EU15, describe real development well. What was the situation in the Soviet Union and in Russia after the dissolution? It is illustrated in Figure 8. The 1960s were characterized by steady GDP growth of about 5%, with intensity between 30% and 50%. The 1970s saw a continuous slight decrease in the rate of GDP growth from 4.6% to 2.2%. During the oil crisis starting in 1973, these decreasing rates of GDP growth were accompanied by negative intensity ranging between −2.9% and −5.8%. The period of 1977–1985 is characterized by steady GDP growth of about 2%, although with a continuous rise in intensity from 0.2% to 42%. From 1985 to 1991, Mikhail Gorbachev was the leader of the Soviet Union. This was a period of significant democratic reforms, which restricted consistent supervision of companies and improved business and other relations with the West. The dynamic parameter of intensity was positive until 1988, but lower than in the previous period. The unstable political environment was not conducive to technological progress. This fact is clearly visible in 1989 and 1990, when the dynamic parameter of intensity took negative values. This marked the start of the real dissolution of the USSR, which was accompanied by local armed conflicts. 1991 saw a plot aimed at toppling President Gorbachev. The USSR ceased to exist on 31 December 1991. The first period of Russian development during Boris Yeltsin's presidency (1992–1999) is characterized by an unconsolidated economy and a continuous recession from 1992 to 1996, with rates of growth of between −3.65% and −12.7% in 1994. GDP growth of 1.4% in 1997 was followed by another recession of −5.3% in 1998, caused by a financial crisis during which inflation soared to 84% and the rouble lost three quarters of its value.[18] The privatization and transformation from a central economy to a market society between 1992 and 1998 were related to the fact that the state only poorly fulfilled its basic functions such as law enforcement. The fact that state property was transferred without proper supervision and various groups of oligarchs and organized criminals emerged clearly had a negative impact on the parameter of intensity. The first presidency of Vladimir Putin (1999–2007), who succeeded in solving the aforementioned problems at least partially, was characterized by steady GDP growth of between 4.7% and 10% in 2000 (GDP increased eightfold in this period). 1999 is interesting in that purely intensive growth was recorded, with intensity of 100%. The whole of Putin's presidency saw mainly intensive growth, with intensity not dropping below 70%. Exports rose by 74% between 2000 and 2006. The country's accumulated debt fell from 60% of GDP in 2000 to only 7.9% of GDP in 2008. Steady GDP growth of between 4.3% and 8.5% was also recorded during Dmitry Medvedev's presidency (2007–2012), amid intensive-extensive development (intensity and extensity both at about 50%). The only exception was 2009, when GDP decreased by about −7.8%, with intensity of −85% and extensity of 15%. The results show that the dynamic parameters of intensity and extensity also proved their informative quality in the case of the USSR and Russia, even though the quality of the input data is debatable. Probably the most discussed period is 1978–1985, the relatively high intensity for which is out of step with the idea of a stagnant Brezhnev and post-Brezhnev USSR. The explanation might lie in the fact that in the case of the USSR, the rates of growth of the input indicators ($G(Y)$, $G(K)$ and $G(L)$) are estimated or calculated subsequently and might be overvalued.

[18] More detailed information about developments in Russia can be found in Hafner (2014, pp. 20–21).

Figure 8 Intensity and extensity of the development of the USSR and Russia, 1961–2011



Source: Authors' calculations based on year-on-year rates of growth of starting data, i.e. *G*(*Y*), *G*(*L*) and *G*(*K*)

CONCLUSION

This article presented a practical example of the analysis of the quality of GDP growth based on the application of intensive factors of development describing the knowledge society over a 50-year period (1961–2011). It turned out that the analysis of development quality can be successfully elaborated by applying a multiplicative aggregate production function where the total input factor is calculated as a weighted geometric mean of labour and capital. The international comparison presented in the article contains only the dynamic role.[19] To extend the analysis to include the static role, it would be necessary to obtain absolute data on the values of *K* and *L* or the national wealth of the relevant countries. The static role can answer the question of whether the current extensive development in the USA is a result of it having reached a high technical level in the past (i.e. before 1960).

Our example comparing the quality of annual development in the USA, China, Russia and the EU15 over the last 50 years demonstrates how much useful information can be obtained from time series of only three economic indicators (*G*(*Y*), *G*(*L*), *G*(*K*)). The analysis showed that China appears to be the most dynamically and intensively developing great power. In the last decade, Russia's development seems to have been very intensive as well. Given the above-mentioned facts, we believe that the method of measuring the effect of intensive and extensive factors on the development of output (GDP in our case) presented in the article can serve as an alternative to growth accounting. As for output growth versus input growth, the results of our method are going to be very similar. Moreover, our method allows the effect of intensive and extensive factors to be quantified even in cases of decreasing GDP and decreasing inputs.

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[19] The meanings of dynamic role and static role are explained in Mihola (2007b, p. 448).

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Constructions of the Average Rate of Return of Pension or Investment Funds Based on Chain Indices

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Abstract

In this paper we consider the problem of the proper construction of the average rate of return of pension (or investment) funds. We refer to some economical postulates given by Gajek and Kaluszka (2000). We present, discuss and compare several measures of the average rate of return of funds. We also present alternative measures based on original chain indices. We take into consideration discrete and continuous time stochastic models.

Keywords	***JEL code***
Average rate of return, stochastic process, martingale, chain indices	*C43, G12, G23*

INTRODUCTION

Open pension funds and investment funds are institutions that should invest their client's money in the most effective way. There is a number of measures for the efficiency of these investments (see Domański et al., 2011; Białek, 2008). The measures should be well defined – it means that all changes of fund's assets, connected with any investment, should have impact on the given measure. The information about the average return of the group of funds is very important both for fund clients and fund managers. Firstly, it allows to compare the result of the given fund to the rest of funds. It may be helpful to clients in making a decision about money allocation. Secondly, having the knowledge about the average returns of investment funds from different sectors (manufacturing, agricultural, service etc.) we have some information about the financial situation within these sectors. And finally, in the case of pension funds we can find law regulations defining the *minimal rate of return* of funds based on the average rate of return. For example, in the Polish law regulations (The Law on Organization and Operation of Pension Funds, Art. 173, Dziennik Ustaw Nr 139 poz. 934, Art. 173; for the English translation see Polish Pension…, 1997) the half of the average return of a group of funds or the averege return minus four percentage points (depending on which of these values is higher) determined (till February 2014) a minimal rate for any pension fund. In the case of deficit the weak fund had to cover it. It was always a very dangerous situation for this fund.[2] In the Polish law the following definition of the average return of a group of pen-

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[2] In Poland, in 2001 and 2002 Bankowy Fund did not reach the minimal rate of return.

sion funds could be found (only from 1st February 2014 the new law regulation has been in effect and according to which there is no need to calculate the minimal rate of return of funds and the average return - see Dziennik Ustaw 2013, poz. 1717):

$$\bar{r}_0(T_1,T_2)=\sum_{i=1}^{n}\frac{1}{2}r_i(T_1,T_2)\cdot\Big(\frac{A_i(T_1)}{\sum_{i=1}^{n}A_i(T_1)}+\frac{A_i(T_2)}{\sum_{i=1}^{n}A_i(T_2)}\Big), \quad (1)$$

where $r_i(T_1,T_2)$ denotes the rate of the i – th fund during a given time period $[T_1,T_2]$ and $A_i(t)$ denotes the value of i – th fund's assets at time t. Since 2004 till the February 2014 the results of funds for the last 36 months had been verified twice a year. Unfortunately, the measure defined in (1) does not satisfy some economic postulates given by Gajek and Kałuszka (2000). Moreover, considering an even number of funds, where half of them have the return rates equal to 50% and the rest of funds have the return rates equal to (–50%), we should get the real average return rate on the level 0%. But using formula (1) we get 12.5%. In our opinion, this in an argument for searching new definitions of the avarege rate of return of a group of funds.

1 POSTULATES FOR THE AVERAGE RATE OF RETURN

At the first sight the problem of constructing the avarege rate of return seems to be straightforward. But if we look at postulates coming from Gajek and Kałuszka (2000), which are quite natural and economical legitimate, we have to verify this opinion. Let us denote by $p_i(t)$ th value of the participation unit of the i – th fund at time t, and $q_i(t)$ - the number of units of the i – th fund at time t. Below we present and disccuss the postulates for the average rate of return of a group of funds ($\bar{r}(T_1,T_2)$).

Postulate 1

In the case when the group consists of one fund ($n = 1$) then:

$$\bar{r}(T_1,T_2)=r_1(T_1,T_2), \quad (2)$$

where: $$r_1(u,u+1)=\frac{p_1(u+1)-p_1(u)}{p_1(u)}. \quad (3)$$

Postulate 2

If all funds have the same values of their accounting units all the time, i.e.

$$p_i(t) = p_j(t), \text{ for } i \neq j,\ t \in [T_1,T_2], \quad (4)$$

then it holds:

$$\bar{r}(T_1,T_2)=\frac{p_1(T_2)-p_1(T_1)}{p_1(T_1)}. \quad (5)$$

It means that if the unit's value changes in time in the same way in all funds then it does not matter if the clients allocate from one fund to another or where the newcomers place themselves; their individual return rates will always be the same.

Postulate 3

If the number of units of every fund is constant during the time interval $[T_1,T_2]$, then:

$$\bar{r}(T_1,T_2)=\frac{\sum_{i=1}^{n}A_i(T_2)-\sum_{i=1}^{n}A_i(T_1)}{\sum_{i=1}^{n}A_i(T_1)}. \quad (6)$$

In fact, when none of the clients change the fund or come into or out of the business, then any change of assets of the i – th fund reflects only the investment results of the i – th fund. Moreover, postulate 3 implies also postulate 3', namely:

Postulate 3'

Under assumptions from postulate 3, if the initial assets at time $t = T_1$ of every fund are the same and for some k ≤ $n/2$ it holds $r_1 = -r_{k+1}$, $r_2 = -r_{k+2}$,..., $r_k = -r_{2k}$, $r_{2k+1} = 0$,..., $r_n = 0$ then

$$\bar{r}(T_1,T_2) = 0. \tag{7}$$

Postulate 4

For every $t \in [T_1,T_2]$ it shoul hold:

$$1 + \bar{r}(T_1,T_2) = [1 + \bar{r}(T_1, t)][1 + \bar{r}(t, T_2)]. \tag{8}$$

Postulate 4 is a multiplication rule that says that the average rate of return since T_1 until T_2 should equal the average return since t until T_2, given the average return since T_1 until t. Let us notice that the individual rate of return defined in (3) satisfies postulate 4.

Postulate 5

Let us assume that i – th fund obtains the highest return rates and the k – th fund obtains the lowest raturn rates on each time interval $[t,t+1] \subseteq [T_1,T_2]$. Then we should observe:

$$r_k(T_1,T_2) \leq \bar{r}(T_1,T_2) \leq r_i(T_1,T_2). \tag{9}$$

Postulate 5 means that the average return rate is not greater than the rate corresponding to the case when all clients allocate at each $t \in [T_1,T_2]$ to the fund obtaining the highest return rate and not lower than the rate corresponding to the case in which all clients allocate to the fund obtaining the lowest return rate.

Postulate 6

If for some $k \in \{1,2,..., n\}$ it holds

$$\max_{i \neq k} A_i(t) \leq \theta A_k(t) \text{ for any } t \in [T_1,T_2], \tag{10}$$

then we observe

$$\lim_{\theta \to 0} \bar{r}(T_1,T_2) = \frac{p_k(T_2) - p_k(T_1)}{p_k(T_1)}. \tag{11}$$

Postulate 6 means that the influence of small funds (with small asstets) on the average return is negligible.

Postulate 7[3]

If funds are grouped and if the average rate of return of each group is calculated over the time interval $[t,t+1] \subseteq [T_1,T_2]$, then the average rate of return of groups equals to the average rate of return of all funds over the the time interval .

[3] In the original paper of Gajek and Kałuszka (2002) authors treat the postulate 7 as one of properties of the proposed average rate of return. In our opinion it has an axiomatic character and should be treated as a postulate.

Remark 1

The above postulates decribe partly a kind of economical intuition and partly mathematical consistency of any good definition of a weighted average rate of return of a group of pension or investment funds. For example the Polish definition presented in (10) does not satisfy postulates 3, 3', 4 and 7 (the proof is easy and thus omitted, see the example 1). But the construction of a proper definition of the average rate of return is not obvious. For instance, even the well known and popular Value Line Composite Index[4] (VLIC index) defined as:

$$VLIC(T_1,T_2) = [(1+r_1(T_1,T_2))(1+r_2(T_1,T_2)\ \cdot \ldots \cdot (1+r_n(T_1,T_2))]^{\frac{1}{n}} - 1 = \Big[\prod_{i=1}^{n} \frac{p_i(T_2)}{p_i(T_1)}\Big]^{\frac{1}{n}} - 1, \quad (12)$$

does not satisfy postulates 3, 3', 6 and 7 (see the example 1). Let us also notice that the *VLIC* formula can be obtained as a value of the unweighted Jevons' index minus one.

Example 1

We show that measures defined in (1) and (12) does not satisfy postulate 7. Let us consider $n = 5$ funds with the same value of asstes during the time interval $[T_1, T_2]$ and their results as follows:

$r_1(T_1,T_2) = 0{,}05$, $r_2(T_1,T_2) = 0{,}07$, $r_3(T_1,T_2) = 0{,}12$, $r_4(T_1,T_2) = -0{,}03$, $r_5(T_1,T_2) = 0$.

We get for the whole group of funds:

$$\bar{r}_0(T_1,T_2) = \frac{1}{5}\sum_{i=1}^{5} r_i(T_1,T_2) = 0.042,$$

$$VLIC(T_1,T_2) = [\prod_{i=1}^{5}(1+r_i(T_1,T_2)]^{\frac{1}{5}} - 1 = 0.040.$$

Let us assume that funds 1 and 2 are in the first group (I), and funds 3, 4 and 5 are in the second group (II). After calculations we get the following results for groups:

$$\bar{r}_0^{I}(T_1,T_2) = \frac{1}{2}(r_1(T_1,T_2) + r_2(T_1,T_2)) = 0.06,$$

$$\bar{r}_0^{II}(T_1,T_2) = \frac{1}{3}(r_3(T_1,T_2) + r_4(T_1,T_2) + r_5(T_1,T_2)) = 0.03,$$

$$VLIC^{I}(T_1,T_2) = \sqrt{(1+r_1(T_1,T_2))(1+r_2(T_1,T_2))} - 1 = 0.060,$$

$$VLIC^{I}(T_1,T_2) = \sqrt[3]{(1+r_1(T_1,T_2))(1+r_2(T_1,T_2))(1+r_3(T_1,T_2))} - 1 = 0.028.$$

Now, let us calculate the average rate of return for joined groups:

$$\bar{r}_0^{I+II}(T_1,T_2) = \frac{1}{2}\cdot(\bar{r}_0^{I}(T_1,T_2)\cdot\frac{4}{5} + \bar{r}_0^{I}(T_1,T_2)\cdot\frac{6}{5}) = 0.040 \neq \bar{r}_0(T_1,T_2),$$

$$VLIC^{I+II}(T_1,T_2) = \sqrt{(1+VLIC^{I}(T_1,T_2))(1+VLIC^{II}(T_1,T_2))} - 1 = 0.043 \neq VLIC(T_1,T_2).$$

Thus, neither $\bar{r}_0$ nor *VLIC* satisfies the postulate 7.

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[4] This index containing approximately 1675 companies from the NYSE, American Stock Exchange, and Nasdaq.

In the next part of this paper we consider discrete and continuous time stochastic models and present several definitions of the average of return that fulfill postulates 1–7.

2 PROPOSITIONS OF THE AVERAGE RATE OF RETURN IN A DISCRETE TIME STOCHASTIC MODEL

2.1 Significations and assumptions

Let us consider a group of n pension or investment funds that start their activity selling accounting units at the same price. We observe them in discrete time moments $\{t = 0,12,....\}$. Let us define a probability space $(\Omega, \Im, P)$. Let $F = \{\Im_t : t = 0,1,2,...\}$ be a filtration, i.e. each $\Im_t$ is an σ – algebra of Ω with $\Im_0 \subseteq \Im_s \subseteq \Im_t \subseteq \Im$ for any $s < t$. Without loss of generality, we assume $\Im_0 = \{\emptyset, \Omega\}$. The filtration F describes how the information about the market is revealed to the observer. We consider the following state-variables:

$p_i(t)$ – value of the participation unit of the i – th fund at time t,

$q_i(t)$ – number of units of the i – th fund at time t,

$A_i(t) = k_i(t)w_i(t)$ – value of i – th fund's assets at time t,

$A(t) = \sum_{i=1}^{n} A_i(t)$,

$A^*_i(t) = A_i(t) / A(t)$ – the percentage of a relative value of assets of the i – th fund at time t.

We assume that:

- All investments are infinitely divisible.
- There are no transaction costs or taxes and the assets pay no dividends.
- Member does not pay for allocation of his/her wealth.
- There is no consumption of funds.

The presented, technical assumptions make the mathematical transformations easier but the assumptions do not influence the general character of the discussion. The presented research on real data shows that there are still some benefits of using the proposed measures although some of the assumptions can not be satisfied (for example a member can pay for allocation of his/her wealth). Thus the properties of the discussed measures do not depend on the above assumptions.

Here and subsequently, the symbol $X = Y$ means that the random variables X,Y are defined on $(\Omega, \Im, P)$ and $P(X = Y) = 1$. We assume that each $p_i(t)$ and $q_i(t)$ is adapted to $F = \{\Im_t : t = 0,1,2,...\}$ which means that each $p_i(t)$ and $q_i(t)$ is measurable with respect to $\Im_t$. Next we consider some time interval of observations given by $[T_1,T_2]$.

2.2 The measure of Gajek and Kałuszka and its connection with chain indices

Under the above assumptions and symbols Gajek and Kałuszka (2001) proposed the following definition of the average rate of return of a group of funds:

$$\bar{r}_{GK}(T_1,T_2) = \prod_{t=T_1}^{T_2-1}(1+\sum_{i=1}^{n} A_i^*(t) r_i(t,t+1)) - 1. \quad (13)$$

The definition (13) satisfies all the economic postulates 1–7 (see Gajek, Kałuszka, 2001). In the mentioned paper the authors proved also the following theorems.

Theorem 1

If the number of units of each of the fund is constant on the time interval then we have:

$$\bar{r}_{GK}(t,t+1) \leq \bar{r}_0(t,t+1) \quad (14)$$

and in the natural case of:

$$\exists i,j\ \frac{p_i(t+1)}{p_i(t)} \neq \frac{p_j(t+1)}{p_j(t)}, \tag{15}$$

we obtain:

$$\bar{r}_{GK}(t,t+1) < \bar{r}_0(t,t+1). \tag{16}$$

The inequality (16) suggests that the average return defined in the Polish law overestimates the real average rate of return of a group of funds.

Theorem 2

If $\{p_i(t) : t = 0,1,2,....\}$ is an F – martingale[5] for each i, then $\{\bar{r}_{GK}(0,t) : t = 0,1,2,...\}$ is also an F – martingale. Moreover, in case when $\{p_i(t) : t = 0,1,2,....\}$ is an F – submartingale (resp. F – supermartingale) for each i, then $\{\bar{r}_{GK}(0,t) : t = 0,1,2,...\}$ is an F – submartingale (resp. F – supermartingale).

Remark 2

The average rate of return defined in the Polish law ($\bar{r}_0$) in general is not a martingale provided the values of units are martingales (see Gajek and Kałuszka, 2001).

In this part of the paper we treat the group of fund as some aggregate that contains n commodities (funds) with prices $p_i(t)$ and quantities $q_i(t)$, where $t \in [T_1,T_2]$. Let us denote by $P^L(t,t+1)$ the Laspeyres price index defined as follows (see von der Lippe, 2007):

$$P^L(t,t+1) = \frac{\sum_{i=1}^{n} q_i(t)p_i(t+1)}{\sum_{i=1}^{n} q_i(t)p_i(t)}. \tag{17}$$

Let us notice that the definition (13) can be written with the use of the Laspeyres chain index $\overline{P}^{LC}$. In fact we have (see Białek, 2011):

$$\begin{aligned}\overline{P}^{LC}(T_1,T_2)-1 &= \prod_{t=T_1}^{T_2-1} P^L(t,t+1) - 1 = \prod_{t=T_1}^{T_2-1} \frac{\sum_{i=1}^{n} q_i(t)p_i(t+1)}{\sum_{i=1}^{n} q_i(t)p_i(t)} - 1 = \\ &= \prod_{t=T_1}^{T_2-1}\Big(\sum_{i=1}^{n}\frac{q_i(t)p_i(t)}{\sum_{i=1}^{n} q_i(t)p_i(t)}\cdot\frac{p_i(t+1)}{p_i(t)}\Big)-1 = \prod_{t=T_1}^{T_2-1}\Big(1+\sum_{i=1}^{n}\frac{q_i(t)p_i(t)}{\sum_{i=1}^{n} q_i(t)p_i(t)}\cdot\frac{p_i(t+1)-p_i(t)}{p_i(t)}\Big)-1 = \\ &= \prod_{t=T_1}^{T_2-1}(1+\sum_{i=1}^{n} A_i^*(t)r_i(t,t+1))-1 = \bar{r}_{GK}(T_1,T_2).\end{aligned} \tag{18}$$

The question is whether we can use another chain indices to obtain the well-constructed average rate of return of funds. The answer is positive and we present such definitions in the next part of this paper.

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5 It implies that $E(p_i(t)) = const$, where $E(X)$ means the expected value of random variable X.

2.3 A general formula of the average return rate and its special cases

According to presented postulates it can be shown that the proper definition of the average rate of return of funds can be written as some chain price index minus one, namely:

$$\bar{r}(T_1,T_2)=\prod_{t=T_1}^{T_2-1}P(t,t+1)-1\,, \tag{19}$$

where the general form of the price index $P(t,t+1)$ is as follows:

$$P(t,t+1)=\prod_{i=1}^{n}\left(\frac{p_i(t+1)}{p_i(t)}\right)^{w_i(A_i^*(t),A_i^*(t+1))}. \tag{20}$$

The weights w_i used in (20) are positive and sum up to one since

$$w_i(A_i^*(t),A_i^*(t+1))=\frac{M(A_i^*(t),A_i^*(t+1))}{\sum_{i=1}^{n}M(A_i^*(t),A_i^*(t+1))}, \tag{21}$$

where $M(x,y)$ is some type of (weighted) mean of variables x and y (arithmetic, geometric, exponential, etc.).

Remark 3

Let us assume that $M(A_i^*(t),\ A_i^*(t+1))=A_i^*(t)$. Then from (19) and (20) we obtain:

$$\bar{r}(T_1,T_2)=\prod_{t=T_1}^{T_2-1}\prod_{i=1}^{n}\left(\frac{p_i(t+1)}{p_i(t)}\right)^{A_i^*(t)}-1=\prod_{t=T_1}^{T_2-1}\exp\left(\sum_{i=1}^{n}A_i^*(t)\ln\frac{p_i(t+1)}{p_i(t)}\right)-1=\bar{r}_B(T_1,T_2)\,, \tag{22}$$

where $\bar{r}_B(T_1,T_2)$ means the avarege rate of return proposed and discussed in the paper of Białek (2008). Let us notice that in this case the $P(t,t+1)$ formula is a logarithmic Laspeyres price index (see von der Lippe, 2007). Taking $M(A_i^*(t),\ A_i^*(t+1))=A_i^*(t)$ we get the measure $\bar{r}_{LP}(T_1,T_2)$ based on the logarithmic Paasche price index (see von der Lippe (2007)). If we assume $M(A_i^*(t),\ A_i^*(t+1))=A_i^*(t)+A_i^*(t+1))/2$ then we can express the average rate of return by the Törnqvist chain price index, namely we obtain:

$$\bar{r}(T_1,T_2)=\bar{r}_T(T_1,T_2)=\prod_{t=T_1}^{T_2-1}P^T(t,t+1)-1\,, \tag{23}$$

where Törnqvist price index is defined for moments (as basis) and as follows (see Balk and Diewert, 2001):

$$P^T(t,t+1)=\prod_{i=1}^{n}\left(\frac{p_i(t+1)}{p_i(t)}\right)^{\frac{1}{2}(A_i^*(t)+A_i^*(t+1))}. \tag{24}$$

Remark 4 (The next step of generalization)

Let us define for any $x, y \in [0,1]$

$$A_i^x(t)=\frac{p_i(t)q_i^{1-x}(t)q_i^x(t+1)}{\sum_{i=1}^{n}p_i(t)q_i^{1-x}(t)q_i^x(t+1)}, \tag{25}$$

$$A_i^y(t+1) = \frac{p_i(t+1)q_i^{1-y}(t)q_i^y(t+1)}{\sum_{i=1}^{n} p_i(t+1)q_i^{1-y}(t)q_i^y(t+1)} . \tag{26}$$

Let us assume that $\widetilde{M}(x, y)$ denotes the logaritmic mean defined for positive arguments as follows:

$$\widetilde{M}(x, y) = \frac{x - y}{\ln x - \ln y} , \tag{27}$$

if $x \neq y$, and $\widetilde{M}(x, y) = x$ if $x = y$ (see Carlson, 1972).

Let us define the geo-logarithmic family as the class of price indices P_{xy} defined by (see Fattore, 2010):

$$P_{xy}(t,t+1) = \prod_{i=1}^{n} \left(\frac{p_i(t+1)}{p_i(t)}\right)^{\widetilde{w}_i(A_i^x(t), A_i^y(t+1))} , \tag{28}$$

where:

$$\widetilde{w}_i(A_i^x(t), A_i^y(t+1)) = \frac{\widetilde{M}(A_i^x(t), A_i^y(t+1))}{\sum_{i=1}^{n} \widetilde{M}(A_i^x(t), A_i^y(t+1))} . \tag{29}$$

From the axiomatic point of view the general formula (28) is well-constructed. Geo-logarithmic price indices satisfy for example the proportionality, the commensurability or the homogeneity (see Fattore, 2010). In the mentioned paper the author proves that an element of the P_{xy} family is monotonic if and only if $x = y$. It is very interesting that in this case, when just $x = y$, we obtain (see Martini, 1992):

$$P_{xx}(t,t+1) = \frac{\sum_{i=1}^{n} p_i(t+1)q_i^{1-x}(t)q_i^x(t+1)}{\sum_{i=1}^{n} p_i(t)q_i^{1-x}(t)q_i^x(t+1)} . \tag{30}$$

Let us notice that the formula (28) corresponds to the formula (20). In a similar way to (19) we define:

$$\bar{r}_{xy}(T_1,T_2) = \prod_{t=T_1}^{T_2-1} P_{xy}(t,t+1) - 1 = \prod_{t=T_1}^{T_2-1}\prod_{i=1}^{n} \left(\frac{p_i(t+1)}{p_i(t)}\right)^{\widetilde{w}_i(A_i^x(t), A_i^y(t+1))} - 1. \tag{31}$$

It is an interesting, general formula of the average rate of return of funds. Let us notice that from (30) we get that P_{00} is the Laspeyres price index, P_{11} is the Paasche price index and $P_{0.50.5}$ is the Walsh price index (see Białek, 2012). Thus, the $\bar{r}_{00}$ measure is based on the Laspeyres chain index, the $\bar{r}_{11}$ formula is based on the Paasche chain index and the $\bar{r}_{0.50.5}$ measure is based on the Walsh chain index. Let us denote two last formulas by $\bar{r}_P(T_1,T_2)$ and $\bar{r}_W(T_1,T_2)$. The formula $\bar{r}_{00}$ does not need any additional signification since we have:

$$\bar{r}_{00}(T_1,T_2) = \prod_{t=T_1}^{T_2-1} P^L(t,t+1) - 1 = \bar{r}_{GK}(T_1,T_2) . \tag{32}$$

2.4 Comparison of measures $\bar{r}_0$, $\bar{r}_{GK}$ and $\bar{r}_B$

As we know, the process $\{\bar{r}_{GK}(0,t) : t = 0,1,2,...\}$ is a F – martingale provided the processes of prices are also martingales (see Theorem 2). As it was mentioned, the Polish formula $\bar{r}_0(0,t)$ in general does not have this property. In fact, let us consider a group that consists of only $n = 2$ funds. Let us assume $q_1(t) = q_2(t) = q$ and $p_1(0) = p_2(0) = 1$. From (1) we have:

$$\bar{r}_0(0,t)=\sum_{i=1}^{2}\frac{1}{2}r_i(0,t)\cdot\Big(\frac{A_i(0)}{\sum_{i=1}^{n}A_i(0)}+\frac{A_i(t)}{\sum_{i=1}^{n}A_i(t)}\Big)=\sum_{i=1}^{2}\frac{1}{2}(p_i(t)-1)\Big(\frac{1}{2}+\frac{p_i(t)}{p_1(t)+p_2(t)}\Big)=$$

$$=\frac{1}{2}\Big(\frac{1}{2}p_1(t)+\frac{p_1^2(t)}{p_1(t)+p_2(t)}-\frac{p_1(t)}{p_1(t)+p_2(t)}+\frac{1}{2}p_2(t)+\frac{p_2^2(t)}{p_1(t)+p_2(t)}-\frac{p_2(t)}{p_1(t)+p_2(t)}-1\Big)= \quad (33)$$

$$=\frac{1}{4}(p_1(t)+p_2(t))+\frac{1}{2}\frac{p_1^2(t)+p_2^2(t)}{p_1(t)+p_2(t)}-1$$

Let as assume naturally that $P(p_1(t)=p_2(t))<1$ for any $t>0$, which leads to

$$(p_1(t)-p_2(t))^2>0, \quad (34)$$

and equivalently[6]

$$2(p_1^2(t)+p_2^2(t))>p_1(t)+p_2(t))^2. \quad (35)$$

From (33) and (35) we get:

$$E(\bar{r}_0(0,t)=\frac{1}{4}(E(p_1(t))+E(p_2(t)))+\frac{1}{2}E\Big(\frac{p_1^2(t)+p_2^2(t)}{p_1(t)+p_2(t)}\Big)-1>$$

$$>\frac{1}{4}E(p_1(t))+\frac{1}{4}E(p_2(t))+\frac{1}{4}E\Big[\frac{(p_1(t)+p_2(t))^2}{p_1(t)+p_2(t)}\Big]-1=\frac{1}{2}E(p_1(t))+\frac{1}{2}E(p_2(t))-1. \quad (36)$$

Let us notice that in this case, even if $p_1(t)$ and $p_2(t)$ are martingales the average of rate of return is not a martingale. In fact, then we have $E(p_i(t))=E(p_i(0))=1$, but from (36) we obtain:

$$E(\bar{r}_0(0,t))>0=E(\bar{r}_0(0,0)), \quad (37)$$

which confirms that the process $\{\bar{r}_0(0,t):t=0,1,2,...\}$ can not be a martingale (its expected value is not constant in time). The next theorem gives us a condition that allows us to treat the stochastic process $\{\bar{r}_B(0,t):t=0,1,2,...\}$ as a martingale (see Białek, 2005).

Theorem 3

If $\{p_i(0,t):t=0,1,2,...\}$ is a $F-$ martingale, for each i and with the probability one we have:

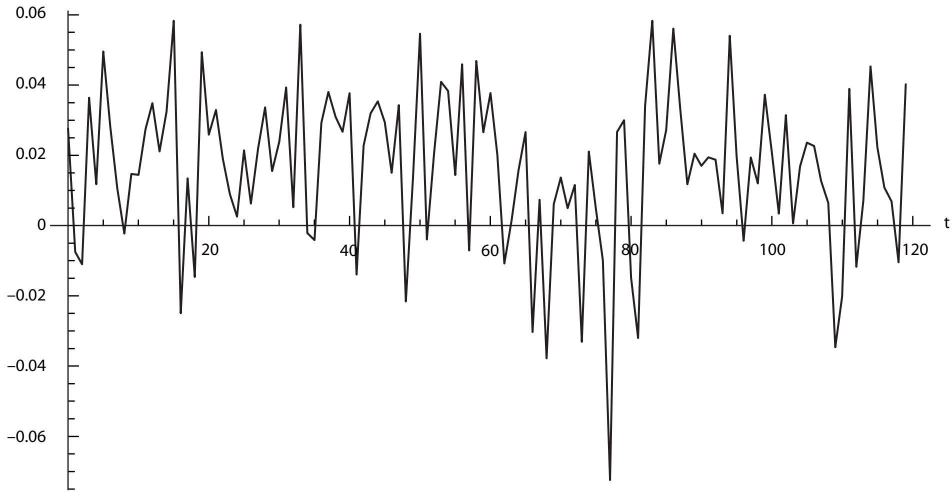
$$\lambda(t,t+1)\overset{def}{=}\sum_{i=1}^{n}A_i^*(t)\ln\frac{p_i(t+1)}{p_i(t)}\geq 0, \text{ for any } t, \quad (38)$$

then $\{\bar{r}_B(0,t):t=0,1,2,...\}$ is also a $F-$ martingale.

The assumption (38) means that, in general, taking into consideration time intevals we observe (within the group of funds) more rises in prices than drops. Thus, during the financial crisis the process may not be a martingale since then it is difficult to fulfill (38). However, in the time of prosperity, as a rule the assumption (38) is satisfied (see Figure 1).

[6] This is a special case of the known enaquality: $(x_1+x_2+...+x_n)^2\leq n(x_1^2+x_2^2+...+x_n^2)$.

Firure 1 Function $\lambda(t,t+1)$ for the case of group of open pension funds in Poland and time interval 06/2002–
–06/2012 *)



*) We consider monthly data and the financial crisis in Poland was (approximately) the strongest for $t \in [62, 83]$.
Source: Own calculations in Mathematica 6.0

The theorem 4 shows some relation between the discussed measures (see Białek, 2005).

Theorem 4
With probability one we have:

$$\bar{r}_B(T_1, T_2) \le \bar{r}_{GK}(T_1, T_2), \tag{39}$$

and if $p_i(t+1) \approx p_i(t)$ for each i and $t \in [T_1, T_2]$, then $\bar{r}_B(T_1, T_2) \approx \bar{r}_{GK}(T_1, T_2)$.

Remark 5
Let us define two random variables $\hat{P}$ and $\hat{Q}$ as follows:

$$\hat{P} = \frac{p_J(t+1)}{p_J(t)}, \tag{40}$$

$$\hat{Q} = \frac{q_J(t+1)}{q_J(t)}, \tag{41}$$

where J is a random variable with distribution

$$P(J=j) = A^*_i(t), j = 1,2,..., n. \tag{42}$$

In the paper of Gajek and Kałuszka (2002) authors prove that:

$$\bar{r}_0(t,t+1) - \bar{r}_{GK}(t,t+1) = \frac{Cov(\hat{P}\hat{Q}, \hat{P})}{2E(\hat{P}\hat{Q})}. \tag{43}$$

Let us notice that in the case $P(\tilde{Q} = const) = 1$ we obtain from (43)

$$\bar{r}_0(t,t+1) - \bar{r}_{GK}(t,t+1) = \frac{Var(\hat{P})}{2(1+\bar{r}_{GK}(t,t+1))} \geq 0\,. \tag{44}$$

From (44) and Theorem 4 we get the following conclusion:

$$\bar{r}_0(t,t+1) \geq \bar{r}_{GK}(t,t+1) \geq \bar{r}_B(t,t+1), \tag{45}$$

and if $p_i(t+1) \approx p_i(t)$ that means $Var(\hat{P}) \approx 0$ we get:

$$\bar{r}_0(t,t+1) \approx \bar{r}_{GK}(t,t+1) \approx \bar{r}_B(t,t+1). \tag{46}$$

The assumption $P(\hat{Q} = const) = 1$ seems to be rather unnatural. In practice, the relatve increment of the number of units of each fund should be proportional to the relative increment of the value of unit, i.e.

$$(\hat{Q} = f(\hat{P}), \tag{47}$$

where $f: R_+ \rightarrow R_+$ is some nondecreasing function.

In the case of (47) we have (see Gajek and Kałuszka, 2002):

$$\bar{r}_0(t,t+1) - \bar{r}_{GK}(t,t+1) = \frac{E(\hat{P}^2 f(\hat{P})) - E(\hat{P}f(\hat{P}))E(\hat{P})}{E(\hat{P}f(\hat{P}))}\,. \tag{48}$$

From the following inequality for nondecreasing functions (see Mitrinovic et al., 1993):

$$E(\hat{P}^2 f(\hat{P})) \geq E(\hat{P}f(\hat{P}))E(\hat{P})\,, \tag{49}$$

from (48) we obtain again:

$$\bar{r}_0(t,t+1) - \bar{r}_{GK}(t,t+1) \geq 0. \tag{50}$$

Thus the formula $\bar{r}_0$ seems to overestimate the real value of the average rate of return.

2.5 Empirical study

Let us consider a group of $n = 14$ Polish open pension funds[7] and time interval of their observations: 06/2002–06/2012. Having monthly data on their numbers of clients and prices of units ($N = 120$ observations) we calculate the discussed measures of the average rate of return for several time intervals from the given period. Our results are presented in Table 1.

Table 1 Considered average rates of return for some time intervals from the period 06/2002–06/2012

Time interval	Measure of the average rate of return [%]						
	$\bar{r}_0(1,12)$	$\bar{r}_{GK}(1,12)$	$\bar{r}_B(1,12)$	$\bar{r}_p(1,12)$	$\bar{r}_{LP}(1,12)$	$\bar{r}_T(1,12)$	$\bar{r}_W(1,12)$
[1, 24]	24.38	24.33	24.32	24.33	24.35	24.33	24.33
[1, 48]	63.42	63.32	63.29	63.33	63.36	63.33	63.33
[1, 72]	83.14	82.96	82.91	82.96	83.00	82.95	82.95
[1, 120]	103.51	103.25	103.17	103.24	103.32	103.25	103.25
[30, 90]	41.41	41.41	41.38	41.40	41.43	41.41	41.41
[60, 120]	6.45	6.37	6.35	6.37	6.38	6.37	6.37

Source: Own calculations in Mathematica 6.0 based on data from <www.parkiet.pl>.

7 Here is the list of open pension funds in Poland in 2012: AIG, Allianz, Bankowy, Aviva, AXA, WARTA, AEGON, Generali, ING, Pekao, Pocztylion, Polsat, PZU, Nordea..

As we can notice, as a rule the Polish measure $\bar{r}_0$ has the highest value (the case of time interval [30, 90] is an exception) and the measure $\bar{r}_B$ has the smallest value (see also the simulation study). This observation seems to confirm the thesis of Theorem 4 and the conclusion from Remark 5. In fact, the Polish formula seems to overestimate the real value of the average rate of return. As it was mentioned, in the Polish law regulations the half of the average return of a group of funds or the average return minus four percentage points (depending on which of these values is higher) determines a minimal rate for any pension fund. In the case of deficit the *weak* pension fund has to cover it and thus it is always a very dangerous situation for this fund. Thus, from the funds' point of view, the definition $\bar{r}_B$ is "the safest". Nevertheless, there is a little difference in values of discussed measures in our research. It is easy to explain this fact because Polish pension funds invest in a very similar way. In other words, the criterion of the mimimal rate of return does not motivate funds to invest more efficiently and thus, funds have very similar portfolios. In such a situation the presented measures of the average return approximate each other (see Postulate 2).

2.6 Simulation study

Let us take into consideration a group of $n = 6$ funds observed at moments $t = 1,2,...,12$ and the following prices of units and numbers of units processes:

$$p_i(t) \sim N(\mu_i(t), \sigma_i(t)),\ i = 1,2,...,6,$$

$$q_i(t) \sim N(\hat{\mu}_i(t), \hat{\sigma}_i(t)),\ i = 1,2,...,6,$$

where $X \sim N(\mu, \sigma)$ denotes a random variable X with a normal (Gaussian) distribution with a mean μ and a standard deviation σ. In our experiment we consider the following functions:

$$\mu_1(t) = 10 + 0.5t,\ \sigma_1(t) = 3,\ \hat{\mu}_1(t) = 200 + 5t,\ \hat{\sigma}_1(t) = 30,$$
$$\mu_2(t) = 100,\ \sigma_2(t) = 10 + t,\ \hat{\mu}_2(t) = 10 - 0.05t,\ \hat{\sigma}_2(t) = 1,$$
$$\mu_3(t) = 20,\ \sigma_3(t) = 2,\ \hat{\mu}_3(t) = 1000 + 50t,\ \hat{\sigma}_3(t) = 30 + 10t,$$
$$\mu_4(t) = 200 - t,\ \sigma_4(t) = 10 + t,\ \hat{\mu}_4(t) = 500 + 5t,\ \hat{\sigma}_4(t) = 30 - t,$$
$$\mu_5(t) = 10,\ \sigma_5(t) = 1 + 0{,}01t,\ \hat{\mu}_5(t) = 100,\ \hat{\sigma}_5(t) = 15,$$
$$\mu_6(t) = 50 + 3t,\ \sigma_4(t) = 4,\ \hat{\mu}_6(t) = 500,\ \hat{\sigma}_4(t) = 30 + 2t.$$

After calculations for k = 10 000 realizations of prices and numbers of units processes we get results presented in Table 2.

Table 2 Basic characteristics of the considered average rates of return for the time interval [1, 12]

Parameter	Measure of rate of return [%]						
	$\bar{r}_0(1,12)$	$\bar{r}_{GK}(1,12)$	$\bar{r}_B(1,12)$	$\bar{r}_p(1,12)$	$\bar{r}_{LP}(1,12)$	$\bar{r}_T(1,12)$	$\bar{r}_W(1,12)$
mean	11.00	7.41	1.50	7.30	13.70	7.30	7.39
standard deviation	8.20	8.79	8.70	8.80	9.50	8.80	8.80
median	10.70	7.20	1.50	7.00	13.40	7.30	7.40
median deviation	5.40	5.63	5.90	5.70	6.09	5.60	5.60
minimum value	−10.08	−16.40	−22.51	−15.63	−11.82	−15.80	−15.80
maximum value	39.32	35.52	30.30	37.90	49.79	36.70	36.70

Source: Own calculations in Mathematica 6.0

As we can notice, the volatilities of all considered measures seem to be similar but there are significant differences between means and medians of some rates of return. Although $\bar{r}_{GK}$, $\bar{r}_P$, $\bar{r}_T$ and $\bar{r}_W$ have very close means and medians, the rest of average rates of return exhibit outstanding values of these parameters. For instance, according to the Theorem 4 and Remark 5 we can notice that the formula $\bar{r}_B$ has the value of mean and median smaller than $\bar{r}_{GK}$ and $\bar{r}_0$. Moreover, the mean and median of $\bar{r}_B$ are the smallest at all. It is also interesting, that the Polish measure $\bar{r}_0$ and the rate $\bar{r}_{LP}$ generates the highest values of mean, median and maximum. We obtain similar conclusions even if we consider a small time interval of observations (see Table 3). Thus in practice, it is very important which measures we use for calculations.

Table 3 Basic characteristics of the considered average rates of return for the time interval [1, 3]

Parameter	Measure of rate of return						
	$\bar{r}_0(1,3)$	$\bar{r}_{GK}(1,3)$	$\bar{r}_B(1,3)$	$\bar{r}_p(1,3)$	$\bar{r}_{LP}(1,3)$	$\bar{r}_T(1,3)$	$\bar{r}_W(1,3)$
mean	2.10	1.50	0.60	1.50	2.40	1.50	1.50
standard deviation	6.30	6.20	6.20	6.30	6.30	6.20	6.30
median	2.21	1.70	1.01	1.70	2.59	1.80	1.70
median deviation	4.30	4.30	4.30	4.30	4.31	4.30	4.30
minimum value	–24.10	–25.00	–26.10	–24.80	–23.50	–24.80	–24.90
maximum value	20.00	19.90	19.50	19.80	21.10	19.90	19.80

Source: Own calculations in Mathematica 6.0

CONCLUSIONS

The Polish definition of the average rate of return of a group of funds does not satisfy some economic postulates given by Gajek and Kałuszka (2000) although it had been in use in Poland for many years. Moreover this measure seems to overestimate the real value of the average return of funds. If funds invest similarly it does not matter which measure we use to calculate the average return of a whole group of funds. In another case the choice of the formula of the average rate of return is significant and important. We observe that the value of the formula $\bar{r}_B$ is the lowest and $\bar{r}_0$ and $\bar{r}_{LP}$ generate the highest values during the considered time interval (see Table 1, Table 2, and Table 3).

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Estimation of Poverty in Small Areas[1]

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Abstract

A qualitative techniques of poverty estimation is needed to better implement, monitor and determine national areas where support is most required. The problem of small area estimation (SAE) is the production of reliable estimates in areas with small samples. The precision of estimates in strata deteriorates (i.e. the precision decreases when the standard deviation increases), if the sample size is smaller. In these cases traditional direct estimators may be not precise and therefore pointless. Currently there are many indirect methods for SAE. The purpose of this paper is to analyze several different types of techniques which produce small area estimates of poverty.

Keywords	***JEL code***
Poverty, small area estimation, Horvitz-Thompson, Generalised Regression, Synthetic, Jack-Knife	*I32, C89*

INTRODUCTION

The focus of this analysis is persons and their income. Estimated parameters are the following: the average household income, the at-risk-of-poverty indicators and their variances. All parameters have been estimated using the Horvitz-Thompson, the Generalised Regression (GREG), and the Synthetic estimation methods. The Jack-Knife method has been used for the estimation of variances to indicate the precision of the estimates. The Absolute Relative Bias (ARB) was applied to compare the performance of the different estimators for 1 000 simulations.

1 DATA AND METHODOLOGY

1.1 Analysed population

Canadian household survey data[3] was used for the simulation. The analysed population $U = (1, ..., i, ..., N)$ consisted of 3 000 individuals from 1 024 households with income values obtained $(y_1,…,y_N)$. The gender[4] and age[5] of individuals have been used as auxiliary information. This population is actually a simple

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[1] This article represents the personal views of the author. It does not reflect in any way Eurostat's position or view in this topic. The article is based on a presentation at the European Conference on Quality in Official Statistics (Q2014) in Vienna, Austria, on 3–5 June 2014.

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[3] Canadian household survey data of the 1991 has been taken from Statvillage data base <http://www.lenato.eu/StatVillage/index.html>.

[4] The population has been divided into two gender groups: males and females.

[5] The population has been divided into seven age groups: less than 5, from 5 to 17, from 18 to 24, from 25 to 59, from 60 to 64, from 65 to 74, from 75 years old.

random sample but was treated as a population and has been divided into seven mutually exclusive strata of different size (see Table 1) for simulation purposes.

1.2 Stratified sampling

A simple random sample drawn from the population can be homogeneous. In order to have more precise estimates of the population the data set has been divided into $H = 7$ mutually exclusive strata $U_1, U_2, \ldots, U_H$ randomly.

For the analysis a stratified simple random sample s composed of seven strata with n_h elements in each has been drawn and y_h values observed. The size of the sample s is $n = n_1 + \ldots + n_h$.

Table 1 Strata size

Number of strata	The population size N_h	The sample size n_h
1	496	50
2	333	33
3	177	18
4	119	12
5	92	9
6	794	79
7	989	99
Total	3 000	300

Source: Own computations

The sample design probability when element i belongs to strata h is $\pi_{ih} = \frac{n_h}{N_h}$; the sampling weight for selected person i from the h^{th} strata is $w_{ih} = \frac{1}{\pi_{ih}} = \frac{N_h}{n_h}$. The value of the observed variable y into h^{th} strata of the i^{th} element is y_{hi}, i = 1,2,..., N_h, $h = 1,2,\ldots, H$. Then the sum of observed values y in h^{th} strata is $t_h = \sum_{i=1}^{N_h} y_{hi}$ and the mean $\mu_h = \frac{t_h}{N_h} = \frac{1}{N_h}\sum_{i=1}^{N_h} y_i$.

The sum and the mean of y values observed through the whole population are accordingly $t = \sum_{h=1}^{H} t_h$ and $\mu = \frac{t}{N}$. (Krapavickaitė, Plikusas, 2005)

1.3 Estimated parameters

The average incomes, the at-risk-of-poverty threshold, the at-risk-of-poverty rate, the at-risk-of-poverty gap index, and the variances of these indicators have been calculated. 1 000 samples have been drawn to verify the best of three applied methods for small area estimation. The estimated indicators and variances have been compared with the real values. Parameters have been estimated for every strata separately and also for the total the population.

1.4 At-risk-of-poverty indicators

Persons or households with disposable income lower than at-risk-of-poverty threshold are considered as living in poverty or social exclusion because there is no possibility of participating fully in society life. In countries with high quality of life conditions not all residents below the at-risk-of-poverty threshold lack money. However, they have a significantly lower potential to meet their needs compared with the rest of community but they may live in good enough conditions.

The at-risk-of-poverty rate and the at-risk-of-poverty gap index are focused on those individuals below the at-risk-of-poverty threshold. The at-risk-of-poverty rate P_0 shows which part of society is below the poverty threshold. The at-risk-of-poverty gap shows the average lack of finance and how much the income has to increase so that the poverty threshold is reached.

1.4.1 The at-risk-of-poverty threshold

The at-risk-of-poverty threshold is defined as 60% of the median equivalised disposable income[6] $z = 60\%M$. This indicator depends on the income distribution in society and varies according to the changes of the general living conditions in the area.

1.4.2 The at-risk-of-poverty threshold estimation

To estimate the at-risk-of-poverty threshold, the median $\hat{M}$ of the income has to been estimated. Firstly units $y_1,\dots,y_n$ of s^{th} sample have been sorted in ascending order $y_{1:s} \le y_{2:s} \le \dots \le y_{n:s}$ and inclusion into s^{th} sample probabilities accordingly $\pi_{1:s};\pi_{2:s};\dots;\pi_{n:s}$. Accumulative totals of sampling weights have been counted $B_1 = \frac{1}{\pi_{1:s}}$, $B_2 = \frac{1}{\pi_{1:s}} + \frac{1}{\pi_{2:s}}$, …, $B_l = \sum_{j=1}^{l} \frac{1}{\pi_{j:s}}$ while one of the l satisfied the following condition $B_{l-1} < 0{,}5\hat{N}$ *and* $B_l > 0{,}5\hat{N}$.

Then the estimated number of the population is $\hat{N} = B_n = \sum_{j=1}^{n} \frac{1}{\pi_{j:s}} = \sum_{s} \frac{1}{\pi_i}$ and the median estimate is $\hat{M} = \begin{cases} y_{l:s}, & \text{if } B_{l-1} < 0{,}5\hat{N} < B_l \\ \frac{1}{2}(y_{l:s} + y_{l+1:s}), & \text{if } B_l = 0{,}5\hat{N} \end{cases}$.

Then the estimate of the poverty threshold is defined by formula $\hat{z} = 60\%\hat{M}$.

1.4.3 The at-risk-of-poverty rate

The at-risk-of-poverty rate is defined as the number of persons below the at-risk-of-poverty threshold divided by the population number $P_0 = \frac{1}{N}\sum_{i=1}^{N} I_{(y_i<z)} = \frac{N_q}{N}$, here $I_{(y_i<z)} = \begin{cases} 1, & y_i < z \\ 0 & y_i \ge z \end{cases}$. N_q defines the number of individuals whose income is below the at-risk-of-poverty threshold $N_q = \sum_{i=1}^{N} I_{(y_i<z)}$.

1.4.4 The at-risk-of-poverty rate estimation

The at-risk-of-poverty rate estimator is $\hat{P}_0 = \frac{1}{\hat{N}}\sum_{i=1}^{n} w_i I_{(y_i<z)} = \frac{\hat{N}_q}{\hat{N}}$, here $\hat{N}$ is the estimated number of the population elements; $\hat{N}_q$ is the estimated number of individuals in the population living in poverty or social exclusion.

1.4.5 The at-risk-of-poverty gap index

The at-risk-of-poverty gap G_n is defined as an amount of difference between the at-risk-of-poverty threshold and income value y_i of i^{th} person living in poverty or social exclusion $G_i = (z - y_i)\, I_{(y_i < z)}$. The at-risk-of-poverty gap index is a proportion of the at-risk-of-poverty gap and the at-risk-of-poverty threshold $P_1 = \frac{1}{N}\sum_{i=1}^{q} \frac{G_i}{z} = \frac{1}{N}\sum_{i=1}^{N} \frac{z - y_i}{z} I_{(y_i<z)}$, here q is number of individuals in poverty or social exclusion.

1.4.6 The at-risk-of-poverty gap index estimation

Then the direct estimate of the at-risk-of-poverty gap index is defined by formula:

$$\hat{P}_1 = \frac{1}{\hat{N}}\sum_{i=1}^{n} \frac{\hat{z} - y_i}{\hat{z}} w_i I_{(y_i<\hat{z})}.$$

---

[6] Equivalised disposable income of person is calculated by dividing the disposable household income by the equivalised household size. All members of the same household are assigned the same equivalised disposable income.

1.5 Direct and indirect estimators

1.5.1 Small Area Estimation

An area is regarded as large if the sample drawn from that area is large enough to get direct estimates of adequate precision. An area is regarded as small if the sample is not large enough to get simple direct estimates of adequate precision. The variance of the estimate decreases through enlarging the size of the sample (Rao, 2010).

In order to have better quality estimates in areas, unbiased auxiliary variables have to be used from the same areas. This kind of estimation is defined as direct. For indirect estimation the auxiliary information has to be taken from adjacent areas.

1.5.2 The Horvitz-Thompson estimator

The Horvitz-Thompson estimator of the sum is $\hat{t}_\pi = \sum_{i=1}^{n} \frac{y_i}{\pi_i} = \sum_{i=1}^{n} w_i y_i$.

For a stratified simple random sample the Horvitz-Thompson variance of the sum estimate is $D\hat{t}_\pi = \sum_{i,j \in s} (\pi_{ij} - \pi_i \pi_j) \frac{y_i y_j}{\pi_i \pi_j}$. The Horvitz-Thompson variance estimate of the sum estimate is $\hat{D}\hat{t}_\pi = \sum_{i,j \in s} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_{ij}} \frac{y_i y_j}{\pi_i \pi_j}$, here $\pi_i = \frac{n_i}{N_i}, i \in U_h$; $\pi_{ij} = \frac{n_h}{N_h} \cdot \frac{n_h - 1}{N_h - 1}$, *when* $i, j \in U_h$ and $\pi_{ij} = \frac{n_h}{N_h} \cdot \frac{n_s}{N_s} = \pi_i \pi_j$, when $i \in U_h$, $j \in U_s$. π_{ij} is the inclusion into the sample probability of two elements *(i, j)* . If $i = j$ *then* $\pi_{ii} = \pi_i$ (Krapavickaitė, Plikusas, 2005).

1.5.3 The Generalised Regression Model (GREG)

y_i, is the values of the income and the value of the vector $\boldsymbol{x}$ is defined as the auxiliary information $\mathbf{x}_i = (x_{1i}, \dots, x_{ji}, \dots, x_{Ji})'$.

The sum of the dominant elements y is the GREG estimator of the sum t_y defined by the following formula $\hat{t}_{y,GREG} = \hat{t}_{y\pi} + \sum_{j=1}^{J} \hat{B}_j \left(t_{X_j} - \hat{t}_{X_j \pi} \right)$, where j is the number of several auxiliary pieces of information about the individual. The Horvitz-Thompson estimator of the sum t_{x_j} is $\hat{t}_{x_j \pi} = \sum_{i=1}^{n} \frac{\mathbf{x}_j}{\pi_i}$. $\hat{B}_1, \hat{B}_2, \dots, \hat{B}_J$ are the estimated components of the vector $\mathbf{x}$ $\hat{\mathbf{B}} = \left(\sum_{i=1}^{n} \frac{\mathbf{x}_i \mathbf{x}_k' q_k}{\pi_i} \right)^{-1} \sum_{i=1}^{n} \frac{\mathbf{x}_i y_i q_i}{\pi_i}$.

The GREG estimation method is appropriate to estimate parameters in non-responses. Then the GREG estimator of the sum is $\hat{t}_{wy} = \sum_r w_i y_i$, where r is the set of the respondents. The calibrated weights are $w_i = \left(1 + \left(\mathbf{t}_x - \hat{\mathbf{t}}_x \right)' \left(\sum_r \frac{\mathbf{x}_i \mathbf{x}_i' q_i}{\pi_i \hat{\theta}_i} \right)^{-1} \mathbf{x}_i q_i \right) \times \frac{1}{\pi_i \hat{\theta}_i} = \frac{g_i}{\pi_i \hat{\theta}_i}$, where $\hat{\theta}_k$ is the estimator of element's i response to the survey probability.

The calibrated estimate of the sum $\hat{t}_{wy}$ is biased. When N is large but sampling rate $\frac{n}{N}$ small then the bias estimate is slight.

1.5.4 Simple Synthetic estimator

The stratified population U_h splits up into k mutually exclusive groups $G_1, \dots, G_K$, $U_h = G_{h1} \cup \dots \cup G_{hK}$ and $U = G_1 \cup \dots \cup G_K$.

The mean of the elements from h^{th} strata and k^{th} group is $\mu_{yhk} = \frac{\sum_{i=0}^{n_{hk}} w_i y_i}{\sum_{i=0}^{n_{hk}} w_i}$, here $w_0 y_0 = 0$ when $n_{hk} = 0$, i.e. if the element from h^{th} strata and k^{th} group in the population does not exist then the sum is . The sum of population in strata h is $t_{yh} = \sum_{k=0}^{n_k} \mu_{yhk} N_{hk}$. The sum estimator of the sample is $\hat{t}_{yh}^{\sin t} = \sum_{k=0}^{K} \hat{\mu}_{yhk} N_{hk}$ and its variance is $D\hat{t}_{yh}^{\sin t} = D(\sum_{k=1}^{K} \hat{\mu}_{yhk} N_{hk})$.

The synthetic estimator is unbiased when $\mu_{yhk} = \mu_{yh}$, here $h = 1,\ldots,H$, $k = 1,\ldots K$. If this is the opposite, it is biased.

1.6 The variance estimation

To estimate the precision of estimated parameters the Jack-Knife variance estimation method has been used.

The Jack-Knife method's idea is to divide stratified sample S_h into K_h mutually exclusive subgroups. If $\hat{\theta}_h$ is the estimate of the parameter θ_h of the primary stratified sample s_h, then $\hat{\theta}_{(hk)}$ is parameter's θ estimator obtained by estimating the sample composed of h^{th} strata elements apart units from k^{th} ($k = 1,\ldots,K_h$) group. The modified sampling weights were used to estimate $\hat{\theta}_{(hk)}$:

$$w_{i(hk)} = \begin{cases} w_i, & \text{when } i^{th} \text{ element does not belong to } h^{th} \text{ stratum}, \\ 0, & \text{when } i^{th} \text{ element belongs to } h^{th} \text{ stratum and } k^{th} \text{ subgroup}, \\ \frac{n_i}{n_i - 1} w_i, & \text{when } i^{th} \text{ element belongs to } h^{th} \text{ stratum}. \end{cases} \quad (1)$$

Then the Jack-Knife variance estimator of θ estimate is equal to

$$\hat{D}_{JACK}\hat{\theta}_{(hk)} = \sum_{h=1}^{H} \left[\frac{K_h - 1}{K_h} \right] \sum_{k=1}^{K_h} \left(\hat{\theta}_{(hk)} - \hat{\bar{\theta}}_{(hk)} \right)^2, \text{ here } \hat{\bar{\theta}}_{(hk)} = \frac{1}{K_h} \sum_{k=1}^{K_h} \hat{\theta}_{(hk)}$$

1.7 The Absolute Relative Bias

The Absolute Relative Bias (ARB) assessed the accuracy of the estimates $ARB = \left| \frac{1}{K} \sum_{k=1}^{K} \frac{\hat{\theta}_h - \theta_h}{\theta_h} \right|$, where K is the number of drawn samples; $\hat{\theta}_h$ is the estimate of the parameter in the strata h; θ_h is real value of parameter in the strata h.

2 RESULTS

2.1 Estimates of parameters

The real values of the average income and the at-risk-of-poverty indicators have been calculated. All parameters have been estimated using Horvitz-Thompson, Generalised Regression, and Synthetic methods (see Tables 2, 4, and 6).

Table 2 Estimates of the average income

Strata	Sample size	Real average income	Horvitz–Thompson estimate of average income	Generalised Regression estimate of average income	Synthetic estimate of average income	The minimum value of average income estimate	The maximum value of average income estimate
Population	300	19 377.85	19 365.35	19 396.90	19 361.57	17 413.25 (H-T) 17 443.45 (GREG) 17 493.11 (S)	21 829.21 (H-T) 21 867.56 (GREG) 21 857.35 (S)
1	50	20 594.44	20 528.29	20 529.53	20 523.78	15 433.70 (H-T) 15 201.54 (GREG) 14 996.18 (S)	27 192.90 (H-T) 27 567.32 (GREG) 27 576.73 (S)
2	33	17 484.20	17 479.58	17 481.74	17 472.13	12 126.88 (H-T) 11 768.48 (GREG) 11 871.44(S)	23 512.79 (H-T) 23 345.83 (GREG) 23 316.08 (S)
3	18	18 513.46	18 599.66	18 615.62	18 588.24	11 272.39 (H-T) 10 939.35 (GREG) 10 954.14 (S)	26 250.29 (H-T) 26 672.41 (GREG) 26 764.36 (S)
4	12	19 184.40	19 027.93	19 015.18	19 058.99	10 000.23 (H-T) 90 672.53 (GREG) 8 738.67 (S)	30 455.87 (H-T) 30 431.29 (GREG) 30 625.30 (S)
5	9	17 757.50	17 843.75	17 848.21	17 844.91	9 201.61 (H-T) 93 491.07 (GREG) 8 953.184 (S)	33 616.91 (H-T) 33 693.56 (GREG) 33 730.83 (S)
6	79	20 150.24	20 121.66	20 123.23	20 120.18	16 389.60 (H-T) 16 401.59 (GREG) 16 420.89 (S)	26 257.12 (H-T) 26 635.74 (GREG) 26 294.45 (S)
7	99	19 113.91	19 129.06	19 131.30	19 149.96	15 538.40 (H-T) 15 573.20 (GREG) 15 350.62 (S)	22 981.10 (H-T) 23 139.04 (GREG) 23 316.41(S)

Note: Here (H–T) value of Horvitz-Thompson estimate, (GREG) value of Generalised Regression estimate; (S) value of Synthetic estimate.
Source: Own computations

The best ARB, estimating the average income and the at-risk-of-poverty gap index for the whole population, was through using the Horvitz-Thompson method. The at-risk-of-poverty rate estimates obtained the least ARB applying the GREG method.

The purpose of the paper was to choose the most accurate method for the estimation in small areas. The results show that in the smallest, third and fourth strata which consist accordingly of 9 and 12 elements in the sample, the Synthetic estimates of the average income are closest to the real values (see Table 3).

Table 3 The ARB of the average income estimates

Strata	Horvitz-Thompson estimate's ARB (%)	Generalised Regression estimate's ARB (%)	Synthetic estimate's ARB (%)
Population	**–0.06447544**	0.098310539	–0.08398375
1	–0.3211974	**–0.31518106**	–0.34310121
2	–0.02643092	**–0.014056**	–0.06902109
3	0.465571393	0.551799055	**0.403882282**
4	–0.81562095	–0.88208503	**–0.65375062**
5	**0.485715332**	0.510841272	0.492216146
6	–0.1417938	**–0.13401672**	–0.14913289
7	**0.079252793**	0.090945055	0.188597999

Source: Own computations

The Synthetic at-risk-of-poverty rate estimate's ARB in the smallest fifth strata is least (see Table 5).

Table 5 The ARB of the at-risk-of-poverty rate estimates

Strata	Horvitz-Thompson estimate's ARB (%)	Generalised regression estimate's ARB (%)	Synthetic estimate's ARB (%)
Population	0.36396329	**0.147665664**	0.152247869
1	**–3.51959494**	–3.7958481	–3.8266288
2	1.468493151	1.192029888	**1.003491015**
3	**4.644761905**	4.757144543	5.058255185
4	2.859782609	**2.601086957**	2.877924901
5	–2.80634921	–2.90370419	**–1.68042706**
6	**–0.63675717**	–0.78252971	–0.8622043
7	1.344097079	**1.068357786**	1.298860988

Source: Own computations

Table 4 Estimates of the at-risk-of-poverty rate

Strata	Sample size	Real value of the at-risk-of-poverty rate	Horvitz-Thompson estimate of the at-risk-of-poverty rate	Generalised Regression estimate of the at-risk-of-poverty rate	Synthetic estimate of the at-risk-of-poverty rate	The minimum value of the at-risk-of-poverty rate estimate	The maximum value of the at-risk-of-poverty rate estim
Population	300	0.194333333	0.193626	**0.1940464**	0.194037465	0.133518 (HT) 0.133518 (GREG) 0.136865 (S)	0.250143 (HT) 0.253482 (GREG) 0.251659 (S)
1	50	0.159274194	0.16488	0.16532	0.165369026	0.04 (HT) 0.04 (GREG) 0.040064 (S)	0.36 (HT) 0.36 (GREG) 0.40 (S)
2	33	0.219219219	0.216	0.2166061	0.217019374	0.030303 (HT) 0.030303 (GREG) 0.029412 (S)	0.454545 (HT) 0.454545 (GREG) 0.469924812 (S)
3	18	0.197740113	0.188556	0.1883333	0.187737913	0.03 (HT) 0.0 (GREG) 0.0 (S)	0.611111 (HT) 0.611111 (GREG) 0.60 (S)
4	12	0.193277311	0.18775	0.18825	0.187714935	0.0 (HT) 0.0 (GREG) 0.0 (S)	0.666667 (HT) 0.666667 (GREG) 0.6875 (S)
5	9	**0.22826087**	0.234667	0.2348889	0.232096627	0.0 (HT) 0.0 (GREG) 0.0 (S)	0.666667 (HT) 0.666667 (GREG) 0.75 (S)
6	79	0.164987406	0.166038	0.1662785	0.166409934	0.063291 (HT) 0.063291 (GREG) 0.060401 (S)	0.303797 (HT) 0.303797 (GREG) 0.303273427 (S)
7	99	0.223458038	0.220455	0.2210707	0.220555629	0.10101 (HT) 0.10101 (GREG) 0.100816 (S)	0.323232 (HT) 0.323232 (GREG) 0.330827068 (S)

Note: Here (H–T) value of Horvitz-Thompson estimate, (GREG) value of Generalised Regression estimate; (S) value of Synthetic estimate.
Source: Own computations

Table 6 Estimates of the at-risk-of-poverty rate

Strata	Sample size	Real value of the at-risk-of-poverty gap index	Horvitz-Thompson estimate of the at-risk-of-poverty gap index	Generalised Regression estimate of the at-risk-of-poverty gap index	Synthetic estimate of the at-risk-of-poverty gap index	The minimum value of the at-risk-of-poverty gap index estimate	The maximum value of the at-risk-of-poverty gap index estimate
Population	300	0.07252193	0.072638	0.0727797	0.072819745	0.045454 (HT) 0.045542 (GREG) 0.046188 (S)	0.106624 (HT) 0.107743 (GREG) 0.109839 (S)
1	50	0.052977463	0.053704	0.0538235	0.053812347	0.004695624 (HT) 0.00481718 (GREG) 0.005271356 (S)	0.136808961 (HT) 0.13717423 (GREG) 0.138750613 (S)
2	33	0.06671773	0.06736	0.0675437	0.067818474	0.0041682 (HT) 0.00417481 (GREG) 0.004052023 (S)	0.21001472 (HT) 0.21065701 (GREG) 0.217354998 (S)
3	18	0.073036323	0.0734	0.0735076	0.073541574	0.00 (HT) 0.00 (GREG) 0.00 (S)	0.304812082 (HT) 0.30481208 (GREG) 0.312076845 (S)
4	12	0.102809342	0.104061	0.1041447	0.10430057	0.00 (HT) 0.00 (GREG) 0.00 (S)	0.34929022 (HT) 0.34967569 (GREG) 0.502667938 (S)
5	9	0.088601259	0.089251	0.0894485	0.088580889	0.00 (HT) 0.00 (GREG) 0.00 (S)	0.336332409 (HT) 0.33700796 (GREG) 0.406565083 (S)
6	79	0.058100608	0.057692	0.0578231	0.057892628	0.008760654 (HT) 0.00877026 (GREG) 0.010559962 (S)	0.135791463 (HT) 0.13738105 (GREG) 0.13721441 (S)
7	99	0.090623887	0.090446	0.0906023	0.090371823	0.036089369 (HT) 0.03669861 (GREG) 0.036437574 (S)	0.175506287 (HT) 0.17515759 (GREG) 0.175058725 (S)

Note: Here (H–T) value of Horvitz-Thompson estimate, (GREG) value of Generalised Regression estimate; (S) value of Synthetic estimate.
Source: Own computations

In the same fifth strata the Synthetic at-risk-of-poverty gap index estimate has the smallest ARB (0.02%) compared with the Horvitz-Thompson and the GREG estimation methods.

Table 7 ARB of the at-risk-of-poverty gap index estimate

Strata	Horvitz-Thompson estimate's ARB (%)	Generalised regression estimate's ARB (%)	Synthetic estimate's ARB (%)
Population	**–0.1594528**	–0.35543944	–0.41065525
1	**–1.37126072**	–1.59705543	–1.57592157
2	**–0.9619282**	–1.23793553	–1.64985282
3	**–0.49766038**	–0.6453229	–0.69178013
4	**–1.21749012**	–1.2989069	–1.45047831
5	–0.73358855	–0.95628486	**0.02299011**
6	0.702989553	0.477610719	**0.357964671**
7	0.19625962	**0.02379632**	0.278143276

Source: Own computations

2.2 Estimated variances of parameters estimates

The largest over-estimations of the variance coefficients of averaged income estimates are in the smallest strata. Significantly better variance coefficients are obtained through the Horvitz-Thompson estimation (see Table 8). While the GREG and the Synthetic estimates are equally worse.

Table 8 Estimated variance coefficients of averaged income estimates

Strata	Sample size	Variance coefficient of the population	Horvitz-Thompson estimate's variance coefficient	GREG estimate's variance coefficient	Synthetic estimate's variance coefficient
Total	300	0.035	**0.039**	0.040	0.040
1	50	0.094	0.102	0.102	**0.101**
2	33	0.095	**0.104**	0.112	0.111
3	18	0.141	**0.135**	0.156	0.156
4	12	0.163	**0.181**	0.208	0.211
5	9	0.239	**0.252**	0.307	0.307
6	79	0.064	**0.068**	0.069	0.069
7	99	0.067	**0.072**	0.073	0.074

Source: Own computations

Concerning the variance coefficients of the at-risk-of-poverty rate and the at-risk-of-poverty gap index estimates, in most strata Horvitz-Thompson also produced the smallest overestimation (see Tables 9 and 10).

Table 9 Estimated variance coefficients of the at-risk-of-poverty rate estimates

Strata	Sample size	Real variation coefficient	Horvitz-Thompson variation coefficient's estimate	GREG variation coefficient's estimate	Synthetic variation coefficient's estimate
Total	300	0.104	**0.110**	0.115	0.117
1	50	0.422	**0.415**	0.410	0.440
2	33	0.408	0.477	0.477	**0.475**
3	18	0.544	0.483	0.484	**0.532**
4	12	0.698	0.602	**0.624**	0.818
5	9	0.172	**0.156**	0.206	0.186
6	79	0.232	**0.228**	0.266	0.284
7	99	0.171	0.217	0.217	**0.203**

Source: Own computations

Table 10 Estimated variance coefficients of the at-risk-of-poverty gap index estimates

Strata	Sample size	Real variation coefficient	Horvitz-Thompson variance coefficient's estimate	GREG variance coefficient's estimate	Synthetic variance coefficient's estimate
Total	300	0.141	**0.151**	0.162	0.166
1	50	0.420	**0.458**	0.461	0.472
2	33	0.421	**0.451**	0.462	0.467
3	18	0.645	**0.638**	0.637	0.613
4	12	0.666	**0.697**	0.747	0.733
5	9	0.792	**0.873**	0.982	1.051
6	79	0.332	0.362	**0.361**	0.362
7	99	0.226	**0.246**	0.247	0.256

Source: Own computations

CONCLUSIONS

Consequently we can see that to get good precise estimates would be better to apply different estimation methods for large and for small areas. Horvitz-Thompson method produces reliable estimates in large areas, but in most of the cases it does not suit for the poverty estimation in areas where sample size is small.

It is therefore suggested that if poverty estimation in small areas is to be made and if auxiliary information from the adjacent areas can be taken into account, the Synthetic method should be used. If, however, that auxiliary information is not available, then given the simulation results in general, the most appropriate estimation method for the analysed data would be Horvitz-Thompson.

The SAS programs text prepared for this simulation could be easily adjusted for other data to check how each of analysed techniques copes with your specific data taken from specific areas.

When comparing estimated variances of parameters estimates with real variances, large ARBs have been obtained. The best results of poverty indicator's estimation of population in small and in large areas are achieved by the Horvitz-Thompson method. This technique must be quite reliable enlarging the sample size, but in opposite, when sample size is reduced and goes to 0 the calculated estimates applying any direct method would be pointless.

Estimating the Jack-Knife variances calculation takes more time but the precision of the estimates increases when the group size is extremely small.

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What is about Development of Wages in the Czech Education and Healthcare Sectors During and after the Economic Downturn?

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Abstract

Wages in the Czech education and healthcare sectors have been a widely debated issue. The present paper deals with the wage development in the above two areas compared to that on the national scale before, during and after the global recession, focusing on recent earnings of Czech teaching and medical staff. Since the income of the latter is notably affected by overtime pay, the structure of wages in the health service sector is given proper attention. The development of the wage levels and concentrations in both the sectors and their comparison with those in other areas is also considered, the professions with the lowest and highest earnings being highlighted.

Keywords

Wage level, wage concentration, wage distribution, education sector, healthcare sector, worst-paid/best-paid professions

JEL code

J31, G01, E24, D31

INTRODUCTION

The level of earnings in public education and health sectors has been a widely debated issue in the Czech Republic in recent times in particular. The present paper deals with the development of wage differentiation in the period 2003–2013 with a focus on the changes during the global economic recession. The gross monthly wage in CZK (nominal wage) was the research variable of interest, 22 wage distributions in the education and healthcare areas having been examined over the period. The analyzed wage distributions were compared with those for all employees in the Czech Republic. Basic data used in this study were drawn from the official website of the Czech Statistical Office (the numbers and percentages of employees in the brackets of gross monthly income according to economic sectors and age, see <http://www.czso.cz/csu/2014edicniplan.nsf/p/110026-14>). Certain problems arose due to the changes in the classification of economic activities during the research period, the wages between 2003 and 2008

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being classified within the ISIC standards, those in the period 2009–2013 according to the NACE nomenclature ("health care" being included in "health and social care" category). For this reason, consistent time series are not available for the whole period, thus some caution is appropriate in assessing the development of wages in time. This may distort the comparison since the worldwide economic downturn began just at the end of 2008. Additional data were taken from the websites of the Ministry of Education, Youth and Sport and the Ministry of Labor and Social Affairs (for the years 2012 and 2013) and Trexima Ltd. (2013).

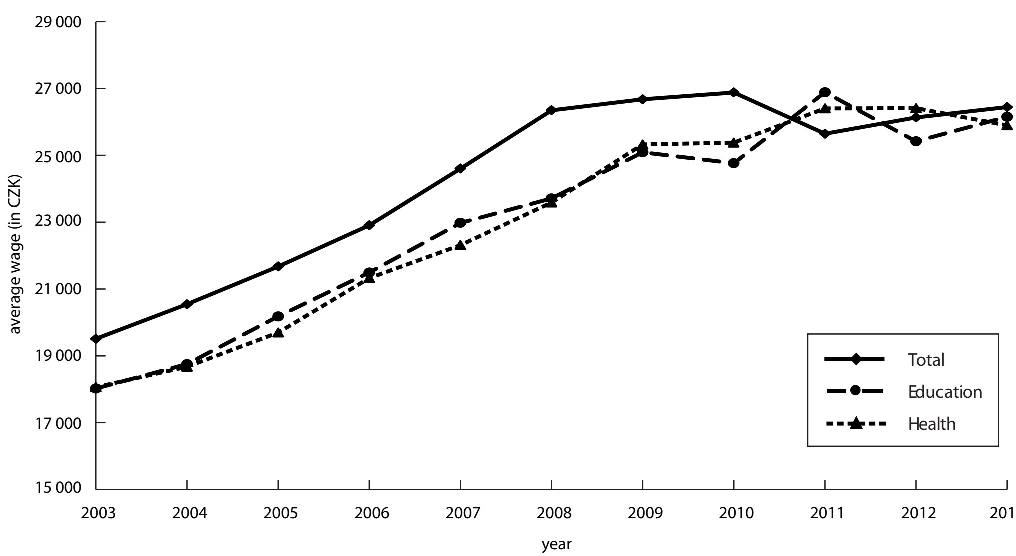
In the statistical literature, numerous Czech and foreign authors address the issue of wage or income development, among the former ones, see, e.g. Bílková (1995), Bílková (2012), Bílková (2013), Malá (2013), Marek (2010), Pacáková (2007), the latter being represented by, for example, Behr (2007), Kaasa (2006), Mallick (2008), Monti (2009), Rothschild (2005), Wessels (2008) and Wolff (2009), publications of an Italian author Camilo Dagum being widely cited in particular see Dagum (1997) or Dagum (1999). The issue of wages is also closely related to the unemployment (see, e.g. Franta, 2010), as well as other macroeconomic aggregates. (Minor discrepancies from the article by Marek (2010) are likely due to different sources of data and the frequency-interval distribution used in the present paper, other data not being currently available.)

The theoretical nature of the methods applied is not addressed here due to the focus of this journal. Descriptive characteristics of the wage distribution are explained, for example, in Triola (2003). Three-parametric lognormal curves represent a basic model probability distribution, their nature being dealt with, e.g. in Bartošová (2006), and the parameters are estimated by the method of L-moments; see Hosking (1997) or Kyselý (2007).

1 DEVELOPMENT OF THE WAGES IN THE CZECH REPUBLIC BETWEEN 2003 AND 2013

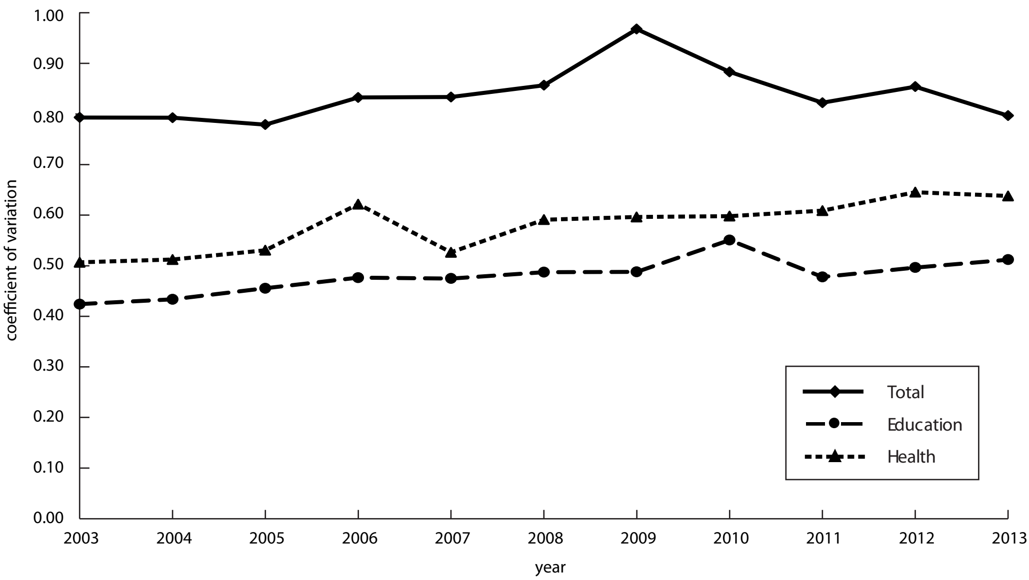
Figure 1 compares the development of the average gross monthly wage in both education and healthcare sectors with that of an aggregate of all employees in the Czech Republic. Figure 2 shows the development of the coefficient of variation of gross monthly wage in both the analyzed sectors, again with the comparison of the coefficient of total wage variation.

Figure 1 Development of the average wage in education and health (and social) care sectors in the Czech Republic between 2003 and 2013



Source: Own research

Figure 2 Development of the wage coefficient of variation in education and health (and social) care sectors in the Czech Republic between 2003 and 2013



Source: Own research

Table 1 The growth rate and average growth rate of gross monthly wage median in the Czech Republic between 2003 and 2013

Year	Set			
	Total	**Tertiary education**	**Education**	**Health care**
2003	–	–	–	–
2004	1.0582	1.0671	1.0432	1.0270
2005	1.0501	1.0550	1.0700	1.0511
2006	1.0493	1.0579	1.0539	1.0846
2007	1.0715	1.0725	1.0588	1.0440
2008	1.0629	1.0718	1.0271	1.0502
2009	1.0004	1.0167	1.0537	1.0573
2010	1.0172	0.9986	0.9738	1.0052
2011	0.9652	0.9589	1.1140	1.0270
2012	1.0189	0.9787	0.9397	0.9943
2013	1.0142	1.0188	1.0203	0.9803
Ø 2003–2009	1.0485	1.0566	1.0510	1.0522
Ø 2009–2011	0.9909	0.9785	1.0416	1.0160
Ø 2011–2013	1.0165	0.9985	0.9792	0.9873
Ø 2003–2013	1.0303	1.0288	1.0344	1.0317

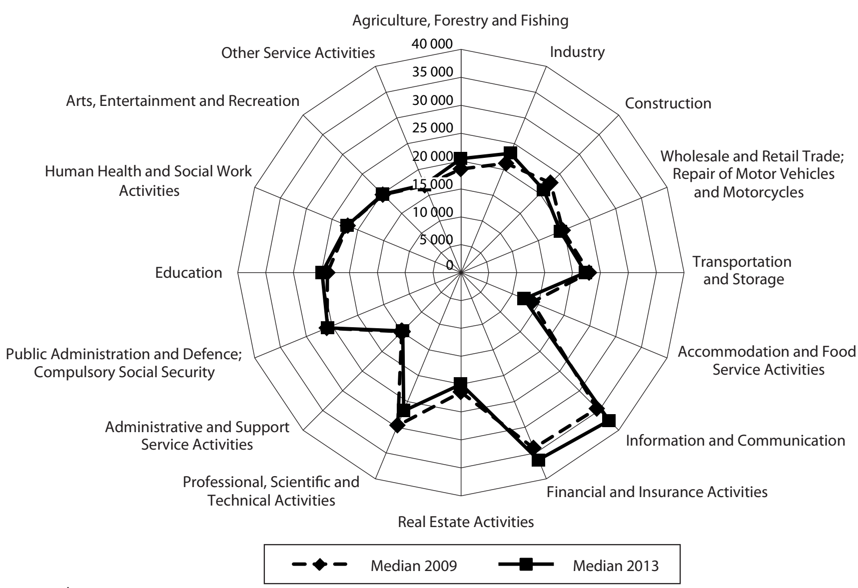
Source: Own research

From Figure 1, we can see a relatively sharp growth in the level of wages until 2009, while the average wage in both the sectors is markedly lower than that on a national scale. Wage growth in education and health care virtually stopped between 2009 and 2010, probably due to the global economic downturn. The wages in the two sectors increased again, in contrast to the national average wage, in the following period eventually reaching and even exceeding (in 2011) the national average wage in the Czech

Republic. It is evident from Figure 2 that the relative variability of gross monthly wage in school and health systems is deeply under that of aggregate wages in the Czech Republic. However, certain caution is necessary when drawing conclusions from Figures 1 and 2 owing to some changes in methodology during the monitored period.

The average wage not being earned by approximately two-thirds of employees, Table 1 gives an overview of the growth of wage medians in the periods before, during and after the crisis. It shows the growth rate and the average growth rate of the median of gross monthly wage in the period 2003–2013, indicating a substantial decline in wage growth during the economic recession in the Czech Republic. Moreover, in 2011, the middle gross monthly wage decreased by 3.48%, falling noticeably in the group of higher educated employees between 2010 and 2012. In the area of education and health care, the situation varies considerably. We can observe in Table 1 that at the beginning of the global downturn in 2009, the middle gross monthly wage increased by 5.37% and 5.73 % in education and healthcare sector, respectively, wage growths being still comparable. In 2010, however, the wages in the education system decreased by 2.62%, while still slightly increasing (by 0.52%) in the health service. The former sector showed a dramatic development in 2011 when the middle gross monthly wage rose by 11.40%, while increasing only by 2.70% in the latter area. In 2012, on the other hand, the median of gross monthly wage went down by 6.03% in the educational sphere, while in the health sector it declined by less than 1 %. In the school system, the middle wage rose by 2.03 % in 2013, while it decreased by 1.97% in the health sector. The table also indicates that between 2009 and 2011, the middle gross monthly wage decreased by 0.91% a year on average, that of university-educated employees declining by 2.15%. In this period, the level of wages was increasing in both the analyzed sectors – by 4.16 % and 1.60% annually on average in the education and healthcare sectors, respectively. In the following period 2011–2013, the middle gross monthly wage decreased in both these areas, on average by 2.08% per year in the former and 1.27% in the latter sector, while the level of wages increased nationwide. The gross monthly wage in all four studied groups rose by around 3% a year on average throughout the research period 2003–2013.

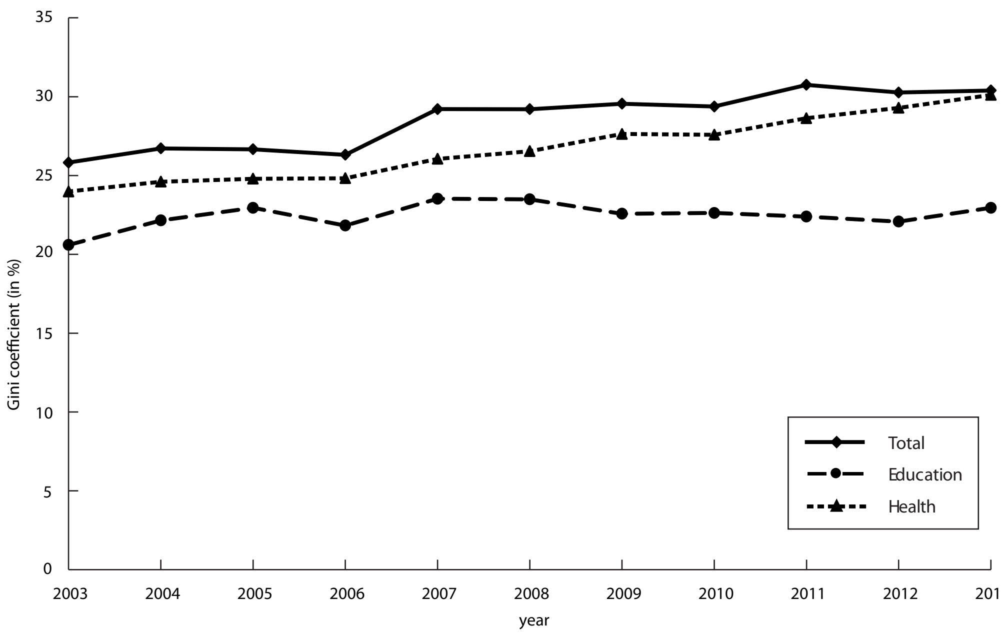
Figure 3 The median of gross monthly wage (in CZK) in all NACE sectors at the beginning of economic crisis (2009) and in 2013



Source: Own research

Figure 3 allows a comparison of the middle gross monthly wage in education and health service with that in other sectors. We can see that the highest wages are earned in the sector of information and communication, the middle gross monthly wage being 34 483 CZK in 2009 and 37 539 CZK in 2013; certain downward bias may have been caused by the use of the interval frequency wage distribution with the same intervals for all sectors including those with the highest level of wage. The financial and insurance sector reports the second highest wage level with the median of 34 055 CZK and 36 344 CZK in the years 2009 and 2013, respectively. The employees in accommodation and food services, on the other hand, have the lowest level of wages, the median being 13 813 CZK and 12 276 CZK in the respective years. The second lowest middle gross monthly wage is recorded in administrative and support services, namely 14 980 CZK in 2009 and 14 783 CZK in 2013. In comparison with the above mentioned high- and low-paid sectors, the wages in education and healthcare areas are in the center of the scale – the middle gross monthly wage in the former being 23 928 CZK and 24 889 CZK and that in the latter 21 949 CZK and 22 087 CZK in 2009 and 2013, respectively. The above mentioned figures are rather low in view that the majority of people employed in the two sectors are university graduates, the middle gross monthly wage of those with tertiary (2nd) degree being 35 220 CZK in 2009 and 33 626 CZK in 2013. This disproportion in earnings has been a constant focus of criticism by the Czech media and general public. (It is also observable from Figure 3 that the wage level declined in eight out of all sixteen sectors between 2009 and 2013.)

Figure 4 Development of the Gini coefficient of concentration in education and health (and social) care sectors in % in the Czech Republic between 2003 and 2013

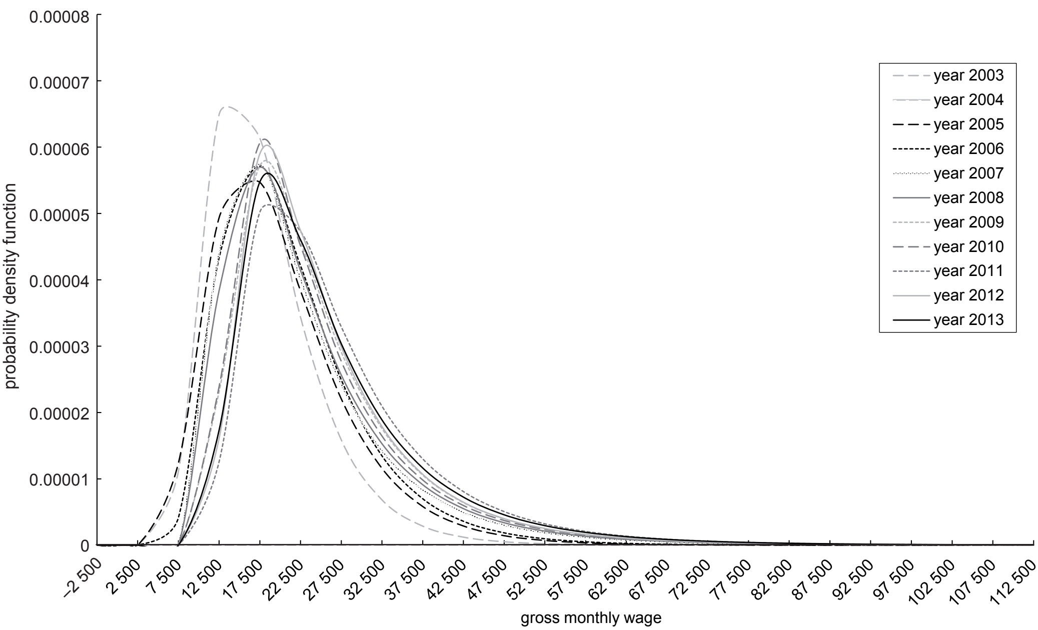


Source: Own research

Figure 4 presents the development of the Gini coefficient of concentration in both monitored sectors over the period. The value of the Gini coefficient (in per cent) ranges from zero (extreme leveling – zero concentration – when all employees earn the same wage) to a hundred (extreme differentiation – maximum possible concentration – when one employee receives the entire wage). We can see from this figure that the employees in both these sectors have relatively evenly distributed wages compared to those earned nationwide. It is apparent that the pay of employees in the school system is spread more evenly than that of health care workers over the research period 2003–2013. While the concentration of wages in the healthcare area was growing steadily from 23.98% to 30.11% throughout the period, the development in the education sector was different. The concentration of wages in the latter area was increasing gradually (with a single dip) until the economic downturn. Then it started to slightly decline with the exception of the last year under review. On a national scale, the wage concentration shows an increasing tendency with a linear trend. Since the concentration of wages in the health sector was growing faster, it almost reached the national level in 2013.

Figures 5 and 6 indicate the development of the model probability distribution in both the analyzed sectors in the monitored period. Because the data in the form of the interval frequency distribution with unequal interval widths were the source for calculations in this study, it was impossible to show the development of empirical frequency distribution. This is why the model probability distributions based on a three-parametric lognormal curve were constructed. The parameters of these curves were estimated using the L-moment method of point parameter estimation, which is known for its high accuracy; see Hosking (1997) or Kyselý (2007).

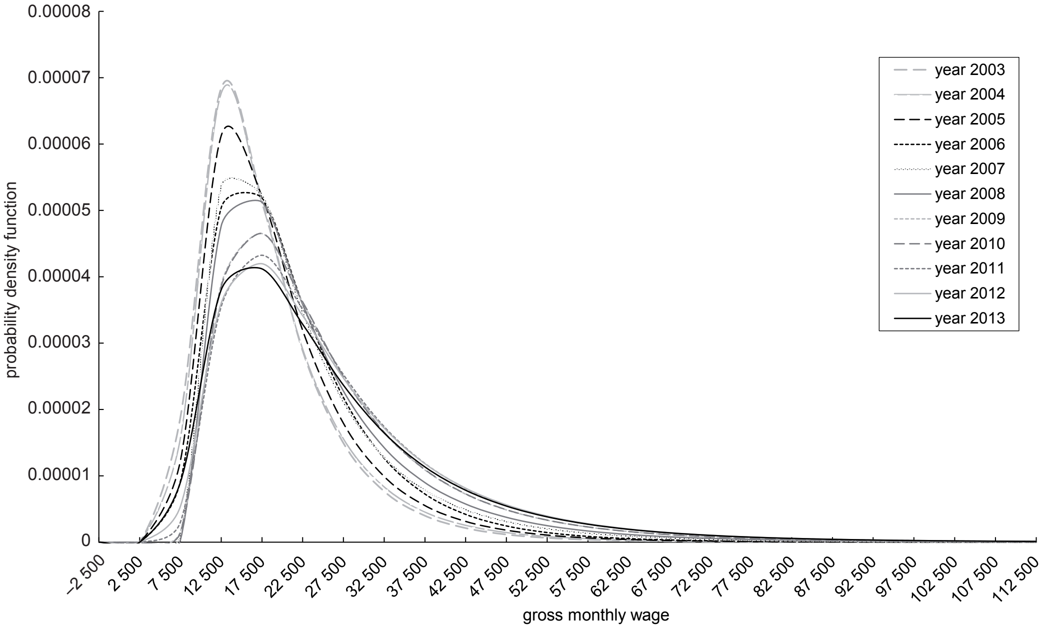
Figure 5 Development of the model probability distribution of gross monthly wage (in CZK) in the education sector in 2003–2013



Source: Own research

It is evident from Figures 5 and 6 that the wage distributions are moving slightly to the right, which is presumably due to an overall increasing wage level in both the sectors. The absolute wage variability increases gradually in time, skewness and kurtosis having a declining tendency.

Figure 6 Development of the model probability distribution of gross monthly wage (in CZK) in the healthcare sector between 2003 and 2013



Source: Own research

2 THE WORST- AND BEST-PAID JOBS IN THE CZECH REPUBLIC

Czech employers are currently (i.e. in 2014) lacking about four thousand people who would be willing to do less qualified or unskilled jobs. Since they are badly paid ones, the vacancies distinctively exceed the number of applicants. Cleaners, waiters/waitresses and guards are the worst-paid jobs in the Czech Republic; see Table 2 presenting the middle wage of ten worst-paid occupations. The lowest middle gross monthly wage of cleaners was 10 125 CZK in 2013 including bonuses and sick leave compensations, a tenth of them having received only 8 836 CZK. (84 200 people are employed as cleaners in the Czech Republic, doing an energy-consuming, physically demanding job). However, despite the low wage and high work intensity, it is not an occupation that belongs among those with vacancies exceeding an interest of candidates. For example, in July 2014, employment agencies offered only 436 cleaner vacancies, which is the 16th highest number; the most (nearly 2 800) vacancies being offered to truck, bus and tram drivers. One of the reasons, why cleaner's work attracts more applicants compared to, for example, truck drivers, are markedly lower job and personal requirements. Apart from basic education or vocational certificate, employers sometimes require manual skills, three years of work experience, the knowledge of Russian or "sense of cleanliness and order". Nevertheless, the job of a cleaner has relatively demanding performance targets – an hourly quota being two hundred square meters. Thus, the cleaning staff have only six minutes for twenty square meters of office floor, including dusting the furniture and emptying the waste, the same time for cleaning the toilets, bathrooms and kitchens. Moreover, their job also includes long-term maintenance of the property, requiring proper application of cleansers and detergents. Generally, the job of a cleaner is undervalued by both the public and employers and, consequently, underpaid. Another badly paid job is that of a waiter/waitress – despite its demands on a long specialized training and experience, communication skills and shift work. The middle gross monthly wage was only 10 956 CZK in 2013. Again, a general underestimation of the job – sometimes increased by the amateurishness and prejudices of some restaurant and pub keepers distrusting their staff – is widespread. The same applies to the third lowest-paid occupation – security guards and watchmen. Although it is a physically and mentally demanding job with a threat of injuries or even permanent disability caused during the performance of the duties, security and surveillance staff belong among those with the lowest hourly wages and the highest job uncertainty since they are mostly hired by security agencies offering temporary employment. The median of their gross monthly wage was only 10 957 CZK in 2013. Lower than 13 000 CZK wage median was earned by kitchen staff, tailors and dressmakers, truck drivers and sellers of food, jewellery, furniture and housing goods as well.

Air traffic controllers, on the other hand, have the largest earnings in the Czech Republic, their wage median being 114 977 CZK a month in 2013. However, there is quite a big difference between the best- and worst-paid employees in this field, their monthly gross wages ranging from 24 093 to 250 017 CZK in 2013. The positions of senior managers in large companies and institutions are the second most remunerative jobs, the wage median amounting to 102 617 CZK in 2013, the earnings ranging from 31 479 to 337 545 CZK. Senior doctors, financial and PR executives also rank among the top earners; see Table 3.

Table 2 The median of gross monthly wage (in CZK) of the worst-paid jobs in the Czech Republic in 2013

Order	Profession	Median
1	Cleaners at the premises of personal services	10 125
2	Cleaners and helpers in health and social care facilities	10 601
3	Waiters and waitresses	10 956
4	Security staff, watchmen	10 957
5	Kitchen maid	11 009
6	Cleaners of production areas (excluding food and pharmaceutical manufacturing) and stores	11 121
7	Security staff and security agencies	11 127
8	Doormen	11 203
9	Cleaners and helpers in hotels, industrial and other buildings	11 310
10	Cleaners and helpers in administrative buildings	11 403

Source: Trexima, own research

Table 3 The median of gross monthly wage (in CZK) of the best-paid professions in the Czech Republic in 2013

Order	Profession	Median
1	Air traffic controllers	114 977
2	Highest representatives of large companies and institutions	102 617
3	Senior doctors in the area of health	89 594
4	Executives in the financial services	87 146
5	Executives in public relations	83 300

Source: Trexima, own research

Czech hourly labor costs are 10.3 EUR per employee on average, i.e. the tenth lowest in the European Union, the second highest, however, among the post-communist countries.

In the first quarter of 2014, the wage median was 20 764 CZK in the Czech Republic, the average wage being 24 806 CZK. The latter is higher than the median because the earnings of the best-paid employees push it up, about two thirds of all employees receiving less than the national average. The median is therefore more adequate than the average since it halves the employees on the poorer and the richer half.

3 THE WAGES OF TEACHERS AND MEDICAL DOCTORS

Teachers rank among the lowest-paid tertiary-qualified professionals in the Czech Republic, 70–90% of university-educated employees in other professions earning on average more money than teachers. The average gross monthly wage of teaching staff was 25 996 CZK in 2013, having improved by 163 CZK in comparison with 2012. The wages of the rank-and-file teachers were around 24 500 CZK, headmasters and other managing staff receiving 35 000 CZK on average, as indicated by the data of the Ministry of Education, Youth and Sport. Its recent statistics also show that while the number of secondary school teachers is declining, the demand for nursery school teachers is rising due to the demographic development. In 2013, almost 206 000 people worked in regionally maintained educational establishments, i.e. in nursey, primary, secondary and higher vocational schools, conservatories or after-school care centers. They were paid 56.5 billion CZK, which was an increase of 0.8% compared to 2012, the average amount of a discretionary bonus rising from 1 884 to 2 103 CZK compared to 2012. Teachers in higher vocational schools earn the most – about 29 500 CZK on average. Grammar school teachers and special education centers staff get over 28 000 CZK of gross income per month, secondary vocational school teachers having by about 100 CZK less. Primary school teachers had less than 27 000 CZK on a monthly payroll last year. Nursery school teachers are at the opposite end of the wage scale, earning about 23 200 CZK a month on average in 2013. Boarding school educators still took about 300 CZK less, after-school assistants earning 21 700 CZK on average in 2013. Non-teaching staff, i.e. caretakers, cooks and administrative staff are the worst-paid in the educational sector. In 2013, they earned 14 500 CZK on average, adjusted statistics indicating even lower (13 471 CZK) average wage. In private and church schools, teachers earned an average of 25 200 CZK, non-teaching staff 18 200 CZK. Wage differences can be detected on the basis of the regional division as well. While teachers earned around 26 600 CZK on average in the regions of Usti nad Labem, Liberec and Central-Bohemia, in those of Zlín and Hradec Králové, the average wage without other personal premiums was lower than 25 300 CZK in 2013. As for the numbers of teachers, the largest reduction was registered in secondary vocational schools (by 672, i.e. 4.6%), the highest increase in staffing levels being recorded in nursery schools (838, 3.2%).

Table 4 The average gross monthly wage of employees in the healthcare and education sectors in the Czech Republic broken down by categories in 2012 and 2013

Category of staff	Average gross monthly wage		Annual increment	
	2012	2013	(Kč)	(%)
Doctors and dentists	61 078	60 635	–443	–0.7
Pharmacists	43 213	42 271	–942	–2.2
General nurses and midwives	29 150	28 706	–444	–1.5
Other paramedical workers with professional qualifications	28 878	28 825	–53	–0.2
Paramedical workers with professional and specialized qualifications	29 016	28 831	–185	–0.6
Paramedical workers under expert supervision or direct guidance	19 510	19 281	–229	–1.2
Other specialists and dentists	26 366	26 175	–191	–0.7
Teaching staff	29 128	26 459	–2 669	–9.2
Technical and administrative staff	15 694	15 577	–117	–0.7
Workers and operational personnel	30 403	30 174	–229	–0.8

Source: <www.mpsv.cz>, own research

As for medical doctors and dentists, their total average gross monthly wage was 60 635 CZK in 2013, general nurses and midwives earning 28 706 CZK, the average wage of the former going down by 0.7% and that of the latter by 1.5% compared to 2012. A decline in the wage level is apparent in all categories of both medical and teaching staff between 2012 and 2013; see Table 4.

In the area of health care, particular wage components may be of interest because of the differences between the genuine wage and the one which would be earned according to the contract terms excluding overtime payments. This is indicated in Table 5 for the year 2013. Average earnings of doctors and dentists paid on the basis of standard wage regulations amounted to 58 837 CZK, the contractual wage being 30 031 CZK. Those of nurses and midwives amounted to 28 707 CZK, of which the standard wage is 18 178 CZK.

Table 5 The structure of the average gross monthly wage of professional healthcare workers in the Czech Republic in 2013

Wage components	Doctors and dentists	Pharmacists	General nurses and midwives	Other paramedical workers with professional qualifications	Paramedical workers with professional and specialized qualifications	Paramedical workers under expert supervision or direct guidance	Other specialists and dentists
Wage tariff	30 031	22 433	18 178	17 450	19 382	12 033	16 756
Personal allowance	6 105	7 536	1 588	2 070	2 861	1 043	3 678
Total remunerations	5 633	3 676	978	1 243	1 774	466	1 439
Overtime	5 818	1 836	1 020	1 866	493	979	373
Operational readiness	1 490	177	113	168	117	53	129
Other	9 759	6 201	6 829	6 029	4 018	4 707	3 737
Total wage	58 837	41 859	28 707	28 825	28 645	19 281	26 112

Source: <www.mpsv.cz>, own research

Differences between the wages of medical doctors in various regions of the Czech Republic may reach up to 30 thousand CZK a month, as it follows from the data of the Ministry of Labour and Social Affairs. In 2013, in the public and government sector, medical specialists (fully certified doctors) in Olomouc Region received the highest average gross monthly wage amounting to 79 108 CZK, followed by those in Pilsen Region with 76 392 CZK. The lowest average gross monthly wage of these professionals was recorded in Liberec (40 020 CZK) and Zlín (49 593 CZK) regions, respectively. No dramatic basic-wage differences have been registered. For specialists, the basic wage oscillates around 40 000 CZK a month, the differences being caused by a high proportion of overtime pay. A closer look at the 2013 data of the Ministry of Labour and Social Affairs shows that the medical specialists in Olomouc Region were paid for more than 200 overtime hours on average, those in Pilsen, Liberec and Zlín regions being remunerated for 191, 179 and 181 extra hours per month, their overtime pay reaching more than 44% of the total wage amount in Olomouc Region and over 30% in Liberec and Zlin regions. Such a large number of hours worked is bad for both patients and doctors, despite the latter earning more money, since the real threat of overwork increases the risk of errors. Moreover, this traditional harmful practice discourages both medical graduates and experienced doctors from working (staying) in the Czech Republic (the former group criticising an inflexible system of further education as well). According to the Czech Medical Chamber statistics, there are 1 050–1 100 general medicine graduates each year, approximately 200 of them leaving immediately abroad instead of starting to work in the Czech health care system and another 200 fully certified doctors – more than half of them aged 30–40 years – quitting their job in the Czech Republic every year in order to get better paid and less stressed.

Doctors are not the only profession whose wage level differs from region to region. Qualified secretarial staff, for example, earned 29 074 CZK of gross monthly wage on average in the Central Bohemian Region, which was 8 653 CZK more than in the Moravian-Silesian Region in 2013. Also, sales representatives' monthly wage was around 35 743 CZK in Prague, i.e. 11 402 CZK higher than in Zlin Region. Similarly, the genuine wages of elementary and secondary school teachers were different from the contractual ones. Teachers earned 28 250 CZK monthly on average in Prague, but only 26 274 CZK in the Vysočina Region, the differences being likely due to the level of teaching experience and expertise.

SUMMARY AND CONCLUSIONS

It is apparent from the results of previous studies that wage growth virtually stopped during the economic recession in the Czech Republic. It is clear from Table 1 that the middle wage increased by only 0.04% nationwide in 2009, yet increasing by 5.37% and 5.73% in education and healthcare sectors, respectively. While the national middle wage fell by 3.48% in 2011, in the above two sectors, the middle wage rose by 11.40% and 2.70%, respectively. In 2012, it increased by 1.89% in the Czech Republic, having declined in the fields of education (by 6.03%) and healthcare. In the latter area, it kept decreasing (by almost 2%) in 2013. The effect of the worldwide economic downturn and its aftermath upon the wage levels in the two analyzed areas was different from that in the whole Czech Republic.

The sectors recording the highest wage level are those of ICT and financial and insurance activities. The lowest wage level, on the other hand, is recorded in the sectors of accommodation and food service and administrative and support service activities. The sectors of education and healthcare are approximately in the middle of the scale.

The three best-paid professions are air traffic controllers, top representatives of large organizations and senior doctors. The worst-paid jobs, on the other hand, are cleaners, waiters/waitresses and security staff. The differences between the two groups are on the order of tens of thousands CZK.

It is to be noted that the term "wage" includes both the salaries of employees in budget-funded (state, public and non-business) organizations and the wages of employees in the private (business) sector, which is in line with the data provided by the Czech Statistical Office.

The present paper also addresses the issue of wage concentration in education and healthcare sectors. In both of them, the level of wage concentration lower than that in the whole Czech Republic was detected during the years 2003–2013, the concentration of wages in the latter sector being higher than that in the former. This means that the wages of employees in the education sector are more comparable than those in the health service. This is an expected outcome, as the level of wages of doctors and dentists is well above that of teachers (university-qualified employees working in both the sectors), while the wage levels of less qualified workers in both these sectors are close to each other. The wage concentration in the healthcare sector rises throughout the analyzed period and in 2013 almost reaches that in the whole Czech Republic. This means that the wages of employees in the health sector are increasingly different from each other.

Since not only university-educated people are employed in both the analyzed areas, special attention was paid to the wage level of individual job positions within each of the two sectors. It turned out that the average gross monthly wage of a Czech teacher was only 25 996 CZK, while that of a doctor or dentist was 60 635 CZK, a substantial proportion of the latter amount being overtime pay.

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Economic Statistics or Statistical Methods in Economics?[1]

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Abstract

In teaching statistics to economists, it often happens that insufficient distinction is drawn between statistical methods applied on economic data on the one hand, and economic statistics as a special discipline with its own theoretic basis, fundamental notions and a specific concept of indices on the other hand. The authors endeavour to point out pitfalls of this insufficient distinction and introduce didactic ways to resolve this problem.

Keywords	***JEL code***
Statistical methods in economics, economic and social statistics, teaching statistics	*E01, C02, N01*

INTRODUCTION

Teaching statistics at universities of economic orientation is subject to a number of popular misconceptions, whose consequences are harmful to both statistics and economics. The worst of it is that students are discouraged by them. It is no secret that, for quite a large proportion of students of economics, mathematics and statistics pose an arduous challenge; they try to evade these disciplines as long as they can in the hope that later, when they work in firms and corporations, they will not need them too badly.[4] What a mistake this is!

For the purposes of the present paper, let us leave aside teaching mathematics and focus on teaching statistics in economics. Our experience is based on forty years of practical teaching statistical methods used in both economics and economic and social statistics at Czech and foreign universities.[5] Although the last two mentioned general areas of understanding statistics in economic fields (economic statistics

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4 This is for example discussed in Gatusso (2011).
5 Statistical methods courses in the faculties of economics are generally two-semester long (lectures + exercises). Economic statistics course is usually one semester long.

and statistical methods in economics) are quite dissimilar to each other, they are often not sufficiently distinguished by either teachers or students of economics at universities.

At the very root of this lack of distinction between statistical methods used in economics on the one hand and economic and social statistics on the other hand is the fact that among statisticians we can find many more of those who specialise in applications of various statistical methods than those who are professionally and primarily interested in economic statistics, national accounts, etc. As a logical consequence, teaching statistics for economists is often reduced to an inorganic combination of learning certain statistical methods with getting some economic data, and applying (more or less mechanically) the former on the latter.

A tempting support of this approach stems from the wide range of statistical software, so that you simply "input" the collected data into suitable software procedures and then just wait for the outcome. And sometimes you do not need even take such a complicated course: instead, simply use Microsoft Excel. At many universities or faculties, that kind of exercise is taken for mastering statistics in economics. At these cases, it is the teachers' fault, because it is unfortunately true that some professors who teach of statistics for economists do not have enough experience from business nor sufficient contacts with the business community or the providers of macroeconomic data.[6]

We can see many instances in which lecturers perseveringly think up pseudo-examples from the national economy, trying to make an impression that real quantitative-economic analysis of certain phenomena is thus achieved. In fact, such pseudo-applications are quite a long way from a realistic economic analysis and, in the long run, discourage both statisticians from economics and economists from statistics. To the detriment of both.

In other words, the relationship between statistical methods (or general quantitative methods in the broad sense) and economics is far more complex in the real world (as opposed to the isolated and virtual realm of pseudo-examples). On the whole, the above-described trivial approach to teaching is shallow and insufficient and leads to irreversible didactic errors. However, problems lie not only in the approach to the application of statistical methods in economics, but also in the fact that such applications are mistakenly considered as economic statistics. There are several reasons – let us go through some of them.

1 HAZARDS HIDDEN IN ECONOMIC DATA

Little attention is given to the quality of economic data. Such data are often easily accessible at websites of various institutions. When applying statistical methods, few people carefully study the potential methodological pitfalls hidden in the data, to what extent they are affected by the methods of data collection used in corporations and state administration, the organisational structures and their changes, or the pricing, taxation, exchange-course and other aspects. The procedure is then completed by an accurate and quickly available calculation of the desired characteristics. A more detailed study, however, shows the lack of any informative value of such characteristics; and their deeper analysis by both theoretically and practically oriented economists leads to disappointment and scepticism. The ultimate result is distrust in the potential of statistics in economics felt by both statisticians and economists.

A bridge between statistics and economics is thus destroyed, or even not built at all, from the very beginning of possible cooperation.

The reader might object that nothing like that should happen and that everybody is aware of such risks. However, the opposite is true. This fact is proved by many years of our experience from various universities in the Czech Republic and abroad and from contacts with the economic practice. It is implied by a strong preference teachers put on the formal side of statistical methods, suppressing the parallel

6 Compare for example Gelman, Loken (2012) or Groth (2013).

need for critical assessment of the economic data to be analysed with the aid of this or that statistical method.

Let us mention a small example from time series analysis: students learn quite a few modelling methods, such as time series trends (the deterministic approach, adaptive methods, the Box-Jenkins methodology, etc.), equip themselves with the corresponding software, get some data and carry out their calculations.

How much information have they obtained about problems that may occur in the respective data environment of the time series in question, and what should their attention be focused on? We have in mind, for example, the issues of spatial, factual and temporal comparability in the data (constant and current prices, methodology of collection, calendar variations, etc.). What about the length of the time series? We keep telling our students that statistics deals with mass phenomena – the more observations the better. It is even the categorical imperative when using, say, the Box-Jenkins methodology. On the other hand, the longer the time series the better for the methods but the higher contamination, often fatal, of the data (due to the changing methodologies, factual discrepancies, pricing recalculation coefficients, etc.). If a trend analysis is mechanically applied to such a time series, total doom follows. Do students know such facts and are they persuaded about them within their study? Well, often they do not and are not.

A solution is: knowledge of economics by teachers of statistics, as well as knowledge of statistics by teachers of economics, should be elevated. More time should be given to quality of data, with less preference given to describing mere methods and tools. This is the only way of reducing the risk of unsuccessful applications of statistics in economics, and also reducing the risk of economists' relying on feeble verbal declarations they are unable to support with relevant numerical illustrations, i.e., to use particular facts in strengthening their argumentations.

2 SIMPLIFIED APPLICATION OF STATISTICAL METHODS – A SOURCE OF DUBIOUS INTERPRETATIONS

Statistical methods and tools applied in economics are often taught in their simplified versions, presenting just basic principles, properties, and utilisation. Why not – after all, most students at economic universities are not specialists in statistics and in the future they are going to become practical users of statistics; very often of just a very restricted range of statistical techniques. Hence this approach is correct. But only until real economic and social situations are encountered. Nothing was explained incorrectly in the teaching, all aspects were given adequate attention, but in confrontations with real situations something seems not to work.

Let us present another trivial story, this time of a correlation coefficient. Students in non-statistical fields (i.e., non-statisticians) are honestly explained what the correlation coefficient is, what its uses are, what the regression concept of its origin is, what the coefficient of determination is, etc. What students actually remember from such explanations is a simple interpretation of the resulting value of the correlation coefficient (hardly anyone deals with the calculations nowadays, a simple MS Excel procedure is sufficient for getting the value): if it holds $|r_{yx}| \geq 0.7$ for its absolute value, we will say that the dependency is strong.

So far so good. But in social sciences a characteristic feature is that they are based on an objectified outcome of people's subjective efforts and motivations; human behaviour is by far not linear and the resulting data may, due to considerable differences in people's characteristics and abilities, have a high level of variability and a low level of consistency. In other words, real data will, to a great extent, be contaminated with subjective features of human behaviour. As an example we can mention an outcome of an opinion poll or consumer behaviour in marketing. Do we really encounter in these – rather usual – applications values of dependency leading to correlation coefficient values in the area of $|r_{yx}| \geq 0.7$? We dare say that it is practically never the case. Its value is much more likely to achieve something like

$r_{yx} = 0.3$. A student not specialised in statistics, equipped with formalism and lacking real knowledge of social data, will conclude that the dependency between the analysed phenomena is weak. But regarding the data quality, even 0.3 may be quite enough for strong dependency. Students, however, rarely learn about such a conclusion, unless they go into a deeper analysis of the underlying problem and, possibly, employ some methods of qualitative survey, as usual in marketing, to name one example. If they do, it may turn out that even $|r_{yx}| \leq 0.3$ is not such a small value in the given situation. We usually do not encumber non-statistician students with such explanations, leaving them at the mercy of simplified techniques for interpretations of statistical results.

Teaching statistical methods for economists is limited by the relatively small number of hours (usually 4 hours per week for 2 semesters), which does not allow to introduce to students all application possibilities of statistics in depth. This leads on the side of the teachers to superficial interpretation of the nature and conditions of applicability of each method, and on the side of the students to their misunderstanding and consequently to their improper use of a simplified interpretations. The solution to this situation can only to reduce the number of topics and methods presented in the basic course of statistics for economists. This will allow to deepen the explanation of selected methods with the emphasis on the conditions of their applicability and interpretation of the findings.[7]

3 ECONOMIC AND SOCIAL STATISTICS

What we said above was concerned with the use of statistical methods in economics. But there is also economic and social statistics, as a special and quite large part of statistics, which requires a different approach. It is the one that is perhaps most neglected by economists. A similar observation is valid for national accounts, which should be taught to every student of economics to provide them with a plastic view of what is globally going on in both national and worldwide economics.

Both these disciplines "sit on the fence" between statistics and economics. In order to be able to cope with them, students must have good knowledge of economics (both theoretical and practical); and it is impossible without a good command of quantitative techniques and the ability to interpret economic data (not only form the viewpoint of statistics but even that of accounting). Let us mention a small example here – the Keynesian economic theory, which is the economic basis of the national accounts. And vice versa: the national accounts as a statistical model of the national economy lead students to understanding mutual relationships between major aggregates and, more generally, how these aggregates work. Moreover: the national accounts cannot be understood without explanations of the fundamentals of statistics because, without those, students cannot get a proper insight into the data provided by the national accounts as a system of economic information and cannot process such data. The circle is thus closed and we are back at the beginning.

The story of the economic and social statistics is quite similar. Economic theory often employs notions such as inflation, unemployment (or employment) production and productivity, etc. Of course. But are we able to quantify and estimate such notions, or are we to rely on mere theoretical meditation?[8]

Therefore, the economic and social statistics is a special discipline of statistics (similar to testing hypotheses or regression and correlation analysis). It has its theoretical basis, a system of fundamental notions, methods and tools and, above all, a specific concept of indices viewed as variables, including definitions of their contents.

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7 Similar consideration can be found for example in Hernandez (2006) or Brown, David (2010).

8 Please note that we have deliberately avoided the term of "measuring" any of these economic variables. Not much can actually be measured in economics; hence we leave measurements to physics, anatomy and similar, more "measurable" fields of human knowledge and endeavour.

In no case can it be reduced to mechanical applications of selected statistical methods (as mentioned above – e.g., time series analysis, statistical inference, regression and correlation analysis, descriptive statistics, and multidimensional statistical methods) to real economic data, and such a reduced version must not be passed off as economic statistics, even though we sometimes see exactly that within teaching at economically oriented universities.

It is the specific concept of the indices and definitions of their contents that make up the crucial framework for the economic and social statistics. Going back to the above-mentioned relationships between theoretical notions of the science called economics on the one hand and possibilities of their relevant quantification on the other hand, we necessarily come to a notion called adequation gap. This adequation gap lies in the core of the matter: many notions utilised within theoretical economics cannot simply be fully quantified and a certain quantitative approximation to such notions must be accepted.

This "approximation" is thus a necessary trade-off between the theoretical economics' needs for quantification of its notions and our practical ability to quantify them as desired. This trade-off between "possible and required" is the structural content of the above-mentioned adequation gap. To provide a tangible example of this gap, we can mention inflation as a theoretical economic category and the index of consumer prices as a quantification of this theoretical notion.

These considerations are not, however explained to students of economy with sufficient emphasis; and if it comes to the worst, they are not mentioned at all. In consequence, students are at a loss when looking for the "proper" statistical data, they do not understand the data they get, use them in inadequate ways and, finally, interpret the results incorrectly.

CONCLUSIONS

Teaching statistics to economists, or more generally at universities and faculties with economic orientation, more attention should be given to interrelation between statistics and economics. This attention should, above all, be demonstrated by using real data from the economy (whether national or corporate) with a strong emphasis on understanding the substance of such data. Similarly, teaching economics should be more attentive to quantifying theoretical notions – we can hardly prove what we cannot quantify, having to rely on mere hypothetical claims which may later – in the light of real data – turn out to be disputable and unprovable, or even doubtful. By no means should applications of statistical methods in economics be confused with economic and social statistics; unfortunately, this is often the case within the teaching process.

A way to solve the problems outlined in the article is certainly not only to reduce the number of topics contained in the basic course of statistics for economists and thus to allow more profound explanation of a smaller number of methods, but also the inclusion of the basic course of economic and social statistics in the mandatory curriculum of students of economics. A separate course of economic and social statistics will enable students to understand the nature and characteristics of statistical data and allow them to avoid some errors in the application of the methods and the interpretation of the conclusions of the analysis. To increase the quality in perception of statistics for students of economics would also undoubtedly contribute deeper economic and economic-statistical knowledge of statistics teachers.

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Development of the Measurement of Product

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Abstract

The measurement of economy is a very long issue closely connected with the development of economic theories and the level of knowledge. Origin of modern measurement can be found in 17th century when Francois Quesnay compiled his Economic Table. Lots of successors developed economic concepts that are currently represented by the system of national accounting. Although national accounting has not a long history since it was established in 1950s, it has become known as a general tool for the description of economy. Development of national accounts was in line with development of economic theory and it respected the changes in economy and society. Both western and socialist countries we looking for methods and rules for the measurement of economy. The West relied on the System of National Accounts and the East focused on Material Product System based on Marx theories. After the collapse of communist regimes in the East, there remained only one universal measurement standard – national accounting. National accounts have been developed for more than 60 years and with several milestones. Currently, the new milestone – SNA 2008 / ESA 2010 is going to be put into practice. And the consequences will be very significant for many users.

Keywords

Production, national accounts, ESA 2010

JEL code

E01, N01

INTRODUCTION

National accounts currently represent universally adopted tool for the measurement of economy. National accounts can be described as a macroeconomic statistical model with two main branches. The first leads to the description of creation of product and the second leads to the generation of income, distribution and redistribution. The complete description of national accounts far exceeds the possibilities of this paper and therefore I focus on the product measurement only.

Gross domestic product (GDP) is probably the most important macroeconomic indicator (aggregate) that is used for many purposes. Unfortunately, national accounts are very often simplified into GDP and all other parts are neglected. Only complete analysis of national accounts can provide sufficient information about living conditions, economic power and wealth. GDP represents created product and it pro-

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vides content to economic definition of product (Samuelson and Nordhaus, 2009). Unfortunately, many times the product is not correctly distinguished from income. From statistical perspective, the difference is crucial. Product is a result of productive activity laying in production of goods and services. Income is generated by both production and distribution. Even income and the balance of incomes with the rest of the world may be more important than production; the position of product is on the top. There can be found many reasons why product is regarded as the most important indicator. Among them, estimation of product is easier than income and the level of international comparison is higher. Product can be estimated even for countries with less developed statistical systems. Product is also emphasised by many international bodies ranging from gross domestic product per capita at purchasing power parity to the share of government deficit and debt in gross domestic product.

Current state of art in macroeconomic statistics is focused on strengthening of international comparability and quality. Gross domestic product as the main indicator is under a deep control of international bodies, analysts and researchers. The role of official statistical authorities increased since statistical outcomes have become a part of international agreements or domestic law system like Stability and Growth Pact, countries' contribution to EU budget based on gross national income etc.

1 HISTORICAL DEVELOPMENT

Historically, the development of measurement of product is connected with the existing productive activity. This condition is still valid and therefore the definition of product is not fixed for ever. In the 17th century, Gregory King defined national income that represented an important milestone, see Frits. Besides, French physician Francois Quesnay compiled his economic table (Tableau Économique).[2] Economic table can be regarded as the first input-output table where only agriculture producers create product and all other processing activities are regarded as sterile. It means that productive sphere contained agriculture producers only, the rest of economy was not created value added. Adam Smith's theoretical description and Karl Marx theories influenced economics and scientists dealing with the measurement of economy had to react to it.

In 1930s, Wassily Leontief presented input-output tables for U.S. economy, see Miller and Blair (2009). Besides the input-output table, independent economic discipline was set up – input-output analysis. Wassily Leontief is a Nobel Prize economist (1973) and his ideas and approach to production have many successors.[3] The first U.S. national accounts were published in 1947 and they were called National income and products accounts (NIPA).[4]

During 1930s, Keynes' theories and Leontief's structural model became the main foundations of national accounts. The two concepts prevailed in the construction of national accounts. The first was devoted to national income and other macro-aggregates and the second concept lied in the application of accounting procedures commonly used on the level of businesses. This resulted in ongoing creation of national accounts in the West (Bos, 1992). The East focused on application of Marx's Labour theory of value that resulted in Material Product System (MPS), see Sixta and Fischer, 2014.

After the end of the Second World War, UN expert Richard Stone[5] prepared a system of accounts and this is regarded as a begging of national accounts. The first system of national accounts was issued in 1952. The process of standardisation of national accounting went on in 1960s and it resulted in complex and deep national accounts' standard – SNA 1968. Reaction to the changes in economy in 1980s resulted in a completely new standard SNA 1993. Implementation of updated national accounts' standards

[2] Explanation can be found at: <https://www.marxists.org/reference/subject/economics/quesnay/1759/tableau.htm>.
[3] See <www.iioa.org>.
[4] See <http://www.bea.gov>.
[5] See <http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1984/stone-bio.html>.

takes always some time. It means that international comparability is temporarily reduced before the new standards are worldwide adopted. Currently, new standard SNA 2008 is going to be put into practice and it brings some fundamental changes in the measurement of product.

Even SNA 1952, SNA 1968, SNA 1993 and SNA 2008 are universal standards issued by the UN, the EU issues its own standards that are focused on higher level of comparability and standardisation of both results and procedures in member countries. The EU member countries' data is used for administrative purposes and therefore deeper standardisation is necessary. The main frame is taken over from the UN standards. Hence, the EU standards are usually issued few years after the UN and they are obligatory for member states, Council Regulation (EC) No 2223/96. Current ESA 1995 (modification of SNA 1993) is going to be replaced by ESA 2010 (modification of SNA 2008). After September 2014, all figures should be published in ESA 2010 methodology.[6] Implementation of SNA 2008/ESA 2010 is process that is significantly harmonised in the EU. Fortunately, this process is in line with the implementation of the 6th Manual of Balance of Payment (IMF, 2009) and there is a high possibility that this will encourage countries to implement rapidly SNA 2008 around the world.

2 PRODUCT AND PRODUCTION

The role of output in national accounts is crucial Output is closely linked to the classification of units into institutional sectors (IS) that represent the key players in national accounts. IS are formed by groups of institutional units (companies, government units, households, non-profit organisations, etc.) with similar behaviour that is determined by the type of output.

Measurement of product is related to the definition of productive activity. This crucial phenomenon is not uniform in all branches of statistics.[7] Gross domestic product is regarded as final product. It means that the definition of the key indicator of economy depends on many aspects. Final product means that intermediates are excluded and the borderline between final user and intermediate user affects the results. The system of national accounts distinguishes between production and output. The main difference between production and output lies in the definition of productive activities. ESA 2010 (3.07) defines production as "an activity carried out under the control, responsibility and management of an institutional unit that uses inputs of labour, capital and goods and services to produce outputs of goods and services." The key difference subsists in ancillary activities like marketing, accounting, etc. The principal activity and secondary activity are defined by so-called kind-of-activity unit (KAU).[8]

Modern approach to output covers three main categories:

a. Market output.
b. Output for own use.
c. Other non-market output.

Market output covers goods and services (products) sold on the market at economically significant prices. Output for own use includes both household production of selected products (e.g. imputed rent, self-supply) and own capital formation (e.g. individual housing construction). Other non-market output represents government and non-profit institutions output that is given by the sum of costs (total output of other non-market producers) less sales. Contrary to market output, both output for own use and other non-market output have the same user and producer.

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6 According to the EU law, member states can ask for derogations that allow them to postpone the transmission of data according to new methodology. Eurostat allows member states to ask for derogation and the last derogations will expire 1 January 2020.

7 There are different approaches to the measurement of production of households products, intra-company sales etc.

8 Besides national accounts, regional accounts are based on local units or local kind-of-activity units. Generally, production approach to GDP is preferred and officially published. Expenditure approach is very scarce, details can be found in Kramulova and Musil (2013).

Modern statistics rely on the complex approach to output. It is not important whether economic events are easy to measure. It is not even important if the activity is legal, hidden or unethical. The emphasis is put on the complete estimate of economic transactions. That is why prostitution, drug production, black market trade, thefts, etc. should be statistically estimated (Fischer and Fischer, 2005). It is connected with "statistical measurement" that in fact means estimate. For example, only about 87% of Czech GDP is surveyed, see GNI Inventory (CZSO, 2012). The development of national accounts is related to continual increase of imputed (not measured) items. SNA 1993 (ESA 1995) improved production and assets boundary. It also clearly recognised formal and informal sectors are also clearly defined Ongoing development of national accounts standards (currently SNA 2008) means that more activities are covered by output (e.g. research and development). Definitely, it is only a convention. Some activities like home planting of tomatoes are regarded as productive while other activities like home sewing of clothes or cleaning are not. This approach tries to focus on the most important issues in current society but on the other hand, it is not rigid or stable.

Even the society is changing very fast in recent times, frequent changes of statistical standards and regulation has always been a problem for the users. Hence the implementation of ESA 2010 / SNA 2008 represents significant changes in national accounts, GDP will be affected very seriously.[9] Actually, SNA 2008 and ESA 2010 are very well developed statistical standards that try to meet recent development in both economics and economy. Unfortunately, general preparation of both national accounts producers (statistical offices) and universities is not sufficient. The theory is far ahead of routine praxis.

3 IMPACT OF ECONOMIC THEORIES ON PRODUCT MEASURENT

Very often it seems that economic theories are far away from statistical praxis. But this is not absolutely true. It should be honestly admitted that economic theory defines the framework of statistical measurement. In other words, total level of product is influenced by generally accepted economic theory. National accounts were built up on the basis of the work of J. M. Keynes combined with W. Leontief and his production function and other economists. Obvious example is Francois Quesnay who regarded agriculture production as productive activity and all other activities were sterile in terms of production.

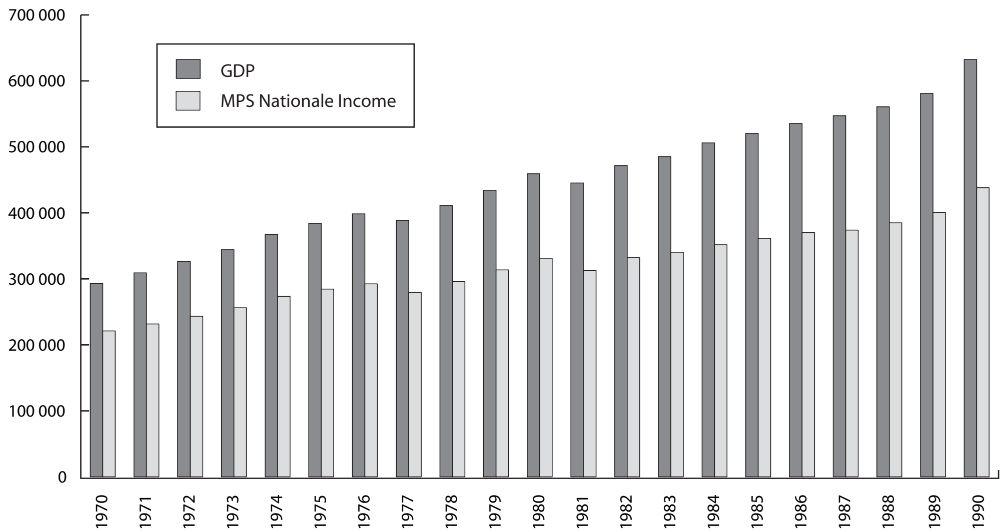
Western economic thinking based mainly on Keynes work was transformed into national accounts after 1950s. Countries of socialist bloc led by the Soviet Union relied on Marx labour theory of value, see Marx (1975). The main difference subsisted in the definition of productive activities. National accounts distinguish three types of output and corresponding types of producers. Socialist system called Material Product System splits the economy into two main parts. Productive sphere and non-productive sphere. Productive sphere contained selected market industries – agriculture, mining, energy, manufacturing, construction and services for productive sphere. Other industries that covered government activities and services provided to households were recorded as non-productive. Decision which industries belong to productive and non-productive was rather arbitrary. Moreover, what was applicable in 1950s, was hardly possible in 1980s. A good example represents telecommunications. Usually big and monopolistic company had to be split between productive and non-productive sphere. Since majority of the costs are fixed costs, it is impossible to split them. Practical solution when 50% belongs to productive and 50% belongs to non-productive was far from the theory. Moreover, telecommunications were very profitable and they can be hardly regarded as non-productive.

This led to the situation when socialist countries were not able to measure significant part of the economy and economic growth. The following Figure 1 compares the level of product for the Czech Republic measured by two systems since 1970, see Sixta and Fischer (2014). It is obvious that with increased de-

[9] Updated estimate of impact of ESA 2010 on Czech GDP is more about 3%, see <www.czso.cz>.

velopment of services and IT products in 1980s, original Marx based system relying on physical products (goods) could not be sufficient. It resulted in the increased difference between National Accounts' GDP and Material Product Systems' national income. In 1990, the difference between both concepts was about 30%. Correct measurement of economy is not a fixed issue forever. Statistical concepts based on more or less harmonised standards are always developing, improving and updating. Both development of economic theory and changes in the society have to be taken into account. Globalisation and new phenomena connected with knowledge society were implemented into revised National Accounts' standards.

Figure 1 Comparison of GDP (ESA 1995) and MPS' national income, CZK mil.



Source: Own computations

4 ESA 2010 AND ESA 1995

Fast development of society in 1990s caused that national accounts' standards issued in 1993 (SNA 1993) and in 1995 (ESA 1995) became obsolete soon. While SNA 1968 was valid 25 years, SNA 1993 only 15 years. Last year (2013), the European Union issued regulation No 549/2013 that put in practice ESA 2010 (European modification of SNA 2008) where new data have to be transmitted to European Commission from 1st September.

As well as previous updates, ESA 2010 keeps the basic logic of accounts in the same form but substantial changes in the definition of assets boundaries were made. From the point of the product measurement, the following changes can be regarded as the most important:

a. Capitalisation of expenditures for research and development.

b. Capitalisation of small tools.

c. Capitalisation of military expenditures.

The most changes are connected with output for own use and other non-market output. Market output is affected very slightly.

There are lots of other changes connected with SNA 2008 / ESA 2010 that affect sector accounts and balance sheets. Among them, the most important changes are found in financial institutions where new sectorisation is put into practice and in financial assets. Significant impact is on Input-Output Tables (IOT) where new concepts in foreign trade change technical coefficients.

There are lots of factors that initiated the changes in ESA 2010 and lots of them are connected with economic research conducted in the U.S. or by the OECD. For example, neoclassical theories mostly reflected in capital services (Jorgenson, 1963) finally stay outside of the core framework and they remained as supplementary tables but as a part of SNA 1993.

4.1 Capitalisation of research and development expenditures

Expenditures on research and development (R&D) were recorded mainly in intermediate consumption and compensation of employees in line with common practice in business accounting. The issue of R&D is connected with the measurement of economy. If we take into account production function, it is supposed that:

$$Y = f(K, L), \tag{1}$$

where Y is product, K represents capital and L represents labour (Fischer and Sixta, 2009). When R&D expenditures are recorded as intermediates then these expenditures do not contribute to the future benefits (product increase). For example, when using Cobb-Doublas production function[10] (Formula 2), the role of total factor productivity is overestimated since capital does not include R&D assets.

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{2}$$

This theoretical construction has empirical evidence in U.S. data. Practically it has two impacts on the users of statistics. First, logical explanation of the development in hi-tech industries. Second, recording R&D as gross fixed capital formation means the increase of the level of GDP. Current approach used in the EU consists in the use of FRASCATI based data (OECD, 2002) and it means the use of intramural expenditures. Investments into R&D can originate from two resources:

a. Purchased R&D services.
b. Own-account production of R&D.

The impacts on product measurement are different in both cases. Purchased R&D services are reclassified from intermediate consumption into gross fixed capital formation. Own account production of R&D subsists in capitalisation of all expenditures connected with research and development. These expenditures covers mainly intermediate consumption (e.g. electricity), compensation of employees (wages for researches), consumption of fixed capital of assets used for R&D and mark-up factor for market producers that ensures the same valuation of market R&D and own account produced R&D.

There is a different approach to market and non-market producers covering government institutions (S.13) and non-profit institutions serving households (S.15). Both output and consumption expenditures are given by the sum of their costs less sales and output for own use. Practically it means that mainly universities have to be split into at least two kind-of-activity units The first provides standard non-market services (education) and the second produces R&D. Total output will be slightly changed and the increase of gross fixed capital formation (own account produced) will be compensated by the decrease of consumption expenditures. The following Table 1 shows approximate impact of capitalisation of research and development on gross domestic product.

[10] Alternative interpretation can be found in Čadil (2007).
[11] This resulted from work of the OECD, for example see <www.oecd.org>.
[12] Output is measured by sum of cost and because consumption of fixed capital will be increased by depreciation of R&D assets, the total output will rise. Such computed output is split between consumption expenditures (other non-market output), gross fixed capital formation (own account production) and sales (from products sold to different customers).

Table 1 Capitalisation of R&D, CZK billion

	1995	2000	2005	2010
Output	19.4	32.8	40.6	63.0
GFCF	13.3	26.5	36.8	47.8
Impact on GDP	**13.1**	**28.1**	**36.6**	**45.4**
- capitalisation of market R&D	1.0	4.1	5.1	2.6
- capitalisation of own account	12.3	22.4	31.7	45.1
- decrease of consumption expenditures in S13	–3.3	–8.1	–12.6	–18.0
- consumption of fixed capital in S13	3.2	9.7	12.4	15.7

Note: S13 – sector of general government.
Source: Czech Statistical Office

Estimation of impact of R&D on GDP is not straightforward. For example, in 2010 total output of both market R&D and R&D for own use is about 63 CZK billion. The impact on GDP has four components. It includes capitalisation of market R&D products (2.6), own account production (45.1), decrease of government consumption expenditures (–18.0) caused by own account production of R&D mainly at universities. Final impact comes from holding of government assets. If government institutions hold assets, their consumption of fixed capital is a part of both output and consumption expenditures. Besides roads, railways, building etc. government intuitions also own R&D products and therefore depreciation of these products is part of government output (15.7).

Capitalisation of R&D products represents the most important effect in updated SNA standards. Since R&D issue is conceptually the most important, it enjoys a great attention in statistical community. However, even a long discussion about freely available R&D services or research with no success, there are still lots of outstanding issues connected with R&D. It will take long time for both users and producers to get familiar with it.

4.2 Capitalisation of small tools

Asset boundary has a crucial impact on the measurement of product. The distinction between intermediates and capital (assets) defines recording of transactions. Intermediates are recorded in intermediate consumption and they do not create wealth. On the contrary, purchases of assets are recorded as capital formation. ESA 1995 determines fixed assets as products used in the production process for more than one year and with the price over 500 ECU at prices of 1995.[13] Currently, ESA 2010 removed the price criteria and only the requirement for service-life remained. Practically it means, that the difference between business and national accounts increased. Even relatively cheap assets that are kept for more than one year should be capitalised. This group includes wide ranges of IT products (laptops, printers, tablets, cell phones), small machineries (e.g. grass cutters) etc. Besides, these criteria are applied on intangible assets, as well. Currently, software has a specific position in national accounts. Capitalisation covers software purchased below accounting (tax) limits[14] and own account production of software. Since capitalisation of own account software was conducted in line with ESA 1995, the implementation of ESA 2010 requires

[13] Czech national accounts used equivalent of CZK 20 000.
[14] Czech tax limit is CZK 60 000.

additional capitalisation of software bellow the limit only. The following Table 2 describes the impact of capitalisation of small tools. Overall impact on Czech GDP in 2010 is about CZK 59.6 billion. It is obvious that software was relatively negligible in early 1990s. Nowadays software represents significant part of capital formation ranging from package to specialised software.

Table 2 Capitalisation of small tools, CZK billion

	1995	2000	2005	2010
Impact on GDP	**32.3**	**27.9**	**48.7**	**59.6**
- tangible assets	28.8	26.2	36.8	43.1
- intangible assets (software)	3.5	1.7	11.9	16.5

Source: Czech Statistical Office

4.3 Capitalisation of military expenditures

Military expenditures were treated as current expenditures that do not create wealth and services in the future. They were recorded in intermediate consumptions of defence industry in the government institution sector. ESA 2010 brought a different concept of treatment of military expenditures. Even the change of the value of GDP is not significantly affected, the key difference lies in the concept. It is assumed that purchases of different kind of weapons provide services of defence regardless of its use. Investments into weapons deter potential enemies and these services of deterrence can be measured by consumption of fixed capital.[15]

Government defence services are measured by the sum of the costs since ministry of defence (including the army) is treated as non-market produces. The costs of defence consist of intermediate consumption (material, energy and services for the army), wages of soldiers and depreciation (consumption of fixed capital) of fixed assets. When ESA 2010 is applied, purchases of weapons are recorded in capital formation. Therefore, government output is decreased due to the decrease of intermediate consumption. On the contrary, government gross value added is higher because of inclusion of consumption of fixed capital of weapons.

The influence of capitalisation of military assets is similar to R&D for non-market producers. The impact on the product is given by previous investments that are currently expressed by consumption of fixed capital. Table 3 illustrates consumption of fixed capital of weapons and its impact on GDP. Since the measurement of other components of defence services have not changed (compensation of employees, intermediate consumption, consumption of fixed capital of other assets, etc.), the impact is given only by weapons.

Table 3 Consumption of fixed capital of weapons, CZK billion

	1995	2000	2005	2010
Impact on GDP (CFC of weapons)	**4.4**	**5.7**	**6.7**	**5.2**

Source: Czech Statistical Office

[15] It is in line with computation of non-market output. Since ministry of defence is regarded as government unit (other non-market producer), the output is estimated by the cost approach. It is in line with computation of non-market output. Since ministry of defence is regarded as government unit (other non-market producer), the output is estimated by the cost approach.

The development of CFC of weapons is influenced by the stock of weapons. It means that socialist Czechoslovakia (and subsequently the Czech Socialist Republic) had a plenty of weapons. Even, the quality of some of them (e.g. old soviet models of tanks) was disputable, the depreciation of such assets was relatively higher than today. After 1991, lots of these old and unused assets were sold or discarded. Investment into military assets could not compensate such decrease of stocks. Since CFC is computed from existing stocks (it is assumed that these assets provide services), it was relatively decreasing throughout the whole period 1990–2010.[16]

5 UPDATED GDP

ESA 2010 changes GDP in order to react to the changes in society. All the main impacts mentioned above were driven by the effort for capturing economic development in modern world. The selection of issues for updates of national accounts' standards corresponds to the importance of these phenomena. The existence of knowledge-based economy is undisputable and old procedures that reflect traditional business accounting cannot record the complexity of economic development. In 1990s, the emphasis was put on software and IT services. Currently it is clear that it was not enough. Knowledge is also incorporated into procedures, techniques, manuals etc. and know-how became a leading factor for progress and wealth. Within all the changes given by the ESA 2010, asset boundary is the most important. It is reflected in R&D, small tools and change in classification of IT (both hardware and software) assets. Special emphasis was also put on databases as a collector of information with significant value.

Different reasons can be found behind the concept of capitalisation of military assets. The key issue lies in the factual accuracy of expenditures with their recording in national accounts. Weapons have usually service-life longer than one year and according to ESA 2010 they bring benefits to holder even not used. The benefit from holding weapons can be expressed by consumption of fixed capital that represents the service provided by the weapons.

Overview of all mentioned changes[17] is presented in Table 4. Since the impact is estimated on nominal GDP (at current prices) and price relations changed significantly over the whole period, it is necessary to emphasise relative comparison (in %). In 1995, the overall impact of these changes is about 3.25% and in 2010 only 2.91%.

Table 4 Impact of selected ESA 2010 changes on Czech GDP, CZK billion (%)

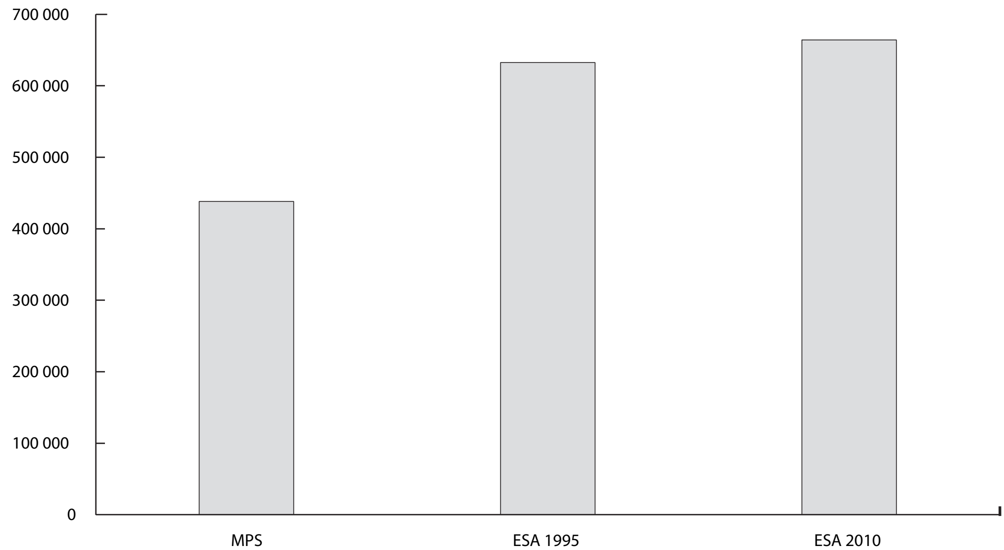
	1995	2000	2005	2010
R&D	13.1	28.1	36.5	45.4
Small tools	32.3	27.9	48.7	59.6
CFC of military assets	4.4	5.7	6.7	5.2
Total	49.8	61.6	91.9	110.2
% of original GDP	3.25	2.72	2.95	2.91

Source: Czech Statistical Office

[16] Data for 1990 has not been processed yet.
[17] There can also be found other changes in concepts that influence product measurement (e.g. insurance services) but with lower impact on GDP. The overall impact of ESA 2010 on GDP for 2010 is 3.2%, see <http://www.czso.cz/csu/tz.nsf/i/narodni_ucty_implementace_esa_2010_20141001>.

When comparing these adjustments throughout different statistical standards, it is obvious that product is crucially affected. While MPS national income was about CZK 438 billion in 1990, ESA 95 GDP was 44% higher.[18] Comparison of ESA 2010 and MPS indicates very high difference (53%) given by the pure change of statistical standards representing mainly assets boundaries, see Figure 2.

Figure 2 Comparison of product between MPS, ESA 1995 and ESA 2010, CZK mil.



Source: Own computations

The update of GDP is not only European issue. Countries that used SNA 1993 or SNA 1968 are currently switching to SNA 2008. There are no significant differences between SNA 2008 and ESA 2010. For example, Australia[19] increased its GDP by 4.4% and France[20] 3.2%. It means that international comparison in purchasing power parity will be affected, as well.

6 FUTURE DEVELOPMENT

Even the revision of national accounts' rules was finished; it can be assumed that this was not the last revision. There are different factors that prove this assumption. First, the economy is still changing very fast and there is no reason to neglect it. Second, national accounts serve to many users ranging from official institutions EU, IMF, OECD, etc. to analysts. The users' demands are developing, as well. Finally, it is always recognised that previous revision brought some outstanding issues that should have be corrected.

One of the conceptual issues with its place in SNA 2008 is capital services. Capital services remained in SNA 2008 as voluntary item. Originally, it was supposed that capital services should become an integral part of national accounts with significant influence on the computation of output for non-market

[18] National income from MPS presented within this paper is based on gross basis for better comparison with national accounts. Within Balances of National Economy, national product was usually preferred on net basis.
[19] See <http://www.abs.gov.au>.
[20] See <http://www.insee.fr/en/themes/comptes-nationaux/default.asp?page=base-2010.htm>.

producers, see Sixta and Fischer (2009). Capital services represent benefits from using assets as production factor. Prevailing concepts links capital services with gross operating surplus, see Harrison (2004). Since many countries opposed to that concept, final decision on capital services classified them as voluntary or satellite item.

It is generally known that current statistics serves for administrative purposes very often. In the EU, statistics is connected mainly with the measurement of government deficit and debt representing Maastricht criteria.[21] For the EU budget, countries' contributions are from 85% based on national accounts' figures namely gross national income (GNI) and weighted average rate (WAR) of value added tax. On one hand, administrative use of statistics promotes its importance and provides some guarantees for statistical surveys. On the other hand, statistics can never be 100% precise or very high rate of precision is inefficient, costly or unachievable. It is clearly seen on the Maastricht criteria, the share of government net lending/borrowing (surplus/deficit) on GDP about 2.9% is considered as correct. Government deficit about 3.1% is considered as incorrect with legal and practical consequences in many EU countries even nobody can guarantee very small differences in statistical measurement.

As the society is developing, the pressure on statistics is rising. A group of qualified users is rising and tools for advanced data analysis are freely available. It means that statistics has many everyday users ranging from general public to the most skilled users at universities. It all leads to the higher pressure on official statistics. In the area of product measurement, there is a strong EUROSTAT effort on shortening publication deadlines and increasing of published detail. Currently, flash estimate of quarterly GDP is published 45 days after reference quarter. EUROSTAT intends to shorten it to 30 days after reference quarter in next two or three years.

CONCLUSION

The development of the measurement of economy is significantly influenced by economic theory and by the level of understanding of society. When the economy started to be discussed in complex in the 17th century, the quantification was aimed at the most important issues. Tableau Économique compiled by Francois Quesnay was focused on agriculture as the main source of the product. Since then, the list of activities that are regarded as productive and lead to the creation of product is still expanding. In 1930s, the quantification of economy resulted in input-output tables and later with preparation of the basis for further national accounts. The division of the world given by the cold war resulted in different development of economic measurement in the West (national accounts) and in the East (material product). Time to time, the efforts for strengthening cooperation and looking for the compromise between two different statistical systems was not successful. After the collapse of communist regimes, countries started to switch to more developed system of national accounts. System of national accounts is currently the only worldwide accepted system that is being still under the development. SNA 1993 and European modification ESA 1995 introduced exhausting and complex approach to the measurement. The principles set by SNA 1993 lasted for long time even the world has been changing. The changes in economy connected with the fast development and wide spread of computers, software and intellectual assets led statistical community to the preparation of updated system of national accounts. In September 2014, SNA 2008 and ESA 2010 come into force in the EU. Even the main principles remained unchanged, significant changes in gross domestic product can be observed due to the different approach to the productive activity. Obviously, the most important change in terms of domestic product is capitalisation of research and development expenditures. Similarly to the previous changes, the aim of SNA 2008/ESA 2010 updates is the effort to keep statistical measurement of economy in touch with reality.

[18] See the Maastricht Treaty – the Treaty on European Union signed in 1992 in Maastricht, Netherlands.

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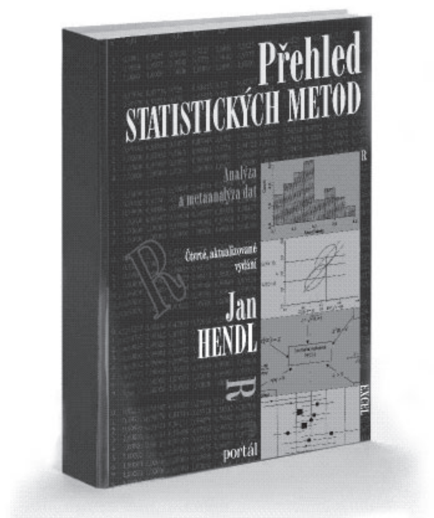
Beyond Statistical Methods – Compendium of Statistical Methods for Researchers

Ondřej Vozár[1] | *Czech Statistical Office, Prague, Czech Republic*

HENDL, J. *Přehled statistických metod: Analýza a metaanalýza dat* (Overview of Statistical Methods: Data Analysis and Metaanalysis). 4th extended edition. Prague: Portál, 2012. ISBN 978-80-262-0200-4



The book serves as a compendium of statistical methods for researchers in a very broad sense. In addition to an overview and explanation of the broad scope of statistical methods it also provides exposition to the principles of empirical research, research plans, ethical considerations, meta-analysis and a practical guide on how to write or read statistical reports.

The first part deals with principles of empirical research, how to formulate and verify hypotheses, write a proposal of research project and also ethics of the research. Ideas on population, sampling, types of variables, measurement errors, various research plans (census, sample survey, experiment, observational study) are very clearly explained. Also practical considerations of data collection, processing and dealing with missing values are given. Statisticians have to learn these issues mostly by trial and error in their practice.

The second part provides a deep but very readable introduction to descriptive statistics, statistical graphs, analysis of outlying values, transformation of the variables, probability, common probability distributions, central limit theorem and sample distributions of the relative frequency, mean and variance.

The third part on statistical induction and statistical methods forms the core of the book. The author combines clear notation in formulae, graphs and solved real life numerical examples to explain the ideas behind the methods. The reader is always informed on possible traps, misinterpretations and ways how to fix them. Author aptly uses decision trees and block diagrams for cases when it is necessary to make choice of the method or decide on the interpretation of the data.

The fifth chapter provides a lucid explanation of point estimates, interpretation of confidence intervals, steps of hypotheses testing, characteristics of the test (power and significance level), simultaneous hypothesis testing and ways how to avoid misinterpretation of the null hypothesis. These vital issues

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are not usually explained in such a detail in the most of available textbooks. The sixth chapter discusses standard situations in the hypotheses testing (testing mean or variance in one sample, two samples or in paired data). Then non-parametric tests on mean and normality are presented.

The seventh chapter deals with analysis of bivariate data including graphs, correlation and regression analysis. In the subsequent chapters standard methods of categorical analysis, analysis of variance and multivariate regression are presented. Reader is kept informed on possible traps (Simpson's paradox, randomization) in application of the methods.

The eleventh chapter guides statisticians in how to impartially assess the size of effects in a statistical study. For example it means to find which value of the sample correlation coefficient means a high effect with given significance level, power of the test and sample size. The twelfth chapter provides guidelines on the choice of the appropriate statistical method based on clear classification. The second part explains common misinterpretations of the results of hypotheses testing and how to avoid or corrects them. Bayesian approach to the statistical induction and computer intensive methods (bootstrap, jackknife, crossvalidation) are explained.

The thirteenth chapter explains multivariate statistical methods including logistic regression analysis, generalized linear models, regression trees, survival analysis, cluster analysis, principal component analysis, factor analysis, discrimination analysis, multivariate scaling, multivariate contingency tables etc. The scope of the methods and the part on structural equations models using latent variables is unique in the Czech statistical literature.

The fourteenth chapter is the first systematic treatise on meta-analysis in the Czech statistical literature. It explains all the phases of this kind of study, presents relevant statistical methods, gives a guide on how to write a research report and discusses its pros and cons.

The fifteenth chapter gives clear guidelines on the structure of a final report and corresponding ethical considerations.

The last chapter gives an overview of the statistical packages, criteria of the method selection and a brief introduction to the language R ("lingua franca" of the statisticians). The only weak point is that the overview of the packages was a bit outdated even in 2012.

Annexes contain the model structure of a final report, statistical tables, basic definitions and formulae of matrix algebra, description of the basic commands in R language, etc.

As mentioned above the book is unique in its scope, as it provides the only up-to-date presentation of some topics in the Czech language. The approach to the presentation combining formulae, graphs, decision trees and solved numerical examples enables understanding of the complex issues for a broad audience from various backgrounds. In my view this book can serve as a bridge between statisticians and researchers, but also between the authors of the scientific papers and reports and their readers.

Having said that I can recommend this book as a reference on the statistical methods and empirical research for professional statisticians, researchers and students in general.

Recent Publications and Events

New Publications of the Czech Statistical Office

Demographic Yearbook of the Czech Republic 2013. Prague: CZSO, 2014.

External trade of the Czech Republic in 2013. Prague: CZSO, 2014.

External trade of the Czech Republic since its joining the European Union till 2013. Prague: CZSO, 2014.

Food Consumption in 2013. Prague: CZSO, 2014.

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Other Selected Publications

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Rozdíly v konkurenceschopnosti mezi státy EU – předpoklady a bariéry jejich překonání (The differences in competitiveness between EU States – assumptions and barriers to their overcoming). Brno: Newton College, 2014.

Conferences

The ***15th Conference of the Association de Comptabilité Nationale*** took place in ***Paris, France*** during ***19–21 November 2014***. More information available at: *http://www.insee.fr*.

The ***60th World Statistics Congress ISI 2015*** will be held between ***26–31 July 2015*** in ***Rio de Janeiro, Brazil***. The congress will bring together members of the statistical community to present, discuss, promote and disseminate research and best practice in every field of Statistics and its applications. More information available at: *http://www.isi2015.ibge.gov.br*.

Papers

We publish articles focused at theoretical and applied statistics, mathematical and statistical methods, conception of official (state) statistics, statistical education, applied economics and econometrics, economic, social and environmental analyses, economic indicators, social and environmental issues in terms of statistics or economics, and regional development issues.

The journal of *Statistika* has the following sections:

The *Analyses* section publishes high quality, complex, and advanced analyses based on the official statistics data focused on economic, environmental, and social spheres. Papers shall have up to 12 000 words or up to twenty (20) 1.5-spaced pages.

The *Discussion* section brings the opportunity to openly discuss the current or more general statistical or economic issues; in short, with what the authors would like to contribute to the scientific debate. Discussions shall have up to 6 000 words or up to 10 1.5-spaced pages.

The *Methodology* section gives space for the discussion on potential approaches to the statistical description of social, economic, and environmental phenomena, development of indicators, estimation issues, etc. Papers shall have up to 12 000 words or up to twenty (20) 1.5-spaced pages.

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